

Materials 9 - Still the overshooting

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1 Model summary with interest rate smoothing ρi_{t-1}

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (1)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (2)$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \rho i_{t-1} + \bar{i}_t \quad (3)$$

$$\hat{\mathbb{E}}_t z_{t+h} = \begin{bmatrix} \bar{\pi}_{t-1} \\ 0 \\ 0 \end{bmatrix} + b h x^{h-1} s_t \quad \forall h \geq 1 \quad b = g x \quad h x \quad \text{PLM} \quad (4)$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \underbrace{(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))}_{\text{fcst error using (4)}} \quad (b_1 \text{ is the first row of } b) \quad (5)$$

$$k_t = \mathbb{I} \times (k_{t-1} + 1) + (1 - \mathbb{I}) \times \bar{g}^{-1} \quad (6)$$

$$\mathbb{I} = \begin{cases} 1 & \text{if } \theta_t \leq \bar{\theta} \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

$$\theta_t = |\hat{\mathbb{E}}_{t-1} \pi_t - \mathbb{E}_{t-1} \pi_t| / \sigma_s \quad \text{CEMP criterion for the gain} \quad (8)$$

The alternative criterion for the choice of gain is a recursive variant of the CUSUM-test (Brown, Durbin, Evans 1975):

1. Let FE_t denote the short-run forecast error, and ω_t firms' estimate of the FE variance.
2. Let $\kappa \in (0, 1)$ and $\tilde{\theta}$ be the new threshold value for the criterion.
3. Then for initial (ω_0, θ_0) , firms in every period estimate the criterion and the FEV as:

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1} (FE_t^2 - \omega_{t-1}) \quad (9)$$

$$\theta_t = \theta_{t-1} + \kappa k_{t-1}^{-1} (FE_t^2 / \omega_t - \theta_{t-1}) \quad (10)$$

$$k_t = \mathbb{I} \times (k_{t-1} + 1) + (1 - \mathbb{I}) \times \bar{g}^{-1} \quad (11)$$

$$\mathbb{I} = 1 \quad \text{if } \theta_t \leq \tilde{\theta} \quad (12)$$

2 Compact notation - with lagged interest rate term in TR

$$z_t = A_p^{RE} \mathbb{E}_t z_{t+1} + A_s^{RE} s_t \quad (13)$$

$$z_t = A_a^{LH} f_a(t) + A_b^{LH} f_b(t) + A_s^{LH} s_t \quad (14)$$

$$s_t = P s_{t-1} + \epsilon_t \quad \rightarrow \quad s'_t = hx s'_{t-1} + \epsilon'_t \quad (15)$$

$$\text{where } s'_t \equiv \begin{pmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{pmatrix} \quad hx \equiv \begin{pmatrix} \rho_r & 0 & 0 & 0 \\ 0 & \rho_i & 0 & 0 \\ 0 & 0 & \rho_u & 0 \\ gx_{3,1} & gx_{3,2} & gx_{3,3} & gx_{3,4} \end{pmatrix} \quad \epsilon'_t \equiv \begin{pmatrix} \varepsilon_t^r \\ \varepsilon_t^i \\ \varepsilon_t^u \\ 0 \end{pmatrix} \quad \text{and} \quad \Sigma' = \begin{pmatrix} \sigma_r & 0 & 0 & 0 \\ 0 & \sigma_i & 0 & 0 \\ 0 & 0 & \sigma_u & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (16)$$

i_{t-1} is an endogenous state and breaks the link that previously had $P = hx$; now this is no longer true. In particular, using Matlabby notation, $P = hx(1 : 3, 1 : 3)$.

And the A_s^{RE} and A_s^{LH} are given by:

$$A_s^{RE} = \begin{pmatrix} \frac{\kappa\sigma}{w} & -\frac{\kappa\sigma}{w} & 1 - \frac{\kappa\sigma\psi_\pi}{w} & 0 \\ \frac{\sigma}{w} & -\frac{\sigma}{w} & -\frac{\sigma\psi_\pi}{w} & 0 \\ \psi_x(\frac{\sigma}{w}) + \psi_\pi(\frac{\kappa\sigma}{w}) & \psi_x(-\frac{\sigma}{w}) + \psi_\pi(-\frac{\kappa\sigma}{w}) + 1 & \psi_x(-\frac{\sigma\psi_\pi}{w}) + \psi_\pi(1 - \frac{\kappa\sigma\psi_\pi}{w}) & \rho \end{pmatrix} \quad (17)$$

$$A_s^{LH} = \begin{pmatrix} g_{\pi s} \\ g_{xs} \\ \psi_\pi g_{\pi s} + \psi_x g_{xs} + \begin{bmatrix} 0 & 1 & 0 & \rho \end{bmatrix} \end{pmatrix} \quad (18)$$

$$g_{\pi s} = (1 - \frac{\kappa\sigma\psi_\pi}{w}) \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} (I_4 - \alpha\beta hx)^{-1} - \frac{\kappa\sigma}{w} \begin{bmatrix} -1 & 1 & 0 & \rho \end{bmatrix} (I_4 - \beta hx)^{-1} \quad (19)$$

$$g_{xs} = \frac{-\sigma\psi_\pi}{w} \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} (I_4 - \alpha\beta hx)^{-1} - \frac{\sigma}{w} \begin{bmatrix} -1 & 1 & 0 & \rho \end{bmatrix} (I_4 - \beta hx)^{-1} \quad (20)$$

3 Recap of timing

Define some objects: (*I usually let t denote the time in which the variable is formed.*)

$$f_t^j = \hat{\mathbb{E}}_t(z_{t+1}) \quad \text{one-period-ahead forecast formed at time } t, j = m, e \text{ (morning or evening)} \quad (21)$$

$$FE_t = z_{t+1} - f_t \quad \text{one-period-ahead forecast error realized at time } t + 1 \quad (22)$$

$$= ALM(t + 1) - PLM(t) \quad (23)$$

$$\theta_t = \hat{\mathbb{E}}_{t-1}(z_t) - \mathbb{E}_{t-1}(z_t) \quad \text{CEMP's criterion} \quad (24)$$

$$= PLM(t - 1) - \mathbb{E}_{t-1} ALM(t) \quad (25)$$

$$PLM(t) : \hat{\mathbb{E}}_t z_{t+1} = \bar{z}_{t-1} + bs_t$$

Morning: morning of time t available: $\mathcal{I}_t^m = \{\bar{z}_{t-1}, s_t, k_{t-1}, FE_{t-2}\}$

1. Form all future expectations using $PLM(t)$ (morning forecast) $\rightarrow z_t$ realized, $\rightarrow FE_{t-1}$ realized
2. Form $\theta_t \rightarrow k_t$ realized
3. **Evening:** Update $\bar{z}_t = \bar{z}_{t-1} + k_t^{-1}(FE_{t-1}^e)$

where $FE_{t-1}^e = z_t - f_{t-1}^e = z_t - (\bar{z}_{t-1} + bs_{t-1})$ is the most recent realized FE, so:

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1}(z_t - (\bar{z}_{t-1} + bs_{t-1}))$$

\rightarrow evening of time t available: $\mathcal{I}_t^e = \{\bar{z}_t, s_t, k_t, FE_{t-1}\}$

4 Current set of baseline parameters

β	0.99	stochastic discount factor	standard (Woodford 2003/2011)
σ	1	IES	consistent with balanced growth
α	0.5	Calvo probability of not adjusting	match 6-month duration of prices (can increase to 0.75)
ψ_π	1.5	coefficient of inflation in Taylor rule	Taylor
ψ_x	0	coefficient of output gap in Taylor rule	focus on π
\bar{g}	0.145	value of the constant gain	CEMP
$\bar{\theta}$	1	threshold deviation between $\hat{\mathbb{E}}$ & \mathbb{E}	CEMP: 0.029
ρ_r	0	persistence of natural rate shock	n.a.
ρ_i	0.6	persistence of monetary policy shock	CEMP: 0.877 (can increase to 0.78 if $\alpha = 0.75$)
ρ_u	0	persistence of cost-push shock	CEMP
σ_r	0.1	standard deviation of natural rate shock	n.a.
σ_i	0.359	standard deviation of mon. policy shock	CEMP
σ_u	0.277	standard deviation of cost-push shock	CEMP
θ	10	price elasticity of demand	Woodford 2003/2011, Chari, Kehoe & McGrattan 2000
ω	1.25	elasticity of marginal cost to output	Woodford 2003/2011, Chari, Kehoe & McGrattan 2000

5 Cross-sectional IRFs, mon. pol shock only, cgain & dgain only ◀

Figure 1: IRF for observables, shock imposed at t

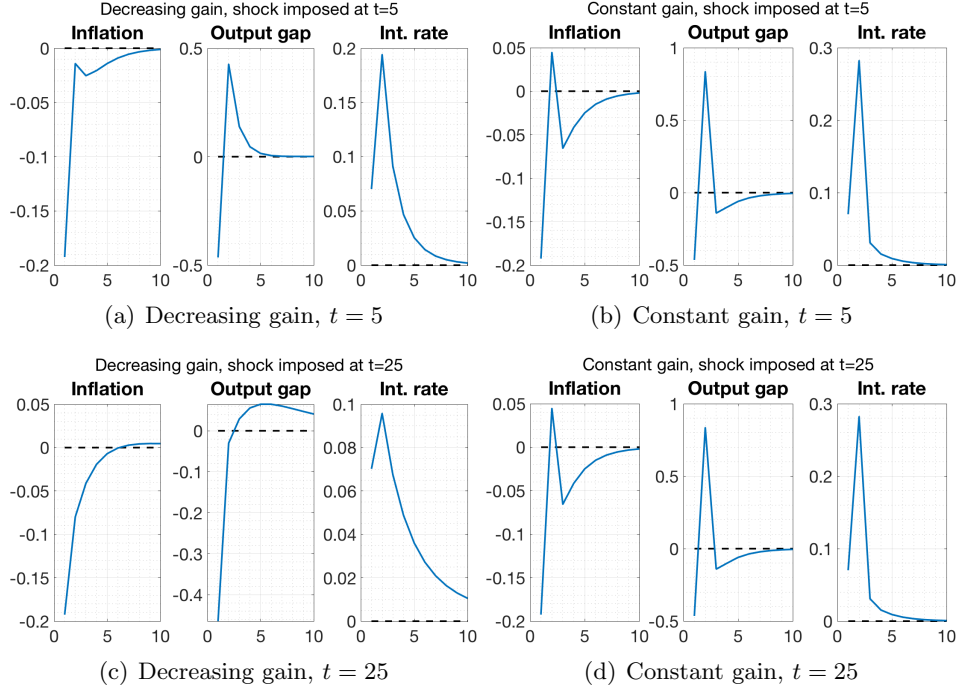


Figure 2: IRF for 1-period ahead forecasts and FEs, together, morning and evening, shock imposed at t

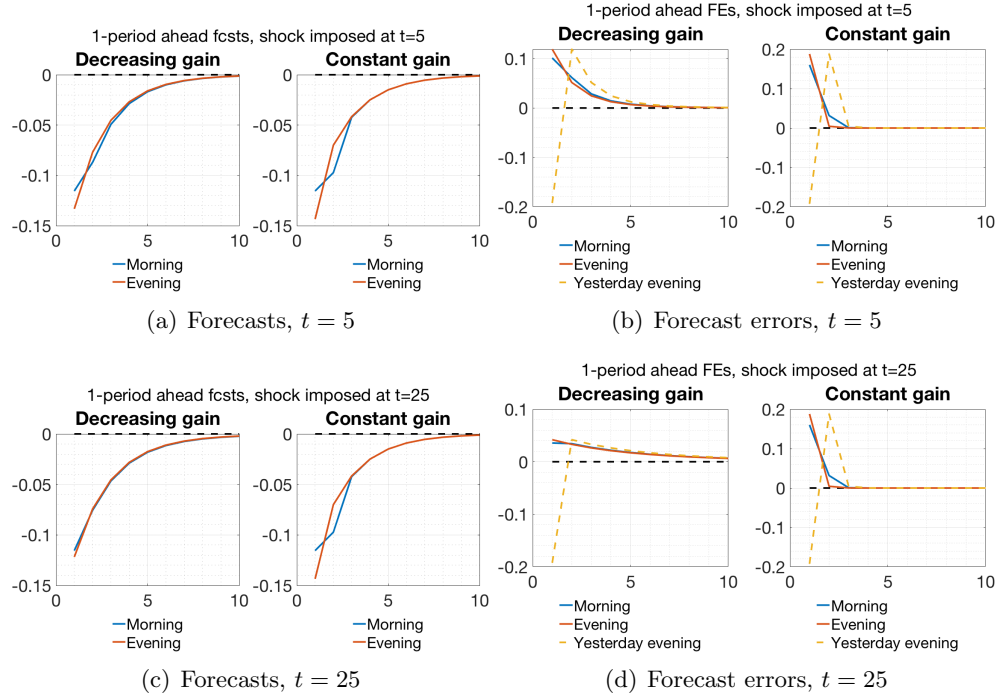
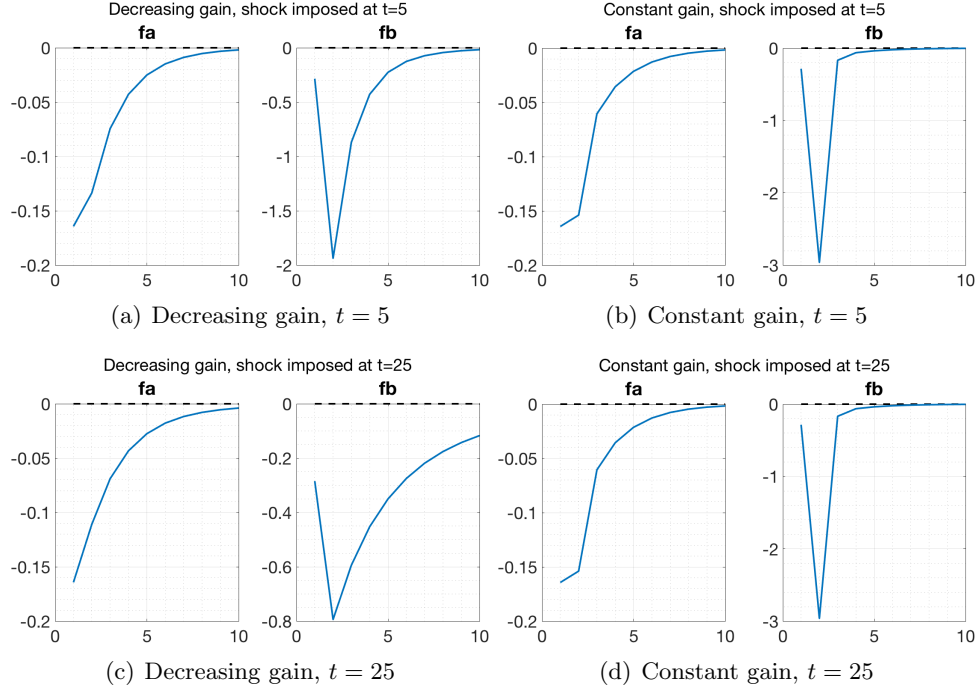


Figure 3: IRF for LH forecasts, shock imposed at t



4 puzzling things, 4 attempts to explain them:

1. Criss-crossing of expectations

If the gain is high enough, expectations overupdate, FE switch signs. (Can make extremely small with lower gain, e.g. $\bar{g} = 0.02$, the value of the dgain after 50 periods.)

Note also that f_a, f_b are mainly driven by the intercept, not the slope. The intercept term is $\frac{\bar{\pi}}{1-\alpha\beta}$ and $\frac{\bar{\pi}}{1-\beta}$. So when α is far from 1, f_b is much bigger

- both in absolute terms,
- as well as relatively to f_a .

2. $i \uparrow$ at $t = 2$ after shock

$i = \pi \downarrow + \rho_i^{t-1} \delta \uparrow$. So b/c of correction in π at $t = 2$, $|\rho_i^{t-1} \delta| - |\pi|$ becomes larger.

3. Overshooting later than $t = 2$

IR decomposed into two effects pulling opposite ways: $IR \approx \mathbb{E}(\pi) + \delta$. Overshooting happens when the expectation effect is larger. As the shock recedes faster than the expectation, x, π

overshoot. Testable implication: if shock iid, overshooting should happen at $t = 2$, and indeed it does.

4. Responses to expectations in RE vs learning (recursive vs. LH-horizon)

Here's the stylized representation of how endogenous variables respond to expectations in the two formulations (once you've plugged in the interest rate):

$$\begin{aligned}
 & \text{RE} \\
 & x_t = \mathbb{E}(\pi) + \mathbb{E}^+(x) \\
 & \pi_t = \mathbb{E}^+(\pi) + \mathbb{E}^+(x) \\
 & \text{Learning} \\
 & x_t = \mathbb{E}(\pi) + \mathbb{E}^{\text{red}}(x) \\
 & \pi_t = \mathbb{E}^+(\pi) + \mathbb{E}^+(x)
 \end{aligned}$$

The difference, marked in red, comes from the fact in learning, expectations are more explicit. Ignoring shocks and setting $\psi_x = 0$, so the Taylor rule is just $i_t = \psi_\pi \pi_t$, the two systems are

$$\begin{aligned}
 & \text{RE} \\
 & x_t = -\sigma i_t + \mathbb{E}_t x_{t+1} + \sigma \mathbb{E}_t \pi_{t+1} \\
 & \pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} \\
 & \text{Learning} \\
 & x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma \beta i_{T+1} + \sigma \pi_{T+1}) \\
 & \pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} (\kappa \alpha \beta x_{T+1} + (1-\alpha) \beta \pi_{T+1})
 \end{aligned}$$

When you plug in the interest rate, you see that the recursive representation hides the negative part of x_t 's dependance on its own future values into current inflation and its dependance on

future inflation:

RE

$$x_t = -\sigma\psi_\pi\pi_t + \mathbb{E}_t x_{t+1}^+ + \sigma \mathbb{E}_t \pi_{t+1}$$

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1}$$

Learning

$$x_t = -\sigma\psi_\pi\pi_t^- + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1}^+ + \sigma(1-\beta\psi_\pi)\pi_{T+1})$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1})$$

To align with this interpretation, it must be the case that π_t 's dependance on future inflation is stronger under RE than learning. Comparing coefficients for my baseline parameterization, this is true.