

PHYSICS 84 | QUANTUM INFORMATION  
SKILLS DIAGNOSTIC REDO

2015-01-29

**0.1 Eigenvalues and Eigenvectors**

Find the eigenvalues and eigenvectors of the following matrix:

$$\begin{bmatrix} 2 & i \\ -i & 2 \end{bmatrix}.$$

**0.2 Digital Logic Circuits**

Design and draw a simple logic circuit that takes two classical bits as input and is described by the following truth table:

A	B	Out
0	0	1
0	1	1
1	0	0
1	1	1

**0.3 Qubit States and Measurement**

A qubit is prepared in the quantum state  $|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + i\sqrt{\frac{2}{3}}|1\rangle$ .

- (a) What is the probability that an ideal projective measurement in the  $\{|a\rangle, |a_\perp\rangle\}$  basis will find the qubit in the state  $|a\rangle$  if  $|a\rangle = |0\rangle$ ?
- (b) What if  $|a\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ ?

**0.4 Matrix Operations**

Write down a matrix  $\vec{U}_\chi$  such that  $\vec{U}_\chi \begin{bmatrix} a \\ be^{i\varphi} \end{bmatrix} = \begin{bmatrix} a \\ be^{i(\varphi+\chi)} \end{bmatrix}$  for any real numbers  $a, b$ , and  $\varphi$ . Is  $\vec{U}_\chi$  a unitary matrix?