Materials 12c - A general solution method for the learning model that works

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The goal is to have a flexible, automated method so one can tweak the model and avoid errors.
See Notes 11 Dec 2019

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1 Model equations and goal

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \rho i_{t-1} + \bar{i}_t \tag{3}$$

Goal: obtain endogenous stuff as a function of expectations and states:

$$z_t = \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = A_a f_a + A_b f_b + A_s s_t \tag{4}$$

where I already have expectations f_a, f_b and the state vector can vary by model, but in this default case it is

$$s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{bmatrix} \tag{5}$$

That is, we want the matrices A_a, A_b and A_s.

2 Step 1 - eliminate the interest rate and introduce LH expectations for observables and states

$$\underbrace{\begin{bmatrix} \sigma\psi_\pi & 1+\sigma\psi_x \\ 1 & -\kappa \end{bmatrix}}_{\equiv M} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \underbrace{\begin{bmatrix} c_{x,b} f_b + c_{x,s} s_t \\ c_{\pi,a} f_a + c_{\pi,s} s_t \end{bmatrix}}_{\equiv N} \tag{6}$$

where M and the c are model-specific. In the baseline model they are given by

$$c_{x,b} = \begin{bmatrix} \sigma(1-\beta\psi_\pi), & 1-\beta-\sigma\beta\psi_x, & 0 \end{bmatrix} \tag{7}$$

$$c_{x,s} = -\sigma \begin{bmatrix} -1 & 1 & 0 & \rho \end{bmatrix} (I_{nx} - \beta h_x)^{-1} \tag{8}$$

$$c_{\pi,a} = \begin{bmatrix} (1-\alpha)\beta, & \kappa\alpha\beta, & 0 \end{bmatrix} \tag{9}$$

$$c_{\pi,s} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} (I_{nx} - \alpha\beta h_x)^{-1} \tag{10}$$

$$c_{i,s} = \begin{bmatrix} 0 & 1 & 0 & \rho \end{bmatrix}$$

3 Step 2 - solve for observables as a function of cs, expectations and states

Now unleash Mathematica (`materials12.nb`, after "Restart 12 Dec 2019") to solve for x and π as

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = M^{-1} N \tag{11}$$

where Mathematica will output the g-matrices, which can be copied directly into Matlab (`matrices_A_intrate_smoothing3.m`).

The A-matrices will just be a stacking of the g-vectors:

$$\underbrace{A_a}_{ny\times ny} = \begin{pmatrix} g_{\pi,a} \\ g_{x,a} \\ g_{i,a} \end{pmatrix} \quad \underbrace{A_b}_{ny\times ny} = \begin{pmatrix} g_{\pi,b} \\ g_{x,b} \\ g_{i,b} \end{pmatrix} \quad \underbrace{A_s}_{ny\times nx} = \begin{pmatrix} g_{\pi,s} \\ g_{x,s} \\ g_{i,s} \end{pmatrix} \tag{12}$$