

Materials 14 - Maybe a last attempt to get rid of the overshooting

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1 Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (1)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (2)$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \quad (3)$$

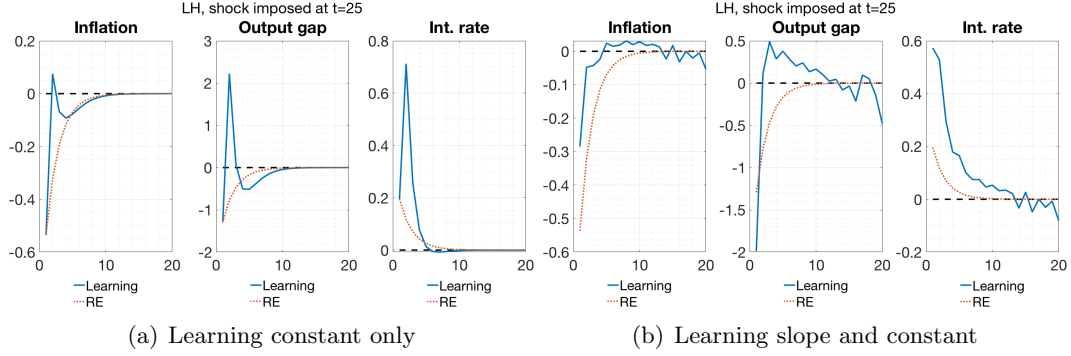
$$\hat{\mathbb{E}}_t z_{t+h} = \bar{z}_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \quad b = g_x h_x \quad \text{PLM} \quad (4)$$

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} \underbrace{(z_t - (\bar{z}_{t-1} + b s_{t-1}))}_{\text{fcst error using (4)}} \quad (5)$$

(Vector learning. For scalar learning, $\bar{z} = \begin{pmatrix} \bar{\pi} & 0 & 0 \end{pmatrix}'$. I'm also not writing the case where the slope b is also learned.)

$$k_t = \begin{cases} k_{t-1} + 1 & \text{for decreasing gain learning} \\ \bar{g}^{-1} & \text{for constant gain learning.} \end{cases} \quad (6)$$

Figure 1: Reference: baseline model



2 Regime-switching

Figure 2: Markov-switching Taylor rule, baseline, learning initialized at active state, conditional on mixed regime sequence

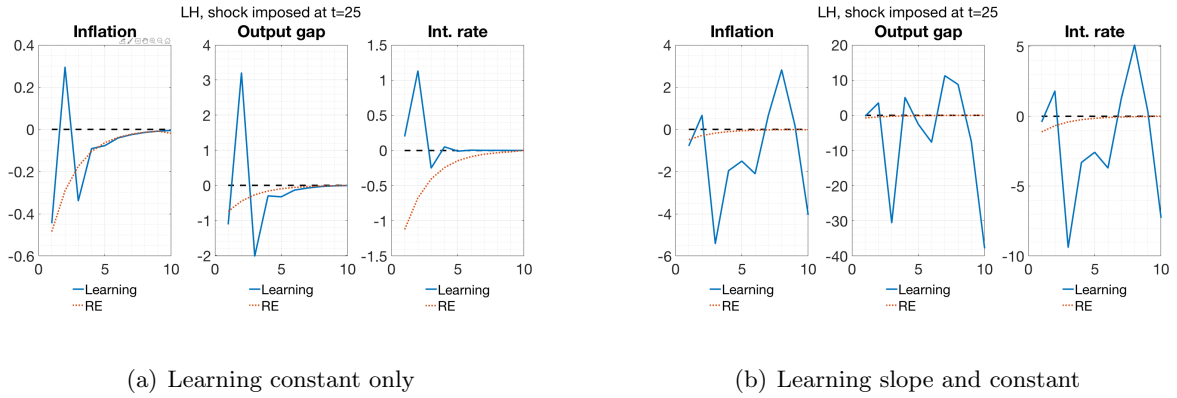
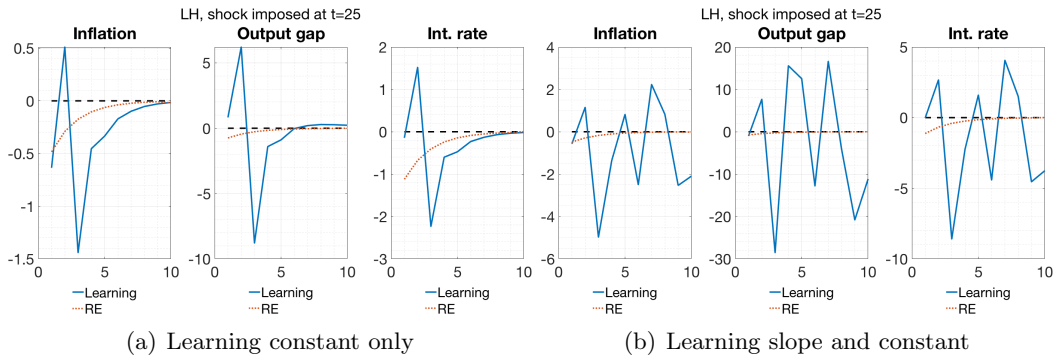


Figure 3: Markov-switching Taylor rule, baseline, learning initialized at passive state, conditional on mixed regime sequence



- Different initialization of learning doesn't make a whole lot of difference.

- It just changes where you start, but doesn't fundamentally affect dynamics.

Figure 4: Markov-switching Taylor rule, baseline, learning initialized at passive state, conditional on passive regime only

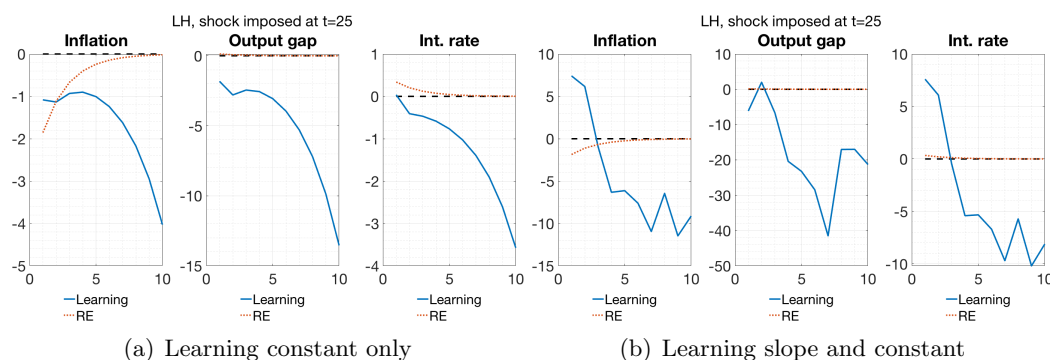
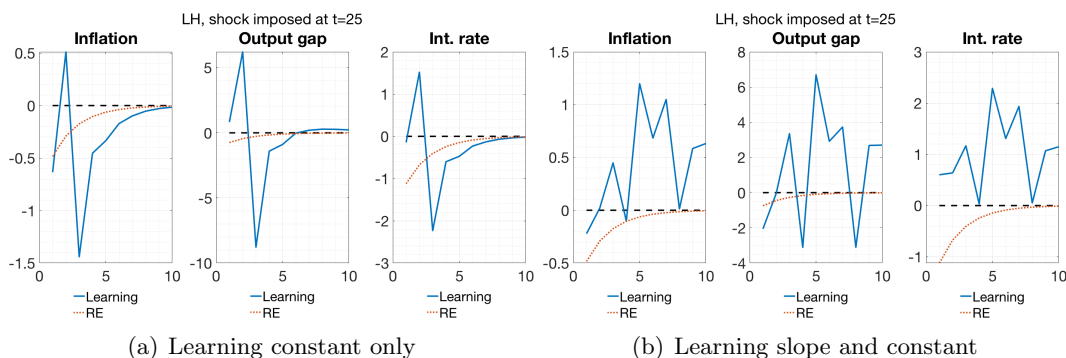


Figure 5: Markov-switching Taylor rule, baseline, learning initialized at passive state, conditional on active regime only



- I'm surprised that the all-passive state is unstable. I've checked and it's not E-stable: the difference in the learning matrix ϕ grows over time, even with decreasing gain learning.
- The all-active is very volatile.

3 Projection facility: checking $\text{eig}(\phi)$ when ϕ isn't square?

What I do now is I check $\text{eig}(R)$ because that is always square, and when ϕ explodes, usually R does too. Of course I can't do this for learning the constant only, but according to my experience, that's where the projection facility is least likely to ever be needed. Of course, this doesn't always work - for interest rate smoothing, it doesn't.

4 Endogenous states don't evolve as they should

Now they do!

5 Bringing anchoring back

I've reworked the choice functions for the endogenous gain, CEMP's criterion and the CUSUM test, to better incorporate vector learning ("out of the difference between PLM and E(ALM) or out of forecast error variances respectively, pick the largest (or the mean) and test whether that's bigger than the threshold value").

CEMP's criterion was

$$\theta_t = |\hat{\mathbb{E}}_{t-1}\pi_t - \mathbb{E}_{t-1}\pi_t|/(\text{Var}(\text{shocks})) \quad (7)$$

$$\text{i.e. PLM- E[ALM], scaled by shocks} \quad (8)$$

To extend this to vector learning and in general to a model with a vector of observables I first rewrite the ALM

$$z_t = A_a f_a + A_b f_b + A_s s_t \quad (9)$$

$$\text{as } z_t = F + G s_t \quad (10)$$

$$\Leftrightarrow z_t = \begin{bmatrix} F & G \end{bmatrix} \begin{bmatrix} 1 \\ s_t \end{bmatrix} \quad (11)$$

Then, since the PLM is $z_t = \phi \begin{bmatrix} 1 \\ s_t \end{bmatrix}$, the generalized CEMP criterion becomes

$$\theta_t = \max |\Sigma^{-1}(\phi - \begin{bmatrix} F & G \end{bmatrix})| \quad (12)$$

where Σ is the VC matrix of shocks. As for the CUSUM criterion, what I did in Materials 5 was

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1} (F E_t^2 - \omega_{t-1}) \quad (13)$$

$$\theta_t = \theta_{t-1} + \kappa k_{t-1}^{-1} (F E_t^2 / \omega_t - \theta_{t-1}) \quad (14)$$

where $F E_t$ is the most recent short-run forecast error ($ny \times 1$), and ω_t is the agents' estimate of the forecast error variance ($ny \times ny$). To take into account that these are now matrices, I now write

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1} (F E_t F E_t' - \omega_{t-1}) \quad (15)$$

$$\theta_t = \theta_{t-1} + \kappa k_{t-1}^{-1} \text{mean}((\omega_t^{-1} F E_t F E_t' - \theta_{t-1})) \quad (16)$$

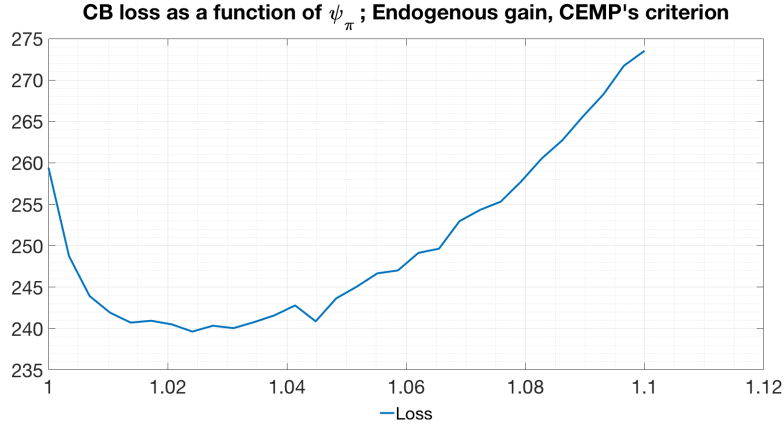
Both work but require quite different threshold parameters to behave similarly.

6 The central bank's loss function

Relying on Clarida, Gali and Gertler (1999), I start with the simple quadratic loss:

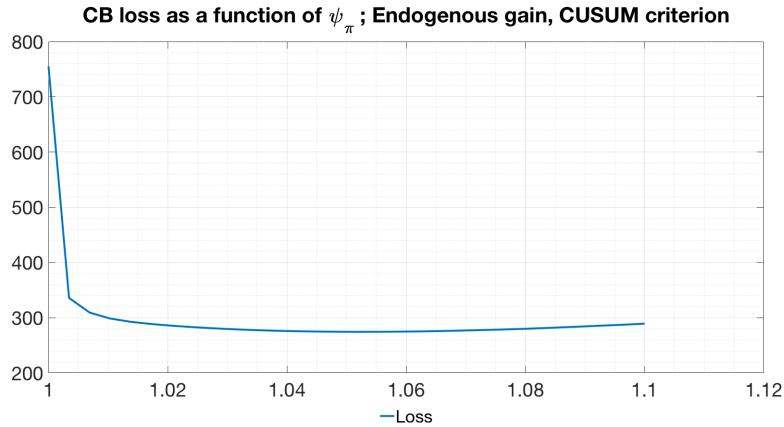
$$\mathcal{L} = \frac{1}{2} \mathbb{E}_t \sum_{T=t}^{\infty} \left(\alpha^{CB} x_T^2 + \pi_T^2 \right) \quad (17)$$

Figure 6: Central bank loss for $\psi_x = 0, \alpha^{CB} = 0.5$, computed as a cross-sectional average, $N = 100, T = 400$.



Fmincon says $\psi_\pi^* = 1.0724, \psi_x^* = 0.0490$.

Figure 7: Central bank loss for $\psi_x = 0, \alpha^{CB} = 0.5$, computed as a cross-sectional average, $N = 100, T = 400$.



Fmincon says $\psi_\pi^* = 1.0646, \psi_x^* = 0.0451$.

An issue with CUSUM: ω often becomes singular, so for fmincon to be able to run, I took the average element value of ω . But this seems to really matter: it never produces anchoring for anything.

Striking is that these optimal values do not produce anchored sequences. But of course this is super-sensitive to threshold criterion values which are completely arbitrary!

7 Anchoring as a function of ψ_π

Figure 8: Inverse gains, $\psi_\pi = 1.01, \psi_x = 0$

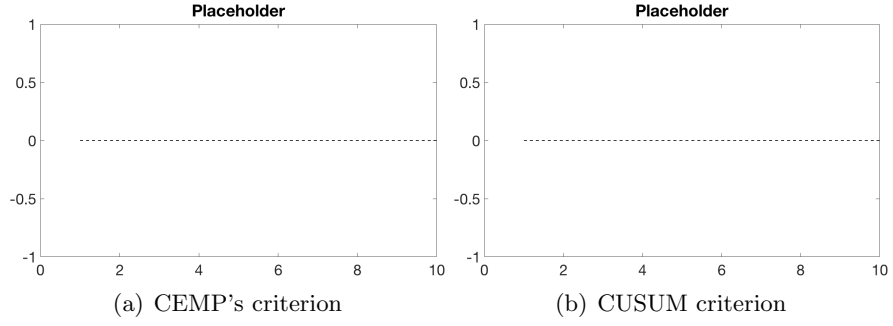


Figure 9: Inverse gains, $\psi_\pi = 1.1, \psi_x = 0$

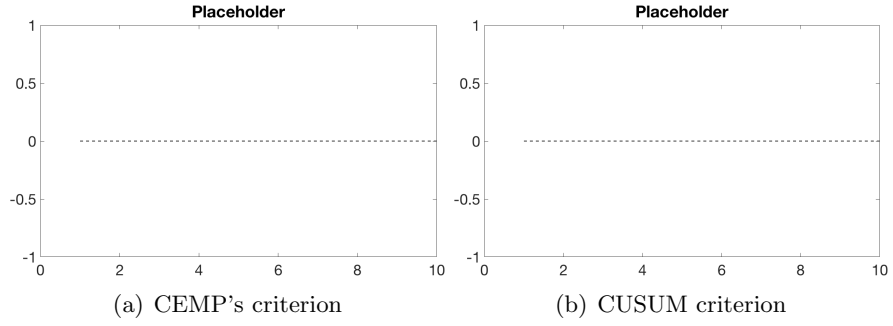


Figure 10: Inverse gains, $\psi_\pi = 1.2, \psi_x = 0$

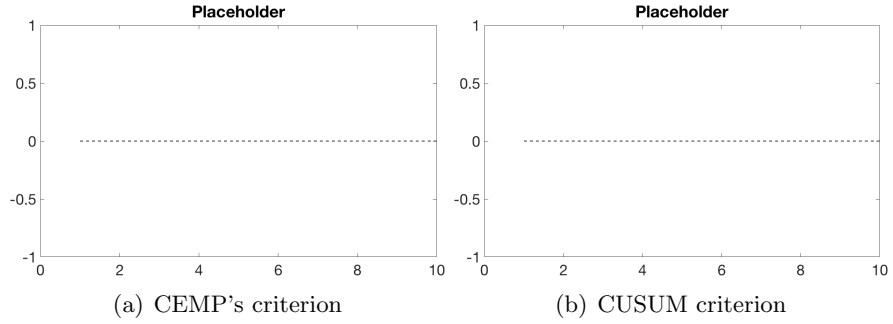


Figure 11: Inverse gains, $\psi_\pi = 1.5, \psi_x = 0$

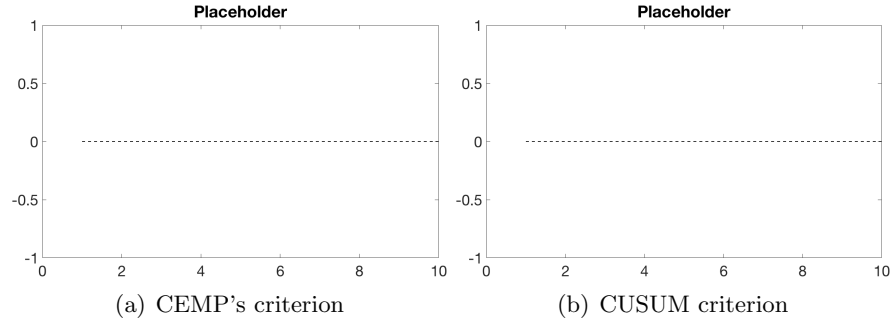


Figure 12: Inverse gains, $\psi_\pi = 1.8, \psi_x = 0$

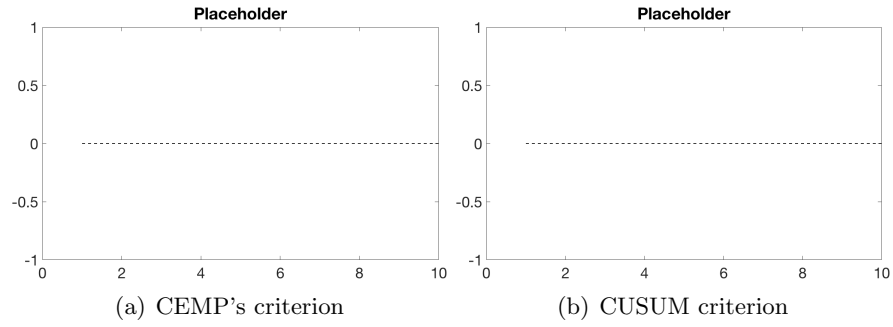
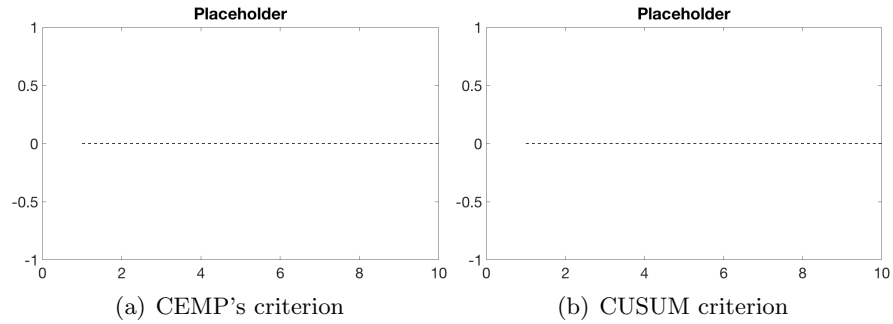


Figure 13: Inverse gains, $\psi_\pi = 2, \psi_x = 0$



8 Reference plots

Figure 14: Baseline

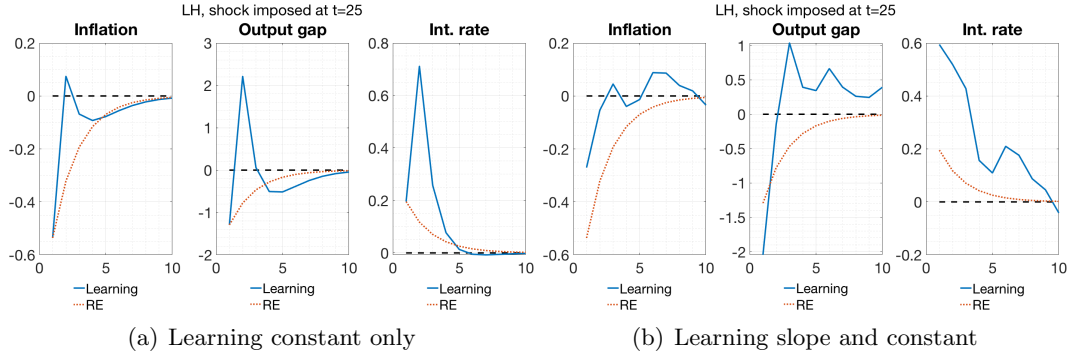


Figure 15: Lagged inflation in Taylor rule, “suboptimal forecasters” info assumption

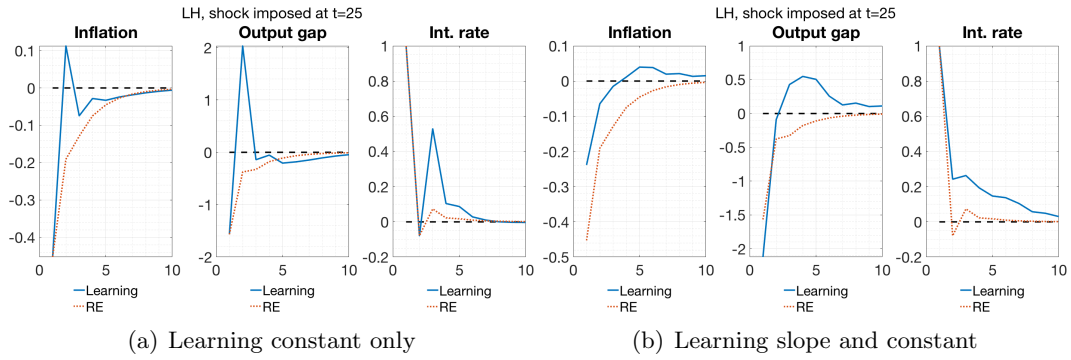


Figure 16: Interest rate smoothing, “suboptimal forecasters” info assumption

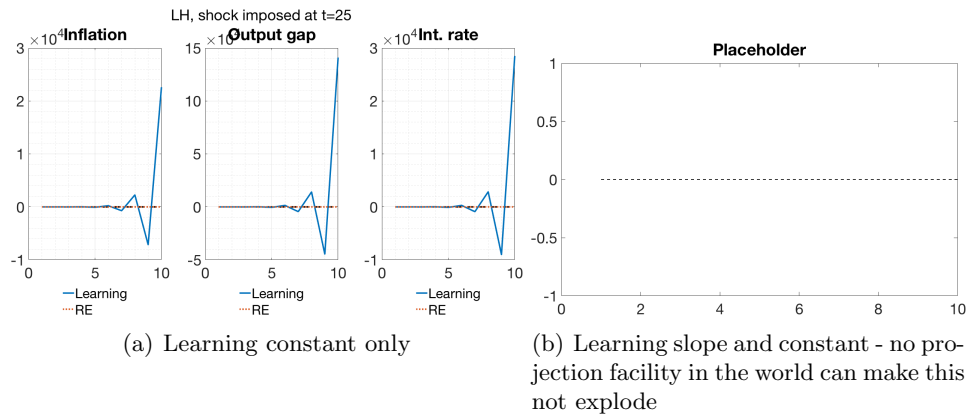


Figure 17: Expected inflation in Taylor rule

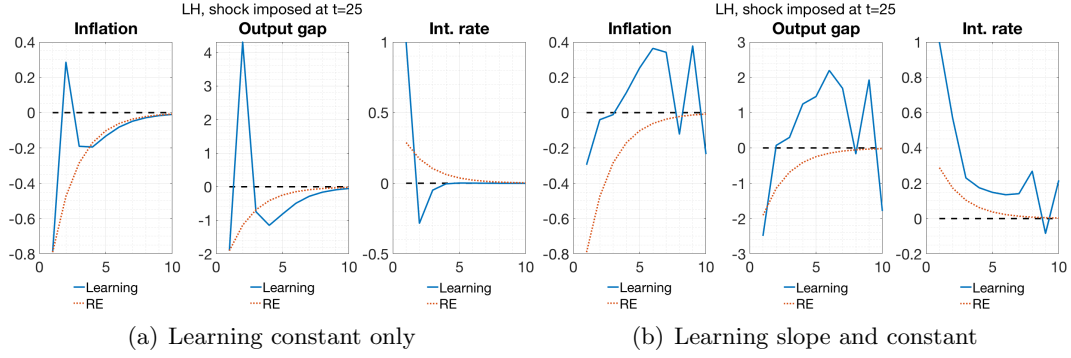


Figure 18: Indexation in NKPC, “suboptimal forecasters” info assumption

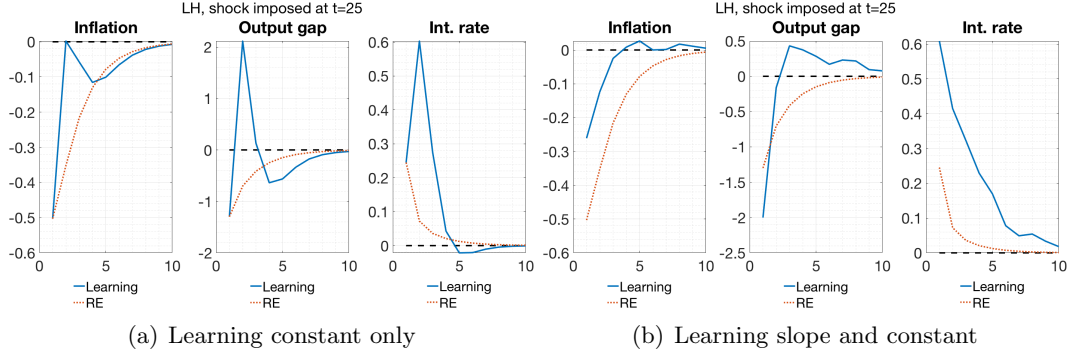


Figure 19: Learn Taylor rule

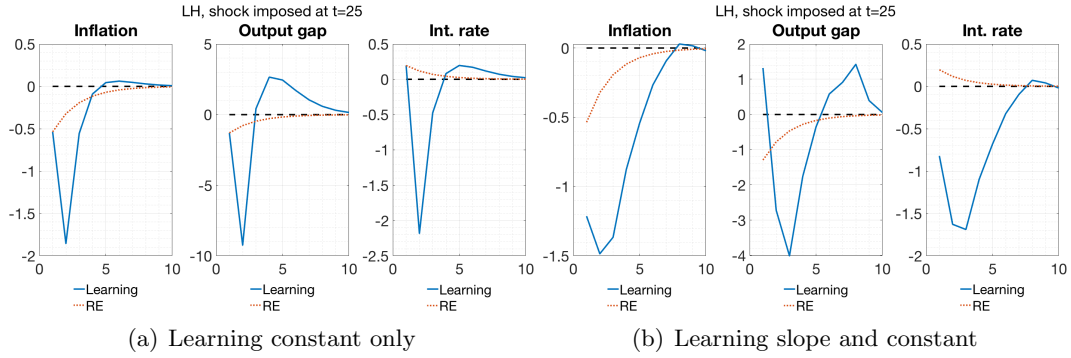


Figure 20: Learn h_x

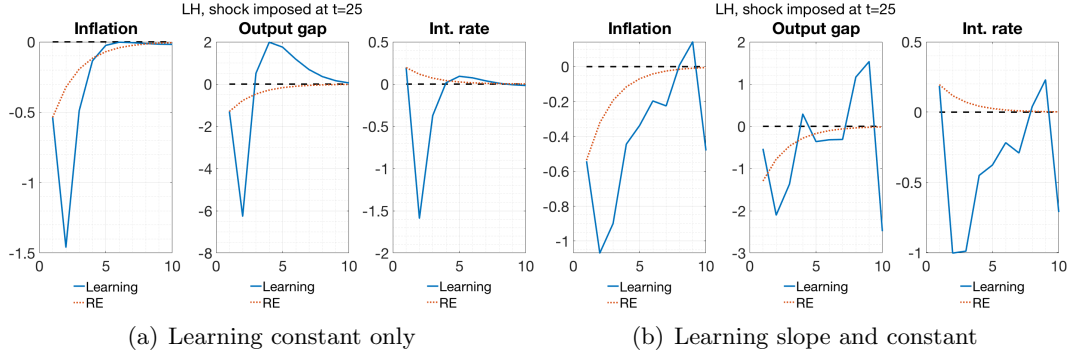


Figure 21: Markov-switching Taylor rule, conditional on passive regime only, learning initialized at passive regime

