Cont'-ing w/ the estimation 25 July 2020

- despite adding SPF, still not identified
so I'm really thinking it must be a code issue
E.g. the avg moments still depend on shocks
when N=100? Can that be?

Also start taking notes on cleaning out
the TIPS from a liq. premium in blue,

and notes on how to include the welfare
mistake you make when you ass. RE
instead of anchoring to the paper in brown.

Let's define some terms. 26 July 2020

Breakeven inflation = difference btw nominal & real yields
of the same maturity

Fisher: $r = i - \pi \Rightarrow \pi = i - r$

$\Rightarrow \pi^{be} = i^{Tbill} - r^{TIPS}$

Andreasen et al say that

1. pos. bias in r^{TIPS} $\rightarrow$ neg. bias in π^{be}.

So the idea is

$$\pi^{be,\,true} = i^{Tbill} - (r^{TIPS} - \text{liq premium}^{TIPS})$$

Now, the FRED π^{be} series (T10YIE) is constructed as

$$\text{T10YIE} = \underbrace{\text{WGS10YR}}_{\text{10-year Treasury}} - \underbrace{\text{DFII10}}_{\text{10-Year Treasury Inflation-Indexed Security}}$$

(both constant maturity)

The question is if DFII10 is filtered for the liq. premium or not. Likely not.

A quick check for the liq premium could be

what Andreasen's model is able to match well:
the "model-free measure of the TIPS liq. premium, ... the difference between inflation-swap rates and TIPS-breakeven inflation."

<u>inflation-swap</u> := an agreement btwn 2 counter-parties to swap fixed-rate payments for a floating-rate payment linked to inflation. i.e. to swap a fixed-rate to an inflation-indexed rate payment!

It sounds like the swap rate is a measure of expected inflation too, which is why

$$\pi^{be} \stackrel{!}{=} \pi^{swap}$$

so that if they're not equal, then

$$(\pi^{be} + \text{liq prem}) = \pi^{swap} \Rightarrow \pi^{swap} - \pi^{be}$$

$=$ liq. premium.

This would be great if π^{swap} was a good measure of π-Exp. But it's not b/c the swap market is, although growing, very small. Still it's quite liquid (ho?) (Fleming & Sporn, 2013)

Investopedia: a zero-coupon inflation swap (ZCIS) — simplest form — is also known as a breakeven inflation swap

But it doesn't even seem like 27 July 2020

π-swap data is publicly accessible.
(A Cleveland Fed paper has it from Bloomberg, but I guess you need an account.)

Haubrich et al emphasize that risk premia may also contaminate things, and I'm scared that Andreasen et al ignore that.

So cont. w/ Andreasen et al.

liq. risk = identified as the difference between prices of principal & coupon payments

The Andreasen et al ATSM model (Section 3)

(affine term structure model

$$r_t^N = \rho_0^N + (\rho_x^N)' X_t$$

nominal short rate — scalar — $N \times 1$ — N pricing factors

X_t evolves as

$$dX_t = \underset{N \times N}{K_x^Q} (\underset{N \times 1}{\theta_x^Q} - X_t)\,dt + \underset{N \times N}{\Sigma_x} \underset{N \times N}{\sqrt{S_{x,t}}}\, d\bar{W}_t^Q$$

where

$\bar{W}_t^Q$ is a standard Wiener process

$$[S_{x,t}]_{k,k} = \underset{N \times 1}{\delta_{0\,k,k}} + \underset{N \times 1}{\delta_{x,k}'} X_t$$

Price of nominal zero-coupon bond maturing at time $t+\tau$

$$P_t^N = \exp\{A^N(\tau) + B^N(\tau)' X_t\} \qquad (3)$$

some known ODEs

In principle, TIPS (or other real bonds) could be priced like this, but that's not a good assumption given the low liquidity of the tips market.

So instead they ass that liq. costs are present, and in fact in the following form:

$$r_t^{R,i} = \rho_0^R + (\rho_x^R)' X_t + \underbrace{h(t-t_0;\, i)}_{\text{increasing fn of time since issuance, to liq costs (latent factor)}} X_t^{liq} \qquad (4)$$

a time-varying

$Z_t \equiv [X_t', X_t^{liq}]$ we have an extended state vector which evolves according to a Wiener process. (5)

→ Price of a real zero-coupon bond maturing at T is:

$$P^{R,i}(t_0, t, T) = \exp\{A^{R,i}(t_0, t, T) + B^{R,i}(t_0, t, T)' Z_t\} \quad (6)$$

where A & B are given ODEs.

⇒ implied breakeven inflation rate from (3) & (6)

$$-\frac{1}{\tau} \log P_t^N(\tau) - \Big(-\frac{1}{\tau} \log P_t^{R,i}(t_0, t, \underbrace{t+\tau}_{=:T})\Big)$$

(which is a fancy way of saying $E\pi = i - r$)

$$= \frac{1}{\tau}\left[A^{R,i}(t_0, t, T) - A(\tau) - B(\tau)' X_t + B^{R,i}(t_0, t, T) \begin{bmatrix} X_t \\ X_t^{liq} \end{bmatrix} \right]$$

Section 3.2. A Gaussian version of the ATSM w/ liquidity risk w/ closed-form expressions (!) for liquidity adjusted real prices

$$r_t^N = \underbrace{L_t^N}_{\text{level factor}} + \underbrace{S_t}_{\text{slope factor}} \quad (5)$$

for the real rate:

$$r_t^{R,i} = L_t^R + \alpha^R S_t + \underbrace{\underset{>0}{\beta^i}\left(1 - e^{-\underset{\geq 0}{\lambda^{L,i}}(t-t_0)}\right)}_{\text{functional form for } h(t-t_0;\, i)} X_t^{liq} \quad (10)$$

α^R: scaled EIR

Interpretation of β^i and $\lambda^{L,i}$

Trading of TIPS happens in 2 phases:

Phase 1: bond i just issued, high supply but also high demand (low liquidity risk)

Phase 2: buy-and-hold investors have acquired their share of TIPS i, are sitting on them contentedly and the supply of bonds i for trading is scarce (high liq. risk)

$\lambda^{L,i}$ = determines length of Phase 1; a low $\lambda^{L,i}$ implies a long Phase 1, less exposure to X_t^{liq}.

β^i = determines maximal exposure of i to X_t^{liq} in Phase 2.

Now, $z_t = [L_t^N, S_t, C_t, L_t^R, X_t^{liq}]'$

$$dz_t = \underbrace{\begin{bmatrix} K_x^Q & 0_{4\times 1} \\ 0_{1\times 4} & K_{liq}^Q \end{bmatrix}}_{\substack{K_z^Q \\ 4\times 5}} \left(\underbrace{\begin{bmatrix} 0_{4\times 1} \\ \theta_{liq}^Q \end{bmatrix}}_{\substack{\theta_z^Q \\ 5\times 1}} - z_t \right) + \Sigma_z \, dW_t^Q \quad (11)$$

$$\underset{4\times 4}{K_x^Q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda & -\lambda & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Actually, then the model (β^i, $\lambda^{L,i}$ and a bunch of other things for all TIPS $i = 1, \dots n_{TIPS}$) is estimated w/ an extended Kalman filter where you back out X_t^{liq} as a filtered state..

na ne!

→ So let's try to understand the behavior of the liq premium and try to argue that it's not driving the dynamics of my figure 1.

p. 26 Mac

1. The liq premium on TIPS is low and falling in general b/c i) the market is growing ii) dealers are expanding their TIPS-trading
2. The TIPS liq premium is higher in recessions, e.g. 2001 9/11 and 2008, but stabilizes afterwards (also partly thanks to QE)
3. The 10-year TIPS exhibits a lower average liq premium and a less volatile one than the rest of the TIPS market

$$\text{mean}\left(\psi_t^{10y}\right) = 30 \text{ basis points} \quad \text{vs} \quad \text{mean}(\psi_t) = 38$$

$$\text{sd}\left(\psi_t^{10y}\right) = 13 \text{ bp} \qquad \text{sd}(\psi_t) = 34$$

4. Older TIPS have a higher liq premium, but QE

mainly lowers their liq premium since the Fed mainly bought TIPS which were issued long ago.

Their dataset goes till Dec. 27, 2013.

Basis points $= \frac{1}{100}\%$, i.e. 0.01%

So mean$(\gamma_t^{10y}) = 30$ bp $\rightarrow 0.3\%$

If I add a sd $= 13$ bp to it, 43 bp $= 0.43\%$

So supp at 2020 Covid-shock, γ^{10y} rose by 2sd, i.e by 26 bp to 56 bp $= 0.56\%$, then π^E is downward biased by 0.56 pp, so instead of π-Exp$(\cdot)$ of 0.5%, it'd be 1%.

1pp = 100 bp

Since int. rates are usually changed by 25 bp, this liq premium is significant, but it doesn't change the message.

⑤ Corr (Ψ, VIX index) = 0.67

avg. liq premium of Andreasen et al

between 0 & 100.

And: regressing Ψ on VIX gives a sig. effect of 0.85**

$1 \uparrow$ in VIX $\rightarrow$ 0.85 bp $\uparrow$ in Ψ

In 2020: VIX $\uparrow$ by 60 $\rightarrow$ 51 bp $\uparrow$ in Ψ.

Here is a VAR(1) <u>28 July 2020</u>

$$y_t = \rho y_{t-1} + \varepsilon_t$$

What's its autocovariance at lag k?

$$E(y_t y_{t-k}) = ?$$

$$E(y_t y_{t-k}) = E(\rho y_{t-1} + \varepsilon_t)(\rho y_{t-1-k} + \varepsilon_{t-k})$$

$$E(y_t y_{t-k}) = \rho^2 E(y_{t-1} y_{t-1-k}) + \underbrace{E(\varepsilon_t \varepsilon_{t-k})}_{=0 \text{ if iid}}$$

$$\Gamma_k = \rho^2 \Gamma_k \quad \ddot{\sim}$$

Its VC-matrix is

$$E(y_t y_t) = (\rho y_{t-1} + \varepsilon_t)(\rho y_{t-1} + \varepsilon_t)$$
$$= \rho^2 E(y_{t-1} y_{t-1}) + E(\varepsilon_t \varepsilon_t)$$
$$\Sigma = \rho^2 \Sigma + Q$$

In this scalar case $\Sigma = (1-\rho^2)^{-1} Q$

→ Hamilton p. 67 Mac: j^{th} autocor of AR(1)

$$\gamma_j = E(Y_t - \mu)(Y_{t-j} - \mu)$$

Hamilton uses the MA(∞)-representation as

$$Y_t = \rho Y_{t-1} + \varepsilon_t$$
$$= \rho^2 Y_{t-2} + \rho \varepsilon_{t-1} + \varepsilon_t$$
$$\ldots = \varepsilon_t + \rho \varepsilon_{t-1} + \rho^2 \varepsilon_{t-2} + \ldots$$

$$\gamma_j = (\varepsilon_t + \rho \varepsilon_{t-1} + \rho^2 \varepsilon_{t-2} + \ldots) \qquad j=1$$
$$(\quad \underset{j}{\varepsilon_{t-1}} + \rho \underset{j-1}{\varepsilon_{t-2}} + \ldots)$$
$$= (\varepsilon_t + \rho(\varepsilon_{t-1} + \rho \varepsilon_{t+2} + \ldots))(\varepsilon_{t-1} + \rho \varepsilon_{t-2} + \ldots)$$
$$= 0 + \rho(\varepsilon_{t-1} + \rho \varepsilon_{t+2} + \ldots)^2$$
$$= \rho^j(\sigma^2 + \rho\sigma^2 + \rho^2\sigma^2 + \ldots) = \rho^j \sigma^2 (1 + \rho + \rho^2 + \ldots)$$

$$= \frac{\rho^j \sigma^2}{1-\rho^2}$$

For AR(1), the j^{th} autocor is

$$\gamma_j = \rho^j \left(\frac{\sigma^2}{1-\rho^2} \right) = F^j \cdot \Sigma \quad \checkmark$$

in the VAR(1)-notation of Hamilton,

eq. [10.2.21]

→ so it seems again that my Matlab code is fine.

→ in the multivariate VAR(1) case this is

$$\left(I_{np^2} - \underset{\uparrow \atop \rho}{F} \otimes F\right)^{-1} \cdot \underbrace{vec(Q)}_{\uparrow \sigma^2}$$

The point is, the moments seem to be computed properly.

Peter meeting 28 July 2020

1) F has to have eig (F) < 1

2) synthetic data: might be an issue of scaling
The logic of GMM $W^{-1} = \begin{bmatrix} 2 \cdot 10^{-6} & & \\ & 1 \cdot 10^{-6} & \\ & & \ddots \end{bmatrix}$

then what matters is that 1 is two times that of 2. But in Matlab, numerically, this can be so close that it screws up.
Take the smallest order of magnitude on diag and rescale w/ that so all diag el's are bigger > 1.

Could also scale the moment vector, altho careful there b/c it might cancel the multiple of W.

3) Take artificial data

4) Try a VAR(1) b/c then the VAR is more misspecied

5) Can instead of estimating the autocov & fit a parametric model to the data

↳ paper by Hansen, Hodrick, Singleton about how to compute the asymptotic std. error (i.e. W) w/o bootstrapping
↳ would require a lot of coding

6) Try again to plot the loss fct as a fct of a single α, when the others are fixed at the truth.

Then you can iteratively to do it with two, and for three just do the blue-line-yellow-line plots.

7) W^{-1}

In Matlab $\left(\begin{pmatrix} 1^{-10} & 0 \\ 0 & 100 \end{pmatrix}^{-1}\right)^{-1} \neq \begin{pmatrix} 1^{-10} & 0 \\ 0 & 100 \end{pmatrix}$

haha! So that could easily cause problems.

<u>Ryan meeting</u> <u>29 July 2020</u>

(Materials 38)

- Makes sense to explore the calibration of measurement error b/c they shouldn't be visible on the ACF.
  Shut off the moments for $E(\pi)$ and just add the meas. e. → compare ACF and see if meas. er. made things worse on its own.

- Maybe the meas. error causes a wedge between N_{estim} & N_{simul}

- Fig 7. Don't reflect (re-)scaling of W in certain respects, e.g. 10^{-5} is smaller in magn. than the others, vs. bottom panel.

Top panel rejects that model is ID b/c of flat regions.
Bottom panel ?!

Do Fig 7. of Mat 38 w/o expectations
- Why does the loss hit a hard 0?
- Why does rescaling change the shape?

↳ based on this picture and the others: a bug.

- Can only have as many moments as parameters in F & Q → so p=4 is better than p=1

- Model is non-linear, so when you est a VAR w/ fixed coeffs it might <u>not</u> be stoch. singular

The other reason is that observables depend on past lags (of shocks) which the VAR doesn't see, and those act like new shocks

What could happen though is that F is very volatile. (And close to singular.)

What you can do?

Ridge $\lambda = 0.01$ small compared to $X'X$ then singularity

meas error: add a little.