

POINT NORMAL TRIANGLES

Rick van Veen Laura Baakman

December 14, 2015

Advanced Computer Graphics

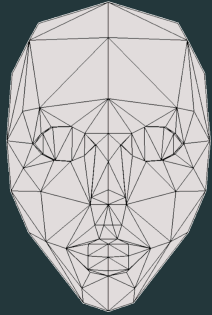
2015-12-09

Point Normal triangles

POINT NORMAL TRIANGLES

Rick van Veen Laura Baakman
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Advanced Computer Graphics

[Rick] Welcome everybody. Tell people that PN means Point Normal triangles.



INPUT MESH



GOURAUD



PN GEOMETRY



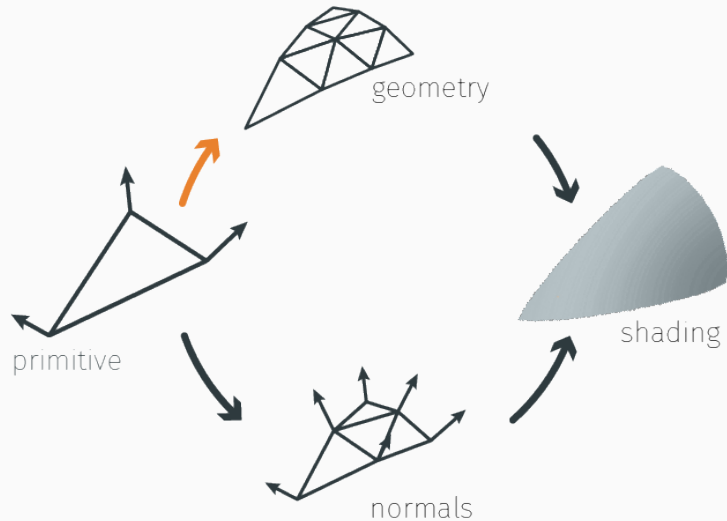
PN TRIANGLES

[Name] Why PN triangles? Look at the nice result it gives :-)) and we will see that it easy to extend it to the 'existing' pipeline.

SINGLE PN TRIANGLE

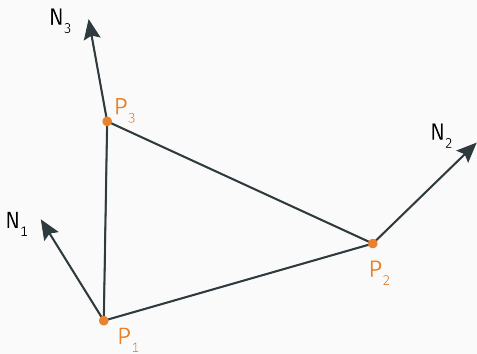
[Name] How does one construct a single PN triangle?

Overview on the next slide



[Name] Why PN triangles? Look at the nice result it gives :-) and we will see that it is easy to extend it to the 'existing' pipeline. Story about Bezier patches...

enhancement: emphasize vertices better



input primitive

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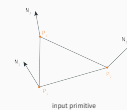
Point Normal triangles

└ Single PN Triangle

└ Geometry

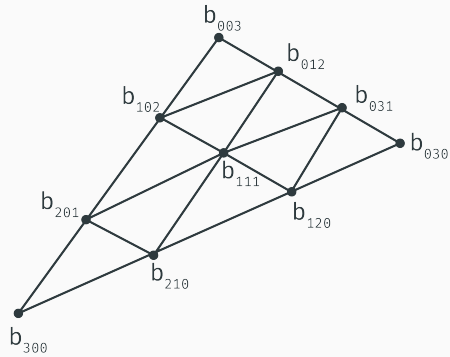
GEOMETRY

enhancement: emphasize vertices better



[Name] This a standard triangle primitive, defined by its vertices and normals.

Focus on getting the different control primitives.



control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$

$$b_{030} = P_2,$$

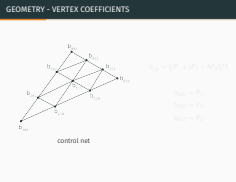
$$b_{003} = P_3$$

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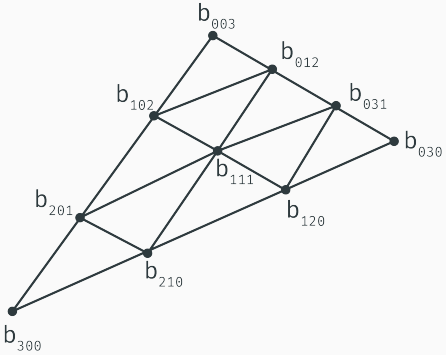
Point Normal triangles

└ Single PN Triangle

└ Geometry - Vertex Coefficients



[Name] These are all the initial control point. Evenly divided on the triangle. -> formula



control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$

$$b_{030} = P_2,$$

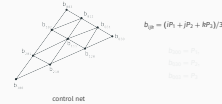
$$b_{003} = P_3$$

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Vertex Coefficients

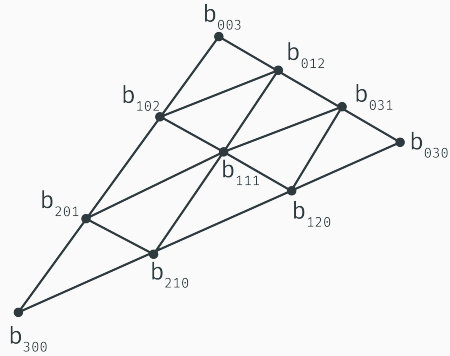


$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$
$$b_{030} = P_2,$$
$$b_{003} = P_3$$

control net

[Name] Nice formula



control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1$$

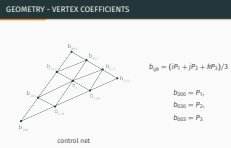
$$b_{030} = P_2$$

$$b_{003} = P_3$$

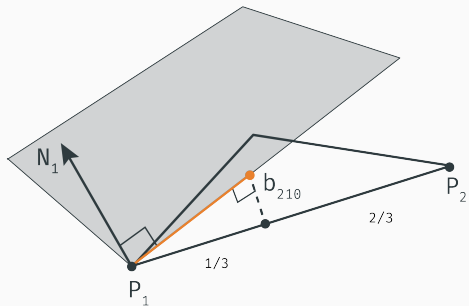
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Point Normal triangles
└ Single PN Triangle

└ Geometry - Vertex Coefficients



[Name] Stress that the vertex coefficients/control points are the one on the original vertices and that they do not move.



normal projection

$$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3},$$

$$\vdots$$

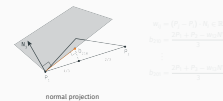
$$b_{201} = \frac{2P_1 + P_3 - w_{13}N_1}{3}$$

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Point Normal triangles

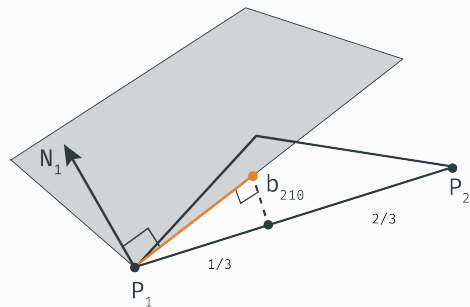
└ Single PN Triangle

└ Geometry - Tangent Coefficients



normal projection

[Name] How to get the tangent coefficient (the ones on the edge but now curvy)



normal projection

$$w_i = (P_i - P_1) \cdot N_1 \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3}$$

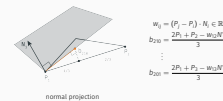
$$b_{201} = \frac{2P_1 + P_2 - w_{11}N_1}{3}$$

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Tangent Coefficients



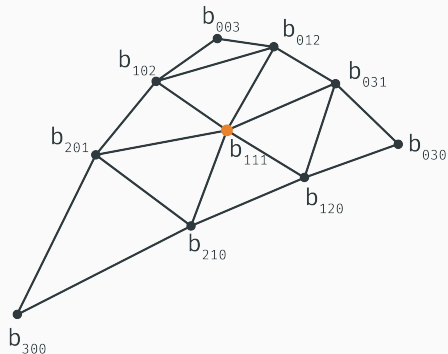
$$w_i = (P_i - P_1) \cdot N_1 \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3}$$

$$\vdots$$

$$b_{201} = \frac{2P_1 + P_2 - w_{11}N_1}{3}$$

[Name] Projection of the initial control points on the normal plane of a vertex.



center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Center Coefficient



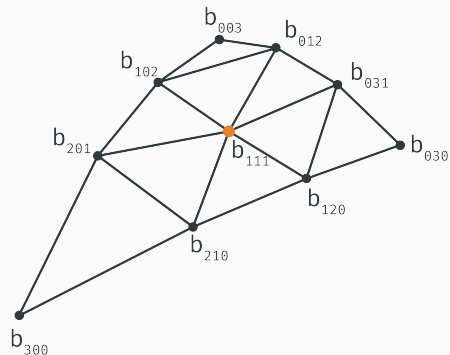
center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$

[Name] Note that this is the result of the previous step -> now only center coefficient is left.



center control point

$$E = (b_{210} + b_{120} + b_{201} + b_{012} + b_{102} + b_{021})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$

2015-12-09

Point Normal triangles

└ Single PN Triangle

└ Geometry - Center Coefficient



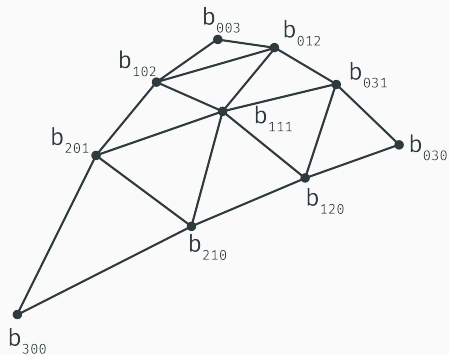
$$E = (b_{210} + b_{120} + b_{201} + b_{012} + b_{102} + b_{021})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$

[Name] Average of the tangent coefficients plus half the difference between the tangent and vertex coefficients. -> why?

enhancement: Set result slide to plain



Point Normal triangles

└ Single PN Triangle

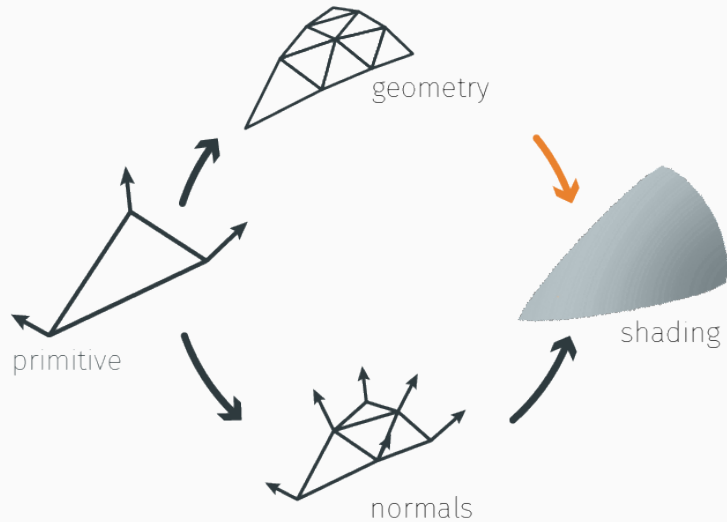
└ Geometry - Result

2015-12-09

enhancement: Set result slide to plain

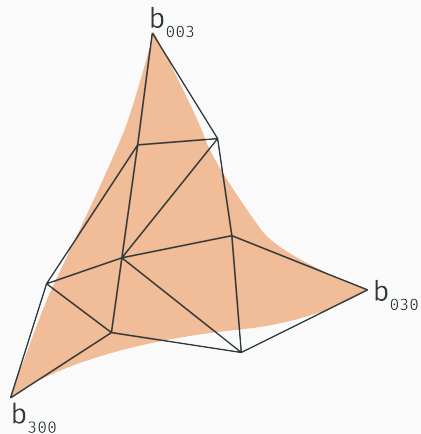


[Name] Results



[Name] Overview -> how to get from this to shading.
Sample/subdivide with formula on following slide.

CUBIC PATCH



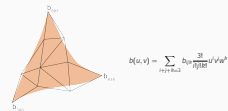
$$b(u, v) = \sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k$$

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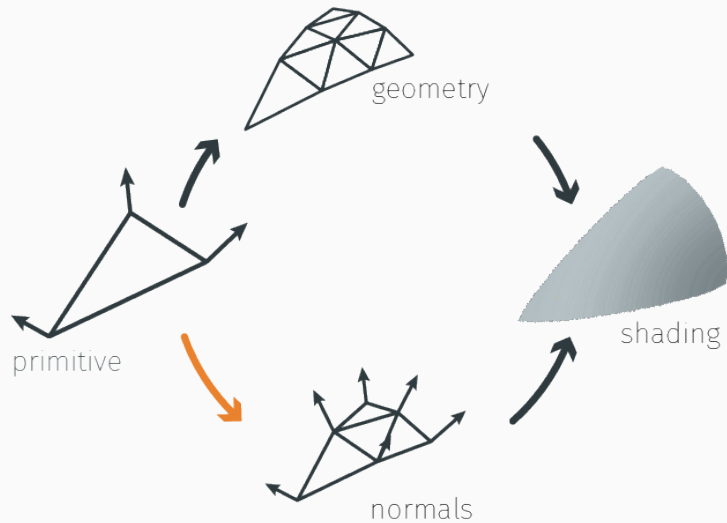
Point Normal triangles

└ Single PN Triangle

└ Cubic patch

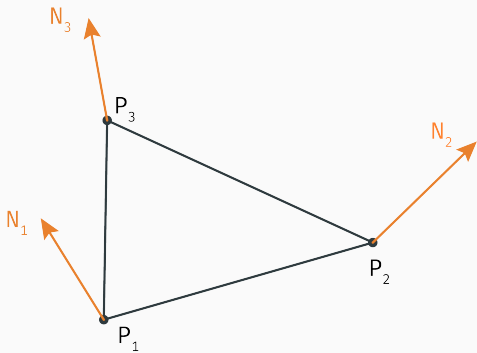


u, v, w are a convex combination **[Name]** Very nice formula with a nice picture.



[Name] From the primitive normals the the PN triangle normals

enhancement: emphasize normals more



input primitive

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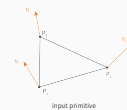
Point Normal triangles

└ Single PN Triangle

└ Normals

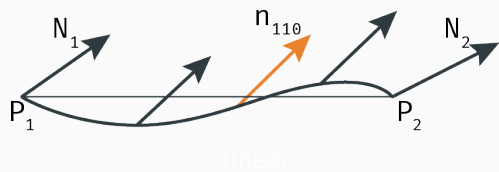
NORMALS

enhancement: emphasize normals more



input primitive

[Name] Recap input primitive and with emphasis on the normals.



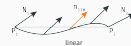
quadratic

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Point Normal triangles

└ Single PN Triangle

└ Normals - theory



quadratic

[Name] Stress that there is a need to capture the cubic bezier curve (inflection points) and that this cannot be

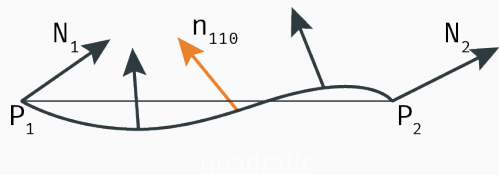
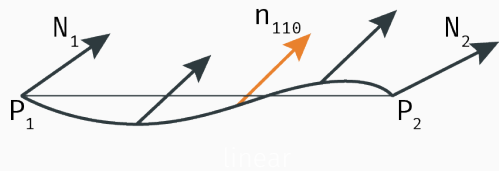
2015-12-09

Point Normal triangles

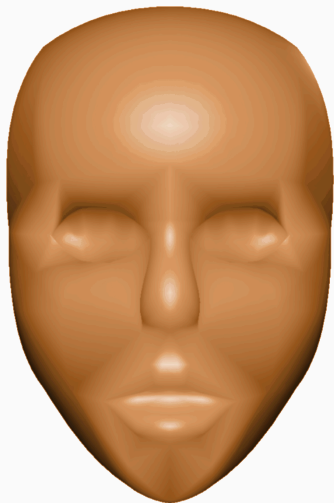
└ Single PN Triangle

└ Normals - theory

NORMALS - THEORY



[Name] Quadratic does capture inflection points. Trade off between performance and result (maybe?)



linear



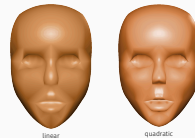
quadratic

2015-12-09

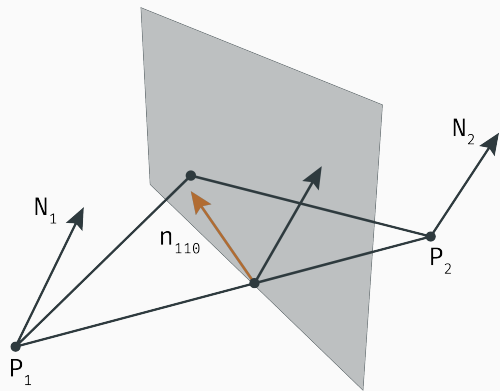
Point Normal triangles

└ Single PN Triangle

└ Normals - example



[Name] Look how pretty.



$$v_{ij} = 2 \frac{(P_j - P_i) \cdot (N_i + N_j)}{(P_j - P_i) \cdot (P_j - P_i)} \in \mathbb{R}$$

$$h_{110} = N_1 + N_2 - v_{12}(P_2 - P_1)$$

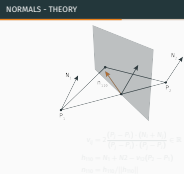
$$n_{110} = h_{110} / ||h_{110}||$$

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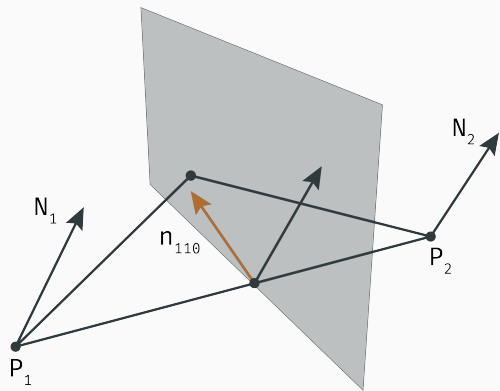
Point Normal triangles

└ Single PN Triangle

└ Normals - theory



[Name] Formula in words: reflect the averaged normal (average of N1 and N2) on the plane orthogonal/perpendicular the the edge at the mid point.



$$v_0 = 2 \frac{(P_2 - P_1) \cdot (N_1 + N_2)}{(P_2 - P_1) \cdot (P_2 - P_1)} \in \mathbb{R}$$

$$h_{110} = N_1 + N_2 - v_0(P_2 - P_1)$$

$$n_{110} = h_{110} / \|h_{110}\|$$

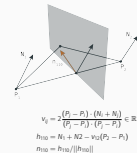
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Point Normal triangles

└ Single PN Triangle

└ Normals - theory

NORMALS - THEORY



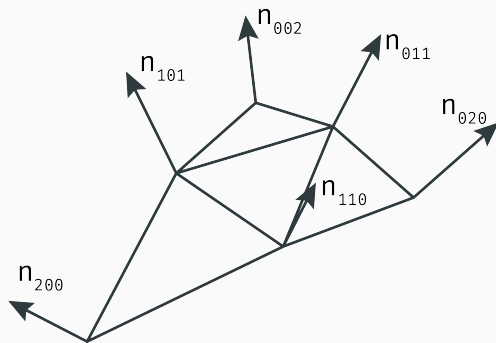
$$v_0 = 2 \frac{(P_2 - P_1) \cdot (N_1 + N_2)}{(P_2 - P_1) \cdot (P_2 - P_1)} \in \mathbb{R}$$

$$h_{110} = N_1 + N_2 - v_0(P_2 - P_1)$$

$$n_{110} = h_{110} / \|h_{110}\|$$

[Name] Formula in words: reflect the averaged normal (average of N1 and N2) on the plane orthogonal/perpendicular the the edge at the mid point.

enhancement: Set result slide to plain



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Point Normal triangles

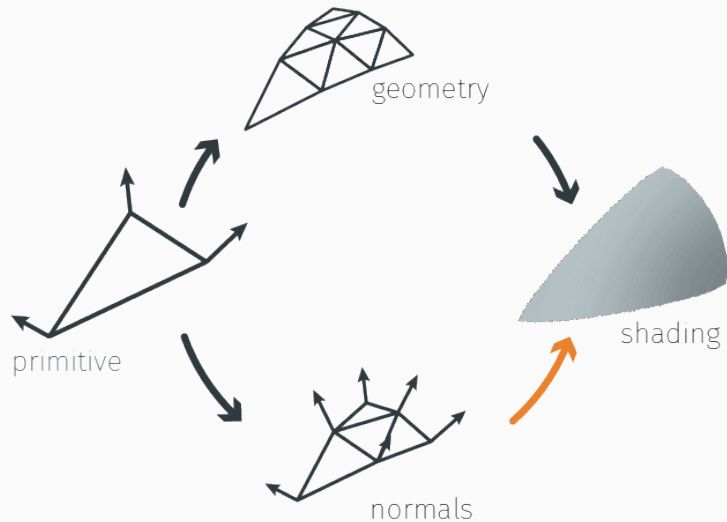
└ Single PN Triangle

└ Normals - result

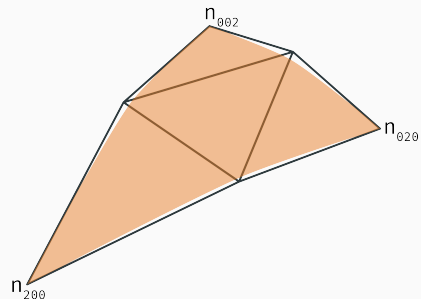
enhancement: Set result slide to plain



[Name] Result



[Name] Why PN triangles? Look at the nice result it gives :-) and we will see that it easy to extend it to the 'existing' pipeline.



$$n(u,v) = \sum_{i+j+k=2} n_{ijk} u^i v^j w^k$$

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Point Normal triangles

└ Single PN Triangle

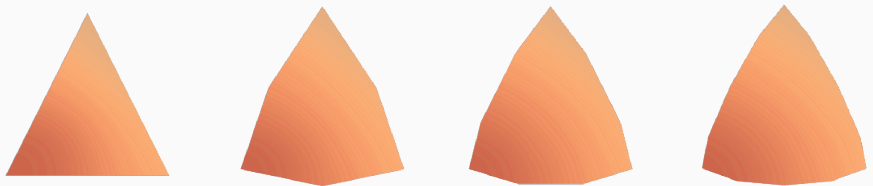
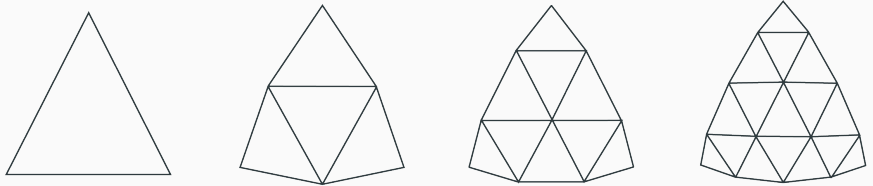
└ Quadratic Patch



$$n(u,v) = \sum_{i+j+k=2} n_{ijk} u^i v^j w^k$$

u, v and w are a convex combination

LEVEL OF DETAIL



0

1

2

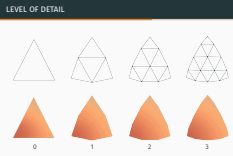
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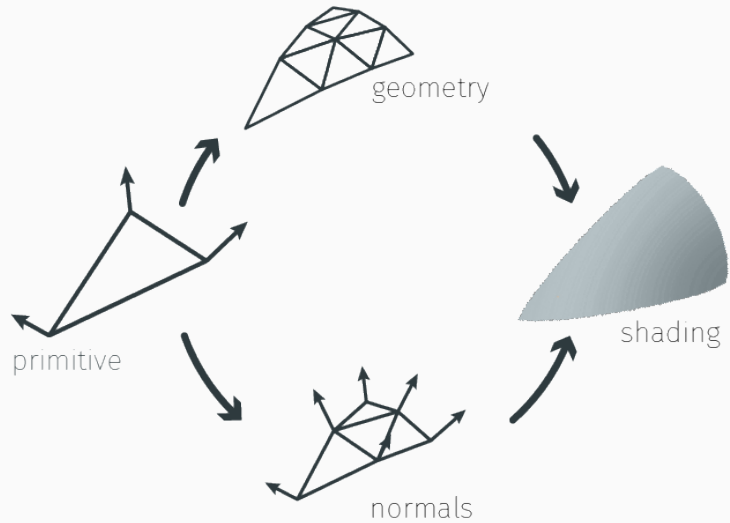
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Point Normal triangles

└ Single PN Triangle

└ Level Of Detail





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Point Normal triangles
└ A Triangle Mesh

A TRIANGLE MESH

A TRIANGLE MESH

“PN triangles should not deviate too much from the original triangle to preserve the shape and avoid interference with other curved triangles.”¹

¹Vlachos et al.

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Point Normal triangles

└ A Triangle Mesh

└ Properties

“PN triangles should not deviate too much from the original triangle to preserve the shape and avoid interference with other curved triangles.”¹

¹Vlachos et al.

- C^1 continuity in the vertex points
- C^0 continuity everywhere else

PN triangles have:²

- C^1 continuity in the vertex points
- C^0 continuity everywhere else

²Jiao and Alexander

SHARP EDGES



mesh



blunt



mesh

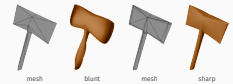


sharp

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- Point Normal triangles
- └ A Triangle Mesh
- └ Sharp Edges

SHARP EDGES



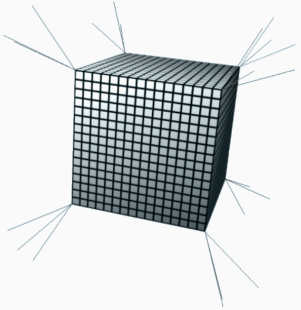
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Point Normal triangles

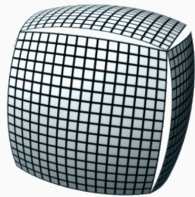
└ A Triangle Mesh

└ Separate Normals

SEPARATE NORMALS



normals



cracks

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Point Normal triangles
└ Graphics Pipeline

GRAPHICS PIPELINE

GRAPHICS PIPELINE

2015-12-09

Point Normal triangles
└ Graphics Pipeline

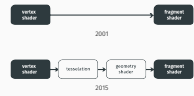
└ Hardware - Pipelines



2001

2015

2015-12-09



2001



2015

2015-12-09

Point Normal triangles
└ Conclusion

CONCLUSION

CONCLUSION

CONCLUSION

2015-12-09

Point Normal triangles

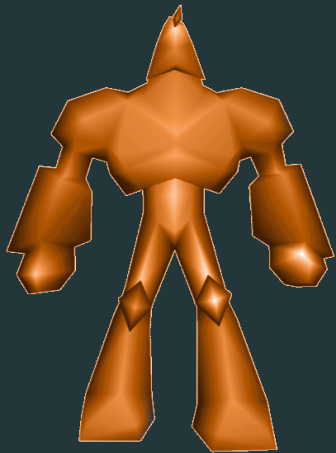
└ Conclusion

└ conclusion

Some conclusion?

Some conclusion?

QUESTIONS?



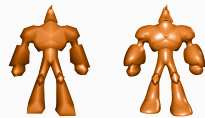
TRIANGLES






PN TRIANGLES

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Point Normal triangles
└ Conclusion



-  Xiangmin Jiao and Phillip J Alexander. “Parallel feature-preserving mesh smoothing”. In: *Computational Science and Its Applications–ICCSA 2005*. Springer, 2005, pp. 1180–1189.
-  J McDonald and M Kilgard. *Crack-free point-normal triangles using adjacent edge normals*. 2010.
-  Alex Vlachos et al. “Curved PN triangles”. In: *Proceedings of the 2001 symposium on Interactive 3D graphics*. ACM. 2001, pp. 159–166.

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Point Normal triangles

└ Conclusion

└ References

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