

Apuntes del For

Demuestre que es un teorema en Hoare:

```
⊢ {N ≥ 1}
BEGIN
  PROD = 0;
  FOR X := 1 UNTIL N DO PROD := PROD + M
END
{PROD = M × N}
```

Solución

1) Demuestre que es un teorema en Hoare

E_1 : 1

E_2 : N

V: X

$P[E_2 + 1/V]$:	$P[N + 1/X]$: PROD = N * M
$P[V + 1/V]$:	$P[X + 1/X]$: PROD = X * M
$P:P[V]$:	$P[X]$: PROD = (X - 1) * M
$P[E_1/V]$:	$P[1/X]$: PROD = (1 - 1) * M

Verificando mediante el Ax3

$P:P[V]$:	$P[X]$: PROD = (X - 1) * M
	$P[X + 1/X]$: PROD = X * M
	$P[N + 1/X]$: PROD = N * M
	$P[1/X]$: PROD = (1 - 1) * M

1) $\vdash \{PROD + M = X * M\} PROD := PROD + M \{PROD = X * M\}$ Ax3

2) $\vdash (PROD = (X - 1) * M) \wedge (1 \leq X) \wedge (X \leq N) \rightarrow (PROD + M = X * M)$

$\vdash (PROD = (X - 1) * M) \wedge (1 \leq X) \wedge (X \leq N) \rightarrow (PROD = X * M - M)$

$\vdash (PROD = (X - 1) * M) \wedge (1 \leq X) \wedge (X \leq N) \rightarrow (PROD = M * (X - 1))$

$\vdash (P \wedge Q \wedge R) \rightarrow P$ por relación lógica

3) $\vdash \{PROD = (X - 1) * M\} \wedge (1 \leq X) \wedge (X \leq N) \{PROD := PROD + M\} \{PROD = X * M\}$ Por RFP (2 en (1

4) $\vdash \{PROD = (1 - 1) * M \wedge (1 \leq N)\} \text{ FOR } X := 1 \text{ UNTIL } N \text{ DO } PROD := PROD + M$
 $\{PROD = N * M\}$ RFOR en (3

5) $\vdash \{0 = (1 - 1) * M \wedge (1 \leq N)\} PROD := 0 \{PROD = (1 - 1) * M \wedge (1 \leq N)\}$ Ax3

6) $\vdash (N \geq 1) \rightarrow 0 = (1-1) * M \wedge (1 \leq N)$
 $\vdash (N \geq 1) \rightarrow 0 = 0 \wedge (1 \leq N)$
 $\vdash (N \geq 1) \rightarrow T \wedge (1 \leq N)$
 $\vdash (N \geq 1) \rightarrow (1 \leq N) \vdash P \rightarrow P$ por relación lógica

7) $\vdash \{N \geq 1\} \text{ PROD} := 0 \{ \text{PROD} = (1-1) * M \wedge (1 \leq N) \}$ RFP (6 en (5

8) $\{N \geq 1\}$
 $\text{PROD} := 0;$
 $\text{FOR } X := 1 \text{ UNTIL } N \text{ DO}$
 $\text{PROD} := \text{PROD} + M$
 $\{ \text{PROD} = N * M \}$ RS en (7 y (4

9) $\{N \geq 1\}$
 BEGIN
 $\text{PROD} := 0;$
 $\text{FOR } X := 1 \text{ UNTIL } N \text{ DO}$
 $\text{PROD} := \text{PROD} + M$
 END
 $\{ \text{PROD} = N * M \}$ RB en (8 lqqd