POINT NORMAL TRIANGLES

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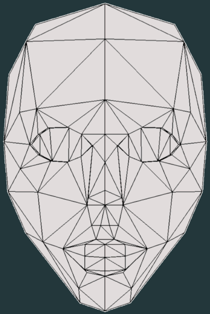
December 14, 2015

Advanced Computer Graphics



INPUT MESH

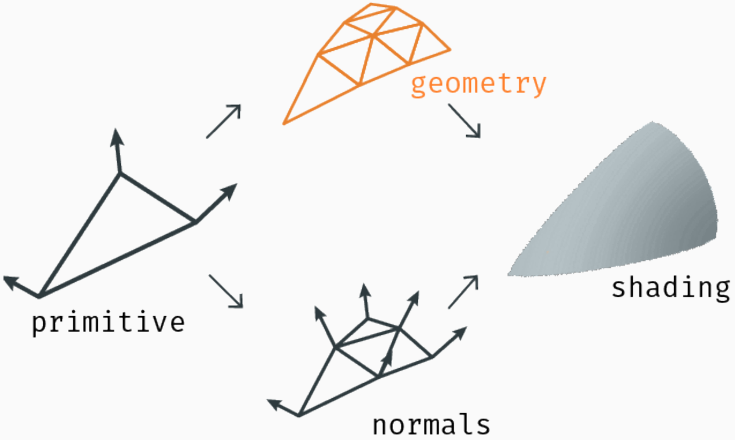
GOURAUD

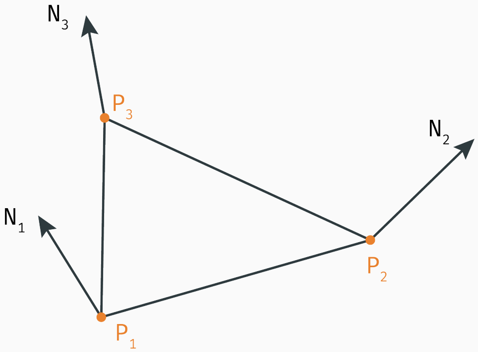
PN GEOMETRY

PN TRIANGLES

SINGLE PN TRIANGLE

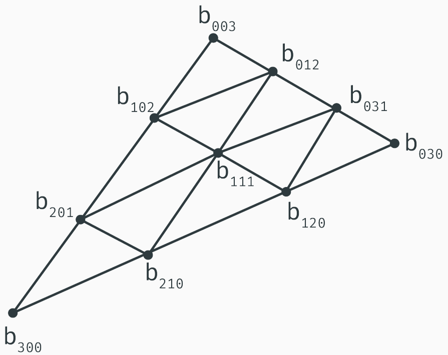
OVERVIEW





Input primitive

GEOMETRY - VERTEX COEFFICIENTS

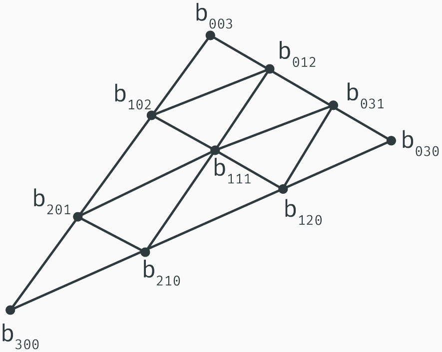


Control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$
$$b_{030} = P_2,$$
$$b_{003} = P_3$$

GEOMETRY - VERTEX COEFFICIENTS



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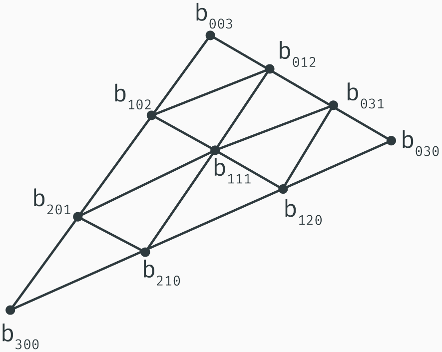

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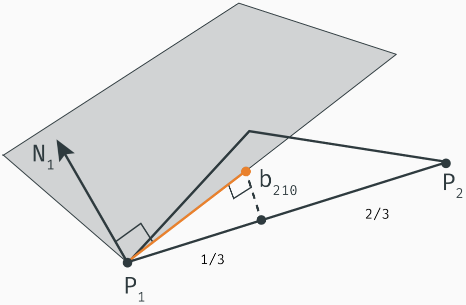
GEOMETRY - TANGENT COEFFICIENTS



Normal projection

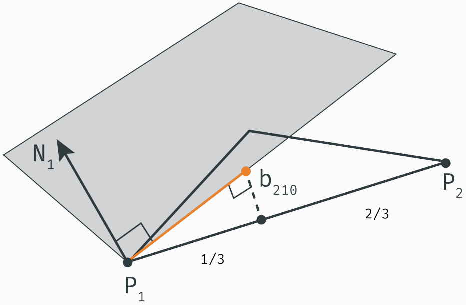
$$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N1}{3},$$

$$\vdots$$

$$b_{201} = \frac{2P_1 + P_3 - w_{13}N1}{3}$$

GEOMETRY - TANGENT COEFFICIENTS



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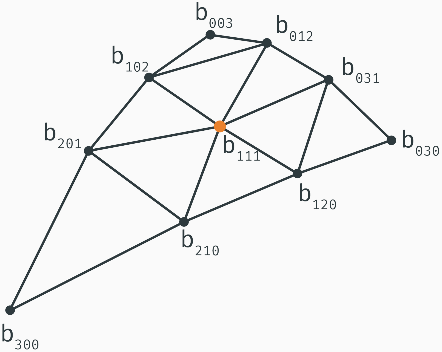
GEOMETRY - CENTER COEFFICIENT



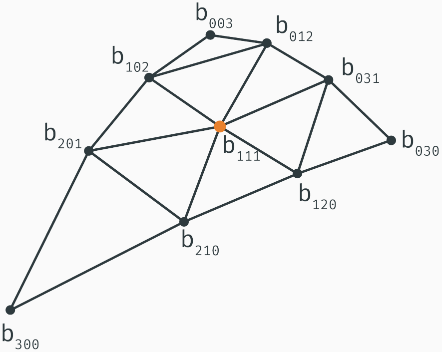
Center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$

GEOMETRY - CENTER COEFFICIENT



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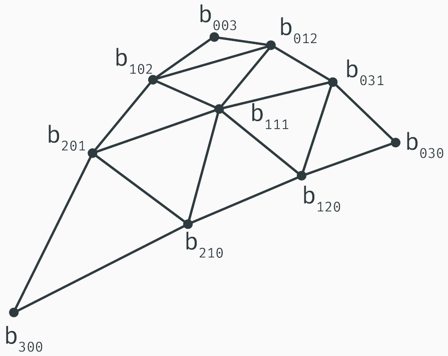
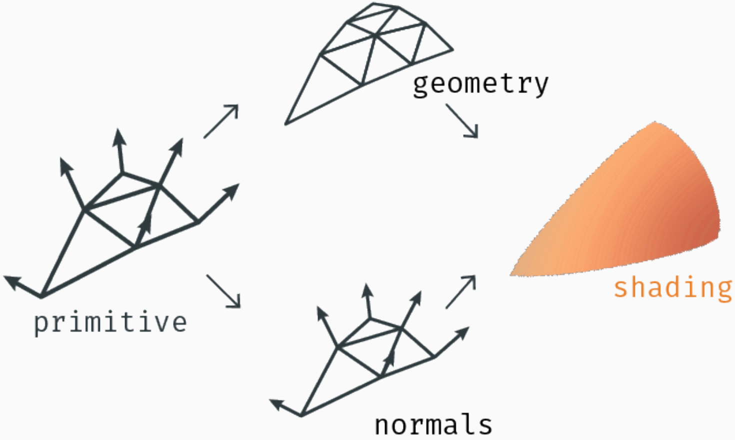
GEOMETRY



OVERVIEW

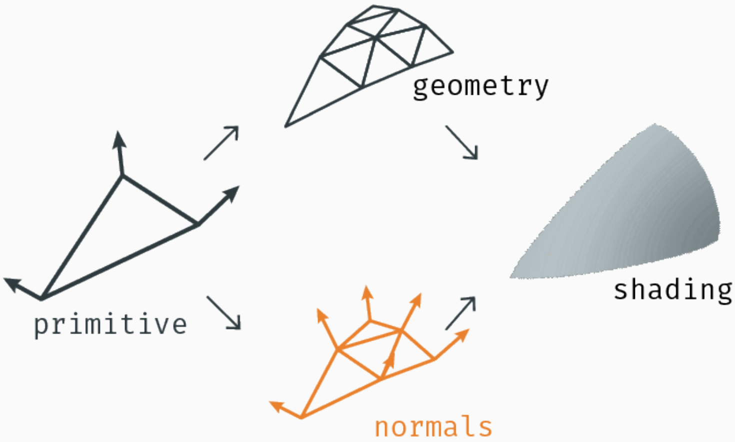


CUBIC PATCH

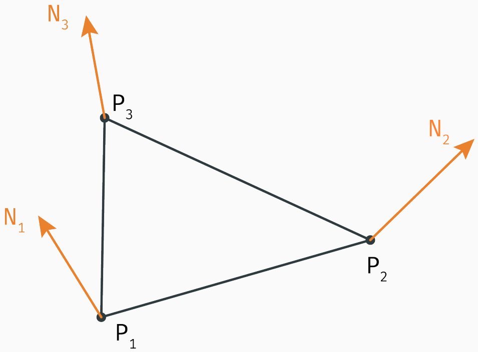
$$b : \mathbb{R}^2 \to \mathbb{R}^3, \text{ for } w = 1 - u - v, u, v, w \geq 0$$

$$\begin{aligned}
b(u, v) &= \sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k \\
&= b_{300} w^3 + b_{030} u^3 + b_{003} v^3 \\
&+ b_{210} 3w^2 u + b_{120} 3wu^2 + b_{201} 3w^2 v \\
&+ b_{021} 3u^2 v + b_{102} 3wv^2 + b_{012} 3uv^2 \\
&+ b_{111} 6wuv.
\end{aligned}$$

OVERVIEW

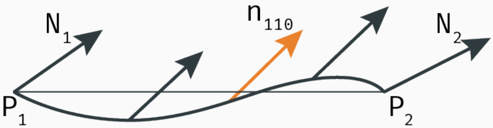


NORMALS



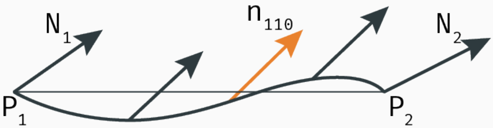
Input primitive

NORMALS - THEORY

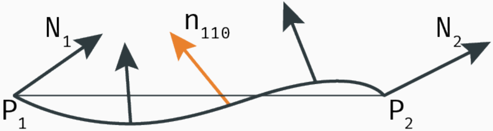


Linear

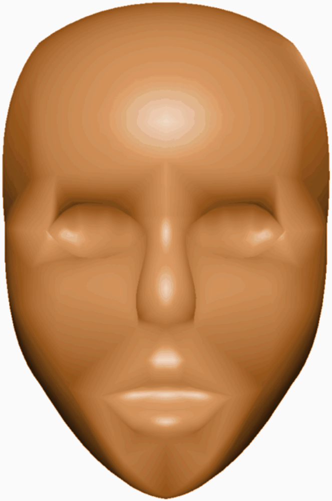
Quadratic



Linear



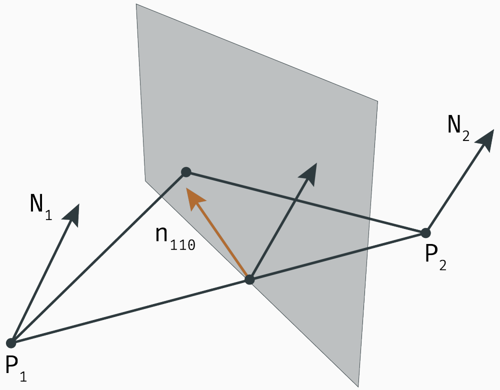
Quadratic



Linear

Quadratic

NORMALS - THEORY

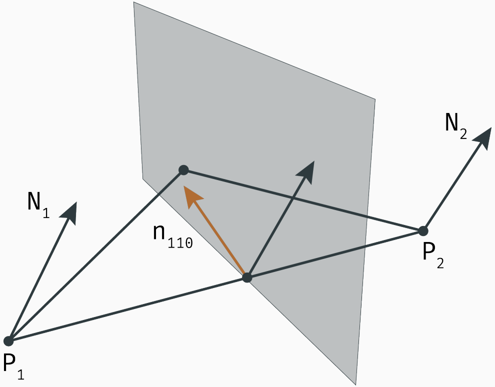


$$v_{ij} = 2\frac{(P_j - P_i)\cdot(N_i + N_j)}{(P_j - P_i)\cdot(P_j - P_i)} \in \mathbb{R}$$

$$h_{110} = N_1 + N2 - v_{12}(P_2 - P_1)$$

$$n_{110} = h_{110}/||h_{110}||$$

NORMALS - THEORY



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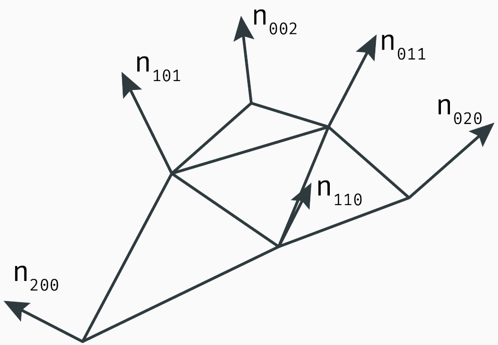
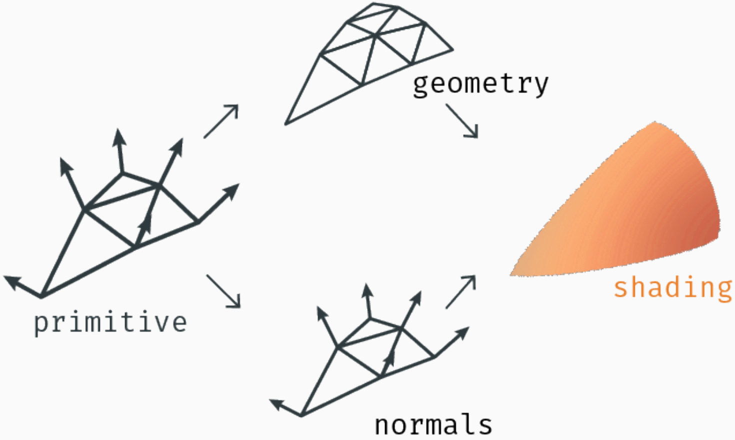
NORMALS - RESULT



OVERVIEW

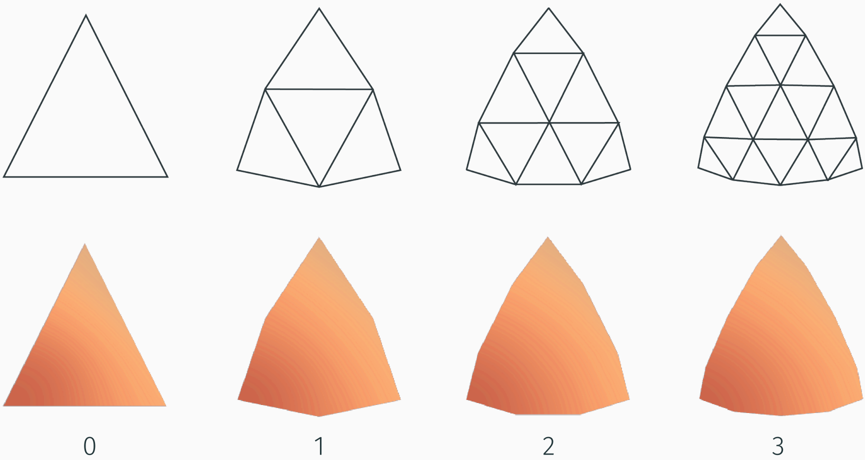


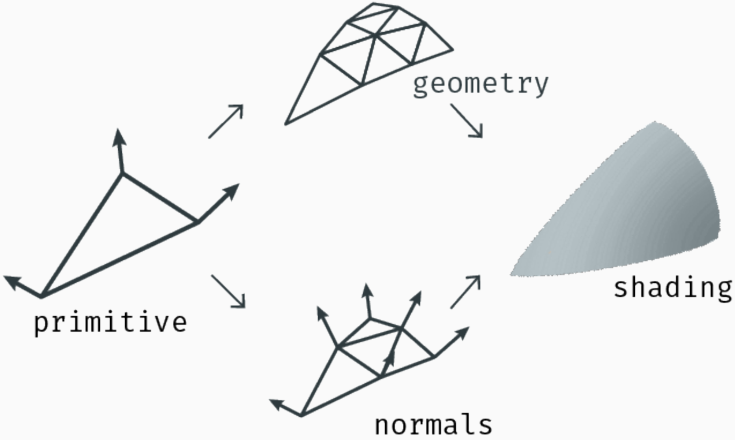
QUADRATIC PATCH

$$n : \mathbb{R}^2 \to \mathbb{R}^3, \text{ for } w = 1 - u - v,\ u, v, w \geq 0$$

$$\begin{aligned} n(u, v) &= \sum_{i+j+k=2} n_{ijk} u^i v^j w^k \\ &= n_{200} w^2 + n_{020} u^2 + n_{002} v^2 \\ &+ n_{110} wu + n_{011} uv + n_{101} wv \end{aligned}$$

LEVEL OF DETAIL




geometry
primitive
shading
normals

A TRIANGLE MESH

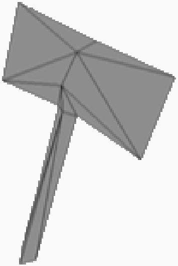
PROPERTIES

“Pn triangles should not deviate too much from the original triangle to preserve the shape and avoid interference with other curved triangles.” Vlachos et al.

CONTINUITY

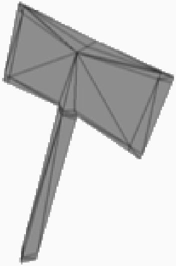
“PN triangles do not usually join with tangent continuity except at the corners.” Vlachos et al.

SHARP EDGES



Blunt

Blunt



Sharp

Sharp

GRAPHICS PIPELINE

2001

HARDWARE - PIPELINES

2001

CONCLUSION

FIN.

REFERENCES

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Alex Vlachos et al. “Curved PN triangles”. In: *Proceedings of the 2001 symposium on Interactive 3D graphics*. ACM. 2001, pp. 159–166.