"Proof That Properly Anticipated Prices Fluctuate Randomly" by Paul A. Samuelson
Summary
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Samuelson states that competitive prices follow a random walk with no predictable bias by displaying a change over time. The uncorrelated and quasi-random walk followed by actual prices is a way of "mother nature" to show that she's present in this process, which is a summation of several small and independent sources of variation. An important argument is that next period's price differences are uncorrelated (if not completely independent of) previous period's price differences.

Today's price works as a signal that carries all past information, becoming the most updated value about the past. When talking about martingales, Samuelson says that if we suppose that there were no martingales, then we could predict the direction of tomorrow's interest rates and respectively buy/sell bonds today.

There is a probability distribution for any future price, whose form depends solely on the number of periods ahead over which we are trying to forecast prices. The probability of an event happening is the sum of the probabilities of different mutually exclusive events and a non-normal distribution is assumed. An ergodic state will emerge as time goes to infinity, we can use the binomial distribution for each period and connect with the central-limit theorem to show that a normal distribution is approached for the log of the next period over the current period as we have an infinite number of periods. This will avoid negative prices.

The relevant spot price of a futures price is given by the spot price of the present time plus the number of prevailing periods. This can be denoted by $Y(T, t)$. When another period passes, we will have the information of that period and change our function to be defined by $Y(T-1, t+1)$. It will continue like this, so we can find a sequence to define it: $Y(t-n, t+n)$. When the due date of the futures contract arrives, arbitrage will ensure that the price quoted by the last period will equal the spot price of that moment. Ignoring interest and risk-aversion, could make sellers and buyers abide by the law of supply and demand and follow the probability distribution to the mean of tomorrow's price.

If one observes different sequences of future prices explained by the basic theorem model until its end, on average, there would be no upward or downward trend. There is no way of making an expected profit by looking at past changes in futures prices, the market quotation $Y(T,t)$ already contains in itself all that can be known about the future. It only implies that the Pearsonian correlation between the spot price of today knowing the terminal number of periods is not correlated with the spot price of tomorrow with the terminal number of periods minus one. As the expiration date approaches, so does variability and riskiness. Within the defined model, all chart methods attempting to read out the past sequence of known prices or any profitable pattern of prediction is doomed to failure. This is because the market has already discounted all knowable information.

Samuelson concludes that the change in Y doesn't necessarily have to be zero, it's even more likely to be negative than positive. The theorem doesn't prove that markets work well or that speculation or randomness of prices is a good thing,