

Monetary Policy & Anchored Expectations An Endogenous Gain Learning Model

Laura Gáti

Boston College

Green Line Macro Meeting

September 18, 2020

*Inflation that runs below its desired level can lead to an unwelcome fall in **longer-term inflation expectations**, which, in turn, can pull actual inflation even lower, resulting in an adverse cycle of ever-lower inflation and inflation expectations. [...] **Well-anchored inflation expectations** are critical[.]*

*Jerome Powell, Chairman of the Federal Reserve ¹
(Emphases added.)*

¹“New Economic Challenges and the Fed’s Monetary Policy Review,” August 27, 2020.

Anchored expectations

$$\mathbb{E}_t \pi_{t+1} = \bar{\pi} + f(\textit{shocks}) \quad (1)$$

$\bar{\pi}$ = long-run expectations

Anchored expectations

$$\mathbb{E}_t \pi_{t+1} = \bar{\pi} + f(\text{shocks}) \quad (1)$$

$\bar{\pi}$ = long-run expectations

Anchored expectations: long-run expectations at Fed's target (π^*)

Anchored expectations

$$\mathbb{E}_t \pi_{t+1} = \bar{\pi} + f(\text{shocks}) \quad (1)$$

$\bar{\pi}$ = long-run expectations

Anchored expectations: long-run expectations at Fed's target (π^*)

→ short-run expectations “anchored” to stable mean:

$$\mathbb{E}_t \pi_{t+1} = \pi^* + f(\text{shocks}) \quad (2)$$

Anchored expectations

$$\mathbb{E}_t \pi_{t+1} = \bar{\pi} + f(\text{shocks}) \quad (1)$$

$\bar{\pi}$ = long-run expectations

Anchored expectations: long-run expectations at Fed's target (π^*)

→ short-run expectations “anchored” to stable mean:

$$\mathbb{E}_t \pi_{t+1} = \pi^* + f(\text{shocks}) \quad (2)$$

Unanchored expectations: long-run expectations deviate systematically from the target

$$\mathbb{E}_t \pi_{t+1} = \bar{\pi}_{t-1}(\text{shocks}) + f(\text{shocks}) \quad (3)$$

Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t \quad (4)$$

$$\pi_t = \kappa x_t + \beta(2\% + f(\text{shocks})) + u_t \quad (5)$$

Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t \quad (4)$$

$$\pi_t = \kappa x_t + \beta(2\% + f(\text{shocks})) + u_t \quad (5)$$

The Fed's uneasiness: public thinks inflation averages only 1%

$$\pi_t = \kappa x_t + \beta(\underset{\downarrow}{1\%} + f(\text{shocks})) + u_t \quad (6)$$

Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t \quad (4)$$

$$\pi_t = \kappa x_t + \beta(2\% + f(\text{shocks})) + u_t \quad (5)$$

The Fed's uneasiness: public thinks inflation averages only 1%

$$\pi_t = \kappa x_t + \beta(\underset{\downarrow}{1\%} + f(\text{shocks})) + u_t \quad (6)$$

Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t \quad (4)$$

$$\pi_t = \kappa x_t + \beta(2\% + f(\text{shocks})) + u_t \quad (5)$$

The Fed's uneasiness: public thinks inflation averages only 1%

$$\pi_t = \kappa x_t + \beta(\underset{\text{red } \downarrow}{1\%} + \underset{\text{green } \downarrow\downarrow}{f(\text{shocks})}) + u_t \quad (6)$$

Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t \quad (4)$$

$$\pi_t = \kappa x_t + \beta(2\% + f(\text{shocks})) + u_t \quad (5)$$

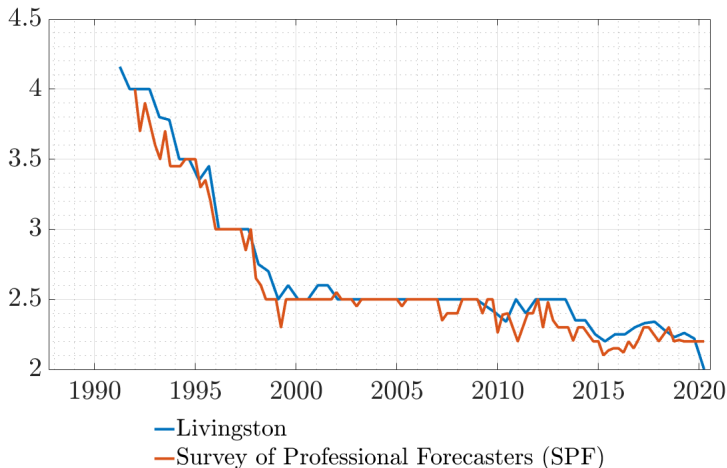
The Fed's uneasiness: public thinks inflation averages only 1%

$$\pi_t = \kappa x_t + \beta(\underset{\text{red } \downarrow}{1\%} + \underset{\text{green } \downarrow\downarrow}{f(\text{shocks})}) + u_t \quad (6)$$

→ unanchored expectations can cause a deflationary spiral

Long-run expectations drifting down?

Figure: Expectations of average inflation over 10 years ($\bar{\pi}$ in data)



Long-run expectations moving systematically?

SPF: for 1999-Q1 onward, estimate

$$\bar{\pi}_t = \beta_0 + \beta_1 fe_{t|t-1} + \epsilon_t \quad (7)$$

where $\bar{\pi}_t \equiv \mathbb{E}_t(\pi_{t+10})$ and $fe_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1}(\pi_t)$ (forecast error)

Long-run expectations moving systematically?

SPF: for 1999-Q1 onward, estimate

$$\bar{\pi}_t = \beta_0 + \beta_1 fe_{t|t-1} + \epsilon_t \quad (7)$$

where $\bar{\pi}_t \equiv \mathbb{E}_t(\pi_{t+10})$ and $fe_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1}(\pi_t)$ (forecast error)

$\Rightarrow \hat{\beta}_1 = 0.06$ (p-value: 0.000017)

$\Rightarrow 1$ pp forecast error $\rightarrow 6$ bp revision in long-run expectations

► Further evidence

This project

- How to conduct monetary policy in interaction with the anchoring expectation formation?
- Model of anchoring expectation formation as an extension to adaptive learning
 \hookrightarrow twist: systematic fluctuations in long-run expectations
- Estimation of the anchoring function: when do expectations become unanchored?

Preview of results

- Optimal policy aggressive when expectations unanchor, accommodates when anchored
 - Taylor rule policy less aggressive on inflation than under rational expectations
- ↪ Anchoring-optimal Taylor rule eliminates 75% of loss from volatility

Related literature

- **Optimal monetary policy in New Keynesian models**

Clarida, Gali & Gertler (1999), Woodford (2003)

- **Adaptive learning**

Evans & Honkapohja (2001, 2006), Eusepi & Preston (2011), Milani (2007, 2014), Lubik & Matthes (2018), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Mele et al. (2019)

- **Anchoring and the Phillips curve**

Sargent (1999), Williams (2006), Svensson (2015), Afrouzi et al. (2015), Hooper et al. (2019), Hebden et al. (2020), Afrouzi & Yang (2020), Reis (2020), Gobbi et al (2019), Carvalho et al (2019)

Today's talk

1. Model of anchoring expectations: key elements
2. Features of optimal policy under anchoring expectation formation

Today's talk

1. Model of anchoring expectations: key elements
2. Features of optimal policy under anchoring expectation formation

The New Keynesian backbone

- IS- and Phillips curve under rational expectations:

$$x_t = \mathbb{E}_t x_{t+1} - \sigma i_t + \mathbb{E}_t \pi_{t+1} + \sigma r_t^n \quad (8)$$

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t \quad (9)$$

The New Keynesian backbone

- IS- and Phillips curve under rational expectations:

$$x_t = \mathbb{E}_t x_{t+1} - \sigma i_t + \mathbb{E}_t \pi_{t+1} + \sigma r_t^n \quad (8)$$

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t \quad (9)$$

- \mathbb{E} : Agents know they are identical and can compute (8)-(9)

The New Keynesian backbone

- IS- and Phillips curve under rational expectations:

$$x_t = \mathbb{E}_t x_{t+1} - \sigma i_t + \mathbb{E}_t \pi_{t+1} + \sigma r_t^n \quad (8)$$

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t \quad (9)$$

- \mathbb{E} : Agents know they are identical and can compute (8)-(9)
- Here instead: adaptive learning

$$\hat{\mathbb{E}}_t \pi_{t+1} = \bar{\pi}_{t-1} + b^{RE} s_t \quad (10)$$

(s_t = exogenous states)

Recursive least squares

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \underbrace{k_t}_{\in (0,1), \text{ gain}} \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b^{RE} s_{t-1}) \right)}_{f_{e_t|t-1}, \text{ forecast error}} \quad (11)$$

Recursive least squares

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \underbrace{k_t}_{\in (0,1), \text{ gain}} \underbrace{(\pi_t - (\bar{\pi}_{t-1} + b^{RE} s_{t-1}))}_{fe_{t|t-1}, \text{ forecast error}} \quad (11)$$

Decreasing gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} fe_{t|t-1} \quad (12)$$

→ sample mean of full sample of forecast errors

Recursive least squares

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \underbrace{k_t}_{\in (0,1), \text{ gain}} \underbrace{(\pi_t - (\bar{\pi}_{t-1} + b^{RE} s_{t-1}))}_{fe_{t|t-1}, \text{ forecast error}} \quad (11)$$

Decreasing gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} fe_{t|t-1} \quad (12)$$

→ sample mean of full sample of forecast errors

Constant gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k fe_{t|t-1} \quad (13)$$

→ sample mean of most recent observations only

Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})) \quad (14)$$

$k_t = \mathbf{g}(fe_{t|t-1})$: anchoring function

Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})) \quad (14)$$

$k_t = \mathbf{g}(fe_{t|t-1})$: anchoring function

$$\mathbf{g}(fe_{t|t-1}) = \sum_i \alpha_i b_i(fe_{t|t-1}) \quad (15)$$

$b_i(fe_{t|t-1})$ = basis, here: second order spline (piecewise linear)

α_i = approximating coefficients, here: use $\hat{\alpha}$ from estimation

► Estimate

► Functional forms in literature

Today's talk

1. Model of anchoring expectations: key elements
2. Features of optimal policy under anchoring expectation formation

Optimal policy - responding to unanchoring

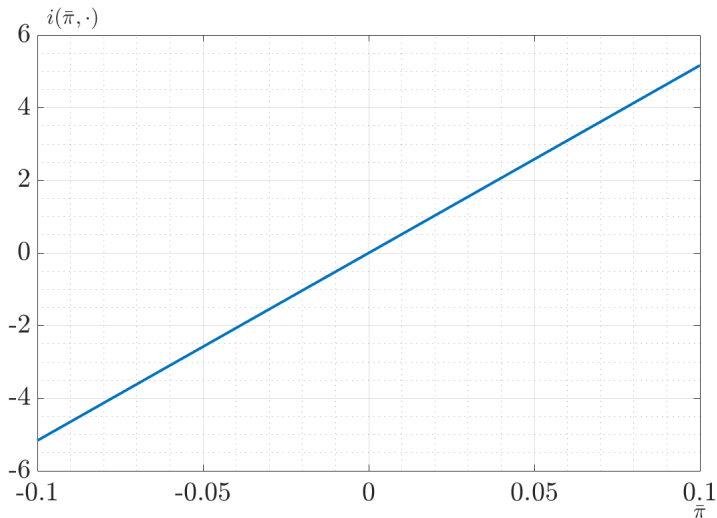


Figure: Policy function: $i(\bar{\pi}, \text{all other states at their means})$

Unanchoring raises volatility

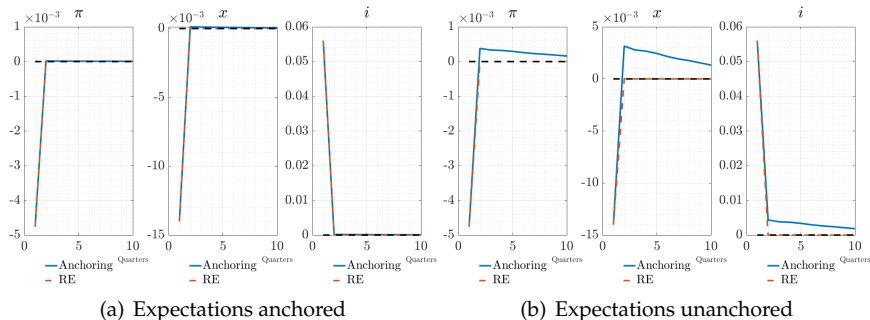
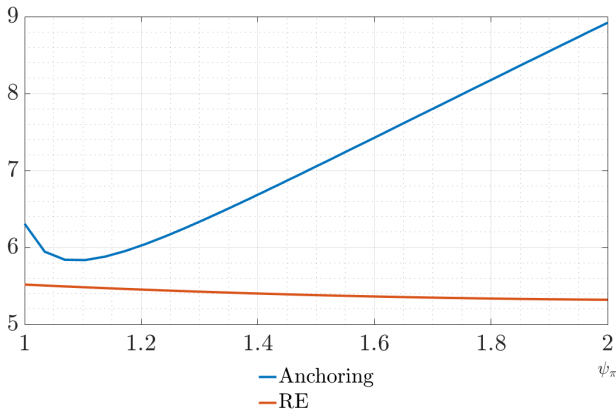


Figure: Impulse responses after a contractionary monetary policy shock

Optimal Taylor-coefficient on inflation

$$i_t = \psi_\pi \pi_t + 0.3x_t \quad (16)$$

Figure: Central bank loss as a function of ψ_π



Anchoring-optimal coefficient: $\psi_\pi^A = 1.09$. RE-optimal coefficient: $\psi_\pi^{RE} = 2.21$.

Losses for various policies

Table: Loss for RE and anchoring models for choice of RE- or anchoring-optimal ψ_π

Anchoring, ψ_π^{RE}	Anchoring, ψ_π^A	RE, ψ_π^{RE}	Optimal policy
9.6901	5.8296	5.3148	≈ 5

→ If model is anchoring, anchoring-optimal ψ_π^A gets 75% of the distance to RE-optimal ψ_π^{RE} under RE

Conclusion

- First theory of monetary policy for potentially unanchored expectations
- Optimal policy frontloads aggressive interest rate response to suppress potential unanchoring
- Matters: already anchoring-optimal Taylor rule reduces losses by 75%
- Future work: how to anchor at zero-lower bound?

Appendix

Calibration - parameters from the literature

β	0.98	stochastic discount factor
σ	1	intertemporal elasticity of substitution
α	0.5	Calvo probability of not adjusting prices
κ	0.0842	slope of the Phillips curve
ψ_π	1.5	coefficient of inflation in Taylor rule*
ψ_x	0.3	coefficient of the output gap in Taylor rule*
\bar{g}	0.145	initial value of the gain
λ_x	0.05	weight on the output gap in central bank loss
ρ_r	0	persistence of natural rate shock
ρ_i	0	persistence of monetary policy shock*
ρ_u	0	persistence of cost-push shock

* pertains to sections where Taylor rule is in effect

Calibration - matching moments

σ_r	0.01	standard deviation, natural rate shock
σ_i	0.01	standard deviation, monetary policy shock*
σ_u	0.5	standard deviation, cost-push shock
$\hat{\alpha}_i$	(0.33; 0.25; 0.001; 0.24; 0.33)	coefficients in anchoring function

Calibrated (σ_j , $j = r, i, u$) or estimated ($\hat{\alpha}_i$) to match the autocovariances of inflation, output gap, interest rate and one-period ahead inflation expectations for lags $0, \dots, 4$.

* pertains to sections where Taylor rule is in effect

Breakeven inflation



Figure: Market-based inflation expectations, various horizons, %

Correcting the TIPS from liquidity risk

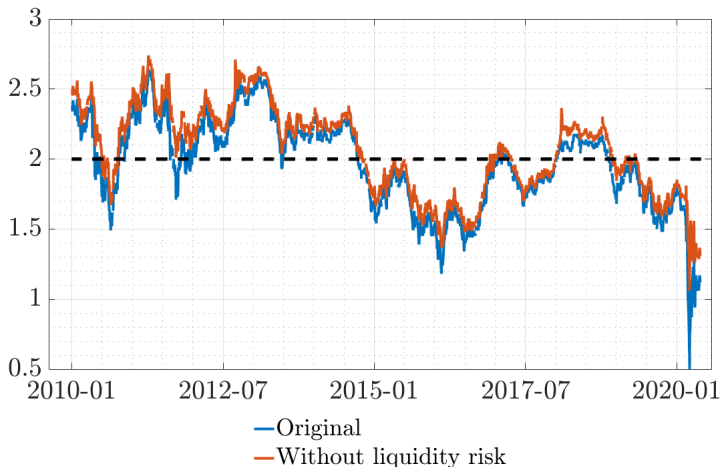


Figure: Market-based inflation expectations, 10 year, %

Further evidence

Figure: Livingston Survey of Firms:
Interquartile range of 10-year ahead inflation expectations

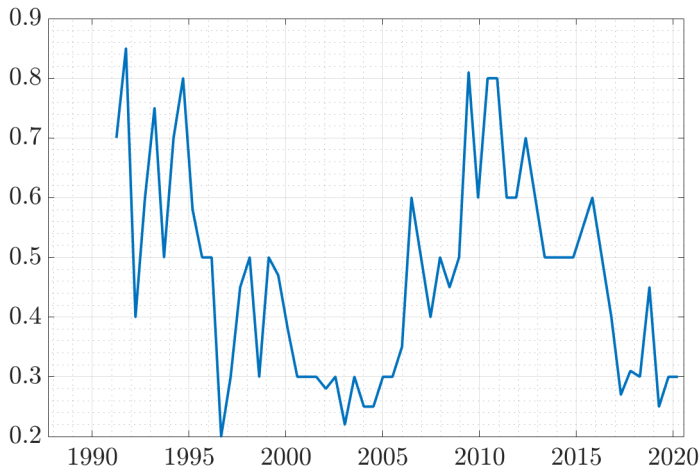
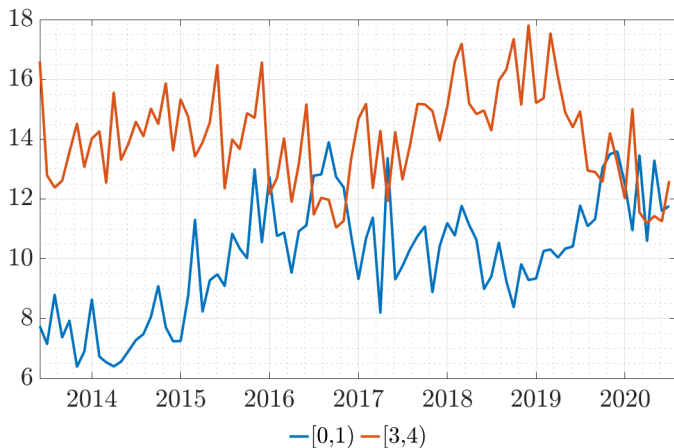


Figure: New York Fed Survey of Consumers:
Percent of respondents indicating 3-year ahead inflation will be in a particular range



Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \quad (17)$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \quad (18)$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \quad (19)$$

Solve for forecast error $fe_t \equiv \pi_t - f_{t-1}$:

$$fe_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k \rightarrow 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \quad (20)$$

Functional forms for g in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \quad (21)$$

$p \equiv \text{Prob}(\text{liquidity trap regime})$
 y_{t-1} output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when } \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \quad (22)$$

θ_t criterion, $\bar{\theta}$ threshold value

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \quad (23)$$

Σ variance-covariance matrix of shocks

$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1} (fe_{t|t-1} fe'_{t|t-1} - \omega_{t-1}) \quad (24)$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1} (fe'_{t|t-1} \omega_t^{-1} fe_{t|t-1} - \theta_{t-1}) \quad (25)$$

ω_t estimated forecast-error variance

Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right) \right)' \quad (26)$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} [1 \quad s_{t-1}] - R_{t-1} \right) \quad (27)$$

Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \quad (28)$$

$$s_t = h s_{t-1} + \epsilon_t \quad (29)$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \quad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \quad (30)$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} y_{T+1} \quad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \quad (31)$$

Anchoring function in the data

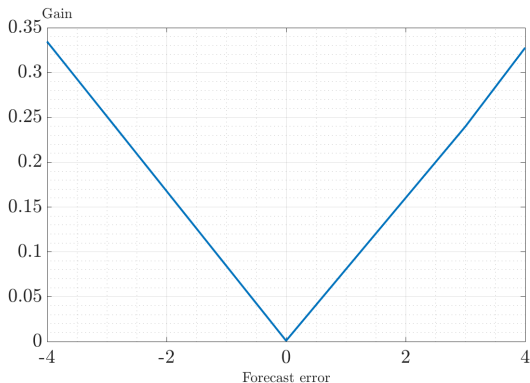


Figure: Learning gain as a function of forecast errors in inflation (pp)

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t) \right. \\ \left. + \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \right\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa}x_t - \varphi_{2,t}(k_t + \mathbf{g}_{\pi}(\pi_t - f_{t-1})) = 0 \quad (32)$$

$$-2\beta\frac{\lambda}{\kappa}x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_f(\pi_{t+1} - f_t)) = 0 \quad (33)$$

Target criterion system for anchoring function as changes of the gain

$$\begin{aligned} \varphi_{6,t} = & -cfe_{t|t-1}x_{t+1} + \left(1 + \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1}) - fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\varphi_{6,t+1} \\ & - \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1})\varphi_{6,t+2} \end{aligned} \quad (34)$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{fe_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{fe_{t|t-1}}\varphi_{6,t+1} \quad (35)$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (35) is given by:

$$\varphi_{6,t} = -2\mathbb{E}_t \sum_{i=0}^{\infty} (\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{fe_{t+j|t+j-1}}}{\frac{k_{t+j}}{fe_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \quad (36)$$

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (37)$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \quad (38)$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \quad (39)$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \quad (40)$$

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \quad (41)$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \quad (42)$$

where ‘hats’ denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (41) backward to some date t , take expectations at t
 2. Sub in (42)
 3. Aggregate over households i
- Obtain (8)

Target criterion

Proposition

In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_{\pi,t} \right)$$
$$\left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\pi,t+j}) \right)$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ at t , $\prod_{j=0}^0 \equiv 1$ and b_1 is the first row of b .

Two layers of intertemporal stabilization tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ - \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j} + fe_{t+1+j|t+j} \mathbf{g}_{\pi,t+j})$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

► Why no commitment?

Corollary

Optimal policy under adaptive learning is time-consistent.

↪ Foreshadow: optimal policy aggressiveness time-varying