Adaptive Learning and Survey Expectations of Inflation

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Abstract

The use of survey information on inflation expectations as an observable in a DSGE model can refine substantially the identification of the shocks that drive the inflation process. An optimal integration of the survey information improves the model forecast for both inflation and the other macroeconomic variables. Models with expectations based on an Adaptive Learning setup can exploit the survey information more efficiently than their Rational Expectations counterparts. The resulting time-variation in the perceived inflation target, in the inflation persistence and in the sensitivity of inflation to various shocks provide a rich and consistent description of the joint dynamics in realized and expected inflation.

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The objective of this paper is to illustrate how the observation of survey evidence on inflation expectations, as measured by the Survey of Professional Forecasters, affects the estimation outcomes of the standard new-Keynesian DSGE model. After documenting that the survey expectations deviate substantially from the implied model expectations, particularly during periods with important trend changes in the inflation dynamics, we propose a simple re-specification of the price and wage mark-up shocks in the Smets and Wouters (2007) model that allows to reconcile the survey and the model forecasts. In particular, the survey data on inflation expectations help to identify separately the innovations in the persistent component of the inflation markup process. These innovations constitute only a small fraction of the high frequency volatility in inflation and are hard to distinguish from the noisy transitory component without observing the additional and timely information present in the survey expectations. By resolving this filtering problem, the model forecast for inflation tend to converge to the survey forecast and the overall fit and forecasting performance of the model improve substantially. Models with expectations based on the Adaptive Learning setup can exploit the survey information more efficiently than their Rational Expectations counterparts. With an appropriate specification of the forecasting or belief models that incorporate the signals from the survey evidence, agents will update their beliefs about the role of observed and latent signals for future inflation in function of the systematic forecast errors. The resulting time-variation in the perceived inflation target, in the inflation persistence and in the sensitivity of inflation to various shocks provide a rich and consistent description of the dynamics in realized and expected inflation.

Inflation expectations have become the most important indicator for monetary policy. Since the financial crisis, realised inflation has been below target in most advanced countries. Recently, declining oil and commodity prices have further pushed headline inflation down and caused repeated negative surprises in the inflation process. In this situation, attention of monetary policymakers has been concentrated on inflation expectations that are observed in surveys or distilled from financial yields. Policy makers stress that it is absolutely necessary for the inflation expectations to remain anchored around the long run inflation objective. The following quote from a speech by C. Evans (2016) illustrates the crucial role of inflation expectations for monetary policy makers: "However, there are some downside risks to this forecast. We might see further declines in energy prices or greater appreciation of the dollar. In addition, undershooting our 2 percent inflation target for as long as we have invites the risk of the public beginning to expect persistently low inflation in the future. If this mindset becomes embedded in decisions regarding wages and prices, then getting inflation back to 2 percent will be that much more difficult. Here, I find it troubling that the compensation for prospective inflation built into a number of financial market asset prices has drifted down considerably over the past two years. More recently, some survey-based measures of inflation expectations, which had previously seemed unmovable, have also edged down. So to achieve our inflation target — and to provide a buffer against downside risks — it is appropriate that we follow a gradual path to policy normalization."

Inflation expectations also play a crucial role in modern macro models: the forward looking New Keynesian Phillips curve is the central equation in the monetary DSGE models. Inflation expectations drive actual price and wage setting; via the expected real return and the intertemporal substitution, they also have the potential to steer aggregate demand, in particular when the nominal rate is constrained by the zero lower bound. Recent literature gave inflation expectations a crucial role in understanding the behaviour of inflation over the Great Recession and the recent recovery period. Coibion and Gorodnichenko (2015) suggest that stability in expectations, in particular on the household side, led to the relative stable inflation realisations during the Great Recession. As households inflation expectations, which may serve as close proxy for firm expectations as well, are more responsive to oil prices than those of professional forecasters, the increase in oil prices between 2009 and 2012 contributed to avoiding the onset of deflationary dynamics. Del Negro, Giannoni and Schorfheide (2015) argue that inflation expectations remained anchored during the Great Recession because monetary policy managed to maintain expectations of rising future marginal costs. High nominal stickiness and a monetary policy that reacts strongly to inflation reinforce the control of policy over expectations.

The behaviour of inflation expectations is also central in the criticism of the low interest rate policies, such as e.g. Cochrane (2015), arguing that prolonged low nominal interest rates and the central bank promises to maintain such policy, may bring about lower, rather than higher, inflation. Evans et al (2016) and Garcia, Schmidt, and Woodford (2015) explain how these outcomes depend on the way agents formulate their expectations and perceive the policy reaction function. Understanding the way inflation expectations are formed and how they develop over time is crucial for evaluating the risk of convergence to a low inflation-low interest rate steady state. Survey expectations might contain signals relevant for interpreting how agents formulate their beliefs and which solution path they select.

Survey expectations on inflation are very informative. The review of various inflation forecasting models and survey data by Ang, Bekaert and Wei (2007) has documented the superior forecasting performance of the survey expectations for inflation. The survey expectations probably reflect a large amount of information that is processed in an efficient way with sufficient flexibility to adjust over time. Survey expectations about future inflation have the advantage that they provide unambiguous information on agents' forecasts. Financial market evidence on inflation compensation, on the other hand, requires further processing to correct for risk premiums. In practice, central bank forecasts combine model forecasts and judgemental forecasts, with the latter heavily influenced by survey or market evidence. By integrating survey evidence in the standard dataset that is processed by the model, the need for additional judgemental adjustments to the model forecast will become less pressing.

Therefore, there are many important reasons for the inclusion of data on inflation expectations into the standard datasets on which our macromodels are estimated. In this way, the dynamics of these inflation expectations are analysed together with realized inflation data in order to pin down more precisely the transmission mechanism of the various shocks. The overall information results in a consistent estimate of the state of the economy and of the expectations, inflation expectation process in particular. Rational or model-consistent expectations is one hypothesis for explaining the way expectations are formed, but this hypothesis is evaluated against alternatives that allow for more flexibility and provide more insight on how the agents formulate their beliefs and how they adjust them when confronted with new data and changes in their environments.

Given the strong arguments for introducing survey data in the information set on which our models are estimated, relatively few papers have actually implemented this approach.[1] A paper that comes closest to ours is Ormeno and Molnar (2015). These authors use survey data on inflation expectations as an observable for the model expectations via an additional measurement equation. Their results illustrate that the survey contains information not present in macro data, and that this can improve the model forecast. They also show how an adaptive learning approach based on small forecasting models is more flexible in exploiting the information than a fully rational expectations model. The paper, however, does not explain what type of information is revealed by the survey and what changes in the model specification can optimize the integration of the survey data in the model. Eusepi and Del Negro (2011) introduce inflation target shocks in their RE-model to improve the match between model and survey forecasts. This exogenous target shock captures the additional information provided by the survey and transmits it to the model forecast. While improving the model fit, the approach does not succeed in completely bridging the gap between model and survey forecasts. Similarly, De Graeve et al (2009) illustrate how an inflation target shock is necessary for matching inflation expectations in the yield structure. Del Negro and Schorfheide (2013) use long run inflation expectations as the observable and an inflation target shock as the modelling device. Eusepi et al (2015), on the other hand, use short term survey expectations and learning about the long run inflation target to model the inflation expectations.

These examples illustrate how Rational Expectation models typically resort to exogenous shocks in the inflation target to match the survey expectations, while Adaptive Learning models can explain the long run drift in expectations by the updating of agents expectation process, in particular their beliefs about the inflation target. We review and compare these alternative approaches systematically and come up with an optimal modelling device. We concentrate on the nature of the shocks that drive the expectation as in Milani and Rajbhandari (2012): they use survey data on various macrovariables to identify a set of news shocks in addition to the contemporaneous innovations and show that these news shocks explain a major part of business cycle, but they do not focus

[1] There is broader literature that investigates what type of price setting models are consistent with the inflation expectations in survey data: i.e., models with sticky information or heterogeneous beliefs, such as Mankiw and Reis (2002) and Branch (2007). See also Coibion and Goradnicienko (2015) for more evidence on information rigidities in survey data and their theoretical interpretation.

on inflation.[2]

Survey expectations have been used successfully as proxy for inflation expectations in single-equation estimates of the NKPC, see, e.g. Roberts (1995). In these exercises, survey data are treated as exogenous and there is no need for explaining how these survey expectations are formed. Another literature has emphasized the limitations of the survey expectations by illustrating how they deviate from the full information rational expectations hypothesis, see, e.g., Roberts (1997), Mankiw et al (2004) and Coibion and Gorodnicienko (2015). These observations led to models with information rigidities that have also the potential to explain the dispersion in survey expectations across agents. Fuhrer (2015), following Adam and Padula (2011) and Branch and McGrough (2009), endogenizes the survey expectations in a macromodel with minimal assumptions based on the law of iterated expectations. We start with a standard rational expectations macromodel and concentrate on the optimal exploitation of the information in the survey forecasts to improve the overall fit and forecast of the model. Three features of our approach are helpful to overcome the above mentioned limitations of the survey data. First, we use short run SPF survey forecasts for inflation as observable variable. These one-quarter ahead forecasts are less subject to all type of inefficiencies typically observed in survey data that cover longer forecast horizons. At the same time, they contain timely information that is complementary to the standard macro dataset and which is crucial for our objective. Second, we consider model specifications that have sufficient flexibility to exploit the information in the survey forecasts. The introduction of a more general markup shock process is one ingredient that is well identified by the observations of the survey forecasts. Finally, we consider a model version in which agents use an Adaptive Learning scheme as alternative to the Rational Expectations approach. This assumption relaxes the full information and model consistent expectation restrictions. Instead, agents will formulate their expectations dependent on the historical realisations and a broad range of signals that they observe and that they try to interpret in real time given their prior information on the model as background structure.

The outline of the paper is as follows. First, we document the main features of the survey and real-time data and illustrate the discrepancy between inflation expectations in the surveys and the expectations implied by standard DSGE models. We also demonstrate that including survey expectations in the model by just adding a measurement equation is not sufficient. In section two, we explain how the introduction of two markup shocks, one *i.i.d.* and another persistent, is extremely helpful for an efficient integration of the survey data in the model. We show this first in a standard Rational Expectations model and discuss the remaining issues in this context. Then we present briefly our Adaptive Learning approach and show how the updating of beliefs accounts well for the time-varying properties of the joint dynamics in realised and expected inflation. Finally, we illustrate the robustness of our results.

[2] See Monti (2010) and Smets, Warne and Wouters (2014) for other examples on how survey information can be related to structural shocks in a forecasting context.

1 Survey expectations versus model expectations

First, we document some properties of the SPF-inflation forecasts and their relation with the real-time releases for the realized inflation data. Then, we show that the survey expectations deviate substantially from the expectations that are implicitly present in two DSGE models, namely, the Smets and Wouters (2007) model estimated with rational or model-consistent expectations (RE) and the Slobodyan and Wouters (2012) model where the estimation assumes that agents are using adaptive learning (AL).[3] The latter model is using an adaptive learning setup in which agents update their perceived forecasting models over time as new data become available with a Kalman filter learning scheme. In the original paper, it was shown that beliefs based on simple AR(2) forecasting models capture the time-varying persistence in the inflation process well. We document the difference between the expectations implied in these models by plotting these forecasts against the SPF forecasts and by computing the statistical properties of the forecasts errors. The SPF typically outperforms the model forecasts for inflation. Therefore, we re-estimate the models using the Survey expectations as an observable for the model expectations allowing for measurement error in the observation equations. This exercise results in substantial and systematic measurement error and the model forecasts are only marginally improved. Clearly, the original model specification is missing the flexibility to exploit efficiently the information that is available in the survey forecasts.

1.1 Comparing model expectations and SPF forecasts.

It is important to note that we re-estimated our models using real-time data. Including SPF-forecasts in the model requires specifying the model in real-time, so that model expectations are based on the same information set that was available to survey participants when they formulated their expectations. As illustrated in Table 1, over the sample since 1971q1, the revision in the second release for the inflation rate in the quarter-to-quarter GDP-deflator relative to the first release has a standard deviation of 0.11 and the final revision has a standard deviation of 0.22. The magnitude of these revisions is of the same order of magnitude as the one-quarter-ahead SPF forecast error which has a standard error of 0.26. The magnitude of the data revisions is significant, and therefore this real-time data issue cannot be ignored when including survey forecasts in the model.[4]

[3] We refer to the original articles for the detailed model specification and estimation results. We provide more information on the learning setup in section 3 and in appendix A.

[4] See also Croushore (2010) for an evaluation of survey forecasts of inflation using real-time data.

Table 1: Statistical properties of the inflation revisions and SPF forecasts errors

	1971q1-2015q3			1996q1-2015q3		
Revisions	bias	mad	rmse	bias	mad	rmse
π_t^{r2} to π_t^{r1}	-0.03	0.08	0.11	-0.02	0.05	0.07
π_t^{rf} to π_t^{r1}	-0.02	0.17	0.22	-0.04	0.12	0.16
π_t^{rf} to π_t^{r2}	0.01	0.16	0.23	-0.02	0.10	0.13
SPF statistics						
$\pi_{t+1\|t}^{SPF} - \pi_{t+1}^{r1}$	0.03	0.21	0.26	0.03	0.17	0.21
$\pi_{t+1\|t}^{SPF} - \pi_{t+1}^{r2}$	0.01	0.21	0.27	0.01	0.16	0.19
$\pi_{t+1\|t}^{SPF} - \pi_{t+1}^{rf}$	0.01	0.18	0.24	-0.01	0.15	0.20
SPF for longer horizons						
$\pi_{t+2\|t}^{SPF} - \pi_{t+2}^{r1}$				0.04	0.19	0.23
$\pi_{t+3\|t}^{SPF} - \pi_{t+3}^{r1}$				0.07	0.19	0.23
$\pi_{t+4\|t}^{SPF} - \pi_{t+4}^{r1}$				0.07	0.21	0.25

Note: π^{r1}, π^{r2}, and π^{rf} are the first, second, and final available quarterly releases for GDP deflator inflation. π^{SPF} is the SPF nowcast. Period 1971q1-2015q3 starts with the period for which the GDP deflator inflation and GDP forecasts of sufficient quality are available in the Survey of Professional Forecasters. The sample 1996q1-2015q3 is the typical period that we use in the out-of-sample model forecast tests presented in the paper.

The models are re-estimated with real time data for inflation in the GDP-deflator and the GDP growth rates.[5] For the other five observables (growth rates of consumption, investment, and real wages, total hours worked, and the Fed-funds rate) we still use the final data because of various issues with real-time counterparts, and the fact that the survey forecasts for these variables start later as well.[6] Agents in the model are assumed to observe the first and the second release of these series: the second release is considered here as the 'true' measure for inflation and GDP growth. The first release is assumed to contain a simple i.i.d. measurement error $\xi^{\pi r}$. For inflation the measurement equations are:

$$\begin{aligned} \pi_t^{r1} &= \overline{\pi} + \widetilde{\pi}_t + \xi_t^{\pi r}, \\ \pi_t^{r2} &= \overline{\pi} + \widetilde{\pi}_{t-1}, \end{aligned}$$

and similarly for GDP growth:

$$\begin{aligned} dy_t^{r1} &= \overline{\gamma} + \widetilde{y}_t - \widetilde{y}_{t-1} + \xi_t^{yr}, \\ dy_t^{r2} &= \overline{\gamma} + \widetilde{y}_t - \widetilde{y}_{t-1}. \end{aligned}$$

When the agents in the model form their expectations for quarter $t+1$, the information set includes the first release of the data for quarter t and the

[5] Our exercise is based on inflation expectations as measured by the GDP-deflator series. This choice corresponds with the data used in the original Smets and Wouters (2007) and Slobodyan and Wouters (2012) models. We intend to test the robustness of our results with CPI and PCE expectations.

[6] Real-time data and SPF data are downloaded from the Philadelphia Fed web-site https://www.philadelphiafed.org/research-and-data/real-time-center/

second release for quarter $t-1$. This timing assumption is an approximation of the information structure available for the SPF participants: survey forecasts for $t+1$ are collected after the first release of data for quarter t is published. Of course data processing and publication takes time, and surveys are collected when quarter $t+1$ is already ongoing, more precisely during the first half of the second month of quarter $t+1$. That is why the forecast for quarter $t+1$ is also called *nowcast*. Nowcasts can reflect information that became available only after the end of the quarter t. This timely nature of the information set available to SPF-participants might contribute to the excellent forecasting performance of the surveys. Table 1 also provides descriptive statistics for the forecasting performance of the SPF. The forecasting errors' RMSE of 0.26 for the complete sample (1971q1-2015q3) and 0.21 for our out-of-sample prediction sample (1996q1-2015q3) will be important benchmarks for the model forecasts later on. For comparison, the RMSE of a no-change forecast for the first release is equal to 0.37 for the complete sample and 0.31 for the out-of-sample period.

Table 2 collects the outcomes of the standard rationality tests applied on these survey forecasts. Following Mankiw, Reis and Wolfers (2003) and Coibion and Gorodnichenko (2015), we test whether the survey forecast errors are persistent and predictable by the forecast or by other information available at the time of forecasting (actual inflation and interest rates) and by the forecast revisions. We run the tests using inflation expectations over the next four quarters as is usually done in the literature but we also consider the one-quarter-ahead expectations. We consider the complete sample over which the survey data are available and our shorter out-of-sample prediction sample.

For the annual inflation forecast errors, we reproduce the well-documented deviations from Full Information Rational Expectations hypothesis. Note however, that these results are not necessary robust over shorter samples: we do not find significant deviations from rationality for the annual SPF errors since 1996. More important for our exercise, we do not observe any deviation from rationality in the one-quarter-ahead forecasts. One explanation can be that survey participants observe already sufficient information about the ongoing quarter so that the traditional arguments that explain the forecast limitations do not apply: easily observable public information might dominate disperse private signals and model uncertainty in the production of the so-called nowcast. The exceptional prediction quality of the nowcast provides an additional argument for concentrating on this concept when we integrate SPF survey information in our DSGE models.[7] This approach is also consistent with the standard forecasting practice in many policy institutions where the model forecast is typically augmented with judgemental - read: survey based - interventions mainly for the very short horizon.[8]

[7] See also Del Negro and Schorfheide (2013) for an illustration of how short term model forecasts can be improved by conditioning on nowcasts for inflation, output and interest rates. Carvalho et al (2015) also use one- and two-quarter ahead survey forecasts to produce long run model forecasts that are consistent with their survey counterpart.

[8] *In the robustness exercise in section 4, we use an alternative timing assumption. All results are robust when we use the next quarter survey forecast as observable but the prediction*

Table 2: Test statistics for SPF forecast errors

	average annual inflation forecast		one quarter ahead forecast	
	1969q1-2015q3	1996q1-2015q3	1969q1-2015q3	1996q1-2015q3
persistence: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t} - \pi^{r1}_{t\|t-h}\right)$				
α	-0.03 (-0.75)	-0.02 (-0.76)	-0.03 (-1.22)	-0.03 (-0.91)
β	0.39 (2.28)	0.23 (0.23)	0.04 (0.34)	-0.24 (-1.31)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t}\right)$				
α	-0.08 (-1.42)	-0.05 (-0.46)	-0.06 (-1.49)	-0.05 (-0.38)
β	0.06 (0.75)	0.03 (0.03)	-0.04 (0.71)	0.06 (0.22)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t}\right) + \gamma\left(\pi^{r1}_{t-1}\right) + \delta\left(r_{t-1}\right)$				
α	-0.01 (-0.26)	-0.03 (-0.34)	-0.06 (-1.55)	-0.09 (-0.71)
β	0.18 (1.07)	0.02 (0.07)	0.06 (0.46)	0.43 (1.51)
γ	0.06 (0.39)	0.04 (0.20)	0.00 (0.03)	-0.30 (-2.09)
δ	-0.15 (-3.70)	-0.03 (-0.52)	-0.02 (-0.50)	0.02 (0.35)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t} - \pi^{r1}_{t\|t-h}\right)$				
α	-0.03 (-0.75)	-0.02 (-0.76)	-0.03 (-1.22)	-0.02 (-0.91)
β	0.39 (2.28)	0.23 (1.50)	0.04 0.34)	-0.25 (-1.31)

Note: For annual inflation forecasts $\pi^{r1}_{t+h|t}$ is representing $\sum_{h=1,4}(E_t\pi^{r1}_{t+h})$. The t-statistics between brackets are based on Newey-West corrected standard errors.

The estimated parameters for the real-time versions of the two DSGE models are documented in Table A1. We will refer to these models as the RE-SW07-9obs and AL-SW12-9obs: relative to the original versions with seven observables, these versions include two additional real-time data series as observables. The estimated parameter values are reasonably comparable to the original Smets and Wouters (2007) and Slobodyan and Wouters (2012) models, despite the use of real-time data and the longer sample. The estimated standard errors for the measurement errors $\xi^{\pi r}$ and ξ^{yr} are almost identical under RE and under AL: 0.12 for GDP inflation and 0.19 for GDP growth.

Figures 1 and 2 illustrate the inflation expectations that are implicitly present in these real-time models. The forecasts for $t+1$ are of particular interest, as they appear directly in the first-order conditions that describe the decision rules of the agents. The plots in the upper panels of these figures present the projected inflation trajectories at each point in time for the next 4 quarters. These are true out-of-sample forecasts, as the underlying models are estimated on the datasets that are available at the moment that the forecast is made. Each forecast starts from the last available observation for the first release π^{r1}_t: thus, the figure also provides information on the ex-post realisations of inflation. In RE-SW07-9obs, these model forecasts are consistent with agents' expectations. In AL-SW12-9obs, the model forecasts produced by the Actual Law of Motion process (ALM-forecasts) are plotted; in general, these are not equivalent to the

errors for inflation increases and the model forecasting gains from survey data decreases as the information content of the survey data declines.

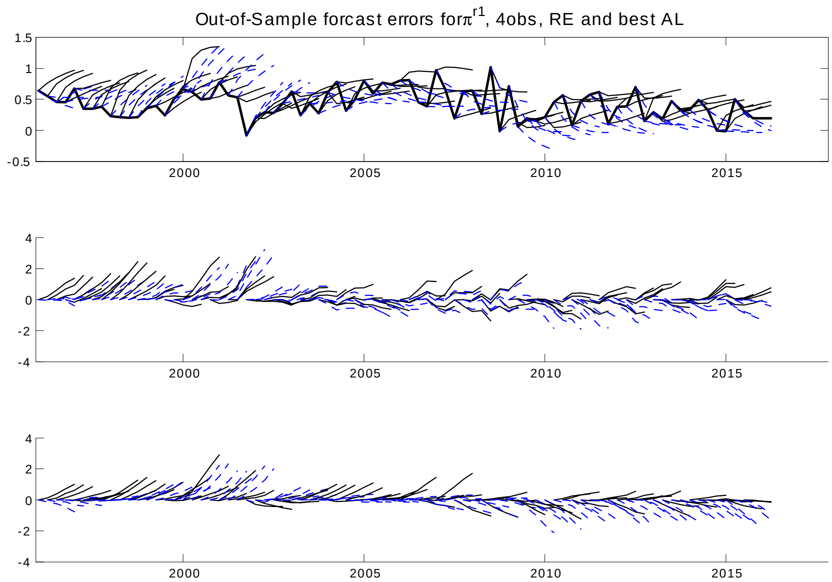
agents' perceived forecasts (PLM forecasts) but the deviations remain modest.[9]

In the middle panels, actual inflation realisations π^{r1} are subtracted from the model forecasts, and the *cumulative* difference is plotted. Around 2000, one could observe that 1Q model forecasts are under-predicting first releases while longer horizon forecasts are over-predicting them. In general, however, the forecast errors tend to be of the same sign at different horizons, which is clearly seen before year 2000.

The lower panels of the figures display the cumulative deviations between the model and SPF nowcasts. Here, we can observe that there are often large deviations between the two forecasts. Both models, but RE-SW07-9obs in particular, tend to predict higher inflation than the SPF nowcast for most of the period between 1996 and 2002, and again from 2004 to 2007. Since the start of the Great Recession, both models, and the AL-SW12-9obs most notably, produce lower forecasts than the survey. At the end of the sample the models also tend to produce forecasts below the SPF. The deviations between model and survey forecasts are very persistent for most of the projection trajectories, as well as across projections as time proceeds.

[9] With our solution procedures, ALM forecast are easily rolled forward for longer horizons. Depending on the specification of the belief models, it is less evident how to produce longer horizon PLM forecasts in the models.

Figure 1: Model and SPF forecasts for 1 to 4 quarters ahead versus realized inflation (first release)



Note: The graph in the middle reproduce the cumulative forecast errors of the two models relative to the realized inflation. The figure at the bottom contains the cumulative deviations of the two model forecasts relative to the SPF nowcasts. The black line corresponds with the RE-forecast, the blue dashed line represents the AL-forecast.

The statistics reported in Table 2 further document the inflation forecasting performance of the RE-SW2007-9obs model re-estimated with real-time data. The forecast errors of the model are large compared to the SPF nowcasts on all three criteria that we consider (bias, MAD, and RMSE). The bias increases systematically with the forecast horizon in the out-of-sample forecasts that covers the more recent sub-period since 1996. The inflation target that is estimated in the RE model is probably biased upward in the first part of the sample. It is important to note at this point is that the RE forecast deviates substantially from the SPF nowcast. The RMSE of the difference between the two forecasts for quarter $t+1$ is 70% of the SPF nowcast error's RMSE: the difference between the two forecasts is almost as large as the forecast error itself. Testing the equivalence of the two forecasts using the Diebold-Mariano test clearly rejects the hypothesis that the two forecasts are equivalent for horizons from one to four. Moreover, the SPF significantly outperforms the model forecast.

Table 2: Statistical properties of the RE-SW2007-9obs model forecast errors and comparison to SPF

	1971q1-2015q3			1996q1-2015q3		
$t+1$ forecast	bias	mad	rmse	bias	mad	rmse
$\pi^{RE}_{t+1\|t} - \pi^{r1}_{t+1}$	0.02	0.27	0.34	0.06	0.23	0.28
$\pi^{RE}_{t+1\|t} - \pi^{r2}_{t+1}$				0.05	0.23	0.27
$\pi^{RE}_{t+1\|t} - \pi^{rf}_{t+1}$				0.02	0.21	0.25
longer horizons						
$\pi^{RE}_{t+2\|t} - \pi^{r1}_{t+2}$	0.02	0.29	0.39	0.12	0.25	0.31
$\pi^{RE}_{t+3\|t} - \pi^{r1}_{t+3}$	0.02	0.32	0.41	0.17	0.27	0.35
$\pi^{RE}_{t+4\|t} - \pi^{r1}_{t+4}$	0.019	0.33	0.45	0.21	0.29	0.36
RE versus SPF	rel. RMSE%		DM-test	rel. RMSE%		DM-test
$horizon = 1$	70.86		4.61	84.22		4.46
$horizon = 2$	56.70		3.10	91.91		3.07
$horizon = 3$	47.22		2.37	95.08		3.28
$horizon = 4$	44.04		1.22	93.16		2.29

Note: Statistics for the ful sample 1971q1-2015q3 are based on in-sample predictions, while the results for 1996q1-2015q3 are based on out-of-sample predictions with recursively estimated models. Rel RMSE is defined as $\frac{RMSE(\pi^{RE}_{t+1|t} - \pi^{SPF}_{t+1|t})}{RMSE(\pi^{SPF}_{t+1|t} - \pi^{r1}_{t+1})}$x100. DM-test is the Diebold-Mariano test for equal accuracy between RE and SPF forecast.

Table 3 provides the same statistics for the AL-SW2012-9obs model re-estimated with real-time data. In this model as well, the inflation forecast errors are large and worse than the SPF in terms of MAD and RMSE. The RMSE of the difference between the two forecasts is up to 78% of the SPF forecast error. The Diebold-Mariano test indicates the superiority of the SPF forecasts over all forecast horizons considered. The longer horizon forecasts deteriorate more under AL than under RE: for long term forecasts, the structure imposed on the RE forecasts, seems to pay off while the flexibility of the AL beliefs can become costly. These results apply for the ALM-forecasts in the AL-model. However, in this model, the PLM forecasts are also relevant. It is these PLM expectations that enter into the agents decision rules when they make the actual price decision. The PLM based on the small forecasting model does a good out-of-sample forecasting job in this model, at least compared to the ALM-forecast.

Table 3: Statistical properties of the AL-SW2012-9obs model forecasts errors and comparison to SPF

	1971q1-2015q3			1996q1-2015q3		
$t+1$ horizon	bias	mad	rmse	bias	mad	rmse
$\pi^{AL\_PLM}_{t+1\|t} - \pi^{r1}_{t+1}$				0.02	0.19	0.26
$\pi^{AL\_ALM}_{t+1\|t} - \pi^{r1}_{t+1}$	-0.02	0.26	0.35	-0.01	0.23	0.29
$\pi^{AL\_ALM}_{t+1\|t} - \pi^{r2}_{t+1}$				-0.03	0.22	0.28
$\pi^{AL\_ALM}_{t+1\|t} - \pi^{rf}_{t+1}$				-0.05	0.21	0.27
longer horizon						
$\pi^{AL\_ALM}_{t+2\|t} - \pi^{r1}_{t+2}$	-0.03	0.30	0.41	-0.00	0.27	0.33
$\pi^{AL\_ALM}_{t+3\|t} - \pi^{r1}_{t+3}$	-0.04	0.33	0.47	0.01	0.31	0.37
$\pi^{AL\_ALM}_{t+4\|t} - \pi^{r1}_{t+4}$	-0.04	0.37	0.53	0.03	0.34	0.42
AL versus SPF	rel. RMSE%		DM-test	rel. RMSE%		DM-test
$horizon = 1$	78.56		3.94	89.97		3.94
$horizon = 2$	70.80		2.10	110.85		2.82
$horizon = 3$	69.20		2.21	130.62		2.70
$horizon = 4$	69.77		2.14	137.41		2.03

Note:

In defense of the two models, one should note that the RMSE forecast errors measured against the final data for the GDP deflator are smaller than when calculated on the first and second releases. These results confirm the good inflation forecast performance that was reported in the original published versions of these models. The DSGE forecasts outperformed various VAR models also in terms of inflation forecast but these gains were most visible in the longer horizon forecasts. Note also that the inflation forecast errors of the models estimated on the real-time data are still smaller than the naive no-change RMSE of 0.37 and 0.31 for the one-quarter-ahead forecast over respectively the complete and the shorter sample.

In Table A5 and A6 in the Appendix we also report the test statistics for rationality test on the model forecasts for inflation. Compared to the corresponding SPF-statistics that were reported in Table 2, the model forecast errors display similar degrees of predictability over the complete sample. For the shorter - more recent - sub-period, the RE model fails on all tests. This finding might be explained as a result of model misspecification in the RE model that is unable to adjust to the time-varying dynamics in the inflation expectations. But the results also suggest that one should be careful with the interpretation of these tests when applied over short intervals.

1.2 Including SPF nowcast as an observable with a measurement error

Here we present results for the two models re-estimated with the SPF nowcast for $t+1$ as an additional observable.[10] We integrate the SPF survey data as observable for the expected variable as follows:

$$\pi^{f}_{t+1|t} = \overline{\pi} + E_t \widetilde{\pi}_{t+1} + \xi^{\pi f1}_{t}, \tag{1}$$

where $\xi^{\pi f1}_{t}$ is an i.i.d. measurement error (ME) between the observed SPF nowcast $\pi^{f}_{t+1|t}$ and the model forecast $E_t \widetilde{\pi}_{t+1}$, which is expressed in deviation from the inflation target $\overline{\pi}$.

The estimated parameters for these model versions, denoted RE-ME-10obs and AL-ME-10obs for estimation under RE and AL, respectively, are available in Table A2. The *i.i.d.* measurement error $\xi^{\pi f1}_{t}$ has a standard error of 0.18 in both the RE and AL versions. Including the survey data in the model in this elementary way does change some of the estimated parameters and shocks' standard errors. The most striking changes are the higher degree of nominal stickiness. In particular, the Calvo-probability for prices is significantly higher with observations on the SPF expectations both under RE and under AL.

Tables 4 and 5 illustrate that the models with the survey observable perform better on all statistics related to the inflation forecast. By minimizing the measurement error on the SPF nowcasts, the model forecasts are forced to resemble more closely the survey forecasts; the model forecast performance measured against the ex-post inflation realisations improves also. Obviously, the model inflation forecasts benefit from the excellent prediction potential of the survey data.

[10] Roberts (1995) was one of the first attempts to use survey forecasts as instruments for the expectations in the estimation of the NKPC. Adam and Pdula (2011), Smith (2009) and Nunes (2010) confirmed that survey forecasts can be used successfully as proxies for the expected inflation but they also point out the limitations of their information content. In this literature, survey expectations are treated as exogenously given observables. Fuhrer (2015) discussed how the survey expectations can be endogenized and observed an important role for intrinsic persistence in expectations. See also Coibion et al (2017) for a discussion of this literature on inflation expectations and surveys. In our approach, actual and survey data are treated as observables in the measurement equation and their dynamics is fully endogenized by the state transitions.

Table 4: Forecast Statistics for the RE-ME-10obs model with SPF observable

	1971q1-2015q3			1996q1-2015q3		
$t+1$ horizon	bias	mad	rmse	bias	mad	rmse
$\pi^{RE}_{t+1\|t}-\pi^{r1}_{t+1}$	0.02	0.25	0.32	0.05	0.20	0.25
$\pi^{RE}_{t+1\|t}-\pi^{r2}_{t+1}$				0.04	0.20	0.23
$\pi^{RE}_{t+1\|t}-\pi^{rf}_{t+1}$				0.01	0.18	0.23
longer horizon...						
$\pi^{RE}_{t+2\|t}-\pi^{r1}_{t+2}$	0.02	0.28	0.37	0.09	0.22	0.27
$\pi^{RE}_{t+3\|t}-\pi^{r1}_{t+3}$	0.02	0.30	0.40	0.12	0.24	0.30
$\pi^{RE}_{t+4\|t}-\pi^{r1}_{t+4}$	0.01	0.32	0.43	0.15	0.26	0.32
RE versus SPF	rel. RMSE%		DM-test	rel. RMSE%		DM-test
$horizon = 1$	56.64		4.26	55.91		3.35
$horizon = 2$	45.16		2.51	60.56		2.56
$horizon = 3$	38.21		1.44	65.71		2.82
$horizon = 4$	38.09		0.66	66.65		1.18

Note:

Table 5: Forecast Statistics for the AL-ME-10obs model with SPF observable

	1971q1-2015q3			1996q1-2015q3		
$t+1$ horizon	bias	mad	rmse	bias	mad	rmse
$\pi^{AL\_PLM}_{t+1\|t}-\pi^{r1}_{t+1}$				0.06	0.19	0.24
$\pi^{AL\_ALM}_{t+1\|t}-\pi^{r1}_{t+1}$	0.02	0.26	0.34	0.06	0.20	0.25
$\pi^{AL\_ALM}_{t+1\|t}-\pi^{r2}_{t+1}$				0.04	0.20	0.24
$\pi^{AL\_ALM}_{t+1\|t}-\pi^{rf}_{t+1}$				0.02	0.17	0.23
longer horizon						
$\pi^{AL\_ALM}_{t+2\|t}-\pi^{r1}_{t+2}$	0.02	0.30	0.40	0.09	0.23	0.27
$\pi^{AL\_ALM}_{t+3\|t}-\pi^{r1}_{t+3}$	0.02	0.33	0.45	0.12	0.23	0.28
$\pi^{AL\_ALM}_{t+4\|t}-\pi^{r1}_{t+4}$	0.02	0.35	0.49	0.14	0.24	0.29
AL versus SPF	rel. RMSE%		DM-test	rel. RMSE%		DM-test
$horizon = 1$	63.11		4.53	57.61		3.03
$horizon = 2$	57.39		2.57	63.47		2.51
$horizon = 3$	50.82		1.85	63.42		2.12
$horizon = 4$	50.12		1.73	65.35		1.16

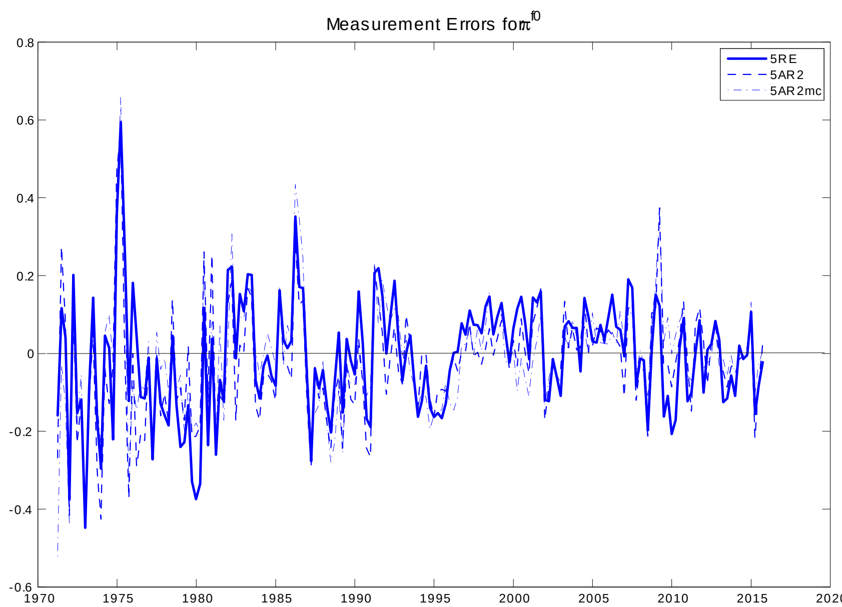
Note:

Figure 2 plots the smoothed estimates of the time series for the measurement error in the RE-ME-10obs and AL-ME-10obs models. The measurement error is substantial, illustrating that the models have a hard time producing a forecast that resembles the survey forecast. During several episodes, the measurement error is also highly persistent with deviations between the two forecasts repeatedly in the same direction for more than a year.[11] The measurement errors for

[11] Add discussion on relation between forecast errors and measurement error and on correlation among shocks.

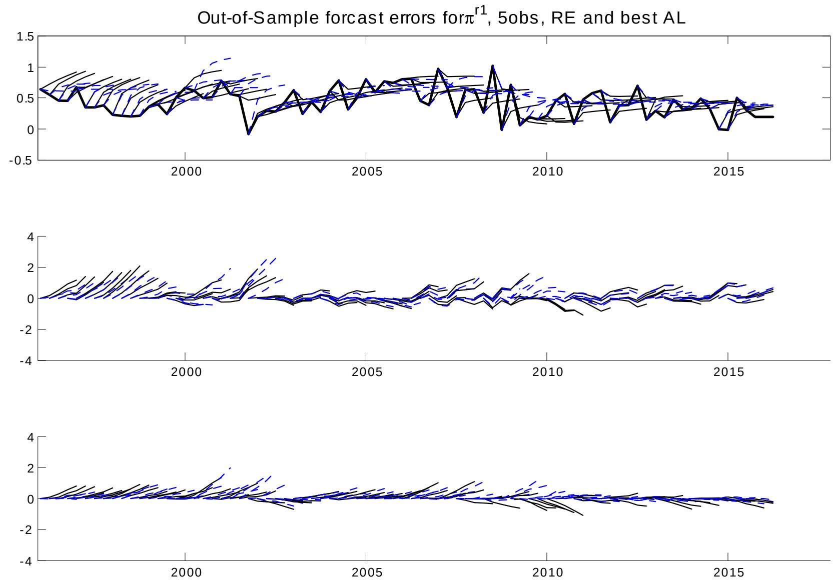
AL-ME-10obs are relatively large during the first half of the sample, but, on average, smaller during the second subsample. This picture confirms the statistics reported in Table 5, as well as the out-of-sample forecast plots presented in Figure 3 below. Compared to RE-ME-10obs, the AL-ME-10obs model does a relatively a good job in the out-of-sample prediction exercise over the period since 1996. The out-of-sample inflation forecasts of AL-ME-10obs are more in line with the survey forecast according to the relative RMSE criteria, but they are still outperformed by the survey according to the DM test. In terms of longer horizon inflation forecasts, the AL (AL-ME-10obs) model is now superior to the RE (RE-ME-10obs) model, while the opposite was true for the 9obs-models.

Figure 2: Estimated measurement error in SPF-data in the RE and AL Model



Note: pexp_dif_pif0_3_5obsme Reverse the sign in these graphs ?

Figure 3: Model forecasts with SPF as observable with measurement error



Note: see Figure 1 for explanation (ferr_pir1_hair_5obs.eps)

This relative success of the AL-ME-10obs model in capturing the overall dynamics of the inflation process is confirmed by the marginal likelihood comparison summarized in the Table 6. Overall, the AL models with 9 and 10 observables have better marginal likelihood than the RE models. By relaxing the RE-restrictions and assuming that expectations are based on small belief models that are updated over time depending on new realisations, the AL-models have some extra flexibility which is useful for forecasting. This flexibility is particularly helpful in replicating the survey forecast observable in the 10obs model. This becomes most obvious when we calculate the marginal likelihood for the original 9 variables implied by the 10obs model. For the AL-models, the marginal likelihood for this common block deteriorates only slightly relative to the 9obs model (-951 versus -943), while for the RE-models, including the survey nowcast in the model deteriorates the marginal likelihood of the common 9 observables significantly more (-999 versus -965). When it has to comply with the survey nowcasts, the AL model is more flexible in delivering predictions consistent with the survey and retains its overall good forecasting performance, while the RE-model is losing in overall performance.

That the AL-models with simple AR(2) PLM belief do a reasonably good job in mimicking the survey expectations was also suggested in Slobodyan and Wouters (2012). This observation is consistent with experimental evidence on expectations forecasting.[12] Small forecasting models provide a good representa-

[12] See Hommes and Zhu (2014) for a review of arguments in favour of small forecasting

tion of how agents formulate their expectations. However, it would be surprising if small models were able to reproduce the potentially rich information set that is underlying the SPF forecasts. To illustrate the role of the belief specification under AL, we also considered slightly more complicated belief model in which the AR(2) specification was augmented with the marginal cost variable in inflation PLM, leaving the remaining PLMs unchanged. This Phillips Curve based specification can capture the basic relation between inflation and its underlying macroeconomic determinants. While thus augmented belief model (AR2+MC) is producing some gain for the standard 9obs model (-934 versus -943), it becomes even more informative for the model with observed survey forecasts. The marginal likelihood of the AL model with the marginal cost (MC) in the beliefs improves by 30 relative to the model with basic AR(2) PLM specification. Also, the estimated standard error for $\xi_t^{\pi f1}$, the measurement error on SPF-expectations, drops from 0.18 to 0.15. The Phillips curve relation seems to have some relevance in forecasting the relatively smooth inflation expectation variable while, it was not too informative for forecasting the highly volatile realized inflation process.

Table 6: Marginal likelihood of alternative model specifications

	71q1-15q3		96q1-15q3	
	9obs	10obs	9obs	10obs
RE-SW07-9obs	-965.22		-361.25	
AL-SW12-9obs (AR2)	-943.42		-340.96	
AL-SW1-9obs2 (AR2+mc)	-934.41		-317.78	
RE-ME-10obs	-999.56	-911.05	-374.78	-302.57
AL-ME-10obs (AR2)	-951.29	-883.46	-337.62	-282.83
AL-ME-10obs (AR2+mc)	-941.46	-857.51	-324.84	-260.00
RE-2MU-10obs	-944.76	-839.96	-344.84	-267.14
AL-2MU-10obs (AR2)	-959.35	-894.51	-338.60	-287.04
AL-2MU-10obs (AR2+mc)	-951.46	-866.45	-333.57	-272.42
AL-2MU-10obs (AR2+mc+UC)	-915.48	-787.18	-312.83	-228.69

Note: We follow Warne A. et al (2016) for calculating the marginal likelihood for a subset of the variables.

We can conclude from this section that just adding SPF as an observable for the expectations in these models is not producing strong gains. Reducing the discrepancy between the SPF nowcast and the model forecasts leads to some interesting changes in the estimated parameters, and the inflation forecasts improve. However, the measurement errors are large and persistent, and are correlated with other structural innovations in the model. There is no evidence that the additional observable leads to better identification of shocks or parameters that could improve the overall model performance. All this suggests that adjustment in the model specification is required in order to bridge the gap between model and survey forecasts more efficiently.

models.

2 Reconciling model and survey expectations

To reconcile the model expectations with the survey expectations, we need more flexibility in the specification of the inflation dynamics. In the Smets and Wouters (2007) model, the price and wage markup shocks are modelled as ARMA processes. For the price markup this process is written as:

$$\mu_t^p = \rho_\mu^p \cdot \mu_{t-1}^p - \theta_\mu^p \cdot \varepsilon_{t-1}^p + \varepsilon_t^p.$$

This specification implies that the same innovation ε_t^p is driving the volatile high frequency MA-component on the one hand and the persistent low frequency AR-component on the other hand. This ARMA specification works well to capture the complex exogenous shock process in the actual price and wage dynamics.[13] As long as the dataset is limited to the standard seven macrovariables, there is no need to distinguish between separate innovations driving the high and the low frequency component. These innovations are simply not identified individually by the standard seven observables. In the AL version of Slobodyan and Wouters (2012), the price and wage markup shocks reduce to *i.i.d.* processese, such that

$$\mu_t^p = \varepsilon_t^p.$$

The time-varying AR(2) beliefs generate the dynamics required for matching the observed price and wage persistence. Importantly, in this setup also, one exogenous innovation is sufficient to describe the exogenous shock process.

Observing the survey expectations which are, most likely, based on a broader and a more timely information set, allows to distinguish precisely the pure *i.i.d.* and the persistent components in the markup processes and the corresponding innovations.[14] Therefore, we specify the price and wage markup processes as a combination of a persistent AR ($\mu_t^{p.ar}$) and a separate *i.i.d.* shocks ($\mu_t^{p.iid}$), each with their own innovation:

$$\begin{aligned} \mu_t^p &= \mu_t^{p.ar} + \mu_t^{p.iid}, \\ \mu_t^{p.ar} &= \rho_\mu^p \cdot \mu_{t-1}^{p.ar} + \varepsilon_{t-1}^{p.ar}, \\ \mu_t^{p.iid} &= \varepsilon_t^{p.iid}. \end{aligned} \tag{2}$$

We further assume that the innovation to the persistent shock process ($\varepsilon_{t-1}^{p.ar}$) is already publicly observed in the quarter prior to its actual impact on price setting. This assumption is not crucial for the results that are presented below, but the model fit improves when using the "news" specification instead of a contemporaneous innovation. Examples of this type of events are oil shocks or other commodity shocks that are observed in world prices before they actually enter into the retail prices, or announced changes in regulated prices and taxes

[13] See also Ang, Bekart and Wei (2007) and Stock and Watson (2007) for more evidence supporting the ARMA specification for forecasting the inflation dynamics.

[14] This specification of the markup process is consistent with the noisy-information model used in Coibion and Gorodnichenko (2015) and therefore also with the observed predictability of ex-post forecast errors by ex-ante forecast revisions in the SPF for inflation.

that are communicated in advance of the actual implementation. The same dual process as defined in equation (2) is used for the wage markup shock to maintain symmetry.

We use the same measurement equation for the SPF survey nowcast as in equation (1). We discuss the implication of this new specification first for the RE-setup of Smets and Wouters (2007) and in the next section in the AL-setup of Slobodyan Wouters (2012).[15]

2.1 Integrating survey data in the augmented RE-SW2007 model

Table A3 summarizes the estimated parameters for this RE model with two markup shocks estimated on ten observables including the SPF nowcast (RE-2MU-10obs model). The estimated standard deviation of the measurement error for the SPF nowcast reduces to 0.04, against 0.18 in the model specification with measurement error only (RE-ME-10obs). The nominal price stickiness parameter is estimated to be high. This tendency for more stickiness was already present in the RE-model with measurement error only. Under RE, high price stickiness seems important for matching the survey expectations. In the wage setting process, the wage markup shock is close to a random walk while nominal stickiness remains relatively low.

Table 7 documents the inflation forecast performance of this model. The inflation forecast of this model improves on all dimensions. The model forecast now resembles very closely the SPF nowcast; the forecast statistics, therefore, are also very similar to those of the SPF. The RMSE of forecast errors for the one-quarter-ahead inflation rate approximates the benchmark SPF performance. The DM-test confirms that the two forecasts are not significantly different. Also at longer horizons, the model forecasts remain very similar to the SPF ones, although these are not observed in the model. Figure 4 provides the corresponding plot of the cumulative forecast deviations for the out-of-sample forecasts. For the RE model, there are almost no obvious differences between the SPF and the model forecasts since 1996. These features of the model forecast are confirmed by the rationality test (reported in Table A7 in the Appendix). The test statistics for the RE-model forecasts are in line with the test outcomes for the SPF forecasts. The model inherits the weaknesses observed in the survey data.

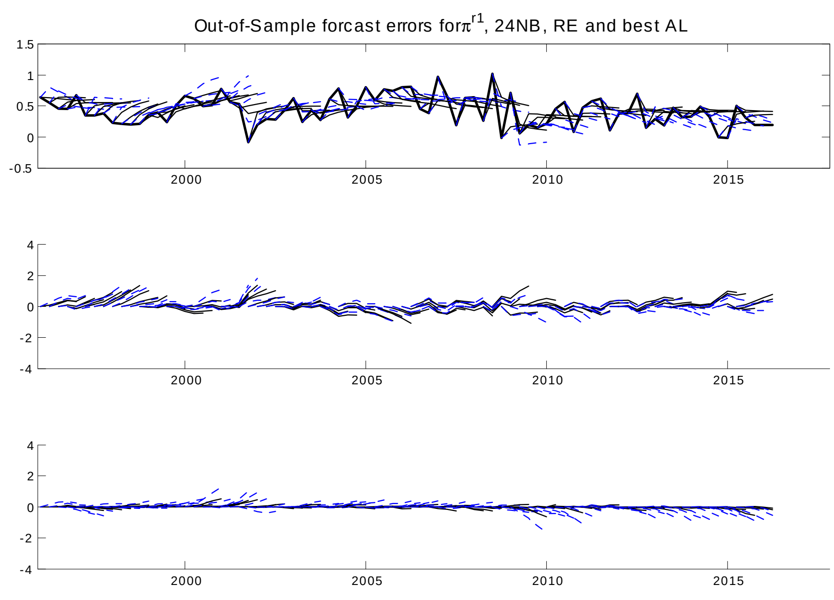
[15] Note that these two separate markup innovations are not identified as long as the survey expectations are not included in the data file. The marginal likelihood of the model with this additional shock is identical to the model with the ARMA structure both under RE and under AL. Both the filtered and the smoothed innovations are highly correlated and only weekly identified.

Table 7: Forecast Statistics for the RE-2MU-10obs model with SPF observable

	1971q1-2015q3			1996q1-2015q3		
$t+1$ forecast	bias	mad	rmse	bias	mad	rmse
$\pi^{RE}_{t+1\|t} - \pi^{r1}_{t+1}$	0.03	0.21	0.26	0.03	0.17	0.21
$\pi^{RE}_{t+1\|t} - \pi^{r2}_{t+1}$				0.02	0.16	0.19
$\pi^{RE}_{t+1\|t} - \pi^{rf}_{t+1}$				-0.01	0.15	0.20
longer horizon...						
$\pi^{RE}_{t+2\|t} - \pi^{r1}_{t+2}$	0.04	0.25	0.34	0.04	0.19	0.23
$\pi^{RE}_{t+3\|t} - \pi^{r1}_{t+3}$	0.04	0.28	0.38	0.05	0.20	0.24
$\pi^{RE}_{t+4\|t} - \pi^{r1}_{t+4}$	0.04	0.30	0.41	0.06	0.21	0.25
RE versus SPF	rel. RMSE%		DM-test	rel. RMSE%		DM-test
$horizon = 1$	4.14		2.47	9.74		1.36
$horizon = 2$	29.28		0.50	29.51		0.45
$horizon = 3$	25.06		-0.75	34.91		1.16
$horizon = 4$	25.84		0.95	31.81		0.06

Note:

Figure 4: Model forecast with SPF observed and two markup shocks



Note: see Figure 1 for explanation

The new structure with two markup shocks gives the model precisely the flexibility necessary for fitting jointly the realised inflation process and the survey nowcast. The highly volatile *i.i.d.* markup component with a standard deviation of 0.24 explains the volatile high-frequency component of actual inflation. As illustrated by the impulse response functions in Figure 5, inflation is only affected by this innovation on impact and returns to its pre-shock level in the next period with a very small negative correction afterwards. This implies that the shock is almost irrelevant for the inflation forecast for $t + 1$, and the spillover effects to the real economy are minimal. On the other hand, the persistent autoregressive price and wage markup shock components with standard errors of 0.03 and 0.01 and persistence of 0.78 and 0.99, respectively, are crucial for capturing the innovations in the survey nowcasts. These "news" shocks have an impact on actual prices already at time t, consistent with the forward-looking nature of the price setting problem. The magnitude of this short run impact effect is substantially smaller than for the iid component: one half for the price and one third for the wage shock's persistent components. Over time, the wage shock starts to dominate as it is a quasi-permanent shock.

The conditional covariance decomposition presented in Table 8 indicates that the four markup shock components have each their own role in the inflation process. The *i.i.d.* price markup explains the one quarter ahead forecast error in realised inflation (77%) but is irrelevant for expectations. The persistent price markup shock is crucial for the short term forecast error in the survey expectations (65%), and consistent with that, also for realized inflation over the medium term horizon of one or two years ahead. The role of the persistent wage markup shock builds up only gradually, but is dominant in the long run and explains 78% of the inflation expectations and 60% of the realized inflation variance at 10 years horizon. Note that this wage shock's component has only minor effects on the short term wage developments. Therefore, it is not surprising that the precise timing of the innovations to this component (in both the "news" and the contemporaneous specification) is difficult to identify. In fact, the persistent wage and price components' innovations are highly correlated (0.79) between themselves, but not with the *i.i.d.* markup innovations. This observation is important as it raises questions about the correct interpretation of the persistent wage markup shock.

Table 8: Conditional variance decomposition for the RE-2MU-10obs model

	ε^a	ε^b	ε^g	ε^{qs}	ε^m	ε^{piid}	ε^{wiid}	ε^{par}	ε^{war}	$\xi^{\pi f1}$
1 quarter horizon										
π_f0	2.44	0.14	0.69	0.00	0.52	0.82	0.29	64.66	24.55	5.89
π_r1	0.65	0.04	0.17	0.00	0.13	76.73	0.09	14.06	6.12	0.00
w	0.05	0.86	0.10	0.05	0.84	9.63	86.60	1.59	0.29	0.00
y	18.71	41.39	20.33	5.04	13.24	0.80	0.33	0.01	0.15	0.00
1 year horizon										
π_f0	3.46	0.18	1.06	0.00	0.73	0.36	0.31	53.97	37.68	2.25
π_r1	1.64	0.09	0.48	0.00	0.35	47.69	0.18	32.65	16.93	0.00
w	0.22	3.74	0.83	0.52	5.47	6.07	72.73	6.16	4.25	0.00
y	17.07	39.17	6.53	10.96	24.28	1.19	0.14	0.45	0.21	0.00
10 year horizon										
π_f0	4.26	0.08	1.54	0.04	0.42	0.11	0.11	14.56	78.33	0.54
π_r1	3.41	0.07	1.22	0.03	0.35	19.07	0.11	15.19	60.56	0.00
w	2.57	5.06	7.78	3.65	16.69	2.50	17.49	10.47	33.78	0.00
y	34.13	15.30	3.20	11.01	24.65	1.14	0.04	4.84	5.68	0.00

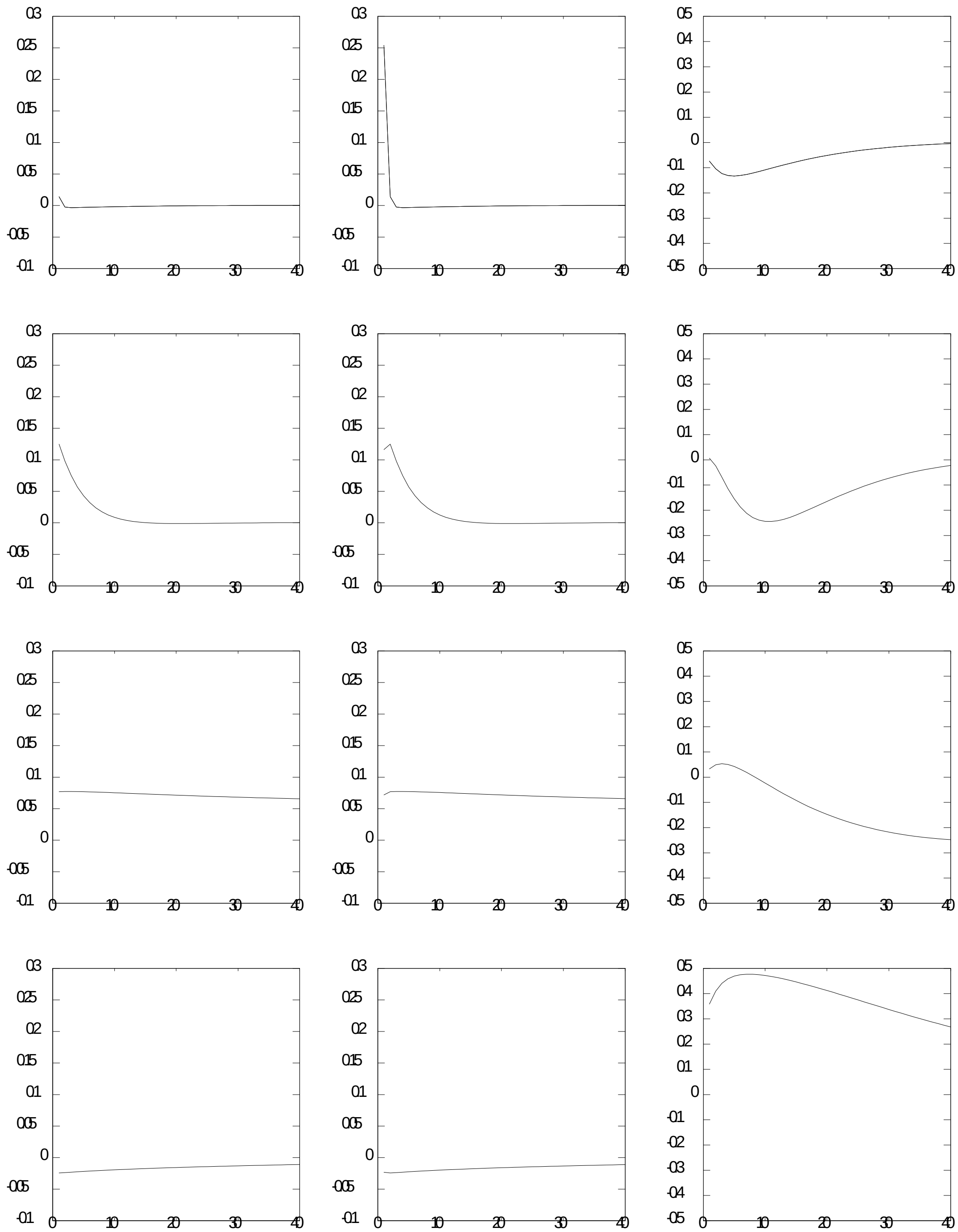
Note:

To illustrate this interpretation problem further, we considered a model with an additional quasi-permanent inflation target shock in the monetary policy reaction function. The model specification is similar to the one used in Smets and Wouters (2003). Del Negro and Eusepi (2011) argued in favour of such a target shock to explain inflation survey expectations in the context of a DSGE model.[16] This exogenous inflation target process has become a popular modelling device to explain the low frequency inflation trend in RE-DSGE models. In Table A4 we report the estimation outcomes for this specification. The marginal likelihood of the model improves slightly from -840 to -834. The target shock substitutes for the persistent wage markup shock: the persistence in the wage markup declines drastically from a quasi-unit root process to a value of 0.40 which curtails its impact. The long term inflation trend which is common to expectations and realisations is now completely explained by the exogenous target shock, while under the wage markup shock interpretation, it is consistent with a severe trade-off problem for monetary policy typical for cost-push shock situations. Without further information from additional labour market variables (as in Gali Smets Wouters 2014) and/or about the monetary policy objective, the RE has a hard time differentiating among these alternative interpretations of the long run inflation component.

[16] De Graeve et al (2009) made a similar argument for the target shock in order to achieve a consistent integration of long term interest rates in these models.

Figure 5: IRF functions for *iid* and AR price markup, AR wage markup, and productivity shocks

	inflation expectations	inflation	output



ε^{piid}

ε^{pap}

ε^{wap}

ε^{a}

Note:

We turn next to discussing this model's performance on other dimensions. Allowing for the two separate markup innovations boosts the log marginal likelihood of the 10obs model by 60 units relative to the RE-ME-10obs model with measurement error only. Of course, the improved fit of inflation expectations constitutes an important contribution to this gain. Still, evaluating the marginal likelihood of the original 9 observables in this model, we observe that on this dimension the model also outperforms both the original RE-SW07-9obs model, and the RE-ME-10obs model with measurement error only. Thus, the overall performance of the model is improved under this specification, and the information from the survey expectations helps to predict the other variables in the economy as well. This result is further documented in Table 9, which presents the forecasting results for the individual variables. Over the entire sample, the in-sample RMSE indicates that the main gains are concentrated in the inflation block. Similarly, in the recent period, the out-of-sample forecasts' gains are concentrated in the price, wage and interest rate block, but the forecasts of other real variables (investment, output and hours) deteriorate slightly.

Table 9: Forecast performance of the RE-2MU-10obs model

	π_r1	π_r2	π_f0	dy_r1	dy_r2	dc	$dinve$	$hours$	dw	r
1971q1-2015q3										
bias	0.03	-0.02	0.01	0.19	-0.01	-0.07	-0.21	0.04	0.01	0.03
MAD	0.21	0.08	0.10	0.48	0.17	0.47	1.27	0.43	0.57	0.15
RMSE	0.26	0.12	0.15	0.63	0.21	0.63	1.71	0.55	0.78	0.23
1996q1-2015q3										
bias	0.03	-0.02	0.01	0.29	-0.00	0.03	0.33	0.30	-0.09	0.08
MAD	0.17	0.05	0.07	0.47	0.17	0.40	1.17	0.47	0.75	0.11
RMSE	0.21	0.07	0.10	0.57	0.21	0.56	1.62	0.60	0.98	0.14
log lik score	0.89	2.00	1.56	-0.09	1.05	0.05	-0.99	0.01	-0.57	1.22
Comparison: RE-ME-10obs model										
RMSE	0.85	0.98	0.57	1.04	1.00	0.98	1.02	1.15	0.97	0.94
log lik score	0.12	0.01	0.43	-0.01	-0.01	0.01	-0.02	-0.12	0.03	0.06
Comparison: RE-SW2007-9obs										
RMSE	0.74	1.10		1.04	1.00	0.97	1.02	1.04	0.99	0.96
log lik score	0.24	0.01		-0.03	0.00	0.02	-0.01	-0.04	0.10	0.05

Note: Results reported for the complete sample 1971q1-2015q3 are calculated as in-sample results, while the results for the sample 1996q1-2015q3 are out-of-sample forecasts with recursively estimation for every period. In the comparison, values for the relative RMSE smaller than one means that the RE-2MU model has smaller forecast errors, while positive values for the relative score means higher log likelihood score for the RE-2MU model

Overall, the nominal block performs very well in this RE-model with two markup shocks. The information of the survey is exploited efficiently, improving the forecast of the nominal variables. However, the RE model imposes a constant variance-covariance structure on the data, while we know from reduced form exercises that the nature of the inflation process has changed over time. The question therefore is whether a structure with two shocks with different persistence but constant variance is optimal. The adaptive learning setup can improve precisely on this dimension.

2.2 Integrating survey data in the AL-SW2012 model

In this section we firstly reiterate the main steps of our Kalman filter based AL-algorithm and we discuss the assumptions that we make on the forecasting models that represent the beliefs of the agents in the AL-model. The simple AR2 specification that we retained in SW2012 is not able to exploit optimally the rich information structure from the survey. We will reformulate the forecasting/belief models so that there is a role for the expectation signals in the agents' beliefs. The estimation results are presented in the second section.

2.2.1 Adaptive Learning and belief specifications

As in Evans & Honkapohja (2001), we assume that the economic agents do not have perfect knowledge of the reduced form parameters of the model when forming expectations about the future. Therefore, they forecast future values of the forward variables in the model (y^f) with a linear functions of endogenous model variables.[17] Only the one-period-ahead forecasts generated by these models are substituted for the expectations in the model.[18] The general logic of this adaptive learning approach works as follows.

The model is represented as

$$\overline{\alpha} + A_0 y_{t-1} + A_1 y_t + A_2 E_t y_{t+1} + B\epsilon_t = 0, \tag{3}$$

where y_t is a vector of endogenous and exogenous model variables. The RE solution of this system is presented as a VAR(1) process,

$$y_t = \mu + T y_{t-1} + R\epsilon_t.$$

Under adaptive learning, agents assume that the forward-looking variables are linear combinations of some variables in the vector y_{t-1}. This assumption is known as a Perceived Law of Motion, or PLM:

$$y_t^f = \beta_{t-1}^0 + \beta_{t-1}^T y_{t-1}. \tag{4}$$

By rolling forward the PLM, we obtain the agents' expectations of forward-looking variables as

$$E_t y_{t+1}^f = \beta_{t|t-1}^0 + \beta_{t|t-1}^T y_t.$$

These expectations are then plugged into the model representation (3), and the resulting purely backward-looking model is solved to produce the Actual Law of Motion, or ALM:

$$y_t = \mu_t + T_t y_{t-1} + R_t \epsilon_t. \tag{5}$$

[17] Our adaptive learning models are realized in a specialized DYNARE toolbox. Therefore, we follow the Dynare notation in our formulae.

[18] This adaptive learning approach is referred to as Euler Equation learning as opposed to the infinite horizon or anticipated utility approach (see Evans et al 2013 and Eusepi and Preston 2015).

The model transmission mechanism (μ, T, and R) thus becomes a time-varying function of coefficients in the agents' forecasting equations (β), called *beliefs*. The beliefs could be updated using any convenient adaptive algorithm. In the literature, Recursive Least Squares (RLS) and Kalman filter proved to be the most popular. In our previous paper (Slobodyan and Wouters 2012) we utilized Bayesian Kalman filter learning as a flexible learning mechanism for a set of forecasting variables.[19] The beliefs models were specified as small forecasting models where the set of variables the agents use for forming their forecasts is much smaller than the Minimum State Variable (MSV) set that is needed to achieve rational expectations forecast. A simple AR(2) specification turned out to be sufficient to capture the time-varying persistence in expectations that is useful to explain the dynamics in the observed macrodata.[20] The forecasting equation for inflation was of the form:

$$\left[\ \pi_t^f\ \right] = \left[\ 1 \quad \pi_{t-1} \quad \pi_{t-2}\ \right] \beta_{\pi,t-1} + \left[u_{\pi,t}\right], \tag{6}$$

We have already mentioned that including the marginal cost as an additional regressor in the prediction model is very useful once we observe the survey expectations in the 10obs model, which suggests that in order for the AL-models to exploit the information from the survey data more efficiently, we must include additional variables in the belief models. A simple AR(2) belief model cannot capture the rich information structure of the survey data that we observed in the analysis of the RE-model. Therefore, we consider a specification for the beliefs that include all independent determinants that affect the inflation dynamics in the structural model equations. This means that we have to include in the belief specification not only the lags of inflation and the marginal cost but also the unobserved innovations in the markup process. It is precisely by including these innovations in the beliefs that identification of the separate markup disturbances becomes possible.[21].

$$\left[\ \pi_t^f\ \right] = \left[\ 1 \quad \pi_{t-1} \quad \pi_{t-2} \quad mc_{t-1} \quad \varepsilon_{t-1}^{p.ar} \quad \varepsilon_{t-2}^{p.iid}\ \right] \beta_{\pi,t-1} + \left[u_{\pi,t}\right], \tag{7}$$

To keep symmetry, a similar belief model was used for beliefs about the wage process (with the marginal rate of substitution replacing the marginal cost):

$$\left[\ w_t^f\ \right] = \left[\ 1 \quad w_{t-1} \quad w_{t-2} \quad mrs_{t-1} \quad \varepsilon_{t-1}^{w.ar} \quad \varepsilon_{t-2}^{w.iid}\ \right] \beta_{w,t-1} + \left[u_{w,t}\right] \tag{8}$$

Agents will use the available information about the markup shocks' components at time t and $t-1$ when forming their expectations for $t+1$. This information is introduced into the model through the observation of the survey nowcasts. While these markup shocks might affect decisions about other variables contemporaneously, these consequences for the traditional macrovariables are modest and are not sufficiently strong for the identification of the precise nature of the shocks in the 9obs models.

[19] See appendix A for more detail on the setup of our learning approach.

[20] See also Hommes and Zhu (2014) for more evidence supporting the use of simple forecasting rules

[21] Note that we must include the iid innovation with two lags in order to secure independence among the RHS-regressors and to avoid singularity in the covariance matrix.

2.2.2 Estimation results with Survey data and two-component markup shocks under AL

The estimated parameters of this AL-2MU-10obs model are standard (see Table A3): the stickiness in both prices and wages is high but not too extreme. The persistent markup shocks' innovations have small standard deviations, 0.04 for prices and 0.03 for wages, and reasonable persistence of respectively 0.78 and 0.65. The measurement error in the inflation expectation is further reduced to 0.01. The model forecast for $t+1$ inflation almost perfectly matches the survey expectations. Of course in this setting the PLM coefficients are also crucial to understand the inflation dynamics both in terms of persistence and volatility. In an AL-context, the transmission via the endogenous belief coefficients is more important for the inflation dynamics than the exogenous persistence in the shocks which is crucial under RE

The one-quarter-ahead inflation forecast of this AL model is almost identical to the equivalent SPF inflation nowcast (see Figure 4 and Table 10). The accuracy of the two forecasts is not significantly different according to the DM-test. For longer horizons, the quality of the inflation forecast is less impressive, both relative to the SPF, although the difference is not significant, and relative to the RE model (RE-2MU-10obs). As suggested before, this can result from the flexibility of the AL coefficients and the lack of parameter restrictions on the belief model. We can partially solve this problem by incorporating longer horizon forecasts into the list of observables, as we illustrate further below. It is also noteworthy that the rationality test for the forecasts produced by this AL model are relatively successful: Table A8 in the appendix display less significant results for persistence or predictability in the forecast errors than the other models considered and than the SPF forecasts both at the one-quarter and the annual horizon.

The marginal likelihood of this model is superior to all previous models. The improvement relative to the AL-model with measurement error only (AL-ME-10obs) is of the order of 66; with respect to the best RE-model (RE-2MU-10obs), the improvement is 50. The model also does an excellent job for the 9 original variables: here the improvement is of the order of 20 to 24. Note that the augmented belief equation for inflation is crucial for this excellent marginal likelihood result: the marginal likelihoods for the models with AR(2) and AR(2)+mc specification for the PLM beliefs are much smaller. Thus, the information about the nature of the markup shocks must be incorporated into the belief equations. In this way we provide the agents in the model with the same timely information that the survey participants possess. When observing the survey forecast for time $t+1$ in the course of time t, the agents in the model can identify correctly the nature of the markup shocks. This information about the persistent components of the shocks determines their contemporaneous actions and their expectations for next period. As in the RE-model, the survey data are extremely informative for distinguishing the more persistent markup shocks from the *i.i.d.* component in the inflation dynamics. When there is a large revision in the one-period-ahead survey forecast, this typically leads to an innovation in the persistent markup shock. The correlation between the change in the forecast and the persistent markup innovation is 0.85. Figure A1 in the Appendix illustrates how changes in the survey forecast determines the interpretation of the markup shock as a persistent or as an iid shock event. This result confirms the usefulness of adding the survey data to the set of observables as we stressed in the Introduction.

These impressive gains in marginal likelihood are explained by two features of the forecasts distribution: the mean forecast precision and the time-varying volatility. The standard out-of-sample prediction statistics of this model (bias, amd and rmse) are excellent. This applies to both the inflation variables and the real variables this time. The AL-model outperforms the RE-model in rmse on all variables except consumption growth. Compared to simpler AL-model, there is an overall gain except for investment and second release data. Given the time-varying covariance structure, the likelihood score of the forecasts becomes informative as it weights the forecast errors by their conditional variances. The impact of this correction on the forecast score is the largest for the inflation expectation variable. While in terms of rmse, the results are marginally worse than for the RE-model, in terms of log likelihood score the AL forecast dominates by far the RE outcome.

In order to illustrate further the impact of the time-varying covariance matrix on the likelihood/posterior evaluation, we did the following experiment. We used the time-varying covariance matrixes for the one-period-ahead forecast errors from the AL-model to evaluate the likelihood of the RE-prediction errors. The log posterior value of the RE-model, evaluated at the parameters corresponding with the RE-mode, improves with this correction for time variation in the prediction uncertainty from -718 to -673. This value can be compared with the log posterior of the AL-model at the mode of -664. Re-evaluating the AL-models with the fixed covariance structure of the RE model results in a deterioration of the log posterior to -716 which is still slightly better than the log posterior mode of the RE-model. Clearly, a large fraction of the improvement in the log posterior value can be attributed to the time-varying covariance matrix that is implied by the updating of the beliefs over time. One could expect that most of this gain is realized in the beginning of the sample which is characterized by large variation in the covariance matrix as will be illustrated below. However, the same result is confirmed when repeating the exercise over the 1996q1-2015q3 period: the log posterior of the RE-model for that sub-sample improves from -268 to -237 when evaluated with the time-varying AL-covariance structure. On the other hand, the AL model deteriorates from -241 to -265 when the constant covariance structure of the RE-model is used. Time-variation in the covariance structure is very important, but the AL model still outperforms the RE-model even when evaluated with the same constant covariance structure.

Table 10: Forecast Statistics for the augmented AL-2MU-10obs model with SPF observable

	1971q1-2015q3			1996q1-2015q3		
$t+1$ horizon	bias	mad	rmse	bias	mad	rmse
$\pi_{t+1\|t}^{AL\_PLM} - \pi_{t+1}^{r1}$				0.03	0.17	0.21
$\pi_{t+1\|t}^{AL\_ALM} - \pi_{t+1}^{r1}$	-0.05	0.21	0.27	0.04	0.17	0.21
$\pi_{t+1\|t}^{AL\_ALM} - \pi_{t+1}^{r2}$				0.02	0.16	0.19
$\pi_{t+1\|t}^{AL\_ALM} - \pi_{t+1}^{rf}$				-0.00	0.15	0.19
longer horizon						
$\pi_{t+2\|t}^{AL\_ALM} - \pi_{t+2}^{r1}$	0.04	0.25	0.24	0.04	0.20	0.24
$\pi_{t+3\|t}^{AL\_ALM} - \pi_{t+3}^{r1}$	0.04	0.27	0.37	0.04	0.20	0.26
$\pi_{t+4\|t}^{AL\_ALM} - \pi_{t+4}^{r1}$	0.03	0.30	0.43	0.04	0.22	0.28
AL versus SPF	rel. rmse%		DM-test	rel. rmse%		DM-test
$horizon = 1$	38.58		0.75	40.05		-0.12
$horizon = 2$	55.45		0.13	54.71		1.57
$horizon = 3$	57.42		-0.29	67.00		1.26
$horizon = 4$	59.52		0.13	69.09		0.77

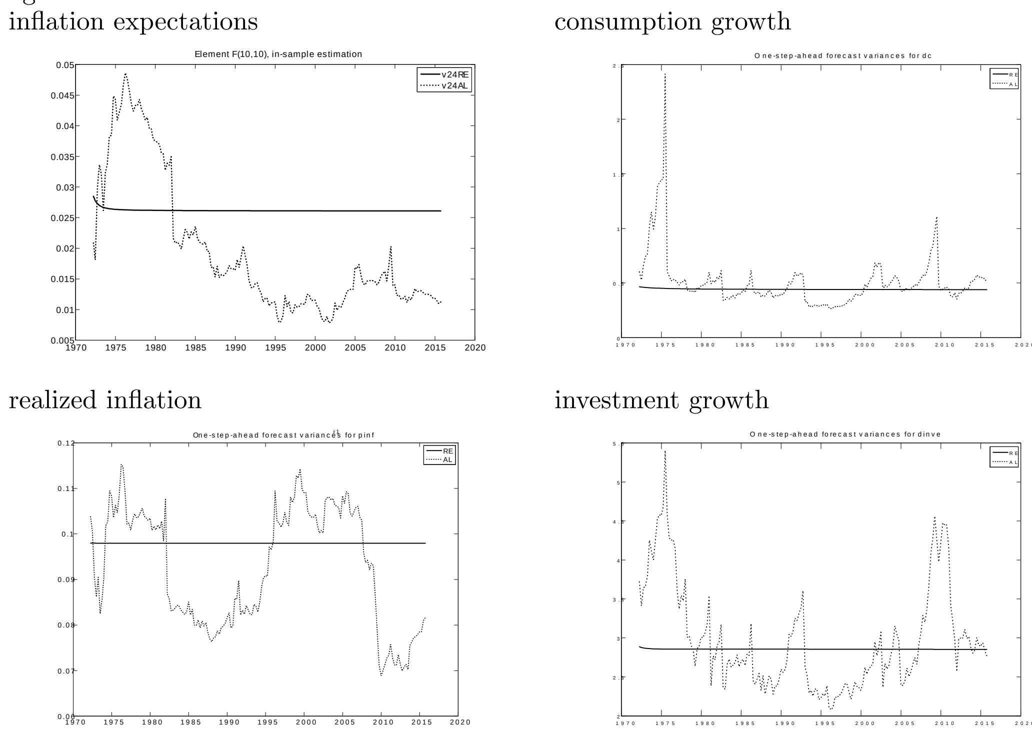
Note:

Table 11: Forecast performance of the Augmented AL-2MU-10obs model

	π_r1	π_r2	π_f0	dy_r1	dy_r2	dc	$dinve$	$hours$	dw	r
1971q1-2015q3										
bias	-0.05	0.02	0.00	-0.17	0.01	0.05	0.21	-0.04	0.02	-0.03
MAD	0.21	0.08	0.11	0.50	0.16	0.53	1.24	0.42	0.55	0.13
RMSE	0.27	0.12	0.16	0.68	0.21	0.72	1.68	0.54	0.75	0.22
1996q1-2015q3										
bias	0.04	-0.01	-0.01	0.15	-0.01	0.08	-0.36	0.14	0.00	0.04
MAD	0.17	0.05	0.08	0.39	0.16	0.42	1.15	0.36	0.73	0.09
RMSE	0.21	0.07	0.11	0.50	0.21	0.61	1.51	0.46	0.94	0.12
log lik score	0.87	1.99	1.71	-0.08	1.06	0.00	-0.92	0.20	-0.49	1.27
Comparison: AL-ME-10obs										
RMSE	0.82	1.06	0.80	0.92	1.01	0.99	1.03	0.90	0.97	0.93
log lik score	0.18	-0.02	0.57	0.01	-0.01	-0.03	-0.04	0.06	0.03	0.07
Comparison: AL-SW2012-9obs										
RMSE	0.72	1.03		0.96	1.01	0.98	1.10	1.01	0.97	1.05
log lik score	0.19	0.03		0.01	-0.01	0.06	-0.09	0.05	0.17	0.04

Note: See Table 9

Figure 6: Conditional Variance for inflation forecast in the RE and AL Model



Note: pexp_dif_pif0_3_5obsme

2.2.3 Analysis of the time-variation in the AL model

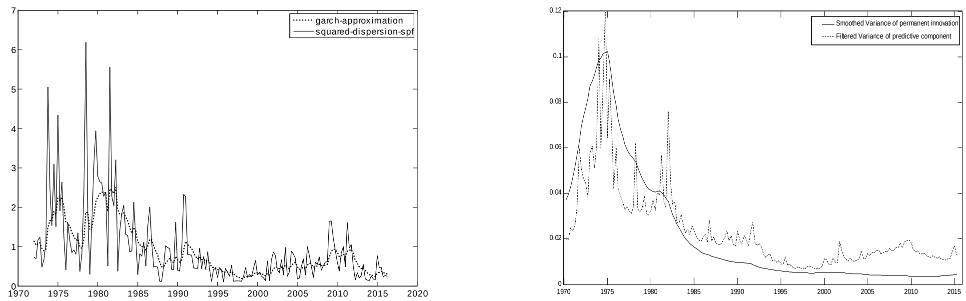
Figure 6 illustrates the time profile of the conditional variance for a selected number of variables. Crucial variation is observed in the variance of the one-period forecast error in the inflation expectations. This variance was up to three times higher in the seventies than in the period since 1995. Interesting is that the variance increased again slightly from 2005 onwards with a peak in 2010, but a decline since then. Uncertainty in inflation expectations has reduced recently despite the remaining uncertainty about the speed of the recovery and corresponding monetary policy reaction. The profile in this uncertainty is also consistent with other indicators of the forecast uncertainty: for instance the squared 1Q-dispersion among individual forecasts in the SPF survey follows a similar historical development. It also resembles the stochastic volatility process of the variance for the persistent unobserved component in the Stock and Watson UC-SV model for inflation (Stock and Watson 2007-JMCB) as illustrated in Figure 7.

The conditional variance in actual inflation realisations follows a more complex profile: it inherits the uncertainty peak in the seventies from the inflation expectations component, but it has an additional peak during the period 1995-2007. The conditional variance in output growth is representative for all other real variables: its profile is strongly affected by a cyclical updating process with a positive outlier in the mid-seventies.

Figure 7: Conditional Variance for inflation forecast in the RE and AL Model

squared IQ-range - spf dispersion

Stock-Watson (2007) variance of permanent inflation component

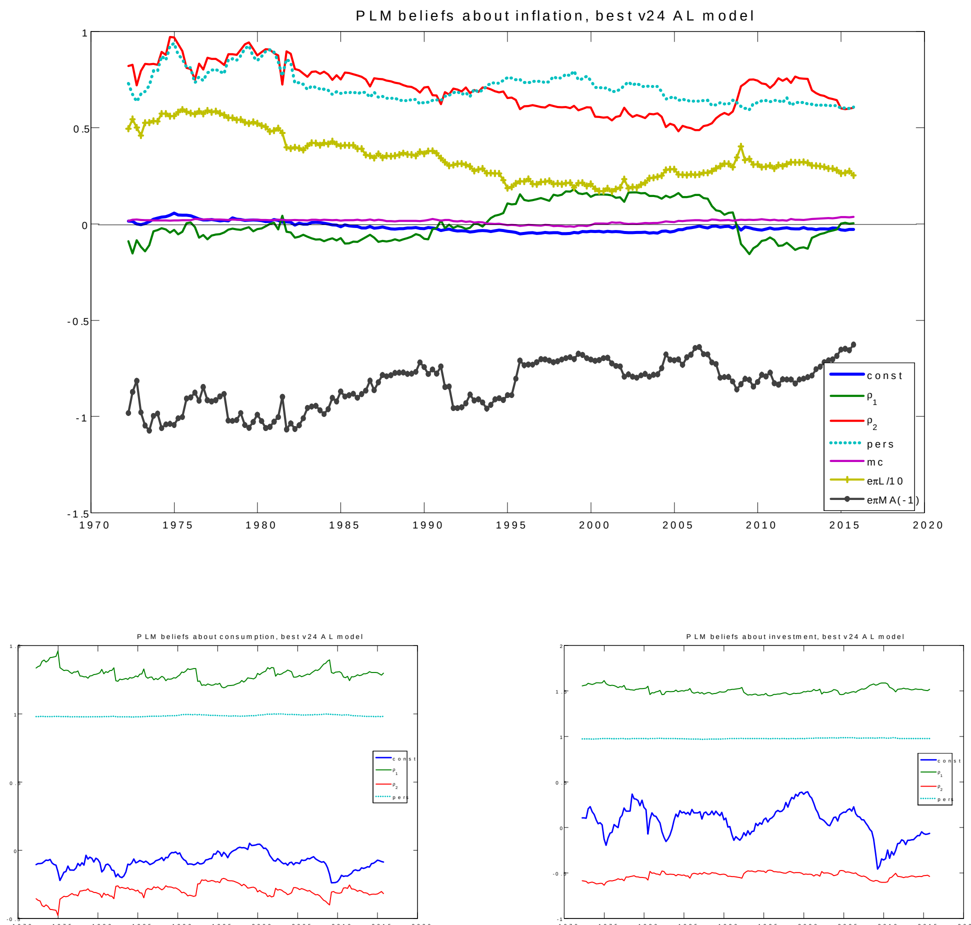


Note: pexp_dif_pif0_3_5obsme, Philadelphiafed.org SPF IQ-dispersion data, update of Stock-Watson decomposition on first release inflation data

This time-variation in the conditional variances is explained by the updating in the belief models. In Figure 8, we plot the time variation in the belief coefficients of the forecasting model for inflation, consumption and investment. Starting with the inflation beliefs, the persistence parameter and the constant follow the same profile as documented in SW2012. The updating in the constant follows systematically the surprise in the inflation realisation: unexpected higher inflation leads to a positive updating in the constant and vice-versa. The updating in the constant is very important for the long run inflation trend, and the scale of this coefficient in Figure 8 is therefore misleading. The updating in the persistence parameter (the sum of ρ_1 and ρ_2, the coefficients of the two lagged inflation terms in the beliefs) reacts in a slightly more complicated way as it depends on the forecast error and on the level of inflation: in periods where inflation is higher than the long-run mean implied by belief coefficients, a positive inflation surprise will generate an upward adjustment in the perceived persistence of inflation. However, when inflation is low, a positive surprise in realised inflation leads to lower perceived persistence. Note also the opposite adjustment in the first (ρ_1) and second (ρ_2) autocorrelation coefficient: ρ_1 is in particular important for the impact effect of all shocks including the highly volatile *i.i.d.* markup shock. The beliefs on the markup shocks are also adjusting in a similar direction: the coefficient of the persistent markup mimics the updating in the constant, while the changes in the i.i.d. markup innovation are somewhere in between the updating in the constant and the persistence.

Clearly big and repeated inflation surprises in the same direction affect the belief coefficients substantially. Understanding this time-variation in the long run perceived inflation target and the perceived persistence and shock sensitivity of inflation is highly relevant for the correct interpretation of inflation expectations in the monetary policy analysis.

Figure 8: Time-varying belief coefficients for inflation, consumption and investment



Note:PLM_beliefs_pinf_v24AL

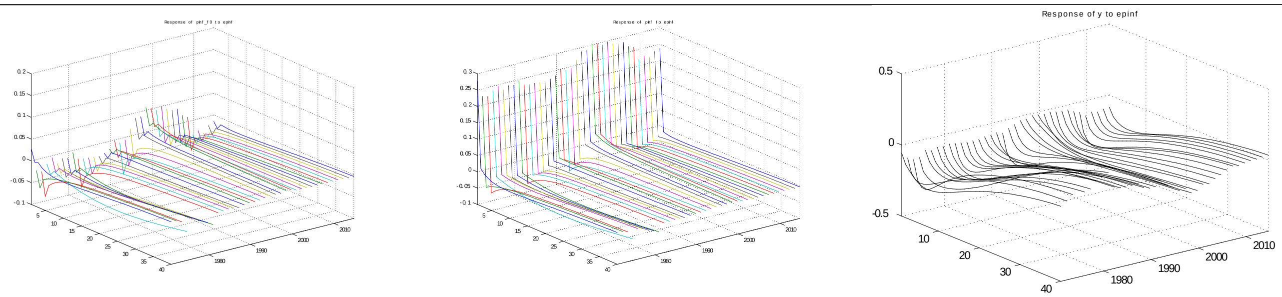
The adjustment in the simple beliefs for consumption and investment are also very interesting: these variables are perceived as almost unit root processes with a highly time-varying drift in the "constant" and a time-varying autocorrelation term in the growth process. These beliefs generate a strong cyclical and skewed accelerator process in investment and consumption. In booms, both the constant drift coefficient and the first order autocorrelation coefficient in the growth rate tend to adjust positively reflecting higher confidence and optimism in the expectations. Once a negative shock interrupts the growth cycle, the beliefs about the constant growth rate decrease rapidly but the second order persistence parameter adjust only with a delay, which means that the negative shocks are perceived as relatively persistent shocks and their impact is extrapolated into the future. Negative shocks in the beginning of the recession are therefore amplified and contribute to the asymmetry in the growth rate over the cycle.[22] The first order autoregressive parameter in the consumption beliefs is of particular importance in that it interacts with the habit coefficient in the consumption Euler equation. When this coefficient approaches the habit parameter, habits and growth rate extrapolation become reinforcing mechanisms that make consumption extremely sensitive to interest rate fluctuations. This explains the peak in the conditional variance of consumption in the mid-seventies. Note also how the Great Recession has a huge impact on the drift growth factor in consumption and investment beliefs and how long it took for these beliefs to adjust again in the recovery.

The time-varying impulse response functions in Figure 9 confirm this amplifying or attenuating effect of the belief coefficients on the transmission mechanism of the various shocks over the cycle

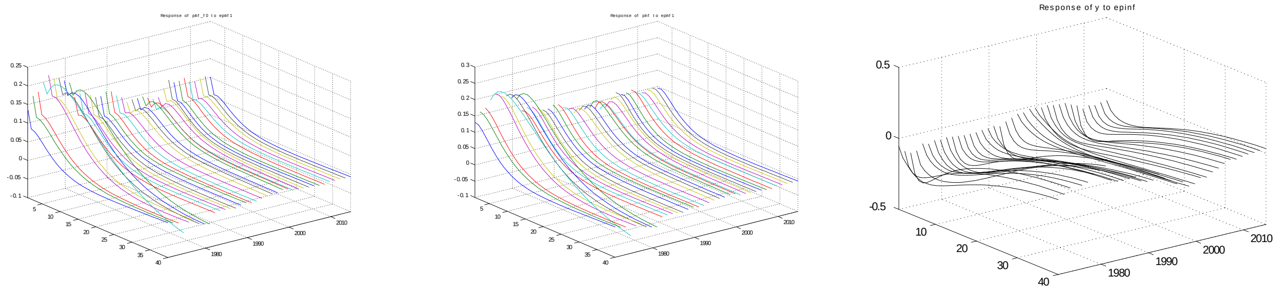
[22] These implications for consumption and investment expectations should be verified by the survey expectations as well. We plan to do this in a follow up paper.

Figure 9: Time-varying impulse response functions in the AL-model: inflation expectations, inflation and output growth to shocks in persistent and iid price markups, persistent wage markup and productivity

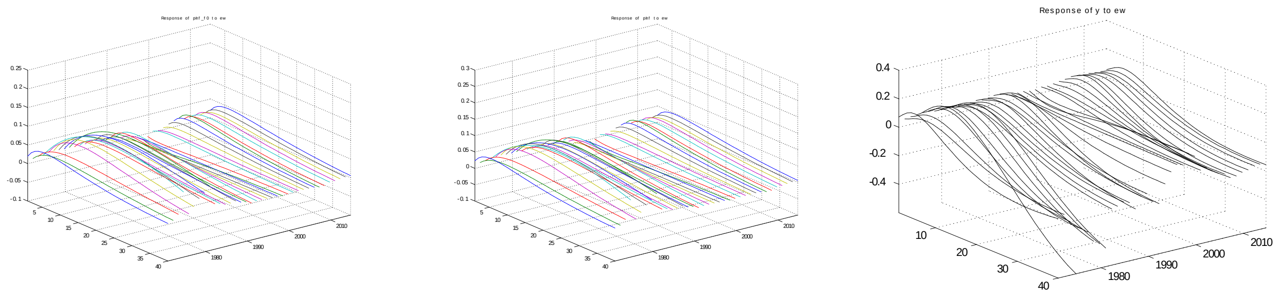
inflation expectations	inflation	output



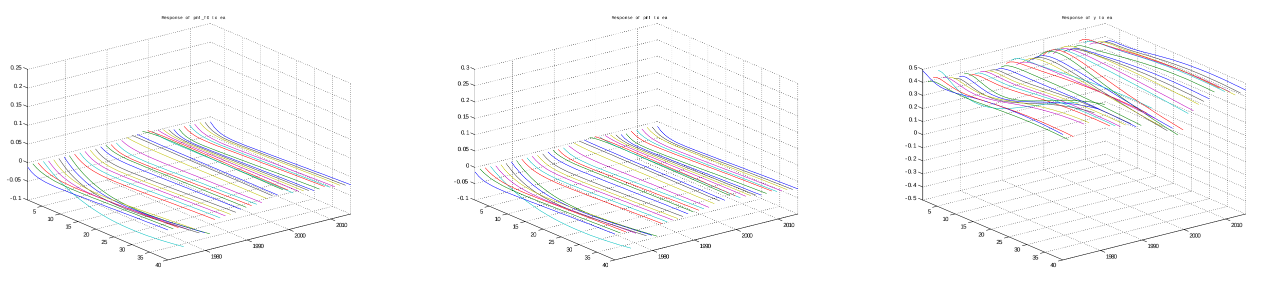
ε^{piid}



ε^{pap}



ε^{wap}



ε^{a}

Note:

The time variation in the irfs of the various shocks on inflation expectations and inflation realisation follow a similar time profile. The updating in the perceived inflation persistence and in the impact coefficients of the persistent markup innovations - which are updating in the same direction - are crucial for these dynamics. High impact effects and high persistence in the inflation belief models explain the high sensitivity and persistence in the seventies and the gradual moderation in the response later on. Inflation expectations and the actual inflation response are generally consistent with each other which is not guaranteed automatically in an AL-context. The time-profile in the irf of the iid markup shock deviates from the other shocks. The belief coefficient on this shock and the first order autoregressive coefficient (ρ_1) are responsible for this specific time variation. This shock, which is important for high frequency inflation fluctuations, explains the second peak between 1995 and 2005 in the variance of the one-period-ahead inflation uncertainty, see Figure 6.

Table 12: Conditional variance decomposition for the AL-2MU-10obs model

	ε^a	ε^b	ε^g	ε^{qs}	ε^m	ε^{piid}	ε^{wiid}	ε^{par}	ε^{war}	$\xi^{\pi f1}$
1 quarter horizon										
π_f0	0.65	0.08	0.00	0.00	0.02	3.72	1.93	92.94	0.03	0.60
π_r1	0.19	0.02	0.00	0.00	0.00	81.18	0.57	18.03	0.01	0.00
w	0.02	0.34	0.02	0.00	0.07	15.33	82.67	0.06	1.48	0.00
y	15.77	63.96	2.20	0.66	13.26	0.21	0.35	3.59	0.01	0.00
1 year horizon										
π_f0	14.04	1.53	0.00	0.00	0.39	1.63	10.59	81.07	0.47	0.26
π_r1	1.65	0.63	0.00	0.00	0.16	53.60	4.32	39.44	0.20	0.00
w	0.06	3.64	0.04	0.02	0.93	11.73	79.20	0.51	3.89	0.00
y	6.40	71.15	0.81	0.37	18.85	0.62	0.36	1.42	0.02	0.00
10 year horizon										
π_f0	17.29	19.27	0.42	0.44	8.05	3.27	17.55	31.89	1.75	0.08
π_r1	11.66	12.99	0.29	0.30	5.42	29.03	11.84	27.29	1.18	0.00
w	0.55	40.76	0.06	0.03	17.04	6.17	29.00	3.64	2.75	0.00
y	11.19	58.44	2.25	0.14	23.20	0.95	1.13	2.61	0.11	0.00

Note: The decomposition is evaluated for the initial belief parameters.

The impact effects of the wage markup shocks on inflation and inflation expectations are highly reduced relative to the RE-model. Most importantly, the effects of these shocks are now transitory, while they were responsible for the long term inflation trend in the RE-model. In fact, in this AL model, the long term inflation trend is no longer explained by the exogenous shocks. It is the learning about the constant in the belief equation that explains this long term trend. This means that all shocks can contribute to this long term inflation expectations depending on how the updating in the inflation beliefs is affected. This type of non-linear interactions is not taken into account in the conditional variance decomposition reported in Table 12, as it is constructed based on the initial beliefs only.[23] Still, this decomposition illustrates that for the short

[23] As explained in learning setup, these beliefs are consistent with the RE-equilibrium for

and medium term dynamics the AL and the RE model are giving a similar interpretation in terms of shock contributions, except that the wage shocks are less important. At the long forecast horizon, all shocks now contribute to the inflation variance. There is also a non-negligible role for demand shocks such as the risk premium and the monetary policy shocks. However, the learning responses must be added on top of this static analysis, making the contributions of different shocks highly nonlinear and non-additive.[24]

In sum, the AL model provides a more informative analysis of the inflation dynamics than the RE model. Instead of explaining the long term inflation trend by exogenous shocks, it is the expectations and the updating in the beliefs that are now crucial for inflation anchoring. This basic AL-result is robust across various specifications. For instance, adding an inflation target shock in this AL-model does not change the results as it did for the RE-model. The results of this exercise are shown in Table A4: the inflation target shock remains minimal and the marginal likelihood of this model does not improve relative to the model without target shock.

The AL model does relatively worse than the RE on forecasting inflation over multiple quarters. This might suggest that there is too much flexibility in the belief specification and updating. One solution for overcoming that problem is to add survey expectations about future quarters to the list of observables. Up to now, we only experimented with one additional observable for inflation two quarters ahead. The beliefs can either contain two separate belief equations for each of these forecasts (unrestricted model) or the two-quarter forecast can be written as a restricted belief equation, where the restriction imposes consistency with the one period ahead forecast coefficients.[25] In both cases, the results are promising in that they improve on the inflation forecast without distorting the other implications of the model. In future work, we also plan to add long term inflation expectations so as to further discipline the updating in the belief coefficients. By doing so, long run inflation expectations will no longer be purely backward looking via the updating process, but there might be some role left for forward-looking expectation shocks related for instance to changes in the monetary or fiscal policy context. This type of model extension can improve the precision of the long term inflation forecasts, but it should be noted that the long term inflation forecasts of our best model are not significantly different, according to the Diebold-Mariano test, from the survey forecasts already in their present form.

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the given structural parameters. The first line in the IRF figure is reproducing exactly this situation. Evaluating the decomposition around this point is still representative as an approximation.

[24] In SW2012, we illustrated how the learning response react to various shocks depending on the state of the economy.

[25] The RHS-variable in the two period ahead forecast can be substituted consistent with the other PLM equations for π_t^f, w_t, r_{Kt} (both enter via mc_t) and the exogenous shocks processes (for productivity and price markup shocks)

3 Robustness exercises and extension

Until now we used the survey forecast for the current quarter ($\pi^f_{t+1|t}$) as the observable for our model expectations ($E_t\pi_{t+1}$) as presented in Equation(1). This survey forecast or nowcast is collected around the end of the first month of the quarter $t+1$ and so it might contain information that is not available for agents when making their decision for quarter t. This provides the survey with a time advantage relative to the model forecast which is based only on the first releases for quarter t and second releases of $t-1$ data for the macro-aggregates. Therefore we experienced also with an alternative setup in which we use the survey forecast for the next quarter ($\pi^f_{t+1|t-1}$) as observable for the model expectations ($E_t\pi_{t+1}$). This means that we used data collected in a previous survey as observable for the model forecast.

As documented in Table 13, the time disadvantage of this survey measure has a clear impact on the forecast performance. Over the complete sample, the forecast errors with respect to the different releases and for different horizons are much larger than for the nowcast timing that was documented in Table 1. But remarkably, for the sample since 1996, the forecast performance remains very similar to the outcomes with the nowcast timing. This result reflects the change in dynamic properties of the inflation process over our sample period.

Table 13: Statistical properties of the SPF forecasts errors with alternative timing

	1971q1-2015q3			1996q1-2015q3		
	bias	mad	rmse	bias	mad	rmse
SPF statistics						
$\pi^{SPF}_{t+1\|t-1}-\pi^{r1}_{t+1}$	0.03	0.25	0.33	0.04	0.19	0.22
$\pi^{SPF}_{t+1\|t-1}-\pi^{r2}_{t+1}$	0.01	0.26	0.35	0.02	0.17	0.21
$\pi^{SPF}_{t+1\|t-1}-\pi^{rf}_{t+1}$	0.02	0.23	0.30	0.00	0.17	0.22
SPF for longer horizons						
$\pi^{SPF}_{t+2\|t-1}-\pi^{r1}_{t+2}$	0.04	0.28	0.38	0.07	0.19	0.23
$\pi^{SPF}_{t+3\|t-1}-\pi^{r1}_{t+3}$	0.05	0.31	0.42	0.07	0.21	0.25
$\pi^{SPF}_{t+4\|t-1}-\pi^{r1}_{t+4}$	0.05	0.33	0.46	0.09	0.22	0.26

From the marginal likelihood results in Table 14, we can first observe from the results for the models with 10 observables that the model specification with two markup shocks, an i.i.d. process and a persistent autoregressive component, appears again as a very flexible structure to match simultaneously the realized and expected inflation data. There is a big improvement in the log marginal likelihood when going from the specification with measurement error for the survey expectations to the setup with two structural markup components in particular in the RE setup. But we do not observe the same big gain in forecasting power under AL when agents are allowed to use the detailed shock information in their belief models as we did when the nowcast expectations were included in the dataset. Our interpretation is that the lagged survey forecasts do not contain the same timely information than the nowcast data and so the identification of the shocks based on this outdated information is not helpful for improving the forecasting performance. This is consistent with the observation that the marginal likelihood of the common 9 variables is worse in the models estimated on 10 observables than in the models estimated on these 9 observables only. Restricting the model expectations to be consistent with the lagged survey forecast has a cost for the overall model performance even in the model with the flexible model specification and AL. This finding suggests that the survey forecasts are useful to improve the model forecast mainly because they incorporate new information. The exercise also confirms the importance of the inflation expectations for the overall model performance: correctly identifying these expectations allow to improve the dynamic interactions between nominal and real variables in the economy.

Table 14: Marginal likelihood of models with alternative timing

	71q1-15q3		96q1-15q3	
	9obs	10obs	9obs	10obs
RE-SW07-9obs	-965.22		-361.25	
AL-SW12-9obs (AR2)	-943.42		-340.96	
AL-SW12-9obs (AR2+mc)	-934.41		-317.78	
RE-ME-10obs	-1016.24	-896.61	-367.65	-288.06
AL-ME-10obs (AR2)	-978.38	-896.37	-343.66	-295.82
AL-ME-10obs (AR2+mc)	-954.71	-836.91	-329.92	-256.46
RE-2MU-10obs	-979.76	-852.09	-353.77	-269.17
AL-2MU-10obs (AR2)				
AL-2MU-10obs (AR2+mc)				
AL-2MU-10obs (AR2+mc+UC)	-962.65	-823.94	-336.92	-250.89

Note: see Table 6.

The robustness of the results was also successfully tested on datasets with real-time data for all observed series and on datasets with survey expectations for several horizons.

4 Conclusions

A proper integration of survey expectations - as measured by the SPF - in a DSGE model makes it possible to identify separately the transitory and the persistent shocks in inflation. By improving the efficiency of the model filter, the forecasts improve both for inflation and for other macrovariables. Under AL, the updating of the belief models that are augmented with the timely information signals from the survey data, generate time-varying estimates of the perceived inflation target, persistence and sensitivity to shocks. In this way, the model captures the joint dynamics in the first and second moments of realized and expected inflation.

Our exercise has some interesting methodological implications for the efficient application of AL in empirical macromodels. In Slobodyan and Wouters 2012, we argued that small belief models were sufficient to capture the important time-variation in inflation expectations and persistence. Such simple expectations models are also favoured by experimental studies (Hommes and Zhu 2014). The results in this paper suggest that expectations contain detailed information on the precise nature of the latest inflation developments. Belief models must be sufficient flexible to capture this information and therefore it might be necessary to augment the belief specifications with a minimum set of latent factors that summarize and transmit all relevant signals.

More research is necessary to test alternative specifications of the belief models in AL. There seems to be a trade-off between simple specifications on the one hand and sufficiently informative specifications on the other hand. Another issue is whether agents update their beliefs gradually over time or whether a regime switching setup that allows for sharp adjustments in beliefs is more appropriate. It also remains an issue whether expectations are determined by a backward looking updating process only or whether there is also a role for forward looking expectation shocks. Finally, we would like to test whether the time-variation delivered by the adaptive learning process is consistent with the time-variation as detected in reduced form models with time-variation in coefficients and volatilities.

This paper was focused on inflation expectations. In follow up work, we will test whether survey data on other variables, such as consumption, investment and wages, are equally important for the model performance.

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Appendix A: Technical details on Kalman Filter learning procedure of Slobodyan and Wouters (2012)

The precise learning procedure is defined as follows. Agents estimate the forecasting model at each point in time given the information set available at that time. We assume that they use an efficient Kalman filter updating mechanism[26]. They believe that the coefficients β (a vector obtained by stacking all β_j) follow a vector autoregressive process around $\overline{\beta}$ (which will be specified later): $vec\left(\beta_t - \overline{\beta}\right) = F \cdot vec\left(\beta_{t-1} - \overline{\beta}\right) + v_t$, where F is a diagonal matrix with $\rho \leq 1$ on the main diagonal[27]. Errors v_t are assumed to be *i.i.d.* with variance-covariance matrix V.

We can write the forecasting model in the following SURE format[28]:

$$\begin{bmatrix} y^f_{1t} \\ y^f_{2t} \\ \vdots \\ y^f_{mt} \end{bmatrix} = \begin{bmatrix} X_{1,t-1} & 0 & \dots & 0 \\ 0 & X_{2,t-1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & X_{m,t-1} \end{bmatrix} \begin{bmatrix} \beta_{1,t-1} \\ \beta_{2,t-1} \\ \vdots \\ \beta_{m,t-1} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{m,t} \end{bmatrix}, \quad (9)$$

The errors $u_{j,t}$ depend on a linear combination of the true model innovations ϵ_t and therefore they are likely to be correlated, making the variance-covariance matrix non-diagonal: $\Sigma = E\left[u_t \cdot u_t^T\right]$. With the above notation, the Kalman filter updating and transition equations for the belief coefficients and the corresponding covariance matrix are given by

$$\begin{aligned} \beta_{t|t} &= \beta_{t|t-1} + P_{t|t-1}X_{t-1}\left[\Sigma + X^T_{t-1}P_{t|t-1}X_{t-1}\right]^{-1} \times \left(y^f_t - X^T_{t-1}\beta_{t|t-1}\right), && (10a) \\ & \quad \textit{with } (\beta_{t+1|t} - \overline{\beta}) = F \cdot (\beta_{t|t} - \overline{\beta}). \\ P_{t|t} &= P_{t|t-1} - P_{t|t-1}X_{t-1}\left[\Sigma + X^T_{t-1}P_{t|t-1}X_{t-1}\right]^{-1} \times X^T_{t-1}P_{t|t-1}, && (10b) \\ & \quad \textit{with } P_{t+1|t} = F \cdot P_{t|t} \cdot F^T + V. \end{aligned}$$

These best estimates for the beliefs ($\beta_{t|t-1}$) are then substituted for β_t in (9) to generate expectations of forward-looking variables, $E_t y^f_{t+1}$. Plugging these expectations into (3), we obtain a purely backward-looking representation of the model (5).[29] The resultant time-dependent matrices μ_t, T_t, and R_t replace the

[26]Sargent and Williams (2005) showed that even if Kalman filter and constant gain learning are asymptotically equivalent on average, their transitory behaviour may differ a lot. In particular, Kalman filter tends to result in much faster adjustment of agents' beliefs. With faster adjustment of beliefs, we are able to better understand whether the initial beliefs or time–varying coefficients matter more for the improved model fit.

[27]ρ is restricted to be the same for the seven variables that are forecasted. Allowing for a variable specific autocorrelation provides some extra flexibility but also larger parameter uncertainty.

[28]The SURE format and the corresponding GLS estimator are necessary to get an efficient estimator of the complete forecasting model because the variables appearing on the RHS in each equation are not identical.

[29]Note that we expand the state vector y in this representation with additional lags that occur in the forecasting models.

constant equivalents in the RE solution. These matrices depend now on both the structural parameters of the decision problem (Θ) and on the best estimates of the forecasting model ($\beta_{t|t-1}$), and contain all necessary information to describe the dynamics and the propagation of the shocks in the model under learning. In terms of adaptive learning literature, the equation (5) represents the Actual Law of Motion (ALM) of the model.

In order to initialize this Kalman filter for the belief coefficients, we need to specify $\beta_{1|0} = \overline{\beta}$, $P_{1|0}$, Σ, and V. In our baseline approach, all these expressions are derived from the correlations between the model variables implied by the RE Equilibrium evaluated for the corresponding structural parameter vector Θ. In other words, the initial beliefs are assumed to be model consistent.[30]

Using the fact that $\widehat{\beta}_{OLS} = \left(X^T X\right)^{-1} X^T y$ is unbiased, we use the theoretical moment matrices $E\left[X^T X\right]$ and $E\left[X^T y\right]$ from the RE solution and set $\beta_{1|0} = (E\left[X^T X\right])^{-1} \cdot E\left[X^T y\right]$. Given $\beta_{1|0}$, we calculate Σ as

$$\Sigma = E\left[\left(y_t^f - X_{t-1}^T \beta_{1|0}\right)\left(y_t^f - X_{t-1}^T \beta_{1|0}\right)^T\right],$$

again using the RE theoretical moments. Finally, $P_{1|0}$, the initial guess about the mean square forecast error of the belief coefficients, and V, the variance–covariance matrix of shocks v_t to these coefficients, are both taken to be proportional to $\left(X^T \Sigma^{-1} X\right)^{-1}$:[31] $P_{1|0} = \sigma_0 \cdot \left(X^T \Sigma^{-1} X\right)^{-1}$, and $V = \sigma_v \cdot \left(X^T \Sigma^{-1} X\right)^{-1}$. This initialization leaves just three parameters, σ_0, σ_v, and ρ, to fully describe the learning dynamics, but in practice, we can keep σ_0, σ_v fixed and optimize over ρ only.

[30] An alternative approach would be to derive the initial beliefs and the underlying moment matrixes from the restricted expectations equilibrium. Given our under-parameterized beliefs, this equilibrium deviates from the REE and requires the solution of the underlying ODE. Computationally this procedure was not feasible in the estimation context.

[31] $\left(X^T \Sigma^{-1} X\right)^{-1}$ is equal to $Var\left[\widehat{\beta}_{GLS}\right]$ where $\widehat{\beta}_{GLS} = \left(X^T \Sigma^{-1} X\right)^{-1} X^T \Sigma^{-1} y$, which gives an efficient estimator for the SURE model. Given knowledge of theoretical moments and of Σ, the matrix $\left(X^T \Sigma^{-1} X\right)^{-1}$ could be readily calculated.

Appendix B: Additional Tables and Figures

Table A1: Prior and posterior distributions 9obs models.

Parameter		Prior distribution			9obs-RE Metropolis Chain			SW2007 Posterior	9obs-AL Metropolis Chain			SW2012 Posterior
		type	mean	std.dev.	mean	5%	95%	mode	mean	5%	95%	mode
Calvo prob. wages	ξ_w	B	0.50	0.10	0.78	0.68	0.88	0.73	0.80	0.75	0.84	0.84
Calvo prob. prices	ξ_p	B	0.50	0.10	0.70	0.62	0.78	0.65	0.74	0.68	0.80	0.65
Indexation wages	ι_w	B	0.50	0.15	0.56	0.33	0.77	0.59	0.25	0.12	0.38	0.21
Indexation prices	ι_p	B	0.50	0.15	0.16	0.06	0.25	0.22	0.46	0.27	0.65	0.19
Gross price markup	ϕ_p	N	1.25	0.12	1.58	1.46	1.71	1.61	1.53	1.40	1.66	1.56
Capital production share	α	N	0.30	0.05	0.19	0.16	0.22	0.19	0.18	0.15	0.22	0.17
Capital utilization cost	ψ	B	0.50	0.15	0.71	0.56	0.87	0.54	0.56	0.34	0.76	0.56
Investment adj. cost	φ	N	4.00	1.50	4.25	2.58	5.82	5.48	3.24	2.11	4.33	3.23
Habit formation	$\varkappa$	B	0.70	0.10	0.65	0.51	0.79	0.71	0.64	0.53	0.76	0.68
Int elast of subst.cons.	σ_c	N	1.50	0.37	1.51	1.08	1.97	1.59	1.76	1.24	2.16	1.58
Labor supply elast.	σ_l	N	2.00	0.75	1.77	0.97	2.57	1.92	2.22	1.58	2.89	1.77
Log hours worked in S.S.	$\bar{l}$	N	0.00	2.00	1.27	-1.01	3.57	-0.10	2.69	1.04	4.54	0.83
Discount factor	$100(\beta^{-1}-1)$	G	0.25	0.10	0.18	0.08	0.28	0.16	0.18	0.07	0.28	0.17
Quarterly Growth in S.S.	$\bar{\gamma}$	N	0.40	0.10	0.32	0.29	0.35	0.43	0.40	0.35	0.45	0.41
Stationary tech. shock	ρ_a	B	0.50	0.20	0.98	0.97	0.99	0.95	0.99	0.98	1.00	0.99
Risk premium shock	ρ_b	B	0.50	0.20	0.47	0.23	0.70	0.18	0.58	0.38	0.74	0.55
Invest. spec. tech. shock	ρ_i	B	0.50	0.20	0.84	0.77	0.92	0.71	0.48	0.38	0.58	0.51
Gov't cons. shock	ρ_g	B	0.50	0.20	0.99	0.99	1.00	0.97	0.99	0.99	1.00	0.97
Price markup shock	ρ_p	B	0.50	0.20	0.96	0.93	0.99	0.90				
Wage markup shock	ρ_w	B	0.50	0.20	0.94	0.90	0.99	0.97				
Response of g_t to ε_t^a	ρ_{ga}	B	0.50	0.20	0.59	0.43	0.74	0.52	0.64	0.49	0.78	0.54
Stationary tech. shock	σ_a	G	0.20	0.15	0.44	0.39	0.48	0.45	0.44	0.40	0.50	0.46
Risk premium shock	σ_b	G	0.20	0.15	0.18	0.11	0.25	0.24	0.20	0.15	0.24	0.15
Invest. spec. tech. shock	σ_i	G	0.20	0.15	0.37	0.31	0.43	0.45	0.43	0.37	0.48	0.45
Gov't cons. shock	σ_g	G	0.20	0.15	0.53	0.48	0.58	0.52	0.51	0.46	0.56	0.50
Price markup shock	σ_p	G	0.20	0.15	0.18	0.14	0.21	0.14	0.20	0.19	0.23	0.15
MA(1) price markup shock	ϑ_p	B	0.50	0.20	0.82	0.73	0.90	0.74				
Wage markup shock	σ_w	G	0.20	0.15	0.40	0.35	0.44	0.24	0.36	0.32	0.41	0.23
MA(1) wage markup shock	ϑ_w	B	0.50	0.20	0.89	0.83	0.97	0.88				
Quarterly infl. rate. in S.S.	$\bar{\pi}$	G	0.62	0.10	0.76	0.59	0.93	0.81	0.59	0.44	0.73	0.64
Inflation response	r_π	N	1.50	0.25	1.59	1.35	1.84	2.03	1.55	1.19	1.87	1.75
Output gap response	r_y	N	0.12	0.05	0.05	0.03	0.08	0.08	0.10	0.05	0.15	0.15
Diff. output gap response	$r_{\Delta y}$	N	0.12	0.05	0.17	0.13	0.20	0.22	0.12	0.09	0.15	0.14
Mon. pol. shock std	σ_r	G	0.20	0.15	0.24	0.21	0.26	0.24	0.22	0.20	0.24	0.22
Mon. pol. shock pers.	ρ_r	B	0.50	0.20	0.09	0.02	0.16	0.12	0.12	0.03	0.19	0.10
Interest rate smoothing	ρ_R	B	0.75	0.10	0.81	0.78	0.85	0.81	0.93	0.89	0.96	0.89
m.e. π_r1	$\sigma_{\pi r1}$	G	0.20	0.15	0.11	0.10	0.12		0.11	0.10	0.12	
m.e. dy_r1	σ_{yr1}	G	0.20	0.15	0.21	0.19	0.22		0.21	0.19	0.23	
Learning persistence	φ	U	0.00	1.00					0.99	0.98	1.00	0.97
Log marginal likelihood					MCMC	-965.22			MCMC	-943.42		

Note: models are evaluated over the period 1971Q1 - 2015Q3 using the first four observations as presample.

Table A2: Prior and posterior distributions 10obs-ME models.

Parameter		Prior distribution			10obs-RE Metropolis Chain			10obs-AL Metropolis Chain		
		type	mean	std.dev.	mean	5%	95%	mean	5%	95%
Calvo prob. wages	ξ_w	B	0.50	0.10	0.80	0.74	0.87	0.84	0.78	0.89
Calvo prob. prices	ξ_p	B	0.50	0.10	0.87	0.80	0.93	0.76	0.69	0.83
Indexation wages	ι_w	B	0.50	0.15	0.50	0.29	0.71	0.26	0.10	0.41
Indexation prices	ι_p	B	0.50	0.15	0.13	0.05	0.21	0.47	0.32	0.63
Gross price markup	ϕ_p	N	1.25	0.12	1.50	1.37	1.63	1.49	1.36	1.62
Capital production share	α	N	0.30	0.05	0.17	0.14	0.20	0.17	0.14	0.21
Capital utilization cost	ψ	B	0.50	0.15	0.65	0.47	0.82	0.52	0.27	0.77
Investment adj. cost	φ	N	4.00	1.50	3.86	2.12	5.82	2.65	1.66	3.84
Habit formation	$\varkappa$	B	0.70	0.10	0.58	0.44	0.71	0.60	0.51	0.69
Int elast of subst.cons.	σ_c	N	1.50	0.37	1.05	0.73	1.41	1.77	1.45	2.11
Labor supply elast.	σ_l	N	2.00	0.75	1.60	0.79	2.42	1.96	1.24	2.69
Log hours worked in S.S.	$\bar{l}$	N	0.00	2.00	0.18	-2.22	2.73	2.87	1.20	4.62
Discount factor	$100(\beta^{-1}-1)$	G	0.25	0.10	0.18	0.08	0.28	0.18	0.07	0.28
Quarterly Growth in S.S.	$\bar{\gamma}$	N	0.40	0.10	0.35	0.31	0.38	0.40	0.36	0.43
Stationary tech. shock	ρ_a	B	0.50	0.20	0.98	0.97	0.99	0.99	0.98	1.00
Risk premium shock	ρ_b	B	0.50	0.20	0.91	0.86	0.99	0.65	0.54	0.76
Invest. spec. tech. shock	ρ_i	B	0.50	0.20	0.69	0.54	0.84	0.38	0.23	0.54
Gov't cons. shock	ρ_g	B	0.50	0.20	0.99	0.98	1.00	0.99	0.99	1.00
Price markup shock	ρ_p	B	0.50	0.20	0.93	0.87	0.98			
Wage markup shock	ρ_w	B	0.50	0.20	0.99	0.98	1.00			
Response of g_t to ε_t^a	ρ_{ga}	B	0.50	0.20	0.61	0.46	0.76	0.62	0.47	0.76
Stationary tech. shock	σ_a	G	0.20	0.15	0.35	0.31	0.38	0.45	0.41	0.49
Risk premium shock	σ_b	G	0.20	0.15	0.08	0.04	0.10	0.17	0.13	0.22
Invest. spec. tech. shock	σ_i	G	0.20	0.15	0.38	0.31	0.45	0.42	0.36	0.47
Gov't cons. shock	σ_g	G	0.20	0.15	0.52	0.47	0.57	0.51	0.47	0.55
Price markup shock	σ_p	G	0.20	0.15	0.21	0.19	0.23	0.24	0.21	0.26
MA(1) price markup shock	ϑ_p	B	0.50	0.20						
Wage markup shock	σ_w	G	0.20	0.15	0.43	0.39	0.47	0.37	0.34	0.41
MA(1) wage markup shock	ϑ_w	B	0.50	0.20						
Quarterly infl. rate. in S.S.	$\bar{\pi}$	G	0.62	0.10	0.66	0.52	0.81	0.68	0.55	0.79
Inflation response	r_π	N	1.50	0.25	1.73	1.45	2.04	1.69	1.40	1.97
Output gap response	r_y	N	0.12	0.05	0.05	0.00	0.09	0.09	0.06	0.13
Diff. output gap response	$r_{\Delta y}$	N	0.12	0.05	0.19	0.15	0.22	0.13	0.09	0.16
Mon. pol. shock std	σ_r	G	0.20	0.15	0.24	0.22	0.27	0.22	0.20	0.24
Mon. pol. shock pers.	ρ_r	B	0.50	0.20	0.05	0.01	0.09	0.11	0.03	0.19
Interest rate smoothing	ρ_R	B	0.75	0.10	0.83	0.79	0.87	0.89	0.86	0.93
m.e. π_r1	$\sigma_{\pi r1}$	G	0.20	0.15	0.11	0.10	0.12	0.11	0.10	0.12
m.e. dy_r1	σ_{yr1}	G	0.20	0.15	0.21	0.19	0.23	0.21	0.19	0.22
m.e. π_f1	$\sigma_{\pi f1}$	G	0.20	0.15	0.15	0.14	0.17	0.18	0.16	0.19
Learning persistence	φ	U	0.00	1.00				0.98	0.97	1.00
Log marginal likelihood					MCMC	-910.87		MCMC	-885.94	

Note: see Table A1.

Table A3: Prior and posterior distributions 2MU-10obs models.

Parameter		Prior distribution			10obs-RE Metropolis Chain			10obs-AL Metropolis Chain		
		type	mean	std.dev.	mean	5%	95%	mean	5%	95%
					.		.			
Calvo prob. wages	ξ_w	B	0.50	0.10	0.79	0.73	0.86	0.89	0.85	0.92
Calvo prob. prices	ξ_p	B	0.50	0.10	0.91	0.87	0.94	0.83	0.78	0.88
Indexation wages	ι_w	B	0.50	0.15	0.38	0.18	0.57	0.25	0.10	0.39
Indexation prices	ι_p	B	0.50	0.15	0.07	0.03	0.11	0.06	0.03	0.10
Gross price markup	ϕ_p	N	1.25	0.12	1.45	1.33	1.56	1.50	1.38	1.61
Capital production share	α	N	0.30	0.05	0.20	0.17	0.23	0.18	0.15	0.21
Capital utilization cost	ψ	B	0.50	0.15	0.69	0.453	0.85	0.72	0.57	0.87
Investment adj. cost	φ	N	4.00	1.50	1.85	1.00	2.67	1.72	1.24	2.17
Habit formation	$\varkappa$	B	0.70	0.10	0.46	0.35	0.57	0.52	0.45	0.59
Int elast of subst.cons.	σ_c	N	1.50	0.37	1.35	1.03	1.66	1.43	1.26	1.59
Labor supply elast.	σ_l	N	2.00	0.75	1.58	0.75	2.39	1.83	1.06	2.60
Log hours worked in S.S.	$\bar{l}$	N	0.00	2.00	0.54	-1.15	2.25	3.37	2.37	4.37
Discount factor	$100(\beta^{-1}-1)$	G	0.25	0.10	0.17	0.07	0.27	0.18	0.09	0.27
Quarterly Growth in S.S.	$\bar{\gamma}$	N	0.40	0.10	0.36	0.32	0.39	0.39	0.35	0.42
Stationary tech. shock	ρ_a	B	0.50	0.20	0.98	0.97	0.99	0.99	0.98	1.00
Risk premium shock	ρ_b	B	0.50	0.20	0.83	0.74	0.94	0.80	0.73	0.88
Invest. spec. tech. shock	ρ_i	B	0.50	0.20	0.86	0.77	0.95	0.39	0.29	0.49
Gov't cons. shock	ρ_g	B	0.50	0.20	0.99	0.98	0.99	0.99	0.99	1.00
Price markup shock	ρ_{par}	B	0.50	0.20	0.77	0.59	0.93	0.79	0.68	0.91
Wage markup shock	ρ_{war}	B	0.50	0.20	1.00	0.99	1.00	0.43	0.09	0.77
Response of g_t to ε_t^a	ρ_{ga}	B	0.50	0.20	0.65	0.50	0.80	0.65	0.50	0.79
Stationary tech. shock	σ_a	G	0.20	0.15	0.45	0.40	0.49	0.44	0.39	0.48
Risk premium shock	σ_b	G	0.20	0.15	0.11	0.08	0.15	0.12	0.11	0.14
Invest. spec. tech. shock	σ_i	G	0.20	0.15	0.44	0.40	0.54	0.32	0.27	0.24
Gov't cons. shock	σ_g	G	0.20	0.15	0.52	0.47	0.57	0.51	0.46	0.55
Price markup shock-iid	σ_{piid}	G	0.20	0.15	0.24	0.22	0.26	0.26	0.23	0.28
Price markup shock-ar	σ_{par}	G	0.20	0.15	0.03	0.01	0.05	0.03	0.02	0.04
Wage markup shock-iid	σ_{wiid}	G	0.20	0.15	0.43	0.39	0.47	0.39	0.35	0.43
Wage markup shock-ar	σ_{wae}	G	0.20	0.15	0.01	0.00	0.01	0.04	0.01	0.08
Quarterly infl. rate. in S.S.	$\bar{\pi}$	G	0.62	0.10	0.62	0.46	0.77	0.60	0.50	0.69
Inflation response	r_π	N	1.50	0.25	1.46	1.17	1.73	1.65	1.37	1.93
Output gap response	r_y	N	0.12	0.05	0.13	0.08	0.17	0.05	0.02	0.08
Diff. output gap response	$r_{\Delta y}$	N	0.12	0.05	0.23	0.19	0.27	0.15	0.12	0.19
Mon. pol. shock std	σ_r	G	0.20	0.15	0.24	0.22	0.27	0.22	0.20	0.24
Mon. pol. shock pers.	ρ_r	B	0.50	0.20	0.06	0.01	0.10	0.11	0.03	0.19
Interest rate smoothing	ρ_R	B	0.75	0.10	0.87	0.84	0.91	0.90	0.87	0.94
m.e. π_r1	$\sigma_{\pi r1}$	G	0.20	0.15	0.11	0.10	0.12	0.11	0.10	0.12
m.e. dy_r1	σ_{yr1}	G	0.20	0.15	0.21	0.19	0.22	0.21	0.19	0.22
m.e. π_f1	$\sigma_{\pi f1}$	G	0.20	0.15	0.04	0.01	0.06	0.02	0.01	0.04
Learning persistence	φ	U	0.00	1.00				0.97	0.96	0.98
Log marginal likelihood					MCMC	−840.90		MCMC	-790.51	

Note: see Table A1.

Table A4: Prior and posterior distributions 2MU-10obs + inflation objective shock.

Parameter		Prior distribution			RE-10obs Metropolis Chain			AL-10obs Metropolis Chain		
		type	mean	std.dev.	mean	5%	95%	mean	5%	95%
Calvo prob. wages	ξ_w	B	0.50	0.10	0.89	0.86	0.92	0.89	0.86	0.92
Calvo prob. prices	ξ_p	B	0.50	0.10	0.90	0.87	0.93	0.83	0.79	0.87
Indexation wages	ι_w	B	0.50	0.15	0.39	0.18	0.58	0.24	0.08	0.39
Indexation prices	ι_p	B	0.50	0.15	0.11	0.06	0.16	0.06	0.03	0.09
Gross price markup	ϕ_p	N	1.25	0.12	1.43	1.32	1.54	1.50	1.39	1.62
Capital production share	α	N	0.30	0.05	0.19	0.16	0.21	0.19	0.16	0.23
Capital utilization cost	ψ	B	0.50	0.15	0.64	0.47	0.83	0.70	0.56	0.85
Investment adj. cost	φ	N	4.00	1.50	2.04	1.00	2.89	1.47	1.04	1.82
Habit formation	$\varkappa$	B	0.70	0.10	0.52	0.41	0.63	0.50	0.44	0.56
Int elast of subst.cons.	σ_c	N	1.50	0.37	1.29	0.95	1.60	1.50	1.30	1.68
Labor supply elast.	σ_l	N	2.00	0.75	2.35	1.62	3.08	1.90	1.21	2.61
Log hours worked in S.S.	$\bar{l}$	N	0.00	2.00	-0.06	-2.04	1.87	3.20	-0.17	4.87
Discount factor	$100(\beta^{-1}-1)$	G	0.25	0.10	0.19	0.08	0.30	0.24	0.08	0.38
Quarterly Growth in S.S.	$\bar{\gamma}$	N	0.40	0.10	0.34	0.30	0.37	0.39	0.35	0.42
Stationary tech. shock	ρ_a	B	0.50	0.20	0.98	0.97	0.99	0.99	0.98	1.00
Risk premium shock	ρ_b	B	0.50	0.20	0.83	0.75	0.93	0.79	0.73	0.86
Invest. spec. tech. shock	ρ_i	B	0.50	0.20	0.91	0.86	0.97	0.34	0.23	0.45
Gov't cons. shock	ρ_g	B	0.50	0.20	0.99	0.99	0.99	0.99	0.99	1.00
Price markup shock	ρ_{par}	B	0.50	0.20	0.25	0.03	0.46	0.77	0.66	0.91
Wage markup shock	ρ_{war}	B	0.50	0.20	0.40	0.09	0.69	0.41	0.11	0.73
Response of g_t to ε_t^a	ρ_{ga}	B	0.50	0.20	0.63	0.49	0.77	0.67	0.53	0.80
Stationary tech. shock	σ_a	G	0.20	0.15	0.45	0.41	0.50	0.44	0.39	0.48
Risk premium shock	σ_b	G	0.20	0.15	0.10	0.08	0.13	0.12	0.10	0.14
Invest. spec. tech. shock	σ_i	G	0.20	0.15	0.48	0.33	0.62	0.34	0.28	0.39
Gov't cons. shock	σ_g	G	0.20	0.15	0.51	0.47	0.56	0.51	0.46	0.55
Price markup shock-iid	σ_{piid}	G	0.20	0.15	0.24	0.22	0.26	0.26	0.23	0.29
Price markup shock-ar	σ_{par}	G	0.20	0.15	0.06	0.04	0.08	0.04	0.02	0.05
Wage markup shock-iid	σ_{wiid}	G	0.20	0.15	0.42	0.38	0.45	0.39	0.34	0.43
Wage markup shock-ar	σ_{wae}	G	0.20	0.15	0.04	0.00	0.07	0.06	0.00	0.12
Quarterly infl. rate. in S.S.	$\bar{\pi}$	G	0.62	0.10	0.61	0.45	0.77	0.59	0.47	0.71
Inflation response	r_π	N	1.50	0.25	1.62	1.26	1.96	1.78	1.40	2.17
Output gap response	r_y	N	0.12	0.05	0.07	0.02	0.13	0.04	0.01	0.06
Diff. output gap response	$r_{\Delta y}$	N	0.12	0.05	0.22	0.18	0.26	0.16	0.13	0.19
Mon. pol. shock std	σ_r	G	0.20	0.15	0.22	0.19	0.25	0.22	0.20	0.24
Mon. pol. shock pers.	ρ_r	B	0.50	0.20	0.07	0.01	0.10	0.10	0.02	0.18
Interest rate smoothing	ρ_R	B	0.75	0.10	0.91	0.88	0.95	0.90	0.88	0.93
m.e. π_r1	$\sigma_{\pi r1}$	G	0.20	0.15	0.11	0.10	0.12	0.11	0.10	0.12
m.e. dy_r1	σ_{yr1}	G	0.20	0.15	0.21	0.19	0.22	0.21	0.19	0.22
m.e. π_f1	$\sigma_{\pi f1}$	G	0.20	0.15	0.03	0.00	0.06	0.02	0.00	0.04
Learning persistence	φ	U	0.00	1.00				0.97	0.96	0.98
Inflation target shock	$\sigma_{\overline{\pi}}$	G	0.20	0.15	0.12	0.10	0.13	0.01	0.00	0.02
Log marginal likelihood					MCMC		−833.52	MCMC		−803.93

Note: see Table A1.

Table A5: Test statistics for RE-9obs model forecast errors

	average annual inflation forecast		one quarter ahead forecast	
	1969q1-2015q3	1996q1-2015q3	1969q1-2015q3	1996q1-2015q3
persistence: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t} - \pi^{r1}_{t\|t-h}\right)$				
α	-0.02 (-0.45)	-0.11 (-3.09)	-0.02 (-0.68)	-0.06 (-2.34)
β	0.05 (0.24)	-0.42 (-5.02)	-0.09 (-0.59)	-0.44 (-2.02)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t}\right)$				
α	-0.07 (-0.92)	0.27 (-6.59)	-0.01 (-0.14)	0.28 (-3.97)
β	0.06 (0.56)	-0.70 (-7.27)	-0.01 (-0.21)	-0.67 (-4.77)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t}\right) + \gamma\left(\pi^{r1}_{t-1}\right) + \delta\left(r_{t-1}\right)$				
α	-0.18 (-0.26)	0.11 (1.10)	0.12 (1.42)	0.31 (2.51)
β	-0.38 (-2.37)	-0.73 (-5.49)	-0.40 (-1.73)	-0.57 (-2.67)
γ	0.57 (3.05)	0.29 (1.45)	0.24 (1.36)	-0.167 (-0.70)
δ	-0.15 (-2.95)	-0.06 (-1.13)	0.08 (1.91)	0.02 (0.45)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t} - \pi^{r1}_{t\|t-h}\right)$				
α	-0.02 (-0.48)	-0.08 (-2.65)	-0.02 (-0.82)	-0.07 (-2.26)
β	0.33 (1.80)	0.19 (1.20)	-0.03 (-0.30)	-0.27 (-1.66)

Note: see Table A5.

Table A6: Test statistics for AL-9obs model forecast errors

	average annual inflation forecast		one quarter ahead forecast	
	1969q1-2015q3	1996q1-2015q3	1969q1-2015q3	1996q1-2015q3
persistence: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t} - \pi^{r1}_{t\|t-h}\right)$				
α	-0.02 (-0.43)	-0.06 (-1.51)	-0.02 (-0.57)	-0.04 (-1.39)
β	0.18 (0.68)	0.06 (0.30)	-0.04 (-0.35)	-0.33 (-0.91)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t}\right)$				
α	-0.10 (1.27)	0.27 (5.65)	0.05 (0.98)	0.27 (4.52)
β	-0.13 (-1.58)	-0.68 (-5.59)	-0.08 (-1.31)	-0.65 (-4.91)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t}\right) + \gamma\left(\pi^{r1}_{t-1}\right) + \delta\left(r_{t-1}\right)$				
α	-0.10 (-0.26)	0.02 (0.19)	0.18 (2.01)	0.26 (1.57)
β	-0.65 (-4.39)	-0.31 (-1.94)	-0.61 (-2.45)	-0.39 (-0.95)
γ	0.73 (3.77)	-0.13 (-0.65)	0.39 (1.99)	-0.26 (-0.83)
δ	-0.15 (-3.03)	-0.17 (-3.16)	0.09 (1.85)	-0.01 (-0.08)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t} - \pi^{r1}_{t\|t-h}\right)$				
α	-0.02 (-0.54)	-0.04 (-1.10)	-0.02 (-0.73)	-0.05 (-1.42)
β	0.37 (1.80)	0.29 (1.96)	0.00 (0.01)	-0.19 (-1.06)

Note: see Table A5.

Table A7: Test statistics for RE-2MU-10obs model forecast errors

	average annual inflation forecast		one quarter ahead forecast	
	1969q1-2015q3	1996q1-2015q3	1969q1-2015q3	1996q1-2015q3
persistence: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t} - \pi^{r1}_{t\|t-h}\right)$				
α	-0.03 (-1.04)	-0.04 (-1.46)	-0.03 (-1.37)	-0.03 (-1.35)
β	0.48 (2.28)	0.01 (0.06)	0.53 (2.58)	-0.18 (-0.32)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t}\right)$				
α	-0.04 (-0.73)	0.07 (-1.00)	-0.07 (-1.62)	-0.01 (-0.10)
β	0.01 (0.12)	-0.24 (-1.41)	0.05 (0.85)	-0.03 (-0.14)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t}\right) + \gamma\left(\pi^{r1}_{t-1}\right) + \delta\left(r_{t-1}\right)$				
α	-0.20 (-3.62)	-0.03 (-0.29)	-0.11 (-2.25)	-0.06 (-0.33)
β	0.29 (1.79)	-0.18 (-1.08)	0.16 (1.47)	0.34 (1.18)
γ	-0.07 (-0.57)	0.11 (0.67)	-0.06 (-0.70)	-0.29 (-1.97)
δ	-0.14 (-3.72)	-0.06 (-1.11)	-0.03 (-1.12)	0.00 (0.09)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t} - \pi^{r1}_{t\|t-h}\right)$				
α	-0.03 (-0.92)	-0.03 (-1.07)	-0.03 (-1.21)	-0.04 (-1.38)
β	0.29 (1.80)	0.23 (1.49)	0.09 (0.79)	-0.25 (-1.33)

Note: see Table A5.

Table A8: Test statistics for AL-2MU-10obs model forecast errors

	average annual inflation forecast		one quarter ahead forecast	
	1969q1-2015q3	1996q1-2015q3	1969q1-2015q3	1996q1-2015q3
persistence: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t} - \pi^{r1}_{t\|t-h}\right)$				
α	-0.04 (-1.32)	-0.05 (-2.01)	-0.05 (-2.93)	-0.05 (-2.27)
β	0.19 (1.16)	-0.13 (-1.24)	0.29 (1.72)	-0.12 (-0.56)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t}\right)$				
α	0.03 (0.66)	0.15 (3.81)	-0.04 (-1.13)	0.06 (0.94)
β	-0.08 (-1.22)	-0.42 (-4.69)	-0.01 (-0.30)	-0.23 (-1.65)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t}\right) + \gamma\left(\pi^{r1}_{t-1}\right) + \delta\left(r_{t-1}\right)$				
α	-0.05 (-0.69)	0.04 (0.49)	-0.05 (-1.04)	-0.01 (-0.11)
β	-0.18 (-1.55)	-0.22 (-2.46)	-0.02 (-0.21)	-0.02 (-0.10)
γ	0.20 (1.37)	-0.09 (-0.74)	0.01 (0.17)	-0.14 (-0.92)
δ	-0.08 (-1.75)	-0.10 (-2.42)	-0.00 (-0.06)	-0.05 (-1.24)
predictability: $\left(\pi^{r1}_{t+h} - \pi^{r1}_{t+h\|t}\right) = \alpha + \beta\left(\pi^{r1}_{t+h\|t} - \pi^{r1}_{t\|t-h}\right)$				
α	-0.04 (-1.18)	-0.04 (-1.74)	-0.05 (-2.46)	-0.05 (-2.32)
β	0.01 (0.08)	0.13 (0.91)	0.09 (1.04)	-0.11 (-0.70)

Note: see Table A5.

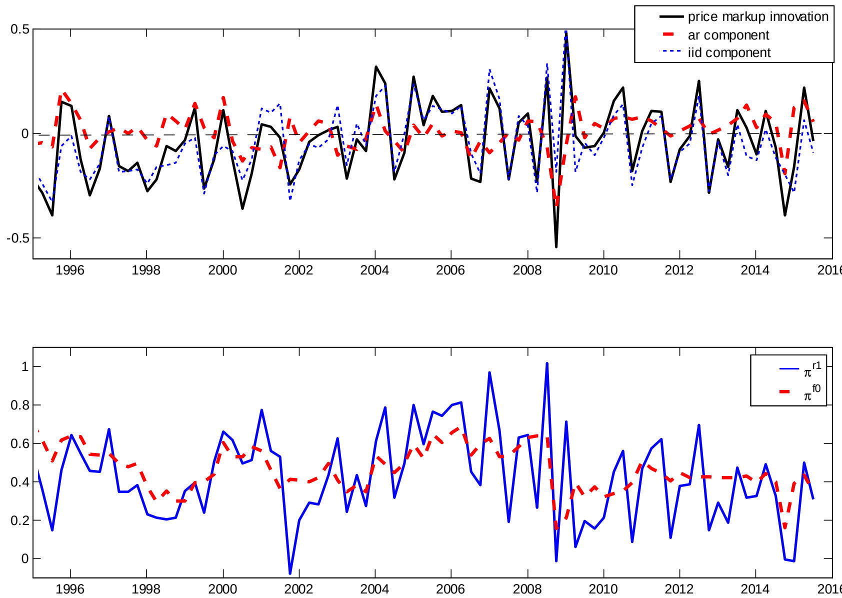

Figure A1 : Identification of markup shock and survey forecast
Note: Upper panel contains smoothed estimates of markup innovations from AL-2MU-10obs model. Lower panel contains first release of inflation (π_t^{r1}) and one-period ahead SPF forecast ($\pi_{t+h|t}^{f0}$).