b. $y' = 1 + (t - y)^2, \quad 2 \leq t \leq 3, \quad y(2) = 1, \quad \text{com } h = 0{,}5$

Solução Analítica

$$y' = 1 + (t - y)^2$$

$$\frac{dy}{dt} = 1 + (t-y)^2 \Rightarrow u = t - y \Rightarrow \frac{du}{dt} = \frac{d}{dt}(t-y) = 1 - \frac{dy}{dt}$$

$$\left(1 - \frac{du}{dt}\right) = 1 + u^2 \Rightarrow \frac{du}{dt} = u^2 \Rightarrow \frac{du}{u^2} = dt \quad \therefore \text{ Agora Integramos}$$

$$\int \frac{du}{u^2} = \int -dt \Rightarrow -1/u = -t + C \Rightarrow 1/u = t - C = \frac{1}{t-y} = t - C \Rightarrow$$

$$\Rightarrow t - y = \frac{1}{t+K} \Rightarrow \boxed{y_{(t)} = t - \frac{1}{t+K}}$$

$$\therefore y_{(2)} = 2 - \frac{1}{2+K} \Rightarrow 2 - 1 = \frac{1}{2+K} \Rightarrow \frac{1}{2+K} = 1 \Rightarrow 2 + K = 1 \quad \boxed{K = -1}$$

Taylor de Ordem 2

$$f = 1 + (t-y)^2$$

$$f' = \frac{d}{dt}\left[1 + (t-y)^2\right] \Rightarrow \frac{d(1)}{dt} + \frac{d}{dt}(t-y)^2 \Rightarrow \cancel{0} + \overbrace{\frac{d}{dt}(t-y)^2}^{\text{Regra da Cadeia}}$$

$$\therefore \frac{du^2}{dt} \circ \frac{d(t-y)}{dt} \Rightarrow \left[2.(t-y)\right].\left[\frac{d}{dt}(t) - \frac{d(y)}{dt}\right] \Rightarrow \boxed{2(t-y).\left[1 - y'\right]}$$

$$f' = 2(t-y).\left[1 - (1 + (t-y)^2\right] = -2(t-y).\left[t-y\right]^2 = \boxed{-2(t-y)^3}$$

$$\therefore \boxed{T^{(2)} = w_i + h.f + \frac{h^2}{2}.f'}$$