POINT NORMAL TRIANGLES

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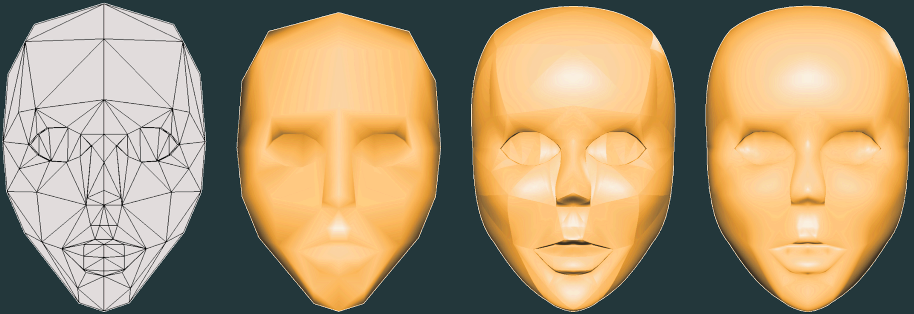
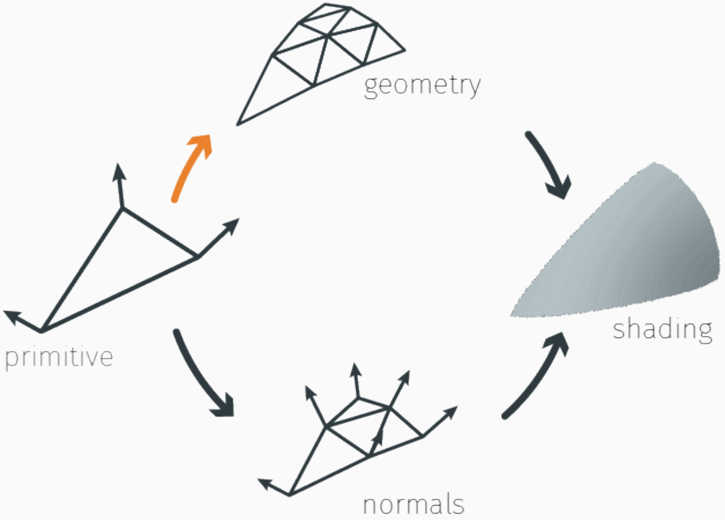
Advanced Computer Graphics



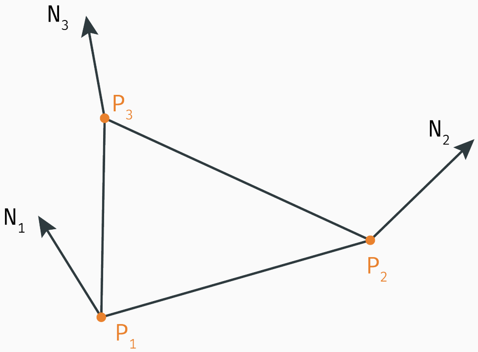
INPUT MESH GOURAUD PN GEOMETRY PN TRIANGLES

SINGLE PN TRIANGLE

OVERVIEW

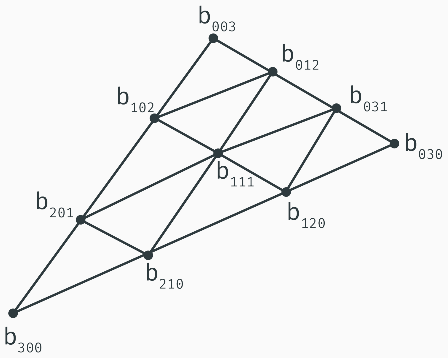


From input to geometry control net



Input primitive

GEOMETRY - VERTEX COEFFICIENTS

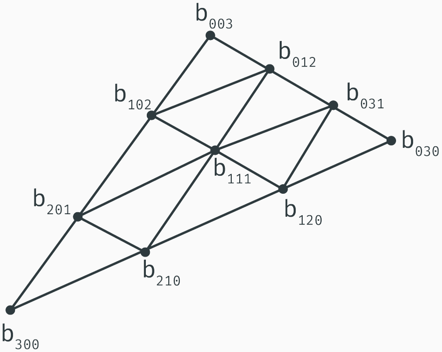


Control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$
$$b_{030} = P_2,$$
$$b_{003} = P_3$$

GEOMETRY - VERTEX COEFFICIENTS



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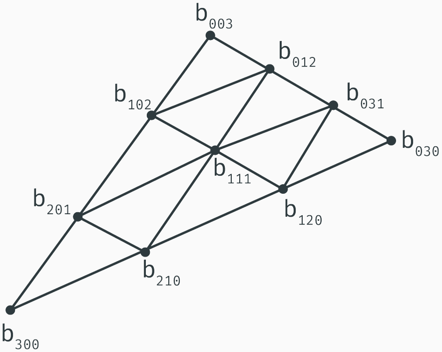

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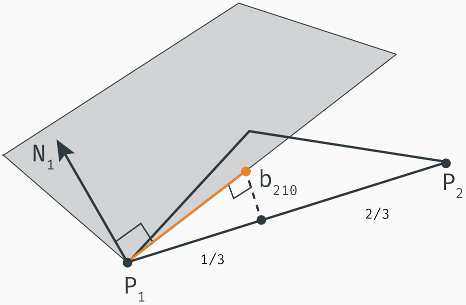
GEOMETRY - TANGENT COEFFICIENTS



Normal projection

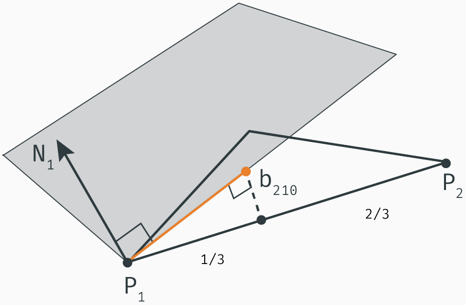
$$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N1}{3},$$

$$\vdots$$

$$b_{201} = \frac{2P_1 + P_3 - w_{13}N1}{3}$$

GEOMETRY - TANGENT COEFFICIENTS



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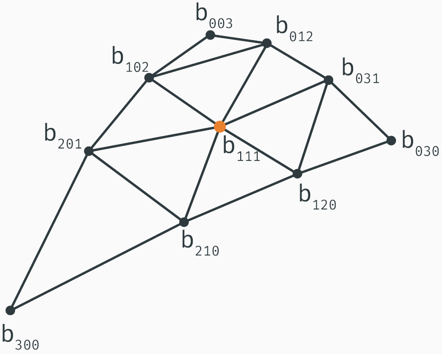


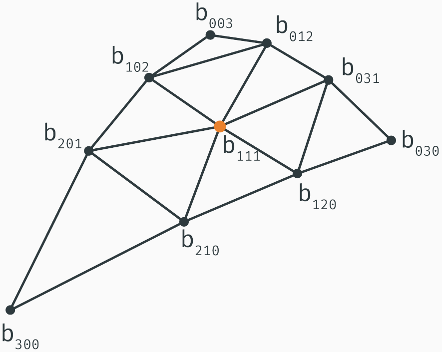
Center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$

GEOMETRY - CENTER COEFFICIENT

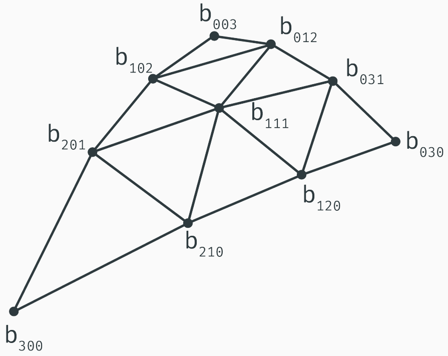


Center control point

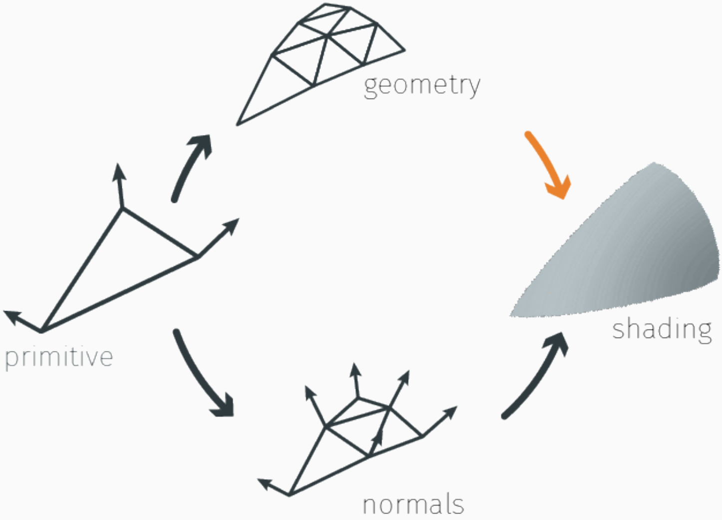
$$E = (b_{210} + b_{120} + b_{021}$$
$$+ b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$
$$b_{111} = E + (E - V)/2$$

with control net point to curve (shading)



OVERVIEW

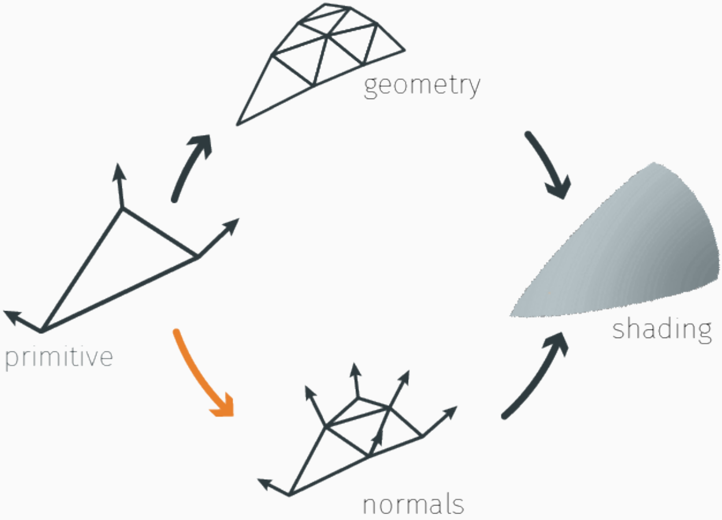


CUBIC PATCH

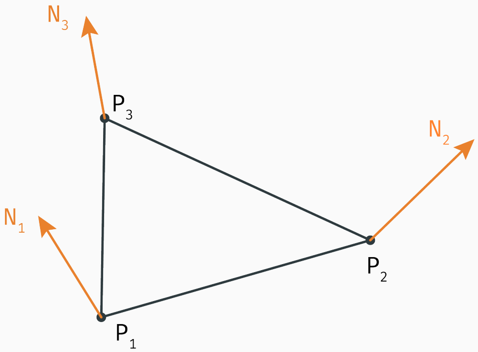
$$b : \mathbb{R}^2 \to \mathbb{R}^3, \text{ for } w = 1 - u - v, u, v, w \geq 0$$

$$\begin{aligned}
b(u, v) = &\sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k \\
&= b_{300} w^3 + b_{030} u^3 + b_{003} v^3 \\
&+ b_{210} 3w^2 u + b_{120} 3wu^2 + b_{201} 3w^2 v \\
&+ b_{021} 3u^2 v + b_{102} 3wv^2 + b_{012} 3uv^2 \\
&+ b_{111} 6wuv.
\end{aligned}$$

OVERVIEW



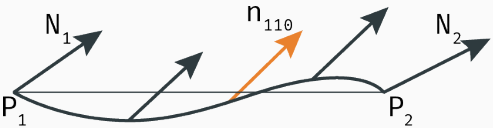
from input to more normals



Input primitive

NORMALS - THEORY

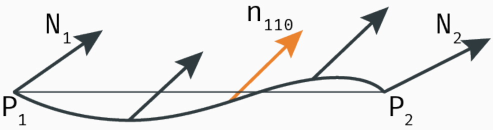
Why do we want to compute these normals?



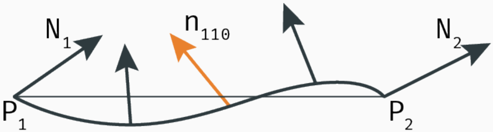
Linear

Quadratic

Why do we want to compute these normals?

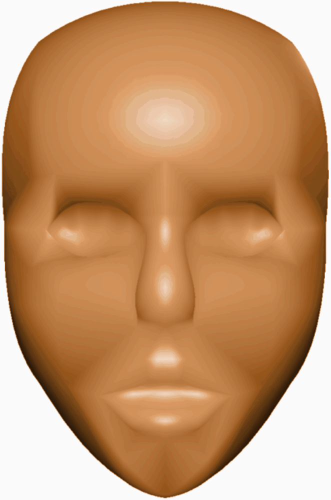


Linear



Quadratic

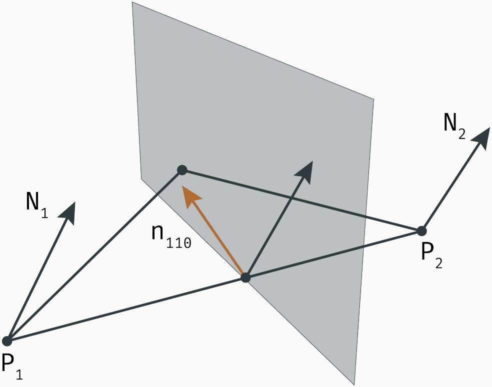
NORMALS - EXAMPLE



Linear

Quadratic

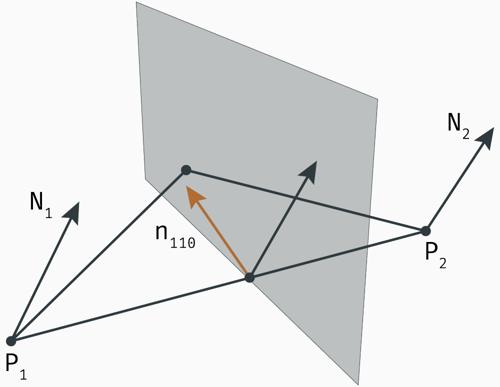
How to compute them



$$v_{ij} = 2\frac{(P_j - P_i)\cdot(N_i + N_j)}{(P_j - P_i)\cdot(P_j - P_i)} \in \mathbb{R}$$

$$h_{110} = N_1 + N2 - v_{12}(P_2 - P_1)$$

How to compute them



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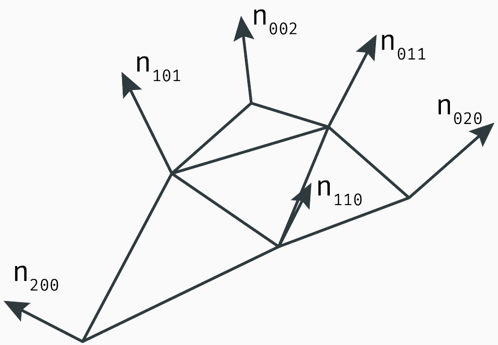
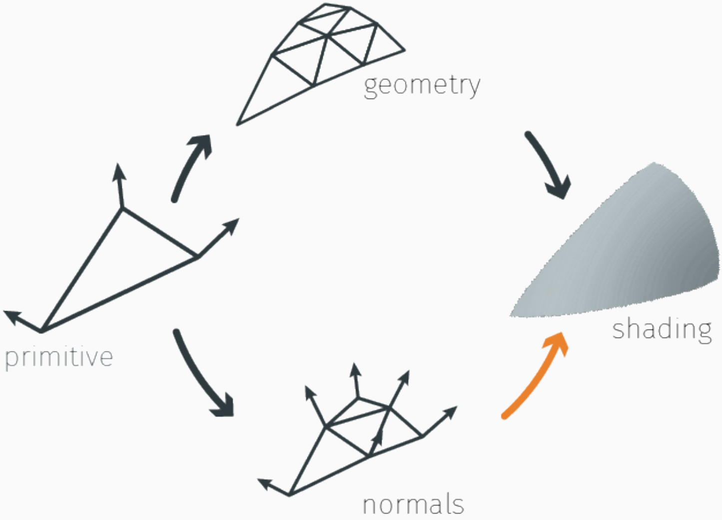
$$h_{110} = N_1 + N2 - v_{12}(P_2 - P_1)$$

NORMALS - RESULT



OVERVIEW

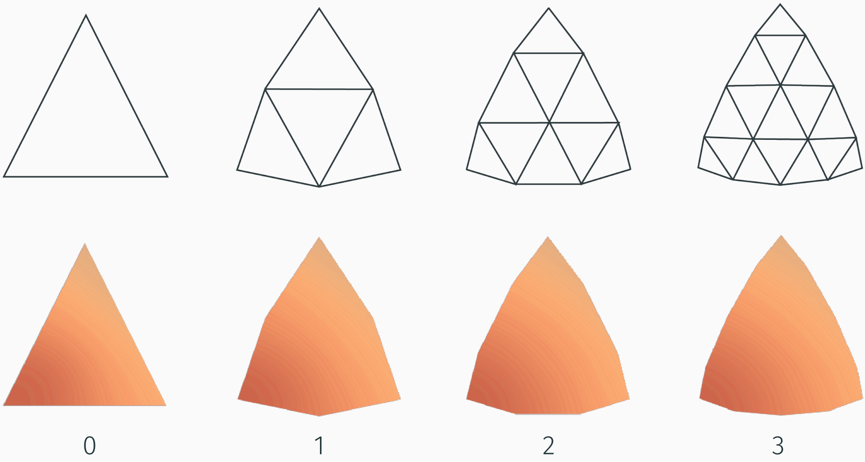


QUADRATIC PATCH

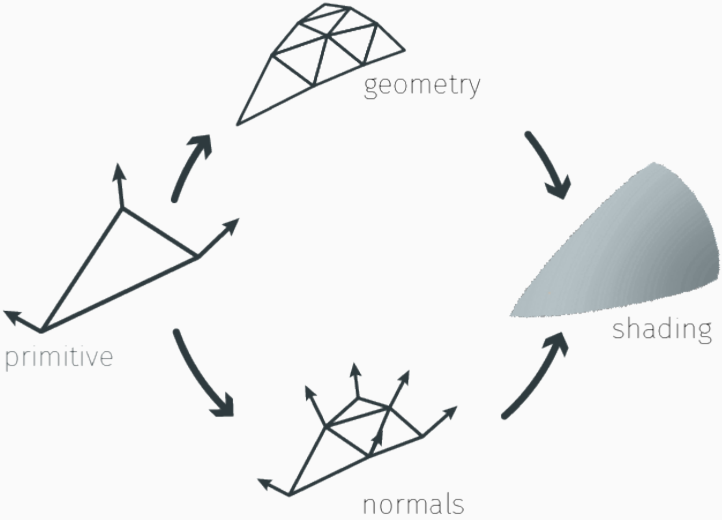
$$n : \mathbb{R}^2 \to \mathbb{R}^3, \text{ for } w = 1 - u - v,\ u, v, w \geq 0$$

$$\begin{aligned} n(u,v) &= \sum_{i+j+k=2} n_{ijk} u^i v^j w^k \\ &= n_{200} w^2 + n_{020} u^2 + n_{002} v^2 \\ &\quad + n_{110} wu + n_{011} uv + n_{101} wv \end{aligned}$$

LOD verhaal



OVERVIEW



A TRIANGLE MESH

PROPERTIES

"Pn triangles should not deviate too much from the original triangle to preserve the shape and avoid interference with other curved triangles." [1]

[1] Vlachos et al.

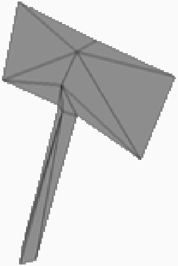
Continuity reference book.

PN triangles have:[2]

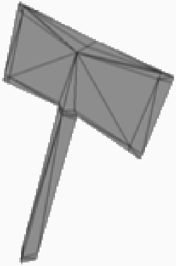
- C^1 continuity in the vertex points
- C^0 continuity everywhere else

[2] Jiao and Alexander

SHARP EDGES

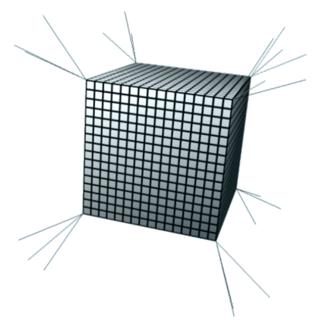

Blunt


Blunt

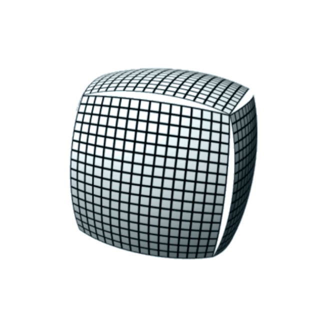

Sharp


Sharp

SEPARATE NORMALS



Normals



Cracks

GRAPHICS PIPELINE

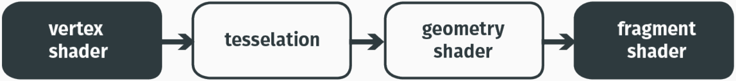
HARDWARE - PIPELINES

2001

2015

HARDWARE - PIPELINES

2001



2015

CONCLUSION

FIN.

Questions?

REFERENCES

- Xiangmin Jiao and Phillip J Alexander. “Parallel feature-preserving mesh smoothing”. In: *Computational Science and Its Applications–ICCSA 2005*. Springer, 2005, pp. 1180–1189.
- J McDonald and M Kilgard. *Crack-free point-normal triangles using adjacent edge normals.* 2010.
- Alex Vlachos et al. “Curved PN triangles”. In: *Proceedings of the 2001 symposium on Interactive 3D graphics*. ACM. 2001, pp. 159–166.