

POINT NORMAL TRIANGLES

Rick van Veen Laura Baakman

December 14, 2015

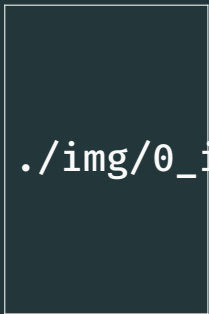
Advanced Computer Graphics

2015-12-09

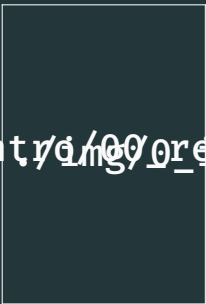
Point Normal triangles

POINT NORMAL TRIANGLES

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INPUT MESH



GOURAUD



PN GEOMETRY



PN TRIANGLES

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Point Normal triangles

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Point Normal triangles
└ Single PN Triangle

SINGLE PN TRIANGLE

SINGLE PN TRIANGLE

OVERVIEW

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Point Normal triangles

└ Single PN Triangle

└ Overview

`./img/1_single/recap_inputToGeom.png`

`./img/1_single/recap_inputToGeom.png`

enhancement: emphasize vertices better

img/1_single/inputPrimitive_emphGeometry.

input primitive

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Point Normal triangles

└ Single PN Triangle

└ Geometry

enhancement: emphasize vertices better



img/1_single/geometry_1.png

control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$

$$b_{030} = P_2,$$

$$b_{003} = P_3$$

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Vertex Coefficients



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img/1_single/geometry_2.png

normal projection

$$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3},$$

$$\vdots$$

$$b_{201} = \frac{2P_1 + P_3 - w_{13}N_1}{3}$$

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Tangent Coefficients



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GEOMETRY - TANGENT COEFFICIENTS



img/1_single/geometry_2.png

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└ Geometry - Tangent Coefficients



img/1_single/geometry_2.png

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img/1_single/geometry_3.png

center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$
$$b_{111} = E + (E - V)/2$$

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Center Coefficient

img/1_single/geometry_3.png

center control point

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img/1_single/geometry_3.png

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Center Coefficient

img/1_single/geometry_3.png

center control point

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$$b_{111} = E + (E - V)/2$$

enhancement: Set result slide to plain

img/1_single/geometry_4.png

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Result

enhancement: Set result slide to plain

img/1_single/geometry_4.png

`./img/1_single/recap_geomToShading.png`

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Point Normal triangles

└ Single PN Triangle

└ Overview

`./img/1_single/recap_geomToShading.png`

CUBIC PATCH

Spacing van de for all

Plaatje?

$b: \mathbb{R}^2 \rightarrow \mathbb{R}^3$, for $w = 1 - u - v$, $u, v, w \geq 0$

$$\begin{aligned} b(u, v) &= \sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k \\ &= b_{300} w^3 + b_{030} u^3 + b_{003} v^3 \\ &\quad + b_{210} 3w^2 u + b_{120} 3wu^2 + b_{201} 3w^2 v \\ &\quad + b_{021} 3u^2 v + b_{102} 3wv^2 + b_{012} 3uv^2 \\ &\quad + b_{111} 6wuv. \end{aligned}$$

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Point Normal triangles

└ Single PN Triangle

└ Cubic patch

CUBIC PATCH

Spacing van de for all

Plaatje?

$$\begin{aligned} b: \mathbb{R}^2 &\rightarrow \mathbb{R}^3, \text{ for } w = 1 - u - v, u, v, w \geq 0 \\ b(u, v) &= \sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k \\ &= b_{300} w^3 + b_{030} u^3 + b_{003} v^3 \\ &\quad + b_{210} 3w^2 u + b_{120} 3wu^2 + b_{201} 3w^2 v \\ &\quad + b_{021} 3u^2 v + b_{102} 3wv^2 + b_{012} 3uv^2 \\ &\quad + b_{111} 6wuv. \end{aligned}$$

`./img/1_single/recap_inputToNormals.png`

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Point Normal triangles

└ Single PN Triangle

└ Overview

`./img/1_single/recap_inputToNormals.png`

NORMALS

enhancement: emphasize normals more

img/1_single/inputPrimitive_emphNormal.png

input primitive

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- Point Normal triangles
 - Single PN Triangle
 - Normals



img/1_single/linearVsQuadraticNormals_lin

linear

quadratic

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Point Normal triangles

└ Single PN Triangle

└ Normals - theory

img/1_single/linearVsQuadraticNormals_lin

linear

quadratic

img/1_single/linearVsQuadraticNormals_linear

linear

img/1_single/linearVsQuadraticNormals_quadratic

quadratic

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Point Normal triangles

└ Single PN Triangle

└ Normals - theory

img/1_single/linearVsQuadraticNormals_linear

linear

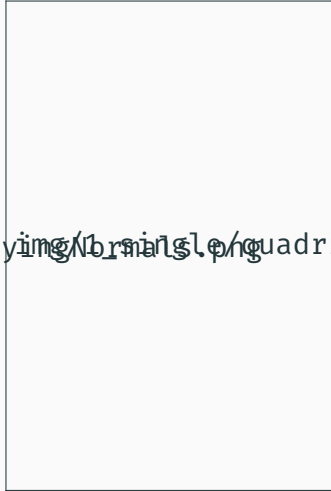
img/1_single/linearVsQuadraticNormals_quadratic

quadratic

NORMALS - EXAMPLE



linear



quadratic

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Point Normal triangles

└ Single PN Triangle

└ Normals - example

NORMALS - EXAMPLE



linear



quadratic

NORMALS - THEORY

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Point Normal triangles

└ Single PN Triangle

└ Normals - theory

img/1_single/computingNormals.png

$$v_{ij} = 2 \frac{(P_j - P_i) \cdot (N_i + N_j)}{(P_j - P_i) \cdot (P_j - P_i)} \in \mathbb{R}$$

$$h_{110} = N_1 + N_2 - v_{12}(P_2 - P_1)$$

img/1_single/computingNormals.png

$$N_i = 2 \frac{(P_j - P_i) \cdot (N_i + N_j)}{(P_j - P_i) \cdot (P_j - P_i)} \in \mathbb{R}$$

$$N_{12} = N_1 + N_2 - v_{12}(P_2 - P_1)$$

$$N_{12} = N_{12} / |N_{12}|$$

img/1_single/computingNormals.png

$$v_i = 2 \frac{(P_i - P_j) \cdot (N_i + N_j)}{(P_i - P_j) \cdot (P_j - P_i)} \in \mathbb{R}$$

$$h_{112} = N_1 + N_2 - v_{12}(P_2 - P_1)$$

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Point Normal triangles

└ Single PN Triangle

└ Normals - theory

img/1_single/computingNormals.png

$$v_i = 2 \frac{(P_i - P_j) \cdot (N_i + N_j)}{(P_i - P_j) \cdot (P_j - P_i)} \in \mathbb{R}$$

$$h_{112} = N_1 + N_2 - v_{12}(P_2 - P_1)$$

$$n_{112} = h_{112} / \|h_{112}\|$$

enhancement: Set result slide to plain

img/1_single/normals.png

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Point Normal triangles

└ Single PN Triangle

└ Normals - result

enhancement: Set result slide to plain

img/1_single/normals.png

OVERVIEW

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Point Normal triangles

└ Single PN Triangle

└ Overview

`./img/1_single/recap_normalsToShading.png`

`./img/1_single/recap_normalsToShading.png`

Plaatje

$n: \mathbb{R}^2 \rightarrow \mathbb{R}^3$, for $w = 1 - u - v, u, v, w \geq 0$

$$\begin{aligned} n(u, v) &= \sum_{i+j+k=2} n_{ijk} u^i v^j w^k \\ &= n_{200} w^2 + n_{020} u^2 + n_{002} v^2 \\ &\quad + n_{110} w u + n_{011} u v + n_{101} w v \end{aligned}$$

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Point Normal triangles

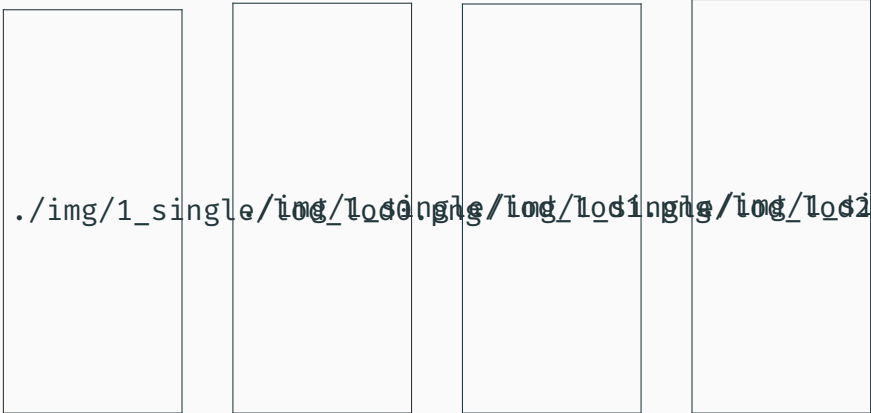
└ Single PN Triangle

└ Quadratic Patch

Plaatje

$$\begin{aligned} n: \mathbb{R}^2 &\rightarrow \mathbb{R}^3, \text{ for } w = 1 - u - v, u, v, w \geq 0 \\ n(u, v) &= \sum_{i+j+k=2} n_{ijk} u^i v^j w^k \\ &= n_{200} w^2 + n_{020} u^2 + n_{002} v^2 \\ &\quad + n_{110} w u + n_{011} u v + n_{101} w v \end{aligned}$$

LEVEL OF DETAIL



0

1

2

3

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Point Normal triangles

└ Single PN Triangle

└ Level Of Detail

LEVEL OF DETAIL



0

1

2

3

`./img/1_single/recap_overview.png`

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Point Normal triangles

└ Single PN Triangle

└ Overview

`./img/1_single/recap_overview.png`

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Point Normal triangles
└ A Triangle Mesh

A TRIANGLE MESH

A TRIANGLE MESH

"PN triangles should not deviate too much from the original triangle to preserve the shape and avoid interference with other curved triangles."¹

¹Vlachos et al.

- C^1 continuity in the vertex points
- C^0 continuity everywhere else

PN triangles have:²

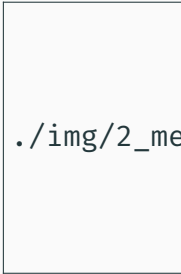
- C^1 continuity in the vertex points
- C^0 continuity everywhere else

²liao and Alexander

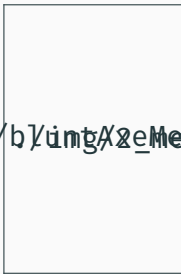
SHARP EDGES

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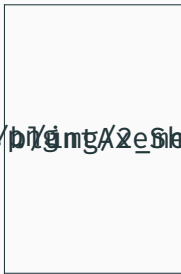
- Point Normal triangles
- └ A Triangle Mesh
- └ Sharp Edges



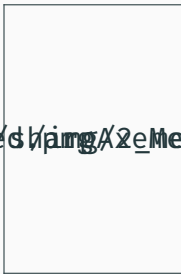
mesh



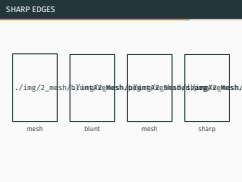
blunt



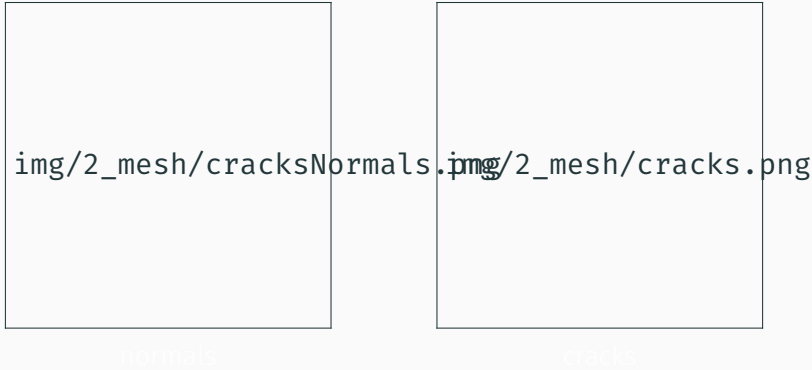
mesh



sharp



SEPARATE NORMALS



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Point Normal triangles

└ A Triangle Mesh

└ Separate Normals

SEPARATE NORMALS



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Point Normal triangles
└ Graphics Pipeline

GRAPHICS PIPELINE

GRAPHICS PIPELINE

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- Point Normal triangles
- └ Graphics Pipeline
- └ Hardware - Pipelines

img/3_pipeline/pipelineDifferences_oldOpenGL.png

2001

2015

img/3_pipeline/pipelineDifferences_oldOpenGL.png

2001

img/3_pipeline/pipelineDifferences_newOpenGL.png

2015

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Point Normal triangles
└ Conclusion

CONCLUSION

CONCLUSION

CONCLUSION

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Point Normal triangles

└ Conclusion

└ conclusion

Some conclusion?

Some conclusion?



FIGUUR 13 UIT PAPER

QUESTIONS?

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Point Normal triangles

└ Conclusion



FIGUUR 13 UIT PAPER

REFERENCES

- 
[Bibliography article](#) pdf J Alexander. “Parallel feature-preserving mesh smoothing”. In: *Computational Science and Its Applications–ICCSA 2005*. Springer, 2005, pp. 1180–1189.
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[Bibliography article](#) pdf J Alexander and M Klinger. “Crack-free point-normal triangles using adjacent edge normals”. 2010.
- 
[Bibliography article](#) pdf J Alexander. “Crack-free PN triangles”. In: *Proceedings of the 2001 symposium on Interactive 3D graphics*. ACM, 2001, pp. 159–166.

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Point Normal triangles

└ Conclusion

└ References


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