ECCOMAS Congress, Oslo 2022

Stabilized mixed formulation for an implicit MPM for viscoplastic fluids by using a variational subgrid-scale framework

Laura Moreno, Alessandro Contri and Antonia Larese

June, 2022

Outline

Motivation

The climatic change is one of the main causes of **water-related extreme events**.

- What is the effect of extreme flow events on existing structures?
- How to properly design new structures?

Bridge failed during Washington **floods** in 2009

Landslide in Wenchuan area of China in 2019

In recent years, **Material Point Method** (MPM) receive attention in geotechnical engineering.

Goals of the work

A damaged bridge in Bad Neuenahr-Ahrweiler, Germany in 2021

Long term goal: Simulating the interaction between a flow-like debris and a structure.
A lot of ingredients are needed to achieve it:

- Implementation of different materials.
- Developing a FSI interaction schemes.
- **Simulating granular flows**.
- ...

Objectives

1. Development a stabilized formulation for **incompressible solid mechanics in mixed formulation** for simulating large deformation regimes.
2. Development of a **Newtonian law** for a linearized displacement-based formulation.

All implementations are performed using **KRATOS Multiphysics**, an open source and high performance simulation software.

Why using the Material Point Method (MPM)?

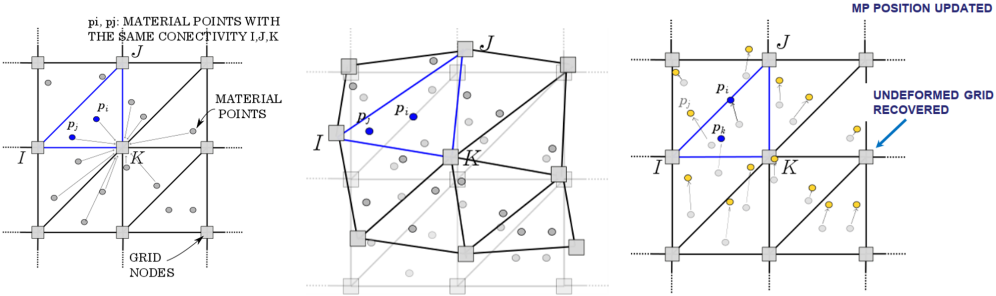
1. Classical FEM is not suitable for large deformations.
2. Discrete approaches are:
   - not suitable for large scale problems,
   - not easy to simulate complex material laws.

Hokkaido **landslide** after earthquake, Japan in 2018

MPM is a particle-in-cell (PIC) method which takes advantage of all the potential and the well-established knowledge reached in **FE technology**.

The Material Point Method

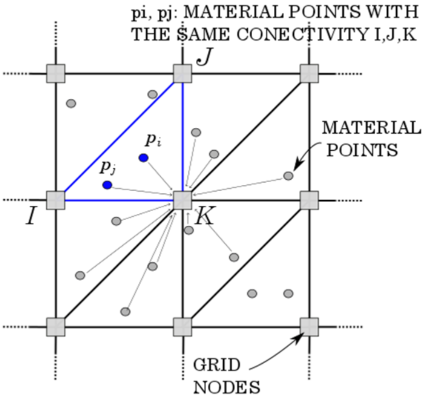
1. A background grid (fixed) used for the FEM solution system.
2. A collection of Material Points (MP) (Lagrangian).
   - Moving integration points.
   - MPs store historical information.
   - Every step is a FEM-like Lagrangian step.
   - The grid information is reset at the end of the step.



Stage 1. Initialization Stage 2. Calculation Stage 3. Convection

Stage 1. Initialization phase

Definition of the initial conditions on the FE grid's nodes.

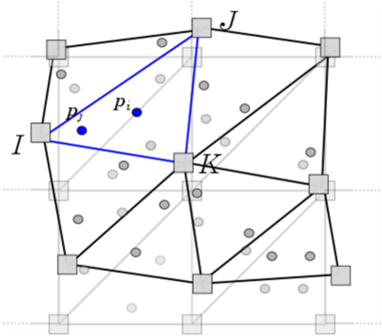


It is composed by:

1. Extrapolation on the nodes.
2. Prediction of nodal displacement, velocity and acceleration using a Newmark scheme.

Stage 2. Calculation phase

Update Lagrangian-FEM solution on the nodes of the background grid.



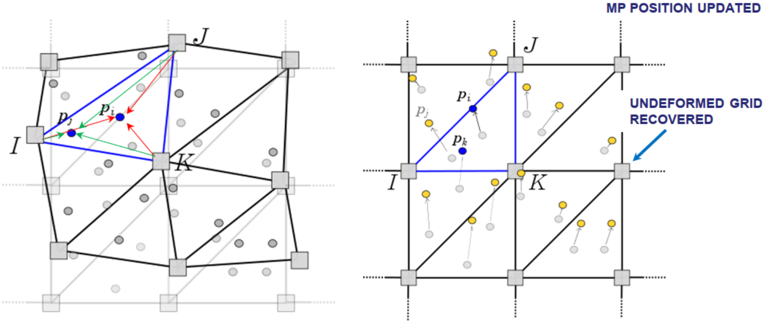
Ω: Initial configuration.
$\varphi(\Omega)$: Current configuration.

Calculation of the solution of an algebraic system of equations (implicit formulation) from t_n to t_{n+1} step. It is obtained following **the FE traditional steps**:

1. Construction of the elemental system. System is evaluated in the current configuration $\varphi(\Omega)$ locally.
2. Assembling of the elemental system.
3. Solving the system through an iterative scheme.

Stage 3. Convective phase

Information is interpolated and stored on the particles which are moved on the calculated positions.



1. Nodal information at time t_{n+1} are interpolated back onto the material points.
2. MP position is updated.
3. The undeformed FE grid is recovered.
4. The material points connectivities are updated (identify the element in which each MP falls).

Mixed formulation u-p for hyperelastic materials

A standard Galerkin displacement-based formulation (u-formulation) fails when Poisson's ratio $\nu \longrightarrow 0.5$ ($\kappa \longrightarrow \infty$) or when the plastic flow is constrained by the volume conservation condition.

Updated Lagrangian Formulation

$$\begin{cases} \rho \dfrac{\partial^2 \boldsymbol{u}}{\partial t^2} - \nabla \cdot \boldsymbol{\sigma} = \rho \boldsymbol{b} & \text{in } \varphi(\Omega)(t), \\ \dfrac{p}{\kappa} - \dfrac{dG}{dJ} = 0 & \text{in } \varphi(\Omega)(t), \\ \boldsymbol{u} = \bar{\boldsymbol{u}} & \text{on } \varphi(\partial\Omega_{\mathrm{D}})(t), \\ \boldsymbol{\sigma} \cdot \boldsymbol{n} = \bar{\boldsymbol{t}} & \text{on } \varphi(\partial\Omega_{\mathrm{N}})(t), \end{cases}$$

$$\boxed{\boldsymbol{\sigma} = \boldsymbol{\sigma}^{\mathrm{dev}} + p\mathbf{I}}$$

Cauchy stress tensor

Hyperelasticity

$$\mathbf{S} = 2\frac{\partial \Psi(\mathbf{C})}{\partial \mathbf{C}} \longrightarrow \boldsymbol{\sigma} = \frac{1}{J}\mathbf{F}\mathbf{S}\mathbf{F}^{\mathrm{T}}.$$

$$\boxed{\mathbf{S} := 2\frac{\partial \Psi_{\mathrm{dev}}}{\partial \mathbf{C}} + \frac{d\Psi_{\mathrm{vol}}}{dJ} J\mathbf{C}^{-1}}$$

- Deviatoric model: Neo-Hookean
  $\Psi_{\mathrm{dev}}(\bar{I}_1) = \dfrac{\mu}{2}(\bar{I}_1 - 3)$
- Volumetric model: Miehe et al.
  $\Psi_{\mathrm{vol}}(J) = \kappa(J - log(J) - 1)$
  $\longrightarrow \dfrac{\partial \Psi_{\mathrm{vol}}(J)}{\partial J} = \kappa\left(1 - \frac{1}{J}\right) = \kappa \dfrac{dG}{dJ}$

Variational form

The weak form problem consists in finding $\boldsymbol{U} = [\boldsymbol{u}, p] :]0, T[\longrightarrow \boldsymbol{\mathcal{W}}$, such that the initial conditions are satisfied and for all $\boldsymbol{V} = [\boldsymbol{w}, q] \in \boldsymbol{\mathcal{W}}_0$,

$$\boxed{\left(\mathcal{D}_t\left(\boldsymbol{U}\right), \boldsymbol{V}\right) + B\left(\boldsymbol{U}, \boldsymbol{V}\right) = \mathcal{F}\left(\boldsymbol{V}\right)}$$

where:

$$\left(\mathcal{D}_t\left(\boldsymbol{U}\right), \boldsymbol{V}\right) = \left(\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2}, \boldsymbol{w}\right)$$

$$B\left(\boldsymbol{U}, \boldsymbol{V}\right) = \left(\boldsymbol{\sigma}^{\mathrm{dev}}(\boldsymbol{u}), \nabla^{\mathrm{s}} \boldsymbol{w}\right) + \left(p\mathbf{I}, \nabla^{\mathrm{s}} \boldsymbol{w}\right) + \left(\frac{p}{\kappa}, q\right) - \left(\frac{dG}{dJ}, q\right)$$

$$\mathcal{F}\left(\boldsymbol{V}\right) = \langle \rho \boldsymbol{b}, \boldsymbol{w} \rangle + \langle \bar{\boldsymbol{t}}, \boldsymbol{w} \rangle_{\varphi(\partial\Omega_{\mathrm{N}})}$$

Linearization

- Newton-Raphson's iterative procedure.
- Taylor's series expansion evaluated at the last known equilibrium configuration $\boldsymbol{U}^* = [\boldsymbol{u}^*, p^*]$.
- B must be allows to compute a correction $\delta\boldsymbol{U} = [\delta\boldsymbol{u}, \delta p]$. Note that $\boldsymbol{U} = \delta\boldsymbol{U} + \boldsymbol{U}^*$.

$$B(\boldsymbol{U}, \boldsymbol{V}) \approx B(\boldsymbol{U}^*, \boldsymbol{V}) + B_{\mathrm{d}}(\delta\boldsymbol{U}, \boldsymbol{V}) + o(\delta\boldsymbol{u}) + o(\delta p)$$

Find the correction $\delta\boldsymbol{U} = [\delta\boldsymbol{u}, \delta p] :]0, T[\longrightarrow \boldsymbol{\mathcal{W}}_0$ such that

$$\boxed{(\mathcal{D}_t(\delta\boldsymbol{U}), \boldsymbol{V}) + B_{\mathrm{d}}(\delta\boldsymbol{U}, \boldsymbol{V}) = \mathcal{F}(\boldsymbol{V}) - (\mathcal{D}_t(\boldsymbol{U}^*), \boldsymbol{V}) - B(\boldsymbol{U}^*, \boldsymbol{V})}$$

$$\mathcal{D}_t(\delta\boldsymbol{U}) = \left[\rho\frac{\partial^2\delta\boldsymbol{u}}{\partial t^2}, 0\right]$$

$$\begin{aligned} B_{\mathrm{d}}(\delta\boldsymbol{U}, \boldsymbol{V}) = & (\nabla\delta\boldsymbol{u}\cdot(\boldsymbol{\sigma}(\boldsymbol{u}^*) + p^*\mathbf{I}), \nabla\boldsymbol{w}) \\ & + \left(\nabla^{\mathrm{s}}\boldsymbol{w}, \mathbb{C}^{\mathrm{dev}}(\boldsymbol{u}^*) + p^*(\mathbf{I}\otimes\mathbf{I} - 2\mathbb{I}) : \nabla^{\mathrm{s}}\delta\boldsymbol{u}\right) \\ & + (\delta p, \nabla\cdot\boldsymbol{w}) + \left(\frac{\delta p}{\kappa}, q\right) - (\mathrm{f}(J(\boldsymbol{u}^*))\nabla\cdot\delta\boldsymbol{u}, q) \end{aligned}$$

Mixed formulation for an incompressible Newtonian fluid material

It is a displacement based formulation, an it is assumed finite strains.

Updated Lagrangian Formulation

$$\begin{cases} \rho \dfrac{\partial^2 \boldsymbol{u}}{\partial t^2} - \nabla \cdot \boldsymbol{\sigma} = \rho \boldsymbol{b} & \text{in } \varphi(\Omega)(t), \\ 1 - \dfrac{1}{J} = 0 & \text{in } \varphi(\Omega)(t), \\ \boldsymbol{u} = \bar{\boldsymbol{u}} & \text{on } \varphi(\partial\Omega_{\mathrm{D}})(t), \\ \boldsymbol{\sigma} \cdot \boldsymbol{n} = \bar{\boldsymbol{t}} & \text{on } \varphi(\partial\Omega_{\mathrm{N}})(t), \end{cases}$$

$$\boxed{\boldsymbol{\sigma} = \underbrace{2\mu \nabla^s \boldsymbol{v}}_{\boldsymbol{\sigma}^{\mathrm{dev}}(\boldsymbol{u})} + p\mathbf{I}}$$

Cauchy stress tensor

Incompressible Newtonian fluid material

$$\boldsymbol{\sigma} = \frac{1}{J} \mathbf{F} \mathbf{S} \mathbf{F}^{\mathrm{T}}$$

$$\boxed{\mathbf{S} := J \left(\mu \mathbf{C}^{-1} \dot{\mathbf{C}} \mathbf{C}^{-1} \right) + pJ\mathbf{C}^{-1}}$$

where

$$\dot{\mathbf{C}} = \frac{D\mathbf{C}}{Dt} = 2\mathbf{F}^T \nabla^S \boldsymbol{v} \mathbf{F}; \; \mathbf{C} = \mathbf{F}^T \mathbf{F}$$

Variational form

The weak form problem consists in finding $\boldsymbol{U} = [\boldsymbol{u}, p] :]0, T[\longrightarrow \boldsymbol{\mathcal{W}}$, such that the initial conditions are satisfied and for all $\boldsymbol{V} = [\boldsymbol{w}, q] \in \boldsymbol{\mathcal{W}}_0$,

$$\boxed{\left(\mathcal{D}_t\left(\boldsymbol{U}\right), \boldsymbol{V}\right) + B\left(\boldsymbol{U}, \boldsymbol{V}\right) = \mathcal{F}\left(\boldsymbol{V}\right)}$$

where:

$$\left(\mathcal{D}_t\left(\boldsymbol{U}\right), \boldsymbol{V}\right) = \left(\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2}, \boldsymbol{w}\right)$$

$$B\left(\boldsymbol{U}, \boldsymbol{V}\right) = \left(\boldsymbol{\sigma}^{\mathrm{dev}}(\boldsymbol{u}), \nabla^{\mathrm{s}} \boldsymbol{w}\right) + \left(p\mathbf{I}, \nabla^{\mathrm{s}} \boldsymbol{w}\right) + \left(1 - \frac{1}{J}, q\right)$$

$$\mathcal{F}\left(\boldsymbol{V}\right) = \langle \rho \boldsymbol{b}, \boldsymbol{w} \rangle + \langle \bar{\boldsymbol{t}}, \boldsymbol{w} \rangle_{\varphi(\partial\Omega_{\mathrm{N}})}$$

Linearization

- Analogous to the hyperelastic materials.
- **Difficulty**: finding the spatial constitutive tensor $\mathbb{c}^{\mathrm{dev}}(\boldsymbol{u}^*)$.

Spatial and temporal discretizations

Spatial discretization of the linearized form.

Galerkin finite element approximation. It consists in finding $\delta \boldsymbol{U}_h \in \boldsymbol{\mathcal{W}}_{h,0}$ such that:

$$\underbrace{\left(\mathcal{D}_t\left(\delta \boldsymbol{U}_h\right), \boldsymbol{V}_h\right)}_{\text{Temporal terms}} + \underbrace{B_{\mathrm{d}}\left(\delta \boldsymbol{U}_h, \boldsymbol{V}_h\right)}_{\text{Semi-linear form}} = \mathcal{F}\left(\boldsymbol{V}_h\right) - \left(\mathcal{D}_t\left(\boldsymbol{U}_h^*\right), \boldsymbol{V}_h\right) - B\left(\boldsymbol{U}_h^*, \boldsymbol{V}_h\right)$$

for all $\boldsymbol{V}_h \in \boldsymbol{\mathcal{W}}_{h,0}$. To discretise the continuum body $\mathcal{B}$ by a set of n_p material points and Ω_p a finite volume of the body to each of those material points.

$$\mathcal{B} \approx \mathcal{B}_h = \bigcup_{p=1}^{n_p} \Omega_p.$$

Time discretization

Monolithic time discretization using a Newmark scheme.

Stabilization technique: Variational Multi-Scale (VMS) Methods

- **Objective**: to approximate the components of the continuous problem solution that cannot be resolved by the finite element mesh.
- Unknown splitting: $\delta \boldsymbol{U} = \underbrace{\delta \boldsymbol{U}_h}_{\in \boldsymbol{\mathcal{W}}_{h,0}} + \underbrace{\tilde{\boldsymbol{U}}}_{\in \tilde{\boldsymbol{\mathcal{W}}}}$ and $\boldsymbol{\mathcal{W}} = \boldsymbol{\mathcal{W}}_{h,0} \bigoplus \tilde{\boldsymbol{\mathcal{W}}}$.

$$\overbrace{\left(\mathcal{D}_t\left(\delta \boldsymbol{U}_h\right), \boldsymbol{V}_h\right) + B_{\mathrm{d}}\left(\delta \boldsymbol{U}_h, \boldsymbol{V}_h\right)}^{\text{Galerkin terms}} + \overbrace{\cancel{\langle \mathcal{D}_t(\tilde{\boldsymbol{U}}), \boldsymbol{V}_h \rangle} + \sum_K \langle \tilde{\boldsymbol{U}}, \overbrace{\mathcal{L}_{\mathrm{d}}^*(\boldsymbol{u}_h; \boldsymbol{V}_h)}^{\text{adjoint operator of } \mathcal{L}_{\mathrm{d}}} \rangle_K}^{\text{Stabilization terms}}$$
$$= \underbrace{\mathcal{F}\left(\boldsymbol{V}_h\right) - B\left(\boldsymbol{U}_h^*, \boldsymbol{V}_h\right) - \left(\mathcal{D}_t\left(\boldsymbol{U}_h^*\right), \boldsymbol{V}_h\right)}_{\text{Galerkin terms}}$$

$$\boxed{\cancel{\frac{\partial \tilde{\boldsymbol{U}}}{\partial t}} + \boldsymbol{\tau}^{-1} \tilde{\boldsymbol{U}} = \tilde{P} \underbrace{\left[\mathfrak{F} - \mathcal{L}\left(\boldsymbol{U}_h^*\right) - \mathcal{D}_t\left(\boldsymbol{U}_h^*\right) - \mathcal{D}_t\left(\delta \boldsymbol{U}_h\right) - \mathcal{L}_{\mathrm{d}}\left(\delta \boldsymbol{U}_h\right)\right]}_{\text{Residual from the linearization}}}$$

Sub-grid scale

Main operators of the problem

1. For computing the residual and obtaining $\tilde{\boldsymbol{U}}$:

$$\mathcal{L}(\boldsymbol{U}^*) := \begin{pmatrix} -\nabla\cdot\left(\boldsymbol{\sigma}^{\text{dev}}(\boldsymbol{u}^*) + p^*\mathbf{I}\right) \\ \dfrac{p^*}{\kappa} - \dfrac{dG}{dJ} \end{pmatrix}, \qquad \mathcal{D}_t(\boldsymbol{U}^*) := \begin{pmatrix} \rho\dfrac{\partial^2\boldsymbol{u}^*}{\partial t^2} \\ 0 \end{pmatrix}, \qquad \mathcal{D}_t(\delta\boldsymbol{U}) := \begin{pmatrix} \rho\dfrac{\partial^2\delta\boldsymbol{u}}{\partial t^2} \\ 0 \end{pmatrix},$$

$$\mathfrak{F} = [\rho\boldsymbol{b}, 0],$$

$$\mathcal{L}_{\text{d}}(\delta\boldsymbol{U}) := \begin{pmatrix} -\nabla\cdot\left(\nabla\delta\boldsymbol{u}\cdot\left(\cancel{\boldsymbol{\sigma}^{\text{dev}}(\boldsymbol{u}^*)} + p^*\mathbf{I}\right)\right) - \nabla\cdot\left(\left(\cancel{\mathbb{C}^{\text{dev}}(\boldsymbol{u}^*)} + p^*\left(\mathbf{I}\otimes\mathbf{I} - 2\mathbb{I}\right)\right) : \nabla^{\text{s}}\delta\boldsymbol{u}\right) - \nabla\delta p \\ \dfrac{\delta p}{\kappa} + \text{f}(J(\boldsymbol{u}^*))\nabla\cdot\delta\boldsymbol{u} \end{pmatrix}$$

2. The adjoint operator is:

$$\mathcal{L}_{\text{d}}^*(\boldsymbol{V}) :=$$

$$\begin{pmatrix} -\nabla\cdot\left(\nabla\boldsymbol{w}\cdot\left(\cancel{\boldsymbol{\sigma}^{\text{dev}}(\boldsymbol{u}^*)} + p^*\mathbf{I}\right)\right) - \nabla\cdot\left(\nabla^{\text{s}}\boldsymbol{w} : \left(\cancel{\mathbb{C}^{\text{dev}}(\boldsymbol{u}^*)} + p^*\left(\mathbf{I}\otimes\mathbf{I} - 2\mathbb{I}\right)\right) - \text{f}(J(\boldsymbol{u}^*))\nabla q\right) \\ \nabla\cdot\boldsymbol{w} + \dfrac{q}{\kappa} \end{pmatrix}$$

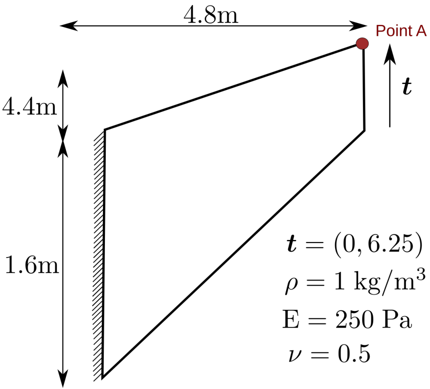
Final expression for linear elements

$$\overbrace{\left(\mathcal{D}_t\left(\delta \boldsymbol{U}_h\right), \boldsymbol{V}_h\right) + B_{\mathrm{d}}\left(\delta \boldsymbol{U}_h, \boldsymbol{V}_h\right)}^{\text{Galerkin terms}} + \overbrace{S_1\left(\boldsymbol{U}_h^*; \delta \boldsymbol{U}_h, \boldsymbol{V}_h\right) + S_2\left(\boldsymbol{U}_h^*; \delta \boldsymbol{U}_h, \boldsymbol{V}_h\right)}^{\textbf{Stabilization terms}}$$
$$= \underbrace{\mathcal{F}\left(\boldsymbol{V}_h\right) - B\left(\boldsymbol{U}_h^*, \boldsymbol{V}_h\right) - \left(\mathcal{D}_t\left(\boldsymbol{U}_h^*\right), \boldsymbol{V}_h\right)}_{\text{Galerkin terms}} + \underbrace{R_1\left(\boldsymbol{U}_h^*, \boldsymbol{V}_h\right) + R_2\left(\boldsymbol{U}_h^*, \boldsymbol{V}_h\right)}_{\textbf{Stabilization terms}}$$

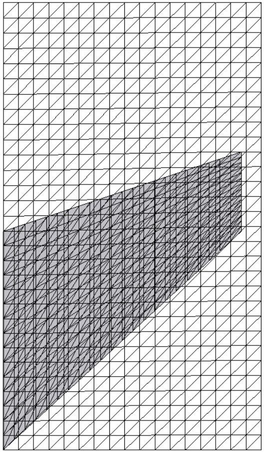
$$S_1\left(\boldsymbol{U}_h^*; \delta \boldsymbol{U}_h, \boldsymbol{V}_h\right) =$$
$$\sum_K \tau_1 \left\langle \tilde{P}\left[-\rho \frac{\partial^2 \delta \boldsymbol{u}_h}{\partial t^2} + \left(\nabla \delta \boldsymbol{u}_h \cdot \nabla p_h^*\right) + \left(\nabla \cdot p_h^* \left(\mathbf{I} \otimes \mathbf{I} - 2\mathbb{I}\right) : \nabla^{\mathrm{s}} \delta \boldsymbol{u}_h\right) + \nabla \delta p_h\right], \right.$$
$$\left. - \left(\nabla \boldsymbol{w}_h \cdot \nabla p_h^*\right) - \left(\nabla^{\mathrm{s}} \boldsymbol{w}_h : \nabla \cdot p_h^* \left(\mathbf{I} \otimes \mathbf{I} - 2\mathbb{I}\right)\right) - \mathrm{f}(J(\boldsymbol{u}_h^*)) \nabla q_h \right\rangle_K$$
$$S_2\left(\boldsymbol{U}_h^*; \delta \boldsymbol{U}_h, \boldsymbol{V}_h\right) = \sum_K \tau_2 \left\langle \tilde{P}\left[-\frac{\delta p_h}{\kappa} + \mathrm{f}(J(\boldsymbol{u}_h^*)) \nabla \cdot \delta \boldsymbol{u}_h\right], \nabla \cdot \boldsymbol{w}_h + \frac{q_h}{\kappa} \right\rangle_K$$

- $\tilde{P}$ is the L^2-projection onto the space of sub-grid scales,
- $\boldsymbol{\tau}$ is a matrix computed within each element

Cook's membrane. Main features.

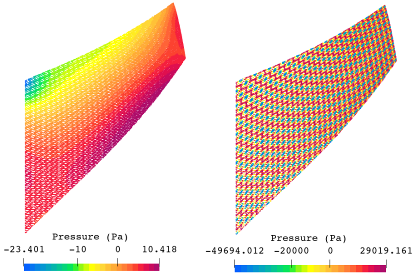


$\boldsymbol{t} = (0, 6.25)$ Pa
$\rho = 1$ kg/m^3
E $= 250$ Pa
$\nu = 0.5$



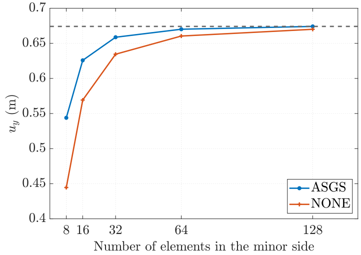
Temporal discretization: $\delta t = 0.0025$ s
Spatial discretization: $h = 0.6, 0.3, 0.15, 0.75, 0.325$ m with 16 material points per element.

Cook's membrane: static and dynamic cases.

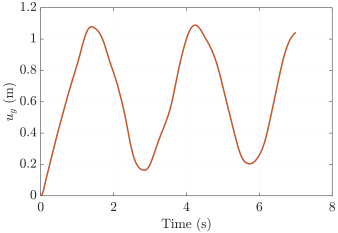


Pressure using stabilization VS without stabilization.

- It is crucial to use a stabilization method to obtain **a pressure distribution without jumps** of pressure values between elements.
- Dynamic case has been impossible to run without stabilization.

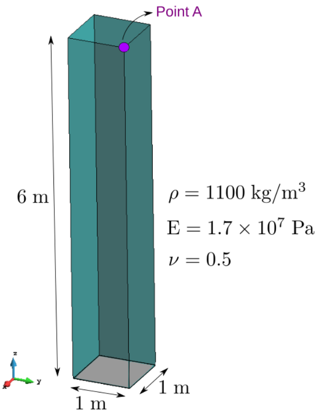


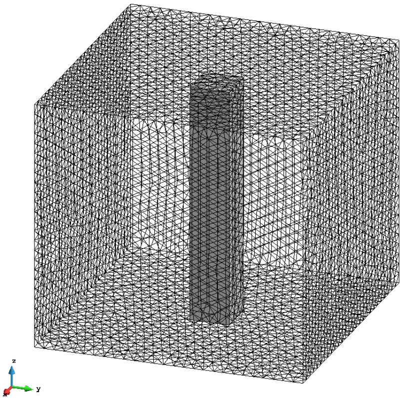
Mesh convergence. Static case.



Evolution of the displacement in time. Dynamic case.

Bending beam problem. Main features

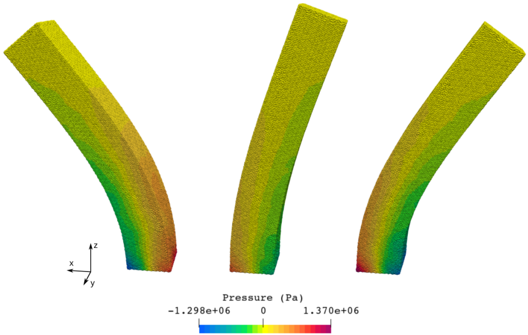




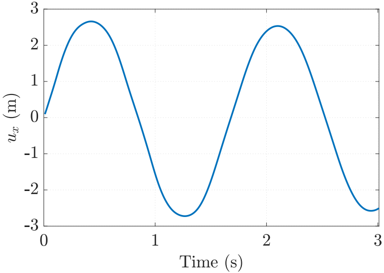
$$\boldsymbol{v}^0(x, y, z) = \frac{5}{3}(z, 0, 0)^T \text{ m/s}$$

Temporal discretization: $\delta t = 0.01$ s, $T_{\mathrm{fin}} = 3$ s,
Spatial discretization: mesh size $h = 0.2$ m, and 16 material points per element in the body.

Bending beam problem. Results



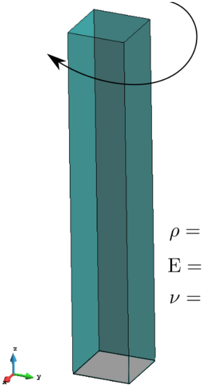
Bending column at $t = 0.5$ s, $t = 1$ s and $t = 1.25$ s.



Evolution of x-displacement in time of the point A.

Deformation is well reproduced. Pressure maximum and minimum peaks are located on the bottom of the beam as it is expected.

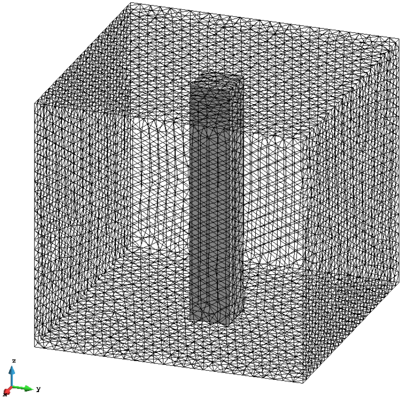
Twisting column problem. Main features



$\rho = 1100 \text{ kg/m}^3$

$\mathrm{E} = 1.7 \times 10^7 \text{ Pa}$

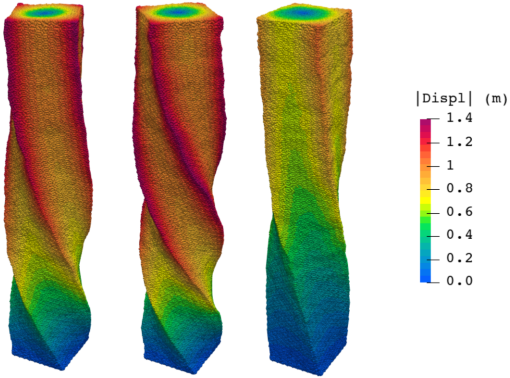
$\nu = 0.5$



$$\boldsymbol{v}^0(x, y, z) = 105 \sin\left(\frac{\pi z}{12}\right)(y, -x, 0)^T \text{ m/s}$$

Temporal discretization: $\delta t = 0.001$ s, $T_{\text{fin}} = 0.5$ s,
Spatial discretization: mesh size $h = 0.2$ m, and 16 material points per element in the body.

Twisting column problem



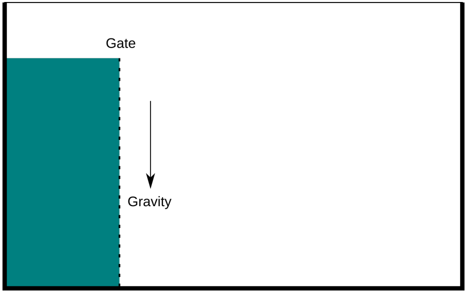
Twisting column at $t = 0.05$ s, $t = 0.1$ s and $t = 0.2$ s.

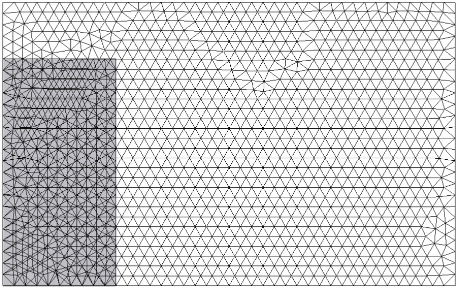
- Displacement field is well captured.
- Simulation is done without remeshing techniques, ALE or level set.

Conclusion

ASGS stabilization method is **more robust than other methods**, such as Polynomial Pressure Method (PPP). Moreover, the later two examples were no able to be computed with PPP.

Dam break problem. Main features

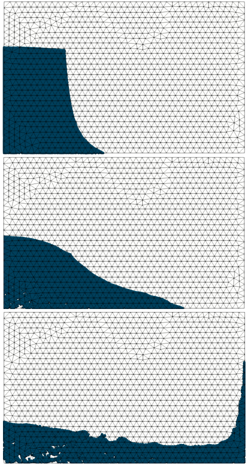




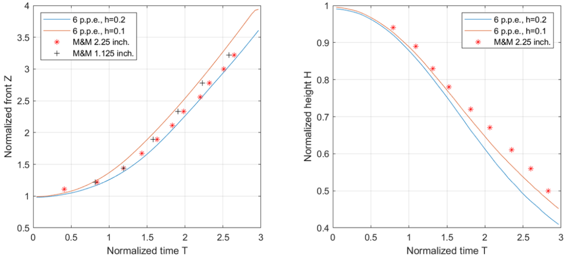
Spatial discretization: Mesh size $h = 0.1$ m and $h = 0.2$ m, and 6 material points per element.

Temporal discretization: $\delta t = 0.001 \text{ s}$, $T_{\text{fin}} = 0.9 \text{ s}$.

Dam break problem. Results



Dam break test at t=0.25, 0.5 and 0.85 s.



Left: distance of the surge front from axis of plane of symmetry; Right: height of the residual column.

- It is one of the ultimate performed tests in free-surface fluid dynamics.
- **Volumetric locking** has been efficiently avoided.
- The results are in agreement with **experimental evidence**.

Conclusions

- VMS-type stabilization have been developed and implemented for a solid mechanics framework and the Material Point Method.
- **ASGS stabilization** is more robust for simulating incompressible hyperelastic materials in comparison with others stabilization techniques such as the Polynomial Pressure Projection.
- This stabilization is able to compute very challenging **large deformation regimes** using the Material Point Method.
- The development of a **Newtonian law** for an Updated Lagrangian linearized **displacement-based formulation**. It is checked for the Material Point Method comparing with an experimental test.

References

1. Iaconeta, I. et al. A stabilized mixed implicit material point method for non-linear incompressible solid mechanics. *Computational Mechanics* 63.6 (2019): 1243-1260.
2. Contri, A. An Updated Lagrangian displacement-based formulation for free surface incompressible fluids using MPM. Master Thesis.
3. Castañar I., Baiges J. and Codina R. A stabilized mixed finite element approximation for incompressible finite strain solid dynamics using a total Lagrangian formulation. *Computer Methods in Applied Mechanics and Engineering* 368 (2020): 113164.
4. Hassan, Osama I. et al. An upwind vertex centred finite volume algorithm for nearly and truly incompressible explicit fast solid dynamic applications: Total and Updated Lagrangian formulations. *Journal of Computational Physics*: X 3 (2019): 100025.
5. Cervera, M. et al. Mixed stabilized finite element methods in nonlinear solid mechanics. Part III: Compressible and incompressible plasticity. *Computer Methods in Applied Mechanics and Engineering* 285 (2015): 752-775.
6. Martin J. C., Moyce W. J., Penney W. G., Price A. T. Thornhill C. K. (1952). Part IV. An experimental study of the collapse of liquid columns on a rigid horizontal plane. Philosophical Transactions of the Royal Society of London. Series A, *Mathematical and Physical Sciences*, 244(882), 312-324.

UNIVERSITÀ DEGLI STUDI DI PADOVA

June 2022

Thank you for your attention!!

Laura Moreno Martínez