Geometric Algorithms
Assignment 1

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A

Figure 1 presents the three possible types of paths that form a straight line through the points $\boldsymbol{P_1}$, $\boldsymbol{P_2}$ and $\boldsymbol{P_3}$. Converting these three 2D points to three dimensional ones, by adding a z-coordinate that is zero, allows us to use the cross product to determine the angle θ, using the definition of the cross product:

$$\boldsymbol{v_1} \times \boldsymbol{v_2} = \|\boldsymbol{v_1}\| \cdot \|\boldsymbol{v_2}\| \cdot \boldsymbol{n} \cdot \sin\theta. \tag{1}$$

Where the $\boldsymbol{v_1} = \boldsymbol{P_2} - \boldsymbol{P_1}$, $\boldsymbol{v_2} = \boldsymbol{P_3} - \boldsymbol{P_1}$ and $\boldsymbol{n} = [0, 0, 1]$ represents the normal of the plane that contains the points.

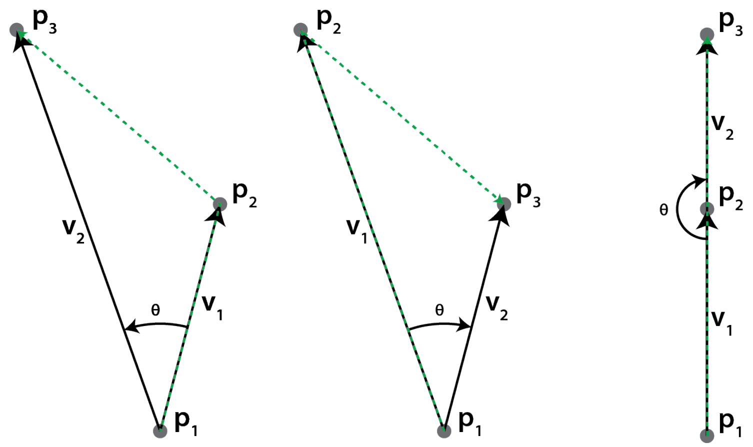
Since all values but the last of the vector $\boldsymbol{n}$ are zero the result of (1) will be of the form $[0, 0, q]$, where q is influenced by $\sin\theta$. The norms of the vectors $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ are greater than or equal to zero by definition. And does not influence the value of q. From this we can conclude that the sign, or the lack thereof of q is completely dependent upon $\sin\theta$. As we are only interested in the direction of the angle, not in its size we can use this to determine the type of path.

If the points are collinear the angle θ is π and which results in $q = 0$. If q is negative the path makes a left turn, if q is positive the path makes a right turn.

Using Mathematica, see Listing 1, we get an expression for the value of q:

$$q = -\boldsymbol{P_1}_y\boldsymbol{P_2}_x + \boldsymbol{P_1}_x\boldsymbol{P_2}_y + \boldsymbol{P_1}_y\boldsymbol{P_3}_x - \boldsymbol{P_2}_y\boldsymbol{P_3}_x - \boldsymbol{P_1}_x\boldsymbol{P_3}_y + \boldsymbol{P_2}_x\boldsymbol{P_3}_y, \tag{2}$$

where $\boldsymbol{P_1}_y$ represents the y-coordinate of the point $\boldsymbol{P_1}$.



(a) left turn **(b)** right turn **(c)** no turn

Figure 1: A path, the green dashed line, through the points $\boldsymbol{P_1}$, $\boldsymbol{P_2}$ and $\boldsymbol{P_3}$ making (a) a left, (b) a right turn and (c) no turn.

Listing 1: Mathematica code used to compute the value of q.

```
p1 = {p1x, p1y, 0};
p2 = {p2x, p2y, 0};
p3 = {p3x, p3y, 0};

v1 = p2 - p1;
v2 = p3 - p2;

Cross[v1, v2]
```