

# ONE- AND TWO-DIMENSIONAL ISING MODEL

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## A. RESULTS

**Table 1:** Results of the 1D simulation for  $N = 10$ ,  $N_{\text{samples}} = 1000$ .

$T$	$\beta$	Numerical		Analytical		accuracy
		$U/N$	$C/N$	$U/N$	$C/N$	
0.20	5.00	-0.900	0.	-1.000	0.004 54	$-\infty$
0.40	2.50	-0.900	0.002 50	-0.987	0.166	-32
0.60	1.67	-0.833	0.421	-0.931	0.370	0.88
0.80	1.25	-0.771	0.450	-0.848	0.438	0.94
1.0	1.00	-0.707	0.419	-0.762	0.420	0.96
1.2	0.833	-0.643	0.269	-0.682	0.371	0.78
1.4	0.714	-0.538	0.241	-0.613	0.318	0.77
1.6	0.625	-0.467	0.223	-0.555	0.270	0.80
1.8	0.556	-0.432	0.248	-0.505	0.230	0.88
2.0	0.500	-0.428	0.198	-0.462	0.197	0.96
2.2	0.455	-0.369	0.137	-0.426	0.169	0.81
2.4	0.417	-0.343	0.131	-0.394	0.147	0.86
2.6	0.385	-0.316	0.124	-0.367	0.128	0.91
2.8	0.357	-0.312	0.106	-0.343	0.113	0.92
3.0	0.333	-0.321	0.0754	-0.322	0.0996	0.84
3.2	0.313	-0.291	0.0926	-0.303	0.0887	0.96
3.4	0.294	-0.292	0.0824	-0.286	0.0794	0.97
3.6	0.278	-0.234	0.0593	-0.271	0.0715	0.82
3.8	0.263	-0.227	0.0563	-0.257	0.0647	0.86
4.0	0.250	-0.203	0.0520	-0.245	0.0588	0.83

**Table 2:** Results of the 1D simulation for  $N = 100$ ,  $N_{\text{samples}} = 1000$ .

$T$	$\beta$	Numerical		Analytical		accuracy
		$U/N$	$C/N$	$U/N$	$C/N$	
0.20	5.00	-0.990	0.	-1.000	0.004 54	$-\infty$
0.40	2.50	-0.990	0.003 93	-0.987	0.166	-20
0.60	1.67	-0.989	0.0142	-0.931	0.370	-12
0.80	1.25	-0.889	0.0989	-0.848	0.438	-0.74
1.0	1.00	-0.821	0.180	-0.762	0.420	0.30
1.2	0.833	-0.790	0.146	-0.682	0.371	0.16
1.4	0.714	-0.613	0.290	-0.613	0.318	0.95
1.6	0.625	-0.499	0.214	-0.555	0.270	0.81
1.8	0.556	-0.542	0.0760	-0.505	0.230	-0.049
2.0	0.500	-0.419	0.129	-0.462	0.197	0.69
2.2	0.455	-0.407	0.181	-0.426	0.169	0.94

**Table 2:** *continued*

$T$	$\beta$	Numerical		Analytical		accuracy
		$U/N$	$C/N$	$U/N$	$C/N$	
2.4	0.417	-0.366	0.0847	-0.394	0.147	0.60
2.6	0.385	-0.384	0.0739	-0.367	0.128	0.61
2.8	0.357	-0.326	0.125	-0.343	0.113	0.92
3.0	0.333	-0.293	0.0890	-0.322	0.0996	0.89
3.2	0.313	-0.327	0.0840	-0.303	0.0887	0.93
3.4	0.294	-0.220	0.0893	-0.286	0.0794	0.79
3.6	0.278	-0.267	0.0392	-0.271	0.0715	0.58
3.8	0.263	-0.293	0.0608	-0.257	0.0647	0.91
4.0	0.250	-0.203	0.0425	-0.245	0.0588	0.71

**Table 3:** *Results of the 1D simulation for  $N = 1000$ ,  $N_{samples} = 1000$ .*

$T$	$\beta$	Numerical		Analytical		accuracy
		$U/N$	$C/N$	$U/N$	$C/N$	
0.20	5.00	-0.999	0.	-1.000	0.00454	$-\infty$
0.40	2.50	-0.999	0.	-0.987	0.166	$-\infty$
0.60	1.67	-0.999	0.	-0.931	0.370	$-\infty$
0.80	1.25	-0.983	0.102	-0.848	0.438	-0.72
1.0	1.00	-0.956	0.280	-0.762	0.420	0.65
1.2	0.833	-0.877	0.383	-0.682	0.371	0.87
1.4	0.714	-0.794	0.208	-0.613	0.318	0.62
1.6	0.625	-0.696	0.409	-0.555	0.270	0.73
1.8	0.556	-0.583	0.157	-0.505	0.230	0.70
2.0	0.500	-0.508	0.129	-0.462	0.197	0.69
2.2	0.455	-0.460	0.0419	-0.426	0.169	-0.56
2.4	0.417	-0.399	0.108	-0.394	0.147	0.82
2.6	0.385	-0.356	0.0345	-0.367	0.128	-0.37
2.8	0.357	-0.371	0.0357	-0.343	0.113	-0.11
3.0	0.333	-0.359	0.0296	-0.322	0.0996	-0.23
3.2	0.313	-0.351	0.0140	-0.303	0.0887	-1.7
3.4	0.294	-0.279	0.0241	-0.286	0.0794	-0.16
3.6	0.278	-0.268	0.0125	-0.271	0.0715	-1.4
3.8	0.263	-0.240	0.0112	-0.257	0.0647	-1.4
4.0	0.250	-0.230	0.0197	-0.245	0.0588	-0.025

**Table 4:** Results of the 1D simulation for  $N = 10$ ,  $N_{samples} = 10000$ .

$T$	$\beta$	Numerical		Analytical		accuracy
		$U/N$	$C/N$	$U/N$	$C/N$	
0.20	5.00	-0.900	0.	-1.000	0.004 54	$-\infty$
0.40	2.50	-0.871	0.366	-0.987	0.166	0.66
0.60	1.67	-0.826	0.395	-0.931	0.370	0.90
0.80	1.25	-0.771	0.393	-0.848	0.438	0.89
1.0	1.00	-0.663	0.396	-0.762	0.420	0.90
1.2	0.833	-0.624	0.321	-0.682	0.371	0.88
1.4	0.714	-0.568	0.294	-0.613	0.318	0.92
1.6	0.625	-0.496	0.236	-0.555	0.270	0.87
1.8	0.556	-0.441	0.201	-0.505	0.230	0.86
2.0	0.500	-0.428	0.169	-0.462	0.197	0.88
2.2	0.455	-0.377	0.143	-0.426	0.169	0.84
2.4	0.417	-0.361	0.136	-0.394	0.147	0.92
2.6	0.385	-0.324	0.108	-0.367	0.128	0.84
2.8	0.357	-0.314	0.0971	-0.343	0.113	0.88
3.0	0.333	-0.296	0.0871	-0.322	0.0996	0.88
3.2	0.313	-0.278	0.0767	-0.303	0.0887	0.88
3.4	0.294	-0.253	0.0741	-0.286	0.0794	0.90
3.6	0.278	-0.258	0.0655	-0.271	0.0715	0.93
3.8	0.263	-0.227	0.0594	-0.257	0.0647	0.89
4.0	0.250	-0.216	0.0534	-0.245	0.0588	0.88

**Table 5:** Results of the 1D simulation for  $N = 100$ ,  $N_{samples} = 10000$ .

$T$	$\beta$	Numerical		Analytical		accuracy
		$U/N$	$C/N$	$U/N$	$C/N$	
0.20	5.00	-0.990	0.	-1.000	0.004 54	$-\infty$
0.40	2.50	-0.984	0.0547	-0.987	0.166	-0.021
0.60	1.67	-0.942	0.102	-0.931	0.370	-0.33
0.80	1.25	-0.835	0.595	-0.848	0.438	0.86
1.0	1.00	-0.785	0.379	-0.762	0.420	0.93
1.2	0.833	-0.683	0.296	-0.682	0.371	0.87
1.4	0.714	-0.583	0.302	-0.613	0.318	0.95
1.6	0.625	-0.549	0.253	-0.555	0.270	0.96
1.8	0.556	-0.497	0.193	-0.505	0.230	0.90
2.0	0.500	-0.444	0.179	-0.462	0.197	0.93
2.2	0.455	-0.423	0.138	-0.426	0.169	0.89
2.4	0.417	-0.396	0.127	-0.394	0.147	0.92
2.6	0.385	-0.359	0.128	-0.367	0.128	0.99
2.8	0.357	-0.343	0.110	-0.343	0.113	0.99

**Table 5:** *continued*

$T$	$\beta$	Numerical		Analytical		accuracy
		$U/N$	$C/N$	$U/N$	$C/N$	
3.0	0.333	-0.319	0.136	-0.322	0.0996	0.86
3.2	0.313	-0.292	0.0805	-0.303	0.0887	0.93
3.4	0.294	-0.277	0.0802	-0.286	0.0794	0.98
3.6	0.278	-0.266	0.0744	-0.271	0.0715	0.97
3.8	0.263	-0.253	0.0735	-0.257	0.0647	0.93
4.0	0.250	-0.242	0.0643	-0.245	0.0588	0.95

**Table 6:** *Results of the 1D simulation for  $N = 1000$ ,  $N_{samples} = 10000$ .*

$T$	$\beta$	Numerical		Analytical		accuracy
		$U/N$	$C/N$	$U/N$	$C/N$	
0.20	5.00	-0.999	0.	-1.000	0.00454	$-\infty$
0.40	2.50	-0.999	0.	-0.987	0.166	$-\infty$
0.60	1.67	-0.988	0.0726	-0.931	0.370	-1.1
0.80	1.25	-0.937	0.479	-0.848	0.438	0.91
1.0	1.00	-0.805	0.928	-0.762	0.420	0.70
1.2	0.833	-0.685	0.193	-0.682	0.371	0.54
1.4	0.714	-0.629	0.173	-0.613	0.318	0.57
1.6	0.625	-0.520	0.406	-0.555	0.270	0.80
1.8	0.556	-0.511	0.243	-0.505	0.230	0.97
2.0	0.500	-0.476	0.190	-0.462	0.197	0.97
2.2	0.455	-0.430	0.187	-0.426	0.169	0.95
2.4	0.417	-0.389	0.181	-0.394	0.147	0.90
2.6	0.385	-0.368	0.0987	-0.367	0.128	0.85
2.8	0.357	-0.350	0.0720	-0.343	0.113	0.71
3.0	0.333	-0.323	0.0659	-0.322	0.0996	0.74
3.2	0.313	-0.304	0.0913	-0.303	0.0887	0.98
3.4	0.294	-0.281	0.0695	-0.286	0.0794	0.92
3.6	0.278	-0.267	0.0773	-0.271	0.0715	0.96
3.8	0.263	-0.258	0.0544	-0.257	0.0647	0.90
4.0	0.250	-0.221	0.0854	-0.245	0.0588	0.79