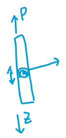
CH3

2023年3月9日 上午 11:35

§3-1 描述应力&应变



$\sigma_z = \frac{P}{A}$

$\epsilon_z = \frac{\delta l}{l}$

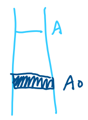
$\epsilon = \frac{\delta}{L_0}$ 变形量

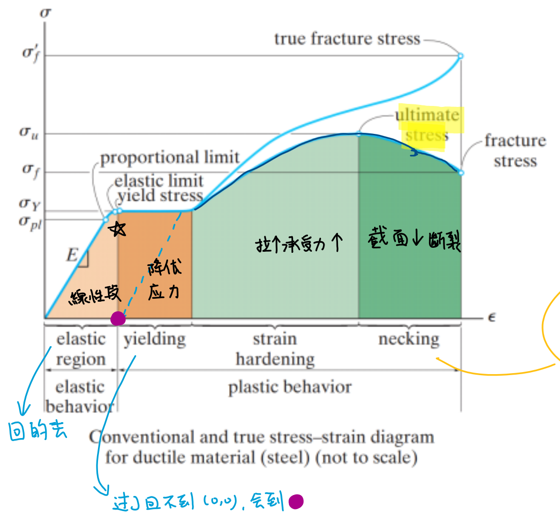
应变：单位長度 变化量

§3-2 stress-strain diagram

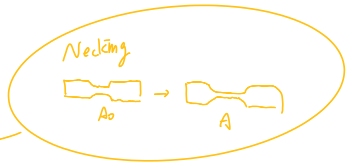
True stress $\sigma = \frac{P}{A}$ $(\neq \frac{P}{A_0})$

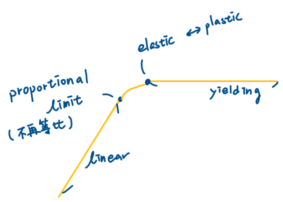
Engineering stress $\sigma = \frac{P}{A_0}$

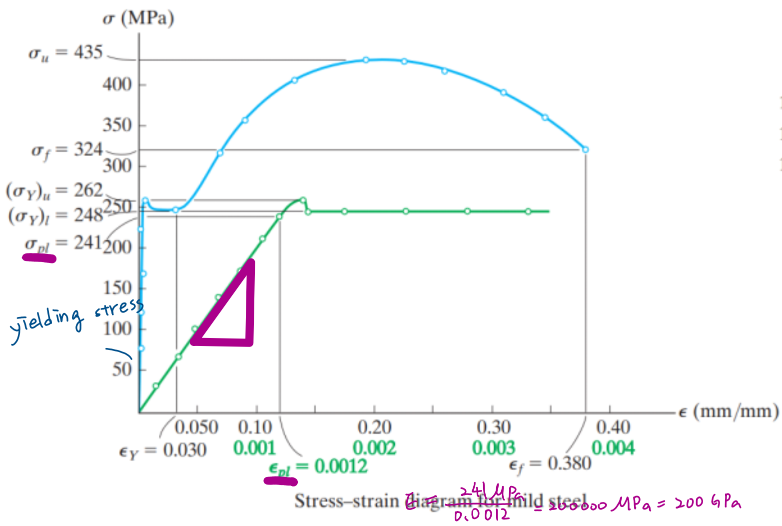




Conventional and true stress–strain diagram for ductile material (steel) (not to scale)



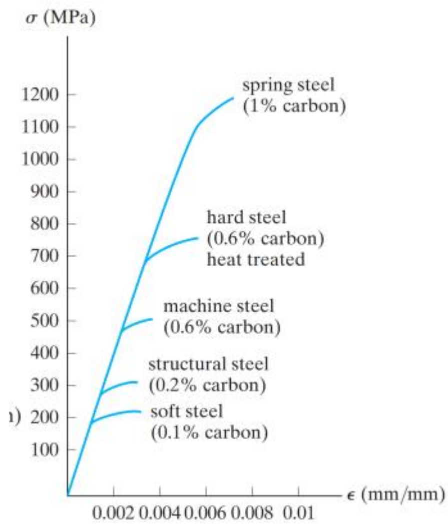




Stress–strain diagram for mild steel

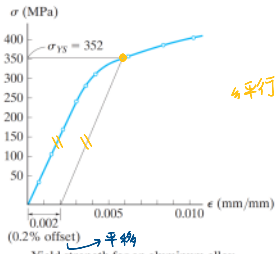
$E = \frac{241\ MPa}{0.0012} = 200000\ MPa = 200\ GPa$

Fig. 3–6



§ 3-3 Behavior

看不出 yielding ωω

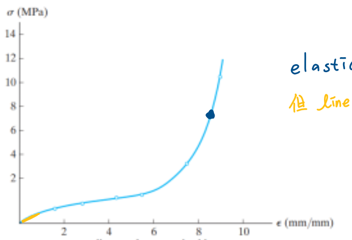


Yield strength for an aluminum alloy

Fig. 3–8

与平行 ① 在 0.002 外画一平行线性段的線
② 交点即所求 σ_y

$\times$ 0.002 = 2000 micron



σ–ϵ diagram for natural rubber

Fig. 3–9

elastic 偏長
但 linear 短

e.g. 混凝土，抗压不抗拉

§ 3.4 Hook's Rule

§ 3.4 Hooke's Law

用 GPa ，MPa 太小

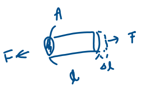
§ strain Energy 3.5

$$\boxed{u = \frac{\partial U}{\partial V} = \frac{1}{2}\sigma\epsilon}$$

$= \frac{1}{2}\sigma\epsilon = \frac{1}{2}E\epsilon^2$

$\sigma = \frac{F}{A}$

$\frac{\Delta l}{l}$ = 应变 (ϵ)



$$\frac{\frac{1}{2}F\Delta l}{A\,l} = \frac{1}{2}\sigma\epsilon$$

$\sigma \quad \epsilon$

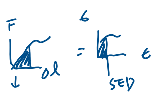
$\sigma\epsilon$ → $E\epsilon$

$= \frac{1}{2}E\epsilon^2$

$= \frac{1}{2}\sigma\epsilon$

$= \frac{1}{2}\frac{\sigma^2}{E}$ #

$= \frac{1}{2}\frac{\sigma^2}{E}$ ※



作功 $= \frac{1}{2}F\Delta l$

☆ 单位体积存的功

↓ 单位体积

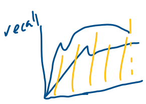
$$= \frac{\frac{1}{2}F\Delta l}{A\,l}$$

elastic string energy density (SED)

Model of resilience

不出线性段可存的 Energy

Module of toughness (不考)



断裂前可储存的 Energy

★ Ex. 3.1

EXAMPLE 3.1

A tension test for a steel alloy results in the stress–strain diagram shown in Fig. 3–18. Calculate the ① modulus of elasticity and the ② yield strength based on a 0.2% offset. Identify on the graph the ③ ultimate stress and the fracture stress ④.

E

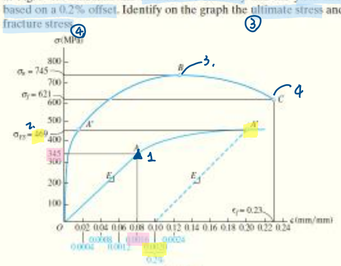


Fig. 3–18

SOLUTION

① **Modulus of Elasticity.** We must calculate the *slope* of the initial straight-line portion of the graph. Using the magnified curve and scale shown in blue, this line extends from point O to an estimated point A, which has coordinates of approximately (0.0016 mm/mm, 345 MPa). Therefore,

$$E = \frac{345\text{ MPa}}{0.0016\text{ mm/mm}} = 216\text{ GPa} \qquad \textit{Ans.}$$

$\frac{F}{A}$

Note that the equation of line OA is thus $\sigma = [216(10^3)\epsilon]$ MPa.

② **Yield Strength.** For a 0.2% offset, we begin at a strain of 0.2% or 0.0020 mm/mm and graphically extend a (dashed) line parallel to OA until it intersects the $\sigma-\epsilon$ curve at A'. The yield strength is approximately

$$\sigma_{YS} = 469\text{ MPa} \qquad \textit{Ans.}$$

③ **Ultimate Stress.** This is defined by the peak of the $\sigma-\epsilon$ graph, point B in Fig. 3–18.

$$\sigma_u = 745\text{ MPa} \qquad \textit{Ans.}$$

④ **Fracture Stress.** When the specimen is strained to its maximum of $\epsilon_f = 0.23$ mm/mm, it fractures at point C. Thus,

$$\sigma_f = 621\text{ MPa} \qquad \textit{Ans.}$$

EXAMPLE 3.2

The stress–strain diagram for an aluminum alloy that is used for making aircraft parts is shown in Fig. 3–19. If a ① specimen of this material is stressed to $\sigma = 600$ MPa, determine the permanent set that remains in the specimen when the load is released. Also, find the modulus of resilience ② both before and after the load application. ③

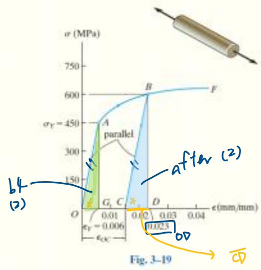


Fig. 3–19

SOLUTION

① **Permanent Strain.** When the specimen is subjected to the load, it strain hardens until point B is reached on the $\sigma-\epsilon$ diagram. The strain at this point is approximately 0.023 mm/mm. When the load is released, the material behaves by following the straight line BC, which is parallel to line OA. Since both of these lines have the same slope, the strain at point C can be determined analytically. The slope of line OA is the modulus of elasticity, i.e.,

$$E = \frac{450\text{ MPa}}{0.006\text{ mm/mm}} = 75.0\text{ GPa}$$

斜率 $= \frac{450}{0.006}$

From triangle CBD, we require

$$E = \frac{BD}{CD}; \qquad 75.0(10^9)\text{ Pa} = \frac{600(10^6)\text{ Pa}}{CD}$$

$$CD = 0.008\text{ mm/mm}$$

$75\times10^3\text{ MPa} = \frac{600\text{ MPa}}{\overline{CD}}$, $\overline{CD} = \frac{600}{75000} = 0.008$ #

This strain represents the amount of *recovered elastic strain*. The permanent set or strain, ϵ_{OC}, is thus

$\overline{OC} = \overline{OD} - \overline{CD}$

$= 0.023 - 0.008$

$$\epsilon_{OC} = 0.023\text{ mm/mm} - 0.008\text{ mm/mm}$$
$$= 0.0150\text{ mm/mm} \qquad \textit{Ans.}$$

NOTE: If gage marks on the specimen were originally 50 mm apart, then after the load is *released* these marks will be 50 mm + (0.0150) (50 mm) = 50.75 mm apart.

$= 0.0150 \text{ mm/mm}$ *Ans.*

NOTE: If gage marks on the specimen were originally 50 mm apart, then after the load is *released* these marks will be 50 mm + (0.0150) (50 mm) = 50.75 mm apart.

② **Modulus of Resilience.** Applying Eq. 3–8, the areas under *OAG* and *CBD* in Fig. 3–19 are*

b4 $$(u_r)_{\text{initial}} = \frac{1}{2}\sigma_{pl}\epsilon_{pl} = \frac{1}{2}(450 \text{ MPa})(0.006 \text{ mm/mm})$$

$$= 1.35 \text{ MJ/m}^3 \rightarrow 10^6 \frac{N \cdot m}{m^2 \cdot m} = \frac{MJ}{m^3}$$ *Ans.*

after $$(u_r)_{\text{final}} = \frac{1}{2}\sigma_{pl}\epsilon_{pl} = \frac{1}{2}(600 \text{ MPa})(0.008 \text{ mm/mm})$$

$$= 2.40 \text{ MJ/m}^3$$ *Ans.*

NOTE: By comparison, the effect of strain hardening the material has caused an increase in the modulus of resilience; however, note that the modulus of toughness for the material has decreased, since the area under the original curve, *OABF*, is larger than the area under curve *CBF*.

*Work in the SI system of units is measured in joules, where 1 J = 1 N·m.

Poisson's Ratio

EXAMPLE 3.4

$E = 200$ GPa (from 查表)

A bar made of A-36 steel has the dimensions shown in Fig. 3–22. If an axial force of $P = 80$ kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section. The material behaves elastically.

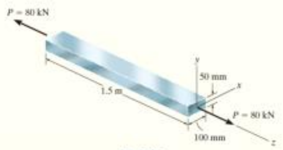


Fig. 3–22

補充 (可能考)!

$l = (1+\epsilon) l_0$

$\frac{\sigma}{E} = \frac{F/A}{E} = \frac{16 \text{ MPa}^{①}}{200 \times 10^3} = 80 \times 10^{-6}$

SOLUTION

The normal stress in the bar is

① $$\sigma_z = \frac{N}{A} = \frac{80(10^3) \text{ N}}{(0.1 \text{ m})(0.05 \text{ m})} = 16.0(10^6) \text{ Pa}$$

① $\frac{80}{100 \times 50} = 16 \text{ MPa}$

From the table given in the back of the book for A-36 steel $E_{st} = 200$ GPa, and so the strain in the z direction is

② $$\epsilon_z = \frac{\sigma_z}{E_{st}} = \frac{16.0(10^6) \text{ Pa}}{200(10^9) \text{ Pa}} = 80(10^{-6}) \text{ mm/mm}$$

The axial elongation of the bar is therefore

$$\delta_z = \epsilon_z L_z = [80(10^{-6})](\cancel{1.5 \text{ m}}\ 1500\text{mm}) = 120\ \mu\text{m}$$ *Ans.*

③ $80 \times 10^{-6} \times 1500 = 120\ \mu m \rightarrow \delta_z$

$\boxed{\frac{\delta l}{l} = \epsilon}$

$\Delta x = x_0 \epsilon_x$

$\Delta y = y_0 \epsilon_y$

Using Eq. 3–9, where $\nu_{st} = 0.32$ as found in the back of the book, the lateral contraction strains in *both* the x and y directions are

给定

$$\epsilon_x = \epsilon_y = -\nu_{st}\epsilon_z = -0.32[80(10^{-6})] = -25.6\ \mu\text{m/m}$$

Thus the changes in the dimensions of the cross section are

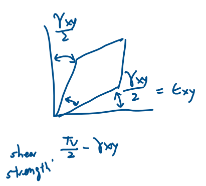
改变量

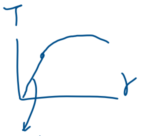
$$\delta_x = \epsilon_x L_x = -[25.6(10^{-6})](0.1 \text{ m}) = -2.56\ \mu\text{m}$$ *Ans.*

$$\delta_y = \epsilon_y L_y = -[25.6(10^{-6})](0.05 \text{ m}) = -1.28\ \mu\text{m}$$ *Ans.*

$$\boxed{\frac{\epsilon_x}{\epsilon_z} = \nu \qquad \epsilon_x = \nu\epsilon_z}$$

§ 3.7





斜率G = shear modules

$\Rightarrow \tau = G\gamma$

☆補充公式：$G = \frac{E}{2(1+\nu)}$

EXAMPLE 3.5

A specimen of titanium alloy is tested in torsion and the shear stress–strain diagram is shown in Fig. 3–25*a*. Determine the shear modulus *G*, the proportional limit, and the ultimate shear stress. Also, determine the maximum distance *d* that the top of a block of this material, shown in Fig. 3–25*b*, could be displaced horizontally if the material behaves elastically when acted upon by a shear force **V**. What is the magnitude of **V** necessary to cause this displacement?

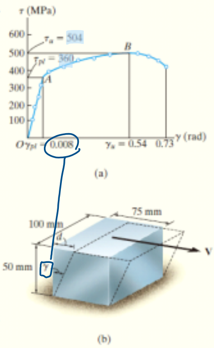


Fig. 3–25

SOLUTION

Shear Modulus. This value represents the slope of the straight-line portion *OA* of the τ–γ diagram. The coordinates of point *A* are (0.008 rad, 360 MPa). Thus,

$$G = \frac{360 \text{ MPa}}{0.008 \text{ rad}} = 45(10^3) \text{ MPa} = 45 \text{ GPa} \qquad \textit{Ans.}$$

τ_{pl} γ_{pl}

The equation of line *OA* is therefore $\tau = G\gamma = [45(10^3)\gamma]$ MPa, which is Hooke's law for shear.

Proportional Limit. By inspection, the graph ceases to be linear at point *A*. Thus,

$$\tau_{pl} = 360 \text{ MPa} \qquad \textit{Ans.}$$

Ultimate Stress. This value represents the maximum shear stress, point *B*. From the graph,

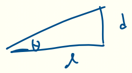
$$\tau_u = 504 \text{ MPa} \qquad \textit{Ans.}$$

Maximum Elastic Displacement and Shear Force. Since the maximum elastic shear strain is 0.008 rad, a very small angle, the top of the block in Fig. 3–25*b* will be displaced horizontally:

$$\tan(0.008 \text{ rad}) \approx 0.008 \text{ rad} = \frac{d}{50 \text{ mm}}$$

最大位移

$$d = 0.4 \text{ mm} \qquad \textit{Ans.}$$



$\theta \to 0$

$\sin\theta \to \theta$

$\cos\theta \to 1$

$\tan\theta \to \theta$

$\Rightarrow d = l\tan\theta$

$= l\theta$ ＃

The corresponding *average* shear stress in the block is $\tau_{pl} = 360$ MPa. Thus, the shear force *V* needed to cause the displacement is

$$\tau_{avg} = \frac{V}{A}; \qquad 360(10^6) \text{ N/m}^2 = \frac{V}{(0.075 \text{ m})(0.1 \text{ m})}$$

$$V = 2700 \text{ kN} \qquad \textit{Ans.}$$

最大剪力

§3.8 Creep 潛变 & Fatigue 疲勞

S-N curve