University of California, Davis

ECS 132 Homework 2

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February 23, 2021

Abstract

This files includes the solutions to HW 2 from ECS 132 Winter Quarter. There are 4 attached .R files for problems 2, 3, 5, and 7.

1 Question 1

For question 1, we are given the following information to find the Var(XYZ).
X is a indicator random variable with probability p
Y is a indicator random variable with probability q
Z is a indicator random variable with probability r

$$\begin{aligned}
\text{Var(XYZ)} &= E(X^2Y^2Z^2) - [E(XYZ)]^2 \\
&= [E(X^2) \cdot E(Y^2) \cdot E(Z^2)] - [EX \cdot EY \cdot EZ]^2 \\
&= ([Var(X) + (EX)^2] \cdot [Var(Y) + (EY)^2] \cdot [Var(Z) + (EZ)^2]) - [EX \cdot EY \cdot EZ]^2 \\
&= ([p(1-p) + p^2] \cdot [q(1-q) + q^2] \cdot [r(1-r) + r^2]) - [p \cdot q \cdot r]^2 \qquad \text{Using 3.78 and 3.79} \\
&= ([p - p^2 + p^2] \cdot [q - q^2 + q^2] \cdot [r - r^2 + r^2]) - [p \cdot q \cdot r]^2 \\
&= (p \cdot q \cdot r) - [p \cdot q \cdot r]^2 \\
&= pqr - (pqr)^2 = pqr(1 - pqr)
\end{aligned} \tag{1.1}$$

2 Question 2

Please see the file Code2.R

3 Question 3

Please see the file Code3.R

4 Question 4

For question 4, we are given the following information to find the Var(X).

A double geometric distribution family has support of all real integers.

The family is indexed by p and q and we are given the P(X=K) for all the possible Ks.

First, we are going to find an equation to calculate Var(X)

$$\begin{aligned} Var(X) &= E(X^2) - EX^2 \\ &= E(X^2) - EX + EX - EX^2 && \text{- EX + EX = + 0} \\ &= E(X^2 - X) + EX - EX^2 && \text{Factored into EX} \\ &= E[X(X-1)] + EX - EX^2 && \text{Algebra} \end{aligned} \tag{4.1}$$

So, now we will solve for EX

$$\begin{aligned} EX &= \sum_{k=-\infty}^{\infty} k \cdot P(X=k) \\ &= \sum_{k=-\infty}^{-1} k \cdot P(X=k) + \sum_{k=0}^{0} k \cdot P(X=k) + \sum_{k=1}^{\infty} k \cdot P(X=k) \\ &= (\sum_{i=-\infty}^{-1} k \cdot c(1-p)^{|k|-1} \cdot p) + (0 \cdot q) + (\sum_{k=1}^{\infty} k \cdot c(1-p)^{|k|-1} \cdot p) \\ &= 2 \cdot cp \cdot \frac{1}{[1-(1-p)]^2} \\ &= \frac{2cp}{p^2} = \frac{2c}{p} \end{aligned} \tag{4.2}$$

Now, we will solve for E[X(X-1)]

$$\begin{aligned} E[X(X-1)] &= (c \sum_{X=-\infty}^{-1} X(X-1) \cdot (1-p)^{X-1} \cdot p) + (0 \cdot q) + (c \sum_{X=1}^{\infty} X(X-1) \cdot (1-p)^{X-1} \cdot p) \\ &= 2 \cdot (c \sum_{X=1}^{\infty} X(X-1) \cdot (1-p)^{X-1} \cdot p) \\ &= 2 \cdot (c \sum_{X=2}^{\infty} p(p-1) \cdot X(X-1) \cdot (1-p)^{X-2}) && \text{Factor out (1-p)} \\ &= 2 \cdot (c \frac{2p(1-p)}{(1-(1-p))^3}) && \text{2nd Derivative of Geometric Series} \\ &= \frac{4c \cdot (1-p)}{p^2} && \text{Algebra} \end{aligned} \tag{4.3}$$

Since we know EX and E[X(X-1)], we can solve for Var(X)

$$\begin{aligned} Var(X) &= E[X(X-1)] + EX - EX^2 \\ &= \frac{4c \cdot (1-p)}{p^2} + \frac{2c}{p} - (\frac{2c}{p})^2 \end{aligned} \tag{4.4}$$

5 Question 5

Please see the file Code5.R

6 Question 6

For question 1, we are given the following information.

X is a random variable with exponential distribution

We need to prove that cX will have exponential distribution when c is greater than 0 First, we will find the cumulative distribution function in terms of X

$$\begin{aligned}
\mathrm{F_Y(t)} &= P(Y \leq t) && \text{definition of } \mathrm{F_Y} \\
&= P(cX \leq t) && \text{definition of Y} \\
&= P(X \leq \frac{t}{c}) && \text{algebra} \\
&= F_{\mathrm{X}}(\frac{t}{c}) && \text{definition of } \mathrm{F_X}
\end{aligned} \tag{6.1}$$

Then, to find the probability density function, we do the following

$$\begin{aligned}
\mathrm{f_Y(t)} &= \frac{d}{dt}F_{\mathrm{Y}}(t) && \text{definition of } \mathrm{F_y} \\
&= \frac{d}{dt}F_{\mathrm{x}}(\frac{t}{c}) && \text{from (6.1)} \\
&= f_{\mathrm{x}}(\frac{t}{c}) \cdot \frac{d}{dt}(\frac{t}{c}) && \text{definition of } \mathrm{f_x} \text{ and chain rule} \\
&= \frac{\lambda}{c}e^{-\frac{\lambda}{c}\cdot t}. && \text{using 7.40}
\end{aligned} \tag{6.2}$$

Here, we have the final density in the form of of the exponential family

$$\lambda e^{-\lambda t} \text{ where } \lambda = \frac{\lambda}{c}, 0 < t < \infty, \text{and } c > 0 \tag{6.3}$$

This proves that if X has exponential distribution, cX does as well if c is greater than 0.

7 Question 7

Please see the file Code7.R