Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

Laura Gáti

Boston College

Federal Reserve Bank of Boston

September 22, 2020

Inflation that runs below its desired level can lead to an unwelcome fall in ***longer-term inflation expectations****, which, in turn, can pull actual inflation even lower, resulting in an adverse cycle of ever-lower inflation and inflation expectations. [...]* ***Well-anchored inflation expectations*** *are critical[.]*
Jerome Powell, Chairman of the Federal Reserve [1]
(Emphases added.)

[1] "New Economic Challenges and the Fed's Monetary Policy Review," August 27, 2020.

Anchored expectations

$$\mathbb{E}_t\,\pi_{t+1} = \bar{\pi} + f(shocks) \tag{1}$$

$\bar{\pi}$ = long-run expectations

Anchored expectations

$$\mathbb{E}_t\, \pi_{t+1} = \bar{\pi} + f(shocks) \tag{1}$$

$\bar{\pi}$ = long-run expectations

Anchored expectations: long-run expectations at Fed's target (π^*)

Anchored expectations

$$\mathbb{E}_t\, \pi_{t+1} = \bar{\pi} + f(shocks) \tag{1}$$

$\bar{\pi}$ = long-run expectations

Anchored expectations: long-run expectations at Fed's target (π^*)

$\rightarrow$ short-run expectations "anchored" to stable mean:

$$\mathbb{E}_t\, \pi_{t+1} = \pi^* + f(shocks) \tag{2}$$

Anchored expectations

$$\mathbb{E}_t \, \pi_{t+1} = \bar{\pi} + f(shocks) \tag{1}$$

$\bar{\pi}$ = long-run expectations

Anchored expectations: long-run expectations at Fed's target (π^*)

$\rightarrow$ short-run expectations "anchored" to stable mean:

$$\mathbb{E}_t \, \pi_{t+1} = \pi^* + f(shocks) \tag{2}$$

Unanchored expectations: long-run expectations deviate systematically from the target

$$\mathbb{E}_t \, \pi_{t+1} = \bar{\pi}_{t-1}(shocks) + f(shocks) \tag{3}$$

Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \, \mathbb{E}_t \, \pi_{t+1} + u_t$$

Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta\,\mathbb{E}_t\,\pi_{t+1} + u_t$$
$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t$$

Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta\, \mathbb{E}_t\, \pi_{t+1} + u_t$$
$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t$$

The Fed's uneasiness: public thinks inflation averages only 1%

$$\pi_t = \kappa x_t + \beta(\underset{\downarrow}{1\%} + f(shocks)) + u_t$$

Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta\, \mathbb{E}_t\, \pi_{t+1} + u_t$$
$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t$$

The Fed's uneasiness: public thinks inflation averages only 1%

$$\underset{\downarrow}{\pi_t} = \kappa x_t + \beta(\underset{\downarrow}{1\%} + f(shocks)) + u_t$$

Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta\, \mathbb{E}_t\, \pi_{t+1} + u_t$$
$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t$$

The Fed's uneasiness: public thinks inflation averages only 1%

$$\underset{\downarrow}{\pi_t} = \kappa x_t + \beta(\underset{\downarrow\downarrow}{1\%} + f(shocks)) + u_t$$

Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta\, \mathbb{E}_t\, \pi_{t+1} + u_t$$
$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t$$

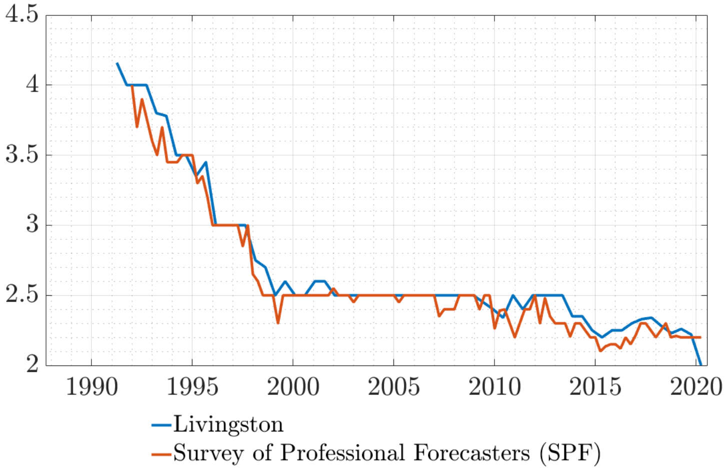
The Fed's uneasiness: public thinks inflation averages only 1%

$$\underset{\downarrow}{\pi_t} = \kappa x_t + \beta(\underset{\downarrow\downarrow}{1\%} + f(shocks)) + u_t$$

$\rightarrow$ unanchored expectations can cause a deflationary spiral

Long-run expectations drifting down?

Figure: Expectations of average inflation over 10 years ($\bar{\pi}$ in data)



TIPS

Long-run expectations moving systematically?

Individual SPF forecasts: for 1991-Q4 onward, estimate rolling regression

$$\Delta\bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{4}$$

where w indexes windows of 19 quarters length and $f_{t|t-1} = \pi_t - \mathbb{E}_{t-1}\,\pi_t$

Long-run expectations moving systematically?

Individual SPF forecasts: for 1991-Q4 onward, estimate rolling regression

$$\Delta\bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{4}$$

where w indexes windows of 19 quarters length and $f_{t|t-1} = \pi_t - \mathbb{E}_{t-1}\,\pi_t$

Table: Rolling regression coefficients

Window (start quarter)	$\hat{\beta}_1^w$
1991-Q4	0.0687^{**}
1996-Q3	0.0464^{***}
2001-Q2	0.0116
2006-Q1	0.0061
2010-Q4	0.0135
2015-Q3	0.0496^{***}

**** significance at 1% ** significance at 5%*

▸ Further evidence

This project

- How to conduct monetary policy in interaction with the anchoring expectation formation?

- Model of anchoring expectation formation as an extension to adaptive learning

  $\hookrightarrow$ twist: fluctuations in long-run expectations responsive to economic conditions

- Estimation of the anchoring function: when do expectations become unanchored?

Preview of results

- Optimal policy aggressive when expectations unanchor, accommodates when anchored

- Taylor rule policy less aggressive on inflation than under rational expectations

$\hookrightarrow$ Anchoring-optimal Taylor rule eliminates 84% of loss from volatility

Related literature

- **Optimal monetary policy in the rational expectations New Keynesian model**
  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Adaptive learning**
  Evans & Honkapohja (2001, 2006), Sargent (1999), Primiceri (2006), Lubik & Matthes (2018), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Mele et al (2019), Eusepi & Preston (2011), Milani (2007, 2014), Marcet & Nicolini (2003)

- **Anchoring and the Phillips curve**
  Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Reis (2020), Gobbi et al (2019), Carvalho et al (2019)

- **Alternatives to rational expectations**
  Coibion & Gorodnichenko (2015), Mankiw & Reis (2002), Sims (2003), Angeletos & Pavan (2009), Gabaix (2014), Bordalo et al (2018)

- **Reputation**
  Barro (1986), Cho & Matsui (1995)

Structure of talk

1. Model of anchoring expectations
2. Solving the Ramsey problem
3. Implementing optimal policy
4. Approximating optimal policy with a Taylor rule

Structure of talk

1. Model of anchoring expectations

2. Solving the Ramsey problem

3. Implementing optimal policy

4. Approximating optimal policy with a Taylor rule

Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j))dj \right] \tag{5}$$

Budget constraint

$$B_t^i \leq (1+i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_t C_t^i \tag{6}$$

▸ Consumption, price level

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \tag{7}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{8}$$

▸ Profits, stochastic discount factor

Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- If use $\mathbb{E}$ (rational expectations, RE)

  Model solution

$$s_t = h s_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \tag{9}$$

$$y_t = g s_t \tag{10}$$

$s_t \equiv$ states
$y_t \equiv$ jumps
$\epsilon_t \equiv$ disturbances

Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- If use $\mathbb{E}$ (rational expectations, RE)

  Model solution

$$s_t = h s_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \tag{9}$$

$$y_t = g s_t \tag{10}$$

  $s_t \equiv$ states
  $y_t \equiv$ jumps
  $\epsilon_t \equiv$ disturbances

- If use $\hat{\mathbb{E}} \rightarrow$ private sector does not know (10)

  $\hookrightarrow$ estimate using observed states & knowledge of (9)

Adaptive learning

- Postulate linear functional relationship instead of (10):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{11}$$

Adaptive learning

- Postulate linear functional relationship instead of (10):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{11}$$

$a \rightarrow$ concept of long-run expectations in the model

Adaptive learning

- Postulate linear functional relationship instead of (10):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{11}$$

$a \rightarrow$ concept of long-run expectations in the model

In RE, $a_{t-1} = \mathbf{0}, \quad b_{t-1} = g\, h \quad \forall t$

Adaptive learning

- Postulate linear functional relationship instead of (10):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{11}$$

$a \rightarrow$ concept of long-run expectations in the model

In RE, $a_{t-1} = \mathbf{0}, \quad b_{t-1} = g\, h \quad \forall t$

$\rightarrow$ In RE, expectations anchored by assumption

Adaptive learning

- Postulate linear functional relationship instead of (10):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{11}$$

$a \rightarrow$ concept of long-run expectations in the model

In RE, $a_{t-1} = \mathbf{0}, \quad b_{t-1} = g\,h \quad \forall t$

$\rightarrow$ In RE, expectations anchored by assumption

- Estimate a, b using recursive least squares (RLS)

Recursive least squares

Jumps are: $(\pi, x, i)'$

Recursive least squares

Jumps are: $(\pi, x, i)'$

Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\,h \quad \forall t \tag{12}$$

Recursive least squares

Jumps are: $(\pi, x, i)'$

Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\, h \quad \forall t \tag{12}$$

$\bar{\pi}_{t-1}$: long-run inflation expectations $\rightarrow$ anchoring

Recursive least squares

Jumps are: $(\pi, x, i)'$

Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\, h \quad \forall t \tag{12}$$

$\bar{\pi}_{t-1}$: long-run inflation expectations $\rightarrow$ anchoring

$\rightarrow$ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\right)}_{\equiv f_{t|t-1}\text{, forecast error}} \tag{13}$$

$k_t \in (0, 1)$ gain
b_1 first row of b

▸ General RLS algorithm

Decreasing versus constant gain

Decreasing gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f_{t|t-1} \tag{14}$$

$\rightarrow$ consider sample mean of full sample of forecast errors

Decreasing versus constant gain

Decreasing gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f_{t|t-1} \tag{14}$$

$\rightarrow$ consider sample mean of full sample of forecast errors

Constant gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k\, f_{t|t-1} \tag{15}$$

$\rightarrow$ consider sample mean of most recent observations only

Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big) \tag{16}$$

$k_t = \mathbf{g}(f_{t|t-1})$: anchoring function

Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}) \big) \tag{16}$$

$k_t = \mathbf{g}(f_{t|t-1})$: anchoring function

$$\mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}) \tag{17}$$

$b_i(f_{t|t-1})$ = basis, here: second order spline (piecewise linear)

γ_i = approximating coefficients

$\hookrightarrow$ estimate $\hat{\gamma}$ via simulated method of moments (SMM)

▸ Functional forms in literature

Calibration - parameters from the literature

β	0.98	stochastic discount factor
σ	1	intertemporal elasticity of substitution
α	0.5	Calvo probability of not adjusting prices
κ	0.0842	slope of the Phillips curve
λ_x	0.05	weight on the output gap in central bank loss
$\bar{g}$	0.145	initial value of the gain
ρ_r	0	persistence of natural rate shock
ρ_u	0	persistence of cost-push shock
ρ_i	0	persistence of monetary policy shock*
ψ_π	1.5	coefficient of inflation in Taylor rule*

* pertains to sections where Taylor rule is in effect

Calibration - matching moments

ψ_x	0.3	coefficient of the output gap in Taylor rule*
σ_r	0.01	standard deviation, natural rate shock
σ_u	0.5	standard deviation, cost-push shock
σ_i	0.01	standard deviation, monetary policy shock*
$\hat{\gamma}_i$	(0.82; 0.61; 0; 0.33; 0.45)	coefficients in anchoring function

- Calibrate $\sigma_j,\ j = r, i, u$ to match the autocovariances of inflation, output gap, interest rate and one-period ahead inflation expectations for lags $0, \ldots, 4$
- Estimate $\hat{\gamma}_i$ to match the same moments

Anchoring function in the data

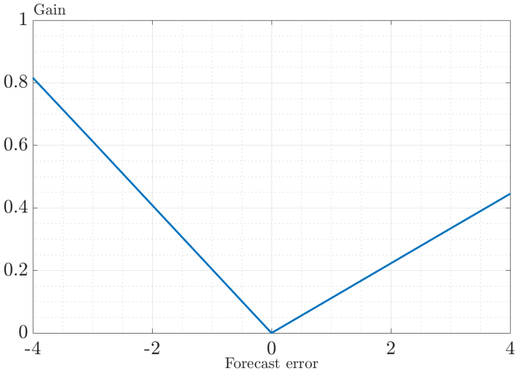


Figure: Learning gain as a function of forecast errors in inflation in %

Anchoring function in the data

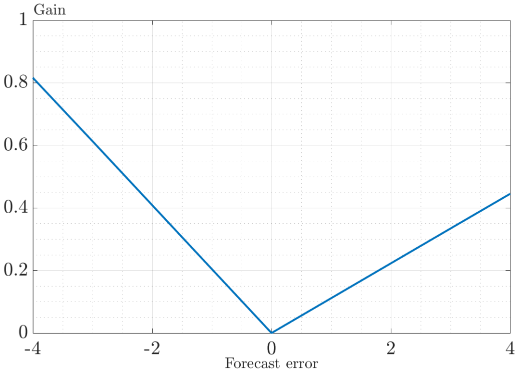
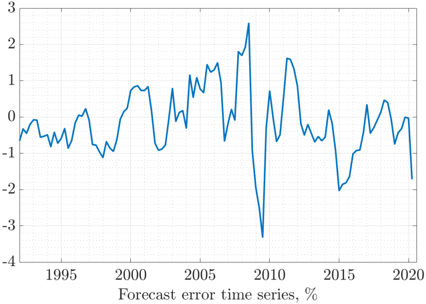


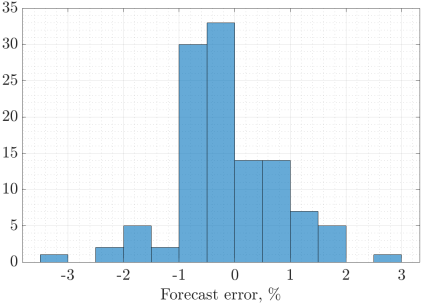
Figure: Learning gain as a function of forecast errors in inflation in %

- Estimates of the gain suggest ≈ 0.05
- Carvalho et al (2019)'s gain time series is max 0.145

Forecast errors in the data

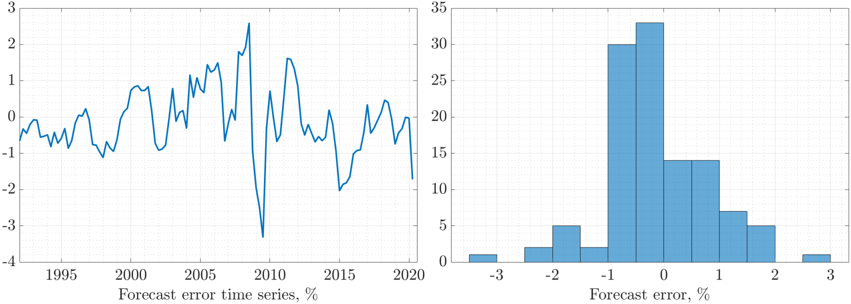
Figure: Time series and distribution of 1-year ahead forecast errors in the SPF





Forecast errors in the data

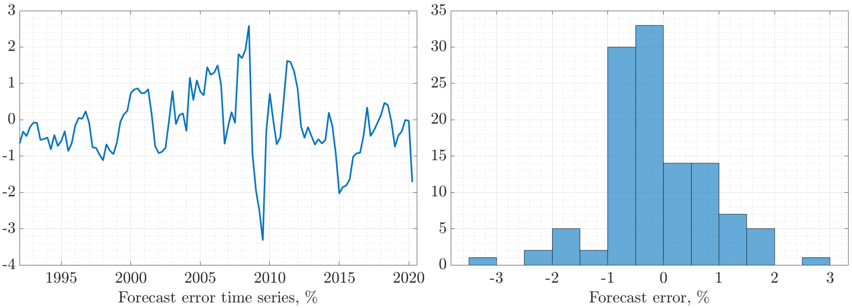
Figure: Time series and distribution of 1-year ahead forecast errors in the SPF



Typical forecast error ≈ -0.5 pp

Forecast errors in the data

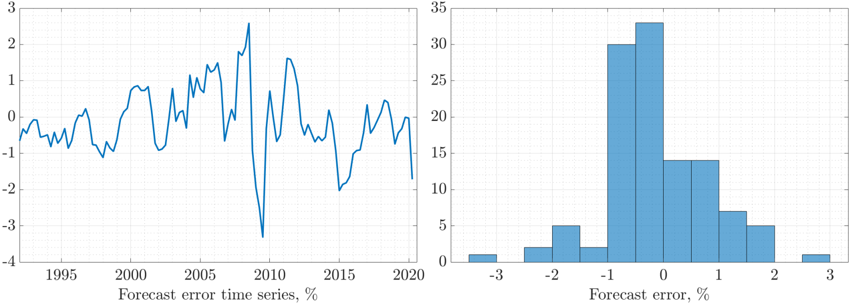
Figure: Time series and distribution of 1-year ahead forecast errors in the SPF



Typical forecast error ≈ -0.5 pp $\quad \rightarrow$ gain of 0.1

Forecast errors in the data

Figure: Time series and distribution of 1-year ahead forecast errors in the SPF



Typical forecast error ≈ -0.5 pp $\quad \rightarrow$ gain of 0.1

$\Rightarrow$ 5 bp shift down in long-run expectations

Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \Big) \tag{18}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \Big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \Big) \tag{19}$$

▸ Derivations ▸ Actual laws of motion

- Expectations evolve according to RLS with the endogenous gain given by (17)

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1} \tag{16}$$

$$k_t = \mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}) \tag{20}$$

→ How should $\{i_t\}$ be set?

Structure of talk

1. Model of anchoring expectations

2. Solving the Ramsey problem

3. Implementing optimal policy

4. Approximating optimal policy with a Taylor rule

Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

- $\mathbb{E}$ is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

Target criterion

Proposition

In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_{\pi,t} \right)$$

$$\left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi},\mathbf{t+j}}) \right)$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t, $\prod_{j=0}^{0} \equiv 1$ *and* b_1 *is the first row of* b.

General case

Two layers of intertemporal stabilization tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i}$$

$$- \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j} + f_{t+1+j|t+j} \mathbf{g}_{\bar{\pi},\mathbf{t+j}})$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

▸ Why no commitment?

Corollary

Optimal policy under adaptive learning is time-consistent.

$\hookrightarrow$ Foreshadow: optimal policy aggressiveness time-varying

Structure of talk

1. Model of anchoring expectations

2. Solving the Ramsey problem

3. Implementing optimal policy

4. Approximating optimal policy with a Taylor rule

Solution procedure

Solve system of model equations + target criterion

$\hookrightarrow$ solve using parameterized expectations (PEA)

$\hookrightarrow$ obtain a cubic spline approximation to optimal policy function

Optimal policy - responding to unanchoring

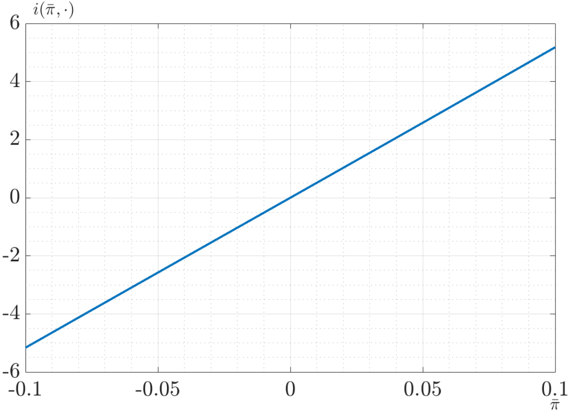


Figure: Policy function: $i(\bar{\pi},$ all other states at their means)

Optimal policy - responding to unanchoring

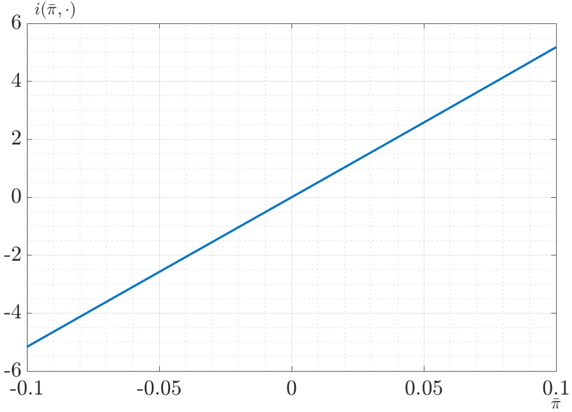
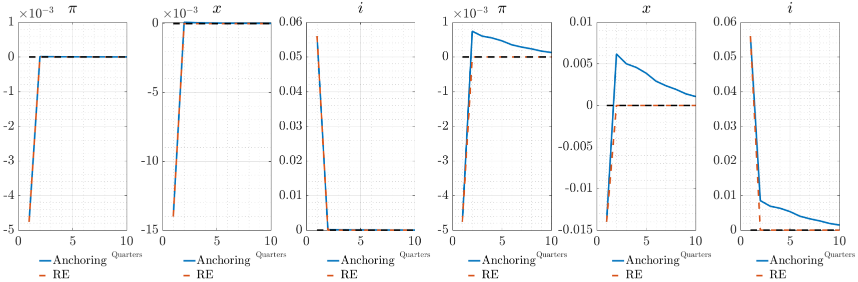


Figure: Policy function: $i(\bar{\pi}$, all other states at their means)

$\rightarrow$ For 5 bp drop in $\bar{\pi}$, lower i by 2.5 pp

Unanchoring causes volatility



(a) Expectations anchored (b) Expectations unanchored

Figure: Impulse responses after a contractionary monetary policy shock when policy follows a Taylor rule

Why so volatile? Term structure of expectations

When very unanchored, gain is large:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1} \tag{16}$$

$\rightarrow \bar{\pi}$ moves a lot

Why so volatile? Term structure of expectations

When very unanchored, gain is large:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1} \tag{16}$$

$\rightarrow \bar{\pi}$ moves a lot $\quad \Rightarrow$ shifts all short-run expectations

$$\hat{\mathbb{E}}_t \pi_{t+1} = \bar{\pi}_{t-1} + b_1^{RE} s_t$$

Why so volatile? Term structure of expectations

When very unanchored, gain is large:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1} \tag{16}$$

$\rightarrow \bar{\pi}$ moves a lot $\quad \Rightarrow$ shifts all short-run expectations

$$\hat{\mathbb{E}}_t \pi_{t+1} = \bar{\pi}_{t-1} + b_1^{RE} s_t$$

Positive feedback: IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \Big)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \Big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \Big)$$

Why oscillatory? Intertemporal anticipation effects

IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \big)$$

Why oscillatory? Intertemporal anticipation effects

IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big)$$

- Additional channel of policy

Why oscillatory? Intertemporal anticipation effects

IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \big)$$

- Additional channel of policy
- Only if policy reaction function internalized

Structure of talk

1. Model of anchoring expectations

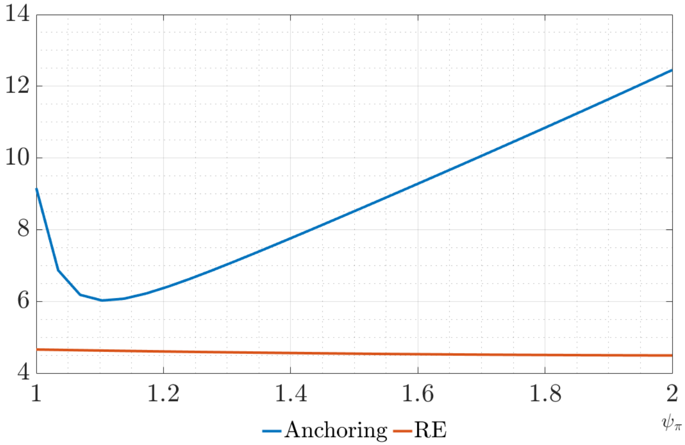
2. Solving the Ramsey problem

3. Implementing optimal policy

4. Approximating optimal policy with a Taylor rule

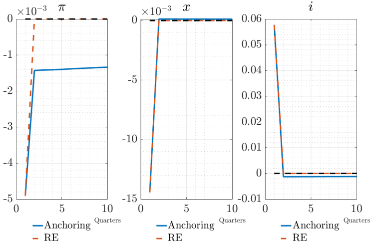
Optimal Taylor-coefficient on inflation

Figure: Central bank loss as a function of ψ_π

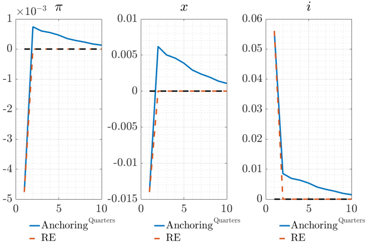


Anchoring-optimal coefficient: $\psi_\pi^A = 1.09$ RE-optimal coefficient: $\psi_\pi^{RE} = 2.21$

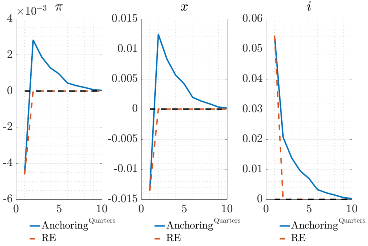
Respond but not too much



(a) $\psi_\pi = 1.01$



(b) $\psi_\pi = 1.5$



(c) $\psi_\pi = 2$

Losses for optimal Taylor-rule coefficient on inflation

Table: Loss for RE and anchoring models for choice of RE- or anchoring-optimal ψ_π

Anchoring, ψ_π^{RE}	Anchoring, ψ_π^{A}	RE, ψ_π^{RE}
14.2633	6.0228	4.4866

Losses for optimal Taylor-rule coefficient on inflation

Table: Loss for RE and anchoring models for choice of RE- or anchoring-optimal ψ_π

Anchoring, ψ_π^{RE}	Anchoring, ψ_π^{A}	RE, ψ_π^{RE}
14.2633	6.0228	4.4866

Anchoring-optimal ψ_π^{A} gets 84% of the distance to RE-optimal ψ_π^{RE} under RE

Conclusion

- First theory of monetary policy for potentially unanchored expectations
- Optimal policy frontloads aggressive interest rate response to suppress potential unanchoring
- Matters: already anchoring-optimal Taylor rule reduces losses by 84%
- Future work
  - $\hookrightarrow$ Better approximation to optimal policy than Taylor rule?
  - $\hookrightarrow$ How to anchor at zero-lower bound?

Appendix

Breakeven inflation

Figure: Market-based inflation expectations, various horizons, %

Return

Correcting the TIPS from liquidity risk

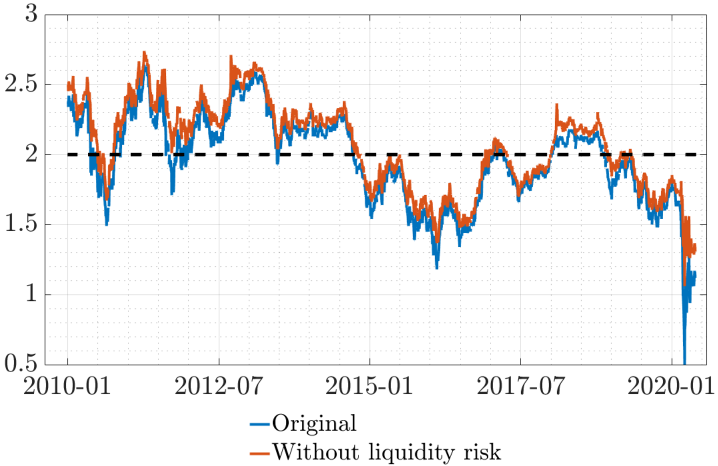
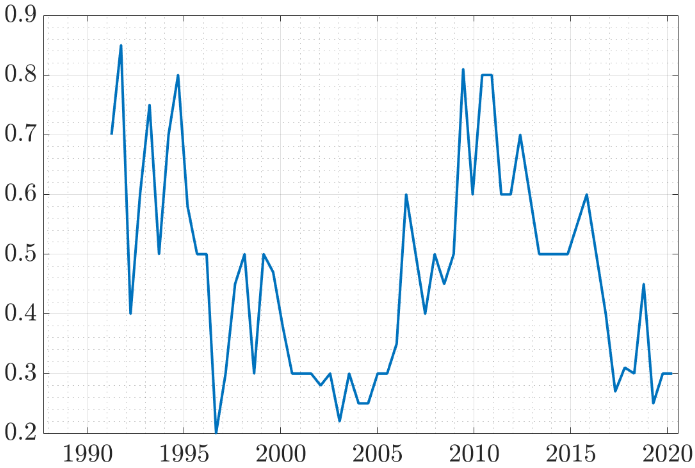


Figure: Market-based inflation expectations, 10 year, %

Return

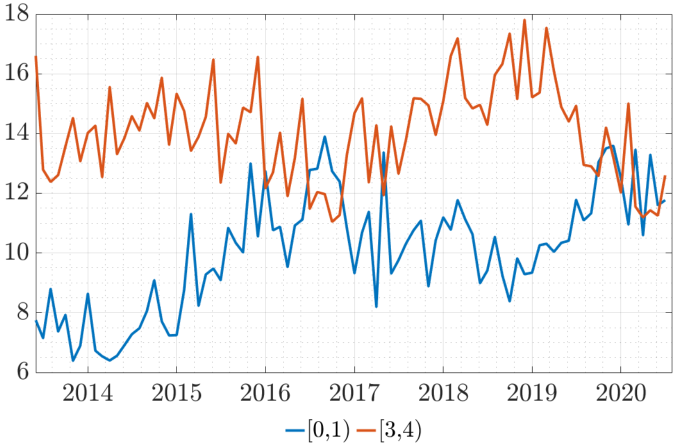
Further evidence

Figure: Livingston Survey of Firms:
Interquartile range of 10-year ahead inflation expectations



Return

Figure: New York Fed Survey of Consumers:
Percent of respondents indicating 3-year ahead inflation will be in a particular range



Return

Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{20}$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{21}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1-k^{-1}}{1-k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1-k^{-1}\beta} u_t \tag{22}$$

Solve for forecast error $f_t \equiv \pi_t - f_{t-1}$:

$$f_t = \underbrace{-\frac{1-\beta}{1-k\beta}}_{\lim_{k\to 1}=-1} f_{t-1} + \frac{1}{1-k\beta} u_t \tag{23}$$

Functional forms for **g** in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \tag{24}$$

$p \equiv Prob(\text{liquidity trap regime})$
y_{t-1} output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{25}$$

θ_t criterion, $\bar{\theta}$ threshold value

Return

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max \left| \Sigma^{-1} (\phi_{t-1} - T(\phi_{t-1})) \right| \tag{26}$$

Σ variance-covariance matrix of shocks
$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1} (f_{t|t-1} f'_{t|t-1} - \omega_{t-1}) \tag{27}$$
$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1} (f'_{t|t-1} \omega_t^{-1} f_{t|t-1} - \theta_{t-1}) \tag{28}$$

ω_t estimated forecast-error variance

Return

Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}\right)'\right)' \tag{29}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{30}$$

Return

Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{31}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{32}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \tag{33}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{34}$$

Return

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t)$$

$$+ \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \Big\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t + \mathbf{g}_\pi(\pi_t - f_{t-1})) = 0 \tag{35}$$

$$-2\beta\frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_{\mathbf{f}}(\pi_{t+1} - f_t)) = 0 \tag{36}$$

Return

Target criterion system for anchoring function as changes of the gain

$$\varphi_{6,t} = -cf_{t|t-1}x_{t+1} + \left(1 + \frac{f_{t|t-1}}{f_{t+1|t}}(1 - k_{t+1}) - f_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1} - \frac{f_{t|t-1}}{f_{t+1|t}}(1 - k_{t+1})\varphi_{6,t+2} \tag{37}$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{f_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{f_{t|t-1}}\varphi_{6,t+1} \tag{38}$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (38) is given by:

$$\varphi_{6,t} = -2\,\mathbb{E}_t \sum_{i=0}^{\infty}(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{f_{t+j|t+j-1}}}{\frac{k_{t+j}}{f_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \tag{39}$$

Return

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{40}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{41}$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \tag{42}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{43}$$

Return

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{44}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{45}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (44) backward to some date t, take expectations at t
2. Sub in (45)
3. Aggregate over households i

$\rightarrow$ Obtain (18)

◂ Return