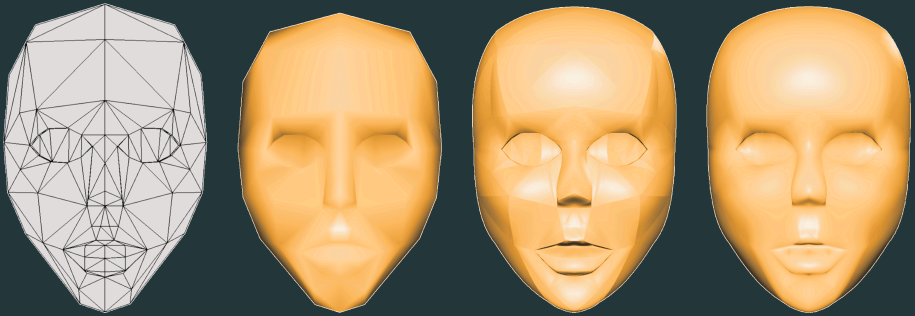
POINT NORMAL TRIANGLES

Rick van Veen   Laura Baakman

December 14, 2015

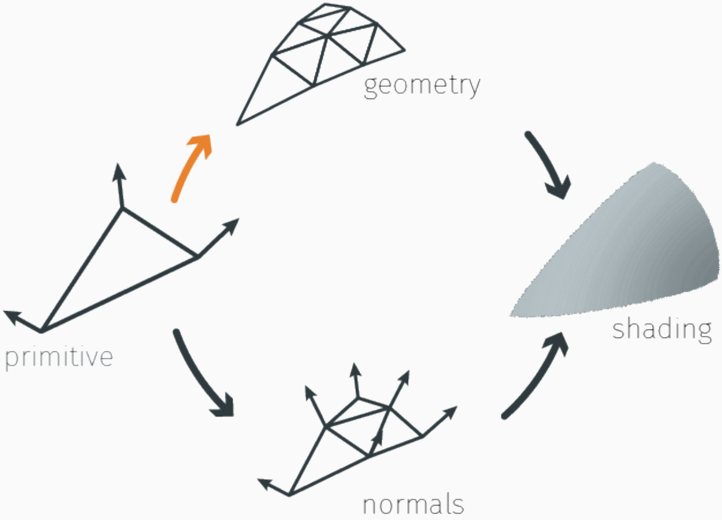
Advanced Computer Graphics



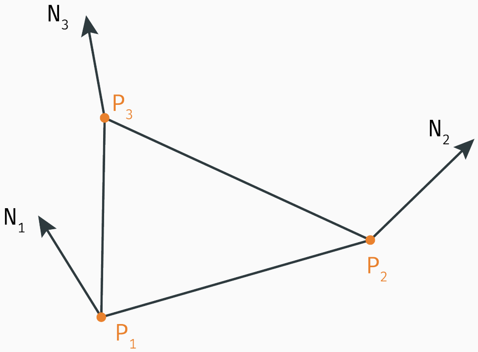
INPUT MESH GOURAUD PN GEOMETRY PN TRIANGLES

SINGLE PN TRIANGLE

OVERVIEW

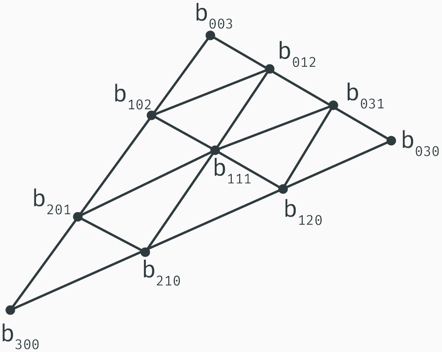


GEOMETRY



Input primitive

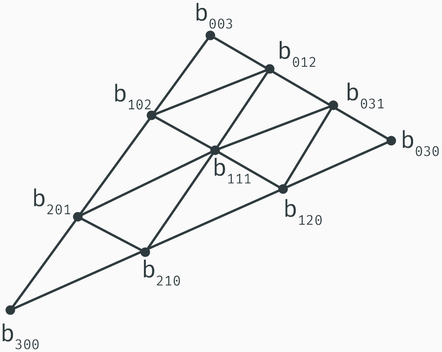
GEOMETRY - VERTEX COEFFICIENTS



Control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$
$$b_{030} = P_2,$$
$$b_{003} = P_3$$

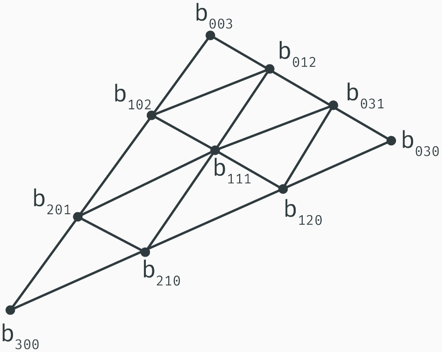


Control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$
$$b_{030} = P_2,$$
$$b_{003} = P_3$$

GEOMETRY - VERTEX COEFFICIENTS

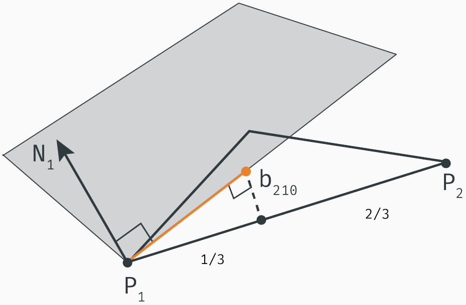


Control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$
$$b_{030} = P_2,$$
$$b_{003} = P_3$$

GEOMETRY - TANGENT COEFFICIENTS



Normal projection

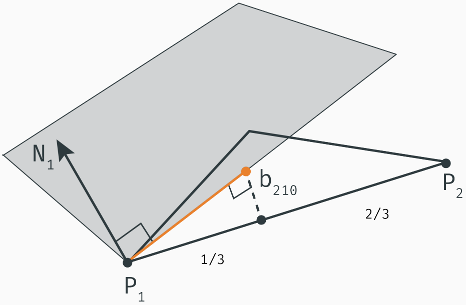
$$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N1}{3},$$

$$\vdots$$

$$b_{201} = \frac{2P_1 + P_3 - w_{13}N1}{3}$$

GEOMETRY - TANGENT COEFFICIENTS



Normal projection

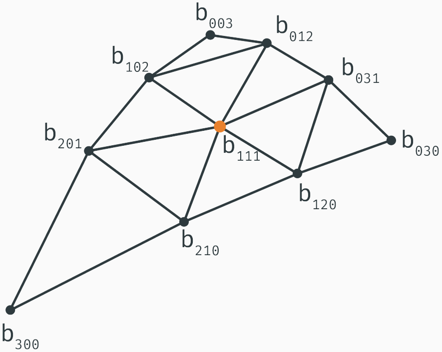
$$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N1}{3},$$

$$\vdots$$

$$b_{201} = \frac{2P_1 + P_3 - w_{13}N1}{3}$$

GEOMETRY - CENTER COEFFICIENT

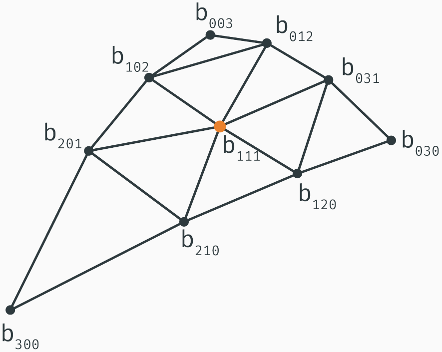


Center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$



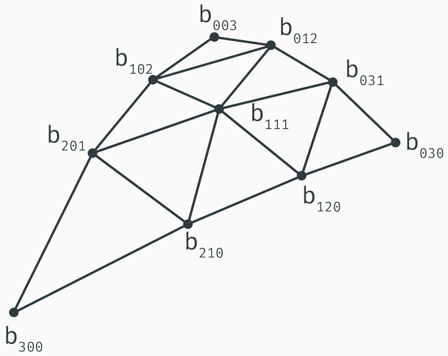
Center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

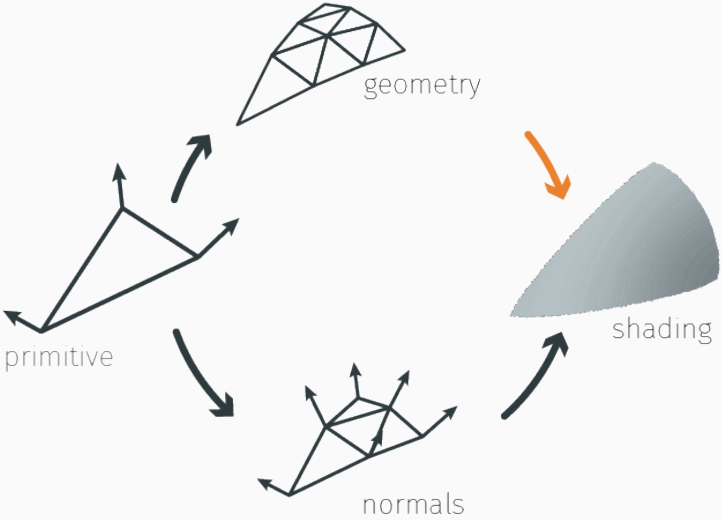
$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$

GEOMETRY



OVERVIEW

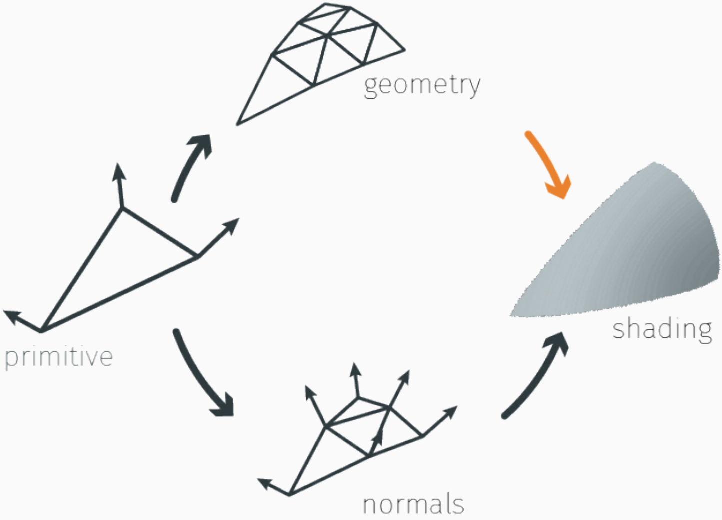


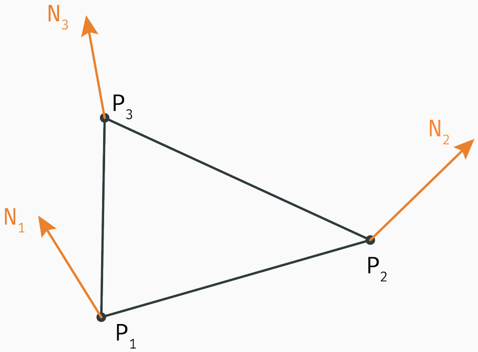
CUBIC PATCH

$$b : \mathbb{R}^2 \to \mathbb{R}^3, \text{ for } w = 1 - u - v, u, v, w \geq 0$$

$$\begin{aligned} b(u,v) &= \sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k \\ &= b_{300} w^3 + b_{030} u^3 + b_{003} v^3 \\ &+ b_{210} 3w^2 u + b_{120} 3wu^2 + b_{201} 3w^2 v \\ &+ b_{021} 3u^2 v + b_{102} 3wv^2 + b_{012} 3uv^2 \\ &+ b_{111} 6wuv. \end{aligned}$$

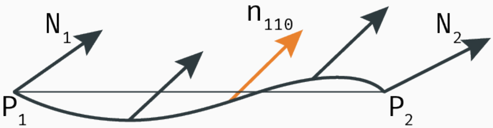
OVERVIEW





Input primitive

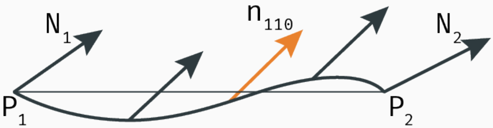
NORMALS - THEORY



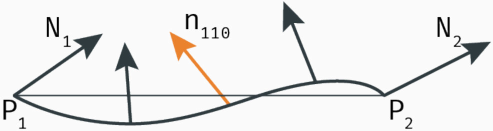
Linear

Quadratic

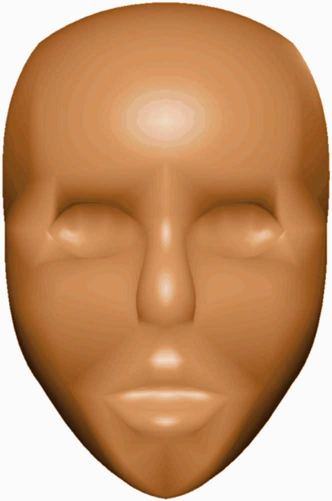
NORMALS - THEORY



Linear



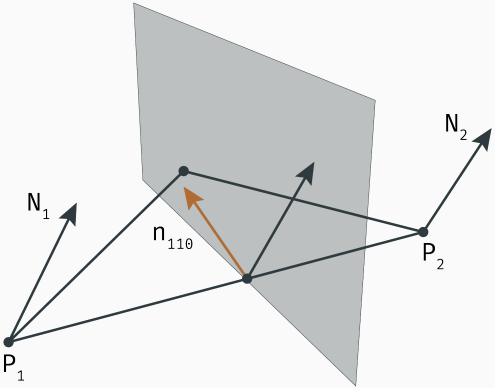
Quadratic



Linear

Quadratic

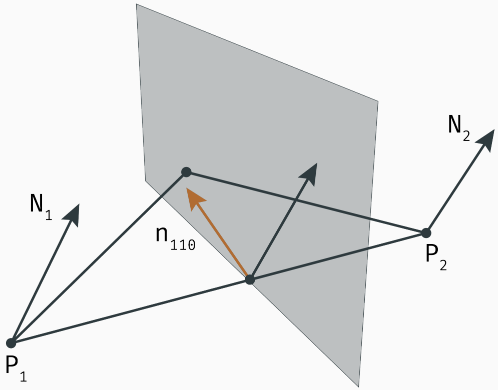
NORMALS - THEORY



$$v_{ij} = 2\frac{(P_j - P_i) \cdot (N_i + N_j)}{(P_j - P_i) \cdot (P_j - P_i)} \in \mathbb{R}$$

$$h_{110} = N_1 + N2 - v_{12}(P_2 - P_1)$$

$$n_{110} = h_{110}/||h_{110}||$$

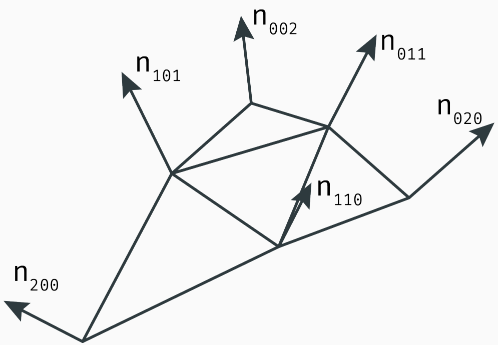


$$v_{ij} = 2\frac{(P_j - P_i) \cdot (N_i + N_j)}{(P_j - P_i) \cdot (P_j - P_i)} \in \mathbb{R}$$

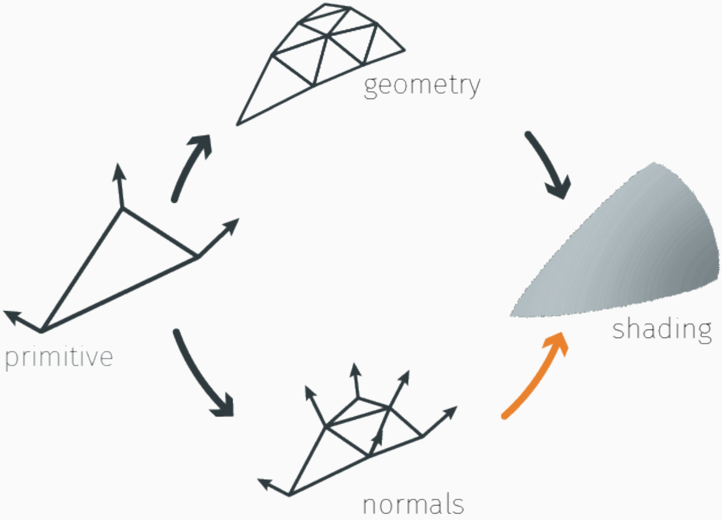
$$h_{110} = N_1 + N2 - v_{12}(P_2 - P_1)$$

$$n_{110} = h_{110}/||h_{110}||$$

NORMALS - RESULT



OVERVIEW

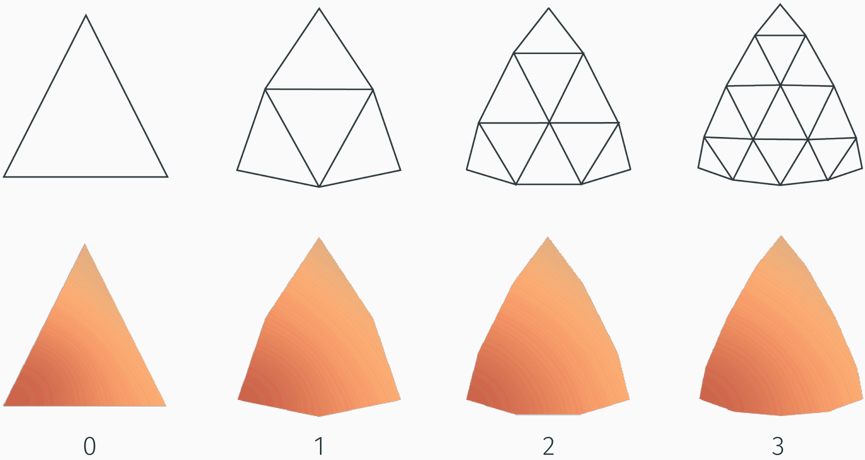


QUADRATIC PATCH

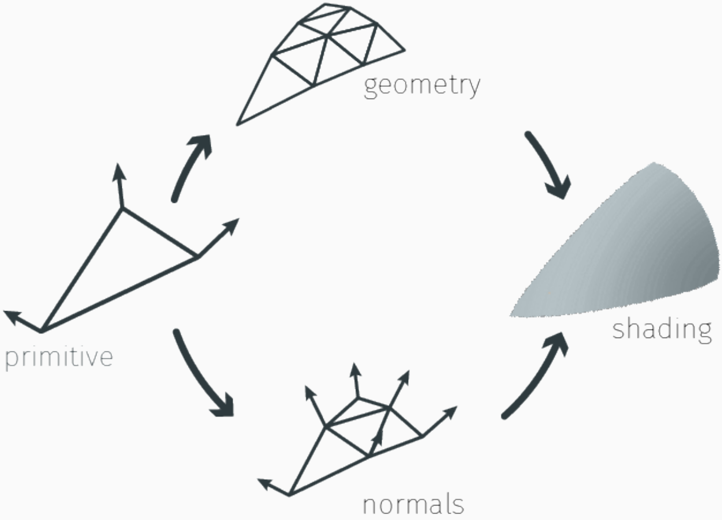
$$n : \mathbb{R}^2 \to \mathbb{R}^3, \text{ for } w = 1 - u - v,\ u, v, w \geq 0$$

$$\begin{aligned} n(u, v) &= \sum_{i+j+k=2} n_{ijk} u^i v^j w^k \\ &= n_{200} w^2 + n_{020} u^2 + n_{002} v^2 \\ &+ n_{110} wu + n_{011} uv + n_{101} wv \end{aligned}$$

LEVEL OF DETAIL



OVERVIEW



A TRIANGLE MESH

PROPERTIES

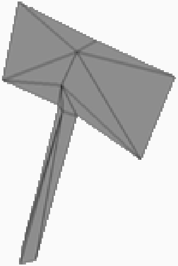
“Pn triangles should not deviate too much from the original triangle to preserve the shape and avoid interference with other curved triangles.” Vlachos et al.

CONTINUITY

PN triangles have:

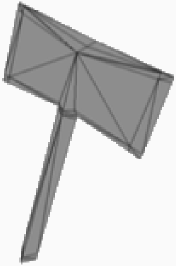
- C^1 continuity in the vertex points
- C^0 continuity everywhere else

SHARP EDGES



Blunt

Blunt



Sharp

Sharp

NVIDIA

GRAPHICS PIPELINE

2001

2015

HARDWARE - PIPELINES

2001

2015

CONCLUSION

FIN.

REFERENCES

REFERENCES

Alex Vlachos et al. “Curved PN triangles”. In: *Proceedings of the 2001 symposium on Interactive 3D graphics*. ACM. 2001, pp. 159–166.