

POINT NORMAL TRIANGLES

Rick van Veen Laura Baakman

December 14, 2015

Advanced Computer Graphics

2015-12-09

Point Normal triangles

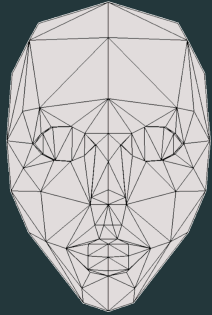
POINT NORMAL TRIANGLES

Rick van Veen Laura Baakman
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Advanced Computer Graphics

[Rick] Welcome everybody. Tell people that PN means Point Normal triangles.

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Point Normal triangles



INPUT MESH



GOURAUD



PN GEOMETRY



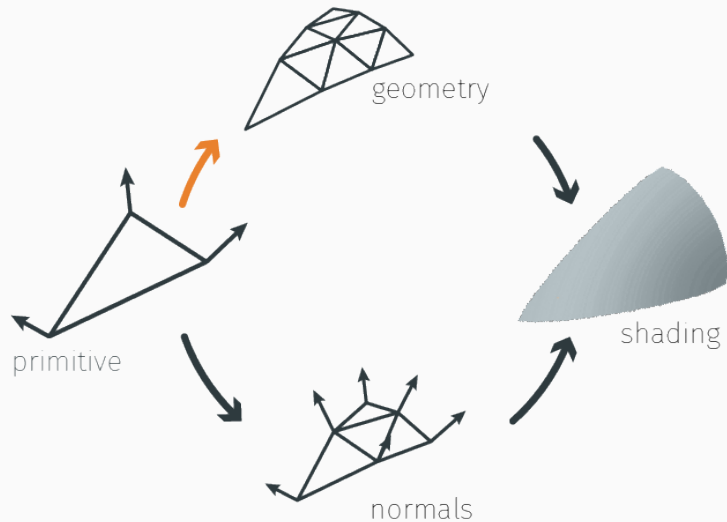
PN TRIANGLES

[Name] Why PN triangles? Look at the nice result it gives :-)) and we will see that it easy to extend it to the 'existing' pipeline.

SINGLE PN TRIANGLE

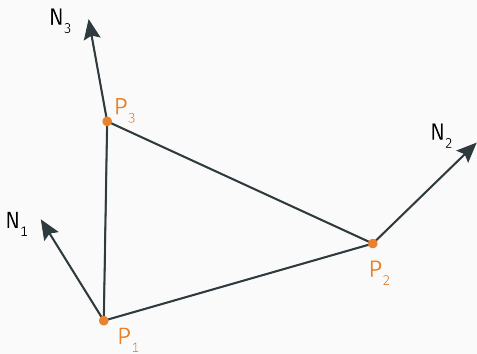
[Name] How does one construct a single PN triangle?

Overview on the next slide



[Name] Why PN triangles? Look at the nice result it gives :-) and we will see that it is easy to extend it to the 'existing' pipeline. Story about Bezier patches...

enhancement: emphasize vertices better



input primitive

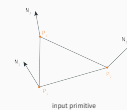
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Point Normal triangles

└ Single PN Triangle

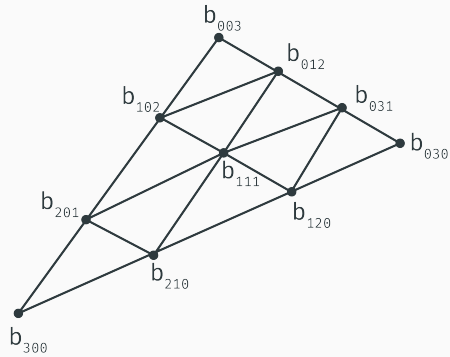
└ Geometry

enhancement: emphasize vertices better



[Name] This a standard triangle primitive, defined by its vertices and normals.

Focus on getting the different control primitives.



control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$

$$b_{030} = P_2,$$

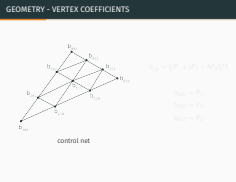
$$b_{003} = P_3$$

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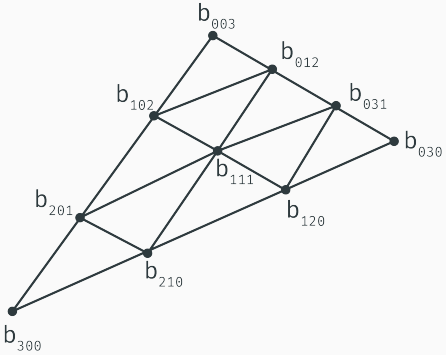
Point Normal triangles

└ Single PN Triangle

└ Geometry - Vertex Coefficients



[Name] These are all the initial control point. Evenly divided on the triangle. -> formula



control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1,$$

$$b_{030} = P_2,$$

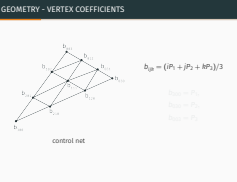
$$b_{003} = P_3$$

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Point Normal triangles

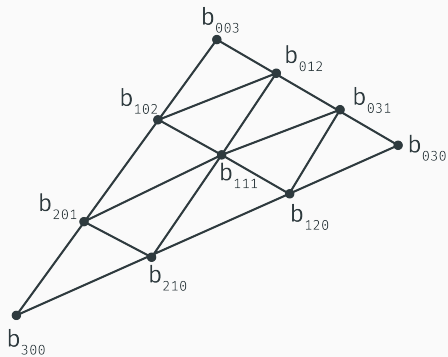
└ Single PN Triangle

└ Geometry - Vertex Coefficients



[Name] Nice formula

GEOMETRY - VERTEX COEFFICIENTS



control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{300} = P_1$$

$$b_{030} = P_2$$

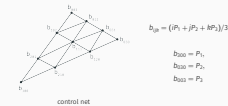
$$b_{003} = P_3$$

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Vertex Coefficients



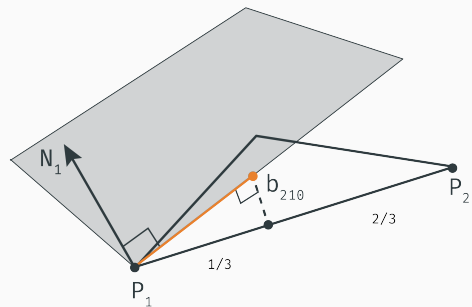
$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

$$b_{100} = P_1$$

$$b_{010} = P_2$$

$$b_{001} = P_3$$

[Name] Stress that the vertex coefficients/control points are the one on the original vertices and that they do not move.



normal projection

$$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3},$$

$$\vdots$$

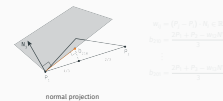
$$b_{201} = \frac{2P_1 + P_3 - w_{13}N_1}{3}$$

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Point Normal triangles

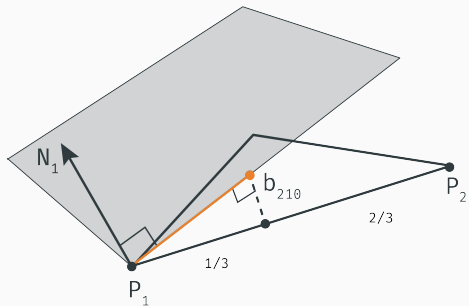
└ Single PN Triangle

└ Geometry - Tangent Coefficients



normal projection

[Name] How to get the tangent coefficient (the ones on the edge but now curvy)



normal projection

$$w_i = (P_i - P_1) \cdot N_1 \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3}$$

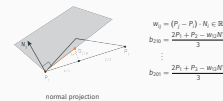
$$b_{201} = \frac{2P_1 + P_2 - w_{11}N_1}{3}$$

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Point Normal triangles

└ Single PN Triangle

└ Geometry - Tangent Coefficients



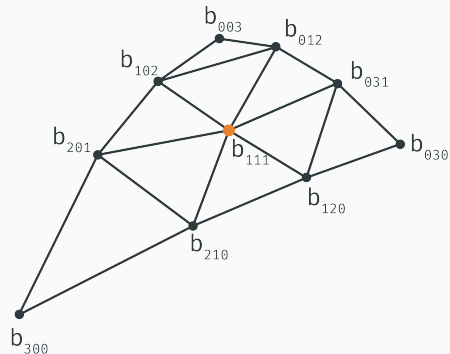
$$w_i = (P_i - P_1) \cdot N_1 \in \mathbb{R}$$

$$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3}$$

$$\vdots$$

$$b_{201} = \frac{2P_1 + P_2 - w_{11}N_1}{3}$$

[Name] Projection of the initial control points on the normal plane of a vertex.



center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

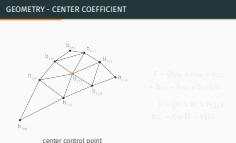
$$b_{111} = E + (E - V)/2$$

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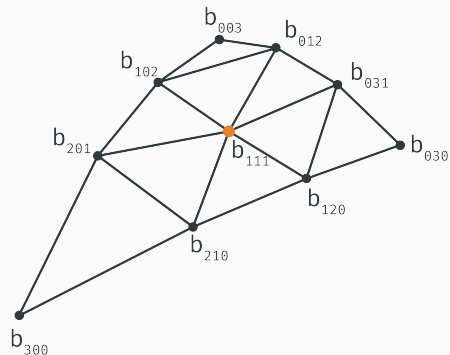
Point Normal triangles

└ Single PN Triangle

└ Geometry - Center Coefficient



[Name] Note that this is the result of the previous step -> now only center coefficient is left.



center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$

2015-12-09

Point Normal triangles

└ Single PN Triangle

└ Geometry - Center Coefficient



center control point

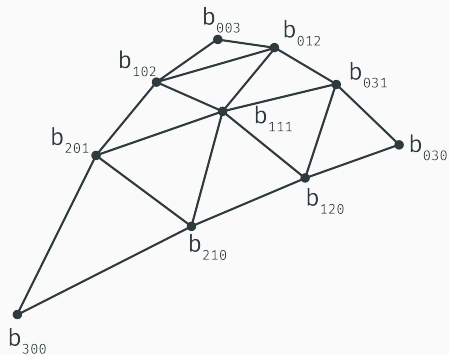
$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$

[Name] Average of the tangent coefficients plus half the difference between the tangent and vertex coefficients. -> why?

enhancement: Set result slide to plain



Point Normal triangles

└ Single PN Triangle

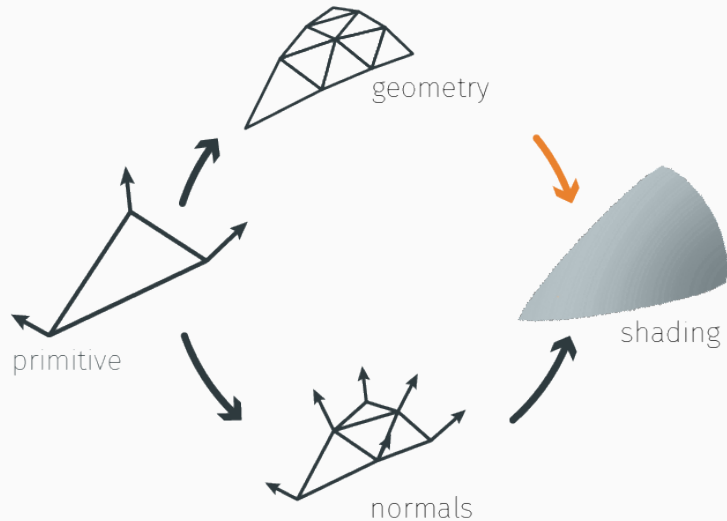
└ Geometry - Result

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enhancement: Set result slide to plain



[Name] Results



[Name] Overview -> how to get from this to shading.
Sample/subdivide with formula on following slide.

Spacing van de for all

Plaatje?

$b: \mathbb{R}^2 \rightarrow \mathbb{R}^3$, for $w = 1 - u - v$, $u, v, w \geq 0$

$$\begin{aligned}
 b(u, v) &= \sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k \\
 &= b_{300}w^3 + b_{030}u^3 + b_{003}v^3 \\
 &\quad + b_{210}3w^2u + b_{120}3wu^2 + b_{201}3w^2v \\
 &\quad + b_{021}3u^2v + b_{102}3wv^2 + b_{012}3uv^2 \\
 &\quad + b_{111}6wuv.
 \end{aligned}$$

Point Normal triangles

└ Single PN Triangle

└ Cubic patch

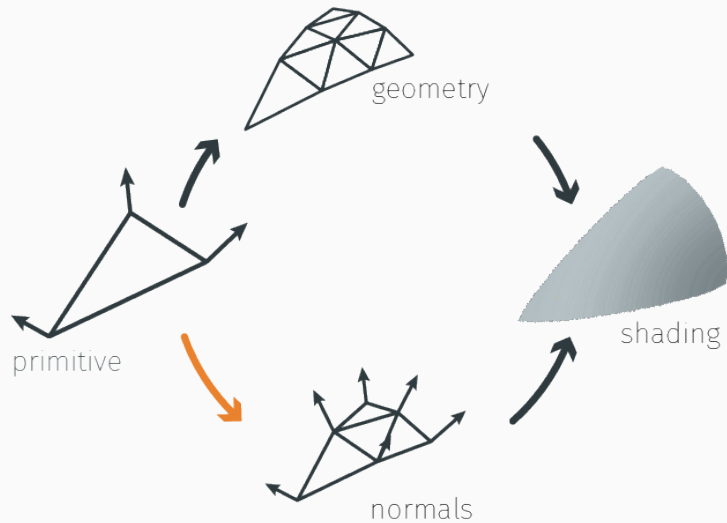
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[Name] Very nice formula with a nice picture.

Spacing van de for all

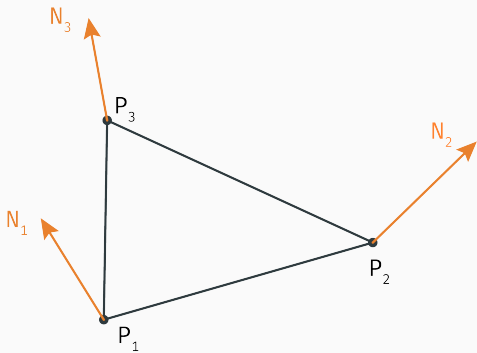
Plaatje?

$$\begin{aligned}
 b: \mathbb{R}^2 \rightarrow \mathbb{R}^3, \text{ for } w = 1 - u - v, u, v, w \geq 0 \\
 b(u, v) &= \sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k \\
 &= b_{300}w^3 + b_{030}u^3 + b_{003}v^3 \\
 &\quad + b_{210}3w^2u + b_{120}3wu^2 + b_{201}3w^2v \\
 &\quad + b_{021}3u^2v + b_{102}3wv^2 + b_{012}3uv^2 \\
 &\quad + b_{111}6wuv.
 \end{aligned}$$



[Name] From the primitive normals the the PN triangle

enhancement: emphasize normals more



input primitive

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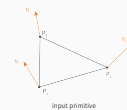
Point Normal triangles

└ Single PN Triangle

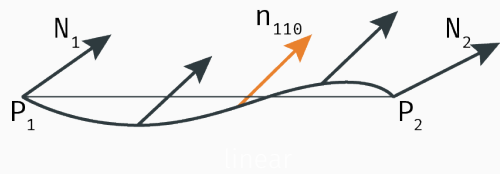
└ Normals

NORMALS

enhancement: emphasize normals more



input primitive



quadratic

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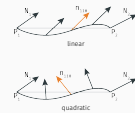
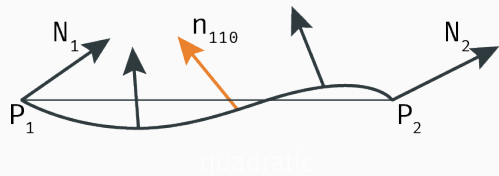
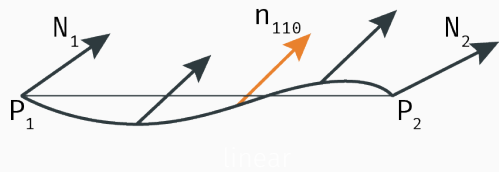
Point Normal triangles

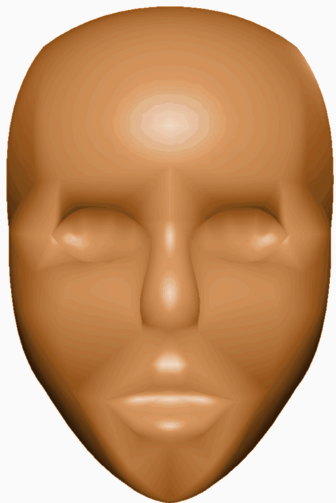
└ Single PN Triangle

└ Normals - theory



quadratic





linear



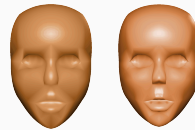
quadratic

2015-12-09

Point Normal triangles

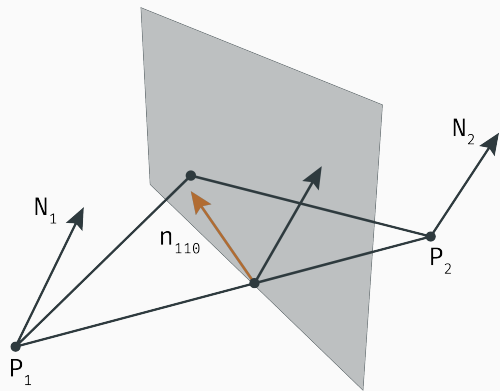
└ Single PN Triangle

└ Normals - example



linear

quadratic



$$v_{ij} = 2 \frac{(P_j - P_i) \cdot (N_i + N_j)}{(P_j - P_i) \cdot (P_j - P_i)} \in \mathbb{R}$$

$$h_{110} = N_1 + N_2 - v_{12}(P_2 - P_1)$$

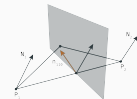
$$n_{110} = h_{110} / ||h_{110}||$$

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Point Normal triangles

└ Single PN Triangle

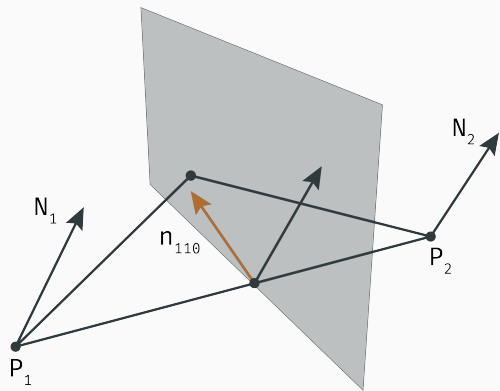
└ Normals - theory



$$N_i = \frac{(P_j - P_i) \times (P_k - P_i)}{\| (P_j - P_i) \times (P_k - P_i) \|}$$

$$N_j = \frac{(P_k - P_j) \times (P_i - P_j)}{\| (P_k - P_j) \times (P_i - P_j) \|}$$

$$N_k = \frac{(P_i - P_k) \times (P_j - P_k)}{\| (P_i - P_k) \times (P_j - P_k) \|}$$



$$v_0 = 2 \frac{(P_1 - P_2) \cdot (N_1 + N_2)}{(P_1 - P_2) \cdot (P_1 - P_2)} \in \mathbb{R}$$

$$h_{110} = N_1 + N_2 - v_0(P_2 - P_1)$$

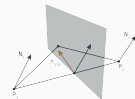
$$n_{110} = h_{110} / \|h_{110}\|$$

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Point Normal triangles

└ Single PN Triangle

└ Normals - theory

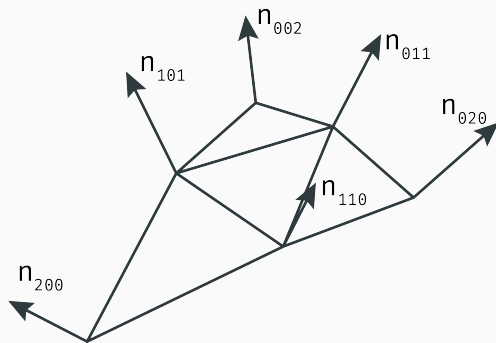


$$v_0 = 2 \frac{(P_1 - P_2) \cdot (N_1 + N_2)}{(P_1 - P_2) \cdot (P_1 - P_2)} \in \mathbb{R}$$

$$h_{110} = N_1 + N_2 - v_0(P_2 - P_1)$$

$$n_{110} = h_{110} / \|h_{110}\|$$

enhancement: Set result slide to plain



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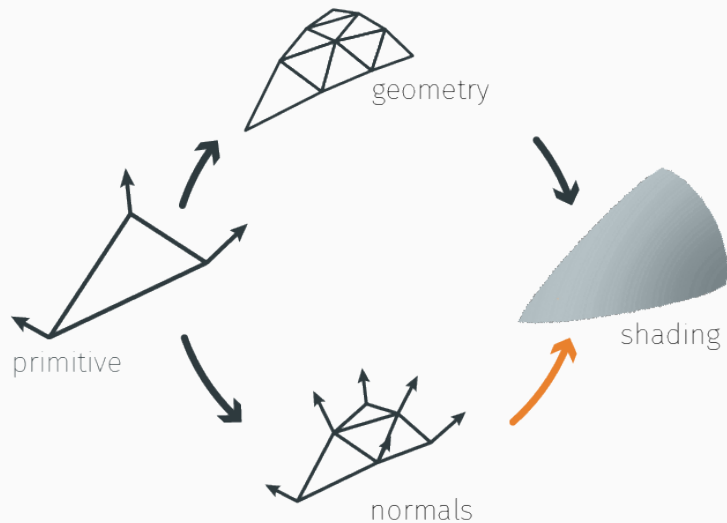
Point Normal triangles

└ Single PN Triangle

└ Normals - result

enhancement: Set result slide to plain





[Name] Why PN triangles? Look at the nice result it gives :-) and we will see that it is easy to extend it to the 'existing' pipeline.

Plaatje

$n: \mathbb{R}^2 \rightarrow \mathbb{R}^3$, for $w = 1 - u - v, u, v, w \geq 0$

$$\begin{aligned} n(u, v) &= \sum_{i+j+k=2} n_{ijk} u^i v^j w^k \\ &= n_{200} w^2 + n_{020} u^2 + n_{002} v^2 \\ &\quad + n_{110} w u + n_{011} u v + n_{101} w v \end{aligned}$$

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Point Normal triangles

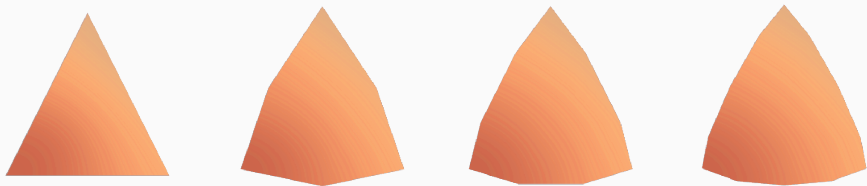
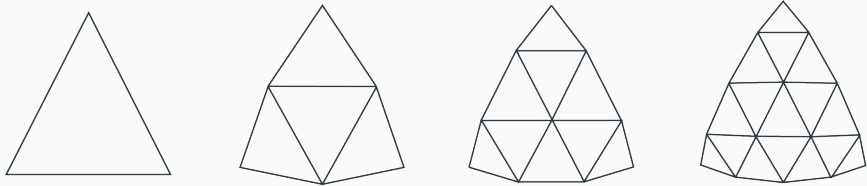
└ Single PN Triangle

└ Quadratic Patch

Plaatje

$$\begin{aligned} n: \mathbb{R}^2 \rightarrow \mathbb{R}^3, \text{ for } w = 1 - u - v, u, v, w \geq 0 \\ n(u, v) &= \sum_{i+j+k=2} n_{ijk} u^i v^j w^k \\ &= n_{200} w^2 + n_{020} u^2 + n_{002} v^2 \\ &\quad + n_{110} w u + n_{011} u v + n_{101} w v \end{aligned}$$

LEVEL OF DETAIL



0

1

2

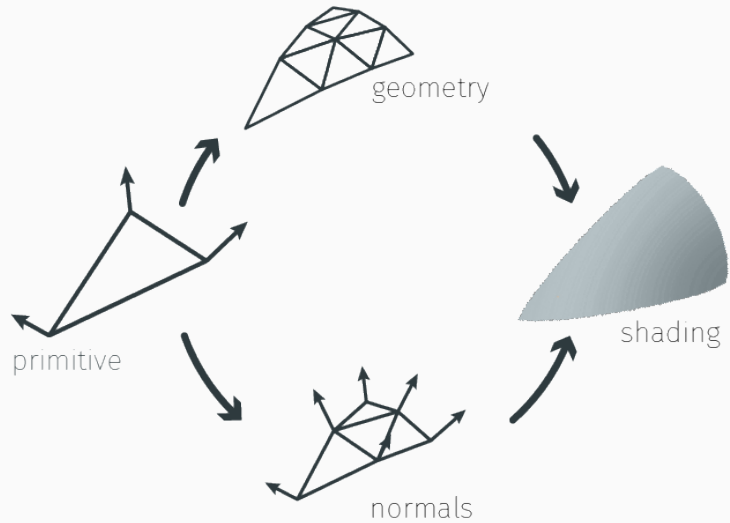
3

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Point Normal triangles
└ Single PN Triangle

└ Level Of Detail





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Point Normal triangles
└ A Triangle Mesh

A TRIANGLE MESH

A TRIANGLE MESH

“PN triangles should not deviate too much from the original triangle to preserve the shape and avoid interference with other curved triangles.”¹

¹Vlachos et al.

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Point Normal triangles

└ A Triangle Mesh

└ Properties

“PN triangles should not deviate too much from the original triangle to preserve the shape and avoid interference with other curved triangles.”¹

¹Vlachos et al.

- C^1 continuity in the vertex points
- C^0 continuity everywhere else

PN triangles have:²

- C^1 continuity in the vertex points
- C^0 continuity everywhere else

²Jiao and Alexander

SHARP EDGES



mesh



blunt



mesh

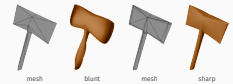


sharp

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- Point Normal triangles
- └ A Triangle Mesh
- └ Sharp Edges

SHARP EDGES



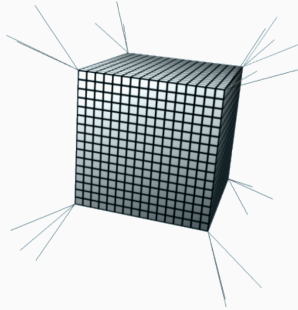
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Point Normal triangles

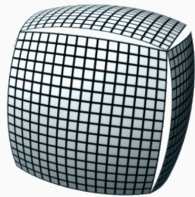
└ A Triangle Mesh

└ Separate Normals

SEPARATE NORMALS



normals



cracks

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Point Normal triangles
└ Graphics Pipeline

GRAPHICS PIPELINE

GRAPHICS PIPELINE

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Point Normal triangles
└ Graphics Pipeline

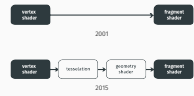
└ Hardware - Pipelines



Point Normal triangles
└ Graphics Pipeline

└ Hardware - Pipelines

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2001



2015

2015-12-09

Point Normal triangles
└ Conclusion

CONCLUSION

CONCLUSION

CONCLUSION

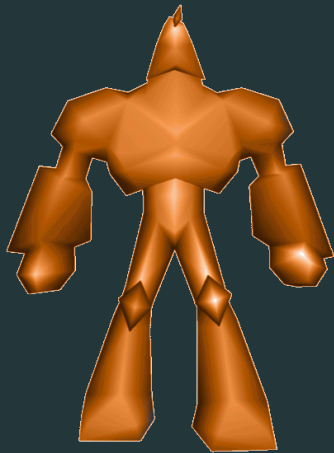
2015-12-09

- Point Normal triangles
 - └ Conclusion
 - └ conclusion

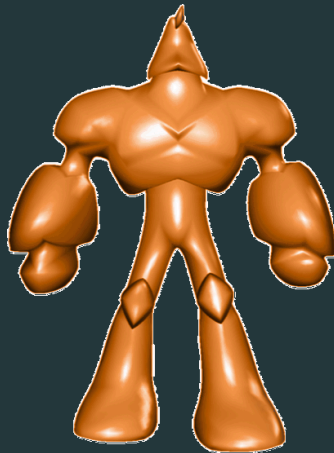
Some conclusion?

Some conclusion?

QUESTIONS?



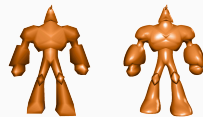
TRIANGLES






PN TRIANGLES

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Point Normal triangles
└ Conclusion



-  Xiangmin Jiao and Phillip J Alexander. “Parallel feature-preserving mesh smoothing”. In: *Computational Science and Its Applications–ICCSA 2005*. Springer, 2005, pp. 1180–1189.
-  J McDonald and M Kilgard. *Crack-free point-normal triangles using adjacent edge normals*. 2010.
-  Alex Vlachos et al. “Curved PN triangles”. In: *Proceedings of the 2001 symposium on Interactive 3D graphics*. ACM. 2001, pp. 159–166.

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Point Normal triangles

└ Conclusion

└ References

-  Xiangmin Jiao and Phillip J Alexander. “Parallel feature-preserving mesh smoothing”. In: *Computational Science and Its Applications–ICCSA 2005*. Springer, 2005, pp. 1180–1189.
-  J McDonald and M Kilgard. *Crack-free point-normal triangles using adjacent edge normals*. 2010.
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