Materials 8 - Massaging RE IRFs and interpreting learning IRFs

Laura Gáti

November 7, 2019

Overview

1 Model summary with interest rate smoothing ρi_{t-1}

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \rho i_{t-1} + \bar{i}_t \tag{3}$$

$$\hat{\mathbb{E}}_t z_{t+h} = \begin{bmatrix} \bar{\pi}_{t-1} \\ 0 \\ 0 \end{bmatrix} + b hx^{h-1} s_t \quad \forall h \geq 1 \qquad b = gx\ hx \qquad \text{PLM} \tag{4}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \underbrace{\big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big)}_{\text{fcst error using (4)}} \qquad (b_1 \text{ is the first row of } b) \tag{5}$$

$$k_t = \mathbb{I} \times (k_{t-1} + 1) + (1 - \mathbb{I}) \times \bar{g}^{-1} \tag{6}$$

$$\mathbb{I} = \begin{cases} 1 & \text{if } \theta_t \leq \bar{\theta} \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

$$\theta_t = |\hat{\mathbb{E}}_{t-1}\pi_t - \mathbb{E}_{t-1}\,\pi_t|/\sigma_s \qquad \text{CEMP criterion for the gain} \tag{8}$$

The alternative criterion for the choice of gain is a recursive variant of the CUSUM-test (Brown, Durbin, Evans 1975):

1. Let FE_t denote the short-run forecast error, and ω_t firms' estimate of the FE variance.
2. Let $\kappa \in (0, 1)$ and $\tilde{\theta}$ be the new threshold value for the criterion.
3. Then for initial (ω_0, θ_0), firms in every period estimate the criterion and the FEV as:

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1}(FE_t^2 - \omega_{t-1}) \tag{9}$$

$$\theta_t = \theta_{t-1} + \kappa k_{t-1}^{-1}(FE_t^2/\omega_t - \theta_{t-1}) \tag{10}$$

$$k_t = \mathbb{I} \times (k_{t-1} + 1) + (1 - \mathbb{I}) \times \bar{g}^{-1} \tag{11}$$

$$\mathbb{I} = 1 \quad \text{if} \quad \theta_t \leq \tilde{\theta} \tag{12}$$

2 Compact notation - with lagged interest rate term in TR

$$z_t = A_p^{RE}\,\mathbb{E}_t\, z_{t+1} + A_s^{RE} s_t \tag{13}$$

$$z_t = A_a^{LH} f_a(t) + A_b^{LH} f_b(t) + A_s^{LH} s_t \tag{14}$$

$$s_t = P s_{t-1} + \epsilon_t \qquad \rightarrow \quad s'_t = hx\; s'_{t-1} + \epsilon'_t \tag{15}$$

$$\text{where} \quad s'_t \equiv \begin{pmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{pmatrix} \quad hx \equiv \begin{pmatrix} \rho_r & 0 & 0 & 0 \\ 0 & \rho_i & 0 & 0 \\ 0 & 0 & \rho_u & 0 \\ gx_{3,1} & gx_{3,2} & gx_{3,3} & gx_{3,4} \end{pmatrix} \quad \epsilon'_t \equiv \begin{pmatrix} \varepsilon_t^r \\ \varepsilon_t^i \\ \varepsilon_t^u \\ 0 \end{pmatrix} \quad \text{and} \quad \Sigma' = \begin{pmatrix} \sigma_r & 0 & 0 & 0 \\ 0 & \sigma_i & 0 & 0 \\ 0 & 0 & \sigma_u & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{16}$$

i_{t-1} is an endogenous state and breaks the link that previously had $P = hx$; now this is no longer true. In particular, using Matlabby notation, $P = hx(1:3, 1:3)$.

And the A_s^{RE} and A_s^{LH} are given by:

$$A_s^{RE} = \begin{pmatrix} \frac{\kappa\sigma}{w} & -\frac{\kappa\sigma}{w} & 1 - \frac{\kappa\sigma\psi_\pi}{w} & 0 \\ \frac{\sigma}{w} & -\frac{\sigma}{w} & -\frac{\sigma\psi_\pi}{w} & 0 \\ \psi_x(\frac{\sigma}{w}) + \psi_\pi(\frac{\kappa\sigma}{w}) & \psi_x(-\frac{\sigma}{w}) + \psi_\pi(-\frac{\kappa\sigma}{w}) + 1 & \psi_x(-\frac{\sigma\psi_\pi}{w}) + \psi_\pi(1 - \frac{\kappa\sigma\psi_\pi}{w}) & \rho \end{pmatrix} \tag{17}$$

$$A_s^{LH} = \begin{pmatrix} g_{\pi s} \\ g_{xs} \\ \psi_\pi g_{\pi s} + \psi_x g_{xs} + \begin{bmatrix} 0 & 1 & 0 & \rho \end{bmatrix} \end{pmatrix} \tag{18}$$

$$g_{\pi s} = (1 - \frac{\kappa\sigma\psi_\pi}{w}) \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} (I_4 - \alpha\beta hx)^{-1} - \frac{\kappa\sigma}{w} \begin{bmatrix} -1 & 1 & 0 & \rho \end{bmatrix} (I_4 - \beta hx)^{-1} \tag{19}$$

$$g_{xs} = \frac{-\sigma\psi_\pi}{w} \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} (I_4 - \alpha\beta hx)^{-1} - \frac{\sigma}{w} \begin{bmatrix} -1 & 1 & 0 & \rho \end{bmatrix} (I_4 - \beta hx)^{-1} \tag{20}$$

3 Recap of timing

Define some objects: *(I usually let t denote the time in which the variable is formed.)*

$$
\begin{aligned}
f_t^j &= \hat{\mathbb{E}}_t(z_{t+1}) \quad \text{one-period-ahead forecast formed at time } t,\ j = m, e \text{ (morning or evening)} && (21)\\
FE_t &= z_{t+1} - f_t \quad \text{one-period-ahead forecast error realized at time } t+1 && (22)\\
&= ALM(t+1) - PLM(t) && (23)\\
\theta_t &= \hat{\mathbb{E}}_{t-1}(z_t) - \mathbb{E}_{t-1}(z_t) \quad \text{CEMP's criterion} && (24)\\
&= PLM(t-1) - \mathbb{E}_{t-1}\, ALM(t) && (25)
\end{aligned}
$$

$PLM(t) : \hat{\mathbb{E}}_t z_{t+1} = \bar{z}_{t-1} + bs_t$

Morning: morning of time t available: $\mathcal{I}_t^m = \{\bar{z}_{t-1}, s_t, k_{t-1}, FE_{t-2}\}$

1. Form all future expectations using $PLM(t)$(morning forecast) $\rightarrow z_t$ realized, $\rightarrow FE_{t-1}$ realized

2. Form $\theta_t \rightarrow k_t$ realized

3. **Evening**: Update $\bar{z}_t = \bar{z}_{t-1} + k_t^{-1}(FE_{t-1}^e)$

   where $FE_{t-1}^e = z_t - f_{t-1}^e = z_t - (\bar{z}_{t-1} + bs_{t-1})$ is the most recent realized FE, so:

   $\bar{z}_t = \bar{z}_{t-1} + k_t^{-1}(z_t - (\bar{z}_{t-1} + bs_{t-1}))$

   $\rightarrow$ evening of time t available: $\mathcal{I}_t^e = \{\bar{z}_t, s_t, k_t, FE_{t-1}\}$

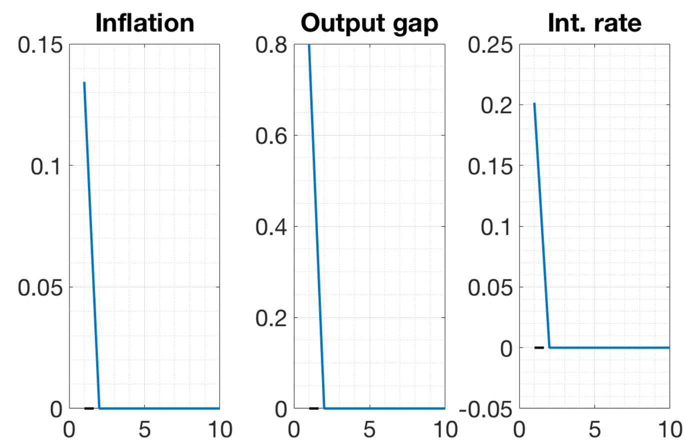
4 Current set of baseline parameters

β	0.99	stochastic discount factor	standard (Woodford 2003/2011)
σ	1	IES	consistent with long-run growth
α	0.5	Calvo probability of not adjusting	match 6-month duration of prices (can increase to 0.75)
ψ_π	1.5	coefficient of inflation in Taylor rule	Taylor
ψ_x	0	coefficient of output gap in Taylor rule	focus on π
$\bar{g}$	0.145	value of the constant gain	CEMP
$\bar{\theta}$	1	threshold deviation between $\hat{\mathbb{E}}$ & $\mathbb{E}$	CEMP: 0.029
ρ_r	0	persistence of natural rate shock	n.a.
ρ_i	0.6	persistence of monetary policy shock	CEMP: 0.877 (can increase to 0.78 if $\alpha = 0.75$)
ρ_u	0	persistence of cost-push shock	CEMP
σ_r	0.1	standard deviation of natural rate shock	n.a.
σ_i	0.359	standard deviation of mon. policy shock	CEMP
σ_u	0.277	standard deviation of cost-push shock	CEMP
θ	10	price elasticity of demand	Woodford 2003/2011, Chari, Kehoe & McGrattan 2000
ω	1.25	elasticity of marginal cost to output	Woodford 2003/2011, Chari, Kehoe & McGrattan 2000

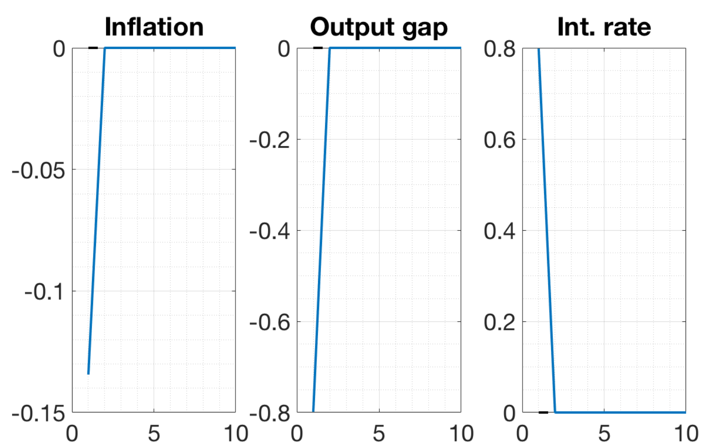
5 IRFs: RE only

5.1 RE: all shocks at a glance

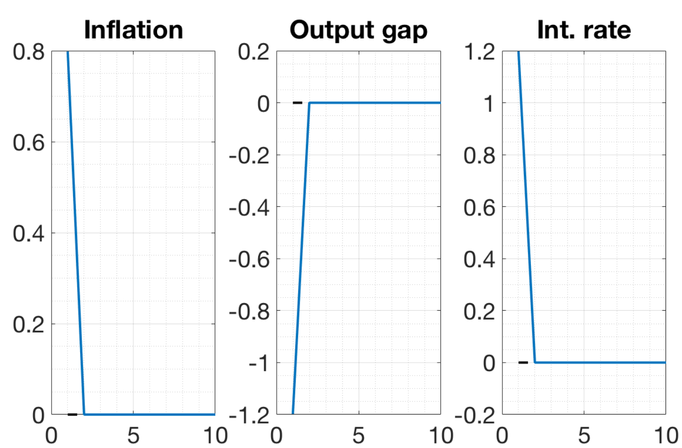
Figure 1: RE: all shocks at a glance



(a) Natural rate shock



(b) Monetary policy shock



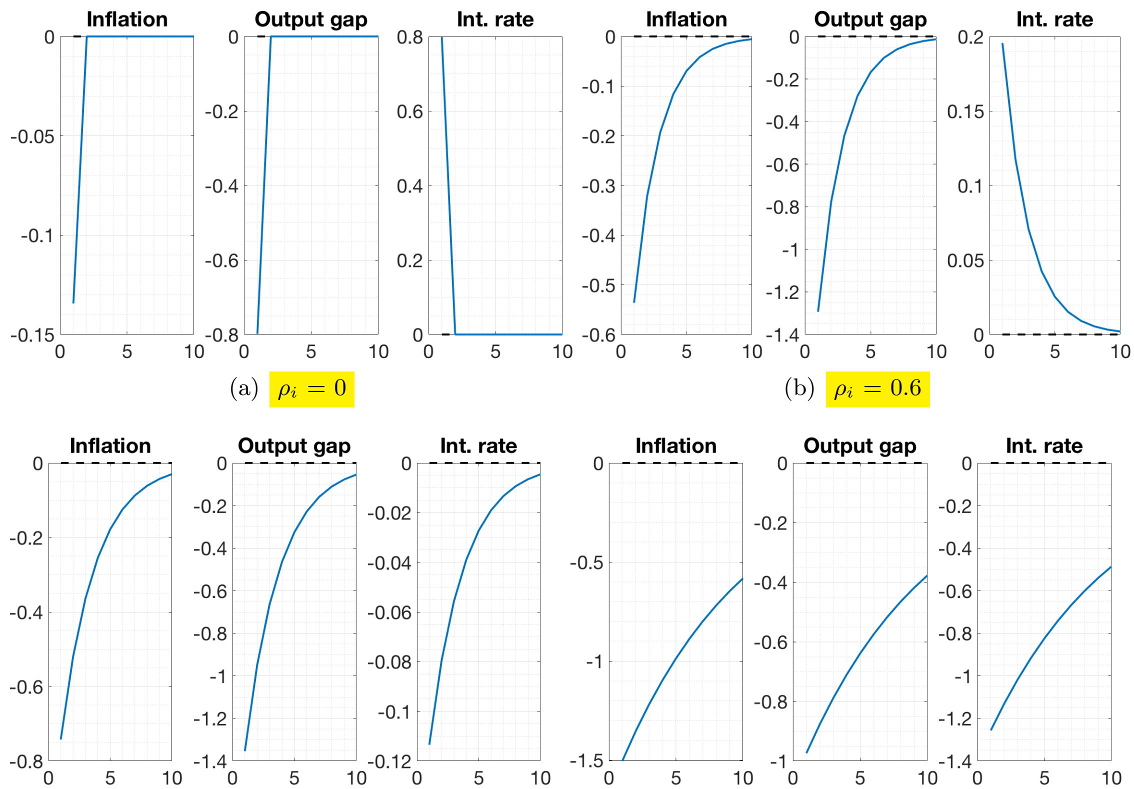
(c) Cost-push shock

5.2 RE: monetary policy shock for various ρ_i ◀

Go to learning analysis.

Come back.

Figure 2: Mon.pol. shock, moving ρ_i

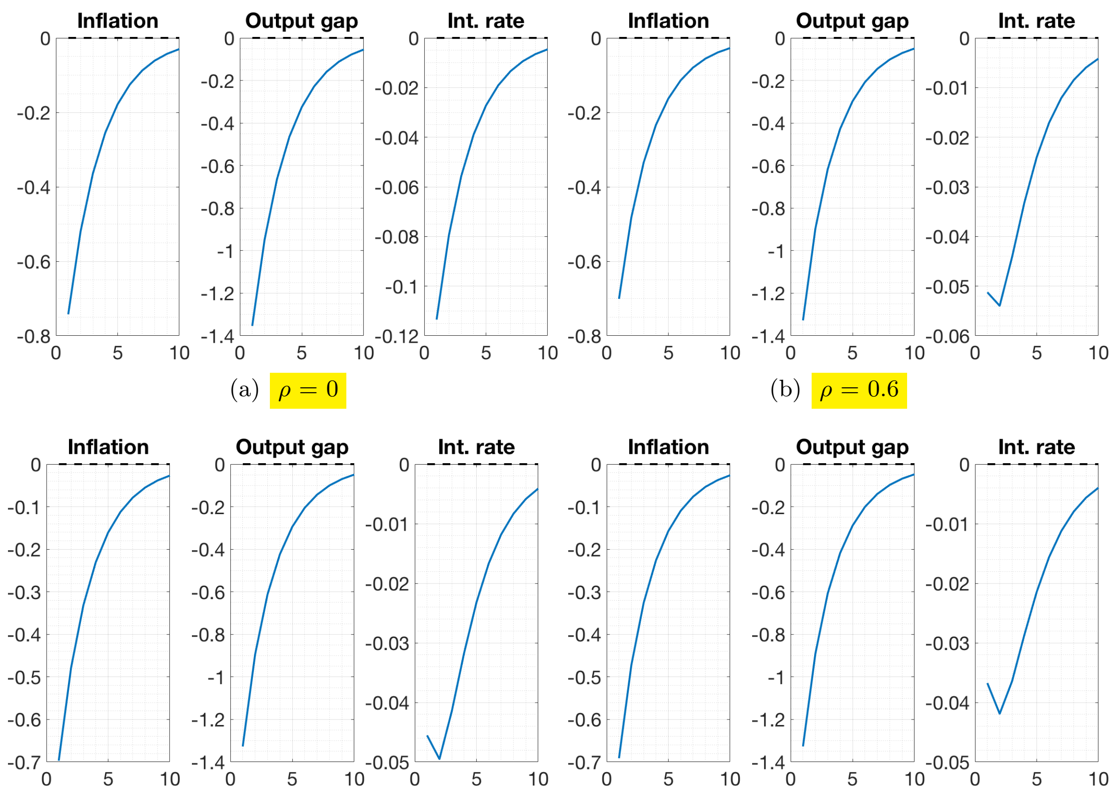


(a) $\rho_i = 0$ (b) $\rho_i = 0.6$

(c) $\rho_i = 0.7$ (d) $\rho_i = 0.9$

5.3 RE: monetary policy shock for various ρ (keep $\rho_i = 0.7$)

Figure 3: Mon.pol. shock, moving ρ

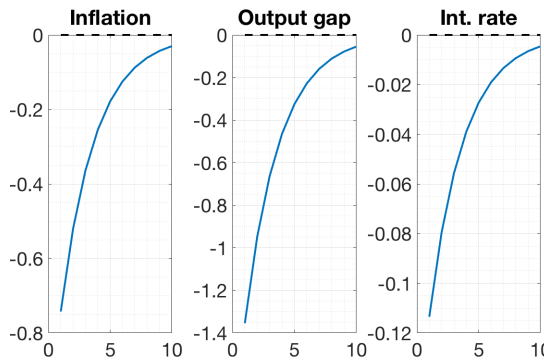


(a) $\rho = 0$ (b) $\rho = 0.6$

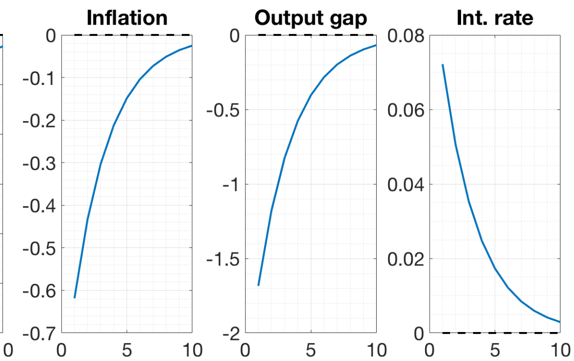
(c) $\rho = 0.7$ (d) $\rho = 0.9$

5.4 RE: monetary policy shock for various α (keep $\rho_i = 0.7$)

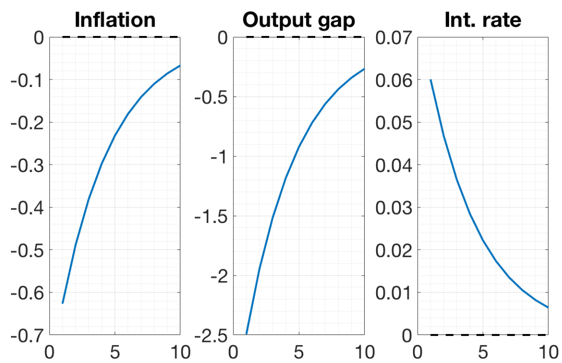
Figure 4: Mon.pol. shock, moving α



(a) $\alpha = 0.5$



(b) $\alpha = 0.6$



(c) $\alpha = 0.75$, $\rho_i = 0.78$

6 Further analysis plots - what are expectations doing? ◀

Understand expectations.

Go to back to RE mon. pol IRFs.

Figure 5: A constant gain learning IRF for a monetary policy shock

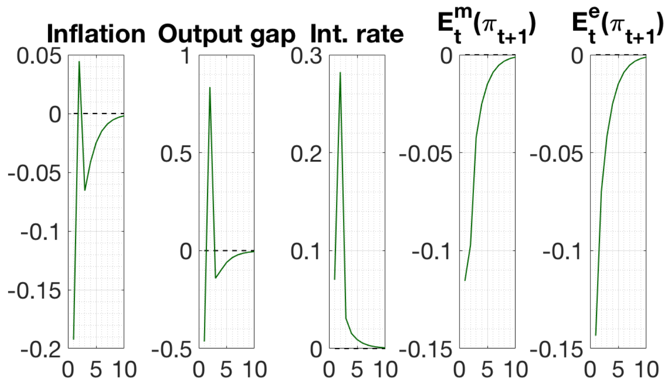
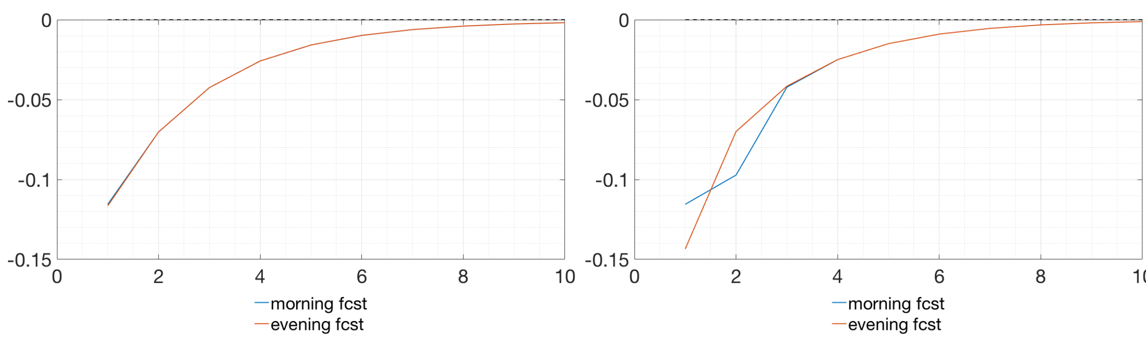
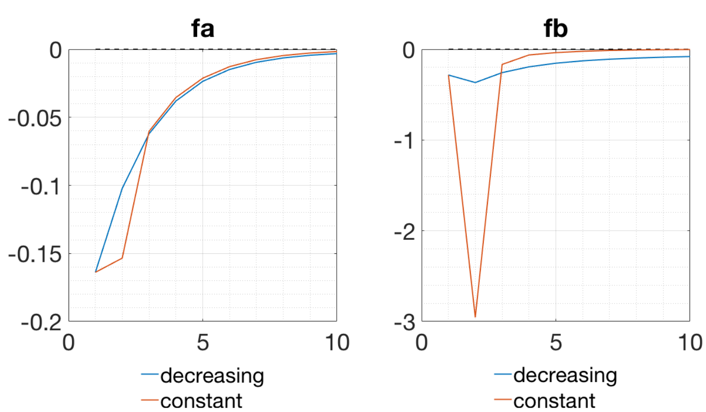


Figure 6: Morning and evening forecasts



(a) Decreasing gain learning (b) Constant gain learning

Figure 7: Long-horizon expectations, mean IRF



7 GE effects of long-horizon expectations ◀

Plugging in the interest rate rule into the equations for the output gap and inflation, equations (1) and (2), and ignoring the exogenous states, I can write the system as a function of discounted sums of expectations (f_a, f_b):

$$x_t = \frac{-\sigma\psi_\pi}{w}\begin{bmatrix}(1-\alpha)\beta & \kappa\alpha\beta & 0\end{bmatrix} f_a + \frac{1}{w}\begin{bmatrix}\sigma(1-\beta\psi_\pi) & 1-\beta & 0\end{bmatrix} f_b \tag{26}$$

$$\pi_t = (1 - \frac{\kappa\sigma\psi_\pi}{w})\begin{bmatrix}(1-\alpha)\beta & \kappa\alpha\beta & 0\end{bmatrix} f_a + \frac{\kappa}{w}\begin{bmatrix}\sigma(1-\beta\psi_\pi) & 1-\beta & 0\end{bmatrix} f_b \tag{27}$$

Let's analyze the signs. Everything is positive, except for the red and blue stuff.

- f_a appears in the x_t-equation because we plugged in the Taylor-rule, and with it, π_t. So this is saying that x_t depends negatively on future expectations of itself and inflation because those lead to increased interest rates.

- $-\beta\psi_\pi$ comes from plugging in the nominal interest rate. Thus, again, x_t depends negatively on expected inflation through the anticipated interest rate response. This also introduces a negative dependence of π_t on own future values via the expected interest rate's effect on x_t.

  Note: $1 - \beta\psi_\pi > 0$ if $\psi_\pi < 1/\beta$. For $\beta = 0.99 \rightarrow \psi_\pi < 1.0101$, or for $\beta = 0.98 \rightarrow \psi_\pi < 1.0204$. So monetary policy can only limit the "GE effect long-horizon expectations" by being more passive on inflation.

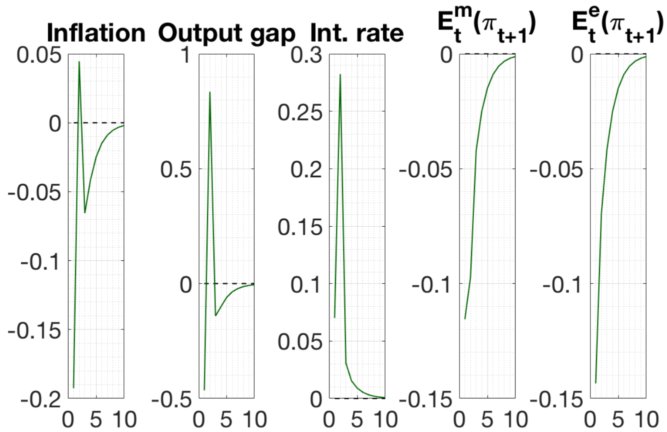
This explains the overshooting despite expectations moving in the right direction (down), and also why overshooting is more pronounced for the output gap.

The reason it's only happening for constant gain learning is because expectations are updated a lot more in that case.

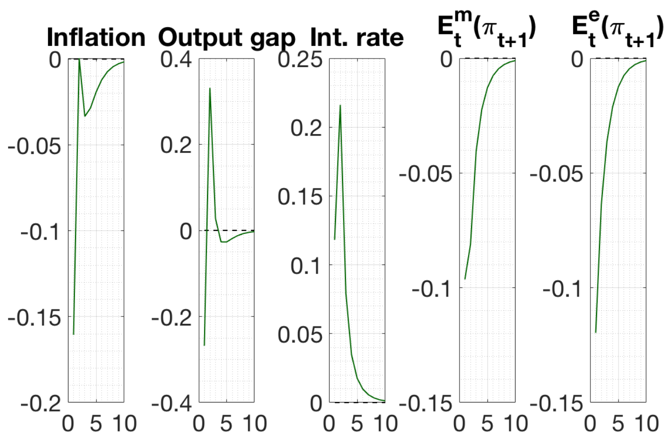
So this is suggesting that monetary policy wants to be aggressive on inflation in order to keep expectations anchored, but once they are unanchored, then being aggressive on inflation induces a lot of volatility because oscillatory belief updating becomes very volatile, and this emphasizes the counterintuitive GE effects from expectations.

8 Decreasing σ ◀

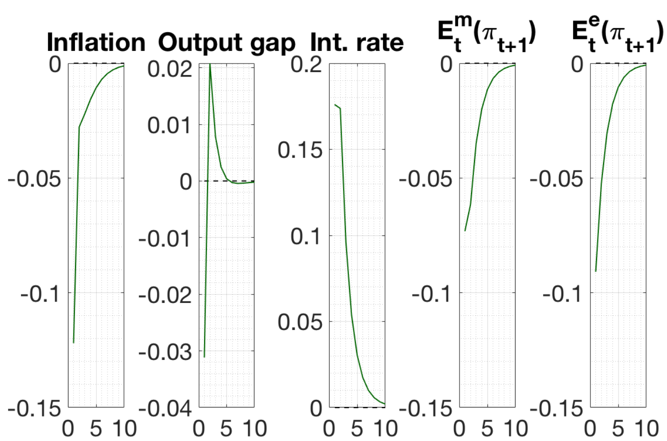
Figure 8: A constant gain learning IRF for a monetary policy shock



(a) $\sigma = 1$



(b) $\sigma = 0.5$

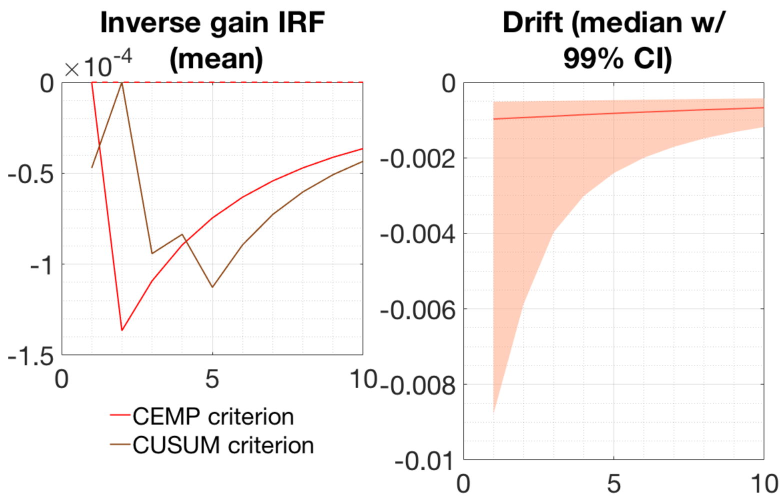


(c) $\sigma = 0.05$

Note: if I set $\beta = 0.98$ instead of 0.99, I can get rid of the period 2 spike in x.

9 A note on shocks and anchoring ◀

Figure 9: An IRF of the endogenous gain and drift for a monetary policy shock

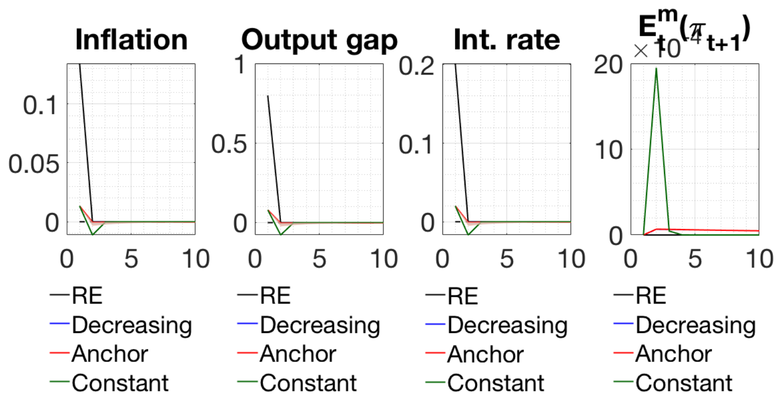


Not just this figure, but also previous ones of the gain indicate that even if a shock knocks expectations out of being anchored, this doesn't lead to huge deviations in the mean path of the gain or the drift. Partly this happens because this particular shock only unanchors very few times (only once - increasing δ increases this number), which is why the mean becomes tiny and the CIs are so tight. Note also that it's tough to compute CIs for the inverse gain because when there's no difference between the gain with the shock δ and without (which is the vast majority of cases), you end up with a bunch of zeros. The inverse doesn't help either because it makes deviations smaller.

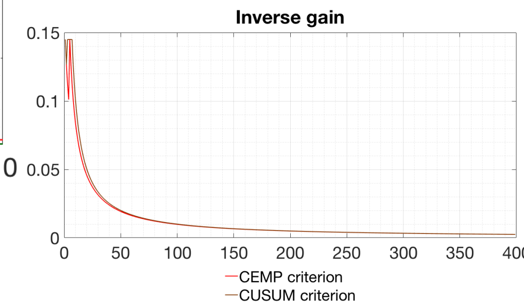
10 IRFs for learning, $\rho_i = 0.6, \alpha = 0.5$

10.1 Natural rate shock, varying ρ

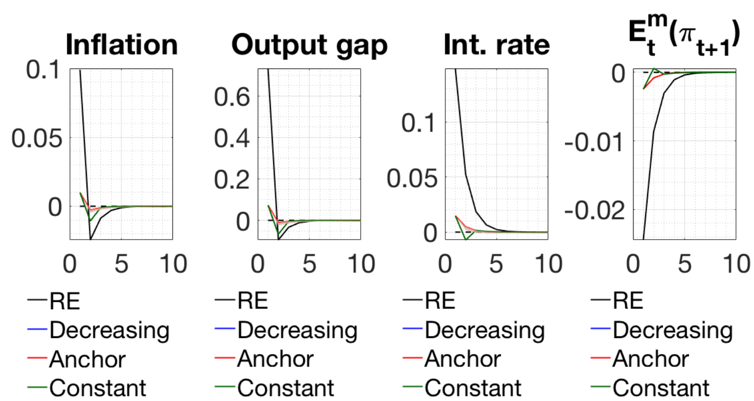
Figure 10: Moving ρ



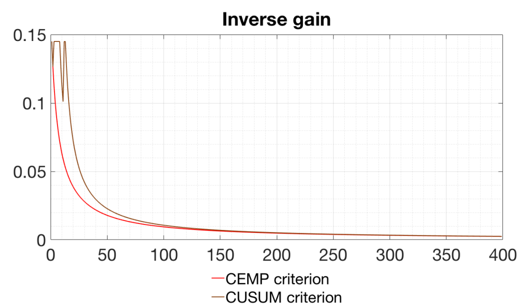
(a) IRFs, $\rho = 0$



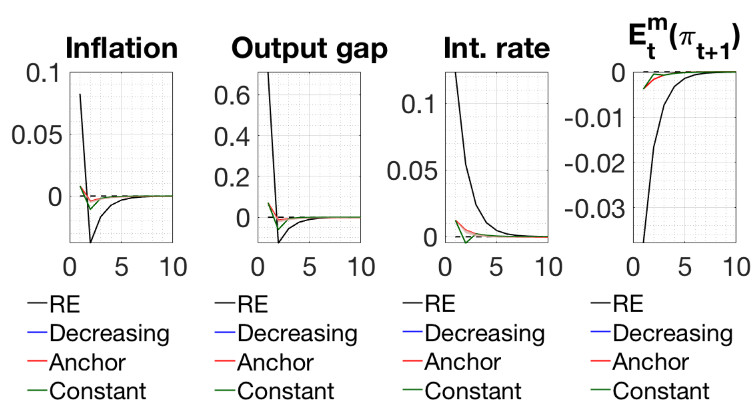
(b) Gain, $\rho = 0$



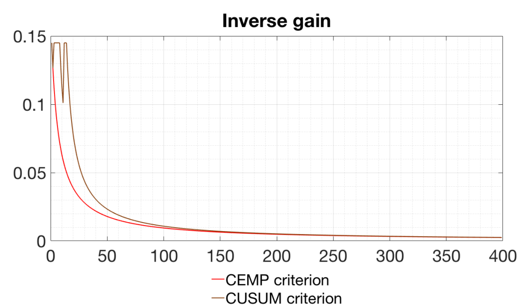
(c) IRFs, $\rho = 0.6$



(d) Gain, $\rho = 0.6$



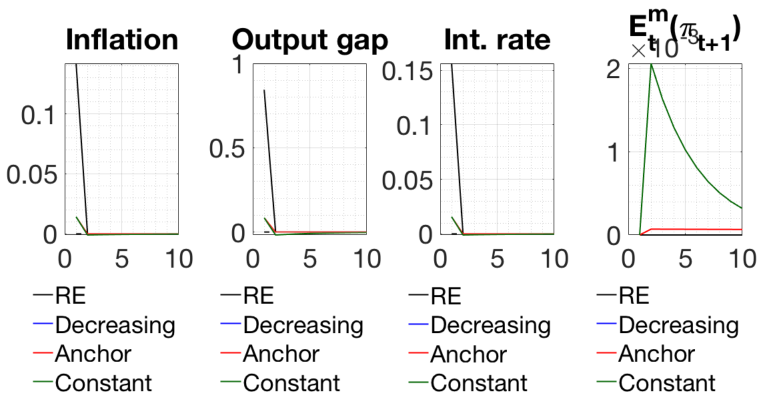
(e) IRFs, $\rho = 0.9$



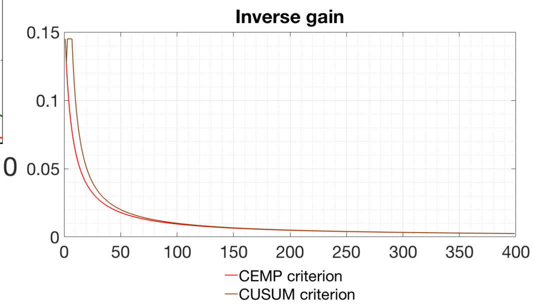
(f) Gain, $\rho = 0.9$

10.2 Natural rate shock, varying ψ_π

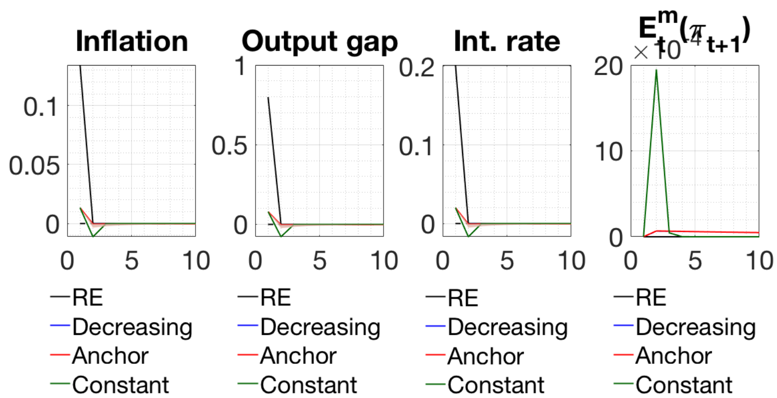
Figure 11: Moving ψ_π



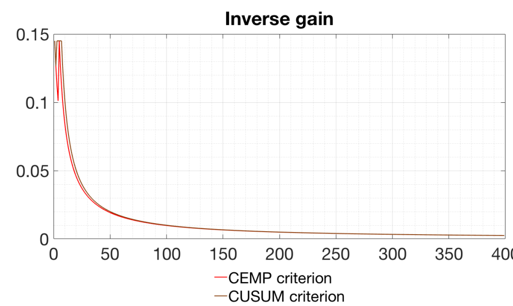
(a) IRFs, $\psi_\pi = 1.1$



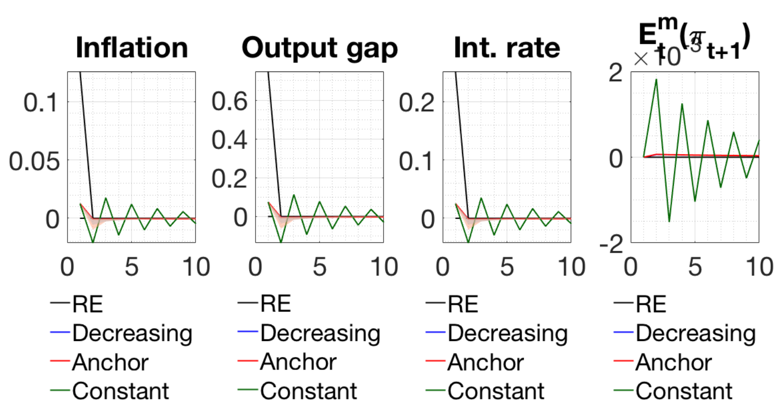
(b) Gain, $\psi_\pi = 1.1$



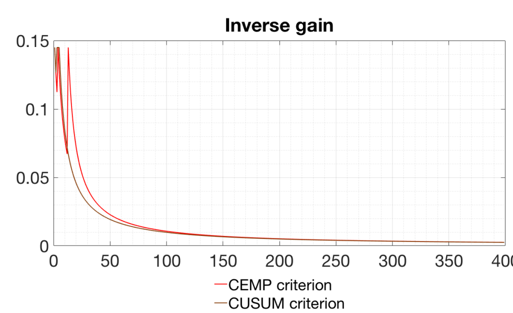
(c) IRFs, $\psi_\pi = 1.5$



(d) Gain, $\psi_\pi = 1.5$



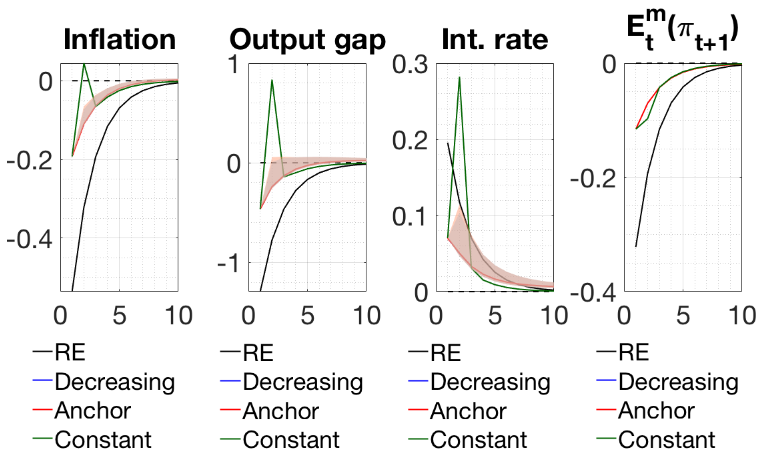
(e) IRFs, $\psi_\pi = 2$



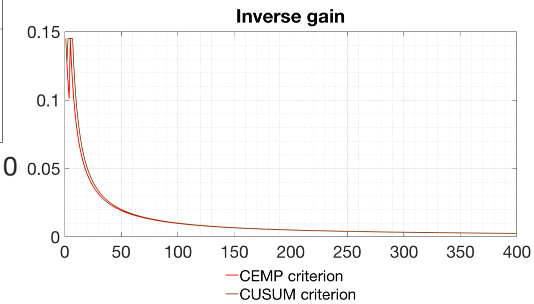
(f) Gain, $\psi_\pi = 2$

10.3 Monetary policy shock, varying ρ

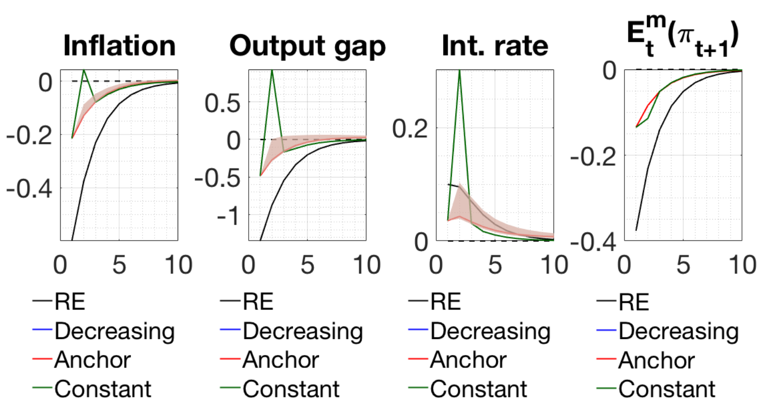
Figure 12: Moving ρ



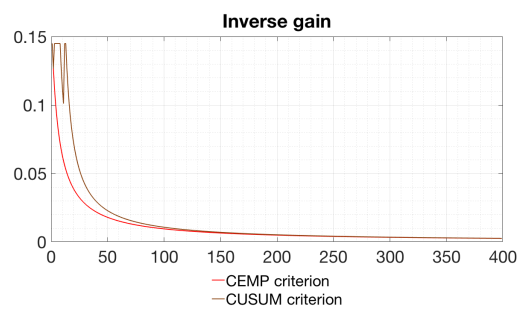
(a) IRFs, $\rho = 0$



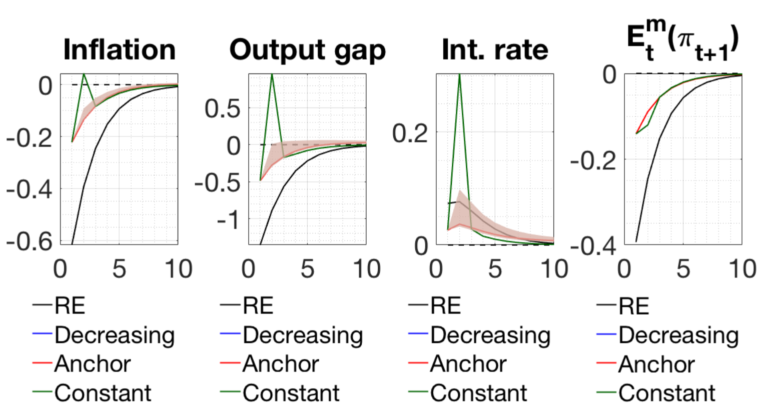
(b) Gain, $\rho = 0$



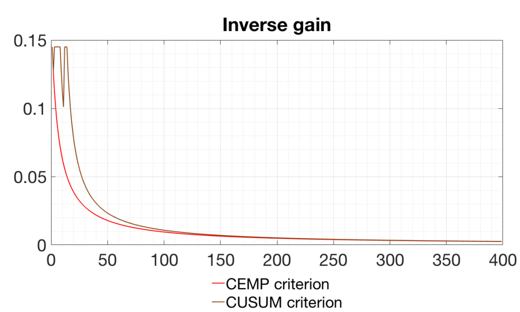
(c) IRFs, $\rho = 0.6$



(d) Gain, $\rho = 0.6$



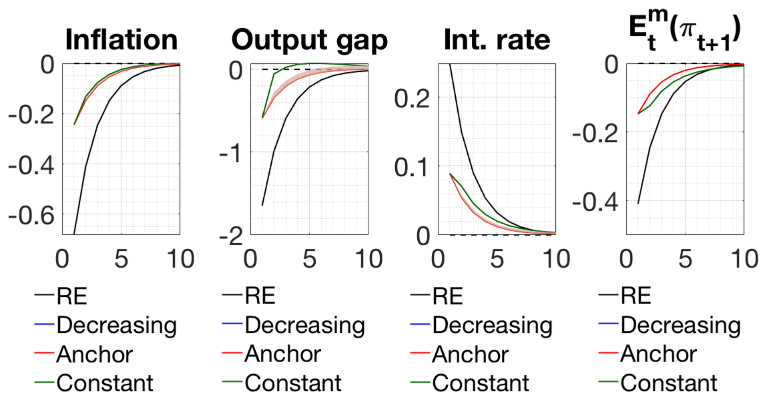
(e) IRFs, $\rho = 0.9$



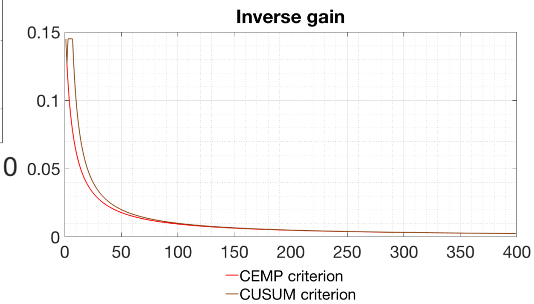
(f) Gain, $\rho = 0.9$

10.4 Monetary policy shock, varying ψ_π

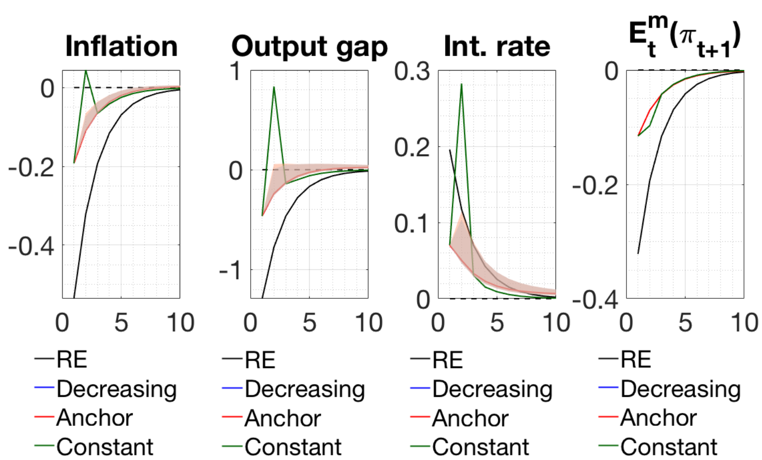
Figure 13: Moving ψ_π



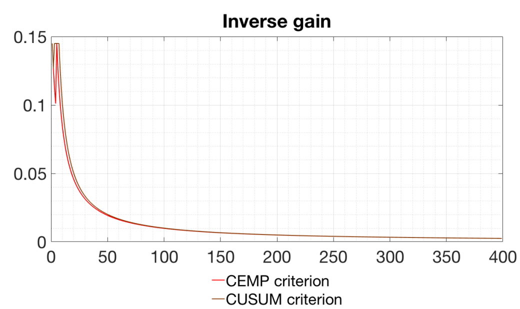
(a) IRFs, $\psi_\pi = 1.1$



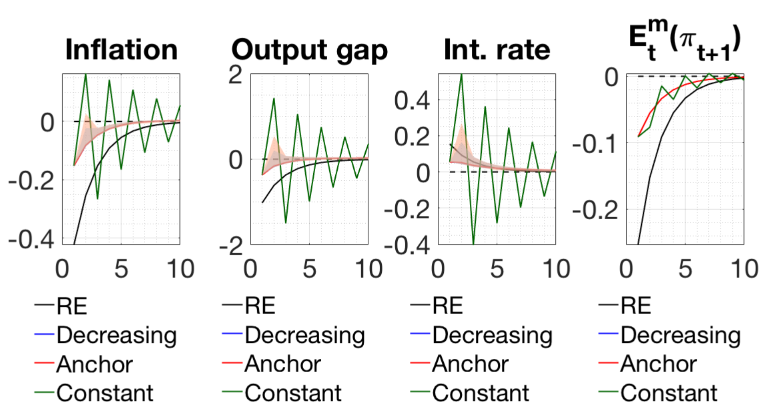
(b) Gain, $\psi_\pi = 1.1$



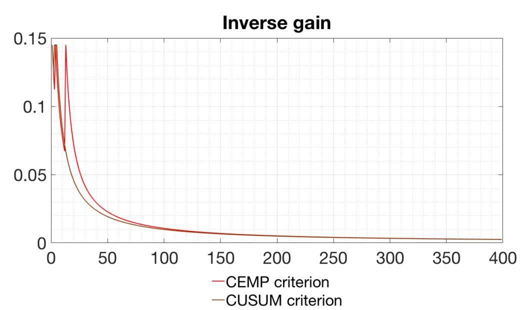
(c) IRFs, $\psi_\pi = 1.5$



(d) Gain, $\psi_\pi = 1.5$



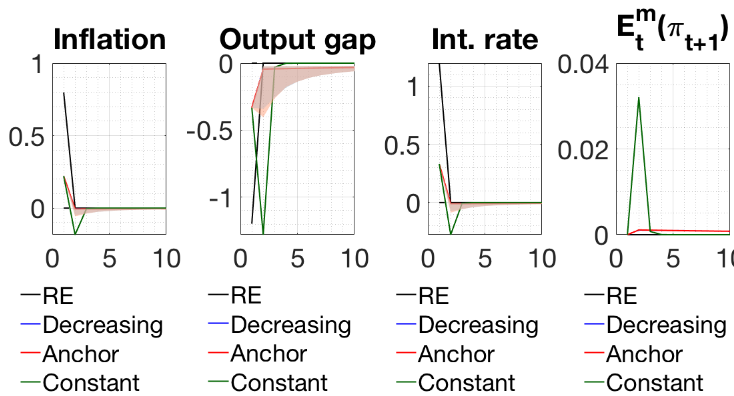
(e) IRFs, $\psi_\pi = 2$



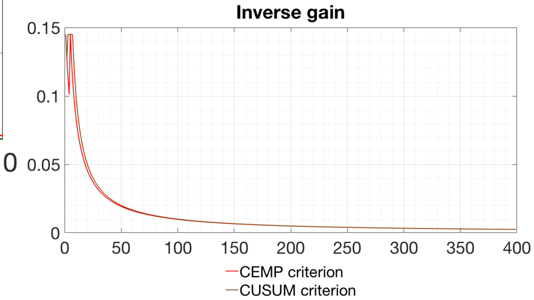
(f) Gain, $\psi_\pi =2$

10.5 Cost-push shock, varying ρ

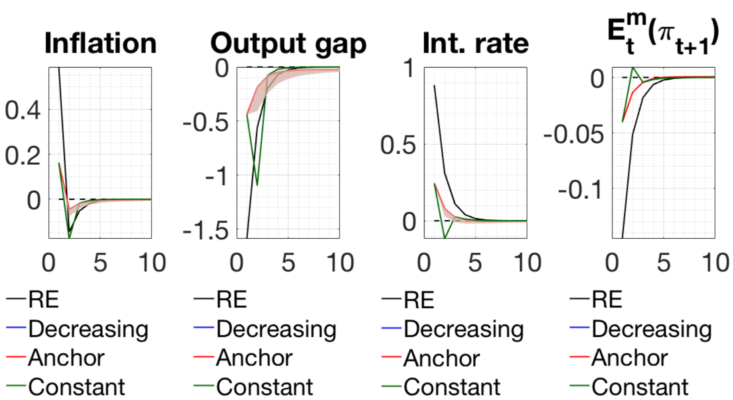
Figure 14: Moving ρ



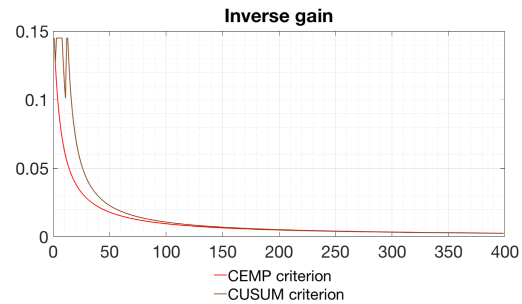
(a) IRFs, $\rho = 0$



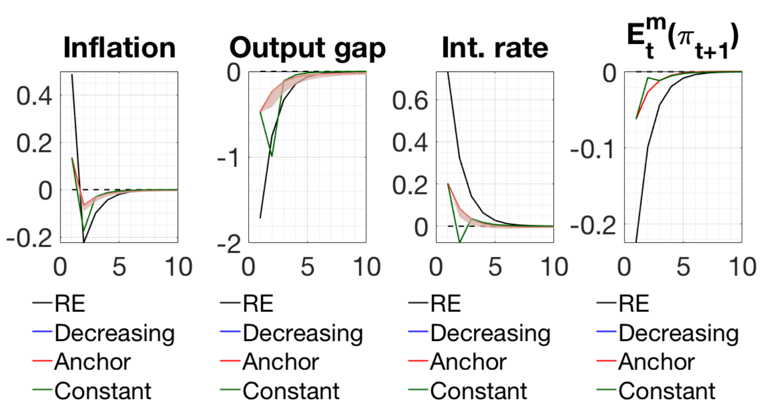
(b) Gain, $\rho = 0$



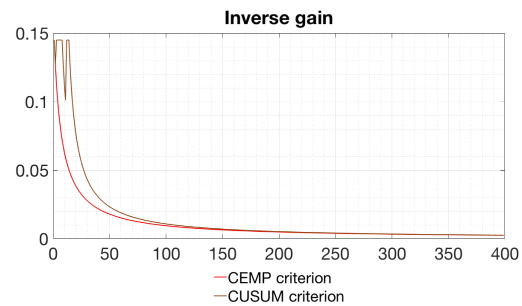
(c) IRFs, $\rho = 0.6$



(d) Gain, $\rho = 0.6$



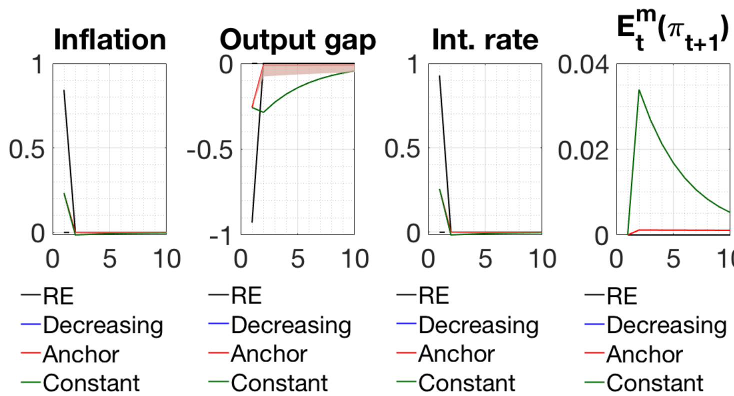
(e) IRFs, $\rho = 0.9$



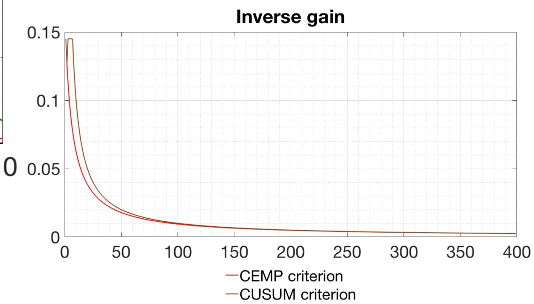
(f) Gain, $\rho = 0.9$

10.6 Cost-push shock, varying ψ_π

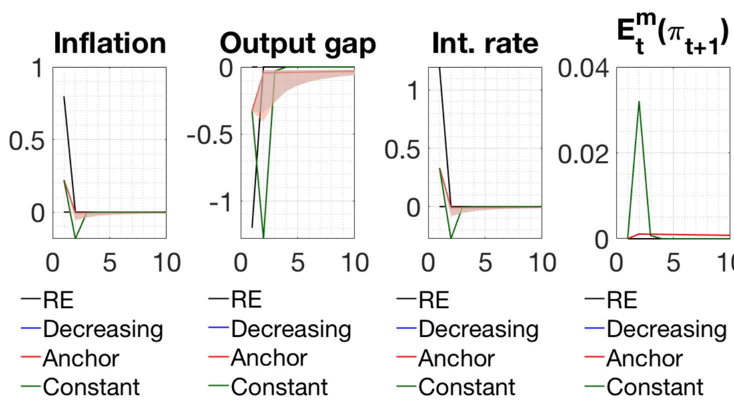
Figure 15: Moving ψ_π



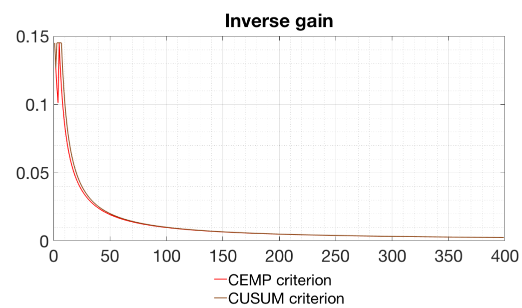
(a) IRFs, $\psi_\pi = 1.1$



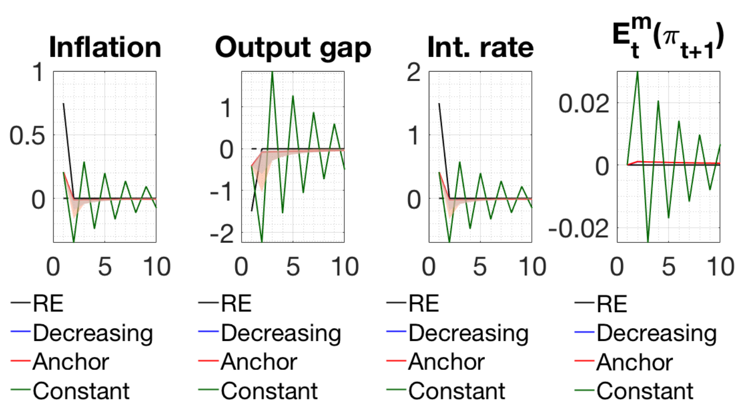
(b) Gain, $\psi_\pi = 1.1$



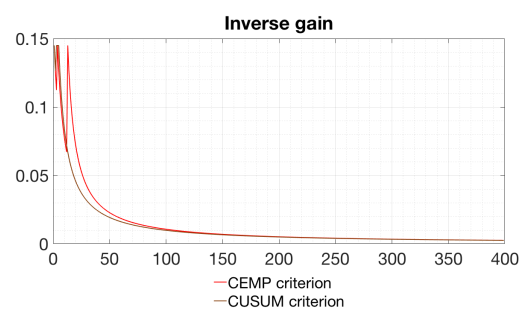
(c) IRFs, $\psi_\pi = 1.5$



(d) Gain, $\psi_\pi = 1.5$



(e) IRFs, $\psi_\pi = 2$



(f) Gain, $\psi_\pi = 2$