

Demostración 2

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Sea

$$E[x] = \frac{1}{2} \sum_{n=1}^N (t_n - f(x_n))^2$$

donde $f(x) = \sum_{r=1}^R C_r \cdot 1(x \in R_r)$

Por demostrar: Si $\frac{\partial E}{\partial C_k} = 0$, entonces

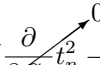
$$\widehat{C}_k = \frac{1}{N_k} \sum_{n|x_n \in R_k} t_n$$

Demostración:

$$\frac{\partial E(x)}{\partial C_k} = \frac{\partial}{\partial C_k} \left[\frac{1}{2} \sum_{n=1}^N (t_n - f(x))^2 \right] \quad (1)$$

$$= \frac{1}{2} \sum_{n=1}^N \frac{\partial}{\partial C_k} (t_n - f(x_n))^2$$

$$= \frac{1}{2} \sum_{n=1}^N \frac{\partial}{\partial C_k} (t_n^2 - 2t_n f(x) + f(x_n)^2)$$

$$= \frac{1}{2} \sum_{n=1}^N \left[\cancel{\frac{\partial}{\partial C_k}} t_n^2 - 2t_n \frac{d}{dC_k} f(x) + \frac{\partial}{\partial C_k} f(x)^2 \right]$$


$$= \frac{1}{2} \sum_{n=1}^N \left[-2t_n \frac{\partial}{\partial C_k} f(x) + \frac{d}{dC_k} f(x)^2 \right]$$

donde:

$$\begin{aligned} \frac{\partial}{\partial C_k} f(x) &= \frac{\partial}{\partial C_k} \sum_{r=k}^R C_r \cdot 1(x \in R_r) \\ &= \frac{\partial}{\partial C_k} [C_k \cdot 1(x \in R_k)] + \frac{\partial}{\partial C_k} \left[\sum_{r \neq k}^R C_r \cdot 1(x \in R_r) \right] \xrightarrow{0} \\ &= 1(x \in R_k) \end{aligned}$$

Por lo tanto,

$$\begin{aligned} &\frac{1}{2} \sum_{n=1}^N \left[-2t_n \frac{d}{dC_k} f(x) + \frac{d}{dC_k} f(x)^2 \right] \\ &= \frac{1}{2} \sum_{n=1}^N \left[-2t_n \cdot 1(x \in R_k) + 2f(x) \frac{d}{dC_k} f(x) \right] \\ &= \frac{1}{2} \sum_{n=1}^N \left[-2t_n \cdot 1(x \in R_k) + 2f(x) 1(x \in R_k) \right] \\ &= \sum_{n=1}^N \left[-t_n 1(x \in R_k) + f(x) 1(x \in R_k) \right] \\ &= \sum_{n=1}^N \left[-t_n 1(x \in R_k) + N_k \cdot \widehat{C_k} \right] = 0 \end{aligned}$$

$$\implies N_k \cdot \widehat{C}_k = \sum_{n=1}^N t_n 1(x \in R_k)$$

$$\implies \widehat{C}_k = \frac{1}{N_k} \sum_{n=1}^N t_n$$