POINT NORMAL TRIANGLES

Rick van Veen Laura Baakman

December 14, 2015

Advanced Computer Graphics

2015-12-09

Point Normal triangles

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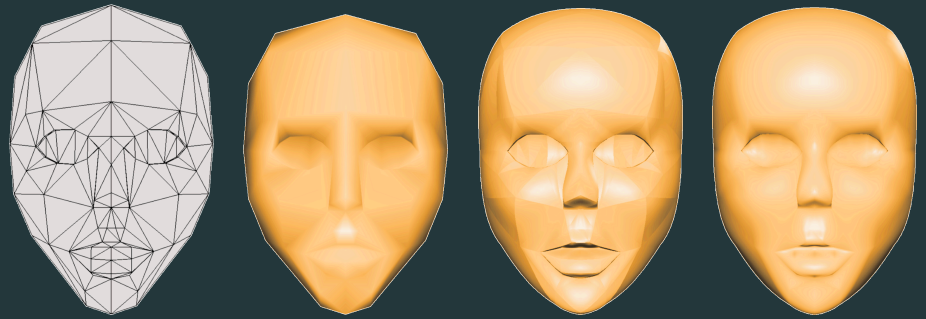
[Rick] Welcome everybody. Tell people that PN means Point Normal triangles.



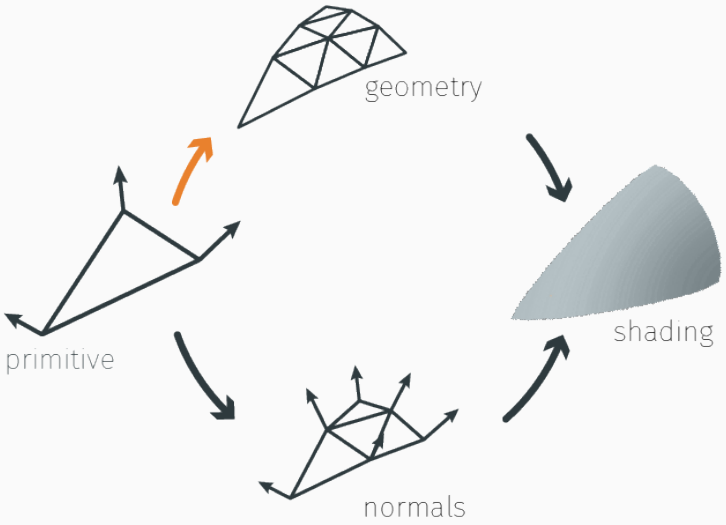
Point Normal triangles

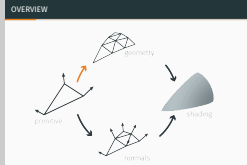
[Name] Why PN triangles? Look at the nice result it gives :-) and we will see that it easy to extend it to the ‘existing’ pipeline.

SINGLE PN TRIANGLE

[Name] How does one construct a single PN triangle?
Overview on the next slide

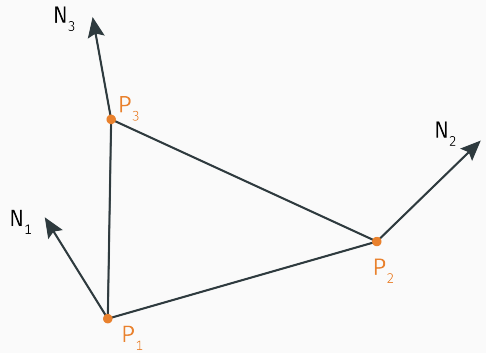
OVERVIEW



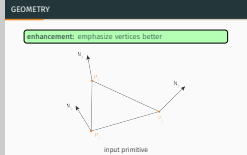


[Name] Why PN triangles? Look at the nice result it gives :-) and we will see that it easy to extend it to the ‘existing’ pipeline. Story about Bezier patches...

enhancement: emphasize vertices better



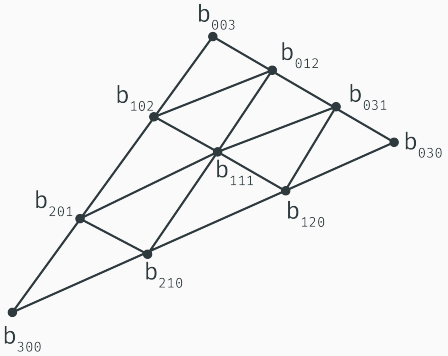
input primitive



[Name] This a standard triangle primitive, defined by its vertices and normals.

Focus on getting the different control primitives.

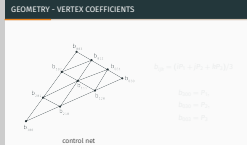
GEOMETRY - VERTEX COEFFICIENTS



control net

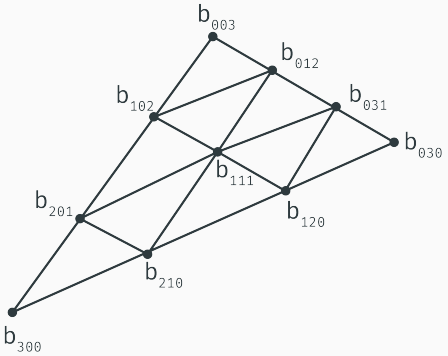
$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$
$$b_{030} = P_2,$$
$$b_{003} = P_3$$

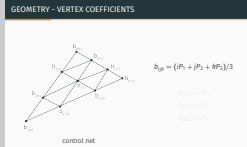


[Name] These are all the initial control point. Evenly divided on the triangle. -> formula

GEOMETRY - VERTEX COEFFICIENTS

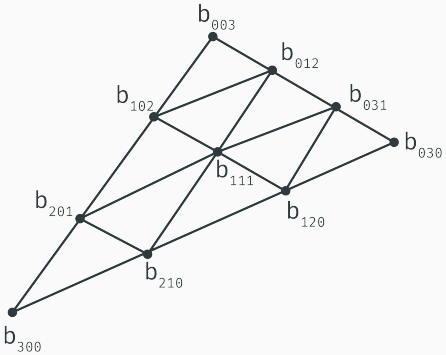


control net



[Name] Nice formula

GEOMETRY - VERTEX COEFFICIENTS



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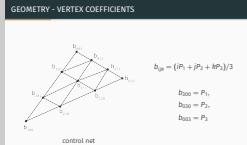
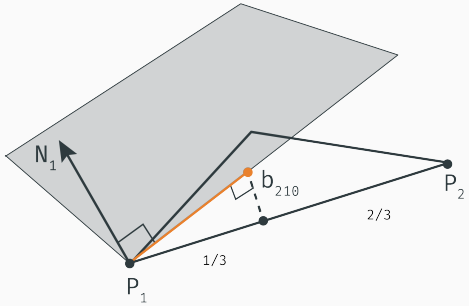


[Name] Stress that the vertex coefficients/control points are the one on the original vertices and that they do not move.

GEOMETRY - TANGENT COEFFICIENTS



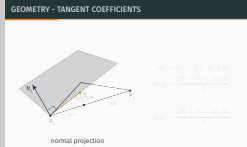
normal projection

$$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N1}{3},$$

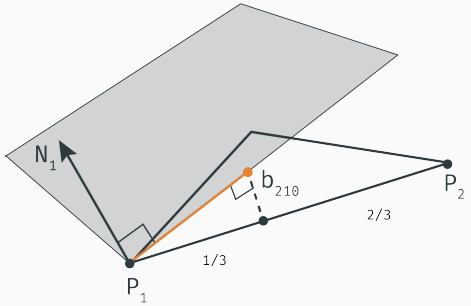
$$\vdots$$

$$b_{201} = \frac{2P_1 + P_3 - w_{13}N1}{3}$$



[Name] How to get the tangent coefficient (the ones on the edge but now curvy)

GEOMETRY - TANGENT COEFFICIENTS



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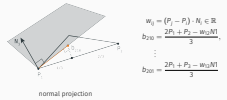
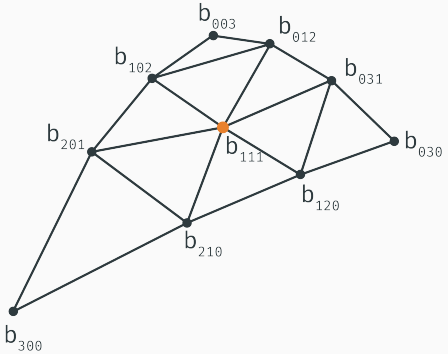
GEOMETRY - TANGENT COEFFICIENTS



$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$
$b_{210} = \frac{2P_1 + P_2 - w_{12}N1}{3},$
$\vdots$
$b_{201} = \frac{2P_1 + P_3 - w_{13}N1}{3}$
normal projection

[Name] Projection of the initial control points on the normal plane of a vertex.

GEOMETRY - CENTER COEFFICIENT

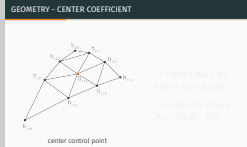


center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

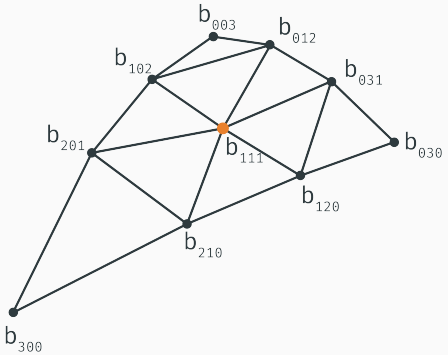
$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$



[Name] Note that this is the result of the previous step -> now only center coefficient is left.

GEOMETRY - CENTER COEFFICIENT



center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

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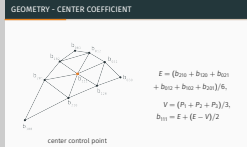
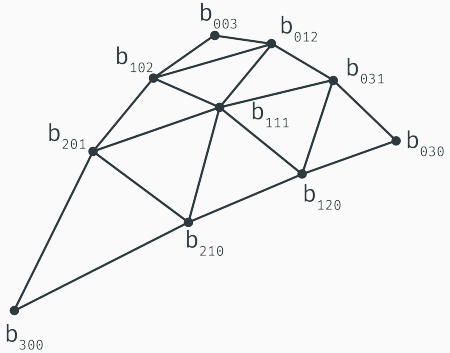
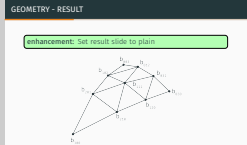


[Name] Average of the tangent coefficients plus half the difference between the tangent and vertex coefficients. -> why?

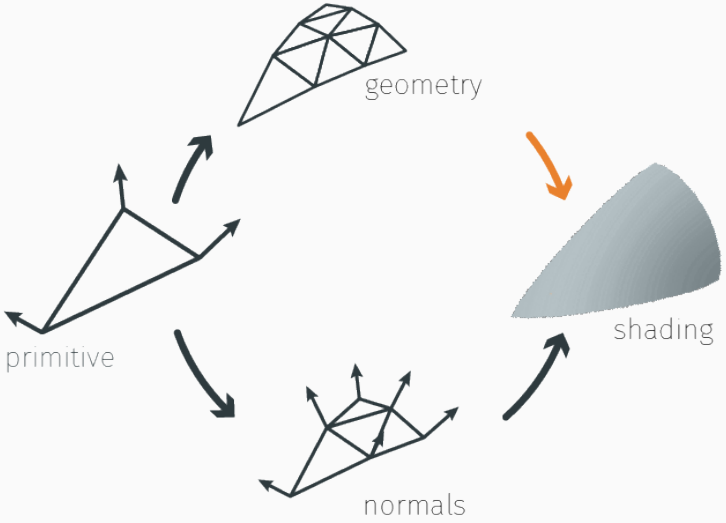
enhancement: Set result slide to plain

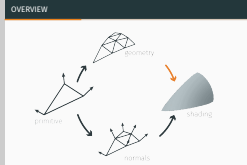




[Name] Results

OVERVIEW





[Name] Overview -> how to get from this to shading.
Sample/subdivide with formula on following slide.

CUBIC PATCH

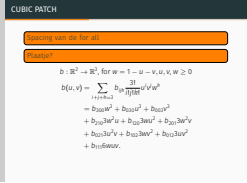
Spacing van de for all

Plaatje?

$$b : \mathbb{R}^2 \to \mathbb{R}^3, \text{ for } w = 1 - u - v, u, v, w \geq 0$$

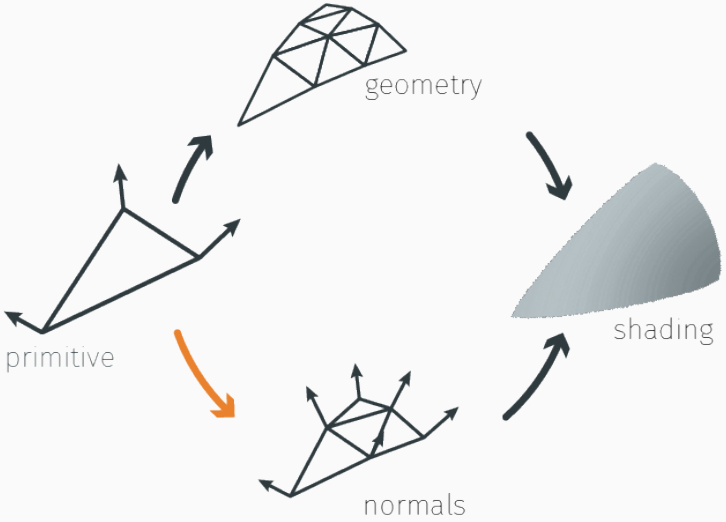
$$b(u,v) = \sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k$$

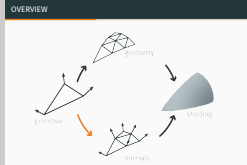
$$= b_{300} w^3 + b_{030} u^3 + b_{003} v^3$$
$$+ b_{210} 3w^2 u + b_{120} 3wu^2 + b_{201} 3w^2 v$$
$$+ b_{021} 3u^2 v + b_{102} 3wv^2 + b_{012} 3uv^2$$
$$+ b_{111} 6wuv.$$



[Name] Very nice formula with a nice picture.

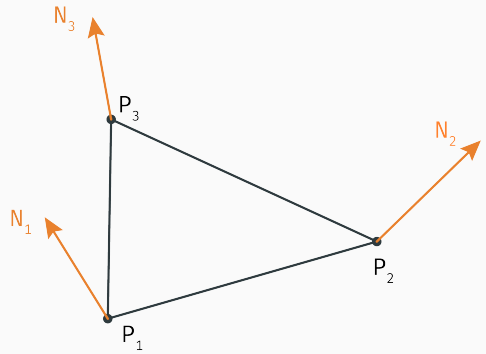
OVERVIEW



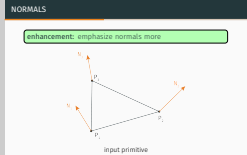


[Name] From the primitive normals the the PN triangle normals

enhancement: emphasize normals more

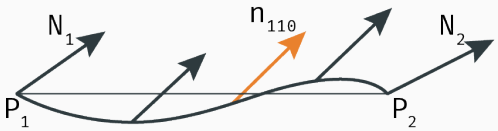


input primitive



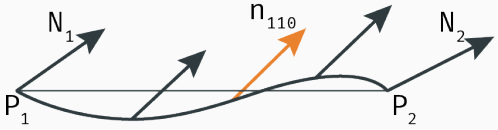
[Name] Recap input primitive and with emphasis on the normals.

NORMALS - THEORY

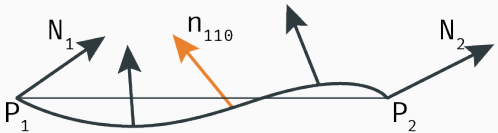


[Name] Stress that there is a need to capture the cubic bezier curve (inflection points) and that this cannot be

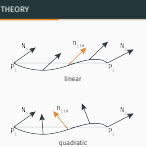
NORMALS - THEORY



linear



quadratic

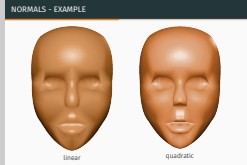


[Name] Quadratic does capture inflection points. Trade off between performance and result (maybe?)

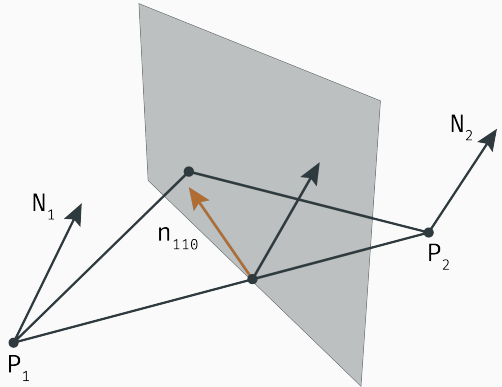
NORMALS - EXAMPLE

linear

quadratic



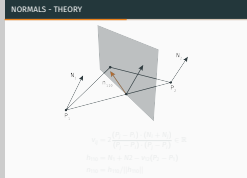
[Name] Look how pretty.



$$v_{ij} = 2\frac{(P_j - P_i) \cdot (N_i + N_j)}{(P_j - P_i) \cdot (P_j - P_i)} \in \mathbb{R}$$

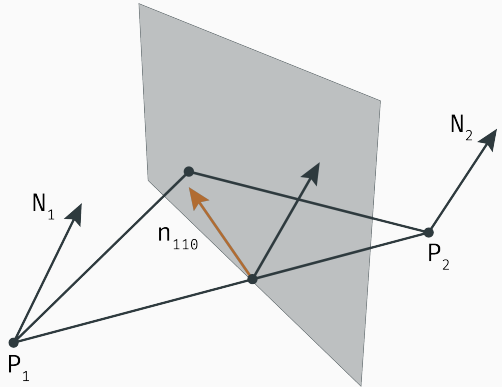
$$h_{110} = N_1 + N2 - v_{12}(P_2 - P_1)$$

$$n_{110} = h_{110}/||h_{110}||$$



[Name] Formula in words: reflect the averaged normal (average of N1 and N2) on the plane orthogonal/perpendicular the the edge at the mid point.

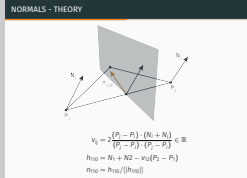
NORMALS - THEORY



$$v_i = 2\frac{(P_j - P_i)\cdot(N_i + N_j)}{(P_j - P_i)\cdot(P_j - P_i)} \in \mathbb{R}$$

$$h_{110} = N_1 + N2 - v_{i1}(P_2 - P_1)$$

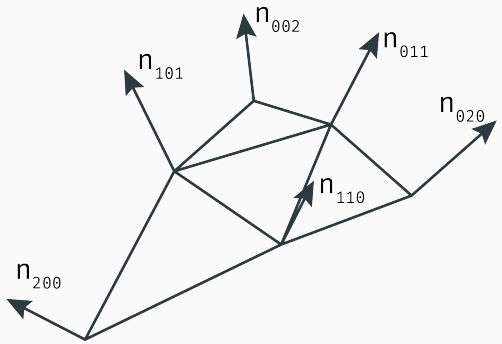
$$n_{110} = h_{110}/\|h_{110}\|$$

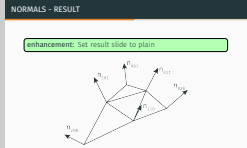


[Name] Formula in words: reflect the averaged normal (average of N1 and N2) on the plane orthogonal/perpendicular the the edge at the mid point.

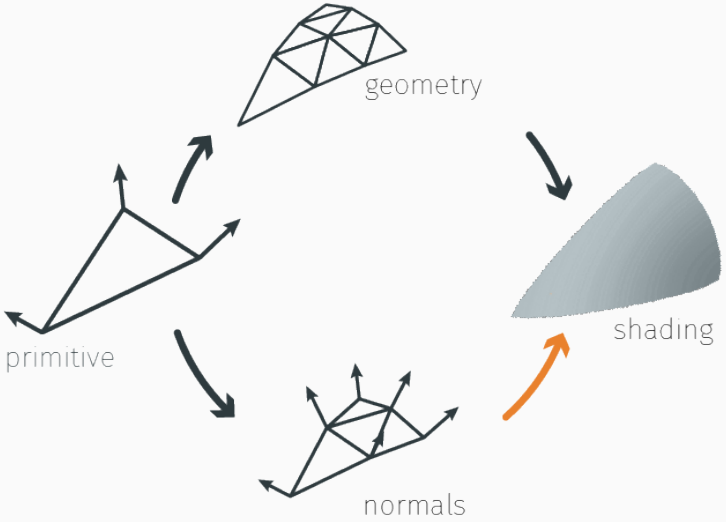
NORMALS - RESULT

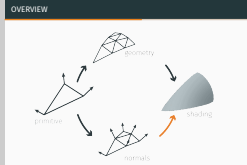
enhancement: Set result slide to plain





OVERVIEW





[Name] Why PN triangles? Look at the nice result it gives :-) and we will see that it easy to extend it to the ‘existing’ pipeline.

Plaatje

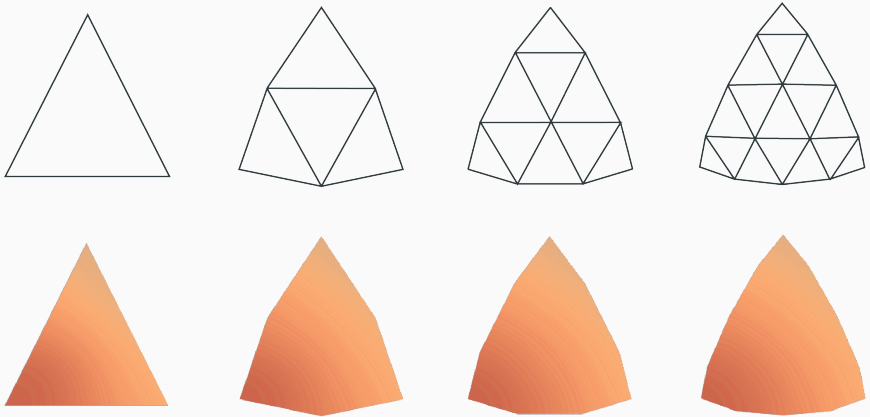
$$n : \mathbb{R}^2 \to \mathbb{R}^3, \text{ for } w = 1 - u - v, u, v, w \geq 0$$

$$n(u,v) = \sum_{i+j+k=3} n_{ijk} u^i v^j w^k$$

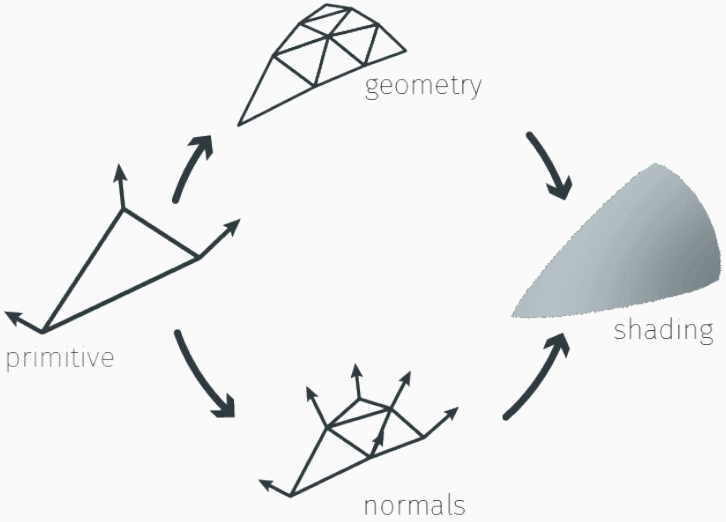
$$= n_{200} w^2 + n_{020} u^2 + n_{002} v^2 + n_{110} wu + n_{011} uv + n_{101} wv$$

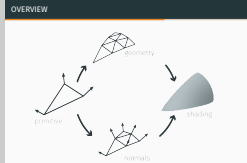
QUADRATIC PATCH
Plaatje
$n : \mathbb{R}^2 \to \mathbb{R}^3$, for $w = 1 - u - v, u, v, w \geq 0$
$n(u,v) = \sum_{i+j+k=3} n_{ijk} u^i v^j w^k$
$= n_{200} w^2 + n_{020} u^2 + n_{002} v^2 + n_{110} wu + n_{011} uv + n_{101} wv$

LEVEL OF DETAIL



OVERVIEW





A TRIANGLE MESH

PROPERTIES

"PN triangles should not deviate too much from the original triangle to preserve the shape and avoid interference with other curved triangles." [1]

[1] Vlachos et al.

CONTINUITY

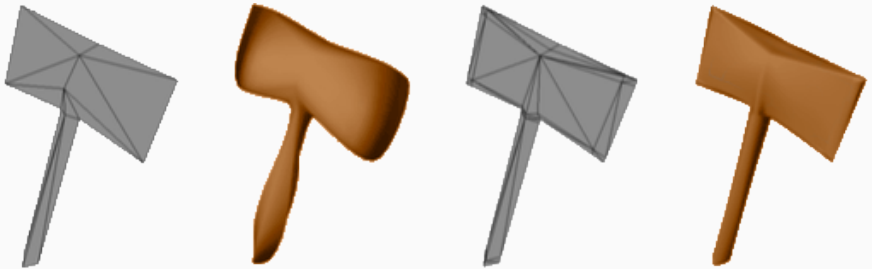
PN triangles have[2]

- C^1 continuity in the vertex points
- C^0 continuity everywhere else

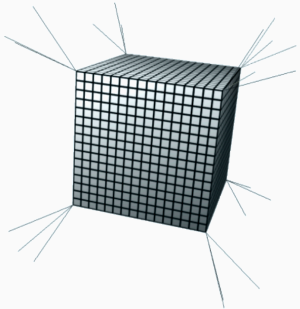
[2] Jiao and Alexander

CONTINUITY
PN triangles have:[2]
- C^1 continuity in the vertex points
- C^0 continuity everywhere else
[2] Jiao and Alexander

SHARP EDGES



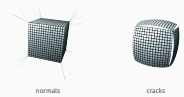
SEPARATE NORMALS



normals

cracks

SEPARATE NORMALS

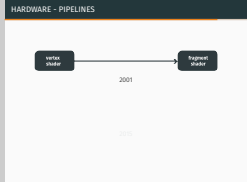


GRAPHICS PIPELINE

HARDWARE - PIPELINES

2001

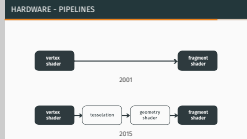
2015



HARDWARE - PIPELINES

2001

2015



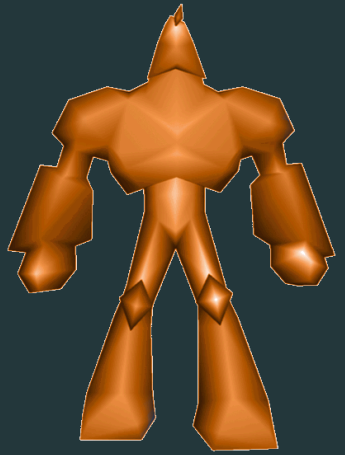
CONCLUSION

CONCLUSION

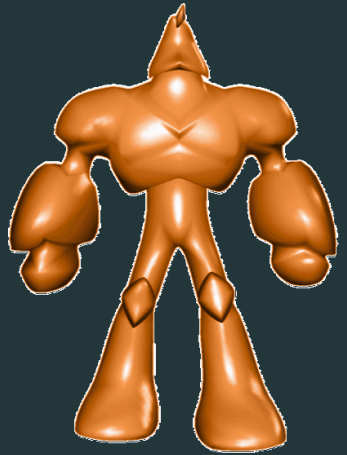
Some conclusion?

CONCLUSION
Some conclusion?

QUESTIONS?



TRIANGLES



PN TRIANGLES

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