

# Materials 33 - Estimating the anchoring function

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## 1 Estimation procedure

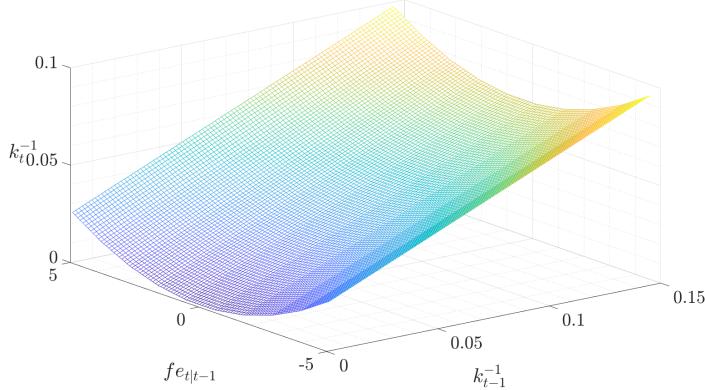
Instead of the AR(1) anchoring function used so far (Equation A.6), I use the following equation

$$k_t^{-1} = \alpha s(X) \quad (1)$$

where  $X = (k_{t-1}^{-1}, fe_{t|t-1})$  and I use piecewise linear interpolation. Now I initialize  $\alpha_0$  by specifying a grid for  $X$ , passing the grid through Equation (A.6) to generate  $k_t^{-1}$ -values, and approximating by fitting the grid to the  $k_t^{-1}$ -values. This means that if the functional relationship in Fig. 1 is the right one, then the AR(1) specification of Equation (A.6) isn't a bad description of the anchoring function.

Then I estimate  $\alpha$  using GMM, targeting the autocovariance structure of inflation, the output gap and the nominal interest rate (federal funds rate) in the data.

**Figure 1:** Initialization via Equation (A.6) implies this functional relationship



$T = 233$  before BK-filtering,  $T = 209$  after BK-filtering. Using the “constant-only, inflation-only” learning PLM. I drop the  $ndrop = 5$  initial values.

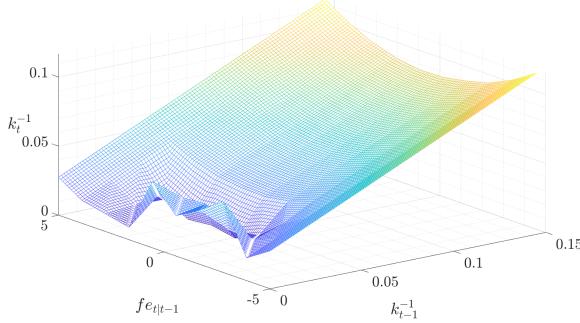
## 2 Estimation issues

1. Different bounds on  $\alpha$  yield different estimates, see below.

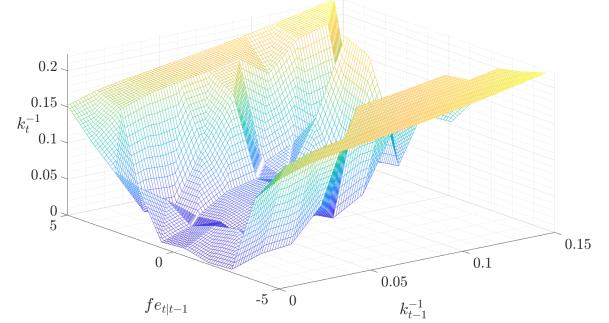
2. The estimation is also very sensitive to the choice of the grids for the gain and for forecast errors. For example, choosing  $n_{grid} = 11$  instead of 10 leads to massively different estimates. In particular, odd vs. even seems to matter.

3. Given  $\hat{\alpha}$ , PEA has a negative gain problem. I don't understand how!

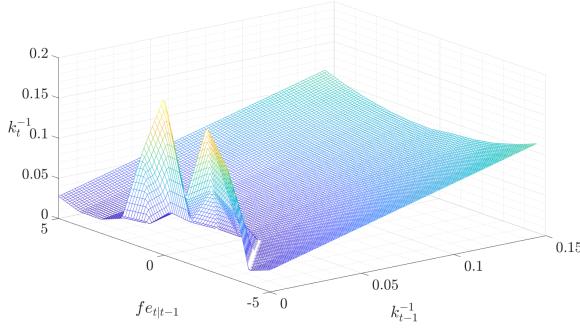
**Figure 2:**  $k_t^{-1}$  as a function of  $k_{t-1}^{-1}$  and  $\tilde{f}e_{t|t-1}$  given  $\hat{\alpha}^{GMM}$



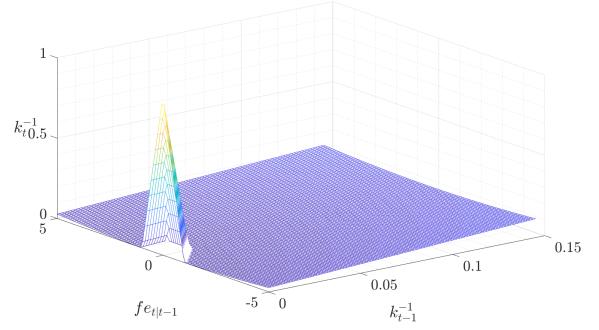
(a) Coefficients unrestricted,  $n_{grid} = 10$



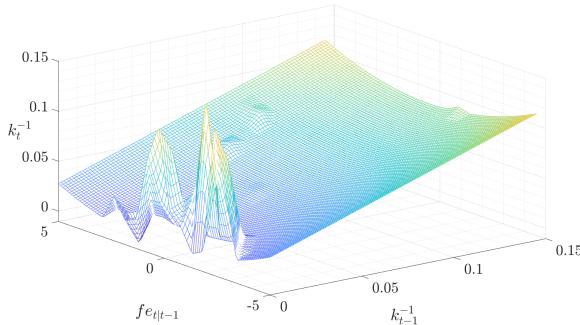
(b) Coefficients restricted  $> 0$ ,  $n_{grid} = 10$



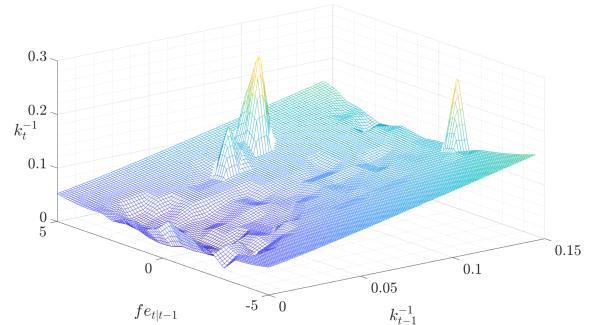
(c) Coefficients unrestricted,  $n_{grid} = 11$



(d) Coefficients restricted  $> 0$ ,  $n_{grid} = 11$



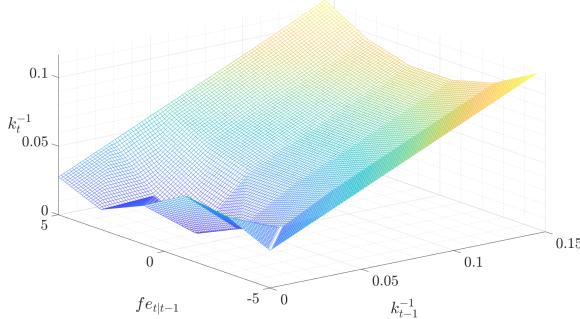
(e) Coefficients unrestricted,  $n_{grid} = 20$ . This actually goes negative!



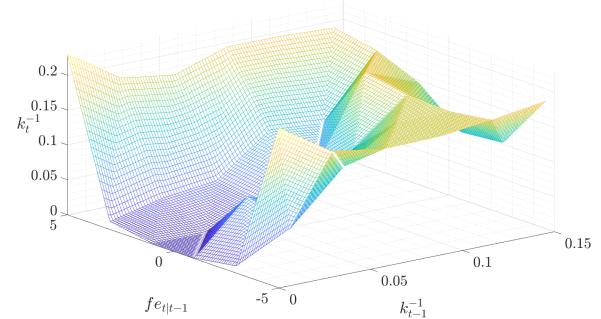
(f) Coefficients restricted  $> 0$ ,  $n_{grid} = 20$

I thought of something: I have  $ngrid^2$  parameters and 45 moments. This means my grid can at most have 6 elements (or I need more autocovariance lags to have more moments).

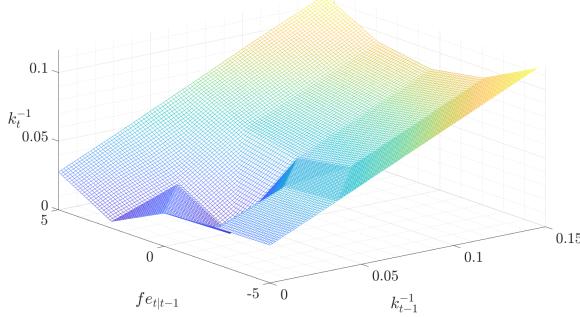
**Figure 3:**  $k_t^{-1}$  as a function of  $k_{t-1}^{-1}$  and  $fe_{t|t-1}$  given  $\hat{\alpha}^{GMM}$



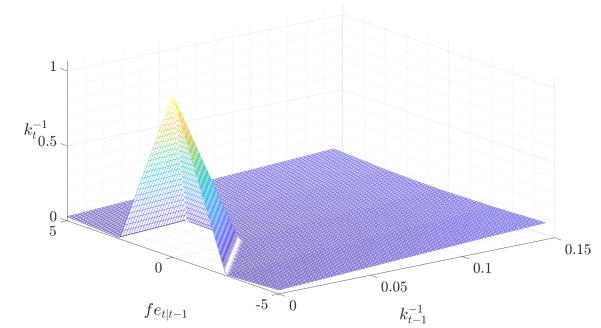
(a) Coefficients unrestricted,  $n_{grid} = 6$



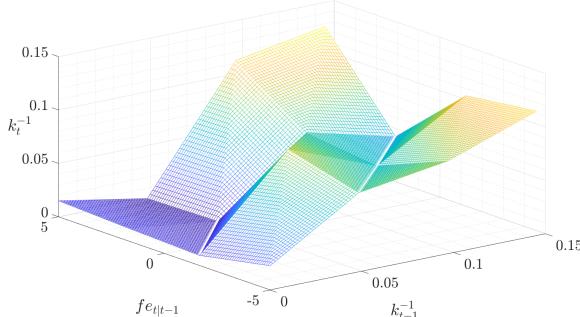
(b) Coefficients restricted  $> 0$ ,  $n_{grid} = 6$



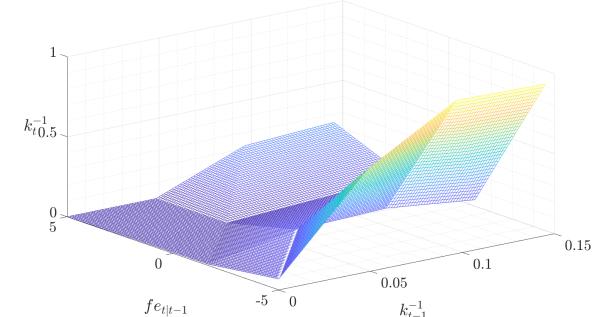
(c) Coefficients unrestricted,  $n_{grid} = 5$



(d) Coefficients restricted  $> 0$ ,  $n_{grid} = 5$



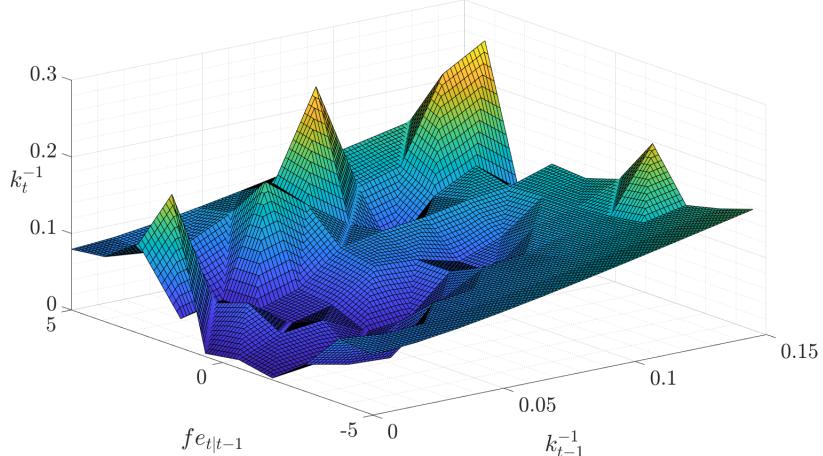
(e) Coefficients unrestricted,  $n_{grid} = 4$ .



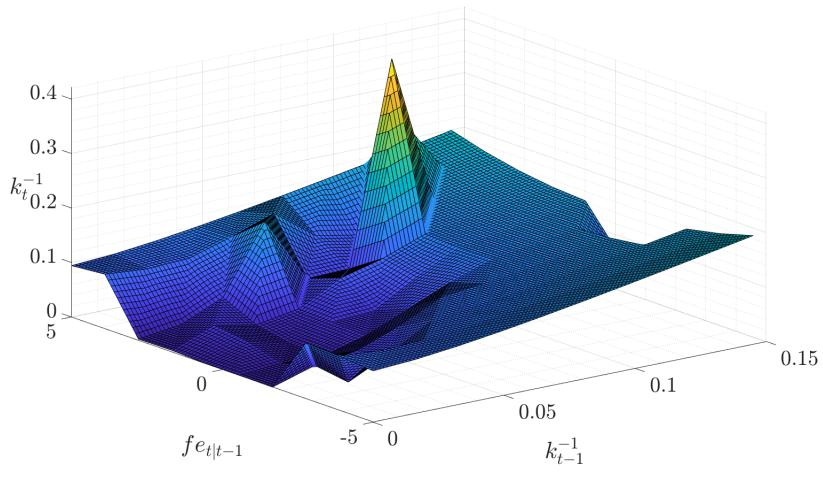
(f) Coefficients restricted  $> 0$ ,  $n_{grid} = 4$

### 3 Older plots for me

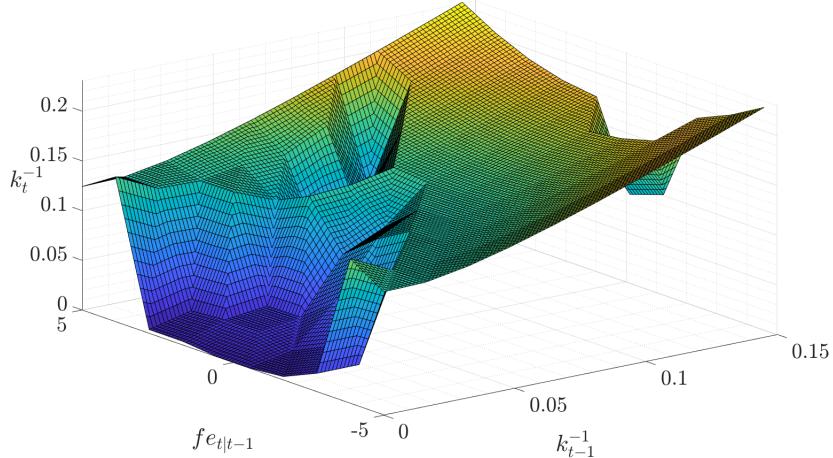
**Figure 4:**  $k_t^{-1}$  as a function of  $k_{t-1}^{-1}$  and  $fe_{t|t-1}$  given  $\hat{\alpha}^{GMM} > 0$



(a)  $N_{burn} = 0$

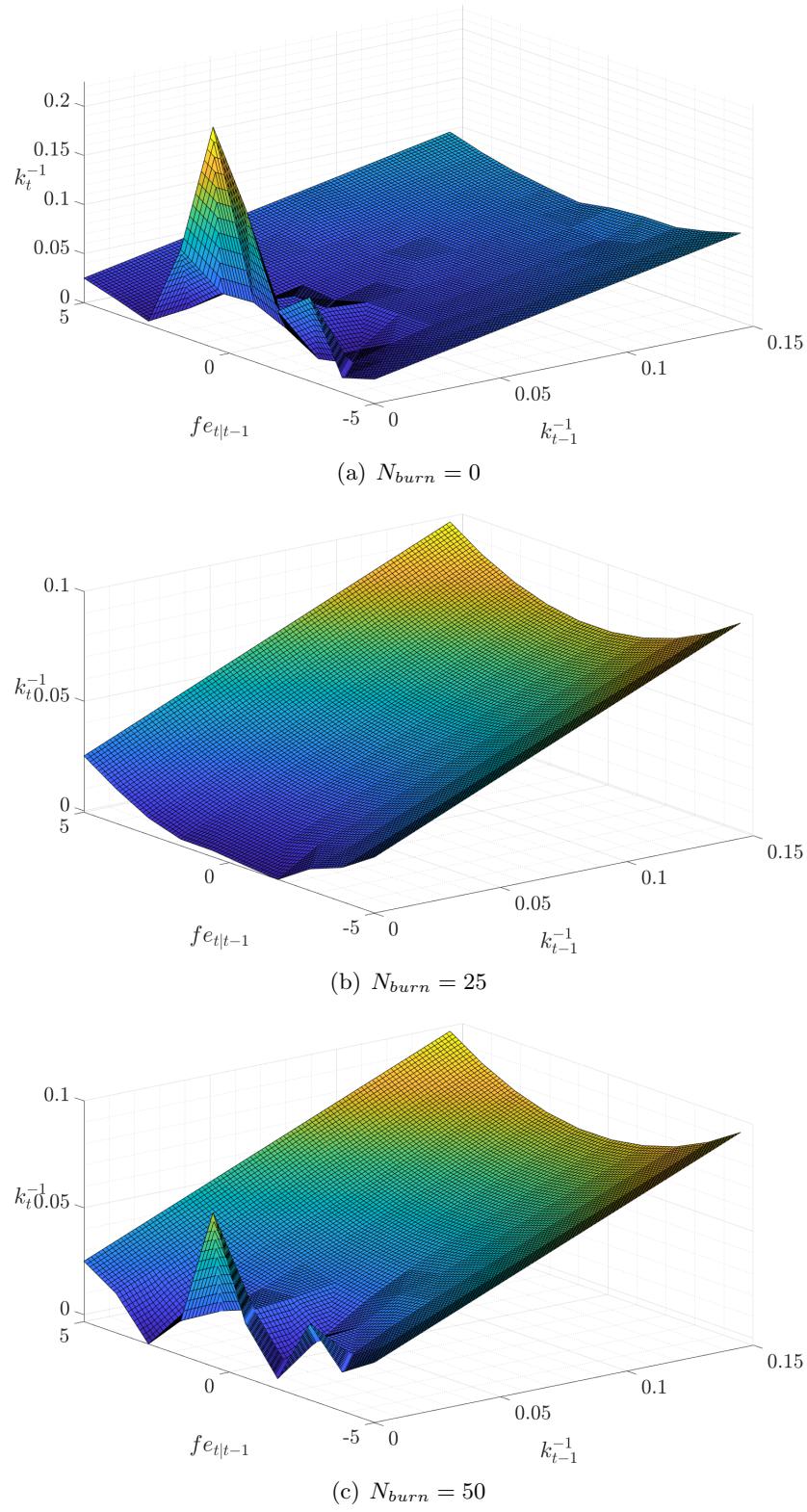


(b)  $N_{burn} = 25$

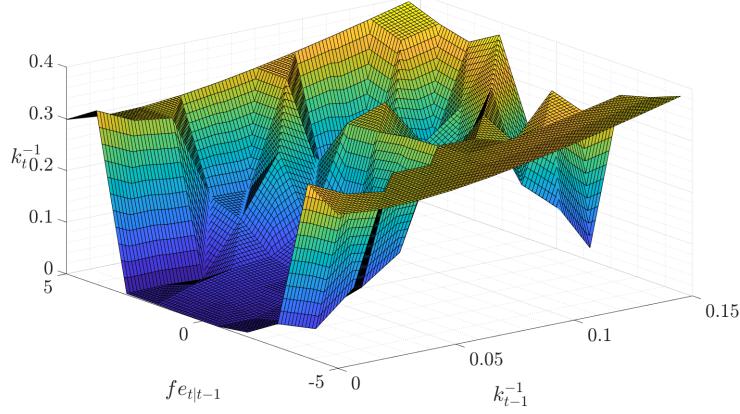


(c)  $N_{burn} = 50$

**Figure 5:**  $k_t^{-1}$  as a function of  $k_{t-1}^{-1}$  and  $fe_{t|t-1}$  given  $\hat{\alpha}^{GMM}$  (not restricted  $> 0$ )

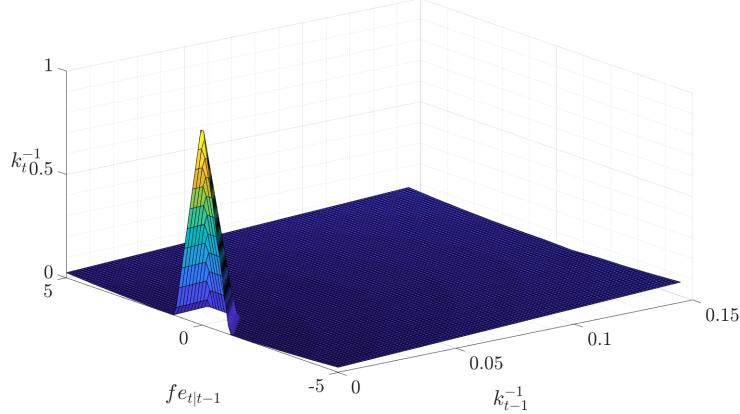


**Figure 6:**  $\min(k^{-1}) = 0$  instead of 0.00001



$N_{burn} = 50$ , PLM = constant only, inflation only

**Figure 7:**  $n_{grid}=11$



$\min(k^{-1}) = 0, \alpha > 0, N_{burn} = 50$ , constant-only, inflation-only PLM

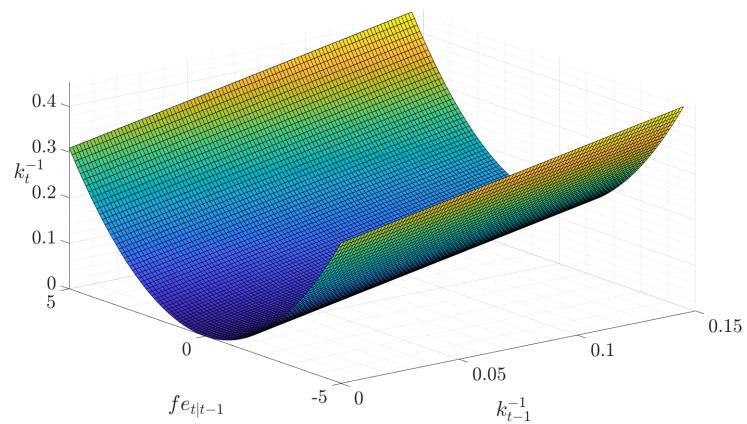
## 4 Is the estimated LOM gain in the nonpermissible range for PEA?

If I fit an AR(1) (Equation A.6) to the estimated gain evolution, I obtain  $(\rho_k, \gamma_k) = (1, 0.0123)$  which results in the following relationship:

But these are very high values; the model isn't E-stable for such high values. Rerunning PEA with the AR(1) specification and the values  $(1, 0.0123)$  is able to solve, but involves a gain that's increasing over time!

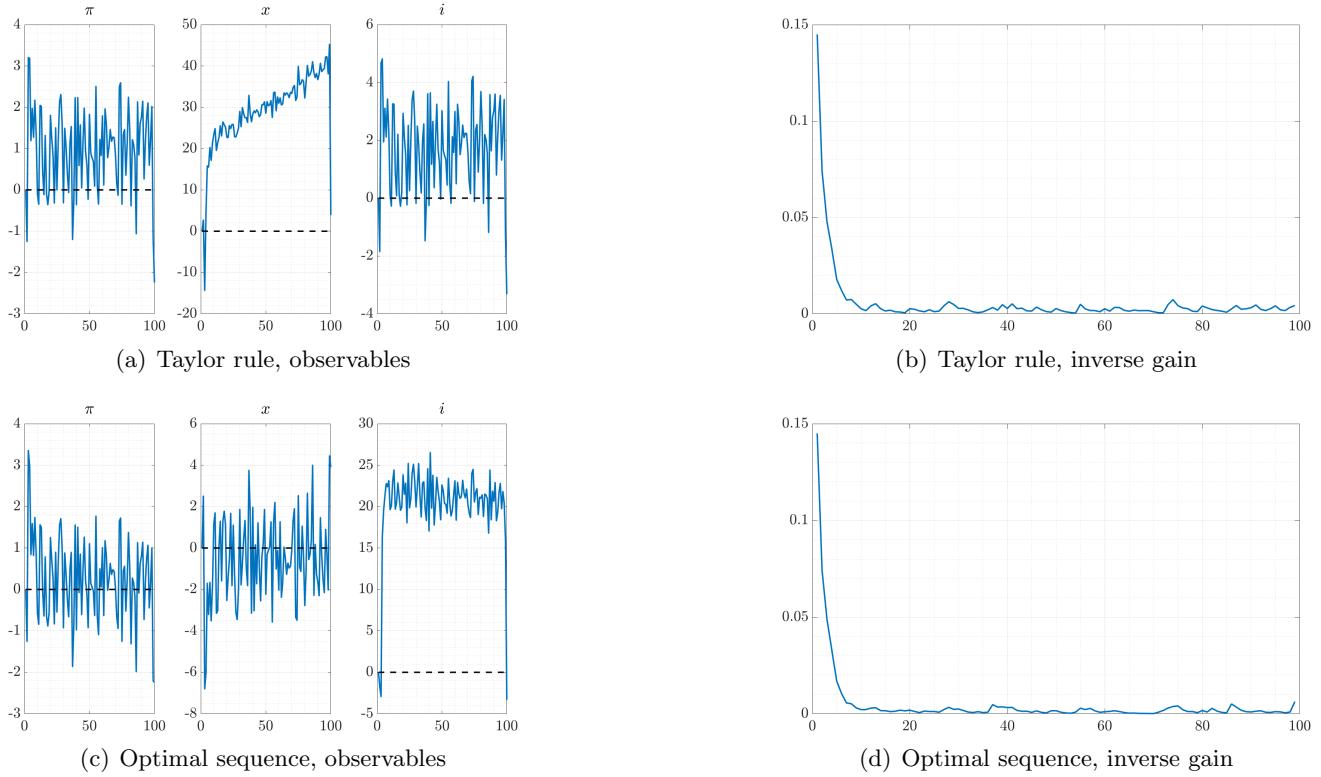
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**Figure 8:** AR(1) approximation to the estimated functional form of the gain

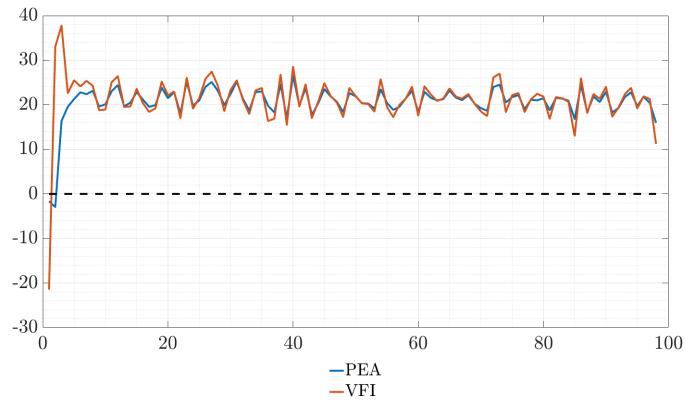


## 5 PEA plots with constant only, $\pi$ only learning

**Figure 9**



**Figure 10:** VFI-PEA comparison using the “constant only,  $\pi$  only” PLM



## A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (\text{A.1})$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (\text{A.2})$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \quad (\text{if imposed}) \quad (\text{A.3})$$

$$\text{PLM: } \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \quad b = g_x \ h_x \quad (\text{A.4})$$

$$\text{Updating: } a_t = a_{t-1} + k_t^{-1} (z_t - (a_{t-1} + b s_{t-1})) \quad (\text{A.5})$$

$$\text{Anchoring function: } k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k f e_{t-1}^2 \quad (\text{A.6})$$

$$\text{Forecast error: } f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \quad (\text{A.7})$$

$$\text{LH expectations: } f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \quad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \quad (\text{A.8})$$

This notation captures vector learning ( $z$  learned) for intercept only. For scalar learning,  $a_t = (\bar{\pi}_t \ 0 \ 0)'$  and  $b_1$  designates the first row of  $b$ . The observables  $(\pi, x)$  are determined as:

$$x_t = -\sigma i_t + [\sigma \ 1-\beta \ -\sigma\beta] f_b + \sigma [1 \ 0 \ 0] (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \quad (\text{A.9})$$

$$\pi_t = \kappa x_t + [(1-\alpha)\beta \ \kappa\alpha\beta \ 0] f_a + [0 \ 0 \ 1] (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \quad (\text{A.10})$$

## B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only,  $k_t^{-1} = \mathbf{g}(f e_{t-1})$ ):

$$\begin{aligned} \pi_t &= -\frac{\lambda_x}{\kappa} \left\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_\pi(t) \right) \right. \\ &\quad \left. \left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\bar{\pi}}(t+j)) \right) \right\} \end{aligned} \quad (\text{B.1})$$

where I'm using the notation that  $\prod_{j=0}^0 \equiv 1$ . For interpretation purposes, let me rewrite this as follows:

$$\begin{aligned} \pi_t &= -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ &\quad - \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j}^{-1} + f e_{t+1+j|t+j}^{eve} \mathbf{g}_{\bar{\pi}}(t+j)) \right) \end{aligned} \quad (\text{B.2})$$

Interpretation: tradeoffs from discretion in RE + effect of current level and change of the gain on future tradeoffs + effect of future expected levels and changes of the gain on future tradeoffs