One- and Two-Dimensional Ising Model

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1. Results

This section presents the results of the experiments with the one- and two- dimensional model discussed in **??**, in section 1.1 and section 1.2, respectively.

1.1. One-Dimensional Model

Tables 1 to 6 in appendix A present the results of the experiment with the one dimensional model.

In tables 1 and 2 we observe that the accuracy of the 1D simulation is reasonable for temperatures greater than 0.4. In the simulation with N = 10. The simulation with N = 100 starts being accurate at $T = 1.4$ if $N_{samples} = 1000$ and at $T = 0.8$ if $N_{samples} = 10000$. If we increase the number of particles to N = 1000 we only find reasonable accuracy with $N_{samples} = 10000$ for $T > 1.6$.

For all values of N we find that the accuracy improves in general as the number of samples increases, although this effect is stronger in simulations with more spins.

1.2. Two-Dimensional Model

The average energy, specific heat and average magnetization per spin for the different combinations of N and $N_{samples}$ can be found in figure 1.

In figure 1a we observe that the average energy per spin is not hardly influenced by the number of samples for $N_{samples} = 10$. As the number of spins in the simulation increases, the difference between the simulation with $N_{samples} = 1 \times 10^3$ and $N_{samples} = 1 \times 10^4$ increases. In general we observe that the average energy per spin increases as the temperature increases.

In figure 1b we observe a bell-shaped curve in the specific heat per sin around $T = 2$. The curve is more defined when $N_{samples}$ is higher and when the number of spins in the simulation increases.

Comparing the measured average magnetization per spin with the theoretical value we observe that the curves reflecting the results of the simulation are less steep. Furthermore the smaller simulations seem to give a better approximation than the simulation with a lot of spins.

2. Discussion

What are we going to discuss?

Invloed van de parameters, T, N, NSAMPLES p 192 physics by computer

2.1. One-Dimensional Model

Compare results with the analytical solution

Waarom wordt accuracy beter als temperature beter wordt?

Waarom wordt accuracy beter als systeem groter wordt?

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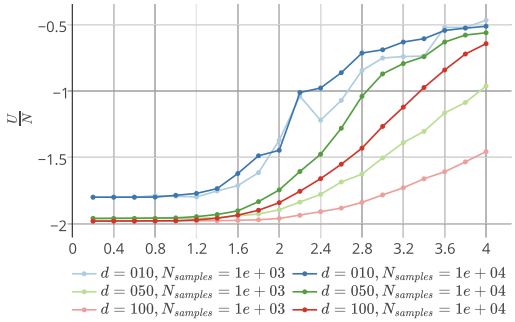
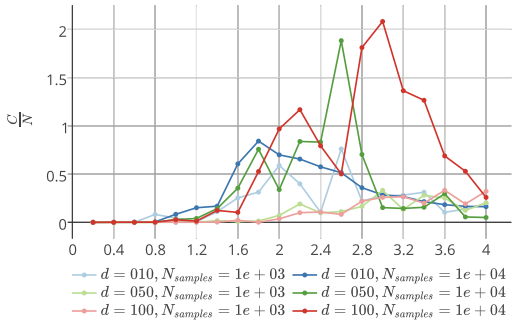
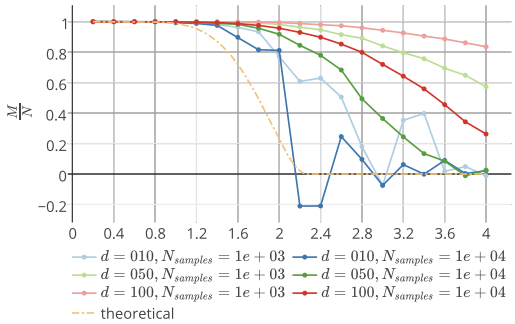


(a) *Average energy per spin.*



(b) *Specific heat per spin.*



(c) *Average magnetization per spin.*

Figure 1: *The (a) average energy, (b) specific heat and (c) average magnetization per spin in a 2D Ising model with* $\mathrm{d} = 10, 50, 100$ *and* $N_{samples} = 1000, 10000$.

2.2. Two-Dimensional Model

Interpret results in terms of a phase transition from a state with magnetization zero to a state with definite magnetization (slide 31)

Compare Average magnetization with the exact result for the infinite system

3. Conclusion

Hoe goed sluit het model aan bij de het exacte resultaat?

Wat hebben we geleerd over de parameters.

A. Results

Table 1: *Results of the 1D simulation for* N = 10, $N_{samples} = 1000$.

		Numerical		Analytical		
T	β	U/N	C/N	U/N	C/N	accuracy
0.20	5	−0.9	0.	−1.0	0.005	$-\infty$
0.40	3	−0.9	0.003	−1.0	0.2	−32
0.60	2	−0.8	0.4	−0.9	0.4	0.88
0.80	1	−0.8	0.5	−0.8	0.4	0.94
1.0	1	−0.7	0.4	−0.8	0.4	0.96
1.2	0.8	−0.6	0.3	−0.7	0.4	0.78
1.4	0.7	−0.5	0.2	−0.6	0.3	0.77
1.6	0.6	−0.5	0.2	−0.6	0.3	0.80
1.8	0.6	−0.4	0.2	−0.5	0.2	0.88
2.0	0.5	−0.4	0.2	−0.5	0.2	0.96
2.2	0.5	−0.4	0.1	−0.4	0.2	0.81
2.4	0.4	−0.3	0.1	−0.4	0.1	0.86
2.6	0.4	−0.3	0.1	−0.4	0.1	0.91
2.8	0.4	−0.3	0.1	−0.3	0.1	0.92
3.0	0.3	−0.3	0.08	−0.3	0.10	0.84
3.2	0.3	−0.3	0.09	−0.3	0.09	0.96
3.4	0.3	−0.3	0.08	−0.3	0.08	0.97
3.6	0.3	−0.2	0.06	−0.3	0.07	0.82
3.8	0.3	−0.2	0.06	−0.3	0.06	0.86
4.0	0.3	−0.2	0.05	−0.2	0.06	0.83

Table 2: *Results of the 1D simulation for* N = 10, $N_{samples} = 10000$.

		Numerical		Analytical		
T	β	U/N	C/N	U/N	C/N	accuracy
0.20	5	−0.9	0.	−1.0	0.005	$-\infty$
0.40	3	−0.9	0.4	−1.0	0.2	0.66
0.60	2	−0.8	0.4	−0.9	0.4	0.90
0.80	1	−0.8	0.4	−0.8	0.4	0.89
1.0	1	−0.7	0.4	−0.8	0.4	0.90
1.2	0.8	−0.6	0.3	−0.7	0.4	0.88
1.4	0.7	−0.6	0.3	−0.6	0.3	0.92
1.6	0.6	−0.5	0.2	−0.6	0.3	0.87
1.8	0.6	−0.4	0.2	−0.5	0.2	0.86
2.0	0.5	−0.4	0.2	−0.5	0.2	0.88
2.2	0.5	−0.4	0.1	−0.4	0.2	0.84

Table 2: *continued*

		Numerical		Analytical		
T	β	U/N	C/N	U/N	C/N	accuracy
2.4	0.4	−0.4	0.1	−0.4	0.1	0.92
2.6	0.4	−0.3	0.1	−0.4	0.1	0.84
2.8	0.4	−0.3	0.10	−0.3	0.1	0.88
3.0	0.3	−0.3	0.09	−0.3	0.10	0.88
3.2	0.3	−0.3	0.08	−0.3	0.09	0.88
3.4	0.3	−0.3	0.07	−0.3	0.08	0.90
3.6	0.3	−0.3	0.07	−0.3	0.07	0.93
3.8	0.3	−0.2	0.06	−0.3	0.06	0.89
4.0	0.3	−0.2	0.05	−0.2	0.06	0.88

Table 3: *Results of the 1D simulation for* $\mathrm{N} = 100$, $N_{samples} = 1000$.

		Numerical		Analytical		
T	β	U/N	C/N	U/N	C/N	accuracy
0.20	5	−1.0	0.	−1.0	0.005	$-\infty$
0.40	3	−1.0	0.004	−1.0	0.2	−20
0.60	2	−1.0	0.01	−0.9	0.4	−12
0.80	1	−0.9	0.10	−0.8	0.4	−0.74
1.0	1	−0.8	0.2	−0.8	0.4	0.30
1.2	0.8	−0.8	0.1	−0.7	0.4	0.16
1.4	0.7	−0.6	0.3	−0.6	0.3	0.95
1.6	0.6	−0.5	0.2	−0.6	0.3	0.81
1.8	0.6	−0.5	0.08	−0.5	0.2	−0.049
2.0	0.5	−0.4	0.1	−0.5	0.2	0.69
2.2	0.5	−0.4	0.2	−0.4	0.2	0.94
2.4	0.4	−0.4	0.08	−0.4	0.1	0.60
2.6	0.4	−0.4	0.07	−0.4	0.1	0.61
2.8	0.4	−0.3	0.1	−0.3	0.1	0.92
3.0	0.3	−0.3	0.09	−0.3	0.10	0.89
3.2	0.3	−0.3	0.08	−0.3	0.09	0.93
3.4	0.3	−0.2	0.09	−0.3	0.08	0.79
3.6	0.3	−0.3	0.04	−0.3	0.07	0.58
3.8	0.3	−0.3	0.06	−0.3	0.06	0.91
4.0	0.3	−0.2	0.04	−0.2	0.06	0.71

Table 4: *Results of the 1D simulation for* N = *100,* $N_{samples}$ = *10000.*

		Numerical		Analytical		
T	β	$^{U}/_{\mathrm{N}}$	$^{C}/_{\mathrm{N}}$	$^{U}/_{\mathrm{N}}$	$^{C}/_{\mathrm{N}}$	accuracy
0.20	5	−1.0	0.	−1.0	0.005	$-\infty$
0.40	3	−1.0	0.05	−1.0	0.2	−0.021
0.60	2	−0.9	0.1	−0.9	0.4	−0.33
0.80	1	−0.8	0.6	−0.8	0.4	0.86
1.0	1	−0.8	0.4	−0.8	0.4	0.93
1.2	0.8	−0.7	0.3	−0.7	0.4	0.87
1.4	0.7	−0.6	0.3	−0.6	0.3	0.95
1.6	0.6	−0.5	0.3	−0.6	0.3	0.96
1.8	0.6	−0.5	0.2	−0.5	0.2	0.90
2.0	0.5	−0.4	0.2	−0.5	0.2	0.93
2.2	0.5	−0.4	0.1	−0.4	0.2	0.89
2.4	0.4	−0.4	0.1	−0.4	0.1	0.92
2.6	0.4	−0.4	0.1	−0.4	0.1	0.99
2.8	0.4	−0.3	0.1	−0.3	0.1	0.99
3.0	0.3	−0.3	0.1	−0.3	0.10	0.86
3.2	0.3	−0.3	0.08	−0.3	0.09	0.93
3.4	0.3	−0.3	0.08	−0.3	0.08	0.98
3.6	0.3	−0.3	0.07	−0.3	0.07	0.97
3.8	0.3	−0.3	0.07	−0.3	0.06	0.93
4.0	0.3	−0.2	0.06	−0.2	0.06	0.95

Table 5: *Results of the 1D simulation for* N = *1000,* $N_{samples}$ = *1000.*

		Numerical		Analytical		
T	β	$^{U}/_{\mathrm{N}}$	$^{C}/_{\mathrm{N}}$	$^{U}/_{\mathrm{N}}$	$^{C}/_{\mathrm{N}}$	accuracy
0.20	5	−1.0	0.	−1.0	0.005	$-\infty$
0.40	3	−1.0	0.	−1.0	0.2	$-\infty$
0.60	2	−1.0	0.	−0.9	0.4	$-\infty$
0.80	1	−1.0	0.1	−0.8	0.4	−0.72
1.0	1	−1.0	0.3	−0.8	0.4	0.65
1.2	0.8	−0.9	0.4	−0.7	0.4	0.87
1.4	0.7	−0.8	0.2	−0.6	0.3	0.62
1.6	0.6	−0.7	0.4	−0.6	0.3	0.73
1.8	0.6	−0.6	0.2	−0.5	0.2	0.70
2.0	0.5	−0.5	0.1	−0.5	0.2	0.69
2.2	0.5	−0.5	0.04	−0.4	0.2	−0.56
2.4	0.4	−0.4	0.1	−0.4	0.1	0.82
2.6	0.4	−0.4	0.03	−0.4	0.1	−0.37
2.8	0.4	−0.4	0.04	−0.3	0.1	−0.11

Table 5: *continued*

		Numerical		Analytical		
T	β	U/N	C/N	U/N	C/N	accuracy
3.0	0.3	−0.4	0.03	−0.3	0.10	−0.23
3.2	0.3	−0.4	0.01	−0.3	0.09	−1.7
3.4	0.3	−0.3	0.02	−0.3	0.08	−0.16
3.6	0.3	−0.3	0.01	−0.3	0.07	−1.4
3.8	0.3	−0.2	0.01	−0.3	0.06	−1.4
4.0	0.3	−0.2	0.02	−0.2	0.06	−0.025

Table 6: *Results of the 1D simulation for* $\mathrm{N} = 1000$, $N_{samples} = 10000$.

		Numerical		Analytical		
T	β	U/N	C/N	U/N	C/N	accuracy
0.20	5	−1.0	0.	−1.0	0.005	$-\infty$
0.40	3	−1.0	0.	−1.0	0.2	$-\infty$
0.60	2	−1.0	0.07	−0.9	0.4	−1.1
0.80	1	−0.9	0.5	−0.8	0.4	0.91
1.0	1	−0.8	0.9	−0.8	0.4	0.70
1.2	0.8	−0.7	0.2	−0.7	0.4	0.54
1.4	0.7	−0.6	0.2	−0.6	0.3	0.57
1.6	0.6	−0.5	0.4	−0.6	0.3	0.80
1.8	0.6	−0.5	0.2	−0.5	0.2	0.97
2.0	0.5	−0.5	0.2	−0.5	0.2	0.97
2.2	0.5	−0.4	0.2	−0.4	0.2	0.95
2.4	0.4	−0.4	0.2	−0.4	0.1	0.90
2.6	0.4	−0.4	0.10	−0.4	0.1	0.85
2.8	0.4	−0.4	0.07	−0.3	0.1	0.71
3.0	0.3	−0.3	0.07	−0.3	0.10	0.74
3.2	0.3	−0.3	0.09	−0.3	0.09	0.98
3.4	0.3	−0.3	0.07	−0.3	0.08	0.92
3.6	0.3	−0.3	0.08	−0.3	0.07	0.96
3.8	0.3	−0.3	0.05	−0.3	0.06	0.90
4.0	0.3	−0.2	0.09	−0.2	0.06	0.79