Optimal Economic Policy and the Problem of Instrument Instability

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It has been said that in a world of perfect knowledge regarding the structure of the economic system and the values of all exogenous variables, policy making becomes a trivial problem.[1] This is true in the sense that it is always possible, with full knowledge and perfect foresight, to choose the appropriate policy for the following period, but it ignores the fact that the correct policy on a period to period basis may turn out to be impractical or even impossible in the longer run. Current policy decisions do not ordinarily have their impact solely in the current period, but rather over a number of periods in the future. Thus, in addition to offsetting the undesired effects of changes in exogenous variables, current policy decisions must offset the current impact of past policy decisions as well. The purpose of this paper is to show that, under quite reasonable assumptions, attempts to offset completely the cumulative impact of past changes in the policy instrument may require ever greater changes in the future value of the instrument, a situation we will characterize as one of "instrument instability." Edward Gramlich has recently discussed the possibility of instrument instability. He suggests that the inclusion of the instrument in the policy maker's welfare function is the appropriate way to handle the difficulty.

This question of instrument instability has not been of great importance in the past, as little or no attempt has been made to control the behavior of the economy within precise limits. As long as policy makers act slowly, merely nudging the economy in one direction or the other, it is unlikely that instrument instability would even be noticed, much less that it would be a serious problem. Now that they are attempting to "fine tune" the economy, however, the question of instrument instability becomes important; if it is found to exist, serious thought must be devoted to the problem of dealing with it.

This paper describes the nature of the problem, presents two simple cases in detail, outlines the general case, cites some empirical evidence, and suggests some remedies. It shows that the question is really one of the magnitude of the weights on current and lagged values of policy changes as they affect current values of the target or goal variables, and that weighting patterns similar to those frequently found in economic analyses can easily give rise to instrument instability.

I

We will use a very simple model, consisting of a single relationship comprising a policy instrument P, an exogenous variable X, and a goal variable Y.

$$(1) \qquad Y_t = X_t + \sum_{i=0}^{n} w_i P_{t-i}$$

We define X to include all current influences on Y_t other than those

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[1] See, for example, Holbrook and Harold Shapiro, William Poole (1970b), and William Brainard. In fairness to these authors, it must be pointed out that they were all working within a short-run framework, while the problem discussed in this paper has to do with policy making over the long run.

contributed by current and lagged values of the instrument, i.e., it includes the net impact of current and lagged values of all policy instruments (other than P) and other truly exogenous variables as well as lagged values of endogenous variables. The units in which P is measured are defined such that $\sum_{i=0}^{n} w_i = 1$, so the long-run multiplier for P is unity.[2]

We assume that the policy maker knows the precise values of X_t and the w_i, that he can control P_t without error, and that he has some means of selecting a desired value for his goal variable, Y_t. With complete knowledge and in the absence of any stochastic or unexplained elements,[3] the policy maker can easily select the optimum value for his instrument as in (2).

$$(2) \qquad P_t = \frac{1}{w_0} Y_t - \frac{1}{w_0} X_t - \frac{1}{w_0} \sum_{i=1}^{n} w_i P_{t-i}$$

It is here that earlier analyses have stopped working with this simple model, on the grounds that policy making in such an environment is trivial. They then introduce stochastic elements, incomplete knowledge, or some other complication and proceed to analyze the impact of its inclusion. While it is true that within the framework of decision making for a single period, equation (2) says all we need to know about the optimal value for P_t, the influence of lagged values of the instrument on the goal variable (instrument instability) can create serious problems for the policy maker in the long run even under the very simple assumptions we are making here.

We might ask whether the fact that an economy is stable in the usual sense does not imply that it will also exhibit instrument stability. Unfortunately, we can readily see that there is no necessary relation between the two types of stability. Let Y_T be a vector of the current values of the goal variables (i.e., the ones whose values appear in the policy maker's welfare function), while Y_L is a matrix of all lagged values of these goal variables which have any impact on their current values. Assume equality in the number of goal variables and instruments, and let P_T and P_L be, respectively, a vector and a matrix of current and lagged values of the instruments. The behavior of the economy can then be represented by (3),

$$(3) \quad AY_T + BP_T + CY_L + DP_L + EZ = 0$$

where Z is a matrix of all other variables which influence the values of the goal variables, and A through E are matrices of coefficients (A and B are square and nonsingular, by assumption). Solving for Y_T, we obtain equation (4),

$$(4) \quad \begin{aligned} Y_T = &- A^{-1}BP_T - A^{-1}CY_L \\ &- A^{-1}DP_L - A^{-1}EZ \end{aligned}$$

and stability in the usual sense is determined by the values of the elements of $A^{-1}C$. The question of instrument stability is not answered in equation (4), but rather in equation (5)

$$(5) \quad \begin{aligned} P_T = &- B^{-1}AY_T - B^{-1}CY_L \\ &- B^{-1}DP_L - B^{-1}EZ \end{aligned}$$

and depends upon the values of the elements of $B^{-1}D$, which have no direct connection with those of $A^{-1}C$. Thus it would be quite possible for an economy to display stability in one sense and instability in the other, and both are matters of importance to a policy maker.

Returning to our initial model, we can simplify further by putting equation (2) into first difference form and assuming, without loss of generality, that the policy

[2] Equation (1) can be viewed as the reduced form of any of a wide variety of linear structural models of macro-economic behavior. In particular, it could equally well have come from a Keynesian or a Monetarist model of the economy.

[3] The policy maker is assumed to be able to forecast all variables without error; nothing is omitted, to appear later in an error term.

maker desires to fix the value of his goal variable at a constant value; this yields equation (6), where ΔY_t is set equal to zero.

$$(6) \quad \Delta P_t = -\frac{1}{w_0}\Delta X_t - \frac{1}{w_0}\sum_{i=1}^{n} w_i \Delta P_{t-i}$$

Let us now assume that, starting from equilibrium, where $\Delta P = \Delta X = 0$, ΔX_t takes on some nonzero value for a single period, and then returns to zero. The question that must be answered is whether the required path of future changes in the instrument converges to zero, or whether it diverges from zero, requiring ever greater policy maneuvers in order to maintain a fixed value for the goal variable. The answer clearly depends on the values of the w_i, but before moving to the general solution, we will first examine a special case.

A very simple weighting scheme encountered frequently in economics is the geometrically declining series b, $b(1-b)$, $b(1-b)^2$, etc., where $0<b<1$. We can examine the stability implications of this set of weights by making the substitution as shown in equation (7),

$$(7) \quad \Delta P_t = -\frac{1}{b}\Delta X_t - \sum_{i=1}^{\infty}(1-b)^i \Delta P_{t-i}$$

and then using the standard transformation to obtain (8).

$$(8) \quad \Delta P_t = -\frac{1}{b}\Delta X_t + \frac{1-b}{b}\Delta X_{t-1}$$

Equation (8) clearly shows that in order to offset the impact on the goal variable of a once for all change in X, the policy maker need only make two adjustments in the policy instrument, one during the period when X_t changes, and one the following period. Thus geometrically declining weights would be very convenient. Nevertheless empirical research with less restrictive weighting functions reveals that a more common result is for the weights to start small, rise to a peak, and then decline, at times becoming negative. Such a set of "humped" weights does not yield so simple a result as the geometrically declining ones, and their implications are not as easily derived.

Simplifying the problem further, we note that the question of instrument instability can be put in the following way: starting from any arbitrary set of past values of the instrument variable, does the implied future path of the instrument converge to a stable value? Thus we can ignore ΔX_t, and rewrite the equation as in (9) or (10).

$$(9) \quad \Delta P_t = -\frac{1}{w_0}\sum_{i=1}^{n} w_i \Delta P_{t-1}$$

$$(10) \quad w_0\Delta P_t + w_1 \Delta P_{t-1} + \ldots + w_n \Delta P_{t-n} = 0$$

In this final form it is revealed as merely a difference equation, and it is a well established result that an equation of this form is stable if, and only if, the roots of the characteristic equation (11) all lie within the unit circle (see Alpha Chiang).

$$(11) \quad w_0 x^n + w_1 x^{n-1} + \ldots + w_{n-1}x + w_n = 0$$

Unfortunately, there is a simple general solution only for the case where $-w_1/w_0 > -w_2/w_0 > -w_3/w_0 > \ldots > -w_n/w_0 > 0$. If this condition holds, the polynomial (11) has a single positive root between 1 and $\sum_{i=1}^{n} -w_i/w_0$ and this root is the one with the greatest modulus (see K. Sydsaeter and Ryuzo Sato). Since we know $\sum_{i=0}^{n} w_i = 1$, it is easy to see that $\sum_{i=1}^{n} -w/w_0 = 1 - 1/w_0$. Thus, in the case where the only positive weight is attached to the current value of the instrument, and where the past weights decline monotonically to zero, the system is stable. This situation is not commonly encountered in this context, and we must ordinarily expect to have to examine each case individually. For this purpose we can use the Schur theorem (see Chiang) which states that the system will exhibit instrument stability if, and only if, each of a set of n determinants composed of ordered arrays of the w_i's is positive.

This theorem provides an answer to the stability question regardless of the complexity of the weighting scheme, but it is difficult to draw any simple or general conclusions from it regarding the patterns of weights which would or would not lead to instrument stability. We will instead examine the special cases of one and two lags, and then look at the actual weights that appear in some models whose form resembles that of equation (1). Suppose that there is only a one-period lag, so all w_i for $i>1$ are zero and (11) reduces to $w_0x+w_1=0$, or $x=-w_1/w_0$. Stability requires that $|x|<1$, or $|w_1|<|w_0|$. If we combine this requirement with our constraint that $w_0+w_1=1$, we can easily see that w_0 must be greater than .5 for stability. If w_0 is less than .5, the policy maker would forever be trying to offset the effect of one period's policy change with a policy change of greater magnitude the following period. Thus, even when the lag in the impact of the instrument is quite short, if the bulk of the impact is not felt until the following period, an attempt to use that tool as a means of maintaining absolute stability in the goal variable will lead eventually to unacceptable changes in the value of the instrument.

The case most frequently encountered in empirical research is the one in which the weights have the same sign, and we will examine that case more closely. Suppose we have the situation depicted in (12)

$$\Delta Y_t = \Delta X_t + w_0 P_t + w_1 \Delta P_{t-1} \tag{12}$$

where $w_0<.5$, implying instrument instability as we have seen. We can then ask: if the policy maker knows that perfect achievement of his goal through the use of his instrument will result in instrument instability, must he avoid using the instrument entirely? The answer is that he can use it, but he may have to accept some fluctuations in his goal variable in the short run in exchange for reduced fluctuations in his instrument variable in the long run. For example, he could set $\Delta P_t=-\Delta X_t$, and the fact that the weights sum to unity will insure that at the end of the second period, the goal variable will be back on its desired path. By taking a two-period horizon the policy maker thus avoids the stability problem entirely.[4]

The preceding is a particular example of a tactic which the policy maker can use if the problem of instrument instability is found to be present. Instead of changing the instrument by the full amount necessary to achieve the desired value for the target, the policy maker can change the instrument by some constant fraction (u) of that amount. The use of this device will limit the required changes in the instrument, but only at the cost of some variation in the value of the target variable. The behavior of this system over time clearly depends on the size of u relative to that of w_0.[5] The device in the preceding pargraph was an example of this procedure with $u=w_0$. Any choice of $u<2w_0$ will produce stability in both the goal and the instrument, while a value of u between $2w_0$ and unity will produce explosive cycles in both

[4] This result is not peculiar to the two-period case, but is true in the general case as well. On the basis of the general form of (12), namely

$$\Delta Y_t = \Delta X_t + w_0\Delta P_t + w_1\Delta P_{t-1} + \ldots + w_n\Delta P_{t-n}$$

we can follow the policy of setting $\Delta P_t=-\Delta X_t$, and be sure that the impact on Y of a given change in X in period t will have been eliminated by period $t+n+1$. Thus the policy maker will always be able to maintain "stability" in his goal variable if he is willing to define the length of the time period to be equal to or greater than the length of the longest lag. While in a theoretical model the length of a period is arbitrary, its choice in the context of this paper is that time interval within which we are willing to ignore variations in the goal variables (i.e., to be concerned only with their average values).

[5] The behavior of the system under different values of u and w_0 can be shown to be related to the behavior of $(1-u/w_0)^n$ as n becomes large.

variables. Thus, if the long-run behavior of the instrument is of no concern to the policy maker, he should make a full adjustment ($u=1$).[6] If he is concerned about both the goal variable and the instrument, he should choose a value for u somewhere between 0 and $2w_0$.[7]

An alternative approach to the problem of instability when $w_0<.5$ is to let $u=1$ whenever possible, but to put an absolute limit on the permissible change in the instrument during a single period. Although such a rule might seem to have some intuitive appeal, it fails to produce the desired result. Under this regime, as long as the required change in the instrument is less than the maximum allowable change, the goal variable is fixed. As soon as the constraint becomes binding (as must inevitably occur), however, both the goal variable and the instrument exhibit cycles of constant amplitude, and the cycles of the goal variable do not even necessarily center on its equilibrium value.

There are many ways in which a policy choice could be made in the unstable two-period lag case, but all will require some sort of compromise between goal and instrument stability. Within the hypothesized structure, if $w_0<.5$, there is no way in which absolute stability in the goal can be achieved without ever increasing changes in the instrument.

Moving on to the case of two lags, such that all $w_i=0$ for $i>2$, Paul Samuelson gives the necessary and sufficient conditions for instrument stability, as shown in (13),

$$1-\frac{w_2}{w_0}>0 \tag{13a}$$

$$1+\frac{w_1}{w_0}+\frac{w_2}{w_0}>0 \tag{13b}$$

$$1-\frac{w_1}{w_0}+\frac{w_2}{w_0}>0 \tag{13c}$$

and from these and our specification that $w_0+w_1+w_2=1$ we can derive[8] the conditions on the individual weights given in (14).

$$w_0>.25 \tag{14a}$$

$$w_1<.5 \tag{14b}$$

$$w_2<w_0 \tag{14c}$$

These conditions are clearly satisfied by any set of declining weights, and they will also be satisfied by weights which first rise and then fall provided that no more than half the weight is on last period's policy, and the impact of policy two periods ago does not exceed the impact of current policy.

Presumably here, too, there is a set of rules for partial adjustment which could turn an unstable situation into a stable one at the cost of some variation in the goal variable, as was the case for the two-period example. Although I have not yet examined this problem closely, it seems reasonable to assume that the same general result would hold, i.e., for zero adjustment in the instrument we lose all control over the goal; for a small adjustment coefficient both goal and instrument exhibit long-run stability; for a full adjustment the instrument is unstable.

II

Rather than explore further the implications of certain special cases, we turn

[6] A choice of $u=1$ implies not only that the instrument fails to appear in the policy maker's utility function, but also that there are no binding constraints on the values it may assume.

[7] A choice of u between $2w_0$ and 1 produces the worst of both worlds, and would never be selected by a rational policy maker.

[8] The derivation is as follows: If $w_0<0$, then from (13b) we obtain $w_0+w_1+w_2<0$, which contradicts our specification that the weights sum to one; therefore $w_0>0$. This result, together with (13a), yields the condition that $w_0>w_2$. Using the fact that $w_0>0$, and adding $2w_1$ to both sides of (13c), we obtain $w_1<.5$. Having derived both (14b) and (14c) we add them together, add w_0 to both sides, and obtain $w_0>.25$.

now to the available empirical evidence on the existence of instrument instability. Our equation (1) can be viewed as a linearized reduced form of any structural model of the economy, and our weights w_i are implicit in the structural coefficients of that model.[9] The reduced forms of such models cannot be easily derived, and their stability characteristics would be most conveniently explored through simulation techniques. There is one exception to this, however, in that Leonall Anderson and Jerry Jordan (A-J) have developed a reduced form model of the *U.S.* economy which differs from our theoretical one only in that it includes more than a single instrument (generally one monetary and one fiscal instrument, but occasionally more than one of each) and has no representation of our X variable. Without attempting to imply either approval or disapproval of the A-J approach, we can examine the weighting patterns which appear in some of the examples of that model for their stability characteristics.

There are more than two dozen published versions of the A-J equation, utilizing a wide variety of independent variables, and fitted over many different time periods (Frank de Leeuw and John Kalchbrenner, Edward Corrigan). While it would be possible to examine them all for their stability characteristics, the outcome tends to be repetitious, and a small sample will be sufficient to convey the results. In a recent version Anderson and Keith Carlson use the quarterly dollar changes in the money stock (ΔM) and full employment federal expenditures (ΔE) as determinants of quarterly changes in the current dollar value of GNP (ΔY) with the results shown in (15).

$$\Delta Y_t = 2.67 + \sum_{i=0}^{4} m_i \Delta M_{t-i} + \sum_{i=0}^{4} e_i \Delta E_{t-i} \tag{15}$$

where

$$\begin{aligned} m_0 &= 1.22 & e_0 &= .56 \\ m_1 &= 1.80 & e_1 &= .45 \\ m_2 &= 1.62 & e_2 &= .01 \\ m_3 &= .87 & e_3 &= -\,.43 \\ m_4 &= .06 & e_4 &= -\,.54 \\ \sum_{i=0}^{4} m_i &= 5.57 & \sum_{i=0}^{4} e_i &= .05 \end{aligned}$$

Applying the Schur theorem to the implied weights we find that while both ΔM and ΔE are stable instruments, ΔE is very nearly unstable (note that e_4 is almost as large as e_0 in absolute value). A small change in one of the weights (e.g., if e_4 were $-.57$ rather than $-.54$) would be sufficient to tip the balance toward instability. The money supply is a much more stable instrument here. This result is similar to those implicit in the earlier versions of the A-J equation. In those equations, ΔE is actually an unstable instrument in most cases, while ΔM is uniformly stable. Full employment federal receipts also appear as a stable instrument in several of the equations, and the monetary base appears as an unstable instrument in one of them.

It would be inappropriate to conclude from these results that the money supply is a stable policy instrument while federal expenditures is an unstable instrument. The evidence can only be taken as sugges-

[9] While the linear form may be a satisfactory tool for examination of the behavior of the model within a narrowly defined region, it is likely to be inappropriate for full analysis of instrument instability. Even if the linearized weights imply instability, these weights may in fact be functions of the amplitude of the changes in the instrument, or of other variables within the system which would be affected by the policy choices. The problem may be amplified or diminished by these interactions, but there is no way to include them in the linearized reduced form. Simulation is thus the only analytical tool likely to lead to acceptable answers regarding the prevalence of instrument instability in an economy (unless one is willing to assume that the structure of the economy is in fact linear).

tive, since the A-J approach is but one of many ways of trying to capture the impact of policy instruments on the goal variables. Other models will have to be tested for instrument instability before we can conclude with any degree of certainty that a particular policy instrument is either stable or unstable within the present structure of the *U.S.* economy. Our analysis of the A-J model does indicate, however, that there is a very real possibility that instrument instability could render the precise achievement of goal stability in the United States impossible over the long run.

III

We have seen that even an omniscient policy maker can be faced with a serious dilemma due to the cumulative effects of past policy choices. The long-run instability of his instrument variable within a regime of full goal achievement may require him to compromise, permitting some goal variability in return for instrument stability.

Far from omniscient, the real world policy maker is in firm command of little if any perfect information, so one might legitimately question the relevance of our conclusions. While it is true that the policy maker must work with imperfect information and rough forecasting models, much current research is designed to improve the quality of our forecasting techniques, and to increase both the quantity and quality of the input data for those forecasts. This research is in part a response to the wide acceptance of the notion that the economy can and should be "finely tuned" to follow precisely the desired path by appropriate adjustments of the policy instruments. Thus, while the problem discussed in this paper may have been academic in the past, it is becoming a question of increasing importance and relevance as our control and forecasting techniques improve.[10] The possibility of instrument instability has not been generally recognized before because policy maneuvers were so grossly in error that the goal variables were highly unstable. The increasing stability of the goal variables (an officially endorsed aim of national policy since World War II, and one which economists have been working to accomplish) may turn out to be unattainable without ever wider and eventually unacceptable fluctuations in some policy instruments.

There are several possible responses to instrument instability, if it should be found to exist. The simplest but perhaps least acceptable approach would be merely to lengthen the period over which we are attempting to stabilize the goal variables. For example, in the one-period lag case discussed earlier, all stability problems disappear if we define a new period equal to two of the old ones, and then tell the policy maker to maintain goal stability on the average over the newly defined period. It would not ordinarily be necessary to define the new period to be so long as to reduce all lagged weights to zero (such a redefinition will always create stability), but merely to be long enough to incorporate the larger lagged weights into the newly defined "current" period. This is an artificial solution, of course, as, in some absolute sense, the goal variable is only apparently stable.

Another technique which bears considerable resemblance to current economic policy decisions in the United States is to choose today's policy so as to achieve the desired goal at some time in the future, on the grounds that today's policy has little impact today, but a growing impact with the passage of time. With the correct choice of the future period for which cur-

[10] Even in the absence of improved control techniques, the problem of instrument instability could arise if the policy maker relied too heavily on a forecasting model which exhibited those characteristics.

rent policy is designed, the instrument stability problem can be overcome, but only at the expense of incomplete goal achievement. Unless the impact of current policy on the current value of the goal variable is actually zero, the continual adjustment of current policy to meet future needs will, in general, prevent the achievement of the goal in the current period. This suggestion, as well as the previous one, touches on the problem of the optimum forecast period, a topic which has not received the attention it deserves.

The policy maker can also avoid the stability problem by making only a partial rather than a full adjustment toward the value of the instrument necessary to maintain complete goal stability. This was demonstrated earlier in the one period lag case, and an appropriate choice of the partial adjustment coefficient would preserve instrument stability in the general case as well.[11]

Finally, it might be that some instruments exhibit instability, while others do not. In this case it would be possible to avoid the problem merely by utilizing the stable instruments. If we have a multiplicity of goals, however, it is likely that discarding one instrument entirely will take us farther from the social optimum than would the judicious use of all instruments.

Each of these remedies (except the last, and only then if the problem instrument is redundant) purchases instrument stability at the expense of greater variability elsewhere in the system, or merely papers over the problem by redefining the period. A complete cure would require a modification in the structure of the system, increasing the impact of current policy and decreasing the impact of past policy.

Much further analysis will be necessary before we can say for certain whether instrument instability is a problem within the structure of the *U.S.* economy. The actual reduced forms of such large scale models as the FRB-MIT, DHLIII, or Wharton would be far more complex than the one represented in (1), and it could be that the complex interactions and nonlinearities present in the real economy are such as to make instrument instability either more or less likely than in the strictly linear case. Experiments with these models are required before we can draw more than tentative conclusions about these issues, but the result of some simulations reported by Poole (1970a) indicate that the money supply in the FRB-MIT model may well be an unstable instrument.

If the resources currently being invested on research in the areas of forecasting and optimal policy are to be of greatest possible benefit, the questions raised in this paper must be fully investigated. Otherwise, we may find ourselves in possession of better forecasting techniques than we know how to use.

[11] The replacement of discretionary policy by a rule for those instruments which are unstable would obviously avoid the problem (this is equivalent to the case of $u=0$ in the example). With a good forecasting model, however, it is possible to do better than this. We can reduce goal variability below that consistent with a rule by choosing some small positive partial adjustment coefficient. Thus, the existence of instrument instability would not eliminate the possibility of discretionary policy, it would only limit its ability to control the economy.

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