# Exact and Heuristic Solutions for Sensor Placement in Large-Scale Irregular Wireless Networks - Complete Results

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Table 1: All results of the MILP model for regular network instances.

Side	Nodes	Objective Function	Num	ber of	Sensors	
l	n	OF	$\overline{sx}$	sy	sz	Time (s)
3	9	12	1	0	8	0,009
4	16	24	2	2	12	0,020
5	25	37	4	0	21	0,018
6	36	48	4	0	32	0,028
7	49	70	6	3	40	0,045
8	64	91	9	0	55	0,040
9	81	108	9	0	72	0,053
10	100	140	12	4	84	0,081
11	121	169	16	0	105	0,090
12	144	192	16	0	128	0,089
13	169	234	20	5	144	0,130
14	196	271	25	0	171	0,177
15	225	300	25	0	200	$0,196 \\ 0,300$
16 17	$\frac{256}{289}$	$\frac{352}{397}$	30 36	6 0	$\frac{220}{253}$	0,300 $0,326$
18	324	432	36	0	288	0,320 $0,344$
19	361	494	42	7	312	0,544 $0,528$
20	400	547	49	ó	351	0.528 0.657
21	441	588	49	0	392	0,778
22	484	660	56	8	420	1,131
$\frac{22}{23}$	529	721	64	0	465	1,297
$\frac{26}{24}$	576	768	64	ŏ	512	1,457
$\frac{21}{25}$	625	850	72	9	544	2,563
26	676	919	81	ő	595	2,151
27	729	972	81	Ő	648	2,405
28	784	1064	90	10	684	3,507
29	841	1141	100	0	741	3,631
30	900	1200	100	0	800	3,606
31	961	1302	110	11	840	7,470
32	1024	1387	121	0	903	6,615
33	1089	1452	121	0	968	5,842
34	1156	1564	132	12	1012	10,433
35	1225	1657	144	0	1081	10,306
36	1296	1728	144	0	1152	9,484
37	1369	1850	156	13	1200	22,057
38	1444	1951	169	0	1275	14,995
39	1521	2028	169	0	1352	14,049
40	1600	2160	182	14	1404	33,857
41 42	1681	2269	196	0	1485	26,604
	1764	2352	196	0	1568	21,573
43 44	1849 1936	2494 2611	$\frac{210}{225}$	15 0	$     \begin{array}{r}       1624 \\       1711     \end{array} $	51,035
45	$\frac{1930}{2025}$	2700	$\frac{225}{225}$	0	1800	$32,593 \\ 35,262$
46	2116	2852	240	16	1860	104,472
47	2209	2977	256	0	1953	52,015
48	2304	3072	$\frac{256}{256}$	0	2048	45,074
49	2401	3234	272	17	2112	132,489
50	2500	3367	289	0	2211	74,958
51	2601	3468	289	0	2312	76,003
52	2704	3640	306	18	2380	145,533
53	2809	3781	324	0	2485	187,932
54	2916	3888	324	ŏ	2592	136,400
55	3025	4070	342	19	2664	325,278
56	3136	4219	361	0	2775	170,915
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	ŗ	Table 1 - continued fi	rom pre	vious	page.	
57	3249	4332	361	0	$\bar{2}888$	126,433
58	3364	4524	380	20	2964	388,231
59	3481	4681	400	0	3081	262,493
60	3600	4800	400	0	3200	152,735
61	3721	5002	420	21	3280	469,948
62	3844	5167	441	0	3403	$422,\!491$
63	3969	5292	441	0	3528	268,005
64	4096	5504	462	22	3612	$827,\!206$
65	4225	5677	484	0	3741	$323,\!110$
66	4356	5808	484	0	3872	$405,\!259$
67	4489	6030	506	23	3960	1305,290
68	4624	6211	529	0	4095	706,038
69	4761	6348	529	0	4232	630,284
70	4900	6580	552	24	4324	$1228,\!480$
71	5041	6769	576	0	4465	1170,010
72	5184	6912	576	0	4608	955,259

Table 2: All results of the GA approach for regular network instances.

Side	Nodes	Objective Function	Number of Sensors		Time	e (s)	
l	n	OF	$\overline{sx}$	sy	sz	Median	Mean
3	9	12	1	0	8	1,05	1,02
4	16	24	2	2	12	1,21	1,22
5	25	37	4	0	21	1,43	1,43
6	36	48	4	0	32	1,60	1,58
7	49	70	6	3	40	1,98	1,96
8	64	91	9	0	55	2,34	2,35
9	81	108	9	0	72	2,85	2,77
10	100	142	12	6	82	3,34	$3,\!35$
11	121	169	16	0	105	4,00	4,00
12	144	201	17	6	121	4,72	4,76
13	169	240	20	11	138	$5,\!61$	5,60
14	196	276	25	5	166	$6,\!55$	$6,\!56$
15	225	318	28	9	188	$7,\!55$	$7,\!57$
16	256	367	34	9	213	8,71	8,78
17	289	417	40	8	241	10,16	10,04
18	324	467	44	11	269	11,54	$11,\!49$
19	361	525	46	26	289	13,02	12,95
20	400	584	54	22	324	14,68	14,65
21	441	645	59	27	355	16,30	16,26
22	484	709	63	36	385	18,04	17,98
23	529	784	73	36	420	20,09	19,95
24	576	855	78	45	453	22,16	22,19
25	625	928	84	51	490	$24,\!22$	24,12
26	676	1013	97	46	533	$26,\!57$	26,69
27	729	1096	107	46	576	$29,\!35$	29,19
28	784	1188	115	59	610	$31,\!88$	31,93
29	841	1279	122	72	647	34,92	34,96
30	900	1367	130	77	693	$36,\!56$	36,50
31	961	1462	138	87	736	39,82	39,18
32	1024	1571	155	82	787	43,49	43,01
33	1089	1673	160	104	825	46,60	$46,\!36$
34	1156	1784	180	88	888	$50,\!25$	49,65
35	1225	1897	191	99	935	54,18	53,81
36	1296	2014	204	106	986	$57,\!49$	$56,\!67$

		Table 2 - cont	inued from	nrovio	us nago		
37	1369	2140	220	111	1038	62,14	61,93
38	1444	2265	229	134	1081	65,72	65,17
39	1521	2396	$\begin{array}{c} 229 \\ 255 \end{array}$	110	1156	70,81	70,07
		$\frac{2590}{2524}$		141			
40	1600		261		1198	75,12	74,10
41	1681	2654	271	160	1250	80,25	79,38
42	1764	2795	294	149	1321	86,64	85,06
43	1849	2949	311	167	1371	92,08	90,95
44	1936	3077	318	187	1431	99,80	99,51
45	2025	3240	345	180	1500	105,64	105,46
46	2116 2209	3397	361	198	1557	113,07	113,73
47		3554	384	193	1632	120,47	119,97
48	2304	3684	391	207	1706	127,33	127,18
49	2401	3880	425	204	1772	132,21	132,63
50	2500	4043	454	181	1865	135,78	137,41
51	2601	4211	466	212	1923	142,76	142,77
52	2704	4392	493	209	2002	149,80	151,01
53	2809	4569	498	266	2045	155,49	156,46
54	2916	4770	527	273	2116	162,07	163,40
55	3025	4944	560	239	2226	168,92	169,79
56	3136	5135	574	277	2285	175,92	176,48
57	3249	5325	607	255	2387	182,17	181,12
58	3364	5526	629	275	2460	190,33	192,00
59	3481	5732	659	274	2548	196,83	197,68
60	3600	5939	672	323	2605	207,10	209,95
61	3721	6169	703	339	2679	213,27	215,12
62	3844	6361	728	333	2783	221,82	223,70
63	3969	6580	765	316	2888	231,92	232,75
64	4096	6798	781	359	2956	241,41	242,72
65	4225	7022	811	364	3050	248,54	251,06
66	4356	7245	844	357	3155	260,42	262,52
67	4489	7504	871	402	3216	267,94	269,47
68	4624	7724	907	379	3338	279,72	281,58
69	4761	7953	932	396	3433	294,36	298,22
70	4900	8210	971	397	3532	296,80	299,98
71	5041	8467	1007	405	3629	305,88	310,38
72	5184	8724	1037	429	3718	316,56	318,40
73	5329	8951	1057	451	3821	333,37	335,22
74	5476	9197	1094	439	3943	344,25	$349,\!17$
75	5625	9488	1149	416	4060	358,00	$361,\!20$
76	5776	9760	1162	498	4116	365,96	372,00
77	5929	10025	1202	490	4237	376,16	378,72
78	6084	10273	1229	502	4353	384,61	387,85
79	6241	10517	1264	484	4493	364,06	365,86
80	6400	10833	1313	494	4593	385,02	383,64
81	6561	11122	1342	535	4684	409,17	409,73
82	6724	11415	1399	494	4831	419,07	419,85
83	6889	11718	1435	524	4930	445,11	441,31
84	7056	12019	1473	544	5039	453,21	450,29
85	7225	12323	1496	610	5119	459,60	461,15
86	7396	12615	1521	656	5219	471,19	$470,\!29$
87	7569	12923	1594	572	5403	494,20	493,97
88	7744	13199	1612	619	5513	507,30	$507,\!28$
89	7921	13552	1657	660	5604	518,22	$520,\!42$
90	8100	13875	1708	651	5741	533,73	535,89
91	8281	14185	1716	756	5809	$534,\!86$	535,68
92	8464	14515	1780	711	5973	544,33	$547,\!28$
93	8649	14837	1848	644	6157	551,24	$555,\!46$
94	8836	15210	1887	713	6236	549,60	$548,\!11$
95	9025	15537	1926	734	6365	$579,\!43$	577,98

		Table 2 - conti	nued from	previo	us page.		
96	9216	15882	1973	747	6496	593,41	593,47
97	9409	16233	2013	785	6611	601,60	600,88
98	9604	16590	2091	713	6800	607,18	607,42
99	9801	16914	2106	795	6900	627,37	635,60
100	10000	17257	2139	840	7021	645,99	$646,\!53$
101	10201	17628	2225	752	7224	641,09	638,49
102	10404	18002	2267	797	7340	667,33	680,17
103	10609	18361	2305	837	7467	697,54	699,45
104	10816	18754	2368	834	7614	713,98	711,40
105	11025	19095	2417	819	7789	725,80	731,69
106	11236	19419	2460	803	7973	757,50	752,90
107	11449	19817	2485	913	8051	728,18	744,79
108	11664	20269	2575	880	8209	783,78	781,69
109	11881	20569	2581	945	8355	783,93	791,41
110	12100	21002	2645	967	8488	812,17	813,99
111	12321	21418	2707	976	8638	807,31	811,24
112	12544	21852	2769	1001	8774	846,33	840,32
113	12769	22197	2825	953	8991	818,38	846,80
114	12996	22613	2863	1028	9105	$892,\!97$	$900,\!59$
115	13225	23030	2923	1036	9266	895,49	898,08
116	13456	23436	2999	983	9474	$908,\!07$	$910,\!45$
117	13689	23851	3044	1030	9615	940,17	950,97
118	13924	24279	3078	1121	9725	$937,\!21$	946,93
119	14161	24738	3152	1121	9888	961,60	979,27
120	14400	25164	3239	1047	10114	$986,\!40$	$990,\!56$
121	14641	25510	3269	1062	10310	999,98	994,59
122	14884	25972	3336	1080	10468	$997,\!78$	991,14
123	15129	26453	3405	1109	10615	$1012,\!48$	1012,75
124	15376	26883	3464	1115	10797	1053,99	$1062,\!05$
125	15625	27321	3552	1040	11033	$1075,\!65$	1072,15
126	15876	27813	3589	1170	11117	1090,61	1084,66
127	16129	28197	3621	1205	11303	1099,03	1095,33
128	16384	28666	3688	1218	11478	1112,33	1108,48
129	16641	29080	3730	1249	11662	1130,63	1131,41
130	16900	29630	3799	1333	11768	1159,08	1163,24
131	17161	30041	3865	1285	12011	1169,80	1183,51
132	17424	30500	3921	1313	12190	1195,02	1218,22
133	17689	31012	4011	1290	12388	1246,31	1225,21
134	17956	31436	4064	1288	12604	1259,52	1259,15
135	18225	31947	4117	1371	12737	1239,75	$1230,\!34$

 $\begin{tabular}{ll} \textbf{Table 3:} & All \end{tabular} \begin{tabular}{ll} \textbf{results} & of the MILP approach for non-regular network instances. \end{tabular}$ 

Topology	Nodes	Objective Function	Number of Sensors		Time (s)	
Topology	n	OF	sx	sy	sz	Time (s)
Semiregular	25	44	5	4	16	0.02
Irregular	25	44	4	7	14	0.02
Semiregular	225	394	42	43	140	0.12
Irregular	225	393	41	45	139	0.14
Semiregular	1225	2143	227	237	761	0.74
Irregular	1225	2149	226	246	753	0.84

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 $\begin{tabular}{lll} \textbf{Table 4:} & All & results & of the GA & approach & for non-regular & network \\ instances. \end{tabular}$ 

Tomology	Nodes	Objective Function	Number of Sensors		Time (s)		
Topology	n	OF	sx	sy	sz	Median	Mean
Semiregular	25	44	5	4	16	1.52	1.52
Irregular	25	44	4	7	14	1.42	1.42
Semiregular	225	431	65	11	149	27.48	27.94
Irregular	225	425	62	14	149	23.67	24.16
Semiregular	1225	2882	465	52	708	772.86	774.82
Irregular	1225	2824	455	38	732	743.11	757.28