Materials 12f - “pil”-extension of baseline model - Lagged inflation in TR using the “optimal forecaster” info assumption

See Notes 9 Jan 2020

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Compare Mathematica (`materials12g2.nb`).

1 Model equations and goal

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t \tag{3}$$

Compact notation

$$z_t = \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = A_a f_a + A_b f_b + A_s s_t \quad \text{with} \quad s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{bmatrix} \tag{4}$$

2 MN matrices

$$\underbrace{\begin{bmatrix} \frac{\sigma}{\beta} & 1+\sigma\psi_x \\ \frac{1}{\alpha} & -\kappa \end{bmatrix}}_{\equiv M} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \underbrace{\begin{bmatrix} \begin{bmatrix} \sigma, & 1-\beta-\sigma\beta\psi_x, & 0 \end{bmatrix} f_b + d_{x,s} s_t \\ \begin{bmatrix} (1-\alpha)\beta, & \kappa\alpha\beta, & 0 \end{bmatrix} f_a + d_{\pi,s} s_t \end{bmatrix}}_{\equiv N} \tag{5}$$

where

$$d_{x,s} = \sigma \begin{bmatrix} 1 & -1 & 0 & \psi_\pi \end{bmatrix} InxBhx \qquad InxBhx \equiv (I_{nx} - \beta h_x)^{-1} \tag{6}$$

$$d_{\pi,s} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} InxABhx \qquad InxABhx \equiv (I_{nx} - \alpha\beta h_x)^{-1} \tag{7}$$

$$d_{i,s} = \begin{bmatrix} 0 & 1 & 0 & \psi_\pi \end{bmatrix} \tag{8}$$

The new thing is the pair of linking equations that tell agents to use h_x to forecast π (they establish a relationship between $f_b(1), f_a(1)$ and the sum of errors):

$$L1' = \frac{1}{(\beta)^2} \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} InxBhx.s_t - \frac{1}{(\beta)^2} \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} s_t \tag{9}$$

$$L2' = \frac{1}{(\alpha\beta)^2} \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} InxABhx.s_t - \frac{1}{(\alpha\beta)^2} \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} s_t \tag{10}$$

3 PQ matrices

$$\underbrace{\begin{bmatrix} \frac{\sigma}{\beta} & 1 & \sigma \\ \frac{1}{\alpha} & -\kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix}}_{\equiv P} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \underbrace{\begin{bmatrix} \begin{bmatrix} \sigma, & 1-\beta, & \beta(-\sigma) \end{bmatrix} f_b + c_{x,s} s_t \\ \begin{bmatrix} (1-\alpha)\beta, & \alpha\beta\kappa, & 0 \end{bmatrix} f_a + c_{\pi,s} s_t \\ c_{i,s} s_t \end{bmatrix}}_{\equiv Q} \tag{11}$$

where

$$c_{x,s} = \sigma \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} .\mathrm{InxBhx}; \tag{12}$$

$$c_{\pi,s} = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} .\mathrm{InxABhx} \tag{13}$$

$$c_{i,s} = \begin{pmatrix} 0 & 1 & 0 & \psi_\pi \end{pmatrix} = d_{i,s} \tag{14}$$

where InxABhx and InxBhx are the same as before.

The (*)-relation needs to be rewritten as

$$f_b(3) = \psi_x f_b(2) + \frac{1}{\beta}\{ \begin{bmatrix} 0 & 1 & 0 & \psi_\pi \end{bmatrix} (I_{nx} - \beta h_x)^{-1} s_t - \begin{bmatrix} 0 & 1 & 0 & \psi_\pi \end{bmatrix} s_t \} \tag{*}$$

and the PQ-solution is subject to the same linking equations $L1'$ and $L2'$.

The Matlab code that uses this is `matrices_A_12g2.m`.