Modelling and Simulation
Practical Assignment 2: Percolation

Rick van Veen (s1883933)* Laura Baakman (s1869140)*

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1. Experiments

This section presents our exploration of the parameter space of the percolation model. In section 1.1 we discuss the influence of the probability parameter p. In this section we often use the size of the cluster to compare the influence of the different parameters. We have defined the cluster size as the number of points that are occupied.

Section 1.2 explores the effect of the size of the system on the generated clusters. In section 1.3 we attempt to determine the fractal dimension of the finite clusters as a function of p. Finally, **??** presents a short analysis of the impact of the used connectivity.

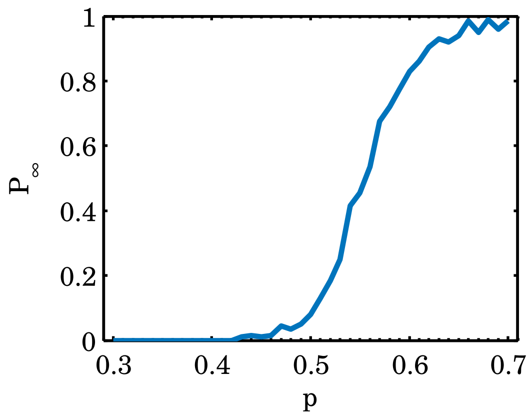

Figure 1: *Ratio of percolating clusters, P_∞, as a function of p. Ratios are calculated over $r_{max} = 200$ runs on a 41×41 grid.*

1.1. Probability

One important property of clusters is their size, and how that size depends on the parameter p. Since one can only determine the size of a finite cluster, we only consider size to be defined for non-percolating clusters. Therefore in the following discussion on the size of clusters we do not consider percolation clusters. Kenzel et al. [2] describe this relation as follows: for small values of p we get a large number of small clusters. As p increases we find positive correlation between p and the average cluster size until p reaches some threshold value p_c. For $p > p_c$ we get either a small finite cluster or a percolating cluster. As $p > p_c$ increases the probability of ending with a finite cluster decreases, until we always get the percolating cluster for $p = 1$. Note that although in theory this cluster should cover the full grid, this is not necessarily the case in our model, since it stops growing as soon as one border site is occupied.

To find an indication of the value of p_c with our model we have let it generate a cluster on a 41×41 grid for $p = 0.31, 0.32, \ldots, 0.7$. For each value of p we grow $r_{max} = 200$ clusters.

Figure 2 presents the mean and standard deviation of the size of the finite clusters as a function of p. In this figure we observe the effect of p on the mean cluster size described by Kenzel et al. Furthermore, based on these data

*These authors contributed equally to this work.

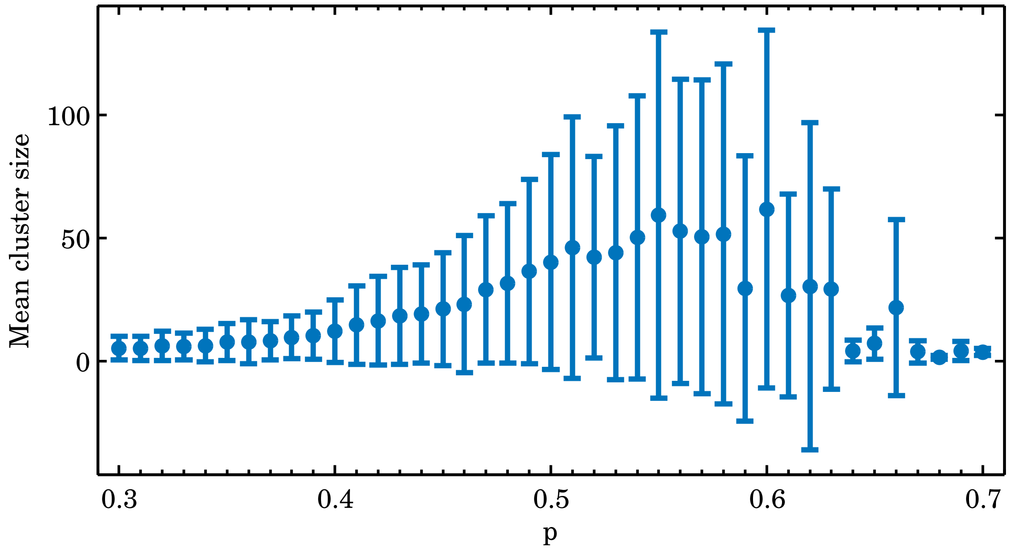

Figure 2: *Mean cluster sizes, represented as points, and standard deviations, indicated by the vertical error bars, as a function of* p*, with a step size of* 0.1*. The mean and standard deviation were calculated over* 200 *runs on a* 41×41 *grid.*

one would estimate p_c to be approximately 0.55.

Kenzel et al. also predicted that the number of percolating clusters relative to the number of finite clusters would grow for $p > p_c$ until $p = 1$, where the only possibility would be a percolating cluster. To observe this effect figure 1 shows P_∞, which is the ratio of the number of percolating clusters to the number of finite clusters. This graph is based on the same data as figure 2. Based on this graph we would say that $p_c \approx 0.4$. This number is lower than the value for p_c based on figure 2. This is probably caused by the relatively small grid sizes, which causes us to classify some clusters as percolating, that are actually finite.

Stauffer [4] has found p_c to be approximately 0.5928 for a square lattice. As stated earlier our lower estimation of p_c is quite likely caused by our small grid.

1.2. System Size

In section 1.1 we postulated that our relatively small grid influenced the found value of p_c. This section qualitatively discusses the relation between the size of the grid and the cluster. We repeat the experiment discussed in section 1.1 but varied $N = 2, 6, \ldots, 60$ instead of p. Since we are mostly interested in the size of finite clusters we choose $p = 0.5 < p_c$. Instead of the size of grid we now measure Q the ratio of the size of finite clusters to the number of sites in the grid.

Figure 3 shows the mean and standard deviation of Q as a function of N. In this graph we observe that average Q decreases as N increases, i.e. the size of the clusters does not grow as hard as the number of sites in the grid.

Figure 4 shows that P_∞ reaches zero for $N > 40$, which indicates that for this value of N there are no more percolating clusters. This fits with the theory discussed in section 1.1, which states that we get only finite clusters for $p < p_c$. We have percolating clusters for $p = 0.5 < p_c$ since the grid is not large enough to hold the finite clusters.

Physics by computer heeft een hoop theorie hiervoer die ik niet helemaal begrijp... of waarvoor we in ieder geval een ander soort plotjes

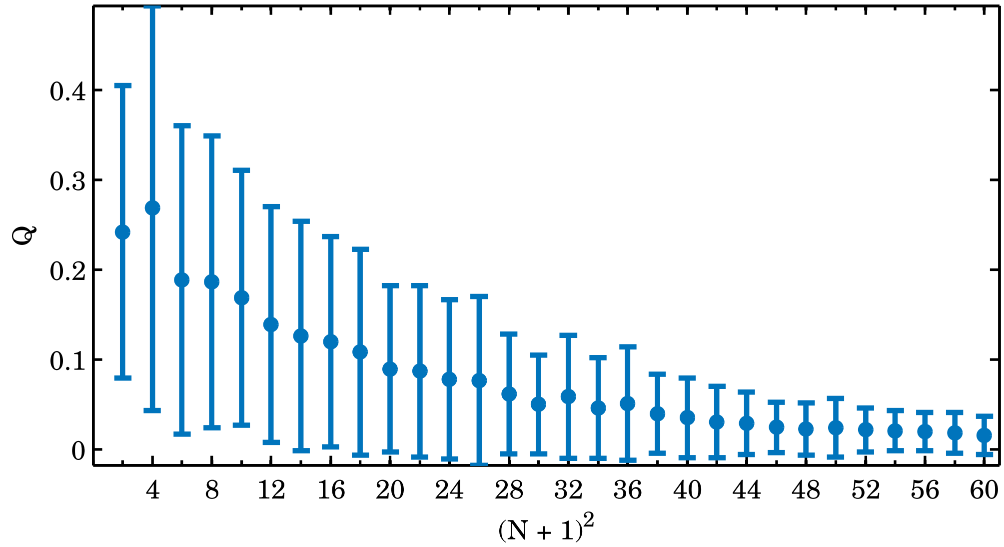

Figure 3: *The mean, represented as points, and standard deviations, indicated by the error bars, of the ratio of the cluster size to number of sites in the grid, i.e.* $(N+1)^2$*.The mean and standard deviation were calculated over 200 runs with* $p = 0.5$*.*

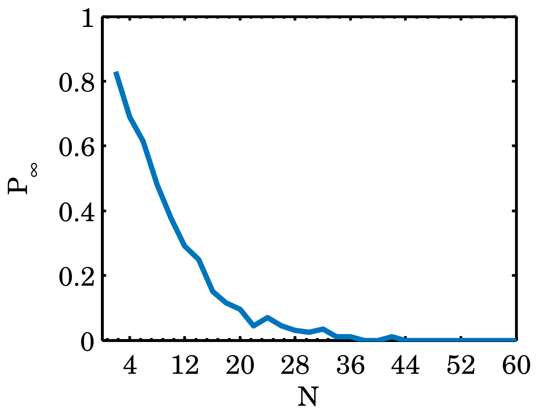

Figure 4: *Ratio of percolating clusters to the number of finite clusters,* P_∞*, as a function of* N*. Ratios are calculated over* $r_{max} = 200$ *runs with* $p = 0.5$*.*

If the lattice size is infinite we are no longer limited by the size of the lattice, in essence we remove one of our stop conditions. In this situation the theory presented in section 1.1 holds, i.e. as long as $p < p_c$ we only have finite clusters. As $p > p_c$ the number of percolating clusters relative to the number of finite clusters increases until we always get a percolating cluster for $p = 1$.

1.3. Fractal Dimension

Hoe verandert de fractal dimension ρ als een functie van p?

Falconer [1] describes the fractal dimension as some number ρ such that

$$(1) \quad M_\varepsilon(\rho) \sim c\varepsilon^{-s}$$

where c and s are constants and $M_\varepsilon(\rho)$ are measurements at different scales ε for $\varepsilon \to 0$. Falconer then shows that the fractal dimension can be estimated “as minus the gradient of a log-log graph plotted over a suitable range of

ε"[1].

One way to get the measurements M_ε is to use box-counting. When box-counting is used the different scales, mentioned in Falconer's definition, are the sizes of the boxes.

We have used the function `box-count` by Moisy [3] to determine the fractal dimension of Percolation clusters. This method uses box sizes that are power of two. Consequently $\varepsilon = 1, 2, 4, \dots 2^q$ where q is the smallest integer such that $q \leq (2N + 1)$.

We have used the box-counting algorithm on a cluster generated with $N = 80$, $p = 0.7$, the used cluster is shown in **??**. **??** presents the number of boxes as a function of the size of the boxes. The box-counting dimension can be read from **??** to be 1.879, which neatly approximates the dimension 1.896 mentioned by Stauffer [4].

Uileggen hoe we dimension uit dit figuur halen.

The small difference between these numbers can be explained by the relatively small size of our cluster.

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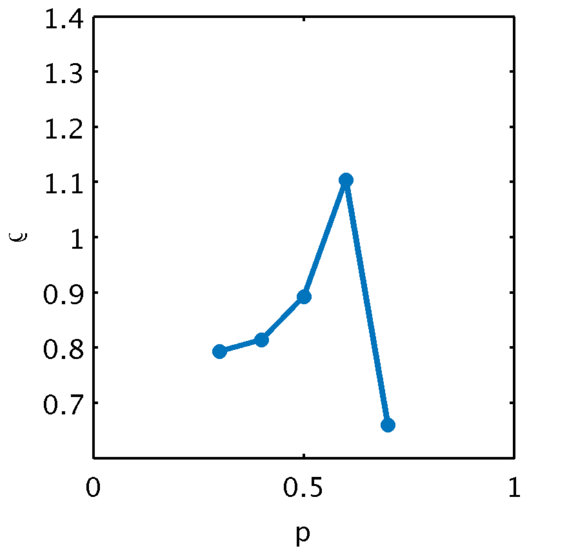

Figure 5: *The average box-counting dimension of 20 finite clusters for each value of p for $p = 0.3, 0.4, \dots, 0.7$.*

References

[1] Kenneth Falconer. *Fractal geometry: mathematical foundations and applications.* John Wiley & Sons, 2004.

[2] Wolfgang Kenzel et al. *Physics by computer.* Springer-Verlag New York, Inc., 1997.

[3] F. Moisy. *Computing a fractal dimension with Matlab: 1D, 2D and 3D Box-counting.* 2008. URL: `http://www.fast.u-psud.fr/~moisy/ml/boxcount/html/demo.html`.

[4] Dietrich Stauffer. *Introduction to percolation theory.* Taylor & Francis, 1985. Chap. 2, pp. 15–58.