# Shadow Price Learning and Expectationally Driven Business Cycles

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May 27th, 2020

## Some Hyperbolic Motivation

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A: The forward-looking nature of decision making by economic agents

# Some Examples

- Life-cycle and permanent income hypothesis (consumption);
- Present-value and asset pricing (investment and savings);
- Ricardian equivalence (fiscal policy);
- Central bank credibility and inflation management (monetary policy);
- Exchange rate pegs and speculative currency crises (trade balance and international finance)

# This Paper

Replace RE with SP-learning (bounded optimality)in news-shock model to determine:

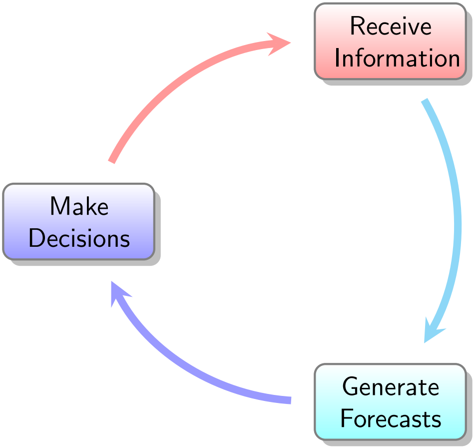
- How macroeconomic models are affected by our decisions regarding expectations formation mechanisms which impinge upon "optimal" behavior
- Whether there any notable interactive effects between information and expectations

# This Paper

Replace RE with SP-learning (bounded optimality)in news-shock model to determine:

- How macroeconomic models are affected by our decisions regarding expectations formation mechanisms which impinge upon "optimal" behavior
  - Resulting equilibria are statistically different from each other
- Whether there any notable interactive effects between information shocks and RE alternatives alternatives
  - The difference is amplified by the presence of anticipated shocks within the model

# Why Consider Information and Expectations-Based Explanations of Business Cycle Activity?



# Related Literature

- Modern macroeconomics is defined by its focus on modeling expectations (e.g. Muth (1961); Lucas (1976))
- Mixed evidence about relative importance of news versus surprise shocks in generating expectationally driven business cycles (e.g. Beaudry and Portier (2006); Ramey (2011); Schmitt-Grohe and Uribe (2012); Barsky and Sims (2011); Khan and Tsoukalas (2012))
- Adaptive learning can generate qualitatively similar expectationally driven business cycles (Eusepi and Preston (2011); Milani (2011, 2012, 2017))

# Key Concepts

1. Timing of economic shocks: "Surprise shocks" versus "news shocks"
2. Macroeconomic dynamic stochastic general equilibrium (DSGE) models and the "co-movement problem"
3. (Boundedly) rational behavior: Rational expectations equilibrium (REE), expectational stability (E-stability), and restricted perceptions equilibrium (RPE)

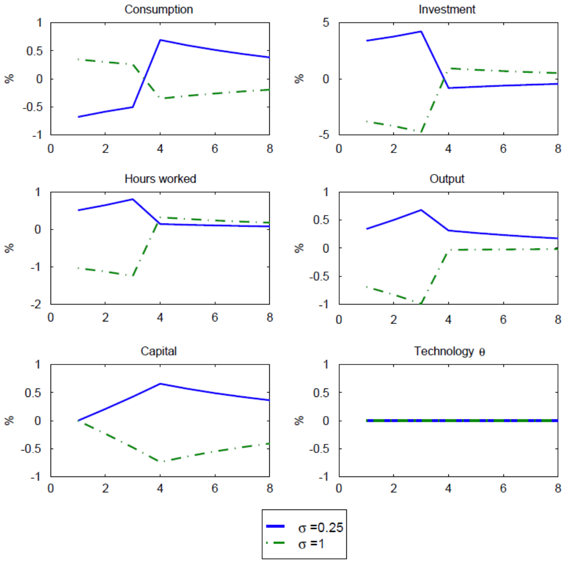
# Key Concept #1: Timing of Economic Shocks

- News shocks represent incomplete (but true) information received today about future innovations to exogenous processes
- Help to predict future economic fundamentals but do not affect current or past fundamentals
- Forward-looking behavior implies news shocks should impact contemporaneous endogenous variables

# Key Concept #2: News-shock Models and the Comovement Problem

- Post-war US data exhibits strong positive comovement in aggregate consumption, investment, employment, and output.
- Standard-bearing modern-day neoclassical and modern sticky price models predict *negative* comovement between consumption and investment, employment, and output in response to anticipated productivity shocks
- "News-shock models" modify economy to produce qualitatively realistic expectationally driven business cycles (EDBCs) in response to news shocks

# Example: RBC and News-shock Challenge



# Key Concept #3: (Boundedly) Rational Behavior

- *Rational Expectations:* Decision makers are endowed with a correctly specified and correctly estimated forecasting model for all relevant variables
- *Boundedly Rational Expectations:* Decisions makers act like "good econometricians" endowed with a forecasting model which is updated based on forecast errors over time

# Organization of Talk

1. Describe the economic environment, information structure, and expectation-formation assumptions
2. Solve the model under different expectations assumptions
3. Calibrate, simulate, and compare results

# Characterizing the Model

Representative agent, news-shock model of Jaimovich and Rebelo (2009) which features:

- Standard household labor-leisure/consumption-savings decision with novel preference structure to solve static comovement problem
- Costly to adjust investment and ability to vary utilization rate of existing physical capital to solve dynamic comovement problem
- Competitive markets for factors and fungible final good
- Exogenous stochastic processes for total factor productivity (TFP) and the conversion-rate of consumption to investment

# News Shocks and Expectations

- Households receive news about future innovations to exogenous stochastic processes zero, four, and eight periods (quarters) in advance
- This information is incorporated into household forecasts of future fundamentals, which affects decisions today
- Two models of forecasting: rational expectations (RE) and shadow price learning (SPL)

# Adaptive Learning and Bounded Rationality/Optimality

- Evans and McGough (2015) consider *shadow price learning* (SP-learning) to explore *bounded optimality*: agents use forecasts of state transition equations and shadow prices to take optimal actions given beliefs.
- Benefits of SP-learning:
  1. Highlights the link between prices, opportunity costs, and behavior
  2. Agents solve a simple two-period problem instead of an infinite-period problem - no dynamic programming!
  3. Forecasting/optimizing mistakes corrected over time allows for structural shifts

# Rational Expectations vs Shadow Price Learning

- RE assumes households know the conditional distribution of all variables i.e. their perceptions about the laws of motion for all variables correspond exactly to the actual laws of motion
  - Implication: Forecast errors are uncorrelated over time
- SPL assumes households forecast the value of the factors which determine optimal decision making today i.e. the endogenous shadow prices which affect expected marginal cost/marginal benefit using a simple linear forecasting model
  - Implication: Speed of updating introduces inertia and allows for systematic mistakes; forecasting endogenous shadow prices suggests boundedly optimal behavior.

# Representing the Model

Choose $C_t$, $I_t$, $h_t$, $u_t$, $K_t$, and $S_t$ to maximize

$$\max \quad \hat{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - \frac{\psi h_t^{1+\frac{1}{\theta}} S_t}{1+\frac{1}{\theta}}\right)^{1-\sigma} - 1}{1-\sigma}$$

$$\text{subject to} \quad S_t = C_t^{\gamma} S_{t-1}^{1-\gamma}$$

$$K_t = (1-\delta(u_t))K_{t-1} + I_t \left(1 - \varphi\left(\frac{I_t}{I_{t-1}}\right)\right)$$

$$C_t + A_t I_t = W_t h_t + R_t \left(u_t K_{t-1}\right) + Profits_t$$

$$Y_t = z_t \left(u_t K_{t-1}\right)^{\alpha_k} \left(h_t\right)^{\alpha_h}$$

$$I_{-1}, K_{-1}, S_{-1} \text{ given}$$

# Technology and Information

Exogenous stochastic processes indexed by $x = \{A, z\}$ evolve according to

$$\ln\left(\frac{x_t}{x}\right) = \rho_x \ln\left(\frac{x_{t-1}}{x}\right) + w^0_{x,t}$$
$$w^{k_x}_{x,t} = w^{k_x+1}_{x,t-1} + \sigma^{k_x}_x \nu^{k_x}_{x,t}$$

where $k_x = \{0, 1, ..., N_x\}$ and $N_x$ is the length of the forecasting horizon for process $x$.

# Example: One-period Ahead News

If agents receive news one and zero periods ahead...

$$\ln\left(\frac{x_t}{x}\right) = \rho_x \ln\left(\frac{x_{t-1}}{x}\right) + w^0_{x,t}$$
$$w^0_{x,t} = w^1_{x,t-1} + \sigma^0_x \nu^0_{x,t}$$
$$w^1_{x,t} = \sigma^1_x \nu^1_{x,t}$$

$$\Rightarrow \ln\left(\frac{x_t}{x}\right) = \rho_x \ln\left(\frac{x_{t-1}}{x}\right) + \left(\underbrace{\sigma^0_x \nu^0_{x,t}}_{\text{Unanticipated}} + \underbrace{\sigma^1_x \nu^1_{x,t-1}}_{\text{Anticipated}}\right)$$

# Solving the Model

Temporary equilibrium is of the form

$$\hat{E}_t G(u_t, x_t, x_{t+1}, \lambda_t, \lambda_{t+1}) = 0$$

where $x$ and $u$ are vectors of states and controls, respectively; $\lambda_t$ is a vector of endogenous state shadow prices.

This paper (currently): replace non-linear transition equations for shadow prices with simple linear model used to forecast.

Compare resulting *restricted perceptions equilibrium* to REE of linearized system.

# Updating the Forecasting Model: Adaptive Learning

The household's forecasting model for endogenous shadow prices is given by

$$\lambda_t = \tilde{H}_t' \tilde{x}_t \tag{2.1}$$

where $\tilde{x}_t$ is data and coefficient estimates $\tilde{H}_t$ are updated via recursive least squares according to

$$R_{H,t} = R^H_{t-1} + g_t \left( \tilde{x}_{t-1} \tilde{x}_{t-1}' - R^H_{t-1} \right) \tag{2.2}$$

$$\tilde{H}_t = \tilde{H}_{t-1} + g_t R^{-1}_{H,t} \tilde{x}_{t-1} \left( \lambda_{t-1} - \tilde{H}_{t-1}' \tilde{x}_{t-1} \right)' \tag{2.3}$$

where $R_H$ is the matrix of sample second moments for regressors.

# Methodology

1. The model solutions from RE and SPL provide a recursive system of equations as a function of exogenous stochastic processes and parameters
2. The system is "calibrated" using previous estimates for the parameters
3. Simulations are conducted by subjecting the complete model to a sequence of random shocks
4. Resulting simulated data is captured and summary statistics analyzed

# Typical Simulation

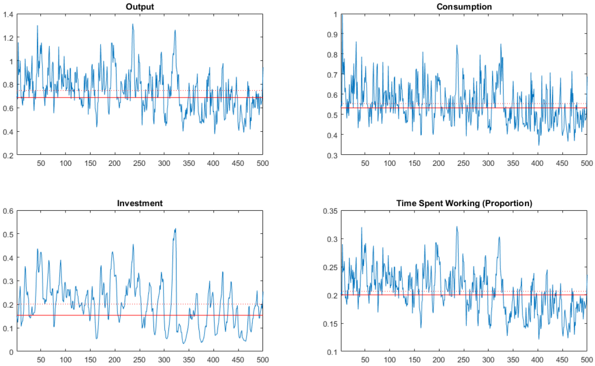



*Figure: SP-learning with constant gain ROLS. Solid red line is non-stochastic RE steady state.*

# Tests for Mean Equality (Model With News)

| | RE Mean | SPL Mean | pval | 95% Low | 95% High |
|---|---|---|---|---|---|
| Consumption | 0.533 | 0.568 | 0.000 | -0.036 | -0.032 |
| Labor Supply | 0.200 | 0.212 | 0.000 | -0.013 | -0.012 |
| Investment | 0.154 | 0.172 | 0.000 | -0.019 | -0.018 |
| Output | 0.687 | 0.736 | 0.000 | -0.052 | -0.046 |
| Investment SP | 0.000 | 0.060 | 0.000 | -0.061 | -0.059 |
| Capital SP | 4.250 | 4.043 | 0.000 | 0.193 | 0.219 |
| Habit-adjustment SP | -154.107 | -155.120 | 0.000 | 0.704 | 1.321 |

*Table: t-tests for Data Generating Process, News, 230 Periods*

# Tests for Mean Equality (Model Without News)

| | RE Mean | SPL Mean | pval | 95% Low | 95% High |
|---|---|---|---|---|---|
| Consumption | 0.534 | 0.563 | 0.000 | -0.031 | -0.027 |
| Labor Supply | 0.200 | 0.210 | 0.000 | -0.011 | -0.010 |
| Investment | 0.154 | 0.169 | 0.000 | -0.016 | -0.014 |
| Output | 0.688 | 0.728 | 0.000 | -0.043 | -0.038 |
| Investment SP | -0.000 | 0.034 | 0.000 | -0.035 | -0.034 |
| Capital SP | 4.248 | 4.068 | 0.000 | 0.167 | 0.192 |
| Habit-adjustment SP | -154.085 | -154.949 | 0.000 | 0.558 | 1.170 |

*Table: t-tests for Data Generating Process, No News, 230 Periods*

# Comparison of Simulated Moments

| | US Data | RE (News) | SPL (News) | RE (No News) | SPL (No News) |
|---|---|---|---|---|---|
| $\sigma_h/\sigma_Y$ | 0.968 | 0.714 | 0.716 | 0.714 | 0.715 |
| $\sigma_I/\sigma_Y$ | 3.103 | 2.386 | 2.167 | 2.282 | 2.023 |
| $\sigma_C/\sigma_Y$ | 0.712 | 0.737 | 0.706 | 0.746 | 0.725 |
| $corr(Y, h)$ | 0.860 | 1.000 | 1.000 | 1.000 | 1.000 |
| $corr(Y, I)$ | 0.890 | 0.850 | 0.879 | 0.922 | 0.912 |
| $corr(Y, C)$ | 0.770 | 0.969 | 0.969 | 0.977 | 0.986 |

*Table: Predicted Business Cycle Statistics*

## Conclusion

- SP-learning causes convergence to an RPE which is statistically different from the model's REE
- Difference is amplified by presence of news shocks due to persistent overestimation of benefit of future investment
- SP-learning as a tool for applied macroeconomists to improve model fit yields mixed results

# Future Work

1. Can agents learn to optimize?
    - If so, does news effect speed of convergence?
2. How does RPE compare to REE at higher order solutions?
3. Can SPL and bounded optimality generate interesting distributions with heterogeneous agents?