CHAPTER 3

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Horizontal ground displacements caused by an earthquake produced fracture of this concrete column. The material properties of the steel and concrete must be determined so that engineers can properly design the column to resist the loadings that caused this failure.

MECHANICAL PROPERTIES OF MATERIALS

CHAPTER OBJECTIVES

- Having discussed the basic concepts of stress and strain, in this chapter we will show how stress can be related to strain by using experimental methods to determine the stress–strain diagram for a specific material. Other mechanical properties and tests that are relevant to our study of mechanics of materials also will be discussed.

3.1 THE TENSION AND COMPRESSION TEST

The strength of a material depends on its ability to sustain a load without undue deformation or failure. This strength is inherent in the material itself and must be determined by *experiment*. One of the most important tests to perform in this regard is the ***tension or compression test***. Once this test is performed, we can then determine the relationship between the average normal stress and average normal strain in many engineering materials such as metals, ceramics, polymers, and composites.

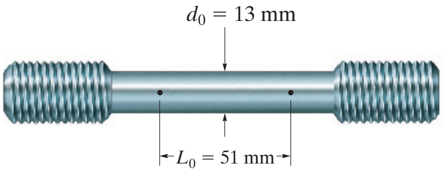

Fig. 3–1

To perform a tension or compression test, a specimen of the material is made into a "standard" shape and size, Fig. 3–1. As shown it has a constant circular cross section with enlarged ends, so that when tested, failure will occur somewhere within the central region of the specimen. Before testing, two small punch marks are sometimes placed along the specimen's uniform length. Measurements are taken of both the specimen's initial cross-sectional area, A_0, and the ***gage-length*** distance L_0 between the punch marks. For example, when a metal specimen is used in a tension test, it generally has an initial diameter of $d_0 = 13$ mm and a gage length of $L_0 = 51$ mm, Fig. 3–1. A testing machine like the one shown in Fig. 3–2 is then used to stretch the specimen at a very slow, constant rate until it fails. The machine is designed to read the load required to maintain this uniform stretching.

At frequent intervals, data is recorded of the applied load P. Also, the elongation $\delta = L - L_0$ between the punch marks on the specimen may be measured, using either a caliper or a mechanical or optical device called an ***extensometer***. Rather than taking this measurement and then calculating the strain, it is also possible to read the normal strain *directly* on the specimen by using an ***electrical-resistance strain gage***, which looks like the one shown in Fig. 3–3. As shown in the adjacent photo, the gage is cemented to the specimen along its length, so that it becomes an integral part of the specimen. When the specimen is strained in the direction of the gage, both the wire and specimen will experience the same deformation or strain. By measuring the change in the electrical resistance of the wire, the gage may then be calibrated to directly read the normal strain in the specimen.

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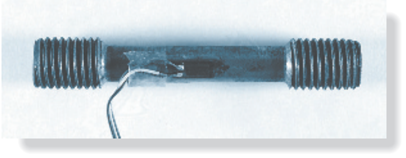


Typical steel specimen with attached strain gage

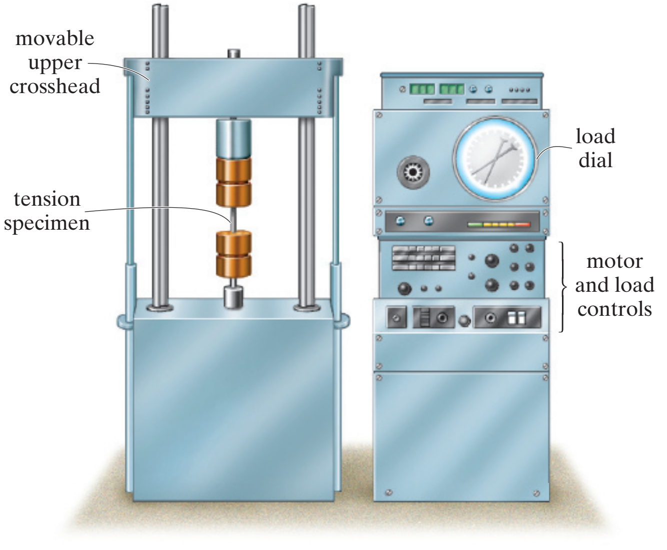
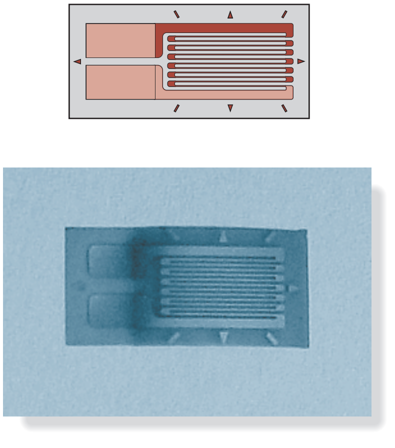


Fig. 3–2



Electrical-resistance strain gage

Fig. 3–3

3.2 THE STRESS–STRAIN DIAGRAM

Once the stress and strain data from the test are known, then the results can be plotted to produce a curve called the ***stress–strain diagram***. This diagram is very useful since it applies to a specimen of the material made of *any* size. There are two ways in which the stress–strain diagram is normally described.

Conventional Stress–Strain Diagram. The ***nominal*** or ***engineering stress*** is determined by dividing the applied load P by the specimen's *original* cross-sectional area A_0. This calculation assumes that the stress is *constant* over the cross section and throughout the gage length. We have

$$\boxed{\sigma = \frac{P}{A_0}} \tag{3–1}$$

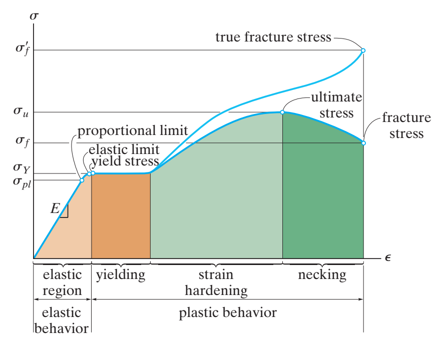
Likewise, the ***nominal*** or ***engineering strain*** is found directly from the strain gage reading, or by dividing the change in the specimen's gage length, δ, by the specimen's *original gage length* L_0. Thus,

$$\boxed{\epsilon = \frac{\delta}{L_0}} \tag{3–2}$$

When these values of σ and ϵ are plotted, where the vertical axis is the stress and the horizontal axis is the strain, the resulting curve is called a ***conventional stress–strain diagram***. A typical example of this curve is shown in Fig. 3–4. Realize, however, that two stress–strain diagrams for a particular material will be quite similar, but will never be exactly the same. This is because the results actually depend upon such variables as the material's composition, microscopic imperfections, the way the specimen is manufactured, the rate of loading, and the temperature during the time of the test.

From the curve in Fig. 3–4, we can identify four different regions in which the material behaves in a unique way, depending on the amount of strain induced in the material.

Elastic Behavior. The initial region of the curve, indicated in light orange, is referred to as the elastic region. Here the curve is a *straight line* up to the point where the stress reaches the ***proportional limit***, σ_{pl}. When the stress slightly exceeds this value, the curve bends until the stress reaches an elastic limit. For most materials, these points are very close, and therefore it becomes rather difficult to distinguish their exact values. What makes the elastic region unique, however, is that after reaching σ_Y, if the load is removed, the specimen will recover its original shape. In other words, no damage will be done to the material.



Conventional and true stress–strain diagram for ductile material (steel) (not to scale)

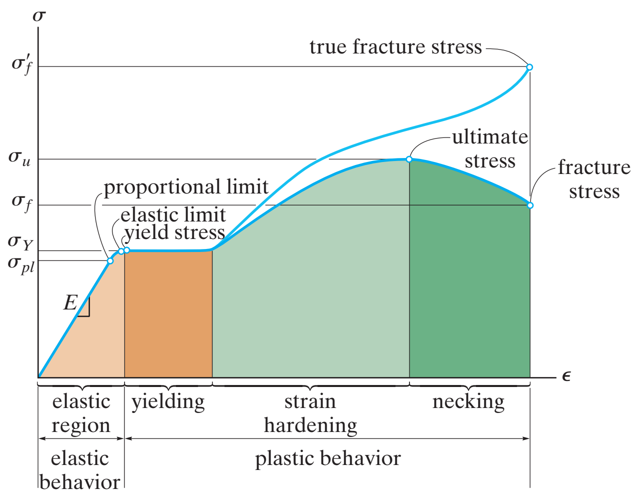
Fig. 3–4

Because the curve is a straight line up to σ_{pl}, any increase in stress will cause a proportional increase in strain. This fact was discovered in 1676 by Robert Hooke, using springs, and is known as ***Hooke's law.*** It is expressed mathematically as

$$\boxed{\sigma = E\epsilon} \tag{3–3}$$

Here E represents the constant of proportionality, which is called the ***modulus of elasticity*** or ***Young's modulus***, named after Thomas Young, who published an account of it in 1807.

As noted in Fig. 3–4, the modulus of elasticity represents the *slope* of the straight line portion of the curve. Since strain is dimensionless, from Eq. 3–3, E will have the same units as stress, such as pascals (Pa), megapascals (MPa), or gigapascals (GPa).



Conventional and true stress–strain diagram
for ductile material (steel) (not to scale)

Fig. 3–4 (Repeated)

Yielding. A slight increase in stress above the elastic limit will result in a breakdown of the material and cause it to *deform permanently*. This behavior is called ***yielding***, and it is indicated by the rectangular dark orange region in Fig. 3–4. The stress that causes yielding is called the ***yield stress*** or ***yield point***, σ_Y, and the deformation that occurs is called ***plastic deformation***. Although not shown in Fig. 3–4, for low-carbon steels or those that are hot rolled, the yield point is often distinguished by two values. The ***upper yield point*** occurs first, followed by a sudden decrease in load-carrying capacity to a ***lower yield point***. Once the yield point is reached, then as shown in Fig. 3–4, *the specimen will continue to elongate (strain)* ***without*** *any increase in load*. When the material behaves in this manner, it is often referred to as being ***perfectly plastic***.

Strain Hardening. When yielding has ended, any load causing an increase in stress will be supported by the specimen, resulting in a curve that rises continuously but becomes flatter until it reaches a maximum stress referred to as the ***ultimate stress***, σ_u. The rise in the curve in this manner is called ***strain hardening***, and it is identified in Fig. 3–4 as the region in light green.

Necking. Up to the ultimate stress, as the specimen elongates, its cross-sectional area will decrease in a fairly *uniform* manner over the specimen's entire gage length. However, just after reaching the ultimate stress, the cross-sectional area will then begin to decrease in a *localized* region of the specimen, and so it is here where the stress begins to increase. As a result, a constriction or "neck" tends to form with further elongation, Fig. 3–5*a*. This region of the curve due to necking is indicated in dark green in Fig. 3–4. Here the stress–strain diagram tends to curve downward until the specimen breaks at the ***fracture stress***, σ_f, Fig. 3–5*b*.

Typical necking pattern which has occurred on this steel specimen just before fracture.

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True Stress–Strain Diagram. Instead of always using the *original* cross-sectional area A_0 and specimen length L_0 to calculate the (engineering) stress and strain, we could have used the *actual* cross-sectional area A and specimen length L at the *instant* the load is measured. The values of stress and strain found from these measurements are called ***true stress*** and ***true strain***, and a plot of their values is called the ***true stress–strain diagram***. When this diagram is plotted, it has a form shown by the upper blue curve in Fig. 3–4. Note that the conventional and true σ–ϵ diagrams are practically coincident when the strain is small. The differences begin to appear in the strain-hardening range, where the magnitude of strain becomes more significant. From the conventional σ–ϵ diagram, the specimen appears to support a *decreasing* stress (or load), since A_0 is constant, $\sigma = N/A_0$. In fact, the true σ–ϵ diagram shows the area A within the necking region is always *decreasing* until fracture, σ'_f, and so the material *actually* sustains *increasing stress*, since $\sigma = N/A$.

Although there is this divergence between these two diagrams, we can neglect this effect since most engineering design is done only within the elastic range. This will generally restrict the deformation of the material to very small values, and when the load is removed the material will restore itself to its original shape. The conventional stress–strain diagram can be used in the elastic region because the true strain up to the elastic limit is small enough, so that the error in using the engineering values of σ and ϵ is very small (about 0.1%) compared with their true values.

Necking

(a)

Failure of a ductile material

(b)

Fig. 3–5

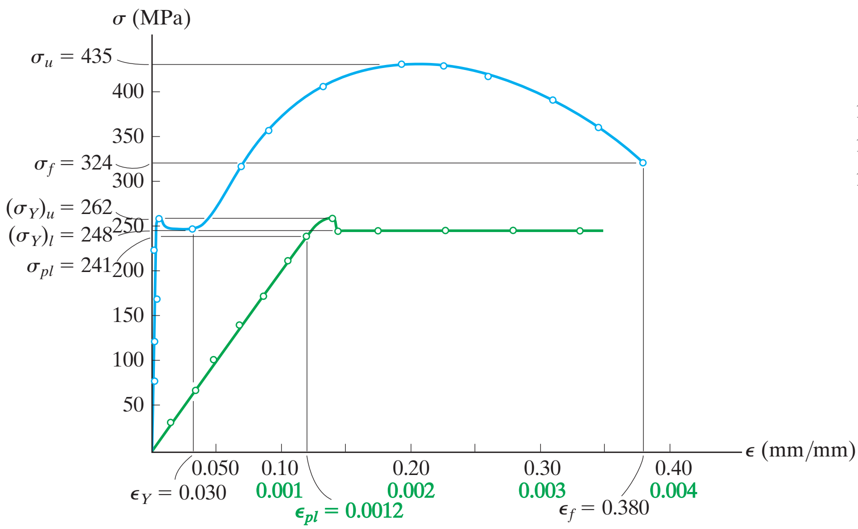
This steel specimen clearly shows the necking that occurred just before the specimen failed. This resulted in the formation of a "cup-cone" shape at the fracture location, which is characteristic of ductile materials.

Steel. A typical conventional stress–strain diagram for a mild steel specimen is shown in Fig. 3–6. In order to enhance the details, the elastic region of the curve has been shown in green using an exaggerated strain scale, also shown in green. Following this curve, as the load (stress) is increased, the proportional limit is reached at $\sigma_{pl} = 241$ MPa, where $\epsilon_{pl} = 0.0012$ mm/mm. When the load is further increased, the stress reaches an upper yield point of $(\sigma_Y)_u = 262$ MPa, followed by a drop in stress to a lower yield point of $(\sigma_Y)_l = 248$ MPa. The end of yielding occurs at a strain of $\epsilon_Y = 0.030$ mm/mm, which is 25 times greater than the strain at the proportional limit! Continuing, the specimen undergoes strain hardening until it reaches the ultimate stress of $\sigma_u = 435$ MPa; then it begins to neck down until fracture occurs, at $\sigma_f = 324$ MPa. By comparison, the strain at failure, $\epsilon_f = 0.380$ mm/mm, is 317 times greater than ϵ_{pl}!

Since $\sigma_{pl} = 241$ MPa and $\epsilon_{pl} = 0.0012$ mm/mm, we can determine the modulus of elasticity. From Hooke's law, it is

$$E = \frac{\sigma_{pl}}{\epsilon_{pl}} = \frac{241(10^6)\text{ Pa}}{0.0012\text{ mm/mm}} = 200\text{ GPa}$$

Although steel alloys have different carbon contents, most grades of steel, from the softest rolled steel to the hardest tool steel, have about this same modulus of elasticity, as shown in Fig. 3–7.



Stress–strain diagram for mild steel

Fig. 3–6

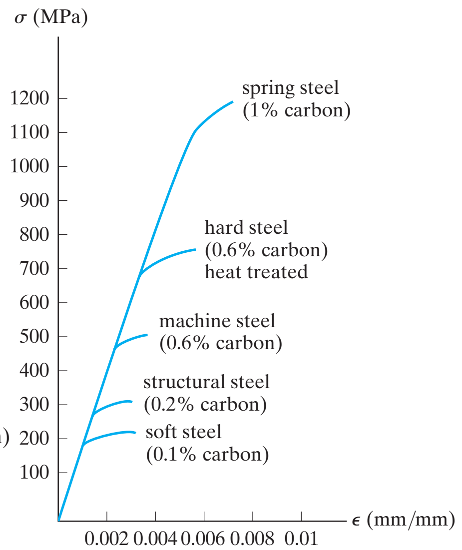


Fig. 3–7

3.3 STRESS–STRAIN BEHAVIOR OF DUCTILE AND BRITTLE MATERIALS

Materials can be classified as either being ductile or brittle, depending on their stress–strain characteristics.

Ductile Materials. Any material that can be subjected to large strains before it fractures is called a ***ductile material***. Mild steel, as discussed previously, is a typical example. Engineers often choose ductile materials for design because these materials are capable of absorbing shock or energy, and if they become overloaded, they will usually exhibit large deformation before failing.

One way to specify the ductility of a material is to report its percent elongation or percent reduction in area at the time of fracture. The ***percent elongation*** is the specimen's fracture strain expressed as a percent. Thus, if the specimen's original gage length is L_0 and its length at fracture is L_f, then

$$\text{Percent elongation} = \frac{L_f - L_0}{L_0}(100\%) \tag{3–4}$$

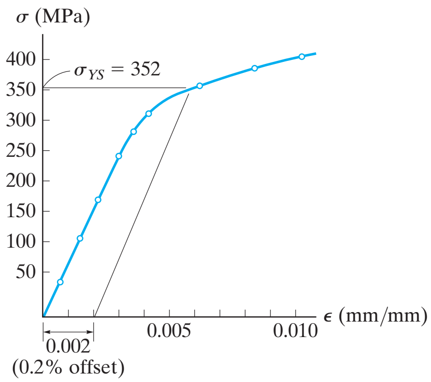
For example, as in Fig. 3–6, since $\epsilon_f = 0.380$, this value would be 38% for a mild steel specimen.

The ***percent reduction in area*** is another way to specify ductility. It is defined within the region of necking as follows:

$$\text{Percent reduction of area} = \frac{A_0 - A_f}{A_0}(100\%) \tag{3–5}$$

Here A_0 is the specimen's original cross-sectional area and A_f is the area of the neck at fracture. Mild steel has a typical value of 60%.

Besides steel, other metals such as brass, molybdenum, and zinc may also exhibit ductile stress–strain characteristics similar to steel, whereby they undergo elastic stress–strain behavior, yielding at constant stress, strain hardening, and finally necking until fracture. In most metals and some plastics, however, constant yielding will *not occur* beyond the elastic range. One metal where this is the case is aluminum, Fig. 3–8. Actually, this metal often does not have a well-defined *yield point*, and consequently it is standard practice to define a ***yield strength*** using a graphical procedure called the ***offset method***. Normally for structural design a 0.2% strain (0.002 mm/mm) is chosen, and from this point on the ϵ axis a line parallel to the initial straight line portion of the stress–strain diagram is drawn. The point where this line intersects the curve defines the yield strength. From the graph, the yield strength is $\sigma_{YS} = 352$ MPa.



Yield strength for an aluminum alloy

Fig. 3–8

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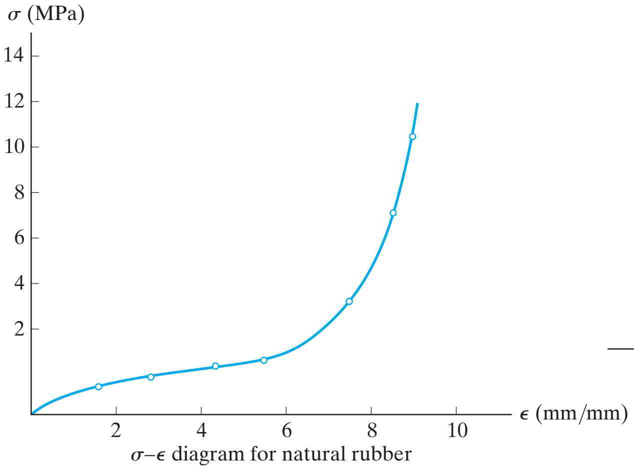
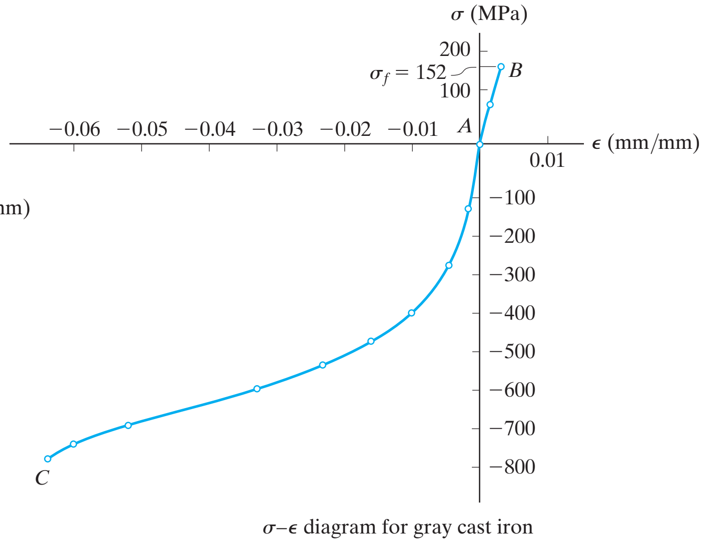


σ–ϵ diagram for natural rubber

Fig. 3–9



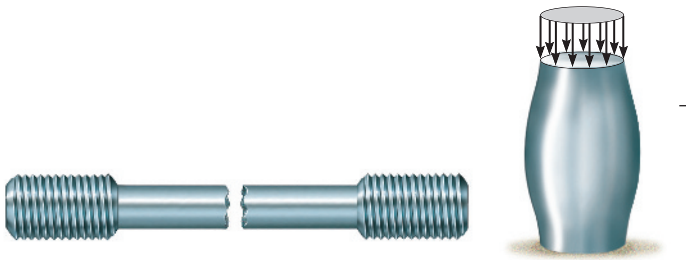
σ–ϵ diagram for gray cast iron

Fig. 3–10

Concrete used for structural purposes must be tested in compression to be sure it reaches its ultimate design stress after curing for 30 days.

Realize that the yield strength is not a physical property of the material, since it is a stress that causes a *specified* permanent strain in the material. In this text, however, we will assume that the yield strength, yield point, elastic limit, and proportional limit all *coincide* unless otherwise stated. An exception would be natural rubber, which in fact does not even have a proportional limit, since stress and strain are *not* linearly related. Instead, as shown in Fig. 3–9, this material, which is known as a polymer, exhibits *nonlinear elastic behavior*.

Wood is a material that is often moderately ductile, and as a result it is usually designed to respond only to elastic loadings. The strength characteristics of wood vary greatly from one species to another, and for each species they depend on the moisture content, age, and the size and arrangement of knots in the wood. Since wood is a fibrous material, its tensile or compressive characteristics parallel to its grain will differ greatly from these characteristics perpendicular to its grain. Specifically, wood splits easily when it is loaded in tension perpendicular to its grain, and consequently tensile loads are almost always intended to be applied parallel to the grain of wood members.



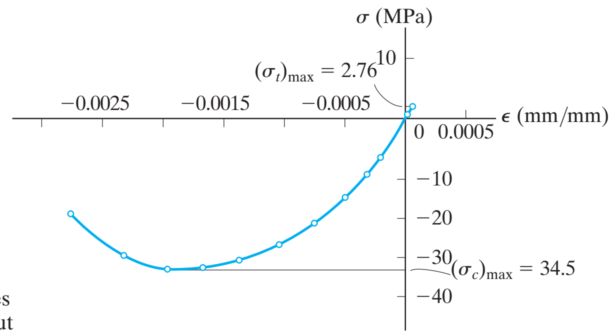
Tension failure of a brittle material

(a)

Compression causes material to bulge out

(b)

Fig. 3–11



$\sigma-\epsilon$ diagram for typical concrete mix

Fig. 3–12

Brittle Materials.

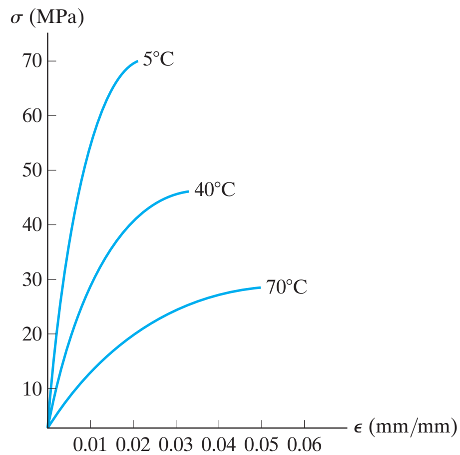
Materials that exhibit little or no yielding before failure are referred to as ***brittle materials***. Gray cast iron is an example, having a stress–strain diagram in tension as shown by the curve AB in Fig. 3–10. Here fracture at $\sigma_f = 152$ MPa occurred due to a microscopic crack, which then spread rapidly across the specimen, causing complete fracture. Since the appearance of initial cracks in a specimen is quite random, brittle materials do not have a well-defined tensile fracture stress. Instead the *average* fracture stress from a set of observed tests is generally reported. A typical failed specimen is shown in Fig. 3–11*a*.

Compared with their behavior in tension, brittle materials exhibit a much higher resistance to axial compression, as evidenced by segment AC of the gray cast iron curve in Fig. 3–10. For this case any cracks or imperfections in the specimen tend to close up, and as the load increases the material will generally bulge or become barrel shaped as the strains become larger, Fig. 3–11*b*.

Like gray cast iron, concrete is classified as a brittle material, and it also has a low strength capacity in tension. The characteristics of its stress–strain diagram depend primarily on the mix of concrete (water, sand, gravel, and cement) and the time and temperature of curing. A typical example of a "complete" stress–strain diagram for concrete is given in Fig. 3–12. By inspection, its maximum compressive strength is about 12.5 times greater than its tensile strength, $(\sigma_c)_{\max} = 34.5$ MPa versus $(\sigma_t)_{\max} = 2.76$ MPa. For this reason, concrete is almost always reinforced with steel bars or rods whenever it is designed to support tensile loads.

It can generally be stated that most materials exhibit both ductile and brittle behavior. For example, steel has brittle behavior when it contains a high carbon content, and it is ductile when the carbon content is reduced. Also, at low temperatures materials become harder and more brittle, whereas when the temperature rises they become softer and more ductile. This effect is shown in Fig. 3–13 for a methacrylate plastic.

Steel rapidly loses its strength when heated. For this reason engineers often require main structural members to be insulated in case of fire.



$\sigma-\epsilon$ diagrams for a methacrylate plastic

Fig. 3–13

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Stiffness. The modulus of elasticity is a mechanical property that indicates the ***stiffness*** of a material. Materials that are very stiff, such as steel, have large values of E ($E_{st} = 200$ GPa), whereas spongy materials such as vulcanized rubber have low values ($E_r = 0.69$ MPa). Values of E for commonly used engineering materials are often tabulated in engineering codes and reference books. Representative values are also listed in the back of the book.

The modulus of elasticity is one of the most important mechanical properties used in the development of equations presented in this text. It must always be remembered, though, that E, through the application of Hooke's law, Eq. 3–3, can be used only if a material has *linear elastic behavior*. Also, if the stress in the material is *greater* than the proportional limit, the stress–strain diagram ceases to be a straight line, and so Hooke's law is no longer valid.

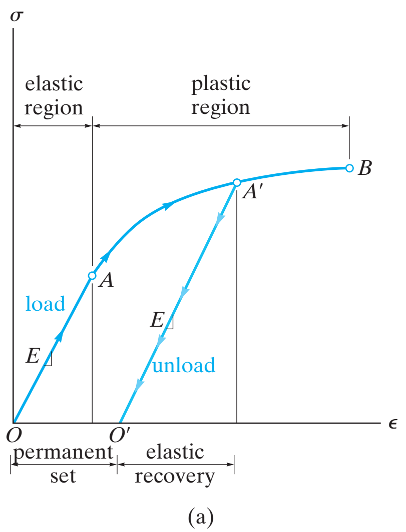


Fig. 3–14

Strain Hardening. If a specimen of ductile material, such as steel, is loaded into the *plastic region* and then unloaded, *elastic strain is recovered* as the material returns to its equilibrium state. The *plastic strain remains*, however, and as a result the material will be subjected to a ***permanent set***. For example, a wire when bent (plastically) will spring back a little (elastically) when the load is removed; however, it will not fully return to its original position. This behavior is illustrated on the stress–strain diagram shown in Fig. 3–14*a*. Here the specimen is loaded beyond its yield point A to point A'. Since interatomic forces have to be overcome to elongate the specimen *elastically*, then these same forces pull the atoms back together when the load is removed, Fig. 3–14*a*. Consequently, the modulus of elasticity, E, is the same, and therefore the slope of line $O'A'$ is the same as line OA. With the load removed, the permanent set is OO'.

If the load is reapplied, the atoms in the material will again be displaced until yielding occurs at or near the stress A', and the stress–strain diagram continues along the same path as before, Fig. 3–14*b*. Although this new stress–strain diagram, defined by $O'A'B$, now has a *higher* yield point (A'), a consequence of strain hardening, it also has *less ductility*, or a smaller plastic region, than when it was in its original state.

This pin was made of a hardened steel alloy, that is, one having a high carbon content. It failed due to brittle fracture.

3.4 STRAIN ENERGY

As a material is deformed by an external load, the load will do external work, which in turn will be stored in the material as internal energy. This energy is related to the strains in the material, and so it is referred to as ***strain energy***. To show how to calculate strain energy, consider a small volume element of material taken from a tension test specimen, Fig. 3–15. It is subjected to the uniaxial stress σ. This stress develops a force $\Delta F = \sigma\,\Delta A = \sigma(\Delta x\,\Delta y)$ on the top and bottom faces of the element, which causes the element to undergo a vertical displacement $\epsilon\,\Delta z$, Fig. 3–15*b*. By definition, ***work*** is determined by the product of a force and displacement in the direction of the force. Here the force is increased uniformly from *zero* to its final magnitude ΔF when the displacement $\epsilon\,\Delta z$ occurs, and so during the displacement the work done on the element by the force is equal to the *average* force magnitude $(\Delta F/2)$ times the displacement $\epsilon\,\Delta z$. The conservation of energy requires this "external work" on the element to be equivalent to the "internal work" or strain energy stored in the element, assuming that no energy is lost in the form of heat. Consequently, the strain energy is $\Delta U = (\frac{1}{2}\Delta F)\,\epsilon\,\Delta z = (\frac{1}{2}\,\sigma\,\Delta x\,\Delta y)\,\epsilon\,\Delta z$. Since the volume of the element is $\Delta V = \Delta x\,\Delta y\,\Delta z$, then $\Delta U = \frac{1}{2}\,\sigma\epsilon\,\Delta V$.

For engineering applications, it is often convenient to specify the strain energy per unit volume of material. This is called the ***strain energy density***, and it can be expressed as

$$u = \frac{\Delta U}{\Delta V} = \frac{1}{2}\sigma\epsilon \qquad (3\text{–}6)$$

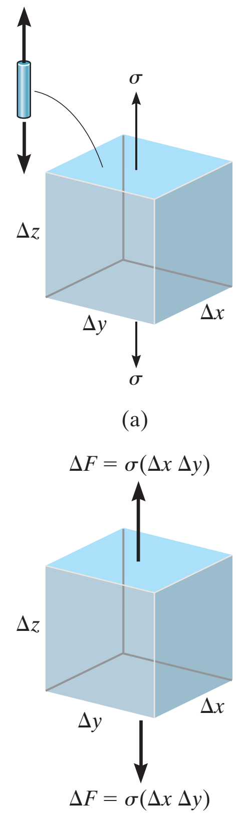
Finally, if the material behavior is *linear elastic*, then Hooke's law applies, $\sigma = E\epsilon$, and therefore we can express the ***elastic strain energy density*** in terms of the uniaxial stress σ as

$$u = \frac{1}{2}\frac{\sigma^2}{E} \qquad (3\text{–}7)$$

Modulus of Resilience. When the stress in a material reaches the proportional limit, the strain energy density, as calculated by Eq. 3–6 or 3–7, is referred to as the ***modulus of resilience***. It is

$$\boxed{u_r = \frac{1}{2}\sigma_{pl}\,\epsilon_{pl} = \frac{1}{2}\frac{\sigma_{pl}^2}{E}} \qquad (3\text{–}8)$$

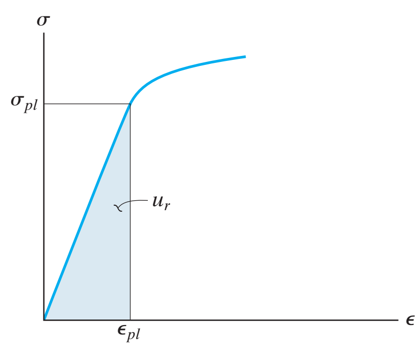
Here u_r is equivalent to the shaded *triangular area* under the elastic region of the stress–strain diagram, Fig. 3–16*a*. Physically the modulus of resilience represents the largest amount of strain energy per unit volume the material can absorb without causing any permanent damage to the material. Certainly this property becomes important when designing bumpers or shock absorbers.



Free-body diagram

(b)

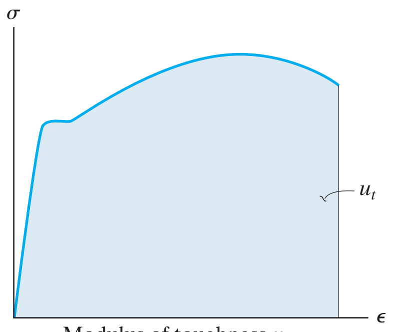
Fig. 3–15



Modulus of resilience u_r

(a)

Fig. 3–16



Modulus of toughness u_t

(b)

Fig. 3–16 (cont.)

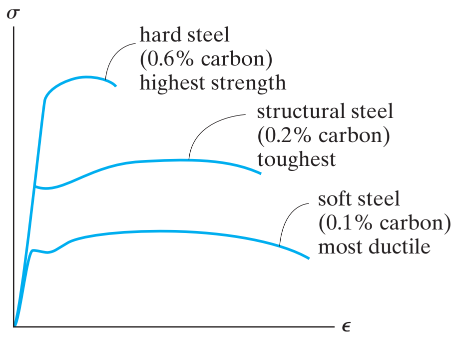


Fig. 3–17

This nylon specimen exhibits a high degree of toughness as noted by the large amount of necking that has occurred just before fracture.

Modulus of Toughness. Another important property of a material is its ***modulus of toughness***, u_t. This quantity represents the *entire area* under the stress–strain diagram, Fig. 3–16*b*, and therefore it indicates the maximum amount of strain energy per unit volume the material can absorb just before it fractures. Certainly this becomes important when designing members that may be accidentally overloaded. By alloying metals, engineers can change their resilience and toughness. For example, by changing the percentage of carbon in steel, the resulting stress–strain diagrams in Fig. 3–17 show how its resilience and toughness can be changed.

IMPORTANT POINTS

- A *conventional stress–strain diagram* is important in engineering since it provides a means for obtaining data about a material's tensile or compressive strength without regard for the material's physical size or shape.
- *Engineering stress and strain* are calculated using the *original* cross-sectional area and gage length of the specimen.
- A *ductile material*, such as mild steel, has four distinct behaviors as it is loaded. They are *elastic behavior, yielding, strain hardening,* and *necking*.
- A material is *linear elastic* if the stress is proportional to the strain within the elastic region. This behavior is described by *Hooke's law*, $\sigma = E\epsilon$, where the *modulus of elasticity E* is the slope of the line.
- Important points on the stress–strain diagram are the *proportional limit, elastic limit, yield stress, ultimate stress*, and *fracture stress*.
- The *ductility* of a material can be specified by the specimen's *percent elongation* or the *percent reduction in area*.
- If a material does not have a distinct yield point, a *yield strength* can be specified using a graphical procedure such as the *offset method*.
- *Brittle materials*, such as gray cast iron, have very little or no yielding and so they can fracture suddenly.
- *Strain hardening* is used to establish a higher yield point for a material. This is done by straining the material beyond the elastic limit, then releasing the load. The modulus of elasticity remains the same; however, the material's ductility *decreases*.
- *Strain energy* is energy stored in a material due to its deformation. This energy per unit volume is called *strain energy density*. If it is measured up to the proportional limit, it is referred to as the *modulus of resilience*, and if it is measured up to the point of fracture, it is called the *modulus of toughness*. It can be determined from the area under the σ–ϵ diagram.

EXAMPLE 3.1

A tension test for a steel alloy results in the stress–strain diagram shown in Fig. 3–18. Calculate the modulus of elasticity and the yield strength based on a 0.2% offset. Identify on the graph the ultimate stress and the fracture stress.

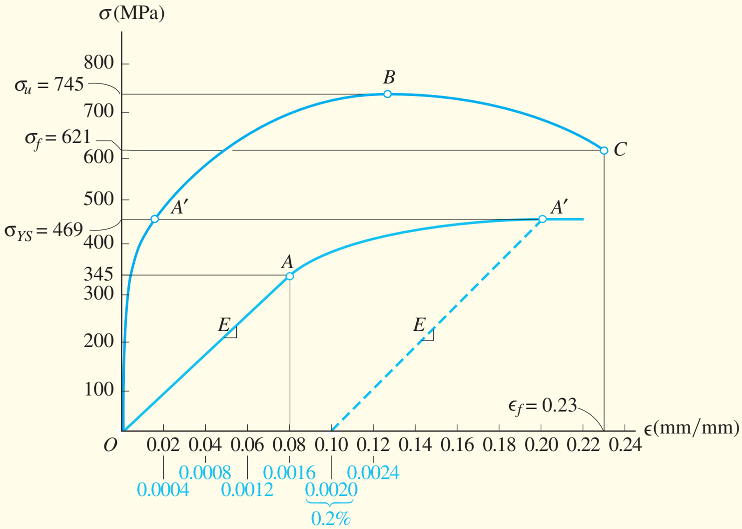


Fig. 3–18

SOLUTION

Modulus of Elasticity. We must calculate the *slope* of the initial straight-line portion of the graph. Using the magnified curve and scale shown in blue, this line extends from point O to an estimated point A, which has coordinates of approximately (0.0016 mm/mm, 345 MPa). Therefore,

$$E = \frac{345\text{ MPa}}{0.0016\text{ mm/mm}} = 216\text{ GPa} \qquad \textit{Ans.}$$

Note that the equation of line OA is thus $\sigma = [216(10^3)\epsilon]$ MPa.

Yield Strength. For a 0.2% offset, we begin at a strain of 0.2% or 0.0020 mm/mm and graphically extend a (dashed) line parallel to OA until it intersects the $\sigma-\epsilon$ curve at A'. The yield strength is approximately

$$\sigma_{YS} = 469\text{ MPa} \qquad \textit{Ans.}$$

Ultimate Stress. This is defined by the peak of the $\sigma-\epsilon$ graph, point B in Fig. 3–18.

$$\sigma_u = 745\text{ MPa} \qquad \textit{Ans.}$$

Fracture Stress. When the specimen is strained to its maximum of $\epsilon_f = 0.23$ mm/mm, it fractures at point C. Thus,

$$\sigma_f = 621\text{ MPa} \qquad \textit{Ans.}$$

EXAMPLE 3.2

The stress–strain diagram for an aluminum alloy that is used for making aircraft parts is shown in Fig. 3–19. If a specimen of this material is stressed to $\sigma = 600$ MPa, determine the permanent set that remains in the specimen when the load is released. Also, find the modulus of resilience both before and after the load application.

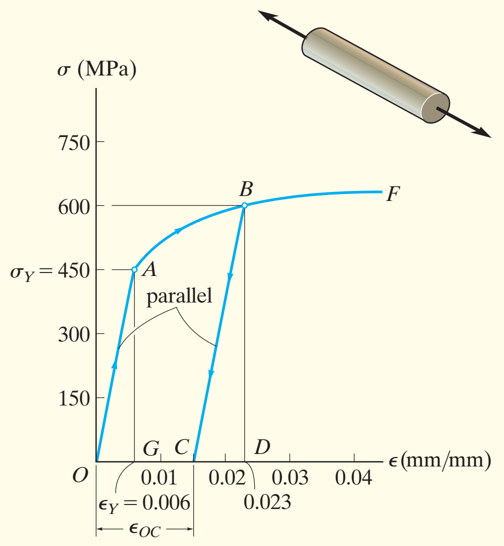


Fig. 3–19

SOLUTION

Permanent Strain. When the specimen is subjected to the load, it strain hardens until point B is reached on the σ–ϵ diagram. The strain at this point is approximately 0.023 mm/mm. When the load is released, the material behaves by following the straight line BC, which is parallel to line OA. Since both of these lines have the same slope, the strain at point C can be determined analytically. The slope of line OA is the modulus of elasticity, i.e.,

$$E = \frac{450 \text{ MPa}}{0.006 \text{ mm/mm}} = 75.0 \text{ GPa}$$

From triangle *CBD,* we require

$$E = \frac{BD}{CD}; \qquad 75.0(10^9)\ \text{Pa} = \frac{600(10^6)\ \text{Pa}}{CD}$$

$$CD = 0.008\ \text{mm/mm}$$

This strain represents the amount of *recovered elastic strain*. The permanent set or strain, ϵ_{OC}, is thus

$$\epsilon_{OC} = 0.023\ \text{mm/mm} - 0.008\ \text{mm/mm}$$

$$= 0.0150\ \text{mm/mm} \qquad \textit{Ans.}$$

NOTE: If gage marks on the specimen were originally 50 mm apart, then after the load is *released* these marks will be 50 mm + (0.0150)(50 mm) = 50.75 mm apart.

Modulus of Resilience. Applying Eq. 3–8, the areas under *OAG* and *CBD* in Fig. 3–19 are*

$$(u_r)_{\text{initial}} = \frac{1}{2}\sigma_{pl}\,\epsilon_{pl} = \frac{1}{2}(450\ \text{MPa})(0.006\ \text{mm/mm})$$

$$= 1.35\ \text{MJ/m}^3 \qquad \textit{Ans.}$$

$$(u_r)_{\text{final}} = \frac{1}{2}\sigma_{pl}\,\epsilon_{pl} = \frac{1}{2}(600\ \text{MPa})(0.008\ \text{mm/mm})$$

$$= 2.40\ \text{MJ/m}^3 \qquad \textit{Ans.}$$

NOTE: By comparison, the effect of strain hardening the material has caused an increase in the modulus of resilience; however, note that the modulus of toughness for the material has decreased, since the area under the original curve, *OABF*, is larger than the area under curve *CBF*.

*Work in the SI system of units is measured in joules, where 1 J = 1 N · m.

EXAMPLE 3.3

The aluminum rod, shown in Fig. 3–20*a*, has a circular cross section and is subjected to an axial load of 10 kN. If a portion of the stress–strain diagram is shown in Fig. 3–20*b*, determine the approximate elongation of the rod when the load is applied. Take $E_{al} = 70$ GPa.

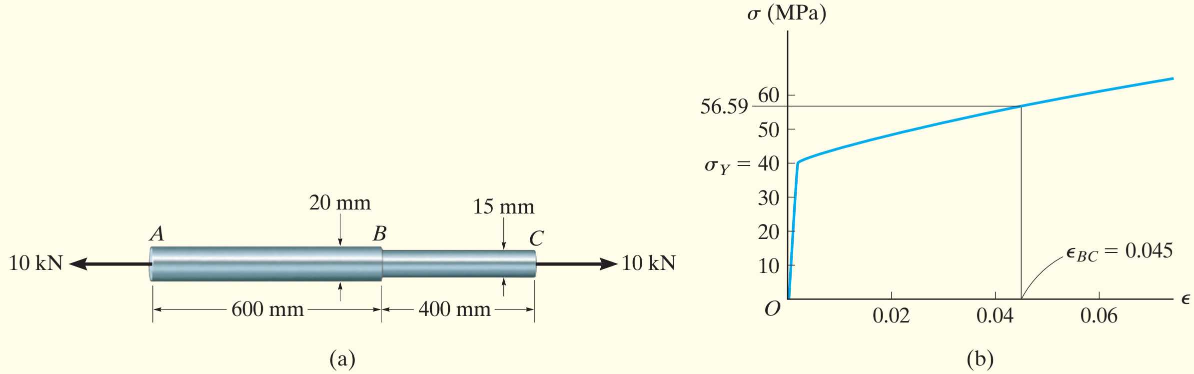


Fig. 3–20

SOLUTION

In order to find the elongation of the rod, we must first obtain the strain. This is done by calculating the stress, then using the stress–strain diagram. The normal stress within each segment is

$$\sigma_{AB} = \frac{N}{A} = \frac{10(10^3)\ \text{N}}{\pi(0.01\ \text{m})^2} = 31.83\ \text{MPa}$$

$$\sigma_{BC} = \frac{N}{A} = \frac{10(10^3)\ \text{N}}{\pi(0.0075\ \text{m})^2} = 56.59\ \text{MPa}$$

From the stress–strain diagram, the material in segment AB is strained *elastically* since $\sigma_{AB} < \sigma_Y = 40$ MPa. Using Hooke's law,

$$\epsilon_{AB} = \frac{\sigma_{AB}}{E_{al}} = \frac{31.83(10^6)\ \text{Pa}}{70(10^9)\ \text{Pa}} = 0.0004547\ \text{mm/mm}$$

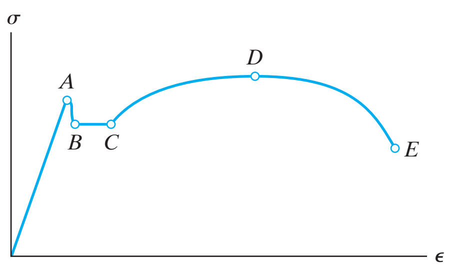
The material within segment BC is strained plastically, since $\sigma_{BC} > \sigma_Y = 40$ MPa. From the graph, for $\sigma_{BC} = 56.59$ MPa, $\epsilon_{BC} \approx 0.045$ mm/mm. The approximate elongation of the rod is therefore

$$\delta = \Sigma\epsilon L = 0.0004547(600\ \text{mm}) + 0.0450(400\ \text{mm})$$
$$= 18.3\ \text{mm} \qquad \textit{Ans.}$$

FUNDAMENTAL PROBLEMS

F3–1. Define a homogeneous material.

F3–2. Indicate the points on the stress–strain diagram which represent the proportional limit and the ultimate stress.



Prob. F3–2

F3–3. Define the modulus of elasticity E.

F3–4. At room temperature, mild steel is a ductile material. True or false?

F3–5. Engineering stress and strain are calculated using the *actual* cross-sectional area and length of the specimen. True or false?

F3–6. As the temperature increases the modulus of elasticity will increase. True or false?

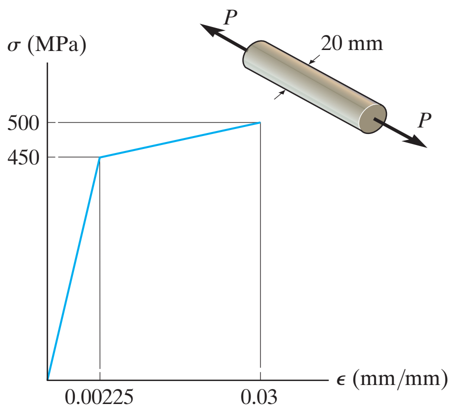
F3–7. A 100-mm-long rod has a diameter of 15 mm. If an axial tensile load of 100 kN is applied, determine its change in length. Assume linear elastic behavior with $E = 200$ GPa.

F3–8. A bar has a length of 200 mm and cross-sectional area of 7500 mm^2. Determine the modulus of elasticity of the material if it is subjected to an axial tensile load of 50 kN and stretches 0.075 mm. The material has linear-elastic behavior.

F3–9. A 10-mm-diameter rod has a modulus of elasticity of $E = 100$ GPa. If it is 4 m long and subjected to an axial tensile load of 6 kN, determine its elongation. Assume linear elastic behavior.

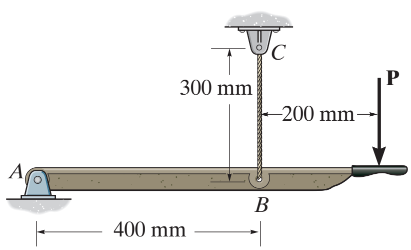
F3–10. The material for the 50-mm-long specimen has the stress–strain diagram shown. If $P = 100$ kN, determine the elongation of the specimen.

F3–11. The material for the 50-mm-long specimen has the stress–strain diagram shown. If $P = 150$ kN is applied and then released, determine the permanent elongation of the specimen.



Prob. F3–10/11

F3–12. If the elongation of wire BC is 0.2 mm after the force **P** is applied, determine the magnitude of **P**. The wire is A-36 steel and has a diameter of 3 mm.



Prob. F3–12

PROBLEMS

3–1. A tension test was performed on a steel specimen having an original diameter of 12.5 mm and gauge length of 50 mm. The data is listed in the table. Plot the stress–strain diagram and determine approximately the modulus of elasticity, the yield stress, the ultimate stress, and the rupture stress. Use a scale of $25\text{ mm} = 140\text{ MPa}$ and $25\text{ mm} = 0.05\text{ mm/mm}$. Redraw the elastic region, using the same stress scale but a strain scale of $25\text{ mm} = 0.001\text{ mm/mm}$.

Load (kN)	Elongation (mm)
0	0
7.0	0.0125
21.0	0.0375
36.0	0.0625
50.0	0.0875
53.0	0.125
53.0	0.2
54.0	0.5
75.0	1.0
90.0	2.5
97.0	7.0
87.8	10.0
83.3	11.5

Prob. 3–1

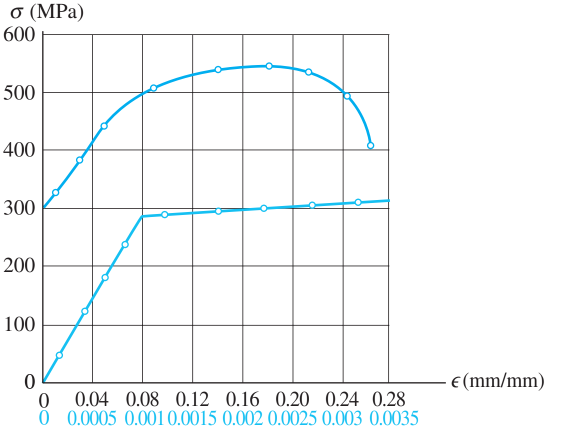
3–2. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine the modulus of elasticity and the modulus of resilience.

3–3. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine approximately the modulus of toughness. The rupture stress is $\sigma_r = 373.8\text{ MPa}$.

σ (MPa)	ϵ (mm/mm)
0	0
232.4	0.0006
318.5	0.0010
345.8	0.0014
360.5	0.0018
373.8	0.0022

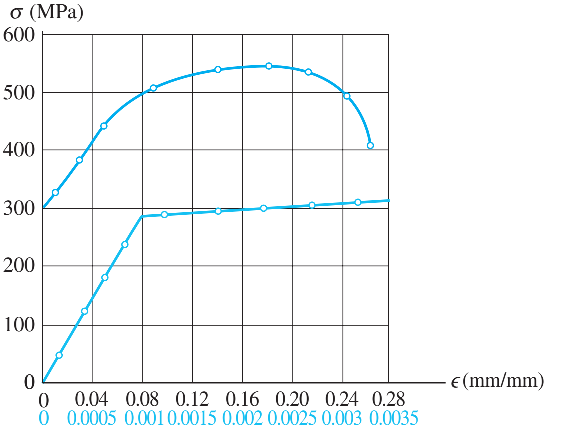
Probs. 3–2/3

***3–4.** The stress–strain diagram for a metal alloy having an original diameter of 12 mm and a gauge length of 50 mm is given in the figure. Determine approximately the modulus of elasticity for the material, the load on the specimen that causes yielding, and the ultimate load the specimen will support.



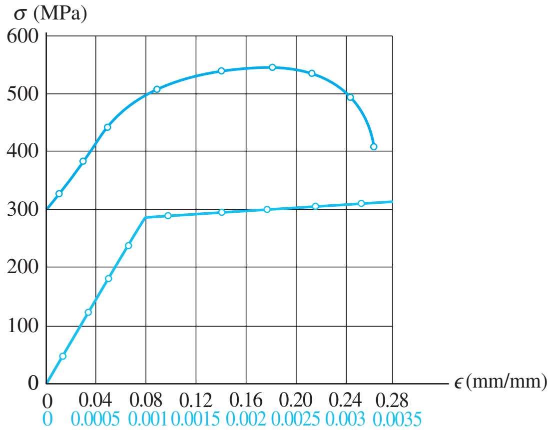
Prob. 3–4

3–5. The stress–strain diagram for a steel alloy having an original diameter of 12 mm and a gauge length of 50 mm is given in the figure. If the specimen is loaded until it is stressed to 500 MPa, determine the approximate amount of elastic recovery and the increase in the gauge length after it is unloaded.



Prob. 3–5

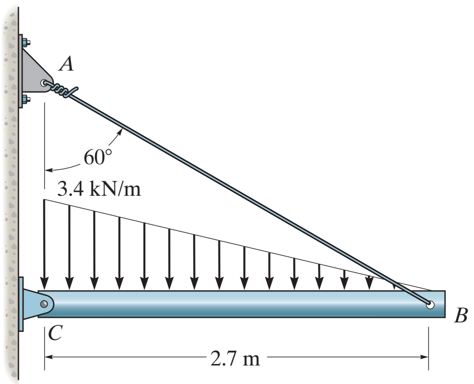
3–6. The stress–strain diagram for a steel alloy having an original diameter of 12 mm and a gauge length of 50 mm is given in the figure. Determine approximately the modulus of resilience and the modulus of toughness for the material.



Prob. 3–6

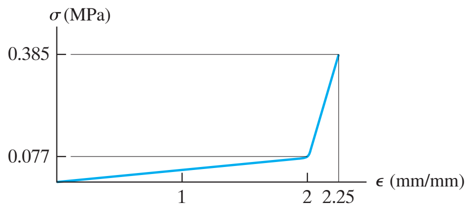
3–7. A specimen is originally 300 mm long, has a diameter of 12 mm, and is subjected to a force of 2.5 kN. When the force is increased from 2.5 kN to 9 kN, the specimen elongates 0.225 mm. Determine the modulus of elasticity for the material if it remains linear elastic.

***3–8.** The strut is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 5 mm, determine how much it stretches when the distributed load acts on the strut.



Prob. 3–8

3–9. The $\sigma-\epsilon$ diagram for elastic fibers that make up human skin and muscle is shown. Determine the modulus of elasticity of the fibers and estimate their modulus of toughness and modulus of resilience.

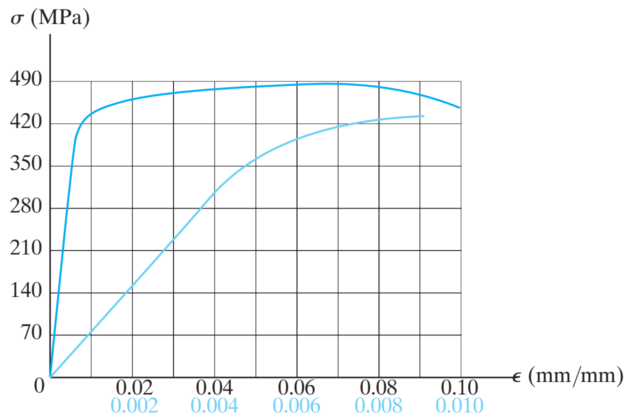


Prob. 3–9

3–10. A structural member in a nuclear reactor is made of a zirconium alloy. If an axial load of 20 kN is to be supported by the member, determine its required cross-sectional area. Use a factor of safety of 3 relative to yielding. What is the load on the member if it is 1 m long and its elongation is 0.5 mm? $E_{zr} = 100$ GPa, $\sigma_Y = 400$ MPa. The material has elastic behavior.

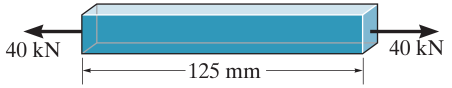
3–11. A tension test was performed on an aluminum 2014-T6 alloy specimen. The resulting stress–strain diagram is shown in the figure. Estimate (a) the proportional limit, (b) the modulus of elasticity, and (c) the yield strength based on a 0.2% strain offset method.

***3–12.** A tension test was performed on an aluminum 2014-T6 alloy specimen. The resulting stress–strain diagram is shown in the figure. Estimate (a) the modulus of resilience; and (b) modulus of toughness.



Probs. 3–10/11/12

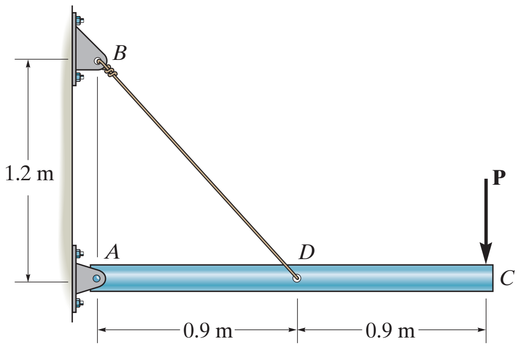
3–13. A bar having a length of 125 mm and cross-sectional area of 437.5 mm^2 is subjected to an axial force of 40 kN. If the bar stretches 0.05 mm, determine the modulus of elasticity of the material. The material has linear-elastic behavior.



Prob. 3–13

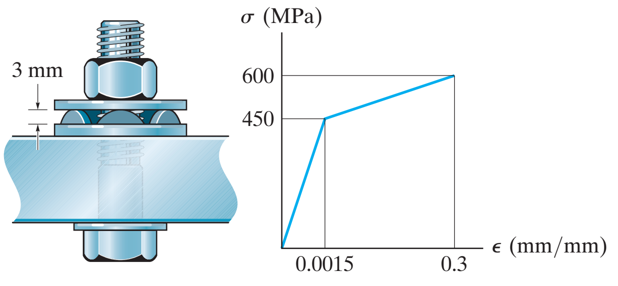
3–14. The rigid pipe is supported by a pin at A and an A-36 steel guy wire BD. If the wire has a diameter of 6.5 mm, determine how much it stretches when a load of $P = 3$ kN acts on the pipe.

3–15. The rigid pipe is supported by a pin at A and an A-36 guy wire BD. If the wire has a diameter of 6.5 mm, determine the load P if the end C is displaced 1.675 mm downward.



Probs. 3–14/15

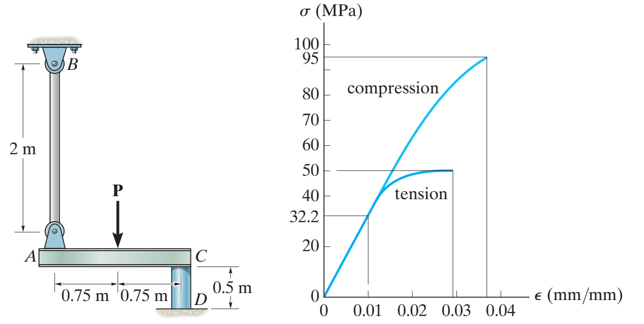
***3–16.** Direct tension indicators are sometimes used instead of torque wrenches to ensure that a bolt has a prescribed tension when used for connections. If a nut on the bolt is tightened so that the six 3-mm high heads of the indicator are strained 0.1 mm/mm, and leave a contact area on each head of 1.5 mm^2, determine the tension in the bolt shank. The material has the stress–strain diagram shown.



Prob. 3–16

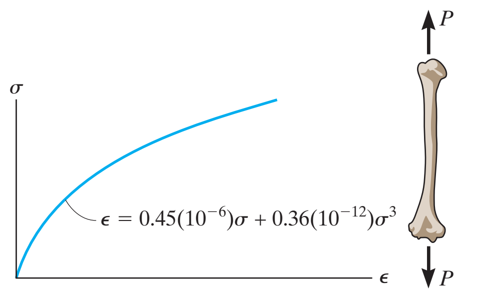
3–17. The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD, both made from this material, and subjected to a load of $P = 80$ kN, determine the angle of tilt of the beam when the load is applied. The diameter of the strut is 40 mm and the diameter of the post is 80 mm.

3–18. The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD made from this material, determine the largest load P that can be applied to the beam before it ruptures. The diameter of the strut is 12 mm and the diameter of the post is 40 mm.



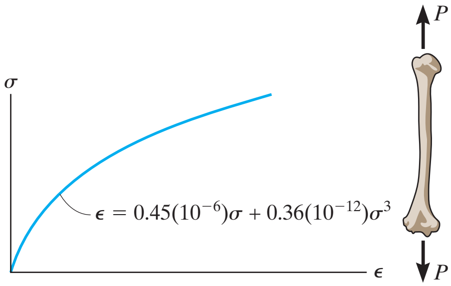
Probs. 3–17/18

3–19. The stress–strain diagram for a bone is shown, and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the yield strength assuming a 0.3% offset.



Prob. 3–19

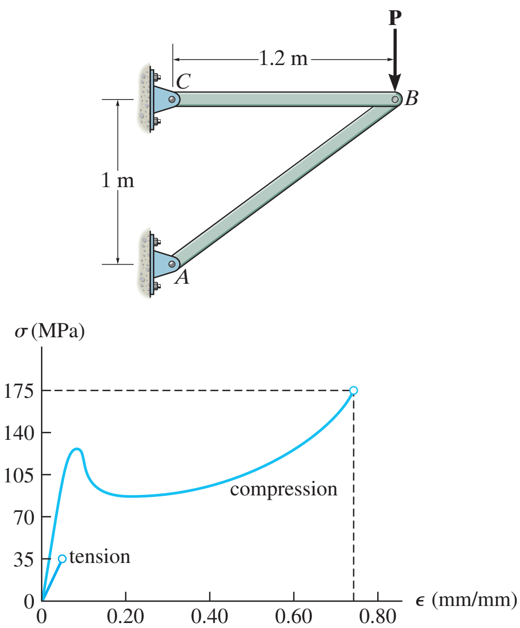
***3–20.** The stress–strain diagram for a bone is shown and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the modulus of toughness and the amount of elongation of a 200-mm-long region just before it fractures if failure occurs at $\epsilon = 0.12$ mm/mm.



Prob. 3–20

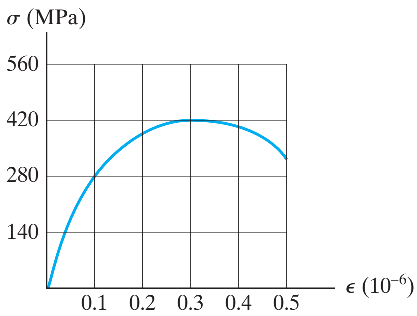
3–21. The two bars are made of polystyrene, which has the stress–strain diagram shown. If the cross-sectional area of bar AB is 975 mm^2 and BC is 2600 mm^2, determine the largest force P that can be supported before any member ruptures. Assume that buckling does not occur.

3–22. The two bars are made of polystyrene, which has the stress–strain diagram shown. Determine the cross-sectional area of each bar so that the bars rupture simultaneously when the load $P = 13.5$ kN. Assume that buckling does not occur.



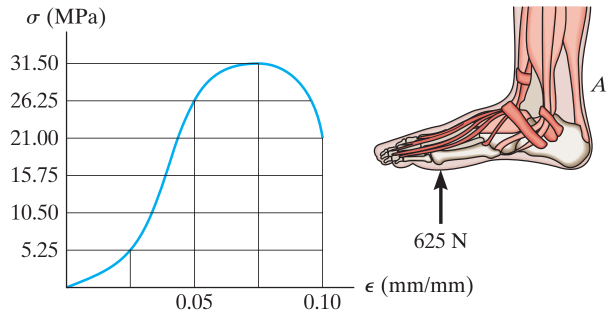
Probs. 3–21/22

3–23. The stress–strain diagram for many metal alloys can be described analytically using the Ramberg-Osgood three parameter equation $\epsilon = \sigma/E + k\sigma^n$, where E, k, and n are determined from measurements taken from the diagram. Using the stress–strain diagram shown in the figure, take $E = 210$ GPa and determine the other two parameters k and n and thereby obtain an analytical expression for the curve.



Prob. 3–23

***3–24.** The $\sigma - \epsilon$ diagram for a collagen fiber bundle from which a human tendon is composed is shown. If a segment of the Achilles tendon at A has a length of 165 mm and an approximate cross-sectional area of 145 mm^2, determine its elongation if the foot supports a load of 625 N, which causes a tension in the tendon of 1718.75 N.



Prob. 3–24

3.5 POISSON'S RATIO

When a deformable body is subjected to a force, not only does it elongate but it also contracts laterally. For example, consider the bar in Fig. 3–21 that has an original radius r and length L, and is subjected to the tensile force P. This force elongates the bar by an amount δ, and its radius contracts by an amount δ'. The strains in the longitudinal or axial direction and in the lateral or radial direction become

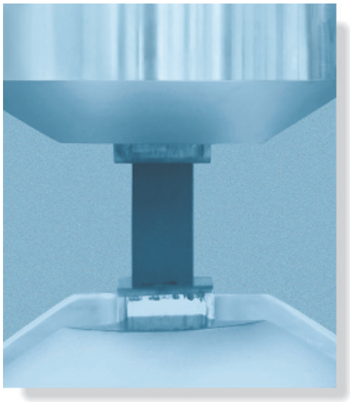
$$\epsilon_{\text{long}} = \frac{\delta}{L} \quad \text{and} \quad \epsilon_{\text{lat}} = \frac{\delta'}{r}$$

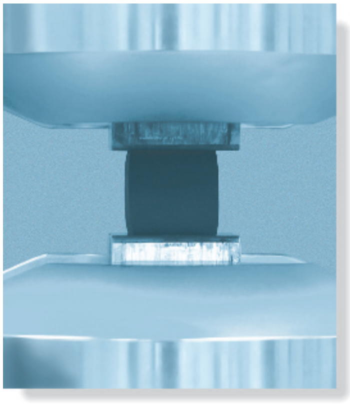
In the early 1800s, the French scientist S. D. Poisson realized that *within the elastic range* the *ratio* of these strains is a *constant*, since the displacements δ and δ' are proportional to the same applied force. This ratio is referred to as ***Poisson's ratio***, ν (nu), and it has a numerical value that is unique for any material that is both *homogeneous and isotropic*. Stated mathematically it is

$$\boxed{\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}} \tag{3–9}$$

The negative sign is included here since *longitudinal elongation* (positive strain) causes *lateral contraction* (negative strain), and vice versa. Keep in mind that these strains are caused only by the single axial or longitudinal force P; i.e., no force acts in a lateral direction in order to strain the material in this direction.

Poisson's ratio is a *dimensionless* quantity, and it will be shown in Sec. 10.6 that its *maximum* possible value is 0.5, so that $0 \le \nu \le 0.5$. For most nonporous solids it has a value that is generally between 0.25 and 0.355. Typical values for common engineering materials are listed in the back of the book.





When the rubber block is compressed (negative strain), its sides will expand (positive strain). The ratio of these strains remains constant.

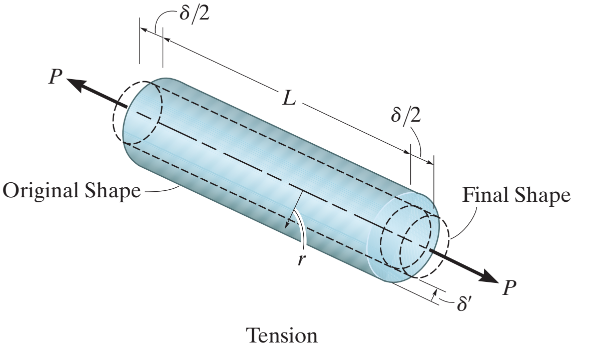


Fig. 3–21

3

EXAMPLE 3.4

A bar made of A-36 steel has the dimensions shown in Fig. 3–22. If an axial force of $P = 80$ kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section. The material behaves elastically.

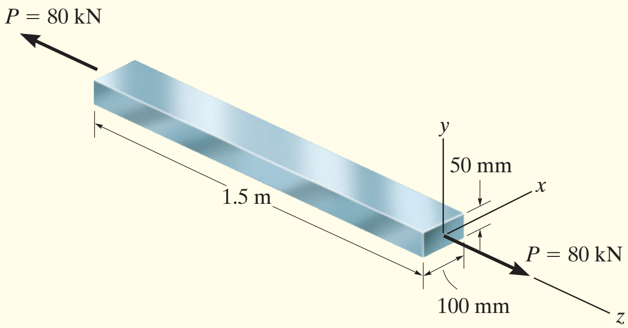


Fig. 3–22

SOLUTION

The normal stress in the bar is

$$\sigma_z = \frac{N}{A} = \frac{80(10^3)\ \text{N}}{(0.1\ \text{m})(0.05\ \text{m})} = 16.0(10^6)\ \text{Pa}$$

From the table given in the back of the book for A-36 steel $E_{st} = 200$ GPa, and so the strain in the z direction is

$$\epsilon_z = \frac{\sigma_z}{E_{st}} = \frac{16.0(10^6)\ \text{Pa}}{200(10^9)\ \text{Pa}} = 80(10^{-6})\ \text{mm/mm}$$

The axial elongation of the bar is therefore

$$\delta_z = \epsilon_z L_z = [80(10^{-6})](1.5\ \text{m}) = 120\ \mu\text{m} \qquad \textit{Ans.}$$

Using Eq. 3–9, where $\nu_{st} = 0.32$ as found in the back of the book, the lateral contraction strains in *both* the x and y directions are

$$\epsilon_x = \epsilon_y = -\nu_{st}\,\epsilon_z = -0.32[80(10^{-6})] = -25.6\ \mu\text{m/m}$$

Thus the changes in the dimensions of the cross section are

$$\delta_x = \epsilon_x L_x = -[25.6(10^{-6})](0.1\ \text{m}) = -2.56\ \mu\text{m} \qquad \textit{Ans.}$$

$$\delta_y = \epsilon_y L_y = -[25.6(10^{-6})](0.05\ \text{m}) = -1.28\ \mu\text{m} \qquad \textit{Ans.}$$

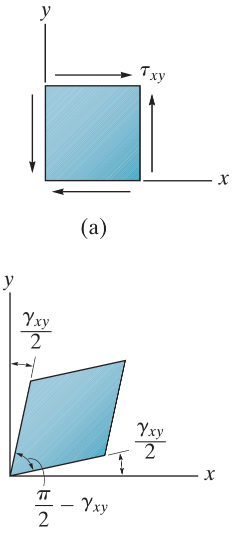


Fig. 3–23

3.6 THE SHEAR STRESS–STRAIN DIAGRAM

In Sec. 1.5 it was shown that when a small element of material is subjected to *pure shear*, equilibrium requires that equal shear stresses must be developed on four faces of the element, Fig. 3–23*a*. Furthermore, if the material is *homogeneous* and *isotropic*, then this shear stress will distort the element *uniformly*, Fig. 3–23*b*, producing shear strain.

In order to study the behavior of a material subjected to pure shear, engineers use a specimen in the shape of a thin tube and subject it to a torsional loading. If measurements are made of the applied torque and the resulting angle of twist, then by the methods to be explained in Chapter 5, the data can be used to determine the shear stress and shear strain within the tube and thereby produce a shear stress–strain diagram such as shown in Fig. 3–24. Like the tension test, this material when subjected to shear will exhibit linear elastic behavior and it will have a defined *proportional limit* τ_{pl}. Also, strain hardening will occur until an *ultimate shear stress* τ_u is reached. And finally, the material will begin to lose its shear strength until it reaches a point where it fractures, τ_f.

For most engineering materials, like the one just described, the elastic behavior is *linear*, and so Hooke's law for shear can be written as

$$\boxed{\tau = G\gamma} \tag{3–10}$$

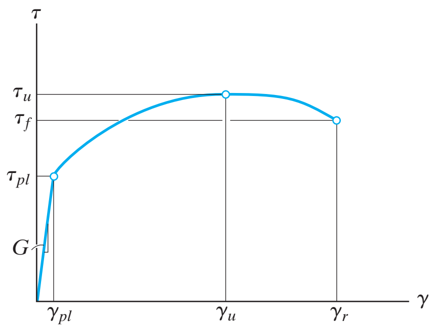


Fig. 3–24

Here G is called the ***shear modulus of elasticity*** or the ***modulus of rigidity***. Its value represents the slope of the line on the τ–γ diagram, that is, $G = \tau_{pl}/\gamma_{pl}$. Units of measurement for G will be the *same* as those for τ (Pa), since γ is measured in radians, a dimensionless quantity. Typical values for common engineering materials are listed in the back of the book.

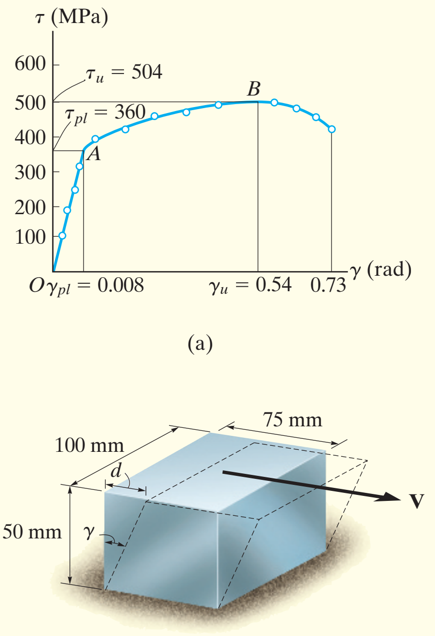
Later it will be shown in Sec. 10.6 that the three material constants, E, ν, and G can all be *related* by the equation

$$\boxed{G = \frac{E}{2(1 + \nu)}} \tag{3–11}$$

Therefore, if E and G are known, the value of ν can then be determined from this equation rather than through experimental measurement.

EXAMPLE 3.5

A specimen of titanium alloy is tested in torsion and the shear stress–strain diagram is shown in Fig. 3–25*a*. Determine the shear modulus G, the proportional limit, and the ultimate shear stress. Also, determine the maximum distance d that the top of a block of this material, shown in Fig. 3–25*b*, could be displaced horizontally if the material behaves elastically when acted upon by a shear force **V**. What is the magnitude of **V** necessary to cause this displacement?



(a)

(b)

Fig. 3–25

SOLUTION

Shear Modulus. This value represents the slope of the straight-line portion OA of the τ–γ diagram. The coordinates of point A are (0.008 rad, 360 MPa). Thus,

$$G = \frac{360 \text{ MPa}}{0.008 \text{ rad}} = 45(10^3) \text{ MPa} = 45 \text{ GPa} \qquad \textit{Ans.}$$

The equation of line OA is therefore $\tau = G\gamma = [45(10^3)\gamma]$ MPa, which is Hooke's law for shear.

Proportional Limit. By inspection, the graph ceases to be linear at point A. Thus,

$$\tau_{pl} = 360 \text{ MPa} \qquad \textit{Ans.}$$

Ultimate Stress. This value represents the maximum shear stress, point B. From the graph,

$$\tau_u = 504 \text{ MPa} \qquad \textit{Ans.}$$

Maximum Elastic Displacement and Shear Force. Since the maximum elastic shear strain is 0.008 rad, a very small angle, the top of the block in Fig. 3–25*b* will be displaced horizontally:

$$\tan(0.008 \text{ rad}) \approx 0.008 \text{ rad} = \frac{d}{50 \text{ mm}}$$

$$d = 0.4 \text{ mm} \qquad \textit{Ans.}$$

The corresponding *average* shear stress in the block is $\tau_{pl} = 360$ MPa. Thus, the shear force V needed to cause the displacement is

$$\tau_{\text{avg}} = \frac{V}{A}; \qquad 360(10^6) \text{ N/m}^2 = \frac{V}{(0.075\text{m})\,(0.1 \text{ m})}$$

$$V = 2700 \text{ kN} \qquad \textit{Ans.}$$

3

EXAMPLE 3.6

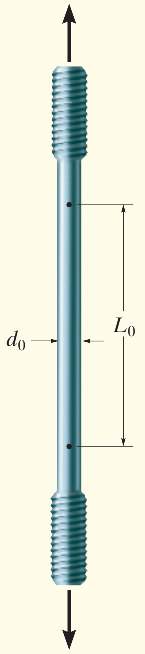


Fig. 3–26

An aluminum specimen shown in Fig. 3–26 has a diameter of $d_0 = 25$ mm and a gage length of $L_0 = 250$ mm. If a force of 165 kN elongates the gage length 1.20 mm, determine the modulus of elasticity. Also, determine by how much the force causes the diameter of the specimen to contract. Take $G_{al} = 26$ GPa and $\sigma_Y = 440$ MPa.

SOLUTION

Modulus of Elasticity. The average normal stress in the specimen is

$$\sigma = \frac{N}{A} = \frac{165(10^3)\text{ N}}{(\pi/4)(0.025\text{ m})^2} = 336.1\text{ MPa}$$

and the average normal strain is

$$\epsilon = \frac{\delta}{L} = \frac{1.20\text{ mm}}{250\text{ mm}} = 0.00480\text{ mm/mm}$$

Since $\sigma < \sigma_Y = 440$ MPa, the material behaves elastically. The modulus of elasticity is therefore

$$E_{al} = \frac{\sigma}{\epsilon} = \frac{336.1(10^6)\text{ Pa}}{0.00480} = 70.0\text{ GPa} \qquad \textit{Ans.}$$

Contraction of Diameter. First we will determine Poisson's ratio for the material using Eq. 3–11.

$$G = \frac{E}{2(1+\nu)}$$

$$26\text{ GPa} = \frac{70.0\text{ GPa}}{2(1+\nu)}$$

$$\nu = 0.347$$

Since $\epsilon_{long} = 0.00480$ mm/mm, then by Eq. 3–9,

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$$

$$0.347 = -\frac{\epsilon_{lat}}{0.00480\text{ mm/mm}}$$

$$\epsilon_{lat} = -0.00166\text{ mm/mm}$$

The contraction of the diameter is therefore

$$\delta' = (0.00166)(25\text{ mm})$$

$$= 0.0416\text{ mm} \qquad \textit{Ans.}$$

*3.7 FAILURE OF MATERIALS DUE TO CREEP AND FATIGUE

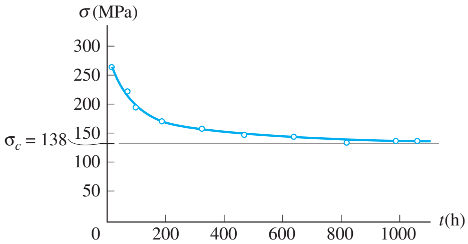
The mechanical properties of a material have up to this point been discussed only for a static or slowly applied load at constant temperature. In some cases, however, a member may have to be used in an environment for which loadings must be sustained over long periods of time at elevated temperatures, or in other cases, the loading may be repeated or cycled. We will not cover these effects in this book, although we will briefly mention how one determines a material's strength for these conditions, since in some cases they must be considered for design.

Creep. When a material has to support a load for a very long period of time, it may continue to deform until a sudden fracture occurs or its usefulness is impaired. This time-dependent permanent deformation is known as ***creep***. Normally creep is considered when metals and ceramics are used for structural members or mechanical parts that are subjected to high temperatures. For some materials, however, such as polymers and composite materials–including wood or concrete–temperature is *not* an important factor, and yet creep can occur strictly from long-term load application. As a typical example, consider the fact that a rubber band will not return to its original shape after being released from a stretched position in which it was held for a very long period of time.

The long-term application of the cable loading on this pole has caused the pole to deform due to creep.

For practical purposes, when creep becomes important, a member is usually designed to resist a specified creep strain for a given period of time. An important mechanical property that is used in this regard is called the ***creep strength***. This value represents the highest stress the material can withstand during a specified time without exceeding an allowable creep strain. The creep strength will vary with temperature, and for design, a temperature, duration of loading, and allowable creep strain must all be specified. For example, a creep strain of 0.1% per year has been suggested for steel used for bolts and piping.

Several methods exist for determining the allowable creep strength for a particular material. One of the simplest involves testing several specimens simultaneously at a constant temperature, but with each subjected to a different axial stress. By measuring the length of time needed to produce the allowable creep strain for each specimen, a curve of stress versus time can be established. Normally these tests are run to a maximum of 1000 hours. An example of the results for stainless steel at a temperature of 650°C and prescribed creep strain of 1% is shown in Fig. 3–27. As noted, this material has a yield strength of 276 MPa at room temperature (0.2% offset) and the creep strength at 1000 h is found to be approximately $\sigma_c = 138$ MPa.



σ–t diagram for stainless steel at 650°C and creep strain at 1%

Fig. 3–27

For longer periods of time, extrapolations from the curves must be made. To do this usually requires a certain amount of experience with creep behavior, and some supplementary knowledge about the creep properties of the material. Once the material's creep strength has been determined, however, a factor of safety is applied to obtain an appropriate allowable stress for design.

The design of members used for amusement park rides requires careful consideration of cyclic loadings that can cause fatigue.

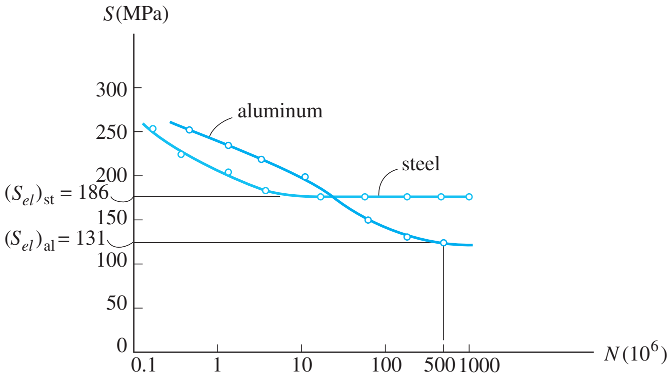
Fatigue. When a metal is subjected to repeated cycles of stress or strain, it causes its internal structure to break down, ultimately leading to fracture. This behavior is called ***fatigue***, and it is usually responsible for a large percentage of failures in connecting rods and crankshafts of engines; steam or gas turbine blades; connections or supports for bridges, railroad wheels, and axles; and other parts subjected to cyclic loading. In all these cases, fracture will occur at a stress that is *less* than the material's yield stress.

The nature of this failure apparently results from the fact that there are microscopic imperfections, usually on the surface of the member, where the localized stress becomes *much greater* than the average stress acting over the cross section. As this higher stress is cycled, it leads to the formation of minute cracks. Occurrence of these cracks causes a further increase of stress at their tips, which in turn causes a further extension of the cracks into the material as the stress continues to be cycled. Eventually the cross-sectional area of the member is reduced to the point where the load can no longer be sustained, and as a result sudden fracture occurs. The material, even though known to be ductile, behaves as if it were brittle.

In order to specify a safe strength for a metallic material under repeated loading, it is necessary to determine a limit below which no evidence of failure can be detected after applying a load for a specified number of cycles. This limiting stress is called the ***endurance*** or ***fatigue limit***. Using a testing machine for this purpose, a series of specimens are each subjected to a specified stress and cycled to failure. The results are plotted as a graph representing the stress S (or σ) on the vertical axis and the number of cycles-to-failure N on the horizontal axis. This graph is called an ***S–N diagram*** or ***stress–cycle diagram***, and most often the values of N are plotted on a logarithmic scale since they are generally quite large.

Engineers must account for possible fatigue failure of the moving parts of this oil-pumping rig.

Examples of S–N diagrams for two common engineering metals are shown in Fig. 3–28. The endurance limit is usually identified as the stress for which the S–N graph becomes horizontal or asymptotic. As noted, it has a well-defined value of $(S_{el})_{\text{st}} = 186$ MPa for steel. For aluminum, however, the endurance limit is not well defined, and so here it may be specified as the stress having a limit of, say, 500 million cycles, $(S_{el})_{\text{al}} = 131$ MPa. Once a particular value is obtained, it is often assumed that for any stress below this value the fatigue life will be infinite, and therefore the number of cycles to failure is no longer given consideration.



S–N diagram for steel and aluminum alloys
(*N* axis has a logarithmic scale)

Fig. 3–28

3

IMPORTANT POINTS

- *Poisson's ratio*, ν, is a ratio of the lateral strain of a homogeneous and isotropic material to its longitudinal strain. Generally these strains are of opposite signs, that is, if one is an elongation, the other will be a contraction.

- The *shear stress–strain diagram* is a plot of the shear stress versus the shear strain. If the material is homogeneous and isotropic, and is also linear elastic, the slope of the straight line within the elastic region is called the modulus of rigidity or the shear modulus, G.

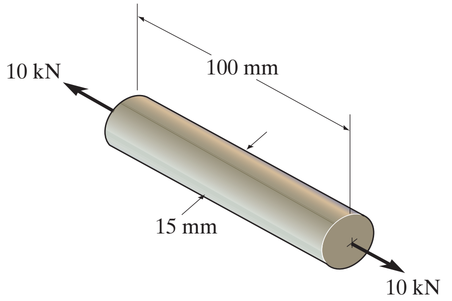
- There is a mathematical relationship between G, E, and ν.

- *Creep* is the time-dependent deformation of a material for which stress and/or temperature play an important role. Members are designed to resist the effects of creep based on their material creep strength, which is the largest initial stress a material can withstand during a specified time without exceeding a specified creep strain.

- *Fatigue* occurs in metals when the stress or strain is cycled. This phenomenon causes brittle fracture of the material. Members are designed to resist fatigue by ensuring that the stress in the member does not exceed its *endurance* or *fatigue limit*. This value is determined from an *S–N* diagram as the maximum stress the material can resist when subjected to a specified number of cycles of loading.

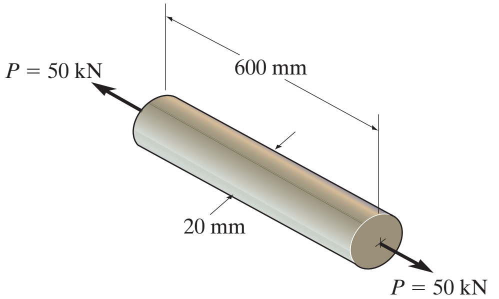
FUNDAMENTAL PROBLEMS

F3–13. A 100-mm-long rod has a diameter of 15 mm. If an axial tensile load of 10 kN is applied to it, determine the change in its diameter. $E = 70$ GPa, $\nu = 0.35$.



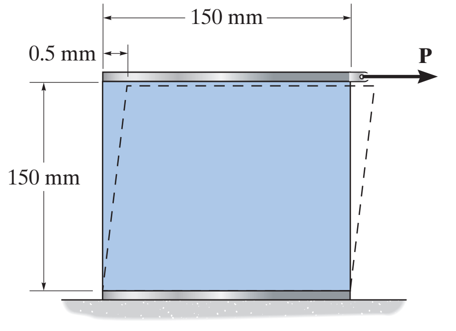
Prob. F3–13

F3–14. A solid circular rod that is 600 mm long and 20 mm in diameter is subjected to an axial force of $P = 50$ kN. The elongation of the rod is $\delta = 1.40$ mm, and its diameter becomes $d' = 19.9837$ mm. Determine the modulus of elasticity and the modulus of rigidity of the material. Assume that the material does not yield.



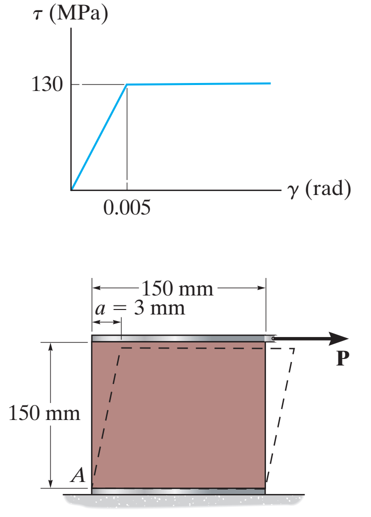
Prob. F3–14

F3–15. A 20-mm-wide block is firmly bonded to rigid plates at its top and bottom. When the force **P** is applied the block deforms into the shape shown by the dashed line. Determine the magnitude of **P**. The block's material has a modulus of rigidity of $G = 26$ GPa. Assume that the material does not yield and use small angle analysis.



Prob. F3–15

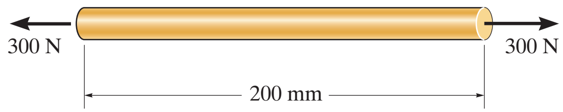
F3–16. A 20-mm-wide block is bonded to rigid plates at its top and bottom. When the force **P** is applied the block deforms into the shape shown by the dashed line. If $a = 3$ mm and **P** is released, determine the permanent shear strain in the block.



Prob. F3–16

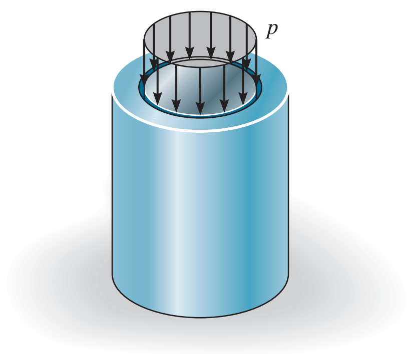
PROBLEMS

3–25. The acrylic plastic rod is 200 mm long and 15 mm in diameter. If an axial load of 300 N is applied to it, determine the change in its length and the change in its diameter. $E_p = 2.70$ GPa, $\nu_p = 0.4$.



Prob. 3–25

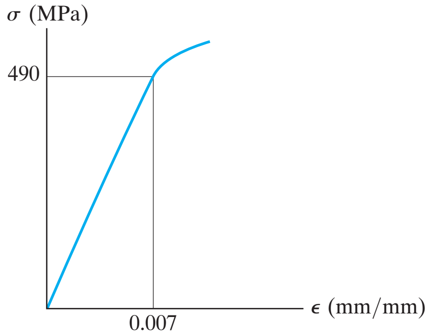
3–26. The plug has a diameter of 30 mm and fits within a rigid sleeve having an inner diameter of 32 mm. Both the plug and the sleeve are 50 mm long. Determine the axial pressure p that must be applied to the top of the plug to cause it to contact the sides of the sleeve. Also, how far must the plug be compressed downward in order to do this? The plug is made from a material for which $E = 5$ MPa, $\nu = 0.45$.



Prob. 3–26

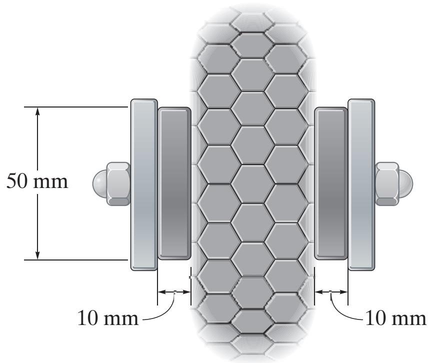
3–27. The elastic portion of the stress–strain diagram for an aluminum alloy is shown in the figure. The specimen from which it was obtained has an original diameter of 12.7 mm and a gage length of 50.8 mm. When the applied load on the specimen is 50 kN, the diameter is 12.67494 mm. Determine Poisson's ratio for the material.

***3–28.** The elastic portion of the stress–strain diagram for an aluminum alloy is shown in the figure. The specimen from which it was obtained has an original diameter of 12.7 mm and a gage length of 50.8 mm. If a load of $P = 60$ kN is applied to the specimen, determine its new diameter and length. Take $\nu = 0.35$.



Probs. 3–27/28

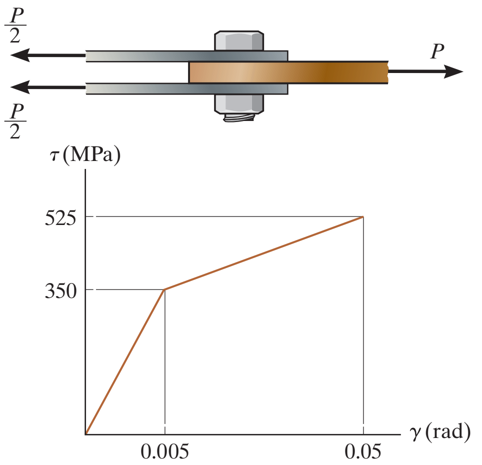
3–29. The brake pads for a bicycle tire are made of rubber. If a frictional force of 50 N is applied to each side of the tires, determine the average shear strain in the rubber. Each pad has cross-sectional dimensions of 20 mm and 50 mm. $G_r = 0.20$ MPa.



Prob. 3–29

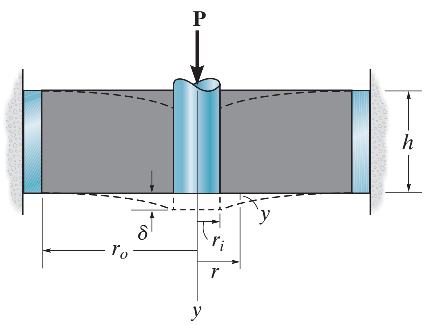
3–30. The lap joint is connected together using a 30 mm diameter bolt. If the bolt is made from a material having a shear stress–strain diagram that is approximated as shown, determine the shear strain developed in the shear plane of the bolt when $P = 340$ kN.

3–31. The lap joint is connected together using a 30 mm diameter bolt. If the bolt is made from a material having a shear stress-strain diagram that is approximated as shown, determine the permanent shear strain in the shear plane of the bolt when the applied force $P = 680$ kN is removed.



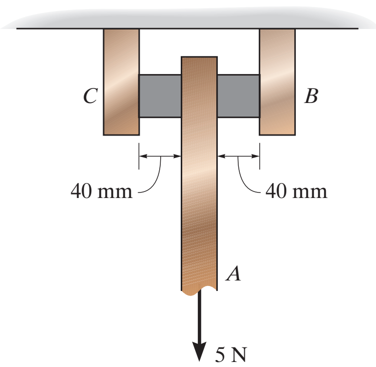
Probs. 3–30/31

*__3–32.__ A shear spring is made by bonding the rubber annulus to a rigid fixed ring and a plug. When an axial load **P** is placed on the plug, show that the slope at point y in the rubber is $dy/dr = -\tan\gamma = -\tan(P/(2\pi hGr))$. For small angles we can write $dy/dr = -P/(2\pi hGr)$. Integrate this expression and evaluate the constant of integration using the condition that $y = 0$ at $r = r_o$. From the result compute the deflection $y = \delta$ of the plug.



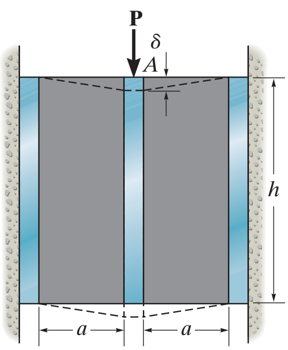
Prob. 3–32

3–33. The support consists of three rigid plates, which are connected together using two symmetrically placed rubber pads. If a vertical force of 5 N is applied to plate A, determine the approximate vertical displacement of this plate due to shear strains in the rubber. Each pad has cross-sectional dimensions of 30 mm and 20 mm. $G_r = 0.20$ MPa



Prob. 3–33

3–34. A shear spring is made from two blocks of rubber, each having a height h, width b, and thickness a. The blocks are bonded to three plates as shown. If the plates are rigid and the shear modulus of the rubber is G, determine the displacement of plate A when the vertical load **P** is applied. Assume that the displacement is small so that $\delta = a\tan\gamma \approx a\gamma$.



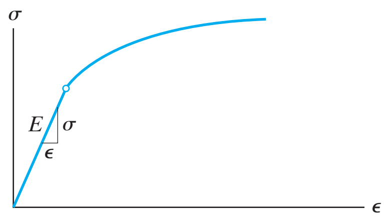
Prob. 3–34

CHAPTER REVIEW

One of the most important tests for material strength is the tension test. The results, found from stretching a specimen of known size, are plotted as normal stress on the vertical axis and normal strain on the horizontal axis.

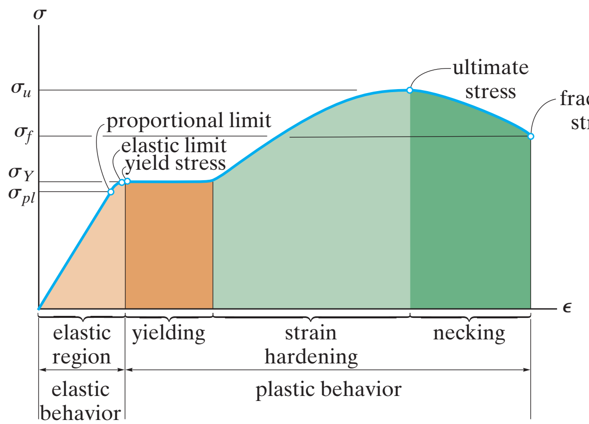
Many engineering materials exhibit initial linear elastic behavior, whereby stress is proportional to strain, defined by Hooke's law, $\sigma = E\epsilon$. Here E, called the modulus of elasticity, is the slope of this straight line on the stress–strain diagram.

$$\sigma = E\epsilon$$



ductile material

When the material is stressed beyond the yield point, permanent deformation will occur. In particular, steel has a region of yielding, whereby the material will exhibit an increase in strain with no increase in stress. The region of strain hardening causes further yielding of the material with a corresponding increase in stress. Finally, at the ultimate stress, a localized region on the specimen will begin to constrict, forming a neck. It is after this that the fracture occurs.

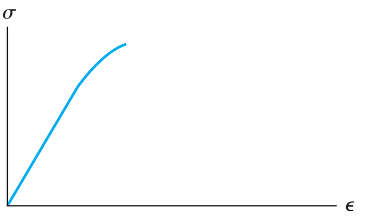


Ductile materials, such as most metals, exhibit both elastic and plastic behavior. Wood is moderately ductile. Ductility is usually specified by the percent elongation to failure or by the percent reduction in the cross-sectional area.

$$\text{Percent elongation} = \frac{L_f - L_0}{L_0}(100\%)$$

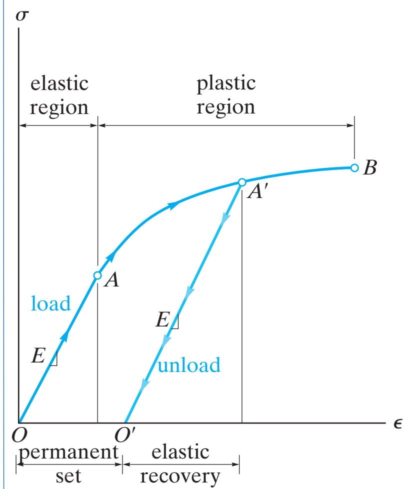
$$\text{Percent reduction of area} = \frac{A_0 - A_f}{A_0}(100\%)$$

Brittle materials exhibit little or no yielding before failure. Cast iron, concrete, and glass are typical examples.

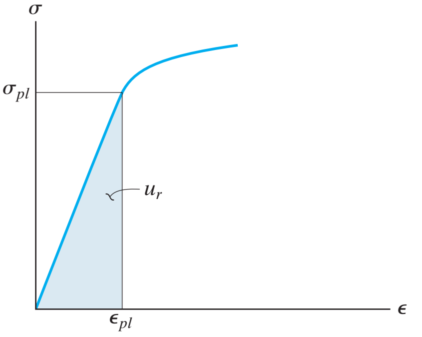


brittle material

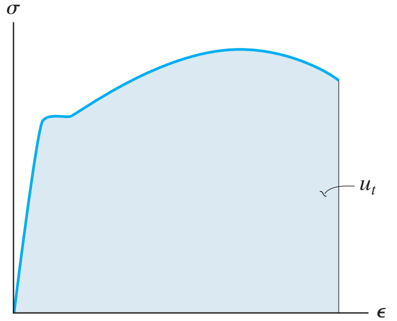
The yield point of a material at A can be increased by strain hardening. This is accomplished by applying a load that causes the stress to be greater than the yield stress, then releasing the load. The larger stress A' becomes the new yield point for the material.



When a load is applied to a member, the deformations cause strain energy to be stored in the material. The strain energy per unit volume, or strain energy density, is equivalent to the area under the stress–strain curve. This area up to the yield point is called the modulus of resilience. The entire area under the stress–strain diagram is called the modulus of toughness.



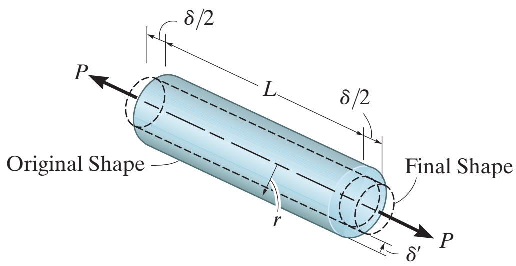
Modulus of resilience u_r



Modulus of toughness u_t

Poisson's ratio ν is a dimensionless material property that relates the lateral strain to the longitudinal strain. Its range of values is $0 \le \nu \le 0.5$.

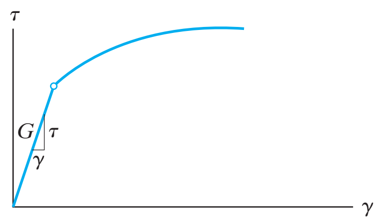
$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$



Tension

Shear stress–strain diagrams can also be established for a material. Within the elastic region, $\tau = G\gamma$, where G is the shear modulus, found from the slope of the line. The value of ν can be obtained from the relationship that exists between G, E, and ν.

$$G = \frac{E}{2(1 + \nu)}$$



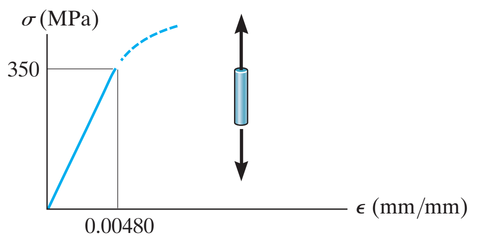
When materials are in service for long periods of time, considerations of creep become important. Creep is the time rate of deformation, which occurs at high stress and/or high temperature. Design requires that the stress in the material not exceed an allowable stress which is based on the material's creep strength.

Fatigue can occur when the material undergoes a large number of cycles of loading. This effect will cause microscopic cracks to form, leading to a brittle failure. To prevent fatigue, the stress in the material must not exceed a specified endurance or fatigue limit.

REVIEW PROBLEMS

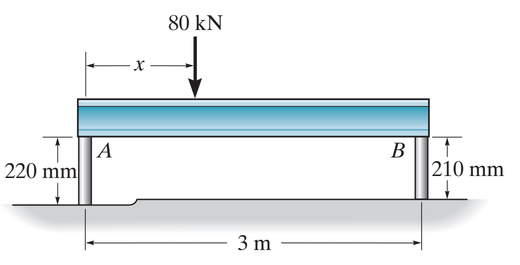
R3–1. The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 50 mm and a diameter of 12.5 mm. When the applied load is 45 kN, the new diameter of the specimen is 12.4780 mm. Compute the shear modulus G_{al} for the aluminum.

R3–2. The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 50 mm and a diameter of 12.5 mm. If the applied load is 40 kN, determine the new diameter of the specimen. The shear modulus is $G_{al} = 27$ GPa.



Prob. R3–1/2

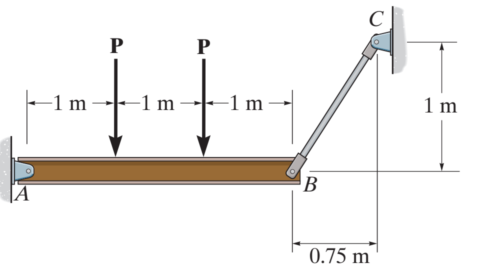
R3–3. The rigid beam rests in the horizontal position on two 2014-T6 aluminum cylinders having the *unloaded* lengths shown. If each cylinder has a diameter of 30 mm, determine the placement x of the applied 80-kN load so that the beam remains horizontal. What is the new diameter of cylinder A after the load is applied? $\nu_{al} = 0.35$.



Prob. R3–3

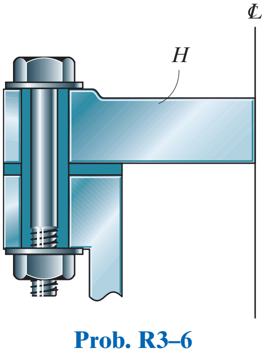
***R3–4.** When the two forces are placed on the beam, the diameter of the A-36 steel rod BC decreases from 40 mm to 39.99 mm. Determine the magnitude of each force **P**.

R3–5. If $P = 150$ kN, determine the elastic elongation of rod BC and the decrease in its diameter. Rod BC is made of A-36 streel and has a diameter of 40 mm.



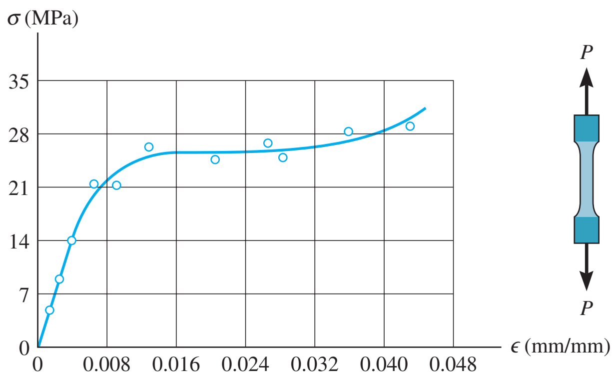
Prob. R3–4/5

R3–6. The head H is connected to the cylinder of a compressor using six steel bolts. If the clamping force in each bolt is 4 kN, determine the normal strain in the bolts. Each bolt has a diameter of 5 mm. If $\sigma_Y = 280$ MPa and $E_{st} = 200$ GPa, what is the strain in each bolt when the nut is unscrewed so that the clamping force is released?



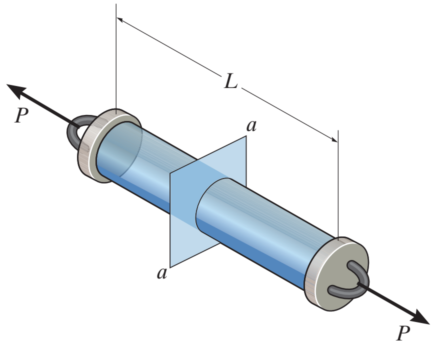
Prob. R3–6

R3–7. The stress–strain diagram for polyethylene, which is used to sheath coaxial cables, is determined from testing a specimen that has a gauge length of 250 mm. If a load P on the specimen develops a strain of $\epsilon = 0.024$ mm/mm, determine the approximate length of the specimen, measured between the gauge points, when the load is removed. Assume the specimen recovers elastically.



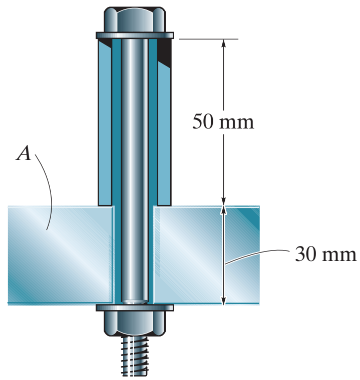
Prob. R3–7

***R3–8.** The solid rod, of radius r, with two rigid caps attached to its ends is subjected to an axial force P. If the rod is made from a material having a modulus of elasticity E and Poisson's ratio ν, determine the change in volume of the material.



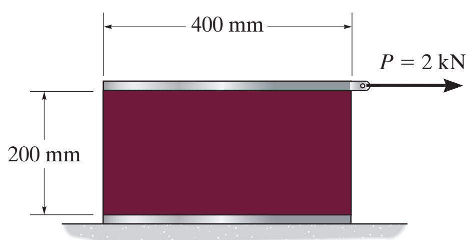
Prob. R3–8

R3–9. The 8-mm-diameter bolt is made of an aluminum alloy. It fits through a magnesium sleeve that has an inner diameter of 12 mm and an outer diameter of 20 mm. If the original lengths of the bolt and sleeve are 80 mm and 50 mm, respectively, determine the strains in the sleeve and the bolt if the nut on the bolt is tightened so that the tension in the bolt is 8 kN. Assume the material at A is rigid. $E_{al} = 70$ GPa, $E_{mg} = 45$ GPa.



Prob. R3–9

R3–10. An acetal polymer block is fixed to the rigid plates at its top and bottom surfaces. If the top plate displaces 2 mm horizontally when it is subjected to a horizontal force $P = 2$ kN, determine the shear modulus of the polymer. The width of the block is 100 mm. Assume that the polymer is linearly elastic and use small angle analysis.



Prob. R3–10

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