One- and Two-Dimensional Ising Model

L.E.N. Baakman*
l.e.n.baakman@student.rug.nl

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Table 1: *Average accuracies of the one-dimensional simulation.*

	Number of Spins (N)		
$N_{samples}$	10	100	1000
1000	0.888	0.994	−38 700.942
10 000	0.890	0.981	−2.395

1. Results

This section presents the results of the experiments with the one- and two- dimensional model discussed in **??**, in section 1.1 and section 1.2, respectively.

1.1. One-Dimensional Model

??–**??** in **??** present the results of the experiment with the one dimensional model. The average accuracies, without the $\pm\infty$ are presented in table 1.

In **????** we observe that the accuracy of the 1D simulation is reasonable for temperatures greater than 0.4. In the simulation with N = 10. The simulation with N = 100 starts being accurate at $T = 1.4$ if $N_{samples} = 1000$ and at $T = 0.8$ if $N_{samples} = 10000$. If we increase the number of spins to N = 1000 we only find reasonable accuracy with $N_{samples} = 10000$ for $T > 1.6$.

For all values of N we find that the accuracy improves we find that the accuracy improves as the number of samples increases.

1.2. Two-Dimensional Model

The average energy, specific heat and average magnetization per spin for the different combinations of N and $N_{samples}$ can be found in figure 1.

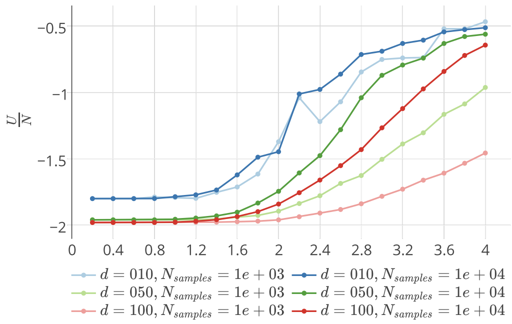
In figure 1a we observe that the average energy per spin is hardly influenced by the number of samples for $N_{samples} = 10$. As the number of spins in the simulation increases, the difference between the simulation with $N_{samples} = 1.00 \times 10^3$ and $N_{samples} = 1.00 \times 10^4$ increases. In general we observe that the average energy per spin increases as the temperature increases. In **??** the phase transition that we expect at $T_c \approx 2.20$ is indicated by an increase in the average energy per spin, after ${}^{U}/_{\mathrm{N}}$ has been constant for $T << T_c$.

In figure 1b we observe a bell-shaped curve in the specific heat per spin around $T = 2$. The curve is more defined when $N_{samples}$ is higher and when the number of spins in the simulation increases. The highest points of these bell curves are found near $T = T_c$, i.e. at the phase transition.

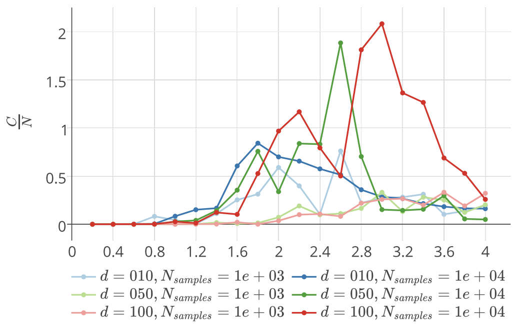
Comparing the measured average magnetization per spin in figure 1c with the theoretical value in the same figure we observe that the curves reflecting the results of the simulation are less steep. Furthermore the smaller simulations seem to give a better approximation than the simulation with a lot of spins. In figure 1c the phase transition is clearly illustrated by the theoretical line. Nearly all lines in this figure illustrate that ${}^{\mathrm{M}}/_{\mathrm{N}}$ is positve for low temperatures, it then decreases around

*Master Profile: Computing Science
Student Number: s1869140

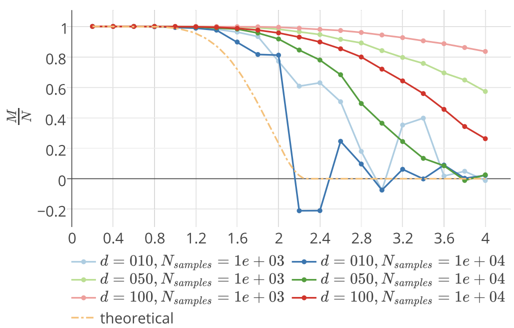
$T = T_c$ and stays zero for temperatures that are a bit higher than the critical temperature. The last observation only holds for the theoretical infinite system. Most of the simulated systems do not reach $^{\mathrm{M}}/_{\mathrm{N}} = 0$.



(a) *Average energy per spin.*



(b) *Specific heat per spin.*



(c) *Average magnetization per spin.*

Figure 1: *The (a) average energy, (b) specific heat and (c) average magnetization per spin in a 2D Ising model with* $\mathrm{d} = 10, 50, 100$ *and* $N_{samples} = 1000, 10000$.