Materials 5c - Evening forecasts

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October 17, 2019

Define some objects: *(I usually let t denote the time in which the variable is formed.)*

$$f_t = \hat{\mathbb{E}}_t(z_{t+1}) \quad \text{one-period-ahead forecast formed at time } t \tag{1}$$

$$FE_t = z_{t+1} - f_t \quad \text{one-period-ahead forecast error realized at time } t+1 \tag{2}$$

$$= ALM(t+1) - PLM(t) \tag{3}$$

$$\theta_t = \hat{\mathbb{E}}_{t-1}(z_t) - \mathbb{E}_{t-1}(z_t) \quad \text{CEMP's criterion} \tag{4}$$

$$= PLM(t-1) - \mathbb{E}_{t-1}\, ALM(t) \tag{5}$$

$PLM(t) : \hat{\mathbb{E}}_t z_{t+1} = \bar{z}_{t-1} + b s_t$

Morning: morning of time t available: $\mathcal{I}_t^m = \{\bar{z}_{t-1}, s_t, k_{t-1}, FE_{t-2}\}$

1. Form all future expectations using $PLM(t) \to z_t$ realized, $\to FE_{t-1}$ realized
2. Form $\theta_t \to k_t$ realized
3. **Evening**: Update $\bar{z}_t = \bar{z}_{t-1} + k_t^{-1}(FE_{t-1})$

   where $FE_{t-1} = z_t - f_{t-1} = z_t - (\bar{z}_{t-2} + b s_{t-1})$, so:

   $\bar{z}_t = \bar{z}_{t-1} + k_t^{-1}(z_t - (\bar{z}_{t-2} + b s_{t-1}))$

   $\to$ evening of time t available: $\mathcal{I}_t^e = \{\bar{z}_t, s_t, k_t, FE_{t-1}\}$

Issue # 1: Updating of $\bar{z}$ is a function of last period's $\bar{z}$, $\bar{z}_{t-2}$, (i.e. not the one available to use this morning). The formulation I've had so far, updating based on $\bar{z}_{t-1} + b s_{t-1}$ is what I've called an "assessment forecast": it's yesterday evening's forecast that is distinct from yesterday morning's forecast. Is that legitimate?

The second issue will be about the criterion. Recall:

$$
\begin{aligned}
\theta_t &= \hat{\mathbb{E}}_{t-1}(z_t) - \mathbb{E}_{t-1}(z_t) \\
&= PLM(t-1) - \mathbb{E}_{t-1}\, ALM(t)
\end{aligned}
$$

Recall: $PLM(t) : \hat{\mathbb{E}}_t z_{t+1} = \bar{z}_{t-1} + bs_t$

$$
\begin{aligned}
ALM_t &= \text{stuff} \times \bar{z}_{t-1} + \text{stuff} \times s_t \\
\theta_t &= \bar{z}_{t-2} + bs_{t-1} - \mathbb{E}_{t-1}(\text{stuff} \times \bar{z}_{t-1} + \text{stuff} \times s_t)
\end{aligned}
$$

Issue #2: I had this issue before, but it's not clear what the RE of $\bar{z}$ is. In particular, I don't know what the index of $\mathbb{E}_{t-1}$ refers to: the morning of $t-1$ or the evening?

- If it's the morning, then $\mathbb{E}_{t-1}(\bar{z}_{t-1}) = \bar{z}_{t-2}$

  $\rightarrow \theta_t = \mathcal{F}(\bar{z}_{t-2}, s_{t-1})$ where $\mathcal{F}$ denotes "function"

- If it's the evening, then $\mathbb{E}_{t-1}(\bar{z}_{t-1}) = \bar{z}_{t-1}$

  $\rightarrow \theta_t = \mathcal{F}(\bar{z}_{t-2}, \bar{z}_{t-1}, s_{t-1})$

The "evening" assumption isn't cool because the criterion depends on the intercept at several time periods, the "morning" assumption isn't cool because just like in Issue #1, we need access to yesterday morning's estimate of the intercept.

What I'm doing right now is $\theta_t = \mathcal{F}(\bar{z}_{t-1}, s_{t-1})$ which amounts to assuming that both expectations, $\hat{\mathbb{E}}_{t-1}, \mathbb{E}_{t-1}$, are taken with respect to the information set of $t-1$ evening, $\mathcal{I}^e_{t-1} = \{\bar{z}_{t-1}, s_{t-1}, \dots\}$. Again: legitimate?

And actually, going back to CEMP reveals that they aren't consequent either:

$$\pi_t = \gamma\pi_{t-1} + (1-\gamma)\bar{\pi}_t + \rho\varphi_{t-1} \quad \text{PLM, i.e. } \hat{\mathbb{E}}_{t-1}\pi_t$$

$\rightarrow$ Clearly $\bar{\pi}_t$ is formed at time $t-1$ morning, before evaluation of the PLM, or $t-2$ evening

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1}(f_{t-1})$$

$\rightarrow$ This means k_t too is formed at $t-1$ morning, or $t-2$ evening

$$k_t = \mathbb{I}_{\theta_t \leq \bar{\theta}}(k_{t-1}+1) + (1-\mathbb{I}_{\theta_t \leq \bar{\theta}})\bar{g}^{-1}$$

$\rightarrow$ This means θ_t too is formed at $t-1$ morning, or $t-2$ evening

$$\theta_t = |\hat{\mathbb{E}}_{t-1}\pi_t - \mathbb{E}_{t-1}\,\pi_t|$$

But θ_t is a function of $\hat{\mathbb{E}}_{t-1}\pi_t$, which we haven't evaluated yet!

$$f_{t-1} = \pi_{t-1} - \hat{\mathbb{E}}_{t-2}\pi_{t-1}$$

Note also that this FE corresponds to my FE_{t-2}.

An alternative timing for the CEMP world is that all of the above takes place at time t, not $t-1$ (so agents are forming $\hat{\mathbb{E}}_t\pi_t$ - which is weird...)

$$\pi_t = \gamma\pi_{t-1} + (1-\gamma)\bar{\pi}_t + \rho\varphi_{t-1} \quad \text{PLM, i.e. } \hat{\mathbb{E}}_{t-1}\pi_t$$

$\rightarrow$ Now $\bar{\pi}_t$ is formed at time t morning, before evaluation of the PLM, or $t-1$ evening

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1}(f_{t-1})$$

$\rightarrow$ This means k_t too is formed at t morning, or $t-1$ evening

$$k_t = \mathbb{I}_{\theta_t \leq \bar{\theta}}(k_{t-1}+1) + (1-\mathbb{I}_{\theta_t \leq \bar{\theta}})\bar{g}^{-1}$$

$\rightarrow$ This means θ_t too is formed at t morning, or $t-1$, after PLM $\hat{\mathbb{E}}_{t-1}\pi_t$ was formed

$$\theta_t = |\hat{\mathbb{E}}_{t-1}\pi_t - \mathbb{E}_{t-1}\,\pi_t|$$

$$f_{t-1} = \pi_{t-1} - \hat{\mathbb{E}}_{t-2}\pi_{t-1}$$

In this case f_{t-1} makes sense, it just raises Issue #1.