

Materials 36 - Convince that estimation is robust

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1 Calibration issues

1.1 $\alpha = \text{Prob}(\text{keep same price}) = ?$

So far I've used 0.5.

- Nikolay Hristov notes: expected duration of contract = $\frac{1}{1-\alpha}$ periods.
- Evidence on average duration of prices:
 - Bils & Klenow (2004): 4.3 months
 - Klenow & Kryvstov (2008): mean 7-9 months, median 4-7 months
 - Nakamura & Steinsson (2008): 7-9 months
 - Klenow & Malin (2010): 6.9 months
 - Eichenbaum, Jaimovitch & Rebelo (2008, published as 2011): 10.6 months

→ On average this gives us 7.56 months, a little more than two quarters. The implied $\alpha \approx 0.6$.

Rotemberg & Woodford (1997) calibrate $\alpha = 0.66$. To err on the flexible price side, I set $\alpha = 0.5$.

1.2 The composite parameter κ once and for all

I've used $\kappa = \frac{(1-\alpha\beta)}{\alpha}\zeta$ where I should have used $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\zeta$. In Preston (2005), and I think this is also Woodford's favorite specification, $\zeta = \frac{\omega+\sigma^{-1}}{1+\omega\theta}$. Let's define terms:

- α : Prob(keep price unchanged)
- β : discount factor
- ζ : measure of strategic complementarity in price setting. The smaller ζ , the more complementarity. This depends on a bunch of things:
 - homogenous vs. specific factor markets ($s_y = 0$ or not)
 - constant vs. variable desired markup ($\epsilon_\mu = 0$ or not)
 - no vs. intermediate inputs ($s_m = 0$ or not)

In particular (Prop 3.3 in Woodford 2011, equation 1.43, Chapter 3, p. 171):

$$\zeta = \frac{(1 - \mu s_m)(s_y + s_Y)}{1 + \theta[\epsilon_\mu + (1 - \mu s_m)s_y]} \quad (1)$$

with

- θ : price elasticity of demand
- $\mu(x)$: markup function
- ϵ_μ : elasticity of markup function (how much do target markups change at different levels of output)
- $s(y, Y, \xi)$: real marginal cost function
- s_y : elasticity of real marginal cost function wrt firm i's output, $y_t(i)$
- s_Y : elasticity of real marginal cost function wrt aggregate output, y_t
- s_m : elasticity of real marginal cost function wrt intermediate inputs, $m_t(i)$

Then expression $\zeta = \frac{\omega + \sigma^{-1}}{1 + \omega\theta}$ is obtained by assuming no intermediate inputs, constant desired markups wrt. output levels and specific factor markets, so that

$$\zeta = \frac{s_y + s_Y}{1 + s_y\theta} \quad (2)$$

What is ω ? It's the derivative of the MC function wrt own output, but this only coincides with s_y for specific factor markets. Woodford shows that for specific factor markets, $s_y = \omega$, $s_Y = \sigma^{-1}$, while for common factor markets $s_Y = \omega + \sigma^{-1}$ because for the latter, there is no distinction between own and aggregate output for the purpose of wage setting and thus marginal cost. So, more broadly, ω is a measure of how marginal cost reacts to some wage-relevant measure of output. Woodford, Chapter 3, (1.16), p. 152:

$$\omega = \underbrace{\omega_w}_{= \eta, \text{ Frisch elasticity of disutility of labor wrt output}} + \underbrace{\omega_p}_{\text{elasticity of MPL wrt output}} \quad (3)$$

Denoting the Frisch elasticity as η , and noting that for Cobb-Douglas, $\partial MPL / \partial y_t = 0$,

$$\omega = \eta \quad (4)$$

- Chari, Kehoe & McGrattan (2000) and Woodford (2011) values: $\theta = 10, \sigma = 1, \omega = 1.25, \beta = 0.99$
These values are not controversial. I just want to check that the Frisch elasticity is Kosher, because by setting $\omega = 1.25$, we are implicitly setting the Frisch. According to Susanto (compare my summaries Part 1, p. 56 Mac and Part 2 p. 46-47 Mac), the inverse Frisch elasticity, $\varepsilon_{H,W} = \eta^{-1}$ needs to be 4 to flatten the labor supply curve (Barro-King comovement puzzle), but estimates suggests it's < 1 . Here, $\omega = 1.25 = \eta$ implies $\eta^{-1} = 4/5 < 1$. I guess that's at least in line with estimates.

- The value of κ

Susanto suggests (somewhere in my notes) that for a NK model to display reasonable dynamics, κ needs to be below 0.1, but preferably even less than 0.01.

Chari, Kehoe & McGrattan (2000) and Woodford (2011) values with $\alpha = 0.5 \rightarrow \kappa = 0.0842$.

Chari, Kehoe & McGrattan (2000) and Woodford (2011) values with $\alpha = 0.6 \rightarrow \kappa = 0.0451$.

Chari, Kehoe & McGrattan (2000) and Woodford (2011) values with $\alpha = 0.66 \rightarrow \kappa = 0.0298$.

Chari, Kehoe & McGrattan (2000) and Woodford (2011) values with $\alpha = 0.7 \rightarrow \kappa = 0.0219$.

2 Back to estimation

2.1 Simulated data with different seed

Figure 1: A seed for shocks of `rng(1)` when true data was generated using `rng(0)`

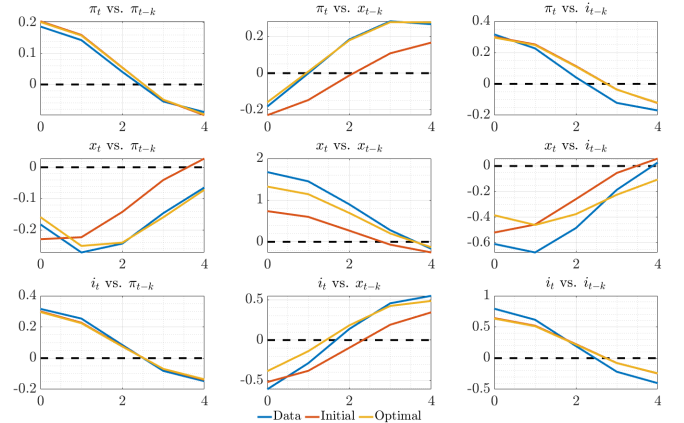
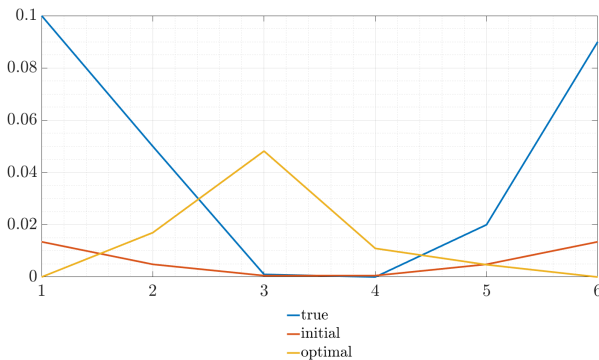
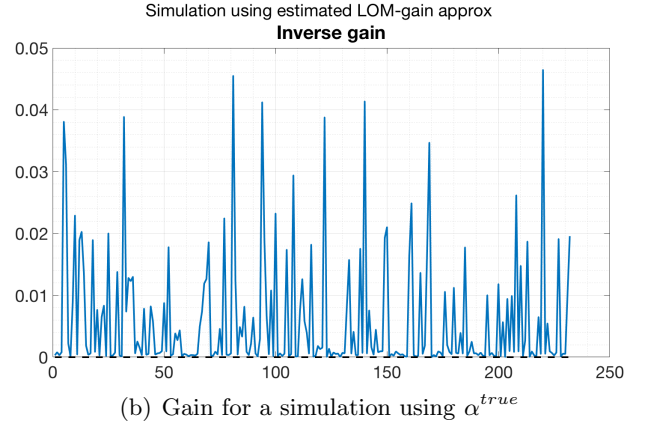
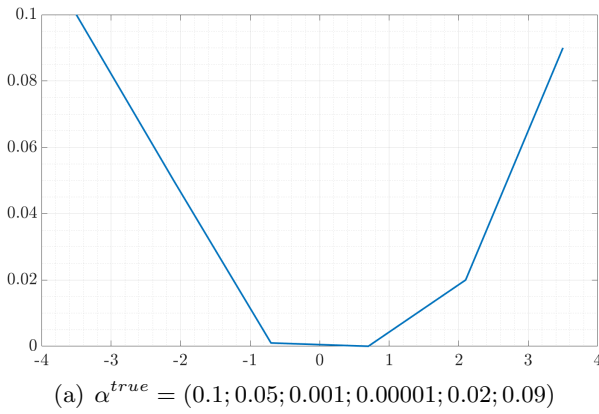
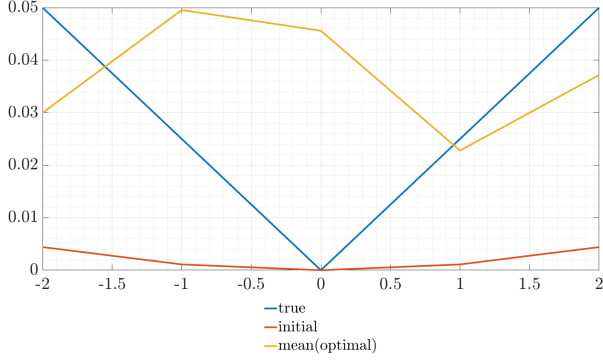
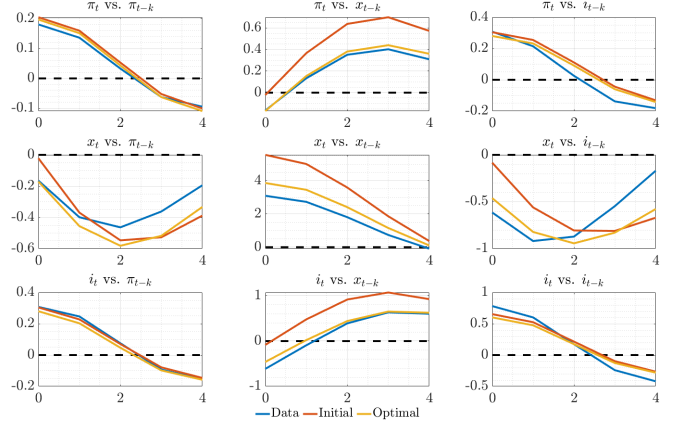


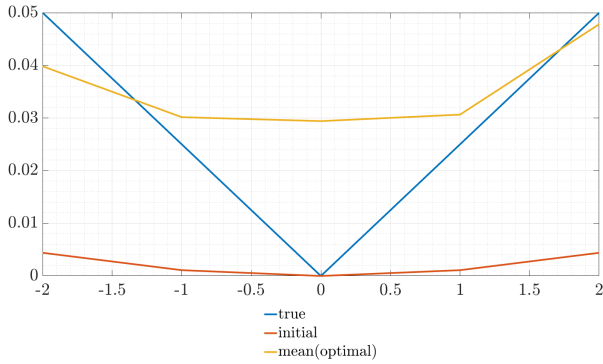
Figure 2: Mean of top 10 estimates for 20 draws of shock histories when true data was generated using `rng(0)` and $\alpha^{true} = (0.05; 0.025; 0; 0.025; 0.05)$, $f \in (-2, 2)$



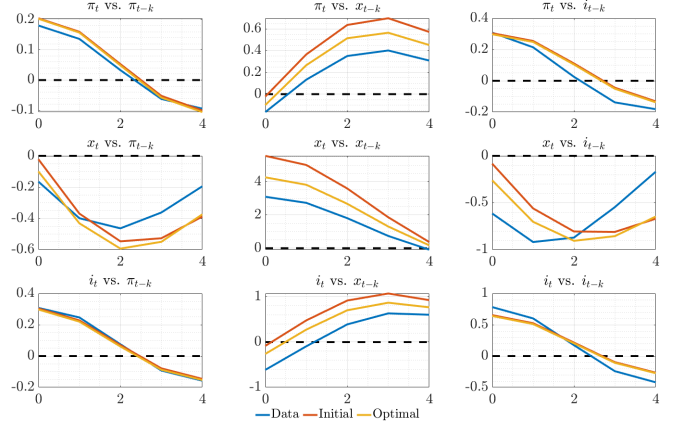
(a) $\alpha^{true}, \alpha_0, \text{mean}(\hat{\alpha}^{top10})$, no additional moments



(b) Autocovariogram, no additional moments

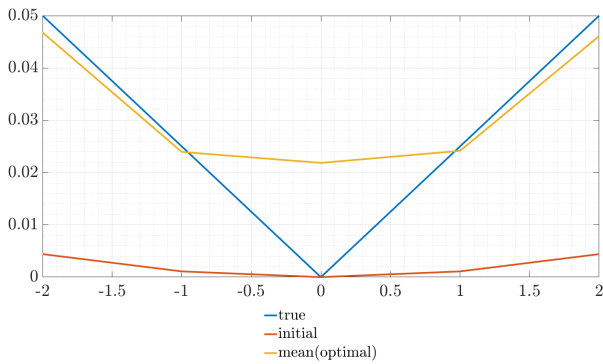


(c) $\alpha^{true}, \alpha_0, \text{mean}(\hat{\alpha}^{top10})$, convexity moments imposed with weight 10M

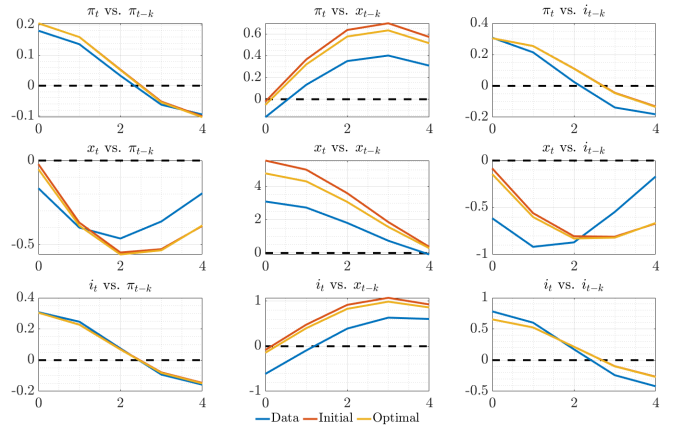


(d) Autocovariogram, convexity moments imposed with weight 10M

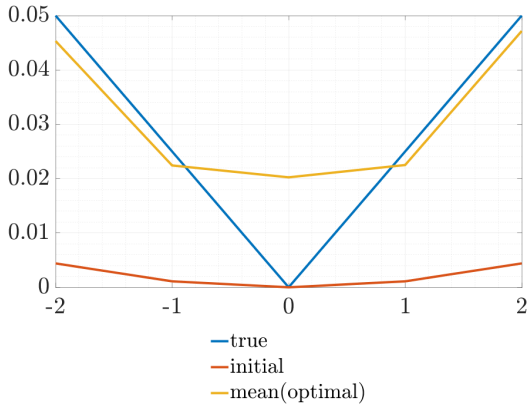
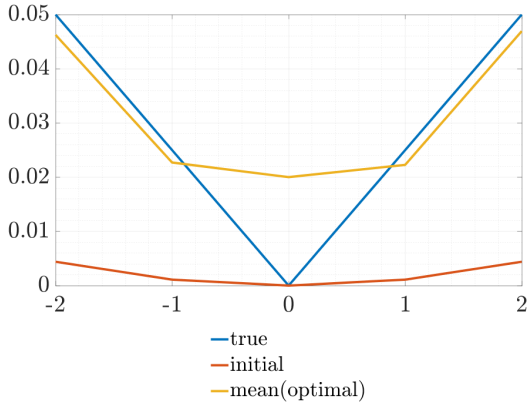
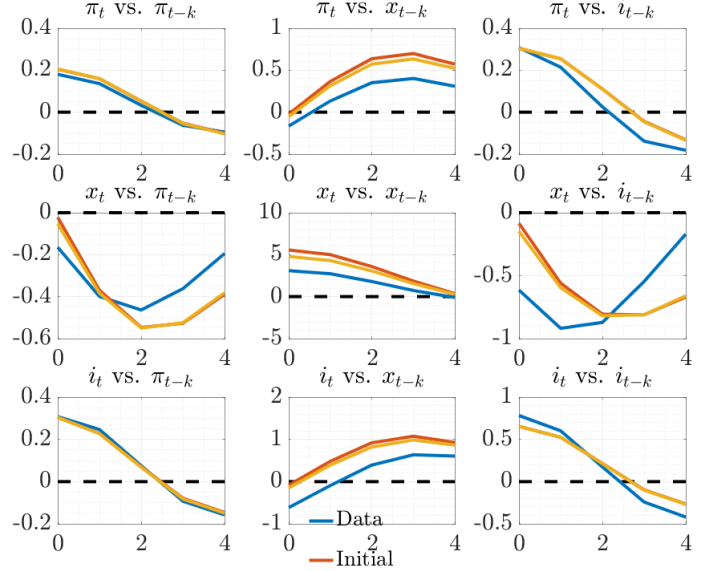
Figure 3: Mean estimates for increasing N , imposing convexity



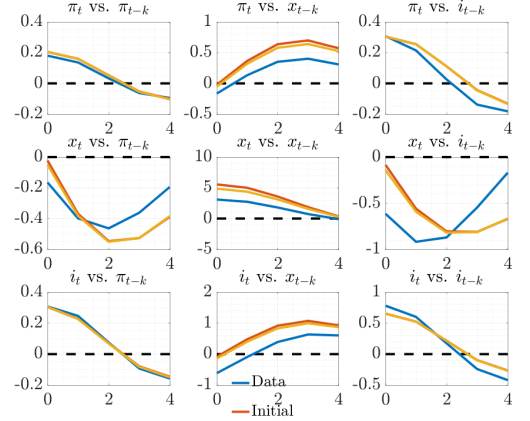
(a) $\alpha^{true}, \alpha_0, \text{mean}(\hat{\alpha})$, $N=1000$, convexity moments imposed with weight 10M



(b) Autocovariogram, convexity moments imposed with weight 10M

Figure 4: Mean estimates for increasing N , imposing convexity, continued

 (a) $\alpha^{true}, \alpha_0, \text{mean}(\hat{\alpha})$, $N=2000$, convexity moments imposed with weight 10M

 (c) $\alpha^{true}, \alpha_0, \text{mean}(\hat{\alpha})$, $N=10000$, convexity moments imposed with weight 10M, $\text{mean}(\hat{\alpha}) = 0.0463; 0.0227; 0.02; 0.0223; 0.0469$


(b) Autocovariogram, convexity moments imposed with weight 10M



(d) Autocovariogram, convexity moments imposed with weight 10M

2.1.1 The range $fe \in (-1, 1)$ seems ill identified

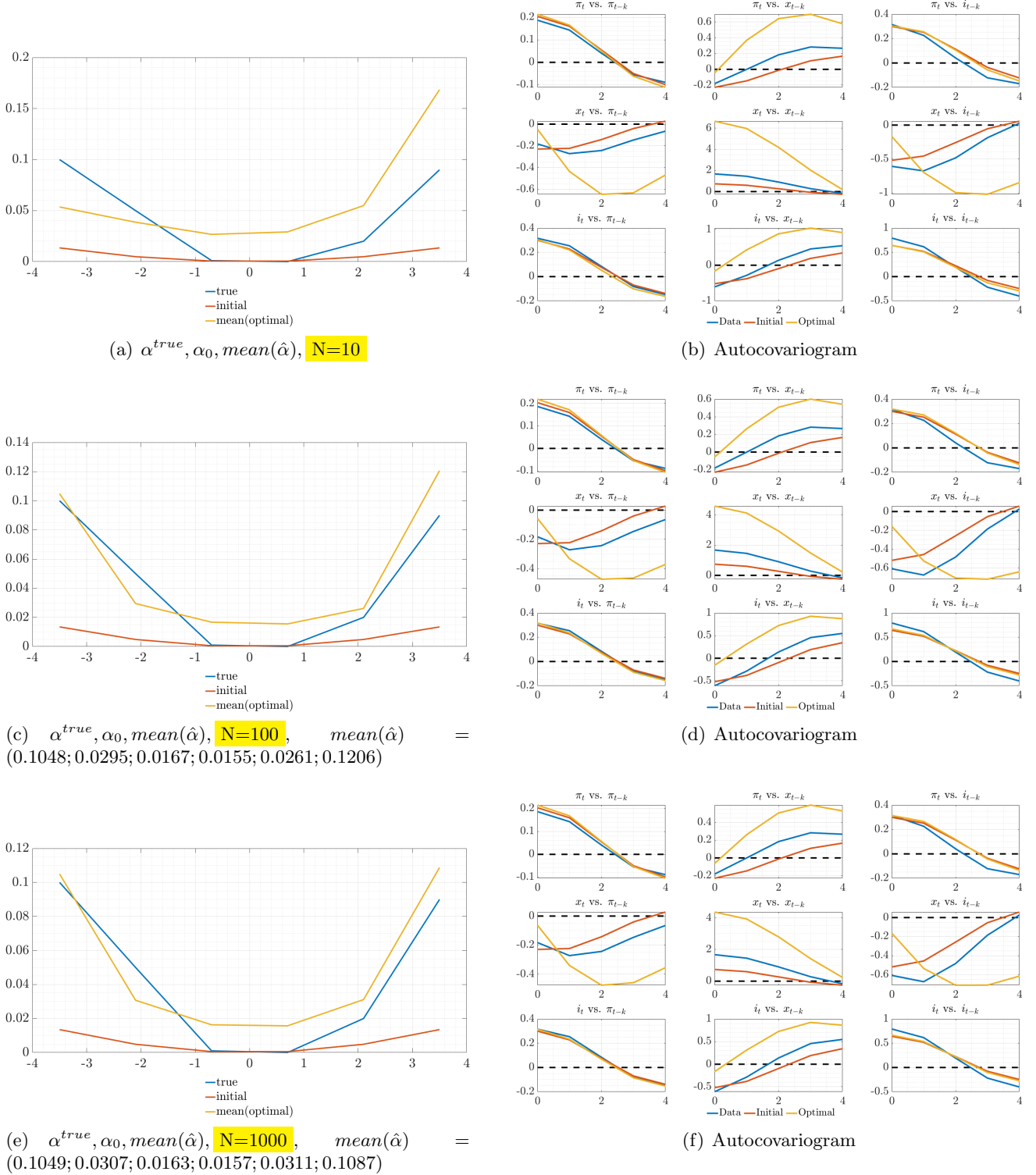
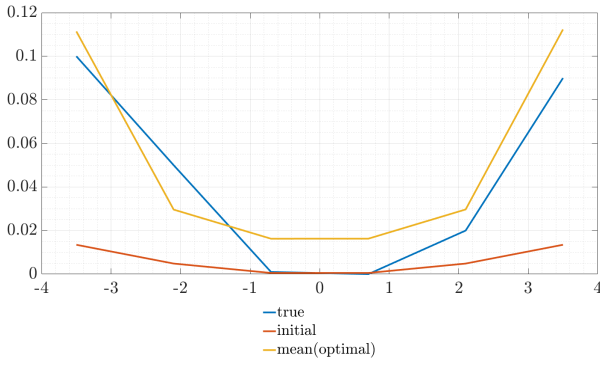
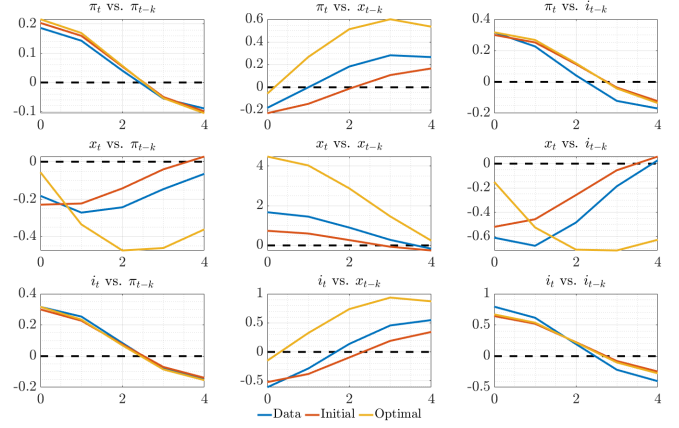
Figure 5: Mean estimates for increasing N , imposing convexity with weight 100K, truth with $nfe = 6, fe \in (-3.5, 3.5)$


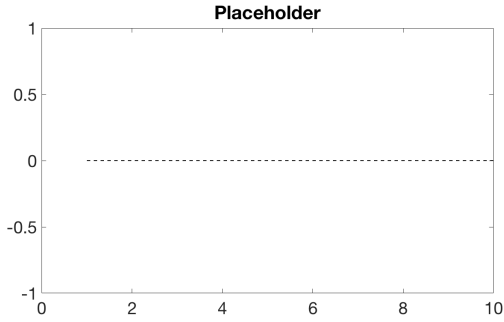
Figure 6: Mean estimates for increasing N , imposing convexity with weight 100K, truth with $nfe = 6$, $fe \in (-3.5, 3.5)$, continued



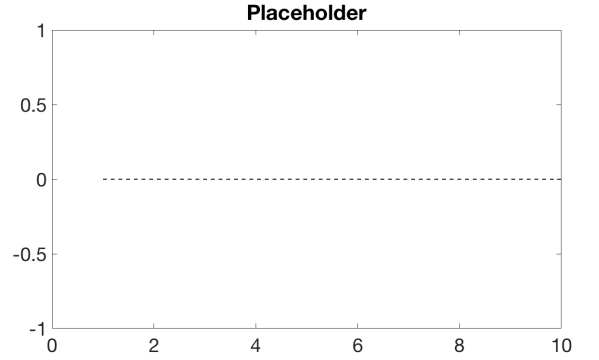
(a) $\alpha^{true}, \alpha_0, mean(\hat{\alpha}), N=10000$, $mean(\hat{\alpha}) = (0.1114; 0.0296; 0.0163; 0.0163; 0.0297; 0.1123)$



(b) Autocovariogram



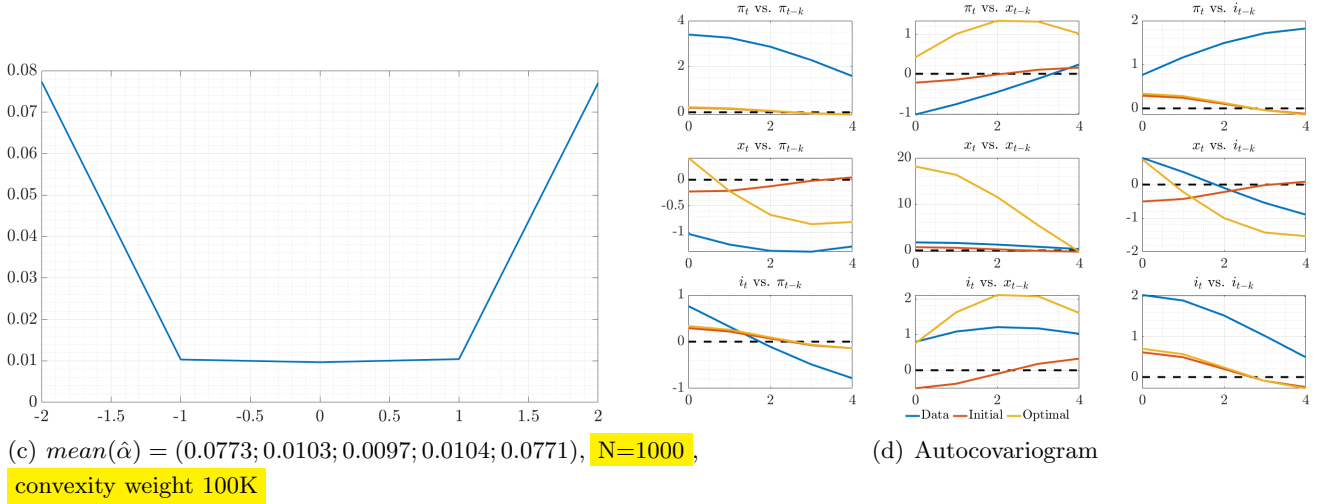
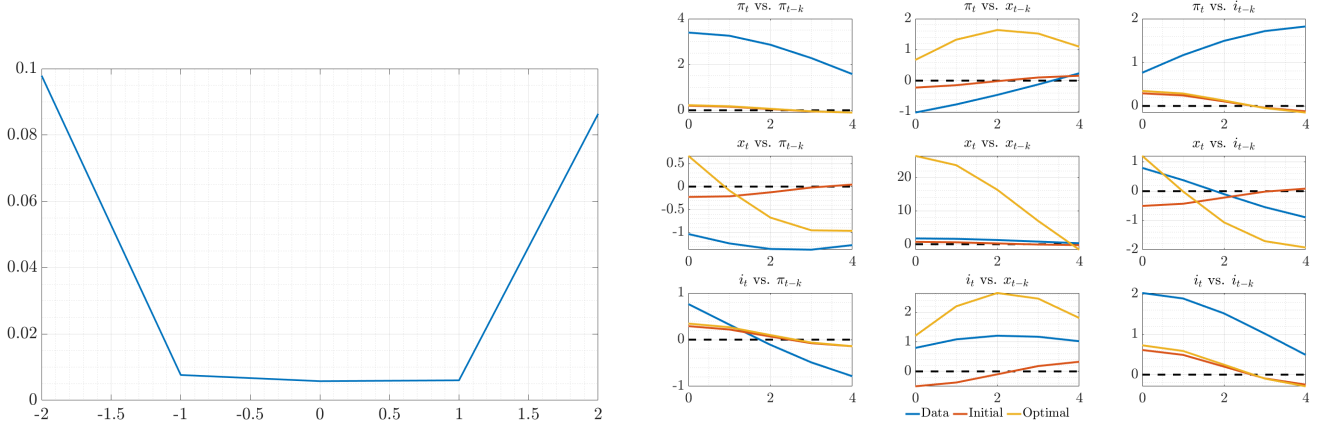
(c) $\alpha^{true}, \alpha_0, mean(\hat{\alpha}), N=100000$, $mean(\hat{\alpha}) = ()$



(d) Autocovariogram

2.2 Autocovariogram for real data

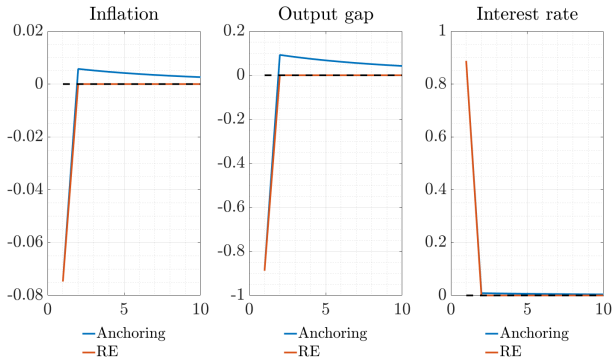
Figure 7: Mean estimated parameters over the cross-section of size N , convexity imposed, mean moment not imposed



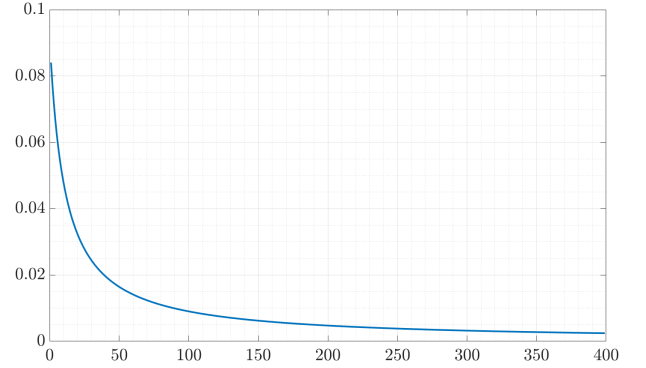
3 Impulse responses to iid monopol shocks across a wide range of learning models

$T = 400, N = 100, n_{drop} = 5$, shock imposed at $t = 25$, calibration as above, Taylor rule assumed to be known, PLM = learn constant only, of inflation only.

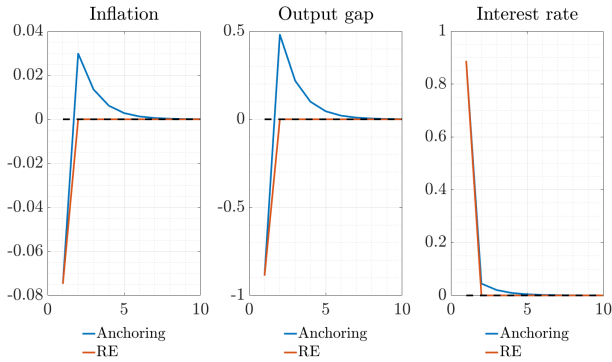
Figure 8: IRFs and gain history (sample means)



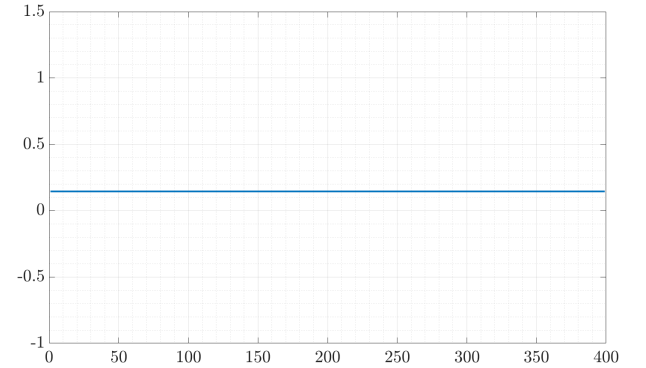
(a) Decreasing gain learning



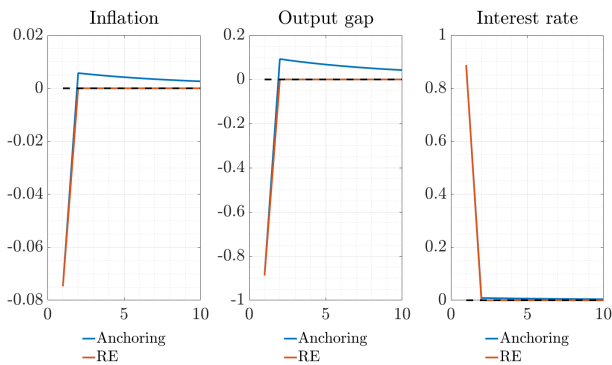
(b) Mean gain



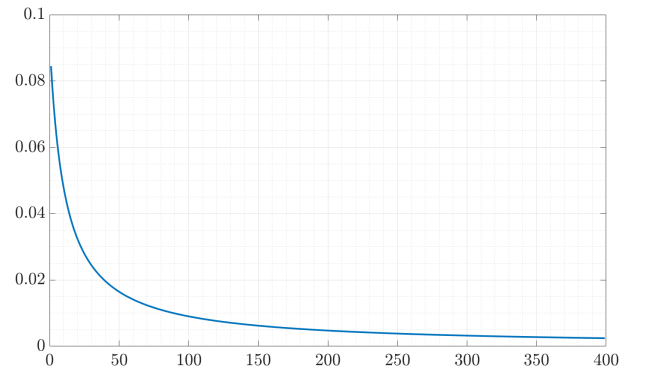
(c) Constant gain learning



(d) Mean gain

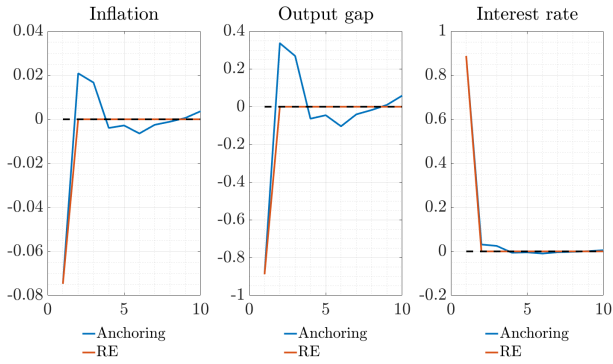


(e) CEMP criterion (vector)

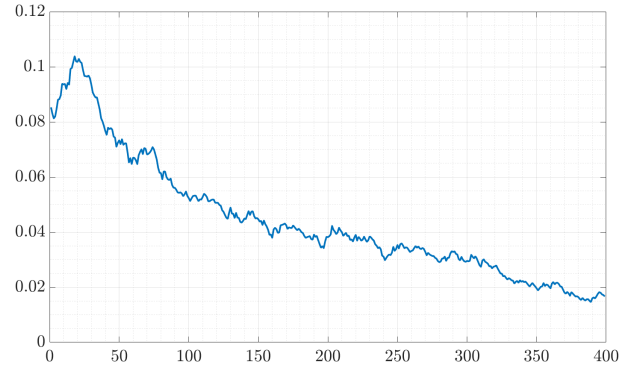


(f) Mean gain

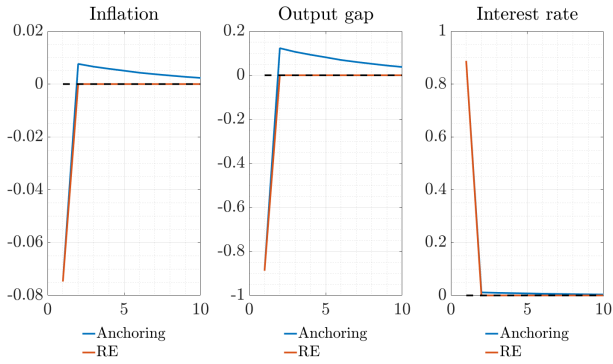
Figure 9: IRFs and gain history (sample means), continued



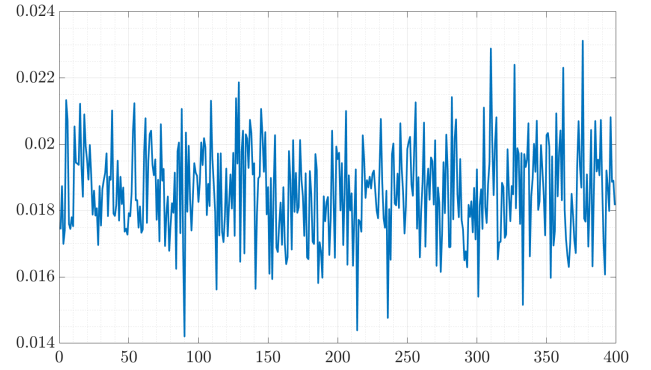
(a) CUSUM criterion (vector)



(b) Mean gain



(c) Smooth criterion, approximated, using $\alpha^{true} = (0.05; 0.025; 0; 0.025; 0.05)$, on $fe \in (-2, 2)$.



(d) Mean gain

A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (\text{A.1})$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (\text{A.2})$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \quad (\text{if imposed}) \quad (\text{A.3})$$

$$\text{PLM:} \quad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \quad b = g_x h_x \quad (\text{A.4})$$

$$\text{Updating:} \quad a_t = a_{t-1} + k_t^{-1} (z_t - (a_{t-1} + b s_{t-1})) \quad (\text{A.5})$$

$$\text{Anchoring function:} \quad k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k f e_{t-1}^2 \quad (\text{A.6})$$

$$\text{Forecast error:} \quad f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \quad (\text{A.7})$$

$$\text{LH expectations:} \quad f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \quad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \quad (\text{A.8})$$

This notation captures vector learning (z learned) for intercept only. For scalar learning, $a_t = (\bar{\pi}_t \ 0 \ 0)'$ and b_1 designates the first row of b . The observables (π, x) are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix} \sigma & 1-\beta & -\sigma\beta \end{bmatrix} f_b + \sigma \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \quad (\text{A.9})$$

$$\pi_t = \kappa x_t + \begin{bmatrix} (1-\alpha)\beta & \kappa\alpha\beta & 0 \end{bmatrix} f_a + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \quad (\text{A.10})$$

B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only, $k_t^{-1} = \mathbf{g}(f e_{t-1})$):

$$\pi_t = -\frac{\lambda_x}{\kappa} \left\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_\pi(t) \right) \right. \\ \left. \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\bar{\pi}}(t+j)) \right) \right\} \quad (\text{B.1})$$

where I'm using the notation that $\prod_{j=0}^0 \equiv 1$. For interpretation purposes, let me rewrite this as follows:

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ - \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j}^{-1} + f e_{t+1+j|t+j}^{eve} \mathbf{g}_{\bar{\pi}}(t+j)) \right) \quad (\text{B.2})$$

Interpretation: **tradeoffs from discretion in RE** + **effect of current level and change of the gain on future tradeoffs** + **effect of future expected levels and changes of the gain on future tradeoffs**