Materials 14 - Maybe a last attempt to get rid of the overshooting

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Overview

1 Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \tag{3}$$

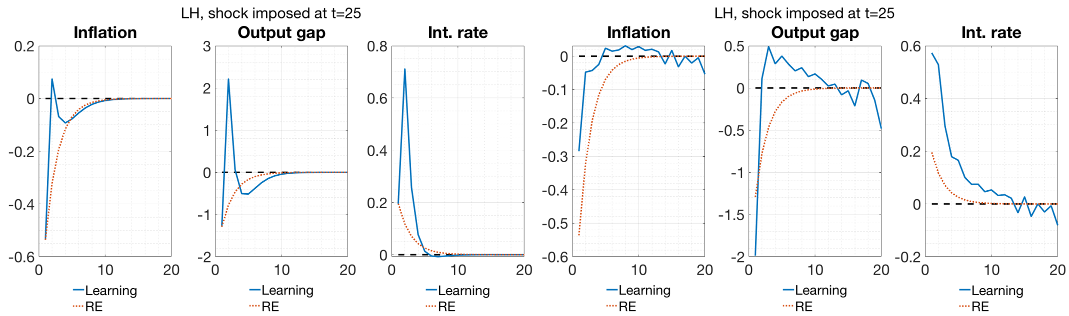
$$\hat{\mathbb{E}}_t z_{t+h} = \bar{z}_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x \; h_x \qquad \text{PLM} \tag{4}$$

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} \underbrace{\big(z_t - (\bar{z}_{t-1} + b s_{t-1})\big)}_{\text{fcst error using (4)}} \tag{5}$$

(Vector learning. For scalar learning, $\bar{z} = \begin{pmatrix} \bar{\pi} & 0 & 0 \end{pmatrix}'$. I'm also not writing the case where the slope b is also learned.)

$$k_t = \begin{cases} k_{t-1} + 1 & \text{for decreasing gain learning} \\ \bar{g}^{-1} & \text{for constant gain learning.} \end{cases} \tag{6}$$

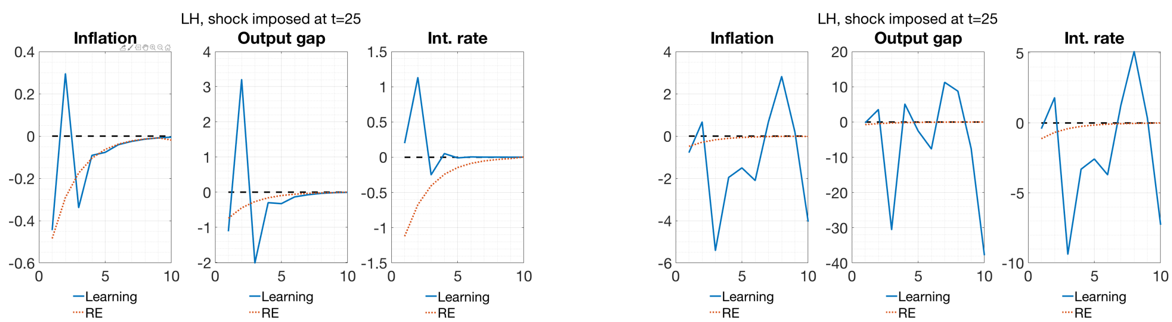
Figure 1: Reference: baseline model



(a) Learning constant only (b) Learning slope and constant

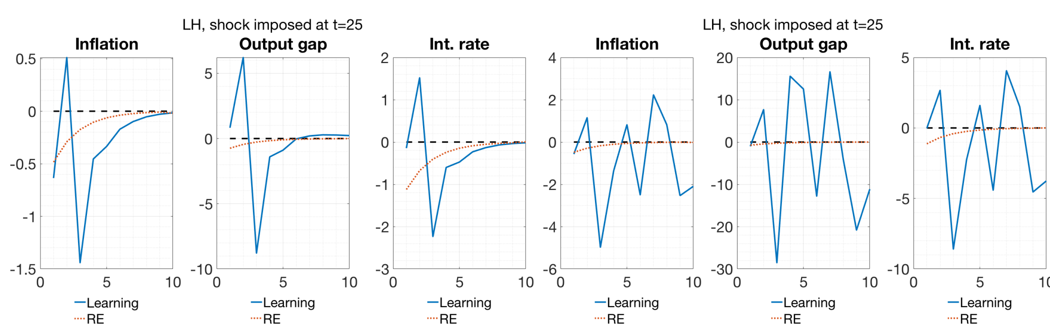
2 Regime-switching

Figure 2: Markov-switching Taylor rule, baseline, learning initialized at active state, conditional on mixed regime sequence



(a) Learning constant only (b) Learning slope and constant

Figure 3: Markov-switching Taylor rule, baseline, learning initialized at passive state, conditional on mixed regime sequence

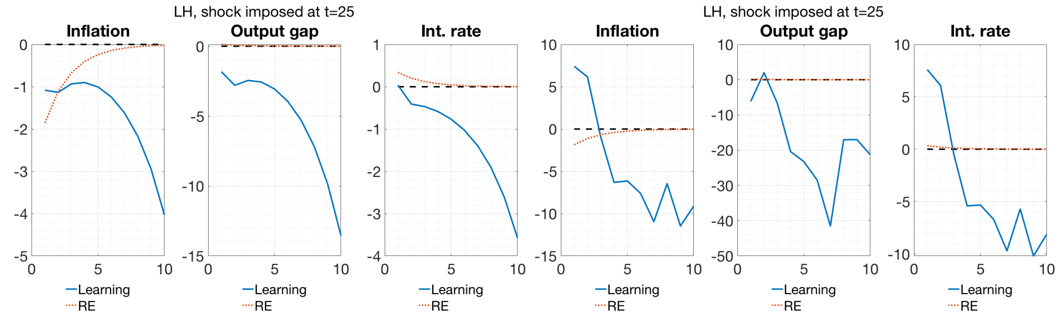


(a) Learning constant only (b) Learning slope and constant

- Different initialization of learning doesn't make a whole lot of difference.

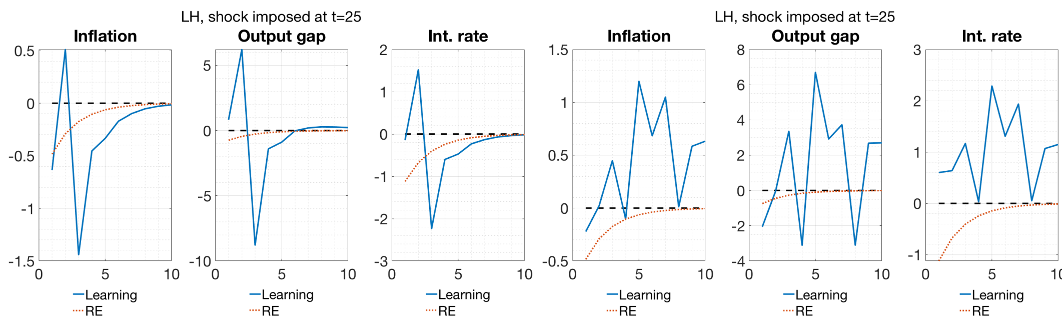
- It just changes where you start, but doesn't fundamentally affect dynamics.

Figure 4: Markov-switching Taylor rule, baseline, learning initialized at passive state, conditional on passive regime only



(a) Learning constant only (b) Learning slope and constant

Figure 5: Markov-switching Taylor rule, baseline, learning initialized at passive state, conditional on active regime only



(a) Learning constant only (b) Learning slope and constant

- I'm surprised that the all-passive state is unstable. I've checked and it's not E-stable: the difference in the learning matrix ϕ grows over time, even with decreasing gain learning.
- The all-active is very volatile.

3 Projection facility: checking `eig(phi)` when ϕ isn't square?

What I do now is I check `eig(R)` because that is always square, and when ϕ explodes, usually R does too. Of course I can't do this for learning the constant only, but according to my experience, that's where the projection facility is least likely to ever be needed. Of course, this doesn't always work - for interest rate smoothing, it doesn't.

4 Endogenous states don't evolve as they should

Now they do!

5 Bringing anchoring back

I've reworked the choice functions for the endogenous gain, CEMP's criterion and the CUSUM test, to better incorporate vector learning ("out of the difference between PLM and E(ALM) or out of forecast error variances respectively, pick the largest (or the mean) and test whether that's bigger than the threshold value").

CEMP's criterion was

$$\theta_t = |\hat{\mathbb{E}}_{t-1}\pi_t - \mathbb{E}_{t-1}\,\pi_t|/(\mathrm{Var(shocks)}) \tag{7}$$

$$\text{i.e.} \quad \text{PLM- E[ALM], scaled by shocks} \tag{8}$$

To extend this to vector learning and in general to a model with a vector of observables I first rewrite the ALM

$$z_t = A_a f_a + A_b f_b + A_s s_t \tag{9}$$

$$\text{as} \quad z_t = F + G s_t \tag{10}$$

$$\Leftrightarrow \quad z_t = \begin{bmatrix} F & G \end{bmatrix} \begin{bmatrix} 1 \\ s_t \end{bmatrix} \tag{11}$$

Then, since the PLM is $z_t = \phi \begin{bmatrix} 1 \\ s_t \end{bmatrix}$, the generalized CEMP criterion becomes

$$\theta_t = \max|\Sigma^{-1}(\phi - \begin{bmatrix} F & G \end{bmatrix})| \tag{12}$$

where Σ is the VC matrix of shocks. As for the CUSUM criterion, what I did in Materials 5 was

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1}(FE_t^2 - \omega_{t-1}) \tag{13}$$

$$\theta_t = \theta_{t-1} + \kappa k_{t-1}^{-1}(FE_t^2/\omega_t - \theta_{t-1}) \tag{14}$$

where FE_t is the most recent short-run forecast error ($ny \times 1$), and ω_t is the agents' estimate of the forecast error variance ($ny \times ny$). To take into account that these are now matrices, I now write

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1}(FE_t FE_t' - \omega_{t-1}) \tag{15}$$

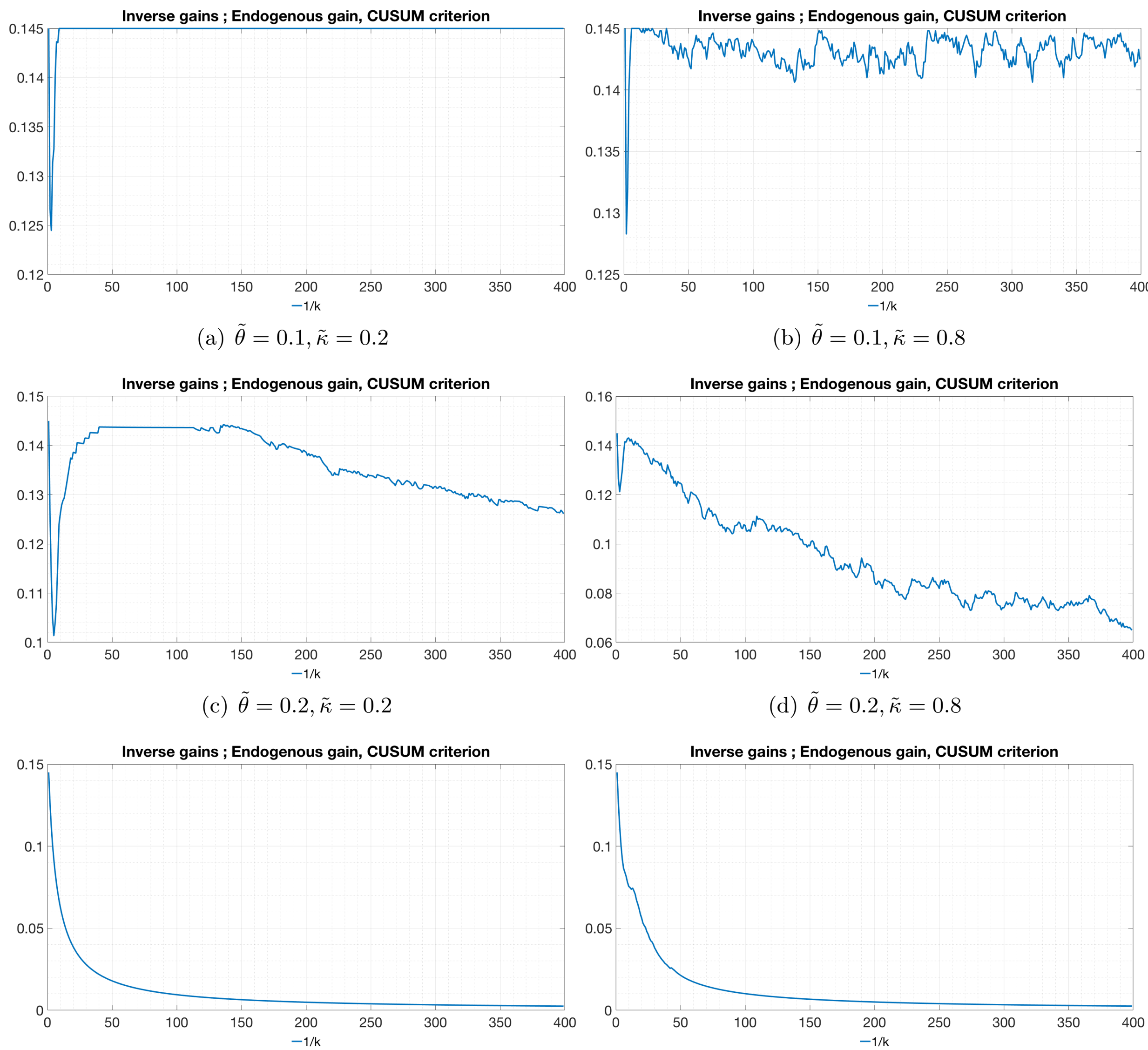
$$\theta_t = \theta_{t-1} + \kappa k_{t-1}^{-1}\mathrm{mean}((\omega_t^{-1} FE_t FE_t' - \theta_{t-1})) \tag{16}$$

Both work but require quite different threshold parameters to behave similarly.

6 Behavior of CEMP and CUSUM criteria as functions of their parameters

$\psi_\pi = 1.5, \psi_x = 0.$

Figure 6: Inverse gains CUSUM, learning the constant only



(a) $\tilde{\theta} = 0.1, \tilde{\kappa} = 0.2$

(b) $\tilde{\theta} = 0.1, \tilde{\kappa} = 0.8$

(c) $\tilde{\theta} = 0.2, \tilde{\kappa} = 0.2$

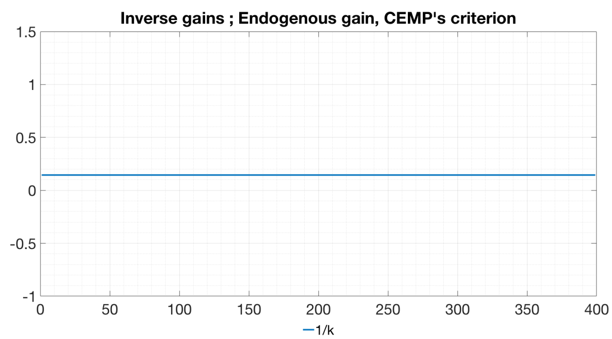
(d) $\tilde{\theta} = 0.2, \tilde{\kappa} = 0.8$

(e) $\tilde{\theta} = 0.4, \tilde{\kappa} = 0.2$

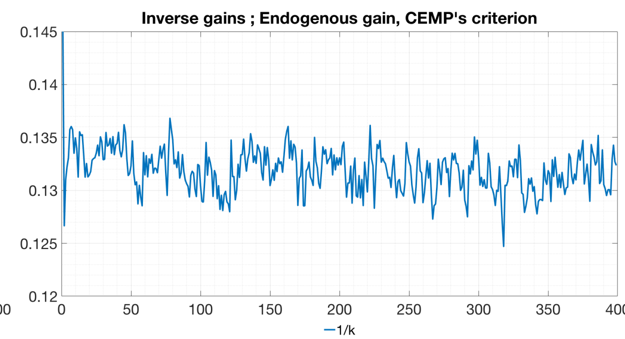
(f) $\tilde{\theta} = 0.4, \tilde{\kappa} = 0.8$

A higher $\tilde{\kappa}$ just increases the action.

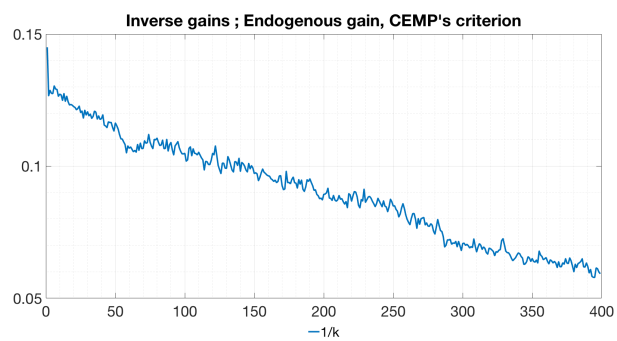
Figure 7: Inverse gains CEMP, learning the constant only



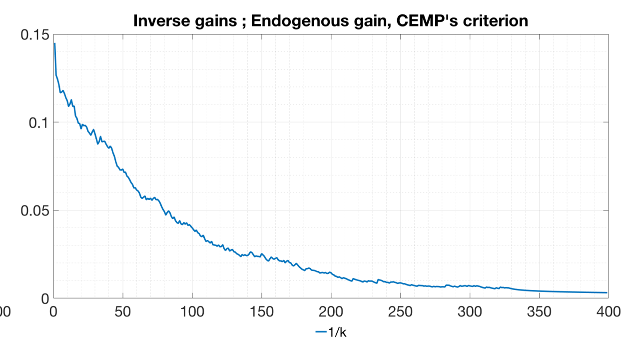
(a) $\bar{\theta} = 1$



(b) $\bar{\theta} = 3$



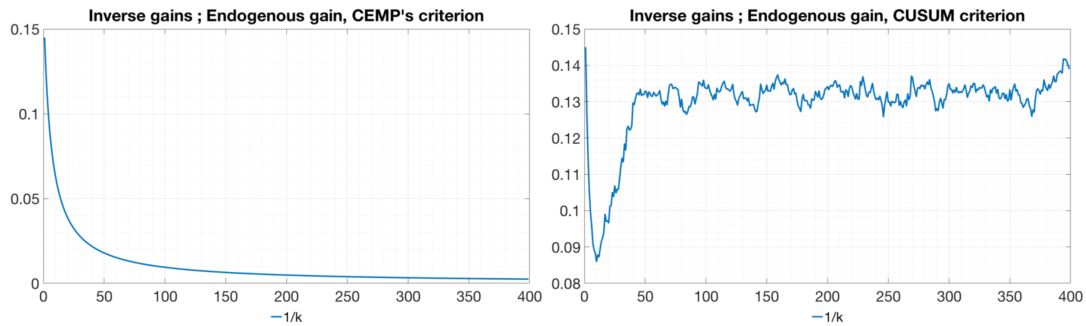
(c) $\bar{\theta} = 4$



(d) $\bar{\theta} = 5$

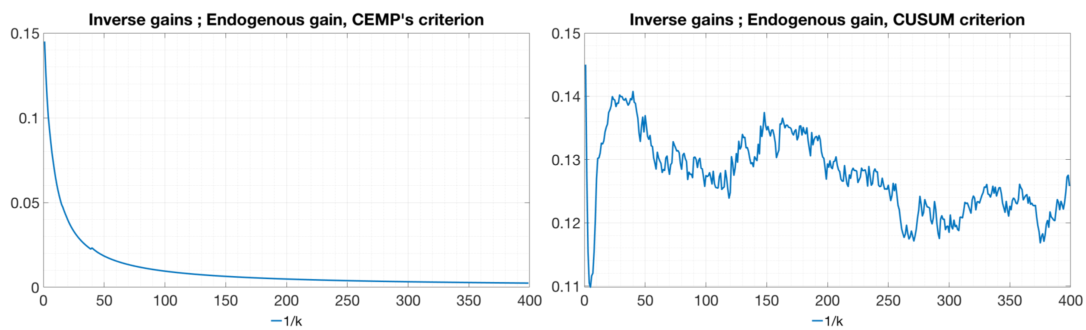
7 Anchoring as a function of ψ_π

Figure 8: Inverse gains, $\psi_\pi = 1.01, \psi_x = 0$



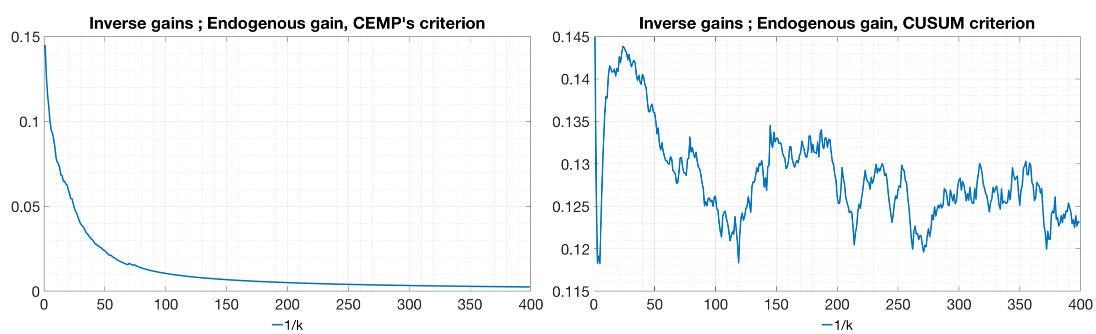
(a) CEMP's criterion, $\bar{\theta} = 4$ (b) CUSUM criterion, $\tilde{\theta} = 0.2$

Figure 9: Inverse gains, $\psi_\pi = 1.1, \psi_x = 0$



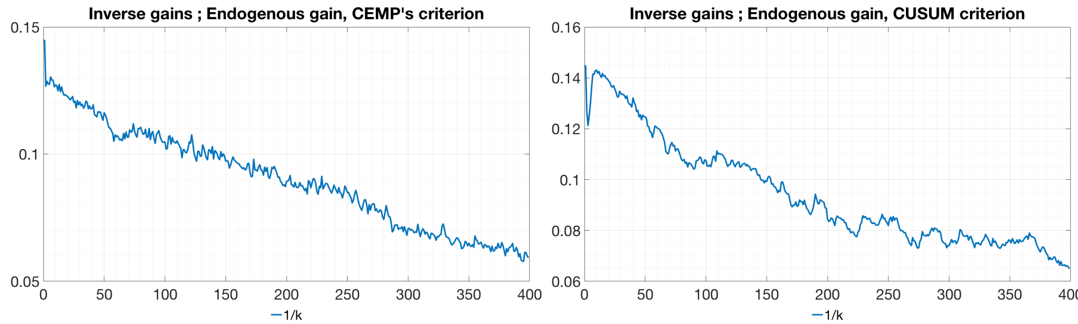
(a) CEMP's criterion, $\bar{\theta} = 4$ (b) CUSUM criterion, $\tilde{\theta} = 0.2$

Figure 10: Inverse gains, $\psi_\pi = 1.2, \psi_x = 0$



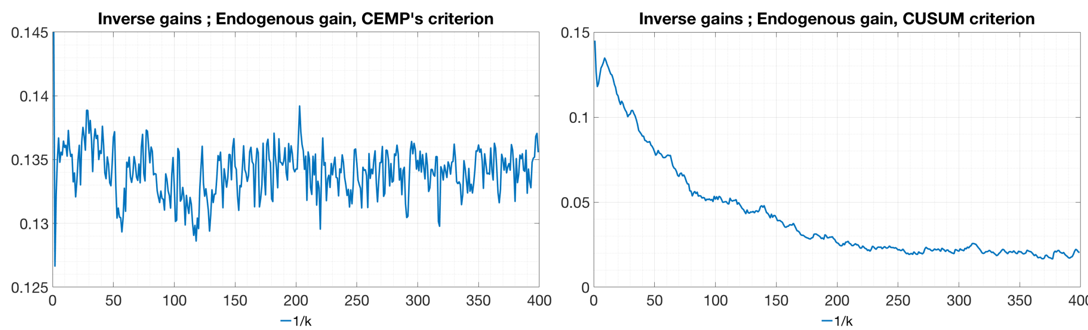
(a) CEMP's criterion, $\bar{\theta} = 4$ (b) CUSUM criterion, $\tilde{\theta} = 0.2$

Figure 11: Inverse gains, $\psi_\pi = 1.5, \psi_x = 0$



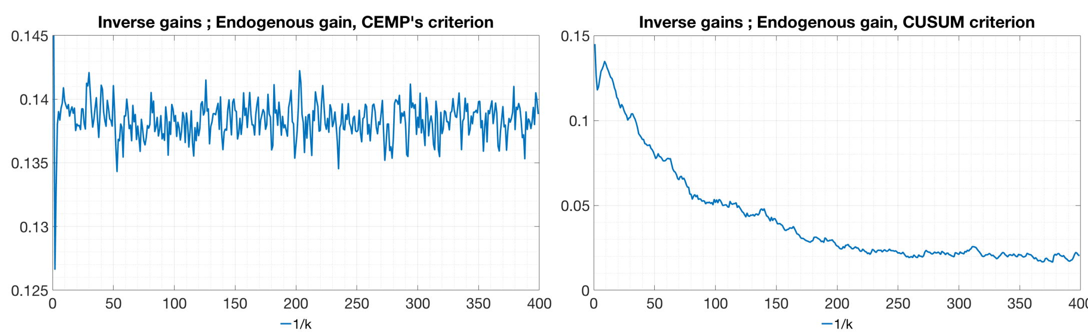
(a) CEMP's criterion, $\bar{\theta} = 4$ (b) CUSUM criterion, $\tilde{\theta} = 0.2$

Figure 12: Inverse gains, $\psi_\pi = 1.8, \psi_x = 0$



(a) CEMP's criterion, $\bar{\theta} = 4$ (b) CUSUM criterion, $\tilde{\theta} = 0.2$

Figure 13: Inverse gains, $\psi_\pi = 2, \psi_x = 0$



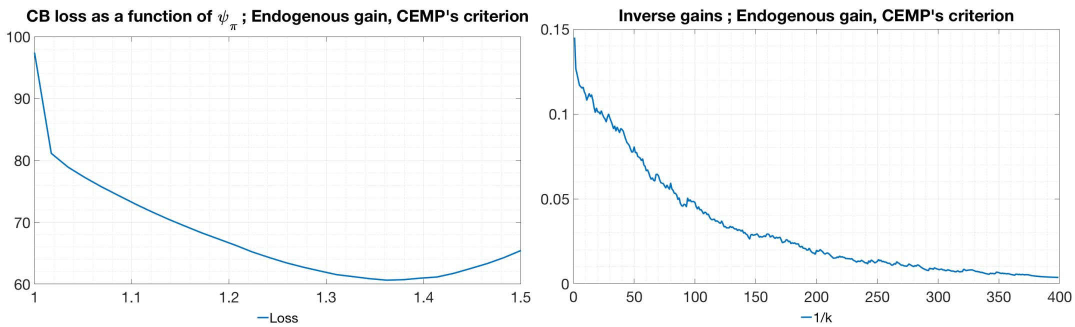
(a) CEMP's criterion, $\bar{\theta} = 4$ (b) CUSUM criterion, $\tilde{\theta} = 0.2$

8 The central bank's loss function

Relying on Clarida, Gali and Gertler (1999), I start with the simple quadratic loss:

$$\mathcal{L} = \frac{1}{2}\,\mathbb{E}_t \sum_{T=t}^{\infty} \left(\alpha^{CB} x_T^2 + \pi_T^2\right) \tag{17}$$

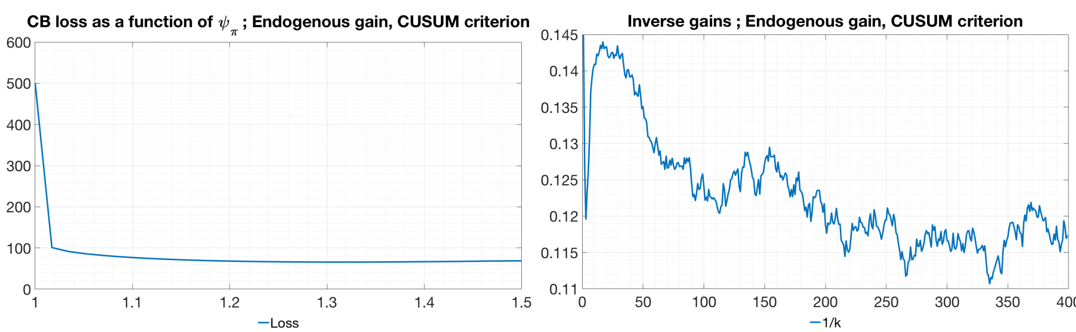
Figure 14: Central bank loss for $\psi_x = 0, \alpha^{CB} = 0, \bar{\theta} = 4$, computed as a cross-sectional average, $N = 100, T = 400$.



(a) Loss (b) Inverse gains associated with ψ_π^*

Fmincon says $\psi_\pi^* = 1.3650$.

Figure 15: Central bank loss for $\psi_x = 0, \alpha^{CB} = 0, \tilde{\theta} = 0.2$, computed as a cross-sectional average, $N = 100, T = 400$.



(a) Loss (b) Inverse gains associated with ψ_π^*

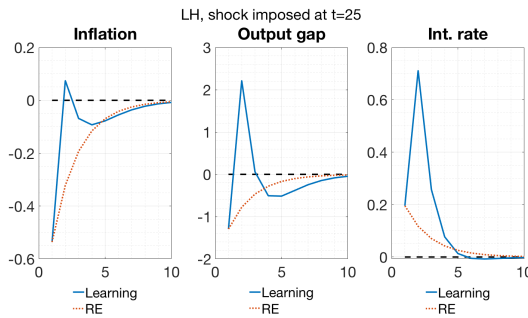
Fmincon says $\psi_\pi^* = 1.3$.

An issue with CUSUM: ω often becomes singular, so for fmincon to be able to run, I took the average element value of ω.

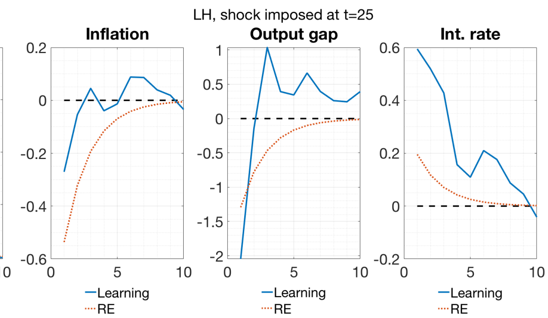
A potential problem is that these optimal values are super-sensitive to threshold criterion values which are completely arbitrary!

9 Reference plots

Figure 16: Baseline

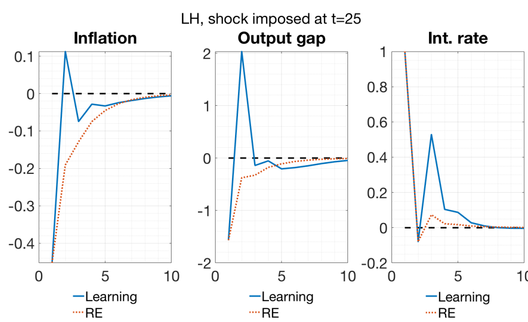


(a) Learning constant only

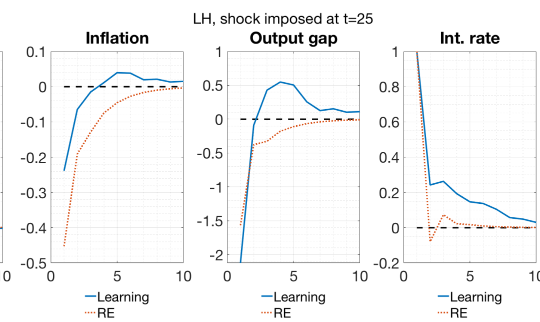


(b) Learning slope and constant

Figure 17: Lagged inflation in Taylor rule, "suboptimal forecasters" info assumption

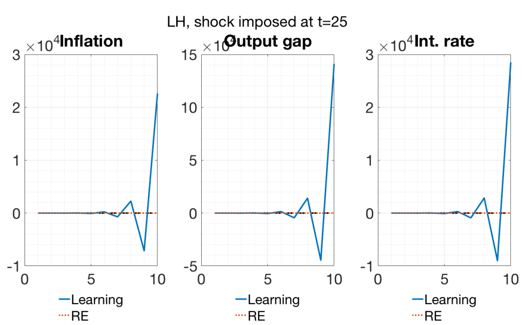


(a) Learning constant only

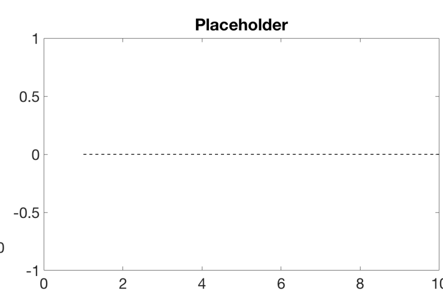


(b) Learning slope and constant

Figure 18: Interest rate smoothing, "suboptimal forecasters" info assumption

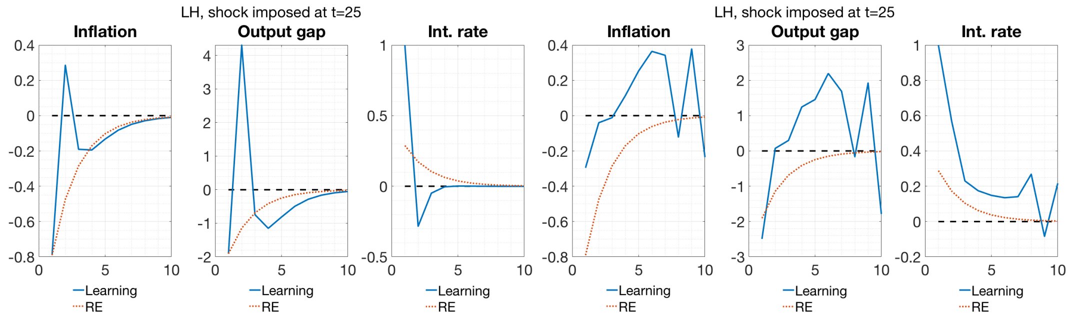


(a) Learning constant only



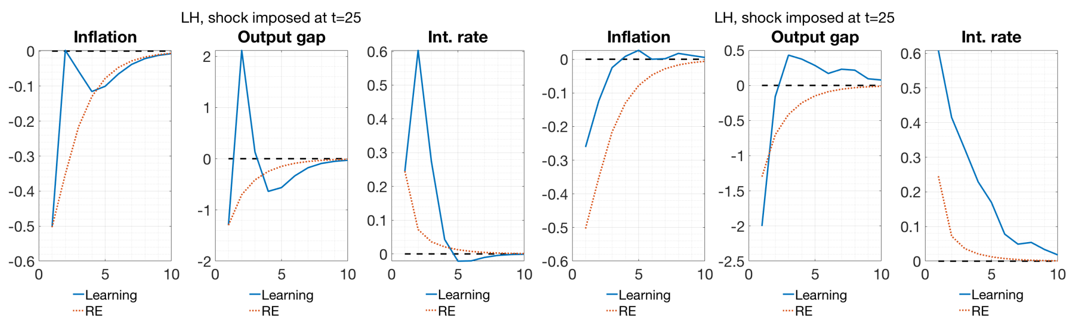
(b) Learning slope and constant - no projection facility in the world can make this not explode

Figure 19: Expected inflation in Taylor rule



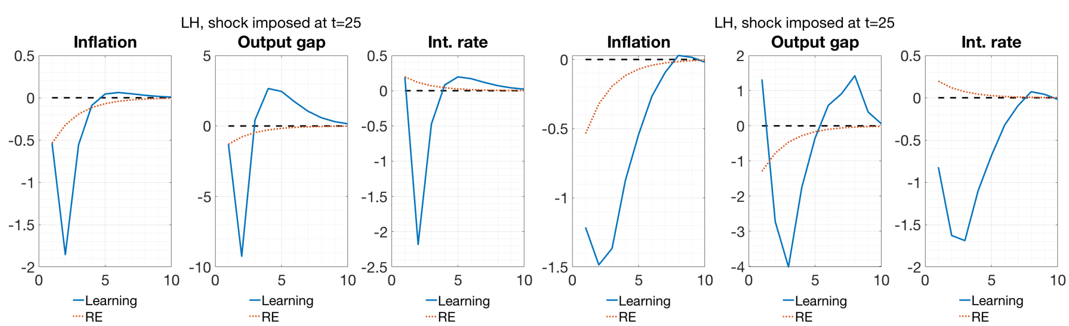
(a) Learning constant only (b) Learning slope and constant

Figure 20: Indexation in NKPC, "suboptimal forecasters" info assumption



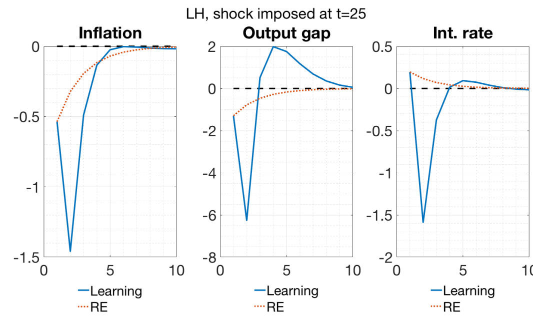
(a) Learning constant only (b) Learning slope and constant

Figure 21: Learn Taylor rule

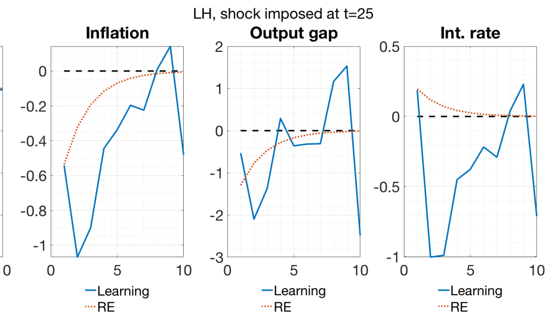


(a) Learning constant only (b) Learning slope and constant

Figure 22: Learn h_x

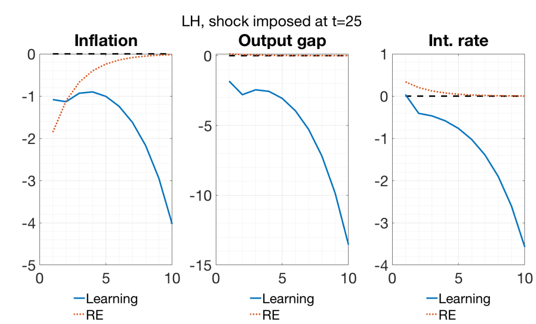


(a) Learning constant only

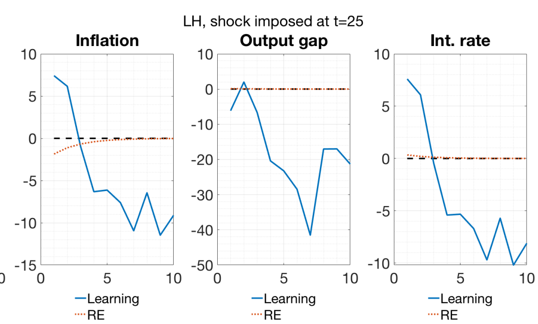


(b) Learning slope and constant

Figure 23: Markov-switching Taylor rule, conditional on passive regime only, learning initialized at passive regime



(a) Learning constant only



(b) Learning slope and constant