EXERCISE 3

a) Y_i = number of breaks

$$x_{i1} = \begin{cases} 1 & \text{if material}_i = B \\ 0 & \text{if material}_i = A \end{cases}$$

$$x_{i2} = \begin{cases} 1 & \text{if tension}_i = M \\ 0 & \text{if tension}_i = \{L, H\} \end{cases} \qquad x_{i3} = \begin{cases} 1 & \text{if tension}_i = H \\ 0 & \text{if tension}_i = \{L, M\} \end{cases}$$

model: $Y_i \sim \text{Poisson}(\mu_i)$ indep for $i = 1, \dots, 54$

$\eta_i = \beta_1 + \beta_2 x_{i1} + \beta_3 x_{i2} + \beta_4 x_{i3}$

$\log(\mu_i) = \eta_i \iff \mu_i = e^{\eta_i}$

b) consider two experiments with same tension and different material

exp. i : material A

exp. j : material B

$\log \mu_i = \beta_1 + \cancel{\beta_2 \overset{0}{\overset{\shortparallel}{x_{i1}}}} + \beta_3 x_{i2} + \beta_4 x_{i3}$

$\log \mu_j = \beta_1 + \beta_2 \overset{1}{\overset{\shortparallel}{x_{j1}}} + \beta_3 x_{j2} + \beta_4 x_{j3}$

$\begin{pmatrix} x_{i2} = x_{j2} \\ x_{i3} = x_{j3} \end{pmatrix}$

$\Rightarrow \log \mu_j - \log \mu_i = \cancel{\beta_1} + \beta_2 + \cancel{\beta_3 x_{j2}} + \cancel{\beta_4 x_{j3}} - \cancel{\beta_1} - \cancel{\beta_3 x_{i2}} - \cancel{\beta_4 x_{i3}}$
$= \beta_2$

β_2 is the difference in the log of the expected counts if I consider material B instead of material A, for fixed level of tension.

c) model B

$Y_i \sim \text{Pois}(\mu) \qquad i = 1, \dots, n$

μ is common $\log(\mu) = \beta_1$

hence

$$\begin{cases} H_0: \beta_2 = \beta_3 = \beta_4 = 0 \\ H_1: \text{at least one is} \neq 0 \end{cases}$$

I use the LRT

$W = 2[\hat{\ell}(\text{model A}) - \tilde{\ell}(\text{model B})] \overset{H_0}{\sim} \chi^2_3$

since model B is the null model

$w^{obs} = D(\text{null}) - D(\text{model A}) = 297.37 - 210.39 = 86.98$

Since $w^{obs} > \chi^2_{3, 1-\alpha}$ for all usual α, I reject H_0

(Reject region is $R = (\chi^2_{3,1-\alpha} ; +\infty)$)