Notation and Abbreviations

The following list specifies general notational conventions used in the book. Occasionally in the text, a symbol has a meaning that differs from the one specified in this list when confusion is unlikely.

General Symbols and Notation

$=$	equals
$\equiv$	equals by definition, is defined as
$\propto$	is proportional to
$\Rightarrow$	implies
$\Leftrightarrow$	is equivalent to
$\sim$	is distributed as
$\overset{a}{\sim}$	is approximately distributed in large samples
$\overset{iid}{\sim}$	is independently, identically distributed as
$\forall$	for all
$\exists$	there exists
$\in$	element of
$\subset$	subset of
$\cup$	union
$\cap$	intersection
$\sum$	summation sign
$\prod$	product sign
$\rightarrow$	converges to, approaches
$\overset{p}{\rightarrow}$	converges in probability to
$\overset{a.s.}{\rightarrow}$	converges almost surely to
$\overset{q.m.}{\rightarrow}$	converges in quadratic mean to
$\overset{d}{\rightarrow}$	converges in distribution to
iid	independently, identically distributed
lim	limit
plim	probability limit

max	maximum
min	minimum
sup	supremum, least upper bound
log	natural logarithm
exp	exponential function
$\lvert z \rvert$	absolute value or modulus of z
K	dimension of a stochastic process or time series
T	sample size, time series length
$\mathbb{R}$	real numbers
$\mathbb{R}^m$	m-dimensional Euclidean space
$\mathbb{C}$	complex numbers
$\mathbb{Z}$	integers
$\mathbb{N}$	positive integers
$\mathbb{P}$	probability
$\mathbb{H}_0$	null hypothesis
$\mathbb{H}_1$	alternative hypothesis
$\mathbb{I}(\cdot)$	indicator function
L	lag operator
Δ	differencing operator
$\mathbb{E}$	expectation operator
$l(\cdot)$	likelihood function
$\log l$	log-likelihood function
$[x]$	largest integer smaller or equal to $x \in \mathbb{R}$
1968m10	October 1968
1968q3	third quarter of 1968

Abbreviations

AD	aggregate demand
AIC	Akaike information criterion
AR	autoregression
ARCH	autoregressive conditional heteroskedasticity
ARMA	autoregressive moving average
AS	aggregate supply
AVAR	asymmetric vector autoregression
BVAR	Bayesian vector autoregression
Corr	correlation, correlation matrix
Cov	covariance, covariance matrix
CPI	consumer price index
CUSUM	cumulative sum
d.f.	degrees of freedom
DFM	dynamic factor model
DGP	data generating process
DSEM	dynamic simultaneous equations model
DSGE	dynamic stochastic general equilibrium
EM	expectation maximization
FAVAR	factor-augmented vector autoregression
FAVARMA	factor-augmented vector autoregressive moving average

FECM	factor error correction model
FIML	full information maximum likelihood
FOMC	Federal Open Market Committee
GARCH	generalized autoregressive conditional heteroskedasticity
GDFM	generalized dynamic factor model
GDP	gross domestic product
GIRF	generalized impulse response function
GLS	generalized least squares
GMM	generalized method of moments
GNP	gross national product
GO-GARCH	generalized orthogonal GARCH
GVAR	global vector autoregression
HP	Hodrick-Prescott
HPD	highest posterior density
HQC	Hannan-Quinn criterion
ICA	independent component analysis
IV	instrumental variables
LM	Lagrange multiplier
LR	likelihood ratio
LS	least squares
M1	narrow money stock
MA	moving average
MCMC	Markov Chain Monte Carlo
MGARCH	multivariate generalized autoregressive conditional heteroskedasticity
ML	maximum likelihood
MS	Markov switching
MSE	mean squared error
MSPE	mean squared prediction error
MS-VAR	Markov-switching vector autoregression
NBER	National Bureau of Economic Research
NBR	nonborrowed reserves
PC	principal components
RBC	real business cycle
SIC	Schwarz information criterion
SNP	semi-nonparametric
ST-VAR	smooth-transition vector autoregression
SVAR	structural vector autoregression
S&P 500	Standard and Poor's 500 stock price index
TFP	total factor productivity
TR	total reserves
TRAMO-SEATS	seasonal adjustment method
TVAR	threshold vector autoregression
TVC-VAR	time-varying coefficient vector autoregression
Var	variance
VAR	vector autoregression
VARMA	vector autoregressive moving average

VARX	VAR with exogenous variables
VECM	vector error correction model
X-12-ARIMA	seasonal adjustment method

Vector and Matrix Operations

M'	transpose of matrix M
M^{-1}	inverse of square matrix M
$M_\perp$	orthogonal complement of matrix M
$M^{1/2}$	square root of symmetric positive definite matrix M
M^k	k^{th} power of matrix M
MN	matrix product of matrices M and N
$\otimes$	Kronecker product
$\text{chol}(M)$	Cholesky factor of positive definite matrix M
$\det(M)$	determinant of matrix M
$\|M\|$	Euclidean norm of matrix M
$\text{rk}(M)$	rank of matrix M
$\text{tr}(M)$	trace of matrix M
vec	column stacking operator
vech	column stacking operator for square matrices (stacks the elements on and below the main diagonal only)
veck	column stacking operator for square matrices (stacks the elements above the main diagonal only)
$\dfrac{\partial\varphi}{\partial\beta'}$	vector or matrix of first order partial derivatives of function φ with respect to vector β
$\dfrac{\partial^2\varphi}{\partial\beta\partial\beta'}$	matrix of second order partial derivatives of φ with respect to β (Hessian matrix)

General Matrices

$\mathbf{D}_m$	$m^2 \times \frac{1}{2}m(m+1)$ duplication matrix
I_m	$m \times m$ unit or identity matrix
$\mathcal{I}(\cdot)$	information matrix
$\mathcal{I}_a(\cdot)$	asymptotic information matrix
$\mathbf{K}_{mn}$	$mn \times mn$ commutation matrix
$\mathbf{L}_m$	$\frac{1}{2}m(m+1) \times m^2$ elimination matrix
0	zero or null matrix or vector of suitable dimension
$0_{m\times n}$	zero matrix of dimension $m \times n$
$\mathcal{O}(K)$	set of $K \times K$ orthogonal matrices

Distributions and General Stochastic Processes

$\mathcal{N}(\mu, \Sigma)$	(multivariate) normal distribution with mean (vector) μ and variance (covariance matrix) Σ
$\chi^2(m)$	χ^2 distribution with m degrees of freedom
$F(m, n)$	F distribution with m numerator and n denominator degrees of freedom

$t(m)$	t distribution with m degrees of freedom
$\mathcal{W}_K(\Sigma, n)$	K-dimensional Wishart distribution with parameters Σ and n
$\mathcal{IW}_K(\Sigma, n)$	K-dimensional inverse Wishart distribution with parameters Σ and n
$\mathcal{U}(a, b)$	uniform distribution on the interval (a, b)
$\mathbf{W}_K$	K-dimensional standard Brownian motion or Wiener process
$I(d)$	stochastic process integrated of order d
$I(1)$	stochastic process integrated of order 1
$I(0)$	stationary stochastic process
mds	martingale difference sequence

Alternative VAR Specifications

$y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t$	reduced-form VAR
$B_0 y_t = B_1 y_{t-1} + \cdots + B_p y_{t-p} + w_t$	structural-form VAR
$\Delta y_t = \mathbf{\Pi} y_{t-1} + \mathbf{\Gamma}_1 \Delta y_{t-1} + \cdots + \mathbf{\Gamma}_{p-1} \Delta y_{t-p+1} + u_t$	reduced-form VECM
$B_0 \Delta y_t = \alpha^\dagger \beta y_{t-1} + \mathbf{\Gamma}_1^\dagger \Delta y_{t-1} + \cdots + \mathbf{\Gamma}_{p-1}^\dagger \Delta y_{t-p+1} + w_t$	structural-form VECM

Vectors and Matrices Related to VAR Models

$u_t = \begin{pmatrix} u_{1t} \\ \vdots \\ u_{Kt} \end{pmatrix}$	K-dimensional white noise process, reduced-form error
$U \equiv [u_1, \ldots, u_T]$	$K \times T$ matrix
$U_t \equiv \begin{pmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$	Kp-dimensional vector
$w_t = \begin{pmatrix} w_{1t} \\ \vdots \\ w_{Kt} \end{pmatrix}$	K-dimensional structural error vector
$y_t = \begin{pmatrix} y_{1t} \\ \vdots \\ y_{Kt} \end{pmatrix}$	K-dimensional stochastic process of observed variables
$y_{t+h\|t}$	h-step forecast of y_{t+h} made at point t
$Y \equiv [y_1, \ldots, y_T]$	$K \times T$ matrix

$Y_t \equiv \begin{pmatrix} y_t \\ \vdots \\ y_{t-p+1} \end{pmatrix}$ Kp-dimensional vector

$Z_t \equiv \begin{pmatrix} 1 \\ y_t \\ \vdots \\ y_{t-p+1} \end{pmatrix}$ $(Kp+1)$-dimensional vector

$Z \equiv [Z_0, \dots, Z_{T-1}]$ $(Kp+1) \times T$ matrix or $[Y_0, \dots, Y_{T-1}]$ $Kp \times T$ matrix

Matrices and Vectors Related to VARs and VECMs

$A_i = \begin{bmatrix} a_{11,i} & \dots & a_{1K,i} \\ \vdots & \ddots & \vdots \\ a_{K1,i} & \dots & a_{KK,i} \end{bmatrix}$ reduced-form VAR coefficient matrix

$A \equiv [A_1, \dots, A_p]$ or $[\nu, A_1, \dots, A_p]$

$\boldsymbol{\alpha} \equiv \mathrm{vec}(A)$

$\mathbf{A} \equiv \begin{bmatrix} A_1 & \dots & A_{p-1} & A_p \\ I_K & & 0 & 0 \\ & \ddots & \vdots & \vdots \\ 0 & \dots & I_K & 0 \end{bmatrix}$

$A(L) \equiv I_K - A_1 L - \dots - A_p L^p$

$B_i = \begin{bmatrix} b_{11,i} & \dots & b_{1K,i} \\ \vdots & \ddots & \vdots \\ b_{K1,i} & \dots & b_{KK,i} \end{bmatrix}$ structural-form VAR coefficient matrix

$B_0^{-1} = \begin{bmatrix} b_0^{11} & \dots & b_0^{1K} \\ \vdots & \ddots & \vdots \\ b_0^{K1} & \dots & b_0^{KK} \end{bmatrix}$ matrix of impact effects of structural shocks

$B(L) \equiv B_0 - B_1 L - \dots - B_p L^p$

α loading matrix of VECM

β cointegration matrix of VECM

$\boldsymbol{\Pi} \equiv \alpha\beta'$

$\boldsymbol{\Gamma}_i$ coefficient matrix on i^{th} lagged difference of VECM

$\Phi_i = \begin{bmatrix} \phi_{11,i} & \dots & \phi_{1K,i} \\ \vdots & \ddots & \vdots \\ \phi_{K1,i} & \dots & \phi_{KK,i} \end{bmatrix}$ coefficient matrix of canonical MA representation

$\Phi(L)$ $= I_K + \sum_{i=1}^{\infty} \Phi_i L^i$

Θ_i $= \begin{bmatrix} \theta_{11,i} & \dots & \theta_{1K,i} \\ \vdots & \ddots & \vdots \\ \theta_{K1,i} & \dots & \theta_{KK,i} \end{bmatrix}$ matrix of structural impulse responses

$\Theta(L)$ $= \sum_{i=0}^{\infty} \Theta_i L^i$

$\boldsymbol{\Xi}$ matrix of long-run effects of reduced-form shocks in VECM

$\boldsymbol{\Upsilon}$ $= \begin{bmatrix} \zeta_{11} & \dots & \zeta_{1K} \\ \vdots & \ddots & \vdots \\ \zeta_{K1} & \dots & \zeta_{KK} \end{bmatrix}$ matrix of long-run effects of structural shocks

Moment Matrices

Γ $\equiv \text{plim } ZZ'/T$

$\Gamma_y(h)$ $\equiv \text{Cov}(y_t, y_{t-h})$ for a stationary process y_t

Σ_u $\equiv \mathbb{E}(u_t u_t')$ (reduced-form white noise covariance matrix)

Σ_w $\equiv \mathbb{E}(w_t w_t')$ (structural-form white noise covariance matrix)

Σ_y $\equiv \mathbb{E}[(y_t - \mu)(y_t - \mu)']$ (covariance matrix of a stationary process y_t)

P lower-triangular Cholesky decomposition of Σ_u

$\Sigma_{\widehat{\alpha}}$ covariance matrix of the asymptotic distribution of $\sqrt{T}(\widehat{\alpha} - \alpha)$

$\Sigma_y(h)$ MSE or forecast error covariance matrix of h-step forecast of y_t

$\Sigma_{\widehat{y}}(h)$ approximate MSE matrix of h-step forecast of estimated process y_t