

Materials 28 - Putting approximating functions and value function iteration together

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The setup of the problem makes the separate issues I'm facing clear:

$$V(x) = \max_u p(u, x) + \beta \mathbb{E} V(x') \quad (1)$$

1. Interpolate instead of discretize
2. Interpolation may have to be shape-preserving
3. Compute expectation on RHS \rightarrow quadrature
4. x is a vector \rightarrow multivariate approximation

ALGORITHM: PARAMETRIC VALUE FUNCTION ITERATION

(Judd, *Numerical Methods*, Algorithm 12.5)

- Objective: Solve Bellman equation \rightarrow find coefficients b^* such that the approximation $\hat{V}(x, b)$ is close enough.
- Initialization: Choose a functional form for $\hat{V}(x, b^0)$ and choose a grid of n interpolation nodes $X = \{x_1, \dots, x_n\}$. Choose initial vector of coefficients b^0 and stopping criterion $\varepsilon > 0$.

Step 1 Maximization step

Compute $v_j = T\hat{V}(\cdot, b^i)$ for $x_j \in X$.

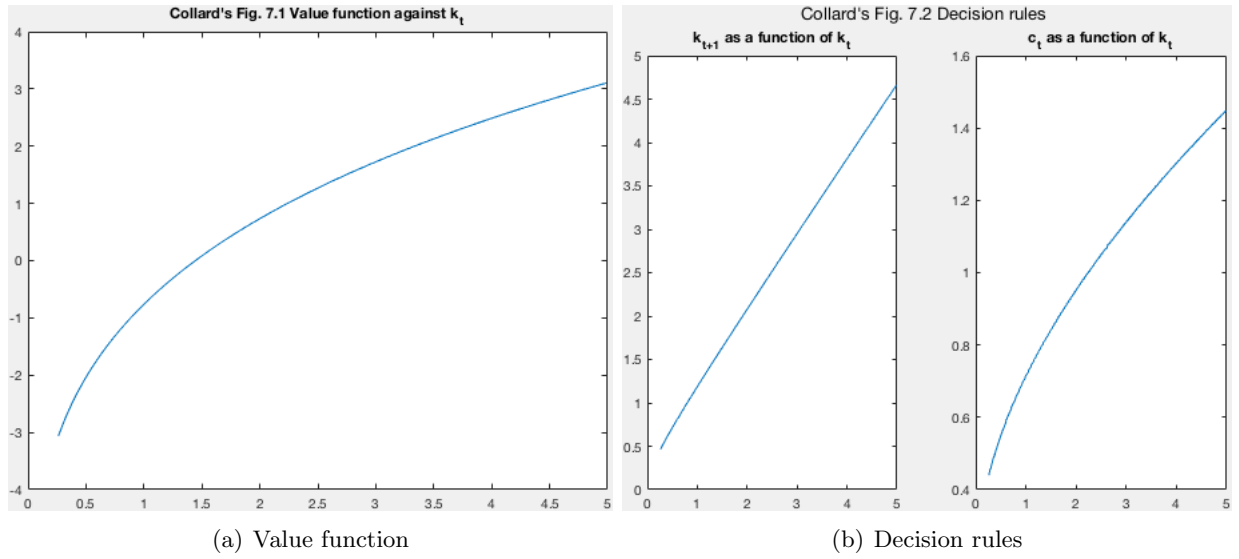
Step 2 Fitting step

Using your choice of approximation method, compute the updated vector of coefficients b^{i+1} such that $\hat{V}(x, b^{i+1})$ approximates the (v_i, x_i) data.

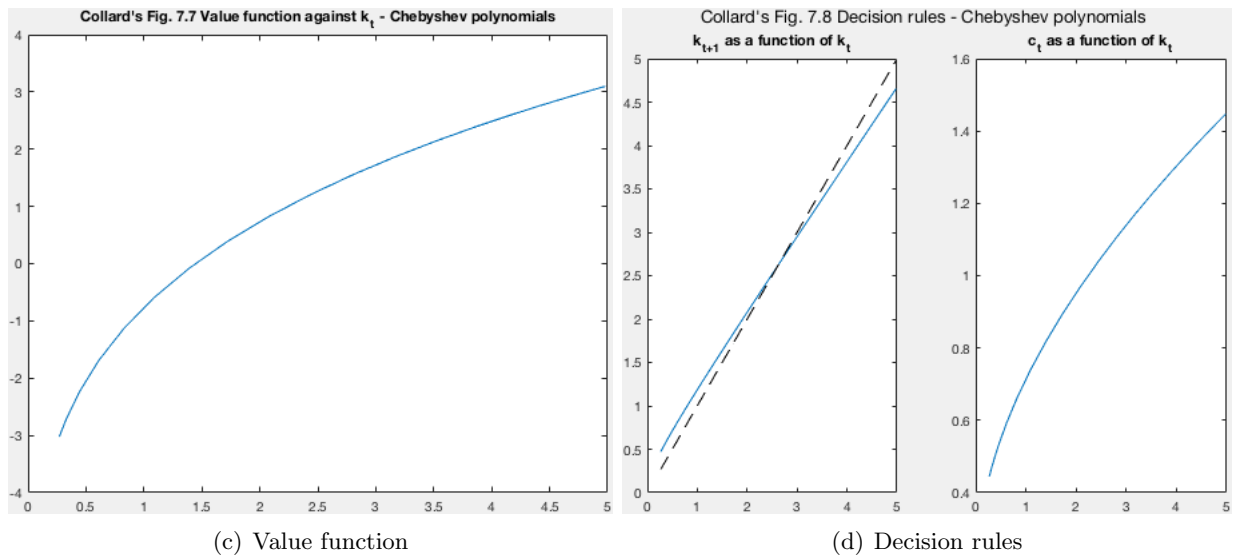
Step 3 If $\|\hat{V}(x, b^i) - \hat{V}(x, b^{i+1})\| < \varepsilon$, stop; else go to Step 1.

1 Optimal growth model

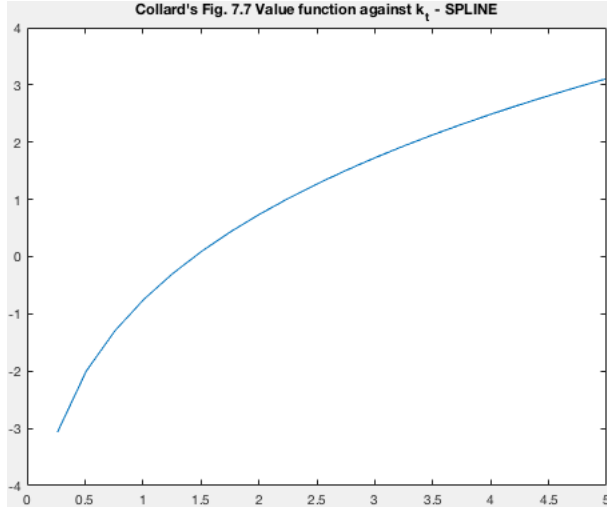
1.1 Optimal growth - value function iteration with discretization



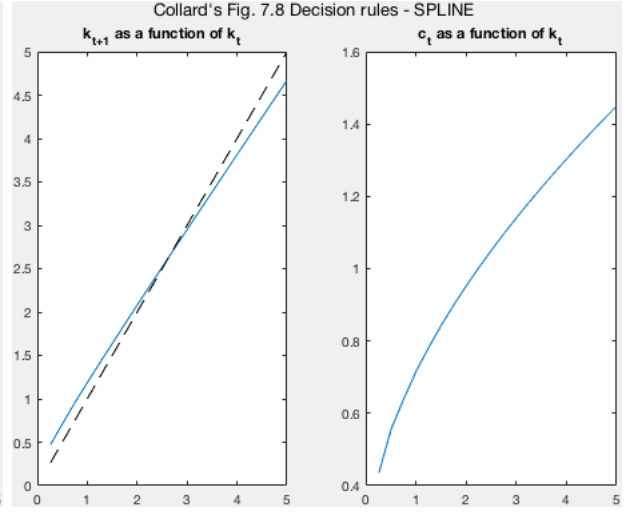
1.2 Optimal growth - value function iteration with Chebyshev polynomial interpolation



1.3 Optimal growth - value function iteration with cubic spline interpolation

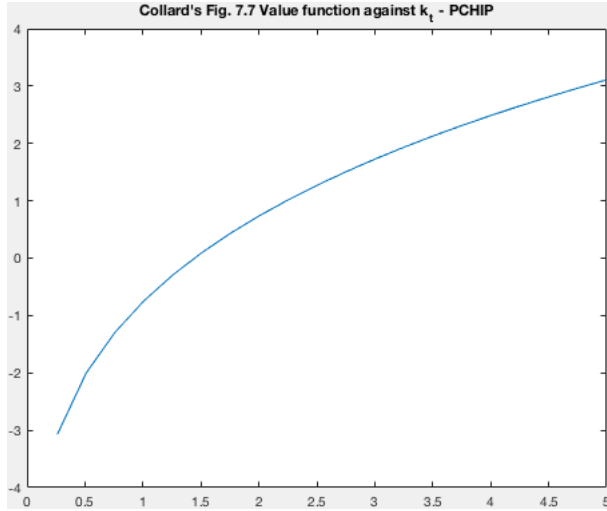


(e) Value function

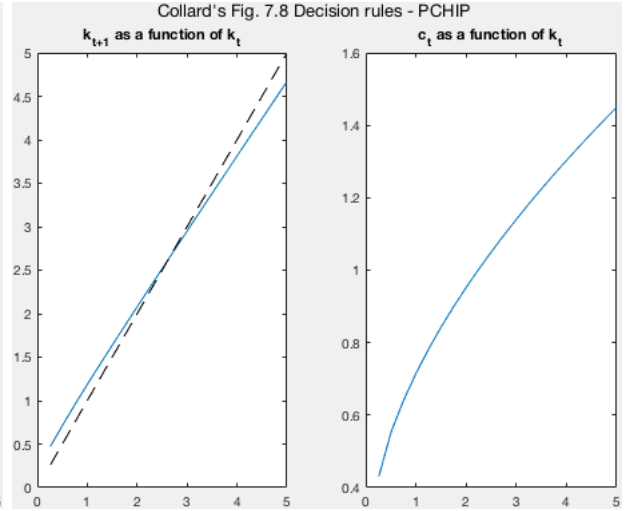


(f) Decision rules

1.4 Optimal growth - value function iteration with piecewise cubic Hermite interpolation (shape-preserving)

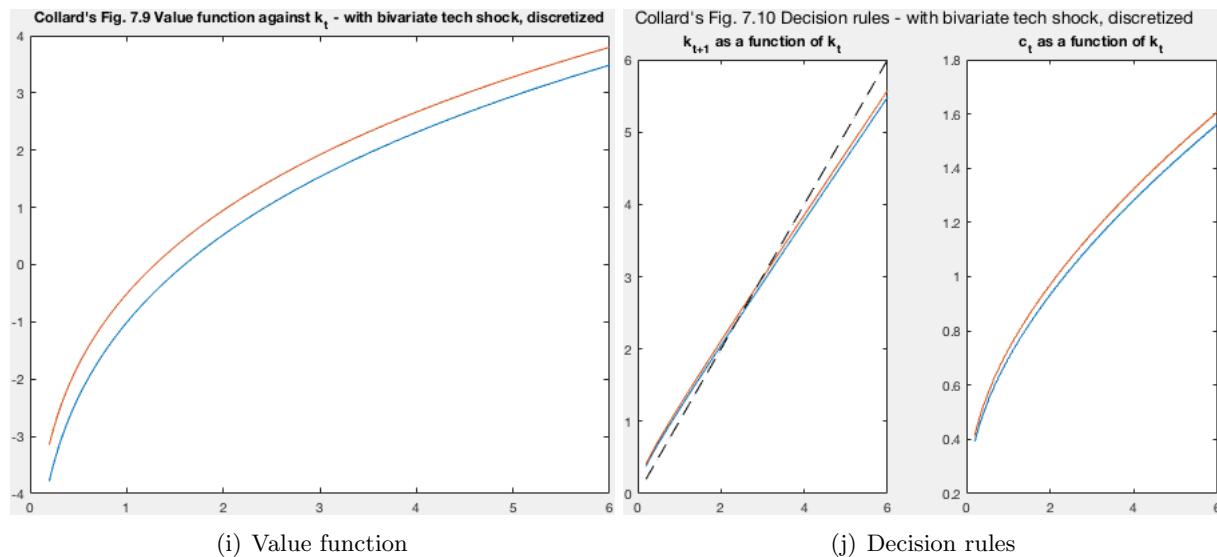


(g) Value function



(h) Decision rules

1.5 Optimal growth - stochastic value function iteration with bivariate tech shock, discretized



This one is not a 100% what Collard gets but hey.

1.6 Optimal growth in Ryan PS6 - stochastic value function iteration with two states, capital and tech, policy function interpolation and Gauss-Hermite quadrature for expectations

- I wonder if this is policy function iteration
- Why is he not actually iterating?
- It's not... it's solving the optimal growth model via projection methods (specifically with collocation)...