5
Adaptive Expectations (1950's)

They cannot look out far.
They cannot look in deep.
But when was that ever a bar
To any watch they keep?

Robert Frost

Adaptive expectations

This chapter describes the Cagan-Friedman adaptive expectations hypothesis, how Phelps used it to formulate the natural-rate hypothesis, and its role in the initial econometric tests of the natural-rate hypothesis. Results in this early literature form building blocks for my later work with a modern form of adaptation.

The original Phelps problem

Edmund Phelps (1967) formulated a control problem for a natural-rate model. He dropped rationality for the public, but not for the government,[1] and assigned the public a particular mechanical forecasting rule known to the government.[2]

The economy repeats forever, and the government evaluates outcome sequences according to (18). When $\delta = 1$, we interpret (18) in the usual limit of means (Cesaro sum) sense:

[1] Note the reversal of Lucas's (1976) recommendation.

[2] Stanley Fischer (1986) and Peter Ireland (1997) describe how imputing to the government such a model of the public's expectations promotes behavior that resembles a concern for reputation.

$\liminf_{T\to\infty} T^{-1}\sum_{t=1}^{T} r(x_t, y_t)$. The public uses the adaptive expectations scheme of Milton Friedman and Philip Cagan:

$$x_t - x_{t-1} = (1-\lambda)(y_{t-1} - x_{t-1}), \tag{29}$$

where $\lambda \in (0,1)$. Notice how (29) is a constant coefficient or constant gain version of the least squares learning algorithm (6), where $(1-\lambda)$ takes the role of t^{-1} in (6). Equation (29) can be represented in the form

$$x_t = (1-\lambda)\sum_{i=1}^{\infty} \lambda^{i-1} y_{t-i}. \tag{30}$$

Equation (30) possesses what Cho and Matsui (1995) call an induction property: if the government keeps repeating a constant $y_t = \tilde{y}$ policy, eventually the public comes nearly to set $x = \tilde{y}$. Solow and Tobin imposed this property in testing the natural unemployment rate hypothesis.

Let $p(U_t, y_t) = -.5(U_t^2 + y_t^2)$. The government's problem is to maximize

$$V^g(U,y) = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} p(U_t, y_t), \quad \delta \in (0,1) \tag{31}$$

(i.e., (18)) by choice of a rule for y_t, subject to (29) and the Phillips curve (3) $U_t = U^* - \theta(y_t - x_t)$, given initial conditions for (y_{-1}, x_{-1}). The state for the government's control problem is $[\,1 \quad y_{t-1} \quad x_{t-1}\,]'$. The solution of the government's problem takes the form $y_t = f_1 + f_2[\lambda x_{t-1} + (1-\lambda)y_{t-1}] = f_1 + f_2 x_t$, with $f_1 \neq 0, f_2 \neq 1$, inequalities that reflect that the public does not use an optimal forecasting rule in setting x_t.[3]

[3] Note that $f_1 = 0, f_2 = 1$ implies that $y_t = x_t$ for all histories of x_t and y_t. Imposing the induction property sets $y_t \approx x_t$ only for particular histories of y_t, and what may be very untypical ones at that. This is the start of Lucas's (1972) and Sargent's (1971) criticisms of the Solow-Tobin way of testing the natural-rate hypothesis. Despite criticizing Tobin and Solow's identification scheme in 1972, Lucas (1980) used it. Charles Whiteman (1983) criticized Lucas's 1980 procedure.

Our interest in this control problem stems from the following result:

PROPOSITION ($\delta = 1$ EVENTUALLY SUSTAINS RAMSEY): In the absence of discounting, the government drives y_t to 0, the Ramsey outcome.

When $\delta = 1$, λ governs the speed of convergence to the Ramsey outcome. When $\delta < 1$, the limit point of y_t depends on a comparison of λ with δ. For $\lambda < \delta$ and δ close to 1, the government's policy eventually approximates the Ramsey outcome. The public's expectations are wrong along the transition path, but are correct in the steady state, by virtue of the induction property.

For parameter values $\theta = 1, U^* = 5$, Tables 1a and 1b summarize optimal disinflation paths for $\delta = .96$ (Table 1a) and $\delta = 1$ (Table 1b) for the two values $\lambda = .7, .9$. We started the government's control problem from the late 1970's initial conditions $x_{-1} = y_{-1} = 12$. These initial conditions imply that $U = U^* = 5$. Notice how for each of the four parameter settings, the government engineers a major recession and immediately brings inflation down more than half way toward its eventual limiting value. This happens even for the long lag specification $\lambda = .9$. Notice also how in the discounted ($\delta = .96$) case, the government accepts a longer but milder recession when $\lambda = .9$ than when $\lambda = .7$.

Some dynamics in these tables resurfaces in later chapters in more sophisticated adaptive schemes that alter the assumption that λ is a free parameter. The self-confirming and forecast misspecification types of equilibria, presented in Chapter 7, transform λ from a free parameter to an equilibrium outcome. Nevertheless, the beneficial role of the induction hypothesis will survive.

Table 1a.
Paths of unemployment and inflation starting from $y_{-1} = 12, x_{-1} = 12$ with $\delta = .96$ and $\lambda = .7$ and .9.

	$\lambda = .7$		$\lambda = .9$	
lags	U	y	U	y
1	12.6	4.4	11.9	5.1
5	8.1	2.2	10.2	4.3
20	5.1	.7	6.9	2.5
50	5.0	.6	5.2	1.6

Table 1b.
Paths of unemployment and inflation starting from $y_{-1} = 12, x_{-1} = 12$ with $\delta = 1$ and $\lambda = .7$ and .9.

	$\lambda = .7$		$\lambda = .9$	
lags	U	y	U	y
1	13.2	3.8	13.4	3.6
5	8.3	1.5	11.3	2.7
20	5.1	0.1	7.1	0.9
50	5.0	0.0	5.2	0.1

Phelps problem: general version

For Phelps's control problem, the reduced form of the Phillips curve matters, not the underlying structure identifying x_t. It is useful to state a more general version of the Phelps problem that assumes that the government's model is a reduced form distributed lag Phillips curve. Define the vectors $X_{Ut-1} = [\,U_{t-1} \quad \cdots \quad U_{t-m_U}\,]'$, $X_{yt-1} = [\,y_{t-1} \quad \cdots \quad y_{t-m_y}\,]'$, $X_t = [\,X'_{Ut} \quad X'_{yt} \quad 1\,]'$. Notice that X_{Ut-1} and X_{yt-1}, and therefore X_{t-1} also, are defined to include information dated $t-1$ and earlier; X_{t-1} is the state vector for the general Phelps problem. I specify alternative reduced form (distributed lag) Phillips curves whose regressors are $X_{Ct} = [\,y_t \quad X'_{t-1}\,]'$ and $X_{Kt} = [\,U_t \quad X'_{t-1}\,]'$, respectively. The subscripts C and K stand for Classical and Keynesian. For convenience, Table 2 records various objects that occur frequently in the Phelps problem. These same objects reappear in the definitions of self-confirming equilibria and in the 1990's adaptive models of Chapters 8 and 9.

Table 2

Objects in the Phelps problem

Object	Meaning
X_{Ut}	$[U_{t-1}, \ldots, U_{t-m_u}]'$
X_{yt}	$[y_{t-1}, \ldots, y_{t-m_y}]'$
X_t	$[X'_{Ut}, X'_{yt}, 1]'$
X_{Ct}	$[y_t \; X'_{t-1}]'$
X_{Kt}	$[U_t \; X'_{t-1}]'$
Class. Phillips curve	$U_t = [\gamma_1 \; \gamma'_{-1}]' X_{Ct} + \varepsilon_{Ct}$
Keynes Phillips curve	$y_t = [\beta_1 \; \beta - 1']' X_{Kt} + \varepsilon_{Kt}$
γ	coeffnts (classical)
β	coeffnts (Keynesian)
$h(\gamma)$	coeffnts of govt rule

We define the Phelps problem in terms of the classical Phillips curve:

$$U_t = \gamma' X_{Ct} + \varepsilon_{Ct}, \tag{32a}$$

or

$$U_t = [\,\gamma_1 \quad \gamma'_{-1}\,] \begin{bmatrix} y_t \\ X_{t-1} \end{bmatrix} + \varepsilon_{Ct} \tag{32b}$$

where ε_{Ct} is perceived to be random noise. The government ranks outcome paths according to the mathematical expectation of (31). The government believes that it sets y_t, apart from a random term, so that

$$y_t = \hat{y}_t + v_{2t}, \tag{33}$$

where v_{2t} is a white noise that is beyond the government's control and is orthogonal to information dated $t-1$ and earlier; $\hat{y}_t$ can be set by the government as a function of information known at $t-1$.

DEFINITION: The *Phelps problem* is to choose a control law $\hat{y}_t = hX_{t-1}$ to maximize the expected value of (31) subject to (32) and (33).

The Phelps problem induces a mapping from the government's beliefs γ to the government's decision rule for setting inflation h:

$$h = h(\gamma). \tag{34}$$

The control problem in the previous section is a special case of the Phelps problem that imposed the restrictions on γ in (32) that come from substituting the adaptive expectations hypothesis (29) into (3). Notice that once that substitution has been made, the state variable x_t remains hidden for the rest of the analysis.

In the more general Phelps problem, the induction hypothesis would make the weights on current and lagged y's in (32) sum to zero. We can express the induction hypothesis in terms of its implications for the inverse relationship obtained by solving (32) for y_t:

$$y_t = \beta' X_{Kt} + \varepsilon_{Kt}, \tag{35a}$$

or

$$y_t = \begin{bmatrix} \beta_1 & \beta'_{-1} \end{bmatrix} \begin{bmatrix} U_t \\ X_{t-1} \end{bmatrix} + \varepsilon_{Kt} \tag{35b}$$

where ε_{Kt} is another random disturbance. We can compute γ from β (and vice versa) using the relations

$$\gamma_1 = \beta_1^{-1}, \quad \gamma_{-1} = -\beta_{-1}/\beta_1. \tag{36}$$

When the Phillips curve is expressed as (35), the induction hypothesis implies that the sum of weights on lagged y's equals unity. This brings us to Solow's and Tobin's early proposal for testing the natural-rate hypothesis.

Testing the natural-rate hypothesis

Solow (1968) and Tobin (1968) exploited the induction hypothesis. They substituted (30) into an inverted version of the Phillips curve (3) to get

$$y_t = (1-\lambda)\sum_{i=1}^{\infty} \lambda^{i-1} y_{t-i} + \theta^{-1}(U^* - U_t). \tag{37}$$

They proposed to test the natural-rate hypothesis by running a regression of the form

$$y_t = b_0 + b_1(1-\lambda)\sum_{i=1}^{\infty} \lambda^{i-1} y_{t-i} + b_2 U_t + \varepsilon_t, \tag{38}$$

where ε_t is a least squares residual. They interpreted a finding that $b_1 < 1$ as indicating a long-run trade-off between inflation and unemployment of slope $b_1 - 1$. An alternative was to fit a less restricted distributed lag

$$y_t = \tilde{\beta}_0 + \tilde{\beta}_1(L) y_{t-1} + \tilde{\beta}_2(L) U_t + \varepsilon_{Kt}, \tag{39}$$

where $\tilde{\beta}_1(L) = \sum_{j=1}^{m_y} \tilde{\beta}_{1,j} L^j$, $\tilde{\beta}_2(L) = \sum_{j=0}^{m_U} \tilde{\beta}_{2,j} L^j$, and L is the lag operator. In this version, the natural-rate hypothesis is taken to be $\tilde{\beta}_1(1) = 1$. Early implementations of the tests found that $b_1 < 1$ (or $\tilde{\beta}_1(1) < 1$) and so rejected the natural-rate hypothesis in favor of a long-run trade-off between inflation and unemployment.[4] Later implementations of the same tests failed to reject the natural-rate hypothesis. According to an argument of Sargent (1971) and King and Watson (1994), that pattern is consistent with the tendency of inflation to have exhibited a unit root after the 1960's but not before.[5]

[4] Lucas (1972) and Sargent (1971) argued that those rejections did not bear on the adequacy of the natural-rate hypothesis. Tobin and Solow deduced the restriction $b_1 = 1$ in effect by using the induction hypothesis to achieve econometric identification of expected inflation in (38). Lucas and Sargent described specific stationary stochastic processes for which the hypothesis that the w_i's add up to one in $x_t = \sum_{i=1}^{\infty} w_i y_{t-i}$ contradicted the hypothesis of rational expectations. They showed how rational expectations imposed restrictions across the w_i's and the stochastic process governing the inflation process in the sample period. The induction hypothesis imposes a restriction on $\sum_i w_i$ from a particular experiment (i.e., a permanent and fixed inflation) that could be very unlikely for many stochastic processes. King and Watson (1994) have revisited this issue, and emphasized that the unit-sum restriction is compatible with rational expectations where y_t has a unit root.

[5] See King and Watson (1994).

Disappearance of beliefs as state variable

The private sector's expectations about inflation, x_t, disappear from the Phelps problem. They are replaced by technical conditions on the zero-frequency characteristics of the distributed lag of y_t appearing on the right side of (38) or (39). The ideas of adaptation and induction are also de-emphasized, as is the natural-rate hypothesis itself. From the viewpoint of Phelps's control problem, it is incidental whether the natural-rate hypothesis holds. The Phelps problem imparts interesting dynamics to the inflation-unemployment choice, whether or not $b_1 = 1$ in (38).

Subversion of Phelps's model

This observation recalls a reaction that accompanied the increasing tendency of tests administered during the 1970's to confirm the natural-rate hypothesis in U.S. data. That reaction was based on thinking about the Phelps problem in the context of large λ's in (30) or long mean lags on lagged y's in (35). In the Phelps problem under (30), for a fixed $\delta < 1$, it is always possible to find a λ close enough to 1 so that high inflationary expectations x_{t-1} will make a government want to avoid the Ramsey outcome.[6] In the late 1970's, models with long expectations adjustment lags were used to recommend against reducing inflation.[7] Large sacrifice ratios – estimated amounts of foregone GDP required to bring inflation down one percentage point – circulated widely in the U.S. in the late 1970's. Despite its encouraging implications in terms of sustaining the Ramsey

[6] But remember the calculations in Table 1. To push the model away from a recommendation for taking a large recession to earn a quick reduction of a high inflation, one would have to put even longer lags into inflation expectations.

[7] By stressing the cross-equation restrictions embedded in rational expectations versions of the natural-rate hypothesis, the analyses of Lucas (1972) and Sargent (1971) asserted that historical estimates of λ were silent about an optimal speed of disinflation. See Sargent (1986) for an extended treatment of this issue in the context of stabilizations of big inflations.

outcome under the induction hypothesis, Phelps's control problem carries a tattered past. It fortified those in the 1970's who advocated learning to live with high inflation because of the unacceptably high costs in unemployment from disinflating.

Nevertheless, the Phelps problem illustrates how activating the induction hypothesis can eventually lead to better outcomes. In the form that Phelps, Tobin, and Solow used, the induction hypothesis retreats from rational expectations.[8]

In Chapters 7 and 8, I impute to the government and the public more symmetry in their understandings and procedures to form models that apply up-dating schemes like (29) to functions rather than to numbers.

The next chapter takes a detour to introduce a type of approximate equilibrium under adaptive expectations. In this equilibrium, λ in (30) disappears as a parameter and becomes an equilibrium outcome. In the subsequent chapter, I describe a 1970's rational expectations model. These two models contain important elements of the dynamics of our *circa* 1990's adaptive expectations models.

[8] But Marianne Baxter (1985), Stanley Fischer (1986), and Peter Ireland (1997) have noted that the adaptive expectations model (29) responds much like rational expectations models where policy makers are drawn from a probability distribution over types of preferences.