19 Practical Issues Related to Trends, Seasonality, and Structural Change

This chapter reviews selected topics that are relevant to the specification of reduced-form VAR models, but are not covered in Chapters 2 and 3. Our discussion selectively draws on insights from earlier chapters including Chapter 18.

19.1 Alternative Trend Models

It is common to incorporate time trends into the specification of the VAR model. For example, we may include a deterministic trend polynomial in the model or we may allow for a stochastic trend. In Chapter 18, we saw that by relaxing the constraint of linearity, VAR models also can accommodate much richer stochastic trend models. There are situations, however, when it is more convenient to detrend the data upfront before fitting the VAR model. An example is the construction of data for the U.S. output gap by computing the deviation of U.S. real GDP from the level of potential U.S. real GDP estimated by the Congressional Budget Office. In addition, there are alternative trend models that do not allow for the joint estimation of the trend and the stochastic component of the VAR model. Examples include the trend filters commonly used in evaluating business cycle models in macroeconomics. In the latter case, the data must be detrended prior to fitting the VAR model.

Unlike a deterministic linear trend model, these trend filters allow the trend rate of growth to evolve over time. Perhaps the most popular trend filter in modern macroeconomics is the HP filter named after Hodrick and Prescott (1997). Other examples include the band-pass filter of Baxter and King (1999) and the random walk filter of Christiano and Fitzgerald (2003).

19.1.1 Hodrick-Prescott (HP) Filter

The HP filter was originally developed to determine the cyclical (or business cycle) component of fluctuations in U.S. postwar real GDP. It is based on the premise that the rate of economic growth accelerates and decelerates in a fairly regular pattern of several years duration that can be captured by a smooth line. The HP trend is extracted from a scalar time series x_t using a two-sided symmetric moving average filter. Given a time series $x_1, \ldots, x_T$, the trend component $\tau_1, \ldots, \tau_T$ is determined as the solution to the following minimization problem:

$$\min_{\{\tau_t\}_{t=1}^{T}} \sum_{t=1}^{T} (x_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 . \qquad (19.1.1)$$

The objective is to minimize the variance of the cyclical component $c_t \equiv x_t - \tau_t$ subject to a penalty in the second difference of τ_t, which measures the acceleration of the trend line. The parameter λ controls the degree of smoothness of the trend component. In the limit, as $\lambda \to \infty$, the trend component will coincide with a linear deterministic trend. At the other extreme, for $\lambda = 0, x_t = \tau_t$. For quarterly U.S. real GDP data, Hodrick and Prescott recommend $\lambda = 1600$; for the corresponding annual data, $\lambda = 400$. These choices are ad hoc. In practice, it may be useful to experiment with different degrees of smoothness and to verify whether the implied cyclical component matches extraneous information about the business cycle. For further discussion of the choice of λ the reader is referred to Ravn and Uhlig (2002).

There are two important caveats. One is that the HP filter described above is a two-sided MA filter that is not designed to estimate the trend component at the edges of the sample. Standard code available for the HP filter makes some ad hoc adjustments to overcome this problem, but in general we have to be cautious in interpreting results for the first and last observation. Another potential limitation of the HP filter is that it is applied to each series individually. To the extent that often several macroeconomic aggregates share a common trend, as would be the case in modeling output, consumption, and investment, it makes sense to filter these series jointly. Such a multivariate generalization of the HP filter was proposed by Kozicki (1999).

19.1.2 Band-Pass Filters

The HP filter is a low-pass filter in that it allows fluctuations that last less than a certain number of years to pass the filter and prevents longer-lasting fluctuations from passing. Under suitable choices for λ we can focus on fluctuations lasting less than, say, eight years which is commonly considered the maximum length of a business cycle. The cycle in this case is defined as all fluctuations that are not part of the trend. Typically, however, we would not consider fluctuations lasting less than, say, two years to be part of the business cycle. This observation led to the development of band-pass filters. Band-pass filters can be designed to filter out all fluctuations that last less than two years or longer

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than eight years, for example, allowing us to focus on the variability of the data at business cycle frequencies (see, e.g., Canova 1998; Baxter and King 1999). The HP filter may be viewed as a special case of a band-pass filter.

19.1.3 Potential Shortcomings of Trend Filters

All trend filters are inherently ad hoc. They define the trend rather than estimating some prespecified trend component. More importantly, they may produce distortions in the filtered data. A well-known example of the dangers of using trend filters is provided by early business cycle analysis. When researchers, starting in the 1920s, smoothed noisy raw data for real economic activity using simple moving average filters, they detected various apparent cycles and waves of different lengths that received considerable attention in the literature at the time. For example, Kuznets (1961) obtained a business cycle of 15 to 25 years duration, named the Kuznets cycle, after applying a low-order moving average filter to U.S. macroeconomic time series. Subsequently, Howrey (1968) showed that the same cycle arises, when Kuznets' filter is applied to artificially generated white noise data (which have no cycles by construction), calling into question the existence of a Kuznets cycle in the U.S. data. This experience has increased awareness of the potential limitations of applying trend filters to macroeconomic data.

While some degree of distortion is inevitable, one criterion of the quality of a trend filter is that it must provide reasonable trend components when applied to data generated from a range of processes thought to be representative of macroeconomic time series data. The HP filter, while specified in an ad hoc manner, performs about as well in this regard as do other more recently developed trend filters (see Christiano and Fitzgerald 2003). Nevertheless, there is considerable evidence of the use of the HP filter producing spurious dynamic relations among time series (see, e.g., Hamilton 2016).

19.1.4 Trend-Filtered Variables in VAR Models

Trend-filtered variables obtained by using the HP filter or other band-pass filters may be accommodated in VAR analysis as follows. Let x_t denote a trending time series such as the log of real output. Suppose we decompose x_t into its trend component, τ_t, and the cyclical component, $c_t \equiv x_t - \tau_t$, defined as the deviation from trend such that trend and cycle are uncorrelated. Once the trend filter has been applied to x_t and the business cycle component c_t has been extracted, we replace x_t in the set of variables to be modeled as a VAR process by c_t and fit a VAR model to the new set of variables. Of course, it is also possible to detrend more than one time series in this manner.

Although this approach is straightforward, there is increased recognition in the literature that statistical filters that can be represented as a symmetric,

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two-sided moving average of the raw data inevitably distort the estimates of the impulse responses and that applying such filters to some variables in the model but not to others creates further misspecification biases (see Canova 2014). Thus, the use of band-pass filters in VAR analysis should be avoided.

19.1.5 Choosing between Different Trend Models

The choice of the detrending method in a particular application by necessity requires judgement. There rarely is only one correct treatment of the trend. One way of discriminating between different detrending methods is to evaluate the implied cyclical component against extraneous information such as the NBER business cycle dates. A detrending method that implies that the U.S. economy was operating above trend during an NBER recession, for example, would be considered implausible. Alternative data-based business cycle dating methods have been discussed, for example, in Harding and Pagan (2002).

19.1.6 Combining Different Trend Specifications

A common situation in applied work is that some model variables are trending, whereas others are not. Consider, for example, the problem of constructing a VAR model for

$$y_t = \begin{pmatrix} gnp_t \\ \pi_t \\ i_t \end{pmatrix},$$

where gnp_t denotes the log level of real GNP, π_t the inflation rate, and i_t the nominal interest rate. Inspection shows that gnp_t is subject to a linear trend, whereas π_t and i_t are not. One common choice in this situation is to replace gnp_t by detrended gnp_t, using linear detrending or a suitable band-pass filter, and postulating a VAR model with intercept for

$$\begin{pmatrix} \widetilde{gnp}_t \\ \pi_t \\ i_t \end{pmatrix},$$

where $\widetilde{gnp}_t$ denotes detrended log real GNP. Alternatively, in the presence of a stochastic trend in gnp_t, we would replace $\widetilde{gnp}_t$ in this model by Δgnp_t.

In the presence of a deterministic time trend in gnp_t, another choice is to include a deterministic time trend in all three equations of a VAR model for the original y_t. This approach obviously is infeasible when working with band-pass filters.

It may seem that a third choice would be to specify a VAR model for y_t that includes both an intercept and a deterministic time trend in each equation with the coefficients on the time trends in equations two and three restricted to zero in estimation. Such a model could be estimated by restricted GLS or, less efficiently, by equation-by-equation LS. This approach is not valid in general, however, because lags of the trending variable gnp_t appear in equations two and three.

19.2 Seasonality

Seasonality in macroeconomic data refers to systematic and recurrent variation in the data within the year. For example, automobile output in Europe tends to drop every summer, as workers go on vacation and plants all but close down. Likewise, construction activity in the Northern hemisphere slows every winter, while the consumption of natural gas surges. Similarly, monthly retail sales spike every year in December, as Christmas approaches, and airfares and motel rates increase during tourist season. Unmodeled seasonality tends to violate the constant parameter assumption of standard linear VAR models. There are a number of remedies depending on the type of seasonal variation in the data. Typically, the objective is to construct the VAR impulse responses controlling for seasonality, which facilitates comparisons with responses in economic models that do not contain seasonality.

19.2.1 Deterministic Seasonal Variation in VAR Models

Just as we have distinguished between deterministic and stochastic trend models, it is necessary to distinguish between deterministic seasonal variation and stochastic seasonal variation. The most common form of deterministic seasonal variation involves adding seasonal dummies to the VAR model. For example, consider the stable autoregression

$$A(L)y_t = \nu_i + u_t,$$

where ν_i is the intercept associated with the i^{th} season and u_t is white noise. Quarterly models would typically include four seasonal dummies, and monthly models would include twelve seasonal dummies, where each dummy variable takes on a value of 1 for the quarter (or month) in question and zero otherwise.[1] While it is common to include one dummy for each quarter (or month), one dummy would obviously be enough if we knew that there is no seasonality in the other quarters (or months). Models with seasonal dummies impart a deterministic pattern. Seasonality is perfectly predictable and, every year, the seasonal effect is the same. Although seasonal dummies are often used when modeling $I(0)$ processes, it is worth stressing that it is equally possible for an $I(1)$ process in levels to have deterministic seasonals.

[1] If all seasonal dummies are included in the VAR model, the intercept must be dropped to avoid a singularity in the regressor matrix. Alternatively, one may retain the intercept, but drop one of the seasonal dummies.

19.2.2 Stochastic Seasonal Variation in VAR Models

Stationary Seasonal Processes. Alternatively, one can model seasonal effects as random, which allows seasonality to be less than perfectly predictable and permits seasonal effects to evolve over time. Such randomness is appealing because there is no reason to expect seasonality to be time-invariant. For example, one would expect seasonal patterns in airfares to evolve with changes in market structure and seasonal energy consumption patterns to evolve with new technologies. Some forms of stochastic seasonality are easy to model. For example, for a VAR process

$$A_s(L^s)y_t = \nu + u_t,$$

where $A_s(L^s) = I_K - A_{s,1}L^s - \cdots - A_{s,r}L^{r\times s}$ is a matrix operator which involves only lags that are multiples of the seasonal period s, the autocorrelation function of y_t spikes at lags $s, 2s, 3s, \ldots, rs$. The seasonal period is for example $s = 4$ for quarterly data and $s = 12$ for monthly data. To allow for more sophisticated nonseasonal autocovariance, a multiplicative model of the form

$$A(L)A_s(L^s)y_t = \nu + u_t$$

may be considered, where $A(L)$ is a standard VAR operator of order p, say. Multiplying out the operator product shows that a standard VAR process with sufficiently large lag order nests the purely seasonal VAR or multiplicative model, if all roots of the VAR operator are outside the unit circle, but some roots are complex pairs with seasonal periodicities. This means that even a standard VAR model may generate seasonality following a stationary stochastic process, provided we include enough lags. One drawback of such models is that stationary seasonal processes tend not to exhibit the regularity commonly associated with seasonal effects, unless the seasonal roots approach the unit circle. This observation suggests that this seasonal model without the addition of seasonal dummies will be of limited relevance for applied work.

Integrated Seasonal Processes. Further complications arise when the seasonal variation is known to follow unit root processes. In that case, the appropriate transformation of the data is the seasonal-differencing operator:

$$\Delta_s \equiv (1 - L^s),$$

where $s = 4$ for quarterly data and $s = 12$ for monthly data. Rather than expressing the data as quarter-to-quarter growth rates, as one would in nonseasonal quarterly VAR models, applying the seasonal differencing operator to series in logs, one constructs the growth rate for the current quarter or month relative to the same quarter or month one year ago. These transformed data are often referred to as year-on-year growth rates in the press and in government reports. Having transformed the relevant data in this fashion, one proceeds by fitting a standard VAR model with intercept to seasonally differenced data or untransformed data, as appropriate.

Once we allow for a seasonal unit root, an obvious concern is that the seasonal components are likely to be cointegrated across at least some variables in the VAR model. Seasonal cointegration means that shocks in this VAR model have a permanent effect on the seasonal pattern in each variable. In that case, LS estimates of the VAR model for the seasonally differenced data would not enforce the long-run properties of the variables because there is no seasonal error correction term. One way of addressing this problem would be to specify a seasonally cointegrated VAR model. There are different possible specifications, depending on the assumptions regarding the roots on the unit circle and the deterministic terms. For example, Lee (1992) specifies a quarterly model of the form

$$\Delta_4 y_t = \nu + \sum_{i=1}^{4} \Pi_i z_{it-1} + \sum_{j=1}^{p-4} \Gamma_j \Delta_4 y_{t-j} + u_t,$$

where

$$\begin{aligned} z_{1t} &= (1 + L + L^2 + L^3) y_t, \\ z_{2t} &= (1 - L + L^2 - L^3) y_t, \\ z_{3t} &= L(1 - L^2) y_t, \\ z_{4t} &= (1 - L^2) y_t \end{aligned}$$

capture roots on the unit circle of different frequencies (see also Johansen and Schaumburg 1999). A corresponding model with seasonal intercepts is discussed by Franses and Kunst (1999). A brief discussion of seasonal cointegration can also be found in Ghysels and Osborn (2001, chapter 3).

In practice, the existence of seasonal unit roots and seasonal cointegration is rarely, if ever, known, and standard statistical tests lack the ability to discriminate reliably between different seasonal models. This creates a problem because imposing incorrect seasonal structure in estimation will invalidate the VAR model estimates. A convenient alternative is to specify the VAR model allowing for at least 12 monthly lags (or at least four quarterly lags) in levels fit to seasonally unadjusted data in levels with deterministic seasonals and possibly deterministic trends included in the VAR model (see Diebold 1993). Apart from the loss of efficiency and the small-sample bias associated with the addition of redundant deterministic regressors, the LS estimates of that model will remain consistent whether the data exhibit any combination of a deterministic trend, a stochastic trend, cointegration, deterministic seasonality, stochastic seasonality, seasonal integration, or seasonal cointegration, mirroring the discussion in Chapter 3. No knowledge of the nature of the seasonal process is required as long as the seasonal DGP is nested within the VAR model. Of course, such a VAR model may be simplified at the expense of generality to the extent that we have additional information about the data that can be imposed in estimation.

19.2.3 Synthesis

When dealing with seasonal $I(0)$ data, the standard remedy is to allow for a sufficient number of autoregressive lags to capture stochastic seasonal dynamics combined with seasonal dummies to capture deterministic seasonal dynamics. When dealing with seasonal $I(1)$ data, there are two main modeling choices. One is seasonal differencing of the data; the other is imposing deterministic seasonality on the $I(1)$ process. Osborn (1993) stresses that seasonal unit roots imply a failure of cointegration between the four quarters of a given time series. This means that observations for different quarters in a given year may move arbitrarily far apart. This implication of seasonal differencing is a priori implausible and difficult to rationalize from an economic point of view. Consequently, Osborn (1993) recommends against using models of seasonal differences in favor of models that superimpose a deterministic seasonal pattern on a conventional $I(1)$ process. The latter models also seem to provide a better description of typical macroeconomic variables than models based on seasonal differences, although it is unclear whether existing statistical procedures can discriminate between these models. In contrast, Osborn (2002) concludes that the default model for forecasting seasonal economic time series for horizons of one or two years should be one with year-on-year differences. Although a good forecasting model is not necessarily the best model for other purposes, Osborn's analysis shows that the choice between using year-on-year growth rates or deterministic seasonality models for series with strong seasonality is not straightforward and has to be decided case by case.

19.2.4 Periodic Seasonal VAR Models

Both the seasonally differenced model and the model with seasonal dummies impose that the slope parameters of the model and its variance-covariance structure are time invariant across seasons. Some researchers have argued that a more general model would allow all model parameters to vary with the season (see, e.g., Osborn 1991, 1993). The periodic VAR model may be viewed as a generalization of the VAR model with seasonal dummies. That model may alternatively be written as:

$$y_t = n_{1t}\nu_1 + \cdots + n_{st}\nu_s + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t,$$

where $n_{it} \in \{0, 1\}$ and $\sum_{i=1}^{s} n_{it} = 1$. In other words, $n_{it} = 1$ if t coincides with the i^{th} season, and $n_{it} = 0$ otherwise. Now consider relaxing the restriction that only the intercept varies by season. This results in the general periodic VAR model:

$$y_t = \nu_t + A_{1t}y_{t-1} + \cdots + A_{pt}y_{t-p} + u_t = A_t Z_{t-1} + u_t$$

with $Z_{t-1} \equiv (1, y'_{t-1}, \ldots, y'_{t-p})'$,

$$A_t \equiv \left[\nu_t, A_{1t}, \ldots, A_{pt}\right] = n_{1t}A_1 + \cdots + n_{st}A_s,$$

and

$$\Sigma_t = n_{1t}\Sigma_1 + \cdots + n_{st}\Sigma_s,$$

where the n_{it} again denote seasonal dummy variables. Periodic VAR models can be motivated based on economic models in which seasonality is an integral part of agents' decision-making. A detailed discussion of periodic VAR models can be found in section 17.3 of Lütkepohl (2005). Extensions of this model to allow for integrated and cointegrated variables are possible (see Ghysels and Osborn 2001, chapter 6; Franses and Paap 2004).

19.2.5 Seasonal TVC-VAR Models

Periodic models imply deterministic variation in the model parameters. An even less restrictive, periodic VAR model would allow the intercept terms to evolve stochastically according to a latent random walk process:

$$y_t = \nu_{s,t} + A_{1t}y_{t-1} + \cdots + A_{pt}y_{t-p} + u_t,$$

where

$$\nu_{s,t} = \nu_{s,t-1} + \eta_{s,t}$$

and $\eta_{s,t}$ is zero-mean white noise. From an econometric point of view, this model belongs to the class of time-varying coefficient vector autoregressive (TVC-VAR) models discussed in Chapter 18. Periodic VAR models and seasonal TVC-VAR models are rarely used in empirical work, perhaps because they are more difficult to estimate and evaluate than standard VAR models.

19.2.6 Seasonally Filtered Data in VAR Models

Many macroeconomic time series are reported in seasonally adjusted form, making allowance for seasonality in the model unnecessary.[2] Most empirical

[2] It is nevertheless good practice to verify that ostensibly seasonally adjusted data are indeed free of seasonality. Often a plot helps verify the absence or presence of seasonality. Alternatively, plots of the spectral density of the (suitably transformed) data or of their autocorrelations can be helpful. As an additional check, one could regress these data on seasonal dummies and conduct a Wald test for the inclusion of these regressors.

studies are based on such seasonally adjusted data. Statistical agencies reporting seasonally adjusted data tend to rely on procedures such as the Census Bureau's X-12-ARIMA filter or Eurostat's TRAMO-SEATS to remove seasonal effects (see Findley, Monsell, Bell, Otto, and Chen 1998; Gómez and Maravall 1997). At the core of the X-12 procedure (and its X-13 successor) is a set of moving average filters which allow the seasonal pattern to evolve over time. The seasonal filter is typically applied to one time series at a time. Notwithstanding the increasing sophistication of some of these seasonal filters, it remains true that all seasonal adjustment procedures – including those implemented by statistical agencies – have side-effects and may distort the data in ways that can change the economic interpretation of the data or the statistical significance of parameter estimates. Applied researchers have tended to ignore this concern, given the convenience of using seasonally adjusted data, which reduces the number of modeling choices to be made and eliminates the need for modifications of the basic VAR model along the lines discussed earlier.

19.2.7 Combining Seasonally Adjusted and Unadjusted Data in the Same VAR Model

Although typically users of VAR models rely on seasonally adjusted data (or on data not subject to any apparent seasonality such as interest rates), occasionally VAR models include seasonal variables that are not available in seasonally adjusted form. In addition, sometimes data have been labelled incorrectly as *seasonally adjusted* by statistical agencies yet still contain visible seasonality, making further adjustments necessary. The most common response to this problem is to allow for a sufficient number of autoregressive lags and/or to include seasonal dummies in all equations of the VAR model. Alternatively, the data in question can be deseasonalized up front to avoid the overparameterization of the VAR model when not all variables contain a seasonal component. In the latter case, the use of the X-12-ARIMA filter is another option. There is no consensus on which approach works better.

When combining variables requiring different seasonal adjustments within the same VAR model, the same caveats apply as when combining different trend specifications. When dealing with deterministic seasonality, we either must deseasonalize the data upfront, before fitting a VAR model with intercept, or we must include the same deterministic seasonal components in each equation of the VAR model along with any other deterministic terms.

19.2.8 Summary

In structural modeling we are typically not so much interested in modeling seasonality. In that situation, if seasonal data are available, there are two options. First, we may remove the seasonal component from the data by applying some seasonal-adjustment procedure. One drawback of such procedures is that they may lead to distortions of the nonseasonal component. Another drawback is that these procedures are applied to individual series and, hence, cannot take into account the possible interaction between the seasonal components of different series.

The second possibility is to allow for seasonality within the VAR model. We discussed several ways of modeling seasonality and their potential drawbacks. For example, adding deterministic terms to account for seasonality captures only very regular types of seasonality. Incorporating seasonal differences in the model allows for stochastic seasonality, but may have implausible implications for the long-run features of the seasonal components. Yet another possibility is to specify VAR models that encompass deterministic and stochastic seasonality. Such models tend to be very large, however, which limits their usefulness for macroeconometric modeling. Thus, it is difficult to provide general guidelines for coping with seasonality in some or all variables.

Ultimately we are interested in modeling seasonality because of its implications for structural impulse response analysis. As long as modeling the seasonal component leaves the additive reduced-form error term unaffected, the identification of the structural shocks may proceed exactly as discussed in earlier chapters. To the extent that there is seasonal heteroskedasticity, standard identification methods for time-invariant VAR models fail, but we may proceed with identification by heteroskedasticity, as discussed in Chapter 14.

Provided the seasonality leaves the autoregressive coefficients unaffected, as would be the case if the seasonality can be captured by seasonal dummies, by seasonal differencing, or by adding autoregressive lags, the construction of the structural impulse responses may proceed as in the time-invariant model. Otherwise, the methods discussed in Chapter 18 may be used.

19.3 Structural Change in the Stochastic Component of the VAR Model

The premise of linear VAR models is that the model structure is time invariant. Few researchers would contend that this premise is literally correct. There are many reasons why we would expect the stochastic process governing the economy to evolve over time. A more defensible position is that the VAR model serves as a local approximation to the data generating process within a given time window.

19.3.1 Breaks in the Stochastic Component

In some cases, there are compelling reasons to suspect a discrete break in the stochastic structure. For example, we know of important changes in the U.S. monetary policy regime in 1979, when Paul Volcker assigned much higher weight to the inflation objective and lower weight to the employment objective than his predecessors. Fitting the same VAR model of U.S. monetary policy to data from the pre- and post-Volcker era hence does not seem appropriate.

One obvious remedy is to split the sample in 1979 and to fit separate vector autoregressions to each subsample (see, e.g., Kilian and Lewis 2011). This means that, in general, users of VAR models by necessity will have to make do with fairly short samples, requiring special attention to the small-sample properties of estimators. Another challenge is that, following major structural changes the data may be contaminated by transition dynamics for years to come. For example, U.S. inflation and interest rates fell only slowly following the monetary policy shift under Paul Volcker in 1979, and remained persistently high well into the 1980s. As a consequence, it may make sense to exclude the early 1980s from the analysis, when splitting the sample, and to start a VAR analysis of the post-1979 U.S. economy only around 1985. Allowing for such transition dynamics further reduces the available sample. In light of these restrictions, a common situation in applied work is that we are unable to fit a sufficient number of lags on the subsample. In that situation a VAR analysis may not be an option.

An alternative approach is to estimate VAR models based on rolling windows of data. This approach is less appropriate when dealing with well-defined breaks in the model structure, as in the Volcker example, and more appropriate when dealing with smooth structural changes. An obvious caveat is that macroeconomists rarely have enough data for rolling windows that are long enough to allow reliable estimation. Hence, VAR estimates from rolling regressions tend to be noisy and unreliable. An added concern is that impulse responses computed from rolling regressions make no allowance for future changes in the structure of the model. A caveat that applies to both split-sample regressions and rolling regression applications of VAR models is that care must be taken that the sample period in question includes enough variation in the data to allow the identification of the structural shocks in question. Moreover, in reporting the impulse response estimates, it is important to normalize the shocks of interest to be of the same magnitude across all samples.

A third approach would be to specify a TVC-VAR model that allows all model parameters to evolve smoothly over time, as discussed in Chapter 18. An example of this strategy is Primiceri (2005). Such models are much harder to estimate reliably in practice given the profligacy of parameters, especially as the number of variables increases. TVC-VAR models require long samples and quickly become intractable when longer lags and/or additional variables are included in the model. These problems are compounded when dealing with structural representations of TVC-VAR models. For example, such models can allow for time variation in the parameters within a given monetary policy rule, but it would be much harder to allow for a change in the monetary policy rule from targeting monetary aggregates toward targeting interest rates, as discussed in Kozicki and Tinsley (2009). Moreover, TVC-VAR models are nonlinear models, which complicates the interpretation of the evolution of the impulse response functions over time and makes it impossible to construct historical decompositions or conventional forecast error variance decompositions. It also restricts the range of strategies that can be used for identifying structural shocks.

An alternative class of nonlinear models is the regime-switching VAR model we already discussed in Chapter 18 (see, e.g., Sims, Waggoner, and Zha 2008). Such models have been used to model the evolution of monetary policy, in particular (see Sims and Zha 2006b). Regime switching models with alternating regimes evolving stochastically according to a latent Markov-switching process allow for changes in the stochastic component of the VAR much like TVC-VAR models, but imply a greater degree of regularity in that the model alternates between a small number of policy regimes. Whether this degree of regularity is found in the real world is an open question. An alternative view would be that monetary policy regimes change over time without any apparent tendency to revert back to previous regimes. In that case, the use of a regime-switching model would be questionable.

For these reasons, traditional linear structural VAR models estimated on samples chosen to avoid structural breaks will remain one of the primary tools of analysis. An obvious question is how to detect these breaks. Although there are Chow-type tests for structural change that could be used to detect likely breakpoints in the slope parameters of the VAR model, as discussed in Chapter 2, these tests tend to have low power especially after accounting for data mining across possible breakpoints. This problem is compounded when the number of breaks is unknown or the break does not occur all at once. Moreover, tests for structural change tend to confound persistent transitory dynamics arising from unusual sequences of structural VAR shocks with structural change.

Hence, a better strategy in practice is to base the selection of the sample primarily on extraneous information about economic institutions. An example of this strategy is provided in Alquist, Kilian, and Vigfusson (2013) who discuss how VAR models of the relationship between the price of oil and U.S. real GDP may be undermined by the inclusion of pre-1973 data, given the changes in the institutional structure of the oil market in late 1973.

19.3.2 Smooth Structural Change in the Stochastic Component

Perhaps the most pernicious form of structural change in practice is smooth structural change. In many cases, we can view linear VAR models as an approximation to a process that is evolving slowly and gradually over time, as long as the sample is not too long. In some cases, such approximations are not credible. For example, in modeling the effects of energy price shocks on the growth rate of U.S. consumption, economic theory tells us that the magnitude of the effect should depend on how large energy expenditures are as a fraction of total expenditures. This energy share has fluctuated considerably since the 1970s. One approach to modeling this form of nonlinearity would be to specify a TVC-VAR model as in Chapter 18. Alternatively, we can convert this nonlinear model into a linear model by weighting the percent change in energy prices by the nominal expenditure share, as proposed by Edelstein and Kilian (2009). This allows us to rely on standard linear VAR models.

A similar approach was taken by Ramey and Vine (2011) in modeling the nonlinearities arising from the regulation of U.S. gasoline prices in the 1970s and early 1980s. Price ceilings are costly to consumers because they create lines at gas stations. These queuing costs are part of the price of gasoline. If they are ignored, the relationship between gasoline prices and the economy may become nonlinear. Ramey and Vine addressed this problem by quantifying the queuing costs and adding them to the observed gasoline price, allowing them to fit a linear VAR model.

The particular adjustments proposed by Edelstein and Kilian (2009) and Ramey and Vine (2011) could be combined, of course (see, e.g., Baumeister and Kilian 2017). Whether the linear VAR framework can be preserved in this fashion depends on the economic context and on the ingenuity of the researcher. If no suitable transformations are available, the only alternative would be a TVC-VAR model or some other model that allows for changing coefficients.