One- and Two-Dimensional Ising Model

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1. Introduction

A large number of systems change their macroscopic properties at thermal equilibria. For example magnetic atoms align them selves to form a magnetic material at low temperature or high pressure. When modeled mathematically, these phase transitions only occur in infinitely large systems. This paper investigates if a a simulation of a finite system, the Ising ferromagnet to be exact, can exhibit phase transitions such as the one described above [1].

Section 1.1 introduces the Ising model of ferromagnetism, the next section discusses the Metropolis Monte Carlo method that is used to estimate the Ising model numerically.

1.1. Ising Model

A magnet can be modeled as a large collection of electronic spins. In the Ising model spins point either up or down, i.e. $S_i = \pm 1$ for $i = 1, \ldots, N$, where N represents the number of spins [2].

One important property of the modeled magnet is its magnetization m, which is defined as its average spin:

$$m = \left| \frac{1}{N} \sum_{i=1}^{N} S_i \right|, \tag{1}$$

where N is the number of spins. At high temperatures the spins point in random directions, consequently the magnetization is approximately zero. As the temperature decreases the magnetization remains near zero until a critical temperature is reached, at which the material magnetizes, i.e. $m \gg 0$. Due to the symmetry of the spin two ferromagnetic states are possible.

Section 1.1.1 and 1.1.2 introduce the one- and two-dimensional Ising model, respectively.

1.1.1. One-Dimensional Model

Describe 1D Ising Model

Energy of a configuration

Average Energy

Specific heat per spin

Present an prove analytical solution

1.1.2. Two-Dimensional Model

2D Ising Model

Energy of a configuration

Average energy

Average magnetization per spin

Specific heat

Present analytical solution

1.2. Metropolis Monte Carlo Methods

Metropolis MC in general

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Importance sampling

The Metropolis solution

What are we going to discuss in this paper?

2. Method

What are we going to discuss in this section?

beter structureren, splitsen in 1D en 2D?

how have we applied the MMC to the 1D and 2D ising model? Refer to appendix with actual implementation

3. Experiments

What are we going to discuss?

3.1. One-Dimensional Model

Wat gaan testen?

Average Energy

Define average energy for 1D

Report average energy for different values of T, N and NSAMPLES

Specific Heat

Define specific Heat for 1D

Report specific heat for different values of T, N and NSAMPLES

3.2. Two-Dimensional Model

Wat gaan we testen

Average Energy

Define average energy for 1D

Report average energy for different values of T, N and NSAMPLES

Specific Heat

Define specific Heat for 1D

Report specific heat for different values of T, N and NSAMPLES

Average Magnetization

Define magnetization

Report average magnetization for different values of T, N and NSAMPLES

4. Discussion

What are we going to discuss?

Interpret results in terms of a phase transition from a state with magnetization zero to a state with definite magnetization (slide 31)

Invloed van de parameters, T, N, NSAMPLES

4.1. One-Dimensional Model

Present analytical solution i.e. prove whatever is one slide 28

Compare results with the analytical solution

4.2. Two-Dimensional Model

Compare Average magnetization with the exact result for the infinite system

5. Conclusion

Hoe goed sluit het model aan bij de het exacte resultaat?

Wat hebben we geleerd over de parameters.

References

[1] Wolfgang Kenzel et al. *Physics by computer*. Springer-Verlag New York, Inc., 1997.

[2] Steven H Strogatz. *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering*. Westview press, 2014.