11.3 Standard Errors of Anything by Delta Method

One quick application illustrates the usefulness of the GMM formulas. Often, we want to estimate a quantity that is a nonlinear function of sample means,

$$b = \phi[E(x_t)] = \phi(\mu).$$

In this case, the formula (11.2) reduces to

$$\text{var}(b_T) = \frac{1}{T}\left[\frac{d\phi}{d\mu}\right]' \sum_{j=-\infty}^{\infty} \text{cov}(x_t, x'_{t-j}) \left[\frac{d\phi}{d\mu}\right]. \tag{11.11}$$

The formula is very intuitive. The variance of the sample mean is the covariance term inside. The derivatives just linearize the function ϕ near the true b.

For example, a correlation coefficient can be written as a function of sample means as

$$\text{corr}(x_t, y_t) = \frac{E(x_t y_t) - E(x_t)E(y_t)}{\sqrt{E(x_t^2) - E(x_t)^2}\sqrt{E(y_t^2) - E(y_t)^2}}.$$

Thus, take

$$\mu = \left[E(x_t)\ \ E(x_t^2)\ \ E(y_t)\ \ E(y_t^2)\ \ E(x_t y_t)\right]'.$$

A problem at the end of the chapter asks you to take derivatives and derive the standard error of the correlation coefficient. One can derive standard errors for impulse-response functions, variance decompositions, and many other statistics in this way.

11.4 Using GMM for Regressions

By mapping OLS regressions in to the GMM framework, we derive formulas for OLS standard errors that correct for autocorrelation and conditional heteroskedasticity of the errors. The general formula is

$$\text{var}(\hat{\beta}) = \frac{1}{T}E(x_t x_t')^{-1}\left[\sum_{j=-\infty}^{\infty} E(\varepsilon_t x_t x'_{t-j}\varepsilon_{t-j})\right]E(x_t x_t')^{-1},$$

and it simplifies in special cases.

Mapping any statistical procedure into GMM makes it easy to develop an asymptotic distribution that corrects for statistical problems such as serial