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The role of learning for asset prices and business cycles☆

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ABSTRACT

The implications of learning-based asset pricing are examined in a business cycle model with financial frictions. Agents learn about stock prices while firms face credit constraints that depend partly on their market value. Expectations are constrained to remain model-consistent conditional on a subjective belief for stock prices. The combination of financial frictions and learning amplifies shocks through a two-sided feedback mechanism between asset prices and real activity. The model matches not only important asset price and business cycle moments, but also several patterns of forecast error predictability in survey data across a range of variables.

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1. Introduction

Financial frictions are a central mechanism by which asset prices interact with macroeconomic dynamics. To understand this interaction, macroeconomists need models that are able to capture the dynamics of asset prices in the data. Moreover, these dynamics should be generated endogenously, because the interaction between prices and the real economy is likely two-sided.

At the same time, there is evidence that expectations of asset prices do not conform to the rational expectations hypothesis. This hypothesis implies, for example, that investors are fully aware of stock return predictability, expecting low returns when prices are high. Instead, survey data show that they expect high returns (Greenwood and Shleifer, 2014). This discrepancy is easy to detect and at odds with rational expectations-based asset pricing theories such as long-run risk, habit, and disaster risk models, but it is consistent with asset pricing theories based on extrapolative expectations or learning (Adam et al., 2015).

These observations prompt me to examine the implications of a learning-based asset pricing theory for the business cycle. I use an otherwise standard business cycle model with two important ingredients. First, firms' access to credit is

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constrained and depends on their market value. Second, agents do not have rational expectations, but are instead updating a subjective belief system about stock prices, which gives rise to extrapolation bias.

Departing from rational expectations opens up many degrees of freedom. To limit these, I develop a novel restriction on expectations which requires that expectations remain consistent with all equilibrium conditions except those that imply stock market clearing. Together with a subjective law of motion for stock prices, these "conditionally model-consistent expectations" determine a dynamically consistent probability measure over all variables that agents use to evaluate their choices. The restriction allows me to study the effects of asset price learning in isolation, while retaining the familiar logic and parsimony of rational expectations. The concept can be easily applied to other contexts in which a researcher is interested in particular deviations from rational expectations without much computational difficulty.

The interaction of learning and financial frictions gives rise to a two-sided feedback loop between asset prices and the production side of the economy. Optimism in financial markets spurs economic activity, because optimistic asset price expectations raise equilibrium firm valuations and loosen credit constraints. Looser credit constraints increase economic activity, and also lead to higher dividend payments that feed back into optimistic stock price expectations. This two-sided amplification mechanism produces both excessive asset price volatility and amplification of business cycle shocks.

From an asset pricing perspective, this paper shows that learning is a plausible explanation for asset price dynamics even in a production economy. From a business cycle perspective, the paper addresses the Kocherlakota (2000) critique that amplification of shocks through financial frictions is usually quite weak. The model with learning jointly matches key business cycle and asset price moments with standard time-separable preferences and shocks only to productivity.

I then take the confrontation of the model with the data one step further, by taking into account survey data on financial and macroeconomic expectations. Survey data allow to discriminate among models of expectation formation (Coibion and Gorodnichenko, 2012; Manski, 2004). The learning model replicates many patterns of forecast error predictability remarkably well. This includes predictability by forecast revisions (Coibion and Gorodnichenko, 2015), for which the model produces underreaction of macroeconomic expectations but not of stock return expectations. The reason for this good fit is that agents' expectational errors from stock price learning spill over into their other forecasts, even though their expectations are conditionally model-consistent. For example, when agents are too optimistic about future stock prices, they also become too optimistic about the tightness of credit constraints and therefore overpredict real economic activity.

The remainder of the paper is structured as follows. Section 2 discusses related literature. Section 3 presents the model and the construction of expectations under learning. Section 4 discusses a special case of the model that shuts off auxiliary frictions and allows closed-form solutions. Section 5 discusses the quantitative fit of the full model on the asset pricing and business cycle side, while Section 6 compares survey data with model-implied forecasts. Section 7 examines the sensitivity of the results to various model assumptions. Section 8 concludes. Supplementary materials are contained in an Online Appendix.

2. Related literature

This paper builds on the learning-based asset pricing theory developed in Adam et al. (2017, 2015). This paper is the first to apply this learning-based asset pricing theory to a business cycle model.

There are a number of papers in the adaptive learning literature that examine linkages between asset prices and the real economy, including Milani (2011, 2017), Caputo et al. (2010), Gelain et al. (2013) and Rychalovska (2016), to name just a few. In that literature, agents typically learn about all forward-looking variables simultaneously in an otherwise linear model. This "adaptive learning" approach has a long tradition, and has recently been shown to improve the fit to the data relative to rational expectations in a maximum likelihood sense (Milani, 2007; Slobodyan and Wouters, 2012). This paper introduces a different approach, focusing on learning about one variable while keeping expectations close to rational otherwise. This approach preserves much of the parsimony of rational expectations models, and does not require the underlying model to be linear.

The paper also relates to a number of studies that try to explain asset price fluctuations with non-rational beliefs about exogenous fundamentals, including the "natural expectations" of Fuster et al. (2012), as well as Hirshleifer et al. (2015), Collin-Dufresne et al. (2016) and Pintus and Suda (2019). A limitation of this approach is that there is no feedback from equilibirum outcomes to beliefs, as beliefs depend on exogenous variables only. As such, the amount of endogenous amplification is limited (Timmermann, 1996). In this paper, agents instead learn about the endogenous price that depends itself on beliefs. This leads to two-sided feedback between equilibirum outcomes and realizations, considerably magnifying endogenous volatility.

The paper also contributes to the literature on financial frictions as an amplification channel. The early literature on financial frictions emphasized amplification of standard productivity or monetary policy shocks (Bernanke and Gertler, 2001; Kiyotaki and Moore, 1997), but the quantitative importance of the "financial accelerator" mechanism was found to be small (Kocherlakota, 2000; Quadrini, 2011). The more recent literature has found sizable amplification effects in response to shocks to borrowing constraints (e.g. Jermann and Quadrini, 2012) and emphasizes their role as driving forces of the business cycle. Other proposed solutions explore non-linearities which lead to greater amplification in severe crisis states (Brunnermeier and Sannikov, 2014; Mendoza, 2010). Instead, this paper turns back to financial frictions as an amplification mechanism for standard and frequent business cycle shocks.

The dependency of the borrowing constraint on equity valuations is similar to Miao et al. (2015). There, rational liquidity bubbles allow for a sunspot shock that governs the size of the bubble and are an important driver of asset prices and business cycles.[1] In this model, asset price volatility arises endogenously, instead of being generated by an exogenous sunspot.

Finally, the paper relates to the use of survey data to test theories of expectation formation. It is well known that expectations measured in surveys fail to conform to the rational expectations hypothesis because forecast errors are statistically predictable (e.g. Bacchetta et al., 2009; Gennaioli et al., 2016). Cole and Milani (2017) show that learning outperforms rational expectations in a DSGE-VAR when survey data on expectations are included in the observables. While the model in this paper does not have enough degrees of freedom to attempt a maximum-likelihood estimation on survey data, it replicates prominent patterns of forecast error predictability across a range of variables.

3. The model

3.1. Model setup

The economy is closed and operates in discrete time. It is populated by two types of households, lenders and investors. Firms are at the heart of the model and combine capital and labor into intermediate goods subject to a borrowing constraint. In addition, the model contains nominal rigidities and investment adjustment costs.

3.1.1. Lenders

Lending households with time-separable preferences maximize utility as follows:

$$\max_{(C_t, L_t, B_{jt}, B_t^g)_{t=0}^{\infty}} \mathbb{E}^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\theta}}{1-\theta} - \eta \frac{L_t^{1+\phi}}{1+\phi}$$

$$\text{s.t. } C_t = \tilde{w}_t L_t + B_t^g - \frac{1+i_{t-1}}{\pi_t} B_{t-1}^g + \int_0^1 \left(B_{jt} - R_{jt-1} B_{jt-1}\right) dj + \Pi_t \tag{3.1}$$

They consume an amount C_t of a homogeneous final consumption good and supply an amount L_t of labor at the real wage rate $\tilde{w}_t$. B_t^g are real quantities of nominal one-period government bonds (in zero net supply) that pay a nominal interest rate i_t, and π_t is the rate of inflation. They also lend funds B_{jt} to intermediate goods producers indexed by $j \in [0, 1]$ at the real interest rate R_{jt}. These loans are the outcome of a contracting problem described later on. As in Bernanke et al. (1999), lending households do not own firm equity.[2] Π_t represents lump-sum profits received from price- and wage-setting firms and capital goods producers.

The first-order conditions are standard. In particular, the stochastic discount factor of the lending household is given by $\Lambda_{t+1} = \beta (C_{t+1}/C_t)^{-\theta}$. Expectations are evaluated under the probability measure $\mathcal{P}$, which does not necessarily coincide with rational expectations.

3.1.2. Intermediate good producers (firms)

The production of intermediate goods is carried out by a continuum of firms, indexed $j \in [0, 1]$. The number of equity shares in each firm is normalized to one. Firm j enters period t with capital K_{jt-1} and a stock of debt B_{jt-1} which needs to be repaid at the gross real interest rate R_{jt-1}. First, capital is combined with labor L_{jt}, compensated at the wage rate w_t, to produce output:

$$Y_{jt} = \left(K_{jt-1}\right)^{\alpha} \left(A_t L_{jt}\right)^{1-\alpha}, \tag{3.2}$$

where A_t is aggregate productivity. Output is sold competitively to wholesale firms at the price q_t.[3] The capital stock depreciates at the rate δ. Capital trades at the price Q_t.

The firm's net worth is the difference between the value of assets and outstanding debt:

$$N_{jt} = q_t Y_{jt} - w_t L_{jt} + Q_t (1-\delta) K_{jt-1} - R_{jt-1} B_{jt-1}. \tag{3.3}$$

It is assumed that firms exit with probability γ. This probability is exogenous and independent across time and firms. As in Bernanke et al. (1999), exit prevents firms from becoming financially unconstrained. If a firm does not exit, it needs to pay

[1] In principle, such bubbly sunspot equilibria could arise in my model as well, but they do not exist in the range of parameter values considered (as shown in the appendix).

[2] Effectively, this leads to segmentation between bond and equity markets. From an asset pricing perspective, this assumption is somewhat unsatisfying. But from a business cycle perspective, it helps the model fit the data, because fluctuations in beliefs under learning would introduce strong wealth effects if lending households held equity. These wealth effects would induce countercyclical shifts in the supply of labor and savings, which make it difficult to replicate the cyclicality of investment and labor in the data. A similar problem is known in the news shock literature (Beaudry and Portier, 2007).

[3] Wholesale firms then transform intermediate goods into varieties of wholesale goods, which are aggregated back into a homogenous final consumption good by retail firms. The description of these firms is relegated to the appendix.

out a fraction $\zeta \in (0, 1)$ of its earnings as a regular dividend. Specifically:[4]

$$D_{jt} = \zeta \left(Y_{jt} - w_t L_{jt} - \delta Q_t K_{jt-1} - \left(R_{jt-1} - 1 \right) B_{jt-1} \right) \tag{3.4}$$

The firm then decides on the new stock of debt B_{jt} and the new capital stock K_{jt}, maximizing the present discounted value of dividend payments using the discount factor of its owners. Its balance sheet must satisfy $Q_t K_{jt} = B_t^j + N_{jt} - \zeta E_{jt}$. If a firm does exit, it pays out its entire net worth N_{jt} as a terminal dividend.

3.1.3. Investors

Investing households differ from lending households in their capacity to own intermediate firms. They are risk-neutral and trade equity claims on firms indexed by $j \in [0, 1]$. Denote the subset of firms alive at the end of period t by $\Gamma_t \subset [0, 1]$. A firm owner's utility maximization problem is:

$$\max_{\left(C_t^f, S_{jt}\right)_{t=0}^{\infty}} \mathbb{E}^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t C_t^f$$

$$\text{s.t. } C_t^f = \int_j S_{jt-1} D_{jt} dj - \int_{j \in \Gamma_t} P_{jt} \left(S_{jt} - S_{jt-1} \right) dj \tag{3.5}$$

$$S_{jt} \in \left[0, \bar{S} \right] \tag{3.6}$$

where $\bar{S} > 1$. Investors do not trade debt claims. In addition, investors face upper and lower bounds on traded stock holdings. The only reason for this assumption is to render demand for stocks finite under arbitrary beliefs. In equilibrium, the bounds are never binding.

The first term on the right-hand side of the budget constraint deals with continuing firms while the second term deals with exiting firms. In addition, investors face upper and lower bounds on traded stock holdings. The only reason for this assumption is to render demand for stocks finite under arbitrary beliefs. In any equilibrium, the bounds are never binding. The first-order condition of an investor is:

$$\left.\begin{array}{lcl} S_{jt} = & 0 & \text{if } P_{jt} > \\ S_{jt} \in & \left[0, \bar{S}\right] & \text{if } P_{jt} = \\ S_{jt} = & \bar{S} & \text{if } P_{jt} < \end{array}\right\} \beta \mathbb{E}_t^{\mathcal{P}} \left[N_{jt+1} \mathbb{1}_{\{j \notin \Gamma_{t+1}\}} + \left(\zeta E_{jt+1} + P_{jt+1} \right) \mathbb{1}_{\{j \in \Gamma_{t+1}\}} \right] \forall j \in \Gamma_t. \tag{3.7}$$

3.1.4. Borrowing constraint

In choosing their debt holdings, firms are subject to a borrowing constraint in which the availability of credit depends on stock market valuations, similar to Miao et al. (2015). The constraint is the solution to a particular limited commitment problem.

Each period, lenders and firms meet to decide on the lending of funds. Only the size B_{jt} and the interest rate R_{jt} of the loan can be contracted in period t. At the end of the period, but before the realization of next period's shocks, firm j can choose to default and renegotiate its debt. The outside option is bankruptcy of the firm and seizure by the lender. Bankruptcy carries a cost of a fraction $1 - \xi$ of the firm's capital being destroyed. The lender can liquidate the firm's assets, selling the remaining capital in the next period. With some probability x, the lender additionally receives the opportunity to "restructure" the firm: The firm gets partial debt relief but remains operational, as in a U.S. Chapter 11 bankruptcy. The lender does not have the ability to operate the firm, but it can sell it to an investor, retaining a fraction ξ of the initial debt.

The appendix shows that the optimal debt contract takes the form of a leverage constraint with a weighted average of liquidation and market value of the firm:

$$B_{jt} \leq (1 - x) \underbrace{\mathbb{E}_t^{\mathcal{P}} \Lambda_{t+1} Q_{t+1} \xi K_{jt}}_{\text{liquidation value}} + x \underbrace{\xi \left(P_{jt} + B_{jt} \right)}_{\text{market value}} \tag{3.8}$$

3.1.5. Further model elements, market clearing, shocks

The price of capital Q_t moves due to standard quadratic investment adjustment costs with parameter ψ. Also, standard nominal price and wage rigidities are introduced. The Calvo probabilities of price and wage non-adjustment are denoted by κ and κ_w, respectively. The elasticities of substitution is σ between product varieties and σ_w between labor varieties. The complete description of these features of the model are relegated to the appendix.

Monetary policy follows a simple Taylor-type interest rate rule:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left(\beta^{-1} + \phi_\pi \pi_t \right). \tag{3.9}$$

check it out!

[4] The optimal dividend payout in this model would be zero, as firms would always prefer to build up net worth to escape the borrowing constraint over paying out dividends. However, the resulting dividend process would not be nearly as volatile as in the data because aggregate dividends would be proportional to aggregate net worth. The dividend payout is set such that fluctuations in the price of intermediate good q_t do not translate into dividend fluctuations.

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Most market clearing conditions have already been stated. To be clear, the market clearing conditions for final consumption goods and stocks are:

$$\tilde{Y}_t = C_t + I_t + \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 + C_t^f \tag{3.10}$$

$$1 = S_{jt},\ j \in [0, 1] \tag{3.11}$$

where $I_t = K_t - (1-\delta)K_{t-1}$Finally, the only aggregate shock is to productivity A_t, which evolves as an AR(1) process with autoregressive parameter ρ and iid innovations $\varepsilon_t \sim \mathcal{N}(0, \sigma_A^2)$.

3.2. Rational expectations equilibrium

Let's recall the generic definition of a rational expectations equilibrium. Let $y_t \in \mathbb{R}^N$ denote the collection of all endogenous model variables, and by $u_t \in \mathbb{R}^M$ the collection of the exogenous model variables, or "fundamentals", at time t. Stochastic processes for y_t and u_t are defined on the spaces $\Omega_y = \Pi_{t=0}^{\infty}\mathbb{R}^N$ and $\Omega_u = \Pi_{t=0}^{\infty}\mathbb{R}^M$, respectively. Denote by $\Omega_u^{(t)}$ the set of all possible histories of exogenous variables up to period t, and its elements by $u^{(t)} \in \Omega_u^{(t)}$. Let $\mathbb{P}_u$ denote the true probability measure for the exogenous variables defined on $(\Omega_u, \mathcal{S}(\Omega_u))$, where $\mathcal{S}(\cdot)$ is the Borel sigma-algebra on a metric space. The topological support of $\mathbb{P}_u$ is denoted by $\text{supp}(\mathbb{P}_u)$.

Definition 1. A *rational expectations (RE) equilibrium* is a sequence of mappings $g_t : \Omega_u^{(t)} \ni u^{(t)} \mapsto y_t \in \mathbb{R}^N$, $t = 0, 1, 2, \ldots$ such that, for all t and $u^{(t)} \in \text{supp}(\mathbb{P}_u)$:

1. the choices contained in y_t solve the time-t decision problem of each agent in the economy, conditional on decision-relevant[5] past and current outcomes contained in $u^{(t)}$ and $y^{(t)} = (g_0(u^{(0)}), \ldots, g_t(u^{(t)}))$, and evaluating the probability of future external decision-relevant outcomes under the probability measure $\mathbb{P}$ implied[6] by $\mathbb{P}_u$ and the mappings $(g_t)_{t=0}^{\infty}$;
2. the allocations contained in $y_t = g_t(u^{(t)})$ clear all markets.

Under a mild restriction on the exit probability γ, there exists a rational expectations equilibrium characterized by the following properties in a neighborhood of the non-stochastic steady state: (i) all firms choose the same capital-labor ratio K_{jt}/L_{jt}; (ii) the stock market value P_{jt} of any firm is linear in its post-dividend net worth $Q_t K_{jt} - B_{jt}$; (iii) borrowers never default in equilibrium and borrow at the risk-free rate; (iv) firms always exhaust their borrowing limit. These properties are proven in the appendix, which also contains the full set of equations defining the rational expectations equilibrium.

3.3. Learning equilibrium

Under learning, agents' behavior remains rational, but they evaluate expectations under a subjective measure $\mathcal{P}$ that is different from the equilibrium distribution $\mathbb{P}$. Under $\mathcal{P}$, agents are not endowed with knowledge of the equilibrium pricing function for stocks, i.e. the mapping of a history of fundamentals $u^{(t)}$ to prices P_{jt}. Instead, they use a simple subjective model to forecast the aggregate stock market value P_t. According to this subjective model, which follows Adam et al. (2017), P_t evolves as a random walk with a small drift:

$$\log P_t = \log P_{t-1} + \hat{\mu}_{t-1} + z_t \tag{3.12}$$

like the constant updating equation, z = FE

constant gain learning

$$\hat{\mu}_t = \hat{\mu}_{t-1} + g z_t, \tag{3.13}$$

where $\hat{\mu}_t$ is the perceived trend price growth, g is the learning gain, and z_t is the *subjective forecast error*. Under $\mathcal{P}$, z_t is normally distributed white noise with variance σ_z^2, independent of the other exogenous shocks. The belief $\hat{\mu}_t$ is updated in the direction of the last forecast error: When agents see stock prices rising faster than they expected, they will also expect them to rise by more in the future. The belief system above is equivalent to a belief that asset price growth is the sum of a temporary and a permanent component, both of which are unobserved. Bayesian updating of this belief leads to the equations above, with the learning gain g representing the perceived ratio of the standard deviation of the permanent relative to the temporary component.[7]

The market value P_t aggregates the stock prices of all firms $j \in [0, 1]$. To preserve the linear aggregation property, I assume that beliefs about individual stock prices under $\mathcal{P}$ are linear in post-dividend net worth, as under rational expectations:

$$P_{jt} = \frac{Q_t K_{jt} - B_{jt}}{Q_t K_t - B_t} P_t. \tag{3.14}$$

[5] A variable is *decision-relevant* if it enters the agents' decision problem, and a decision-relevant variable is *external* if its value is taken as given by the agent, while it is *internal* if the variable is part of the solution of the agents' decision problem.

[6] This probability measure $\mathbb{P}$ is defined on the space $(\Omega_u \times \Omega_y, \mathcal{S}(\Omega_u \times \Omega_y))$ and satisfies $\mathbb{P}(u^{(t)} \in \mathcal{A}) = \mathbb{P}_u(u^{(t)} \in \mathcal{A})$ for all $\mathcal{A} \in \mathcal{S}(\Omega_u^{(t)})$ and $\mathbb{E}^{\mathbb{P}}[y_t \mid u^{(t)}] = g_t(u^{(t)})$ for all t and $u^{(t)} \in \text{supp}(\mathbb{P}_u)$.

[7] Following Adam et al. (2017), I also impose that the belief about price growth $\hat{\mu}_t$ is updated only at the end of the period, so that agents make their forecasts in period t using $\hat{\mu}_{t-1}$ only. The appendix describes the implementation of this lagged belief updating.

To keep expectations as close as possible to rational expectations given agents' subjective stock price beliefs in (3.12)–(3.14), I present a novel restriction on belies called "conditionally model-consistent expectations". Let $(\Omega_z, \mathcal{S}(\Omega_z), \mathcal{P}_z)$ be the probability space that defines the subjective beliefs for z_t (i.e., the z_t are iid normally distributed with mean zero and variance σ_z^2). Agents' subjective beliefs depend on this perceived stochastic forecast error even though in equilibrium, model outcomes are a function only of fundamentals u_t. The subjective probability measure $\mathcal{P}$ is defined by a mapping from fundamentals u_t and the subjective forecast error z_t to model outcomes y_t.

Definition 2. *Conditionally model-consistent expectations* (CMCE) are a sequence of mappings $h_t : \Omega_u^{(t)} \times \Omega_z^{(t)} \ni (u^{(t)}, z^{(t)}) \mapsto y_t \in \mathbb{R}^N$, $t = 0, 1, 2, \ldots$ such that, for all t and $(u^{(t)}, z^{(t)}) \in \text{supp}(\mathcal{P}_{u,z})$:

1. the choices contained in y_t solve the time-t decision problem of each agent in the economy, conditional on decision-relevant past and current outcomes contained in $u^{(t)}$ and $y^{(t)} = (h_0(u^{(0)}, z^{(0)}), \ldots, h_t(u^{(t)}, z^{(t)}))$, and evaluating the probability of future decision-relevant outcomes under the probability measure $\mathcal{P}$ implied[8] by $\mathbb{P}_u \otimes \mathcal{P}_z$ and the mappings $(h_t)_{t=0}^{\infty}$;
2. the allocations contained in $y_t = h_t(u^{(t)}, z^{(t)})$ clear all markets except the markets for stocks and final consumption goods;
3. stock prices under $\mathcal{P}$ follow the law of motion given by (3.12)–(3.14).

The mappings h_t are called the *subjective* or*perceived law of motion (PLM).* The measure $\mathcal{P}$ is called the*subjective probability measure.*

Imposing CMCE together with the subjective beliefs for stock prices fully determine agents' expectations, thereby limiting the degrees of freedom of expectation formation. The learning equilibrium only has two additional parameters (g and σ_z^2) relative to the rational expectations equilibrium. Moreover, CMCE preserve as much as possible of the logic of rational expectations that macroeconomists are familiar with.

One can interpret Definition 2 as saying that the rationality and decision rules of all agents, as well as most market clearing conditions, are common knowledge. Market clearing for stocks and final consumption goods are not common knowledge.[9] The resulting degrees of freedom are filled with subjective beliefs about stock prices (3.12)–(3.14). But ultimately, it does not matter which parts of the model agents "understand". All that matters is that agents are endowed with expectations that are consistent with the conditions in the definition above.

The equilibrium under learning is a special case of an "internally rational equilibrium" (Adam and Marcet, 2011):

Definition 3. An*equilibrium* with conditionally model-consistent expectations is a sequence of mappings $r_t : \Omega_u^{(t)} \ni u^{(t)} \mapsto z_t \in \mathbb{R}$ and $g_t : \Omega_u^{(t)} \ni u^{(t)} \mapsto y_t \in \mathbb{R}^N$, $t = 0, 1, 2, \ldots$ such that, for all t and $u^{(t)} \in \text{supp}(\mathbb{P}_u)$:

1. $g(u^{(t)}) = h(u^{(t)}, (r_0(u^{(0)}), \ldots, r_t(u^{(t)})))$;
2. the allocations contained in $y_t = g_t(u^{(t)})$ clear the markets for stocks.

The mappings g_t are called the *objective* or*actual law of motion (ALM).* The measure implied by $\mathbb{P}_u$ and the mappings $(g_t)_{t=0}^{\infty}$ is called the *objective probability measure* and is denoted by $\mathbb{P}$.

This definition says that equilibrium outcomes have to be consistent with the PLM h_t, and that the values of the forecast error z_t—which maps one-to-one into aggregate stock prices P_t—have to be such that stock markets clear. By construction of h_t, all other markets then clear as well. Under subjective expectations $\mathcal{P}$, the forecast error z_t is an unpredictable stochastic disturbance, while under the objective measure $\mathbb{P}$, it is a predictable function of fundamentals.[10]

Computing the learning equilibrium is an easy two-step procedure: First, compute the PLM $h_t(u^{(t)}, z^{(t)})$; second, compute the ALM $g_t(u^{(t)})$. Both steps are no more complicated than solving the RE equilibrium. The appendix outlines the procedure for computing perturbation approximations of the learning equilibrium.

the expectations of other variables (given E(stock prices)) are rational.

It is important to stress that CMCE do not imply that agents have rational expectations of all variables other than stock prices. Agents' systematic forecast errors on stock prices do spill over to other forecasts, which is what enables the model to match the predictability of forecast errors in Section 6. Nonetheless, agents endowed with CMCE make the "best" forecasts compatible with their subjective view about the evolution of stock prices, in the following sense:

Proposition 4. *In an equilibrium with conditionally model-consistent expectations,*

$$\mathbb{E}^{\mathcal{P}}[y_{t+1} \mid u_0, P_0, \ldots, u_{t+1}, P_{t+1}] = y_{t+1} \ \mathbb{P}\text{-a.s.} \tag{3.15}$$

[8] The probability measure $\mathcal{P}$ is defined on the space $(\Omega_u \times \Omega_z \times \Omega_y, \mathcal{S}(\Omega_u \times \Omega_z \times \Omega_y))$ and satisfies $\mathcal{P}(u^{(t)} \in \mathcal{A}, z^{(t)} \in \mathcal{B}) = \mathbb{P}_u(u^{(t)} \in \mathcal{A})\mathcal{P}_z(z^{(t)} \in \mathcal{B})$ for all $\mathcal{A} \in \mathcal{S}(\Omega_u^{(t)})$, $\mathcal{B} \in \mathcal{S}(\Omega_z^{(t)})$, and $\mathbb{E}^{\mathcal{P}}[y_t \mid u^{(t)}, z^{(t)}] = h_t(u^{(t)}, z^{(t)})$ for all t, $u^{(t)} \in \text{supp}(\mathbb{P}_u)$, $z^{(t)} \in \text{supp}(\mathcal{P}_z)$.

[9] Why are subjective expectations not required to be consistent with consumption goods market clearing? The answer is that otherwise, expectations satisfying Definition 2 would not exist. Because subjective expectations are consistent with the solutions of agents' decision problems, Walras's law applies under $\mathcal{P}$. At the same time, investors' demand for stocks under subjective stock price beliefs in (3.12)–(3.14) do not equal the aggregate supply of stocks $\mathcal{P}$-almost surely, and therefore excess demand for the stock market as a whole is non-zero. Walras's law then implies that excess demand in at least one other market also has to be non-zero $\mathcal{P}$–almost surely. I remove consistency with final consumption goods market clearing, so as to preserve the logic of most asset pricing models.

[10] Note that the subjective and objective measures are singular: $\mathcal{P} \perp \mathbb{P}$. It is therefore not possible to go from objective to subjective expectations through an equivalent measure change.

Proof. See the appendix. □

4. Inspecting the mechanism

Before discussing the full model, I present a stripped-down version which does away with nominal rigidities, adjustment costs, net worth dynamics and risk aversion. These simplifications allow for a closed-form solution.

4.1. Simplifying the model

Nominal rigidities and investment adjustment costs are removed by setting $\kappa = \kappa_w = 0$, $\sigma = \sigma_w = 0$, and $\psi = 0$. Preferences are simplified by setting the lending household coefficient of relative risk aversion to $\gamma = 0$ and making labor supply perfectly inelastic at some level $\bar{L}$. Productivity innovations are made permanent by setting the autoregressive coefficient for the productivity process to $\rho = 1$. The financial structure of the firm is simplified by setting the exit rate of firms to $\gamma = 0$ and the dividend payout ratio to $\zeta = 1$, which together imply that $B_t = K_t$. The aggregate dividend expression and the borrowing constraint simplify to:

$$D_t = \left(R_t^k - R_{t-1}\right)K_{t-1} \tag{4.1}$$

$$B_t = \xi x K_t + \xi(1-x)(P_t + B_t). \tag{4.2}$$

4.2. Rational expectations equilibrium

In the rational expectations equilibrium of the simplified model, financial frictions do not lead to any amplification of the productivity shock. To see this, start with the case $\xi = 1$, where the borrowing constraint (4.2) is never binding. Because labor supply is fixed, the equilibrium marginal return to capital is:

$$R_t^k = \alpha\left(\frac{\bar{L}}{\tilde{K}_t}\right)^{\alpha} + 1 - \delta \tag{4.3}$$

where the capital stock is scaled by productivity as $\tilde{K}_t = K_t/A_t$. Denote the expected return on capital by the function $R^k\left(\tilde{K}_t\right) = \mathbb{E}_t\left[R_{t+1}^k \mid \tilde{K}_t\right]$. The firm equates this expected return with the interest rate, which is constant at $R = 1/\beta$. At the optimum then, capital is simply proportional to productivity: $K_t = K^* A_t$ for some fixed value K^*.

For values of ξ below one, the borrowing constraint is always binding. Let $\tilde{P}_t = P_t/A_t$. The equilibrium is characterized by the following two equations:

$$\tilde{P}_t = \frac{\left(R^k\left(\tilde{K}_t\right) - R\right)\tilde{K}_t}{R-1} \tag{4.4}$$

$$\tilde{K}_t = \frac{x\xi}{1-\xi}\tilde{K}_t \tag{4.5}$$

The first equation pins down the stock market value of the firm, which depends on the capital stock through dividends and the return on capital in the enumerator. The second equation determines the capital stock that can be reached by exhausting the borrowing constraint.

The solution is of course that $\tilde{K}_t$ is constant. The capital stock is proportional to productivity, just as was the case when $\xi = 1$. In this simplified setup, financial frictions do not lead to *any* amplification or propagation of the productivity shock here. Similarly, the dynamics of asset prices are unchanged by the presence financial frictions: The stock price evolves simply as a random walk, without excess volatility or return predictability.

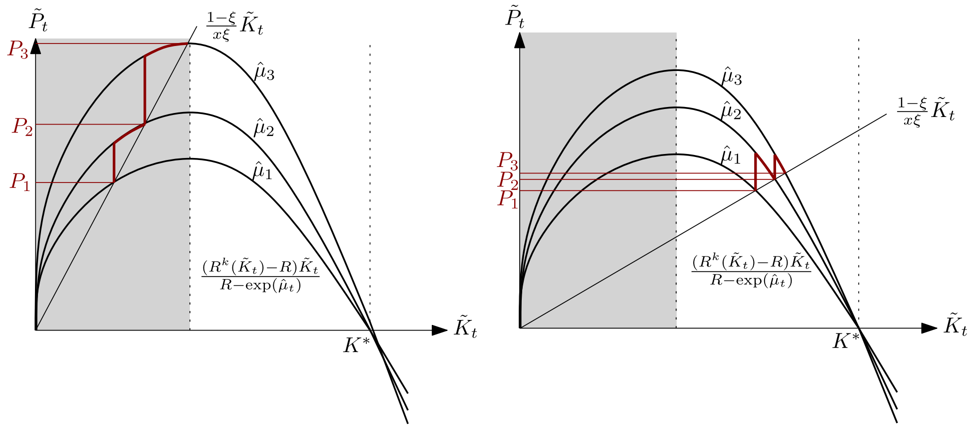
4.3. Learning equilibrium

To find the equilibrium under learning, the first step is to find the subjective law of motion according to Definition 2. Subjective beliefs have to be consistent with optimal lending household behavior. This implies consistency with a fixed labor supply and with an interest rate of $R = 1/\beta$. Next, the firm's labor demand depends on productivity A_t and the wage rate w_t. Definition 2 requires that expectations about the exogenous productivity process be correct, and that expectations of the wage rate be consistent with constant employment. As a result, the belief for the return on capital under $\mathcal{P}$ is given by (4.3). Beliefs also have to be consistent with the firm's dividend policy, which means that $\mathbb{E}_t^{\mathcal{P}}[D_{t+1}] = \left(R^k\left(\tilde{K}_t\right) - R\right)\tilde{K}_t A_t$.

Taken together, we can summarize the *perceived law of motion* of capital and stock prices by

$$\log \tilde{P}_t = \log \tilde{P}_{t-1} + \varepsilon_t + \hat{\mu}_{t-1} + z_t \tag{4.6}$$

$$\tilde{K}_t = \min\left(K^*, \frac{x\xi}{1-\xi}\tilde{P}_t\right) \tag{4.7}$$



(a) Amplification.

(b) Dampening.

Fig. 1. Response to a productivity shock in the simplified model.
The diagram shows the evolution of the simplified learning model at $t = 1, 2, 3$ after a positive productivity shock ε_t at time $t = 1$ starting from the steady state. Capital and stock prices are scaled by productivity A_t as $\tilde{K}_t = K_t/A_t$ and $\tilde{P}_t = P_t/A_t$.

$$\hat{\mu}_t = \hat{\mu}_{t-1} + gz_t. \tag{4.8}$$

The second step is to compute the equilibrium under learning. By Definition 3, this requires finding values for z_t such that $S_t = 1$, i.e. the stock market clears. This is the case if the Euler equation for investors holds: $P_t = \beta \mathbb{E}_t^{\mathcal{P}}[D_{t+1} + P_{t+1}]$. The *actual law of motion* for capital and stock prices is characterized by three equations:

$$\tilde{P}_t = \frac{\left(R^k\left(\tilde{K}_t\right) - R\right)\tilde{K}_t}{R - \exp\left(\hat{\mu}_t + \frac{1}{2}\sigma_z^2\right)}. \tag{4.9}$$

$$\tilde{K}_t = \frac{x\xi}{1-\xi}\tilde{P}_t \tag{4.10}$$

$$\hat{\mu}_{t+1} = \hat{\mu}_t - \frac{\sigma_\nu^2}{2} + g\left(\Delta \log \tilde{P}_t + \varepsilon_t - \hat{\mu}_t\right) \tag{4.11}$$

The first equation is the stock pricing equation under learning. The second equation is the borrowing constraint, which always binds in equilibrium.[11] The third equation is the belief updating equation. Comparing the perceived and the actual law of motion, it becomes clear that the subjective forecast error z_t is in fact predictable. Moreover, systematic forecast errors on stock prices spill over to other forecasts. For example, the forecast error for future capital is $K_{t+1} - \mathbb{E}_t^{\mathcal{P}} K_{t+1} = z_t x\xi/(1-\xi)$.

Starting from the steady state of the model, suppose that a positive productivity innovation $\varepsilon_1 > 0$ hits the economy at $t = 1$. The response to this shock can be worked out from Fig. 1, which depicts the stock pricing equation (4.9) and the credit constraint (4.9). The shock initially raises stock prices and the capital stock proportionally to productivity, just as it would under rational expectations: $\tilde{K}_1 = \tilde{K}_0$ and $\tilde{P}_1 = \tilde{P}_0$. But now, learning investors observe the unexpected rise in asset prices $z_1 = \varepsilon_1$ and are unsure whether their positive forecast error z_1 is due to a transitive shock or a permanent increase in the growth rate of stock prices. By Eq. (4.11), they revise their beliefs $\hat{\mu}_2$ upward. In the next period, the more optimistic beliefs increase the demand for stocks, and $\tilde{P}_2$ in Eq. (4.9) rises. Beliefs continue to rise in subsequent periods as long as observed asset price growth is higher than expected price growth. Ultimately though, the latter will outstrip the former, and prices gradually fall back to their steady-state levels. These learning dynamics produce return volatility and predictability.

Asset price learning affects economic activity because it influences current stock prices and therefore the tightness of the borrowing constraint. But the feedback in this model is two-sided, as real activity affects stock prices through dividend payments. A higher capital stock affects expected dividend payments in two ways. First, because the internal rate of return on capital is higher than the cost of debt, a higher capital stock will increase expected dividends. This is a partial equilibrium effect. Second, higher levels of capital lower the marginal return $R^k(K_t, A_t)$ because of decreasing returns to scale at the aggregate level, increasing wages and decreasing expected dividends. This is a general equilibrium effect.[12]

Panel (a) of Fig. 1 depicts a case when financial frictions are severe (ξ is low). Here, the difference between the return on capital R^k and the return on debt R is high, and the partial equilibrium effect dominates. After the productivity shock, beliefs rise to $\hat{\mu}_2$, and the firm can invest more and increase its dividends. In turn, stock prices rise even more, and the learning dynamics are amplified.

[11] To see this, assume that P_t were sufficiently high so that the borrowing constraint didn't bind. Then $R^k(K_t, A_t) = R$ and therefore $P_t = 0$, a contradiction.
[12] This effect hinges on wage flexibility and inelastic labor supply. Rigid wages or elastic labor supply will mitigate the dampening effects and help amplification.

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Panel (a) of Fig. 1 depicts a case when financial frictions are mild (ξ is high). Here, the difference between R^k and R is small and the general equilibrium effect dominates. A relaxation of the borrowing constraint due to a rise in $\hat{\mu}_2$ still allows the firm to increase the capital stock, but in general equilibrium, wages rise so much that dividends fall. Stock prices do not rise as much, and the learning dynamics are dampened.

5. Quantitative results

The general model is solved using a second-order approximation around the non-stochastic steady state (described in the appendix). First, the parameterization is discussed, followed by business cycle and asset price statistics. Impulse response functions confirm the presence of a strong amplification mechanism.

5.1. Calibration

The capital share in production is set to $\alpha = 0.33$. The depreciation rate $\delta = 0.025$ corresponds to 10% annual depreciation. The persistence of productivity shocks is set to $\rho = 0.95$.[13] The discount factor is set such that the steady-state real interest rate equals 2% per year, implying a discount factor $\beta = 0.9951$. The Frisch elasticity of labor supply is set to 3, implying $\phi = 0.33$, and risk aversion is set to $\theta = 1$ (log utility in consumption).

The structure of financial constraints is governed by x, the probability of restructuring after default; ξ, the tightness of the borrowing constraint; γ, the rate of firm exit; and ζ, the dividend payout ratio. I calibrate the restructuring rate to $x = 0.03$, which is broadly in line with the fraction of U.S. business bankruptcies that files under Chapter 11 and subsequently emerges from bankruptcy with an approved restructuring plan (Flynn and Crewson, 2009; Lawton, 2012). Values of $\xi = 0.3094$ and $\gamma = 0.0155$ are chosen to jointly match an average investment share in output of 18% and an average ratio of debt to assets of one (the sample average in the Fed flow of funds). A value of $\zeta = 0.449$ is chosen to match the sample average of the S&P 500 dividend payout ratio.

The degree of price rigidity is set to $\kappa = 0.7$, in line with most of the New-Keynesian literature, and the degree of wage rigidities is set to $\kappa_w = 0.93$. The degree of wage ridigidy is at the upper end of the DSGE literature, which helps to match the relative volatility of employment in the data, and also helps the amplification mechanism of the learning model: It ensures that changes in subjective expectations generate strong positive comovement of consumption, investment and employment (see Section 7.3). The elasticity of substitution between varieties of the final consumption good, as well as that among varieties of labor used in production, is set to $\sigma = \sigma_w = 4$. The strength of the monetary policy reaction to inflation is set to $\phi_\pi = 1.5$, and the degree of nominal rate smoothing is set to $\rho_i = 0.85$.

The three remaining parameters are set to match volatilities in the data.[14] The standard deviation of the productivity shock is set to $\sigma_A = 0.00614$ to match a volatility of output of 1.43%. The learning gain is set to $g = 0.0048$ to match the volatility of annualized quarterly stock returns in the S&P500 of 32.56%. The rather small value of g implies that agents believe the amount of predictability in stock prices to be small. Finally, the strength of investment adjustment costs is set to $\psi = 7.07$ to match the relative volatility of investment to output of 2.9. Strong adjustment costs are needed because investment is affected by stock price movements, and the amount of stock price volatility under learning is matched to the data. In a sense, the amplification mechanism from learning would be too strong without these adjustment costs.

5.2. Business cycle and asset price moments

Table 1 presents a set of commonly used business cycle statistics. Moments in the data are shown in Column (1). Moments for the learning model are shown in Column (2), while Columns (3) and (4) contain the corresponding moments for the model under rational expectations, and a comparison model without financial frictions.[15] The model parameters are held constant in Columns (2) through (4).

The standard deviation of detrended output is shown in the first row. By construction, it is matched well by the learning model in Column (2). When learning is shut off in Column (3), output volatility halves. This shows the degree of amplification that learning adds to the model. The standard financial accelerator mechanism is present in the model as well, since the volatility of output drops further in Column (4) when financial frictions are shut off. But it is not as powerful as when it is combined with learning.

The next rows report the standard deviation of investment, consumption, hours worked and dividends relative to output. Moving from Column (2) to (3), the removal of learning leads to a drop in the relative volatility of investment and hours worked. This is because the learning model features a high level of investment adjustment costs to match investment volatility. Without large asset price fluctuations generated by learning, investment becomes too smooth, as does the marginal product of capital and hence labor demand. The volatility of dividends is matched well by the learning model.

[13] Qualitatively, the results carry through if one assumes permanent shocks to productivity, $\rho = 1$, as is common in the asset pricing literature.

[14] A different parameterization targeting more data moments, based an SMM estimation, is provided in the appendix.

[15] In the benchmark economy without financial frictions, intermediate goods producers are owned by lending households and face no financial constraint. Also, they take on zero debt, so that the stock price under rational expectations simply equals the value of the capital stock $Q_t K_t$. Learning with CMCE can be introduced to this model similarly to the baseline learning model. See the appendix for details.

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Table 1
Business cycle statistics.

	Moment	(1) Data	(2) Learning	(3) RE	(4) No fin. fric., RE
Output volatility	$\sigma_{hp}(Y_t)$	1.43% (0.14%)	1.43%*	0.72	0.46
Volatility rel.to output	$\sigma_{hp}(I_t)/\sigma_{hp}(Y_t)$	2.90 (0.12)	2.90*	0.77	0.59
	$\sigma_{hp}(C_t)/\sigma_{hp}(Y_t)$	0.60 (0.035)	0.43	0.79	1.33
	$\sigma_{hp}(L_t)/\sigma_{hp}(Y_t)$	1.13 (0.061)	1.12	0.50	0.42
	$\sigma_{hp}(D_t)/\sigma_{hp}(Y_t)$	3.00 (.489)	2.62	2.06	1.84
Correlation with output	$\rho_{hp}(I_t, Y_t)$	0.95 (0.0087)	0.74	0.91	0.32
	$\rho_{hp}(C_t, Y_t)$	0.94 (0.0087)	0.76	0.95	0.99
	$\rho_{hp}(L_t, Y_t)$	0.85 (.035)	0.94	0.88	−0.34
	$\rho_{hp}(D_t, Y_t)$	0.56 (0.080)	0.69	0.73	−0.30
Inflation	$\sigma_{hp}(\pi_t)$	0.27% (0.047%)	0.10%	0.11%	0.13%
Nominal rate	$\sigma_{hp}(i_t)$	0.37% (0.046%)	0.06%	0.05%	0.07%

Quarterly U.S. data 1962Q1–2012Q4. Standard errors in parentheses. π_t is quarterly CPI inflation. i_t is the federal funds rate. All following variables are in logarithms. L_t is total non-farm payroll employment. Consumption C_t consists of services and non-durable private consumption. Investment I_t consists of private non-residential fixed investment and durable consumption. Output Y_t is the sum of consumption and investment. Dividends D_t are four-quarter moving averages of S&P 500 dividends. $\sigma_{hp}(\cdot)$ is the standard deviation and $\rho_{hp}(\cdot,\cdot)$ is the correlation coefficient of HP-filtered data (smoothing coefficient 1600). Moments used in the calibration are marked with an asterisk.

The contemporaneous correlations of consumption, investment, hours worked and dividends with output are broadly in line with the data. Inflation and interest rate volatility are somewhat lower than in the data. It is possible to match these moments better at the expense of choosing somewhat less standard parameter values (see the appendix).

Asset price statistics are presented in Table 2. Starting with excess volatility, the model with learning reproduces the standard deviation of excess returns in the data and also comes close to the volatility of the P/D ratio. The rational expectations version of the model cannot produce a similar amount of volatility (in fact, this is true for any parameterization of the model). The poor asset pricing performance is well known and can of course be addressed with other modelling devices e.g. habit (Boldrin et al., 2001) or long-run risk (Tallarini Jr., 2000). However, neither of these alternative asset pricing models can explain the disconnect of survey expectations from statistically expected stock returns discussed in Section 6.

Stock returns exhibit considerable predictability by the P/D ratio at one- and five-year frequencies. Predictability is not targeted by the calibration, and in fact it is somewhat stronger than in the data, reflected in a persistence of the P/D ratio that is somewhat lower than in the data. The rational expectations model is not able to produce sizable return predictability.

The learning model also produces a distribution of returns that is negatively skewed and heavy-tailed, as in the data, underlining the non-linearities in the asset price dynamics under learning. At the same time, the model delivers a low and smooth risk-free rate. However, the learning model is not able to produce an average equity premium. The reason is that, even though returns are highly volatile, this volatility is not priced because it stems from the updating of subjective beliefs.

The last row of Table 2 shows that the correlation of stock prices with output is only about 0.46 in the data, while in the learning model it is 0.65. One could think that this high correlation arises because of the link between stock prices and investment in the borrowing constraint, but in fact the correlation is even higher in the rational expectations economy without financial frictions. Rather, the high correlation arises because the model is driven only by a single productivity shock. In the data, many movements in stock prices occur without immediate changes in fundamentals, and the model in this paper is not set up to capture those movements.

5.3. Impulse response functions

Impulse response functions reveal the amplification mechanism at play. Figure E.1 plots four sets of impulse responses to a persistent productivity shock. The first and second set of responses depicts the model under learning and RE, respectively. The third and fourth sets depicts the benchmark economy without financial frictions under RE and learning, respectively.[16] In the learning model, output rises persistently after the shock due to both the increased productivity and the relaxation of

[16] For ease of comparison, the learning gain in the “no financial frictions, learning” case has been adjusted to deliver a response of stock prices that is slightly larger than in the baseline learning model.

Table 2
Asset price statistics.

	Moment	(1) Data	(2) Learning	(3) RE	(4) No fin. fric., RE
Excess volatility	$\sigma(R_{t,t+1})$	32.56% (2.44%)	32.56%*	0.35%	0.87%
	$\sigma\left(\frac{P_t}{D_t}\right)$	41.08% (6.11%)	50.38%	2.87%	2.35%
Return predictability	$\rho\left(\frac{P_t}{D_t}, R_{t,t+4}\right)$	−0.297 (0.092)	−0.360	0.541	−0.741
	$\rho\left(\frac{P_t}{D_t}, R_{t,t+20}\right)$	−0.585 (0.132)	−0.795	0.477	−0.647
	$\rho\left(\frac{P_t}{D_t}, \frac{P_{t+4}}{D_{t+4}}\right)$	0.904 (0.056)	0.843	0.836	0.882
negative skewness	$\text{skew}(R_{t,t+1})$	−0.897 (0.154)	−0.342	−0.025	−0.002
heavy tails	$\text{kurt}(R_{t,t+1})$	1.57 (0.62)	2.84	0.07	0.03
risk-free rate	$\mathbb{E}\left(R_t^f\right)$	1.99% (0.61%)	1.99%	1.99%	1.99%
	$\sigma\left(R_t^f\right)$	2.34% (.29%)	0.37%	0.34%	.39%
equity premium	$\mathbb{E}\left(R_{t,t+1} - R_t^f\right)$	4.06% (1.93%)	0.00%	0.00%	0.00%
price correlation with output	$\rho_{hp}(P_t, Y_t)$	0.458 (0.115)	0.646	0.984	0.984

Quarterly U.S. data 1962Q1–2012Q4. Standard errors in parentheses. Dividends D_t are four-quarter moving averages of S&P 500 dividends. The stock price index P_t is the S&P 500. Stock returns $R_{t,t+s}$ are annualized s-quarter ahead real returns of the S&P 500. Risk-free returns R_t^f are 3-month real Treasury yields. $\sigma(\cdot)$ is the standard deviation; $\rho(\cdot, \cdot)$ is the correlation coefficient; $\rho_{hp}(\cdot, \cdot)$ is the correlation coefficient of HP-filtered data (smoothing coefficient 1600); skew$(\cdot)$ is skewness; kurt$(\cdot)$ is excess kurtosis. Moments used in the calibration are marked with an asterisk.

credit constraints from higher asset prices. It also translates into amplification of the responses of investment, consumption, and employment compared to RE. The rise in stock prices in the second row of Figure E.1 is large under learning and accompanied by an initial spike in dividend payments above the level under RE. The nominal interest rate falls less under learning as the monetary authority reacts to the inflationary pressures stemming from the relaxation in credit constraints.

Under rational expectations, the increase in output is larger with financial frictions than without. This is the standard financial accelerator effect, which is known to be quantitatively weak especially for productivitiy shocks. Stock prices display little volatility regardless of the presence of financial frictions. Finally, Figure E.1 also presents impulse responses of the benchmark without financial frictions but with learning. Here, too, learning can generate sizeable fluctuations in stock prices, but these fluctuations do not affect macroeconomic outcomes as much. Learning still causes a positive wealth effect on households and thereby magnifies the responses of output and consumption. But the same wealth effects tends to lower investment because households want to frontload consumption (Fig. 2).

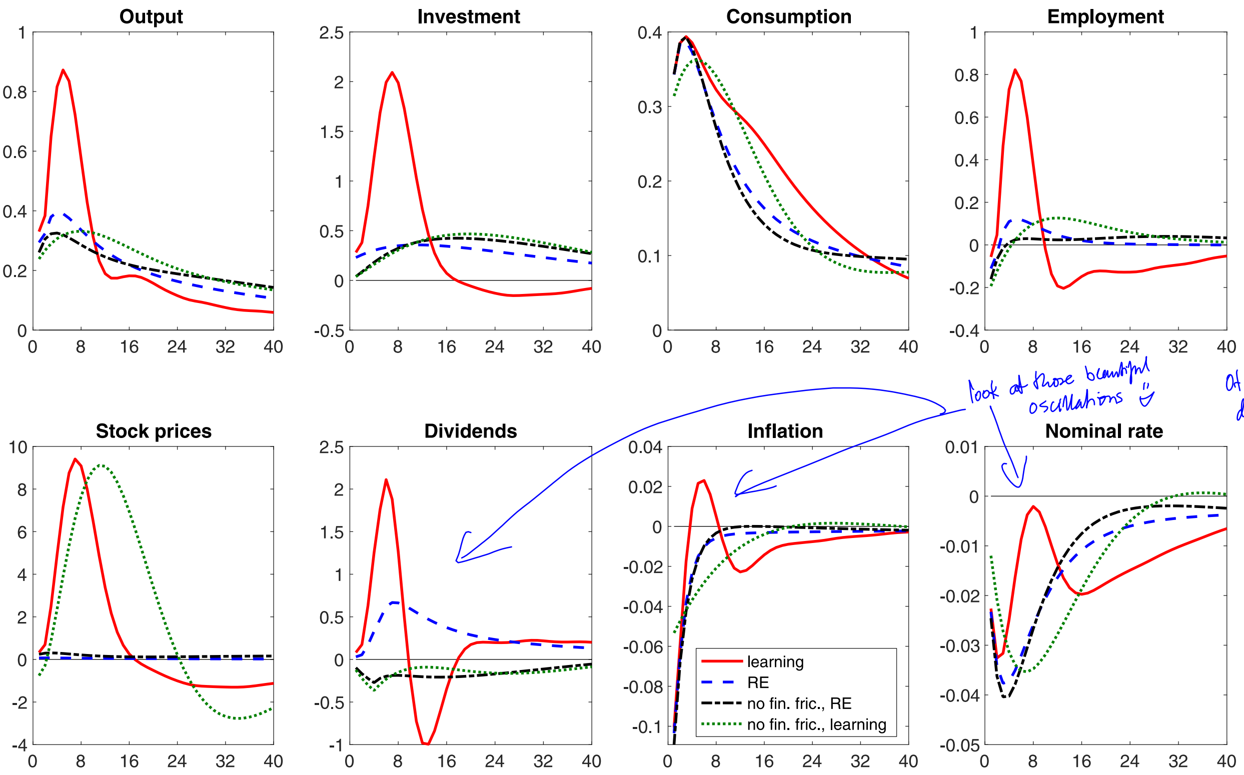
6. Survey data on expectations

Under rational expectations, a forecast error should not be systematically predictable by information available at the time of the forecast. Yet the absence of predictability is easily rejected in the data, which is a longstanding challenge for rational expectations.

In this model, agents also make systematic, predictable forecast errors. This is true not only for stock prices but also for other endogenous model variables, despite the fact that agents' beliefs are model-consistent conditional on stock prices. A systematic mistake in predicting stock prices will spill over into a corresponding mistake in predicting the tightness of credit constraints, and hence investment, output, and so forth.

Fig. 3 compares the predictability of forecast errors in the Federal Reserve's Survey of Professional Forecasters (SPF) as well as the CFO survey to that obtained from simulated model data. Each red dot corresponds to a correlation of the error of the mean survey forecast with a variable that is observable by respondents at the time of the survey. Under the null of rational expectations, all coefficients should be zero, which is clearly at odds with the data. The blue crosses show the corresponding correlation coefficient in the model, together with 95% confidence intervals in small samples of the same length as the data.

Panel (a) of the figure contains the correlations of future forecast errors with the P/D ratio. When stock prices are high, people systematically over-predict future stock returns, which is reproduced by the model. In the model, high stock prices are caused by high return expectations, but they are followed by low returns because of mean reversion in beliefs. The model also reproduces the predictability pattern for output, investment, consumption and unemployment, as agents' optimism about asset prices spills over into optimismn about borrowing constraints. Where the model fails to replicate the data is



look at those beautiful oscillations ;)

Of course he doesn't mention them.

Fig. 2. Impulse responses to a productivity shock.
Impulse responses to a one-standard deviation innovation in ε_t, averaged over 5000 random shock paths with a burn-in of 1000 periods. Stock prices, dividends, output, investment, consumption, and employment are in 100*log deviations. Stock returns and the nominal interest rate are in percentage point deviations.

for inflation: When stock prices are high, forecasters under-predict inflation, while in the model the opposite result obtains. This discrepancy probably arises because stock market booms are inflationary in the model, which seems not to be true in the data (Christiano et al., 2010).

Panel (b) of the figure repeats the exercise for the first difference of the P/D ratio. The growth rate of P/D predicts forecast errors of output, investment and consumption positively and forecast errors of unemployment negatively. The model replicates the patterns in the data remarkably well. In the model, this happens because agents' expectations adjust slowly: They under-predict an expansion in its beginning but then overshoot and over-predict it when it is about to end. The predictability for stock returns, inflation and interest rates is less strong in the data.

Finally, Panel (c) of the figure reports predictability by forecast revisions, as proposed by Coibion and Gorodnichenko (2015). Forecast errors are positively predicted by forecast revisions across all macroeconomic variables. While Coibion and Gorodnichenko interpret this predictability in favor of models of rational inattention, here, the learning model captures this "underreaction" of expectations also remarkably well. The model does not produce predictability of stock return forecast error; in fact, the alternative parameterization provided in the appendix even produces "overreaction" of expectations, i.e. a negative relation between forecast revisions and subsequent forecast errors.

7. Sensitivity

7.1. Belief formation concept

This paper has introduced the concept of conditionally model-consistent expectations (CMCE) to specify beliefs about variables other than stock prices. But there are other ways in which one could specify these expectations. Here, I discuss two alternatives.

The first alternative belief formation process is adaptive learning (AL) along the lines of Evans and Honkapohja (2001): After linearizing the model equations, one replaces all forward-looking variables with a linear forecasting rule, the parameters of which are updated using recursive least squares. Here, I log-linearize the model, keep (3.12)–(3.13) as the forecasting equations for the stock price, and use standard minimum-state-variable forecasting rules for variables other than the stock price, updated using constant-gain learning. The appendix contains a complete description of the algorithm.

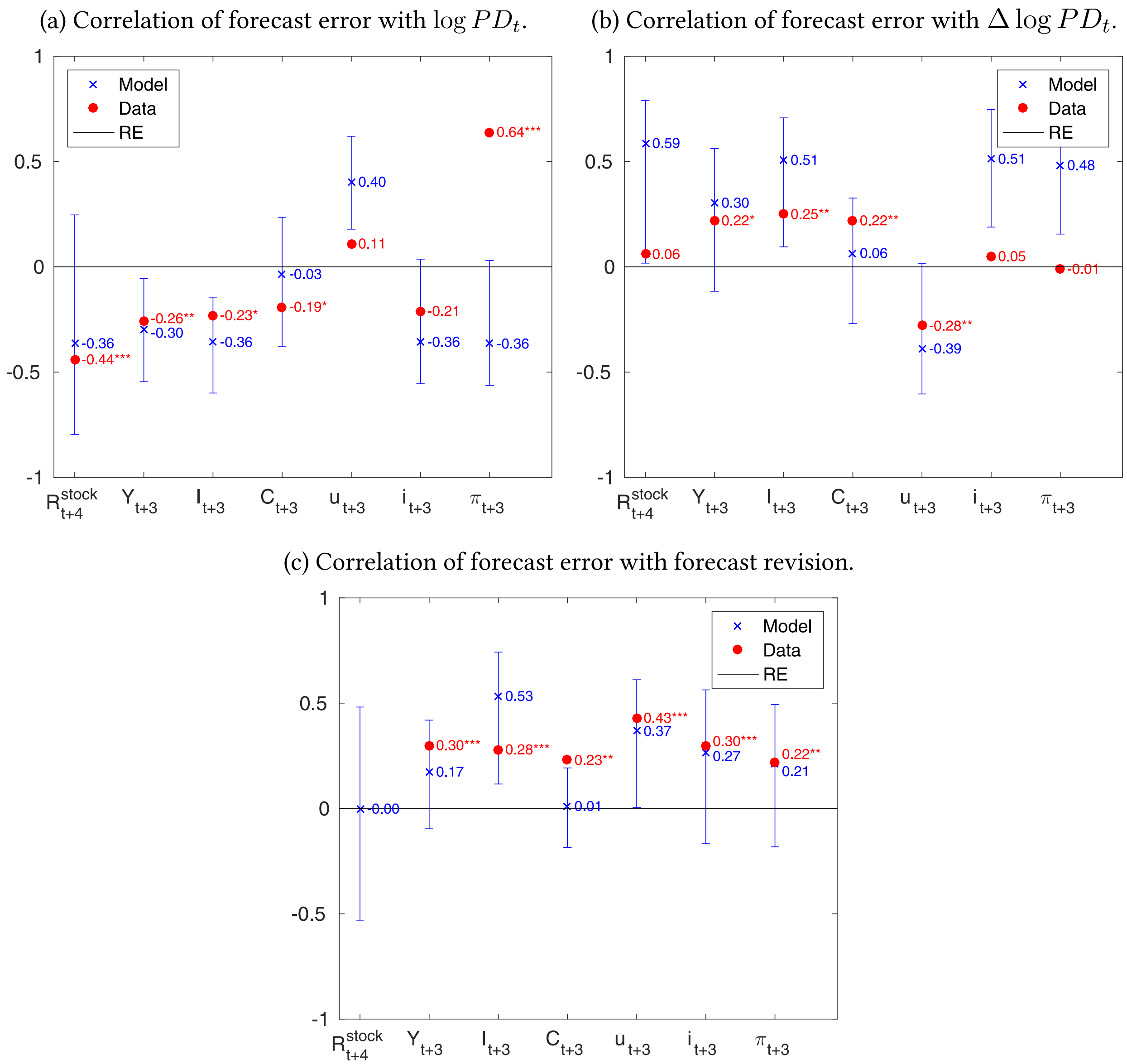


Fig. 3. Forecast error predictability.
Red dots show correlation coefficients for mean forecast errors on one year-ahead nominal stock returns (Graham-Harvey survey) and three quarters-ahead real output growth, investment growth, consumption growth, unemployment rate, CPI inflation and 3-month treasury bill (SPF). Regressors: Panel (a) is the S&P 500 P/D ratio and Panel (b) is its first difference. Panel (c) is the forecast revision as in Coibion and Gorodnichenko (2015), which is only available in the SPF. Data from Graham-Harvey covers 2000Q3–2012Q4. Data for the SPF covers 1981Q1–2012Q4. *, **, and *** indicate significance at the 1, 5, and 10% level, respectively, using Newey–West standard errors. Blue crosses show corresponding correlation coefficients in the model, computed using a simulation of length 50,000, where subjective forecasts are computed using a second-order approximation to the subjective belief system on a path in which no more future shocks occur, starting at the current state in each period. Unemployment in the model is taken to be $u_t = 1 - L_t$. Stock returns in the model $R^{stock}_{t,t+4}$ are quarterly nominal aggregate market returns. Blue lines show 95% confidence bands of the correlation coefficients in the model in small samples of the same size as the data (123 quarters in the SPF and 49 quarters in the Graham-Harvey survey) from 5000 simulations with a burn-in period of 1000 periods.

The second alternative is what could be called “mixed rational expectations” (mixed RE). Here, the forecast of the stock price under learning is substituted into the Euler equation for stocks, after which the resulting set of model equations is solved under rational expectations (again, the appendix contains a complete description). This captures a setting in which agents have fully rational expectations for all variables except stock prices. In particular, they take the consequences of their own systematic prediction errors into account for decisions other than stock holdings. By contrast, under CMCE, agents are genuinely unaware of the predictability of their forecast errors.

Fig. 4 shows impulse responses to a productivity shock for the two alternative belief formation processes. In each case, the stock price learning gain g is adjusted to yield a stock price response that is slightly stronger than the baseline. The

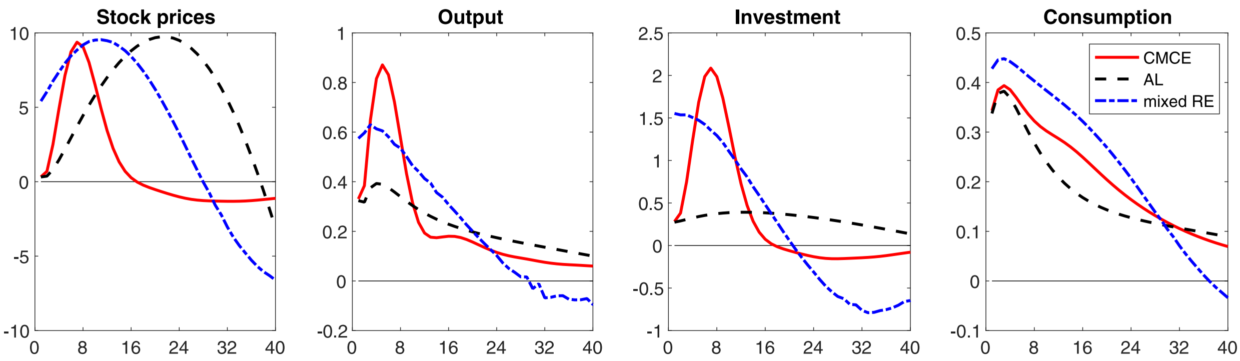


Fig. 4. Impulse responses under alternative belief formation processes.
Impulse responses to a one standard deviation positive productivity shock, averaged over 5000 random shock paths with a burn-in of 1000 periods. Red solid line: Conditionally model-consistent expectations (CMCE). Black dashed line: Adaptive learning (AL). Blue dash-dotted line: Mixed-rational expectations (mixed RE).

Table 3
Sensitivity of selected moments under alternative specifications.

	Moment	$\sigma_{hp}(Y_t)$	$\frac{\sigma_{hp}(I_t)}{\sigma_{hp}(Y_t)}$	$\frac{\sigma_{hp}(L_t)}{\sigma_{hp}(Y_t)}$	$\frac{\sigma_{hp}(\pi_t)}{\sigma_{hp}(Y_t)}$	$\frac{\sigma_{hp}(Y_t)}{\sigma_{hp}(Y_{t,RE})}$	$\sigma(R_{t,t+1})$	$\rho\left(\frac{P_t}{D_t}, R_{t,t+4}\right)$	$\rho\left(\frac{P_t}{D_t}, \frac{P_{t+4}}{D_{t+4}}\right)$
(1)	Baseline	1.43%	2.90	1.12	0.07	1.99	32.56%	−.360	.843
(2)	$g = 0.002$	1.07%	1.10	1.00	0.09	1.47	23.67%	−.559	.325
(3)	$\rho_\mu = .99$	1.35%	2.60	1.07	0.06	1.86	29.44%	−.464	.704
(4)	$\rho_\mu = .85$	1.06%	1.45	0.96	0.08	1.46	22.67%	−.716	−.127
(5)	$a_0 = 0.0001$	1.24%	2.25	1.00	0.06	1.72	25.17%	−.498	.620
(6)	$a_0 = 0.001$	1.10%	2.00	1.11	0.10	1.52	30.48%	−.815	−.240
(7)	$\psi = 1$	3.06%	4.70	1.40	0.07	4.41	37.71%	−.325	.881
(8)	$\kappa = 0.93$	2.19%	2.64	1.49	0.03	11.58	41.03%	−.489	.718
(9)	$\kappa_w = 0.5$	0.83%	2.26	0.74	0.08	1.72	16.76%	−.371	.840
(10)	$\kappa, \kappa_w, \psi = 0$	0.74%	3.15	0.36	0.05	1.08	4.92%	−.184	.962

Data as in Tables E.2 and E.3.

figure shows that despite a slightly weaker and less persistent stock price response, the baseline model with CMCE delivers stronger peak business cycle amplification than either of the alternatives.

The reason is that under CMCE, misperceptions of future stock prices matter beyond their direct effect on current stock prices. When agents overpredict future stock prices, they also overpredict the slackness of the borrowing constraint affecting future firm investment, as well as future household income. This misperception increases aggregate demand and affects the forecast of the future price of capital entering the borrowing constraint (3.8), which both amplify the transmission of misperceptions in financial markets. Under mixed rational expectations, these additional channels are absent by construction, as agents do not make systematic forecast errors for any variables other than stock prices; and under AL, it turns out that such misperceptions are quantitatively unimportant in this model.

7.2. Mean-reverting beliefs

Consider extending the belief system (3.12)–(3.13) as follows:

$$\log P_t = \log P_{t-1} + \hat{\mu}_{t-1} + z_t \tag{7.1}$$

$$\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} - a_0 \log \frac{P_{t-1}}{\bar{P}} + g z_t. \tag{7.2}$$

This specification nests the baseline belief system when $\rho_\mu = 1$ and $a_0 = 0$. If $\rho_\mu < 1$ and $a_0 = 0$, agents believe stock price growth to be mean-reverting, although they still believe the level of stock prices to follow a random walk. If $\rho_\mu < 1$ and $a_0 > 0$, then agents also believe that the level of stock prices is mean-reverting. The top part of Table 3 documents a number of key moments for different specifications of the extended belief process.

Row (1) repeats the moments under the baseline, and Row (2) shows the sensitivity to the learning gain g (keeping $\rho_\mu = 1$ and $a_0 = 0$). The gain governs the speed with which expectations adjust to recent observations of price growth. Asset price volatility as well as the amount of amplification obtained under learning are very sensitive to this parameter. A lower gain reduces asset price volatility because subjective capital gains expectations become less volatile. By consequence, the tightness of the credit constraint becomes less procyclical and output volatility is reduced, as well as the amount of amplification over the rational expectations model.

In Rows (3) and (4), the learning gain is fixed, $a_0 = 0$, and ρ_μ is being varied. Row (3) shows that the value $\rho_\mu = 1$ is not special: Even if agents believe that asset price growth will mean-revert eventually, the properties of the model barely change. However, Row (4) shows that as ρ_μ moves further away from one, some undesirable properties emerge: The 4-quarter autocorrelation of the price-dividend ratio turns negative and return predictability becomes excessive. These properties are the result of deterministic asset price oscillations at yearly frequency, which are clearly at odds with the data.

The same conclusions emerge in Rows (5) and (6), where $\rho_\mu = 1$ and a_0 is being varied. Again, the value $a_0 = 1$ is not special. Row (5) shows that, even if agents believe that asset price *levels* mean-revert in the long-run, the effects of learning change little as long as price growth expectations are persistent. However, as a_0 moves further away from zero in Row (6), deterministic asset price oscillations emerge under learning, which lead to excessive return predictability and a negative autocorrelation of the price-dividend ratio.

7.3. Nominal rigidities and adjustment costs

The analysis of the simplified model in Section 4 has made clear that the main mechanism interacting learning and financial constraints operates without nominal rigidities and adjustment costs. However, these frictions are important for the quantitative fit of the full model. In particular, nominal rigidities help generate positive comovement of consumption, investment and employment in response to subvjective changes in beliefs, for the same reasons that they help with comovement in response to financial shocks (Ajello, 2016).

The lower part of Table 3 displays the sensitivity of key moments to the auxiliary frictions. Other model parameters are kept at their baseline values for comparison.

In Row (7), adjustment costs are lowered by setting $\psi = 1$. The result is a large increase in output volatility and amplification from learning, while investment is more than four times more volatile relative to output. Because the borrowing constraint in the model links investment to stock prices, high adjustment costs are needed to match the stock price volatility in the data without generating counterfactually volatile investment. Even with smaller shocks, it would not be possible to jointly match output and investment volatility.

In Row (8), price rigidities are increased by setting $\kappa = 0.93$, the same value as for wage rigidities κ_w. The consequence is a strong increase in output volatility and amplification. Price rigidities provide additional amplification through aggregate demand (see also Section 7.1): Optimistic asset price beliefs increase aggregate demand, lower markups and increase prices q_t for credit-constrained intermediate goods producers. These firms can then invest more and pay out higher dividends. Price rigidities also dampen the rise in real interest rates unless monetary policy is very aggressive. However, inflation volatility in Row (8) becomes extremely low, as the downward pressure on inflation from the productivity shock and the upward pressure from the wealth effect on households are counteracting forces on inflation.

In Row (9), wage rigidities are lowered by setting $\kappa_w = 0.5$. The effect is that employment volatility drops by a large amount. Real wage fluctuations are a general equilibrium force that typically counteracts expansions that are not primarily driven by TFP shocks. Wage rigidities dampen movements in the real wage and allow employment to co-move more strongly in response to changes in subjective expectations, thereby helping amplification.

Finally, in Row (10), price and wage rigidities as well as adjustment costs are completely shut off. As a result, excess stock return volatility all but disappears and amplification from learning is much lower than in the baseline.

7.4. Monetary policy

Although the model does not contain monetary policy shocks, one can still simulate the effects of an unanticipated negative shock to the nominal interest rate, as shown in Fig. 5. The learning model delivers strong amplification of the monetary policy shock in all business cycle aggregates, demonstrating that the amplification mechanism works for demand-type shocks as well.

This experiment suggests that monetary policy could be quite powerful in mitigating the effects of belief fluctuations. To examine this idea in some more detail, consider extending the interest rate rule (3.9) with reactions to output and asset price growth:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(1/\beta + \phi_\pi \pi_t + \phi_Y \Delta \log Y_t + \phi_P \Delta \log P_t) \tag{7.3}$$

I consider several values for the parameters ϕ_π, $\phi_{\Delta Y}$, $\phi_{\Delta P}$ in Table 4. Column (1) shows the baseline policy rule, under which learning amplifies output fluctuations by 46%. In Column (2), the coefficient on inflation ϕ_π is doubled. This rule achieves a marked reduction in the volatility of inflation, but at the cost of higher output volatility. Column (3) augments the baseline rule by a reaction to output growth. This rule reduces the volatility of output and stock prices, while keeping the volatility of inflation unchanged. Column (4) considers a reaction of interest rates to stock price growth. Such a reaction is effective in stabilizing the economy under learning. Stock price volatility drop markedly and output volatility is further reduced, at the cost of only moderately higher inflation.[17] The amplification mechanism is completely eliminated—output volatility is the same as under rational expectations. Stabilization is achieved primarily because fluctuations in subjective

[17] In the alternative model parameterization provided in the appendix, a reaction to stock price growth even improves both output and inflation volatility.

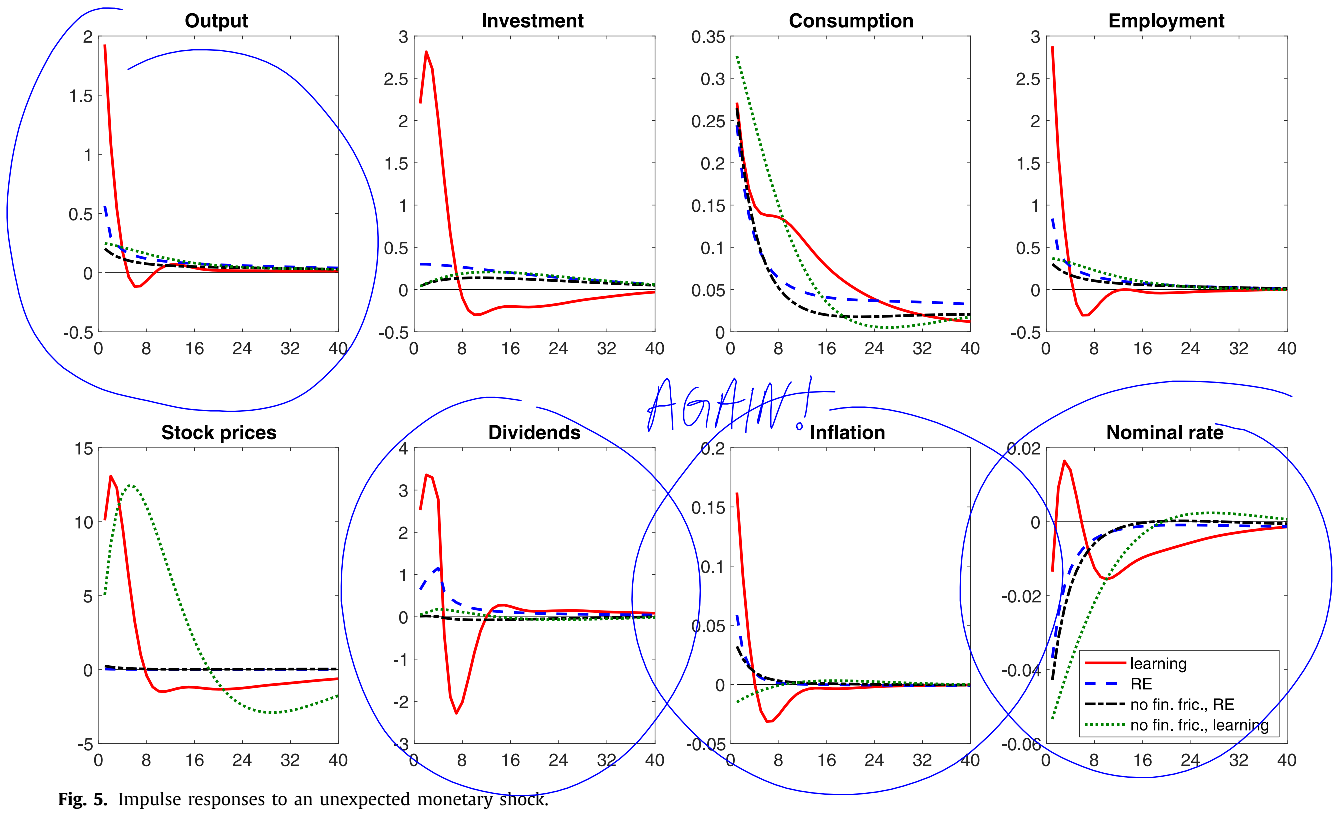


Fig. 5. Impulse responses to an unexpected monetary shock.
Impulse responses to an unexpected negative shock to the interest rate rule (3.9). Responses averaged over 5000 random shock paths with a burn-in of 1000 periods. The size of the innovation is chosen to produce a 10 basis point fall in the equilibrium nominal rate. Stock prices, dividends, output, investment, consumption and employment are in 100*log deviations. Inflation and the nominal interest rate are in percentage point deviations.

Table 4
Alternative monetary policy rules.

	(1)	(2)	(3)	(4)
ϕ_π	1.5	3.0	1.5	1.5
ϕ_Y			0.5	
ϕ_P				0.5
$\sigma(Y)$	2.67%	3.2%	1.97%	1.78%
$\sigma(\pi)$	0.16%	0.09%	0.16%	0.23%
$\sigma(P)$	41.17%	55.07%	26.62%	9.62%
$\sigma(i)$	0.15%	0.17%	0.09%	0.12%
$\sigma(Y)/\sigma(Y_{RE})$	1.56	1.31	1.22	1.01

Standard deviations of output, stock prices, inflation, and interest rates (unfiltered) under learning in percent. The standard deviation of output under RE $\sigma(Y_{RE})$ is calculated at the same parameter values as the learning solution. The interest rate smoothing coefficient is kept at $\rho_i = 0.85$ for all rules considered.

beliefs are dampened in this case: When asset prices rise, monetary policy tightens, thereby increasing borrowing costs and reducing dividend payoutsand preventing subjective investor optimism from building up.

These observations suggests that, in an environment with asset price learning, a positive interest rate reaction to stock price growth could be beneficial. In related work, Caines and Winkler (2018) derive optimal monetary policy in a simpler New-Keynesian model with asset price learning, and find that the policymaker moves nominal interest rates positively with asset price expectations to implement optimal policy.

8. Conclusion

In this paper, I have examined the implications of learning-based asset pricing in a business cycle model with financial frictions. When firms' borrowing constraints depend on their market value, learning interacts with credit frictions to form a two-sided feedback loop between stock prices and firm profits that amplifies learning dynamics and magnifies the financial accelerator. The model jointly matches key business cycle and asset pricing moments.

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To isolate the effects of stock price learning in the presence of several forward-looking variables, the paper developed a particular restriction on expectations called “conditionally model-consistent expectations”. It requires that agents’ expectations remain consistent with the maximum number of equilibrium conditions compatible with subjective asset price beliefs. Despite this restriction, the model replicates several patterns of forecast error predictability in survey data across a broad range of variables. The method is general and can be of interest for other researchers wishing to study other limited departures from rational expectations.[18]

Further research could explore firm heterogeneity, accounting for the fact that not all firms are credit constrained at the same time. Also, the expectation formation mechanism proposed here has several implications, e.g. for the comovement of forecast bias across variables, that can be tested in empirical work.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jmoneco.2019.03.002.

References

Adam, K., Marcet, A., 2011. Internal rationality, imperfect market knowledge and asset prices. J. Econ. Theory 146 (3), 1224–1252.
Adam, K., Marcet, A., Beutel, J., 2017. Stock price booms and expected capital gains. Am. Econ. Rev. 107 (8), 2352–2408.
Adam, K., Marcet, A., Nicolini, J.P., 2015. Stock market volatility and learning. J. Financ. 71 (1), 33–82.
Ajello, A., 2016. Financial intermediation, investment dynamics and business cycle fluctuations. Am. Econ. Rev. 106 (8), 2256–2303.
Bacchetta, P., Mertens, E., van Wincoop, E., 2009. Predictability in financial markets: what do survey expectations tell us? J. Int. Money Financ. 28 (3), 406–426.
Beaudry, P., Portier, F., 2007. When can changes in expectations cause business cycle fluctuations in neoclassical settings? J. Econ. Theory 135 (1), 458–477.
Bernanke, B.S., Gertler, M., 2001. Should central banks respond to movements in asset prices? Am. Econ. Rev. 91 (2), 253–257.
Bernanke, B.S., Gertler, M., Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework. In: Taylor, J.B., Woodford, M. (Eds.), Handbook of Macroeconomics, 1. Elsevier, pp. 1341–1393. 21.
Boldrin, M., Christiano, L.J., Fisher, J.D.M., 2001. Habit persistence, asset returns, and the business cycle. Am. Econ. Rev. 91 (1), 149–166.
Brunnermeier, M.K., Sannikov, Y., 2014. A macroeconomic model with a financial sector. Am. Econ. Rev. 104 (2), 379–421.
Caines, C., Winkler, F., 2018. Asset price learning and optimal monetary policy. International Finance Discussion Paper 1236. Federal Reserve Board.
Caputo, R., Medina, J.P., Soto, C., 2010. The financial accelerator under learning and the role of monetary policy. Working Papers of the Central Bank of Chile 590. Central Bank of Chile.
Christiano, L., Ilut, C., Motto, R., Rostagno, M., 2010. Monetary policy and stock market booms. In: Proceedings of the Jackson Hole Economic Policy Symposium, pp. 85–145.
Coibion, O., Gorodnichenko, Y., 2012. What can survey forecasts tell us about information rigidities? J. Polit. Econ. 120 (1), 116–159.
Coibion, O., Gorodnichenko, Y., 2015. Information rigidity and the expectations formation process: a simple framework and new facts. Am. Econ. Rev. 105 (8), 2644–2678.
Cole, S., Milani, F., 2017. The misspecification of expectations in New Keynesian models: a DSGE-VAR approach. Macroecon. Dyn. 1–34.
Collin-Dufresne, P., Johannes, M., Lochstoer, L.A., 2016. Parameter learning in general equilibrium: the asset pricing implications. Am. Econ. Rev. 106 (3), 664–698.
Evans, G., Honkapohja, S., 2001. Learning and Expectations in Macroeconomics. Princeton University Press.
Flynn, E., Crewson, P., 2009. Chapter 11 filing trends in history and today. Am. Bankruptcy Inst. J. 14, 65 ff.
Fuster, A., Hebert, B., Laibson, D., 2012. Natural expectations, macroeconomic dynamics, and asset pricing. In: Acemoglu, D., Woodford, M. (Eds.), NBER Macroeconomics Annual 2011, 26. National Bureau of Economic Research, NBER Chapters, pp. 1–48.
Gelain, P., Lansing, K.J., Mendicino, C., 2013. House prices, credit growth, and excess volatility: implications for monetary and macroprudential policy. Int. J. Central Bank. 9 (2), 219–276.
Gennaioli, N., Ma, Y., Shleifer, A., 2016. Expectations and investment. NBER Macroecon. Annu. 30 (1), 379–431.
Greenwood, R., Shleifer, A., 2014. Expectations of returns and expected returns. Rev. Financ. Stud. 27 (3), 714–746.
Hirshleifer, D., Li, J., Yu, J., 2015. Asset pricing in production economies with extrapolative expectations. J. Monet. Econ. 76, 87–106.
Jermann, U., Quadrini, V., 2012. Macroeconomic effects of financial shocks. Am. Econ. Rev. 102 (1), 238–271.
Kiyotaki, N., Moore, J., 1997. Credit cycles. J. Polit. Econ. 105 (2), 211–248.
Kocherlakota, N.R., 2000. Creating business cycles through credit constraints. Q. Rev. 2–10.
Lawton, A., 2012. Chapter 11 triage: diagnosing a debtor’s prospects for success. Ariz. Law Rev. 54 (4), 985–1028.
Manski, C.F., 2004. Measuring expectations. Econometrica 72 (5), 1329–1376.
Mendoza, E.G., 2010. Sudden stops, financial crises, and leverage. Am. Econ. Rev. 100 (5), 1941–1966.
Miao, J., Wang, P., Xu, Z., 2015. A Bayesian dynamic stochastic general equilibrium model of stock market bubbles and business cycles. Quant. Econ. 6 (3), 599–635.
Milani, F., 2007. Expectations, learning and macroeconomic persistence. J. Monet. Econ. 54 (7), 2065–2082.
Milani, F., 2011. Expectation shocks and learning as drivers of the business cycle. Econ. J. 121 (552), 379–401.
Milani, F., 2017. Learning about the interdependence between the macroeconomy and the stock market. Int. Rev. Econ. Financ. 49, 223–242.
Pintus, P.A., Suda, J., 2019. Learning financial shocks and the great recession. Rev. Econ. Dyn. 31, 123–146.
Quadrini, V., 2011. Financial frictions in macroeconomic fluctuations. FRB Richmond Econ. Q. 97 (3), 209–254.
Rychalovska, Y., 2016. The implications of financial frictions and imperfect knowledge in the estimated DSGE model of the u.s. economy. J. Econ. Dyn. Control 73, 259–282.
Slobodyan, S., Wouters, R., 2012. Learning in a medium-scale DSGE model with expectations based on small forecasting models. Am. Econ. J. Macroecon. 4 (2), 65–101.
Tallarini Jr., T.D., 2000. Risk-sensitive real business cycles. J. Monet. Econ. 45 (3), 507–532.
Timmermann, A., 1996. Excess volatility and predictability of stock prices in autoregressive dividend models with learning. Rev. Econ. Stud. 63 (4), 523–557.

[18] Code to compute perturbation solutions to custom models with conditionally model-consistent expectations is available as a Dynare add-on at fabianwinkler.com/research.