Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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*Inflation that runs below its desired level can lead to an unwelcome fall in **longer-term inflation expectations**, which, in turn, can pull actual inflation even lower, resulting in an adverse cycle of ever-lower inflation and inflation expectations. [...] **Well-anchored inflation expectations** are critical[.]*
Jerome Powell, Chairman of the Federal Reserve [1]
(Emphases added.)

[1] "New Economic Challenges and the Fed's Monetary Policy Review," August 27, 2020.

Long-run expectations: capturing responsiveness to short-run conditions

Individual SPF forecasts: for 1991-Q4 onward, estimate rolling regression

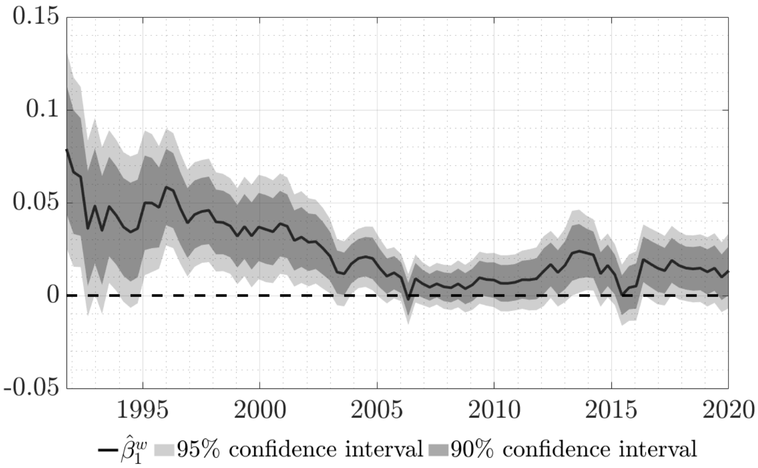
$$\Delta\bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{1}$$

where w indexes windows of 20 quarters length,

$f_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1}\,\pi_t$ individual one-year-ahead forecast error

Time-varying responsiveness

$$\Delta \bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{1}$$



Disagreement measures

This project

- How to conduct monetary policy in interaction with the anchoring expectation formation?

- Model of expectations unanchoring
  $\hookrightarrow$ extension to adaptive learning that captures time-varying responsiveness of long-run expectations

- Estimate how unanchoring takes place in data
  $\hookrightarrow$ quantify novel anchoring channel and use for monetary policy analysis

Preview of results

1. Estimation
   - Larger mistakes unanchor more
   - Overestimating inflation unanchors more than underestimating it (Hebden et al 2020)
   - On average, people discount observations older than 8 quarters
2. Optimal policy
   - Responds aggressively to inflation when unanchored, accommodates inflation when anchored
3. Taylor rule
   - Less aggressive on inflation than under rational expectations

Related literature

- **Optimal monetary policy in the New Keynesian model**

  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Adaptive learning**

  Evans & Honkapohja (2001, 2006), Sargent (1999), Primiceri (2006), Lubik & Matthes (2018), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Mele et al (2019), Eusepi & Preston (2011), Milani (2007, 2014)

- **Anchoring and the Phillips curve**

  Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Reis (2020), Hebden et al 2020, Gobbi et al (2019), Carvalho et al (2019)

- **Reputation**

  Barro (1986), Cho & Matsui (1995)

Structure of talk

1. Model of anchoring expectations
2. Quantification of learning channel
3. Solving the Ramsey problem
4. Implementing optimal policy
5. Approximating optimal policy with a Taylor rule

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Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j))dj \right] \tag{2}$$

Budget constraint

$$B_t^i \leq (1+i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_t C_t^i \tag{3}$$

▸ Consumption, price level

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \tag{4}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{5}$$

▸ Profits, stochastic discount factor

Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- Model implies mapping between exogenous states s_t and observables $y_t \equiv (\pi, x, i)'$

$$y_t = g s_t \tag{6}$$

- Under rational expectations (RE), private sector knows model $\rightarrow$ knows (6)

$$\mathbb{E}_t \, y_{t+1} = g \, \mathbb{E} \, s_{t+1} \tag{7}$$

- $\hat{\mathbb{E}}$: agents do not internalize that identical $\rightarrow$ do not know aggregate model $\rightarrow$ do not know (6)

Adaptive learning

- Agents know exogenous evolution of states

$$s_t = hs_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \tag{8}$$

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- Postulate linear functional relationship instead of (6):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{9}$$

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$a \rightarrow$ concept of long-run expectations in the model

- Estimate a, b using recursive least squares (RLS) using observed states and knowledge of (8)

Recursive least squares (RLS)

Observables are: $(\pi, x, i)'$

Assumption: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\, h \quad \forall t \tag{10}$$

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$\bar{\pi}_{t-1}$: long-run inflation expectations $\rightarrow$ anchoring

$\rightarrow$ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\right)}_{\equiv f_{t|t-1}, \text{ forecast error}} \tag{11}$$

$k_t \in (0,1)$ gain
b_1 first row of b

▸ General RLS algorithm

Alternatives for the gain

1. Decreasing gain:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f_{t|t-1} \tag{12}$$

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Model summary

- New Keynesian core: IS and Phillips curves

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{15}$$

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▸ Derivations ▸ Actual laws of motion

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→ How should $\{i_t\}$ be set?

Structure of talk

1. Model of anchoring expectations

2. Quantification of learning channel

3. Solving the Ramsey problem

4. Implementing optimal policy

5. Approximating optimal policy with a Taylor rule

Estimating form of gain function

$$\mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}) \tag{19}$$

- $b_i(f_{t|t-1})$ = piecewise linear basis
- γ_i = approximating coefficient at node i

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$\hookrightarrow$ Estimate $\hat{\gamma}$ via simulated method of moments
(Duffie & Singleton 1990, Lee & Ingram 1991, Smith 1993)

- Calibrate parameters of New Keynesian core to literature
- Calibrate variances of disturbances to match moments
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- Moments: autocovariances of inflation, output gap, federal funds rate and 1-year ahead SPF inflation expectations at lags $0, \ldots, 4$

Calibration - parameters from the literature

β	0.98	stochastic discount factor
σ	1	intertemporal elasticity of substitution
α	0.5	Calvo probability of not adjusting prices
κ	0.0842	slope of the Phillips curve
ψ_π	1.5	coefficient of inflation in Taylor rule
$\bar{g}$	0.145	initial value of the gain
λ_x	0.05	weight on the output gap in central bank loss

Main sources: Chari et al 2000, Woodford 2003, Nakamura & Steinsson 2008

Calibration - matching moments

ψ_x	0.3	coefficient of the output gap in Taylor rule
σ_r	0.01	standard deviation, natural rate shock
σ_i	0.01	standard deviation, monetary policy shock
σ_u	0.5	standard deviation, cost-push shock
$\hat{\gamma}_i$	(0.82; 0.61; 0; 0.33; 0.45)	coefficients in anchoring function

Estimated form for $\mathbf{g}(\cdot)$

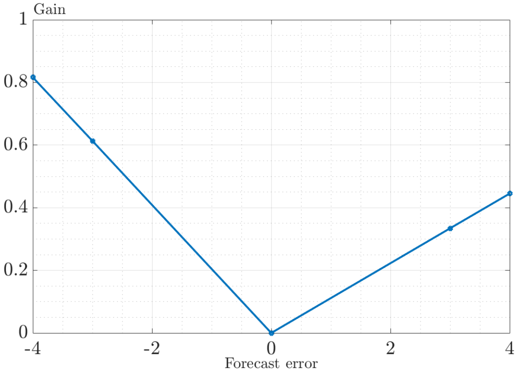
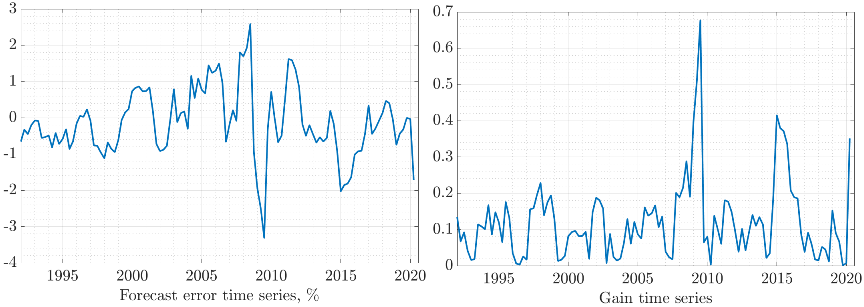


Figure: Gain as a function of forecast errors in inflation in %

Forecast errors in the data

Figure: Time series of 1-year ahead forecast errors and implied gain in the SPF



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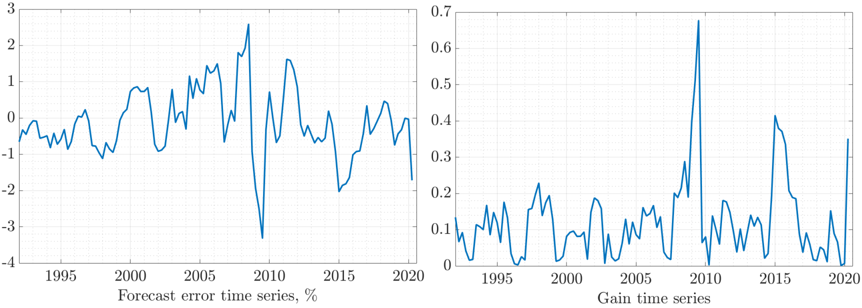

Mean gain ≈ 0.12

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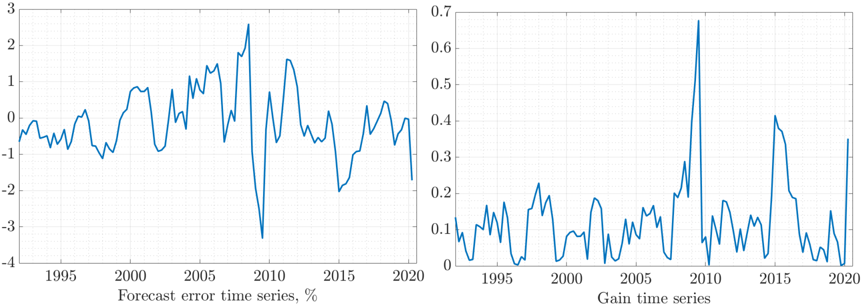

Mean gain ≈ 0.12 $\quad \rightarrow$ discount forecast errors older than 8 quarters

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Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

- $\mathbb{E}$ is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

Target criterion

Proposition

In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t$$
$$+ \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - f_{t+1+j|t+j} \mathbf{g}_{\bar{\pi},\mathbf{t+j}}) \right)$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t*, and* b_1 *is the first row of* b*.*

▸ General case

Responding to cost-push shocks

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - f_{t+1+j|t+j} \mathbf{g}_{\bar{\pi},\mathbf{t+j}}) \right)$$

Intratemporal tradeoffs in RE (discretion)

Responding to cost-push shocks

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Postpone current tradeoff to future as long as gain > 0

Responding to cost-push shocks

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Extent to which can postpone depends on not unanchoring too much in future

Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

▸ Why no commitment?

Corollary

Optimal policy under adaptive learning is time-consistent.

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Solution procedure

Solve system of model equations + target criterion

$\hookrightarrow$ solve using parameterized expectations (PEA)

$\hookrightarrow$ obtain a cubic spline approximation to optimal policy function

Optimal policy - responding to unanchoring

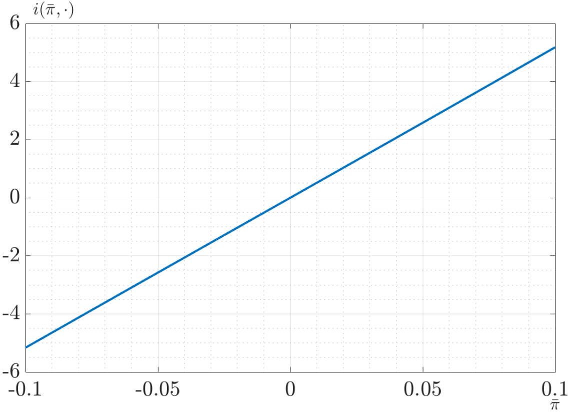

Figure: Policy function: $i(\bar{\pi}$, all other states at their means)

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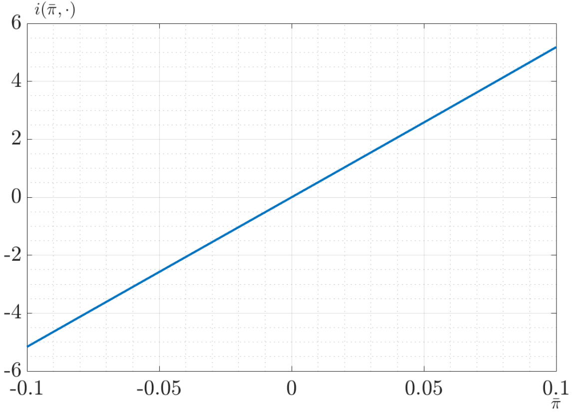
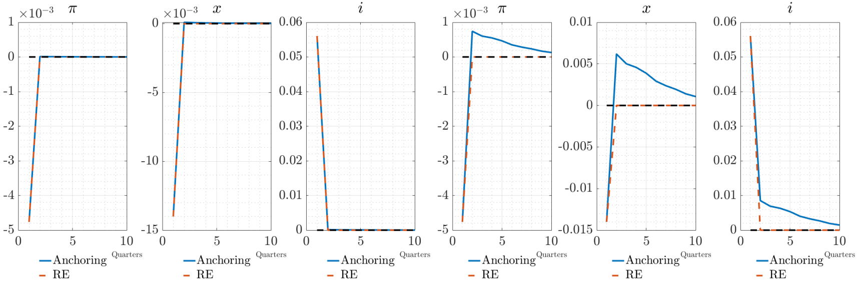


Figure: Policy function: $i(\bar{\pi}$, all other states at their means)

$\rightarrow$ For 5 bp drop in $\bar{\pi}$, lower i by 2.5 pp

Unanchoring causes volatility



(a) Expectations anchored (b) Expectations unanchored

Figure: Impulse responses after a contractionary monetary policy shock when policy follows a Taylor rule

Why so volatile? Term structure of expectations

IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big)$$

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Why oscillatory? Intertemporal anticipation effects

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- Additional channel of policy

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- Additional channel of policy
- Only if policy reaction function internalized

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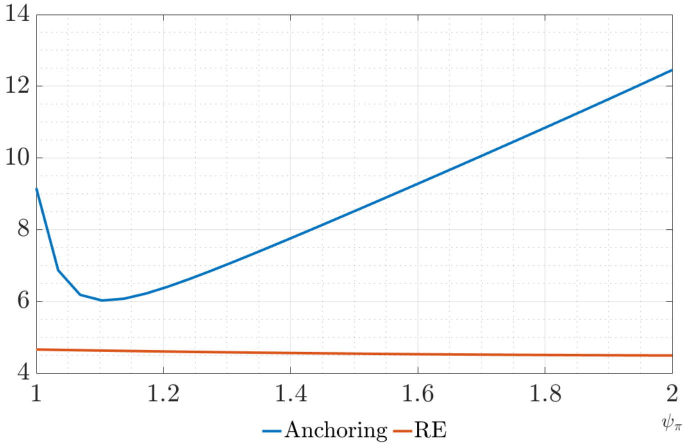
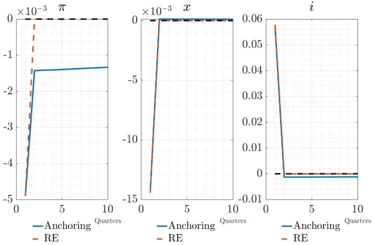
Optimal Taylor-coefficient on inflation

Figure: Central bank loss as a function of ψ_π

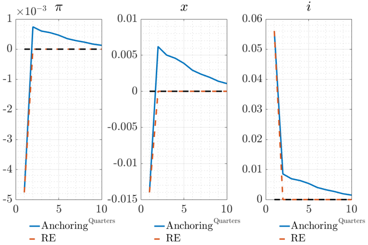


Anchoring-optimal coefficient: $\psi_\pi^A = 1.09$ RE-optimal coefficient: $\psi_\pi^{RE} = 2.21$

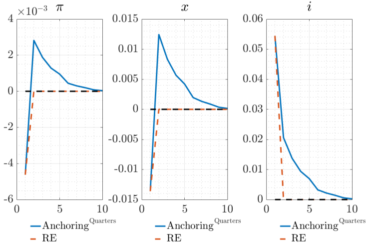
Respond but not too much



(a) $\psi_\pi = 1.01$



(b) $\psi_\pi = 1.5$



(c) $\psi_\pi = 2$

Conclusion

First theory of monetary policy for potentially unanchored expectations

Estimation of unanchoring in the data

- Large and negative surprises unanchor more
- Estimated gain time series: on average, people only use the last 8 quarters of data

Monetary policy

- Expectations unanchoring makes smoothing shocks over time possible
- Optimal policy frontloads aggressive interest rate response to suppress potential unanchoring
- Taylor rule less aggressive than under rational expectations

Future work

$\hookrightarrow$ How to anchor at zero-lower bound?

$\hookrightarrow$ Other applications: currency crises

Appendix

Breakeven inflation

Figure: Market-based inflation expectations, various horizons, %

Return

Correcting the TIPS from liquidity risk

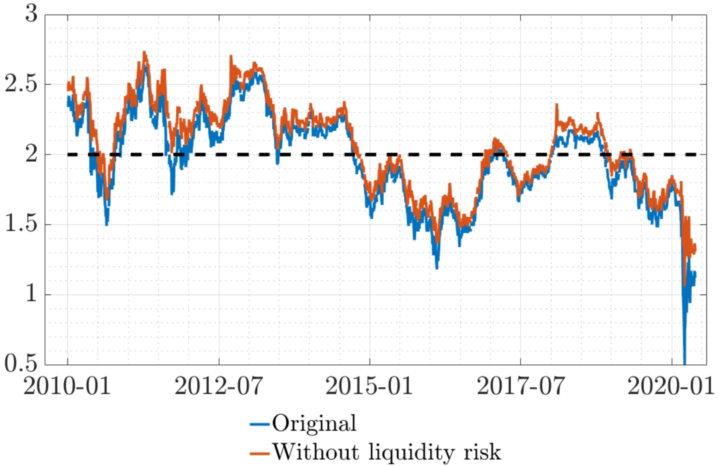


Figure: Market-based inflation expectations, 10 year, %

Return

Further evidence

Figure: Livingston Survey of Firms:
Interquartile range of 10-year ahead inflation expectations

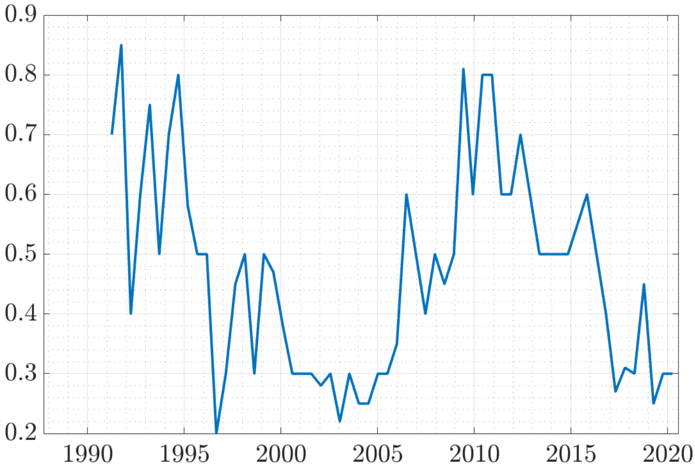


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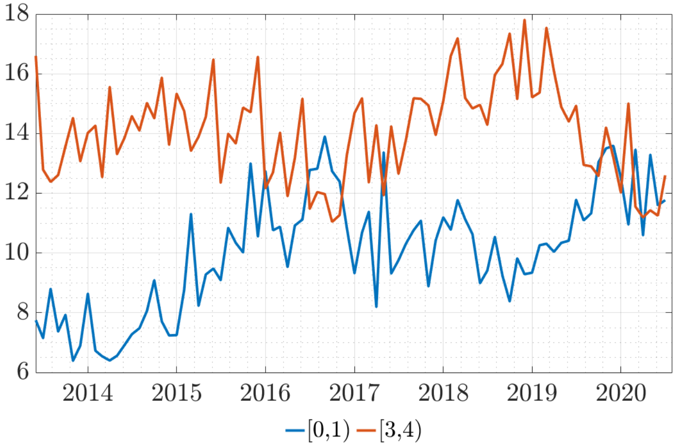
Figure: New York Fed Survey of Consumers:
Percent of respondents indicating 3-year ahead inflation will be in a particular range



Return

Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{20}$$
$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{21}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \tag{22}$$

Solve for forecast error $f_t \equiv \pi_t - f_{t-1}$:

$$f_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k\to 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \tag{23}$$

Functional forms for **g** in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \tag{24}$$

$p \equiv Prob$(liquidity trap regime)
y_{t-1} output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{25}$$

θ_t criterion, $\bar{\theta}$ threshold value

◀ Return

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max|\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \tag{26}$$

Σ variance-covariance matrix of shocks
$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(f_{t|t-1} f'_{t|t-1} - \omega_{t-1}) \tag{27}$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(f'_{t|t-1} \omega_t^{-1} f_{t|t-1} - \theta_{t-1}) \tag{28}$$

ω_t estimated forecast-error variance

Return

Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}\right)'\right)' \tag{29}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{30}$$

Return

Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{31}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{32}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \tag{33}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{34}$$

Return

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t)$$
$$+ \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \Big\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t + \mathbf{g}_\pi(\pi_t - f_{t-1})) = 0 \tag{35}$$
$$-2\beta\frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_\mathbf{f}(\pi_{t+1} - f_t)) = 0 \tag{36}$$

Return

Target criterion system for anchoring function as changes of the gain

$$
\begin{aligned}
\varphi_{6,t} &= -cf_{t|t-1}x_{t+1} + \left(1 + \frac{f_{t|t-1}}{f_{t+1|t}}(1-k_{t+1}) - f_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1} \\
&\quad - \frac{f_{t|t-1}}{f_{t+1|t}}(1-k_{t+1})\varphi_{6,t+2} \qquad (37) \\
0 &= 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{f_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{f_{t|t-1}}\varphi_{6,t+1} \qquad (38)
\end{aligned}
$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (38) is given by:

$$
\varphi_{6,t} = -2\,\mathbb{E}_t \sum_{i=0}^{\infty}(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{f_{t+j|t+j-1}}}{\frac{k_{t+j}}{f_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \qquad (39)
$$

Return

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{40}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{41}$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \tag{42}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{43}$$

Return

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{44}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{45}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (44) backward to some date t, take expectations at t
2. Sub in (45)
3. Aggregate over households i

$\rightarrow$ Obtain (15)

Return