Materials 19

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Take a very simple optimal policy problem where the planner chooses $\{\pi_t, x_t, f_t, k_t^{-1}\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t)$$
$$+ \varphi_{2,t}(f_t - f_{t-1} - k_t^{-1}(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t^{-1} - \mathbf{g}(\pi_t - f_{t-1})) \Big\}$$

where the IS-curve, $x_t = \mathbb{E}_t x_{t+1} + \sigma f_t - \sigma i_t + \sigma r_t^n$, is a non-binding constraint, and $\mathbb{E}_t x_{t+1}$ is rational. After some manipulation, FOCs reduce to:

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t^{-1} + \mathbf{g}_{\pi}(\pi_t - f_{t-1})) = 0 \tag{1}$$
$$-2\beta \frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1}^{-1} - \mathbf{g}_{\mathbf{f}}(\pi_{t+1} - f_t)) = 0 \tag{2}$$

Combining FOCs with the three model equations, I obtain the following system in $\{\pi_t, x_t, f_t, k_t^{-1}, \varphi_t\}_{t=0}^{\infty}$ where I've relabeled $\varphi \equiv \varphi_2$ for simplicity:

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t^{-1} + \mathbf{g}_{\pi}(\pi_t - f_{t-1})) = 0 \tag{3}$$
$$-2\beta \frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1}^{-1} - \mathbf{g}_{\mathbf{f}}(\pi_{t+1} - f_t)) = 0 \tag{4}$$
$$\pi_t - \kappa x_t - \beta f_t + u_t = 0 \tag{5}$$
$$f_t - f_{t-1} - k_t^{-1}(\pi_t - f_{t-1}) = 0 \tag{6}$$
$$k_t^{-1} - \mathbf{g}(\pi_t - f_{t-1}) = 0 \tag{7}$$