Finally back to work! To do 7 Oct 2019

- generate IRFs from all models } Susanto
- think about about slope, b

✓ - change κ (and α) and see if π fluctuates as } Peter
  much as x then

✗ - $\hat{E}$-operator
- θ_t as SR FE
- small π vs. larger x & i → contrast to } Ryan
  rest of Lit to see what mechanism is
  responsible

✓ - think about whether ppl know the TR?

✓ - read Thomas Lubik: Indeterminacy & Learning (JME)
✓ - derive Preston's IS curve → me.

Lubik: has a comment that b/c of anticipated utility (Kreps 1998, Cogley & Sargent 2008), their model can be solved using standard, RE algorithms

→ in Lubik, the CB learns the econ and announces

$i = \psi_\pi \pi_t + \psi_x x_t$ w/ (ψ_π, ψ_x) each period, updated

according to its estimates. Depending on $(\psi_{x,t}, \psi_{\pi,t})$

we can land in the determinacy or indeterminacy region.

Ryan is right: This amounts to ppl not knowing the

TR. And he's doubly right b/c in my setting, they

know the Taylor-rule, the only thing they don't know

are future things.

Let's first try to derive the IS curve in Preston <u>8 Oct 2019</u>

$$\text{HH: } \max \ \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[u(C_T^i; \xi_T) - \int_0^1 v(h_T^i(j), \xi_T)\, dj \right] \quad (1)$$

$$M_t^i + B_t^i \leq (1+i_{t-1}^m) M_{t-1}^i + (1+i_{t-1}) B_{t-1}^i + P_t Y_t^i - T_t - P_t C_t^i \quad (2)$$

$$P_t Y_t^i = \int_0^1 \left[W_t(j) h_t^i(j) + \Pi_t(j) \right] dj$$

So the HH chooses $\{C_t^i(j), h_t^i(j), M_t^i, B_t^i\}$ $\forall j \in [0,1]$ so as to

max (1) s.t. (2), taking as given $\{p_T(j), w_T(j), \Pi_T, i_{T-1}, i_{T-1}^m, \xi_T\}$
$\forall T \geq t$.

In the App, Preston defines $W_{t+1}^i = (1+i_t^m) M_t^i + (1+i_t) B_t^i$ as beginning-of-period wealth at time t+1

$\Rightarrow$ (2) $M_t^i + B_t^i \leq W_t^i + P_t Y_t^i - T_t - P_t C_t^i$

Now let $\Delta_t \equiv \dfrac{i_t - i_t^m}{1+i_t}$ recall: $W_{t+1}^i = (1+i_t^m) M_t^i + (1+i_t) B_t^i$

$$\Leftrightarrow \quad P_t C_t^i + \underbrace{M_t^i + B_t^i}_{(1+i_t^m - i_t^m) M_t^i + \frac{1+i_t}{1+i_t} B_t^i} \leq W_t^i + \left[P_t Y_t^i - T_t\right]$$

$$= -i_t^m M_t^i + (1+i_t^m) M_t^i + \frac{(1+i_t)}{1+i_t} B_t^i$$

$$= -i_t^m M_t^i + \frac{(1+i_t)(1+i_t^m) M_t^i}{1+i_t} + \frac{(1+i_t)}{1+i_t} B_t^i$$

$$= \underbrace{-i_t^m M_t^i + \frac{i_t (1+i_t^m)}{1+i_t} M_t^i}_{= \frac{(1+i_t)(-i_t^m) M_t^i + i_t (1+i_t^m) M_t^i}{1+i_t}} + \underbrace{\frac{(1+i_t^m) M_t^i + (1+i_t) B_t^i}{1+i_t}}_{= \frac{1}{1+i_t} W_{t+1}}$$

$$= \frac{(-i_t^m - i_t i_t^m + i_t + i_t i_t^m) M_t^i}{1+i_t} = \frac{i_t - i_t^m}{1+i_t} M_t^i = \Delta_t M_t^i$$

So $\Rightarrow$ (2) $P_t C_t^i + \Delta_t M_t^i + \dfrac{1}{1+i_t} W_{t+1}^i \leq W_t^i + \left[P_t Y_t^i - T_t\right]$ (32)

Now solve this fwd.

$$W_t^i \geq P_t C_t^i + \Delta_t M_t^i - (P_t Y_t^i - T_t) + \frac{1}{1+i_t} W_{t+1}^i$$

$$\rightarrow W_t^i \geq P_t C_t^i + \Delta_t M_t^i - (P_t Y_t^i - T_t)$$

$$+ \left(\frac{1}{1+i_t}\right)\left[P_{t+1} C_{t+1}^i + \Delta_{t+1} M_{t+1}^i - (P_{t+1} Y_{t+1}^i - T_{t+1})\right]$$

$$+ \dots$$

$$+ \underbrace{\left(\frac{1}{1+i_t}\right) \cdot \dots \cdot \left(\frac{1}{1+i_{t+j-1}}\right)}_{=: R_{t,t+j} = \prod_{s=1}^{j}\left(\frac{1}{1+i_{t+s-1}}\right)}\left[P_{t+j} C_{t+j}^i + \Delta_{t+j} M_{t+j}^i - (P_{t+j} Y_{t+j}^i - T_{t+j})\right]$$

$$W_t^i \geq \sum_{j=0}^{\infty} R_{t,t+j}\left[P_{t+j} C_{t+j}^i + \Delta_{t+j} M_{t+j}^i - (P_{t+j} Y_{t+j}^i - T_{t+j})\right]$$

The flow BC solved fwd to get the intertemporal BC, the IBC.

Let's now take the FOCs using the new flow BC, 32

$$\mathcal{L} = \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t}\left[u(C_T^i, \xi_T) - \int_0^1 v(h_T^i(j), \xi_T)\, dj\right]$$

$$+ \lambda_t\left(-P_t C_t^i - \Delta_t M_t^i - \frac{1}{1+i_t} W_{t+1}^i + W_t^i + \underbrace{[P_t Y_t^i - T_t]}_{w_t^j h_t^j}\right)$$

c 1) $u_c = \lambda_t P_t \rightarrow \lambda_t = \frac{u_c}{P_t}$

h^j 2) $v_{h^j} = \lambda_t P_t w_t^j \Rightarrow \lambda_t = \frac{v_{h^j}}{P_t w_t}$

$$\left.\right\} \frac{u_c}{P_t} = \frac{v_h}{P_t w_t} \Rightarrow \frac{v_h}{u_c} = w_t$$ real wage

B (3) $\lambda_t(1+i_t) = \beta \hat{E}_t \lambda_{t+1} \rightarrow 1+i_t = \beta \hat{E}_t \frac{\lambda_{t+1}}{\lambda_t} = \beta \hat{E}_t \frac{P_t}{P_{t+1}} \frac{u' c_{t+1}}{u' c_t}$

ok so I *almost* get what Preston gets: (difference)

$$\underline{1+i_t} = \beta E_t \frac{P_t}{P_{t+1}} \frac{u_c(c_{t+1})}{u_c(c_t)} \qquad (33)$$

$$\frac{v'_n}{u'_c} = \underline{w_t} \qquad (34)$$

- A cashless econ implies $i_t = i_t^m$ or $M_t^i = 0$

  since $M^s > 0$, we get $i_t = i_t^m \Rightarrow \Delta_t = 0 \;\; \forall t$
- Market clearing implies $y_t(j) = c_t(j) \;\; \forall j \Rightarrow C_t = Y_t$
- Zero debt fiscal policy: $\underbrace{B_t = 0 \Rightarrow T_t = (1+i_{t-1})M_{t-1} - M_t}_{\text{ain't so sure if this is right!}}$

  ↳ thus the IBC becomes

$$W_t^i = \sum_{j=0}^{\infty} R_{t,t+j} \left[P_{t+j} C_{t+j}^i \qquad - (P_{t+j} Y_{t+j}^i - T_{t+j}) \right]$$

$$W_t^i = \sum_{j=0}^{\infty} R_{t,t+j} \left[P_{t+j} C_{t+j}^i - P_{t+j} Y_{t+j}^i - (1+i_{t+j-1}) M_{t+j-1} + M_{t+j} \right]$$

Maybe the point is that there is no diff btwn bonds & money,

so $W_{t+1}^i = (1+i_t)M_t + (1+i_t)B_t = (1+i_t)[M_t + B_t]$

so we can ass that ppl only hold bonds, $M_t = 0$

$\rightarrow W_{t+1}^i = (1+i_t)B_t \quad \rightarrow T_t = 0$ (a little non-kosher but ok)

$$W_t^i \geq \sum_{j=0}^{\infty} R_{t,t+j} \left[P_{t+j} C_{t+j}^i - P_{t+j} Y_{t+j}^i \right]$$

Funnily, the App. stops here.

But it is clear that w/ a specific U-fct ass, loglin of the EE (33) gives rise to (3). But does loglin of the IBC $W_t^i \geq \sum_{j=0}^{\infty} R_{t,t+j} \left[P_{t+j} C_{t+j}^i - P_{t+j} Y_{t+j}^i \right]$ give rise to (4), $\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i = \bar{\omega}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T^i$?

What's the loglin of $R_{t,t+j} = \prod_{s=1}^{j} \left(\frac{1}{1+i_{t+s-1}} \right)$?

$$= \sum_{s=1}^{j} \ln \left(\frac{1}{1+i_{t+s-1}} \right) = \sum_{s=1}^{j} -\ln(1+i_{t+s-1})$$

Take the gross interest rate $1+i_t$ as a unit, call it R_t

$$\Rightarrow \sum_{s=1}^{j} -\ln(R_{t+s-1}) \text{ Total diff } = \sum_{s=1}^{j} -\frac{dR_{t+s-1}}{R} = -\sum_{s=1}^{j} \hat{R}_{t+s-1}$$

where $R = \beta^{-1}$ so $\bar{i} = i^* = \beta^{-1} - 1$

Suppose I write the IBC as

$$W_t^i \geq \sum_{j=0}^{\infty} R_{t,t+j} P_{t+j} \left[C_{t+j}^i - Y_{t+j}^i \right] \quad \Big| : P_t \bar{Y} \qquad \bar{Y} = Y^*$$

$$\bar{\omega}_t^i = \sum_{j=0}^{\infty} R_{t,t+j} \frac{P_{t+j}}{P_t} \left[\frac{C_{t+j}^i}{\bar{Y}} - \frac{Y_{t+j}^i}{\bar{Y}} \right]$$

$$\bar{\omega}_t^i = \sum_{j=0}^{\infty} R_{t,t+j} \Pi_{t,t+j} \left[\hat{C}_{t+j}^i - \hat{Y}_{t+j}^i \right]$$ To get eq (4),

we need the loglin of $R_{t,t+j}\,\pi_{t,t+j}$ to be β^j

Honestly, I don't think there's a way. Or?

$R_{t,t+j}\,\pi_{t,t+j}$ is the sdf. It's just like the firms' sdf in Calvo, $\phi_{t+\tau,t} = \beta \dfrac{P_t C_t}{P_{t+\tau} C_{t+\tau}}$, so then, analogously to there, it cancels and we're left w/ β every time.

→ yes, you can see it from eq (33).

⇒ ok so eq. (4) is good!

The next step is solving (3) backwards.

(3): $\hat{c}_t^i = \hat{E}_t^i \hat{c}_{t+1}^i - \sigma(\hat{i}_t - \hat{E}_t^i \hat{\pi}_{t+1}) + g_t - \hat{E}_t^i g_{t+1}$

$$\hat{E}_t^i \hat{c}_{t+1}^i = \hat{c}_t^i - g_t + \hat{E}_t^i g_{t+1} + \sigma(\hat{i}_t - \hat{E}_t^i \pi_{t+1})$$

$$\hat{E}_t^i \hat{c}_{t+1}^i - \hat{E}_t^i g_{t+1} = \hat{c}_t^i - g_t + \sigma(\hat{i}_t - \hat{E}_t^i \pi_{t+1})$$

Let's call $t+1 = T$

$$\hat{E}_t^i \hat{c}_T^i - \hat{E}_t g_T = \underbrace{\hat{c}_{T-1}^i - g_{T-1}} + \sigma\underbrace{(\hat{i}_{T-1} - \hat{E}_t^i \pi_T)}$$

$$= \hat{c}_{T-2}^i - g_{T-2} + \sigma(\hat{i}_{T-2} - \hat{E}_t^i \pi_{T-1}) + \cdots$$

$$= \ldots \hat{c}_t^i - g_t + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \quad \checkmark$$

$$\hat{E}^i_t \hat{C}^i_T = \hat{E}^i_t g_T + \hat{c}^i_t - g_t + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1})$$

$$= \hat{c}^i_t - g_t + \hat{E}_t \left[g_T + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \right]$$

So now sub $\hat{E}^i_t \hat{C}^i_T$ (I guess) into the IBC, eq (4)

$$\hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}^i_T = \varpi^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}^i_T$$

$$\Rightarrow \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \hat{c}^i_t - g_t + \hat{E}_t \left[g_T + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \right] \right\}$$

$$= \tilde{\omega}^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}^i_T$$

$$\Leftrightarrow \hat{c}^i_t - g_t = \tilde{\omega}^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}^i_T - g_T - \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \right]$$

Conjecture: $\sum_{T=t}^{\infty} \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) = \sum_{T=t}^{\infty} (\hat{i}_T - \hat{\pi}_{T+1})$.

Why? B/c $\sum_{T=t}^{\infty} \sum_{s=t}^{T-1} \text{stuff}_s = \sum_{s=t}^{t-1} \text{stuff}_s + \sum_{s=t}^{t} \text{stuff}_s$

So what if I don't sub the (3) solved fwd into (4), but just (3)?

(3) $\hat{C}^i_t = \hat{E}^i_t \hat{C}^i_{t+1} - \sigma(\hat{i}_t - \hat{E}^i_t \hat{\pi}_{t+1}) + g_t - \hat{E}^i_t g_{t+1}$

(4) $\hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}^i_T = \tilde{\omega}^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}^i_T$

$\rightarrow \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{C}^i_{T+1} - \sigma(\hat{i}_T - \hat{\pi}_{T+1}) + g_T - g_{T+1} \right] = \tilde{\omega}^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}^i_T$

that doesn't seem to do it at all!

Let's pause the issue of eq (5). Supp. we have it.

- Then integrate over i $\rightarrow$ $\tilde{\omega}_t \Rightarrow 0$

  $C_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\hat{Y}_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta(g_T - g_{T+1}) \right]$.

- $\hat{Y}_t = \hat{C}_t$

  $\hat{Y}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\hat{Y}_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta(g_T - g_{T+1}) \right]$.

- $x_t := \hat{Y}_t - \hat{Y}^n_t$

  $x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)x_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta(g_T - g_{T+1}) - (1-\beta)\hat{Y}^n_T \right]$

  $- \hat{Y}^n_t$ even that doesn't work out! damn!

Ryan meeting 8 Oct 2019

- Invite Eusepi / Preston for macro seminar? Who organizes it?
  - ↳ Ryan, Susanto & Pablo for the spring, Rosen, Fabio & Jaromir for fall

Ryan said that Susanto connects b (slope) w/ Missing Deflation b/c b transmits shocks to SR fcsts

Work after: 8 Oct 2019

So supp my previous conjecture is right. Then

$$\hat{c}_t^i - g_t = \tilde{\omega}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}_T^i - g_T + \sigma(\hat{i}_T - \hat{\pi}_{T+1}) \right]$$

and I still don't get (5).

But if I now do the last 3 steps:

$$\underbrace{\hat{Y}_t - \hat{Y}_t^n}_{} \underbrace{- g_t + \hat{Y}_t^n}_{} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}_T - \hat{Y}_T^n - g_T + \hat{Y}_T^n + \sigma(\hat{i}_T - \hat{\pi}_{T+1}) \right]$$

$$\Rightarrow \quad x_t + r_t^n = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[x_t \right.$$

↳ no, the problem is that that ain't right b/c

$$r_t^n = Y_{t+1}^n - Y_t^n \quad \left[+ (g_t - g_{t+1}) \text{ if we have this preference shock} \right]$$

↳ and for me, $r_t^n = \frac{1}{\sigma}(Y_{t+1}^n - Y_t^n)$, in line w/ Basu, SUM part 2, p.58)

So let's set all $g_t = 0 \ \forall t$ b/c I don't have it.

→ Then the EE is $\hat{c}_t^i = \hat{E}_t^i \hat{c}_{t+1} - \sigma(\hat{i}_t - \hat{E}_t^i \hat{\pi}_{t+1})$

Solve backwd: $\hat{E}_t c_{t+1}^i = \hat{c}_t^i + \sigma(\hat{i}_t - \hat{E}_t^i \pi_{t+1})$

$$\hat{E}_t c_T^i = \hat{c}_{T-1}^i + \sigma(\hat{i}_{T-1} - \hat{E}_t^i \pi_T)$$

$$\hat{E}_t \hat{C}^i_T = \sigma(\hat{i}_{T-1} - \hat{E}^i_t \pi_T) + \sigma(\hat{i}_{T-2} - \hat{E}^i_t \pi_{T-1}) + \dots$$
$$+ \sigma(\hat{i}_t - \hat{E}_t \pi_{t+1}) + \hat{c}^i_t$$

$$\hat{E}^i_t \hat{C}^i_T = \hat{c}^i_t + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{E}_t \pi_{s+1})$$ (EE $^{\text{simple}}_{\text{solved bwd}}$)

Sub this into (4)

$$\hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{C}^i_t + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{E}_t \pi_{s+1})\right] = \tilde{\omega}^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}^i_T$$

$$\Leftrightarrow \hat{C}^i_t \sum_{T=t}^{\infty} \beta^{T-t} + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \sigma \sum_{s=0}^{T-1} (\hat{i}_s - \hat{E}_t \pi_{s+1})] = \tilde{\omega}^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}^i_t$$

$$\Leftrightarrow \underset{\checkmark}{\hat{c}^i_t} + \underbrace{(1-\beta)\sigma \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \sum_{s=t}^{T-1} (\hat{i}_s - \hat{E}_t \pi_{s+1})} = \underset{\checkmark}{(1-\beta)\tilde{\omega}^i_t} + \underset{\checkmark}{\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} (1-\beta) \hat{Y}^i_T}$$

now I only need this to equal $\beta\sigma(\hat{i}_T - \hat{\pi}_{T+1})$

But potentially, it should actually be $(1-\beta)(\sigma?)$

Now I'm only missing the $\hat{i}_T - \hat{\pi}_{T+1}$ term <u>9 Oct 2019</u>

in eq (5). So supp. again that I have eq (5), w/o g_s,

and I do the last 3 steps:

· aggregate, set $C_t = Y_t$ and am about to get $x_t = \hat{Y}_t - \hat{Y}^n_t$

$$\hat{Y}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\hat{Y}_T - \beta\sigma(\hat{i}_T - \pi_{T+1})\right]$$

$$\hat{Y}_t - \hat{Y}_t^n = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\hat{Y}_T - \beta b(\hat{i}_T - \hat{\pi}_{T+1}) \right] - \hat{Y}_t^n$$

$$(*)\quad x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)[\hat{Y}_T - \hat{Y}_T^n] - \beta b(\hat{i}_T - \hat{\pi}_{T+1}) + (1-\beta)\hat{Y}_T^n \right] - \hat{Y}_t^n$$

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)(\hat{Y}_{T+1} - \hat{Y}_{T+1}^n) - \beta b(\hat{i}_t - \hat{\pi}_{T+1}) + (1-\beta)\hat{Y}_{T+1}^n \right]$$
$$- \hat{Y}_t^n + (1-\beta)\hat{Y}_t$$

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \beta b(\hat{i}_T - \hat{\pi}_{T+1}) \right]$$
$$+ \underline{x_t - \beta \hat{Y}_t + (1-\beta)\hat{Y}_{t+1}^n + (1-\beta)\beta \hat{Y}_{t+2}^n + (1-\beta)\beta^2 \hat{Y}_{t+3}^n} + \ldots$$

$$\underbrace{x_t - \beta \hat{Y}_t + Y_{t+1}^n}\ \underbrace{-\beta \hat{Y}_{t+1}^n + \beta \hat{Y}_{t+2}^n}_{\beta r_{t+1}^n}\ \underbrace{-\beta^2 \hat{Y}_{t+2}^n + \beta^2 \hat{Y}_{t+3}^n}_{+\ \beta^2 \hat{r}_{t+2}}\ \underbrace{-\beta^3 \hat{Y}_{t+3}^n}_{+\ \ldots\ \text{cool}} + \ldots$$

$$\hat{Y}_t - \hat{Y}_t^n - \beta \hat{Y}_t + \hat{Y}_{t+1}^n$$
$$= \hat{Y}_t - \beta \hat{Y}_t + r_t^n$$
$$= (1-\beta)\hat{Y}_t + r_t^n$$

grr... let's retry, from step (*)

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)[\hat{Y}_T - \hat{Y}_T^n] - \beta b(\hat{i}_T - \hat{\pi}_{T+1}) + (1-\beta)\hat{Y}_T^n \right] - \hat{Y}_t^n$$

$$x_t = \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_T - \beta b(\hat{i}_T - \hat{\pi}_{T+1}) \right]$$
$$- \hat{Y}_t^n + (1-\beta)\hat{Y}_t^n + (1-\beta)\beta \hat{Y}_{t+1}^n + (1-\beta)\beta^2 \hat{Y}_{t+2}^n + \ldots$$

The last series is $-\hat{Y}_t^n + (1-\beta)\hat{Y}_t^n + (1-\beta)\beta\hat{Y}_{t+1}^n + (1-\beta)\beta^2\hat{Y}_{t+2}^n + \dots$

$$= \underbrace{-\beta\hat{Y}_t^n + \beta\hat{Y}_{t+1}^n}\ \underbrace{-\beta^2\hat{Y}_{t+1}^n + \beta^2\hat{Y}_{t+2}^n}\ - \beta^3\hat{Y}_{t+2}^n + \dots$$

$$\beta r_t^n + \beta^2 r_{t+1}^n + \dots$$

$$= \beta\left[r_t^n + \beta r_{t+1}^n + \beta^2 r_{t+2}^n + \dots\right]$$

$$= \beta\sum_{T=0}^{\infty}\beta^{T-t} r_T^n$$

now this guy is superfluous ...

and we have x_T, not x_{T+1} in the Σ, i.e.

$$x_t = \sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta)x_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta r_T^n\right] \quad \Big| -x_t$$

$$\beta x_t = \sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta)x_{T+1} - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta r_T^n\right]$$

$$x_t = \sum_{T=t}^{\infty}\beta^{T-t}\left[\frac{1-\beta}{\beta}x_{T+1} - \sigma(\hat{i}_T - \hat{\pi}_{T+1}) + r_T^n\right]$$

hát ezt nem hiszem el ... mostmár minden stimmel csak ez nem ...

(Még az $r_t^n = \sigma r_t^n$ is jó, ha átdefiniálom $r_t^n = \frac{1}{\sigma}(\hat{Y}_{t+1}^n - \hat{Y}_t^n)$ mint Basu Sem 2 p. 58)

So at the current point in time I'm inclined to say that if eq.(5) is right, then eq.(6) is too, except the coefficient of x_{T+1} should be $\frac{1-\beta}{\beta}$, not $1-\beta$.

So we need to turn back to deriving eq (5) from (3).

$$\hat{c}_t^i = \hat{E}_t c_{t+1}^i - \sigma(i_t^i - \hat{E}_t^i \pi_{t+1}) \qquad (3)$$

Drop i's and hats for simplicity:

$$c_t - E_t c_{t+1} = -\sigma(i_t - E_t \pi_{t+1})$$

$$(1 - L^{-1}) c_t = -\sigma(i_t - E_t \pi_{t+1})$$

$$\text{or} \quad -(1 - L) E_t c_{t+1} = -\sigma(i_t - E_t \pi_{t+1})$$

$$\Leftrightarrow \quad (1 - L) E_t c_\tau = \sigma(i_\tau - E_t \pi_{\tau+1})$$

$$E_t c_\tau = (1 - L)^{-1} \sigma(i_\tau - E_t \pi_{\tau+1})$$

See rotemberg_pricing_peter_
_mynotes.pdf.

not trivial

$$= \sigma \sum_{i=0}^{\infty} 1^i \text{ RHS} = \sigma \sum_{T=t}^{\tau-1} (i_T - E_t \pi_{T+1})$$

I think this is pretty much what I got before, and it's also in accordance w/ Preston's (4.5)

Plugging this into the LHS of (4), we have

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\sigma \sum_{s=t}^{T-1} (i_s - \hat{E}_t \pi_{s+1}) \right]$$

$$= \sigma \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \sum_{s=t}^{T-1} (i_s - \hat{E}_t \pi_{s+1})$$

$$= \sum_{s=t}^{t-1} \text{stuff} + \beta \sum_{s=t}^{t} + \beta^2 \sum_{s=t}^{t+1} + \beta^3 \sum_{s=t}^{t+2}$$

That don't look good.

$$= \mathfrak{z}\, \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \sum_{s=t}^{T-1} \left(i_s - \hat{E}_t \pi_{s+1} \right)$$

Let's try to redefine the indices

$$\mathfrak{z}\, \hat{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{s=0}^{t-1} \text{stuff}_s$$

$$= \mathfrak{z} \cdot \Big[\underset{t=0}{0} + \underset{t=1}{\beta \cdot \sum_{s=0}^{0}} + \underset{t=2}{\beta^2 \cdot \sum_{s=0}^{1}} \quad \ldots$$ mmm... :S

Also I kinda feel that even if I don't take time t, but time 0 expectations, I can't get around the second sum...

$$= \mathfrak{z} \left[\beta \cdot \text{stuff}_0 + \beta^2 \text{stuff}_0 + \beta^2 \text{stuff}_1 + \beta^3 \text{stuff}_0 + \beta^3 \text{stuff}_1 + \beta^3 \text{stuff}_2 + \ldots \right.$$

$$= \mathfrak{z} \left[(\beta + \beta^2 + \ldots) \text{stuff}_0 + (\beta^2 + \beta^3 + \ldots) \text{stuff}_1 + \text{etc.} \right.$$ | Ignore $\mathfrak{z}$

$$= \beta (1 + \beta^2 + \beta^3 + \ldots) \text{stuff}_0 + \beta^2 (1 + \beta^2 + \beta^3 + \ldots) \text{stuff}_1 + \ldots$$

$$= (\beta^1 \text{stuff}_0 + \beta^2 \text{stuff}_1 + \beta^3 \text{stuff}_2 + \ldots) \frac{1}{1-\beta} \quad (!)$$

$$= (\beta^0 \text{stuff}_0 + \beta^1 \text{stuff}_1 + \beta^2 \text{stuff}_2 + \ldots) \frac{\beta}{1-\beta} \quad (!)$$

So the LHS is

$$\mathfrak{z}\, \hat{E}_t \sum_{T=t}^{\infty} \frac{\beta}{1-\beta} \left(i_T - \hat{E}_t \pi_{T+1} \right)$$ Holy cow!

So (5) is

$$\frac{1}{1-\beta} c_t + \sigma \hat{E}_t \sum_{T=t}^{\infty} \frac{\beta}{1-\beta} (i_T - \hat{E}_t \pi_{T+1}) = \hat{E}_t \sum_{T=t}^{\infty} \hat{Y}_T + \bar{w}_t$$

$$\Rightarrow \hat{c}_t^i + \sigma \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \beta (i_T - \hat{E}_t \pi_{T+1}) = (1-\beta)\bar{w}_t + \hat{E}_t \sum_{T=t}^{\infty} (1-\beta) \hat{Y}_T$$

So ...

$$\hat{c}_t^i = (1-\beta)\bar{w}_t^i + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\hat{Y}_T - \beta\sigma (i_T - \hat{E}_t \pi_{T+1}) \right]$$

= Preston's (5). Yay!

The last 3 steps again: aggregate and set $\hat{c}_t = \hat{Y}_t$, drop hats.

$$Y_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) Y_T - \beta\sigma (i_T - \pi_{T+1}) \right]$$

$$Y_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)(Y_T - Y_T^n) - \beta\sigma (i_T - \pi_{T+1}) + (1-\beta) Y_T^n \right]$$

$$Y_t = \underbrace{\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \beta\sigma (i_T - \pi_{T+1}) \right]}_{=: CS} + (1-\beta) x_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} (1-\beta) Y_T^n$$

$$x_t = \text{correct-stuff} - Y_t^n + (1-\beta) x_t + (1-\beta)\left[Y_t^n + \beta Y_{t+1}^n + \beta^2 Y_{t+2}^n + \dots \right.$$

$$\beta x_t = CS \cancel{- Y_t^n} \cancel{+ Y_t^n} - \beta Y_t^n + \beta Y_{t+1}^n - \beta^2 Y_{t+1}^n + \beta^2 Y_{t+2}^n - \beta^3 Y_{t+2}^n + \dots \left. \right]$$

$$\beta x_t = CS + \beta r_t^n + \beta^2 r_{t+1}^2 + \dots$$

$$x_t = \frac{1}{\beta} CS + r_t^n + \beta r_{t+1}^2 + \dots$$

σr_T^N w/ my def.

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\frac{1-\beta}{\beta} x_{T+1} - \sigma (i_T - \pi_{T+1}) + r_T^N \right].$$

Wait... can it be that I did one thing wrong:

when I have $\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta) x_T] = (1-\beta) \hat{E}_t \sum_{T=t}^{T-t} x_T$

$$= (1-\beta) x_t + (1-\beta) \hat{E}_t [\beta x_{t+1} + \beta^2 x_{t+2} + \dots]$$

$$= (1-\beta) x_t + (1-\beta) \hat{E}_t \left[\beta \sum_{T=t}^{\infty} \beta^{T-t} x_{T+1} \right]$$

Yeah!

So that means that my truly correct CS is:

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta) \beta x_{T+1} - \beta \sigma (i_T - \pi_{T+1})]$$

so that when I take $\frac{1}{\beta}$ CS I obtain

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta) x_{T+1} - \sigma(i_T - \pi_{T+1}) + \sigma r_T^N]$$

which is Preston's (6) = (18), and the equation I had in my DW prezi and in materials 4. Yay!

Great! We can go on to investigating the expectation-operator, $\hat{E}_t$. Obs. 1. Preston took derivatives thru it no problem, w/o saying anything. So let's go to Evans & Honkapohja (2001).

Evans & Honk (2001) on the nonrational expectation-operator $\hat{E}$

- p. 68-69 Ramsey model: they derive them at no point.
- Minimum-state-variable (MSV) solution (introduced in McCallum (1983)
- heterogeneous learning p. 223-225 (Evans, Honk & Marimon 2000)
- "regular" vs "irregular" models (Farmer 1999, Pesaran 1987)
  ↳ unique REE (Blanchard-Kahn conditions)
- RE is an eqb concept p. 11
- Expectation is a fct of observables, and so is the updating p. 17-18 → I think it's fair to say that we conjecture $\hat{E}$ to be linear and of the form $\bar{\pi} + bs_t$, and $\bar{\pi}_t = Q(\bar{\pi}_{t-1}, \theta_t)$ nonlinear updating rule and to somehow verify this using undet. coeffs.

Reading the chapter on Nonlinear models 10 Oct 2019
(Chapter 11)

Let the nonlinear univariate model be of the form

$$y_t = \underset{\substack{\uparrow \\ \text{nonlinear}}}{F(y_{t+1})^e} + \underset{\substack{\uparrow \\ \text{shock}}}{v_t} \qquad (11.2)$$

shock $v_t \sim \text{iid}(0, \cdot)$ if $v_t = 0$

$$= E_t\left[F(y_{t+1})\right] \qquad (11.2) = (11.1)$$

A more general nonlinear model is

$$y_t = H\left(G(y_{t+1}, v_{t+1})^e, v_t\right) \qquad (11.3)$$

[Here's an argument (p. 273) for linearity:

1. Supp agents don't know $G(y_{t+1}, v_{t+1})^e$, but have data on its past values $G(y_j, v_j)$ $j = 1, \dots, t$.
2. A natural estimator for $G(y_{t+1}, v_{t+1})^e$ is the sample mean: $\theta_t = \frac{1}{t}\sum_{j=1}^{t} G(y_j, v_j)$ which is then updated using RLS as more data becomes available.
3. The sample mean is a linear operator.

[Peter's argument for differentiating them:

- E is an integral over states
- Differentiation proceeds over different variables, not states.

By the way, Evans & Honk derive them $\hat{E}$ all over the place here w/o remarks

- That's all Evans & Honk had to say about $\hat{E}$.
- Liam Graham also simply differentiates thru $\tilde{E}$ (Macp 5)

Moving on: <u>small π, large x</u>

- well, setting $\alpha = 1$ and $\kappa = 1$ still <u>doesn't</u> get π to move as much as x (and thus i). So what's going on?
- now what I did is on top of $(\alpha = 1, \kappa = 1)$, I shut off all shocks except u_t, so now only π is affected. Now the magnitude of the responses is the same (almost)
  - ↳ But even then, even then x & i fluctuate more!
- shocks shut off except u, α back to 0.5
  - ⇒ same as w/ $\alpha = 1$ ⇒ α doesn't matter
- -11-, κ back to 0.51.
  - ⇒ κ matters a <u>lot</u>: this really brings in the gap between π and x.

Let's note a couple of things

1. You'd think that the higher the shock volatilities, the more unanchoring happens. But that is only partly so:
   - If I shut off $\bar{\imath}$-shocks, c.p., I'm always anchored
   - If I shut off the other shocks, c.p., I'm unanchored

→ What's going on?

3. What matters for the size of gaps in π & x is
   - κ → but this doesn't explain it all

     ↳ Why?
   - size of shocks DOESN'T really matter, α doesn't matter

→ Something else going on?

4. CEMP say about the criterion θ_t that it's a reduced-form way to capture model misspecification tests

   ↳ they simulate a calibrated version where firms instead employ a CUSUM-test of shifting means (Brown, Durbin, Evans 1975) (see CEMP p. 18)

   ↳ get nearly identical results but w/ more parameters to estimate.

Analysis of the issues:

1) κ = slope of NKPC $\quad \pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t$

- If $\kappa = 0$, no rel. btwn π & x $\Rightarrow$ money is strongly non-neutral, x persistently $\neq 0$.
- If $\kappa \to \infty$, then flex prices: changes in x translate immediately and a <u>lot</u> into π $\Rightarrow$ money is neutral

Note that $\frac{\partial \kappa}{\partial \alpha} < 0$ $\Rightarrow$ If firms are stuck w/ a price for longer ($\alpha \uparrow$), then $\kappa \downarrow$ and money becomess less neutral!

- So if $\alpha \uparrow$ (from 0.5 to 1) $\rightarrow$ we get anchoring where we previously didn't
- If $\alpha \downarrow$ (from 0.5 to 0.1) $\rightarrow$ we get deanchoring and the gaps in π are indeed huger

$\Rightarrow$ the lower α (higher κ), the more deanchoring we get and the bigger the gaps in π are relatively to x

$\Rightarrow$ I think what is happening is that when α high (κ low), we have a lot of price stickiness ⇒ so inflation cannot respond a lot to shocks, so the margin of adjustment is x

↳ we need sufficiently flex. prices for inflation to move away from its RE path.

⇒ cool, now we understand that fairly well SOLVED

⇒ a RESULT:

the more price stickiness in a learning world w/ anchoring, the more deanchored periods will show up in output gaps instead of inflation b/c x will be the margin of adjustment!

Let's do a survey of lit on learning w/ price stickiness to see if we have anything similar

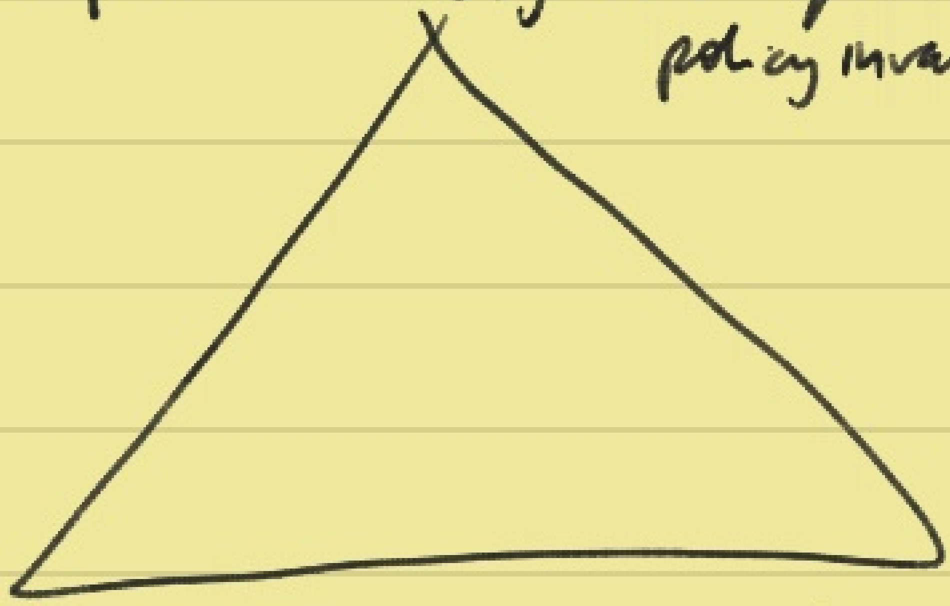
- CEMP: can't adress b/c no x in model
- Graham: no inflation in model
- Preston: doesn't do simulations

- Eusepi et al, Limits: doesn't get it
- Ferrero (2007): has π & x, but doesn't seem to discuss
  the relative role of these.
- Orphanides & Williams (2004) is a little difficult to compare
  b/c the model isn't an NK model
  → there doesn't seem to be a clear mapping btwn
  a parameter and price stickiness (maybe α?)
  They do have a related result: when inflation inertia
  is low (≈ flex prices) then RLS w/ a constant gain
  produces very bad fcsts ⇒ i.e. π moves far
  from π^{RE}.

Angeletos 10 Oct 2019

Eusepi Preston

His main point: Dogmatic agents: econ & beliefs are policy invariant



adaptive learning

bayesian learning

1) Why isn't dogmatic beliefs enough? (i.e. why learning?)

→ b/c w/ dogmatic E, E are policy invariant!

Mon. pol. might depart from TR to correct E, but can't influence E.

2) why adaptive and not Bayesian learning?

It might not matter?

And lastly: what evidence is there in the data for learning?

Send him paper when you have it!