Materials 33 - Estimating the anchoring function

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1 Estimation procedure

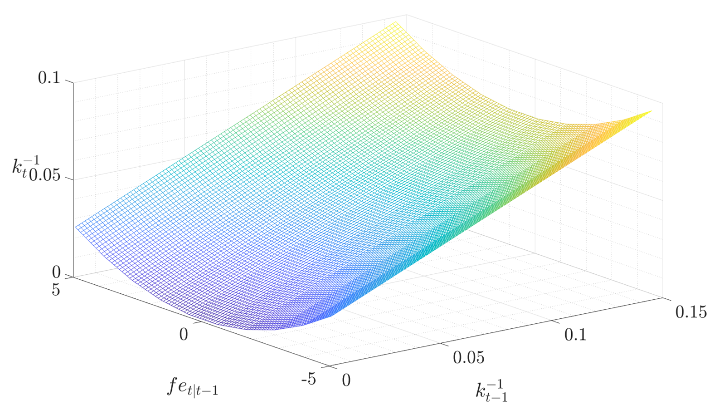
Instead of the AR(1) anchoring function used so far (Equation A.6), I use the following equation

$$k_t^{-1} = \alpha s(X) \tag{1}$$

where $X = (k_{t-1}^{-1}, fe_{t|t-1})$ and I use piecewise linear interpolation. Now I initialize α_0 by specifying a grid for X, passing the grid through Equation (A.6) to generate k_t^{-1}-values, and approximating by fitting the grid to the k_t^{-1}-values. This means that if the functional relationship in Fig. 1 is the right one, then the AR(1) specification of Equation (A.6) isn't a bad description of the anchoring function.

Then I estimate α using GMM, targeting the autocovariance structure of inflation, the output gap and the nominal interest rate (federal funds rate) in the data.

Figure 1: Initialization via Equation (A.6) implies this functional relationship



$T = 233$ before BK-filtering, $T = 209$ after BK-filtering. Using the "constant-only, inflation-only" learning PLM. I drop the $ndrop = 5$ initial values.

2 Estimation issues

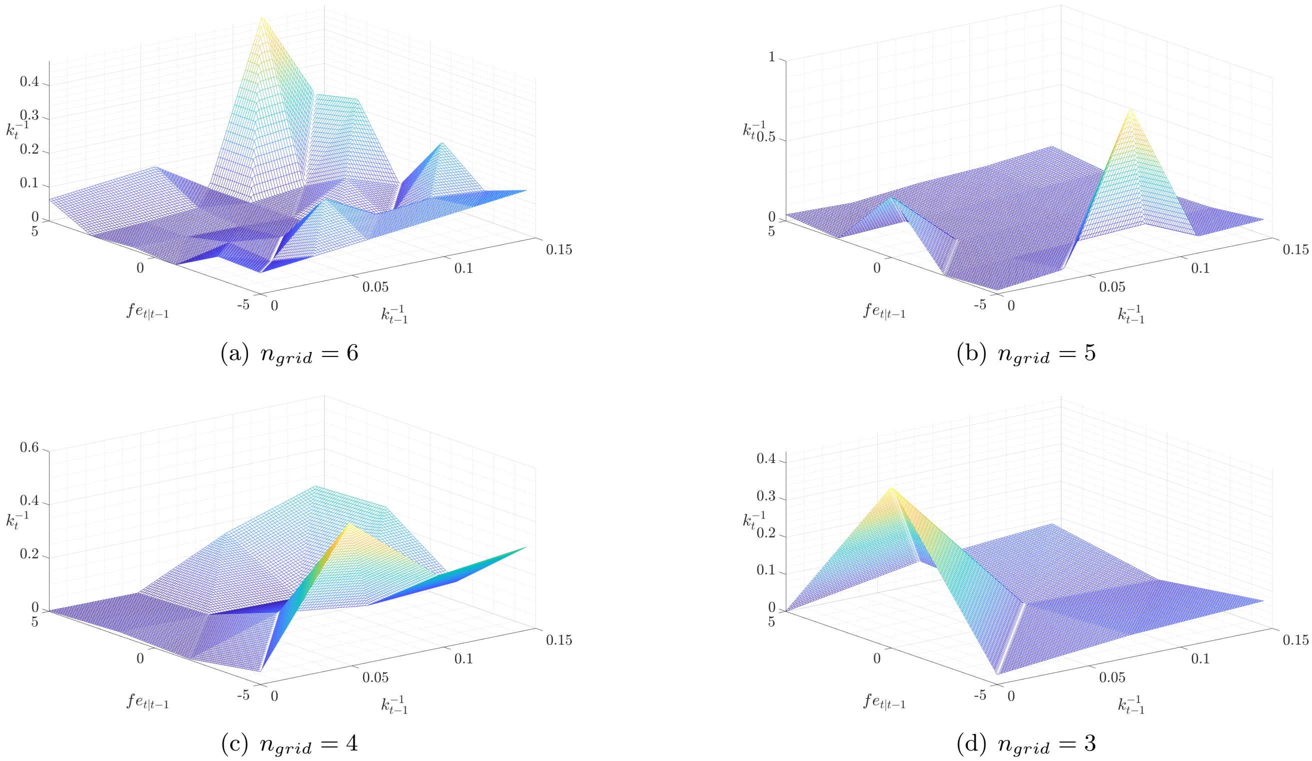
1. Different bounds on α yield different estimates. My understanding of `ndim_simplex` is that the coefficients are the heights at each gridpoint, i.e. that $\alpha(x, y)$ is the z-axis value associated with gridpoint (x, y). Therefore I impose $\alpha \in (0, 1)$ as lower and upper bounds.

2. Certain α still imply explosive model dynamics. (I think that should be fine.) I exclude them by setting the estimation loss very high.

3. I have $ngrid^2$ parameters and 45 moments. This means my grid can at most have 6 elements (or I need more autocovariance lags to have more moments). Therefore I investigate $ngrid = 3, 4, 5, 6$.

4. Target criterion relies on the assumption $k_t^{-1} = \mathbf{g}(fe)$. Need a 1D estimate to implement the target criterion.

Figure 2: k_t^{-1} as a function of k_{t-1}^{-1} and $fe_{t|t-1}$ given $\hat{\alpha}^{GMM} \in (0, 1)$



(a) $n_{grid} = 6$

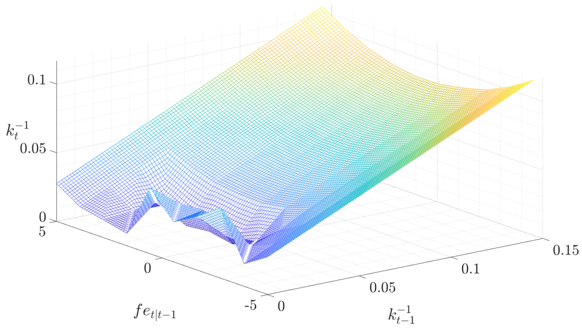
(b) $n_{grid} = 5$

(c) $n_{grid} = 4$

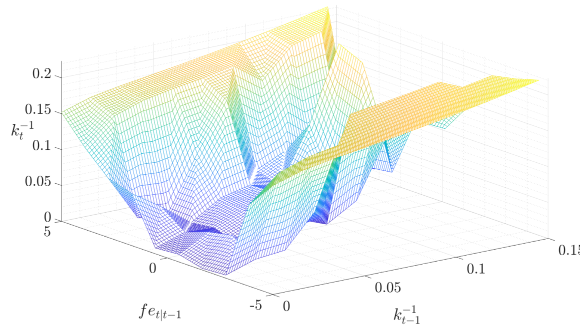
(d) $n_{grid} = 3$

3 Older plots for me

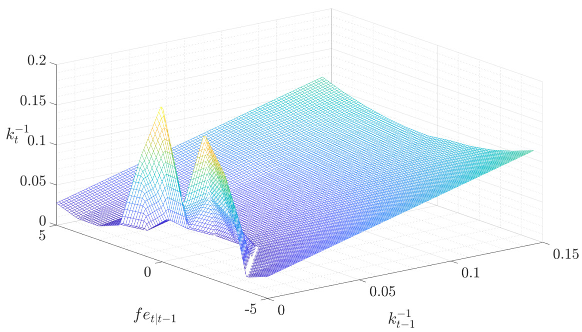
Figure 3: k_t^{-1} as a function of k_{t-1}^{-1} and $fe_{t|t-1}$ given $\hat{\alpha}^{GMM}$



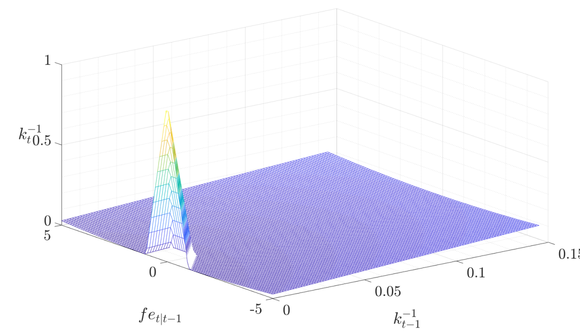
(a) Coefficients unrestricted, $n_{grid} = 10$



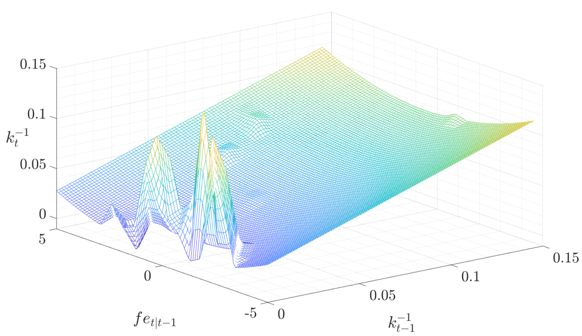
(b) Coefficients restricted > 0, $n_{grid} = 10$



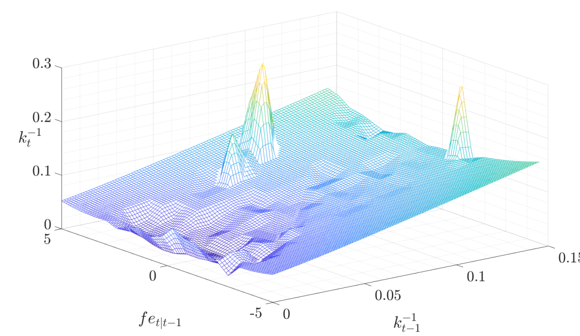
(c) Coefficients unrestricted, $n_{grid} = 11$



(d) Coefficients restricted > 0, $n_{grid} = 11$



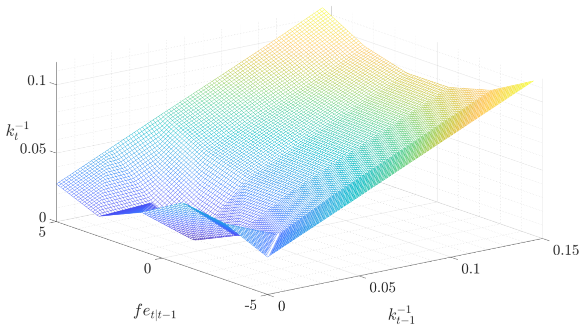
(e) Coefficients unrestricted, $n_{grid} = 20$. This actually goes negative!



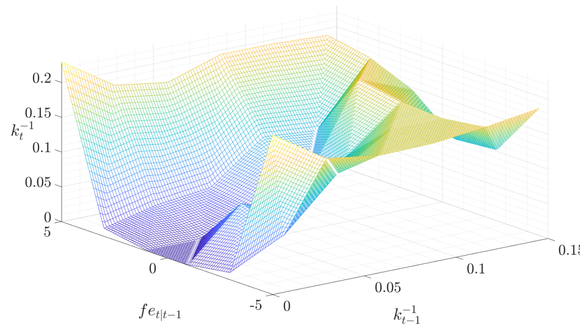
(f) Coefficients restricted > 0, $n_{grid} = 20$

I thought of something: I have $ngrid^2$ parameters and 45 moments. This means my grid can at most have 6 elements (or I need more autocovariance lags to have more moments).

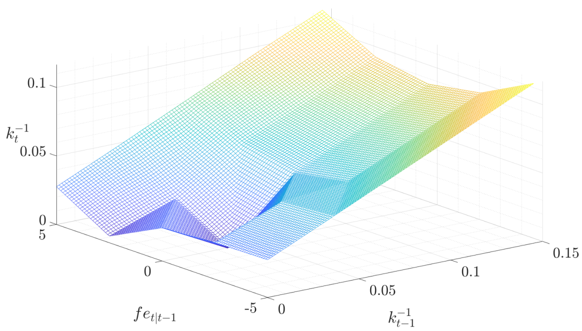
Figure 4: k_t^{-1} as a function of k_{t-1}^{-1} and $fe_{t|t-1}$ given $\hat{\alpha}^{GMM}$



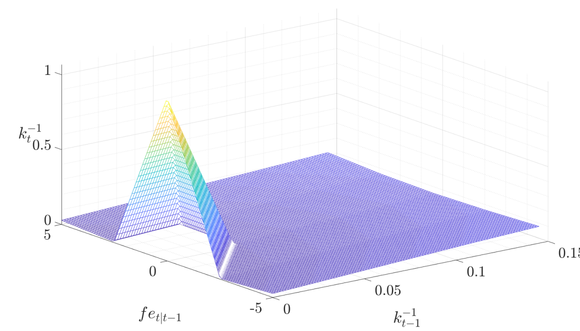
(a) Coefficients unrestricted, $n_{grid} = 6$



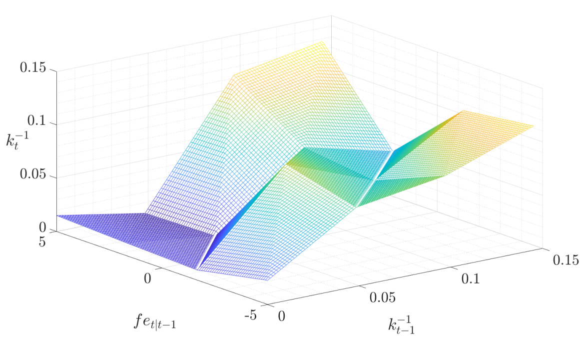
(b) Coefficients restricted > 0, $n_{grid} = 6$



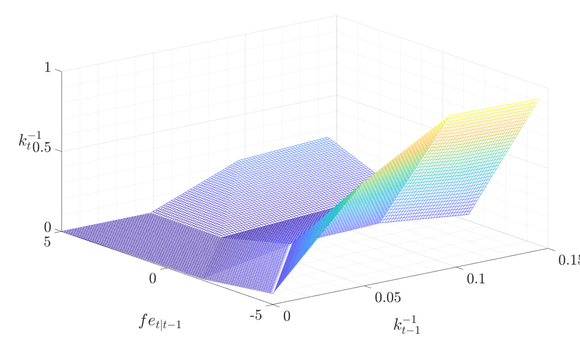
(c) Coefficients unrestricted, $n_{grid} = 5$



(d) Coefficients restricted > 0, $n_{grid} = 5$

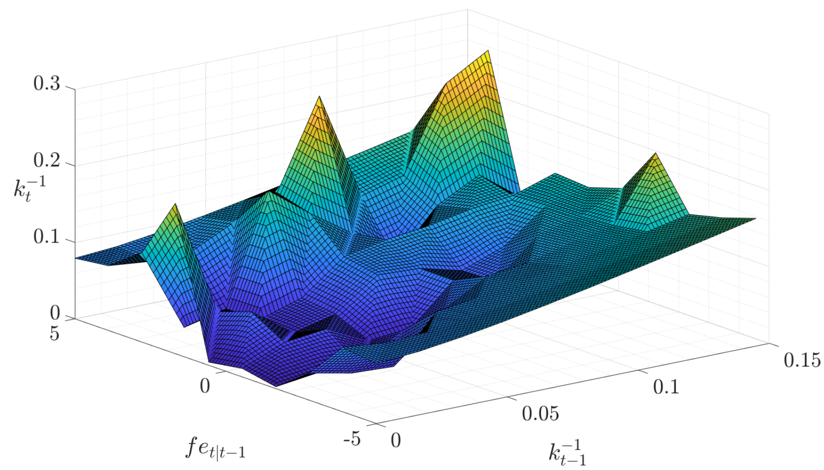


(e) Coefficients unrestricted, $n_{grid} = 4$.

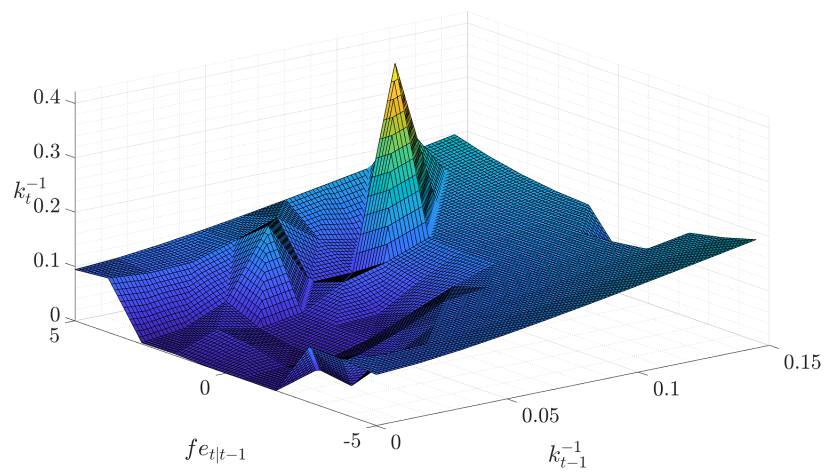


(f) Coefficients restricted > 0, $n_{grid} = 4$

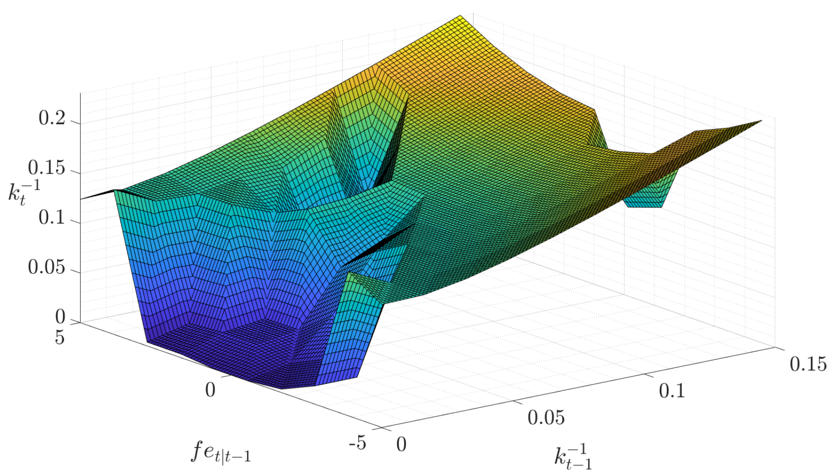
Figure 5: k_t^{-1} as a function of k_{t-1}^{-1} and $fe_{t|t-1}$ given $\hat{\alpha}^{GMM} > 0$



(a) $N_{burn} = 0$

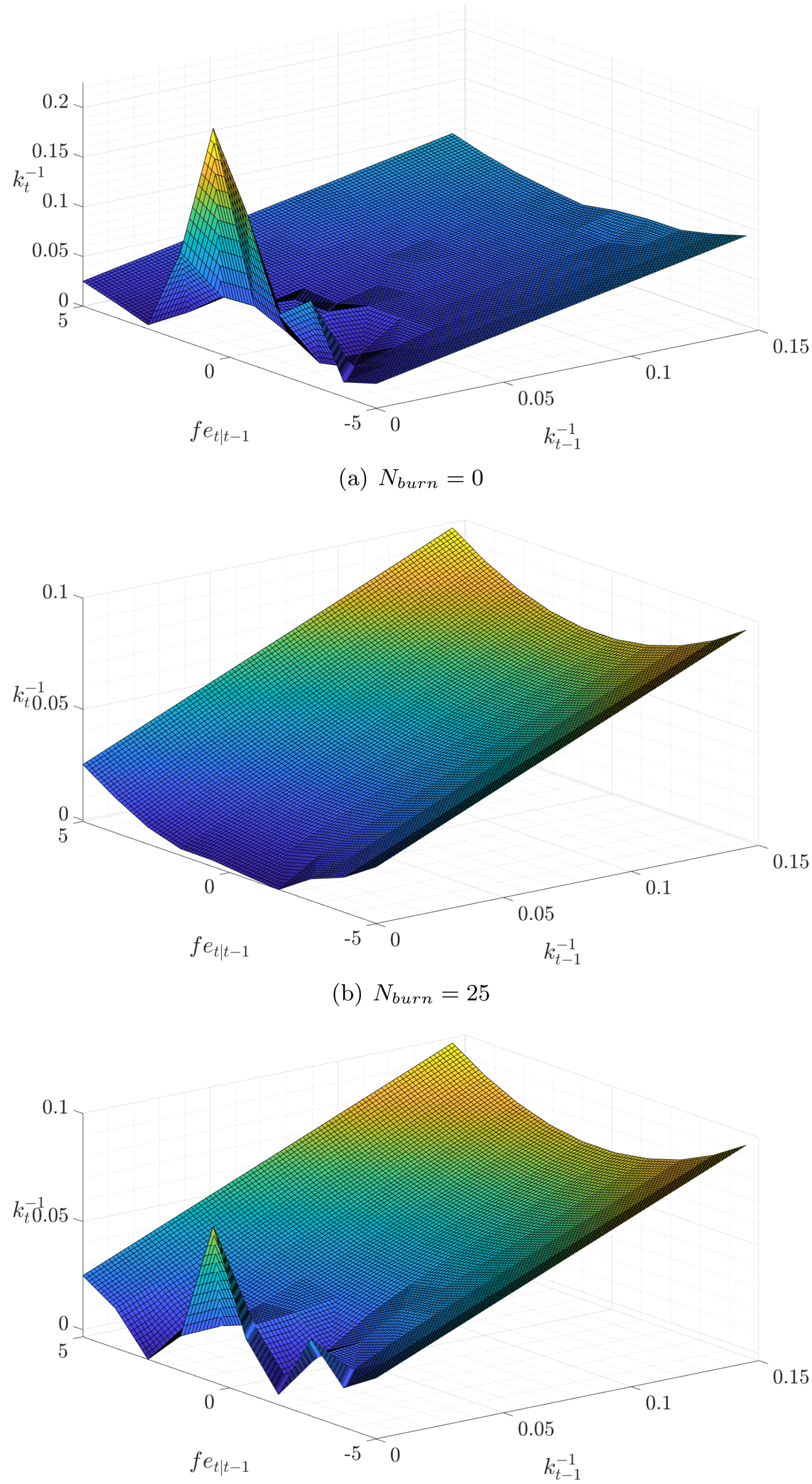


(b) $N_{burn} = 25$



(c) $N_{burn} = 50$

Figure 6: k_t^{-1} as a function of k_{t-1}^{-1} and $fe_{t|t-1}$ given $\hat{\alpha}^{GMM}$ (not restricted > 0)

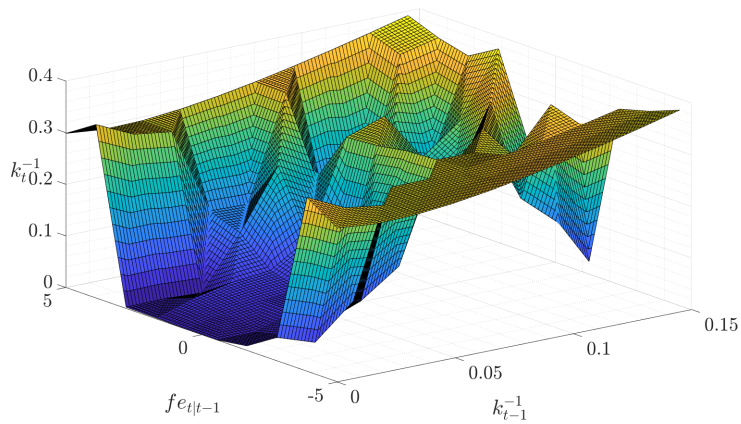


(a) $N_{burn} = 0$

(b) $N_{burn} = 25$

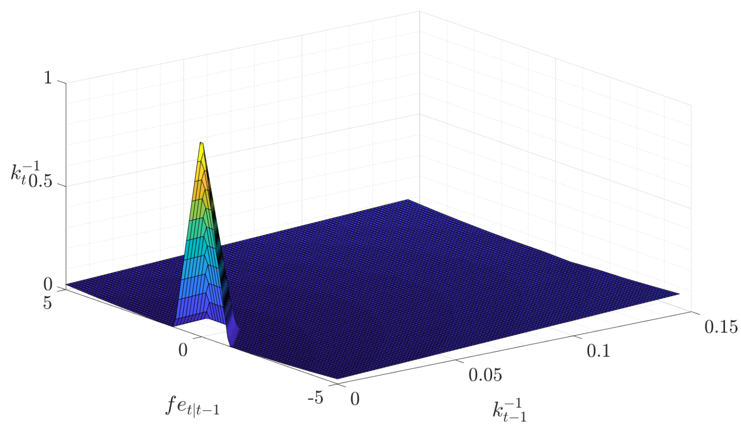
(c) $N_{burn} = 50$

Figure 7: $min(k^{-1}) = 0$ instead of 0.00001



$N_{burn} = 50$, PLM = constant only, inflation only

Figure 8: n_{grid}=11



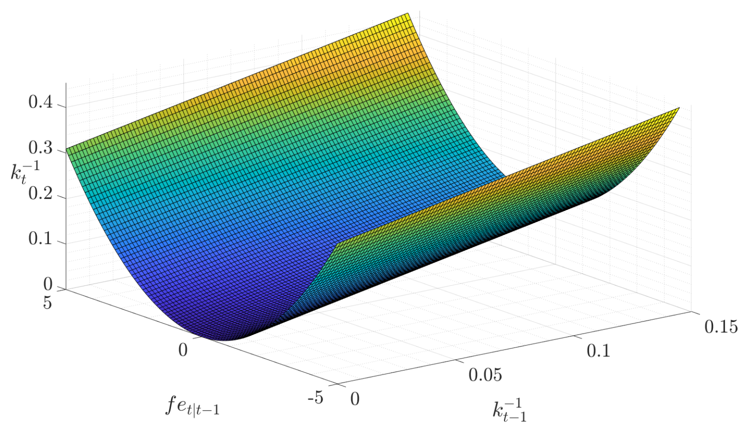
$min(k^{-1}) = 0, \alpha > 0, N_{burn} = 50$, constant-only, inflation-only PLM

4 Is the estimated LOM gain in the nonpermissible range for PEA?

If I fit an AR(1) (Equation A.6) to the estimated gain evolution, I obtain $(\rho_k, \gamma_k) = (1, 0.0123)$ which results in the following relationship:

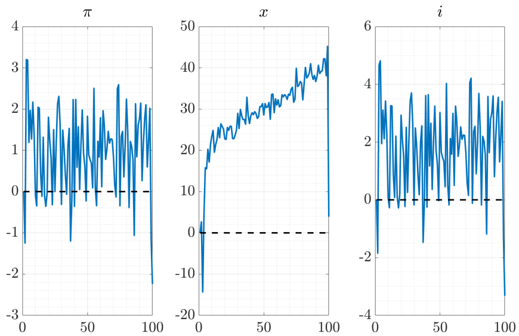
But these are very high values; the model isn't E-stable for such high values. Rerunning PEA with the AR(1) specification and the values (1, 0.0123) is able to solve, but involves a gain that's increasing over time!

Figure 9: AR(1) approximation to the estimated functional form of the gain

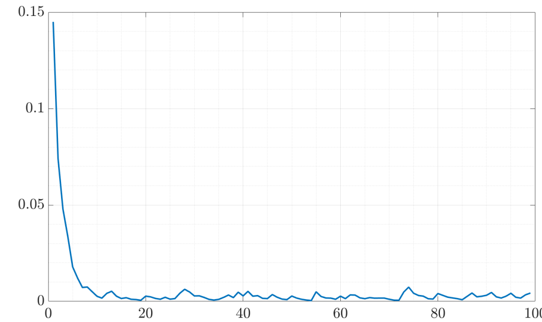


5 PEA plots with constant only, π only learning

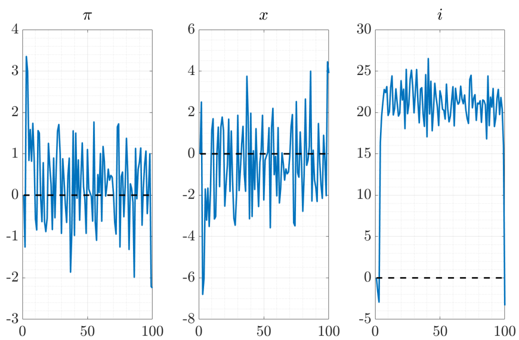
Figure 10



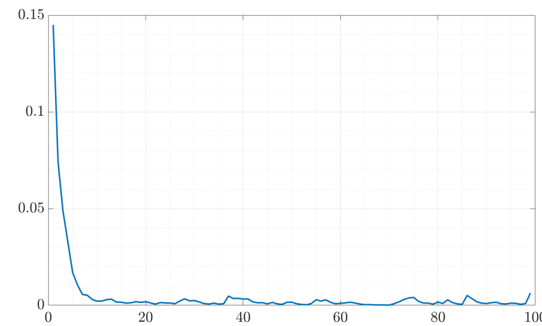
(a) Taylor rule, observables



(b) Taylor rule, inverse gain

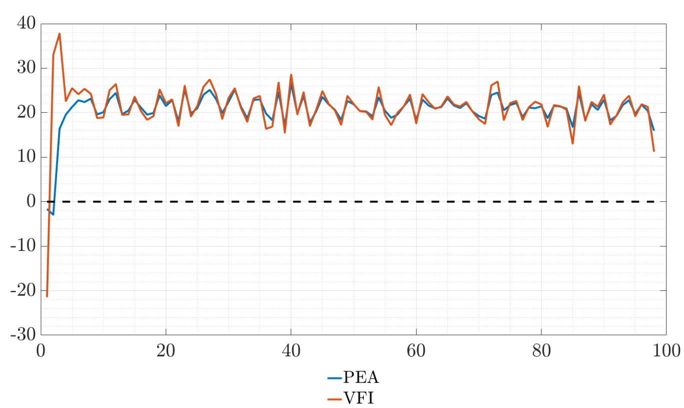


(c) Optimal sequence, observables



(d) Optimal sequence, inverse gain

Figure 11: VFI-PEA comparison using the "constant only, π only" PLM



A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{A.1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{A.2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \qquad \text{(if imposed)} \tag{A.3}$$

$$\text{PLM:} \qquad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\, h_x \tag{A.4}$$

$$\text{Updating:} \qquad a_t = a_{t-1} + k_t^{-1}\big(z_t - (a_{t-1} + b s_{t-1})\big) \tag{A.5}$$

$$\text{Anchoring function:} \qquad k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k f e_{t-1}^2 \tag{A.6}$$

$$\text{Forecast error:} \qquad f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \tag{A.7}$$

$$\text{LH expectations:} \qquad f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \qquad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \tag{A.8}$$

This notation captures vector learning (z learned) for intercept only. For scalar learning, $a_t = \begin{pmatrix}\bar{\pi}_t & 0 & 0\end{pmatrix}'$ and b_1 designates the first row of b. The observables (π, x) are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix}\sigma & 1-\beta & -\sigma\beta\end{bmatrix} f_b + \sigma \begin{bmatrix}1 & 0 & 0\end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \tag{A.9}$$

$$\pi_t = \kappa x_t + \begin{bmatrix}(1-\alpha)\beta & \kappa\alpha\beta & 0\end{bmatrix} f_a + \begin{bmatrix}0 & 0 & 1\end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \tag{A.10}$$

B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only, $k_t^{-1} = \mathbf{g}(f e_{t-1})$):

$$\pi_t = -\frac{\lambda_x}{\kappa}\Bigg\{x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_\pi(t)\Big)$$
$$\Big(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi}}(t+j))\Big)\Bigg\} \tag{B.1}$$

where I'm using the notation that $\prod_{j=0}^{0} \equiv 1$. For interpretation purposes, let me rewrite this as follows:

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + f e_{t|t-1}^{eve}\mathbf{g}_\pi(t)\Big)\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i}$$
$$-\frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + f e_{t|t-1}^{eve}\mathbf{g}_\pi(t)\Big)\Big(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(k_{t+1+j}^{-1} + f e_{t+1+j|t+j}^{eve})\mathbf{g}_{\bar{\pi}}(t+j)\Big) \tag{B.2}$$

Interpretation: tradeoffs from discretion in RE + effect of current level and change of the gain on future tradeoffs + effect of future expected levels and changes of the gain on future tradeoffs