Materials 2

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Overview

1 A CEMP-Preston mix

Suppose we have a NK model with LR forecasts being relevant, as in Preston (2005):

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{Preston, eq. (18)}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{Preston, eq. (19)}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \tag{Preston, eq. (27)}$$

where I've 1) added σ in front of r_T^n, reflecting the derivation of the shock on the NKIS; 2) added u_T, a cost-push shock to the NKPC.

I'm assuming that the innovations can be summarized as:

$$s_t = P s_{t-1} + \epsilon_t \tag{1}$$

$$\text{where} \quad s_t \equiv \begin{pmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{pmatrix} \quad P \equiv \begin{pmatrix} \rho_r & 0 & 0 \\ 0 & \rho_i & 0 \\ 0 & 0 & \rho_u \end{pmatrix} \quad \epsilon_t \equiv \begin{pmatrix} \varepsilon_t^r \\ \varepsilon_t^i \\ \varepsilon_t^u \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_i & 0 \\ 0 & 0 & \sigma_u \end{pmatrix} \tag{2}$$

Let z_t summarize the endogenous variables as

$$z_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \tag{3}$$

Then I can write system (1), (18), (19) and (27) compactly as

$$z_t = A_1 f_a + A_2 f_b + A_3 s_t \tag{4}$$

$$s_t = P s_{t-1} + \epsilon_t \tag{5}$$

where f_a and f_b capture discounted long-run expectations of the endogenous states z, and matrices A_1, A_2 and A_3 gather coefficients as:

$$f_a \equiv \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} z_{T+1} \tag{6}$$

$$f_b \equiv \sum_{T=t}^{\infty} (\beta)^{T-t} z_{T+1} \tag{7}$$

$$A_1 = \begin{pmatrix} g_{\pi a} \\ g_{xa} \\ \psi_\pi g_{\pi a} + \psi_x g_{xa} \end{pmatrix} \quad A_2 = \begin{pmatrix} g_{\pi b} \\ g_{xb} \\ \psi_\pi g_{\pi b} + \psi_x g_{xb} \end{pmatrix} \quad A_3 = \begin{pmatrix} g_{\pi s} \\ g_{xs} \\ \psi_\pi g_{\pi s} + \psi_x g_{xs} + \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \end{pmatrix} \tag{8}$$

$$g_{\pi a} = (1 - \frac{\kappa\sigma\psi_\pi}{w}) \left[(1-\alpha)\beta, \kappa\alpha\beta, 0\right] \tag{9}$$

$$g_{xa} = \frac{-\sigma\psi_\pi}{w} \left[(1-\alpha)\beta, \kappa\alpha\beta, 0\right] \tag{10}$$

$$g_{\pi b} = \frac{\kappa}{w} \left[\sigma(1-\beta\psi_\pi), (1-\beta-\beta\sigma\psi_x, 0\right] \tag{11}$$

$$g_{xb} = \frac{1}{w} \left[\sigma(1-\beta\psi_\pi), (1-\beta-\beta\sigma\psi_x, 0\right] \tag{12}$$

$$g_{\pi s} = (1 - \frac{\kappa\sigma\psi_\pi}{w}) \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (I_3 - \alpha\beta P)^{-1} - \frac{\kappa\sigma}{w} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} (I_3 - \beta P)^{-1} \tag{13}$$

$$g_{xs} = \frac{-\sigma\psi_\pi}{w} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (I_3 - \alpha\beta P)^{-1} - \frac{\sigma}{w} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} (I_3 - \beta P)^{-1} \tag{14}$$

$$w = 1 + \sigma\psi_x + \kappa\sigma\psi_\pi \tag{15}$$

This is where the CEMP bit comes in: let agents form forecasts according to the relation

$$\hat{\mathbb{E}}_t z_{t+1} = \bar{z}_t + \underbrace{C}_{\text{TBD}} s_t + e_{t+1} \tag{PLM}$$

where $\bar{z}_t$ is the LR expectation of all endogenous variables. CEMP would love if we called this the "drift" in beliefs. Let this drift evolve according to CEMP's criterion as:

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} f_{t-1} \tag{16}$$
$$f_{t-1} = z_{t-1} - \hat{\mathbb{E}}_{t-2} z_{t-1} \quad \text{(short-run forecast error)} \tag{17}$$
$$k_t = \mathbb{I}(k_{t-1}+1) + (1-\mathbb{I})\bar{g}^{-1} \tag{18}$$
$$\mathbb{I} = \begin{cases} 1 & \text{if} \quad \theta_t \leq \bar{\theta} \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$
$$\text{where} \quad \theta_t = |\hat{\mathbb{E}}_{t-1} z_t - \mathbb{E}_{t-1}\, z_t| / (\sigma_r + \sigma_i + \sigma_u) \quad \text{(subjective - objective forecast)} \tag{20}$$

Anticipated utility:

$$\hat{\mathbb{E}}_t \bar{z}_T = \bar{z}_t \; \forall \; T > t \Rightarrow \bar{z}_{T|t} = \bar{z}_t \; \forall \; T > t \tag{21}$$

1.1 Deriving the ALM

To get the ALM, we need to write the expectations f_a, f_b based on the PLM. Subbing in the PLM and using the anticipated utility assumption, I get

$$f_a = \frac{1}{1-\alpha\beta} \bar{z}_t + C(I_3 - \alpha\beta P)^{-1} s_t \tag{22}$$
$$f_b = \frac{1}{1-\beta} \bar{z}_t + C(I_3 - \beta P)^{-1} s_t \tag{23}$$

Then the ALM is expression (4), with expectations evaluated using (22) and (23):

$$z_t = \underbrace{\left(A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} \right)}_{\equiv B_1} \bar{z}_t + \underbrace{\left(A_1 C (I_3 - \alpha\beta P)^{-1} + A_2 C (I_3 - \beta P)^{-1} + A_3 \right)}_{\equiv B_2} s_t \tag{ALM}$$
$$z_t = B_1 \bar{z}_t + B_2 s_t \tag{24}$$

1.2 SR forecast error and the criterion

$$f_{t-1} = z_{t-1} - \hat{\mathbb{E}}_{t-2} z_{t-1} \quad \text{(short-run forecast error: ALM - PLM)}$$
$$\theta_t = |\hat{\mathbb{E}}_{t-1} z_t - \mathbb{E}_{t-1}\, z_t| / (\sigma_r + \sigma_i + \sigma_u) \quad \text{(criterion: PLM - } \mathbb{E}_{t-1}\text{ALM)}$$
$$\Rightarrow \quad f_{t-1} = B_1 \bar{z}_{t-1} - \bar{z}_{t-2} + B_2 s_{t-1} - C s_{t-2}$$
$$(\sigma_r + \sigma_i + \sigma_u)\theta_t = |(I_3 - B_1)\bar{z}_{t-1} + (I_3 - B_2 P) s_{t-1}|$$

1.3 Model summary

$$z_t = B_1 \bar{z}_t + B_2 s_t \tag{ALM}$$

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} f_{t-1} \tag{Drift LOM}$$

$$f_{t-1} = B_1 \bar{z}_{t-1} - \bar{z}_{t-2} + B_2 s_{t-1} - C s_{t-2} \tag{SR fcst error}$$

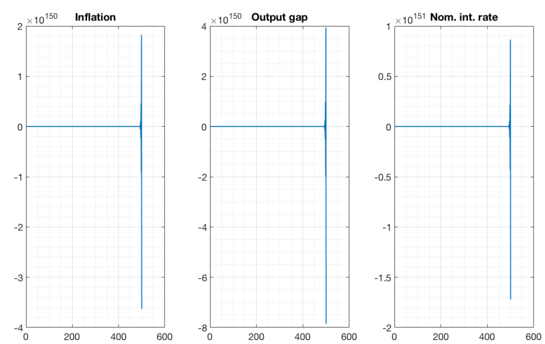
$$k_t = \mathbf{f_k}(\bar{z}_{t-1}, k_{t-1}, s_{t-1}) \quad \text{where } \mathbf{f_k} \text{ evaluates the criterion } \theta_t \tag{Gain LOM}$$

$$(\sigma_r + \sigma_i + \sigma_u)\theta_t = |(I_3 - B_1)\bar{z}_{t-1} + (I_3 - B_2 P) s_{t-1}| \tag{criterion}$$

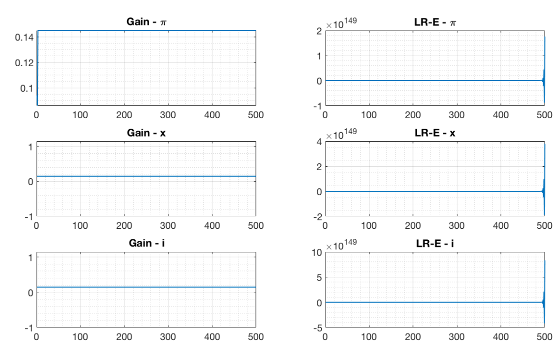
$$\mathbf{f_k} = \mathbb{I}_{\theta_t \le \bar{\theta}}(k_{t-1} + 1) + (1 - \mathbb{I}_{\theta_t \le \bar{\theta}})\bar{g}^{-1} \tag{anchoring}$$

$$s_t = P s_{t-1} + \epsilon_t \tag{exog. process}$$

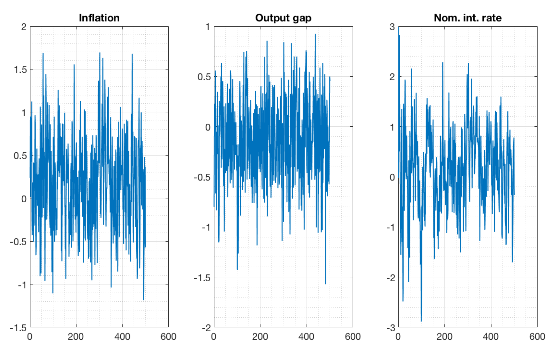
2 Two initial simulations



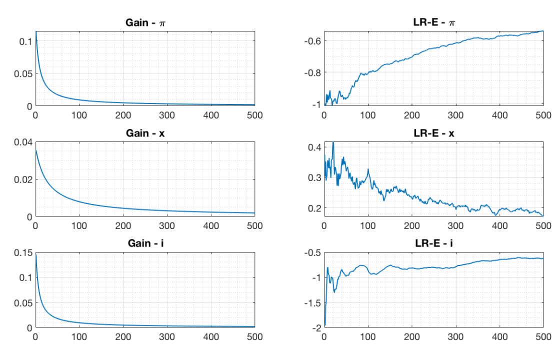
(a) Observables 1



(b) Nonlinear states 1



(c) Observables 2



(d) Nonlinear states 2

Two (potentially connected) issues:

- $\bar{\theta}$ needs to be quite huge for gain to decrease (20 instead of CEMP's 0.029)
- Nonlinear states? Based on CEMP's def of $\bar{\pi}$ being a nonlinear state because the gain is a nonlinear function of it, here $\bar{z}$ and s are both nonlinear states.