

# Monetary Policy & Anchored Expectations

## An Endogenous Gain Learning Model

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*Policymakers came out of the Great Inflation era with a clear understanding that it was essential to anchor inflation expectations at some low level.*

*Jerome Powell, Chairman of the Federal Reserve <sup>1</sup>*

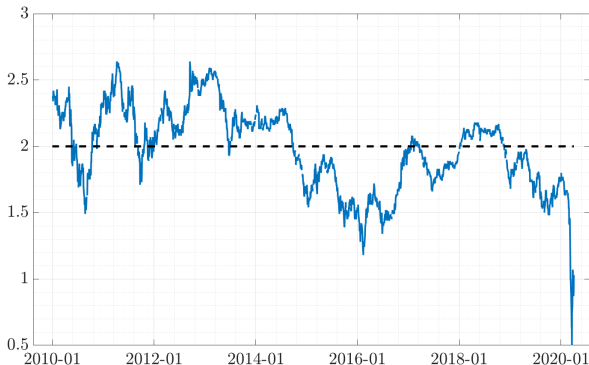


Figure: Market-based inflation expectations, 10 year, average, %

<sup>1</sup>Federal Reserve “Challenges for Monetary Policy,” August 23, 2019.

# This project

- How to conduct optimal monetary policy in interaction with the anchoring expectation formation?
- Model of anchoring expectation formation as an endogenous gain adaptive learning scheme
- Estimation of the anchoring function: when do expectations become unanchored?

# Preview of results

- 1 pp forecast error unanchors expectations
  - Optimal monetary policy responsiveness time-varying
- ↪ Unanchored expectations introduce intertemporal stabilization and volatility tradeoffs, latter matters more
- ↪ Optimal policy aggressive when expectations unanchor, dovish when anchored

# Related literature

- **Optimal monetary policy in New Keynesian models**

Clarida, Gali & Gertler (1999), Woodford (2003)

- **Econometric learning**

Evans & Honkapohja (2001, 2006), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Eusepi & Preston (2011), Milani (2007, 2014), Lubik & Matthes (2018), Mele et al (2019)

- **Anchoring and the Phillips curve**

Sargent (1999), Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Reis (2020), Gobbi et al (2019), Carvalho et al (2019)

# Structure of talk

1. Unanchoring in the data
2. Model of anchoring expectations
3. Solving the Ramsey problem
4. Implementing optimal policy

# Unanchoring in the data

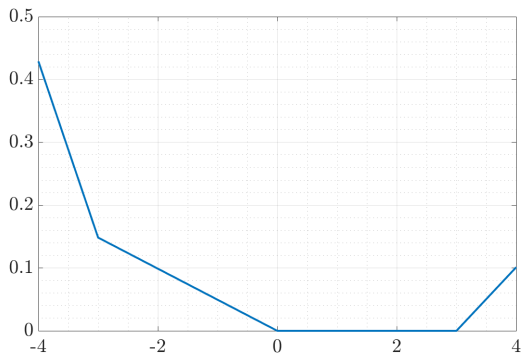


Figure: Learning gain as a function of forecast errors in inflation (pp)

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# Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ U(C_T^i) - \int_0^1 v(h_T^i(j)) dj \right] \quad (1)$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_t C_t^i \quad (2)$$

► Consumption, price level

# Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[ \Pi_t^j(p_t(j)) \right] \quad (3)$$

subject to demand

$$y_t(j) = Y_t \left( \frac{p_t(j)}{P_t} \right)^{-\theta} \quad (4)$$

► Profits, stochastic discount factor

## Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- If use  $\mathbb{E}$  (rational expectations, RE)

Model solution

$$s_t = hs_{t-1} + \epsilon_t \quad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \quad (5)$$

$$y_t = gs_t \quad (6)$$

$s_t \equiv$  states

$y_t \equiv$  jumps

$\epsilon_t \equiv$  disturbances

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- If use  $\hat{\mathbb{E}} \rightarrow$  private sector does not know (6)

$\hookrightarrow$  estimate using observed states & knowledge of (5)

# Adaptive learning

- Postulate linear functional relationship instead of (6):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \quad (7)$$

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- Note: **misspecified**  $\rightarrow$  not model-consistent (not RE)
- Estimate  $a, b$  using recursive least squares (RLS)



# Recursive least squares

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Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g h \quad \forall t \quad (8)$$

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$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g \quad \forall t \quad (8)$$

→ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))}_{\equiv fe_{t|t-1}, \text{ forecast error}} \quad (9)$$

$k_t \in (0, 1)$  gain

$b_1$  first row of  $b$

# Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})) \quad (10)$$

$k_t = \mathbf{g}(fe_{t|t-1})$ : anchoring function

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$k_t = \mathbf{g}(fe_{t|t-1})$ : anchoring function

$$\mathbf{g}(fe_{t|t-1}) = \alpha b(fe_{t|t-1}) \quad (11)$$

$b(fe_{t|t-1})$  = basis, here: second order spline (piecewise linear)

$\alpha$  = approximating coefficients, here: use  $\hat{\alpha}$  from estimation

# Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (12)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (13)$$

► Derivations

► Actual laws of motion

- Expectations evolve according to RLS with the endogenous gain given by (11)

→ How should  $\{i_t\}$  be set?

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# Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

- $\mathbb{E}$  is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model



# Target criterion

## Result

*In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap*

$$\pi_t = -\frac{\lambda_x}{\kappa}x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_{\pi,t} \right)$$
$$\left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\bar{\pi},t+j}) \right)$$

where  $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$  at  $t$ ,  $\prod_{j=0}^0 \equiv 1$  and  $b_1$  is the first row of  $b$ .

# Two layers of intertemporal stabilization tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ - \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j} + fe_{t+1+j|t+j} \mathbf{g}_{\pi,t+j})$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

## Lemma

*The discretion and commitment solutions of the Ramsey problem coincide.*

► Why no commitment?

## Corollary

*Optimal policy under adaptive learning is time-consistent.*

↪ Foreshadow: optimal policy aggressiveness time-varying

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# Solution procedure

Solve system of model equations + target criterion

↪ solve using parameterized expectations (PEA) and value function iteration (VFI)

↪ obtain a cubic spline approximation to optimal policy function

## Optimal policy I - responding to unanchoring

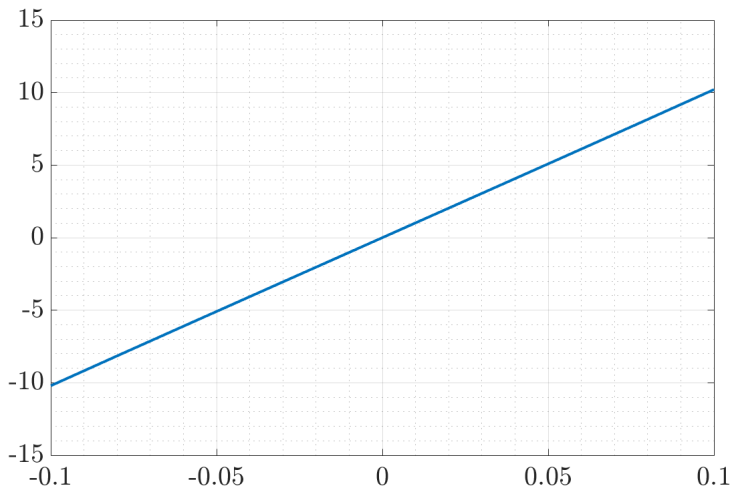
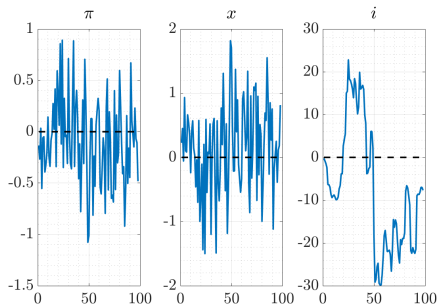
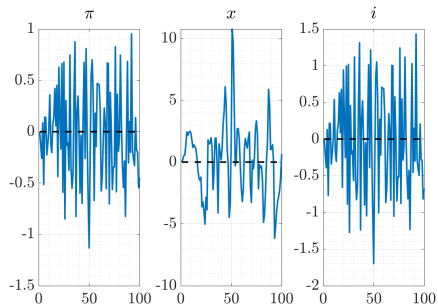


Figure: Comparative statics of the policy function:  $\partial i / \partial \bar{\pi}$

## Optimal policy II - a particular history



(a) Optimal policy



(b) Taylor rule with 1.5 and 0 coefficients on inflation and the output gap

Figure: Observables conditional on a particular evolution of shocks

# The intertemporal volatility tradeoff

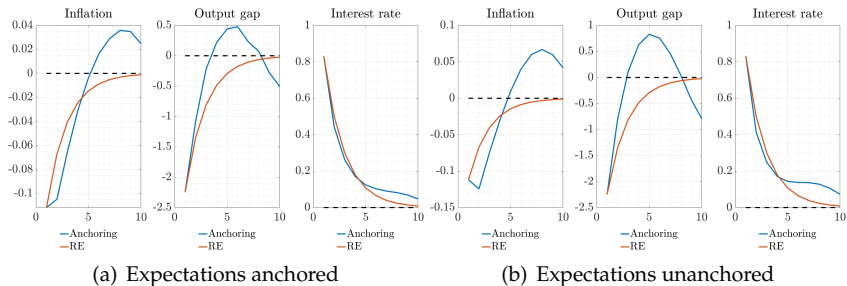
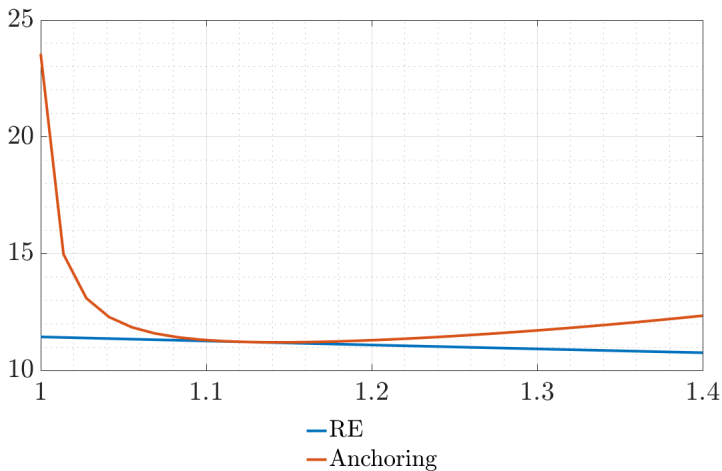


Figure: Impulse responses after a contractionary monetary policy shock



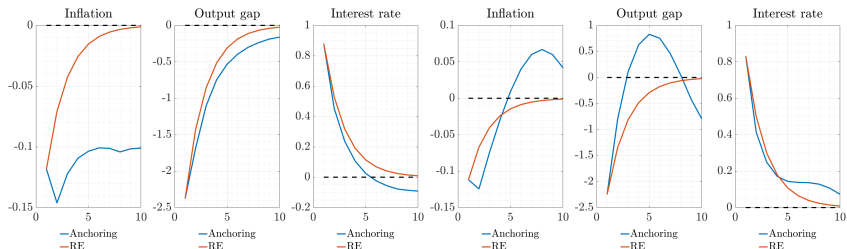
## Optimal policy III - optimal Taylor-rule coefficients



(a)  $\lambda_x = 0, \lambda_i = 0$

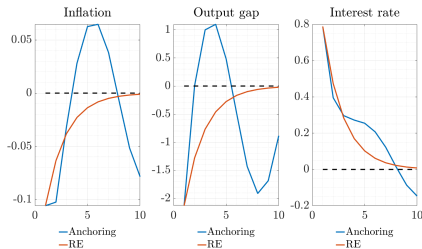
Figure: CB loss as a function of  $\psi_\pi$

# The intertemporal volatility tradeoff - again



(a)  $\psi_\pi = 1.01$

(b)  $\psi_\pi = 1.5$



(c)  $\psi_\pi = 2$

# Conclusion

- Interaction between monetary policy and anchoring
- Optimal policy conditions on stance of current and expected future anchoring
  - ↔ determine intertemporal tradeoffs
- Frontloads aggressive interest rate response to suppress potential unanchoring
- For a 1 pp positive (negative) forecast error, raises (lowers) interest rate by 11.61 pp

## Appendix

# Correcting the TIPS from liquidity risk

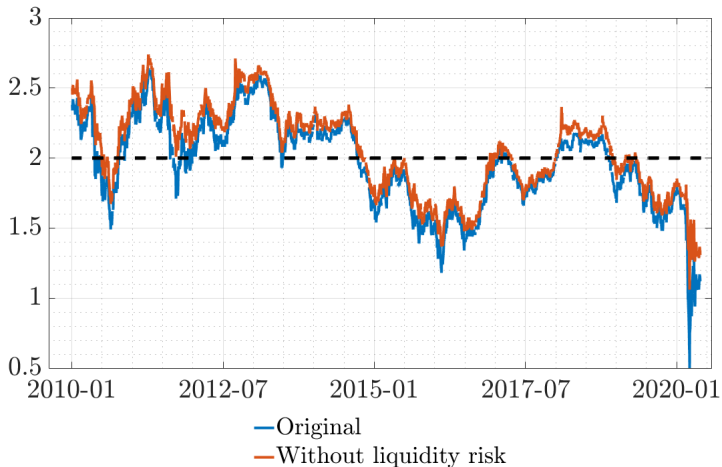


Figure: Market-based inflation expectations, 10 year, average, %

# Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \quad (14)$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \quad (15)$$

Solve for the time series of expectations  $f_t$

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \quad (16)$$

Solve for forecast error  $fe_t \equiv \pi_t - f_{t-1}$ :

$$fe_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k \rightarrow 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \quad (17)$$

# Functional forms for $g$ in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \quad (18)$$

$p \equiv \text{Prob}(\text{liquidity trap regime})$   
 $y_{t-1}$  output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when } \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \quad (19)$$

$\theta_t$  criterion,  $\bar{\theta}$  threshold value

# Choices for criterion $\theta_t$

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \quad (20)$$

$\Sigma$  variance-covariance matrix of shocks

$T(\phi)$  mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1} (fe_{t|t-1} fe'_{t|t-1} - \omega_{t-1}) \quad (21)$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1} (fe'_{t|t-1} \omega_t^{-1} fe_{t|t-1} - \theta_{t-1}) \quad (22)$$

$\omega_t$  estimated forecast-error variance



# Recursive least squares algorithm

$$\phi_t = \left( \phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left( y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right) \right)' \quad (23)$$

$$R_t = R_{t-1} + k_t \left( \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} [1 \quad s_{t-1}] - R_{t-1} \right) \quad (24)$$

# Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \quad (25)$$

$$s_t = h s_{t-1} + \epsilon_t \quad (26)$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \quad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \quad (27)$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} y_{T+1} \quad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \quad (28)$$

# No commitment - no lagged multipliers

Simplified version of the model: planner chooses  $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$  to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t) \right. \\ \left. + \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \right\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa}x_t - \varphi_{2,t}(k_t + \mathbf{g}_{\pi}(\pi_t - f_{t-1})) = 0 \quad (29)$$

$$-2\beta\frac{\lambda}{\kappa}x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_f(\pi_{t+1} - f_t)) = 0 \quad (30)$$

# Target criterion system for anchoring function as changes of the gain

$$\begin{aligned} \varphi_{6,t} = & -cfe_{t|t-1}x_{t+1} + \left(1 + \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1}) - fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\varphi_{6,t+1} \\ & - \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1})\varphi_{6,t+2} \end{aligned} \quad (31)$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{fe_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{fe_{t|t-1}}\varphi_{6,t+1} \quad (32)$$

$\varphi_{6,t}$  Lagrange multiplier on anchoring function

The solution to (32) is given by:

$$\varphi_{6,t} = -2\mathbb{E}_t \sum_{i=0}^{\infty} (\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{fe_{t+j|t+j-1}}}{\frac{k_{t+j}}{fe_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \quad (33)$$

# Details on households and firms

Consumption:

$$C_t^i = \left[ \int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (34)$$

$\theta > 1$ : elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[ \int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \quad (35)$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \quad (36)$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \quad (37)$$

# Derivations

## Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \quad (38)$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \quad (39)$$

where ‘hats’ denote log-linear approximation and  $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$ .

1. Solve (38) backward to some date  $t$ , take expectations at  $t$
  2. Sub in (39)
  3. Aggregate over households  $i$
- Obtain (12)