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0.1 Eigenvalues and Eigenvectors

Find the eigenvalues and eigenvectors of the following matrix:

$$\begin{bmatrix} 2 & i \\ -i & 2 \end{bmatrix}.$$

0.2 Digital Logic Circuits

Design and draw a simple logic circuit that takes two classical bits as input and is described by the following truth table:

A	B	Out
0	0	1
0	1	1
1	0	0
1	1	1

0.3 Qubit States and Measurement

A qubit is prepared in the quantum state $|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + i\sqrt{\frac{2}{3}}|1\rangle$.

(a) What is the probability that an ideal projective measurement in the $\{|a\rangle, |a_\perp\rangle\}$ basis will find the qubit in the state $|a\rangle$ if $|a\rangle = |0\rangle$?

(b) What if $|a\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$?

0.4 Matrix Operations

Write down a matrix $\vec{U}_\chi$ such that $\vec{U}_\chi \begin{bmatrix} a \\ b\mathrm{e}^{\mathrm{i}\varphi} \end{bmatrix} = \begin{bmatrix} a \\ b\mathrm{e}^{\mathrm{i}(\varphi+\chi)} \end{bmatrix}$ for any real numbers a, b, and φ. Is $\vec{U}_\chi$ a unitary matrix?