Geometric Algorithms
Assignment 3

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A

Point in Triangle

We try to determine if the point $\mathbf{Q}$ lies in the triangle defined by the points $\mathbf{P_1}$, $\mathbf{P_2}$ and $\mathbf{P_3}$. To this end we define the vectors $\mathbf{v}_1 = \mathbf{P_2} - \mathbf{P_1}$ and $\mathbf{v_2} = \mathbf{P_3} - \mathbf{P_1}$, see Figure 1. Each point $\mathbf{P}$ inside the grey area in this figure can be described as:

$$\mathbf{P} = \mathbf{P_1} + a \cdot \mathbf{v_1} + b \cdot \mathbf{v_2}. \tag{1}$$

For all points to the right of $\mathbf{P_1}$ $a > 0$ and $b > 0$. The points that define the triangle can all be expressed according to (1):

$$\mathbf{P_1} = \mathbf{P_1} + 0 \cdot \mathbf{v_1} + 0 \cdot \mathbf{v_2} \tag{2}$$
$$\mathbf{P_2} = \mathbf{P_1} + 1 \cdot \mathbf{v_1} + 0 \cdot \mathbf{v_2} \tag{3}$$
$$\mathbf{P_3} = \mathbf{P_1} + 0 \cdot \mathbf{v_1} + 1 \cdot \mathbf{v_2}. \tag{4}$$

Based on these (2) through (4) we find that a point $\mathbf{Q}$ lies on the triangle if it can be expressed according to (1) with $a, b \in (0, 1)$ and with $a + b < 1$.

Solving the resulting equation with Mathematica, see Listing 1, gives us expressions for a and b, namely:

$$a = -\frac{-\mathbf{P_{1,0}P_{3,1}} + \mathbf{P_{1,0}Q1} + \mathbf{P_{1,1}P_{3,0}} - \mathbf{P_{1,1}Q_0} - \mathbf{P_{3,0}} \cdot \mathbf{Q_1} + \mathbf{P_{3,1}} \cdot \mathbf{Q_0}}{-\mathbf{P_{1,0}P_{2,1}} + \mathbf{P_{1,0}} \cdot \mathbf{P_{3,1}} + \mathbf{P_{1,1}} \cdot \mathbf{P_{20}} - \mathbf{P_{1,1}P_{3,0}} - \mathbf{P_{2,0}} \cdot \mathbf{P_{3,1}} + \mathbf{P_{2,1}} \cdot \mathbf{P_{3,0}}} \tag{5}$$
$$b = -\frac{-\mathbf{P_{1,0}} \cdot \mathbf{P_{2,1}} + \mathbf{P_{1,0}} \cdot \mathbf{Q_1} + \mathbf{P_{1,1}} \cdot \mathbf{P_{2,0}} - \mathbf{P_{1,1}Q_0} - \mathbf{P_{2,0}} \cdot \mathbf{Q_1} + \mathbf{P_{2,1}} \cdot \mathbf{Q_0}}{\mathbf{P_{1,0}P_{2,1}} - \mathbf{P_{1,0}} \cdot \mathbf{P_{3,1}} - \mathbf{P_{1,1}} \cdot \mathbf{P_{2,0}} + \mathbf{P_{1,1}P_{3,0}} + \mathbf{P_{2,0}} \cdot \mathbf{P_{3,1}} - \mathbf{P_{2,1}} \cdot \mathbf{P_{3,0}}} \tag{6}$$

where $\mathbf{P_{r,s}}$ represents the s'th element of the point r and $\mathbf{Q_t}$ repsents the t'th element of the point $\mathbf{Q}$.

In (5) and (6) are the same, this is the magnitude of the $\mathbf{v_1} \times \mathbf{v_2}$ if we represent the vectors as three dimensional by adding a z-coordinate of zero. If that cross product is zero $\mathbf{v_1}$ and $\mathbf{v_2}$ are parallel and thus the point $\mathbf{Q}$ cannot be represented according to (1).

A.1 Finding the Triangle

B

Listing 1: Mathematica code used to compute the to compute a and b.

```
p1 = {p10, p11};
p2 = {p20, p21};
p3 = {p30, p31};
v1 = p2 - p1;
v2 = p3 - p1;

p4 = p1 + a * v1 + b * v2;
p41 = Part[p4, 1] == Q0;
p42 = Part[p4, 2] == pp1;
solution = Solve[{p41, p42}, {a, b}]
```

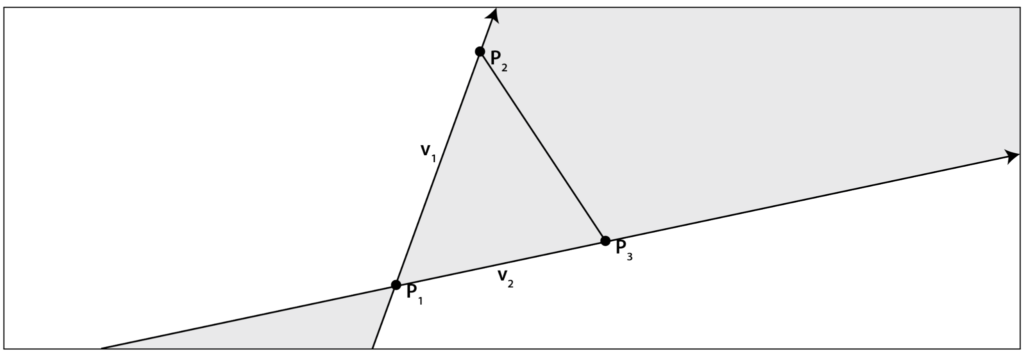

Figure 1: A triangle defined by the points $\mathbf{P_1}$, $\mathbf{P_2}$ and $\mathbf{P_3}$, with the vectors $\mathbf{v_1}$ and $\mathbf{v_2}$. The grey area covers all points that can be described according to (1).