# POINT NORMAL TRIANGLES

Rick van Veen  Laura Baakman

December 14, 2015

Advanced Computer Graphics





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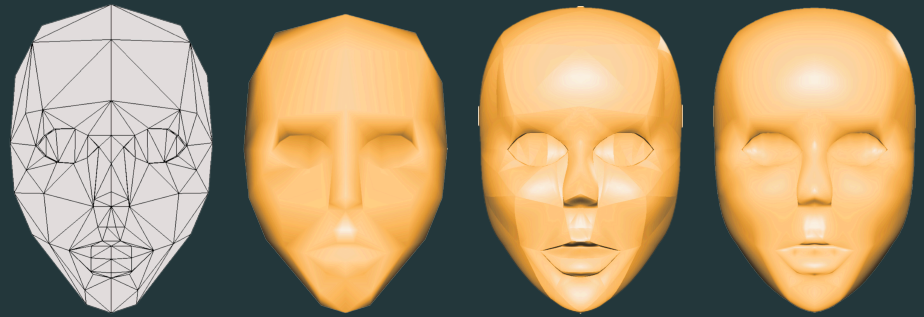
**[Rick]** Welcome everybody. Tell people that PN means Point Normal triangles.




# Point Normal triangles

2015-12-11

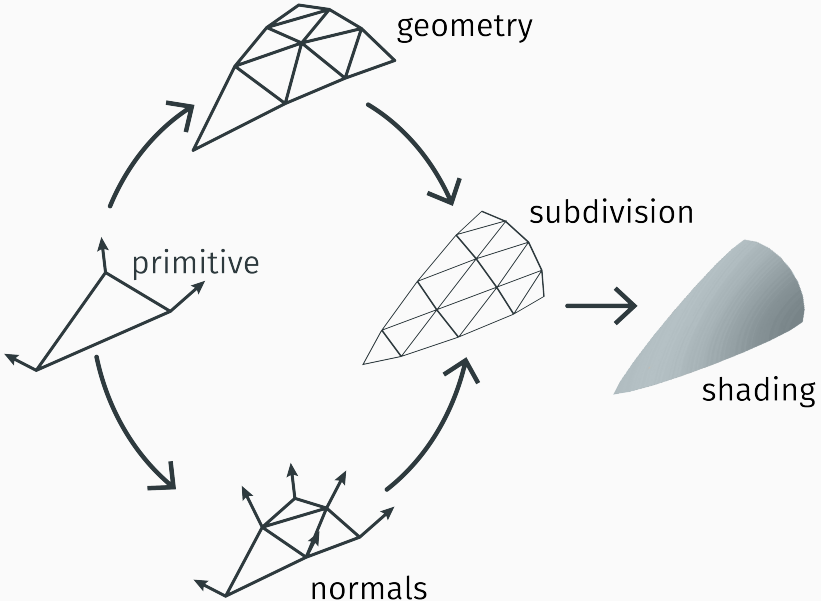
**[Rick]** Why PN triangles? Look at the nice result it gives :-) and we will see that it easy to extend it to the ‘existing’ pipeline.

# SINGLE PN TRIANGLE



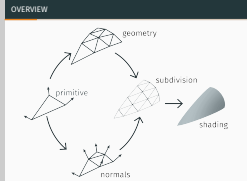
**[Rick]** How does one construct a single PN triangle?
*Overview on the next slide*

# OVERVIEW



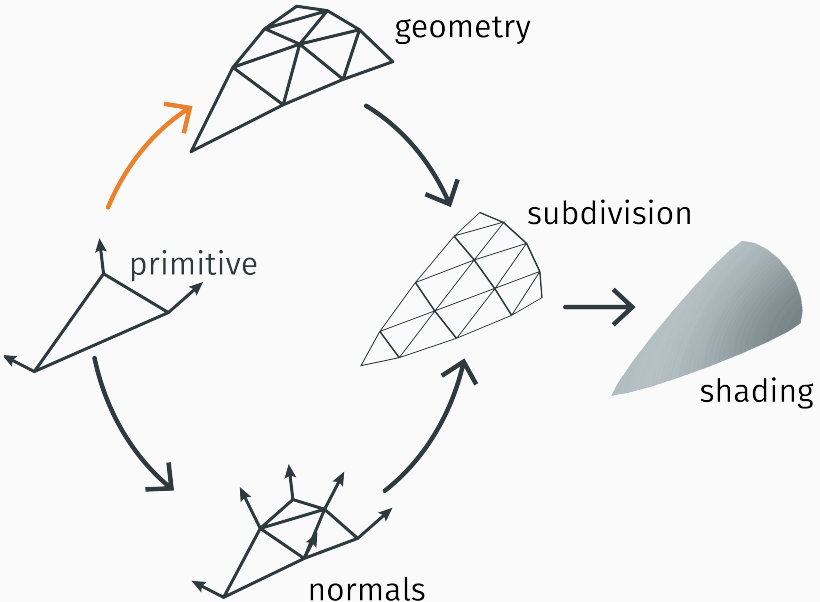




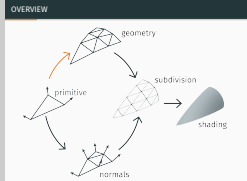

**[Laura]** PN triangle is defined by geometry and normal component.

# OVERVIEW

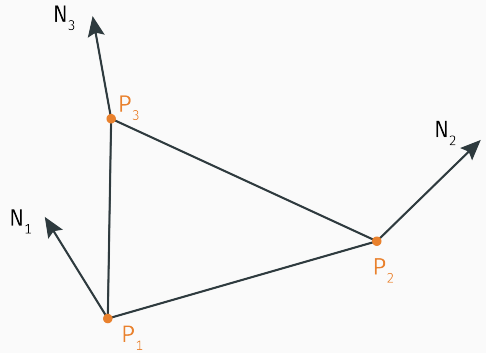






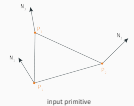

**[Laura]** PN triangle is defined by geometry and normal component.




input primitive

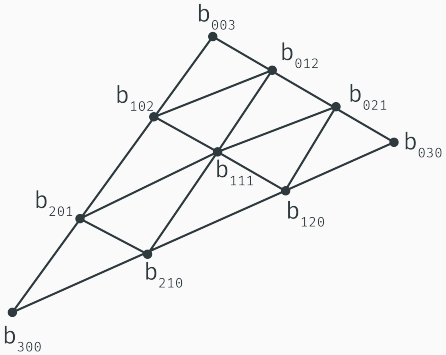







**[Laura]** This a standard triangle primitive, defined by its vertices and normals.
Focus on getting the different control primitives.
Note that we only have this input primite, without information about its neighbours.

# GEOMETRY - VERTEX COEFFICIENTS

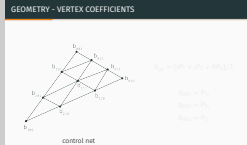



control net

$$b_{ijk} = (iP_1 + jP_2 + kP_3)/3$$

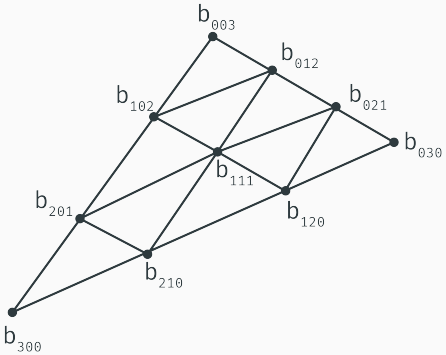
$$b_{300} = P_1,$$
$$b_{030} = P_2,$$
$$b_{003} = P_3$$





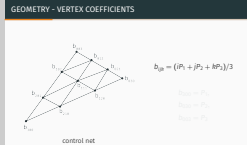

**[Laura]** These are all the initial control point. Evenly divided on the triangle. -> formula

# GEOMETRY - VERTEX COEFFICIENTS



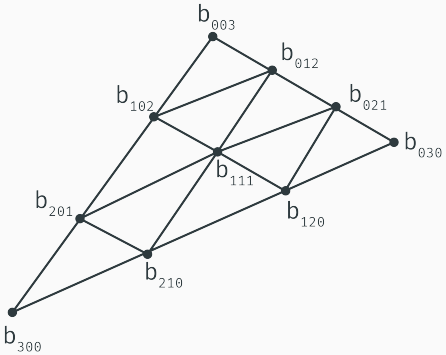

control net






**[Laura]** Nice formula

# GEOMETRY - VERTEX COEFFICIENTS

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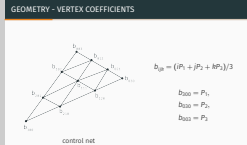
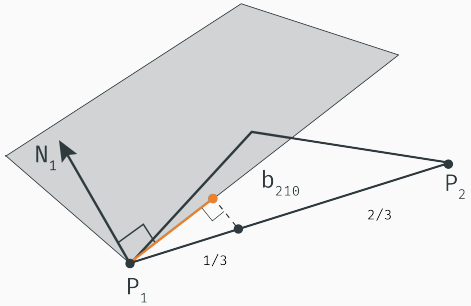



**[Laura]** Stress that the vertex coefficients/control points are the one on the original vertices and that they do not move.

# GEOMETRY - TANGENT COEFFICIENTS




normal projection

$$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$$

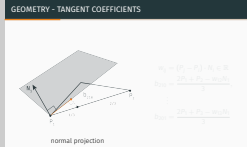
$$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3},$$

$$\vdots$$

$$b_{201} = \frac{2P_1 + P_3 - w_{13}N_1}{3}$$

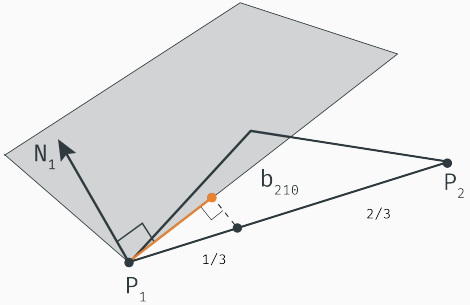







**[Laura]** How to get the tangent coefficient (the ones on the edge but now curvy)

# GEOMETRY - TANGENT COEFFICIENTS

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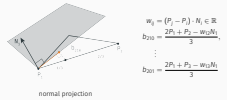
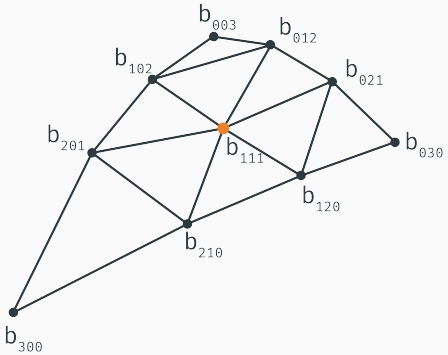
GEOMETRY - TANGENT COEFFICIENTS


$w_{ij} = (P_j - P_i) \cdot N_i \in \mathbb{R}$
$b_{210} = \frac{2P_1 + P_2 - w_{12}N_1}{3},$
$\vdots$
$b_{201} = \frac{2P_1 + P_3 - w_{13}N_1}{3}$
normal projection


**[Laura]** Projection of the initial control points on the normal plane of a vertex.

# GEOMETRY - CENTER COEFFICIENT




center control point

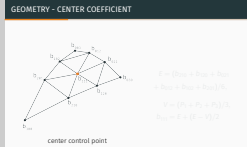
$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

$$V = (P_1 + P_2 + P_3)/3,$$

$$b_{111} = E + (E - V)/2$$

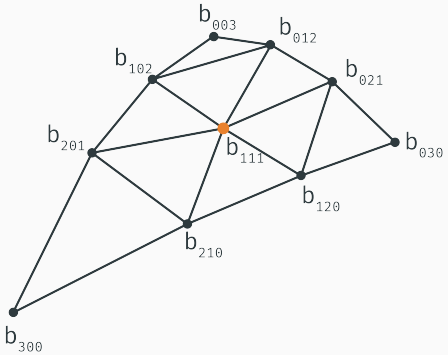







**[Laura]** Note that this is the result of the previous step -> now only center coefficient is left.

# GEOMETRY - CENTER COEFFICIENT




center control point

$$E = (b_{210} + b_{120} + b_{021} + b_{012} + b_{102} + b_{201})/6,$$

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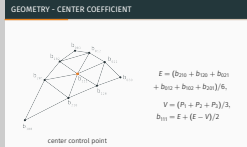
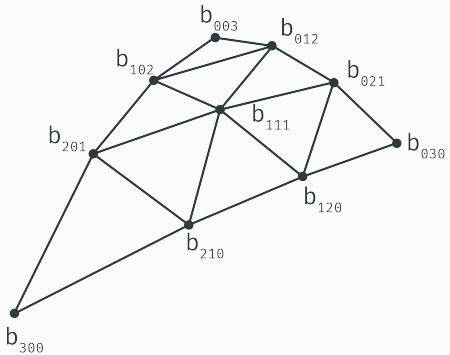



**[Laura]** Average of the tangent coefficients plus half the difference between the tangent and vertex coefficients. -> why?

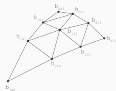
# GEOMETRY - RESULT





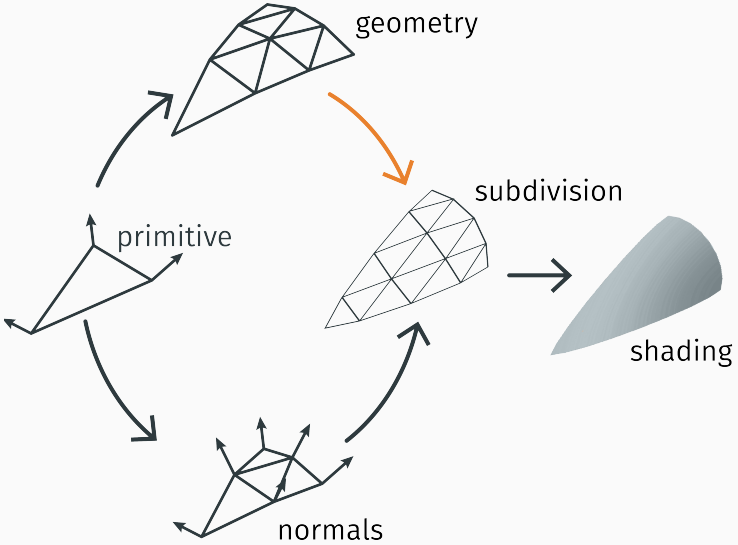



GEOMETRY - RESULT



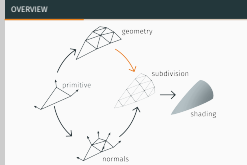
**[Laura]** Results

# OVERVIEW



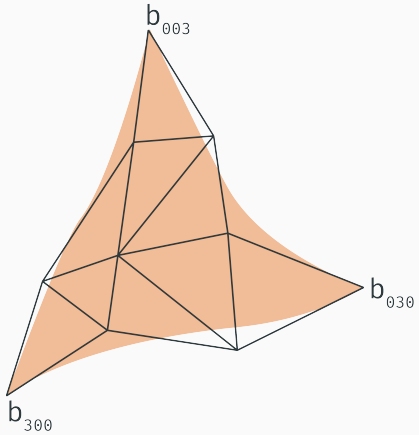





**[Rick]** Overview -> how to get from this to shading.
Sample/subdivide with formula on following slide.

# CUBIC BÉZIER PATCH



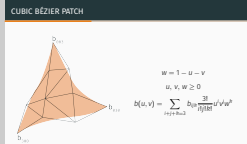

$$w = 1 - u - v$$

$$u, v, w \geq 0$$

$$b(u,v) = \sum_{i+j+k=3} b_{ijk} \frac{3!}{i!j!k!} u^i v^j w^k$$

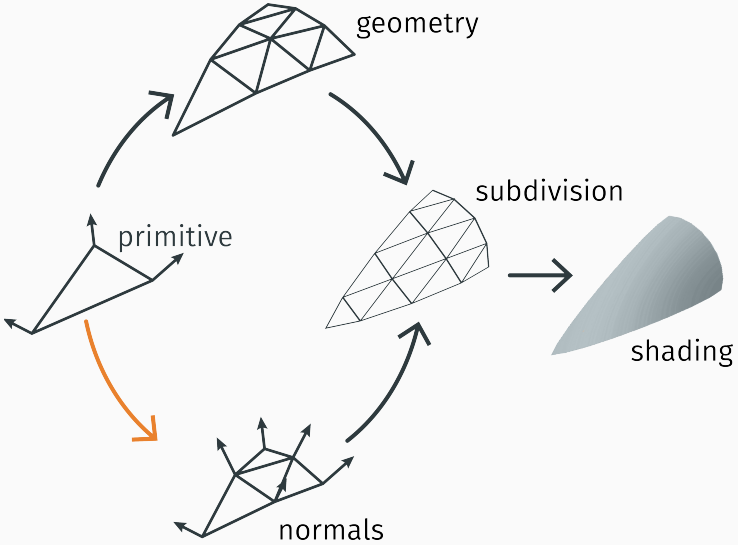






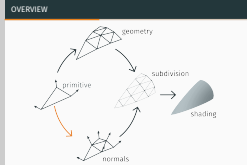

u, v, w are a convex combination**[Rick]** Very nice formula with a nice picture. Menstion **Barycentric coordinates**

# OVERVIEW



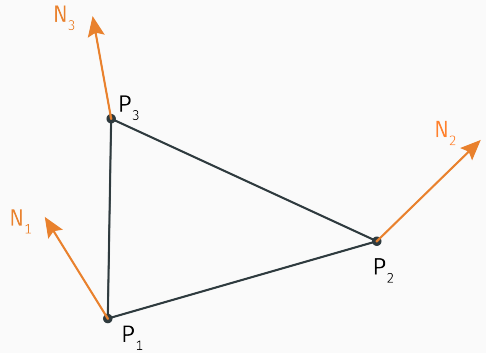





**[Rick]** From the primitive normals the the PN triangle normals

# NORMALS



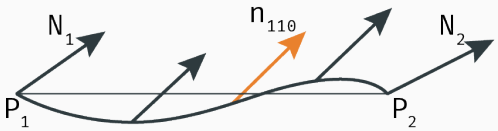

input primitive






**[Rick]** Recap input primitive and with emphasis on the normals.

# NORMALS - THEORY



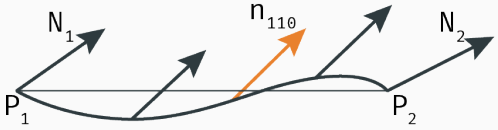




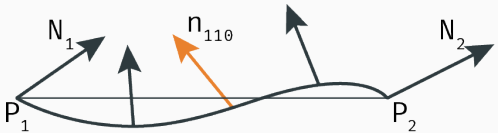


**[Rick]** Stress that there is a need to capture the cubic bezier curve (inflection points) and that this cannot be

# NORMALS - THEORY



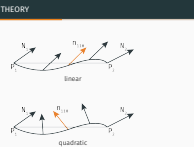

linear




quadratic








**[Rick]** Quadratic does capture inflection points. Trade off between performance and result (maybe?)

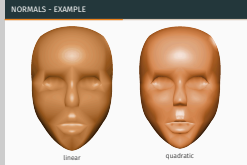
# NORMALS - EXAMPLE

linear

quadratic

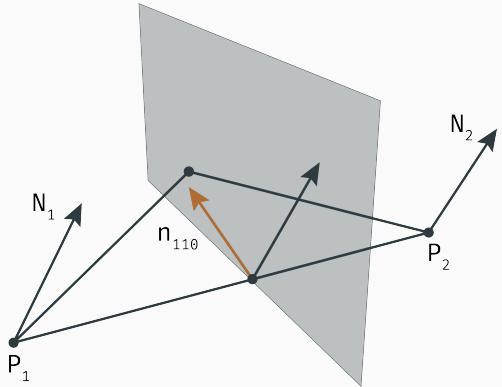







**[Rick]** Look how pretty.

# NORMALS - THEORY



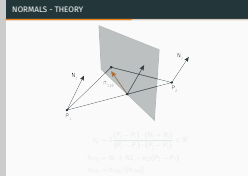

$$v_{ij} = 2\frac{(P_j - P_i)\cdot(N_i + N_j)}{(P_j - P_i)\cdot(P_j - P_i)} \in \mathbb{R}$$

$$h_{110} = N_1 + N2 - v_{12}(P_2 - P_1)$$

$$n_{110} = h_{110}/||h_{110}||$$

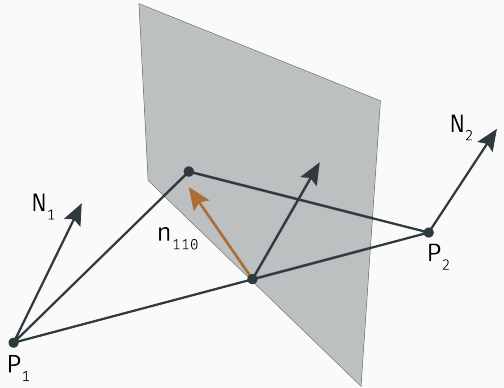






**[Rick]** Formula in words: reflect the averaged normal (average of N1 and N2) on the plane orthogonal/perpendicular the the edge at the mid point.

# NORMALS - THEORY

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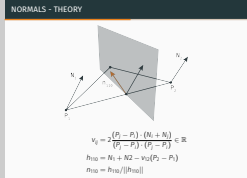
$$h_{110} = N_1 + N2 - v_{12}(P_2 - P_1)$$

$$n_{110} = h_{110}/\|h_{110}\|$$





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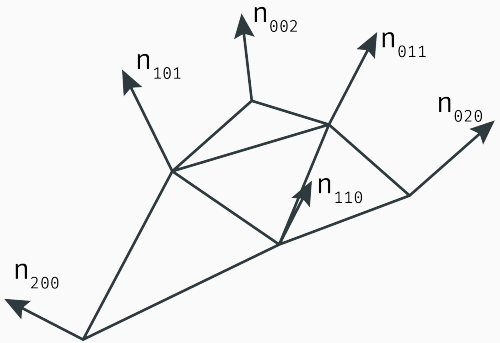
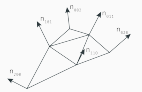
# NORMALS - RESULT





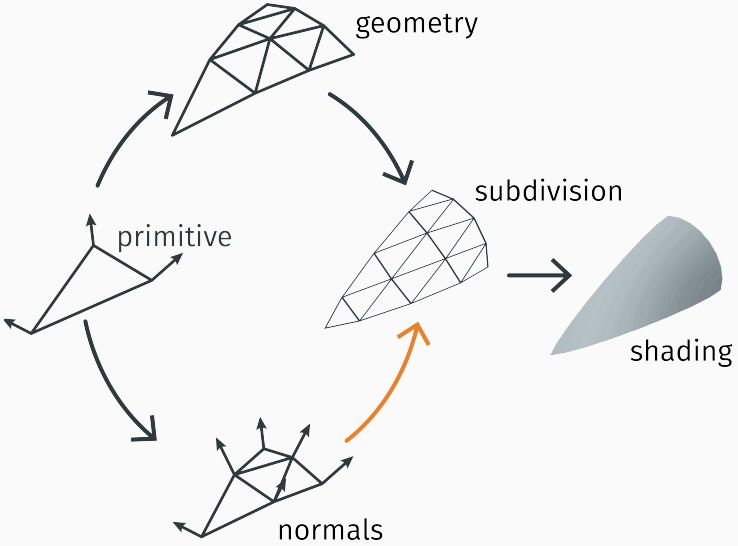



NORMALS - RESULT



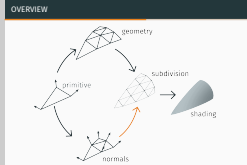
**[Rick]** Result

# OVERVIEW



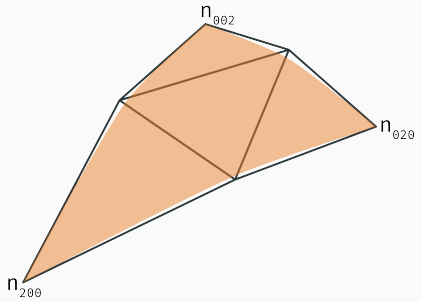





**[Laura]** Why PN triangles? Look at the nice result it gives :-) and we will see that it easy to extend it to the ‘existing’ pipeline.

# QUADRATIC PATCH




$$w = 1 - u - v$$

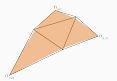
$$u, v, w \geq 0$$

$$r(u, v) = \sum_{i+j+k=2} n_{ijk} u^i v^j w^k$$





QUADRATIC PATCH



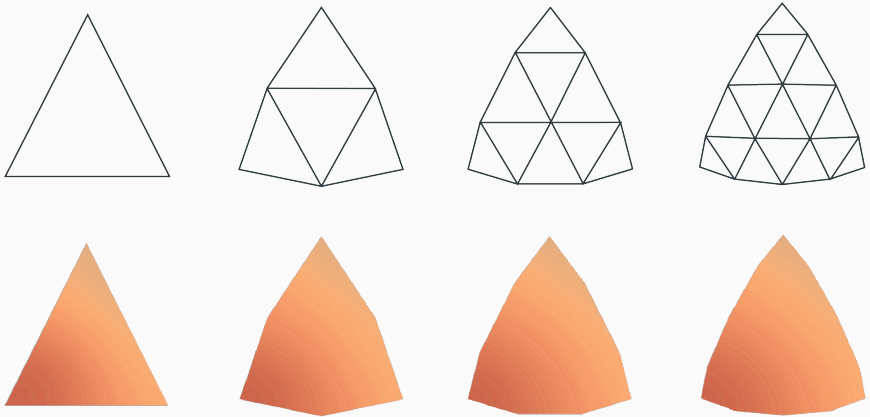
$w = 1 - u - v$

$u, v, w \geq 0$

$r(u, v) = \sum_{i+j+k=2} n_{ijk} u^i v^j w^k$

**[Laura]** u, v and w are an convex combination

# LEVEL OF DETAIL



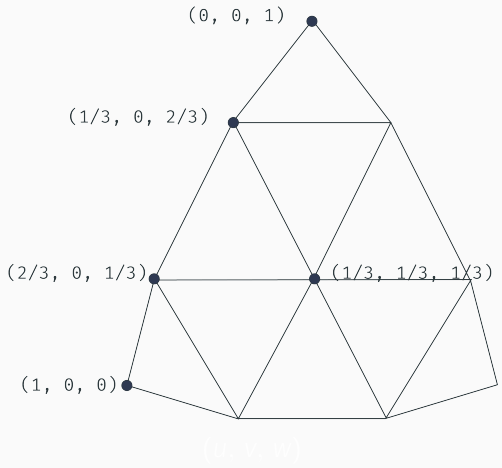






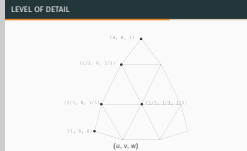
**[Laura]** Level of detail -> subdivision -> how many triangles go through to the next shaders.

# LEVEL OF DETAIL



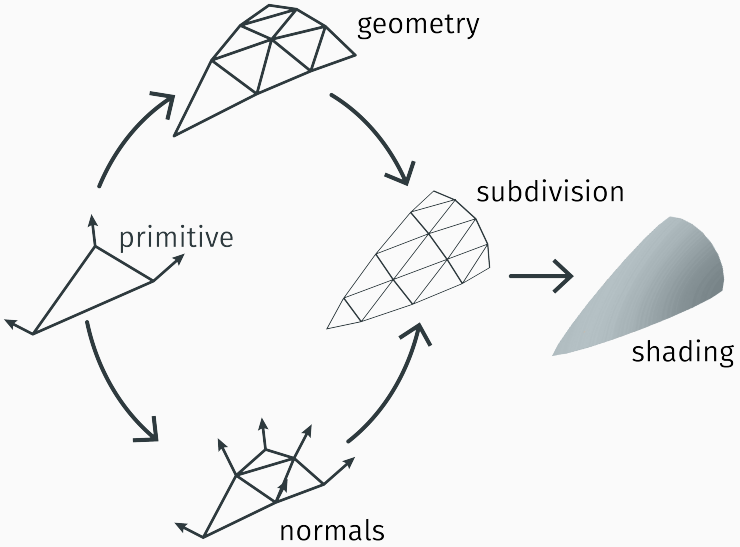




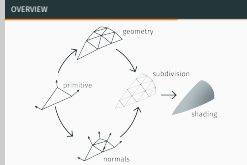
**[Laura]** blaat

# OVERVIEW








**[Laura]** Shading out of the scope of this presentation**[Laura]** Why quadartic patch for normals, why cubic patch for geometry

# A TRIANGLE MESH

[rick] blaat

# PROPERTIES

*"PN triangles should not deviate too much from the original triangle to preserve the shape and avoid interference with other curved triangles."* [1]

[1] Vlachos et al.






**[Rick]** Problem when combining multiple triangles, so this is a important property

# CONTINUITY

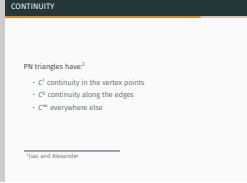
PN triangles have[2]

- $C^1$ continuity in the vertex points
- $C^0$ continuity along the edges
- $C^\infty$ everywhere else

[2] Jiao and Alexander

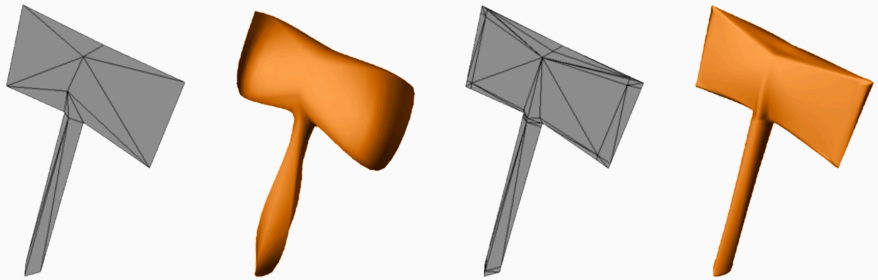







**[Rick]** Continuity C0 is important -> no gaps. Higher is better because this gives a more smooth result.

# SHARP EDGES

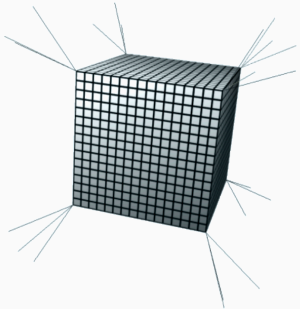




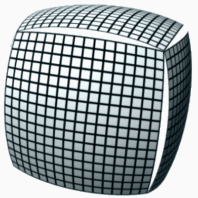


**[Rick]** Curved triangles do not always give the preferred results -> sharp edges. Solution is to insert more triangles at the sharp edges -> model needs to be changed :(

# SEPARATE NORMALS



normals



cracks





SEPARATE NORMALS
normals
cracks

**[Rick]** Beyond the scope of the paper extension exist to overcome the problem what you have when combining multiple meshes. Story about shared vertices.

# GRAPHICS PIPELINE

**[Name]** How does one construct a single PN triangle?

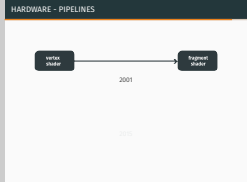
# HARDWARE - PIPELINES


2001

2015



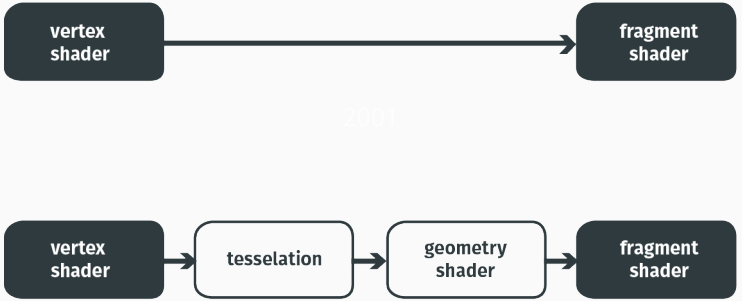





**[Laura]** Great part of the paper stresses the point that it can easily be implemented as a preprocessing step (CPU.

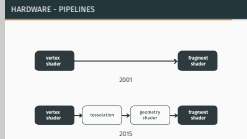
2001 pipeline (OpenGl 1.3)

# HARDWARE - PIPELINES



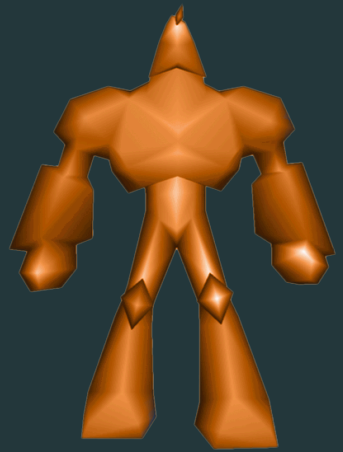




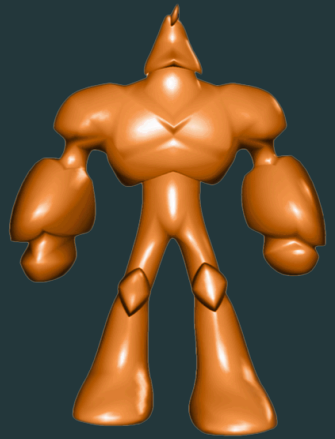




**[Laura]** 2015 we have OpenGL 4.5 with more programmable shaders and the whole process can be done on the GPU. Since PN triangles only usses the primitive, no neighboring primitives, easy in shaders.

# QUESTIONS?



TRIANGLES



PN TRIANGLES



**Laura** Why PN triangles are not suited to rendering for CAD: On real objects, the normal field is determined by the geometry and thus fixed. If you invent fake normals the rendering looks better but the user gets an unpleasant surprise when the product is actually manufactured.

# REFERENCES


- Xiangmin Jiao and Phillip J Alexander. “Parallel feature-preserving mesh smoothing”. In: *Computational Science and Its Applications–ICCSA 2005*. Springer, 2005, pp. 1180–1189.
- J McDonald and M Kilgard. *Crack-free point-normal triangles using adjacent edge normals*. 2010.
- Alex Vlachos et al. “Curved PN triangles”. In: *Proceedings of the 2001 symposium on Interactive 3D graphics*. ACM. 2001, pp. 159–166.



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