Hierarchical models with different priors for between groups standard deviation

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Data

```
regions <- c('Lazio', 'Lombardia', 'Abruzzo', 'Veneto', 'Emilia-Romagna', 'Toscana', 'Campania', 'Friuli

regions
```

```
##  [1] "Lazio"                 "Lombardia"             "Abruzzo"
##  [4] "Veneto"                "Emilia-Romagna"        "Toscana"
##  [7] "Campania"              "Friuli Venezia Giulia" "Sicilia"
## [10] "Calabria"
```

```
hier_data <- get_hier_data(data_it, regions, initial_date = as.Date('2020-08-30') )

p_delay <- get_delay_distribution()

stan_data_hier <- list(J = length(regions),
                       N = nrow(hier_data$exposures),
                       N_nonzero = length(hier_data$nonzero_days),
                       nonzero_days = hier_data$nonzero_days,
                       conv_gt = get_gt_convolution_ln2(nrow(hier_data$exposures)),
                       length_delay = length(p_delay),
                       p_delay = p_delay,
                       exposures = hier_data$exposures,
                       nonzero_positives = hier_data$positives[hier_data$nonzero_days ,]

)
```

Models

Three hierarchical models with different priors on tau are compared to the simple model (that fits a different model for each region).

Hierarchical model 1

$$log(r_t) \sim N(\mu_t, \tau^2)$$
$$\tau \sim HalfCauchy(0, 2.5)$$

Hierarchical model 2

1

$$log(r_t) \sim N(\mu_t, \tau^2)$$
$$\tau \sim HalfCauchy(0, 1)$$

Hierarchical model 3

$$log(r_t) \sim N(\mu_t, \tau^2)$$
$$\tau \sim HalfCauchy(0, 0.5)$$

```r
## Simple model
compiled_grouped <- stan_model('../stan/grouped_model.stan')
```

```
## Trying to compile a simple C file
```

```
## Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
## clang -mmacosx-version-min=10.13 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG   -I
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/incl
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/includ
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/includ
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util/
## namespace Eigen {
## ^
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util/
## namespace Eigen {
##                 ^
##                  ;
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/incl
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/includ
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/Core:96:10: fa
## #include <complex>
##          ^~~~~~~~~
## 3 errors generated.
## make: *** [foo.o] Error 1
```

```r
fit_grouped <- sampling(compiled_grouped, data= stan_data_hier, iter=2000, cores=getOption("mc.cores", 
```

```
## 
## SAMPLING FOR MODEL 'grouped_model' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.022425 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 224.25 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
```

```
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 1351.81 seconds (Warm-up)
## Chain 1:                1075.06 seconds (Sampling)
## Chain 1:                2426.88 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'grouped_model' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0.007134 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 71.34 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 1268.62 seconds (Warm-up)
## Chain 2:                1154.82 seconds (Sampling)
## Chain 2:                2423.45 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'grouped_model' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0.012382 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 123.82 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
```

```
## Chain 3:
## Chain 3:   Elapsed Time: 1189.94 seconds (Warm-up)
## Chain 3:                 1118.24 seconds (Sampling)
## Chain 3:                 2308.19 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'grouped_model' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0.008676 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 86.76 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:   Elapsed Time: 1285.77 seconds (Warm-up)
## Chain 4:                 1075.81 seconds (Sampling)
## Chain 4:                 2361.58 seconds (Total)
## Chain 4:
```

```r
## Hierarchical model 1
compiled_hier <- stan_model('../stan/hier_rt_model.stan')
```

```
## Trying to compile a simple C file

## Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
## clang -mmacosx-version-min=10.13 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG   -I
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/incl
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/inclu
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/inclu
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util/
## namespace Eigen {
## ^
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util/
## namespace Eigen {
##                 ^
##                 ;
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/incl
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/inclu
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/Core:96:10: fa
## #include <complex>
##          ^~~~~~~~~
```

```
## 3 errors generated.
## make: *** [foo.o] Error 1
```

```
fit_hier <- sampling(compiled_hier, data = stan_data_hier, iter= 2000, cores=getOption("mc.cores", 1L))
```

```
##
## SAMPLING FOR MODEL 'hier_rt_model' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.013848 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 138.48 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 1274.6 seconds (Warm-up)
## Chain 1:                948.957 seconds (Sampling)
## Chain 1:                2223.56 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'hier_rt_model' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0.008572 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 85.72 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 1315.35 seconds (Warm-up)
## Chain 2:                938.585 seconds (Sampling)
## Chain 2:                2253.94 seconds (Total)
## Chain 2:
```

```
##
## SAMPLING FOR MODEL 'hier_rt_model' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0.008591 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 85.91 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 1260.59 seconds (Warm-up)
## Chain 3:                944.517 seconds (Sampling)
## Chain 3:                2205.11 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'hier_rt_model' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0.007199 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 71.99 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 1285.94 seconds (Warm-up)
## Chain 4:                978.256 seconds (Sampling)
## Chain 4:                2264.19 seconds (Total)
## Chain 4:
## Hierarchical model 2
compiled_hierC1 <- stan_model('../stan/hier_model_C1.stan')

## Trying to compile a simple C file
```

```
## Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
## clang -mmacosx-version-min=10.13 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG   -I
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/incl
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/inclu
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/inclu
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util/
## namespace Eigen {
## ^
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util/
## namespace Eigen {
##                  ^
##                  ;
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/incl
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/inclu
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/Core:96:10: fa
## #include <complex>
##          ^~~~~~~~~
## 3 errors generated.
## make: *** [foo.o] Error 1
```

```r
fit_C1 <- sampling(compiled_hierC1, data = stan_data_hier, iter=2000,cores=getOption("mc.cores", 1L))
```

```
##
## SAMPLING FOR MODEL 'hier_model_C1' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.014197 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 141.97 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 1257.01 seconds (Warm-up)
## Chain 1:                951.29 seconds (Sampling)
## Chain 1:                2208.3 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'hier_model_C1' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0.007642 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 76.42 seconds.
## Chain 2: Adjust your expectations accordingly!
```

```
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 1377.47 seconds (Warm-up)
## Chain 2:                963.79 seconds (Sampling)
## Chain 2:                2341.26 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'hier_model_C1' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0.007546 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 75.46 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Exception: Exception: poisson_rng: Rate parameter is 0, but must be > 0!  (in 'modeld77356b
##   (in 'modeld77356b4a1c0_hier_model_C1' at line 148)
##
## Chain 3: Exception: Exception: poisson_rng: Rate parameter is 0, but must be > 0!  (in 'modeld77356b
##   (in 'modeld77356b4a1c0_hier_model_C1' at line 148)
##
## Chain 3: Exception: Exception: poisson_rng: Rate parameter is 0, but must be > 0!  (in 'modeld77356b
##   (in 'modeld77356b4a1c0_hier_model_C1' at line 148)
##
## Chain 3: Exception: Exception: poisson_rng: Rate parameter is 0, but must be > 0!  (in 'modeld77356b
##   (in 'modeld77356b4a1c0_hier_model_C1' at line 148)
##
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 1289.43 seconds (Warm-up)
## Chain 3:                978.512 seconds (Sampling)
```

```
## Chain 3:                    2267.94 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'hier_model_C1' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0.007417 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 74.17 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 1423.53 seconds (Warm-up)
## Chain 4:                1016.24 seconds (Sampling)
## Chain 4:                2439.77 seconds (Total)
## Chain 4:

## Warning in validityMethod(object): The following variables have undefined
## values: log_lik[1,1],The following variables have undefined values:
## log_lik[2,1],The following variables have undefined values: log_lik[3,1],The
## following variables have undefined values: log_lik[4,1],The following
## variables have undefined values: log_lik[5,1],The following variables have
## undefined values: log_lik[6,1],The following variables have undefined values:
## log_lik[7,1],The following variables have undefined values: log_lik[8,1],The
## following variables have undefined values: log_lik[9,1],The following
## variables have undefined values: log_lik[10,1],The following variables have
## undefined values: log_lik[11,1],The following variables have undefined values:
## log_lik[12,1],The following variables have undefined values: log_lik[13,1],The
## following variables have undefined values: log_lik[14,1],The following
## variables have undefined values: log_lik[15,1],The following variables have
## undefined values: log_lik[16,1],The following variables have undefined values:
## log_lik[17,1],The following variables have undefined values: log_lik[18,1],The
## following variables have undefined values: log_lik[19,1],The following
## variables have undefined values: log_lik[20,1],The following variables have
## undefined values: log_lik[21,1],The following variables have undefined values:
## log_lik[22,1],The following variables have undefined values: log_lik[23,1],The
## following variables have undefined values: log_lik[24,1],The following
## variables have undefined values: log_lik[25,1],The following variables have
## undefined values: log_lik[26,1],The following variables have undefined values:
## log_lik[27,1],The following variables have undefined values: log_lik[28,1],The
## following variables have undefined values: log_lik[29,1],The following
## variables have undefined values: log_lik[30,1],The following variables have
## undefined values: log_lik[31,1],The following variables have undefined values:
```

```
## log_lik[32,1],The following variables have undefined values: log_lik[33,1],The
## following variables have undefined values: log_lik[34,1],The following
## variables have undefined values: log_lik[35,1],The following variables have
## undefined values: log_lik[36,1],The following variables have undefined values:
## log_lik[37,1],The following variables have undefined values: log_lik[38,1],The
## following variables have undefined values: log_lik[39,1],The following
## variables have undefined values: log_lik[40,1],The following variables have
## undefined values: log_lik[41,1],The following variables have undefined values:
## log_lik[42,1],The following variables have undefined values: log_lik[43,1],The
## following variables have undefined values: log_lik[44,1],The following
## variables have undefined values: log_lik[45,1],The following variables have
## undefined values: log_lik[46,1],The following variables have undefined values:
## log_lik[47,1],The following variables have undefined values: log_lik[48,1],The
## following variables have undefined values: log_lik[49,1],The following
## variables have undefined values: log_lik[50,1],The following variables have
## undefined values: log_lik[51,1],The following variables have undefined values:
## log_lik[52,1],The following variables have undefined values: log_lik[53,1],The
## following variables have undefined values: log_lik[54,1],The following
## variables have undefined values: log_lik[55,1],The following variables have
## undefined values: log_lik[56,1],The following variables have undefined values:
## log_lik[57,1],The following variables have undefined values: log_lik[58,1],The
## following variables have undefined values: log_lik[59,1],The following
## variables have undefined values: log_lik[60,1],The following variables have
## undefined values: log_lik[61,1],The following variables have undefined values:
## log_lik[62,1],The following variables have undefined values: log_lik[63,1],The
## following variables have undefined values: log_lik[64,1],The following
## variables have undefined values: log_lik[65,1],The following variables have
## undefined values: log_lik[66,1],The following variables have undefined values:
## log_lik[67,1],The following variables have undefined values: log_lik[68,1],The
## following variables have undefined values: log_lik[69,1],The following
## variables have undefined values: log_lik[70,1],The following variables have
## undefined values: log_lik[71,1],The following variables have undefined values:
## log_lik[72,1],The following variables have undefined values: log_lik[73,1],The
## following variables have undefined values: log_lik[74,1],The following
## variables have undefined values: log_lik[75,1],The following variables have
## undefined values: log_lik[76,1],The following variables have undefined values:
## log_lik[77,1],The following variables have undefined values: log_lik[78,1],The
## following variables have undefined values: log_lik[79,1],The following
## variables have undefined values: log_lik[80,1],The following variables have
## undefined values: log_lik[1,2],The following variables have undefined values:
## log_lik[2,2],The following variables have undefined values: log_lik[3,2],The
## following variables have undefined values: log_lik[4,2],The following
## variables have undefined values: log_lik[5,2],The following variables have
## undefined values: log_lik[6,2],The following variables have undefined values:
## log_lik[7,2],The following variables have undefined values: log_lik[8,2],The
## following variables have undefined values: log_lik[9,2],The following
## variables have undefined values: log_lik[10,2],The following variables have
## undefined values: log_lik[11,2],The following variables have undefined values:
## log_lik[12,2],The following variables have undefined values: log_lik[13,2],The
## following variables have undefined values: log_lik[14,2],The following
## variables have undefined values: log_lik[15,2],The following variables have
## undefined values: log_lik[16,2],The following variables have undefined values:
## log_lik[17,2],The following variables have undefined values: log_lik[18,2],The
## following variables have undefined values: log_lik[19,2],The following
```

```
## variables have undefined values: log_lik[20,2],The following variables have
## undefined values: log_lik[21,2],The following variables have undefined values:
## log_lik[22,2],The following variables have undefined values: log_lik[23,2],The
## following variables have undefined values: log_lik[24,2],The following
## variables have undefined values: log_lik[25,2],The following variables have
## undefined values: log_lik[26,2],The following variables have undefined values:
## log_lik[27,2],The following variables have undefined values: log_lik[28,2],The
## following variables have undefined values: log_lik[29,2],The following
## variables have undefined values: log_lik[30,2],The following variables have
## undefined values: log_lik[31,2],The following variables have undefined values:
## log_lik[32,2],The following variables have undefined values: log_lik[33,2],The
## following variables have undefined values: log_lik[34,2],The following
## variables have undefined values: log_lik[35,2],The following variables have
## undefined values: log_lik[36,2],The following variables have undefined values:
## log_lik[37,2],The following variables have undefined values: log_lik[38,2],The
## following variables have undefined values: log_lik[39,2],The following
## variables have undefined values: log_lik[40,2],The following variables have
## undefined values: log_lik[41,2],The following variables have undefined values:
## log_lik[42,2],The following variables have undefined values: log_lik[43,2],The
## following variables have undefined values: log_lik[44,2],The following
## variables have undefined values: log_lik[45,2],The following variables have
## undefined values: log_lik[46,2],The following variables have undefined values:
## log_lik[47,2],The following variables have undefined values: log_lik[48,2],The
## following variables have undefined values: log_lik[49,2],The following
## variables have undefined values: log_lik[50,2],The following variables have
## undefined values: log_lik[51,2],The following variables have undefined values:
## log_lik[52,2],The following variables
```

```r
## Hierarchical model 3
compiled_hierC05 <- stan_model('../stan/hier_model_C05.stan')
```

```
## Trying to compile a simple C file

## Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
## clang -mmacosx-version-min=10.13 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG   -I
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/incl
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/includ
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/includ
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util/
## namespace Eigen {
## ^
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/src/Core/util/
## namespace Eigen {
##                 ^
##                  ;
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/StanHeaders/incl
## In file included from /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/includ
## /Library/Frameworks/R.framework/Versions/4.0/Resources/library/RcppEigen/include/Eigen/Core:96:10: fa
## #include <complex>
##          ^~~~~~~~~
## 3 errors generated.
## make: *** [foo.o] Error 1
```

```
fit_C05 <- sampling(compiled_hierC05, data= stan_data_hier, iter=2000,cores=getOption("mc.cores", 1L))
```

```
##
## SAMPLING FOR MODEL 'hier_model_C05' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.015648 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 156.48 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 1340.02 seconds (Warm-up)
## Chain 1:                972.676 seconds (Sampling)
## Chain 1:                2312.69 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'hier_model_C05' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0.009088 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 90.88 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 1300.29 seconds (Warm-up)
## Chain 2:                936.442 seconds (Sampling)
## Chain 2:                2236.73 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'hier_model_C05' NOW (CHAIN 3).
```

```
## Chain 3:
## Chain 3: Gradient evaluation took 0.009058 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 90.58 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 1282.25 seconds (Warm-up)
## Chain 3:                938.121 seconds (Sampling)
## Chain 3:                2220.37 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'hier_model_C05' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0.008213 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 82.13 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 1247.98 seconds (Warm-up)
## Chain 4:                893.731 seconds (Sampling)
## Chain 4:                2141.71 seconds (Total)
## Chain 4:
```

Posterior histograms for tau

```
mcmc_hist(fit_hier, pars = 'tau') + ggtitle('Model 1')
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

Model 1

tau
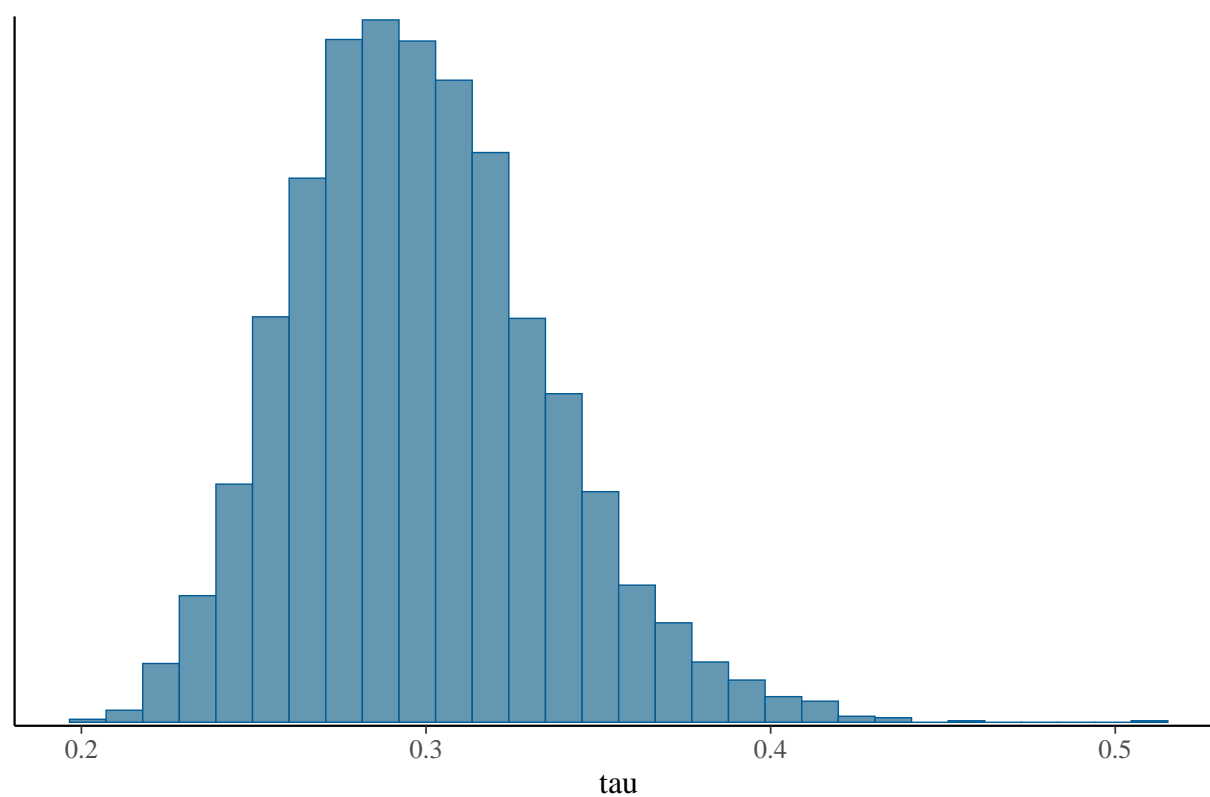
```
mcmc_hist(fit_C1, pars = 'tau') + ggtitle('Model 2')
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

Model 2
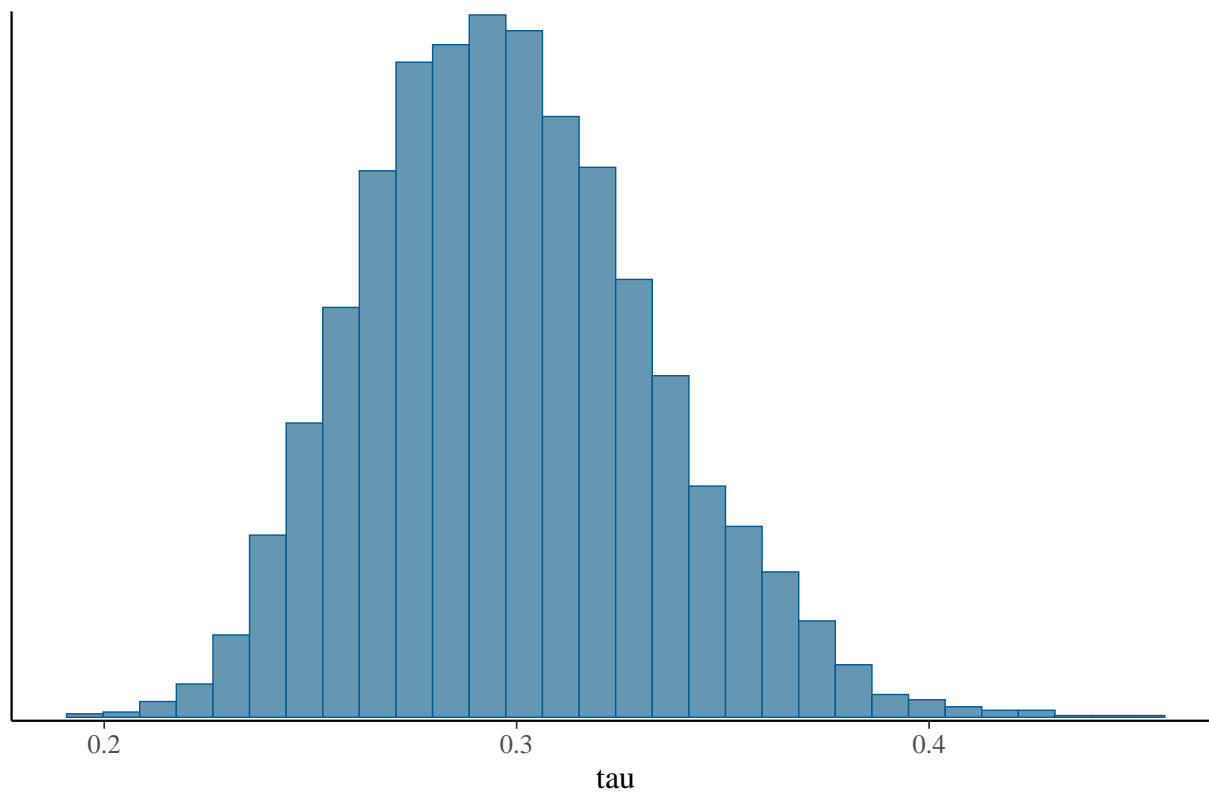

```r
mcmc_hist(fit_C05, pars = 'tau') + ggtitle('Model 3')
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

Model 3
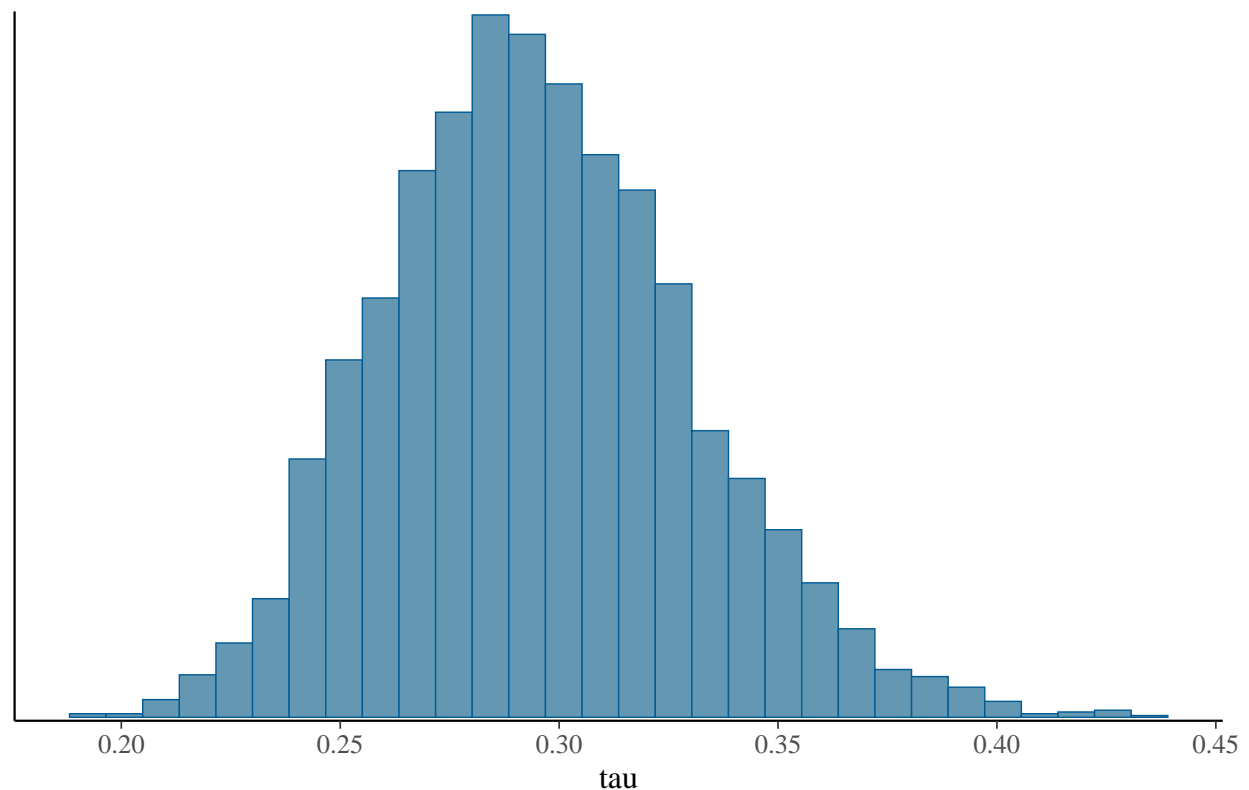
Rt

For each of the three hierarchical models, the resulting Rt curve is overlapped to the Rt curve produced by the single region model, both with 50% and 95% credibility intervals.

The same plots are produced for the regions Lazio, Lombardia, Veneto, Friuli Venezia Giulia and Abruzzo.

```r
### Retrieve rt medians and intervals for a given model and region
rt_intervals <- function(data, fit, regions_vector, region){
  summary <- summary(fit)
  region_idx = which(regions_vector == region)
  rt_1 <- which(rownames(summary$summary) == paste('r_t[1,', region_idx,  ']', sep=''))
  rt_index <- seq(rt_1, rt_1 + length(data$dates) * length(regions_vector)  -1 , by=length(regions_vecto
  rt_median <- summary$summary[rt_index, '50%']
  min_rt_50_interval <- summary$summary[rt_index, '25%']
  max_rt_50_interval <- summary$summary[rt_index, '75%']
  min_rt_95_interval <- summary$summary[rt_index, '2.5%']
  max_rt_95_interval <- summary$summary[rt_index, '97.5%']

  return(list(median = rt_median, min_50 = min_rt_50_interval, max_50 = max_rt_50_interval, min_95 = mi
}

### Plot rt curve for a single region
plot_rt <- function(data, fit, regions_vector, region){

    intervals <- rt_intervals(data, fit, regions_vector, region)

    p <- ggplot(  ) +
```

```r
        geom_line(aes(x = data$dates, y=intervals$median ), color='darkred')+
        xlab('Date') +
        ylab('') +
        geom_hline(yintercept=1, linetype="dashed", color = "red") +
        geom_vline(xintercept = data$dates[1]) +
        geom_ribbon(aes(x = data$dates, ymin = intervals$min_50, ymax = intervals$max_50), alpha= 0.5, fil
        geom_ribbon(aes(x = data$dates,ymin =intervals$min_95, ymax = intervals$max_95), alpha= 0.1, fill

    return(p)
}

### overlay with rt plot of the single region model
plot_overlay <- function(plot, region){

    simple_intervals <- rt_intervals(hier_data, fit_grouped, regions, region)

    p <- plot + geom_line(aes(x=hier_data$dates, y = simple_intervals$median), color = 'navyblue') +
    geom_ribbon(aes(x = hier_data$dates,ymin = simple_intervals$min_50, ymax = simple_intervals$max_50), a
    geom_ribbon(aes(x = hier_data$dates,ymin = simple_intervals$min_95, ymax = simple_intervals$max_95), a
      ggtitle(paste('Rt', region , sep = ' '))

    return(p)

}

get_plot_limits <- function(plot) {
    gb = ggplot_build(plot)
    xmin = gb$layout$panel_params[[1]]$x.range[1]
    xmax = gb$layout$panel_params[[1]]$x.range[2]
    ymin = gb$layout$panel_params[[1]]$y.range[1]
    ymax = gb$layout$panel_params[[1]]$y.range[2]
    list(xmin = xmin, xmax = xmax, ymin = ymin, ymax = ymax)
}
```

Lazio

```r
p1 <- plot_rt(hier_data, fit_hier, regions, 'Lazio')
p1 <- plot_overlay(p1, 'Lazio') + labs(subtitle = 'hierarchical model n.1 (red) and single region model

p2 <- plot_rt(hier_data, fit_C05, regions, 'Lazio')
p2 <- plot_overlay(p2, 'Lazio')+ labs(subtitle = 'hierarchical model n.2 (red) and single region model

p3 <- plot_rt(hier_data, fit_C1, regions, 'Lazio')
p3 <- plot_overlay(p3,'Lazio') + labs(subtitle = 'hierarchical model n.3 (red) and single region model

ymin <- min(get_plot_limits(p1)$ymin, get_plot_limits(p2)$ymin, get_plot_limits(p3)$ymin)
ymax <- min(get_plot_limits(p1)$ymax, get_plot_limits(p2)$ymax, get_plot_limits(p3)$ymax)

p1 + ylim(c(ymin, ymax))
```
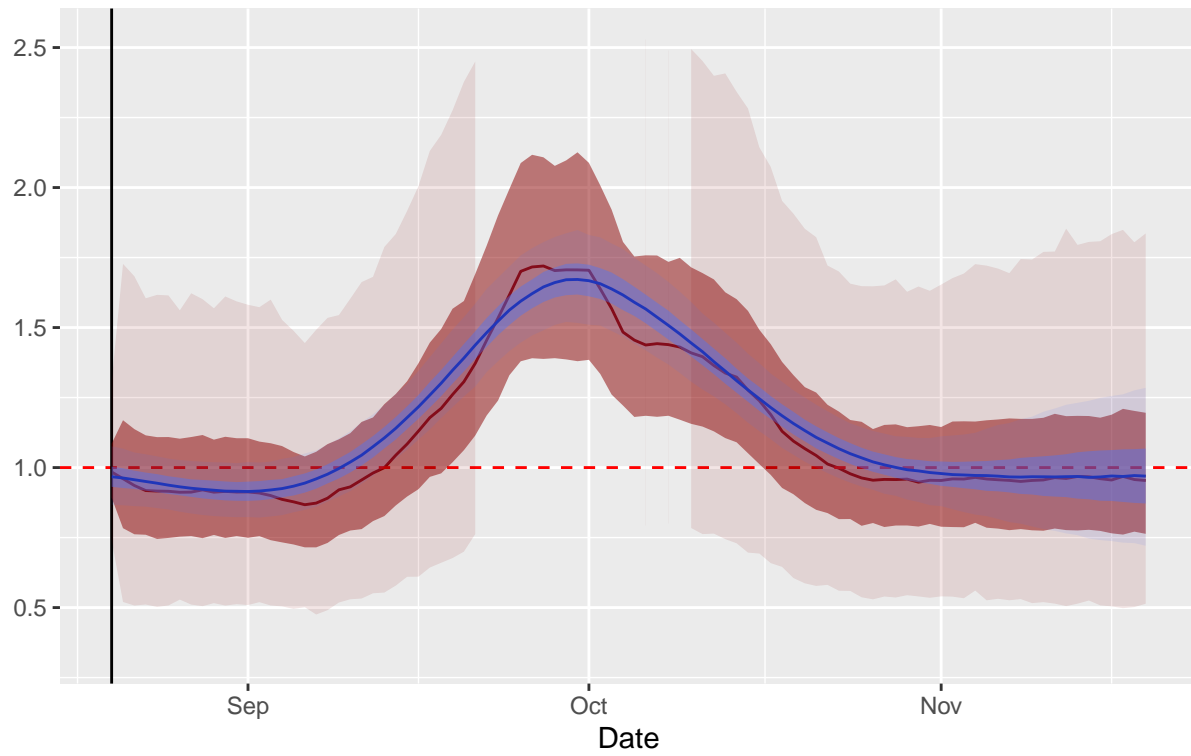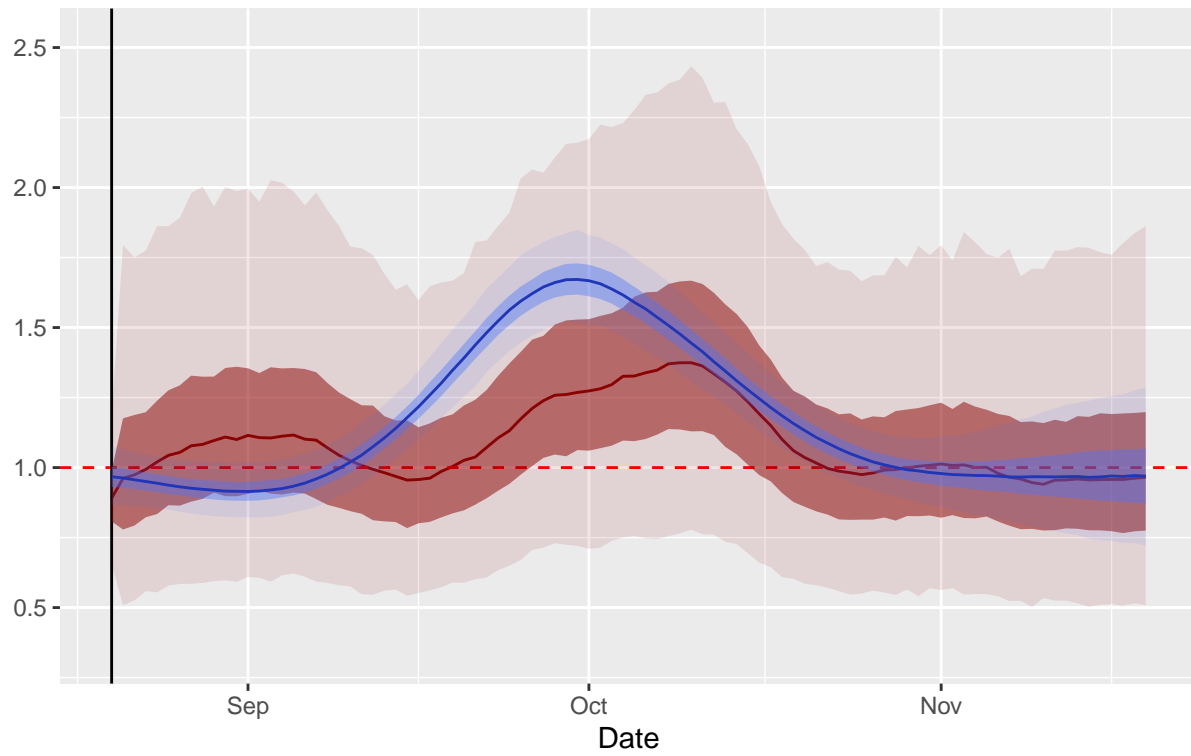
Rt Lazio

hierarchical model n.1 (red) and single region model (blue)
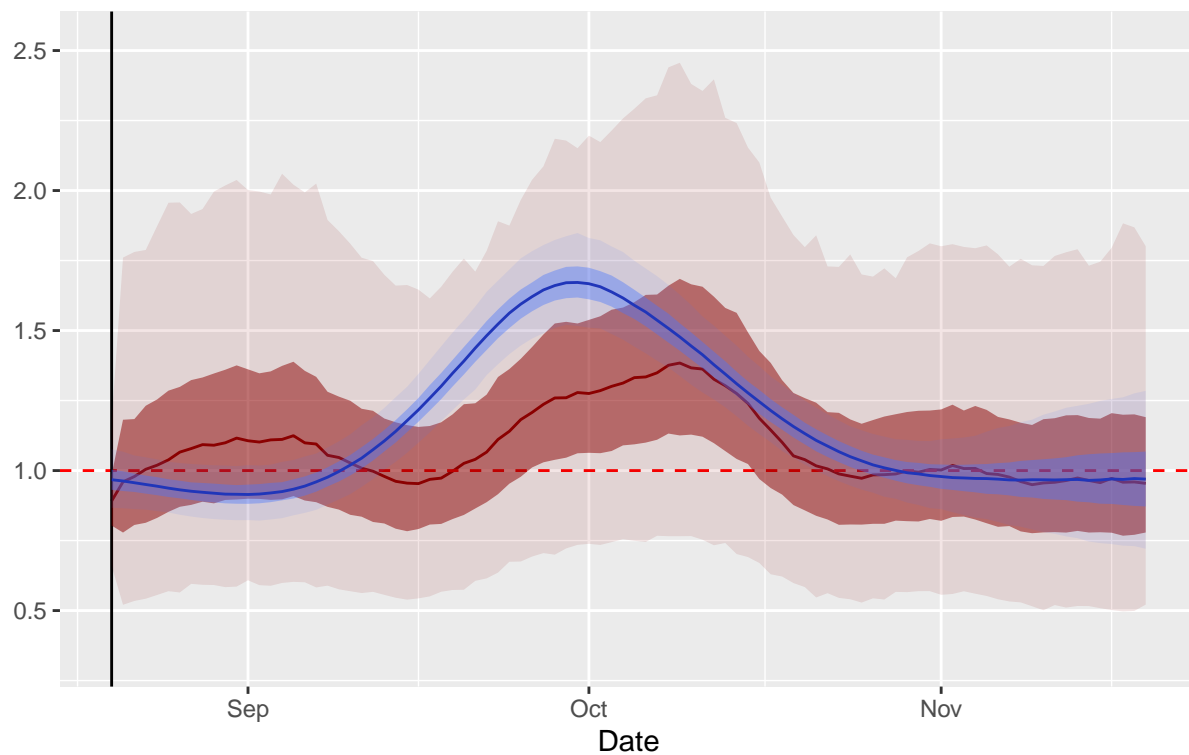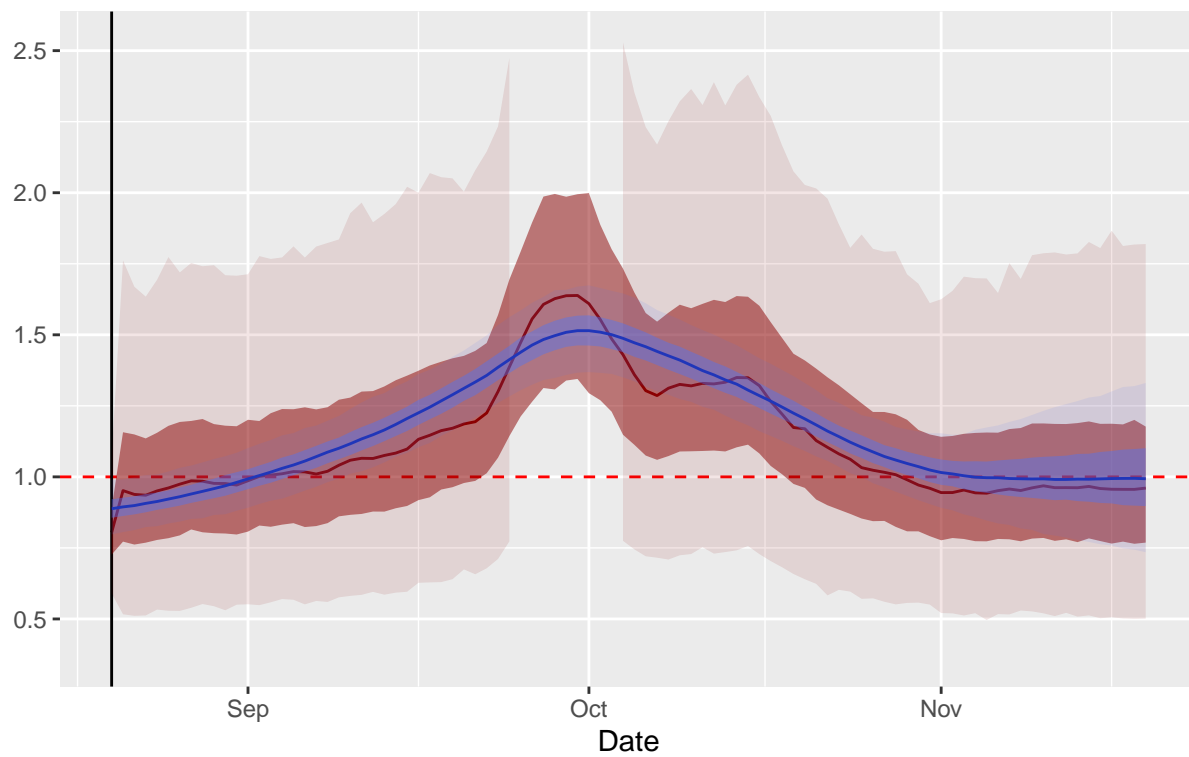


```
p2 + ylim(c(ymin, ymax))
```

Rt Lazio

hierarchical model n.2 (red) and single region model (blue)


```
p3 + ylim(c(ymin, ymax))
```
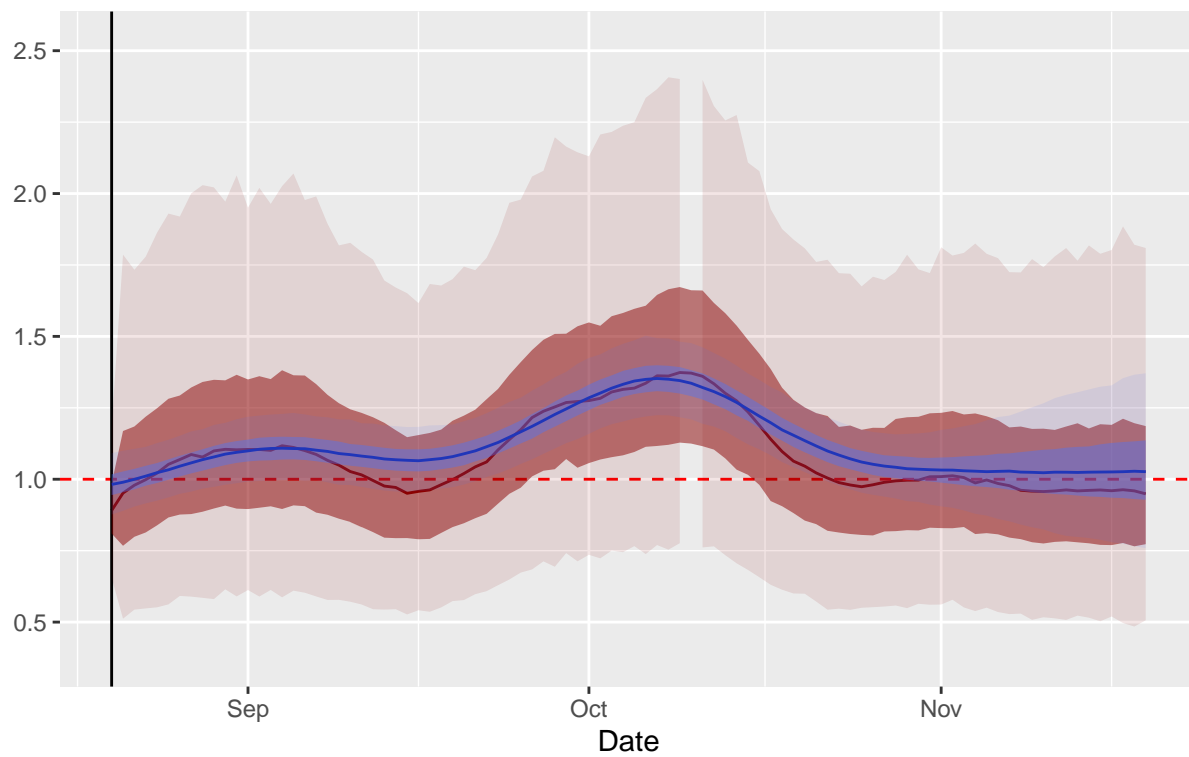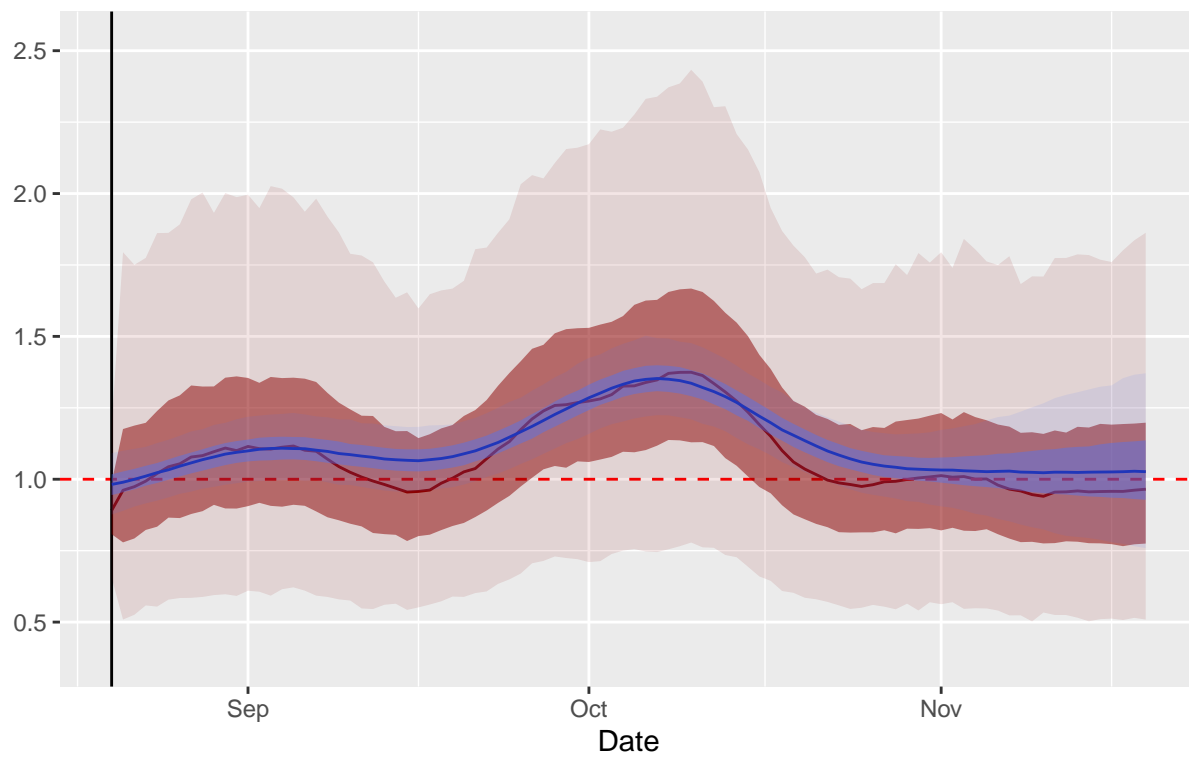
Rt Lazio
hierarchical model n.3 (red) and single region model (blue)
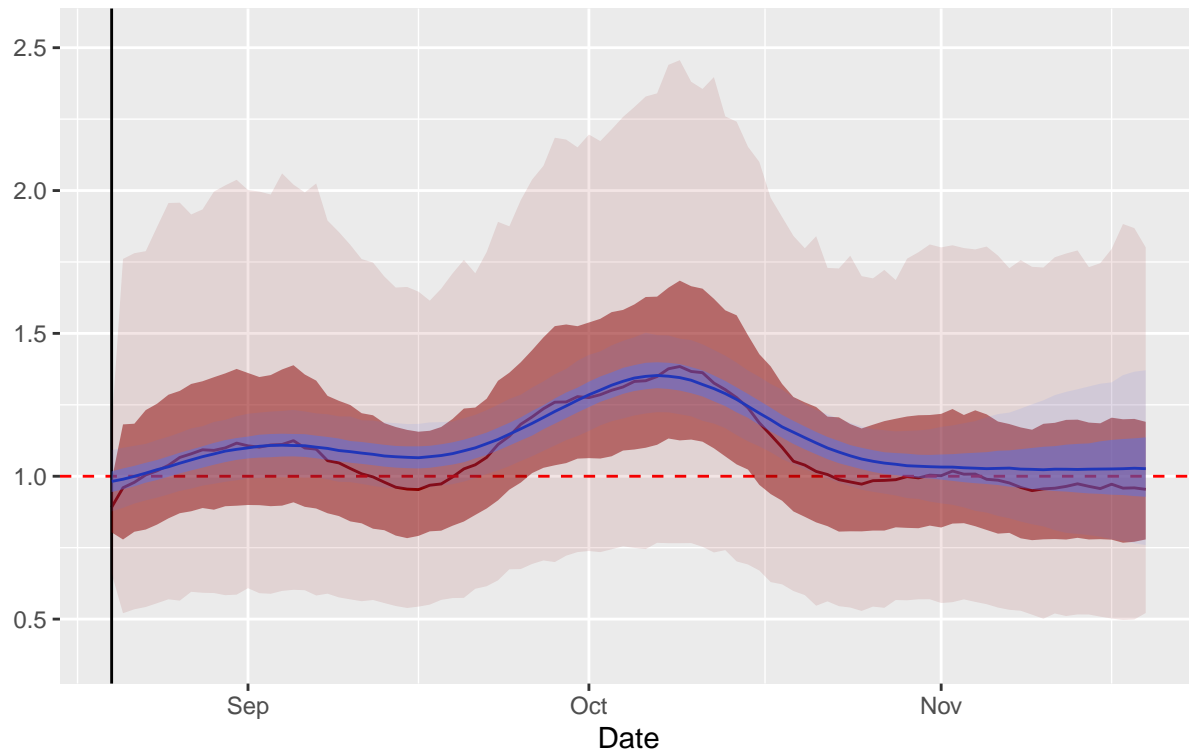
Lombardia

```
p1 <- plot_rt(hier_data, fit_hier, regions, 'Lombardia')
p1 <- plot_overlay(p1, 'Lombardia') + labs(subtitle = 'hierarchical model n.1 (red) and single region m

p2 <- plot_rt(hier_data, fit_C05, regions, 'Lazio')
p2 <- plot_overlay(p2, 'Lombardia')+ labs(subtitle = 'hierarchical model n.2 (red) and single region mo

p3 <- plot_rt(hier_data, fit_C1, regions, 'Lazio')
p3 <- plot_overlay(p3,'Lombardia') + labs(subtitle = 'hierarchical model n.3 (red) and single region mo

ymin <- min(get_plot_limits(p1)$ymin, get_plot_limits(p2)$ymin, get_plot_limits(p3)$ymin)
ymax <- min(get_plot_limits(p1)$ymax, get_plot_limits(p2)$ymax, get_plot_limits(p3)$ymax)

p1 + ylim(c(ymin, ymax))
```
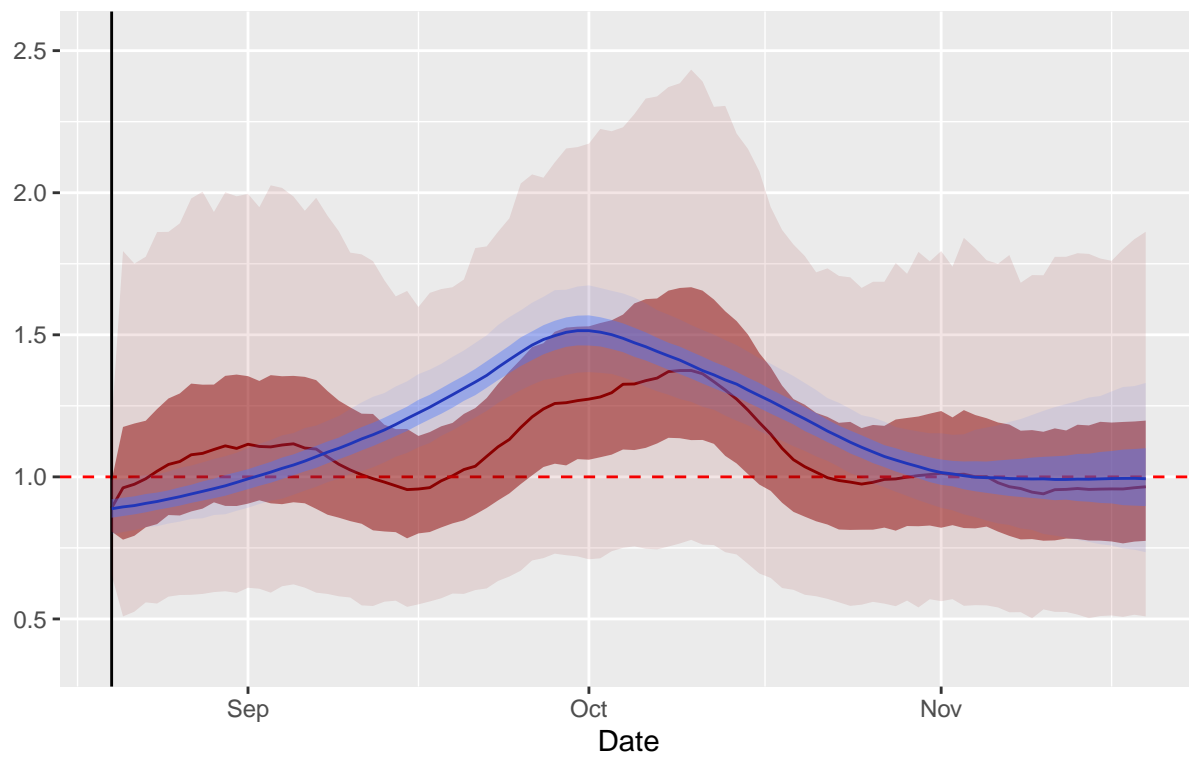
Rt Lombardia

hierarchical model n.1 (red) and single region model (blue)

```
p2 + ylim(c(ymin, ymax))
```
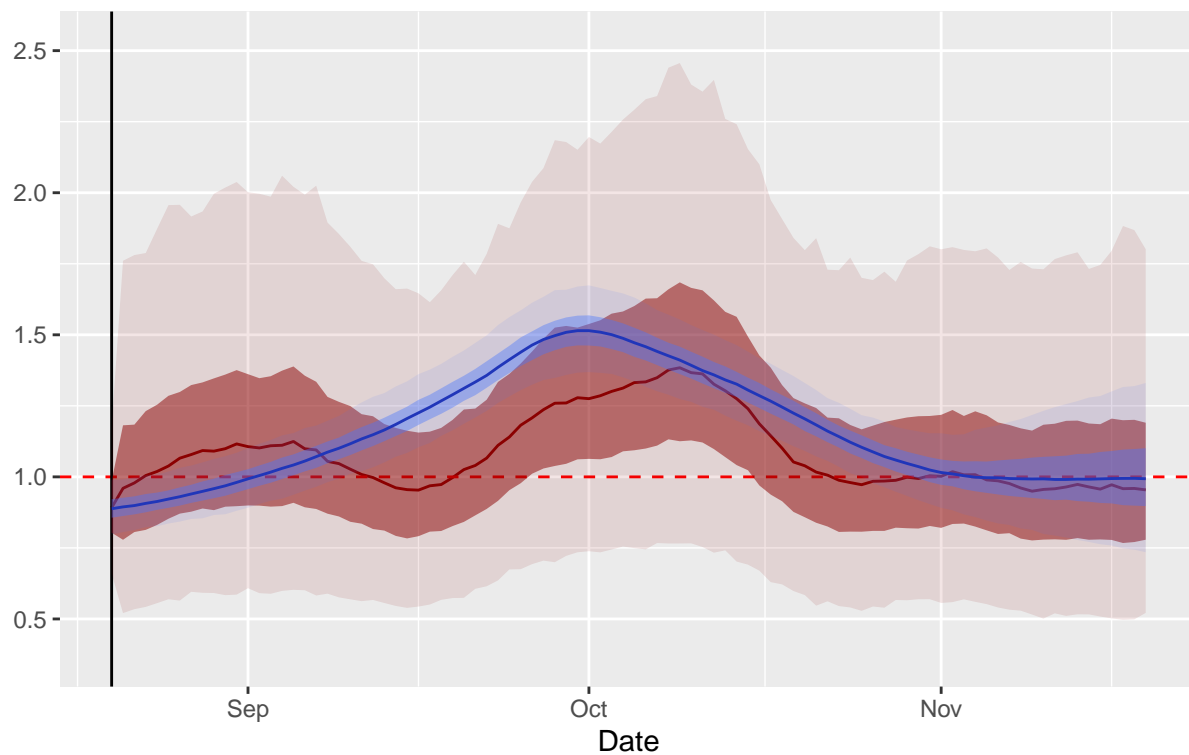
Rt Lombardia
hierarchical model n.2 (red) and single region model (blue)


```
p3 + ylim(c(ymin, ymax))
```

Rt Lombardia

hierarchical model n.3 (red) and single region model (blue)

Veneto

```
p1 <- plot_rt(hier_data, fit_hier, regions, 'Veneto')
p1 <- plot_overlay(p1, 'Veneto') + labs(subtitle = 'hierarchical model n.1 (red) and single region model

p2 <- plot_rt(hier_data, fit_C05, regions, 'Lazio')
p2 <- plot_overlay(p2, 'Veneto')+ labs(subtitle = 'hierarchical model n.2 (red) and single region model

p3 <- plot_rt(hier_data, fit_C1, regions, 'Lazio')
p3 <- plot_overlay(p3,'Veneto') + labs(subtitle = 'hierarchical model n.3 (red) and single region model

ymin <- min(get_plot_limits(p1)$ymin, get_plot_limits(p2)$ymin, get_plot_limits(p3)$ymin)
ymax <- min(get_plot_limits(p1)$ymax, get_plot_limits(p2)$ymax, get_plot_limits(p3)$ymax)

p1 + ylim(c(ymin, ymax))
```
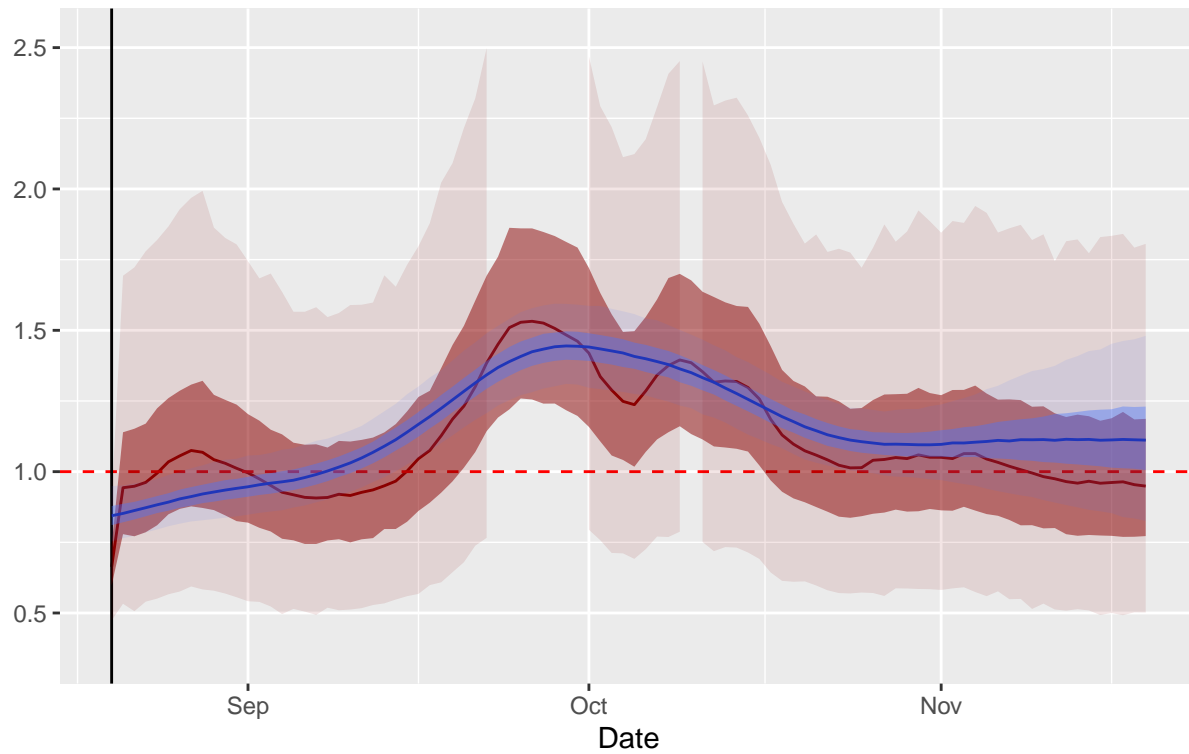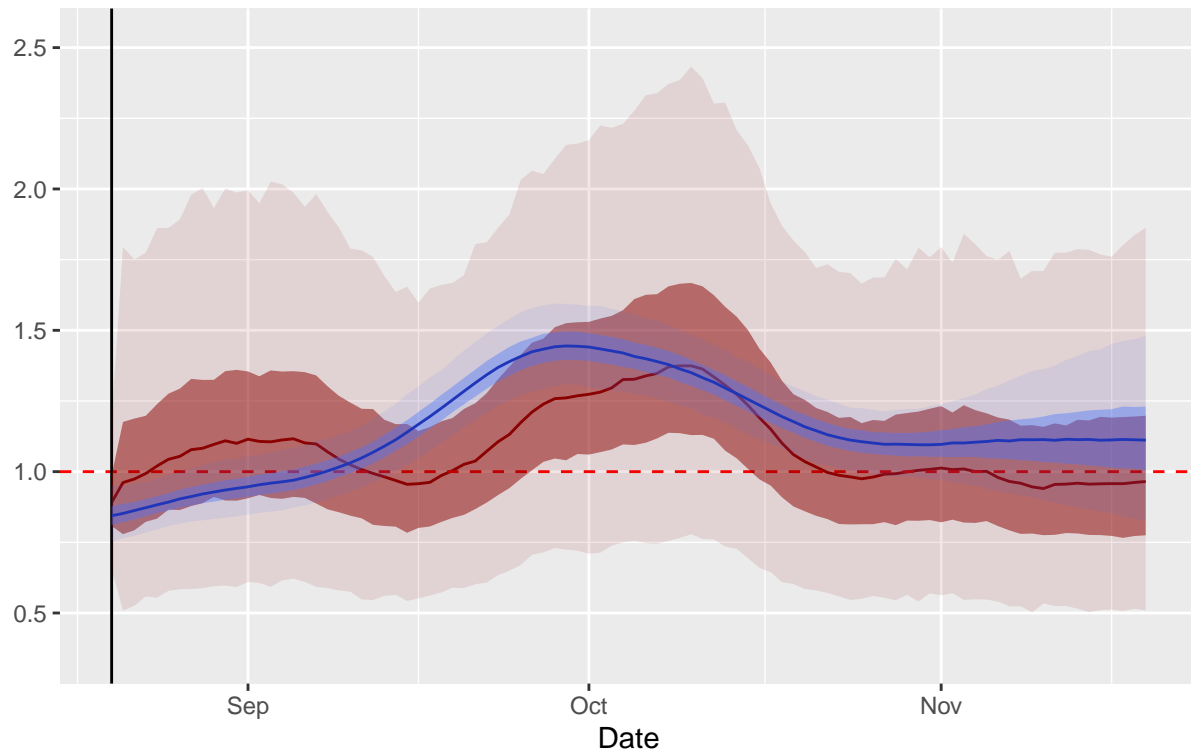
Rt Veneto

hierarchical model n.1 (red) and single region model (blue)


```
p2 + ylim(c(ymin, ymax))
```

Rt Veneto

hierarchical model n.2 (red) and single region model (blue)


```
p3 + ylim(c(ymin, ymax))
```

Rt Veneto

hierarchical model n.3 (red) and single region model (blue)

FVG

```
p1 <- plot_rt(hier_data, fit_hier, regions, 'Friuli Venezia Giulia')
p1 <- plot_overlay(p1, 'Friuli Venezia Giulia') + labs(subtitle = 'hierarchical model n.1 (red) and sin

p2 <- plot_rt(hier_data, fit_C05, regions, 'Lazio')
p2 <- plot_overlay(p2, 'Friuli Venezia Giulia')+ labs(subtitle = 'hierarchical model n.2 (red) and singl

p3 <- plot_rt(hier_data, fit_C1, regions, 'Lazio')
p3 <- plot_overlay(p3,'Friuli Venezia Giulia') + labs(subtitle = 'hierarchical model n.3 (red) and singl

ymin <- min(get_plot_limits(p1)$ymin, get_plot_limits(p2)$ymin, get_plot_limits(p3)$ymin)
ymax <- min(get_plot_limits(p1)$ymax, get_plot_limits(p2)$ymax, get_plot_limits(p3)$ymax)

p1 + ylim(c(ymin, ymax))
```

Rt Friuli Venezia Giulia
hierarchical model n.1 (red) and single region model (blue)


```
p2 + ylim(c(ymin, ymax))
```
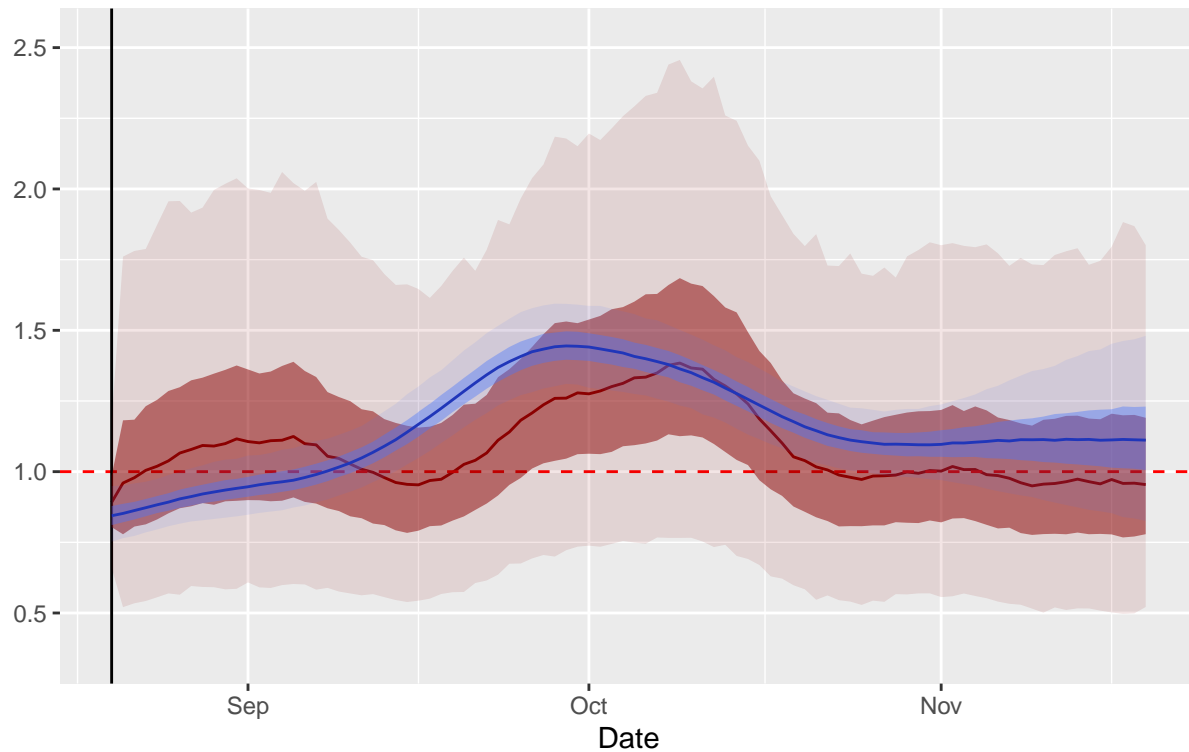
Rt Friuli Venezia Giulia
hierarchical model n.2 (red) and single region model (blue)


```
p3 + ylim(c(ymin, ymax))
```

Rt Friuli Venezia Giulia
hierarchical model n.3 (red) and single region model (blue)

Abruzzo

```r
p1 <- plot_rt(hier_data, fit_hier, regions, 'Abruzzo')
p1 <- plot_overlay(p1, 'Abruzzo') + labs(subtitle = 'hierarchical model n.1 (red) and single region mode

p2 <- plot_rt(hier_data, fit_C05, regions, 'Lazio')
p2 <- plot_overlay(p2, 'Abruzzo')+ labs(subtitle = 'hierarchical model n.2 (red) and single region model

p3 <- plot_rt(hier_data, fit_C1, regions, 'Lazio')
p3 <- plot_overlay(p3,'Abruzzo') + labs(subtitle = 'hierarchical model n.3 (red) and single region model

ymin <- min(get_plot_limits(p1)$ymin, get_plot_limits(p2)$ymin, get_plot_limits(p3)$ymin)
ymax <- min(get_plot_limits(p1)$ymax, get_plot_limits(p2)$ymax, get_plot_limits(p3)$ymax)

p1 + ylim(c(ymin, ymax))
```
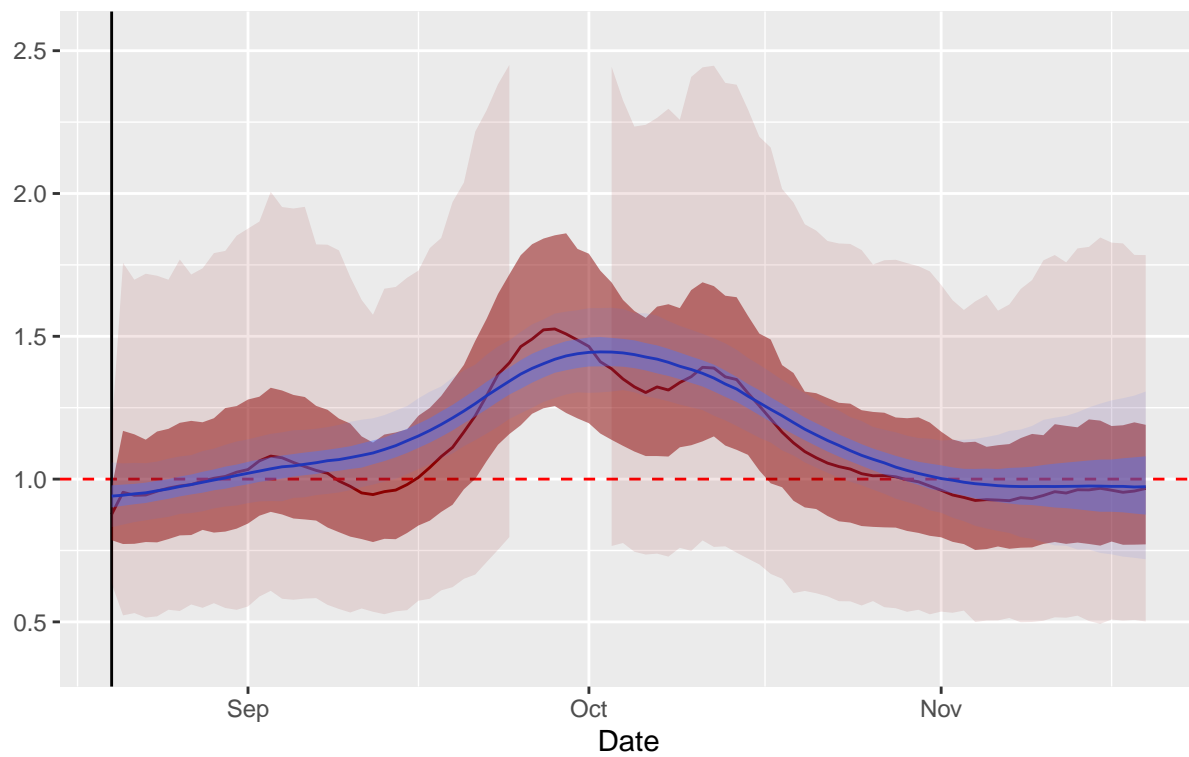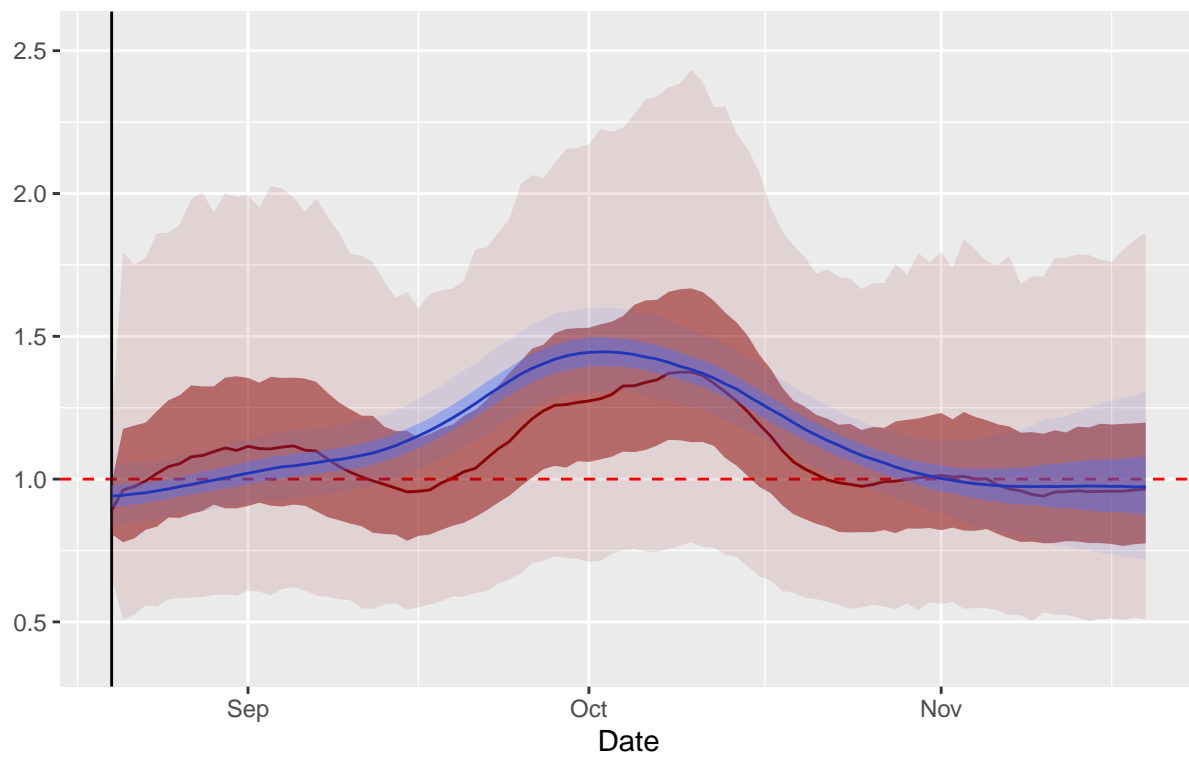
Rt Abruzzo

hierarchical model n.1 (red) and single region model (blue)


```
p2 + ylim(c(ymin, ymax))
```
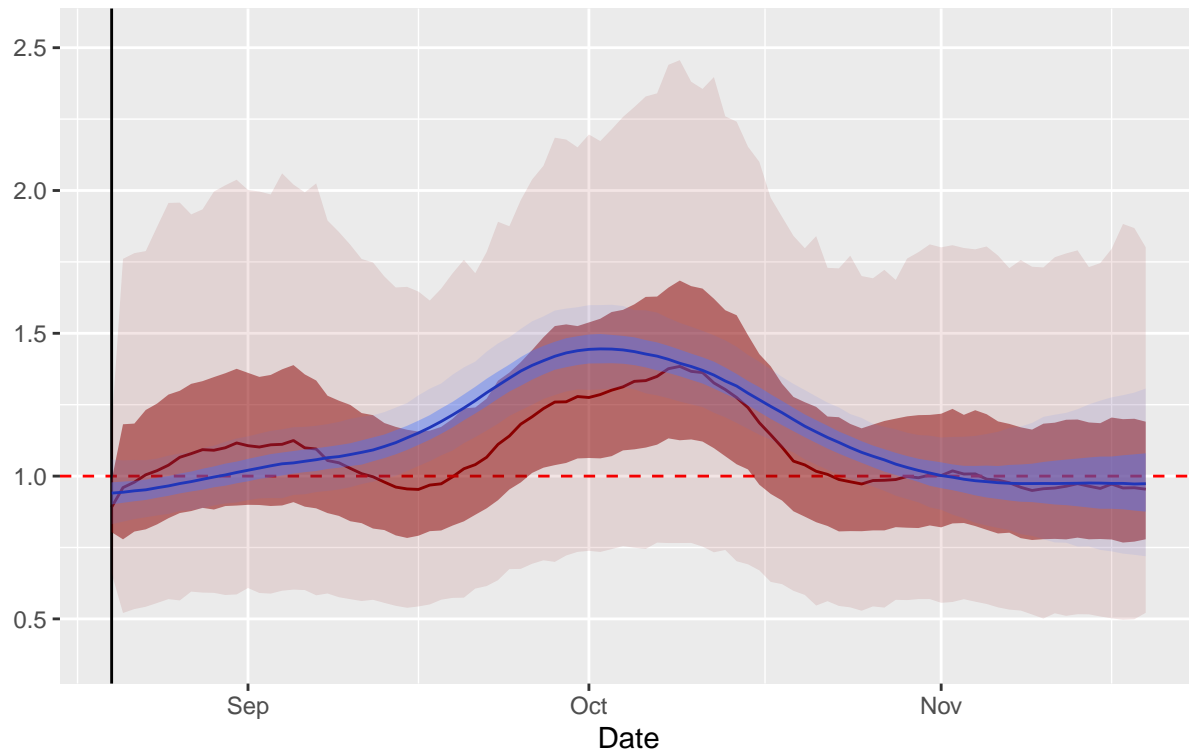
Rt Abruzzo

hierarchical model n.2 (red) and single region model (blue)


```
p3 + ylim(c(ymin, ymax))
```

Rt Abruzzo
hierarchical model n.3 (red) and single region model (blue)

Model comparison

```
log_lik_1 <- extract_log_lik(fit_hier)
loo_1 <- loo(log_lik_1)
waic_1 <- waic(log_lik_1)

log_lik_2 <- extract_log_lik(fit_C1)
loo_2 <- loo(log_lik_2)
waic_2 <- waic(log_lik_2)

log_lik_3 <- extract_log_lik(fit_C05)
loo_3 <- loo(log_lik_3)
waic_3 <- waic(log_lik_3)
```

```
loo_compare(loo_1, loo_2, loo_3)
```

```
##        elpd_diff se_diff
## model2  0.0       0.0
## model3 -0.2       0.2
## model1 -0.6       0.4
```

looic

```
loo_1$estimates[3,1]
```

```
## [1] 9243.193
```

```r
loo_2$estimates[3,1]
```

```
## [1] 9242.059
```

```r
loo_3$estimates[3,1]
```

```
## [1] 9242.554
```

waic

```r
loo_compare(waic_1, waic_2, waic_3)
```

```
##         elpd_diff se_diff
## model2  0.0       0.0
## model3 -0.2       0.3
## model1 -0.5       0.3
```

```r
waic_1$estimates[3,1]
```

```
## [1] 9241.65
```

```r
waic_2$estimates[3,1]
```

```
## [1] 9240.689
```

```r
waic_3$estimates[3,1]
```

```
## [1] 9241.102
```