Classification

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The Stock Market Data

The Smarket data consists of the percentage returns for the S&P 500 stock index over 1,250 days, from the beginning of 2001 until the end og 2005. For each date, we have recorded the percentage returns for each of the five previous trading days, `Lag1` through `Lag5`. We have also recorded `Volume` (the number of shared traded on the previous day, in billions), `Today` (the percentage return on the date in question) and `Direction` (whether the market was "Up" or "Down" on this date).

```
library(ISLR)
names(Smarket)
```

```
## [1] "Year"      "Lag1"      "Lag2"      "Lag3"      "Lag4"      "Lag5"
## [7] "Volume"    "Today"     "Direction"
```

```
dim(Smarket)
```

```
## [1] 1250    9
```

```
summary(Smarket)
```

```
##       Year           Lag1                Lag2                Lag3
##  Min.   :2001   Min.   :-4.922000   Min.   :-4.922000   Min.   :-4.922000
##  1st Qu.:2002   1st Qu.:-0.639500   1st Qu.:-0.639500   1st Qu.:-0.640000
##  Median :2003   Median : 0.039000   Median : 0.039000   Median : 0.038500
##  Mean   :2003   Mean   : 0.003834   Mean   : 0.003919   Mean   : 0.001716
##  3rd Qu.:2004   3rd Qu.: 0.596750   3rd Qu.: 0.596750   3rd Qu.: 0.596750
##  Max.   :2005   Max.   : 5.733000   Max.   : 5.733000   Max.   : 5.733000
##       Lag4                Lag5              Volume           Today
##  Min.   :-4.922000   Min.   :-4.92200   Min.   :0.3561   Min.   :-4.922000
##  1st Qu.:-0.640000   1st Qu.:-0.64000   1st Qu.:1.2574   1st Qu.:-0.639500
##  Median : 0.038500   Median : 0.03850   Median :1.4229   Median : 0.038500
##  Mean   : 0.001636   Mean   : 0.00561   Mean   :1.4783   Mean   : 0.003138
##  3rd Qu.: 0.596750   3rd Qu.: 0.59700   3rd Qu.:1.6417   3rd Qu.: 0.596750
##  Max.   : 5.733000   Max.   : 5.73300   Max.   :3.1525   Max.   : 5.733000
##  Direction
##  Down:602
##  Up  :648
##
##
##
##
```

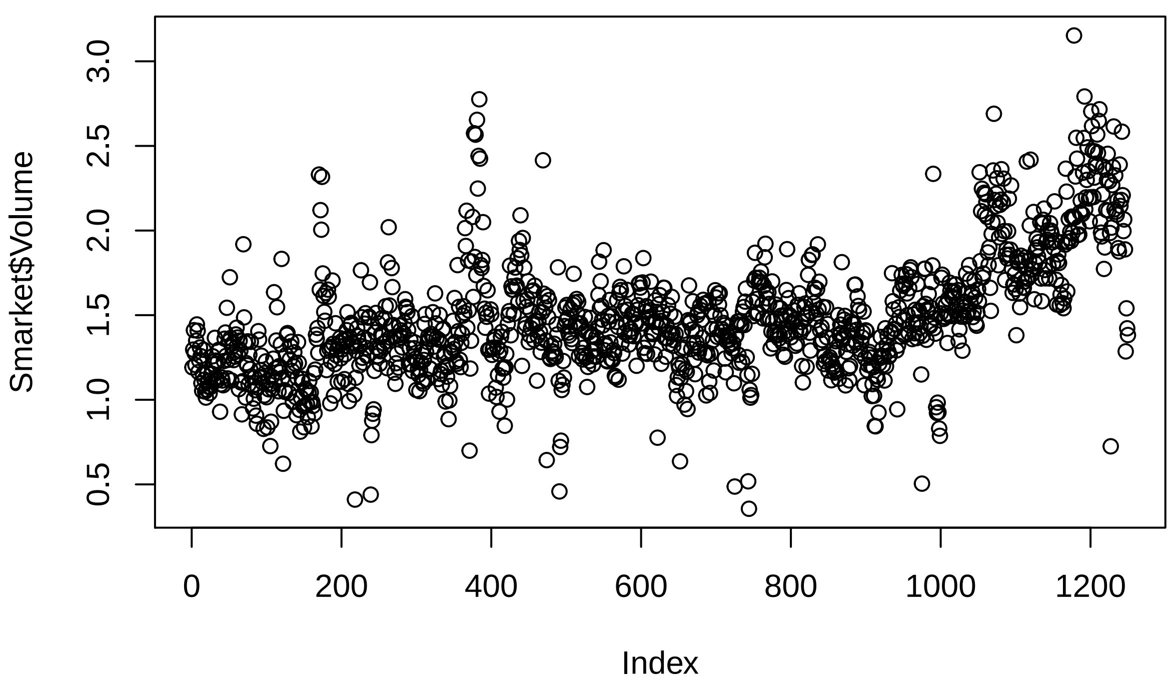
The `cor()` function produces a matrix that contains all of the pairwise correlations among the predictors in a dataset.We need to remove the `Direction` variable because it is qualitative.

```r
cor(Smarket[,-9])
```

```
##              Year         Lag1         Lag2         Lag3         Lag4
## Year   1.00000000  0.029699649  0.030596422  0.033194581  0.035688718
## Lag1   0.02969965  1.000000000 -0.026294328 -0.010803402 -0.002985911
## Lag2   0.03059642 -0.026294328  1.000000000 -0.025896670 -0.010853533
## Lag3   0.03319458 -0.010803402 -0.025896670  1.000000000 -0.024051036
## Lag4   0.03568872 -0.002985911 -0.010853533 -0.024051036  1.000000000
## Lag5   0.02978799 -0.005674606 -0.003557949 -0.018808338 -0.027083641
## Volume 0.53900647  0.040909908 -0.043383215 -0.041823686 -0.048414246
## Today  0.03009523 -0.026155045 -0.010250033 -0.002447647 -0.006899527
##                Lag5      Volume        Today
## Year    0.029787995  0.53900647  0.030095229
## Lag1   -0.005674606  0.04090991 -0.026155045
## Lag2   -0.003557949 -0.04338321 -0.010250033
## Lag3   -0.018808338 -0.04182369 -0.002447647
## Lag4   -0.027083641 -0.04841425 -0.006899527
## Lag5    1.000000000 -0.02200231 -0.034860083
## Volume -0.022002315  1.00000000  0.014591823
## Today  -0.034860083  0.01459182  1.000000000
```

As one would expect, the correlations between the lag variables and today's returns are close to zero. In other words, there appears to be little correlation between today's returns and the previous days' returns. The only substantial correlation is between `Year` and `Volume`. By plotting the data, we see that `Volume` is increasing over time. In other words, the average number of shares traded daily increased from 2001 to 2005.

```r
plot(Smarket$Volume)
```



Logistic Regression

Next, we will fit a logistic regression model in order to predict `Direction` using `Lag1` through `Lag5` and `Volume`. The `glm()` gunction fits *generalized linear models*, a class of models that includes logistic regression. The syntax of the `glm()` function is similiar to that of `lm()`, expect that we must pass in the argument `family = binomial` in order to tell R to run a logistic regression rather than some other type of generalized linear model.

```
glm.fit = glm(Direction~Lag1 + Lag2 + Lag3 + Lag4 + Lag5 + Volume, data=Smarket, family=binomial)
summary(glm.fit)
```

```
##
## Call:
## glm(formula = Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 +
##     Volume, family = binomial, data = Smarket)
##
## Deviance Residuals:
##    Min      1Q  Median      3Q     Max
## -1.446  -1.203   1.065   1.145   1.326
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -0.126000   0.240736  -0.523    0.601
## Lag1        -0.073074   0.050167  -1.457    0.145
## Lag2        -0.042301   0.050086  -0.845    0.398
## Lag3         0.011085   0.049939   0.222    0.824
## Lag4         0.009359   0.049974   0.187    0.851
## Lag5         0.010313   0.049511   0.208    0.835
## Volume       0.135441   0.158360   0.855    0.392
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1731.2  on 1249  degrees of freedom
## Residual deviance: 1727.6  on 1243  degrees of freedom
## AIC: 1741.6
##
## Number of Fisher Scoring iterations: 3
```

The smallest p-value is associated with `Lag1`. The negative coefficient for this predictor suggests that if the market had a positive return yesterday, then it is less likely to go up today. However, at a value of 0.145, the p-value is still relatively large, and there is no clear evidence of a real association between `Lag1` and `Direction`.

We use the `coef()` function in order to access just the coefficients for this fitted model. We can use the `summary()` function to access particular aspects of the fitted model, such as p-values for the coefficients.

```
coef(glm.fit)
```

```
##  (Intercept)         Lag1         Lag2         Lag3         Lag4         Lag5
## -0.126000257 -0.073073746 -0.042301344  0.011085108  0.009358938  0.010313068
##       Volume
##  0.135440659
```

The `predict()` function can be used to predict the probability that the market will go up, given values of the predictors. The `type="response"` option tells R to output probabilities of the form $P(Y = 1|X)$, as opposed to other information such as the logit. If no data set is supplied to the predict()`function, then the probabilities are computed for the training data that was used to fit the logistic regression model. Here we have printed only the first 10 probabilities. We know that these values correspond to the probability of the market going up, rather than down, because the`contrasts()‘ function indicates that R has created a dummy variable with a 1 for "Up".

```
glm.probs = predict(glm.fit, type="response")
glm.probs[1:10]
```

```
##         1         2         3         4         5         6         7         8
## 0.5070841 0.4814679 0.4811388 0.5152224 0.5107812 0.5069565 0.4926509 0.5092292
##         9        10
## 0.5176135 0.4888378
```

```
contrasts(Smarket$Direction)
```

```
##      Up
## Down  0
## Up    1
```

In order to make a prediction as to whether the market will go up or down on a particular day, we must convert these predicted probabilities into class labels,"Up" or "Down". The following two commands create a vector of class predictions based on whether the predicted probability of a market increase is greater than or less than 0.5.

```
glm.pred = rep("Down", 1250)
glm.pred[glm.probs >0.5] = "Up"
```

The first command creates a vector with 1,250 "Down" elements. The second line transforms to "Up" all of the elements for which the predicted probability of a market increase exceeds 0.5. Given these predictions, the table() function can be used to produce a confusion matrix in order to determine how many observations were correctly or incorrectly classified.

```
table(glm.pred, Smarket$Direction)
```

```
##
## glm.pred Down  Up
##     Down  145 141
##     Up    457 507
```

```
mean(glm.pred==Smarket$Direction)
```

```
## [1] 0.5216
```

The diagonal elements of the confusion matrix indicate correct predictions, while the off diagonals represent incorrect predictions. Hence our model correctly predicted that the market would go up on 507 days and that would go down for 145 days, for a total of $507 + 145 = 652$ correct predictions. The `mean()` function can be used to compute the fraction of days for which the prediction was correct. In this case, logistic regression correctly predicted the movement of the market 52.2% of the time.

At first glance, it appears that the logistic regression is working a little better than random guessing. However, this result is misleading because we trained and tested the model on the same set of 1,250 observations. In other words, 100 - 52.2 = 47.8% is the *training* error rate. As we have seen previously, the training error rate is often overly optimistic - it tends to underestimate the test error rate. In order to better assess the accuracy of our logistic regression model in this setting, we can fit the model using part of the data, and then examine how well it predicts the *held out* data. This will yield a more realistic error rate, in the sense that in practice we will be interested in our model's performance not on the data that was used to fit the model, but rather on days in the future for which the market's movements are unknown.

To implement this strategy, we will first create a vector corresponding to the observations from 2001 through 2004. We will then use this vector to create a held out dataset of observations from 2005.

```
train = (Smarket$Year<2005)
Smarket.2005 = Smarket[!train,]
dim(Smarket.2005)
```

```
## [1] 252   9
```

```
Direction.2005 = Smarket$Direction[!train]
```

The object `train` is a vector of 1,250 elements, corresponding to the observations in our dataset. The elements of the vector that correspond to observations that occured before 2005 are set to "TRUE" whereas those that correspond to observations in 2005 are set to "FALSE". The object `train` is a *Boolean* vector, since its elements are "TRUE" and "FALSE". Boolean vectors can be used to obtain a subset of the rows or columns of a matrix. For instance, the command `Smarket[train,]`, would pick out a submatrix of the stock market dataset, corresponding only to dates before 2005, since those are the ones for which the elements of `train` are "TRUE". The `!` symbol can be used to reverse all the elements of a Boolean vector. That is, `!train` is a vector similar to `train`, expect that the elements that are "TRUE" in `train` get swapped to "FALSE" in `!train`. Therefore, `Smarket[!train]` yields a submatrix of the stock market data containing only the observations for which `train` is "FALSE" - that is, the observations with dates in 2005. The output above indicates that there are 252 such observations.

We now fit a logistic regression model using only the subset of the observations that correspond to dates before 2005, using the `subset` argument. We then obtain predicted probabilities of the stock market going up for each of the days in our test set - that is, for the days in 2005.

```
glm.fits = glm(Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 + Volume, data=Smarket, family=binomial, su
glm.probs = predict(glm.fits, Smarket.2005, type="response")
```

Notice that we have trained and tested our model on two completely seperate datasets: training was performed using only the dates before 2005, and testing was performed using only the dates in 2005. Finally, we compute the predictions for 2005 and compare them to the actual movements of the market over that time period.

```
glm.pred = rep("Down", 252)
glm.pred[glm.probs>0.5]="Up"
table(glm.pred, Direction.2005)
```

```
##         Direction.2005
## glm.pred Down Up
##     Down   77 97
##     Up     34 44
```

```
mean(glm.pred==Direction.2005)
```

```
## [1] 0.4801587
```

```
mean(glm.pred!=Direction.2005)
```

```
## [1] 0.5198413
```

The `!=` notation means *not equal to*, ans so the last command computes the test set error rate. The results are rather disappointing: the test error rate is 52%, which is worse than random guessing! Of course this result is not at all surprising, given that one would not generally expect to be able to use the previous days' returns to predict future market performance.

We recall that the logistic regression model had very underwhelming p-values associated with all the predictors, and that the smallest p-value, though not very small, corresponded to `Lag1`. Perhaps by removing the variables that appear not be helpful in predicting `Direction`, we can obtain a more effective model. After all, using the predictors that have no relationship with the response tends to cause a deterioration in the test error rate (since such predictors cause an increase in variance without a corresponding decrease in bias), and so removing such predictors may in turn yield an improvement. Below we have refit the logistic regression using just `Lag1` and `Lag2`, which seemed to have the highest predictive power in the original logistic regression model.

```
glm.fits = glm(Direction ~ Lag1 + Lag2, data=Smarket, family=binomial, subset = train)
glm.probs = predict(glm.fits, Smarket.2005, type="response")
```

```r
glm.pred=rep("Down", 252)
glm.pred[glm.probs>0.5]="Up"
table(glm.pred, Direction.2005)
```

```
##         Direction.2005
## glm.pred Down  Up
##     Down   35  35
##     Up     76 106
```

```r
mean(glm.pred==Direction.2005)
```

```
## [1] 0.5595238
```

Now the results appear to be a little better: 56% of the daily movements have been correctly predicted. It is worth noting that in this case, a much simpler strategy of predicting that the market will increase every day will also be correct 56% of the time! Hence, in terms of overall error rate, the logistic regression method is no better than the naive approach. However, the confusion matrix shows that on days when the logistic regression model predicts an increase in the market, it has a 58% accuracy rate. This suggests a possible trading strategy of buying on days when the model predicts an increase market, and avoiding trades on days when a decrease is predicted. Of course one would need to investigate more carefully whether this small improvement was real or just due to random chance.

Suppose that we want to predict the returns associated with particular values of `Lag1` and `Lag2`. In particular, we want to predict `Direction` on a day when `Lag1` and `Lag2` equal 1.2 and 1.1, respectively and on a day when they equal 1.5 and -0.8. We do this using the `predict()` function.

```r
predict(glm.fits, newdata=data.frame(Lag1=c(1.2,1.5), Lag2=c(1.1, -0.8)), type="response")
```

```
##         1         2
## 0.4791462 0.4960939
```

Linear Discriminant Analysis

Now we will perform LDA on the `Smarket` data. In R, we fit an LDA model using the `lda()` function, which is part of the `MASS` library. Notice that the syntax for `lda()` function is identical to that of `lm()`, and to that of `glm()` expect for the absence of the `family` option. We fit the model using only the observations before 2005.

```r
library(MASS)
lda.fit = lda(Direction ~Lag1 + Lag2, data=Smarket, subset=train)
lda.fit
```

```
## Call:
## lda(Direction ~ Lag1 + Lag2, data = Smarket, subset = train)
##
## Prior probabilities of groups:
##     Down       Up
## 0.491984 0.508016
##
## Group means:
##             Lag1        Lag2
## Down  0.04279022  0.03389409
## Up   -0.03954635 -0.03132544
##
## Coefficients of linear discriminants:
##             LD1
```

```
## Lag1 -0.6420190
## Lag2 -0.5135293
```

The LDA output indicates that $\hat{\pi}_1 = 0.492$ and $\hat{\pi}_2 = 0.508$; in other words, 49.2% of the training observations correspond to days in which the market went down. It also provides the group means; there are the average of each predictor within each class, and are used by LDA as estimates of μ_k. These suggest that there is a tendency for the previous 2 days' returns to be negative on days when the market increases, and a tendency for the previous days' returns to be positive when the market declines. The *coefficients of linear discriminants* output provides the linear combination of `Lag1` and `Lag2` that are used to form the LDA decision rule. In other words, there are the multipliers of the elements of $X = x$ in (4.19). If $-0.642 * `Lag1` - 0.514 * `Lag2`$ is large, then the LDA classifier will predict a market increase, and if it is small then the LDA classifier will predict a market decline. The `plot()` function produces plots of the *linear discriminants*, obtained by computing $-0.642 * `Lag1` - 0.514 * `Lag2`$ for each of the training observations.

The `predict()` function returns a list with three elements. The first element, `class`, contains the LDA's predictions about the movement of the market. The second element, `posterior`, is a matrix whose kth column contains the posterior probability that the corresponding observation belongs to the kth class. Finally, `x` contains the linear discriminants, described earlier.

```
lda.pred = predict(lda.fit, Smarket.2005)
names(lda.pred)
```

```
## [1] "class"     "posterior" "x"
```

As we observed, the LDA and logistic regression predictions are almost identical.

```
lda.class = lda.pred$class
table(lda.class, Direction.2005)
```

```
##          Direction.2005
## lda.class Down  Up
##      Down   35  35
##      Up     76 106
```

```
mean(lda.class==Direction.2005)
```

```
## [1] 0.5595238
```

Applying a 50% threshold to the posterior probabilities allow us to recreate the predictions contained in `lda.pred$class`.

```
sum(lda.pred$posterior[,1]>=.5)
```

```
## [1] 70
```

```
sum(lda.pred$posterior[,1]<=.5)
```

```
## [1] 182
```

Notice that the posterior probability output by the model corresponds to the probability that the market will *decrease*:

```
lda.pred$posterior[1:20,1]
```

```
##       999      1000      1001      1002      1003      1004      1005      1006
## 0.4901792 0.4792185 0.4668185 0.4740011 0.4927877 0.4938562 0.4951016 0.4872861
##      1007      1008      1009      1010      1011      1012      1013      1014
## 0.4907013 0.4844026 0.4906963 0.5119988 0.4895152 0.4706761 0.4744593 0.4799583
##      1015      1016      1017      1018
## 0.4935775 0.5030894 0.4978806 0.4886331
```

```r
lda.class[1:20]
```

```
##  [1] Up   Up   Up   Up   Up   Up   Up   Up   Up   Up   Up   Down Up   Up   Up  
## [16] Up   Up   Down Up   Up  
## Levels: Down Up
```

If we wanted to use a posterior probability threshold other than 50% in order to make predictions, then we could easily do so. For instance, suppose that we wish to predict a market decrease only if we are very certain that the market will indeed decrease on that day - say, if the posterior probability is at least 90%.

```r
sum(lda.pred$posterior[,1]>.9)
```

```
## [1] 0
```

No days in 2005 meet that threshold! In fact, the greatest posterior probability of decrease in all of 2005 was 52.02%.

Quadratic Discriminant Analysis

We will now fit a QDA model to the Smarket data. QDA is implemented in R using the `qda()` function, which is also part of the `MASS` library. The syntax is identical to that of `lda()`.

```r
qda.fit = qda(Direction ~ Lag1 + Lag2, data=Smarket, subset=train)
qda.fit
```

```
## Call:
## qda(Direction ~ Lag1 + Lag2, data = Smarket, subset = train)
## 
## Prior probabilities of groups:
##     Down       Up 
## 0.491984 0.508016 
## 
## Group means:
##             Lag1        Lag2
## Down  0.04279022  0.03389409
## Up   -0.03954635 -0.03132544
```

The output contains the group means. But it does not contain the coefficients of the linear discriminants, because the QDA classifier involves a quadratic, rather than a linear, function of the predictors. The `predict()` function works in exactly the same fashion as for LDA.

```r
qda.class = predict(qda.fit, Smarket.2005)$class
table(qda.class, Direction.2005)
```

```
##          Direction.2005
## qda.class Down  Up
##      Down   30  20
##      Up     81 121
```

```r
mean(qda.class==Direction.2005)
```

```
## [1] 0.5992063
```

Interestingly, the QDA predictions are accurate almost 60% of the time even though the 2005 data was not used to fit the model. The level of accuracy is quite impressive for stock market data, which is known to be quite hard to model accuratly. This suggests that the quadratic form assumed by QDA may capture the true relationship more accurately than the linear forms assumed by LDA and logistic regression. However, we recommend evaluating the method's performance on a larger test set before betting that this approach will consistently beat the market.

K-Nearest Neighbours

We will now perform KNN using the `knn()` function, which is part of the `class` library. This function works rather differently from the other model-fitting functions that we have encountered thus far. Rather than a two-step approach in which we first fit the model and then we use the model to make predictions, `knn()` forms predictions using a single command. The function requires four inputs.

1) A matrix containing the predictors associated with the training data, labeled `train.X` below.
2) A matrix containing the predictors associated with the data for which we wish to make predictions, labeled `test.X` below.
3) A vector containing the class labels for the training observations, labeled `train.Direction` below.
4) A value for K, the number of nearest neighbours to be used by the classifier.

We use the `cbind()` function, short for *column bind*, to bind the `Lag1` and `Lag2` variables together into two matrices, one for the training set and the other for the test set.

```
library(class)
attach(Smarket)
train.X = cbind(Lag1, Lag2)[train,]
test.X = cbind(Lag1, Lag2)[!train,]
train.Direction=Direction[train]
```

Now the `knn()` function can be used to predict the market's movement for the dates in 2005. We set a random seed before we apply `knn()` because if several observations are tied as nearest neighbours, then R will randomly break the tie. Therefore, a seed must be set in order to ensure reproducibility of results.

```
set.seed(1)
knn.pred = knn(train.X, test.X, train.Direction, k=1)
table(knn.pred, Direction.2005)
```

```
##          Direction.2005
## knn.pred Down Up
##     Down   43 58
##     Up     68 83
```

```
(83+43)/252
```

```
## [1] 0.5
```

The results using $K = 1$ are not very good, since only 50% of the observations were correctly predicted. Of course, it may be that $K = 1$ results in an overly flexible fit to the data. Below, we repeat the analysis using $K = 3$.

```
knn.pred = knn(train.X, test.X, train.Direction, k=3)
table(knn.pred, Direction.2005)
```

```
##          Direction.2005
## knn.pred Down Up
##     Down   48 54
##     Up     63 87
```

```
mean(knn.pred==Direction.2005)
```

```
## [1] 0.5357143
```

The results have improved slightly. By increasing K further turns out to provide no further improvements. It appears that for this data, QDA provides the best results of the methods we have examined so far.

An Application to Caravan Insurance Data

Finally, we will apply the KNN approach to the `Caravan` dataset, which is part of the `ISLR` library. This dataset includes 85 predictors that measure the demographic characteristics of 5,822 individuals. The response variable is `Purchase`, which indicates whether or not a given individual purchases a caravan insurance policy. In this dataset, only 6% of people purchased caravan insurance.

```
dim(Caravan)
```

```
## [1] 5822   86
```

```
attach(Caravan)
summary(Purchase)
```

```
##   No  Yes
## 5474  348
```

```
348/5822
```

```
## [1] 0.05977327
```

Because the KNN classifier predicts the class of a given test observation by identifying the observations that are nearest to it, the scale of the variables matters. Any variables that are on a large scale will have a much larger effect on the *distance* between the observations, and hence on the KNN classifier, then variables on the small scale. For instance, imagine a dataset that contains two variables, `salary` and `age` (measured in dollars and years respectively). As far as KNN is concerned, a difference of $1,000 in salary is enormous compared to a difference of 50 years in age. Consequently, `salary` will drive the KNN classification results, and `age` will have almost no effect. This is contrary to our intuition that a salary difference of 1,000 is quite small compared to an age difference of 50 years. Furthermore, the importance of scale to the KNN classifier leads to another issue: if we measured `salary` in Japanese yen, or if we measured `age` in minutes, then we would get quite different classification results from what we get if these two variables are measured in dollars and years.

A good way to handle this problem is to *standardize* the data so that all variables are given a mean of zero and a standard deviation of one. Then, all variables will be comparable on the scale. The `scale()` function does just this. In standardizing the data, we exclude column 86 because that is the qualitative `Purchase` variable.

```
standardized.X = scale(Caravan[,-86])
var(Caravan[,1])
```

```
## [1] 165.0378
```

```
var(Caravan[,2])
```

```
## [1] 0.1647078
```

```
var(standardized.X[,1])
```

```
## [1] 1
```

```
var(standardized.X[,2])
```

```
## [1] 1
```

Now every column of `standardized.X` has a standard deviation of one and a mean of zero.

We now split the observations into a test set, containing the first 1,000 observations and a training set, containing the remaining observations. We fit a KNN model on the training data using $K = 1$, and evaluate its performance on the test data.

```
test = 1:1000
train.X = standardized.X[-test,]
test.X = standardized.X[test,]
train.Y = Purchase[-test]
test.Y = Purchase[test]


set.seed(1)

knn.pred = knn(train.X, test.X, train.Y, k=1)
mean(test.Y!=knn.pred)
```

```
## [1] 0.118
```

```
mean(test.Y!="No")
```

```
## [1] 0.059
```

The vector `test` is numeric, with values from 1 through 1,000. Typing `standardized.X[test,]` yields the submatrix of the data containing the observations whose indices range grom 1 to 1,000, whereas typing `standardized.x[-test,]` yields the submatrix containing the observations whose indices do *not* range from 1 to 1,000. The KNN error rate on the 1,000 test observations is just under 12%. At first glance, this may appear to be fairly good. However, only 6% of customers purchased insurance, we could get the error rate down to 6% by always predicting `No` regardless of the values of the predictors!

Suppose there is some non-trivial cost to trying to sell insurance to a given individual. For instance, perhaps a salesperson must visit each potential customer. If the company tries to sell insurance to a random selection of customers, then the success rate will be only 6% which may be far too low given the costs involved. Instead, the company would like to try to sell insurance only to customers who are likely to buy it. So that the overall error rate is not of interest. Instead, the fraction of individuals that are correctly predicted to buy insurance is of interest.

It turns out that KNN with $K = 1$ does far better than random guessing among the customers that are predicted to buy insurance. Among 77 such customers, 9 or 11.7%, actually do purchase insurance. This is double the rate that one would obtain from random guessing.

```
table(knn.pred, test.Y)
```

```
##         test.Y
## knn.pred  No Yes
##      No  873  50
##      Yes  68   9
```

```
9/(68+9)
```

```
## [1] 0.1168831
```

Using $K = 3$, the success rate increases to 19%, and with $K = 5$ the rate is 26.7%. This is over four times the rate that results from random guessing. It appears that KNN is finding some real patterns in a difficult dataset!

```
knn.pred=knn(train.X, test.X, train.Y, k=3)
table(knn.pred, test.Y)
```

```
##         test.Y
## knn.pred  No Yes
##      No  920  54
##      Yes  21   5
```

```r
5/26
```

```
## [1] 0.1923077
```

```r
knn.pred=knn(train.X, test.X, train.Y, k=5)
table(knn.pred, test.Y)
```

```
##         test.Y
## knn.pred  No Yes
##      No  930  55
##      Yes  11   4
```

```r
4/15
```

```
## [1] 0.2666667
```

As a comparison, we can also fit the logistic regression model to the data. If we use 0.5 as the predicted probability cut-off for the classifier, then we have a problem: only seven of the test observations are predicted to purchase insurance. Even worse, we are wrong about all of these! However, we are not required to use a cut-off of 0.5. If we instead predict a purchase any time the predicted probability of purchase exceeds 0.25, we get much better results: we predict that 33 people will purchase insurance, and we are correct about 33% of these people. This is over five times better than random guessing.

```r
glm.fits = glm(Purchase~., data=Caravan, family=binomial, subset=-test)
```

```
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

```r
glm.probs=predict(glm.fits, Caravan[test,], type="response")
glm.pred=rep("No", 1000)
glm.pred[glm.probs>0.5]="Yes"
table(glm.pred, test.Y)
```

```
##         test.Y
## glm.pred  No Yes
##      No  934  59
##      Yes   7   0
```

```r
glm.pred=rep("No",1000)
glm.pred[glm.probs>=0.25]="Yes"
table(glm.pred, test.Y)
```

```
##         test.Y
## glm.pred  No Yes
##      No  919  48
##      Yes  22  11
```

```r
11/(22+11)
```

```
## [1] 0.3333333
```

Applied Excercises

Classification on the Weekly Dataset

1. Produce some numerical and graphical summaries of the `Weekly` data. Do there appear to be any patterns?

```r
library(ISLR)
library(MASS)
```

```r
library(class)

Direction = Weekly$Direction
Weekly$Direction = NULL
Weekly$NumericDirection = as.numeric(Direction) # Maps Down => 1 and Up>= 2
Weekly$NumericDirection[Weekly$NumericDirection==1] = -1 # Maps Down >= -1 and Up >= 2
Weekly$NumericDirection[Weekly$NumericDirection==1] = +1 # Maps Down >= -1 and Up >=+1
```

```r
# Look at the correlation between the output and the input lags
Weekly.cor = cor(Weekly)
Weekly.cor
```

```
##                         Year         Lag1        Lag2        Lag3         Lag4
## Year              1.00000000 -0.032289274 -0.03339001 -0.03000649 -0.031127923
## Lag1             -0.03228927  1.000000000 -0.07485305  0.05863568 -0.071273876
## Lag2             -0.03339001 -0.074853051  1.00000000 -0.07572091  0.058381535
## Lag3             -0.03000649  0.058635682 -0.07572091  1.00000000 -0.075395865
## Lag4             -0.03112792 -0.071273876  0.05838153 -0.07539587  1.000000000
## Lag5             -0.03051910 -0.008183096 -0.07249948  0.06065717 -0.075675027
## Volume            0.84194162 -0.064951313 -0.08551314 -0.06928771 -0.061074617
## Today            -0.03245989 -0.075031842  0.05916672 -0.07124364 -0.007825873
## NumericDirection -0.02220025 -0.050003804  0.07269634 -0.02291281 -0.020549456
##                          Lag5      Volume        Today NumericDirection
## Year             -0.030519101  0.84194162 -0.032459894      -0.02220025
## Lag1             -0.008183096 -0.06495131 -0.075031842      -0.05000380
## Lag2             -0.072499482 -0.08551314  0.059166717       0.07269634
## Lag3              0.060657175 -0.06928771 -0.071243639      -0.02291281
## Lag4             -0.075675027 -0.06107462 -0.007825873      -0.02054946
## Lag5              1.000000000 -0.05851741  0.011012698      -0.01816827
## Volume           -0.058517414  1.00000000 -0.033077783      -0.01799521
## Today             0.011012698 -0.03307778  1.000000000       0.72002470
## NumericDirection -0.018168272 -0.01799521  0.720024704       1.00000000
```

2. Use the full dataset to perform a logistic regression with `Direction` as the response and the five lag variables plus `Volume` as predictors. Use the summary function to print the results. Do any of the predictors appear to be statistically significant. If so, which ones?

```r
Weekly$NumericDirection = NULL
Weekly$Direction = Direction

five_lag_model = glm(Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 + Volume, data=Weekly, family=binomial
summary(five_lag_model)
```

```
##
## Call:
## glm(formula = Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 +
##     Volume, family = binomial, data = Weekly)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.6949  -1.2565   0.9913   1.0849   1.4579
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.26686    0.08593   3.106   0.0019 **
```

```
## Lag1        -0.04127    0.02641  -1.563   0.1181
## Lag2         0.05844    0.02686   2.175   0.0296 *
## Lag3        -0.01606    0.02666  -0.602   0.5469
## Lag4        -0.02779    0.02646  -1.050   0.2937
## Lag5        -0.01447    0.02638  -0.549   0.5833
## Volume      -0.02274    0.03690  -0.616   0.5377
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1496.2  on 1088  degrees of freedom
## Residual deviance: 1486.4  on 1082  degrees of freedom
## AIC: 1500.4
##
## Number of Fisher Scoring iterations: 4
```

```
print(contrasts(Weekly$Direction))
```

```
##      Up
## Down  0
## Up    1
```

3. Compute the confusion matrix and overall fraction of correct predictions. Explain what the confusion matrix is telling you about the types of mistakes made by logistic regression.

```
p_hat = predict(five_lag_model, newdata=Weekly, type="response")
y_hat = rep("Down", length(p_hat))
y_hat[p_hat > 0.5] = "Up"
CM = table(predicted=y_hat, truth=Weekly$Direction)
print(CM)
```

```
##          truth
## predicted Down  Up
##      Down   54  48
##      Up    430 557
```

```
print(sprintf("LR (all features): overall fraction correxct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "LR (all features): overall fraction correxct =   0.561065"
```

4. Now fit the logistic regression model using a training data period from 1990 to 2008, with `Lag2` as the only predictor. Compute the confusion matrix and the overall fraction of correct predictions for the held out data (that is, the data from 2009 and 2010).

```
# Logistic regression with only Lag2 as the predictor (since it is the most significant predictor)

Weekly.train = (Weekly$Year >= 1990) & (Weekly$Year <= 2008) # Our training set
Weekly.test = (Weekly$Year >= 2009) # Our testing set

lag2_model = glm(Direction ~ Lag2, data=Weekly, family=binomial, subset=Weekly.train)
```

```
# CM on test data
p_hat = predict(lag2_model, newdata=Weekly[Weekly.test,], type="response")
y_hat = rep("Down", length(p_hat))
y_hat[p_hat > 0.5] = "Up"
CM = table(predicted=y_hat, truth=Weekly[Weekly.test,]$Direction)
print(CM)
```

```
##          truth
## predicted Down Up
##      Down    9  5
##      Up     34 56
```

```
print(sprintf("LR (only Lag2): overall fraction correct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "LR (only Lag2): overall fraction correct =   0.625000"
```

5. Use LDA

```
lda.fit = lda(Direction ~ Lag2, data=Weekly, subset=Weekly.train)

lda.predict=predict(lda.fit, newdata=Weekly[Weekly.test,])
CM = table(predicted=lda.predict$class, truth=Weekly[Weekly.test,]$Direction)
print(CM)
```

```
##          truth
## predicted Down Up
##      Down    9  5
##      Up     34 56
```

```
print(sprintf("LDA (only Lag2): overall fraction correct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "LDA (only Lag2): overall fraction correct =   0.625000"
```

6. Use QDA

```
qda.fit = qda(Direction ~ Lag2, data=Weekly, subset=Weekly.train)

qda.predict = predict(qda.fit, newdata=Weekly[Weekly.test,])
CM = table(predicted=qda.predict$class, truth=Weekly[Weekly.test,]$Direction)
print(CM)
```

```
##          truth
## predicted Down Up
##      Down    0  0
##      Up     43 61
```

```
print(sprintf("QDA (only Lag2): overall fraction correct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "QDA (only Lag2): overall fraction correct =   0.586538"
```

7. Use KNN with $K = 1$

```
X.train = data.frame(Lag2=Weekly[Weekly.train,]$"Lag2")
Y.train = Weekly[Weekly.train,]$"Direction"

X.test = data.frame(Lag2=Weekly[Weekly.test,]$"Lag2")
```

```
y_hat_k1 = knn(X.train, X.test, Y.train, k=1, use.all=FALSE)
CM = table(predicted=y_hat_k1, truth=Weekly[Weekly.test,]$Direction)
print(CM)
```

```
##          truth
## predicted Down Up
##      Down   22 31
##      Up     21 30
```

```r
print(sprintf("KNN (k=1) (only Lag2): overall fraction correct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "KNN (k=1) (only Lag2): overall fraction correct =   0.500000"
```

```r
y_hat_k3 = knn(X.train, X.test, Y.train, k=3, use.all=FALSE)
CM = table(predicted=y_hat_k3, truth=Weekly[Weekly.test,]$Direction)
print(CM)
```

```
##          truth
## predicted Down Up
##      Down   16 19
##      Up     27 42
```

```r
print(sprintf("KNN (k=3) (only Lag2): overall fraction correct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "KNN (k=3) (only Lag2): overall fraction correct =   0.557692"
```

Classification of the Auto Dataset

1. Create a binary variable, `mpg01`, that contains a 1 if `mpg` contains a value above the median, and a 0 if `mpg` contains a value below its median. You can compute the median using the `median()` function.

```r
save_plots = F
set.seed(0)

Auto = na.omit(Auto)
Auto$name = NULL
```

```r
mpg01 = rep(0, dim(Auto)[1]) # 0 >= less than the median of mpg
mpg01[Auto$mpg > median(Auto$mpg)] = 1 # 1 >= greater than the median of mpg
```

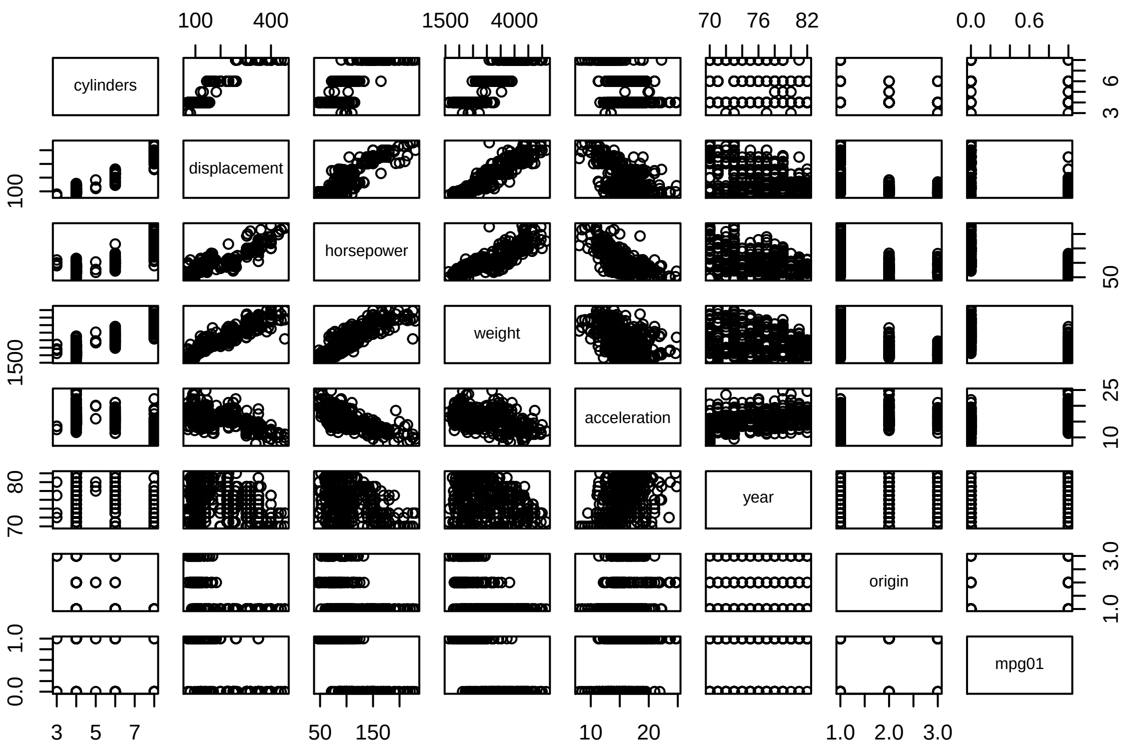
```r
Auto$mpg01 = mpg01
Auto$mpg = NULL
```

2. Explore the data graphically in order to investigate the association between `mpg01` and the other features. Which of the other features seem most likely to be useful in predicting `mpg01`?

```r
print(cor(Auto))
```

```
##               cylinders displacement horsepower     weight acceleration
## cylinders     1.0000000    0.9508233  0.8429834  0.8975273   -0.5046834
## displacement  0.9508233    1.0000000  0.8972570  0.9329944   -0.5438005
## horsepower    0.8429834    0.8972570  1.0000000  0.8645377   -0.6891955
## weight        0.8975273    0.9329944  0.8645377  1.0000000   -0.4168392
## acceleration -0.5046834   -0.5438005 -0.6891955 -0.4168392    1.0000000
## year         -0.3456474   -0.3698552 -0.4163615 -0.3091199    0.2903161
## origin       -0.5689316   -0.6145351 -0.4551715 -0.5850054    0.2127458
## mpg01        -0.7591939   -0.7534766 -0.6670526 -0.7577566    0.3468215
##                    year     origin      mpg01
## cylinders    -0.3456474 -0.5689316 -0.7591939
## displacement -0.3698552 -0.6145351 -0.7534766
## horsepower   -0.4163615 -0.4551715 -0.6670526
## weight       -0.3091199 -0.5850054 -0.7577566
## acceleration  0.2903161  0.2127458  0.3468215
## year          1.0000000  0.1815277  0.4299042
## origin        0.1815277  1.0000000  0.5136984
## mpg01         0.4299042  0.5136984  1.0000000
```

```
pairs(Auto)
```




```
Auto$mpg01 = as.factor(mpg01)
```

3. Split the data into a training set and test set

```
n = dim(Auto)[1]
inds.train = sample(1:n,3*n/4)
Auto.train = Auto[inds.train,]
inds.test = (1:n)[-inds.train]
Auto.test = Auto[inds.test,]
```

4. Perform LDA on the training data in order to predict `mpg01` using the variables that seemed the most associated with `mpg01`. What is the test error of the model obtained?

```
lda.fit = lda(mpg01 ~ cylinders + displacement + weight, data=Auto.train)

lda.predict = predict(lda.fit, newdata=Auto.test)
CM = table(predicted = lda.predict$class, truth=Auto.test$mpg01)
print(CM)
```

```
##          truth
## predicted  0  1
##         0 43  2
##         1 10 43
```

```
print(sprintf("LDA: overall fraction correct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "LDA: overall fraction correct =   0.877551"
```

5. Perform QDA on the training data in order to predict `mpg01` using the variables that seemed most associated with `mpg01`. What is the test error of the model obtained?

```r
qda.fit = qda(mpg01 ~ cylinders + displacement + weight, data=Auto.train)

qda.predict = predict(qda.fit, newdata = Auto.test)
CM = table(predicted = qda.predict$class, truth=Auto.test$mpg01)
print(CM)
```

```
##          truth
## predicted  0  1
##         0 46  4
##         1  7 41
```

```r
print(sprintf("QDA: overall fraction correct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "QDA: overall fraction correct =   0.887755"
```

6. Perform logistic regression on the training data in order to predict `mpg01` using the variables that seemed the most associated with `mpg01`. What is the test error rate of the model obtained?

```r
lr.fit = glm(mpg01 ~ cylinders + displacement + weight, data=Auto.train, family=binomial)

p_hat = predict(lr.fit, newdata=Auto.test, type="response")
y_hat = rep(0, length(p_hat))
y_hat[p_hat > 0.5] = 1
CM = table(predicted = as.factor(y_hat), truth=Auto.test$mpg01)
print(CM)
```

```
##          truth
## predicted  0  1
##         0 43  3
##         1 10 42
```

```r
print(sprintf("LR: overall fraction correct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "LR: overall fraction correct =   0.867347"
```

Classification of the Boston dataset

Using the `Boston` dataset, fit classification models in order to predict whether a given suburb has a crime rate above or below the median. Explore logistic regression, LDA and KNN models using various subsets of the predictors. Describe your findings.

```r
n = dim(Boston)[1]
```

Introduce a fariable whether or not the crime raate is above=1 / below=0 the median.

```r
Boston$crim01 = rep(0,n)
Boston$crim01[Boston$crim >= median(Boston$crim)] =1
Boston$crim = NULL
```

Look to see what features are most strongly correlated with crim01:

```r
Boston.cor = cor(Boston)
print(sort(Boston.cor[,'crim01']))
```

```
##         dis          zn       black        medv          rm        chas
## -0.61634164 -0.43615103 -0.35121093 -0.26301673 -0.15637178  0.07009677
##     ptratio       lstat       indus         tax         age         rad
##  0.25356836  0.45326273  0.60326017  0.60874128  0.61393992  0.61978625
##         nox      crim01
```

```
##  0.72323480  1.00000000
```

Split the data into testing and training parts:

```
inds.train = sample(1:n,3*n/4)
inds.test = (1:n)[-inds.train]
Boston.train = Boston[inds.train,]
Boston.test = Boston[inds.test,]
```

Fit several models to the training data

```
lr_model = glm(crim01 ~ nox + rad + dis, data=Boston.train, family=binomial)

p_hat = predict(lr_model, newdata=Boston.test, type="response")
y_hat = rep(0, length(p_hat))
y_hat[p_hat > 0.5] = 1
CM = table(predicted=y_hat, truth=Boston.test$crim01)
print(CM)
```

```
##          truth
## predicted  0  1
##         0 61 12
##         1  5 49
```

```
print(sprintf("LR: overall fraction correct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "LR: overall fraction correct =   0.866142"
```

Use LDA:

```
lda.fit = lda(crim01 ~ nox + rad + dis, data=Boston.train)

lda.predict = predict(lda.fit, newdata = Boston.test)
CM = table(predicted=lda.predict$class, truth=Boston.test$crim01)
print(CM)
```

```
##          truth
## predicted  0  1
##         0 64 18
##         1  2 43
```

```
print(sprintf("LR: overall fraction correct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "LR: overall fraction correct =   0.842520"
```

Use QDA:

```
qda.fit = qda(crim01 ~ nox + rad + dis, data=Boston.train)

qda.predict = predict(qda.fit, newdata=Boston.test)
CM = table(predicted=qda.predict$class, truth=Boston.test$crim01)
print(CM)
```

```
##          truth
## predicted  0  1
##         0 65 13
##         1  1 48
```

```
print(sprintf("QDA: overall fraction correct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "QDA: overall fraction correct =   0.889764"
```

Use KNN:

```
X.train = Boston.train; X.train$"crim01" = NULL
Y.train = Boston.train$"crim01"

X.test = Boston.test; X.test$"crim01" = NULL

Y_hat_k1 = knn(X.train, X.test, Y.train, k=1)

CM = table(predicted=Y_hat_k1, truth=Boston.test$crim01)
print(CM)
```

```
##          truth
## predicted  0  1
##         0 62  2
##         1  4 59
```

```
print(sprintf("KNN (k=1): overall fraction correct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "KNN (k=1): overall fraction correct =   0.952756"
```

```
Y_hat_k3 = knn(X.train, X.test, Y.train, k=3)

CM = table(predicted=Y_hat_k3, truth=Boston.test$crim01)
print(CM)
```

```
##          truth
## predicted  0  1
##         0 61  6
##         1  5 55
```

```
print(sprintf("KNN (k=3): overall fraction correct = %10.6f", (CM[1,1] + CM[2,2])/sum(CM)))
```

```
## [1] "KNN (k=3): overall fraction correct =   0.913386"
```