

POINT NORMAL TRIANGLES

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Advanced Computer Graphics

./img/0_input.png ./img/00_gouraud.png ./img/000_pn_geometry.png ./img/0000_pn_triangles.png

INPUT MESH

GOURAUD

PN GEOMETRY

PN TRIANGLES

SINGLE PN TRIANGLE

CUBIC BÉZIER TRIANGLES

$b : \mathbb{R}^2 \rightarrow \mathbb{R}^3$, for $w = 1 - u - v$, $u, v, w \geq 0$

$$\begin{aligned} b(u, v) &= \sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k \\ &= b_{300}w^3 + b_{030}u^3 + b_{003}v^3 \\ &\quad + b_{210}3w^2u + b_{120}3wu^2 + b_{201}3w^2v \\ &\quad + b_{021}3u^2v + b_{102}3wv^2 + b_{012}3uv^2 \\ &\quad + b_{111}6wuv. \end{aligned}$$



`img/1_single/inputPrimitive.png`

Input primitive



`img/1_single/geometry_1.png`

Control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$

$$b_{030} = P_2,$$

$$b_{003} = P_3$$



img/1_single/geometry_1.png

Control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$

$$b_{030} = P_2,$$

$$b_{003} = P_3$$



img/1_single/geometry_2.png

Normal projection

$$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$$

$$b_{210} = (2P_1 + P_2 - w_{12}N_1)/3,$$

$$b_{120} = (2P_2 + P_1 - w_{21}N_2)/3,$$

$$b_{021} = (2P_2 + P_3 - w_{23}N_2)/3,$$

$$b_{012} = (2P_3 + P_2 - w_{32}N_3)/3,$$

$$b_{102} = (2P_3 + P_1 - w_{31}N_3)/3,$$

$$b_{201} = (2P_1 + P_3 - w_{13}N_1)/3$$



img/1_single/geometry_2.png

Normal projection

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$$b_{201} = (2P_1 + P_3 - w_{13}N_1)/3$$



img/1_single/geometry_3.png

Center control point

$$\begin{aligned} E &= (b_{210} + b_{120} + b_{021} \\ &\quad + b_{012} + b_{102} + b_{201})/6, \\ V &= (P_1 + P_2 + P_3)/3, \\ b_{111} &= E + (E - V)/2 \end{aligned}$$



img/1_single/geometry_3.png

Center control point

$$\begin{aligned} E &= (b_{210} + b_{120} + b_{021} \\ &\quad + b_{012} + b_{102} + b_{201})/6, \\ V &= (P_1 + P_2 + P_3)/3, \\ b_{111} &= E + (E - V)/2 \end{aligned}$$



`img/1_single/geometry_4.png`



`img/1_single/inputPrimitive.png`

Input primitive



`img/1_single/normals.png`

$$A^2 + B^2 = C^2$$



`img/1_single/normals.png`

$$A^2 + B^2 = C^2$$

img/1_single/linearVsQuadraticNormals_linear

Linear

Quadratic

NORMALS - THEORY

img/1_single/linearVsQuadraticNormals_linear

Linear

img/1_single/linearVsQuadraticNormals_quadratic

Quadratic



img/1_single/computingNormals.png

$$A^2 + B^2 = C^2$$



img/1_single/computingNormals.png

$$A^2 + B^2 = C^2$$

NORMALS - RESULT



Linear



Quadratic

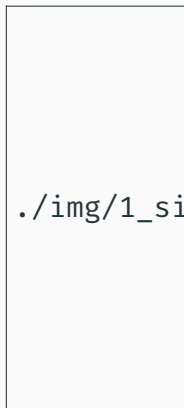
Barycentric coordinates recap

$$n : \mathbb{R}^2 \rightarrow \mathbb{R}^3, \text{ for } w = 1 - u - v, u, v, w \geq 0$$

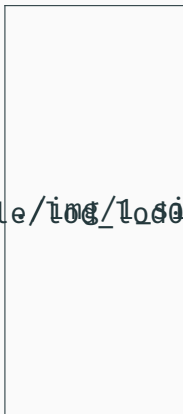
$$\begin{aligned} n(u, v) &= \sum_{i+j+k=2} n_{ijk} u^i v^j w^k \\ &= n_{200} w^2 + n_{020} u^2 + n_{002} v^2 \\ &\quad + n_{110} wu + n_{011} uv + n_{101} wv \end{aligned}$$

LEVEL OF DETAIL

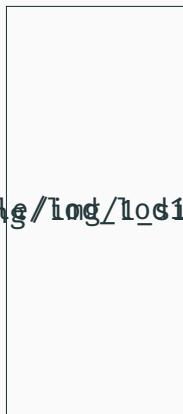
LOD verhaal



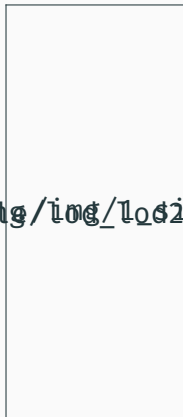
0



1



2



3

CONSTRUCTION

The steps. Recap of everything construct geometry and normals and evaluate less (low lod) or more points (high lod)



A TRIANGLE MESH

PROPERTIES

Shared normals + [Thales of Milet, 500 BC]?

CONTINUITY

Continuity recap?

Continuity

Sharp edges

GRAPHICS PIPELINE

Waarom waren PN triangles hip in 2001? Plus pipeline

Hoe zou je het nu kunnen implementeren? Plus pipeline

CONCLUSION

FIN.

Questions?

REFERENCES

beamer presentation
Alex Martelli et al. "Quilfed PN triangles". In: *Proceedings of the 2001 symposium on Interactive 3D graphics*. ACM. 2001, pp. 159–166.