The Calvo Parameter

Tutorial

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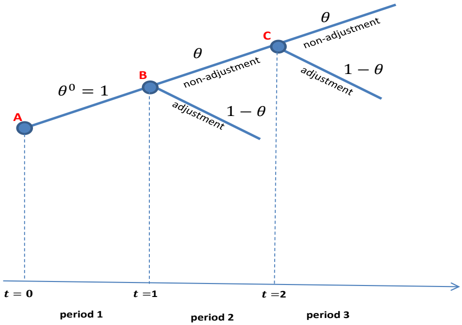

The Calvo Parameter

- Assume that in each period a typical firm is allowed to adjust its price with probability $1-\theta$. Its price remains unchanged with probability θ.
- Now consider a firm that changes its price at the beginning of the current period $t=0$ after observing the realizations of *all* exogenous stochastic variables. By definition, the probability that the price *survives* over the current period (called "period 1" in the figure), which is the probability for the price to remain unchanged between nodes **A** and **B** is $\theta^0 = 1$.[1]
- The probability that the price survives until the end of the current period and gets changed at the beginning of the next period (exactly in node **B**) is $\theta^0(1-\theta) = 1-\theta$. The latter is the probability that this price has a *lifetime* equal to exactly one period (which is the period called "period 1" in the figure).
- Similarly, the probability for the price to survive until the end of period 2 and then to be changed at the beginning of period 3 (in node **C**) is $\theta \cdot (1-\theta)$ The latter is the probability for a *lifetime* equal to exactly two periods. Accordingly the probability for surviving exactly three periods is equal to $\theta^2(1-\theta)$ and so on.
- What is the *expected lifetime* of this particular price? (For how many periods is this price expected to remain unchanged?)
- The expected lifetime of the price can then be computed by employing the familiar formula:

$$E(Lifetime) = \sum_{i=1}^{\infty} (Lifetime = i) \cdot (Probability \quad for \quad Lifetime = i).$$

Equivalently:

$$E(Lifetime) = 1 \cdot \theta^0(1-\theta) + 2 \cdot \theta(1-\theta) + 3 \cdot \theta^2(1-\theta) + \ldots + \overbrace{N}^{\text{Lifetime}} \cdot \underbrace{\theta^{N-1}(1-\theta)}_{\text{Probability for Lifetime}=N} + \ldots$$

[1] In DSGE models it is commonly assumed that at the very beginning of each period (e.g. in node **A**) agents observe all the shocks hitting the economy and then immediately make their optimal decisions. Since no other shocks occur until the end of the period (e.g. until node **B**), the optimal labor supply, consumption and prices chosen in node **A** remain active over the entire period (between **A** and **B**).

- Now let us rewrite the last equation as follows:

$$E(Lifetime) = 1 \cdot \theta^0(1-\theta) + 2 \cdot \theta(1-\theta) + 3 \cdot \theta^2(1-\theta) + \ldots + N \cdot \theta^{N-1}(1-\theta) + \ldots$$

$$E(Lifetime) = (1-\theta)(1 + 2\theta + 3\theta^2 + 4\theta^3 + \ldots N\theta^{N-1} + \ldots)$$

- Then we get:

$$\begin{aligned} E(Lifetime) \quad &= (1-\theta)(1 + 2\theta + 3\theta^2 + 4\theta^3 + \ldots N\theta^{N-1} + \ldots) = \\ \\ &(1-\theta)(1 + \theta + \theta^2 + \theta^3 + \theta^4 + \ldots) \\ &+(1-\theta)(\theta + \theta^2 + \theta^3 + \theta^4 + \theta^5 + \ldots) \\ &+(1-\theta)(\theta^2 + \theta^3 + \theta^4 + \theta^5 + \theta^6 + \ldots) \\ &+\ldots \\ &+(1-\theta)(\theta^N + \theta^{N+1} + \theta^{N+2} + \theta^{N+3} + \ldots) \\ &+\ldots \end{aligned}$$

- The last equation is equivalent to:

$$
\begin{aligned}
E(Lifetime) &= (1-\theta)\left(\frac{1}{1-\theta} + \frac{\theta}{1-\theta} + \frac{\theta^2}{1-\theta} + \frac{\theta^3}{1-\theta} + \ldots\right) \\
&= (1-\theta)\cdot\frac{1}{1-\theta}\cdot(1+\theta+\theta^2+\theta^3+\ldots) \\
&= (1-\theta)\cdot\frac{1}{1-\theta}\cdot\frac{1}{1-\theta} = \boxed{\frac{1}{1-\theta}}.
\end{aligned}
$$

- Hence, if $\theta = 0.75$ and if one period equals one quarter, then the average (or expected) lifetime of a nominal price chosen today is equal to:

  $$\frac{1}{1 - 0.75} = 4 \quad \text{quarters} \quad = \quad 1 \quad \text{year}.$$

  In other words, $\theta = 0.75$ implies that on average prices remain constant (unchanged) for one year.

- Thus, given empirical data on the average number of quarters (months/years) over which firms' prices remain constant, we can calibrate the parameter θ.