Materials 22 - GMM of simple anchoring function

Laura Gáti

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Overview

1 Specifications of anchoring function and estimation

- Anchoring function

$$k_t = k_{t+1} + \frac{1}{(d\ fe)^2} \tag{1}$$

Agents update their PLM using the inverse gain k_t^{-1}. Thus the bigger $\frac{1}{(d\ fe)^2}$, the more the gain is *decreasing*. Higher forecast errors fe or a higher d means closer to constant gains. I tried the inverse formulation with $h_t \equiv k_t^{-1}$ and

$$h_t = h_{t-1} + (d\ fe_{t-1})^2 \tag{2}$$

but it always led to explosive simulations.

- Target: I gather the time series of inflation, output gap and federal funds rate, filter them, and compute empirical autocovariances:

$$ac^{data}(h) \equiv \text{cov}(y_t, y_{t-h}) \tag{3}$$

for $h = 0, \dots, K$, selecting $K = 4$. I gather these autocovariances for the three variables in the matrix AC. The target then is $ac^{data} \equiv \text{vec}(AC)$ (a $n_y(K+1) \times 1$ vector, i.e. 15×1). Thus the objective function can be written as:

$$J \equiv (ac^{data} - ac^{model})' W^{-1} (ac^{data} - ac^{model}) \tag{4}$$

- Initial $d_0 = 10$.

2 Estimation issues

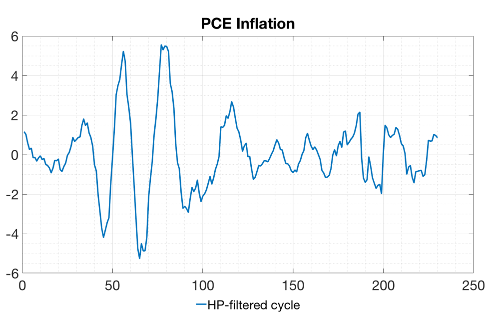
- W: Ideally I'd want to use a weighting matrix with the estimated variances of the target moments on the diagonal:

$$W = \begin{pmatrix} \hat{\sigma}^2_{ac(\pi,0)} & 0 & \dots & & 0 \\ 0 & \hat{\sigma}^2_{ac(x,0)} & 0 & \dots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \dots & & 0 & \hat{\sigma}^2_{ac(i,K)} \end{pmatrix} \tag{5}$$

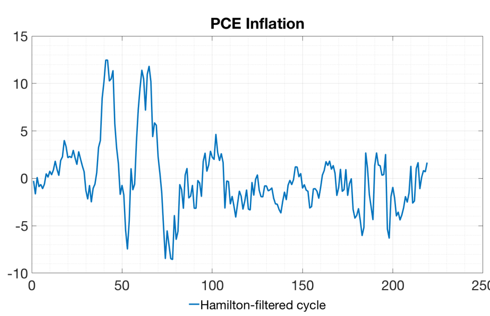
Since I don't fit the data to a time series process, I create bootstrapped samples from the original (filtered) data. This however results in tiny bootstrapped variances, so W^{-1} is huge.

3 Robustness to different filters

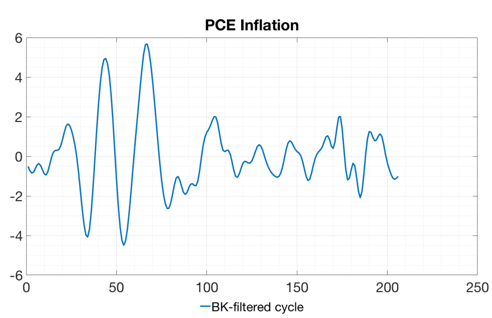
Figure 1: Cyclical component of inflation filtered using different methods



(a) Hodrick-Prescott, $\lambda = 1600$



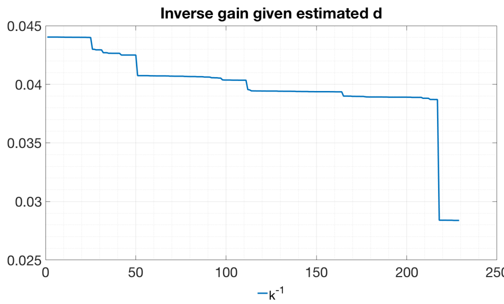
(b) Hamilton, 4 lags, $h = 8$



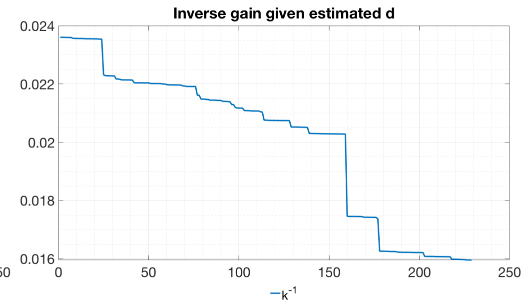
(c) Baxter-King, $(6, 32)$ quarters, truncation at 12 lags

4 Estimates

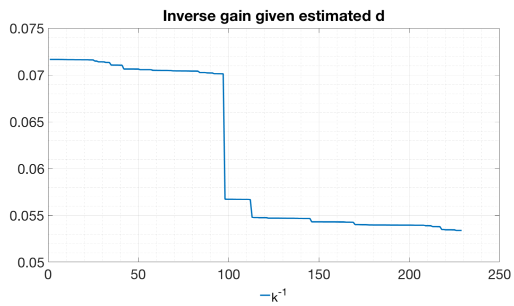
Figure 2: Inverse gain for $\hat{d}$ for the different filters



(a) Hodrick-Prescott, $\lambda = 1600$



(b) Hamilton, 4 lags, $h = 8$



(c) Baxter-King, $(6, 32)$ quarters, truncation at 12 lags

Table 1: $\hat{d}$

	$W = I$	$W = \text{diag}(\hat{\sigma}_{ac(0)}, \ldots, \hat{\sigma}_{ac(K)})$
HP	77.7899	10
Hamilton	32.1649	10
BK	90.3929	10

5 The other thing: numerical implementation of target criterion

The target criterion in the simplified model:

$$\pi_t = -\frac{\lambda_x}{\kappa}\left\{x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_\pi(t)\right)\right.$$
$$\left.\left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=1}^{i-1}(1 - k_{t+j}^{-1}(\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}))\right)\right\} \tag{6}$$

- I think this is the highest priority.