

# POINT NORMAL TRIANGLES

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Rick van Veen   Laura Baakman

December 14, 2015

Advanced Computer Graphics

2015-12-09

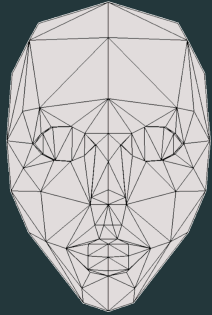
## Point Normal triangles

POINT NORMAL TRIANGLES

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Rick van Veen   Laura Baakman  
December 14, 2015  
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**[Rick]** Welcome everybody. Tell people that PN means Point Normal triangles.



INPUT MESH



GOURAUD



PN GEOMETRY



PN TRIANGLES

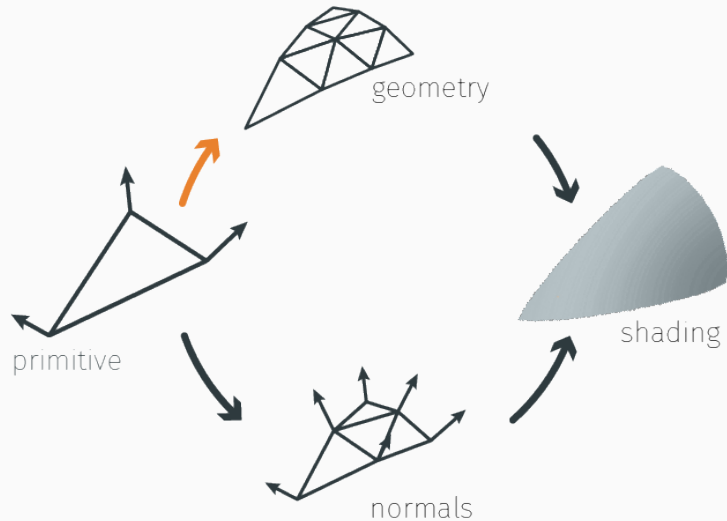
**[Name]** Why PN triangles? Look at the nice result it gives :- ) and we will see that it is easy to extend it to the 'existing' pipeline.

## SINGLE PN TRIANGLE

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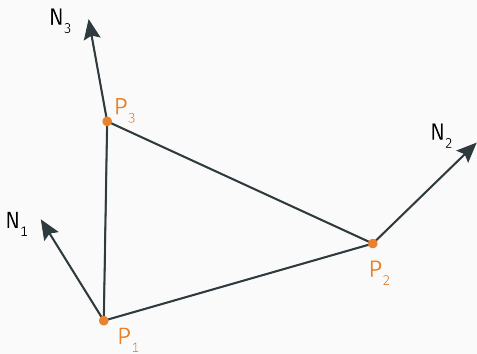
**[Name]** How does one construct a single PN triangle?

*Overview on the next slide*



**[Name]** Why PN triangles? Look at the nice result it gives :- ) and we will see that it is easy to extend it to the 'existing' pipeline. Story about Bezier patches...

enhancement: emphasize vertices better



input primitive

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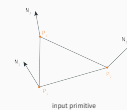
Point Normal triangles

└ Single PN Triangle

└ Geometry

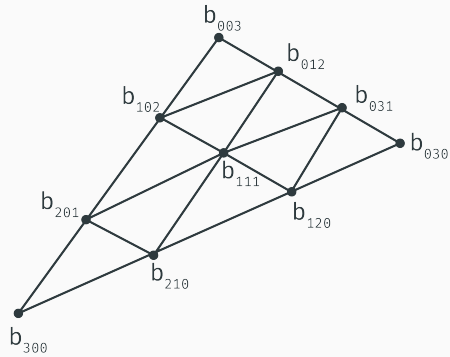
GEOMETRY

enhancement: emphasize vertices better



**[Name]** This a standard triangle primitive, defined by its vertices and normals.

Focus on getting the different control primitives.



control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$

$$b_{030} = P_2,$$

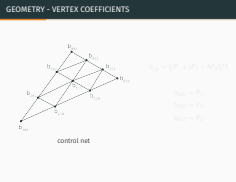
$$b_{003} = P_3$$

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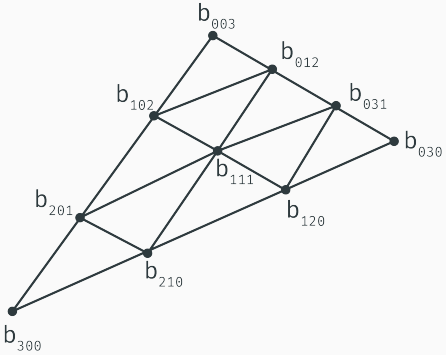
Point Normal triangles

└ Single PN Triangle

└ Geometry - Vertex Coefficients



**[Name]** These are all the initial control point. Evenly divided on the triangle. -> formula



control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$

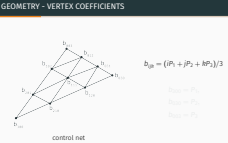
$$b_{030} = P_2,$$

$$b_{003} = P_3$$

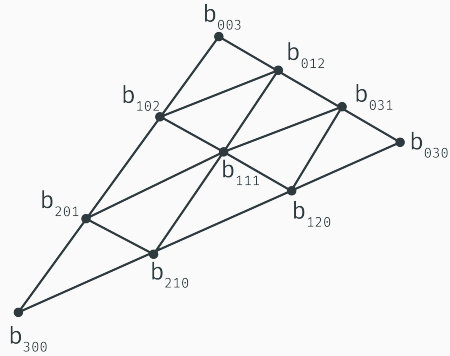
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Point Normal triangles  
└ Single PN Triangle

└ Geometry - Vertex Coefficients



[Name] Nice formula



control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1$$

$$b_{030} = P_2$$

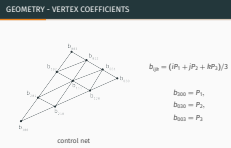
$$b_{003} = P_3$$

2015-12-09

Point Normal triangles

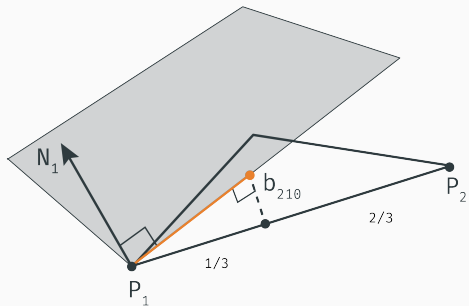
└ Single PN Triangle

└ Geometry - Vertex Coefficients



**[Name]** Stress that the vertex coefficients/control points are the one on the original vertices and that they do not move.





normal projection

$$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3},$$

$$\vdots$$

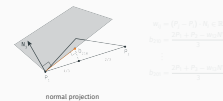
$$b_{201} = \frac{2P_1 + P_3 - w_{13}N_1}{3}$$

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Point Normal triangles

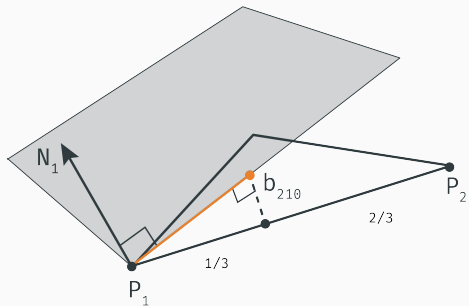
└ Single PN Triangle

└ Geometry - Tangent Coefficients



normal projection

**[Name]** How to get the tangent coefficient (the ones on the edge but now curvy)



normal projection

$$w_i = (P_i - P_1) \cdot N_1 \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3}$$

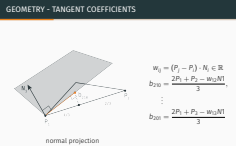
$$b_{201} = \frac{2P_1 + P_2 - w_{11}N_1}{3}$$

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Tangent Coefficients



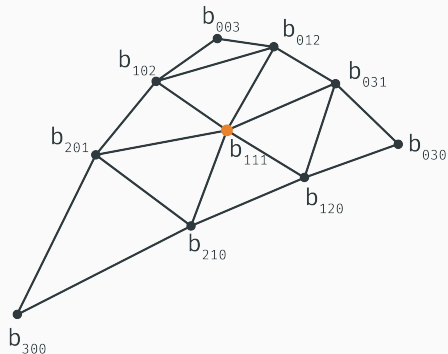
$$w_i = (P_i - P_1) \cdot N_1 \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3}$$

$$\vdots$$

$$b_{201} = \frac{2P_1 + P_2 - w_{11}N_1}{3}$$

**[Name]** Projection of the initial control points on the normal plane of a vertex.



center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

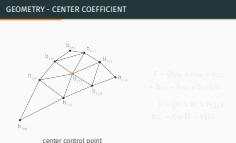
$$b_{111} = E + (E - V)/2$$

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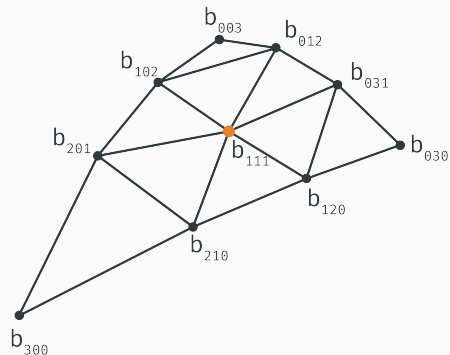
Point Normal triangles

└ Single PN Triangle

└ Geometry - Center Coefficient



**[Name]** Note that this is the result of the previous step -> now only center coefficient is left.



center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$

2015-12-09

Point Normal triangles

└ Single PN Triangle

└ Geometry - Center Coefficient



center control point

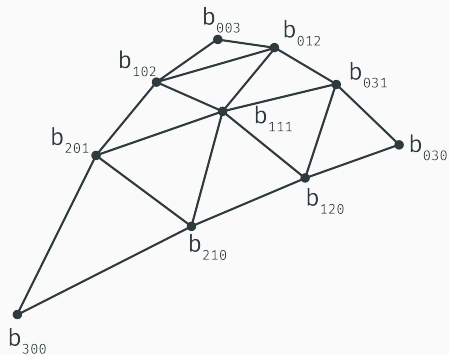
$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$

**[Name]** Average of the tangent coefficients plus half the difference between the tangent and vertex coefficients. -> why?

enhancement: Set result slide to plain



Point Normal triangles

└ Single PN Triangle

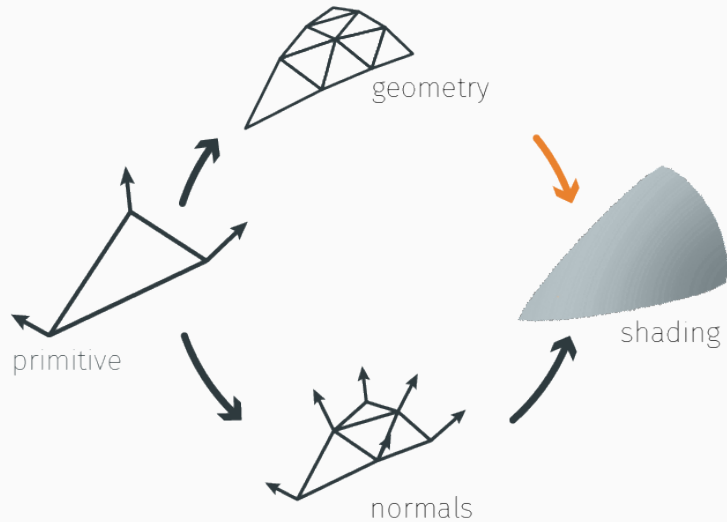
└ Geometry - Result

2015-12-09

enhancement: Set result slide to plain

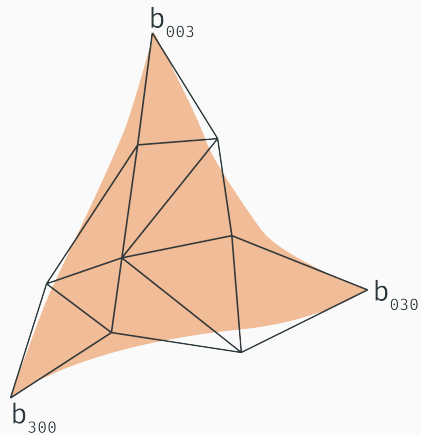


[Name] Results



**[Name]** Overview -> how to get from this to shading.  
Sample/subdivide with formula on following slide.

# CUBIC PATCH



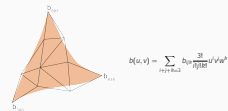
$$b(u, v) = \sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k$$

2015-12-09

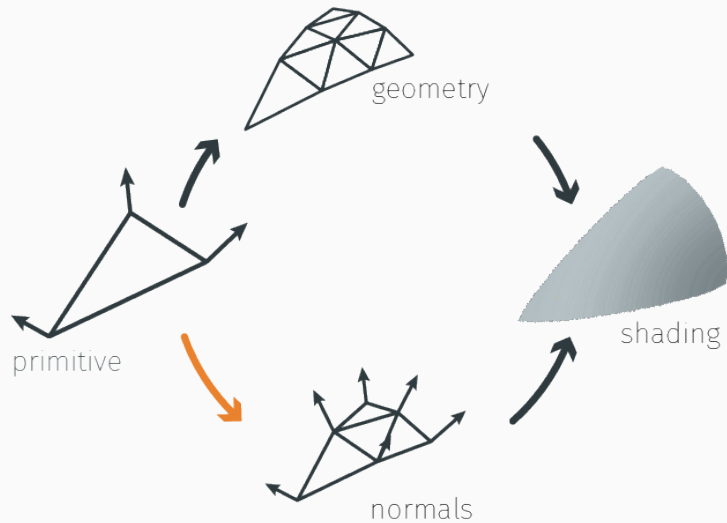
Point Normal triangles

└ Single PN Triangle

└ Cubic patch



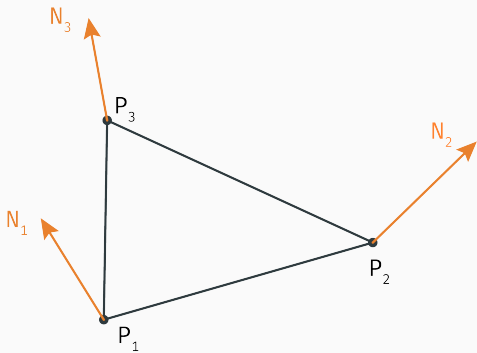
$u, v, w$  are a convex combination **[Name]** Very nice formula with a nice picture.



**[Name]** From the primitive normals the the PN triangle normals



enhancement: emphasize normals more



input primitive

2015-12-09

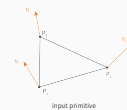
Point Normal triangles

└ Single PN Triangle

└ Normals

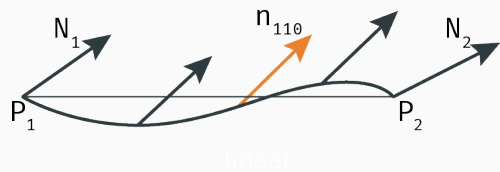
NORMALS

enhancement: emphasize normals more



input primitive

**[Name]** Recap input primitive and with emphasis on the normals.



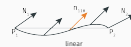
quadratic

2015-12-09

Point Normal triangles

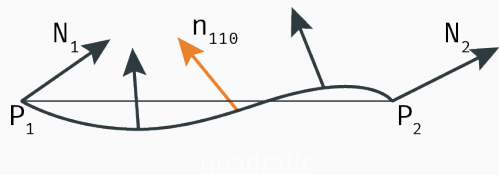
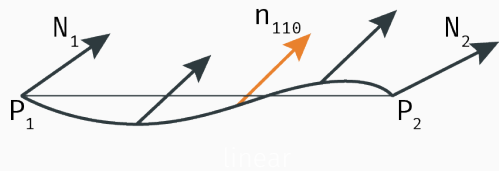
└ Single PN Triangle

└ Normals - theory

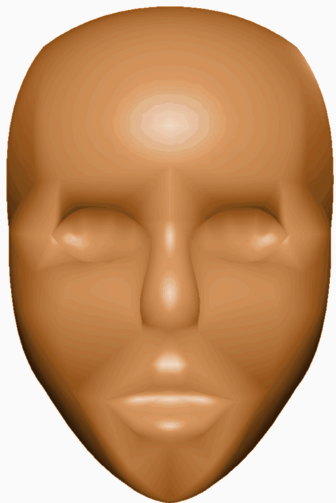


quadratic

**[Name]** Stress that there is a need to capture the cubic bezier curve (inflection points) and that this cannot be



**[Name]** Quadratic does capture inflection points. Trade off between performance and result (maybe?)



linear



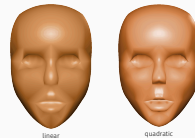
quadratic

2015-12-09

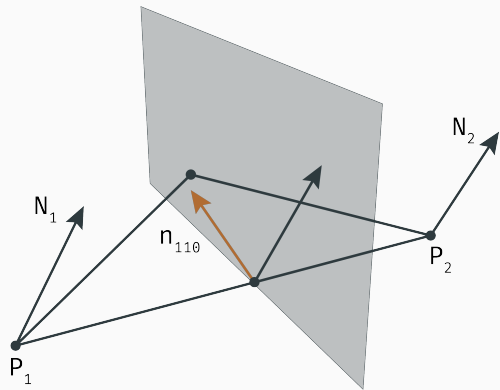
Point Normal triangles

└ Single PN Triangle

└ Normals - example



**[Name]** Look how pretty.



$$v_{ij} = 2 \frac{(P_j - P_i) \cdot (N_i + N_j)}{(P_j - P_i) \cdot (P_j - P_i)} \in \mathbb{R}$$

$$h_{110} = N_1 + N_2 - v_{12}(P_2 - P_1)$$

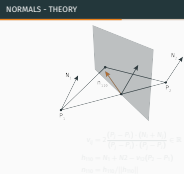
$$n_{110} = h_{110} / ||h_{110}||$$

2015-12-09

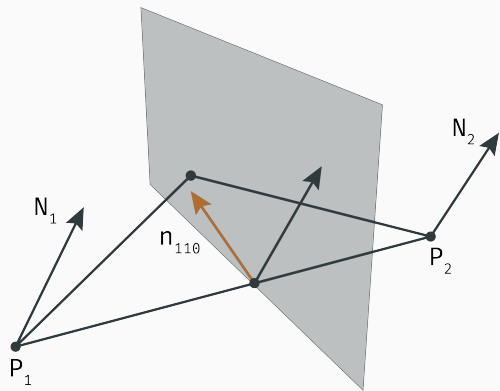
Point Normal triangles

└ Single PN Triangle

└ Normals - theory



**[Name]** Formula in words: reflect the averaged normal (average of N1 and N2) on the plane orthogonal/perpendicular the the edge at the mid point.



$$v_0 = 2 \frac{(P_2 - P_1) \cdot (N_1 + N_2)}{(P_2 - P_1) \cdot (P_2 - P_1)} \in \mathbb{R}$$

$$h_{110} = N_1 + N_2 - v_0(P_2 - P_1)$$

$$n_{110} = h_{110} / \|h_{110}\|$$

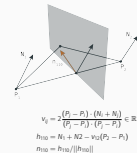
2015-12-09

Point Normal triangles

└ Single PN Triangle

└ Normals - theory

NORMALS - THEORY



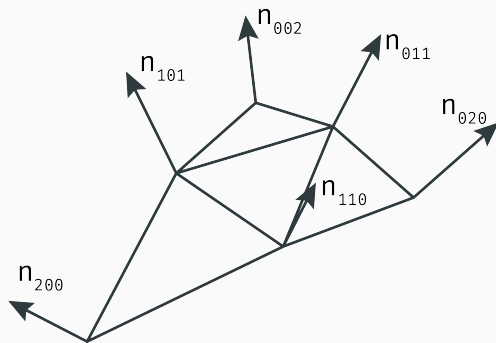
$$v_0 = 2 \frac{(P_2 - P_1) \cdot (N_1 + N_2)}{(P_2 - P_1) \cdot (P_2 - P_1)} \in \mathbb{R}$$

$$h_{110} = N_1 + N_2 - v_0(P_2 - P_1)$$

$$n_{110} = h_{110} / \|h_{110}\|$$

**[Name]** Formula in words: reflect the averaged normal (average of N1 and N2) on the plane orthogonal/perpendicular the the edge at the mid point.

enhancement: Set result slide to plain



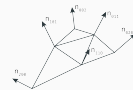
2015-12-09

Point Normal triangles

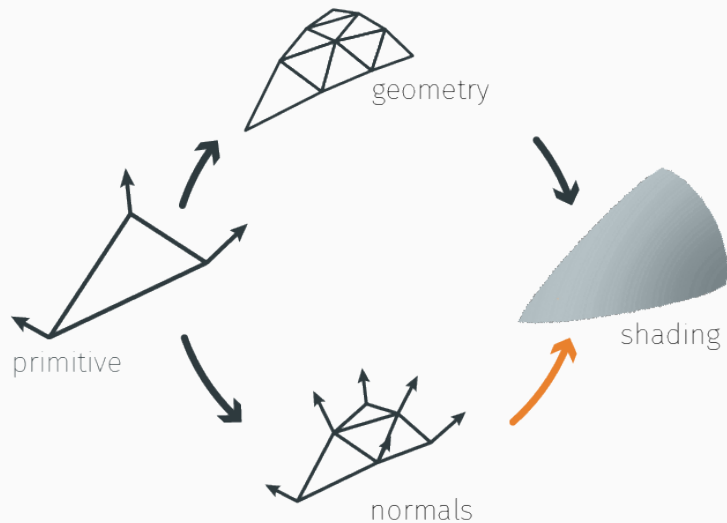
└ Single PN Triangle

└ Normals - result

enhancement: Set result slide to plain



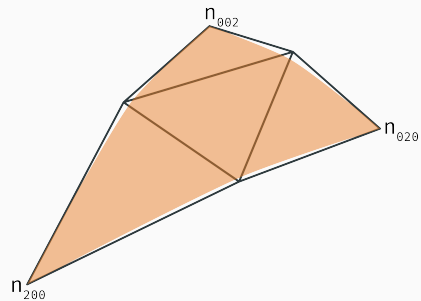
[Name] Result



**[Name]** Why PN triangles? Look at the nice result it gives :- ) and we will see that it is easy to extend it to the 'existing' pipeline.



# QUADRATIC PATCH



$$n(u,v) = \sum_{i+j+k=2} n_{ijk} u^i v^j w^k$$

2015-12-09

Point Normal triangles

└ Single PN Triangle

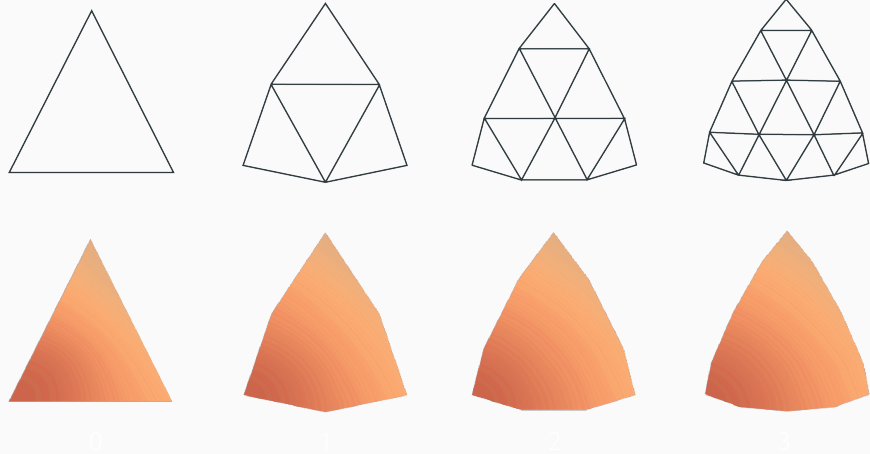
└ Quadratic Patch



$$n(u,v) = \sum_{i+j+k=2} n_{ijk} u^i v^j w^k$$

**[Name]**  $u, v$  and  $w$  are a convex combination

# LEVEL OF DETAIL



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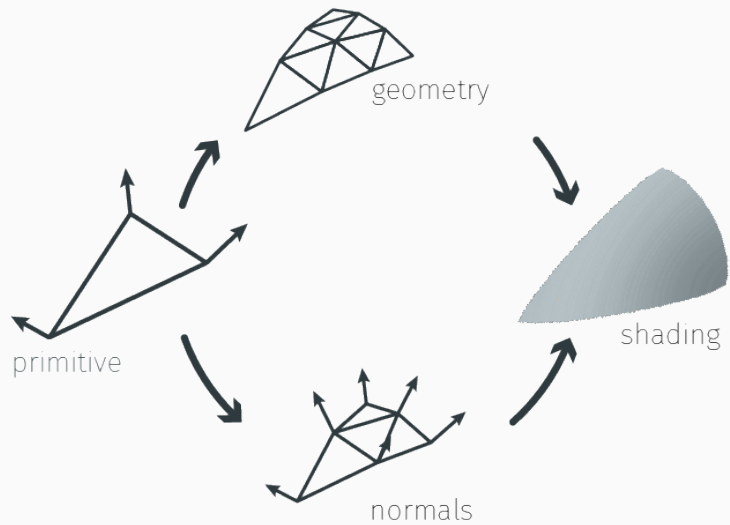
Point Normal triangles

└ Single PN Triangle

└ Level Of Detail



**[Name]** Level of detail -> subdivision -> how many triangles go through to the next shaders.



[Name]

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Point Normal triangles  
└ A Triangle Mesh

A TRIANGLE MESH

## A TRIANGLE MESH

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*“PN triangles should not deviate too much from the original triangle to preserve the shape and avoid interference with other curved triangles.”<sup>1</sup>*

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<sup>1</sup>Vlachos et al.

2015-12-09

Point Normal triangles

└ A Triangle Mesh

└ Properties

*“PN triangles should not deviate too much from the original triangle to preserve the shape and avoid interference with other curved triangles.”<sup>1</sup>*

<sup>1</sup>Vlachos et al.

**[Name]** Problem when combining multiple triangles, so this is a important property

- $C^1$  continuity in the vertex points
- $C^0$  continuity everywhere else

PN triangles have:<sup>2</sup>

- $C^1$  continuity in the vertex points
- $C^0$  continuity everywhere else

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<sup>2</sup>Jiao and Alexander

**[Name]** Continuity  $C^0$  is important -> no gaps. Higher is better because this gives a more smooth result.

# SHARP EDGES

2015-12-09

Point Normal triangles

└ A Triangle Mesh

└ Sharp Edges

SHARP EDGES



mesh



blunt



mesh



sharp

**[Name]** Curved triangles do not always give the preferred results -> sharp edges. Solution is to insert more triangles at the sharp edges -> model needs to be changed :(

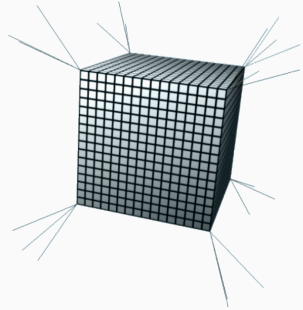
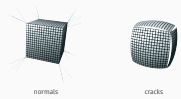
Point Normal triangles

└ A Triangle Mesh

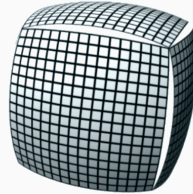
└ Separate Normals

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SEPARATE NORMALS



normals



cracks

**[Name]** Beyond the scope of the paper extension exist to overcome the problem what you have when combining multiple meshes. Story about shared vertices.



2015-12-09

Point Normal triangles  
└ Graphics Pipeline

GRAPHICS PIPELINE

## GRAPHICS PIPELINE

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**[Name]** How does one construct a single PN triangle?

Point Normal triangles

└ Graphics Pipeline

└ Hardware - Pipelines

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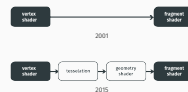
2001



2001

2015

**[Name]** Great part of the paper stresses the point that it can easily be implemented as a preprocessing step (CPU).  
2001 pipeline (OpenGL 1.3)



2001



2015

[Name] 2015 we have OpenGL 4.5 with more programmable shaders and the whole process can be done on the GPU.

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Point Normal triangles  
└ Conclusion

CONCLUSION

## CONCLUSION

# CONCLUSION

Conclusion?

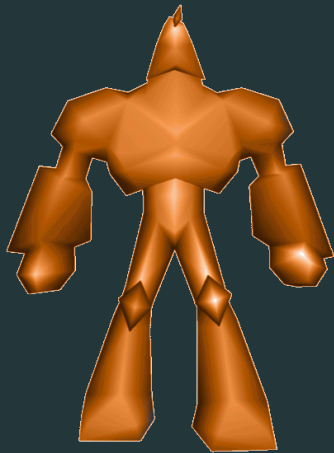
2015-12-09

- Point Normal triangles
  - └ Conclusion
- └ conclusion

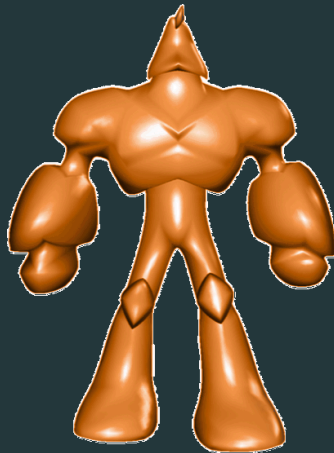
Conclusion?

[Name] Conclusion?

## QUESTIONS?



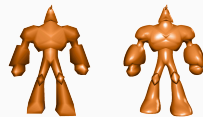
TRIANGLES






PN TRIANGLES

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Point Normal triangles  
└ Conclusion



-  Xiangmin Jiao and Phillip J Alexander. “Parallel feature-preserving mesh smoothing”. In: *Computational Science and Its Applications–ICCSA 2005*. Springer, 2005, pp. 1180–1189.
-  J McDonald and M Kilgard. *Crack-free point-normal triangles using adjacent edge normals*. 2010.
-  Alex Vlachos et al. “Curved PN triangles”. In: *Proceedings of the 2001 symposium on Interactive 3D graphics*. ACM. 2001, pp. 159–166.

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Point Normal triangles

└ Conclusion

└ References

-  Xiangmin Jiao and Phillip J Alexander. “Parallel feature-preserving mesh smoothing”. In: *Computational Science and Its Applications–ICCSA 2005*. Springer, 2005, pp. 1180–1189.
-  J McDonald and M Kilgard. *Crack-free point-normal triangles using adjacent edge normals*. 2010.
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