Sea $\sigma = \frac{1}{1+\exp(-x)}$

P.D $\frac{\partial}{\partial x}\sigma(x) = \sigma(x)(1-\sigma(x))$

$$\frac{\partial \sigma(x)}{\partial x} = \frac{\partial}{\partial x}\left(\frac{1}{1+\exp(-x)}\right)$$

$$= -[1+\exp(-x)]^{-2}[-\exp(-x)]$$

$$= \frac{-\exp(-x)}{[1+\exp(-x)]^2}$$

$$= \frac{1}{1+\exp(-x)}\left[-\frac{\exp(x)}{1+\exp(-x)}\right]$$

$$= \sigma(x)\left[\frac{1}{1-\exp(-x)} + \frac{1-\exp(-x)}{1-\exp(-x)}\right]$$

$$\Rightarrow \sigma(x)\left[1 - \frac{1}{1-\exp(-x)}\right]$$

$$= \sigma(x)[1-\sigma(x)]$$

■.