The Term Structure of Debt Commitments, Liquidity Concerns, and Durable Good Choices

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Abstract

This paper investigates the role of liquidity constraints in shaping loan term choices within the auto loan market, a major component of household debt in the United States. I address two key features of auto loan term lengths in the U.S.: their substantial cross-sectional heterogeneity and the notable rise in average term lengths over time. Using data from the Federal Reserve Bank of New York/Equifax Consumer Credit Panel, along with supplemental income and price data, I establish a causal link between liquidity constraints and loan term lengths, demonstrating that much of the cross-sectional variation in term lengths can be attributed to differences in liquidity constraints among borrowers. To further analyze these patterns, I develop a quantitative model of term length choice, showing that access to longer loan terms enables borrowers to smooth consumption and manage debt more effectively. Through this model, I also demonstrate that while time-series variation in liquidity constraints alone does not fully account for the increase in term lengths, the narrowing gap between interest rates on debt and savings, when interacted with liquidity constraints, has contributed to the observed trend toward longer loan maturities.

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1 Introduction

There has been an explosion of interest in incorporating liquidity constraints into macroeconomic models, marking a shift away from the permanent income hypothesis toward frameworks that capture market incompleteness and consumer heterogeneity. This shift underscores the importance of liquidity constraints in shaping consumer behavior, particularly in relation to the demand for durable goods. Liquidity constraints influence how households navigate large purchases, affecting both the timing and financing of these durable assets. Understanding the role of liquidity constraints in driving marginal propensities to consume (MPCs) is not only essential for capturing household decision-making but also holds policy relevance, as MPC responses to liquidity constraints can shape aggregate demand patterns and potentially inform economic interventions.

In this paper, I study the importance of liquidity constraints within a major asset category: auto loans. I examine how these constraints shape MPCs by analyzing the connections between liquidity, durable debt contract choices, and ultimately consumer durables demand behavior. Focusing on auto loans—a significant component of household debt—offers a unique perspective on these dynamics. Auto loan borrowers are typically younger, lower-income, and lower-asset individuals, who are therefore especially affected by liquidity limitations. By analyzing this context, I explore how liquidity constraints impact contract term choices and durable goods demand, deepening our understanding of the mechanisms through which liquidity constraints influence financial decisions in the durable goods market.

A central feature of auto loan contracts is the term length, which plays a crucial role in shaping the financial aspects of the loan, including monthly payment size, speed of repayment, interest paid, and the borrower's equity accumulation. The choice of loan term length is particularly relevant for liquidity-constrained households, as longer terms generally lower monthly payments, making auto purchases accessible to borrowers with limited liquidity. Unlike the largely standardized 30-year mortgage in housing, auto loans display significant heterogeneity in term lengths, ranging from just a few years to over seven years. Moreover, auto loan term lengths have steadily increased in the U.S. since the post-Great Recession period, driven by both consumer preferences for lower payments and lenders' willingness to extend loan maturities.

Understanding the role of loan term lengths is especially important for households with low liquidity, as liquidity-constrained individuals are often highly responsive to changes in their borrowing capacity. Extending loan terms functions similarly to increasing a borrowing limit, reducing monthly payments and allowing borrowers to finance larger loans without increasing their immediate financial burden. For these households, the monthly car payment represents a minimum consumption commitment: once they enter into an auto loan, they are obligated to make the payment until the loan is fully paid off or renegotiated, often during a subsequent car purchase. Consequently, households choosing a loan term must weigh their current liquidity needs against their anticipated financial flexibility over the life of the loan. Longer terms with lower monthly payments provide an avenue for managing future liquidity constraints. For instance, extending a loan from 60 to 72 months can reduce monthly payments by approximately 13–16% (An et al., 2020), depending on the interest rate. This dynamic is particularly relevant for auto loans, which are more frequently utilized by lower-income, lower-wealth households who are more likely to face liquidity constraints.

This paper addresses two key facts about auto loan term lengths in the U.S.: first, their substantial heterogeneity, and second, the notable rise in average term lengths over time. I demonstrate that liquidity constraints are essential for understanding these patterns. Specifically, I document that cross-sectional heterogeneity in term lengths can largely be attributed to variations in liquidity constraints among borrowers. Additionally, while time-series variation in liquidity constraints alone does not fully explain the upward trend in term lengths, I show that the narrowing gap between interest rates on debt and savings has contributed to this increase. As this interest rate gap has decreased, liquidity considerations have increasingly influenced borrowers to opt for longer loan terms, thereby reducing monthly payments and addressing liquidity needs.

There is suggestive evidence that term lengths are related to liquidity constraints. Longer loan terms are generally associated with more highly leveraged loans, a pattern that holds across various credit scores. Furthermore, individuals with lower credit scores, who are more likely to face liquidity constraints, tend to opt for longer loan terms. This paper aims to establish a causal link between low liquidity and term length choices. To do so, I utilize several data sources to empirically examine this relationship. The primary dataset comes from the Federal Reserve Bank of New York/Equifax Consumer Credit Panel (CCP), which provides comprehensive borrower-level credit data, covering a 5% national sample of anonymized individuals from 1999 to 2023. A subset focuses specifically on auto loans, capturing details such as loan amounts, monthly payments, and delinquency statuses. From these data, I infer key variables such as term lengths and construct a measure of liquidity constraints based on the distance between revolving credit limits and credit usage. Additionally, I incorporate IRS zipcode-level income data and adjust for inflation using CPI and PCE price deflators from Federal Reserve Economic Data (FRED).

The empirical strategy centers on using the measure of liquidity constraints—specifically, the share of revolving credit balance left—as the independent variable, and the term length of auto loans as the dependent variable. This liquidity constraint measure is zero for fully constrained individuals and one for those who are entirely unconstrained. I expect a negative coefficient, as individuals with lower liquidity should opt for longer term lengths. Consistent with this hypothesis, the results of the OLS regression show that moving from fully constrained to fully unconstrained is associated with a 1.625-month decrease in term length, which is approximately 2.6% of the mean term length. Additionally, the OLS results indicate that moving from fully constrained to fully unconstrained reduces the probability of selecting a loan term greater than 60 months by 5.2 percentage points. However, this estimate is likely biased towards zero due to potential endogeneity. Borrowers who anticipate future liquidity constraints may take preemptive actions, such as opening new credit accounts, which makes simple correlations between credit usage and loan terms unreliable.

To address potential endogeneity concerns, I employ an instrumental variables (IV) approach, following Braxton et al. (2024), using the age of the borrower's oldest credit account as an instrument for liquidity constraints. This instrument is valid because credit limits generally increase with account age, and credit scores—which affect credit limits—also reflect account age. By isolating exogenous variation in liquidity constraints, this approach allows for a more accurate estimation of the causal effect of liquidity on loan term length.

The results of the IV specification show a considerably larger effect of liquidity constraints on term length compared to the OLS estimates. Specifically, moving from fully constrained to fully unconstrained is associated with a 12.51-month decrease in term length, which is about 20% of the mean term length. For the likelihood of selecting a loan term over 60 months, the IV estimates show a reduction of 31 percentage points, highlighting a much stronger relationship than the OLS results suggest. These findings are robust across a variety of specifications and controls, including lagged credit usage, time and geographic controls, and individual risk scores.

In the next part of the paper, I examine the heterogeneity in term length choices, as well as the factors that contribute to the observed increase in term lengths over time, using a model.[1] This model focuses on the trade-offs households face when deciding on loan term lengths: the benefits are lower monthly payments, which are especially valuable to individuals who are liquidity constrained or anticipate future liquidity constraints, while the costs are higher interest rates on longer loans and an increased total interest paid over the loan's life. Extending term lengths helps smooth payments over time, providing critical financial flexibility for households that face minimum monthly payment commitments throughout the loan period. Therefore, proximity to liquidity constraints plays a key role, not only in the choice of term length but in how households manage expected liquidity needs going forward.

The model confirms that households closer to liquidity constraints tend to choose

[1] This model is based on a straightforward Aiyagari-Bewley household problem. In it, households are 'born' with a loan and must choose the repayment speed or term length. They can adjust their consumption-savings but are otherwise constrained to the loan terms.

longer term lengths, but it also reveals that individuals who are near liquidity-constrained—those not currently at their borrowing limit but likely to face liquidity constraints in the future—also opt for longer terms. This finding highlights that it is not simply the binary condition of being liquidity-constrained or not, but rather the degree of nearness to liquidity constraints, that drives term length choices.

Additionally, the model shows how the distance to liquidity constraints determine household response to shocks. For instance, a narrowing gap between auto loan interest rates and savings rates—effectively making debt less costly relative to savings—leads to an increase in term lengths. However, this effect is not uniform across all households. The model demonstrates that while those at the most constrained end already choose the longest terms available, households that are near liquidity-constrained (who anticipate potential future constraints) respond more actively to these shifts in relative interest rates by extending their term lengths. This finding emphasizes that changes in borrowing conditions, like the interest rate gap, impact those on the edge of liquidity constraints most strongly, driving them to adjust their debt commitments in anticipation of future liquidity needs.

To further explore the role of relative interest rates in driving term lengths, I develop a quantitative model of term length choice. This model operates in partial equilibrium and incorporates several key features. Households face idiosyncratic income risk, modeled as an AR(1) process, and have Cobb-Douglas preferences over a flexible consumption good and a durable consumption good. Households can save through a risk-free savings technology but cannot borrow against it. However, they can borrow through a loan tied to the value of the durable good, up to a specified loan-to-value (LTV) ratio. Each period, households must decide whether to retain their existing durable good and loan or adjust both. Adjustments come with a fixed cost, and this discrete choice is influenced by a taste shock drawn from an extreme value distribution. If households choose to adjust, they must also select from a menu of loan term lengths, proxied by repayment speeds. This decision is modeled as a discrete choice, and the interest rate on the loan increases with repayment speed.

The model is calibrated using standard targets from the literature, following McKay and Wieland (2021) and Beraja and Zorzi (2024). The predictions align closely with the simple model's predictions: lower asset levels, proximity to a borrowing constraint, and lower income all lead to the selection of longer loan terms. These households value the ability to smooth consumption over time, prioritizing lower monthly payments when their marginal utility of consumption is highest.

However, the quantitative model provides additional insights beyond the simple model. For example, choosing longer term lengths enables households to afford higher-value durable goods and take on more leveraged loans. By spreading their payments over a longer period, households can smooth consumption, making higher levels of debt more manageable. This highlights how access to longer term lengths can increase both borrowing and consumption of durable goods.

In the final part of the paper, I use this quantitative model to compare economies with different relative costs of borrowing and lending. I find that a decrease in the spread between borrowing and lending rates leads to an increase in both loan term lengths and the total amount of debt taken on by households. This occurs because a lower spread reduces the cost of borrowing, making it more attractive for households to take on additional debt. Notably, I find that the increase in durable goods demand associated with the difference in relative rates is partially driven by the availability of a menu of term lengths. This suggests that having the option to extend loan terms plays a crucial role in amplifying demand, compared to a model where only one repayment speed is available.

Related Literature. The results of this paper contribute to several broad literatures. First, this paper contributes to the growing literature on term lengths in auto loans, with a focus on how term length choices relate to both credit risk and liquidity. Studies such as Hertzberg et al. (2018) and An et al. (2020) explore the link between term lengths and credit risk, with Hertzberg et al. (2018) finding that selection into longer loans is associated with higher credit risk, while An et al. (2020) show that although borrowers opting for longer terms are more likely to default, they do so at lower rates than their counterparts with shorter loans. Katcher et al. (2024) also document the increase in term lengths, and look at the relationship between term lengths and likelihood of prepayment. Another strand of work focuses on the responsiveness to automobile debt and demand to increasing term lengths. Attanasio et al. (2008) show that consumers' have a greater responsiveness of debt to changes in loan maturity, rather than interest rates. They interpret this as evidence of liquidity constraints playing a key role in auto decisions, including term length. Argyle et al. (2020) expand on this, showing that consumers tend to increase spending in response to longer loan maturities, interpreting this as evidence of monthly payment targeting. My contribution to this literature is to model term length choice from a household perspective, identifying liquidity as a critical driver of longer loan terms. In contrast to Attanasio et al. (2008) and Argyle et al. (2020), I provide direct empirical evidence of the effect of liquidity constraints on term length choice, rather than interpreting increased spending as indirect evidence of liquidity constraints.

This paper also relates to the broader literature on liquidity constraints and auto demand, a well-established area of research that connects financial conditions with consumer behavior in the auto market. Mishkin (1976) was among the first to link liquidity constraints to demand for durable goods, a connection that has been explored in more detail by Heitfield and Sabarwal (2004) and Attanasio et al. (2008). More recently, Adams et al. (2009) and Mian and Sufi (2012) examined the role of credit availability in shaping auto loan terms and vehicle purchases. Benmelech et al. (2017) show the importance of credit supply for auto purchases; Gavazza and Lanteri (2021) further emphasize the role of credit supply not only for direct purchases, but also for liquidity in secondary car markets. A substantial literature has implemented tests for incomplete markets by showing that current consumption responds to current liquidity, as demonstrated by Johnson et al. (2006) and Zeldes (1989). Ganong and Noel (2020) further contribute to this literature by showing that mortgage modifications that extend term lengths, rather than reduce principal, are more effective in preventing default, highlighting the importance of term extensions over principal reductions for household financial stability. Building on this work, I link liquidity constraints directly with term length choices for auto loans, exploring how shifts in liquidity influence both short- and long-term borrowing decisions.

Finally, understanding the dynamics of expenditures on durable goods has long been an important question in macroeconomics (see, for instance, Mankiw (1982) and Bernanke (1985)). Grossman and Laroque (1990) develop a model of durable goods adjustment subject to transaction costs, which captures the notion of indivisibility: to increase the utility flow from durables, a household must trade their *entire* current durable good and replace it with a new one. Consistent with this, several papers focus on models of lumpy durables adjustment (see, among others, Caballero (1993) and Eberly (1994)). Recent studies embed this lumpy adjustment process in general equilibrium frameworks with uninsurable idiosyncratic risk, including Kaplan and Violante (2014), Berger and Vavra (2015), Guerrieri and Lorenzoni (2017), McKay and Wieland (2021), Beraja and Zorzi (2024), Gavazza and Lanteri (2021), and Berger et al. (2023). This paper examines durable goods demand within a partial equilibrium framework with uninsurable idiosyncratic risk. I extend this literature by incorporating term length choices into a quantitative model of auto demand, providing a novel approach to understanding the interaction between liquidity constraints and durable goods purchasing behavior. This model emphasizes the important

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Outline. The remainder of this paper is structured as follows. Section 2 provides a background on auto loans, including motivating facts for the project. Section 3 provides an overview of the data used in this study, including the Federal Reserve Bank of New York/Equifax Consumer Credit Panel (CCP). Section 4 presents the empirical analysis, focusing on the relationship between liquidity constraints and term length choices in auto loans. Section 5 develops a simple model of term length choice, exploring both the relationship between liquidity and term length choice as well as other factors that driven households to opt for longer terms. Section 6 extends this simple model to a larger quantitative framework, which is used to explore the implications of changes in borrowing and lending rates. Section 7 concludes.

2 Background and motivating facts

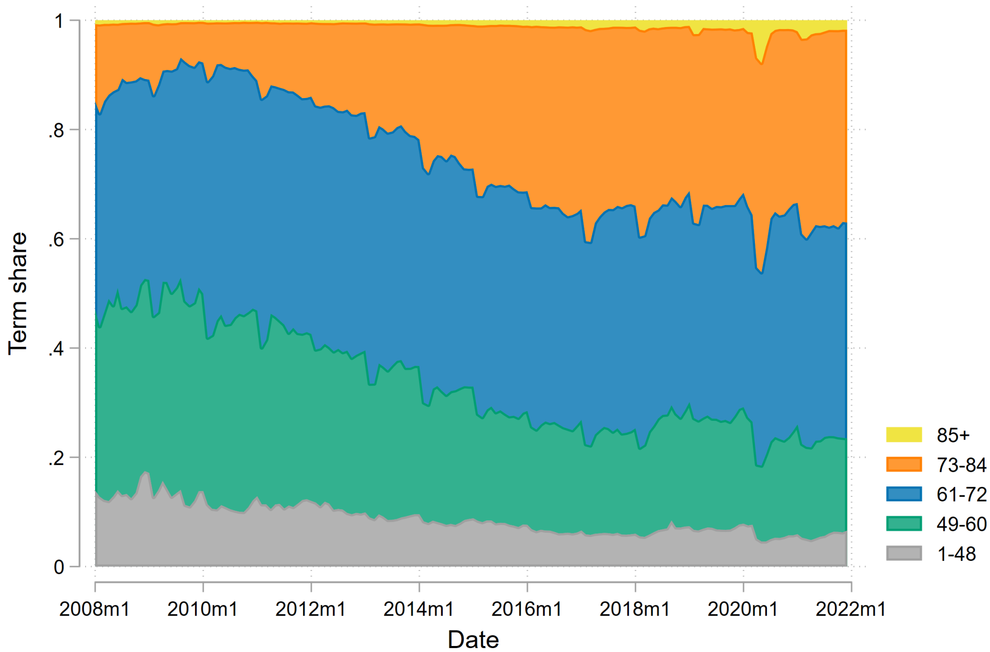
2.1 Motivating Facts

To motivate this study, we begin by examining several key facts about term lengths in auto loans. Figure 1 displays the share of loans originated at each term length over time. Two main insights emerge from this figure. First, there is substantial heterogeneity in term lengths at any given point in time. This is in contrast to mortgages in the U.S., where the vast majority of borrowers opt for a standardized 30-year loan term. Second, we observe a clear trend of increasing term lengths over time. The share of loans with terms over five years has grown from about 50 percent to nearly 80 percent by the end of the sample period. This paper will explore how liquidity considerations play a crucial role in explaining both the cross-sectional heterogeneity in term lengths and the upward trend over time.

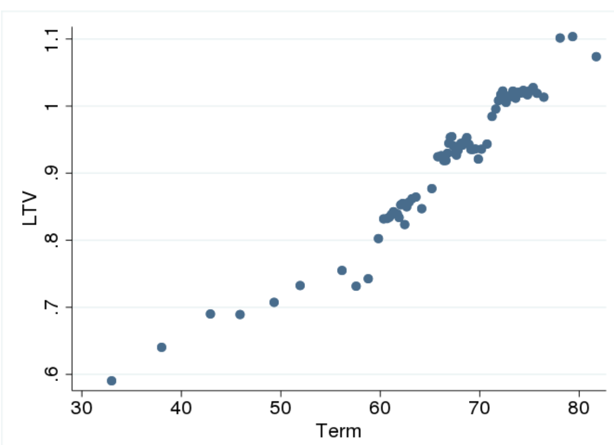
Further evidence of the link between liquidity constraints and term lengths is shown in Figures 2, 7, and 2b. Figure 2 highlights that liquidity and term length are indeed correlated. Figure 2a demonstrates that longer term lengths are associated with more highly leveraged loans, which are more likely to be taken on by liquidity-constrained individuals. Figure 2b shows that individuals with lower credit scores—who are more likely to face liquidity constraints—tend to choose longer loan terms. These observations motivate the need to examine liquidity as a key driver of term length choices. In later sections, this link between liquidity and term lengths will be established more rigorously. Specifically, Section 4 will empirically show that liquidity constraints are a causal factor for term length choices, while Sections 5 and 6 will further substantiate this relationship using theoretical models.

While liquidity constraints provide a compelling explanation for the observed heterogeneity in term lengths, they alone cannot account for the increase in term lengths over time. Figure 3 shows the share of “hand-to-mouth” individuals over time, revealing that

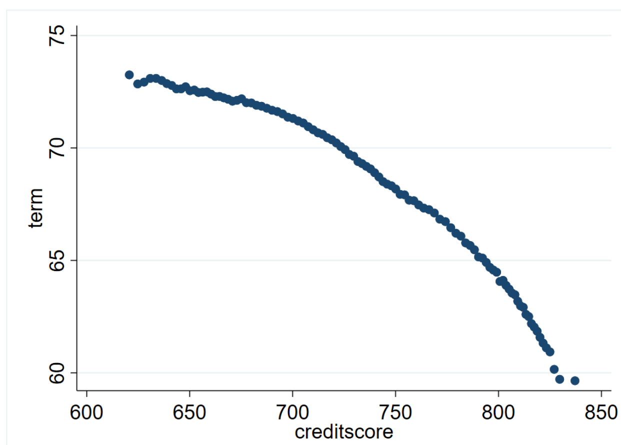
Figure 1: Average loan leverage over term lengths



Source: Experian Autocount data with author's calculations.



(a) Average LTV over term lengths



(b) Average term lengths over credit scores

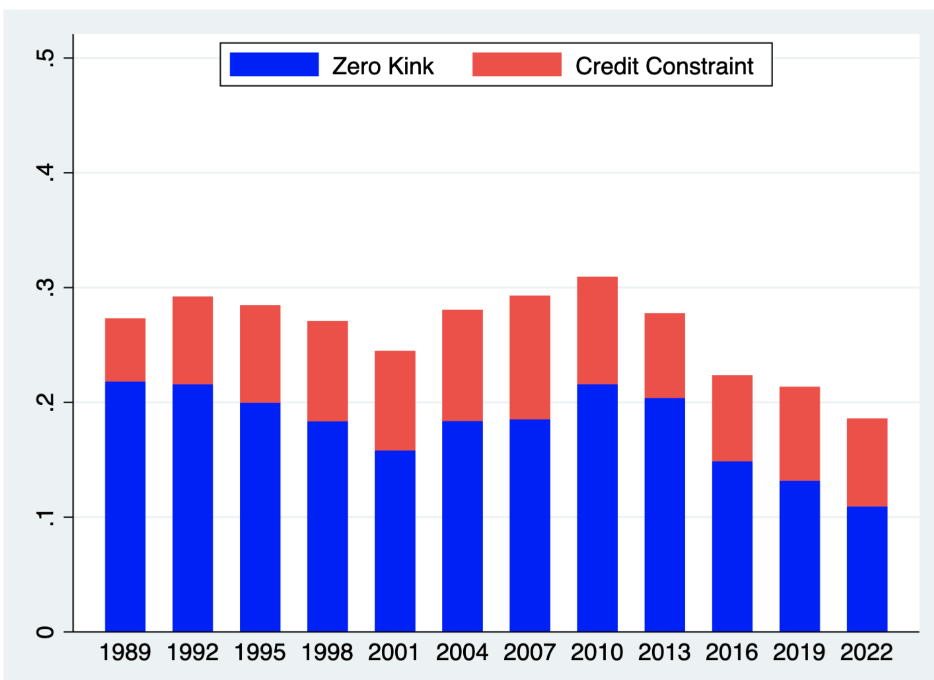
Figure 2: Liquidity and term lengths

Source: Experian Autocount data with author's calculations.

this share has actually declined, rather than increased, since the Great Recession. This decline suggests that changes in the number of liquidity-constrained individuals are unlikely to explain the upward trend in term lengths. However, Figure 4 shows movements in the relative cost of borrowing over time; specifically, the cost of auto borrowing has become cheaper relative to savings. This trend, when combined with the benefits of extended term lengths for liquidity-constrained individuals, may help explain the increase in loan term lengths over time.

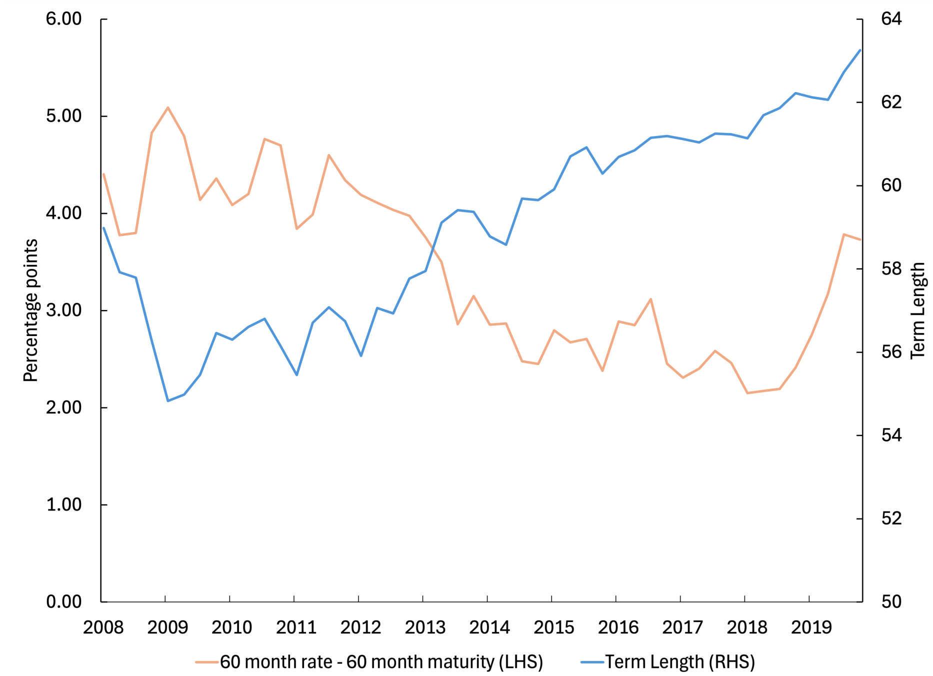
In summary, the facts about auto loan term lengths—both the heterogeneity and the time-series increase—motivate the focus of this paper. The importance of liquidity constraints, in combination with changing relative borrowing costs, provides a framework for understanding these patterns. The rest of this section will provide further background on auto loans, characteristics of the borrower population, and an overview of why liquidity constraints and term lengths interact in shaping auto loan demand.

Figure 3: Hand to Mouth Share over time



Source: SCF data with author's calculations.

Figure 4: Rate gap vs. Term lengths over time



Source: FRED data with author's calculations.

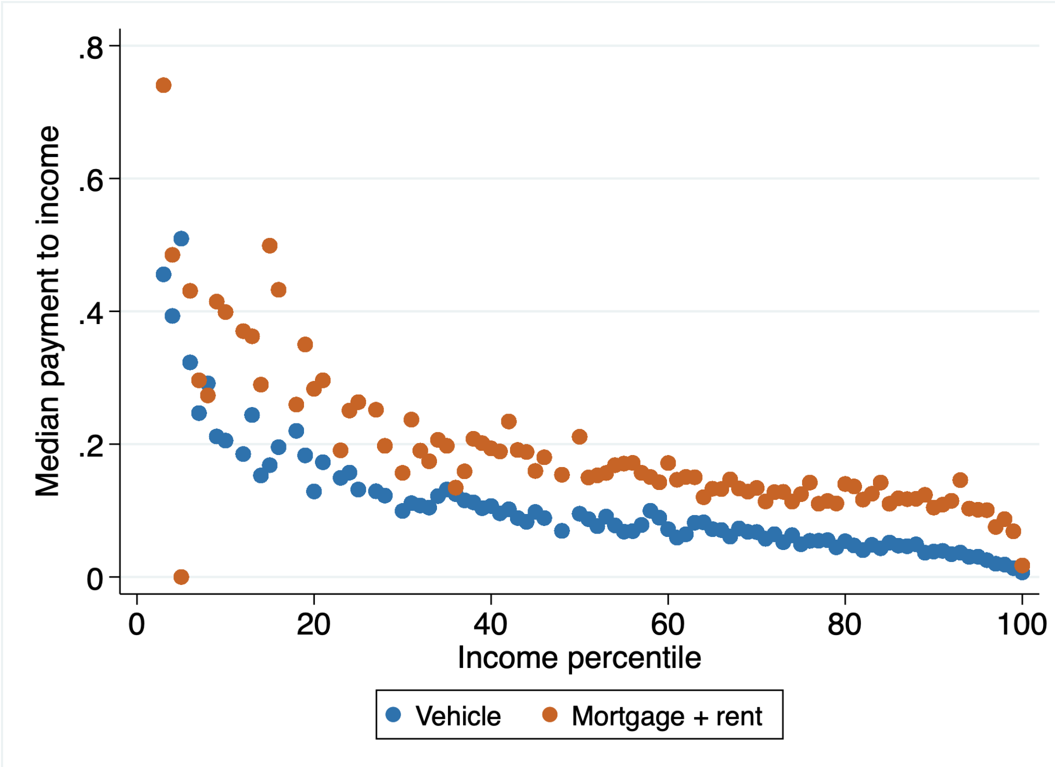
2.2 Background

Auto loan details Auto loans are structured as simple-interest installment loans, where the borrower makes fixed monthly payments over a set term length. These loan contracts offer a schedule of interest rates and term lengths that vary depending on household characteristics, such as the value of the car, the size of the down payment, the borrower's FICO score, and their income level. The monthly payment (denoted as M) covers both interest and principal, calculated to ensure that the total loan amount P is fully repaid by the end of the term length T. The relationship between these terms can be expressed as follows:

$$P = \frac{M}{(1+i)} + \frac{M}{(1+i)^2} + \cdots + \frac{M}{(1+i)^T}$$

where i is the monthly interest rate. Unlike other types of loans, refinancing is uncommon in the auto loan market, and borrowers have the flexibility to make early payments

Figure 5: Payment to Income for Population of Auto Borrowers

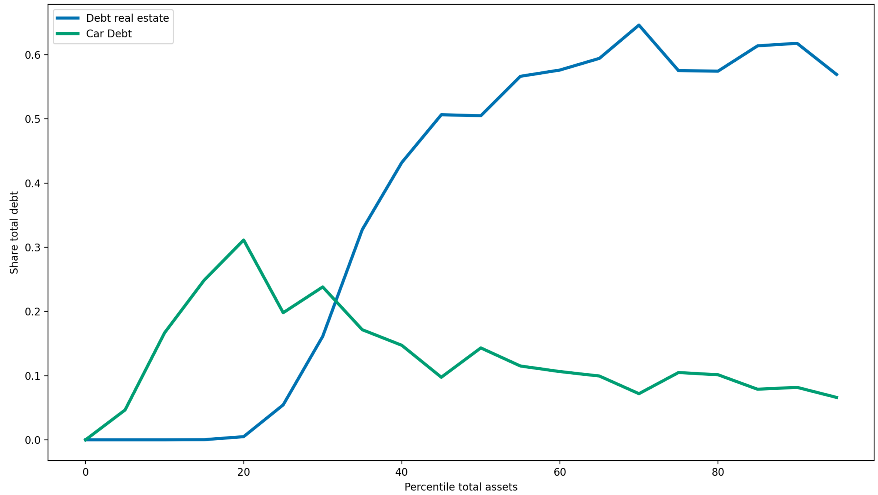


Source: Survey of Consumer Finances and author's calculations.

without incurring prepayment penalties. This feature provides households with the option to pay off the loan more quickly if their financial situation improves, allowing for more flexibility in managing debt.

Population auto borrowers Auto loan borrowers represent a unique segment of the borrowing population, distinct in both their income and wealth profiles. Figure 5 illustrates monthly payment-to-income ratios for auto borrowers across the income distribution, showing the relative burden of auto loan payments alongside housing costs (i.e., rent and mortgage payments). Notably, auto loan payments are a significant financial commitment, especially for low-income households. This suggests that liquidity considerations are a central factor for these households when determining the optimal term length of their loans. For low-income borrowers, term length decisions likely weigh heavily on balancing monthly affordability with longer-term financial stability.

Figure 6: Debt shares across asset distribution



Source: Survey of Consumer Finances and author's calculations.

Figure 6 further highlights the importance of auto loans for low-wealth households by comparing the share of total debt held in auto loans versus mortgages across the asset distribution. For low-asset borrowers, auto loans constitute a much larger proportion of their debt than mortgages, underscoring the critical role auto loans play in their financial portfolios. This stands in contrast to higher-asset households, where mortgages are often the predominant form of debt.

The key insights from these figures are threefold. First, auto debt holds particular significance for low-income and low-asset borrowers, who are more likely to face liquidity constraints and thus experience higher marginal propensities to consume (MPCs). Second, while much of the existing literature focuses on mortgages, auto debt is, in fact, the most critical debt category for these liquidity-constrained, high-MPC borrowers. Lastly, because auto loans represent a substantial and ongoing commitment within the monthly budget, it is not only the immediate liquidity constraints but also the ability to maintain liquidity throughout the life of the loan that influences borrowing decisions. This longer-term perspective on liquidity needs underscores the importance of term length choices in managing financial flexibility for these borrowers.

3 Data

Data Sources

This paper uses several sources of administrative and other data to study the auto market. Each is described in detail below.

Federal Reserve Bank of New York/Equifax Consumer Credit Panel (CCP). I make use of two datasets within the Equifax Consumer Credit Panel (CCP). The first is the 'main' CCP dataset, which covers a 5% national sample of anonymized borrowers quarterly from 2005 to 2021.[2] This dataset includes a wide array of information on borrowers' credit histories, such as account balances and performance. Critically, the available variables allow me to construct a proxy for liquidity constrainedness: credit availability. This information includes both credit limits and usage, which can be used to construct a measure of credit availability. I use distance between revolving credit balances and limits as my measure of credit availability.

The second is the 'auto tradeline' CCP dataset. This dataset covers the same 5% national sample, but includes information on up to four active auto loans per borrower.[3] This dataset includes information on the open date, initial loan amount and monthly payment, as well as evolving balances and deliquency status. Though the term length is observed for some loans, for most observations it must be recovered using the initial loan amount, balance, and monthly payment. Following An et al. (2020), I exclude all auto leases, as well as loans with non-monthly payments, missing monthly payment amounts, and missing or small initial loan amounts.

Other Data. I use Internal Revenue Service (IRS) zipcode-level data on annual adjusted gross income per tax return to get proxies for income at the zipcode level. Data on CPI

[2]While data is available from 1999 and for a few years after 2021, I use this more restricted sample for which zipcode level income data is also available.

[3]Here, active auto loans refers to auto loans with with both a positive balance and activity in the last six months. Less than 0.1% of borrowers have more than four active auto loans, according to Equifax.

and PCE deflators are from FRED.

Sample & Summary Statistics

I now provide details on the analysis sample, including summary statistics and sample construction. For my main analysis on the impact of liquidity constraints on term choice, I use both the 'main' and 'auto tradeline' CCP datasets, as well as the IRS zipcode-level income data, from 2005 – 2021. Data is available for all 50 states and the District of Columbia, but I drop any observation for which I do not have the zipcode income data.[4] I further restrict the sample to observations where individuals are between the ages of 20 and 80, and drop any observation for which I do not have a risk score (not this measure of risk score is *not* a FICO score).

From the 'main' CCP dataset, I use age (calculated from birth year) and risk score. I construct a measure of credit 'type', which is a flag for whether the individual has ever had a derogatory account or bankruptcy. I also construct a measure of credit availability, which I define as the percent limit left of revolving credit.[5] This is my measure of liquidity constrainedness.

I use the 'auto tradeline' dataset to construct a measure of term length for each auto loan. The 'auto tradeline' dataset allows me to both view information on the loan at origination, such as amount borrowed and monthly payment, as well as evolving balance over time. Using an ammortization function, I can back out the term length and interest rate of a simple-interest loan using the initial loan amount, evolving balance, and monthly payment. Since I can do this for each period I observe the evolving balance, I get a set of possible term lengths and interest rates for each loan. I take the median of these to get my measure of term length and interest rate. I exclude leases, and loans with non-monthly payment schedules, missing monthly payment amounts, and missing or small initial loan amounts, as I am unable to approximate term lengths/interest rates for these observa-

[4] I cut the availble CCP data at 2005 and 2021 due to the availability of IRS income data.
[5] i.e. credit usage = 1 - (revolving credit balance / credit limit).

tions. I further drop observations with approximated term lengths above 120 months, as these are exceedingly rare in datasets where term length is directly measured. I also drop observations where the approximated term length is very different across observed dates, as this is likely due to errors in the data. Finally, I drop observations with non-sensical interest rates (above 100% or below 0%). I merge the 'auto tradeline' data with the 'main' CCP data using the unique identifier of each individual. I merge the data using the quarter of the loan origination date.

The resulting sample includes 127,848 individuals and 307,909 auto loans from 2005Q1 to 2021Q4. Summary statistics can be found in Table 1.

Table 1: Summary Statistics

	Obs.	Mean	St. Dev.
Panel A: Individual characteristics			
Age	307,909	45.35	14.05
AGI per tax return	n/a	$ 69.44	$ 44.13
Risk score (not Fico)	307,909	704.88	88.67
Ever bankrupt	307,909	0.155	0.362
Ever derogatory or bankrupt	307,909	0.438	0.496
Panel B: Liquidity Characteristics			
Percent limit left	307,909	0.628	0.389
Percent limit left (if >0)	299,867	0.650	0.311
Panel C: Auto Loan Characteristics			
Term length	307,909	62.49	15.16
Term length above 60 mos.	307,909	0.602	0.489
Interest rate (APR)	307,909	0.067	3.35
Initial balance	307,909	$ 23.97	$ 13.01
Monthly payment	307,909	$ 0.442	$ 0.229

Notes: All dollar values are reported in 2017 dollars, and are reported in thousands.

4 Term choice and liquidity

I begin by empirically estimating the effect of liquidity on term choice in auto loans, using the CCP data. To do this, I rely on two key variables available via the CCP: the term length of the loan and the distance between the revolving credit balance and the credit limit. As discussed in subsection 3, the term length is inferred from the initial loan amount, monthly payment, and balance. My measure for liquidity constrainedness is based on current credit usage. Specifically, I will use percent limit left (i.e. one minus the balance over the limit). When this value is zero, it indicates that the individual is using all of their available credit, and so is likely to be liquidity constrained. Conversely, when this value is one, the individual is not using any available credit, and is likely to be far from their constraint.

Empirical strategy

Let y_{it} denote term length of auto loan, and c_{it} denote the share of credit used (e.g. the revolving credit balance over the credit limit). Let X_{it} be a vector of controls, including age, state by quarter fixed effects, zip-code income, risk score fixed effects, and a credit 'type' control.[6] The empirical model is then:

$$y_{it} = \alpha + \beta c_{it} + X_{it}\gamma + \epsilon_{it}. \tag{1}$$

In equation 1, β is the coefficient of interest, which captures the effect of an additional percentage point of credit availability on term length. If β is negative, this would suggest that individuals closer to their liquidity constraints are more likely to choose longer term lengths.

However, equation 1 cannot be estimated with OLS, as credit usage is likely to be endogenous. For instance, individuals who expect to be constrained may open additional accounts, raising their credit limits and biasing the coefficient on credit usage towards zero. To address this, I use an instrumental variables strategy. Following the work of Braxton et al. (2024), I use the age of oldest account as an instrument for credit usage. This

[6] This is a dummy variable which flags whether individuals have ever had a derogatory account or bankruptcy

strategy is adapted from the work in Gross and Souleles (2002), who show that credit card limits increase automatically as a function of the length of time since prior limit increases. Lenders often using arbitrary time thresholds for the increases such as 6 or 12 months. They use number of months since the last credit increase to instrument for credit limit increases.

Braxton et al. (2024) adjust this instrument for credit usage. They argue that not only are credit limits an arbitrary function of the age of the account, but the size of the credit limit revisions is a function of credit scores, which in turn are a function of account age. Because it is illegal under the Equal Credit Opportunity Act of 1974 to use physical age (and other demographic information) in credit scoring, most credit scoring companies use age of oldest account as an indirect measure of physical age. By using age of oldest account as an instrument, and controlling for physical age, credit limits can be treated as conditionally exogenous as they result from credit-scoring and arbitrary limit-increase timing.

Let Z_i be the age of oldest account. The updated empirical model is then:

$$y_{it} = \alpha + \beta \hat{c}_{it} + X_{it}\gamma + \epsilon_{it} \tag{2}$$

$$c_{it} = \alpha_1 + \beta_1 Z_i + X_{it}\gamma_1 + u_{it}, \tag{3}$$

where $\hat{c}_{it}$ is the predicted value of c_{it} from the second stage regression. For this instrument to be valid, it must satisfy both the relevance and exclusion restrictions. For relevance, the instrument must be correlated with credit usage. This occurs because of the relationship between age of oldest account and credit limits, a key part of credit usage. For exclusion, I need either strict or conditional exogeneity (i.e. $cov(Z_{i,t}, \epsilon_{i,t}) = 0$ or $cov(Z_{i,t}, \epsilon_{i,t}|X_{i,t}) = 0$). Here, I have conditional exogeneity. It is conditional for two reasons. First, as discussed above, I must control for physical age. Second, one may worry that since age of oldest account affects credit score, this in turn may impact other outcomes related to the term choice, such as the term-interest rate schedule. For this reason I also control directly for risk score using both direct risk score fixed effects, as well as risk score interacted with quarter and state fixed effects.

Results

OLS results. Table 2 presents the OLS regression results, showing the relationship between liquidity constraints and loan term choices. Columns (1) and (2) provide estimates for both term length as a continuous variable and as a binary indicator of whether the term length exceeds 60 months. The coefficients for both models are negative and statistically significant, suggesting that individuals with lower credit usage (those closer to their credit limit) are more likely to select longer loan terms, while individuals with higher available credit tend toward shorter terms. These findings align with the hypothesis that liquidity constraints are a key driver of loan term selection.

In Column (1), the analysis includes basic controls such as age, quarter-by-state fixed effects, and log real household income at the zip-code level, with errors clustered at the county level following Braxton et al. (2024). Column (2) extends these controls by incorporating risk score and borrower type indicators, including a "derogatory" flag and fixed effects for credit group by state and quarter. Credit groups are categorized based on a non-FICO risk score from the CCP dataset.

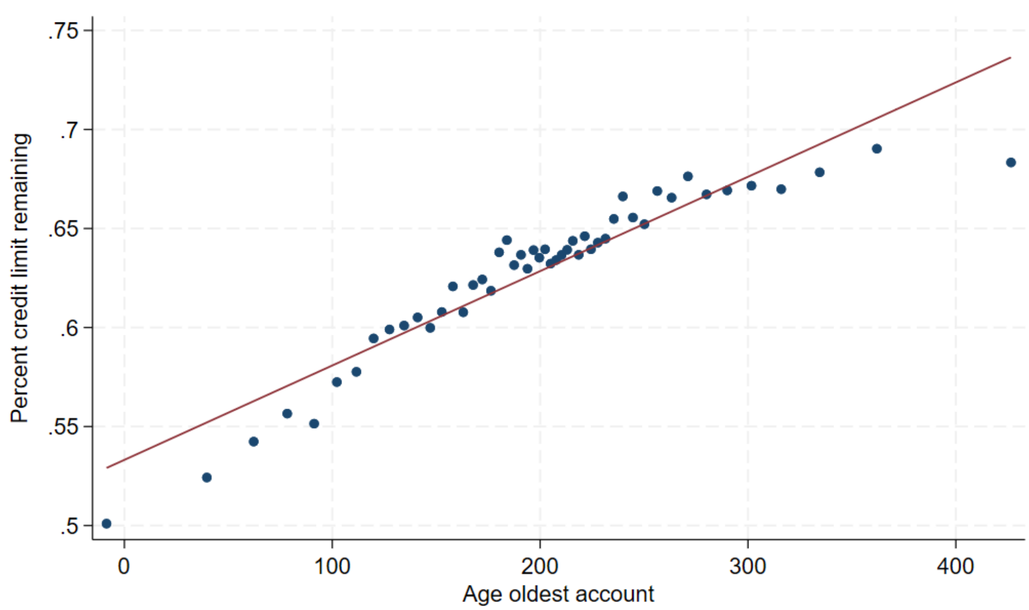
The interpretation of the coefficients indicates that moving from a fully constrained to a fully unconstrained credit limit is associated with a decrease in term length by either 1.625 or 2.939 months, depending on the control variables used. Similarly, the probability of having a loan term above 60 months decreases by 5.2 to 11.4 percentage points, depending on the model specification. While these coefficients are relatively modest, as a large shift in liquidity corresponds to less than a 3 percent change in average term length, they nevertheless reveal a consistent and meaningful relationship between liquidity and loan term choices. However, as previously discussed, these OLS results are expected to be biased downward due to potential endogeneity issues.

Table 2: OLS & IV Model Estimation Results

	Independent variable: Share limit remaining			
	OLS	OLS	IV	IV
	(1)	(2)	(3)	(4)
Term Length	–2.939***	–1.625***	–8.262***	–12.51***
	(0.386)	(0.302)	(0.931)	(1.925)
Term Length Above 60	–0.114***	–0.0519***	–0.401***	–0.310***
	(0.0143)	(0.0091)	(0.0311)	(0.0610)
Observations	307,906	306,165	307,906	306,165
Term mean	62.49	62.50	62.49	62.50
Indep. var. sd.	0.389	0.387	0.389	0.387
F-stat (term length)	–	–	220.2	126.4
F-stat (above 60)	–	–	238.2	131.3
Credit group & type FE	No	Yes	No	Yes

Notes: *** $p<0.001$, ** $p<0.01$, * $p<0.05$. Source: Federal Reserve Bank of New York's Consumer Credit Panel/Equifax data (CCP) with author's calculations.

Figure 7: IV First Stage



Source: Federal Reserve Bank of New York's Consumer Credit Panel/Equifax data (CCP) with author's calculations.

IV results. Before examining the IV results, it is essential to discuss the instrument employed. Figure 7 provides a binscatter plot of the first stage of the IV regression, where the instrument is residualized for the main set of controls. The first-stage coefficients, including those with various control specifications, can be found in Appendix Table [reference pending]. Additionally, the F-statistics for the first stage are consistently high across all specifications, supporting the strength of the instrument.

The IV regression results are presented in columns (3) and (4) of Table 2. Similar to the OLS results, the IV coefficients are negative and statistically significant at the 0.001 level, confirming the expected direction of the relationship. A potential concern with the IV approach is that the age of the oldest account might influence the interest rate-term schedule through its effect on the credit score. To address this, column (4) includes fixed effects for credit group by state and quarter, as well as borrower type. The results in columns (3) and (4) remain consistent, suggesting that the instrument's validity holds under these additional controls.

The IV estimates are larger than the OLS estimates, as anticipated. Specifically, the IV results indicate that moving from fully constrained to fully unconstrained credit usage decreases the loan term length by approximately 8.262 to 12.51 months, depending on the control variables included. This corresponds to a change of 13–20 percent of the mean term length, a notable increase over the OLS estimates. These larger estimates are consistent with previous findings, such as those in Braxton et al. (2024), where similar instruments produced elevated effect sizes.

For the term length indicator above 60 months, the coefficients are -0.401 and -0.310 in columns (3) and (4), respectively, aligning with the hypothesis that greater liquidity constraints lead to longer loan terms. In interpreting these results, it is worth noting that the analysis above compares the extremes of fully constrained versus fully unconstrained scenarios. However, even a one standard deviation change in liquidity is associated with a considerable reduction in average term length of 4.84 months, or 7.7 percent of the mean term length.

Overall, both the OLS and IV results support the hypothesis that liquidity constraints significantly influence loan term decisions. Specifically, these findings suggest that proximity to liquidity constraints plays a critical role in determining term length choices, with constrained individuals more likely to select extended loan terms to alleviate immediate financial pressures.

Robustness The results presented above are robust to various changes in specification, as detailed in the Appendix Section A. The first set of robustness checks re-estimates all models in the main table, restricting the sample to observations where the liquidity measure is positive. This adjustment addresses cases where individuals have balances above their revolving limit, resulting in negative liquidity measures. Such cases often arise when borrowers reach their credit limit and incur additional charges, such as interest or late fees. Re-estimating the models with this restricted sample yields consistent results, confirming that the main findings are not driven by observations with negative liquidity values.

A second set of robustness checks repeats the analysis using lagged values of the percent limit left as the liquidity measure. This approach tests the stability of the results by accounting for potential endogeneity issues in the liquidity measure. Once again, the results are consistent with the main findings, reinforcing the conclusion that liquidity constraints influence loan term choices.

Finally, I test robustness to alternative control specifications, including the use of different geographical controls, linear versus age fixed effects, and variations in credit group classifications for fixed effects. Across all of these alternative specifications, the results for both OLS and IV regressions remain negative and statistically significant, underscoring the stability of the relationship between liquidity constraints and loan term lengths.

Takeaways The empirical findings highlight the importance of liquidity as a key driver of term length choice in auto loans, with liquidity-constrained households more likely to select longer terms. This relationship underscores how heterogeneity in liquidity across households helps explain the variation observed in auto loan term lengths. However, as discussed in Section 2, there has not been a notable change in the proportion of liquidity-constrained households or in their average liquidity levels over time, suggesting that liquidity alone may not fully account for the observed trend toward longer term lengths in recent years.

To further explore these dynamics, the next section develops a simple model of term length choice. This model clarifies why liquidity-constrained households prioritize extended repayment terms and demonstrates how shifts in relative interest rates may contribute to understanding the time-series patterns of term lengths.

5 Simple Model

In this section, I present a simple model to illustrate how an individual might choose the speed of debt repayment. By focusing solely on term choice and excluding other decisions related to debt and durable goods, this model allows for a more isolated examination of the trade-offs associated with repayment speed. To capture the effects of liquidity constraints, the model retains other standard features of an Aiyagari-Bewley framework.

5.1 Simple Model Set-up

Utility. This model is based on a standard Aiyagari-Bewley household setup. Households have a constant relative risk aversion (CRRA) utility function with a momentary utility given by:

$$u(c) = \frac{(c)^{1-\sigma} - 1}{1 - \sigma}, \tag{4}$$

where $\sigma > 0$ and c denotes consumption. Households discount the future at rate β.

Income Process. Each household i receives labor income, denoted y_i, which follows a stochastic process:

$$\log y_i' = \rho_y \log y_i + \epsilon_i, \tag{5}$$

where $|\rho_y| < 1$, and ϵ_i is an idiosyncratic income shock drawn from a normal distribution with standard deviation σ_y.

Risk-Free Assets. Households can invest in one-period ahead risk-free assets at an interest rate r, subject to a non-negativity constraint $a' \geq 0$.

Term Choice. Households in this model begin with an inherited debt balance, b_0, which they must repay over time. They cannot choose the size of the debt or its purpose (such as a durable purchase), but they do choose a repayment speed, represented by a fraction $\mu \in (0, 1)$ of the principal to be repaid each period. A higher μ value corresponds to faster debt repayment, while a lower μ value indicates slower repayment. The repayment speed μ also determines the interest rate $r_b(\mu)$ on the outstanding debt, with slower repayment (lower μ) generally associated with a higher interest rate. Details of the calibration for $r_b(\mu)$ are discussed in Section 5.2.

Household Problem. The household's optimization problem is:

$$\max_{\mu, c_t, a_{t+1}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{6}$$

$$\text{s.t.} \quad c_t + a_{t+1} + (r_b(\mu) + \mu) b_t = y_t + (1 + r_a) a_t, \tag{7}$$

$$a_{t+1} \geq 0, \tag{8}$$

$$b_{t+1} = (1 - \mu) b_t = (1 - \mu)^{t+1} b_0. \tag{9}$$

5.2 Simple Calibration

The model is calibrated at a quarterly frequency to align with the quantitative analysis in later sections. Parameters for the household problem are chosen to match the quantitative model in Section 6.2. The interest rate function $r_b(\mu)$ is calibrated to align with empirical interest rates across different loan terms, as observed in Katcher et al. (2024). Specifically, term groups are divided into ranges such as less than 48 months, 48 months, 60 months, 72 months, and greater than 72 months.

To match these terms to the model's repayment rate μ, I use the duration of a financial asset, defined as the weighted average time until cash flows are received. For a given term length T with interest rate r, the model uses a corresponding μ such that the duration in the model matches that of a real-world loan term:

$$\sum_{t=1}^{\infty} \frac{t \cdot (r+\mu)(1-\mu)^{t-1}}{(1+r)^t} = \frac{1+i}{i+v}. \tag{10}$$

With the estimated values for term length and interest rates from Katcher et al. (2024), I fit a continuous function for $r_b(\mu)$ using:

$$r_b(\mu) = \frac{a}{(\mu - b)^c}. \tag{11}$$

Parameters $a = 0.0270$, $b = 0.0241$, and $c = 0.3347$ are estimated to match the observed data and provide a smooth, monotonic relationship that ensures $r_b(\mu)$ decreases with increasing μ. The fitted function, as shown in Figure 8, provides a close approximation to the empirical interest rate-term length pairs.

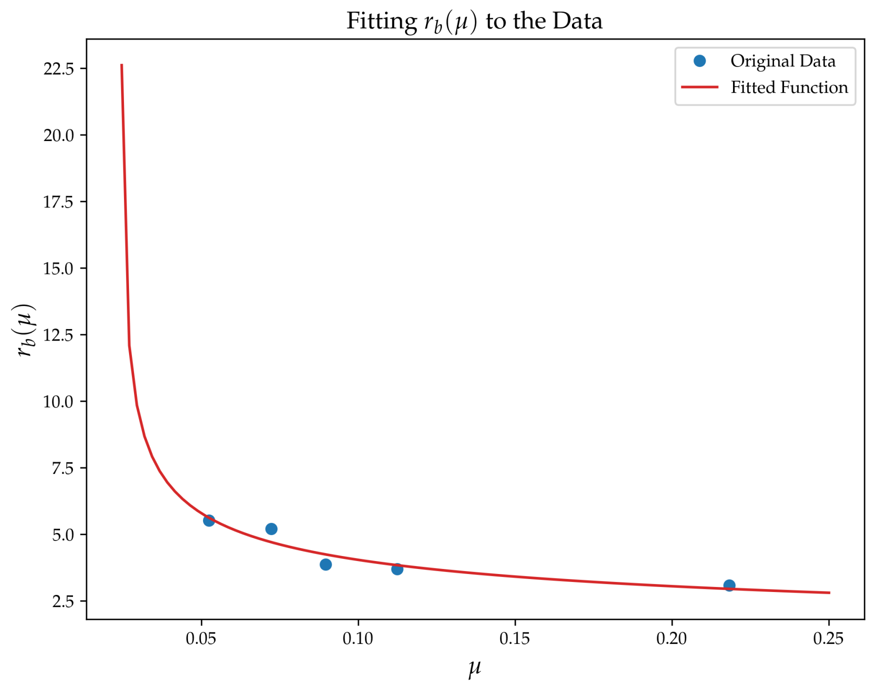


Figure 8: Interest Rate as a Function of Repayment Speed

5.3 Trade-offs of extending term lengths

In the household problem described, the decision of how quickly to repay debt reflects a fundamental trade-off. On one hand, extending the repayment horizon (i.e., choosing a lower μ) reduces monthly payments, making them more affordable in the short term. This effect stems from the fact that $r_b(\mu) + \mu$ is an increasing function of μ, meaning that smaller values of μ correspond to lower periodic payments. However, the cost of a lower μ is higher total interest paid over the life of the loan. There are two reasons for this increased cost: first, a longer repayment horizon results in a higher interest rate due to $r_b'(\mu) > 0$; second, by extending repayment, households carry larger debt balances forward, which increases the overall interest burden.

The first-order condition for the household's repayment speed, given in Equation 12, captures this trade-off:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u'(c_t)(r_b'(\mu)+1)(1-\mu)^t\right]=\mathbb{E}_0\left[\sum_{t=1}^{\infty}\beta^t u'(c_t)t(1-\mu)^{t-1}(r_b(\mu)+\mu)\right]. \qquad (12)$$

The left-hand side of the equation represents the marginal cost of increasing μ (i.e., opting for a shorter repayment horizon). This includes the factor $r_b'(\mu)+1$, which affects payments positively, thus making them more costly. However, choosing a higher μ reduces the future debt balance faster, which is represented on the right-hand side of the equation by the term $t(1-\mu)^{t-1}$.

The weighting of these costs and benefits depends on the household's relative marginal utility of consumption over time and across different states. Liquidity-constrained households, or those close to liquidity constraints, typically experience high marginal utility of consumption in the near term, as they have a strong preference for reducing monthly payments early on. This preference incentivizes choosing a longer repayment horizon (lower μ), allowing them to allocate more resources to consumption when liquidity is most needed. Importantly, it is not only those who are fully constrained who favor longer terms, but also those near the liquidity constraint, as they anticipate the possibility of becoming constrained in the near future. Thus, liquidity constraints shape repayment decisions throughout the loan's duration, not just at initiation.

This dynamic is reflected in the model's results, particularly the optimal choice of μ, illustrated in Figure 9. The heatmap shows the optimal μ for different initial values of income (y) and assets (a), with darker shades indicating longer repayment horizons (lower μ). The results demonstrate that individuals closest to the borrowing constraint (i.e., with $a_0 = 0$) select the longest repayment periods. Moreover, even those near the constraint value longer terms, as they anticipate the likelihood of tighter liquidity in the future. As households accumulate more assets and move further from the constraint, the costs of higher interest associated with longer terms outweigh the immediate benefit of lower payments, leading them to select shorter repayment horizons. This pattern holds consistently across various initial income levels, reinforcing the model's insight that liquidity

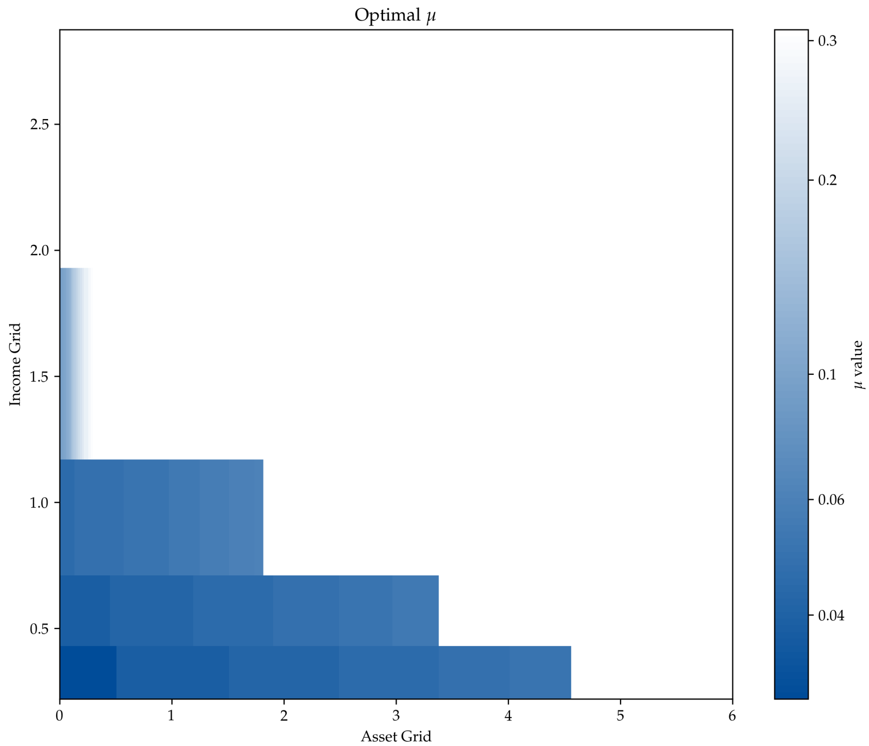


Figure 9: Optimal μ^* as a function of a_0 and y_0

constraints are a critical determinant of repayment behavior.

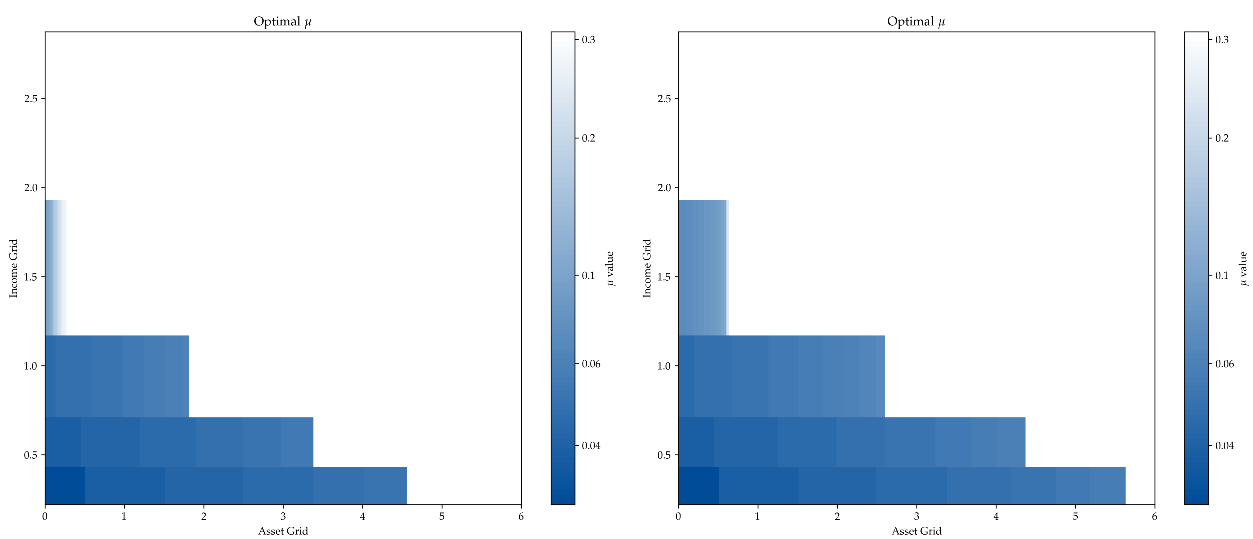
5.4 Rate gap and term length choice

While the empirical evidence and results from the simple model underscore the importance of liquidity in determining term lengths, it is evident that liquidity constraints alone do not explain the observed increase in term lengths over time. Although liquidity affects heterogeneity in term length choices among households, there have been no significant movements in the distribution of liquidity constraints over time, making it an unlikely driver for the trend toward longer term lengths.

The primary benefit of extending term lengths is reduced monthly payments, which is particularly advantageous for liquidity-constrained individuals. However, since this benefit remains constant over time, it does not account for the upward trend in term lengths. On the other hand, the cost associated with increasing term lengths arises from higher interest rates and the compounding of interest over a more extended repayment period. While the benefit side of term length decisions has remained stable, there have been shifts in the relative interest rates for auto debt compared to savings—changing the cost side of this decision.

This section examines how fluctuations in the interest rate gap between debt and savings influence the optimal repayment rate choice, μ, within the simple model. Figure 10 presents the results, with panel 10a illustrating the high rate gap scenario and panel 10b showing the low rate gap scenario. The results indicate that while a change in the relative cost does not alter the term decision for the most liquidity-constrained individuals (who were already choosing the maximum possible term length), it does affect the choices of the *near*-liquidity-constrained. These households not only select lower values of μ (indicating longer term lengths) but also show an expanded state-space where low μ values are optimal.

Ultimately, it is the interaction between the rate gap and liquidity considerations that influences term length choices, with near-liquidity-constrained individuals extending their term lengths in response to a higher rate gap.



(a) Optimal μ^*: high $r_b(\mu) - r$

(b) Optimal μ^*: low $r_b(\mu) - r$

Figure 10: Comparison of optimal μ^* across different $r_b(\mu) - r_a$

Limitations of the Simple Model While the simple model provides valuable insight into how liquidity constraints and the interest rate gap affect term length decisions, it has some limitations. Notably, it does not account for changes in the size of the car purchased or the size of the debt taken on—both of which are likely to vary with liquidity constraints. These factors, such as the scale of auto purchases and financing choices, can also be influenced by shifts in liquidity and rate gaps, potentially altering the decision to extend term lengths.

To ensure that the intuition developed in the simple model holds up in a more comprehensive framework, and to explore how debt levels and overall auto demand respond to rate gap shocks, the next section extends this model to a larger quantitative framework. This extended model incorporates a costly durable goods adjustment choice, allowing households to select both the size of their durable good and the corresponding debt level. This approach enables a deeper understanding of how households adjust both term lengths and durable goods choices in response to interest rate and liquidity constraints.

6 Quantitative Model

This section introduces the quantitative model, which builds on the simple framework by incorporating additional features of household durables' decision making. The additional features include durable adjustment, durable size, and the size of a durable loan. The extended framework is useful for several reasons. First, it allows me to confirm the results of the simple model hold in a richer, more standard, durables' model setting. Further, it allows me to look at two additional results. First, how the addition of term choice to a standard durables' model enhances our understanding of durables choice, and; Second, how the interest rate environment comparison done above affects other variables of interest such as durables' adjustment probability, demand, and debt.

The section is outlined as follows: section 6.1 outlines the problem set-up; section 6.2 discusses the model calibration, and section 6.3 discusses the model results. Information about the solution algorithm can be found in Section Appendix B.

6.1 Set-up

Households. The economy is populated by a continuum of infinitely lived households indexed by i. Households discount the future at rate β. The momentary utility of a household is given by:

$$u(c,d) = \frac{\left(c^{\alpha} d^{1-\alpha}\right)^{1-\sigma} - 1}{1-\sigma}. \tag{13}$$

Where $\sigma > 0$. c and d denote flexible consumption and the stock of durables, respectively.[7] Households cannot freely adjust their durable, but may always freely adjust the other consumption good.

Durable goods. Each period, households must choose whether to adjust their stock of durable goods. If adjusting, they sell their inherited stock of durables. Their inherited stock will be $(1-\delta)d$, where d is the previous period's durable stock and δ is the rate of depreciation. Revenues from the sale are $(1-f)p(1-\delta)d$. Here, p is the price of the durable good and f is a proportional adjustment cost which captures the loss from adjustment. Household purchase a new durable goods' stock d' at price p.

Households who are not adjusting exogenously maintain their durable stock via maintenance. For them, the rate of depreciation is $(1-\chi)\delta$, where they have purchased $\chi\delta$ additional units of the durable. The law of motion for the durable stock of non-adjusters is

$$d' = (1-(1-\chi)\delta)d. \tag{14}$$

Both adjusters and non-adjusters use their updated durable stock d' for the period's

[7] Here, I assume that the service from durables is equal to its stock. This is consistent with similar models in the literature.

utility. They also must pay proportional user fees for the car ν (think of this as gas/insurance for a car). Thus, total costs for adjusters are $(p+\nu)d' - (1-f)p(1-\delta)d$. And for non-adjusters $\nu d' + p\chi\delta d$.

Income process. The labour income of household i is given by:

$$\log y_i' = \rho_y \log y_i + \epsilon_i \tag{15}$$

Where $|\rho_y| < 1$ and ϵ_i is an idiosyncratic income shock drawn from a normal distribution with standard deviation σ_y.

Asset-backed loans. Households may take out loans with their durable purchases as collateral. As in the simple loan introduced in Section 5, these loans have fixed interest rates and are modeled as a proportional repayment plan.[8] That is, each period household pay back a fixed proportion of the remaining balance, μ. A household that inherits a loan level b will make a total payment $(r^b + \mu)b$, where r^b is the rate of interest on the loan. Their loan balances evolve according to:

$$b' = (1-\mu)b$$

Households adjusting their durable stock and/or loan balance may not save with the loan, and the loan balance is limited by a pre-specified loan-to-value cap, λ. That is $b' \in [0, \lambda p d']$. Here, p is the price of the durable and d' is the level of the durable stock.

Term choice. When households adjust their durable stock and/or loan balance, they also make a choice over their repayment speed/term choice, μ. This is modeled as a discrete choice over a menu of possible μ values, each with a corresponding interest rate.

[8]This creates a contrast with the 'true' repayment. Proportional repayment features steep principal payments early on in the contract and slow payments towards the end. True fixed monthly payment contracts will instead of a concave principal balance path.

Risk-free assets. Households can invest in one-period ahead risk-free assets. A household's position in these assets is denoted by a'. These assets pay interest rate r. Households are constrained to save in these assets, that is, $a' \geq 0$.

Value functions. Let s be the vector of household states $\{y, \mu, d, b, a\}$. The value functions associated with adjustment and non-adjustment are denoted by $V^a(s)$ and $V^n(s)$, respectively. The overall value function is

$$V(s) = \max\{V^a(s) - \kappa + \epsilon_a, V^n(s) + \epsilon_n\} \tag{16}$$

Where ϵ_n and ϵ_n are taste shocks drawn from an extreme value one distribution with scale parameter σ_a, and κ is a utility cost of adjustment. The taste shocks are included both for computational and realism purposes. Their computational value is to smooth out the value function around the discrete choice, allowing the use of derivatives in the solution algorithm. The taste shock allows me to match the fact that households with seemingly little incentives to frequently adjust their car.[9]

The probability a household chooses to adjust as a function of their state s is:

$$P(a|s) = \frac{exp\left(\frac{V^a(s)}{\sigma_a}\right)}{exp\left(\frac{V^a(s)}{\sigma_a}\right) + exp\left(\frac{V^n(s)}{\sigma_a}\right)} \tag{17}$$

The non-adjuster's consumption and savings decisions are characterized by the fol-

[9] In this way, it servces a simlar role to the depreciation shock in McKay and Wieland (2021).

lowing value function:

$$V^n(s) = \max_{c,a'} u(c,d') + \beta \mathbb{E}\left[V(s')|y\right] \tag{18}$$
$$\text{s.t. } c + a' + \nu d' + \chi \delta p d \leq y + (1+r)a - (r^b + \mu)b$$
$$d' = (1 - (1-\chi)\delta)d$$
$$b' = (1-\mu)b$$
$$a' \geq 0$$

The adjuster's value function will also be a maximum over the different discrete options for μ:

$$V^a(s) = \max\{V^a_{\mu_1}(s) + \epsilon_{\mu_1}, V^a_{\mu_2}(s) + \epsilon_{\mu_2}\} \tag{19}$$

Where, analogous to above, the ϵ are taste shocks are taste shocks drawn from an extreme value one distribution with scale parameter σ_μ An adjuster's decision, conditional on μ, is characterized by:

$$V^a_\mu(s) = \max_{c,d',b',a'} u(c,d') + \beta \mathbb{E}\left[V(s')|y\right] \tag{20}$$
$$\text{s.t. } c + a' + (\nu + p)d' - b' \leq y + (1+r)a + p(1-f)(1-\delta)d - (1+r^b)b$$
$$b' \in [0, \lambda p d']$$
$$a' \geq 0$$

6.2 Calibration

The following section outlines the targets for a quarterly calibration of the above model. Model parameters and targets are summarized in Table 3.

Set parameters. σ is set to its typical value of 2, so the EIS is 1/2.

In line with the calibration strategy in McKay and Wieland (2021), depreciation is set using the annual auto durable depreciation divided by the auto durable stock in the BEA Fixed Asset tables (averaged 1970 – 2019), which is 20%. Maintenance costs are set using NIPA data on maintenance expenditures. Maintenance expenditures are PCE on motor vehicles maintenance and repairs. Dividing by depreciation gives $\chi = 0.35$. The user fee, ν, is set to match operating costs on cars. Expenditure on motor vehicle fuels/lubricants/fluids are be 22% of the value of the stock of vehicles.

The persistence and standard deviation of the income process are 0.966 and 0.5117, respectively. These are based on estimates from Floden and Lindé (2001), and adjusted for quarterly calibration in McKay et al. (2016).

The real risk-free interest rate to be the average real federal funds rate from 1991–2007, which is 1.5%. Households have a menu of two term lengths, which are calibrated to match the duration of a 5 and a 7 year term length. The interest rates on car loans at these two maturities are taken from Geng et al. For details on how the μ and interest rate on cars is calibrated, refer to the details in Section 5.2. The maximum allowable ltv on car loans, λ, is set at 0.8.[10]

Calibrated parameters. There are five calibrate parameters in the model: the discount rate (β), the weight on durable goods in the utility function (α), the utility adjustment cost (κ), and the dispersion of the taste shocks for adjustment and term length (σ_a and σ_μ, respectively). β is calibrated to target a net liquid asset to aggregate income ratio of 0.26. α is calibrated to match a ratio of auto to flexible consumption spending of 17%. κ is calibrated to match an annual adjustment probability of 29.6%. σ_a is calibrated to match the relative marginal propensity to consume on durables relative to flexible consumption goods. σ_μ is calibrated to match the share of households choosing the high-μ term option in steady-state.

[10]This is consistent with other ltv limits and downpayment requires in the literature, such as McKay and Wieland (2021) and Beraja and Zorzi (2024).

Table 3: Model Calibration

Parameter	Description	Value	Source
Calibrated Parameters			
β	Discount factor	0.96	Liq Asset share = 0.26
α	Non-durable share	0.73	D spending/C = 0.17
κ	Adjustment cost	0.65	Adjustment probability = 29.6%
σ_a	Adjustment taste shock scale	0.08	See text
σ_μ	μ Taste shock scale	0.1	See text
Set Parameters			
σ	EIS	2	See text
δ	Annual depreciation rate	0.2	BEA Fixed Asset
χ	Exogenous maintenance share	0.35	See text
ν	User cost	0.22	See text
ρ	Income persistence	0.966	Floden and Lindé (2001)
σ_y	Income st. dev.	0.5117	Floden and Lindé (2001)
r	Risk-free real rate	0.015	Real Fed. Funds Rate
μ	Exogenous repayment share	[0.07,0.2]	See text
r_b	Real borrowing rate	[5.201,3.08]	Katcher et al. (2024)
λ	Borrowing limit	0.8	See text

6.3 Quantitative Model Results

Policy Functions To validate that the relationship between liquidity and term length choices in the quantitative model aligns with both the simple model and the empirical findings, we first examine the policy functions for households' durable good choices, leverage (measured as debt relative to the durable good's value), and the likelihood of opting for a longer term length (lower repayment rate, μ). Figure 11 provides a visual representation of these policy functions.

In line with the simple model, the quantitative model results show that households closer to their liquidity constraints are more likely to choose longer term lengths, with this probability diminishing as households move further away from liquidity constraints. This pattern is especially evident in the right-most panel of Figure 11, which indicates a higher preference for longer terms among liquidity-constrained households. Additionally, unlike the simple model, this extended framework allows us to see that households close to the liquidity constraint tend to choose smaller cars and maintain more leveraged loans. Despite selecting smaller durable goods, these households still favor longer repayment terms, reinforcing the importance of liquidity constraints in shaping term length preferences.

Moreover, consistent with empirical observations, households that select longer term lengths also tend to choose larger durable goods and carry higher levels of leverage, indicating that the decision to extend repayment horizons is closely tied to the overall financing strategy for durable purchases. Figure 12 further illustrates this connection, demonstrating how the liquidity constraint influences households' broader choices in debt and durable good consumption, with a clear preference for extended terms among those who seek higher leverage.

Rate Gap Comparison In this section, we compare two steady-state scenarios, each with a different calibration for the interest rate gap between the savings rate r and the loan interest rate function $r_b(\mu)$. Specifically, we introduce a shocked scenario in which the rate gap is reduced by one percentage point. This reduction in the rate gap leads to a modest decrease in the proportion of households choosing the high μ (faster repayment) option, with the share declining by about 1-3 percentage points.

As observed in the simple model, this shift is primarily driven by changes in the likelihood that near-liquidity-constrained households will choose the low-μ (longer term length) option. Households in the low-rate-gap environment also tend to take on more debt and exhibit higher demand for durable goods, indicating a link between the cost of borrowing and households' overall willingness to leverage. This dynamic suggests that the lower cost of extended repayment options increases households' willingness to commit to larger loans for durables.

Further analysis is planned to explore how the sensitivity of the share of households choosing low μ varies with adjustments to the taste parameter calibration, which could provide additional insights into the nuanced relationship between rate gaps, liquidity constraints, and term length choices.

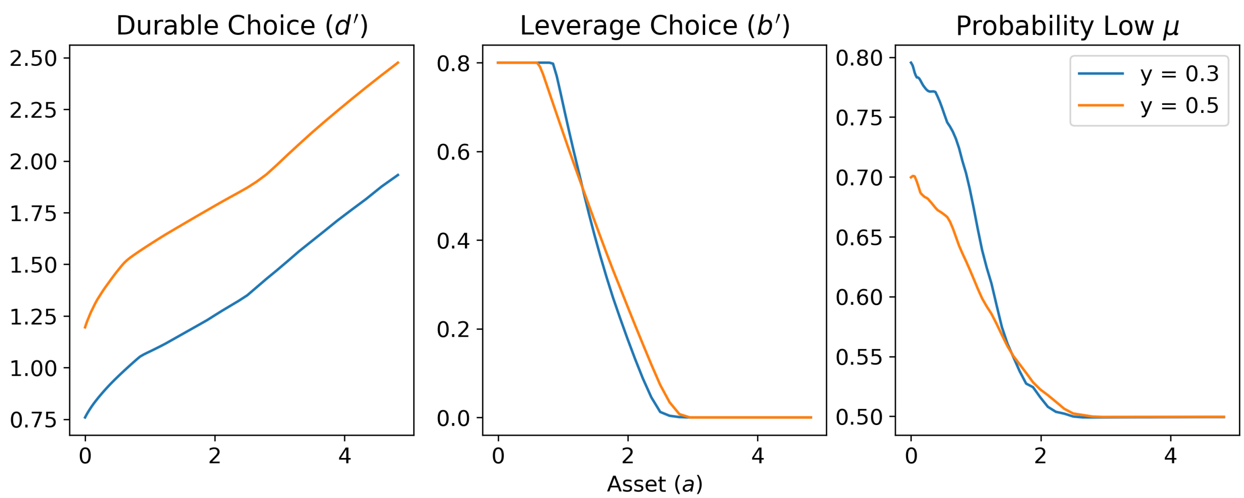


Figure 11: Quantitative Model Adjuster Policies Across Incomes

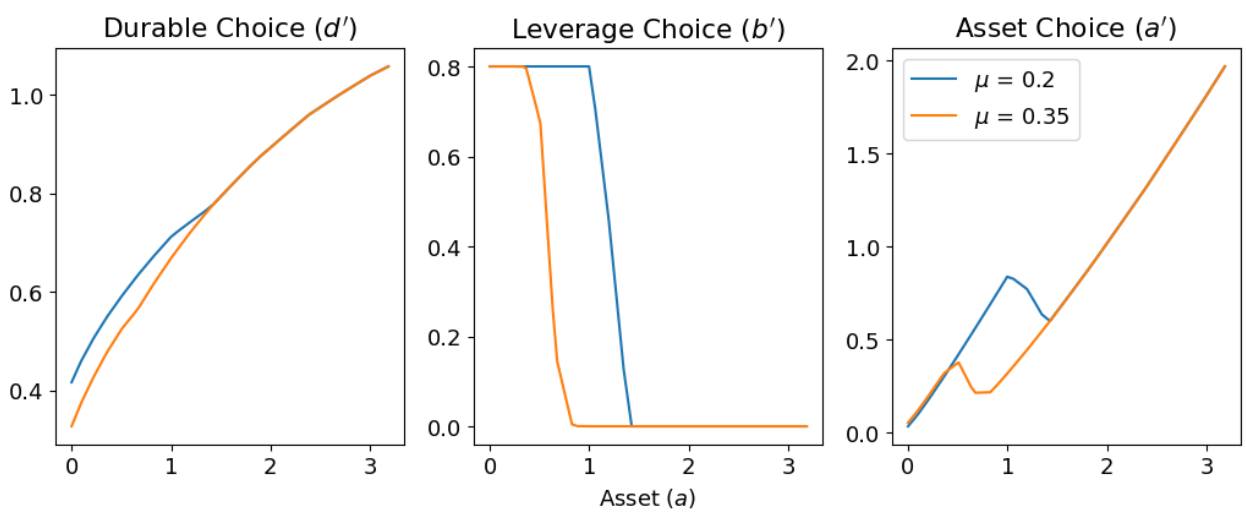


Figure 12: Quantitative Model Adjuster Policies Across μ Choices

7 Conclusion

This paper has explored the significant role that liquidity constraints play in determining loan term lengths, particularly within the context of auto loans, an asset class that is especially relevant to liquidity-constrained households. By examining both empirical evidence and modeling frameworks, this analysis deepens our understanding of how liquidity limitations shape financial decisions regarding debt repayment speeds and, ultimately, durable goods consumption. The findings contribute to a growing literature on consumer heterogeneity and liquidity constraints, emphasizing the need for models that capture the diverse borrowing needs and financial behaviors of households with varying liquidity levels.

The empirical work establishes a clear relationship between liquidity constraints and term length choice, demonstrating that liquidity-constrained households tend to select longer repayment horizons, effectively lowering their monthly payments. This choice aligns with the financial constraints that these households face, as they prioritize affordability in their monthly obligations. The analysis shows that liquidity considerations alone explain much of the cross-sectional variation in auto loan term lengths, as households closer to liquidity limits systematically choose longer terms. Additionally, by employing an instrumental variables (IV) approach to address endogeneity, the empirical analysis uncovers an even stronger relationship between liquidity and term length choice than suggested by ordinary least squares (OLS) estimates. Specifically, moving from fully constrained to unconstrained results in a substantial decrease in term length and a significant reduction in the likelihood of selecting a term over 60 months. These empirical insights highlight the critical role of liquidity constraints in driving term choices and emphasize the importance of addressing endogeneity when assessing the effects of financial limitations on borrowing behavior.

The simple model developed in this paper offers theoretical support for the empirical findings, focusing on the trade-offs households face between lower monthly payments and higher overall interest costs when choosing term lengths. In this model, liquidity-constrained households and those near liquidity constraints exhibit a strong preference for longer terms, as these terms help smooth consumption by reducing monthly payment commitments. The model confirms that it is not merely the presence of liquidity constraints but also the proximity to such constraints that influences term choices. Households on the edge of liquidity constraints tend to select extended repayment horizons, anticipating future liquidity needs. This model illustrates the inherent trade-offs faced by liquidity-constrained households and underscores how they navigate these constraints by adjusting their loan terms.

The quantitative model extends this analysis by incorporating a richer decision environment, where households choose both the size of their durable good and the level of debt financing required. This model further emphasizes the interaction between liquidity constraints and term length decisions, showing that households with limited liquidity also tend to choose smaller durable goods and more leveraged loans. However, even among those selecting smaller durables, the preference for longer term lengths persists, underscoring the strong influence of liquidity constraints on financing choices. The quantitative model aligns with empirical evidence indicating that households selecting longer terms are also those with higher leverage and larger durable goods purchases, thereby linking term choice to broader financial and consumption decisions.

In addition to exploring the role of liquidity constraints, the quantitative model examines how changes in the rate gap—the difference between auto loan interest rates and savings rates—affect term length decisions. Comparing two steady states with different rate gaps, the analysis reveals that a narrower rate gap leads to an increase in long-term loan choices, particularly among households close to liquidity constraints. These households respond to the lower relative cost of debt by extending their repayment horizons, taking on additional debt, and increasing their demand for durable goods. The rate gap thus serves as a powerful driver of term length choice, amplifying the impact of liquidity constraints on financial behavior. This finding highlights the importance of considering both liquidity constraints and borrowing costs in understanding consumer debt choices, as households weigh the relative cost of debt against their immediate financial needs.

While the simple and quantitative models provide valuable insights, they have certain limitations. The simple model abstracts away from variations in the size of the car or debt taken on, both of which are likely influenced by liquidity constraints as well. To ensure that the model's intuition is robust to these factors, and to better understand how term length extension impacts durable goods demand and adjustment probabilities, future work will expand the model to incorporate a costly durables adjustment choice. This extended model will allow households to decide both on the size of their durable good and the debt associated with it, offering a more comprehensive view of how liquidity constraints and term length flexibility shape broader financial decisions.

In sum, this paper highlights the complex interplay between liquidity constraints and loan term length choices, showing that these constraints are critical for understanding both cross-sectional and time-series variation in auto loan terms. However, liquidity considerations alone do not fully explain the observed trends over time; rather, the interaction between liquidity and rate gaps provides a fuller picture of term length dynamics. Future empirical work will further examine the impact of rate gaps on term lengths, going beyond the theoretical framework to quantify these effects in real-world data. Additionally, extending the model to analyze the contribution of term length choices to overall durable goods demand will provide deeper insights into how households manage debt commitments over time. Through this research, the paper contributes to a better understanding of the role of liquidity constraints in household financial behavior, with implications for both economic policy and consumer finance practices.

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A Empirical Section: Additional Tables

Table 4: IV First Stage Results

	Independent variable: Age Oldest Account	
	(1)	(2)
Share Limit Left	0.000477***	-0.000218 ***
	(0.0000140)	(0.0000124)
Observations	307,906	306,165
Age sd.	113.2	113.3
Credit group & type FE	No	Yes

Notes: *** p<0.001, ** p<0.01, * p<0.05. Source: Federal Reserve Bank of New York's Consumer Credit Panel/Equifax data (CCP) with author's calculations.

Table 5: OLS & IV Model Estimation Results: Alternative Controls

	Independent variable: Share limit remaining			
	OLS	OLS	IV	IV
	(1)	(2)	(3)	(4)
Term Length	–2.745***	–1.404***	–9.280***	–12.30***
	(0.339)	(0.229)	(0.974)	(2.116)
Term Length Above 60	–0.109***	–0.0477***	–0.440***	–0.313***
	(0.0125)	(0.00727)	(0.0323)	(0.06664)
Observations	276,477	274,610	276,477	274,610
Term mean	62.30	62.32	62.30	62.32
F-stat (term length)	–	–	227.7	140.3
F-stat (above 60)	–	–	266.9	145.4

Notes: *** p<0.001, ** p<0.01, * p<0.05. Source: Federal Reserve Bank of New York's Consumer Credit Panel/Equifax data (CCP) with author's calculations.

Table 6: OLS & IV Model Estimation Results: Lagged Limit Left

	Independent variable: Lagged share limit remaining			
	OLS	OLS	IV	IV
	(1)	(2)	(3)	(4)
Term Length	–8.360***	–11.70***	–10.62***	–9.359***
	(0.953)	(0.876)	(1.525)	(0.990)
Term Length Above 60	–0.406***	–0.529***	–0.266***	–0.445***
	(0.0312)	(0.0292)	(0.0500)	(0.0324)
Observations	307,909	299,864	298,311	276,480
Term mean	62.49	62.50	62.50	62.30
Indep. var. sd.	0.389	0.311	0.310	0.397
F-stat (term length)	–	–	414.7	223.6
F-stat (above 60)	–	–	346.0	269.7
Credit group & type FE	No	Yes	No	Yes

Notes: *** $p<0.001$, ** $p<0.01$, * $p<0.05$. Source: Federal Reserve Bank of New York's Consumer Credit Panel/Equifax data (CCP) with author's calculations.

B Solution Algorithm

The following section outlines the solution algorithm for the partial equilibrium problem outlined in Section 6.

General set-up Before getting to the 'main' algorithm, the above problem needs to be re-written to have households choosing a loan-to-value ratio, rather than the size of the loan directly. Without this modification, each household would need to have a grid depend on the size of their durable choice. Now, let $\tilde{b} = \frac{b}{pd}$. The re-written problem is then:

$$
\begin{aligned}
V^n(y,d,\tilde{b},a) = & \max_{c,a'} u(c,d') + \beta \mathbb{E}\left[V(y',d',\tilde{b}',a')|y\right] \qquad (21)\\
& \text{s.t. } c + a' + (\chi\delta p)d + \nu d' = y + (1+r)a - (r^b+\mu)\tilde{b}p^- d\\
& \quad a' \geq 0\\
& \quad \tilde{b}' = \frac{(1-\mu)}{(1-(1-\chi)\delta)}\frac{p^-}{p}\tilde{b}\\
& \quad d' = (1-(1-\chi)\delta)d\\
V^a_\mu(y,d,\tilde{b},a) = & \max_{\{c,d',\tilde{b}',a'\}} u(c,d') + \beta \mathbb{E}\left[V(y',d',\tilde{b}',a')|y\right] \qquad (22)\\
& \text{s.t. } c + a' + (p(1-\tilde{b}')+\nu)d' = y + (1+r)a + (1-f)(1-\delta)pd - (1+r^b)\tilde{b}p^- d\\
& \quad a' \geq 0\\
& \quad \tilde{b}' \in [0,\lambda].
\end{aligned}
$$

For notation convenience, hereafter b will refer to the loan-to-value. Further, V^a_μ will be denoted V^a, and V^a will be denoted V^A.

First-order and envelope conditions. For the non-adjustment problem, the first order condition with respect to a' is

$$[a']\colon u_c(c,d') \geq \beta \mathbb{E}\left[V_a(y',d',b',a')|y\right], \qquad (23)$$

and the envelope conditions are

$$
\begin{aligned}
V_a^n(y,d,b,a) &= (1+r)u_c\left(c,d'\right), && (24)\\
V_d^n(y,d,b,a) &= \Big(1-(1-\chi)\delta\Big)\Big(u_d(c,d') + \beta\mathbb{E}\left[V_d(y',d',b',a')|y\right]\Big)\\
&\quad - \Big(\chi\delta p + \nu(1-(1-\chi)\delta)\Big)u_c(c,d'), && (25)\\
V_b^n(y,d,b,a) &= \frac{(1-\mu)}{(1-(1-\chi)\delta)}\frac{p^-}{p}\beta\mathbb{E}\left[V_b(y',d',b',a')|y\right] - (r^b+\mu)u_c(c,d'). && (26)
\end{aligned}
$$

For the adjustment problem (conditioned on a choice of μ), the first order conditions for a', d', and b' are

$$
\begin{aligned}
&[a']\colon u_c(c,d') \geq \beta\mathbb{E}\left[V_a(y',d',b',a')|y\right], && (27)\\
&[d']\colon u_d(c,d') + \beta\mathbb{E}\left[V_d(y',d',b',a')|y\right] = \Big(p(1-b)+\nu\Big)u_c(c,d'), && (28)\\
&[b']\colon \begin{cases} pd'u_c(c,d') + \beta\mathbb{E}\left[V_b(y',d',b',a')|y\right] = 0 & \text{if } b' \in (0,\lambda)\\ pd'u_c(c,d') + \beta\mathbb{E}\left[V_b(y',d',b',a')|y\right] > 0 & \text{if } b' = \lambda\\ pd'u_c(c,d') + \beta\mathbb{E}\left[V_b(y',d',b',a')|y\right] < 0 & \text{if } b' = 0 \end{cases}, && (29)
\end{aligned}
$$

and the envelope conditions are

$$
\begin{aligned}
V_a^a(y,d,b,a) &= (1+r)u_c(c,d') && (30)\\
V_d^a(y,d,b,a) &= \Big((1-f)(1-\delta)p - (1+r^b)bp^-\Big)u_c(c,d') && (31)\\
V_b^a(y,d,b,a) &= -(1+r^b)p^- d u_c(c,d'). && (32)
\end{aligned}
$$

For the algorithm, I will further rewrite the adjustment problem. Since the adjustment problem re-optimizes a', b' and d', the household does not need to know the individual values for a, b and d, but rather the total resources they contribute to their budget. To save time in the computation, I drop dependence on these states, and instead write the value function in terms of assets-on-hand defined as

$$z = (1 + r)a + (1 - f)(1 - \delta)pd - (1 + r^b)\tilde{b}p^- d. \tag{33}$$

Note that it is relatively easy to move from the solution in terms of the state variables y and z, and the solution in terms of the original state variables d, b, and a. For each combination $\{d, b, a\}$ there is a corresponding z, for which the solution is known.

For the algorithm, it is also convenient to re-express the adjustment problem as three staged problems

$$V^a(y, z) = \max_{d'} \underbrace{\left\{ \max_{b' \in [0,\lambda]} \underbrace{\left\{ \max_{a' \geq 0, c} u(y + z - (p(1 - b') + \nu)d' - a', d') + \beta \mathbb{E}\left[V(y', d', b', a')|y\right] \right\}}_{V^{a,(1)}(y,z,d',b')} \right\}}_{V^{a,(2)}(y,z,d')} \tag{34}$$

The innermost problem will solve for c and a', taking decisions for d' and b' as given. This can be written as:

$$\begin{aligned} V^{a,(1)}(y, z, d', b') = \max_{c,a'} u(c, d') + \beta \mathbb{E}\left[V(y', d', b', a')|y\right] \\ \text{s.t. } c + a' = y + z - (p(1 - b') + \nu)d' \\ a' \geq 0. \end{aligned} \tag{35}$$

This has first order condition for a'

$$[a'] \colon u_c(c, d') \geq \beta \mathbb{E}\left[V_a(y', d', b', a')|y\right], \tag{36}$$

and envelope conditions

$$[a']\colon u_c(c,d') \geq \beta \mathbb{E}\left[V_a(y',d',b',a')|y\right], \tag{37}$$

Given the solution for the inner problem, the middle problem will solve for b' taking the decision for d' as given:

$$\begin{aligned} V^{a,(2)}(y,z,d') &= \max_{b'} V^{a,(1)} \\ \text{s.t. } b' &\in [0,\lambda]. \end{aligned} \tag{38}$$

Given the solution for the middle problem, the outer problem will take a decision for d':

$$V^{a}(y,z) = \max_{b'} V^{a,(2)} \tag{39}$$

Note that the first order conditions of the three stages collapse to the first order conditions written above. For convenience, I define the post-decision value function as $W(s) = \beta \mathbb{E}\left[V(s')|y\right]$.

Algorithm. I start with a guess for the value function and its partial derivatives, defined over a discretized grid. I then iterate backward until convergence. Π denotes the transition matrix of the exogenous income state, y.

0. **Preamble**. Create grids for a, b, d, and z, and discretize exogenous income process using the Rouwenhorst method.

1. **Initial guess**. Create guess for V (and also for V_d, V_b, and V_a.).

2. **Common** $\underline{y' \to y}$. By definition

$$W(y,d',b',a') = \beta\Pi V(y',d',b',a') \tag{40}$$

$$W_a(y,d',b',a') = \beta\Pi V_a(y',d',b',a') \tag{41}$$

$$W_d(y,d',b',a') = \beta\Pi V_d(y',d',b',a') \tag{42}$$

$$W_b(y,d',b',a') = \beta\Pi V_b(y',d',b',a') \tag{43}$$

3. **Non-adjustment problem**. Solve the non-adjustment problem, given guesses for V, V_d, V_b, and V_a. Note that I suppress the n superscript on all policy functions in this section for notation convenience. Thus, the $a'(y,d,b,a)$ that I find below is the a' choice *conditional* on the choice of not adjusting.

   (i) **Unconstrained** $\underline{a' \to a}$. Assume that the constraint on assets does not bind. Then (**??**) can be re-written to define c as

   $$c(y,d',b',a') = u_c^{-1}\Big(W_a(y,d',b',a'),d'\Big). \tag{44}$$

   Note that because the guess W_a is defined in terms of d' and b', this problem will initially be defined in terms of these rather than d and b, which are the state variables of the problem. There is a one to one mapping between the two. Using the budget constraint, I get $a^{endo}(y,d',b',a')$, which is the a that implies the household chooses $\{c(y,d',b',a'),a'\}$. This is:

   $$\begin{aligned} a^{endo}(y,d',b',a') = \frac{1}{1+r}\Big(c(y,d',b',a') + a' + p\chi\delta\frac{d'}{1-(1-\chi)\delta} \\ + \frac{p(\mu+r^b)}{1-\mu}b'd' + \nu d' - y\Big). \end{aligned} \tag{45}$$

   (ii) **Upper Envelope**. Normally, $a'(y,d',b',a)$ can be found via interpolation, putting $\big(a^{endo}(y,d',b',a'), \text{agrid}\big) \to (\text{agrid}, a'(y,d',b',a))$. However, because this problem features a discrete choice, there may be discontinuities in W_a that lead to non-unique solutions for the inversion.

To correct for this, I take the upper envelope of the solution. For each non-unique solution of the inversion, the upper envelope algorithm chooses the point for which the value function gives greater utility.

The algorithm starts by initializing an empty value function at minus infinity. Suppose $a(j)$ is j^{th} point on the grid for a. Fixing the other states, for each possible j, I check for all values of a' which $a^{endo} \in [a(j), a(j+1)]$. If the utility of the value function is greater than any past value, I replace this as the solution.

The algorithm gives both the policy function $a'(y, d', b', a)$ as well as an updated value function $V^n(y, d', b', a)$. At the end of this step, I can calculate $W_b(y, d', b', a)$ and $W_d(y, d', b', a)$ by interpolation, evaluating W_b and W_d at the policy function $a'(y, d', b', a)$. For full details on this algorithm, please see Bardóczy (2022).

(iii) **Update state** $\underline{d' \to d}$. Using interpolation, re-write $a'(y, d', b', a)$, $V^n(y, d', b', a)$, $W_b(y, d', b', a)$ and $W_d(y, d', b', a)$ in terms of d rather than d'. Do this by evaluating each at $d' = (1 - (1 - \chi)\delta)$.

(iv) **Update state** $\underline{b' \to b}$. Using interpolation, re-write $a'(y, d, b', a)$, $V^n(y, d, b', a)$, $W_b(y, d, b', a)$ and $W_d(y, d, b', a)$ in terms of b rather than b'. Do this by evaluating each at at $b' = \frac{(1-\mu)}{(1-(1-\chi)\delta)} \frac{p^-}{p} b$.

(v) **Update guesses**. First calculate $c(y, d, b, a)$ as

$$c(y, d, b, a) = y + (1 + r)a - (\mu + r^b)p^- bd - p\chi\delta d - \nu(1 - (1 - \chi)\delta)d - a'(y, d, b, a) \tag{46}$$

Note that there will be some states for which it is impossible to have positive consumption. In particular, states with very low assets but very high durable consumption. For these states, I force consumption to $1e - 9$.

Then, I can use the envelope conditions to update guesses as follows:

$$V_a^n(y,d,b,a) = (1+r)u_c\left(c(y,d,b,a),(1-(1-\chi)\delta)d\right), \tag{47}$$

$$\begin{aligned} V_d^n(y,d,b,a) = &\Big(1-(1-\chi)\delta\Big)\Big(u_d(c(y,d,b,a),(1-(1-\chi)\delta)d)+W_d(y,d,b,a)\Big) \\ &-\Big(\chi\delta p+\nu(1-(1-\chi)\delta)+(r^b+\mu)p^-b\Big)u_c(c,d'), \end{aligned} \tag{48}$$

$$\begin{aligned} V_b^n(y,d,b,a) = &\frac{(1-\mu)}{(1-(1-\chi)\delta)}\frac{p^-}{p}W_b(y,d,b,a) \\ &-(r^b+\mu)p^-du_c(c(y,d,b,a),(1-(1-\chi)\delta)d). \end{aligned} \tag{49}$$

Note that $V^n(y,d,b,a)$ was already obtained in previous steps, and does not need explicit updating in this step.

4. **Conditional Adjustment problem**. Solve the adjustment problem conditioned on a value for μ, given guesses for V, V_d, V_b, and V_a. Note that I suppress the a superscript on all policy functions in this section for notation convenience. Thus, the $a'(y,d,b,a)$ that I find below (and analogous policy functions for d' and b') is the a' choice *conditional* on both the choice of adjusting and a given choice of μ.

   (i) **Unconstrained** $\underline{a' \to z|d',b'}$. Here I solve the first order condition of the 'inner' maximization problem, where I solve for c and a' taking the choice for d' and b', as well as states y and z, as given.

   As above, I use an adjusted version of endogenous gridpoint method, that adds taking the upper envelope, to account for the discrete choice. I use the grid for a' and find the endogenous z households must have had in order to choose that a',

   Assume that the constraint on assets does not bind. Then (27) can be re-written to define c as

$$c(y,d',b',a') = u_c^{-1}\Big(W_a(y,d',b',a'),d'\Big). \tag{50}$$

   Using the budget constraint, I get $z^{endo}(y,d',b',a')$, which is the z that implies the household chooses $\{c(y,d',b',a'),a'\}$. This is

$$z^{endo}(y,d',b',a') = a' + c(y,d',b',a') + \big(p(1-b')+\nu\big)\,d' - y$$

(ii) **Upper envelope**. As above, I use the upper envelope to go from $\big(z^{endo}(y,d',b',a'),\text{agrid}\big) \rightarrow (agrid, a'(y,z,d',b'))$. As the steps are analogous, please refer to the above algorithm. The results of upper envelope step are a policy function $a'(y,z,d',b')$ and a value function, $V^a(y,z,d',b')$ which are in terms of the state variables $\{y,z\}$ and the choice variables $\{d',b'\}$. This steps also enforces the borrowing constraint on a'. At the end of this step, I can calculate $W_b(y,d',b',a)$ and $W_d(y,d',b',a)$ by interpolation, evaluating W_b and W_d at the policy function $a'(y,z,d',b')$.

(iii) **Choose** $b'|d'$. For the next two stages of the adjustment problem, I can no longer employ endogenous gridpoint method and must instead employ a root finding algorithm on the first order condition.

The first order condition for b' taking the choice of d' as given as well as the optimal solution for both c and a' is the envelope condition of the 'inner' problem with respect to b', equation

$$V^{a,(1)}_{b'} = pd'u_c(c(y,z,d',b'),d') + W_b(y,z,d',b') \tag{51}$$

There are three cases of solutions for the above equation. The first, if (**??**) is always positive, then b' takes on the corner solution $b' = \lambda$. The second, if (**??**) is always negative, b' takes the corner solution $b' = 0$. If the above equation crosses zero at least once, there is an interior solution. I use a root finding algorithm to find the grid points between which the equation crosses zero. If there are multiple inflection points, I use the value function to choose the true maximum and pick between multiple inflection points using the value function.

The root finding algorithm also exploits that for some state values of the problem (combinations $\{y, z, d'\}$), there is either no solution for b' such that cash on hand is strictly positive, or there is a further restricted set of b' values for which b' is positive. It searches over this restricted set, and sets b' to its maximum possible value for areas of the state space where there is no solution.

The resulting policy function is $b'(y, z, d')$. At the end of this step, I can calculate $V^a(y, z, d')$, $W_d(y, z, d')$ and $a'(y, z, d')$ by evaluating $V^a(y, z, d', b')$, $W_d(y, z, d', b')$ and $a'(y, z, d','b')$ at the policy function $b'(y, z, d')$. $c(y, z, d')$ can be calculated using the budget constraint

$$c(y, z, d') = y + z - a'(y, z, d') - (p(1 - b'(y, z, d') + \nu)d' \tag{52}$$

Where any negative value of c is replaced with $1e - 9$.

(iv) **Choose** d'. Like with the choice for b', I use a root finder over the first order condition. The first order condition for the outer problem is the envelope condition of the middle problem with respect to d', which in turn is the envelope condition of the inner problem with respect to d'. This is

$$V_{d'}^{a,(2)} = u_d(c(y, z, d'), d') + W_d(y, z, d') - \Big(p(1 - b) + \nu\Big)u_c(c(y, z, d'), d') \tag{53}$$

As above, I use a root finding algorithm to find all local maximum points, and use $V^a(y, z, d')$ to determine the global maximum. The root finding algorithm exploits that for each state value $\{y, z\}$ there are values of d' which push cash-on-hand negative and cannot be solutions.

The resulting policy function is $d'(y, z)$. At the end of this step, I can calculate $V^a(y, z)$, $a'(y, z)$, and $b'(y, z)$ by evaluating $V^a(y, z, d')$, $a'(y, z, d')$, ad $b'(y, z, d')$ at $d'(y, z)$.

5. **Interpolate** $z \rightarrow \{d, b, a\}$. Because I need my guesses for V, V_a, V_b, V_d to be in terms of the original state space $\{y, d, b, a\}$, I interpolate for each combination of $\{y, d, a\}$ to put all policy functions and guesses onto the original grid.

6. **Update guesses**. First calculate $c(y,d,b,a)$ as

$$\begin{aligned} c(y,d,b,a) &= y + (1+r)a + \Big(p(1-f)(1-\delta) - (1+r^b)bp^-\Big)d \\ &\quad - a'(y,d,b,a) - \Big(p(1-b'(y,d,b,a)) + \nu\Big)d'(y,d,b,a) \end{aligned} \tag{54}$$

As in the non-adjustment problem, there may be some states for which it is impossible to have positive consumption. For these states, I force consumption to $1e-9$.

Then, I can use the envelope conditions to update guesses as follows:

$$V_a^a(y,d,b,a) = (1+r)u_c(c(y,d,b,a), d'(y,d,b,a)) \tag{55}$$

$$V_d^a(y,d,b,a) = \Big((1-f)(1-\delta)p - (1+r^b)bp^-\Big)u_c(c(y,d,b,a), d'(y,d,b,a)) \tag{56}$$

$$V_b^a(y,d,b,a) = -(1+r^b)p^- d u_c(c(y,d,b,a), d'(y,d,b,a)). \tag{57}$$

7. **Discrete μ choice**. Given solutions for both the adjustment problem conditional on each value of μ, calculate the adjustment probabilities and solve the discrete choice problem. This will give an updated guess for V^A (V_a^A, V_b^A and V_d^A).

8. **Discrete adjustment choice**. Given solutions for both the adjustment and non-adjustment problem, calculate the adjustment probabilities and solve the discrete choice problem. This will give an updated guess for V (V_a, V_b and V_d).

   Go back to step 2, repeat until convergence.