

# Monetary Policy & Anchored Expectations An Endogenous Gain Learning Model

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*Inflation that runs below its desired level can lead to an unwelcome fall in **longer-term inflation expectations**, which, in turn, can pull actual inflation even lower, resulting in an adverse cycle of ever-lower inflation and inflation expectations. [...] **Well-anchored inflation expectations** are critical[.]*

*Jerome Powell, Chairman of the Federal Reserve <sup>1</sup>  
(Emphases added.)*

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<sup>1</sup>“New Economic Challenges and the Fed’s Monetary Policy Review,” August 27, 2020.

# Long-run expectations: capturing responsiveness to short-run conditions

Individual SPF forecasts: for 1991-Q4 onward, estimate rolling regression

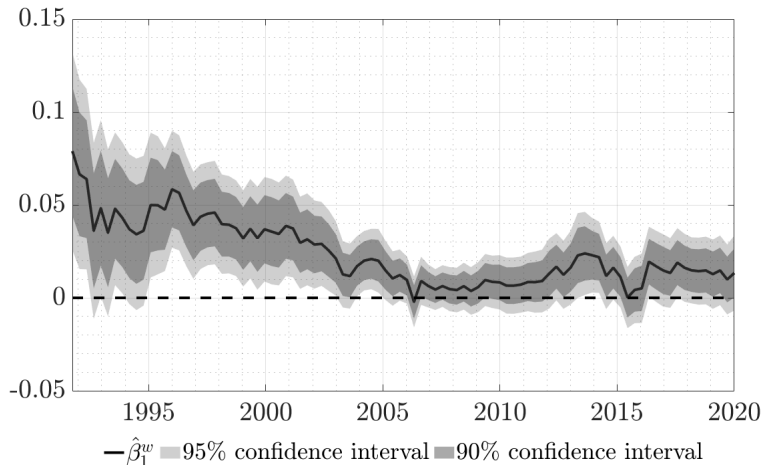
$$\Delta \bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \quad (1)$$

where  $w$  indexes windows of 20 quarters length,

$f_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1} \pi_t$  individual one-year-ahead forecast error

# Time-varying responsiveness

$$\Delta \bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \quad (1)$$



# This project

- How to conduct monetary policy when expectations can become unanchored?
- Model of expectations unanchoring
  - ↪ extension to adaptive learning that captures time-varying responsiveness of long-run expectations
- Estimate how unanchoring takes place in data
  - ↪ quantify novel anchoring channel
- Analyze monetary policy
  - ↪ analytically and numerically using novel model disciplined by data

# Preview of results

## 1. Estimation

- Larger mistakes unanchor more
- Overestimating inflation unanchors more than underestimating it (Hebden et al 2020)

## 2. Optimal policy

- Responds aggressively to inflation when unanchored, accommodates inflation when anchored

## 3. Taylor rule

- Less aggressive on inflation than under rational expectations

## Related literature

- **Optimal monetary policy in the New Keynesian model**

Clarida, Gali & Gertler (1999), Woodford (2003)

- **Adaptive learning**

Evans & Honkapohja (2001, 2006), Sargent (1999), Primiceri (2006), Lubik & Matthes (2018), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Mele et al (2019), Eusepi & Preston (2011), Milani (2007, 2014), Marcet & Nicolini (2003)

- **Anchoring and the Phillips curve**

Goodfriend (1993), Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Reis (2020), Hebden et al 2020, Gobbi et al (2019), Carvalho et al (2019)

# Structure of talk

1. Model of anchoring expectations
2. Quantification of learning channel
3. Solving the Ramsey problem
4. Implementing optimal policy
5. Approximating optimal policy with a Taylor rule



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# Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ U(C_T^i) - \int_0^1 v(h_T^i(j)) dj \right] \quad (2)$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_t C_t^i \quad (3)$$

► Consumption, price level

# Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[ \Pi_t^j(p_t(j)) \right] \quad (4)$$

subject to demand

$$y_t(j) = Y_t \left( \frac{p_t(j)}{P_t} \right)^{-\theta} \quad (5)$$

► Profits, stochastic discount factor

## Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- Model implies mapping between exogenous states  $s_t$  and observables  $y_t \equiv (\pi, x, i)'$

$$y_t = g s_t \tag{6}$$

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$$\mathbb{E}_t y_{t+1} = g \mathbb{E} s_{t+1} \quad (7)$$

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- $\hat{\mathbb{E}}$ : agents do not internalize that identical → do not know aggregate model → do not know (6)

# Adaptive learning

- Agents know exogenous evolution of states

$$s_t = hs_{t-1} + \epsilon_t \quad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \quad (8)$$

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- Estimate  $a, b$  using recursive least squares (RLS) using observed states and knowledge of (8)

# Recursive least squares (RLS)

Observables are:  $(\pi, x, i)'$

Assumption: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g h \quad \forall t \quad (10)$$

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$\bar{\pi}_{t-1}$ : long-run inflation expectations  $\rightarrow$  anchoring

$\rightarrow$  RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))}_{\equiv f_{i|t-1}, \text{ forecast error}} \quad (11)$$

$k_t \in (0, 1)$  gain

$b_1$  first row of  $b$

# Alternatives for the gain

1. Decreasing gain:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f_{t|t-1} \quad (12)$$

2. Constant gain:

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Optimal monetary policy: -

# Model summary

- New Keynesian core: IS and Phillips curves

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1 - \beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (15)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1 - \alpha)\beta\pi_{T+1} + u_T) \quad (16)$$

► Derivations

► Actual laws of motion

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→ How should  $\{i_t\}$  be set?

# Structure of talk

1. Model of anchoring expectations
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# Estimating form of gain function

- Calibrate parameters of New Keynesian core to literature
- Estimate flexible form of expectations process via simulated method of moments  
(Duffie & Singleton 1990, Lee & Ingram 1991, Smith 1993)

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \mathbf{g}(f_{t|t-1}) f_{t|t-1} \quad (18)$$

- Moments: autocovariances of inflation, output gap, federal funds rate and 1-year ahead SPF inflation expectations at lags  $0, \dots, 4$

# Calibration - parameters from the literature

$\beta$	0.98	stochastic discount factor
$\sigma$	1	intertemporal elasticity of substitution
$\alpha$	0.5	Calvo probability of not adjusting prices
$\kappa$	0.0842	slope of the Phillips curve
$\psi_\pi$	1.5	coefficient of inflation in Taylor rule
$\psi_x$	0.3	coefficient of the output gap in Taylor rule
$\sigma_r$	0.01	standard deviation, natural rate shock
$\sigma_i$	0.01	standard deviation, monetary policy shock
$\sigma_u$	0.5	standard deviation, cost-push shock
$\bar{g}$	0.145	initial value of the gain

Main sources: Chari et al 2000, Woodford 2003, Nakamura & Steinsson 2008, Carvalho et al 2019



# Estimated expectations process

$$\bar{\pi}_t - \bar{\pi}_{t-1} = \mathbf{g}(f_{t|t-1}) f_{t|t-1} \quad (18)$$

$\mathbf{g}(\cdot)f_{t|t-1}$  (annualized percentage points)

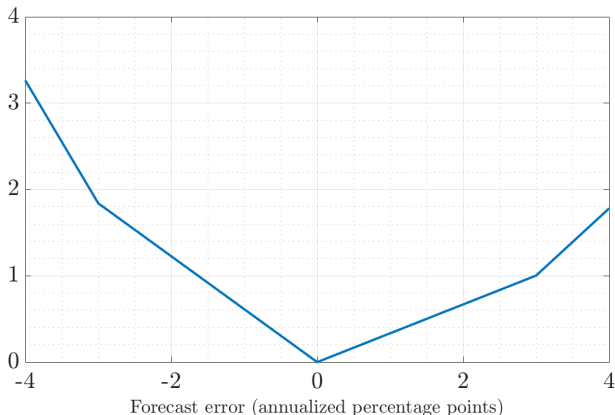


Figure: Changes in long-run inflation expectations as a function of forecast errors

# The expectation process over time

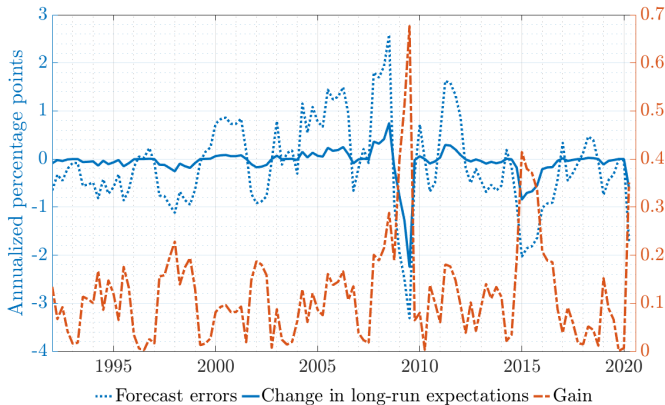


Figure: Time series of forecast errors, changes in long-run expectations and implied gain

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# Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

- $\mathbb{E}$  is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

# Target criterion

## Proposition

*Let  $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial \mathbf{z}}$  at  $t$  and define  $b_1$  as the first row of  $b$ . Then monetary policy optimally brings about the following target relationship between inflation and the output gap*

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t$$

RE (discretion): move  $\pi_t$  and  $x_t$  to offset cost-push shocks

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$$\pi_t - \Gamma(k) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} = -\frac{\lambda_x}{\kappa} x_t$$

Adaptive learning: can move  $\mathbb{E}_t x_{t+i}$  too if  $k > 0$

►  $\Gamma(k)$

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$$\pi_t - \Omega \left( k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - f_{t+1+j|t+j} \mathbf{g}_{\pi,t+j}) \right) = -\frac{\lambda_x}{\kappa} x_t$$

Endogenous gain: ability to move  $\mathbb{E}_t x_{t+i}$  depends on present and future degree of unanchoring

► Full expression,  $\Omega$

► No commitment

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# Numerical solution procedure

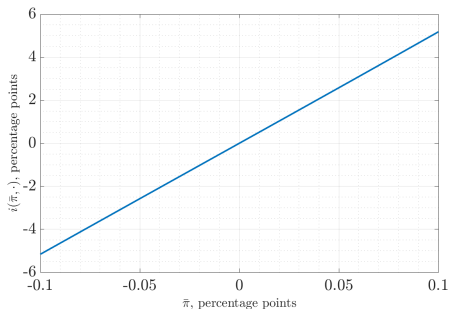
Solve system of model equations + target criterion

For calibrated model with  $\lambda_x = 0.05$  (Rotemberg & Woodford 1997),

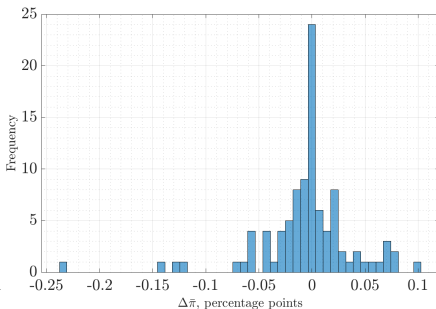
↪ solve using parameterized expectations algorithm

↪ obtain a cubic spline approximation to optimal policy function

# Optimal policy - responding to unanchoring



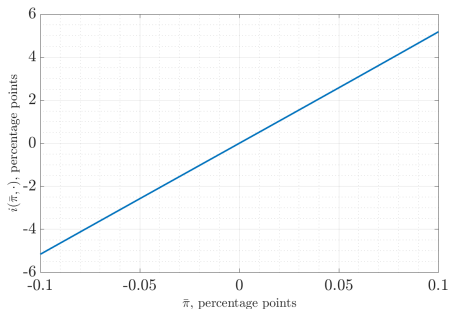
$i(\bar{\pi}, \text{all other states at their means})$



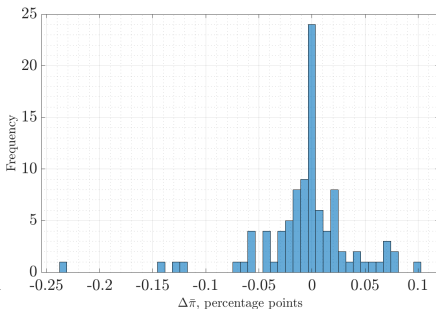
Stabilizing  $\bar{\pi}$

5 bp movement in  $\bar{\pi} \rightarrow 250$  bp movement in  $i$

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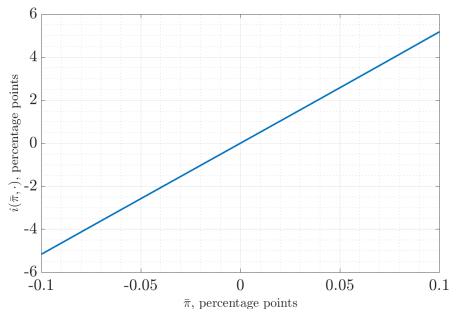
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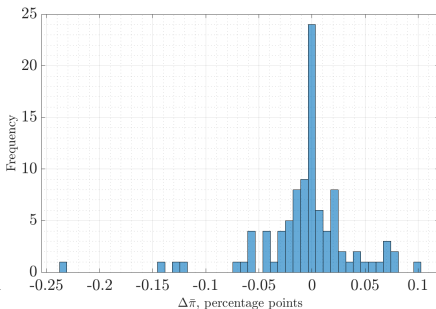
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5 bp movement in  $\bar{\pi} \rightarrow 250$  bp movement in  $i$  (1 standard dev. change in  $\bar{\pi}$ )

Mode: 0.3 bp movement in  $\bar{\pi} \rightarrow 15$  bp movement in  $i$

# Unanchoring causes volatility

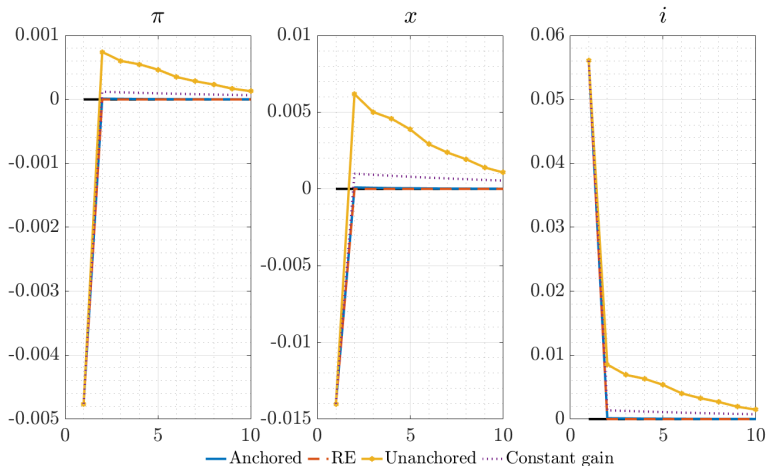


Figure: Impulse responses after a contractionary monetary policy shock when policy follows a Taylor rule

# Volatility comes from endogenous gain

- Constant gain:

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Shocks raise the gain  $\rightarrow$  central bank needs to anchor

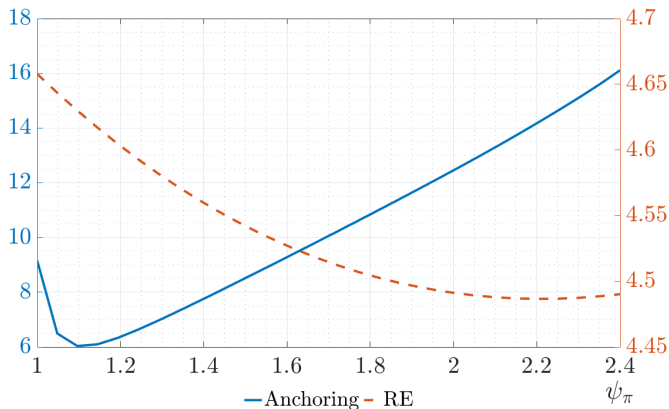
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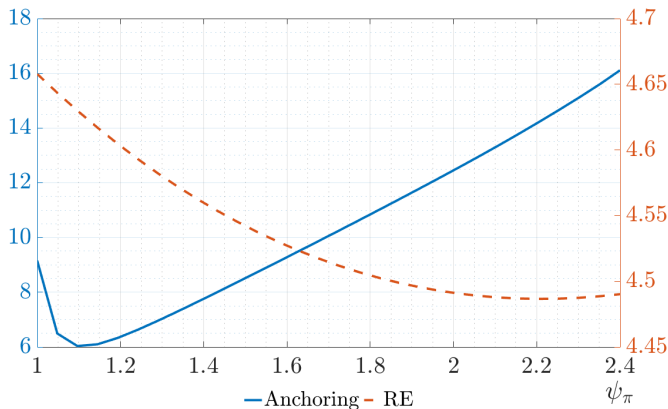
# Optimal Taylor-coefficient on inflation

Figure: Central bank loss as a function of  $\psi_\pi$



# Optimal Taylor-coefficient on inflation

Figure: Central bank loss as a function of  $\psi_\pi$



Anchoring-optimal coefficient:  $\psi_\pi^A = 1.09$

RE-optimal coefficient:  $\psi_\pi^{RE} = 2.21$

# Respond but not too much

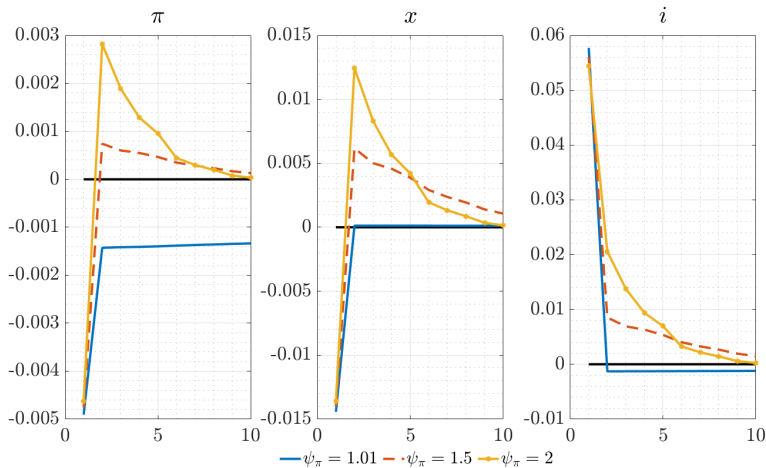


Figure: Impulse responses for unanchored expectations for various values of  $\psi_\pi$

# Why less aggressive? Future interest rate expectations

IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n)$$

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- Current interest rate  $i_t$ : one channel of policy

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- Current interest rate  $i_t$ : one channel of policy
- Expected future interest rates: additional channel of policy
- Only if policy reaction function internalized

# Conclusion

First theory of monetary policy for potentially unanchored expectations

Estimation of novel unanchoring channel

- Large and negative surprises unanchor more
- Estimated gain time series: on average, people only use the last 8 quarters of data

Monetary policy

- Expectations unanchoring makes smoothing shocks over time possible
- Optimal policy aggressive when unanchored, accommodates otherwise
- Taylor rule less aggressive than under rational expectations

Future work

- ↪ How to anchor at zero-lower bound?
- ↪ Other applications: currency crises

## Appendix



# Breakeven inflation

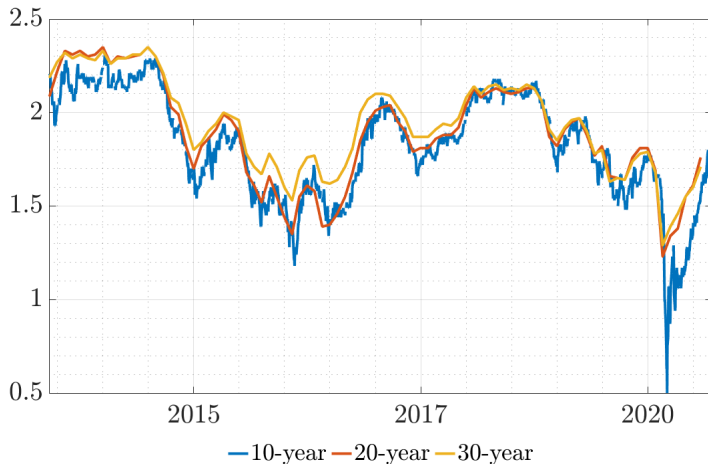


Figure: Market-based inflation expectations, various horizons, %

# Correcting the TIPS from liquidity risk

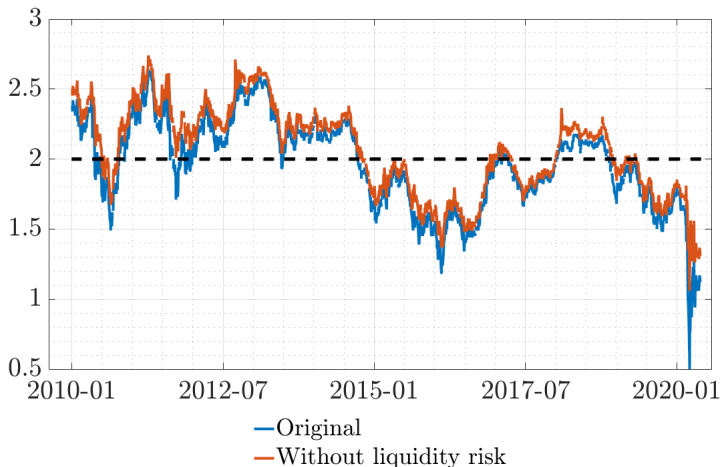


Figure: Market-based inflation expectations, 10 year, %

# Further evidence

Figure: Livingston Survey of Firms:  
Interquartile range of 10-year ahead inflation expectations

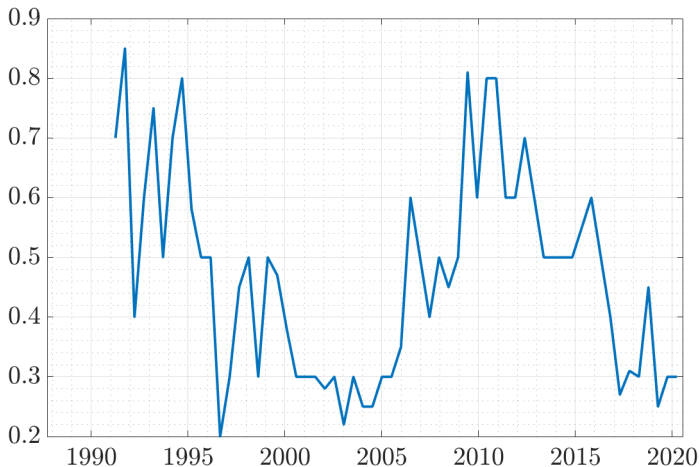
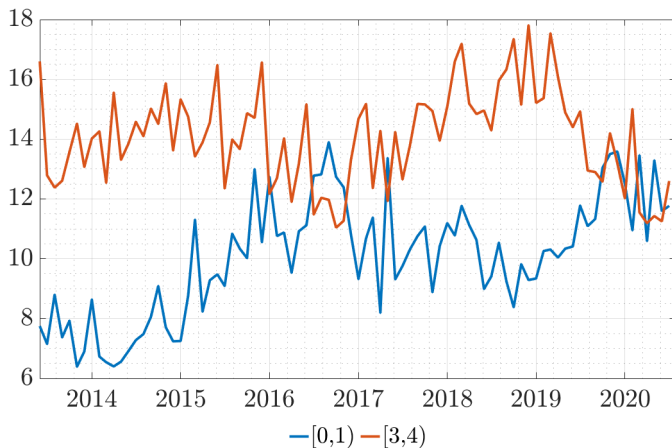


Figure: New York Fed Survey of Consumers:  
Percent of respondents indicating 3-year ahead inflation will be in a particular range



# Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \quad (19)$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \quad (20)$$

Solve for the time series of expectations  $f_t$

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \quad (21)$$

Solve for forecast error  $f_t \equiv \pi_t - f_{t-1}$ :

$$f_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k \rightarrow 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \quad (22)$$

# Functional forms for $g$ in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \quad (23)$$

$p \equiv \text{Prob}(\text{liquidity trap regime})$   
 $y_{t-1}$  output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when } \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \quad (24)$$

$\theta_t$  criterion,  $\bar{\theta}$  threshold value

# Choices for criterion $\theta_t$

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \quad (25)$$

$\Sigma$  variance-covariance matrix of shocks

$T(\phi)$  mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1} (f_{t|t-1} f'_{t|t-1} - \omega_{t-1}) \quad (26)$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1} (f'_{t|t-1} \omega_t^{-1} f_{t|t-1} - \theta_{t-1}) \quad (27)$$

$\omega_t$  estimated forecast-error variance

# Recursive least squares algorithm

$$\phi_t = \left( \phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left( y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right) \right)' \quad (28)$$

$$R_t = R_{t-1} + k_t \left( \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} [1 \quad s_{t-1}] - R_{t-1} \right) \quad (29)$$



# Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \quad (30)$$

$$s_t = h s_{t-1} + \epsilon_t \quad (31)$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \quad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \quad (32)$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} y_{T+1} \quad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \quad (33)$$

# Piecewise linear approximation to gain function

$$\mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}) \quad (34)$$

- $b_i(f_{t|t-1})$  = piecewise linear basis
- $\gamma_i$  = approximating coefficient at node  $i$

↪ Estimate  $\hat{\gamma}$  via simulated method of moments

# Target criterion

## Proposition

*In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap*

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - f_{t+1+j|t+j} \mathbf{g}_{\pi,t+j}) \right)$$

where  $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$  at  $t$ , and  $b_1$  is the first row of  $b$ .

## Lemma

*The discretion and commitment solutions of the Ramsey problem coincide.*

► Why no commitment?

## Corollary

*Optimal policy under adaptive learning is time-consistent.*

# No commitment - no lagged multipliers

Simplified version of the model: planner chooses  $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$  to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t) \right. \\ \left. + \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \right\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa}x_t - \varphi_{2,t}(k_t + \mathbf{g}_{\pi}(\pi_t - f_{t-1})) = 0 \quad (35)$$

$$-2\beta\frac{\lambda}{\kappa}x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_f(\pi_{t+1} - f_t)) = 0 \quad (36)$$

# Target criterion system for anchoring function as changes of the gain

$$\begin{aligned} \varphi_{6,t} = & -cf_{t|t-1}x_{t+1} + \left(1 + \frac{f_{t|t-1}}{f_{t+1|t}}(1 - k_{t+1}) - f_{t|t-1}\mathbf{g}_{\pi,t}\right)\varphi_{6,t+1} \\ & - \frac{f_{t|t-1}}{f_{t+1|t}}(1 - k_{t+1})\varphi_{6,t+2} \end{aligned} \quad (37)$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{f_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{f_{t|t-1}}\varphi_{6,t+1} \quad (38)$$

$\varphi_{6,t}$  Lagrange multiplier on anchoring function

The solution to (38) is given by:

$$\varphi_{6,t} = -2\mathbb{E}_t \sum_{i=0}^{\infty} \left(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}\right) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{f_{t+j|t+j-1}}}{\frac{k_{t+j}}{f_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \quad (39)$$

# Details on households and firms

Consumption:

$$C_t^i = \left[ \int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (40)$$

$\theta > 1$ : elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[ \int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \quad (41)$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \quad (42)$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \quad (43)$$

# Derivations

## Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \quad (44)$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \quad (45)$$

where ‘hats’ denote log-linear approximation and  $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$ .

1. Solve (44) backward to some date  $t$ , take expectations at  $t$
  2. Sub in (45)
  3. Aggregate over households  $i$
- Obtain (15)