BMI 203: Algorithms - Homework 1

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1 Implement BubbleSort and QuickSort

I implemented BubbleSort with a nested for-loop, as follows:

```
def bubblesort(x):
    for n in range(len(x)):
        #now comparing adjacent, so iterate up to 2nd-to-last
        for i in range(len(x)-1):
            if x[i] > x[i+1]:
                saveBit = x[i+1]
                x[i+1] = x[i]
                x[i] = saveBit
    return x
    ## Number of Assignments:
    ## O(N^2)  (x, n*N, i*N*N-1)
    ## Number of Conditionals:
    ## O(N^2)  (pairwise comparison ln3)*N*N-1
```

For QuickSort, I used recursion to sort each of the sub-lists on either side of the pivot:

```
def quicksort(x):
    if len(x) < 2:
        return x
    else:
        pivot = x[0]
        lt = np.array([])
        gt = np.array([])
        for n in x[1:]:
            if n < pivot:
                lt = np.append(lt,n)
            else:
                gt = np.append(gt,n)
    return np.concatenate( (quicksort(lt),
                            np.array([pivot]),
                            quicksort(gt)),axis=0 )
    ##  Number of Assignments:
    ##  O(NlogN)  ((x, pivot, lt*N, gt*N, n*N) * logN)
    ##  Number of Conditionals:
    ##  O(NlogN)   (base case ln1)*logN, (partition ln8)*N*logN
```

When counting the assignments and conditionals, I put the same QuickSort algorithm in a wrapper function that initializes nonlocal count variables, which the inner algorithm then increments as it runs.

2 Testing in Travis

For both BubbleSort and QuickSort, the test set run by Travis includes the following:

- even-length and odd-length vectors
- vectors with one or more repeated entries
- the zero-length and one-length vectors
- a vector containing upper- and lower-case characters

The final build of the code passed all of these tests in Travis.

3 Big-O Complexity

3.1 Expected Complexity

As alluded to in the comments at the end of the code shown in Section 1, we can look at the algorithm structure in order to make an informed prediction as to the algorithm complexity.

In BubbleSort, we must iterate through every element in the list (minus one) to perform the pairwise evaluation and swapping, and then, we must do that same process for every position in the list (imagine that the element that should be last starts out first). This results in a runtime that is $O(N^2)$ for all cases.

In QuickSort, the absolute worst-case scenario requires $O(N^2)$ runtime (for when the list is in opposite-sorted order, which requires $n + n - 1 + n - 2 + ... + 2 + 1 = \frac{n^2}{2}$ steps). However, unlike in BubbleSort, the $O(N^2)$ runtime is not a requirement. The branching tree created by the recursion, when balanced, can reduce the runtime to $O(NlogN)$, which we can see in the code in that, along with the recursion, there is only a single for-loop that iterates through N.

3.2 Test Setup

For each vector length L in the set $[100, 200, 300..., 1000]$, 100 random vectors of integers between 0 and 100 were generated. The number of assignments and conditionals required and the amount of wall-clock time required to sort all 100 vectors of a given length L were measured. The data was then plotted alongside theoretical curves $y_1 = C_1 * N^2$ and $y_2 = C_2 * N * log_2(N)$ for a visual comparison.

3.3 Results

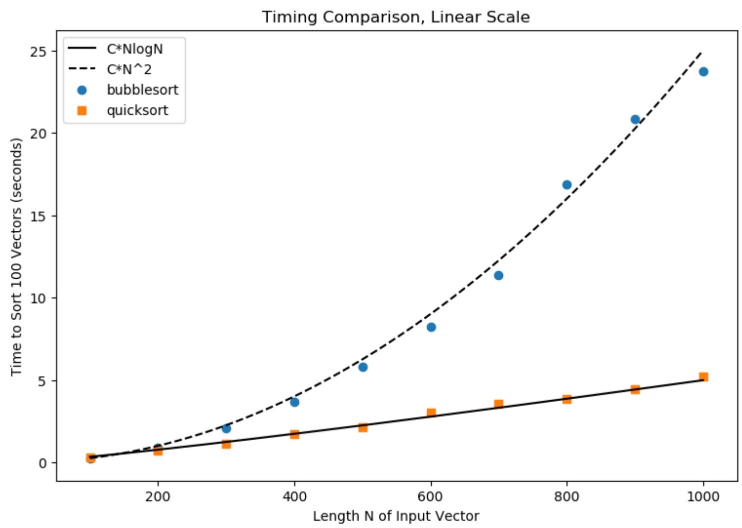
The data for the timing test is as follows:

Length (x100 Vectors)	BubbleSort Time (s)	QuickSort Time (s)
100	0.23	0.30
200	0.90	0.71
300	2.06	1.11
400	3.68	1.72
500	5.80	2.15
600	8.22	3.00
700	11.36	3.58
800	16.87	3.82
900	20.87	4.45
1000	23.73	5.22

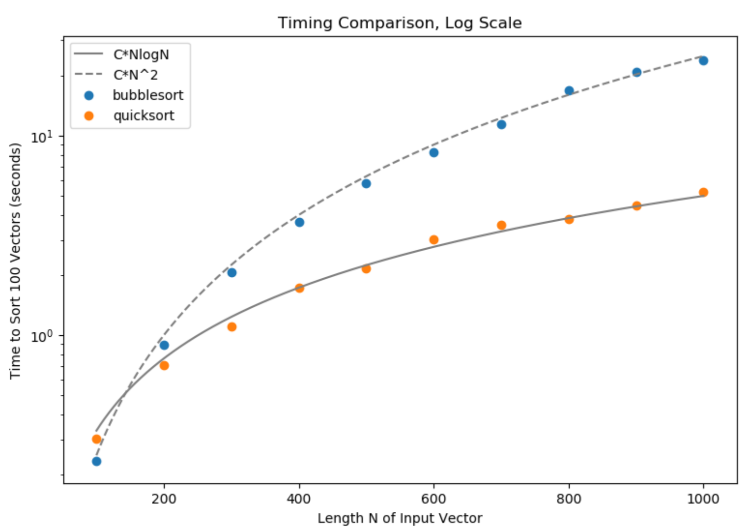
For assignments and conditionals, the average across all 100 vectors in each set is reported:

Length (x100 Vectors)	B, Assign.	Q, Assign.	B, Cond.	Q, Cond.
100	10074	969	25	830
200	40300	2143	100	1866
300	90655	3551	218	3104
400	161124	4874	375	4257
500	251820	7228	607	6407
600	362613	8071	871	7068
700	493653	9919	1217	8720
800	644717	11844	1572	10445
900	816204	14584	2068	12985
1000	1007604	16739	2535	14940

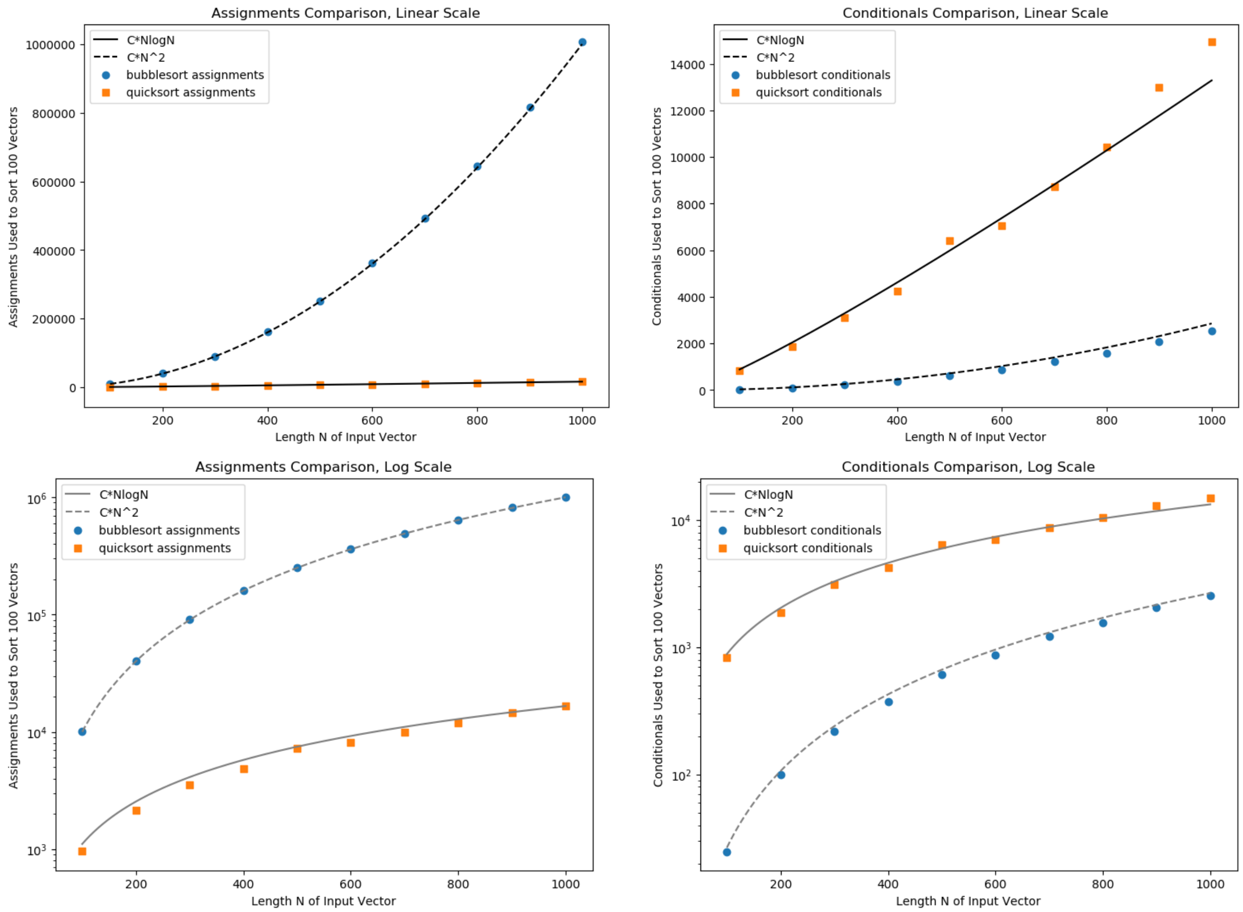
We can see just from the tables that the values for QuickSort grow more slowly than the values for BubbleSort,for both the wall-clock time and the number of operations. When graphed (with specifically chosen values of C_1 and C_2 for illustrative purposes), the shape of the data for QuickSort and BubbleSort align very well with $y = C * N * log_2(N)$ and $y = C * N^2$ respectively:



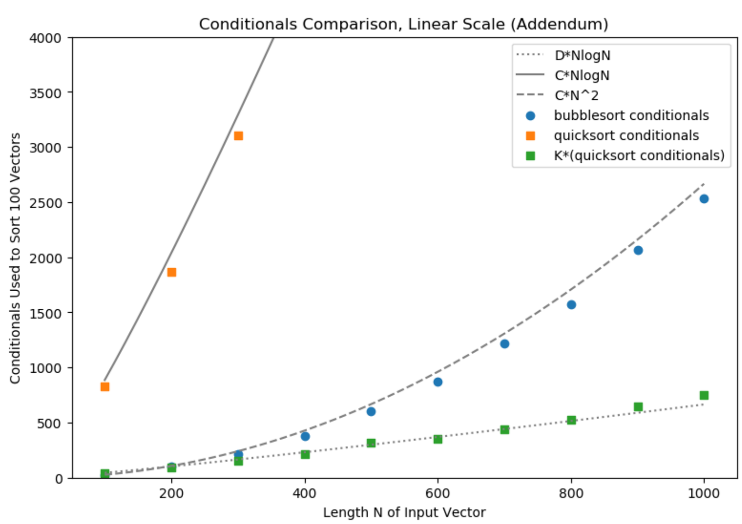
The difference can be seen immediately between the two algorithms, but to highlight the difference in the actual curvature of the two trends, not just their relative scales, we can plot them with a log-scaled yaxis:



For the assignments and conditionals, the same pattern is evident, and again, moving to a log-scaled y-axis makes the differences in curvature more apparent:



Unlike with the raw time or the assignments counts, We note that for the conditionals, QuickSort's counts are higher than that of BubbleSort. This is not counter to our original hypothesis, however, since the curvature is still more closely aligned with a logarithmic function than a quadratic; furthermore, the difference in raw values is easily manipulated with scalar multiplication:



Indeed, looking at the code in Section 1, we can see that while QuickSort costs more conditionals per iteration than BubbleSort, BubbleSort runs more iterations overall (on average), leading to the steeper curve in all three measures of complexity for BubbleSort relative to Quicksort.

4 Github Repository

All of the code used in this assignment is on Github: https://github.com/laueste/algorithms-ucsf-2019