

Latent class analysis

Latent class analysis extensions

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Extension topics

- Local dependence models
- Multiple latent variables
- **Ordinal indicators**
- **Tree-step modelling**

Ordinal indicators

Ordinal indicators

- So far: dichotomous or unordered polytomous indicators
- When indicators are **ordinal**, we can restrict their relationship with the latent variable(s)
- Several options:
 1. Different types of logits: **adjacent-category**, cumulative, continuation-ratio
 2. Inequality restrictions
 3. Binomial count (k out of K)
- LG approach: **adjacent-category logit** models with fixed scores for item categories, for example,
- Assumption: local odds-ratios are category independent

Another “classic” example

General Social Survey 1982 (see McCutcheon, 1987; Magidson & Vermunt, 2001, 2004)

- Evaluation of surveys by respondent (2 questions)
 - Purpose
 - Accuracy
- Evaluation of respondent by interviewer (2 questions)
 - Understanding
 - Cooperation
- Are there different types of survey respondents?

Fit measures GSS example

Model	L ²	prop. red. L ²	df	p-value	BIC	AIC	class. errors
1-Cluster	257.26	0.00	29	0.00	51.60	199.26	0.00
2-Cluster	79.51	0.69	22	0.00	-76.51	35.51	0.08
3-Cluster	22.09	0.91	15	0.11	-84.29	-7.91	0.13
4-Cluster	6.61	0.97	8	0.58	-50.12	-9.39	0.20

Profiles

	Cluster 1	Cluster 2	Cluster 3
	<i>Ideal</i>	<i>Believers</i>	<i>Skeptics</i>
Class Sizes	0.6169	0.2038	0.1793
PURPOSE			
Good	0.8905	0.9157	0.1592
Depends	0.0524	0.0706	0.2220
Waste	0.0570	0.0137	0.6189
ACCURACY			
Mostly True	0.6148	0.6527	0.0426
Not True	0.3852	0.3473	0.9574
UNDERSTAND			
Good	0.9957	0.3241	0.7532
Fair, poor	0.0043	0.6759	0.2468
COOPERATE			
Interested	0.9452	0.6879	0.6432
Cooperative	0.0547	0.2583	0.2559
Impatient/ Hostile	0.0001	0.0538	0.1009

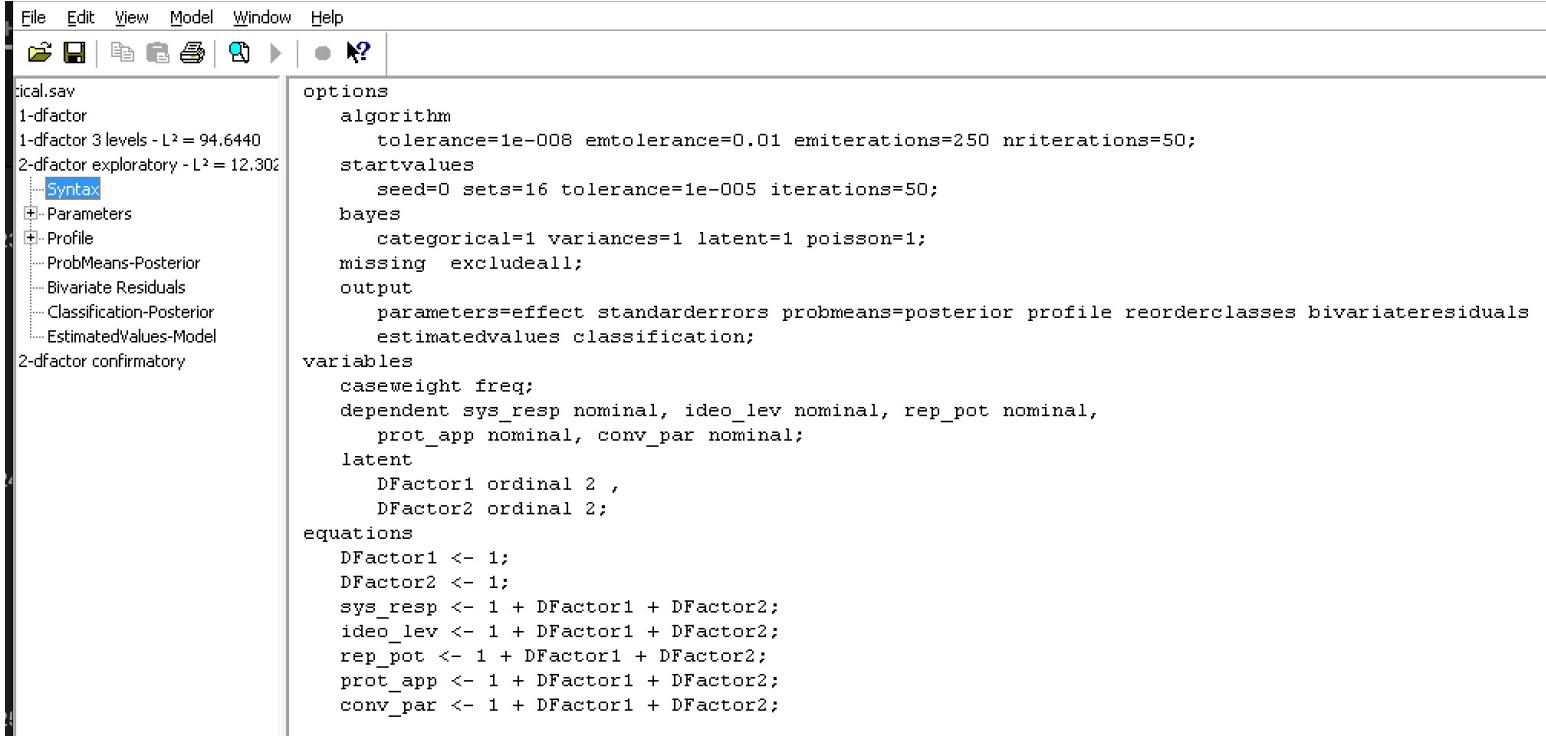
Example GSS (cont.)

Model		L²	BIC	df	p-value
Nominal	1-Cluster	257.26	51.60	29	0.00
	2-Cluster	79.51	-76.51	22	0.00
	3-Cluster	22.09	-84.29	15	0.11
Ordinal	1-Cluster	257.26	51.60	29	0.00
	2-Cluster	82.60	-87.60	24	0.00
	3-Cluster	30.67	-104.07	19	0.04

Profiles ordinal

	Cluster1	Cluster2	Cluster3
Cluster Size	0.6407	0.1925	0.1668
<i>PURPOSE</i>			
Good	0.8792	0.2512	0.9167
Depends	0.0756	0.1475	0.0579
waste	0.0451	0.6013	0.0254
<i>ACCURACY</i>			
mostly true	0.6233	0.0415	0.6753
not true	0.3767	0.9585	0.3247
<i>UNDERSTA</i>			
good	0.9949	0.7483	0.2027
fair/poor	0.0051	0.2517	0.7973
<i>COOPERAT</i>			
interested	0.9362	0.6365	0.6971
cooperative	0.0609	0.2754	0.2412
impatient/hostile	0.0029	0.0881	0.0617

In Latent GOLD - nominal



The screenshot shows the Latent GOLD software interface. The menu bar includes File, Edit, View, Model, Window, and Help. The toolbar contains icons for opening files, saving, printing, and other functions. The left pane displays a project tree with files like 'ical.sav', '1-dfactor', '2-dfactor exploratory', and a 'Syntax' node which is currently selected. The right pane shows the model syntax:

```
options
  algorithm
    tolerance=1e-008 emtolerance=0.01 emiterations=250 nriterations=50;
  startvalues
    seed=0 sets=16 tolerance=1e-005 iterations=50;
  bayes
    categorical=1 variances=1 latent=1 poisson=1;
    missing excludeall;
  output
    parameters=effect standarderrors probmeans=posterior profile reorderclasses bivariate residuals
    estimatedvalues classification;
variables
  caseweight freq;
  dependent sys_resp nominal, ideo_lev nominal, rep_pot nominal,
    prot_app nominal, conv_par nominal;
  latent
    DFactor1 ordinal 2 ,
    DFactor2 ordinal 2;
equations
  DFactor1 <- 1;
  DFactor2 <- 1;
  sys_resp <- 1 + DFactor1 + DFactor2;
  ideo_lev <- 1 + DFactor1 + DFactor2;
  rep_pot <- 1 + DFactor1 + DFactor2;
  prot_app <- 1 + DFactor1 + DFactor2;
  conv_par <- 1 + DFactor1 + DFactor2;
```

In Latent GOLD - nominal

The screenshot shows the Latent GOLD software interface with the following details:

File Menu: File, Open, Save, Print, Exit, Help.

Project List: 32white.sav, hen3.sav.

Model Selection Tree:

- A. 1 cluster model - $L^2 = 257.2604$
- B. 2-cluster model
- C. 3-cluster model - $L^2 = 22.0872$** (selected)
- + Parameters
- + Profile
- + ProbMeans
- Bivariate Residuals
- Iteration Detail
- D. 4-cluster model
- E. 2-cluster model with a local dependency
- F. basic 2-DFactor model

hen3.sav

- A. 1-cluster
- B. 2-cluster
- C. 3-cluster
- D. 4-cluster
- E. 2 ordered clusters
- F. 3 ordered clusters
- G. 4 ordered clusters
- H. Semi-parametric nominal response model (2-level)
- I. Semi-parametric nominal response model (3-level)
- J. Semi-parametric nominal response model (4-level)
- K. Parametric nominal response model (CFactor)

3-Cluster Model Statistics:

	Number of cases	1202
Number of parameters (Npar)	20	
Random Seed	829341	
Best Start Seed	2309733	

Chi-squared Statistics:

	Degrees of freedom (df)	15	p-value
L^2	22.0872	0.11	
X-squared	23.5098	0.074	
Cressie-Read	22.6612	0.091	

Information Criteria:

BIC (based on L^2)	-84.2889
AIC (based on L^2)	-7.9128
AIC3 (based on L^2)	-22.9128
CAIC (based on L^2)	-99.2889
SABIC (based on L^2)	-36.6430

Dissimilarity Index: 0.0272

Total BVR: 2.9898

Log-likelihood Statistics:

Log-likelihood (LL)	-2754.6430
Log-prior	-4.8466

In Latent GOLD - ordinal

The screenshot shows the Latent GOLD software interface. The menu bar includes File, Edit, View, Model, Window, and Help. The toolbar contains icons for opening files, saving, and running models. The left sidebar displays a file tree with several saved projects and model types (A through F). The main window is a syntax editor with the following content:

```
options
  maxthreads=all;
  algorithm
    tolerance=1e-08 emtolerance=0.01 emititerations=250 nriterations=50 ;
  startvalues
    seed=0 sets=16 tolerance=1e-05 iterations=50;
  bayes
    categorical=1 variances=1 latent=1 poisson=1;
  montecarlo
    seed=0 sets=0 replicates=500 tolerance=1e-08;
  quadrature nodes=10;
  missing excludeall;
  output
    parameters=effect standarderrors profile probmeans=posterior loadings
      bivariate residuals iterationdetails reorderclasses;
variables
  caseweight frq;
  dependent purpose ordinal, accuracy nominal, understa nominal,
    cooperat ordinal;
latent
  Cluster nominal 3;
equations
  Cluster <- 1;
  purpose <- 1 + Cluster;
  accuracy <- 1 + Cluster;
  understa <- 1 + Cluster;
  cooperat <- 1 + Cluster;
```

In Latent GOLD - ordinal

The screenshot shows the Latent GOLD software interface with the following details:

File menu: File, Edit, View, Model, Window, Help.

Toolbar: File, Open, Save, Syntax, Parameters, Profile, ProbMeans-Posterior, Bivariate Residuals, Iteration Detail, Model2.

Project Tree:

- gss82white.sav
 - C. 3-cluster model - $L^2 = 30.6695$
 - Syntax
 - Parameters
 - Profile
 - ProbMeans-Posterior
 - Bivariate Residuals
 - Iteration Detail
 - Model2
- gss82white.sav
 - A. 1 cluster model - $L^2 = 257.2604$
 - B. 2-cluster model
 - C. 3-cluster model - $L^2 = 22.0872$
 - D. 4-cluster model
 - E. 2-cluster model with a local dependency
 - F. basic 2-DFactor model
- heinen3.sav
 - A. 1-cluster
 - B. 2-cluster
 - C. 3-cluster
 - D. 4-cluster
 - E. 2 ordered clusters
 - F. 3 ordered clusters
 - G. 4 ordered clusters
 - H. Semi-parametric nominal response model
 - I. Semi-parametric nominal response model

3-Cluster Syntax Model

Number of cases	1202
Number of parameters (Npar)	16
Random Seed	182958
Best Start Seed	2136901

Chi-squared Statistics

Degrees of freedom (df)	19	p-value
L^2	30.6695	0.044
X-squared	31.3307	0.037
Cressie-Read	30.6590	0.044
BIC (based on L^2)	-104.0736	
AIC (based on L^2)	-7.3305	
AIC3 (based on L^2)	-26.3305	
CAIC (based on L^2)	-123.0736	
SABIC (based on L^2)	-43.7221	
Dissimilarity Index	0.0382	
Total BVR	5.0510	

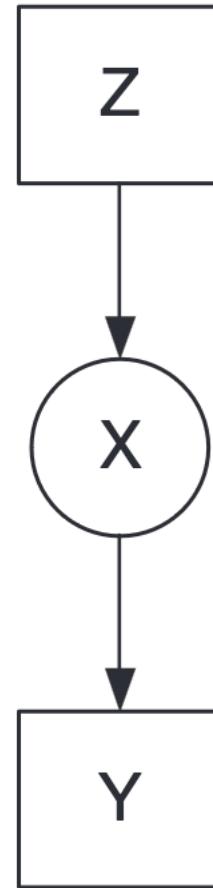
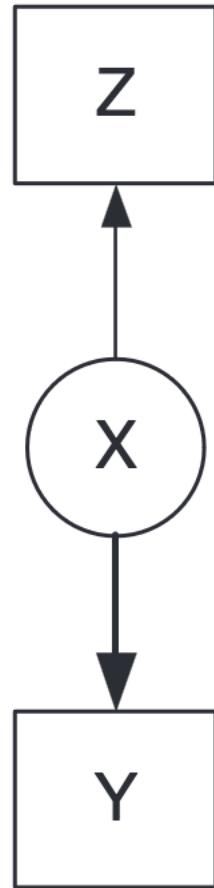
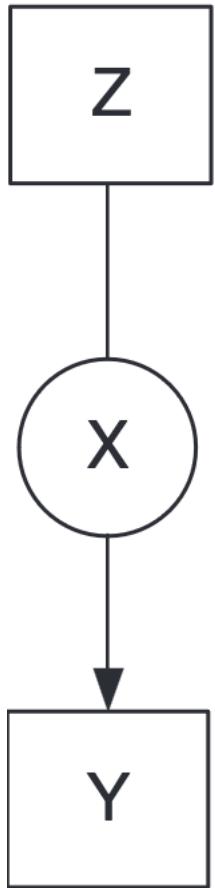
Log-likelihood Statistics

Log-likelihood (LL)	-2758.9342
Log-prior	-4.8370

Three step modeling

Analyzing the assigned class memberships

Correction for misclassification



Three step approach *without* correction (“classical three step”)

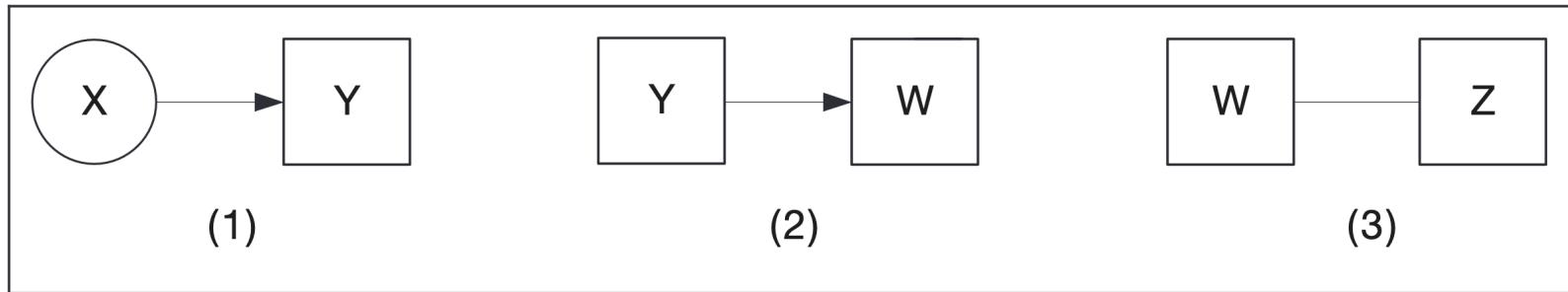


Figure 2. The steps of the standard three-step approach.

Classify-analyze does not work!

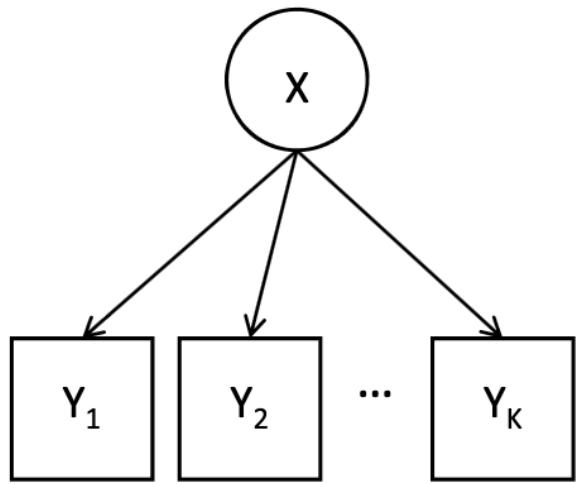
- You might think that after classification it is easy to model people's latent class membership
- “Just take assigned class and analyze it as though observed”
- Unfortunately, -> **biased estimates** and wrong se's
(Still relatively little-known among practitioners?)

“Classical 3step” does not work!

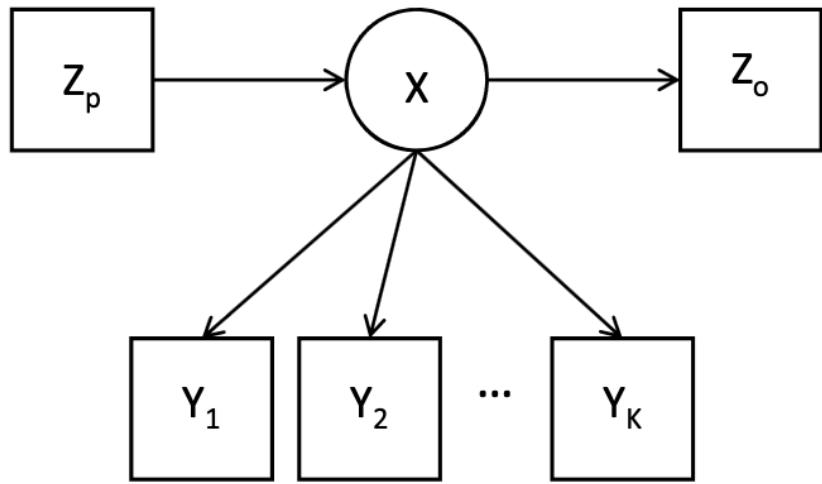
Solutions:

- *Ignore the problem.*
 - This will lead to bias and wrong standard errors.
 - But bias might not be so bad compared with additional variance from alternatives (tradeoff)
 - Hard to know when this will be the case...
- *Model covariates directly* (as in poLCA or loglinear cvam)
 - “circularity” : Feels like “cheating” (even though from model-based perspective nothing is wrong)
 - Can change the class solution sometimes to something less useful
 - Can be tricky when the intended use of the class assignments is not a multinomial regression

“One step approach”



Measurement model



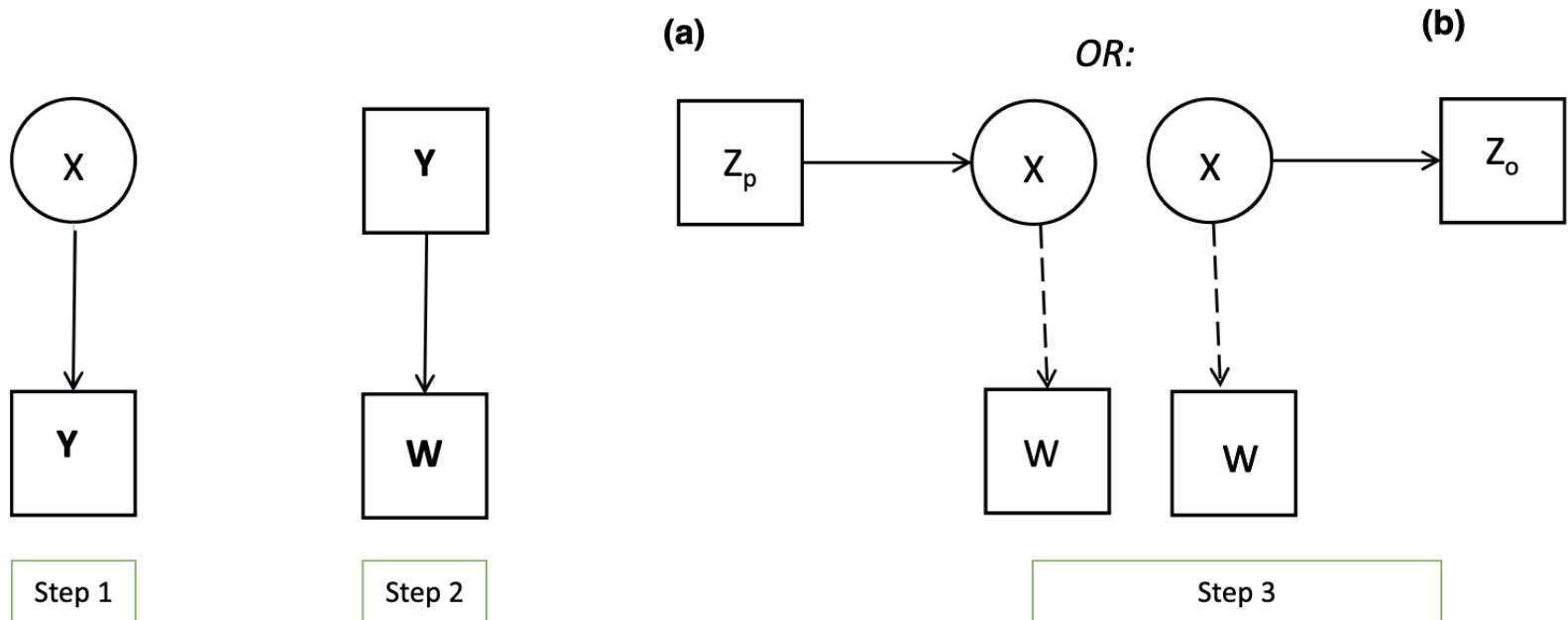
One-step approach

How to model (X, Z)?

Solutions:

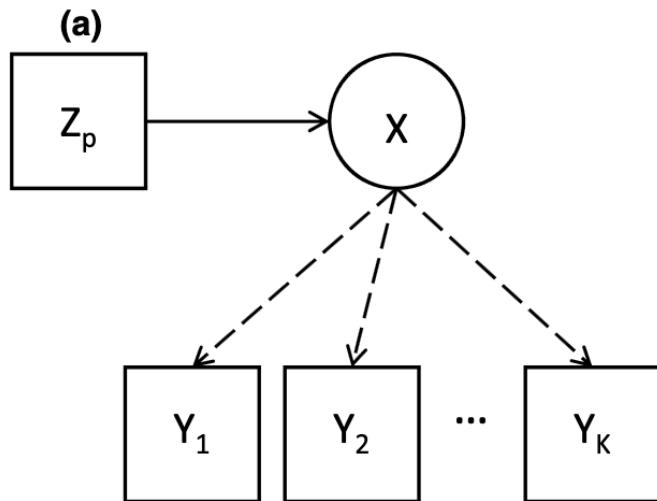
- Three-step modeling: (BCH, 2002; Vermunt 2010; Bakk et al 2013)
 1. Run the latent class model with Y and X only
 2. Compute things on the LC model
 - a. Assign cases to their suspected class W
 - b. Compute the classification matrix E
 3. Correct ($W|Z$) for misclassification in W as a measure of X using E
- Two-step modeling: (Bakk & Kuha 2018)
 1. Run the latent class model with Y and X only
 2. Run another latent class model, now with covariate Z, while fixing all “measurement parameters”, ($Y|X$), to estimates from step 1

Three-step approach *with correction*

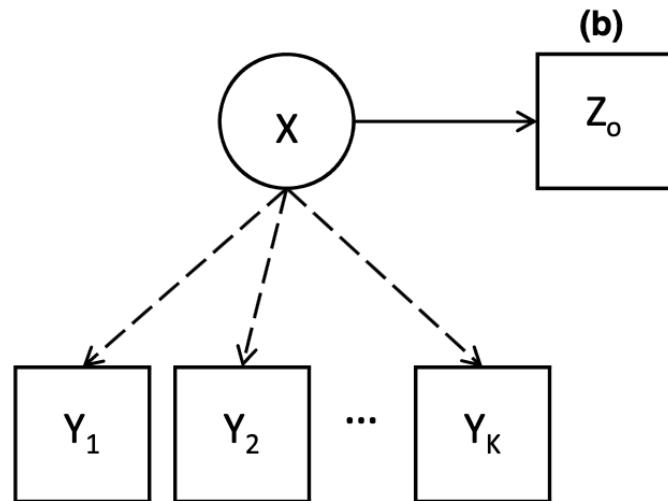


Source: Bakk & Kuha 2020

Two-step approach



OR:



Step 2 of the two-step approach

Table 1. Recommended modelling approaches for different circumstances

Approach	Distal outcome	Covariate	Large models
One-step approach ML	Not recommended	Recommended with reservations	Recommended with reservations
	Recommended only if underlying model assumptions are met for models with continuous and count outcomes. Suitable for categorical outcomes	Recommended	Extendable to models with multiple latent variables, multiple outcomes and covariates. Not possible to (easily) model DIF ^a
BCH	Most robust for continuous and count outcomes. Negative weights can be counterintuitive	Recommended	Currently not possible to extend to complex models, or to model DIF
Two-step approach	Not recommended with continuous or count outcome. Recommended with categorical outcome	Recommended	Recommended, most flexible stepwise approach. Flexible for modelling DIF
LTB	Recommended if no heteroscedasticity is present for continuous and count outcome. Recommended for categorical outcome	Not appropriate	Not appropriate
Classical three-step	Not recommended	Not recommended	Not recommended

Note. ^aBy DIF we mean differential item functioning, also known as direct effect between the covariate and (some) indicator(s).

Correction for misclassification

Which is of these is true?

Measurement error/misclassification:

- Always causes estimates of relationships between variables to be *underestimated*;
- Is something you can only try to *prevent* as much as possible. You can't really do anything about it once it's there;
- Is not relevant when you're only looking at *trends* (over time);
- Is not relevant when you're only looking at *averages*;
- Is taken care of by using assigned classes/factor scores;
- Is a problem with self-reports, but not with administrative registers or other behavioral variables.

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- Is not relevant when you're only looking at *trends* (over time);
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Correction for misclassification

- Standard BCH
- Generalized BCH
- ML

Standard/classic BCH

- Named after Bolck, Croon & Hagenaars (2002)
- Invented by Fuller (1987)
- (Example of Stigler's law of eponymy)

Real example with simple categorical data analysis

True vote (admin)		
Answered	No	Yes
"No"	0.671	0.015
"Yes"	0.329	0.985
Total	1	1

Ideology		
True vote	Dem.	Rep.
No	0.405	0.312
Yes	0.595	0.688
Total	1	1

Ideology		
Answered	Dem.	Rep.
"No"	0.305	0.248
"Yes"	0.695	0.752
Total	1	1

		Ideology	
True vote		Dem.	Rep.
No	0.405	0.312	
Yes	0.595	0.688	
Total	1	1	

		Ideology	
Answered		Dem.	Rep.
"No"	0.305	0.248	
"Yes"	0.695	0.752	
Total	1	1	

A little numerical example

- Proportion Democrats who say "No" is a *mixture* of
 - 0.671 of true No's say "No" and
 - 0.015 of true Yes's who say "No"
- This leads to:
 - $(0.671)(0.405) + (0.015)(0.595) = 0.281.$
- So we *observe* about 30% of nonvoters among Democrats,
- when the *truth* is 40% nonvoters among Democrats.

General result

The observed table is the result of a matrix multiplication between the misclassification rates and the true table:

$$\underbrace{\begin{bmatrix} \text{Misclassification} \\ \text{No} & \text{Yes} \\ 0.671 & 0.015 \\ 0.329 & 0.985 \end{bmatrix}}_{\cdot} \cdot \underbrace{\begin{bmatrix} \text{True} \\ \text{No} & \text{Dem.} & \text{Rep.} \\ 0.405 & 0.312 \\ 0.595 & 0.688 \end{bmatrix}}_{\approx} \underbrace{\begin{bmatrix} \text{Observed} \\ \text{"No"} & \text{Dem.} & \text{Rep.} \\ 0.305 & 0.248 \\ 0.695 & 0.752 \end{bmatrix}}_{\text{Yes}}$$

Know: Observed table and Misclassification table

Want: True table

→ To estimate true table, just invert the equation:

$$\underbrace{\begin{bmatrix} & \text{True} \\ & \text{Dem.} & \text{Rep.} \\ \text{No} & 0.405 & 0.312 \\ \text{Yes} & 0.595 & 0.688 \end{bmatrix}}_{\text{Observed}} \approx \underbrace{\begin{bmatrix} & \text{Misclassification} \\ & \text{No} & \text{Yes} \\ 0.671 & & 0.015 \\ 0.329 & & 0.985 \end{bmatrix}^{-1}}_{\text{Misclassification}} \cdot \underbrace{\begin{bmatrix} & \text{Observed} \\ & \text{"No"} & \text{Dem.} & \text{Rep.} \\ \text{"Yes"} & 0.305 & 0.248 \\ & 0.695 & 0.752 \end{bmatrix}}_{\text{True}}$$

- Note the **negative weights**
- Many people have intuition that M should be used as weights
- It's actually the inverse!

$$\underbrace{\begin{bmatrix} & \text{True} \\ & \text{Dem.} & \text{Rep.} \\ \text{No} & 0.405 & 0.312 \\ \text{Yes} & 0.595 & 0.688 \end{bmatrix}}_{\text{Misclassification inverse}} \approx \underbrace{\begin{bmatrix} & \text{Misclassification inverse} \\ & \text{No} & \text{Yes} \\ 1.502 & -0.023 \\ -0.502 & 1.023 \end{bmatrix}}_{\text{Observed}} \cdot \begin{bmatrix} & \text{Observed} \\ & \text{Dem.} & \text{Rep.} \\ \text{"No"} & 0.305 & 0.248 \\ \text{"Yes"} & 0.695 & 0.752 \end{bmatrix}$$

- For comparison, in our example we did have the true values
- and the true and estimated tables appear similar:

		Estimated		True		Observed		
		Dem.	Rep.	No	Dem.	Rep.	Dem.	Rep.
No	Yes	0.442	0.355	0.405	0.312	0.305	0.248	
	No	0.558	0.645	0.595	0.688	0.695	0.752	

?

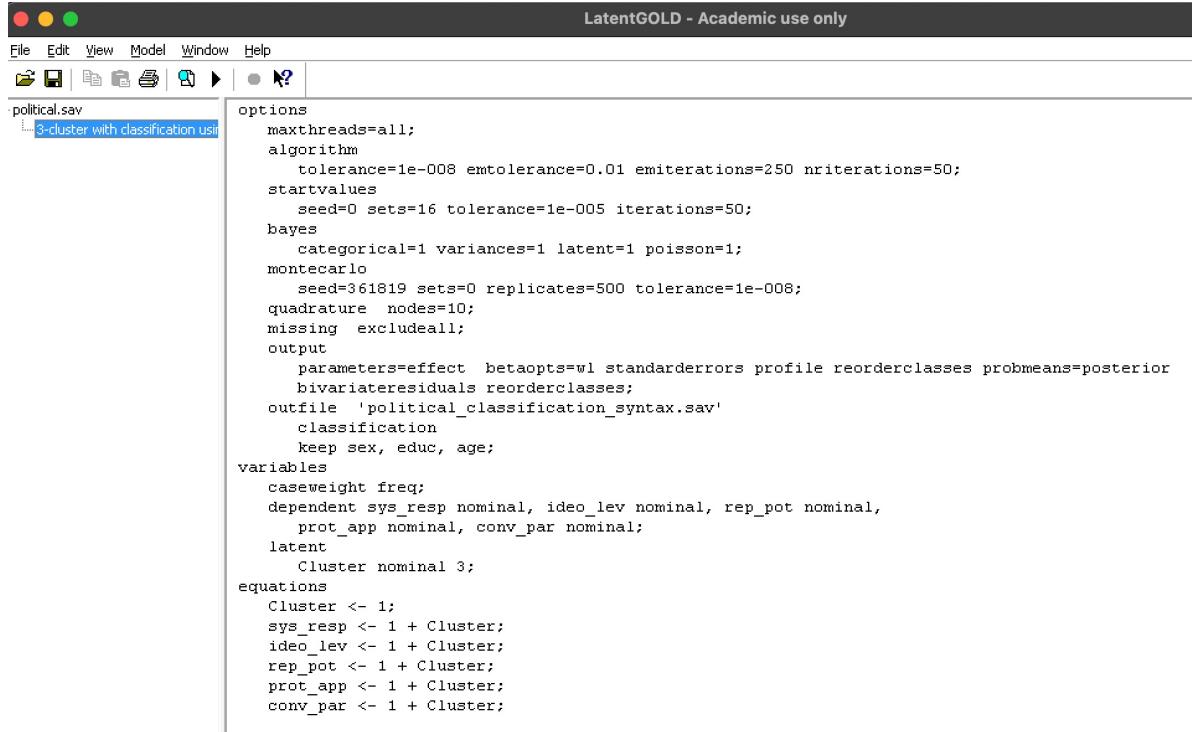
- So in this example the correction of the Turnout-Ideology crosstable for measurement error has worked very well;
- In practice of course we cannot usually do this check!

“Generalized” BCH

(Vermunt 2010)

- BCH impractical for ~more complicated than cross-table
- We saw that the elements of the inverse of the classification matrix are used as weights (some negative)
- It turns out we can extend this idea:
 1. Create a “complete-data” set in which each original observation is replicated K times, one for each value of X
 2. Set the weight of each observation to the appropriate element of the inverse classification matrix
 3. Run any analysis of your choice which allows (neg.) weights.
(using observation ID as cluster corrects standard errors)

3 step in Latent GOLD – step 1 & 2



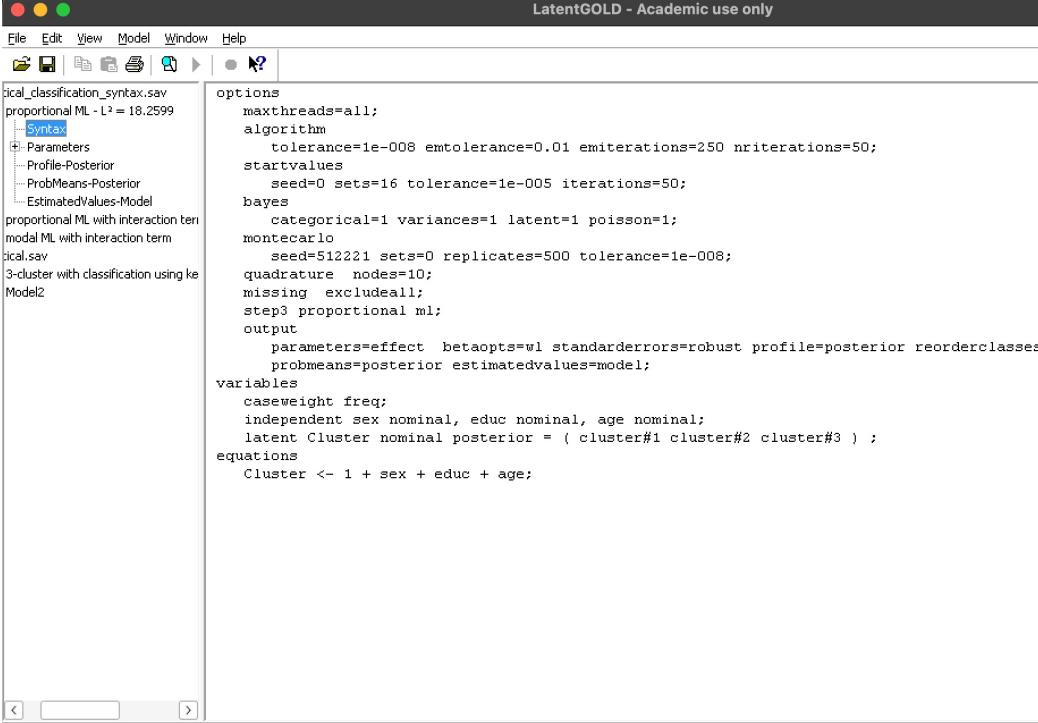
The screenshot shows the LatentGOLD software interface. The title bar reads "LatentGOLD - Academic use only". The menu bar includes File, Edit, View, Model, Window, and Help. Below the menu is a toolbar with icons for opening files, saving, and other functions. The main window displays a syntax file named "political.sav". The syntax code is as follows:

```
options
    maxthreads=all;
    algorithm
        tolerance=1e-008 emtolerance=0.01 emiterations=250 nriterations=50;
    startvalues
        seed=0 sets=16 tolerance=1e-005 iterations=50;
    bayes
        categorical=1 variances=1 latent=1 poisson=1;
    montecarlo
        seed=361819 sets=0 replicates=500 tolerance=1e-008;
        quadrature nodes=10;
        missing excludeall;
    output
        parameters=effect betaopts=w1 standarderrors profile reorderclasses probmeans=posterior
            bivariate residuals reorderclasses;
    outfile 'political_classification_syntax.sav'
        classification
        keep sex, educ, age;
variables
    caseweight freq;
    dependent sys_resp nominal, ideo_lev nominal, rep_pot nominal,
        prot_app nominal, conv_par nominal;
latent
    Cluster nominal 3;
equations
    Cluster <- 1;
    sys_resp <- 1 + Cluster;
    ideo_lev <- 1 + Cluster;
    rep_pot <- 1 + Cluster;
    prot_app <- 1 + Cluster;
    conv_par <- 1 + Cluster;
```

3 step in Latent GOLD

rep_pot Repression Potential	prot_app Protest Approval	conv_par Conventional Participation	freq cell count	sex Sex	educ Education (training)	age Age (generation)	clu#1 Cluster1	clu#2 Cluster2	clu#3 Cluster3	clu# Cluster modal
2	1	1		1 1	2	3	0.22885148	0.582529165	0.1886193499	2
2	1	1		1 2	1	3	0.22885148	0.582529165	0.1886193499	2
2	1	1		1 2	2	2	0.22885148	0.582529165	0.1886193499	2
2	1	2		1 1	1	2	0.70227264	0.085522685	0.2122046782	1
2	1	2		1 1	2	2	0.70227264	0.085522685	0.2122046782	1
2	1	2		1 2	2	2	0.70227264	0.085522685	0.2122046782	1
2	2	1		4 1	1	1	0.05050353	0.060949501	0.8885469688	3
2	2	1		1 1	2	1	0.05050353	0.060949501	0.8885469688	3
2	2	1		1 1	2	3	0.05050353	0.060949501	0.8885469688	3
2	2	1		1 2	1	1	0.05050353	0.060949501	0.8885469688	3
2	2	1		2 2	2	1	0.05050353	0.060949501	0.8885469688	3
2	2	1		1 2	2	2	0.05050353	0.060949501	0.8885469688	3
2	2	2		6 1	1	1	0.13319179	0.007690198	0.8591180074	3
2	2	2		3 1	1	2	0.13319179	0.007690198	0.8591180074	3
2	2	2		1 1	1	3	0.13319179	0.007690198	0.8591180074	3

3 step in Latent GOLD - ML



The screenshot shows the LatentGOLD software interface with the title bar "LatentGOLD - Academic use only". The menu bar includes File, Edit, View, Model, Window, and Help. Below the menu is a toolbar with icons for opening files, saving, and running models. The left sidebar displays a file tree with "dical_classification_syntax.sav" selected, showing sub-nodes like "Syntax", "Parameters", "Profile-Posterior", "ProbMeans-Posterior", and "EstimatedValues-Model". The main window contains the model syntax:

```
options
  maxthreads=all;
  algorithm
    tolerance=1e-008 emtolerance=0.01 emiterations=250 nriterations=50;
  startvalues
    seed=0 sets=16 tolerance=1e-005 iterations=50;
  bayes
    categorical=1 variances=1 latent=1 poisson=1;
  montecarlo
    seed=512221 sets=0 replicates=500 tolerance=1e-008;
    quadrature nodes=10;
    missing excludeall;
  step3 proportional ml;
  output
    parameters=effect betaopts=wl standarderrors=robust profile=posterior reorderclasses
      probmeans=posterior estimatedvalues=model;
variables
  caseweight freq;
  independent sex nominal, educ nominal, age nominal;
  latent Cluster nominal posterior = ( cluster#1 cluster#2 cluster#3 );
equations
  Cluster <- 1 + sex + educ + age;
```

3 step in Latent GOLD – ML & BCH

- step3
 - <proportional | modal>
 - <ml | bch | none>
- The command WriteBCH='filename' creates an expanded data file with one record per latent class for each observation and the weights needed for the various types of non-adjusted and bch-adjusted 3-step analysis methods