Modelling and Simulation
Practical Assignment 2: Percolation

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1. Introduction

Beter inleiding, misschien beginnen met bacterie in petri schaaltje

Kenzel et al. [1] give the following interpretation of the percolation model; it describes the geometry of the randomly generated pores in a porous material through which only certain particles can percolate if the pores form continuous paths. We model this material using a finite lattice, although different lattices are possible, we consider only a square lattice.

The exact percolation model is describes in section 2, this section also presents an implementation of the model in pseudo code. In section 3 we discuss some of the experiments we have performed with the model and their results. Section 4 presents a summary of the findings of our experiment.

Algorithm 1: `percolation`$(mask, N, p)$

```
input  : $N$ size
         $p$ probability
         $mask$ $r \times c$ binary matrix.
output : $grid$ $(N+1) \times (N+1)$ matrix
center := $(N+1,\ N+1)$
push(queue, center)
grid := initGrid(N, N)
while not isEmpty(queue) do
    site = pop(queue)
    sites = grow(grid, site, mask, p)
    if onBorder(site) then
        break
    push(queue, sites)
```

2. Method

Algorithm 1 presents our iterative growth process, the method `percolation` expects three arguments `N`, `probability` and `mask`. Given the size parameter `N`, the grid used for the percolation is $(N+1) \times (N+1)$, since this causes the grid to have an uneven number of rows and columns its center is always clearly defined as (N, N). The parameter $p \in [0, 1]$ is the probability that a given site in the cluster becomes occupied. The `mask` is a binary matrix with r rows and c columns that determines the used connectivity, until section 3.4 we only consider four-connected clusters, which use the mask presented in figure 6a.

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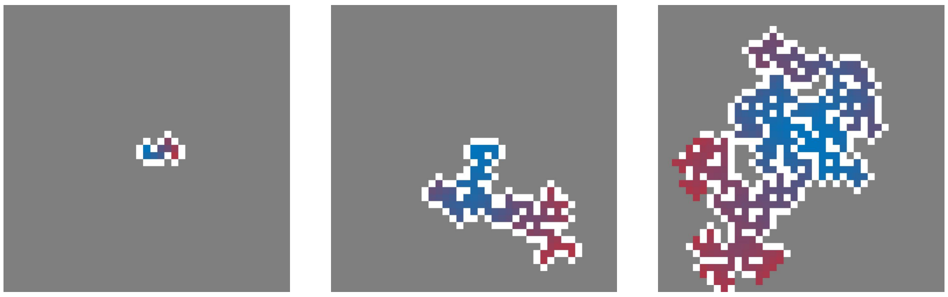
Initially the only site we have to consider is the center site, which is consquently the only site in the queue at the first iteration.

Each iteration we pop the next `site` from the queue. We grow this point, using the function `grow`. This method considers all neighbors that are connected to `site` according to `mask`. For each of these neighbors we determine the value z, which is randomly sampled from an



(a) *$N = 20$ and $p = 0.3$* **(b)** *$N = 20$ and $p = 0.5$* **(c)** *$N = 20$ and $p = 0.6$*

Figure 1: *Examples of (a) a small finite cluster, (b) a larger finite cluster and (c) a percolating cluster using four-connected neighbours. The colours of the elements in the cluster indicate when that point was added to the cluster, the 'colder' the color the earlier in the percolation it was added to the cluster. White cells are empty and gray cells are undetermined.*

uniform distribution with the range $[0, 1]$. If $z \leq p$ we mark the neighbor site as occupied, otherwise it is marked empty. The method `grow` returns the neighbor sites that are occupied, these are added to the queue, as these sites are should also be allowed to grow.

The growth of the clusters stops if it cannot grow anymore or if it has reached one of the borders of the grid. In the first case the cluster is finite, which means that all neighbor sites of the cluster, according to the connectivity defined by the `mask`, are marked as empty. In algorithm 1 we test for this condition via the guard of the loop; if the queue is empty there are no more neighbors to consider, consequently the cluster must be finite.

A percolating cluster is a cluster that has reached the border of the grid, i.e. if there is a occupied site with row or column number 1 or $2N + 1$. We test for this condition with the method `onBorder`. It should be noted that we only check if a site is on the border of the grid after we have already grown the site.

Figure 1 presents three clusters grown with the algorithm. We can clearly see that the finite clusters are completely surround by a white border, which indicates that these sites are empty. The illustration of the percolation cluster, figure 1c, shows that although there are still sites that can grow indicate by the lack of white neighbors, there is one site on the border near the bottom left corner of the image that has stopped the percolation.

3. Experiments

This section presents our exploration of the parameter space of the percolation model. In section 3.1 we discuss the influence of the probability parameter p. Section 3.2 explores the effect of the size of the system on the clusters. In section 3.3 we attempt to determine the fractal dimension of the finite clusters as a function of p. Finally, section 3.4 presents a short analysis of the impact of the connectivity.

3.1. Probability

Discuss cluster size statistics, mean cluster size M and sd as a function p for finite clusters

Determine some vague fraction

To investigate the effect of different p on a lattice with a constant size we perform the following experiment. We opt for a lattice size, with $N = 20$, which results in 41×41 sized grid. We calculate the mean and standard deviation of the finite clusters over $r_{max} = 200$ runs. The probability of growth p is incremented with 0.01 ranging from 0.3 to 0.7. The resulting statistics for all p are shown in figure 2.

We observe that the mean cluster sizes up to approximately $p = 0.55$ generally increase, which is consistent with the definition of p. With $p > 0.55$ we see that the mean cluster sizes start do decrease again. This drop in mean cluster size can be explained with the plot shown in figure 3. Figure 3 shows the P_∞ ratio as a function of p, where the P_∞ is the ratio of 'infinite' clusters. Looking at approximately $p = 0.55$ we see that the number of finite clusters decrease...

Which is not as obvious, as I first thought so need to look at theory...

Look into p_c! Ssy p172 in physics by computer

3.2. System Size

How do the results change when the system size changes. Experiment with different latice sizes

Wat could the behavior be in the limit of infinite lattice sizes

3.3. Fractal Dimension

Definitie van fractal dimension

Welke fractal dimension hebben anderen gevonden?

Boxcounting = MinkowskiâĂŞBouligand dimension uitleggen met source

$$\dim_{\text{box}} = \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log \left[\frac{1}{\epsilon}\right]}. \quad (1)$$

We have used the function `boxcount` by Moisy [2] to determine the fractal dimension of different clusters. This method uses boxsizes that are power of two consequently $\varepsilon = 1, 2, 3, \ldots 2^Q$ where Q is the smallest integer such that $Q \leq (2N + 1)$. We have used the boxcounting algorithm on a cluster generated with $N = 80$, $p = 0.7$, the used cluster is shown in figure 4.

Figure 5 presents the number of boxes as a function of the size of the boxes.

Observatie

Past de gevonden fractal dimension met de theorie?

3.4. Connectivity

How does the connectivity influence the final cluster

4. Conclusion

Vat bevindingen van experiment samen

References

[1] Wolfgang Kenzel et al. *Physics by computer*. Springer-Verlag New York, Inc., 1997.

[2] F. Moisy. *Computing a fractal dimension with Matlab: 1D, 2D and 3D Box-counting*. 2008. URL: `http://www.fast.u-psud.fr/~moisy/ml/boxcount/html/demo.html`.

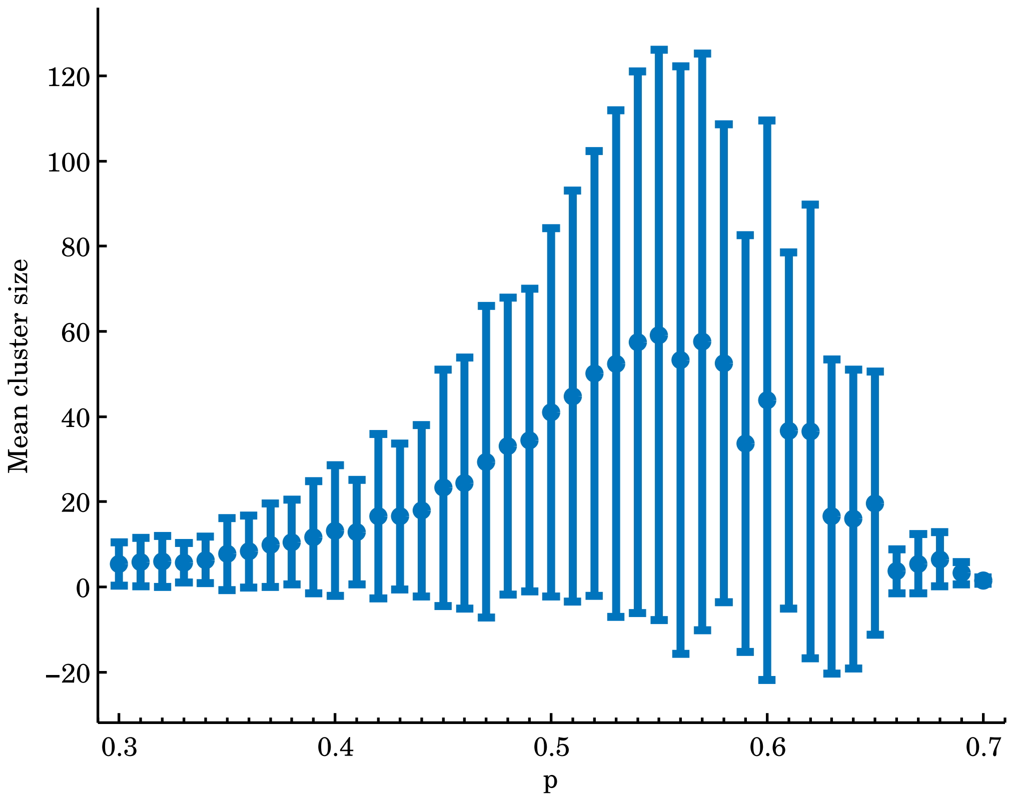


Figure 2: *Mean cluster sizes μ (indicated by the points) and standard deviations σ (vertical error bars) computed as a function of p, with a step size of* 0.01*. Values μ and σ were calculated over* 200 *runs with a grid of size* 41×41.

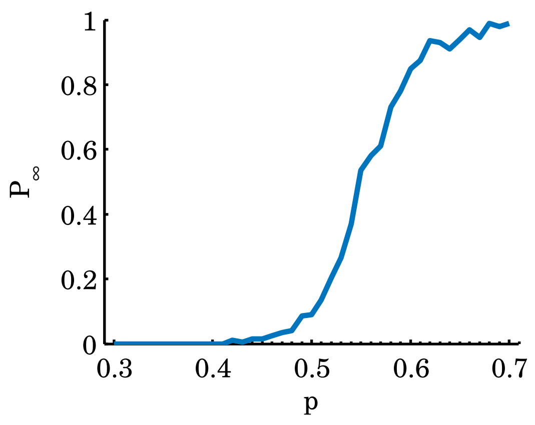


Figure 3: *Ratio of percolating clusters, P_∞, as a function of p. Ratios calculated over $r_{max} = 200$ runs on a grid size of 41×41.*

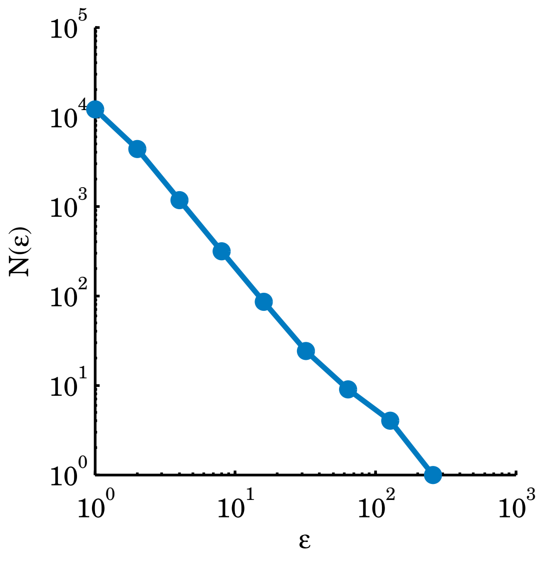


Figure 5: *The number of boxes used to cover a cluster ($N = 80, p = 0.7$) as a function of the box size for the cluster presented in figure 4.*

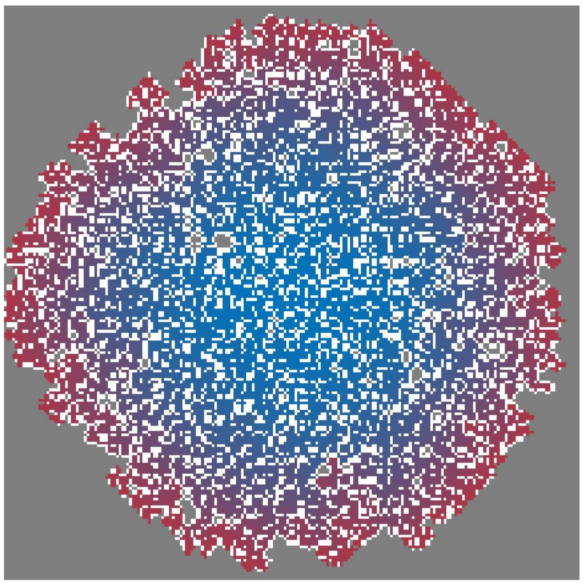
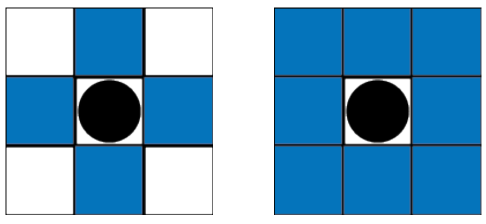


Figure 4: *The cluster used to determine the Minkowski-Bouligand dimension of clusters generated with percolation, the clusters was generated with $N = 80$, $p = 0.7$.*



(a) *4-connectivity* **(b)** *8-connectivity*

Figure 6: *Connectivity masks for (a) four-connectivity and (b) eight-connectivity. The filled-in squares are considered neighbors of the center square, which is indicated with a dot.*