An introduction to digital audio

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ELECTRONICS DIVISION
Tutorial Colloquium on

"AUDIO ENGINEERING"

Organised by
Professional Groups E10 (Circuits and systems) and E14 (Television, radio and data broadcasting)

On Wednesday, 9 March 1994

Digest No: 1994/062

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THE INSTITUTION
OF ELECTRICAL
ENGINEERS

TUTORIAL COLLOQUIUM ON "AUDIO ENGINEERING"

Organised by Professional Groups E10 (Circuits and systems) and
E14 (Television, radio and data broadcasting)

to be held at the University of Essex on

Wednesday, 9 March 1994

PROGRAMME

	10.00 am	Registration and coffee
	Chairman:	R Heylen (University of Essex)
	10.25	Welcoming address: Professor R Johnston, Vice Chancellor, University of Essex
	10.30	Introduction
1	10.40	"An introduction to digital audio": M O Hawksford (University of Essex)
2	11.20	"Mini disc - What? Why? How?": M Risby (Sony Broadcast International)
3	12.00	"High quality audio": J R Stuart (Meridian Audio)
	12.40 pm	LUNCH AND INFORMAL DEMONSTRATIONS
4	1.40	"Digital audio broadcasting (DAB) - radio for tomorrow": M C D Maddocks (BBC)
5	2.20	"Virtual acoustic synthesis": S Bates (Yahama Research & Development)
	3.00	TEA
6	3.20	"PASC data coding for DCC": G Wirtz (Philips Consumer Electronics, Netherlands)
	4.00	DISCUSSION PANEL
	4.30	CLOSE

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An introduction to digital audio

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1 Introduction

In this review paper we shall briefly discus some of the principles of digital audio and then outline example areas that are currently topical within the industry. However, it is assumed that the principles of uniform sampling, uniform amplitude quantization and dither are understood and that when used together, realise a digital channel with analogue-like characteristics where the signal is distorted only by bandlimitation and additive noise.

Central to a digital audio system are the processes of filtering and of noise shaping, where for example, oversampling and noise shaping can influence a system by enabling an interchange of complexity between the analogue and digital domains. This leads to alternative structures for digital-to-analogue and analogue-to-digital conversion where system related errors can be traded. Also, if these processes are combined with auditory data to form models of human hearing, then there is an opportunity to achieve optimum performance for a given data rate, which at one extreme can lead to a performance beyond the requirement for human hearing, or at the other extreme, reasonable audio quality with a low rate of data communication.

This paper reviews some methods of data conversion that depend upon digital signal processing (DSP) and also considers aspects of loudspeaker systems to illustrate how DSP can radically affect an audio system.

2 Oversampling

Oversampling can be used in both analogue-to-digital (ADC) and digital-to-analogue (DAC) converters, where the sampling rate $\{Rf_s\}$ Hz is chosen to be greater than the Nyquist sampling rate f_s Hz and R is the oversampling ratio that is generally selected as a power of 2 for computational efficiency. In digital systems, sampling rates can be either reduced, a process termed *decimation* (where samples are discarded), or increased, a process of *interpolation* (where new samples are created). It is important to note that decimation may represent a real loss of information as inevitably the information bandwidth is reduced, but interpolation can never represent a gain of information. In fact, processing errors may actually imply a slight loss of information even though the sampling rate is increased!

To introduce oversampling, consider an ADC where $R = 4$, $\{Rf_s\} = 176.4$ kHz and where optimal dither is assumed to engender a uniform spectral spread of quantization distortion over the band 0 to 88.2 kHz. There are two principal consequences of x4 oversampling:

(i) The input bandwidth must be restricted to 0 to 154.35 kHz to prevent aliasing distortion entering the audio band 0 to 22.05 kHz. Since, in practice, most audio signals in the absence of ultrasonic interference have minimal spectral content above ≈30 kHz, the oversampled ADC requires virtually no pre-quantizer filtering; hence, degradation by now redundant analogue processing can be eliminated. A degree of aliasing distortion is permissible providing the audio band is not contaminated, where the process is illustrated in Figure 1.

(ii) A second consequence of $R = 4$ oversampling is the location of the quantization spectrum into the band 0 to 88.2 kHz, whereby only 0.25 (assuming a uniform noise spectrum) of the power resides in the range 0 to 22.05 kHz. This is our first encounter with noise shaping and it is equivalent to a ≈ 6 dB reduction in noise power representing a 1 bit improvement in resolution.

As the oversampling ratio is increased above $R = 4$, it may be supposed that further enhancements can be achieved. However, there are limitations with this more conventional approach:

(a) The faster ADC conversion rate may introduce error as a consequence of finite settling times.

(b) The quantization distortion spectrum falls with frequency, thus limiting any significant noise advantage with increasing R.

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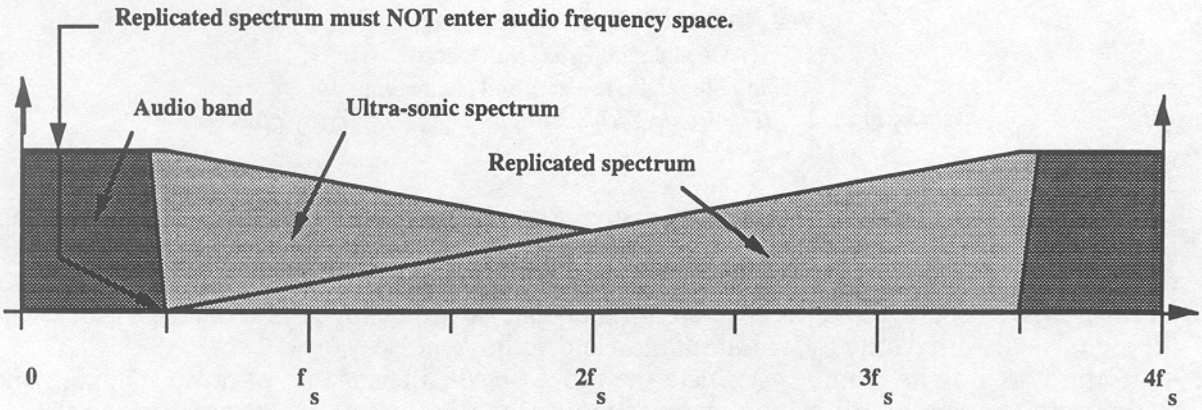


Fig. 1 Spectrum of four-times oversampled signal showing input can extend to 154,35 kHz without aliased components entering the audio band.

To down-convert the 176.4 kHz sampling rate to 44.1 kHz requires decimation by a factor of 4 where the process is illustrated in Figure 2. The digital signal is initially filtered to remove all signal components in the band 22.05 kHz to 154.35 kHz allowing sub-sampling to reduce the sampling rate without aliasing distortion corrupting the audio band.

Figure 2 reveals that the digital, decimation filter performs a similar function to the analogue anti-aliasing filter used with Nyquist sampling. In fact, the oversampling process has enabled the anti-aliasing filter to be transferred from the analogue to the digital domain, otherwise its function is essentially identical. The advantages are that a digital filter can be designed to exhibit a near-ideal transfer function, with low in-band amplitude ripple, rapid rates of out-of-band attenuation, zero group delay distortion, a wide dynamic range, together with time-invariant performance and no aging or thermal problems. These reasons, coupled with exact replication in manufacture, designer specified dynamic range and an insensitivity to power supply, component quality and ground-rail problems associated with analogue circuits, makes the approach attractive and cost-effective for volume production.

A more familiar application of oversampling is in digital-to-analogue conversion, where the process of interpolation is used to increase the sampling rate typically by a factor of 4. The attraction of this technique parallells that used in the ADC example, where processes normally performed in the analogue domain are transferred into the digital domain, thus relaxing the demands on analogue circuitry and simultaneously allowing better signal recovery and enhancement in resolution.

The basic process of interpolation is shown in Figure 3 and consists of two stages:

(i) Zero samples are inserted into the digital code so that the effective data rate is increased from, say, 44.1 kHz to 176.4 kHz.

(ii) Sample values are calculated using a digital, low-pass filter with a pass band ideally 0 to 22.05 kHz, so that the sampling sideband pairs, associated with 44.1 kHz and 88.2 kHz and 132.3 kHz are attenuated.

3 Noise shaping and bitstream conversion (delta-sigma modulation)

Information theory *Shannon 1948* defines the information capacity of a communication channel as a function of bandwidth and signal-to-noise ratio and supports the concept that noise and bandwidth can be interchanged whilst maintaining a constant channel capacity, hence in principle the sample amplitude resolution can be reduced without incurring a loss of source information. There is an undeniable logic to this proposal, where the greater the oversampling ratio, the less dependent the performance becomes upon an individual sample,enabling a coarser quantization to be used.

For example, if an oversampled data sequence is requantized and providing there is either sufficient signal activity or the inclusion of digital dither *Lipshitz and Vanderkooy 1984, 1987*, then the additional requantization distortion will have a near-uniform power spectrum which for $R = 4$ is spread across the band 0 to 88.2 kHz. Consequently, only one quarter of this extra noise power falls within the audio band 0 to 22.05 kHz and corresponds to a modest 1-bit enhancement over a sequence operating at the Nyquist rate of 44.1 kHz.

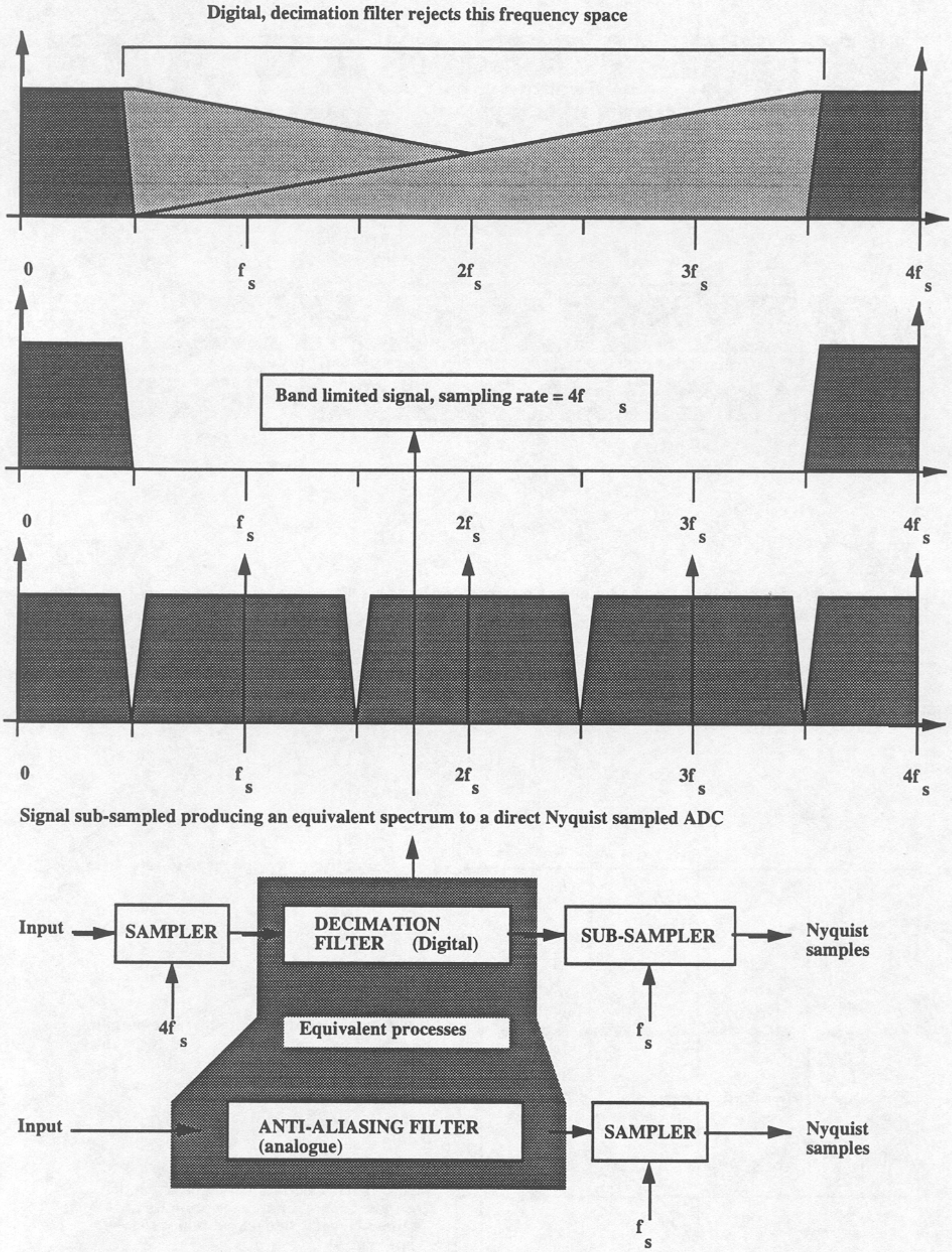


Fig. 2 Times-four decimation and equivalent low-pass filter process.

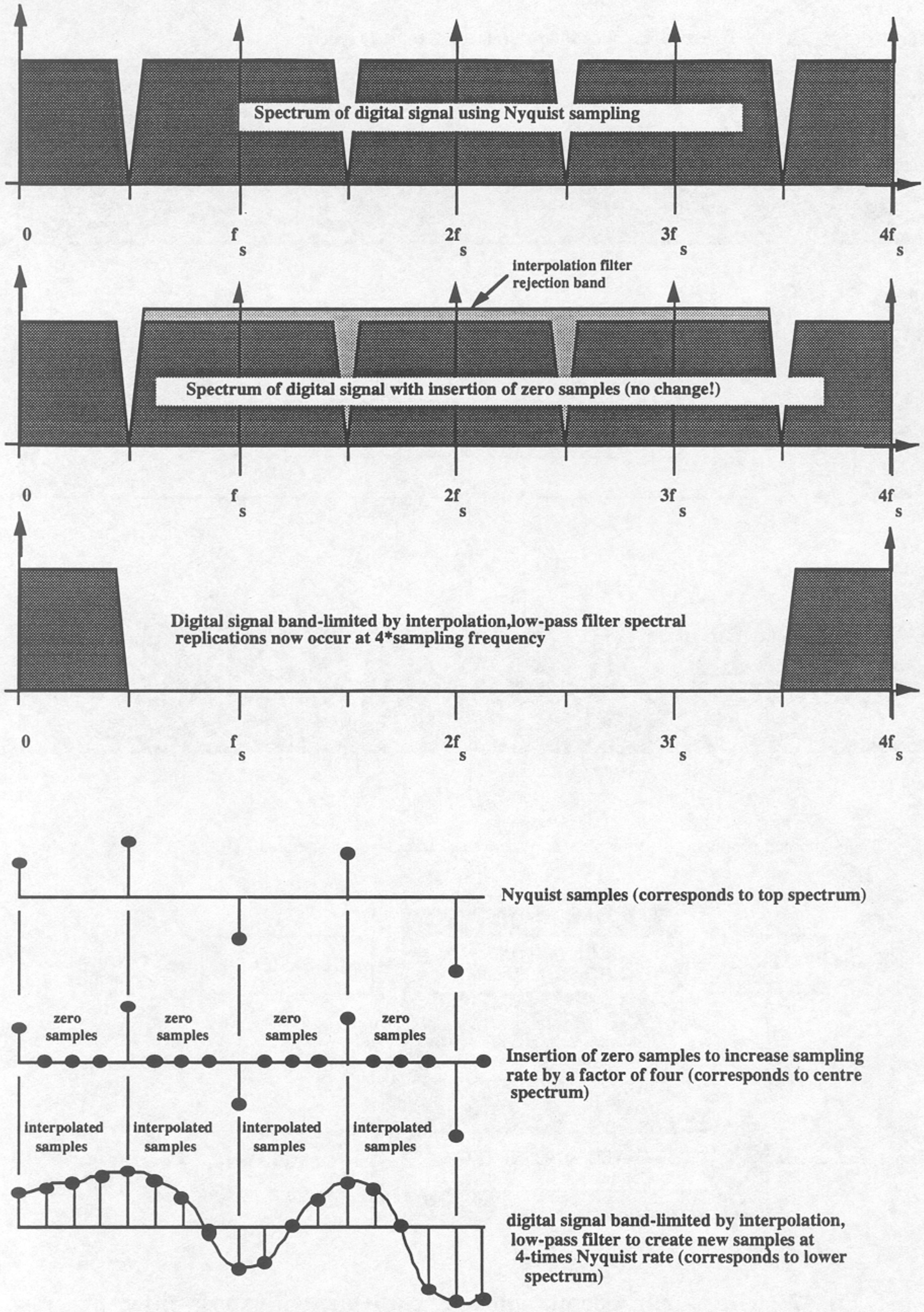


Fig. 3 4-times interpolation illustrated in both frequency and time domains.

In this scheme, the higher the oversampling ratio, the greater is the fractional bandwidth in which no useful information resides. Hence, if samples are more coarsely quantized, the requantization noise can be concentrated into this redundant frequency space. Although in this initial example the noise spectrum was assumed flat this need not be the case where using *noise shaping* the noise can be spectrally shaped to occupy the frequency space created by oversampling. Noise shaping coders *Tewksbury et al 1978* can be implemented in both the analogue and digital domains and therefore function as either an ADC or a DAC. Usually the coders operate with *oversampled data* and achieves shaping of the distortion spectrum by applying local negative feedback to the requantization system as shown in the canonic model of Figure 4, *Rowden et al 1992.*

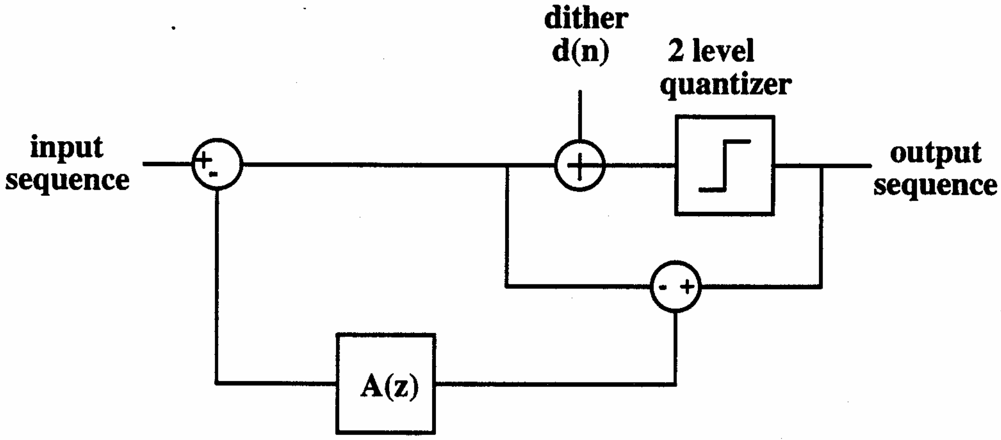


Fig. 4 Canonic form of noise shaper configured about a quantizer.

The non-linear, quantizer Q is shown as a unity-gain amplifier with an additive error sequence q(n) to represent the quantization distortion, while the transfer function A is designed to achieve the desired noise shaping spectrum.

The output sequence y(n) is expressed as a function of the input sequence x(n) and quantization distortion sequence q(n) as,

$$y(n) \quad = \quad x(n) \ + \ \frac{q(n)}{1 + A}$$

Defining a frequency dependent, noise shaping function as $D_f = (1 + A)^{-1}$,

$$y(n) \quad = \quad x(n) \ + \ q(n)\, D_f$$

This succinct expression gives insight into the noise shaping process, where for large values of *A* the input and output sequences are almost identical, differing only by the additive quantization distortion which is frequency shaped by D_f, a factor inversely proportional to *A* . Since the noise shaper is required to suppress noise at low frequency and relocate it to high frequency, the transfer function *A* should exhibit a high, low-frequency gain that progressively falls with increasing frequency, whereby D_f is correspondingly small at low frequency and progressively rises with increased frequency, thus shaping the noise spectrum.

For an ADC, *A* must be selected to form a stable system, where a solution common to unity-gain stable operational amplifiers, is to choose *A* to be an integrator, taking the form $A = 1/j\omega T$, whereby the distortion shaping factor becomes

$$|D_f| = \frac{\omega T}{\left[1 + (\omega T)^2\right]^{\frac{1}{2}}} \quad \approx \omega T \text{ for } \omega T << 1$$

(ω is angular frequency and T is a time constant)

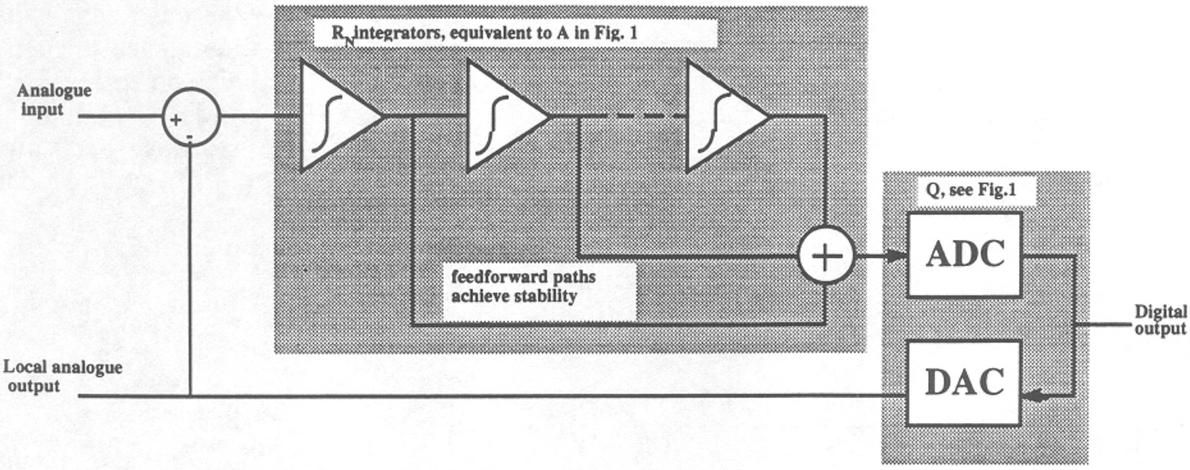


Fig. 5a Noise shaping ADC with ADC/DAC representing quantizer.

At low frequency the noise shaping characteristic has a slope of 6.02 dB/octave which mirrors the slope of *A* of –6.02 dB/octave. However, in practice this slope is found to be rather low and requires extremely high oversampling ratios to attain an acceptable low-frequency SNR. The problem is addressed by increasing the number of cascaded integrators R_N, whereon the noise shaping slope now approaches $6.02R_N$ dB/octave with corresponding improvements in coding performance *Hawksford 1985* However, to form a stable system, the transfer function must be tempered by the inclusion of $(R_N - 1)$ zeros to shape the high-frequency transfer function to a first-order response. The modified system can be implemented using $(R_N - 1)$ feedforward paths *Hawksford 1989*, where in Figure 5a a noise shaper of order R_N is shown configured as an ADC with a quantizer Q using a back-to-back flash converter and DAC, where the flash ADC output forms the digital output signal in a noise-shaped, oversampled format. A more complete structure after *Adams 1986* is shown in Figure 5b that includes the decimation filters to convert the digital data to a more standard (44.1 kHz) sampling format.

The noise shaping coder can also be configured as a DAC, however:
(i) Integration is performed by a digital accumulator using unity-gain, positive feedback around a one-sample delay, thus forming a summation of sequential samples.
(ii) The requantization of data is performed by arithmetic truncation.
The transformed digital noise shaper is shown in Figure 6 where the noise shaping function is,

$$|D_f| = \left[2\sin\left(\pi\frac{f}{Rf_s}\right)\right]^{R_N}$$

$|D_f|$, is shown in Fig. 4 for $R_N = 1$ to 6, where the following points are highlighted:
(i) for $f \ll Rf_s$, $|D_f|$ shows a similar form to the analogue configuration, where

$$|D_f| = \left[\frac{2\pi f}{R\,f_s}\right]^{R_N}$$

(ii) for $f = Rf_s/6$, $|D_f| = 1$ for all R_N
(iii) for $f > Rf_s/6$, $|D_f| > 1$, that is, the noise spectrum is actually amplified
(iv) $|D_f|$ is maximum at $f = Rf_s/2$ where

$$|D_f| = 2^{R_N}$$

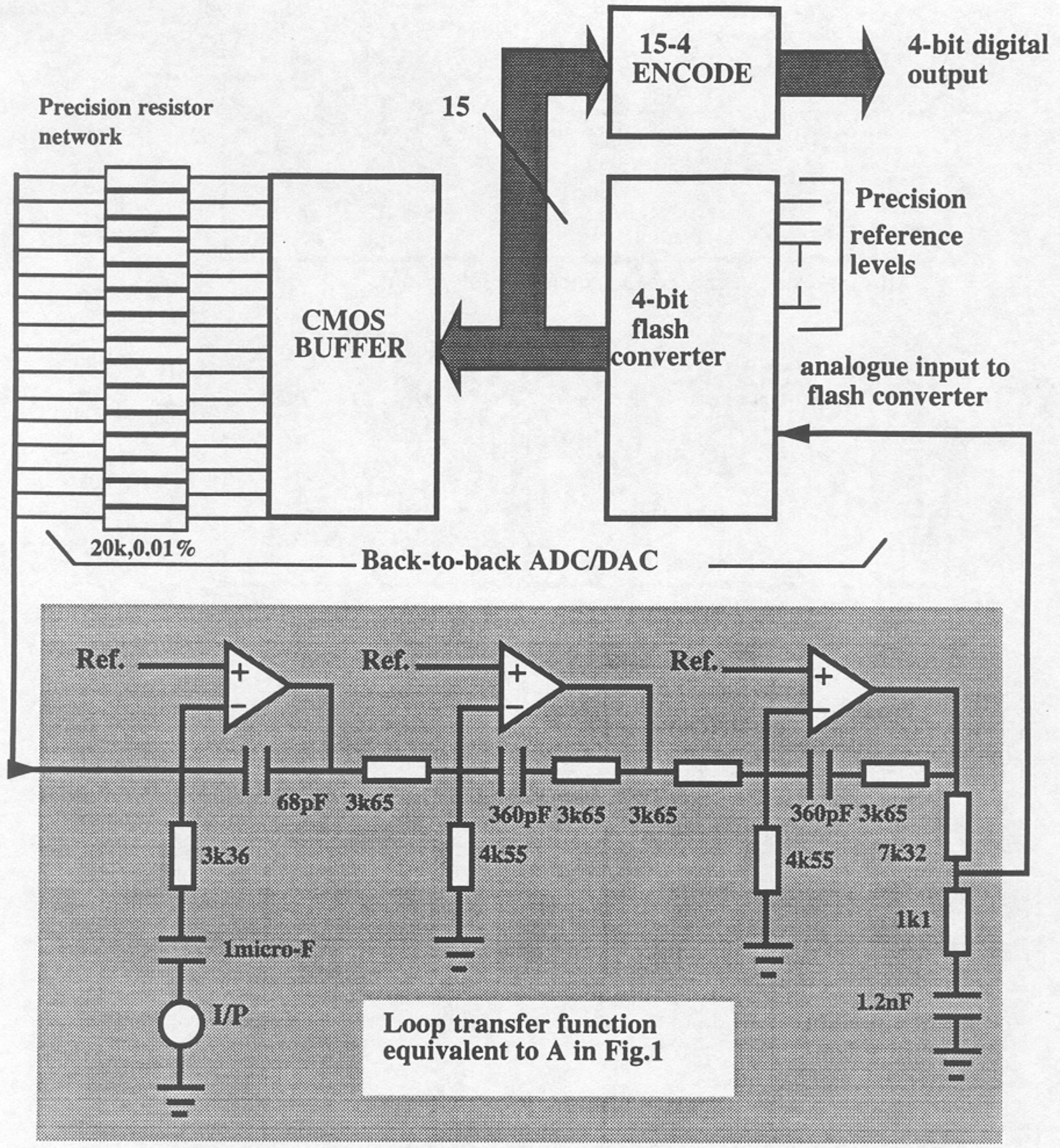


Fig. 5b Front-end of dbx 4-bit ADC after Adams 1986.

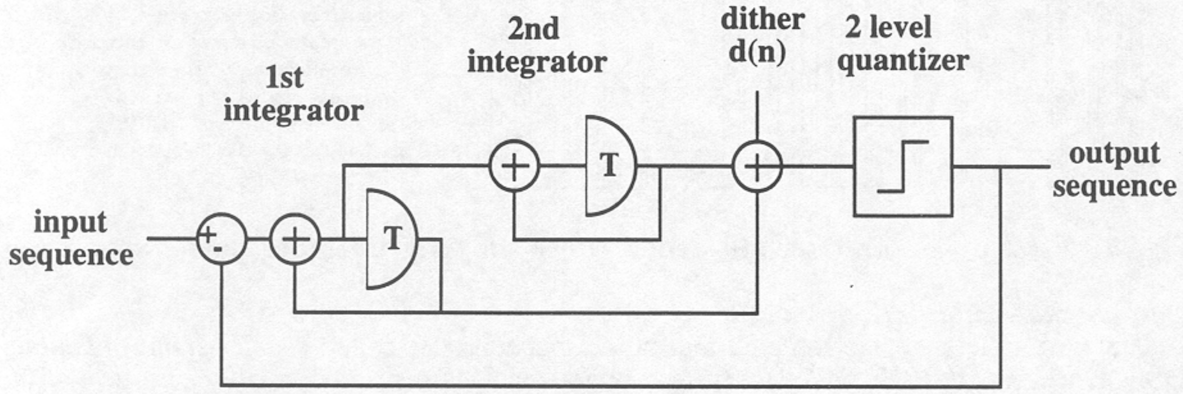
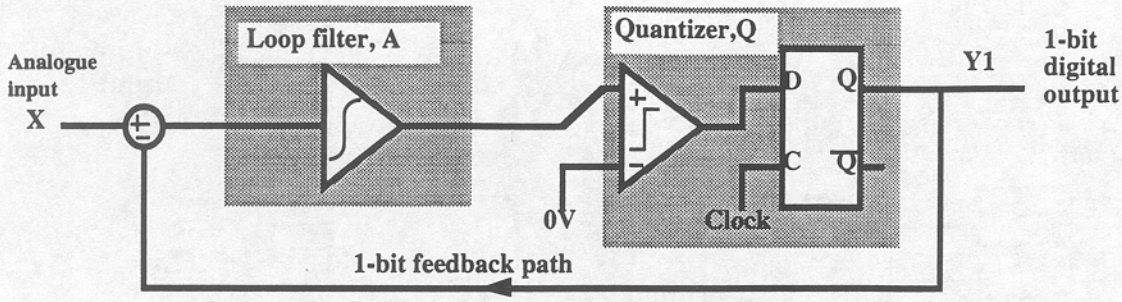
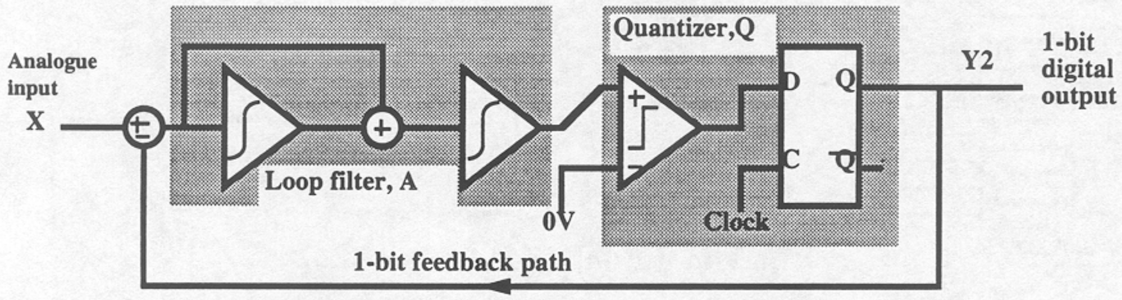


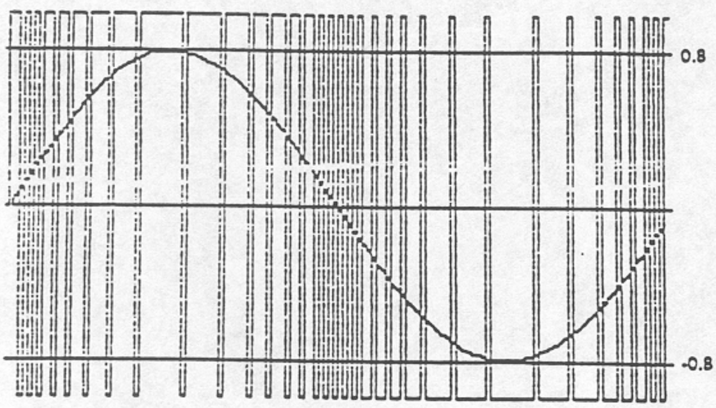
Fig. 6 Second-order delta-sigma modulator configured as a DAC.



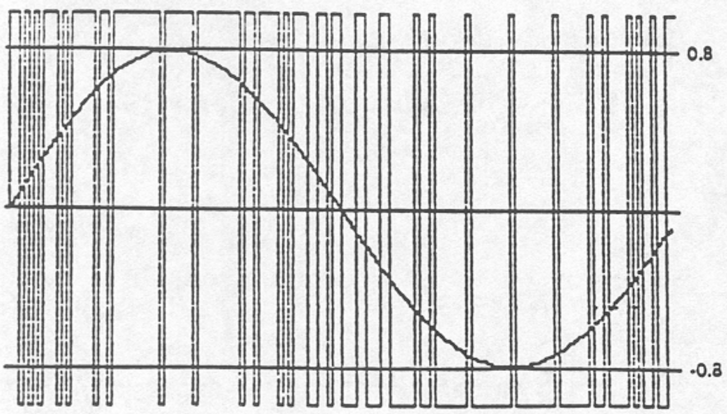
(b) 1st-order Delta-sigma modulator (1 integrator).



(a) 2nd-order Delta-sigma modulator (2 integrators).



First-order delta-sigma modulator output pulse waveform for a sinusoidal input signal. Note that the output pulses are 100% duration, also observe how low-amplitude inputs are poorly coded where the output sequence follows a ...010101... pattern.
Vin = 0.8SIN(2pift), "ONE" = +1, "ZERO" = -1



Second-order delta-sigma modulator output pulse waveform for a sinusoidal input signal.Note that the output pulses are 100% duration, also observe how low-amplitude inputs are encoded more accurately where there is evidence of greater pulse activity by the breakdown of ...010101... sequences.Input is identical to first-order case.

Fig. 7 First-order and second-order DSM showing example pulse waveforms.

Delta-sigma modulation

There is a class of oversampled and noise shaped coder called a *delta-sigma modulator* (DSM) *Inose et al 1962, 1963* where the quantizer Q is restricted to only two levels, thus forming a serial binary output code. DSM was introduced in 1962 and is a derivative of the historically significant delta-modulator circa a patent *Deloraine et al 1947/48* and a subsequent paper *de Jager's 1952*. However, the two-level restriction on Q interacts with the closed-loop transfer function, which should be either first, second, or with care, a third-order loop, otherwise irreversible, level dependent non-linear instability results.

The DSM is representative of the limiting case in transforming a multi-level signal to a more coarse quantization, where the coder generates a sequence of 1 and 0 pulses which, when averaged, yields a close approximately to the audio band data. Since the rate of occurrences *or density* of 1 and 0 pulses is made proportional to the input signal this class of converter is sometimes called *time-quantized pulse density modulation* and because the output sequence is in a coded-binary, serial format, the global name *bitstream* is gaining favour.

To demonstrate the behaviour of DSM, Figure 7 shows both a first- and second-order digital DSM, together with example pulse-output waveforms for a sinusoidal excitation, where the density of 1, 0 pulses is seen to follow the input amplitude, though the time quantization of samples should be noted. Observe how the second-order system introduces more energetic combinations of 1, 0 pulse patterns, which when averaged by the reconstruction filter give improved signal coding particularly at lower signal levels.

To understand the process of DSM it is constructive to draw a parallel with frequency modulation. Imagine the input signal modulates the frequency of a voltage controlled oscillator, where for zero input the frequency is half the DSM sampling rate. At each positive zero crossing of the fm signal a constant-area pulse is produced, thus making the rate of occurrence of pulses directly proportional to the audio signal. If the pulse sequence is now averaged the recovered signal is proportional to the pulse area-hence input signal. However to quantize the signal, the pulses must be constrained in time by relocating them to the nearest sampling instant which occur at a rate equal to the DSM sampling rate. The time axis is divided into a regular array of time slots where if a pulse (from the vco) is produced within a time slot, it is then relocated to an instant coincident with the end of that slot. Consequently, a first-order DSM can be modelled as time-quantized frequency modulation *Flood et al 1971, Hawksford 1972.*

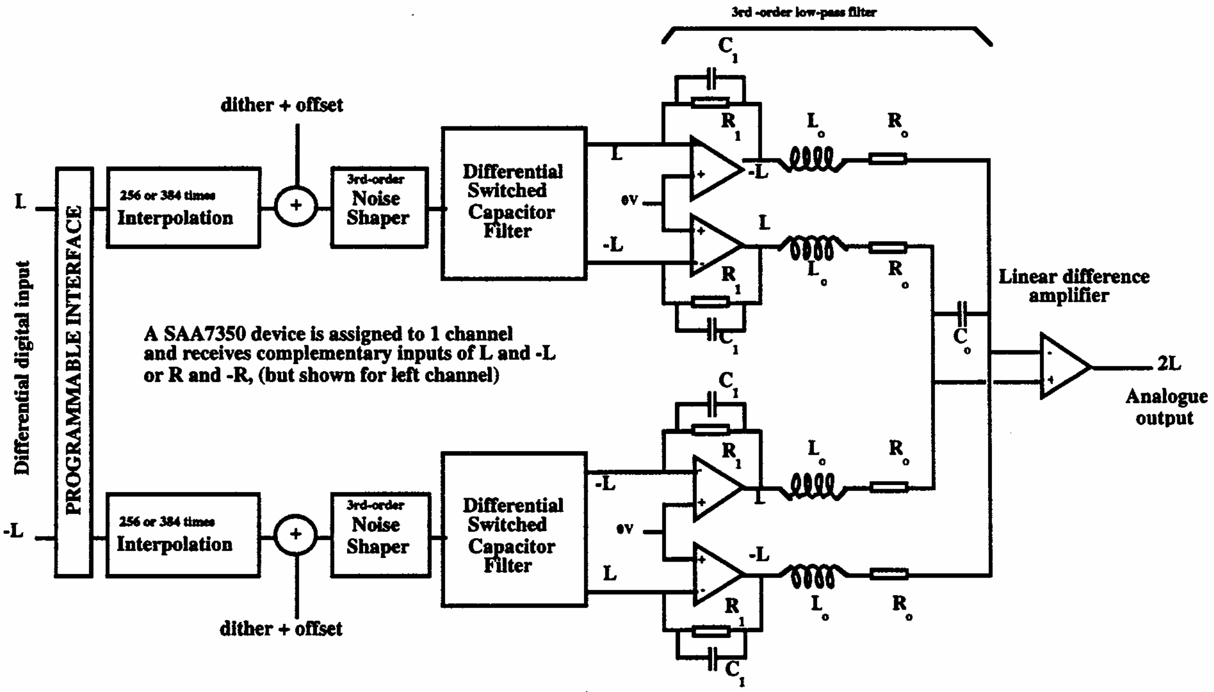


Fig. 8 Basic system of a double differential bitstream DAC.

Providing a high sampling rate and high-order noise shaping can be combined, then DSM offers an extremely high-performance potential in both ADC and DAC that overcomes the accuracy problem of the least significant bits which is a major factor in determining the low-level resolution of a digital audio system. An example of a DAC using the bitstream techniques is shown in Figure 8 and further discussion can be found in a *Hi-Fi News and Record*

Review article *Hawksford 1991.* Although DSP is required to perform interpolation/decimation and noise shaping, the ability to use VLSI to integrate these systems means a sophisticated specification can now be achieved at low cost. It is here that digital audio is gaining the initiative as it represents one of the most demanding applications of conversion technology.

4 Psychoacoustic noise shaping

Although the noise shaper shown in Figure 4 is generally used with oversampling, it has found recent application combining perceptual weighting together with Nyquist sampled digital audio data. As an example, consider digital requantization where an ADC has a nominal resolution of 20 bit but the channel, such as a compact disc system, has a word length of 16 bit. Normally this truncation would be performed in association with triangular pdf digital dither and result in a flat noise spectrum decorrelated from the data sequence. However, if a noise shaper now encloses the requantization process, the noise spectrum can be shaped non uniformly such that the noise is lower in one spectral region while increased in another.

Since the background noise of a well behaved audio system is of low level, it is possible to take advantage of the ear's sensitivity curve close to the threshold of hearing, as depicted in Figure 9. This characteristic shows greatest sensitivity in the 1 to 5 kHz region but with a rapid reduction at both higher and lower frequencies; consequently, noise shaping can be used to relocate the requantization noise mainly to the higher frequency band above 8 kHz. It must stressed however, that the noise shaping process does not reduce the overall noise power *per se*, in fact it may actually increase, it is only when the spectrally shaped noise is weighted by the hearing sensitivity characteristic that there can be a net reduction in subjective sound level. This is an example of where the use of noise shaping and the inclusion of perceptual weighting criteria can match more closely the theoretical information capacity of a digital channel.

It has been shown by *Wannamaker 1992*, that a performance approaching ≈ 20 bit subjective dynamic range is viable even though the channel is limited to 16 bit uniform quantization and that subtractive dither Craven et al 1992 can also be implemented in association with noise shaping. This performance is of cause only achieved if there are no other significant noise sources in the ADC, DAC and associated electronics which inevitably fill in the spectral wells created by noise shaping.

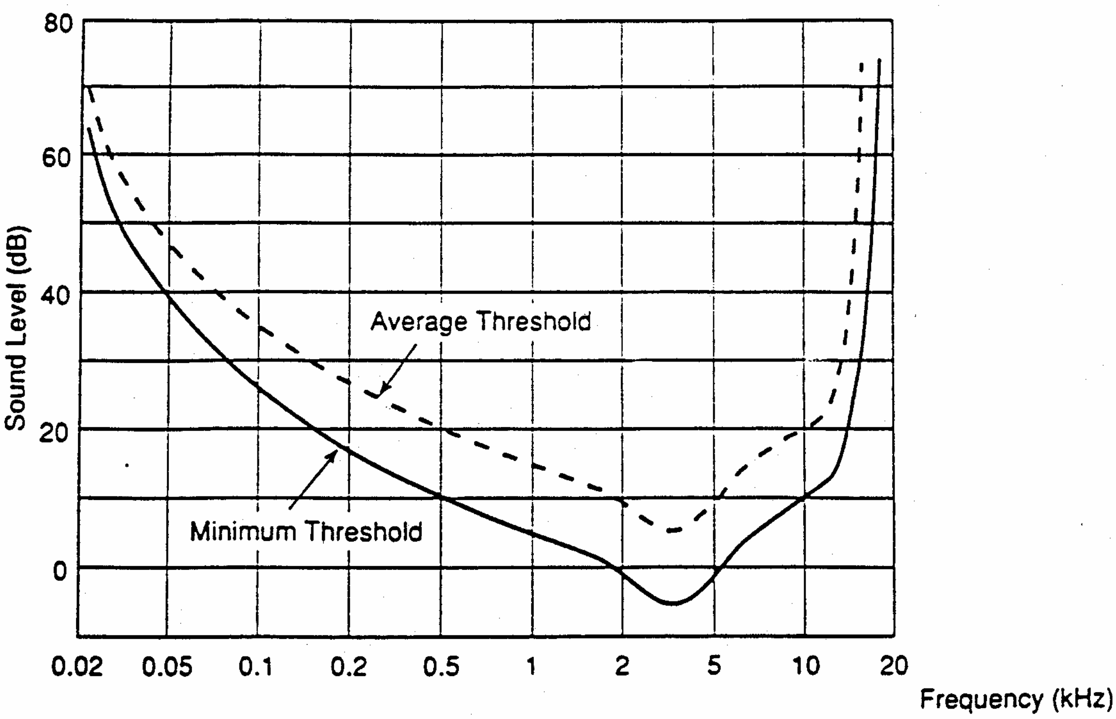


Fig. 9 Threshold of human hearing characteristic.

5 Signal processing for loudspeaker systems

Although active loudspeaker systems require additional electronic circuitry, DSP can rationalise the hardware and overcome many of the problems associated with a purely analogue design. For example:

(a) DSP allows crossover target transfer functions, including amplitude and phase compensation of each drive unit, to be specified to a high degree of accuracy. Thus the polar response can be optimised and the overall frequency response can approach a constant amplitude/linear phase characteristic.

(b) Crossover filters can include options for either sharp frequency transition bands or more gentle frequency response slopes, there is considerable design flexibility, although research has yet to establish the best criteria.

(c) Delay compensation to align acoustic centres of drive units on axis is straightforward to implement using a digital delay.

(d) Simple adjustment for individual drive unit sensitivities without wasting power in passive elements of a crossover thus enabling amplifier power to be used efficiently

(e) Since separate power amplifiers are used for each drive unit, there is a corresponding division of signal power that reduces the voltage and current demand on an individual amplifier, *Hawksford 1986* .

(f) Momentary clipping of a single power amplifier can be softened by preprocessing and only has a localised (in the sense of frequency range) impact on subjective performance, hence the system is more overload tolerant.

(g) Use of current drive technology and/or mixed current drive/voltage drive systems can enhance system performance.

(h) Each DAC (bitstream or conventional) only handles a bandlimited audio signal which should reduce intermodulation distortion, minimise the probability of system overload in drive unit frequency response correction processes, and widen the choice of conversion strategy for subjective optimisation by the designer.

(i) Signals can be routed to the active loudspeaker system via an optical or a conventional digital interface with central commands such as stand-by mode, volume level, equalization programs being remotely down loaded from a central (or indeed distributed) control centre. There is the option here to define a local-area network for digital/video distribution systems.

(j) A digital model of a drive unit can be included to predict overload distortion. This enables signals to be dynamically controlled to maintain the loudspeaker within its performance envelope, yet minimise the subjective impact of overload.

Digital crossover filters

The majority of MC drive units can operate only over a limited bandwidth before either chaotic cone break up in poor designs or controlled but non-pistonic behaviour in better designs limits frequency response smoothness and impairs polar response. Also, it is well documented *Olson, FO, 1967* that even pistonic cones can only operate with reasonable success up to wavelength comparable with the cone diameter. Thus, observing the need for a large cone diameter at low frequency to displace the required volume of air, then the rationale for using at least two drive units with an appropriate crossover filter is evident.

There is a substantial literature on the subject of crossover networks yet keen eared, loudspeaker designers can operate on an intellectual plane that circumvents (or ignores!) much of this work as their experience augmented by a little CAD, has taught them the art of balancing the numerous performance facets that are not easily visualised through numeric documentation. However, once the flexibility of DSP is introduced the art of the design become more precise with the consequence of near-text book results.

As an example, consider the 2-way loudspeaker system illustrated in Figure 10 that incorporates two digital crossover filters with both composite frequency response correction and delay compensation for non-aligned drive unit acoustic centres. An initial design strategy could be based upon a (two-way) passive loudspeaker where by measuring the frequency responses between crossover input and hf and lf drive unit speech coils, digital filters are designed with similar responses. There are several methods that could accomplish this task and in principle a similar performance to the passive system should be achieved though hopefully devoid of the errors induced by non-linear drive unit impedances interacting with the passive crossover. However, by approaching the design in this way it is unlikely that the full potential of DSP will be realised as there exist more general methods which are compromised if the passive loudspeaker alignment is taken as a starting point.

In the period 1981-82, Martin Rutz, a European Degree Scheme student at Essex University, implemented a digital crossover system *Rutz 1982* for a 2-way loudspeaker. Although the hardware at that time was not to audiophile standards being based upon an Intel 2920 8 bit processor that included onboard ADC and DACs, the algorithmic structures were appropriate and the hardware confirmed the technical feasibility of the system. The study was particularly concerned with polar response where it became evident that even with relatively sharp FIR filters, differences in phase response between drive units resulted in polar response asymmetry in the crossover transition region.

However, as well as phase response anomalies, drive units also exhibit a non-uniform frequency response that can be corrected within the digital filter. A further research project *Bews 1987* undertaken by Dr Richard Bews (founder of LFD audio) over the period 1983-87 investigated the design of FIR filters that combined both crossover filter and drive unit amplitude response compensation. The research resulted in a prototype similar in form to Figure 10 and used the new (as then) Texas TMS320 C10 signal processor.

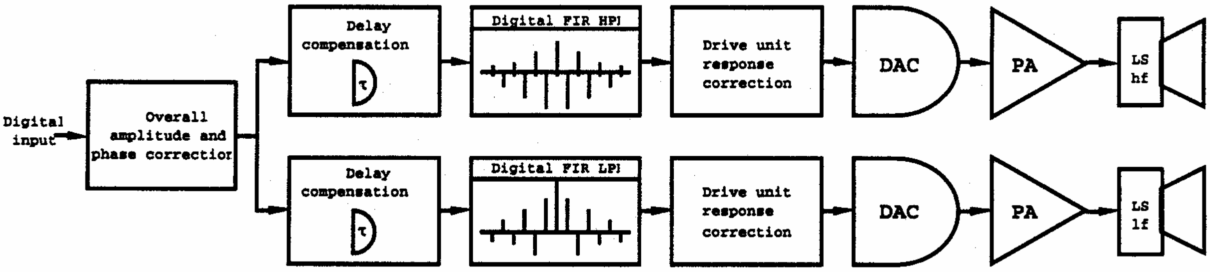


Fig. 10 Two-way digital and active loudspeaker system.

A problem area for passive loudspeakers is non-uniformity in the polar response during a crossover transition band resulting from non-coincident drive units and the individual drive unit transfer functions. A design option using DSP enables rapid crossover transitions which produce corresponding small interference regions. Unfortunately a narrow transition band implies a protracted and oscillatory impulse response for both high-pass and low-pass filters. Of course, on axis the filter responses should combine such that the time dispersion is cancelled, nevertheless, if in the crossover region the polar responses of the drive units are not well matched, then it is the possible for the off-axis response to be coloured by the crossover ringing response not being fully suppressed. This aspect of crossover design is a current topic of research, *Rimell and Hawksford 1993.*

This example shows how DSP can be used to implement crossover filters where the following advantages result:

* universal hardware that can be software tailored
* simple system replication with identical filters
* time synchronised filter responses
* temperature and supply rail insensitive (within operating regime)
* accurate matching to individual drive units and enclosures
* direct interface to CD and other digital sources
* well controlled polar response in crossover transition region
* opportunity to match individual drive units to crossover
* digital gain control with zero nonlinear distortion.

Finally, Figure 11 shows a 3-way system that includes a current-drive subwoofer and an efficient dynamic tracking power amplifier as an example of possible future system configurations. Broadband correction is also included, *Greenfield and Hawksford 1989.*

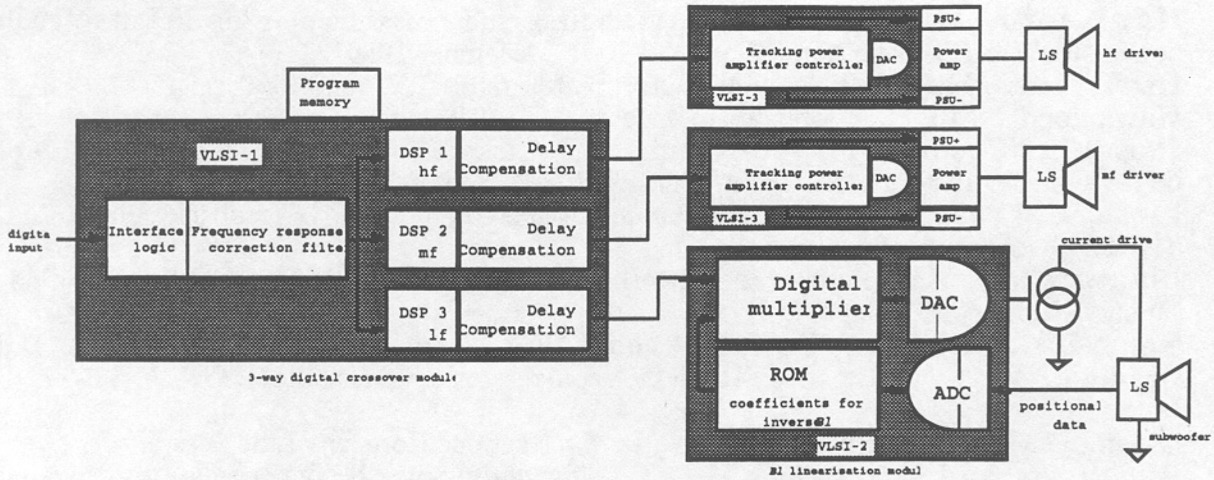


Fig. 11 Two-way digital and active loudspeaker system.

6 Conclusion

This review paper has discussed some of the issues concerning noise shaping and oversampling and also introduced bitstream techniques, psychoacoustic noise shaping and digital techniques for loudspeaker systems. These methods are some of the processes that can be used to aid realisation of high-resolution audio electronics where techniques of VLSI allow powerful DSP to be performed at an economically viable cost and which in some cases can simplify critical analogue electronics.

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