Demostración 2

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Sea

$$E[x] = \frac{1}{2}\sum_{n=1}^{N}(t_n - f(x_n))^2$$

donde $f(x) = \sum_{r=1}^{R} C_r \cdot 1(x \in R_r)$

Por demostrar: Si $\frac{\partial E}{\partial C_k} = 0$, entonces

$$\widehat{C_k} = \frac{1}{N_k}\sum_{n|x_n \in R_k} t_n$$

Demostración:

$$\frac{\partial E(x)}{\partial C_k} = \frac{\partial}{\partial C_k}[\frac{1}{2}\sum_{n=1}^{N}(t_n - f(x)^2] \tag{1}$$

$$= \frac{1}{2}\sum_{n=1}^{N}\frac{\partial}{\partial C_k}(t_n - f(x_n))^2$$

$$= \frac{1}{2}\sum_{n=1}^{N}\frac{\partial}{\partial C_k}(t_n^2 - 2t_n f(x) + f(x_n)^2)$$

$$= \frac{1}{2}\sum_{n=1}^{N}[\cancelto{0}{\frac{\partial}{\partial C_k}t_n^2} - 2t_n\frac{d}{dC_k}f(x) + \frac{\partial}{\partial C_k}f(x)^2]$$

$$= \frac{1}{2}\sum_{n=1}^{N}[-2t_n \frac{\partial}{\partial C_k} f(x) + \frac{d}{dC_k} f(x)^2]$$

donde:

$$\frac{\partial}{\partial C_k} f(x) = \frac{\partial}{\partial C_k} \sum_{r=k}^{R} C_r \cdot 1(x \in R_r)$$

$$= \frac{\partial}{\partial C_k}\left[C_k \cdot 1(x \in R_k)\right] + \cancelto{0}{\frac{\partial}{\partial C_k}\left[\sum_{r \neq k}^{R} C_r \cdot 1(x \in R_r)\right]}$$

$$= 1(x \in R_k)$$

Por lo tanto,

$$\frac{1}{2}\sum_{n=1}^{N}[-2t_n \frac{d}{dC_k} f(x) + \frac{d}{dC_k} f(x)^2]$$

$$= \frac{1}{2}\sum_{n=1}^{N}[-2t_n \cdot 1(x \in R_k) + 2f(x)\frac{d}{dC_k} f(x)]$$

$$= \frac{1}{2}\sum_{n=1}^{N}[-2t_n \cdot 1(x \in R_k) + 2f(x)1(x \in R_k)]$$

$$= \sum_{n=1}^{N}[-t_n 1(x \in R_k) + f(x)1(x \in R_k)]$$

$$= \sum_{n=1}^{N}[-t_n 1(x \in R_k) + N_k \cdot \widehat{C_k} = 0$$

$$\implies N_k \cdot \widehat{C_k} = \sum_{n=1}^{N} t_n 1(x \in R_k)$$

$$\implies \widehat{C_k} = \frac{1}{N_k} \sum_{n=1}^{N} t_n$$