

Materials 13 - Still looking for a version of the model w/o overshooting

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1 Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (1)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (2)$$

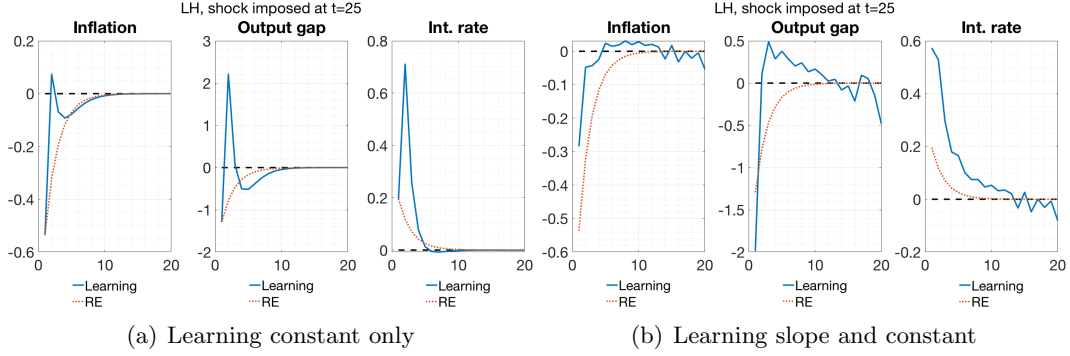
$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \quad (3)$$

$$\hat{\mathbb{E}}_t z_{t+h} = \begin{bmatrix} \bar{\pi}_{t-1} \\ 0 \text{ } (\bar{x}_{t-1}) \\ 0 \text{ } (\bar{i}_{t-1}) \end{bmatrix} + b h_x^{h-1} s_t \quad \forall h \geq 1 \quad b = g_x \ h_x \quad \text{PLM} \quad (4)$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \underbrace{(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))}_{\text{fcst error using (4)}} \quad (b_1 \text{ is the first row of } b) \quad (5)$$

$$k_t = \begin{cases} k_{t-1} + 1 & \text{for decreasing gain learning} \\ \bar{g}^{-1} & \text{for constant gain learning.} \end{cases} \quad (6)$$

Figure 1: Reference: baseline model

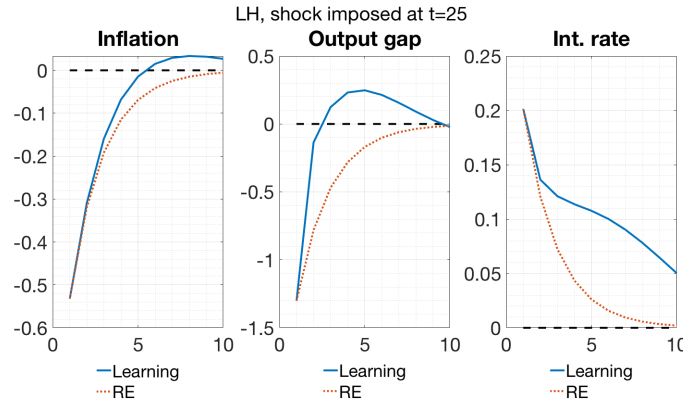


2 Ideas

1. Check ψ_π above but close to 1, or lower β

→ works but only quantitatively; qualitatively, the overshooting is still there, likely because this only cancels out one of the two channels through which $\mathbb{E}\pi$ affects x_t negatively.

Figure 2: Baseline, $\beta = 0.96$ instead of 0.99



2. Fix shock for simulation

Indeed the issue was that for learning, I accidentally scaled down the shock by $\sigma_i < 1$, while for RE I had maintained $\sigma_i = 1$.

3. Interest rate smoothing as $i_t = \rho i_{t-1} + (1 - \rho)(\psi_\pi \pi_t + \psi_x x_t) + \bar{i}_t$

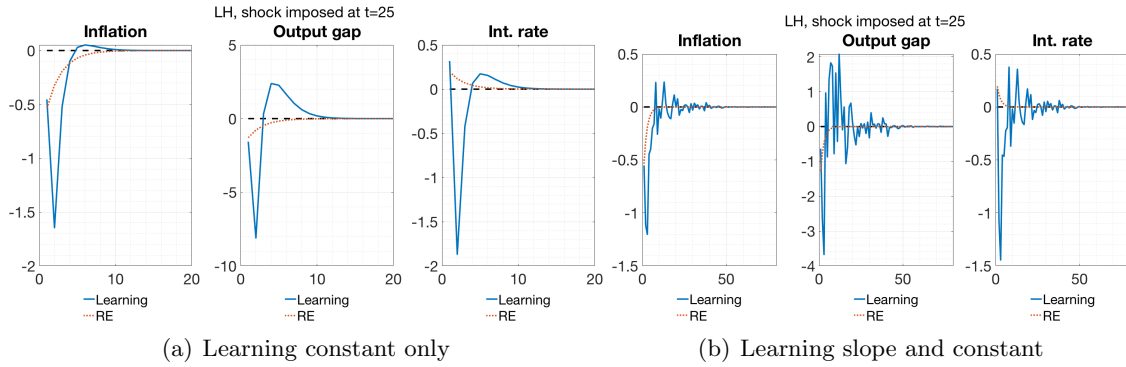
Doesn't work either - it doesn't change the model except reduces ψ_π .

4. Indexation in NKPC

Doesn't work either - same model dynamics.

5. Learn h_x

Figure 3: Learning h_x , baseline



Like learning the Taylor rule b/c agents initially don't know if the shock will continue.

6. Central bank's $\mathbb{E}\pi_{t+1}$ in TR?

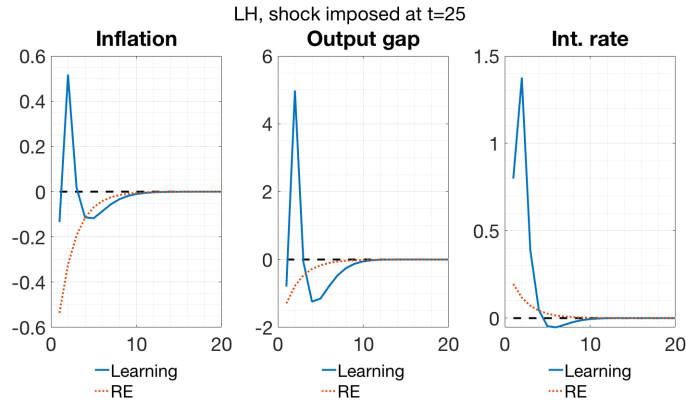
Done a correction for $\hat{\mathbb{E}}\pi_{t+1}$ in TR, now both are stable, but overshooting is still there in both. Not so dissimilar to baseline except that the periods are shifted.

7. Initialize beliefs away from RE somehow

Slobodyan & Wouters do this, but in an estimation context, which I think is necessary because you need pre-sample data to condition priors on.

8. Slobodyan & Wouters' "VAR-learning": use lagged observables to learn from, not from states.

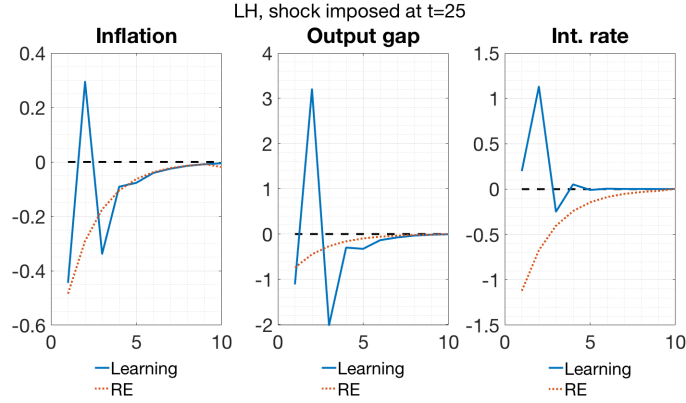
Figure 4: VAR learning, baseline, learning only constant



For learning both slope and constant, not E-stable. Kind of makes sense since I'd think that this amplifies positive feedback.

9. Davig & Leeper-style switching Taylor rule where only long-run Taylor principle holds?

Figure 5: Markov-switching Taylor rule, baseline, learning only constant (slope learning unstable, why?)



10. Some kind of moving average of inflation (or average) in the TR?

A quick question on projection facility: checking $\text{eig}(\phi)$ when ϕ isn't square?

Right now I'm splitting up ϕ as $\mathbf{a} = \phi(:,1)$, $\mathbf{b} = \phi(:, 2:\text{end})$ and then checking $\text{eig}(\mathbf{b})$ and $\text{eig}(\text{diag}(\mathbf{a}))$.

3 Details on the Markov-switching setup

Model equations remain the same, except the Taylor rule now is:

$$i_t = \psi_\pi(r_t)\pi_t + \psi_x x_t + \bar{i}_t \quad (7)$$

$$r_t = \begin{cases} 1 & \text{active regime} & \psi_\pi = \psi_1 = 1.8 \\ 2 & \text{passive regime} & \psi_\pi = \psi_2 = 0.89 \end{cases} \quad (8)$$

$$r_{t+1} = \begin{cases} p_{11}1 + (1 - p_{11})2 & \text{if } r_t = 1 \\ (1 - p_{22})1 + p_{22}2 & \text{if } r_t = 2 \end{cases} \quad \text{where } p_{ji} \equiv \text{Prob}(s_{t+1} = j | s_t = i) \quad (9)$$

So I solve the RE model by introducing the new jump variables π_{it}, x_{it}, i_{it} , $i = 1, 2$ and writing the model equations as

$$x_{it} = (p_{1i} \mathbb{E}_t x_{1t+1} + p_{2i} \mathbb{E}_t x_{2t+1}) - \sigma(i_{it} - (p_{1i} \mathbb{E}_t \pi_{1t+1} + p_{2i} \mathbb{E}_t \pi_{2t+1})) + \sigma r_t^n \quad (10)$$

$$\pi_{it} = \kappa x_{it} + \beta(p_{1i} \mathbb{E}_t \pi_{1t+1} + p_{2i} \mathbb{E}_t \pi_{2t+1}) + u_t \quad (11)$$

$$i_{it} = \psi_i \pi_{it} + \psi_x x_{it} + \bar{i}_t \quad (12)$$

Now I unleash the usual method of solving for the observable and state transition matrix g_x, h_x . The only difference will be that since the number of jumps now is double the old number, g_x will be $2n_y \times n_x$. Is it correct to interpret $g_x(1) \equiv \mathbf{gx}(1:\mathbf{ny}, :)$ as pertaining to regime 1, and $g_x(2) \equiv \mathbf{gx}(\mathbf{ny}+1:\mathbf{end}, :)$ to regime 2?

Then, generating an exogenous regime sequence r , I compute RE IRFs as usual for the state block, but depending on the state, I use the corresponding block of g_x . With x_0 being the impulse, so that $IR_1^x = x_0$, I do the following:

$$IR_t^y = \begin{cases} g_x(1)x_t & \text{if } r_t = 1 \\ g_x(2)x_t & \text{if } r_t = 2 \end{cases} \quad (13)$$

$$IR_{t+1}^x = h_x x_t \quad (14)$$

As for the learning model, the compact notation for the model was:

$$z_t = A_a f_a(t) + A_b f_b(t) + A_s s_t \quad (15)$$

where the A -matrices are functions of the parameters, including ψ_π . The LH expectations f_a and f_b are only functions of the learning coefficients a, b and of the state transition matrix h_x . So the only salient difference is that agents react to expectations differently depending on the regime: the A -matrices become state-dependent.

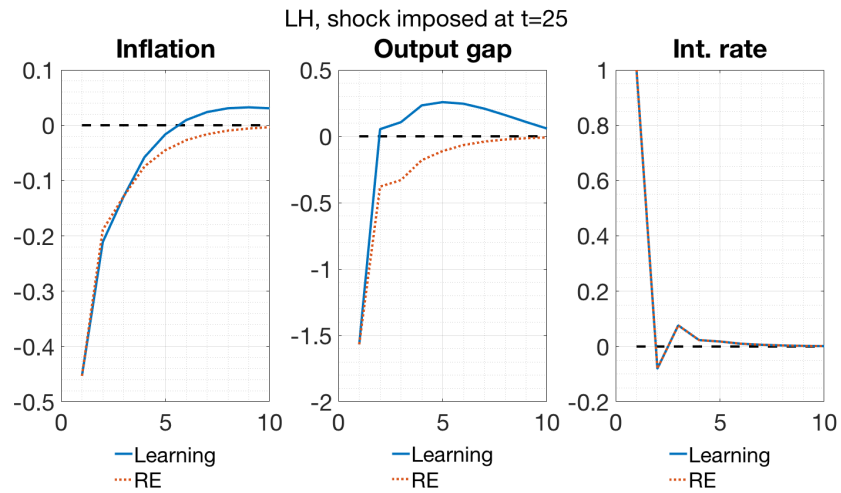
Is it correct then to simulate the model as

$$z_t = A_a(i)f_a(t) + A_b(i)f_b(t) + A_s(i)s_t \quad \text{for } i = 1, 2 \quad (16)$$

?

4 An issue again with those endogenous states!

Figure 6: Lagged inflation in TR, “suboptimal forecasters” info assumption, $\beta = 0.96$ instead of 0.99



Look at the interest rate: it doesn't move differently from the RE model. The code doesn't realize that the 4th state is the lagged 1st jump; my attempts to do the replacement have only lead to instability.