

Comparison of hierarchical models with school variable

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Data

```
regions <- c('Lazio', 'Lombardia', 'Abruzzo', 'Veneto', 'Emilia-Romagna', 'Toscana', 'Campania', 'Friuli Venezia Giulia', 'Sicilia', 'Calabria')

regions

## [1] "Lazio"           "Lombardia"        "Abruzzo"
## [4] "Veneto"          "Emilia-Romagna"   "Toscana"
## [7] "Campania"        "Friuli Venezia Giulia" "Sicilia"
## [10] "Calabria"

hier_data <- get_hier_data(data_it, regions, initial_date = as.Date('2020-08-30') )

p_delay <- get_delay_distribution()

## school effect

school_opening <- as.Date('2020-09-14')
school <- rep(0, length(hier_data$dates))
school[which(hier_data$dates > school_opening +10)] <- 1
grow_school <- which(hier_data$dates>=school_opening & hier_data$dates <= school_opening +10)
school[grow_school] <- (grow_school - which(hier_data$dates ==school_opening))^2 /100

## mask effect

masks_obligation <- as.Date('2020-10-13')
masks <- rep(0, length(hier_data$dates))
masks[which(hier_data$dates > masks_obligation +10)] <- 1
grow_masks <- which(hier_data$dates>=masks_obligation & hier_data$dates <=masks_obligation +10)
masks[grow_masks] <- (grow_masks - which(hier_data$dates ==masks_obligation))^2 /100

stan_data_hier <- list(J = length(regions),
                       N = nrow(hier_data$exposures),
                       N_nonzero = length(hier_data$nonzero_days),
                       nonzero_days = hier_data$nonzero_days,
                       conv_gt = get_gt_convolution_ln2(nrow(hier_data$exposures)),
                       length_delay = length(p_delay),
                       p_delay = p_delay,
                       exposures = hier_data$exposures,
                       nonzero_positives = hier_data$positives[hier_data$nonzero_days ,],
                       school= school)
```

```

)

stan_data_hier_2 <- list(J = length(regions),
                        N = nrow(hier_data$exposures),
                        N_nonzero = length(hier_data$nonzero_days),
                        nonzero_days = hier_data$nonzero_days,
                        conv_gt = get_gt_convolution_ln2(nrow(hier_data$exposures)),
                        length_delay = length(p_delay),
                        p_delay = p_delay,
                        exposures = hier_data$exposures,
                        nonzero_positives = hier_data$positives[hier_data$nonzero_days ,],
                        school= school[hier_data$nonzero_days]

)

compiled_hier <- stan_model('..../stan/hier_model_school.stan')

## Trying to compile a simple C file
## Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
## clang -mmacosx-version-min=10.13 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG -I
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/include
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util
## namespace Eigen {
## ^
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util
## namespace Eigen {
## ^
## ;
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/include
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/Core:96:10: f
## #include <complex>
## ^
## 3 errors generated.
## make: *** [foo.o] Error 1
fit_hier <- sampling(compiled_hier, data= stan_data_hier, iter=2000, cores=getOption("mc.cores", 1L))

##
## SAMPLING FOR MODEL 'hier_model_school' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.017064 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 170.64 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration: 1 / 2000 [  0%] (Warmup)
## Chain 1: Iteration: 200 / 2000 [ 10%] (Warmup)
## Chain 1: Iteration: 400 / 2000 [ 20%] (Warmup)
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## Chain 1: Iteration: 600 / 2000 [ 30%] (Warmup)
## Chain 1: Iteration: 800 / 2000 [ 40%] (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%] (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%] (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%] (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%] (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%] (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%] (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 2559.61 seconds (Warm-up)
## Chain 1: 2043.95 seconds (Sampling)
## Chain 1: 4603.56 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'hier_model_school' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0.009345 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 93.45 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration: 1 / 2000 [ 0%] (Warmup)
## Chain 2: Iteration: 200 / 2000 [ 10%] (Warmup)
## Chain 2: Iteration: 400 / 2000 [ 20%] (Warmup)
## Chain 2: Iteration: 600 / 2000 [ 30%] (Warmup)
## Chain 2: Iteration: 800 / 2000 [ 40%] (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%] (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%] (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%] (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%] (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%] (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%] (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 3153.75 seconds (Warm-up)
## Chain 2: 4698.11 seconds (Sampling)
## Chain 2: 7851.86 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'hier_model_school' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0.012142 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 121.42 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration: 1 / 2000 [ 0%] (Warmup)
## Chain 3: Iteration: 200 / 2000 [ 10%] (Warmup)
## Chain 3: Iteration: 400 / 2000 [ 20%] (Warmup)
## Chain 3: Iteration: 600 / 2000 [ 30%] (Warmup)
## Chain 3: Iteration: 800 / 2000 [ 40%] (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%] (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%] (Sampling)

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## Chain 3: Iteration: 1200 / 2000 [ 60%] (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%] (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%] (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%] (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 3145.7 seconds (Warm-up)
## Chain 3:           4403.1 seconds (Sampling)
## Chain 3:           7548.8 seconds (Total)
## Chain 3:
## 
## SAMPLING FOR MODEL 'hier_model_school' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0.012436 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 124.36 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration: 1 / 2000 [ 0%] (Warmup)
## Chain 4: Iteration: 200 / 2000 [ 10%] (Warmup)
## Chain 4: Iteration: 400 / 2000 [ 20%] (Warmup)
## Chain 4: Iteration: 600 / 2000 [ 30%] (Warmup)
## Chain 4: Iteration: 800 / 2000 [ 40%] (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%] (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%] (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%] (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%] (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%] (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%] (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%] (Sampling)
## Chain 4:
## Chain 4: Elapsed Time: 3580.99 seconds (Warm-up)
## Chain 4:           4288.63 seconds (Sampling)
## Chain 4:           7869.61 seconds (Total)
## Chain 4:
## 
## Warning: The largest R-hat is 1.1, indicating chains have not mixed.
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#r-hat

## Warning: Bulk Effective Samples Size (ESS) is too low, indicating posterior means and medians may be
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#bulk-ess

## Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quantiles
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#tail-ess

compiled_hier_2 <- stan_model('..../stan/hier_model_school_2.stan')

## Trying to compile a simple C file

## Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
## clang -fPIC -mmacosx-version-min=10.13 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG -I
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/include StanHeaders.h
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include RcppEigen.h

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## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util
## namespace Eigen {
## ^
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util
## namespace Eigen {
## ^
## ;
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/include
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/Core:96:10: f
## #include <complex>
## ^
## 3 errors generated.
## make: *** [foo.o] Error 1

fit_hier_2 <- sampling(compiled_hier_2, data= stan_data_hier_2, iter=2000, cores=getOption("mc.cores", 1))

##
## SAMPLING FOR MODEL 'hier_model_school_2' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.024141 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 241.41 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration: 1 / 2000 [ 0%] (Warmup)
## Chain 1: Iteration: 200 / 2000 [ 10%] (Warmup)
## Chain 1: Iteration: 400 / 2000 [ 20%] (Warmup)
## Chain 1: Iteration: 600 / 2000 [ 30%] (Warmup)
## Chain 1: Iteration: 800 / 2000 [ 40%] (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%] (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%] (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%] (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%] (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%] (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%] (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 1612.46 seconds (Warm-up)
## Chain 1: 1065.51 seconds (Sampling)
## Chain 1: 2677.98 seconds (Total)
## Chain 1:
## 
## SAMPLING FOR MODEL 'hier_model_school_2' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0.009356 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 93.56 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration: 1 / 2000 [ 0%] (Warmup)
## Chain 2: Exception: Exception: poisson_rng: Rate parameter is 0, but must be > 0! (in 'modele17245b031f_hier_model_school_2' at line 150)

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##
## Chain 2: Exception: Exception: poisson_rng: Rate parameter is 0, but must be > 0!  (in 'modele17245b031f_hier_model_school_2' at line 150)
##
## Chain 2: Iteration: 200 / 2000 [ 10%] (Warmup)
## Chain 2: Iteration: 400 / 2000 [ 20%] (Warmup)
## Chain 2: Iteration: 600 / 2000 [ 30%] (Warmup)
## Chain 2: Iteration: 800 / 2000 [ 40%] (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%] (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%] (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%] (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%] (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%] (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%] (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 1602.7 seconds (Warm-up)
## Chain 2:           1065.42 seconds (Sampling)
## Chain 2:           2668.12 seconds (Total)
## Chain 2:
## 
## SAMPLING FOR MODEL 'hier_model_school_2' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0.01239 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 123.9 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration: 1 / 2000 [  0%] (Warmup)
## Chain 3: Iteration: 200 / 2000 [ 10%] (Warmup)
## Chain 3: Iteration: 400 / 2000 [ 20%] (Warmup)
## Chain 3: Iteration: 600 / 2000 [ 30%] (Warmup)
## Chain 3: Iteration: 800 / 2000 [ 40%] (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%] (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%] (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%] (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%] (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%] (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%] (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 1586.55 seconds (Warm-up)
## Chain 3:           1059.13 seconds (Sampling)
## Chain 3:           2645.68 seconds (Total)
## Chain 3:
## 
## SAMPLING FOR MODEL 'hier_model_school_2' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0.008663 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 86.63 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration: 1 / 2000 [  0%] (Warmup)

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## Chain 4: Iteration: 200 / 2000 [ 10%] (Warmup)
## Chain 4: Iteration: 400 / 2000 [ 20%] (Warmup)
## Chain 4: Iteration: 600 / 2000 [ 30%] (Warmup)
## Chain 4: Iteration: 800 / 2000 [ 40%] (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%] (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%] (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%] (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%] (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%] (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%] (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%] (Sampling)
## Chain 4:
## Chain 4: Elapsed Time: 1571.76 seconds (Warm-up)
## Chain 4: 1060.64 seconds (Sampling)
## Chain 4: 2632.41 seconds (Total)
## Chain 4:

## Warning in validityMethod(object): The following variables have undefined
## values: log_lik[1,1],The following variables have undefined values:
## log_lik[2,1],The following variables have undefined values: log_lik[3,1],The
## following variables have undefined values: log_lik[4,1],The following
## variables have undefined values: log_lik[5,1],The following variables have
## undefined values: log_lik[6,1],The following variables have undefined values:
## log_lik[7,1],The following variables have undefined values: log_lik[8,1],The
## following variables have undefined values: log_lik[9,1],The following
## variables have undefined values: log_lik[10,1],The following variables have
## undefined values: log_lik[11,1],The following variables have undefined values:
## log_lik[12,1],The following variables have undefined values: log_lik[13,1],The
## following variables have undefined values: log_lik[14,1],The following
## variables have undefined values: log_lik[15,1],The following variables have
## undefined values: log_lik[16,1],The following variables have undefined values:
## log_lik[17,1],The following variables have undefined values: log_lik[18,1],The
## following variables have undefined values: log_lik[19,1],The following
## variables have undefined values: log_lik[20,1],The following variables have
## undefined values: log_lik[21,1],The following variables have undefined values:
## log_lik[22,1],The following variables have undefined values: log_lik[23,1],The
## following variables have undefined values: log_lik[24,1],The following
## variables have undefined values: log_lik[25,1],The following variables have
## undefined values: log_lik[26,1],The following variables have undefined values:
## log_lik[27,1],The following variables have undefined values: log_lik[28,1],The
## following variables have undefined values: log_lik[29,1],The following
## variables have undefined values: log_lik[30,1],The following variables have
## undefined values: log_lik[31,1],The following variables have undefined values:
## log_lik[32,1],The following variables have undefined values: log_lik[33,1],The
## following variables have undefined values: log_lik[34,1],The following
## variables have undefined values: log_lik[35,1],The following variables have
## undefined values: log_lik[36,1],The following variables have undefined values:
## log_lik[37,1],The following variables have undefined values: log_lik[38,1],The
## following variables have undefined values: log_lik[39,1],The following
## variables have undefined values: log_lik[40,1],The following variables have
## undefined values: log_lik[41,1],The following variables have undefined values:
## log_lik[42,1],The following variables have undefined values: log_lik[43,1],The
## following variables have undefined values: log_lik[44,1],The following
## variables have undefined values: log_lik[45,1],The following variables have

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## following variables have undefined values: log_lik[28,2],The following
## variables have undefined values: log_lik[29,2],The following variables have
## undefined values: log_lik[30,2],The following variables have undefined values:
## log_lik[31,2],The following variables have undefined values: log_lik[32,2],The
## following variables have undefined values: log_lik[33,2],The following
## variables have undefined values: log_lik[34,2],The following variables have
## undefined values: log_lik[35,2],The following variables have undefined values:
## log_lik[36,2],The following variables have undefined values: log_lik[37,2],The
## following variables have undefined values: log_lik[38,2],The following
## variables have undefined values: log_lik[39,2],The following variables have
## undefined values: log_lik[40,2],The following variables have undefined values:
## log_lik[41,2],The following variables have undefined values: log_lik[42,2],The
## following variables have undefined values: log_lik[43,2],The following
## variables have undefined values: log_lik[44,2],The following variables have
## undefined values: log_lik[45,2],The following variables have undefined values:
## log_lik[46,2],The following variables

compiled_hier_3 <- stan_model('..../stan/hier_model_school_3.stan')

## Trying to compile a simple C file

## Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
## clang -mmacosx-version-min=10.13 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG -I
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/include
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util
## namespace Eigen {
## ^
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util
## namespace Eigen {
## ^
## ;
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/include
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/Core:96:10: f
## #include <complex>
## ^
## 3 errors generated.
## make: *** [foo.o] Error 1

fit_hier_3 <- sampling(compiled_hier_3, data= stan_data_hier, iter=2000, cores=getOption("mc.cores", 1L))

##
## SAMPLING FOR MODEL 'hier_model_school_3' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.016914 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 169.14 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)

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## Chain 1: Iteration: 600 / 2000 [ 30%] (Warmup)
## Chain 1: Iteration: 800 / 2000 [ 40%] (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%] (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%] (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%] (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%] (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%] (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%] (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 1489.13 seconds (Warm-up)
## Chain 1:           1031.36 seconds (Sampling)
## Chain 1:           2520.49 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'hier_model_school_3' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0.010142 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 101.42 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration: 1 / 2000 [  0%] (Warmup)
## Chain 2: Iteration: 200 / 2000 [ 10%] (Warmup)
## Chain 2: Iteration: 400 / 2000 [ 20%] (Warmup)
## Chain 2: Iteration: 600 / 2000 [ 30%] (Warmup)
## Chain 2: Iteration: 800 / 2000 [ 40%] (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%] (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%] (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%] (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%] (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%] (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%] (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 1552.59 seconds (Warm-up)
## Chain 2:           1037.01 seconds (Sampling)
## Chain 2:           2589.59 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'hier_model_school_3' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0.009977 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 99.77 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration: 1 / 2000 [  0%] (Warmup)
## Chain 3: Iteration: 200 / 2000 [ 10%] (Warmup)
## Chain 3: Iteration: 400 / 2000 [ 20%] (Warmup)
## Chain 3: Iteration: 600 / 2000 [ 30%] (Warmup)
## Chain 3: Iteration: 800 / 2000 [ 40%] (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%] (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%] (Sampling)

```

```

## Chain 3: Iteration: 1200 / 2000 [ 60%] (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%] (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%] (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%] (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 1524.96 seconds (Warm-up)
## Chain 3:           1028.78 seconds (Sampling)
## Chain 3:           2553.74 seconds (Total)
## Chain 3:
## 
## SAMPLING FOR MODEL 'hier_model_school_3' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0.008384 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 83.84 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration: 1 / 2000 [ 0%] (Warmup)
## Chain 4: Iteration: 200 / 2000 [ 10%] (Warmup)
## Chain 4: Iteration: 400 / 2000 [ 20%] (Warmup)
## Chain 4: Iteration: 600 / 2000 [ 30%] (Warmup)
## Chain 4: Iteration: 800 / 2000 [ 40%] (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%] (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%] (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%] (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%] (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%] (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%] (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%] (Sampling)
## Chain 4:
## Chain 4: Elapsed Time: 1534.27 seconds (Warm-up)
## Chain 4:           1038.31 seconds (Sampling)
## Chain 4:           2572.58 seconds (Total)
## Chain 4:

```

School effect

Model 1

```

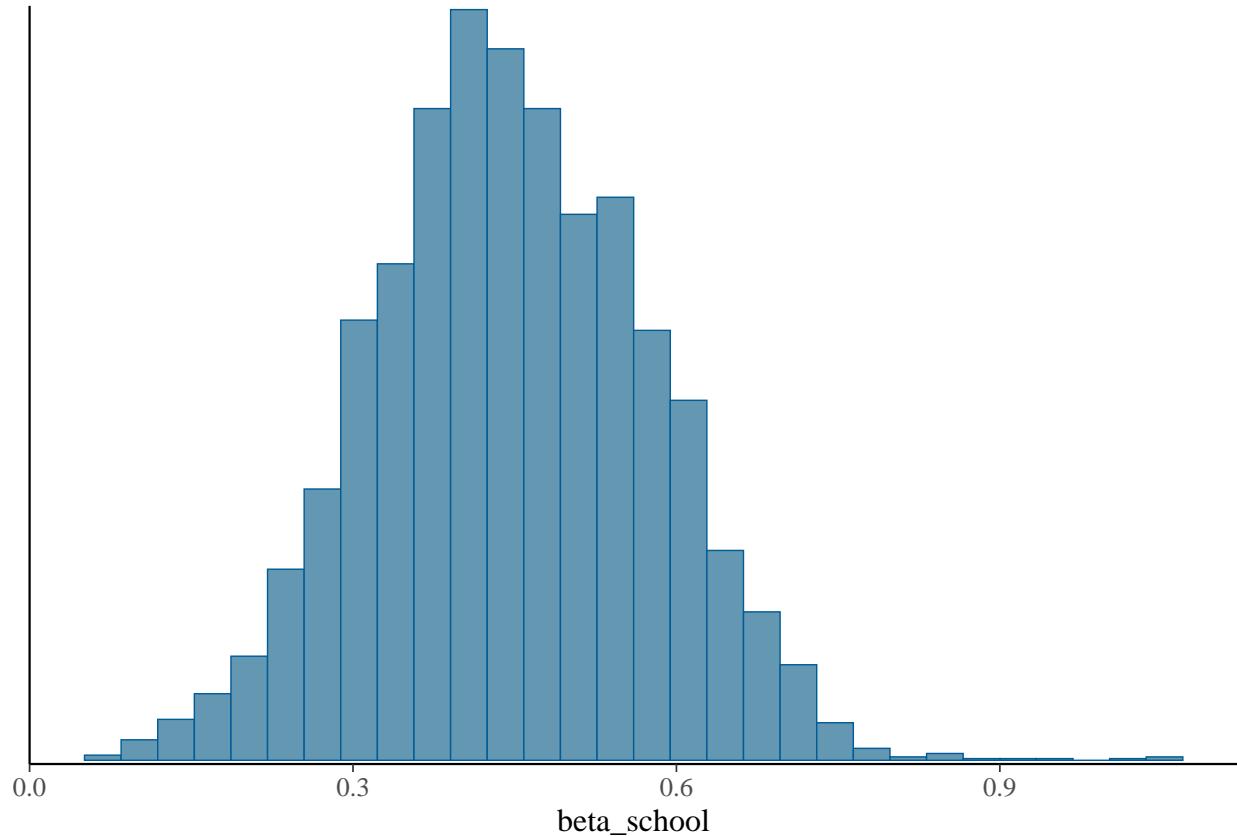
print(fit_hier, pars='beta_school')

## Inference for Stan model: hier_model_school.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##          mean se_mean    sd 2.5%  25%  50%  75% 97.5% n_eff Rhat
## beta_school 0.45    0.02 0.13  0.2 0.36 0.44 0.54  0.69     47   1.1
##
## Samples were drawn using NUTS(diag_e) at Sat Nov 28 13:51:21 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).

```

```
mcmc_hist(fit_hier, pars='beta_school')

## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



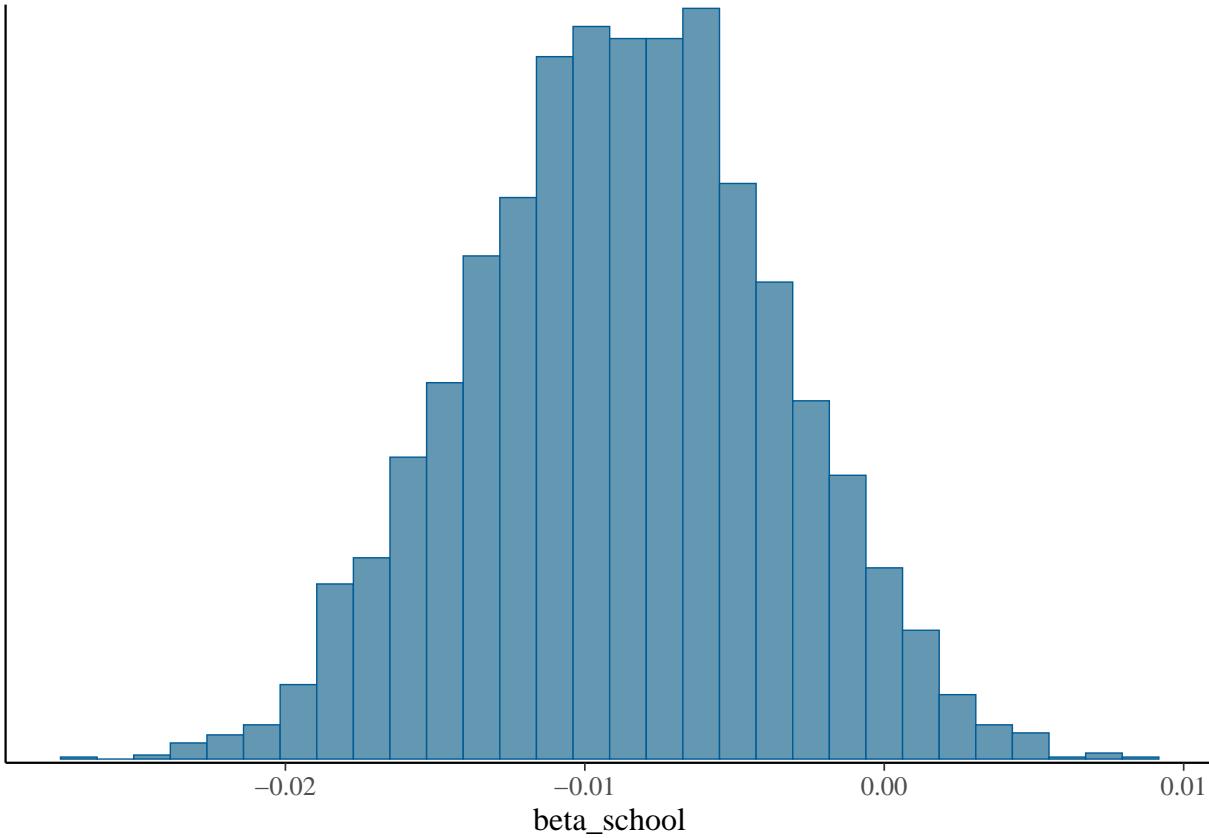
Model 2

```
print(fit_hier_2, pars='beta_school')

## Inference for Stan model: hier_model_school_2.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##           mean se_mean    sd  2.5%  25%  50%  75% 97.5% n_eff Rhat
## beta_school 1.43    0.04 2.06 -2.56  0.02 1.44 2.82  5.51  3250     1
##
## Samples were drawn using NUTS(diag_e) at Sat Nov 28 16:52:57 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).

mcmc_hist(fit_hier_3, pars='beta_school')

## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



Model 3

```

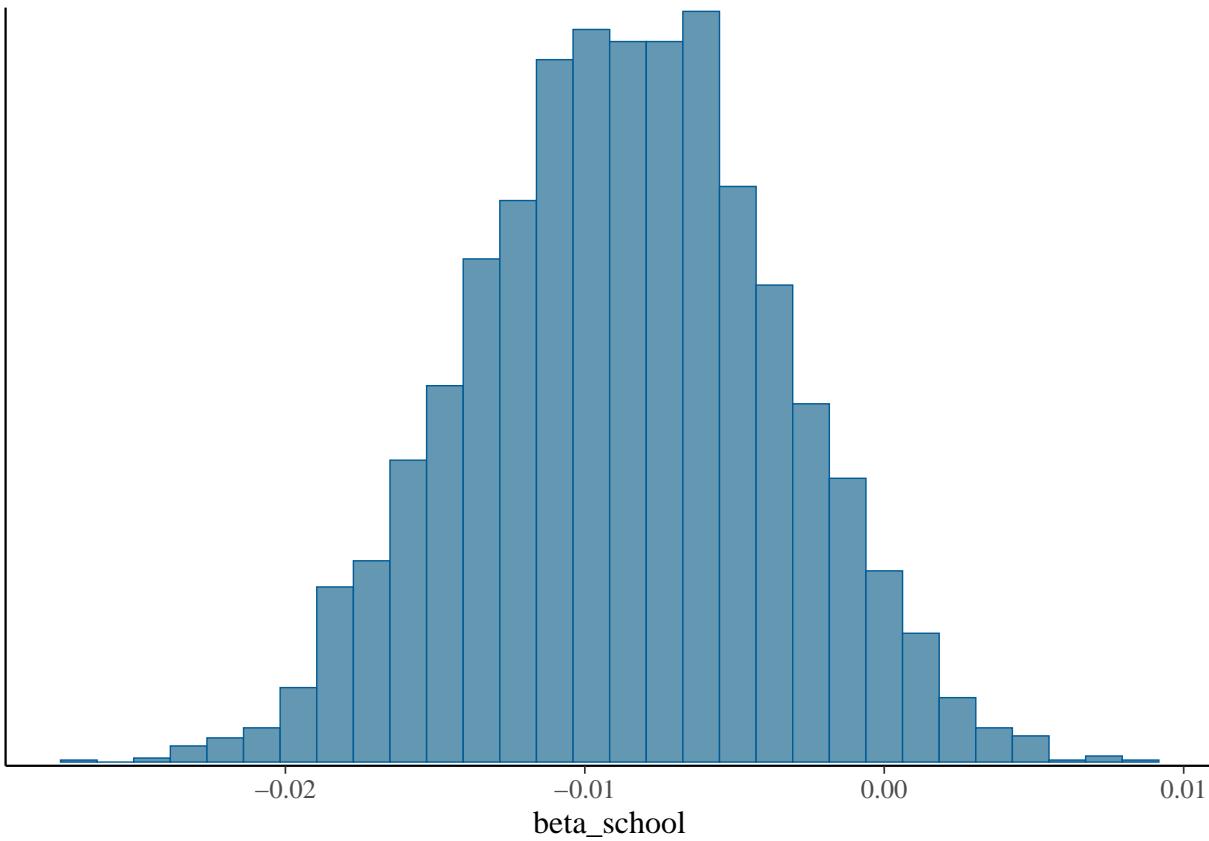
print(fit_hier_3, pars='beta_school')

## Inference for Stan model: hier_model_school_3.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##           mean se_mean    sd  2.5%   25%   50%   75% 97.5% n_eff Rhat
## beta_school -0.01      0 0.01 -0.02 -0.01 -0.01 -0.01      0 1851     1
##
## Samples were drawn using NUTS(diag_e) at Sat Nov 28 19:46:53 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).

mcmc_hist(fit_hier_3, pars='beta_school')

## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.

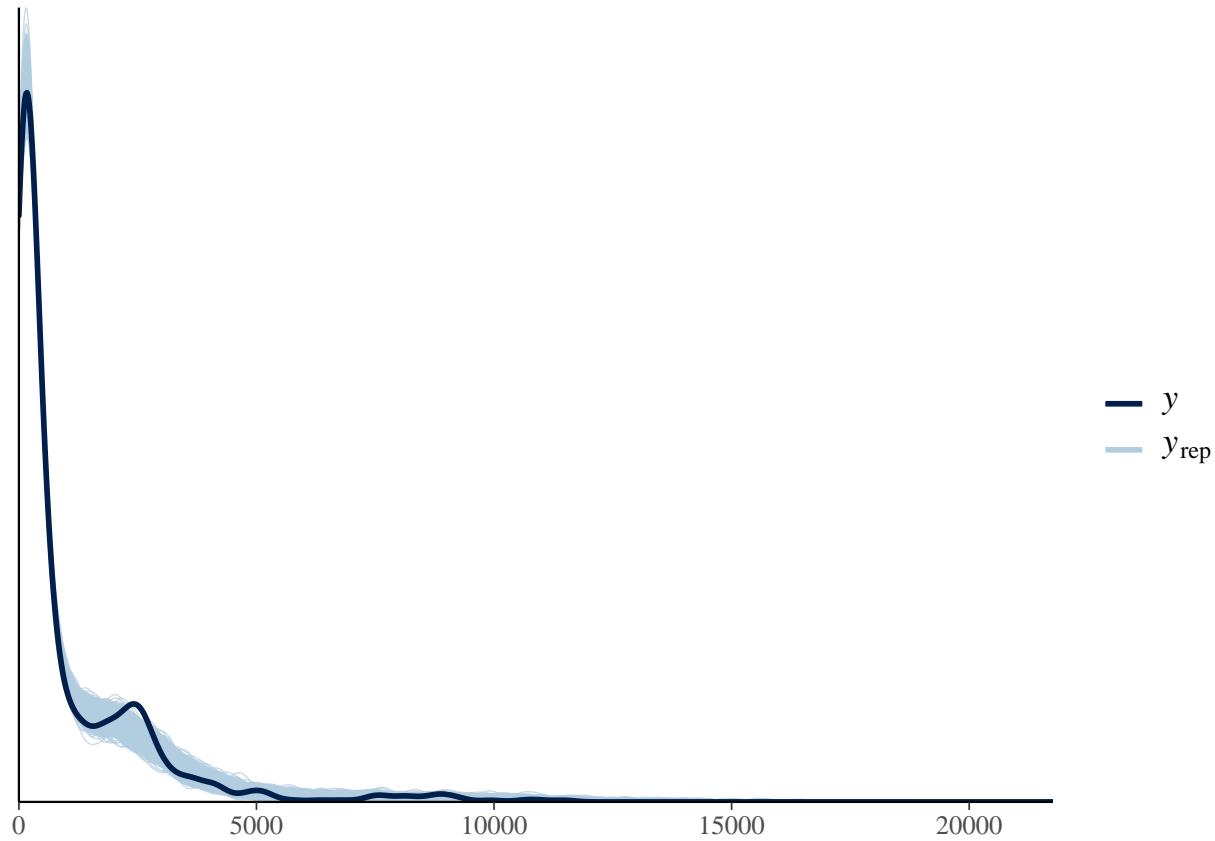
```



Posterior predictive check

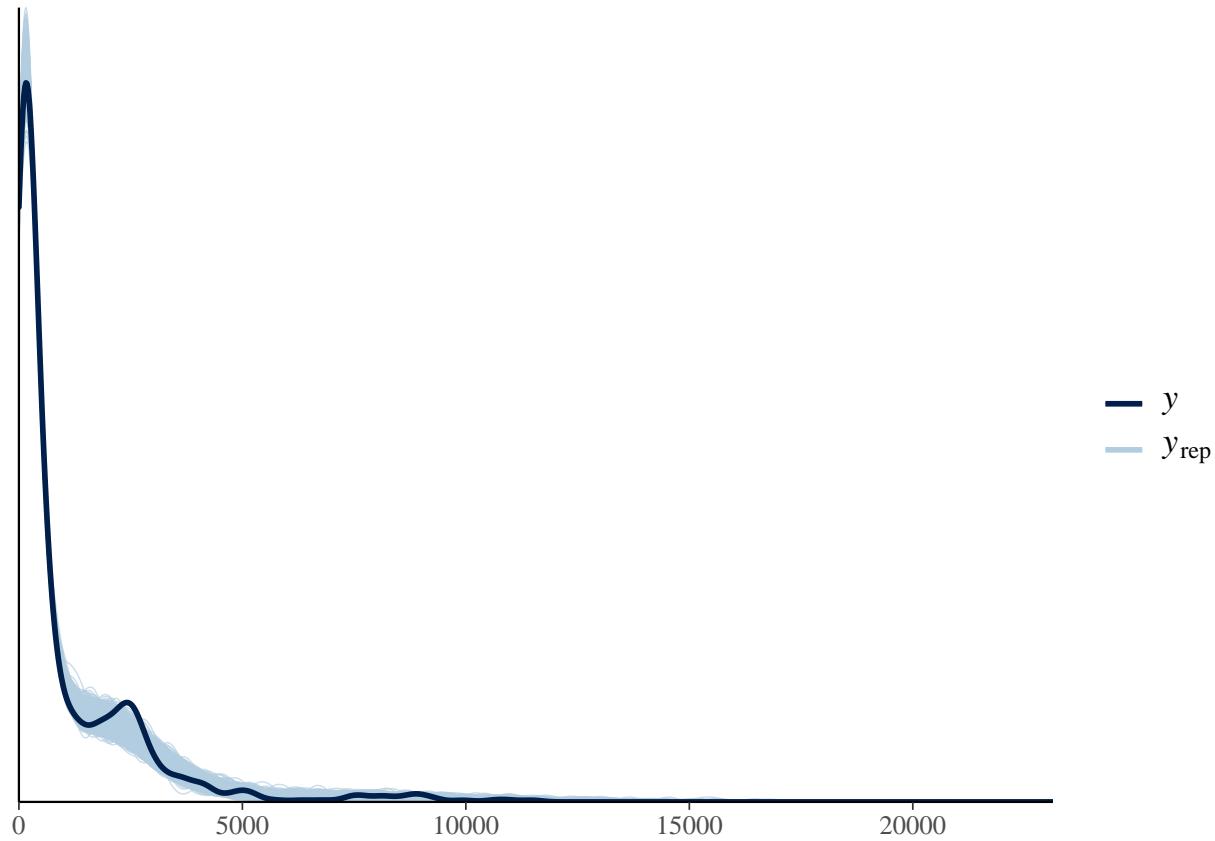
Model 1

```
y_rep <- as.matrix(fit_hier, pars = "y_rep")
ppc_dens_overlay(y = as.vector(stan_data_hier$nonzero_positives), y_rep[1:1000, ])
```



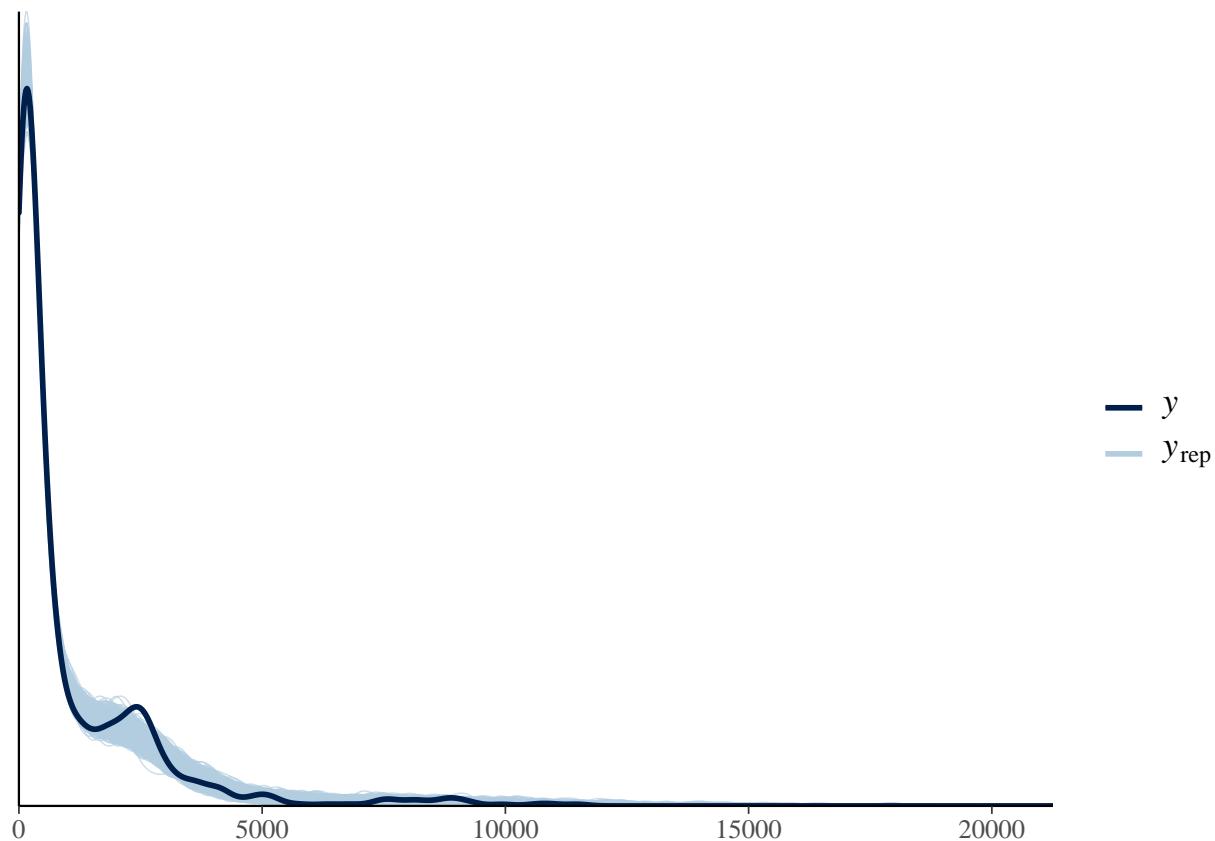
Model 2

```
y_rep_2 <- as.matrix(fit_hier_2, pars = "y_rep")
ppc_dens_overlay(y = as.vector(stan_data_hier_2$nonzero_positives), y_rep_2[1:1000, ])
```



Model 3

```
y_rep_3 <- as.matrix(fit_hier_3, pars = "y_rep")
ppc_dens_overlay(y = as.vector(stan_data_hier$nonzero_positives), y_rep_3[1:1000, ])
```



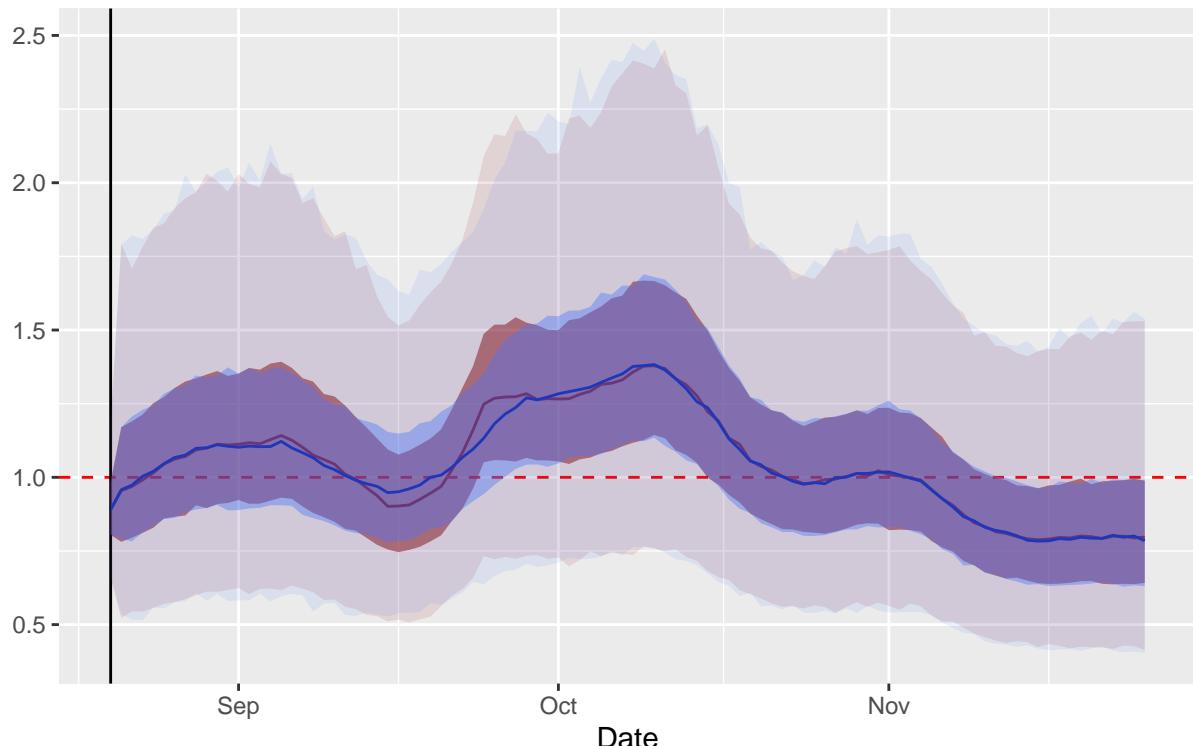
Rt

Lazio

```
p <- plot_rt_hier(hier_data, fit_hier, regions, 'Lazio')
plot_overlay(hier_data, fit_hier_2, regions, p, 'Lazio')+
  labs(subtitle = 'model 1 (red) vs model 2 (blue) ')
```

Rt Lazio

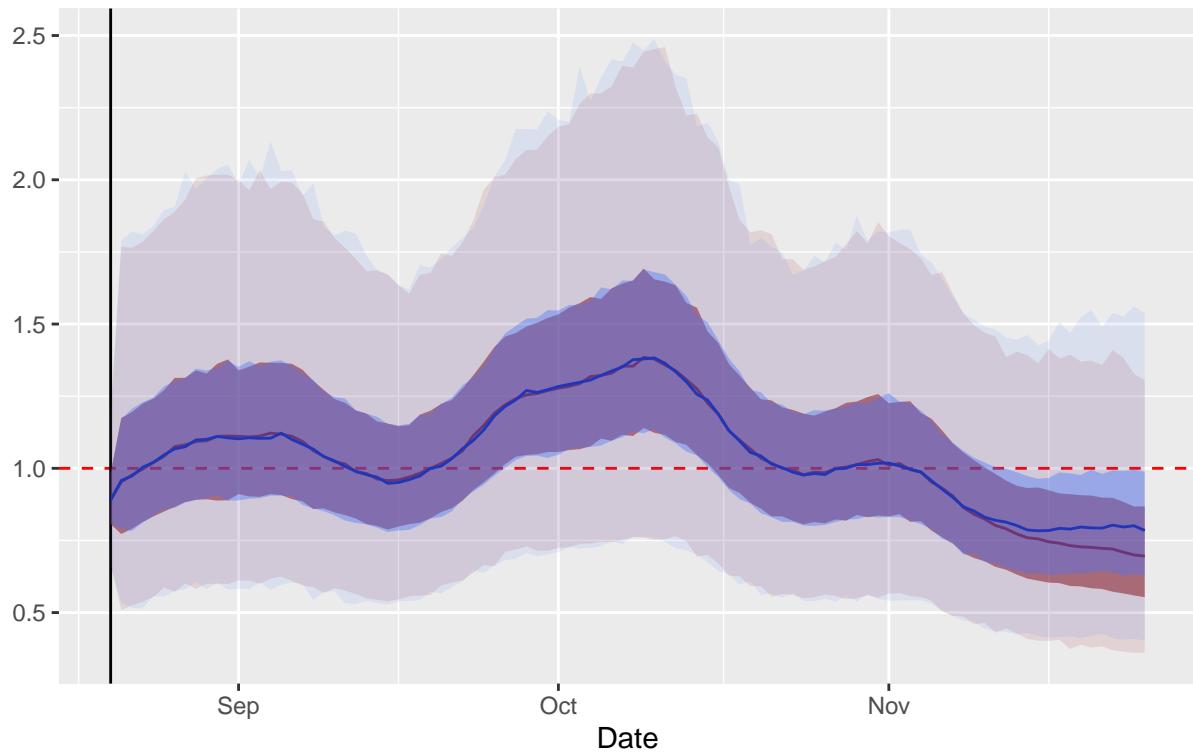
model 1 (red) vs model 2 (blue)



```
p2 <- plot_rt_hier(hier_data, fit_hier_3, regions, 'Lazio')
plot_overlay(hier_data, fit_hier_2, regions, p2, 'Lazio')+
  labs(subtitle = 'model 3 (red) vs model 2 (blue) ')
```

Rt Lazio

model 3 (red) vs model 2 (blue)

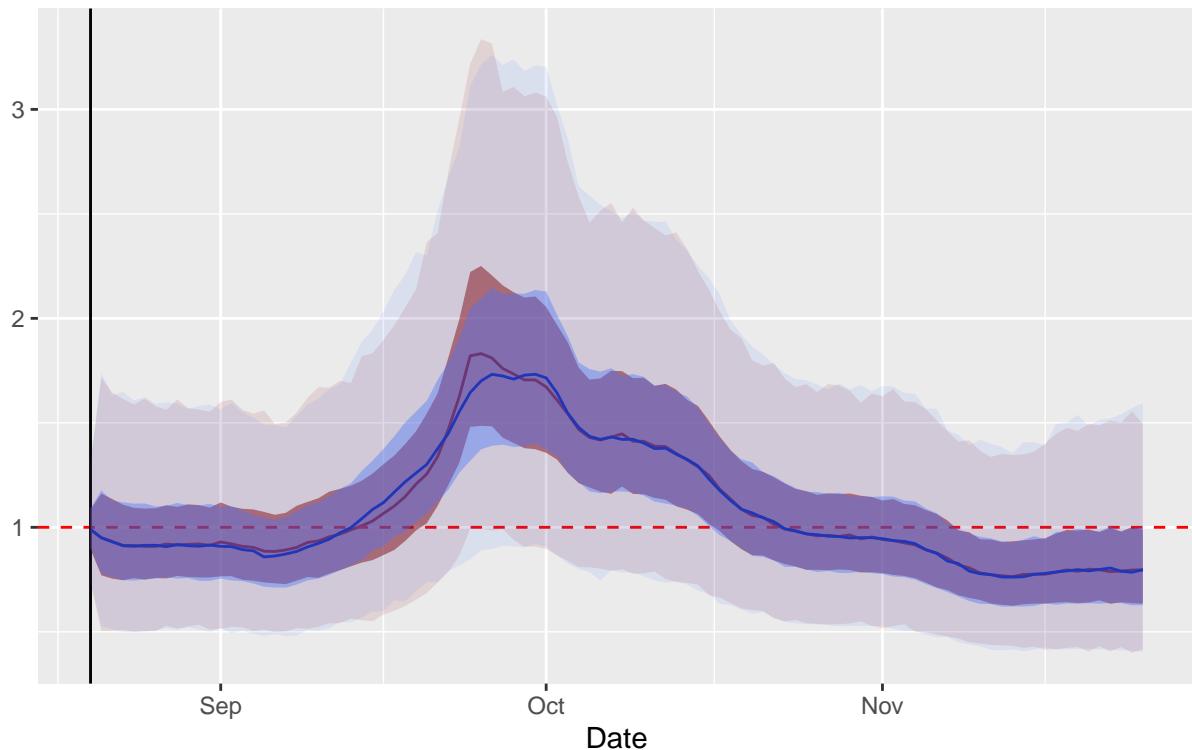


Lombardia

```
p <- plot_rt_hier(hier_data, fit_hier, regions, 'Lombardia')
plot_overlay(hier_data, fit_hier_2, regions, p, 'Lombardia')+
  labs(subtitle = 'model 1 (red) vs model 2 (blue) ')
```

Rt Lombardia

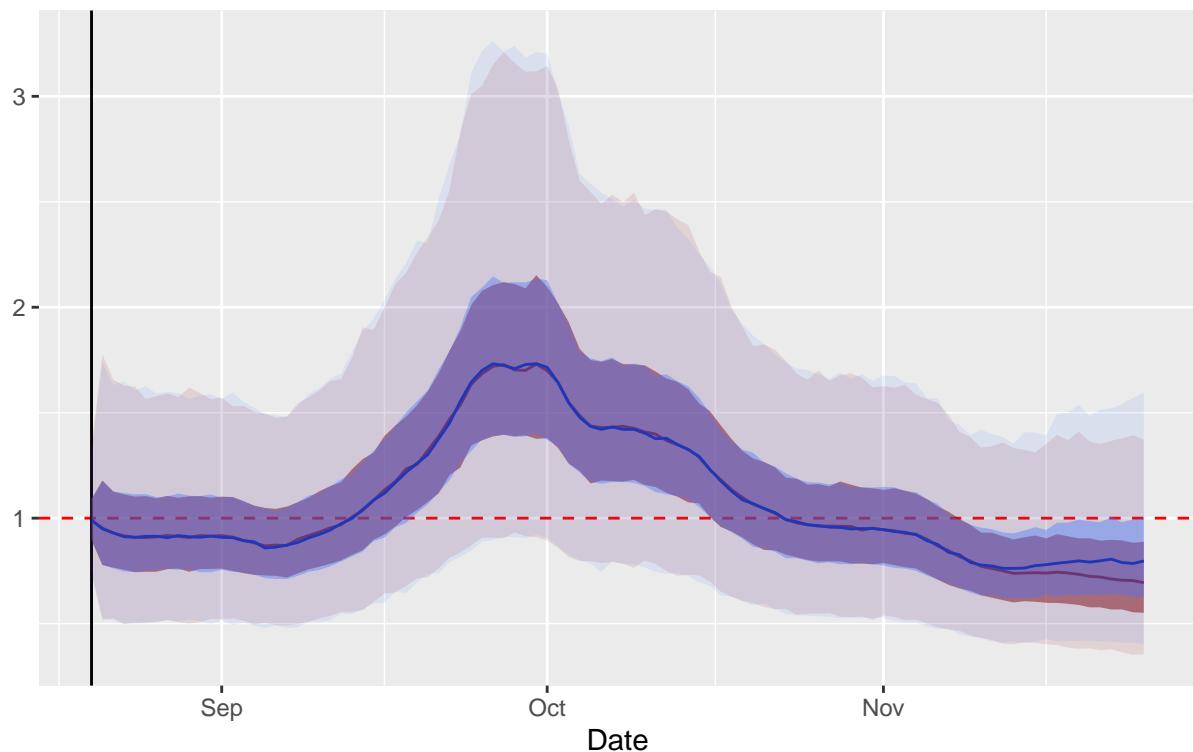
model 1 (red) vs model 2 (blue)



```
p2 <- plot_rt_hier(hier_data, fit_hier_3, regions, 'Lombardia')
plot_overlay(hier_data, fit_hier_2, regions, p2, 'Lombardia')+
  labs(subtitle = 'model 3 (red) vs model 2 (blue) ')
```

Rt Lombardia

model 3 (red) vs model 2 (blue)

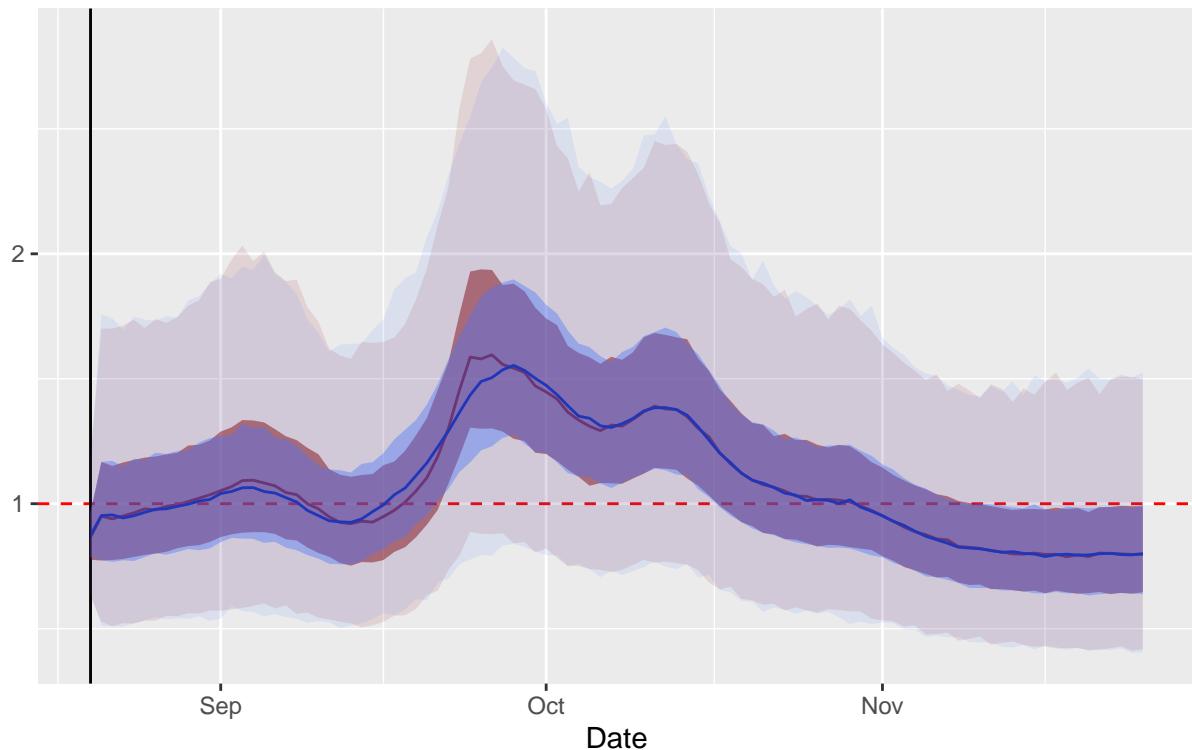


Abruzzo

```
p <- plot_rt_hier(hier_data, fit_hier, regions, 'Abruzzo')
plot_overlay(hier_data, fit_hier_2, regions, p, 'Abruzzo')+
  labs(subtitle = 'model 1 (red) vs model 2 (blue) ')
```

Rt Abruzzo

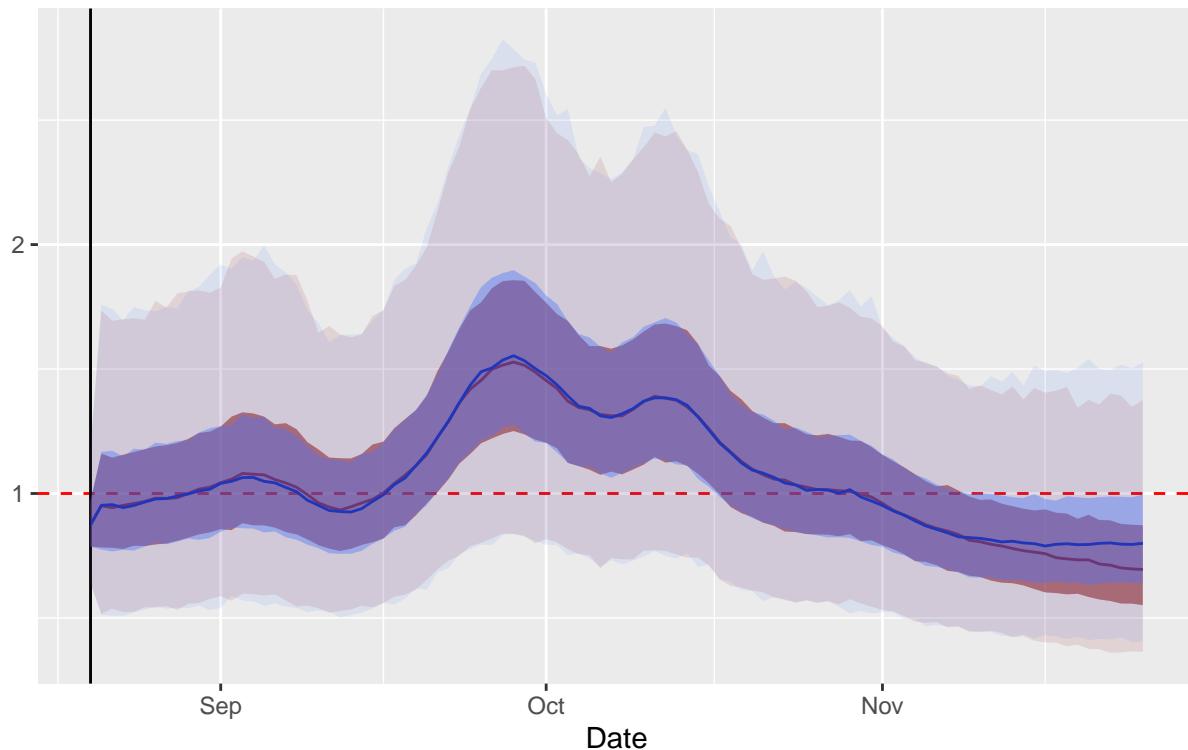
model 1 (red) vs model 2 (blue)



```
p2 <- plot_rt_hier(hier_data, fit_hier_3, regions, 'Abruzzo')
plot_overlay(hier_data, fit_hier_2, regions, p2, 'Abruzzo')+
  labs(subtitle = 'model 3 (red) vs model 2 (blue) ')
```

Rt Abruzzo

model 3 (red) vs model 2 (blue)



Model comparison

```
log_lik_1 <- extract_log_lik(fit_hier)
loo_1 <- loo(log_lik_1)
waic_1 <- waic(log_lik_1)

log_lik_2 <- extract_log_lik(fit_hier_2)
loo_2 <- loo(log_lik_2)
waic_2 <- waic(log_lik_2)

log_lik_3 <- extract_log_lik(fit_hier_3)
loo_3 <- loo(log_lik_3)
waic_3 <- waic(log_lik_3)

loo_compare(loo_1, loo_2, loo_3)

##          elpd_diff se_diff
## model1    0.0      0.0
## model3   -1.4     1.4
## model2   -2.3     2.1

looic
loo_1$estimates[3,1]

## [1] 10089
```

```
loo_2$estimates[3,1]
## [1] 10093.52
loo_3$estimates[3,1]
## [1] 10091.89

waic
loo_compare(waic_1, waic_2, waic_1)

##          elpd_diff se_diff
## model1    0.0      0.0
## model3    0.0      0.0
## model2   -2.1      2.2
```