

# POINT NORMAL TRIANGLES

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Rick van Veen   Laura Baakman

December 14, 2015

Advanced Computer Graphics

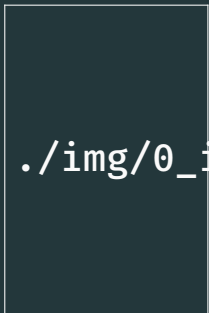
2015-12-09

Point Normal triangles

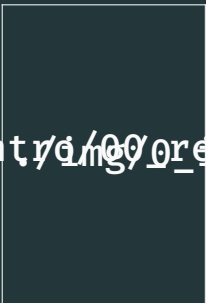
POINT NORMAL TRIANGLES

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INPUT MESH



GOURAUD



PN GEOMETRY



PN TRIANGLES

2015-12-09

Point Normal triangles

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Point Normal triangles  
└ Single PN Triangle

SINGLE PN TRIANGLE

## SINGLE PN TRIANGLE

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# OVERVIEW

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Point Normal triangles

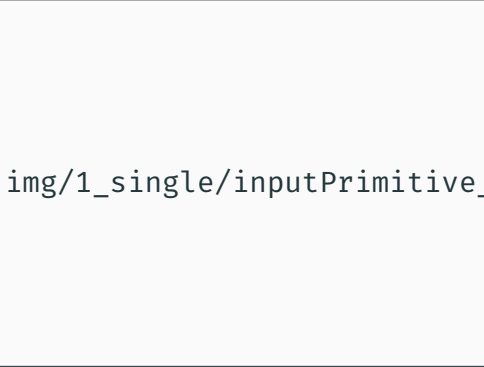
└ Single PN Triangle

└ Overview

./img/1\_single/recap\_inputToGeom.png

./img/1\_single/recap\_inputToGeom.png

enhancement: emphasize vertices better



img/1\_single/inputPrimitive\_emphGeometry.

input primitive

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Point Normal triangles

└ Single PN Triangle

└ Geometry

enhancement: emphasize vertices better

img/1\_single/inputPrimitive

emphGeometry.

input primitive

img/1\_single/geometry\_1.png

control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$

$$b_{030} = P_2,$$

$$b_{003} = P_3$$

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Vertex Coefficients



$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$
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img/1\_single/geometry\_1.png

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└ Geometry - Vertex Coefficients

img/1\_single/geometry\_1.png

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$$b_{300} = P_1,$$

$$b_{030} = P_2,$$

$$b_{003} = P_3$$



img/1\_single/geometry\_2.png

normal projection

$$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3},$$

$$\vdots$$

$$b_{201} = \frac{2P_1 + P_3 - w_{13}N_1}{3}$$

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Tangent Coefficients



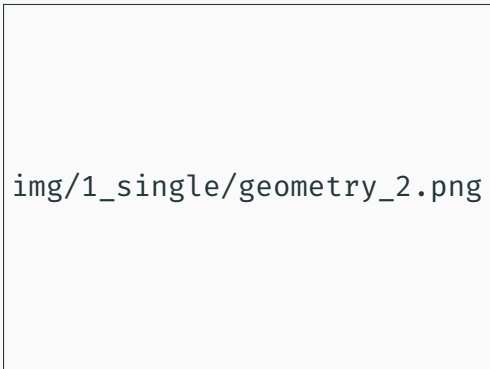
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$$\vdots$$

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# GEOMETRY - TANGENT COEFFICIENTS



normal projection

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Tangent Coefficients



normal projection

$$w_i = (P_j - P_i) \cdot N_i \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3}$$

$$\vdots$$

$$b_{201} = \frac{2P_1 + P_2 - w_{13}N_1}{3}$$

img/1\_single/geometry\_3.png

center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$
$$b_{111} = E + (E - V)/2$$

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Center Coefficient

img/1\_single/geometry\_3.png

center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$
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img/1\_single/geometry\_3.png

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Center Coefficient

img/1\_single/geometry\_3.png

center control point

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$$V = (P_1 + P_2 + P_3)/3,$$
$$b_{111} = E + (E - V)/2$$

enhancement: Set result slide to plain

img/1\_single/geometry\_4.png

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Result

enhancement: Set result slide to plain

img/1\_single/geometry\_4.png

`./img/1_single/recap_geomToShading.png`

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Point Normal triangles

└ Single PN Triangle

└ Overview

`./img/1_single/recap_geomToShading.png`

# CUBIC PATCH

Spacing van de for all

Plaatje?

$b: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ , for  $w = 1 - u - v$ ,  $u, v, w \geq 0$

$$\begin{aligned} b(u, v) &= \sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k \\ &= b_{300} w^3 + b_{030} u^3 + b_{003} v^3 \\ &\quad + b_{210} 3w^2 u + b_{120} 3wu^2 + b_{201} 3w^2 v \\ &\quad + b_{021} 3u^2 v + b_{102} 3wv^2 + b_{012} 3uv^2 \\ &\quad + b_{111} 6wuv. \end{aligned}$$

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Point Normal triangles

└ Single PN Triangle

└ Cubic patch

CUBIC PATCH

Spacing van de for all

Plaatje?

$$\begin{aligned} b: \mathbb{R}^2 &\rightarrow \mathbb{R}^3, \text{ for } w = 1 - u - v, u, v, w \geq 0 \\ b(u, v) &= \sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k \\ &= b_{300} w^3 + b_{030} u^3 + b_{003} v^3 \\ &\quad + b_{210} 3w^2 u + b_{120} 3wu^2 + b_{201} 3w^2 v \\ &\quad + b_{021} 3u^2 v + b_{102} 3wv^2 + b_{012} 3uv^2 \\ &\quad + b_{111} 6wuv. \end{aligned}$$

`./img/1_single/recap_inputToNormals.png`

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Point Normal triangles

└ Single PN Triangle

└ Overview

`./img/1_single/recap_inputToNormals.png`



# NORMALS

enhancement: emphasize normals more

img/1\_single/inputPrimitive\_emphNormal.png

input primitive

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Point Normal triangles

└ Single PN Triangle

└ Normals

NORMALS

enhancement: emphasize normals more

img/1\_single/inputPrimitive\_emphNormal.png

input primitive

img/1\_single/linearVsQuadraticNormals\_lin

linear

quadratic

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Point Normal triangles

└ Single PN Triangle

└ Normals - theory

img/1\_single/linearVsQuadraticNormals\_lin

linear

quadratic

# NORMALS - THEORY

img/1\_single/linearVsQuadraticNormals\_linear

linear

img/1\_single/linearVsQuadraticNormals\_quadratic

quadratic

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Point Normal triangles

└ Single PN Triangle

└ Normals - theory

NORMALS - THEORY

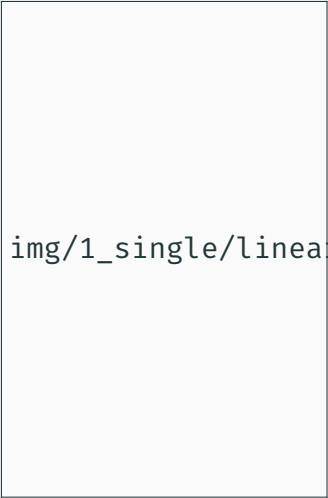
img/1\_single/linearVsQuadraticNormals\_linear

linear

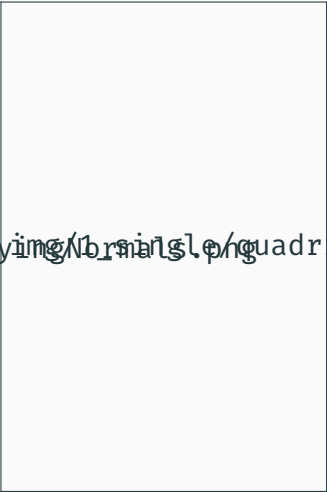
img/1\_single/linearVsQuadraticNormals\_quadratic

quadratic

# NORMALS - EXAMPLE



linear



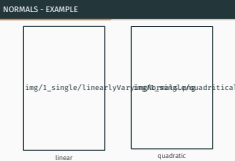
quadratic

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Point Normal triangles

└ Single PN Triangle

└ Normals - example



# NORMALS - THEORY

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Point Normal triangles

└ Single PN Triangle

└ Normals - theory

img/1\_single/computingNormals.png

$$v_{ij} = 2 \frac{(P_j - P_i) \cdot (N_i + N_j)}{(P_j - P_i) \cdot (P_j - P_i)} \in \mathbb{R}$$

$$h_{110} = N_1 + N_2 - v_{12}(P_2 - P_1)$$



$$N_i = 2 \frac{(P_j - P_i) \cdot (N_i + N_j)}{(P_j - P_i) \cdot (P_j - P_i)} \in \mathbb{R}$$

$$h_{110} = N_1 + N_2 - v_{12}(P_2 - P_1)$$

$$h_{101} = h_{110} \cdot (P_2 - P_1)$$

img/1\_single/computingNormals.png

$$v_i = 2 \frac{(P_i - P_j) \cdot (N_i + N_j)}{(P_i - P_j) \cdot (P_j - P_i)} \in \mathbb{R}$$

$$h_{112} = N_1 + N_2 - v_{12}(P_2 - P_1)$$

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Point Normal triangles

└ Single PN Triangle

└ Normals - theory

img/1\_single/computingNormals.png

$$v_i = 2 \frac{(P_i - P_j) \cdot (N_i + N_j)}{(P_i - P_j) \cdot (P_j - P_i)} \in \mathbb{R}$$

$$h_{112} = N_1 + N_2 - v_{12}(P_2 - P_1)$$

$$n_{112} = h_{112} / \|h_{112}\|$$

enhancement: Set result slide to plain

img/1\_single/normals.png

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Point Normal triangles

└ Single PN Triangle

└ Normals - result

enhancement: Set result slide to plain

img/1\_single/normals.png

`./img/1_single/recap_normalsToShading.png`

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Point Normal triangles

└ Single PN Triangle

└ Overview

`./img/1_single/recap_normalsToShading.png`



Plaatje

$n: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ , for  $w = 1 - u - v, u, v, w \geq 0$

$$\begin{aligned} n(u, v) &= \sum_{i+j+k=2} n_{ijk} u^i v^j w^k \\ &= n_{200} w^2 + n_{020} u^2 + n_{002} v^2 \\ &\quad + n_{110} w u + n_{011} u v + n_{101} w v \end{aligned}$$

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Point Normal triangles

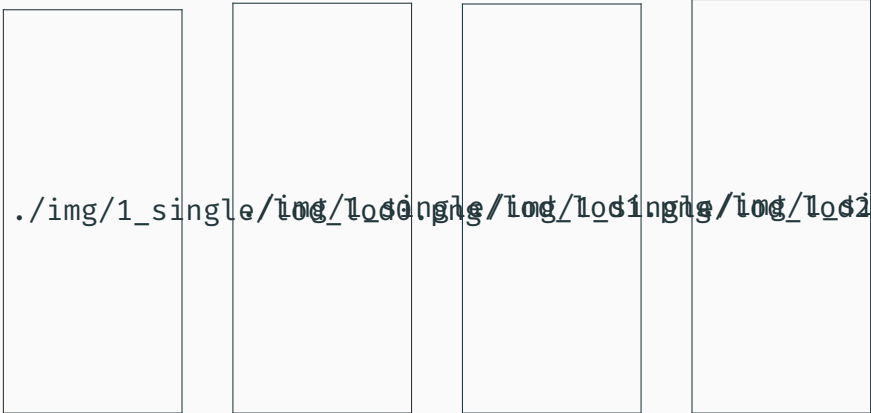
└ Single PN Triangle

└ Quadratic Patch

Plaatje

$$\begin{aligned} n: \mathbb{R}^2 \rightarrow \mathbb{R}^3, \text{ for } w = 1 - u - v, u, v, w \geq 0 \\ n(u, v) &= \sum_{i+j+k=2} n_{ijk} u^i v^j w^k \\ &= n_{200} w^2 + n_{020} u^2 + n_{002} v^2 \\ &\quad + n_{110} w u + n_{011} u v + n_{101} w v \end{aligned}$$

# LEVEL OF DETAIL



0

1

2

3

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Point Normal triangles

└ Single PN Triangle

└ Level Of Detail

LEVEL OF DETAIL



0

1

2

3

`./img/1_single/recap_overview.png`

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Point Normal triangles

└ Single PN Triangle

└ Overview

`./img/1_single/recap_overview.png`

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Point Normal triangles  
└ A Triangle Mesh

A TRIANGLE MESH

## A TRIANGLE MESH

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*"PN triangles should not deviate too much from the original triangle to preserve the shape and avoid interference with other curved triangles."<sup>1</sup>*

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<sup>1</sup>Vlachos et al.

- $C^1$  continuity in the vertex points
- $C^0$  continuity everywhere else

PN triangles have:<sup>2</sup>

- $C^1$  continuity in the vertex points
- $C^0$  continuity everywhere else

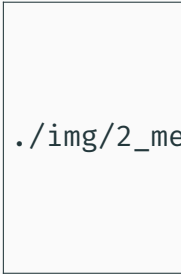
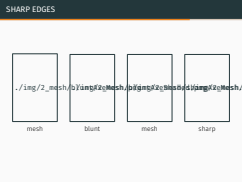
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<sup>2</sup>liao and Alexander

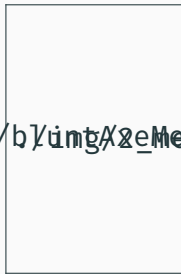
# SHARP EDGES

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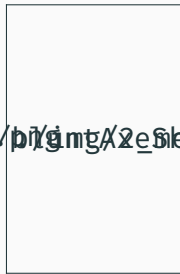
- Point Normal triangles
- └ A Triangle Mesh
- └ Sharp Edges



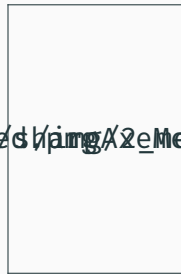
mesh



blunt

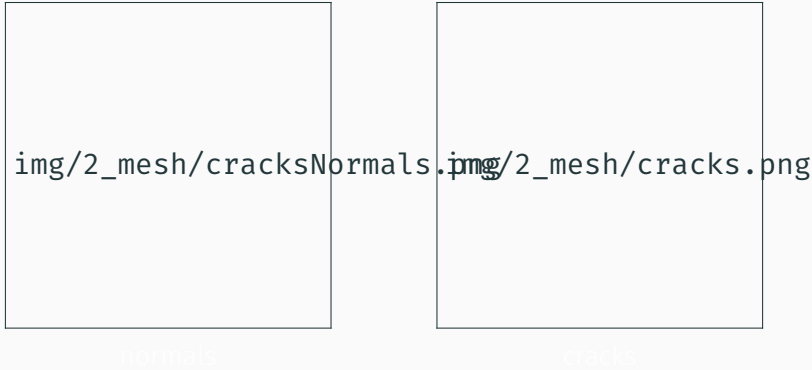


mesh



sharp

# SEPARATE NORMALS



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Point Normal triangles

└ A Triangle Mesh

└ Separate Normals

SEPARATE NORMALS





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Point Normal triangles  
└ Graphics Pipeline

GRAPHICS PIPELINE

## GRAPHICS PIPELINE

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- Point Normal triangles
- └ Graphics Pipeline
- └ Hardware - Pipelines

img/3\_pipeline/pipelineDifferences\_oldOpenGL.png

2001

2015

img/3\_pipeline/pipelineDifferences\_oldOpenGL.png

2001

2015

# HARDWARE - PIPELINES

Point Normal triangles

└ Graphics Pipeline

└ Hardware - Pipelines

2015-12-09

img/3\_pipeline/pipelineDifferences\_oldOpenGL.png

2001

img/3\_pipeline/pipelineDifferences\_newOpenGL.png

2015

img/3\_pipeline/pipelineDifferences\_oldOpenGL.png

2001

img/3\_pipeline/pipelineDifferences\_newOpenGL.png

2015

2015-12-09

Point Normal triangles  
└ Conclusion

CONCLUSION

## CONCLUSION

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# CONCLUSION

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Point Normal triangles

└ Conclusion

└ conclusion

Some conclusion?

Some conclusion?



FIGUUR 13 UIT PAPER

QUESTIONS?

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Point Normal triangles

└ Conclusion



FIGUUR 13 UIT PAPER

## REFERENCES

- 
[Bibliography article](#) pdf J Alexander. “Parallel feature-preserving mesh smoothing”. In: *Computational Science and Its Applications–ICCSA 2005*. Springer, 2005, pp. 1180–1189.
- 
[Bibliography article](#) pdf J Alexander and M Klinger. “Crack-free point-normal triangles using adjacent edge normals”. 2010.
- 
[Bibliography article](#) pdf J Alexander. “Crack-free PN triangles”. In: *Proceedings of the 2001 symposium on Interactive 3D graphics*. ACM, 2001, pp. 159–166.

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Point Normal triangles

└ Conclusion

└ References


[Bibliography article](#) pdf J Alexander. “Parallel feature-preserving mesh smoothing”. In: *Computational Science and Its Applications–ICCSA 2005*. Springer, 2005, pp. 1180–1189.


[Bibliography article](#) pdf J Alexander and M Klinger. “Crack-free point-normal triangles using adjacent edge normals”. 2010.


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