Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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Policymakers came out of the Great Inflation era with a clear understanding that it was essential to anchor inflation expectations at some low level.
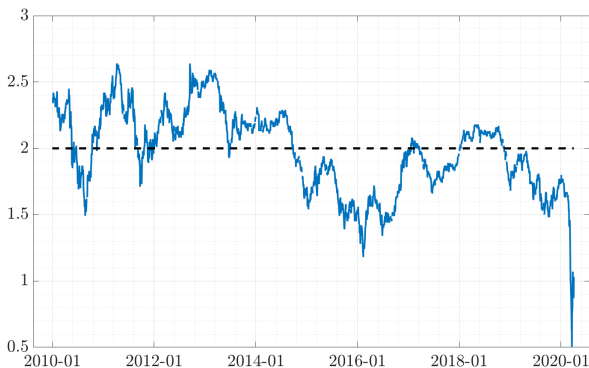*Jerome Powell, Chairman of the Federal Reserve* [1]

Figure: Market-based inflation expectations, 10 year, average, %

This project

- Estimation of the anchoring function: when do expectations become unanchored?

- Model of anchoring expectation formation as an endogenous gain adaptive learning scheme

$\rightarrow$ How to conduct optimal monetary policy in interaction with the anchoring expectation formation?

Preview of results

- A 1%-point forecast error unanchors expectations

- Optimal monetary policy responsiveness time-varying

$\hookrightarrow$ Unanchored expectations introduce an intertemporal volatility tradeoff

$\hookrightarrow$ Illustrate analytically in special case: target criterion

Related literature

- **Optimal monetary policy in New Keynesian models**

  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Econometric learning**

  Evans & Honkapohja (2001, 2006), Preston (2005, 2008), Molnár & Santoro (2014), Ferrero (2007), Eusepi & Preston (2011), Milani (2007, 2014), Lubik & Matthes (2018)

- **Anchoring and the Phillips curve**

  Sargent (1999), Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Gobbi et al (2019), Carvalho et al (2019)

Structure of talk

1. Unanchoring in the data

2. Model of anchoring expectations

3. Solving the Ramsey problem

4. Implications

Structure of talk

Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j)) dj \right] \tag{1}$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1}) B_{t-1}^i + \int_0^1 w_t(j) h_t^i(j) + \Pi_t^i(j) dj - T_t - P_t C_t^i \tag{2}$$

> Consumption, price level

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \tag{3}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{4}$$

▸ Profits, stochastic discount factor

Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- If use $\mathbb{E}$ (rational expectations, RE)

  Model solution

  $$s_t = h s_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \qquad (5)$$

  $$y_t = g s_t \qquad (6)$$

  States: $s_t \equiv (r_t^n, u_t)'$ (states)
  $y_t \equiv (\pi_t, x_t, i_t)'$ (jumps)

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- If use $\hat{\mathbb{E}} \to$ private sector does not know (6)
  $\to$ estimate using observed states & knowledge of (5)

- Households and firms don't know they are identical

Adaptive learning

- Estimate g using recursive least squares (RLS)

  $\rightarrow$ nonrational expectations:

  $$\hat{\mathbb{E}}_t y_{t+1} = \phi_{t-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix} \tag{7}$$

- Note: misspecified

  Can write:

  $$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{8}$$

  In RE, $a_{t-1} = (0,0,0)'$, $\quad b_{t-1} = g\, h \quad \forall t$

Recursive least squares

Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\,h \quad \forall t \qquad (9)$$

$\rightarrow$ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\right)}_{\equiv\, fe_{t|t-1},\ \text{forecast error}} \qquad (10)$$

$k_t \in (0,1)$ gain
b_1 first row of b

Anchoring mechanism: endogenous gain

Gain in literature usually exogenous:

$$k_t = \begin{cases} \frac{1}{t} & \text{decreasing} \\ k & \text{constant} \end{cases}$$

Here instead

$$k_t = k_{t-1} + \mathbf{g}(fe_{t|t-1}) \tag{11}$$

▸ Functional forms

Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{12}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{13}$$

▸ Derivations

- Expectations evolve according to RLS with the endogenous gain given by (11)

$\rightarrow$ How should $\{i_t\}$ be set?

Structure of talk

Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

- $\mathbb{E}$ is the central bank's (CB) expectation

- Assumption: CB observes private expectations and knows the model

Special case

- Only inflation intercept learned

- Anchoring function simplified to

$$k_t = \mathbf{g}(fe_{t|t-1}) \tag{14}$$

Target criterion for special case

Result

In the simplified model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa}\left\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_{\pi,t} \right) \right.$$

$$\left. \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi},\mathbf{t+j}}) \right) \right\}$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ at t, $\prod_{j=0}^{0} \equiv 1$ and b_1 is the first row of b.

Two layers of intertemporal tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa}x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t + fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\mathbb{E}_t\sum_{i=1}^{\infty}x_{t+i}$$

$$-\frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t + fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\mathbb{E}_t\sum_{i=1}^{\infty}x_{t+i}\prod_{j=0}^{i-1}(k_{t+1+j} + fe_{t+1+j|t+j}\mathbf{g}_{\bar{\pi},\mathbf{t+j}})$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

Corollary

Optimal policy under adaptive learning is time-inconsistent.

Lemma

The commitment solution of the Ramsey problem does not exist under adaptive learning.

Structure of talk

Implementation: feedback rule

- Related issue under RE: optimal interest rate sequence implies indeterminate equilibrium

$\Rightarrow$ Reaction function stabilizes expectations

Recall IS-curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big)$$

- E.g. Taylor rule disciplines expectations:

$$\hat{\mathbb{E}}_t i_T = \psi_\pi \hat{\mathbb{E}}_t \pi_T + \psi_x \hat{\mathbb{E}}_t x_T$$

Next steps: form of reaction function

- Model suggests $i_t = \mathbf{f}(\pi_t, k_t, \bar{\pi}_{t-1}; t)$ nonlinear

- Explains deviations from Taylor rule

- However: no commitment makes Taylor rule more viable than under RE as a rough approximation of optimal feedback rule

- If Taylor rule, model prefers being less aggressive on inflation

Conclusion

- Interaction between monetary policy and anchoring

- Optimal policy conditions on stance of current and expected future anchoring

$$\hookrightarrow \text{determine intertemporal tradeoffs}$$

- Explain departures from the Taylor rule like US, fall 2019

- If Fed acted to anchor expectations, then Missing Deflation and Inflation are not "missing"
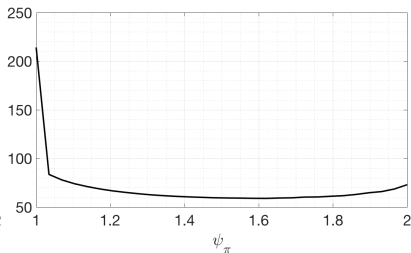
Short-run costs, long-run benefits

Assume Taylor rule and no concern for output gap stabilization

$$i_t = \psi_\pi \pi_t \qquad \lambda_x = 0$$
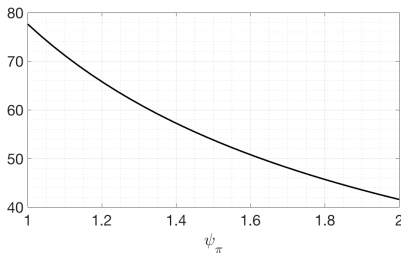
(a) RE

(b) Anchoring

Figure: Central bank loss as a function of ψ_π

Functional forms for **g**

- Smooth anchoring function

$$k_t = k_{t-1} - c + df e^2_{t|t-1} \tag{15}$$

$c, d > 0$

- Kinked anchoring function

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{16}$$

θ_t criterion, $\bar{\theta}$ threshold value

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \tag{17}$$

Σ variance-covariance matrix of shocks
$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(fe_{t|t-1} fe'_{t|t-1} - \omega_{t-1}) \tag{18}$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(fe'_{t|t-1} \omega_t^{-1} fe_{t|t-1} - \theta_{t-1}) \tag{19}$$

ω_t estimated forecast-error variance

Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right)' \right)' \tag{20}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{21}$$

Compact notation

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{22}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{23}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{pmatrix} \tag{24}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{25}$$

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \bigg\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t)$$

$$+ \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \bigg\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa}x_t - \varphi_{2,t}(k_t + \mathbf{g}_\pi(\pi_t - f_{t-1})) = 0 \qquad (26)$$

$$-2\beta\frac{\lambda}{\kappa}x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g_f}(\pi_{t+1} - f_t)) = 0 \qquad (27)$$

Short-run costs from oscillatory dynamics

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{28}$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{29}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \tag{30}$$

Solve for forecast error $fe_t \equiv \pi_t - f_{t-1}$:

$$fe_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k \to 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \tag{31}$$

Target criterion system for anchoring function as changes of the gain

$$\varphi_{6,t} = -cfe_{t|t-1}x_{t+1} + \left(1 + \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1}) - fe_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1}$$

$$- \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1})\varphi_{6,t+2} \tag{32}$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{fe_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{fe_{t|t-1}}\varphi_{6,t+1} \tag{33}$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (33) is given by:

$$\varphi_{6,t} = -2\,\mathbb{E}_t \sum_{i=0}^{\infty}(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i})\prod_{j=0}^{i-1}\frac{\frac{k_{t+j}}{fe_{t+j|t+j-1}}}{\frac{k_{t+j}}{fe_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \tag{34}$$

Details on households and firms

Consumption:
$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{35}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:
$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{36}$$

Profits:
$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \tag{37}$$

Stochastic discount factor
$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{38}$$

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{39}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{40}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (39) backward to some date t, take expectations at t
2. Sub in (40)
3. Aggregate over households i
$\rightarrow$ Obtain (12)