

Hierarchical model with risk level intercept

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Hierarchical model for italian regions

```
regions <- unique(data_it$region)
regions

## [1] "Abruzzo"          "Basilicata"        "Calabria"
## [4] "Campania"         "Emilia-Romagna"   "Friuli Venezia Giulia"
## [7] "Lazio"           "Liguria"          "Lombardia"
## [10] "Marche"           "Molise"            "P.A. Bolzano"
## [13] "P.A. Trento"      "Piemonte"          "Puglia"
## [16] "Sardegna"         "Sicilia"           "Toscana"
## [19] "Umbria"           "Valle d'Aosta"    "Veneto"
```

```
hier_data <- get_hier_data(data_it, regions, y_delay, 0_delay, R_delay, initial_date = as.Date("2020-10-15"), final_date = as.Date("2020-12-22"))

risk_idx <- matrix(data = NA, nrow = nrow(hier_data$yellow_dummies), ncol = ncol(hier_data$yellow_dummies))
for(i in 1:nrow(risk_idx)){
  for(j in 1:ncol(risk_idx)){
    if(hier_data$yellow_dummies[i,j] == 1) risk_idx[i, j] <- 1
    else if(hier_data$orange_dummies[i,j] == 1) risk_idx[i,j] <- 2
    else if(hier_data$red_dummies[i,j]==2) risk_idx[i,j] <- 4
  }
}

risk_idx[is.na(risk_idx)] <- 1

p_delay <- get_delay_distribution()
```

```
stan_data_hier <- list(j = length(regions),
                      k = nrow(hier_data$exposures),
                      k_nonzero = length(hier_data$nonzero_days),
                      nonzero_days = hier_data$nonzero_days,
                      conv_gt = get_gt_convolution_ln2(nrow(hier_data$exposures)),
                      length_delay = length(p_delay),
                      p_delay = p_delay,
                      exposures = hier_data$exposures,
                      nonzero_positives = hier_data$nonzero_positives[, ],
                      risk_idx = risk_idx[hier_data$nonzero_days, ])

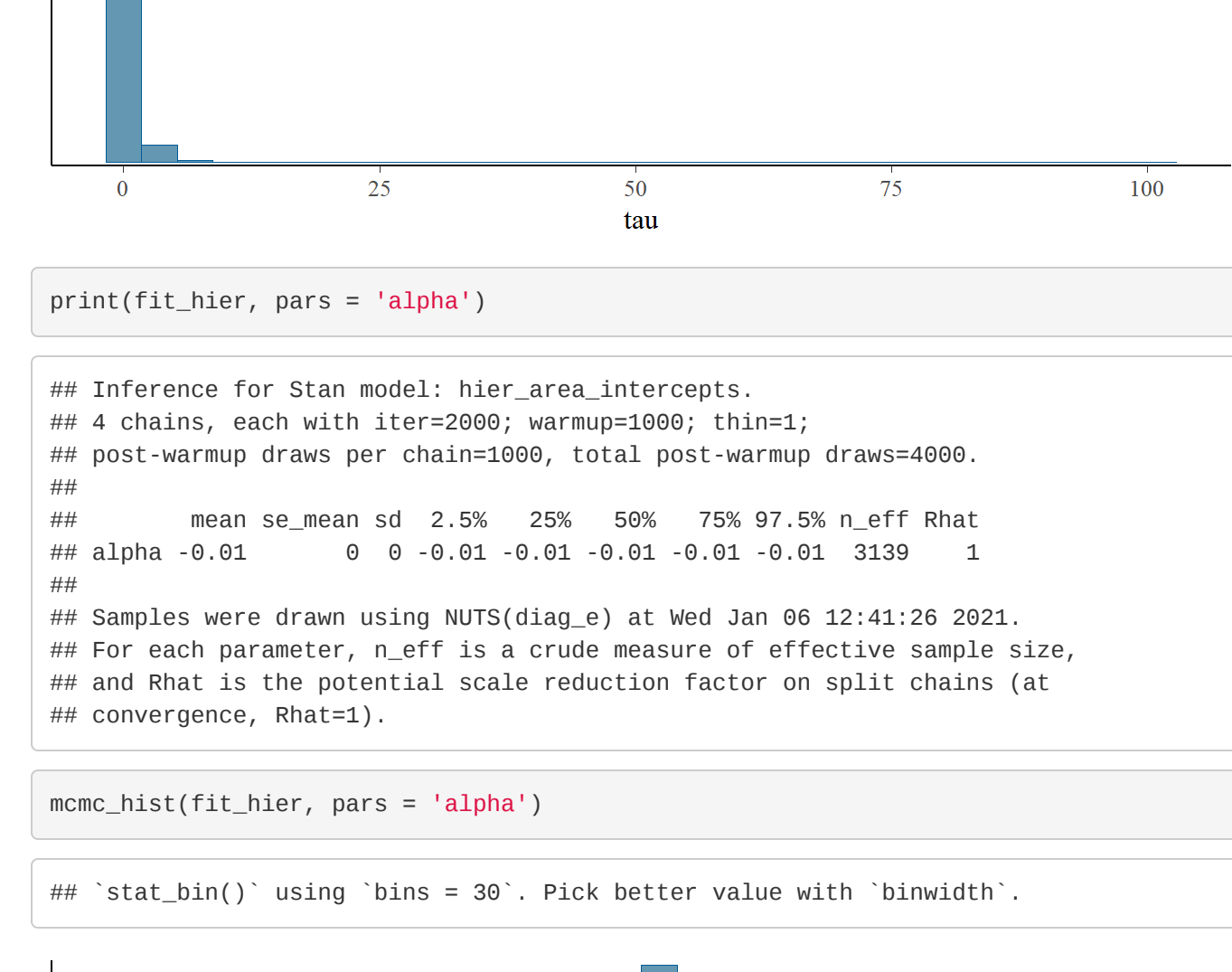
compiled_hier <- stan_model("../stan/hier_area_intercepts.stan")
fit_hier <- sampling(compiled_hier, data = stan_data_hier, iter = 2000)

print(fit_hier, pars="tau")
```

```
## Inference for Stan model: hier_area_intercepts.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##      mean se_mean sd 2.5% 25% 50% 75% 97.5% n_eff Rhat
## tau 0.32   0.03 1.74 0.01 0.03 0.09 0.3 1.77 3772   1
##
## Samples were drawn using NUTS(diag.e) at Wed Jan 06 12:41:26 2021.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```
mcmc_hist(fit_hier, pars="tau")

## 'stat_bin()' using 'bins = 30'. Pick better value with 'binwidth'.
```

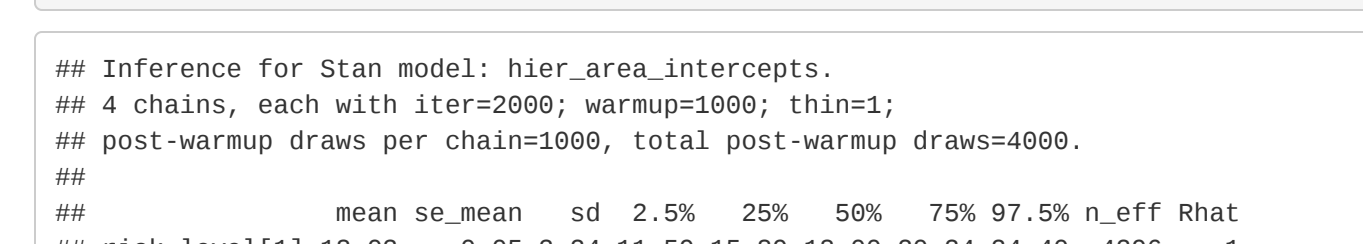


```
print(fit_hier, pars = "alpha")

## Inference for Stan model: hier_area_intercepts.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##      mean se_mean sd 2.5% 25% 50% 75% 97.5% n_eff Rhat
## alpha -0.01   0.00 0.01 -0.01 -0.01 -0.01 -0.01 3339   1
##
## Samples were drawn using NUTS(diag.e) at Wed Jan 06 12:41:26 2021.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```
mcmc_hist(fit_hier, pars="alpha")

## 'stat_bin()' using 'bins = 30'. Pick better value with 'binwidth'.
```



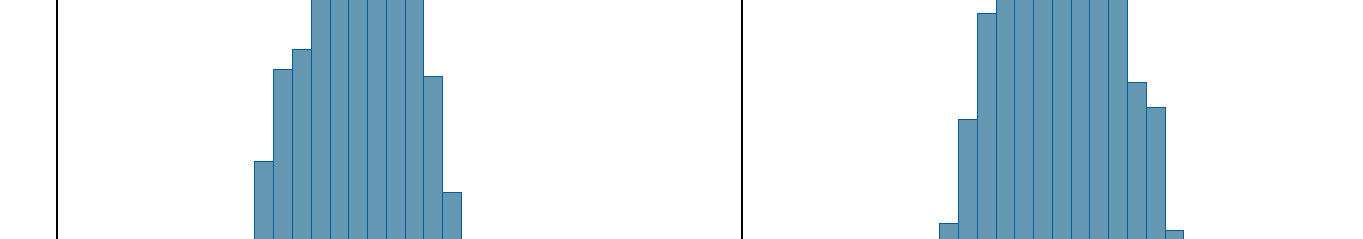
Risk level intercepts

```
print(fit_hier, pars = "risk_level")

## Inference for Stan model: hier_area_intercepts.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##      mean se_mean sd 2.5% 25% 50% 75% 97.5% n_eff Rhat
## risk_level[1] 18.83   0.85 3.24 11.59 15.89 15.89 28.24 24.46 4296   1
## risk_level[2] 32.89   0.89 4.79 23.68 29.54 32.89 36.28 42.25 2918   1
## risk_level[3] 29.43   0.88 4.74 20.13 26.12 29.44 32.58 38.97 3022   1
## risk_level[4] 15.61   0.88 3.10 8.17 14.64 15.62 17.78 20.51 1604   1
##
## Samples were drawn using NUTS(diag.e) at Wed Jan 06 12:41:26 2021.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

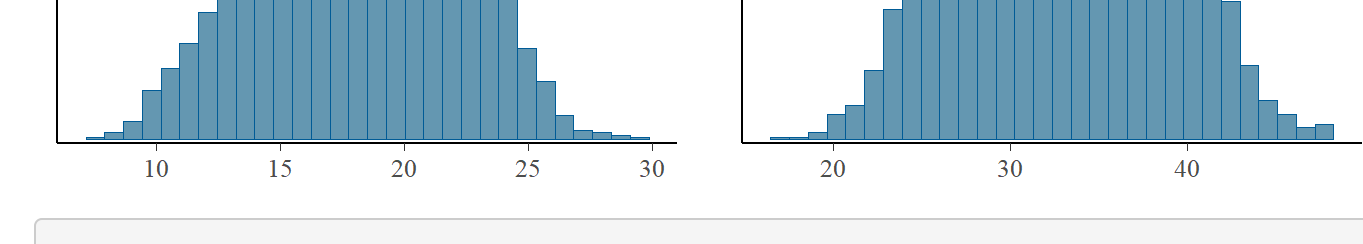
```
mcmc_hist(fit_hier, pars = c("risk_level[1]", "risk_level[2]"))

## 'stat_bin()' using 'bins = 30'. Pick better value with 'binwidth'.
```



```
mcmc_hist(fit_hier, pars = c("risk_level[3]", "risk_level[4]"))

## 'stat_bin()' using 'bins = 30'. Pick better value with 'binwidth'.
```



Trace plots

```
mcmc_trace(as.array(fit_hier, pars = c("tau")),
           np = nuts_params(fit_hier))

## No divergences to plot.
```

```
mcmc_trace(as.array(fit_hier, pars = c("phi")),
           np = nuts_params(fit_hier))

## No divergences to plot.
```

Posterior predictive check

```
y_rep <- as.matrix(fit_hier, pars = "y_rep")
ppc_dens_overlay(y = as.vector(stan_data_hier$nonzero_positives), y_rep[1:1000, ])

## 'stat_bin()' using 'bins = 30'. Pick better value with 'binwidth'.
```

Posterior predictive check by region

```
regional_yrep_idx <- function(region, regions_vector, nonzero_days){
  region_idx <- which(regions_vector == region)
  yrep_idx <- (region_idx-1) * length(nonzero_days) + 1
  range <- yrep_idx : (yrep_idx + length(nonzero_days)-1)
  return(range)}

groups <- function(regions, nonzero_days){
  group <- rep(regions[1], length(nonzero_days))
  for(i in 2:length(regions)){
    group <- c(group, rep(regions[i], length(nonzero_days)))
  }
  return(group)}
```

Lombardia

```
ppc_dens_overlay(y = stan_data_hier$nonzero_positives[, which(regions == "Lombardia")], y_rep[1:1000, regional_yrep_idx("Lombardia", regions, stan_data_hier$nonzero_days)])
```

```
ppc_intervals(y = stan_data_hier$nonzero_positives[, which(regions == "Lombardia")], y_rep[1:1000, regional_yrep_idx("Lombardia", regions, stan_data_hier$nonzero_days)])
```

Veneto

```
ppc_dens_overlay(y = stan_data_hier$nonzero_positives[, which(regions == "Veneto")], y_rep[1:1000, regional_yrep_idx("Veneto", regions, stan_data_hier$nonzero_days)])
```

```
ppc_intervals(y = stan_data_hier$nonzero_positives[, which(regions == "Veneto")], y_rep[1:1000, regional_yrep_idx("Veneto", regions, stan_data_hier$nonzero_days)])
```

Emilia Romagna

```
ppc_dens_overlay(y = stan_data_hier$nonzero_positives[, which(regions == "Emilia-Romagna")], y_rep[1:1000, regional_yrep_idx("Emilia-Romagna", regions, stan_data_hier$nonzero_days)])
```

```
ppc_intervals(y = stan_data_hier$nonzero_positives[, which(regions == "Emilia-Romagna")], y_rep[1:1000, regional_yrep_idx("Emilia-Romagna", regions, stan_data_hier$nonzero_days)])
```

```
ppc_stat_grouped(y=as.vector(stan_data_hier$nonzero_positives), yrep = y_rep, group = groups(regions, stan_data_hier$nonzero_days), stat="mean", binwidth=0.5)
```

```
mean_inv_phi = mean(rstan::extract(fit_hier)$inv_phi)
mean_y_rep = colMeans(y_rep)
std_resid = (as.vector(stan_data_hier$nonzero_positives) - mean_y_rep) / sqrt(mean_y_rep * mean_y_rep * 2 * mean_inv_phi)
qqplot(mean_y_rep, std_resid ~ hline.at(2) ~ hline.at(-2))
```



Rt Lombardia

```
plot_rt_hier(hier_data, fit_hier, regions, "Lombardia")
```

Rt Veneto

```
plot_rt_hier(hier_data, fit_hier, regions, "Veneto")
```

Rt Emilia Romagna

```
plot_rt_hier(hier_data, fit_hier, regions, "Emilia-Romagna")
```

Loaic and WAIC

```
logLik <- extract_logLik(fit_hier)
loo <- loo(logLik)
waic <- waic(logLik)
```

```
looEstimates[3,1]
```

```
## [1] 17880.55
```

```
waicEstimates[3,1]
```

```
## NULL
```