ADAPTIVE LEARNING, PERSISTENCE, AND OPTIMAL MONETARY POLICY

Vitor Gaspar
Banco de Portugal

David Vestin
European Central Bank

Frank Smets
European Central Bank

Abstract
We show that, when private sector expectations are determined in line with adaptive learning, optimal policy responds persistently to cost-push shocks. The optimal response is stronger and more persistent, the higher is the initial level of perceived inflation persistence by the private sector. Such a sophisticated policy reduces inflation persistence and inflation volatility at little cost in terms of output gap volatility. Persistent responses to cost-push shocks and stability of inflation expectations resemble optimal policy under commitment and rational expectations. Nevertheless, it is clear that the mechanism at play is very different. In the case of commitment it relies on expectations of future policy actions affecting inflation expectations; in the case of sophisticated central banking it relies on the reduction in the estimated inflation persistence parameter based on inflation data generated by shocks and policy responses. (JEL: E52)

1. Introduction

Inflation dynamics crucially depends on how inflation expectations are formed. In most modern macroeconomics, expectations are modeled in accordance with rational expectations. Over the last thirty years, researchers have systematically explored the implications of rational expectations for the conduct of policy. However, rational expectations (paraphrasing Evans and Honkapohja 2001) assume economic agents who are extremely knowledgeable. An alternative approach is to limit their knowledge so that, as time goes by and available data changes, so does the agents' forecasting rule. Thus, the alternative can be understood as implying bounded rationality. In fact, it implies limiting agents' knowledge about the true structure of the model. Adaptive learning, as a minimal departure from rational expectations (see Evans and Honkapohja 2001 and also Orphanides and Williams

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E-mail addresses: Gaspar: vgaspar@bportugal.pt; Smets: frank.smets@ecb.int; Vestin: david.vestin@ecb.int

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2005), provides a plausible framework for modeling the behavior of economic agents who are coping with accelerating economic change. Moreover, adaptive learning seems to provide an empirically reasonable way to model the formation of the private sector's expectations (see Orphanides and Williams 2004).

Orphanides and Williams (2005) have shown that adaptive learning matters for the conduct of monetary policy. They show for the case of linear feedback rules that strengthening the policy response to inflation helps to limit the increase in perceived inflation persistence. Thus, a strategy of tight inflation control may reduce both inflation and output gap volatility. Svensson (2003) argues that simple instrument rules fail to capture how central banks actually conduct policy. If adaptive learning is (empirically) a good description of expectations formation, then it is important to characterize optimal monetary policy in such a setting.[1]

Modeling the optimal behavior of policy makers requires specifying their information set. As in Gaspar, Smets, and Vestin (2005), here we consider the (admittedly) extreme case of sophisticated central banking. Specifically, we assume that the central bank has full information about the structure of the economy (this is standard under rational expectations). In our context, this implies that the information set includes knowledge about the precise mechanism generating private sector expectations. Because a similar assumption is also implicit in the rational expectations literature, it should provide a useful benchmark for comparing optimal monetary policy under adaptive learning and rational expectations.

In this paper, we focus on the implications of sophisticated central banking for inflation persistence. Adaptive learning implies that inflation dynamics will be affected by the history of shocks driving the economy. For research on inflation persistence this means that inflation is time varying even if the inflation target remains unchanged. Thus, adaptive learning is relevant when interpreting empirical estimates of inflation persistence.

The remainder of the paper is organized as follows. Section 2 introduces the model, its calibration, and the simulation method. Section 3 presents the results, and Section 4 concludes.

2. The Model, Calibration, and the Simulation Method

2.1. A New Keynesian Model of Inflation Dynamics and Monetary Policy

Our primary interest here is inflation persistence. Therefore, we want a model where there is intrinsic inflation persistence under rational expectations and interaction of persistence with private sector learning. Toward this end, we choose a

1. For example, in Gaspar, Smets, and Vestin (forthcoming) we argue that persistent inflation expectations, like those generated by adaptive learning, help to explain the dynamics of monetary policy regime change associated with the Volcker disinflation (starting in October 1979).

simple extension of the benchmark New Keynesian model in Woodford (2003). A finite elasticity of substitution between goods, θ, leads to monopolistically competitive producers who set prices that are sticky (in the Calvo sense) such that only a fraction $1 - \alpha$ of firms set prices optimally in each period. Furthermore, firms—which are not "drawn" to optimally update their prices—instead partially index their prices at a rate γ to lagged inflation (along the lines of, e.g., Smets and Wouters 2003). Finally, we assume the existence of temporary cost-push shocks that generate a trade-off for monetary policy, which can be formally motivated by a stochastic intratemporal elasticity of substitution between goods or, alternatively, by stochastic taxes.

Woodford (2003) shows that under rational expectations these assumptions lead to a Phillips curve of the form

$$\pi_t - \gamma\pi_{t-1} = \beta(\mathrm{E}_t\pi_{t+1} - \gamma\pi_t) + \kappa x_t + u_t, \tag{1}$$

where π is inflation, x is the output gap, β is the discount rate, u is a cost-push shock (assumed i.i.d.), and κ is a function of the structural parameters including the degree of Calvo price stickiness. Furthermore, up to a second-order approximation, the (negative of the) period social welfare function takes the form

$$L_t = (\pi_t - \gamma\pi_{t-1})^2 + \lambda x_t^2, \tag{2}$$

where $\lambda = \kappa/\theta$ measures the relative weight on output gap stabilization. We will assume here that the central bank uses the social welfare function to guide its policy decisions, both under rational expectations and under private sector learning.[2] If $\gamma \neq 1$, then the optimal rate of inflation is zero (otherwise there will be inefficient dispersion of prices in the steady state), and we therefore assume that the known inflation target (coinciding with the average level of inflation in the absence of an overambitious output gap target) equals this level. To keep the model simple, we assume that the central bank controls the output gap directly.

2.2. The Formation of Inflation Expectations

We consider two assumptions regarding the formation of inflation expectations in equation (1): rational expectations and recursive least-squares learning.

The standard assumption in the literature is to assume rational expectations. In this case, the private sector knows the structure of the economy as shown in (1) as well as the monetary policy reaction function implied by the central bank's loss

2. It is clear that it matters at which stage of the analysis learning is introduced. In this paper, we follow the convention in the adaptive learning literature and assume that the structural relations (besides the expectations operator) remain identical when moving from rational expectations to adaptive learning.

function (2). In this case, it turns out that optimal monetary policy under discretion responds to the exogenous shock but not to lagged inflation (in contrast to when the loss function consists of squared inflation and output; see Clarida, Galí, and Gertler 1999). Optimal discretionary policy is described by

$$x_t = -\frac{\kappa}{\kappa^2 + \lambda} u_t. \tag{3}$$

Under the optimal discretionary policy, the output gap responds only to the current cost-push shock. In particular, following a positive cost-push shock to inflation, monetary policy is tightened and the output gap falls. The strength of the response depends on the slope of the New Keynesian Phillips curve and the weight on output gap stabilization in the loss function. In contrast, if the central bank is able to credibly commit to future policy actions, then optimal policy will feature a persistent "history dependent" response as discussed extensively in Woodford (2003). The relevant mechanism relies on the fact that credible promises of future policy actions help stabilize current inflation through expectations.

Under rational expectations and discretionary monetary policy, the only endogenous state variable is lagged inflation, and hence the equilibrium dynamics of inflation will follow a first-order autoregressive process, where it turns out that the coefficient on lagged inflation equal the degree of indexation:

$$\pi_t = \gamma \pi_{t-1} + \tilde{u}_t. \tag{4}$$

The alternative expectation formation we consider is adaptive learning. Specifically, we assume that the private sector believes the inflation process is well approximated by equation (4). They estimate the equation recursively on the basis of a "constant gain" least-squares algorithm implying perpetual learning.

Thus, the agents estimate the following reduced-form equation for inflation:[3]

$$\pi_t = c_t \pi_{t-1} + \varepsilon_t. \tag{5}$$

Agents are bounded rational because they do not take into account the fact that c_t varies over time. Furthermore, c_t captures the estimated (or perceived) inflation persistence.

The following equations describe the recursive updating of the parameters estimated by the private sector:

$$c_t = c_{t-1} + g R_t^{-1} \pi_{t-1} (\pi_t - \pi_{t-1} c_{t-1}) \tag{6}$$

and

$$R_t = R_{t-1} + g\left(\pi_{t-1}^2 - R_{t-1}\right), \tag{7}$$

3. We assume that the private sector knows the inflation target (equal to zero). In future research, we intend to explore the implications of learning about the inflation target.

where g is the constant gain. Note that because of the learning dynamics the number of state variables are expanded to four $(u_t, \pi_{t-1}, R_t, c_{t-1})$, all known by the central bank when they set policy at time t.

A further consideration with regard to the updating process is the information used by the private sector when updating its estimates and forming its forecast for the next period's inflation. We assume that agents use current inflation when they forecast future inflation (discussed further below) but not in updating the parameters. This implies that expected inflation may be written simply as

$$\mathrm{E}_t\pi_{t+1} = c_{t-1}\pi_t. \tag{8}$$

Generally, there is a simultaneity problem in forward-looking models when combined with learning. In (1), current inflation is determined in part by future expected inflation; but according to (8), expected future inflation is not determined until current inflation is determined. Moreover, in the general case also the estimated parameter c will depend on current inflation if current inflation is used to update the parameter currently used. The literature has taken (at least) three approaches to this problem. The first is to lag the information set so that agents use only $t-1$ inflation when forecasting π_{t+1}, which was the assumption used in Gaspar and Smets (2002). A different and more common route is to look for the fixed point that reconciles both the forecast and actual inflation but to disallow agents from updating the coefficients using current information. This has the benefit of keeping the deviation from the standard model as small as possible (also the rational expectations equilibrium changes if one lags the information set) while keeping the fixed-point problem relatively simple. At an intuitive level, it can also be justified by the assumption that it takes more time to re-estimate a forecasting model than to apply an existing model. Finally, a third approach is to also let the coefficients be updated with current information. This results in a more complicated fixed-point problem.[4]

Substituting equation (8) into (1) gives

$$\pi_t = \frac{1}{1+\beta(\gamma - c_{t-1})}(\gamma\pi_{t-1} + \kappa x_t + u_t). \tag{9}$$

2.3. Calibration of the Model

We are now ready to study the dynamics of inflation, but before doing so we must make specific assumptions about key parameters. In the simulations we use the following set of structural parameters as a benchmark: $\gamma = 0.5$, $\beta = 0.99$,

4. It is possible to solve this problem in the current setting. However, we leave this for future research.

$\theta = 10$, and $\alpha = 0.66$. Coupled with additional assumptions on the intertemporal elasticity of substitution of consumption and the elasticity of labor supply (see the discussion in Woodford 2003) these structural parameters imply that $\kappa = 0.019$ and $\lambda = 0.002$. We choose γ such that there is some inflation persistence in the benchmark calibration; γ at about 0.5 is a value frequently found in empirically estimated new Keynesian Phillips curves (see, e.g., Smets 2003). Our choice of $\theta = 10$ corresponds to a markup of about 10%. Furthermore, α is chosen such that the average duration of prices is three quarters, which is consistent with U.S. evidence. The constant gain, g, is calibrated at 0.03. Orphanides and Williams (2004) found that a value in the range 0.01 to 0.04 is needed to match up the resulting model-based inflation expectations with the Survey of Professional Forecasters. A value of 0.03 corresponds to an average sample length of about 17 quarters.[5] In the limiting case, when the gain approaches zero, the influence of policy on the estimated inflation persistence goes to zero and hence plays no role in the policy problem. Finally, the standard deviation of the shock, u_t, is set to 0.004.

2.4. Monetary Policy and Private Sector Learning

In the context of adaptive learning, we distinguish two alternative assumptions regarding the conduct of monetary policy: a simple rule and "sophisticated" monetary policy. Under the simple rule, the central bank conducts policy according to the same reaction function as under rational expectations (under discretion) characterized by (3). This rule, together with the Phillips curve (1) and the system of equations (6)–(8) determining private sector inflation expectations, uniquely pins down the dynamics of the system. Under the simple rule, if inflation persistence is high then the dynamics of inflation may even become explosive. For such a case the simulation process breaks down. In order to rule this out, we follow Orphanides and Williams (2004) and implement a cutoff point that stops the updating when the estimated persistence parameter exceeds 1. Then we simply assume that the private sector continues to use the unit root process until some shock drives the estimate down.

Sophisticated central banking, in contrast, implies solving the full dynamic optimization problem, where the parameters associated with the estimation process are also state variables. We emphasize that our use of the term "sophisticated" is not to imply that the simple rule is unsophisticated. It is, of course, possible to solve the "sophisticated" problem only by assuming (as we do) that the central bank has full information about how the private sector forms expectations—a stark contrast to what other authors assume about the state of the central banks

5. See Orphanides and Williams (2004). Similarly, Milani (2005) estimates the gain parameter to be 0.03 using a Bayesian estimation methodology.

knowledge. We are quite sympathetic to these alternative assumptions but still think it is interesting to consider what the central bank would do if indeed it were endowed with full information.

Specifically, in this case the central bank solves the following dynamic programming problem:

$$
\begin{aligned}
&V(u_t, \pi_{t-1}, c_{t-1}, R_t) \\
&\quad = \min_{x_t} (\pi_t, -\gamma\pi_{t-1})^2 + \lambda x_t^2 + \mathrm{E}_t \beta V(u_{t+1}, \pi_t, R_{t+1}, c_t), \qquad (10)
\end{aligned}
$$

subject to the expectations-adjusted Phillips curve (1) and, again, the equations determining private sector expectations (6)–(8).

We resort to nonlinear methods in order to solve the policy problem. We employ the collocation methods described in Judd (1998) and Miranda and Fackler (2002); the value function is approximated by a combination of cubic splines, which translates the problem to a root-finding exercise (for details see Gaspar, Smets, and Vestin 2005).

3. Results

Table 1 compares for our benchmark calibration the variance and autocorrelation of output and the quasi-difference of inflation in four cases: Optimal commitment and discretionary policy under rational expectations and optimal policy (sophisticated central banking) and a simple rule under adaptive learning. Note that the simple policy rule is exactly the same as the rule under discretion (equation [3]). The table also shows the expected welfare loss as a proportion of the welfare loss under commitment.

Starting with the comparison between commitment and discretion under rational expectations, it is well known (see Clarida, Galí, and Gertler 1999 and Woodford 2003) that commitment implies a persistent response to cost-push shocks lasting well after the shock has vanished from the economy. The intuition is that the optimal policy under commitment—by generating expectations of a price-level reduction in the face of a positive cost-push shock—reduces the

TABLE 1. Summary of results.

	Rational Expectations		Adaptive Learning	
	Commitment	Discretion	Sophisticated	Simple Rule
$\mathrm{Corr}(x_t, x_{t-1})$	0.65	0	0.54	0
$\mathrm{Corr}(\pi_t, \pi_{t-1})$	0.24	0.50	0.34	0.60
$\mathrm{Var}(x_t)$	1	0.95	1.01	0.95
$\mathrm{Var}(\pi_t - \gamma\pi_{\tau-1})$	1	1.37	1.10	1.55
$E[\mathrm{L_t}]$	1	1.28	1.08	1.43

Notes: $\mathrm{Var}(x_t)$, $\mathrm{Var}(\pi_t - \gamma\pi_{\tau-1})$ and $E[\mathrm{L_t}]$ are measured as ratios relative to commitment.

immediate impact of the shock, spreading it over time. Table 1 shows that the output gap is persistent under commitment, yet not so under the simple rule (assuming i.i.d cost-push shocks). In contrast, inflation is much more persistent under discretion (0.5 against 0.24). The variance of the quasi-difference of inflation is about 37% higher under discretion while output gap volatility is only about 5% lower (illustrating the stabilization bias under discretion). As a result, the expected loss is about 28% higher under discretion.

A comparison of the outcomes under rational expectations and adaptive learning with the same simple policy rule (columns 2 and 4 of Table 1) confirms the findings of Orphanides and Williams (2004). Whereas the autocorrelation of the output gap remains unchanged at zero, the autocorrelation of inflation increases from 0.5 to about 0.6 under adaptive learning. As a result, the variance of the quasi-difference of inflation and the expected welfare loss increase sharply. The intuition for the increase in the variance of the quasi-difference of inflation is that a higher average perceived inflation persistence increases the impact of cost-push shocks on current inflation through inflation expectations.

Finally, optimal policy under adaptive learning allows for sharp reductions in inflation persistence and volatility relative to the simple rule. The variance of the quasi-difference of inflation declines from about 55% above the commitment case to only about 10%. At the same time, the variance of the output gap increases but only a little over 6%, compared with the simple rule and about 1% compared with the commitment case. In terms of expected loss, optimal policy allows for a reduction from about 43% above commitment to about 8% above.

Overall, sophisticated central banking under adaptive learning shares some features of optimal policy under commitment and rational expectations. In both cases, persistent responses to cost-push shocks induce a significant positive autocorrelation in the output gap, leading to lower inflation persistence through stable inflation expectations. Nevertheless, the mechanism under adaptive learning is clearly different, since commitment to future policy actions plays no role. Specifically, sophisticated central banking relies on its ability to influence estimated inflation persistence. This becomes clear from Figure 1, which plots the mean dynamic response of inflation, the output gap and the perceived inflation persistence to a cost push shock of one standard deviation for various initial levels (0.3, 0.5, and 0.7) of the estimated persistence parameter c.

A few observations are worth making. First, the higher the initial inflation persistence estimated by the private sector, the larger the response of a sophisticated central bank to a given cost-push shock. The intuition for this is straightforward: a given cost-push shock will have a larger effect on current inflation through its effect on inflation expectations when agents perceive the persistence of such shocks to be higher. As the central bank is trading off inflation and output gap volatility, it will respond by tightening monetary policy more. Second, the higher the initial perceived inflation persistence, the more persistent the output gap

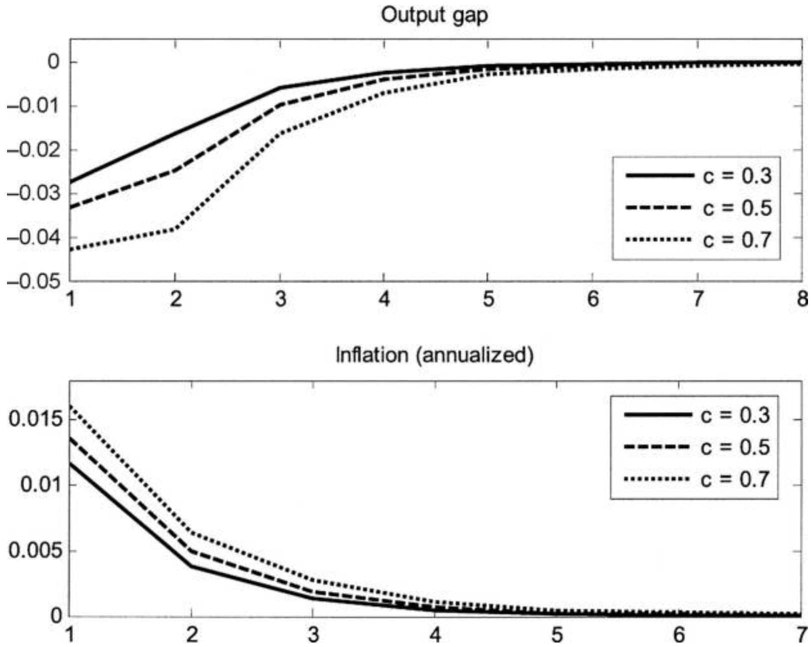

FIGURE 1. Mean dynamic responses to a one–standard deviation cost-push shock for different levels of perceived inflation persistence (c).

response to a cost-push shock. Such a policy succeeds at reducing the estimated degree of inflation persistence quite quickly. Third, on average the estimated inflation persistence falls below its non-stochastic steady state. Over time, the perceived inflation persistence falls to a level close to 0.3 rather than 0.5.

4. Conclusion

In this paper we show that, when private sector expectations are formed in line with adaptive learning, optimal policy responds in a persistent way to cost-push shocks. Through its persistent response to shocks—coupled with optimal response to state variables—sophisticated central banking reduces inflation persistence and inflation volatility at little cost in terms of output gap volatility.

Persistent response to cost-push shocks and stability of inflation expectations resemble optimal policy under commitment and rational expectations. Nevertheless, it is clear that the mechanism at play is very different. In the case of commitment it relies on expectations of future policy actions affecting inflation expectations. Specifically, in the event of a positive cost-push shock, optimal policy under commitment creates an output gap that persists long after the transitory shock has faded away. The intuition is that, by doing so, optimal policy lowers price expectations moderating the current increases in prices; thus the impact of the original shock is spread out over time. In the case of sophisticated central banking the mechanism at play is based on the reduction in the estimated inflation persistence parameter based on data generated by shocks and policy responses. By creating a track record of stable inflation, the central bank anchors inflation expectations, thereby reinforcing its ability to maintain stability.

References

Clarida, Richard, Jordi Galí, and Mark Gertler (1999). "The Science of Monetary Policy: a New Keynesian Perspective." *Journal of Economic Literature*, 37, 1661–1707.

Evans, George W., and Seppo Honkapohja (2001). *Learning and Expectations in Macroeconomics*. Princeton University Press.

Gaspar, Vitor, and Frank Smets (2002). "Monetary Policy, Price Stability and Output Gap Stabilisation." *International Finance*, 5, 193–202.

Gaspar, Vitor, Frank Smets, and David Vestin (forthcoming). "Monetary Policy over Time." *Macroeconomic Dynamics*, forthcoming.

Gaspar, Vitor, Frank Smets, and David Vestin (2005). "Optimal Monetary Policy under Adaptive Learning." Manuscript, European Central Bank. Draft presented at the Workshop "The Implications of Uncertainty and Learning for Monetary Policy," University of California, Santa Cruz, April 1, 2005.

Judd, Kenneth L. (1998). *Numerical Methods in Economics*. MIT Press.

Milani, Fabio (2005). "Expectations, Learning and Macroeconomic Persistence." Manuscript, Princeton University.

Miranda, Mario J., and Paul Fackler (2002). *Applied Computational Economics and Finance*. MIT Press.

Orphanides, Athanasios, and John C. Williams (2004). "Robust Monetary Policy with Imperfect Knowledge." Manuscript, Board of Governors of the Federal Reserve System. Draft presented at the ECB Conference "Monetary Policy and Imperfect Knowledge", Wurzburg, October 14–15, 2004.

Orphanides, Athanasios, and John C. Williams (2005). "Imperfect Knowledge, Inflation Expectations and Monetary Policy." In *Inflation Targeting*, edited by Ben S. Bernanke and Michael Woodford. University of Chicago Press.

Smets, Frank (2003). "Maintaining Price Stability: How Long is the Medium Term?" *Journal of Monetary Economics*, 50, 1293–1309.

Smets, Frank, and Raf Wouters (2003). "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area." *Journal of the European Economic Association*, 1, 1123–1175.

Svensson, Lars (2003). "What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules." *Journal of Economic Literature*, 41, 426–477.

Woodford, Michael (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.