SOLUTION OF TRANSIENT VISCOELASTIC FLOW PROBLEMS APPROXIMATED BY A VMS STABILIZED FINITE ELEMENT FORMULATION USING TIME-DEPENDENT SUBRID-SCALES

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Recent studies indicate that classical residual-base stabilized methods for unsteady incompressible flows may experience difficulties when the time step is small in relation with the spatial grid size. Bochev et al. argue that spatial stabilization in conjunction with finite differencing in time implies destabilizing terms and that $\delta t > Ch^2$ (where δt is the time step size, C a positive constant and h the spatial grid size) is a sufficient condition to avoid these instabilities. Nevertheless, for anisotropic space-time discretizations, this inequality is not necessarily satisfied, and in fact complications in residual-based stabilized methods are reported. These problems can happen, for instance, when small time steps result from the necessity of accuracy to solve transient problems due to the presence of non-linear terms in the differential equations, a very common issue in viscoelastic flow formulations. In particular, the approximations used in Variational Multiscale (VMS) methods [1] usually neglect the time derivative of the sub-grid scales, resulting in the inequality $\delta t > Ch^2$ being required to obtain stable solutions. The aim of this work is the design of finite element stabilized techniques based on the Variational Multiscale (VMS) method that allow to compute time-dependent viscoelastic flow problems with high elasticity and considering an anisotropic space-time discretization. Although the main advantage is achieve stable solutions for anisotropic spacetime discretizations, other benefits related with elastic problems are proved in this study. In particular, the proposed methods are designed for the standard and logarithmic formulations [2] in order to deal with high Weissenberg number problems, ensuring stability in all cases. A comparison between formulations and stabilization techniques will be performed to demonstrate the efficiency of time-dependent sub-grid scales and the term-by-term methodologies.

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