

Materials 34 - Still estimating the anchoring function

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1 Estimation procedure

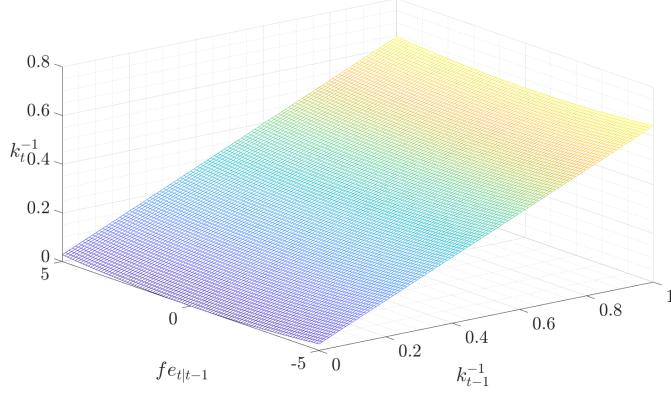
Instead of the AR(1) anchoring function used so far (Equation A.6), I use the following equation

$$k_t^{-1} = \alpha s(X) \tag{1}$$

where $X = (k_{t-1}^{-1}, f e_{t|t-1})$ and I use piecewise linear interpolation. I initialize α_0 by specifying a grid for X , passing the grid through Equation (A.6) to generate k_t^{-1} -values, and approximating by fitting the grid to the k_t^{-1} -values. See Fig. 1.

Then I estimate α using GMM, targeting the autocovariance structure of inflation, the output gap and the nominal interest rate (federal funds rate) in the data.

Figure 1: Initialization via Equation (A.6) implies this functional relationship

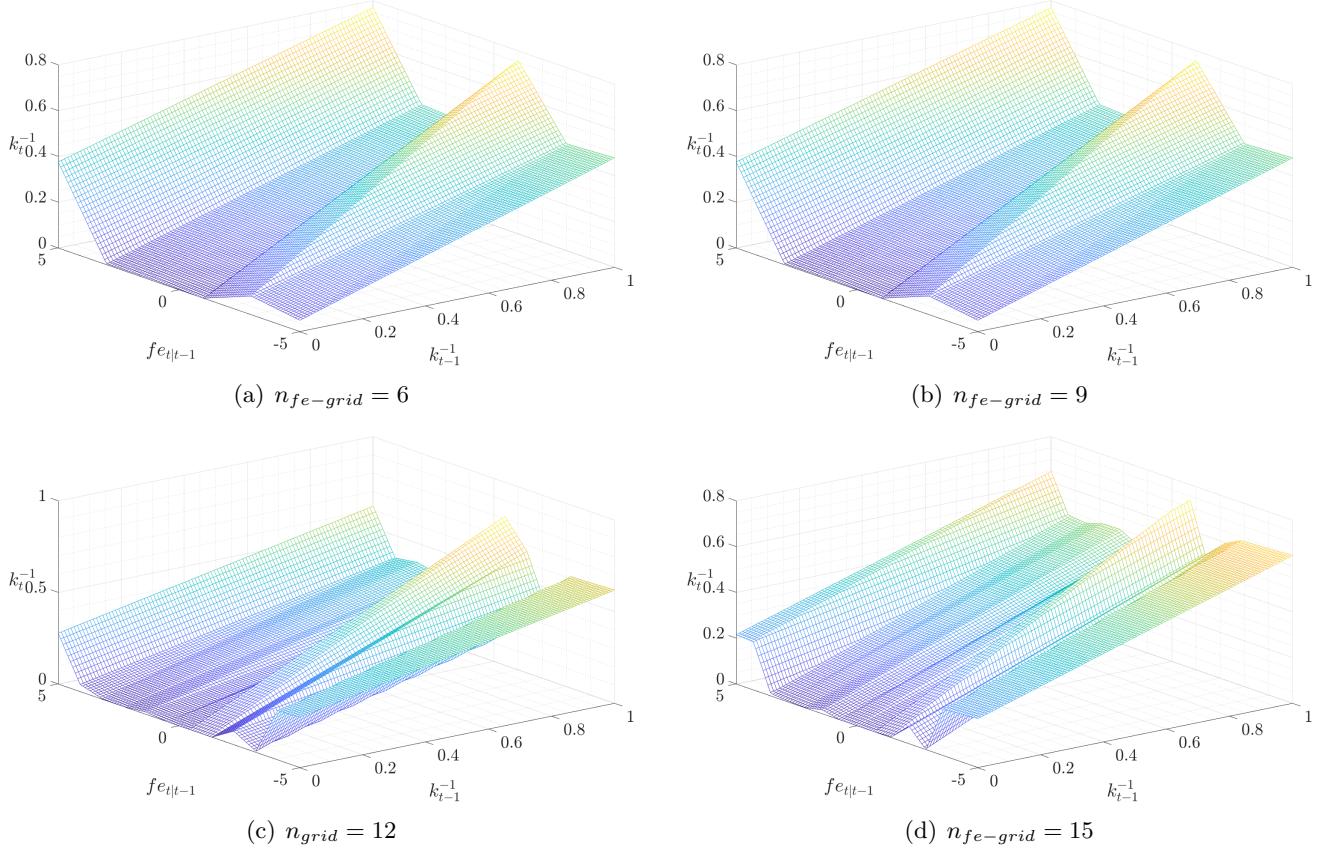


$T = 233$ before BK-filtering, $T = 209$ after BK-filtering. Using the “constant-only, inflation-only” learning PLM. I drop the $ndrop = 5$ initial values. I restrict $\alpha \in (0, 1)$, the support of k^{-1} in the grid. I target the lag $0, \dots, 4$ autocovariance matrices, dropping repeated entries at lag 0, leaving me with 42 moments.

2 Estimating the 2D anchoring function

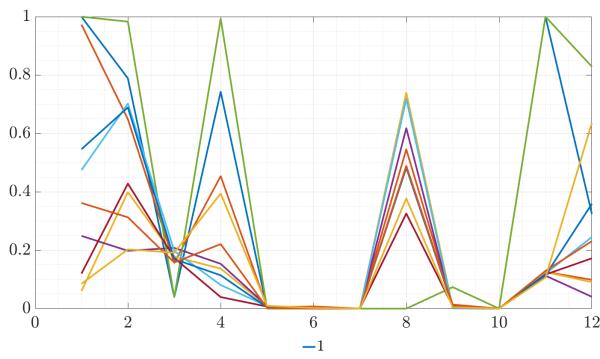
2.1 Real data, 2D function

Figure 2: k_t^{-1} as a function of k_{t-1}^{-1} and $fe_{t|t-1}$ given $\hat{\alpha}^{GMM} \in (0, 1)$. The dimension of the gain-grid is 2.

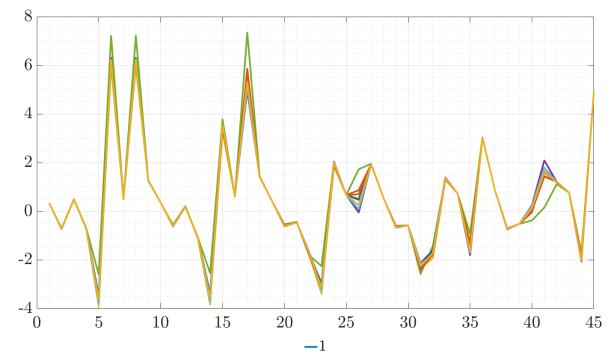


This actually looks quite robust i) to the gridsize, ii) to starting from random points. All I have done at this point is to increase the support of the gain, decrease its gridpoints to 2, increase the forecast error gridpoints, and implement `lsqnonlin`. Increasing the support of the gain also had the feature that now there are no explosions. So all these measures seem to have been beneficial.

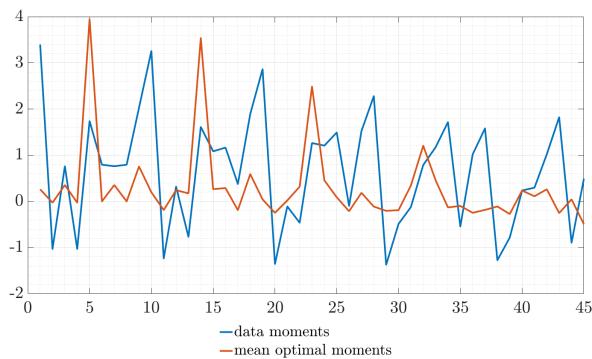
Figure 3: $\hat{\alpha}$ for various random starting points



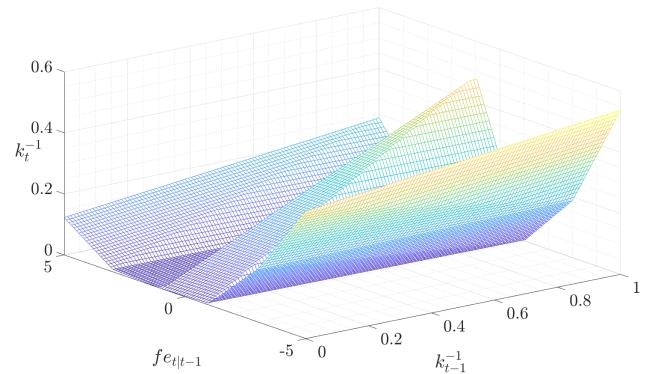
(a) $\hat{\alpha}$



(b) Residuals



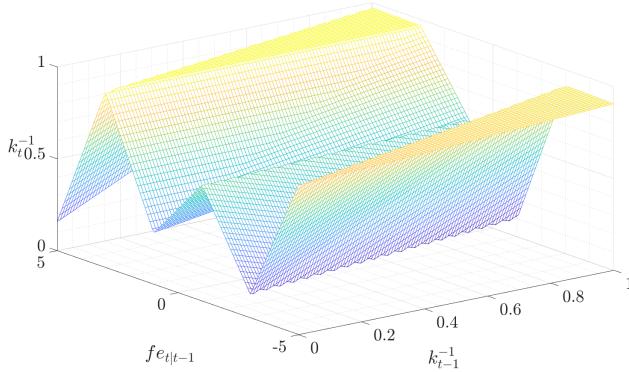
(c) Moments



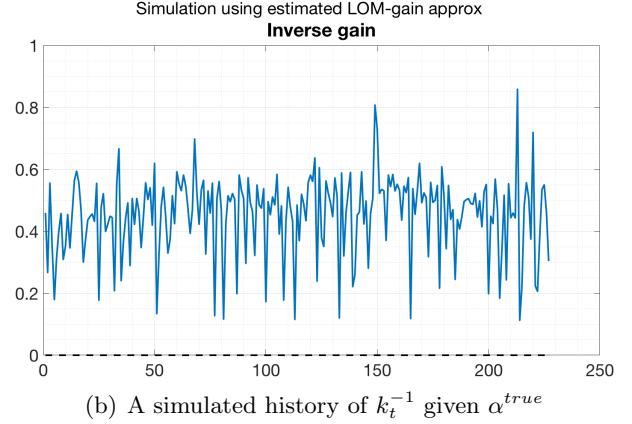
(d) Implied relationship at median estimate

2.2 Simulated data, 2D function

Figure 4: The truth using 2 gridpoints on $[0, 1]$ for the gain, 6 on $[-5, 5]$ for the forecast error

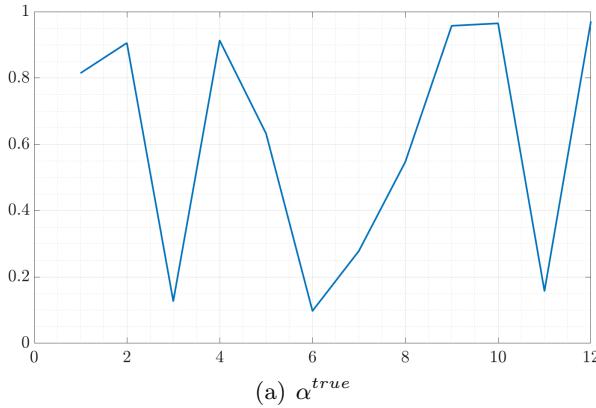


(a) k_t^{-1} as a function of k_{t-1}^{-1} and $fe_{t|t-1}$ given α^{true}

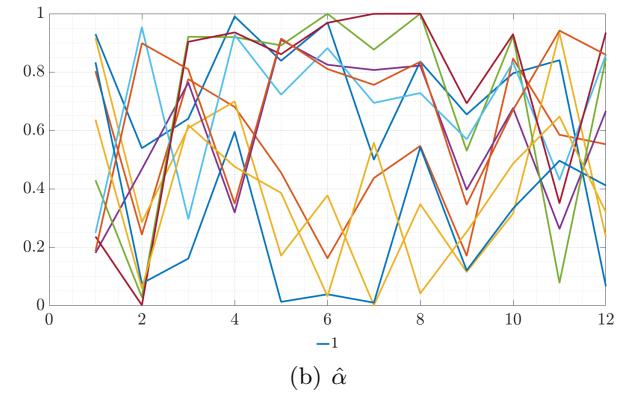


(b) A simulated history of k_t^{-1} given α^{true}

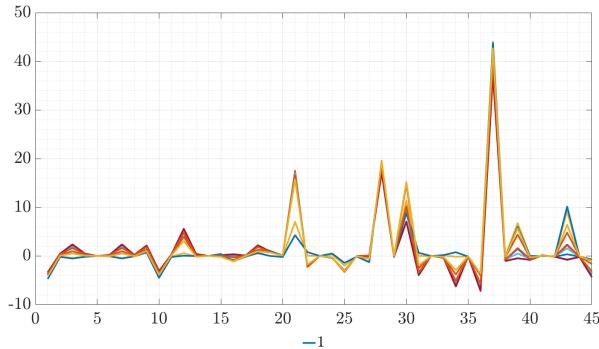
Figure 5: $\hat{\alpha}$ for various random starting points



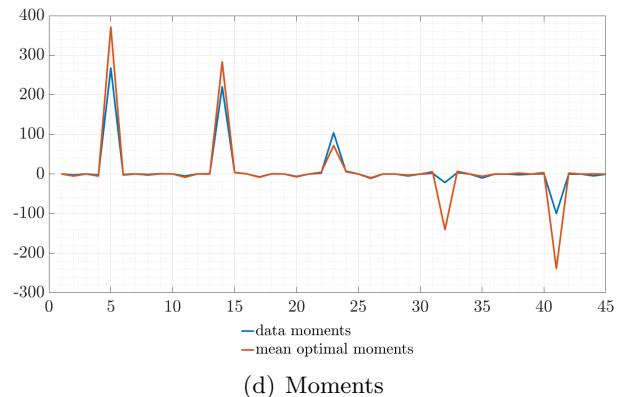
(a) α^{true}



(b) $\hat{\alpha}$



(c) Residuals

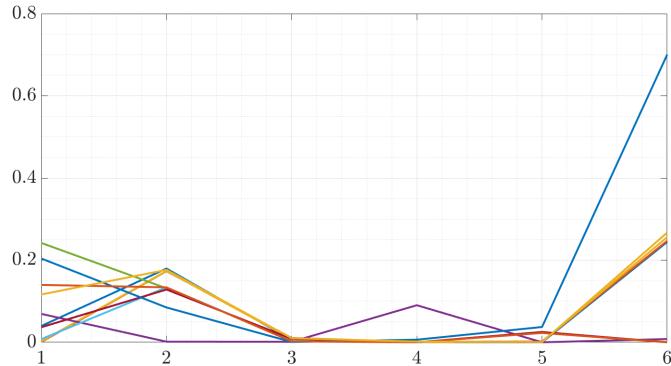


(d) Moments

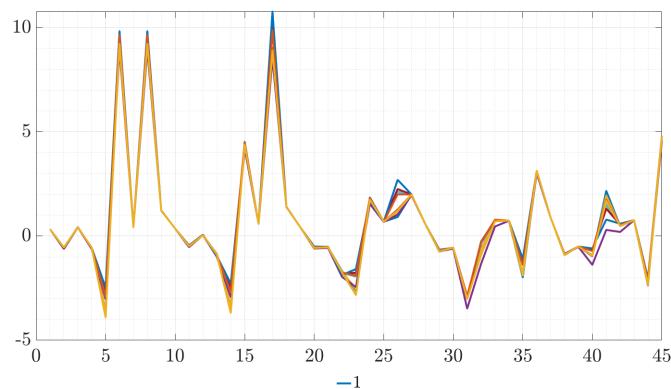
3 Univariate anchoring function

3.1 Real data, 1D function

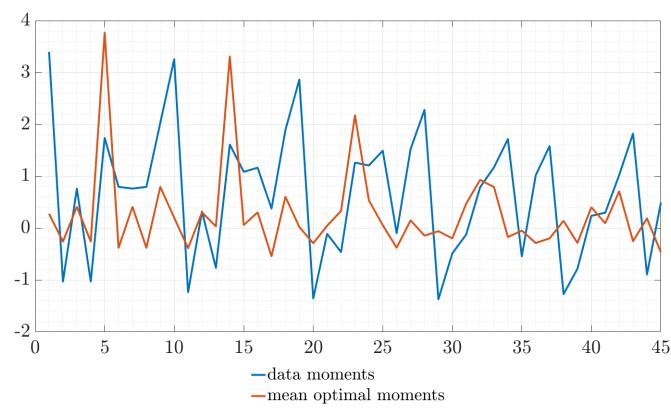
Figure 6: $\hat{\alpha}$ for various random starting points



(a) $\hat{\alpha}$



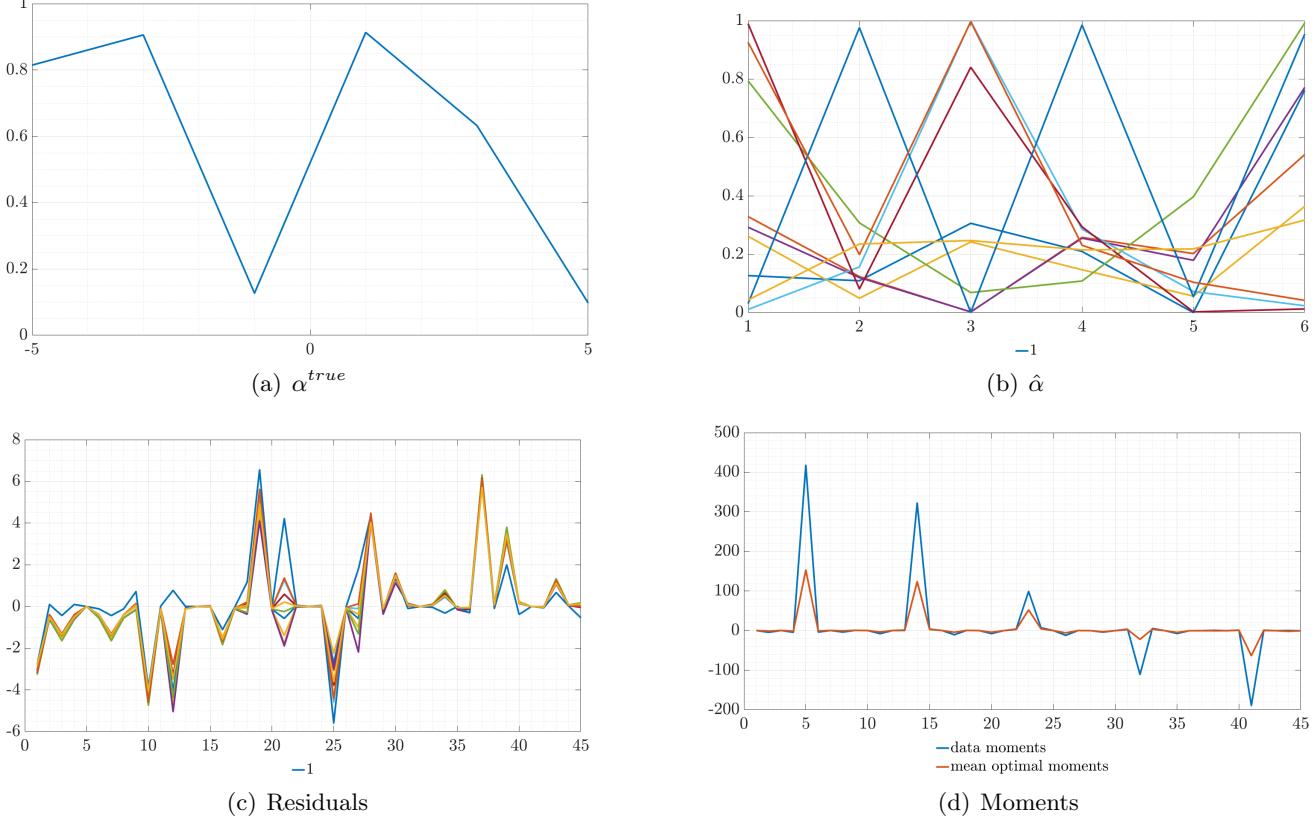
(b) Residuals



(c) Moments

3.2 Simulated data, 1D function

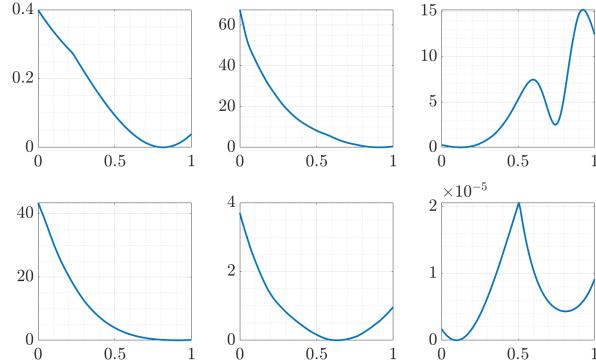
Figure 7: $\hat{\alpha}$ for various random starting points



4 Investigate loss function for 1D case in detail

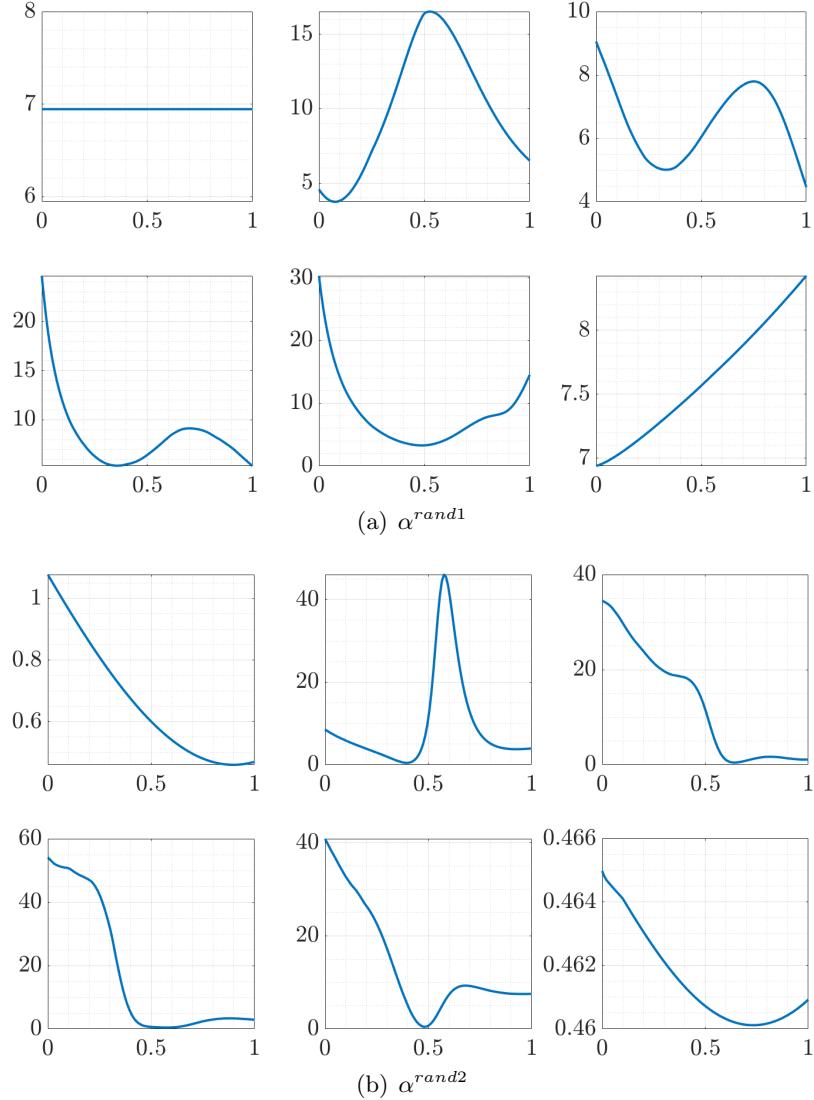
4.1 Loss, changing one parameter at a time

Figure 8: Objective function when I hold all coefficients at α^{true} , move one at a time from 0 to 1



These minima correspond to the true values.

Figure 9: Objective function when I hold all coefficients at α^{rand} , move one at a time from 0 to 1



A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (\text{A.1})$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (\text{A.2})$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \quad (\text{if imposed}) \quad (\text{A.3})$$

$$\text{PLM: } \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \quad b = g_x \ h_x \quad (\text{A.4})$$

$$\text{Updating: } a_t = a_{t-1} + k_t^{-1} (z_t - (a_{t-1} + b s_{t-1})) \quad (\text{A.5})$$

$$\text{Anchoring function: } k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k f e_{t-1}^2 \quad (\text{A.6})$$

$$\text{Forecast error: } f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \quad (\text{A.7})$$

$$\text{LH expectations: } f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \quad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \quad (\text{A.8})$$

This notation captures vector learning (z learned) for intercept only. For scalar learning, $a_t = (\bar{\pi}_t \ 0 \ 0)'$ and b_1 designates the first row of b . The observables (π, x) are determined as:

$$x_t = -\sigma i_t + [\sigma \ 1-\beta \ -\sigma\beta] f_b + \sigma [1 \ 0 \ 0] (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \quad (\text{A.9})$$

$$\pi_t = \kappa x_t + [(1-\alpha)\beta \ \kappa\alpha\beta \ 0] f_a + [0 \ 0 \ 1] (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \quad (\text{A.10})$$

B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only, $k_t^{-1} = \mathbf{g}(f e_{t-1})$):

$$\begin{aligned} \pi_t &= -\frac{\lambda_x}{\kappa} \left\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_\pi(t) \right) \right. \\ &\quad \left. \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+j}^{-1} - (\pi_{t+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})) \mathbf{g}_{\bar{\pi}}(t+j) \right) \right\} \end{aligned} \quad (\text{B.1})$$

where I'm using the notation that $\prod_{j=0}^0 \equiv 1$. For interpretation purposes, let me rewrite this as follows:

$$\begin{aligned} \pi_t &= -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x (1-\alpha)\beta}{\kappa (1-\alpha\beta)} \left(k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ &\quad - \frac{\lambda_x (1-\alpha)\beta}{\kappa (1-\alpha\beta)} \left(k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+j}^{-1} + f e_{t+j|t+j}^{eve} \mathbf{g}_{\bar{\pi}}(t+j)) \right) \end{aligned} \quad (\text{B.2})$$

Interpretation: tradeoffs from discretion in RE + effect of current level and change of the gain on future tradeoffs + effect of future expected levels and changes of the gain on future tradeoffs