Materials for Susanto - IRFs for learning

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Overview

1 Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \rho i_{t-1} + \bar{i}_t \tag{3}$$

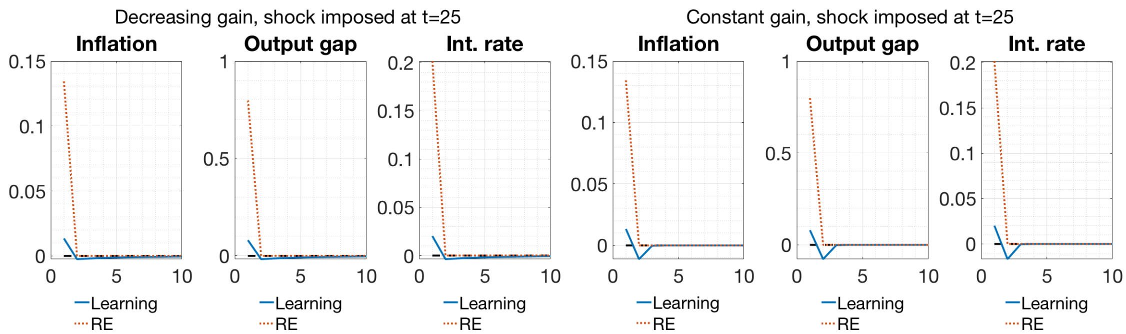
$$\hat{\mathbb{E}}_t z_{t+h} = \begin{bmatrix} \bar{\pi}_{t-1} \\ 0 \\ 0 \end{bmatrix} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\ h_x \qquad \text{PLM} \tag{4}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \underbrace{\big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big)}_{\text{fcst error using (4)}} \qquad (b_1 \text{ is the first row of } b) \tag{5}$$

$$k_t = \begin{cases} k_{t-1} + 1 & \text{for decreasing gain learning} \\ \bar{g}^{-1} & \text{for constant gain learning.} \end{cases} \tag{6}$$

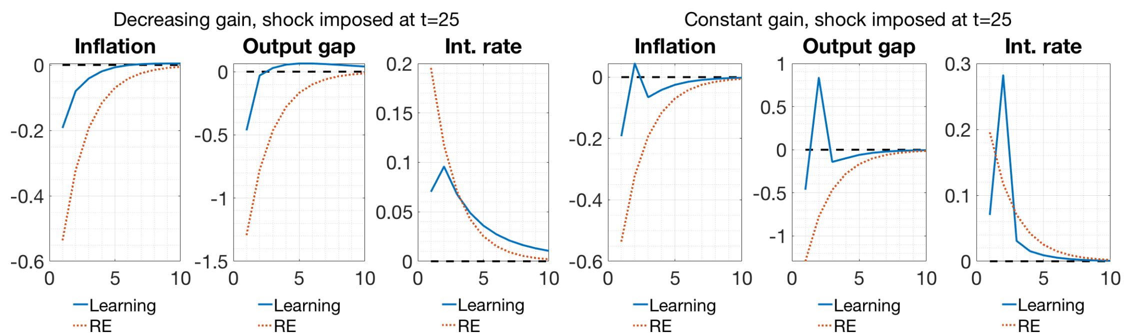
2 IRFs for 3 shocks - persistence and overshooting

Figure 1: Natural rate shock



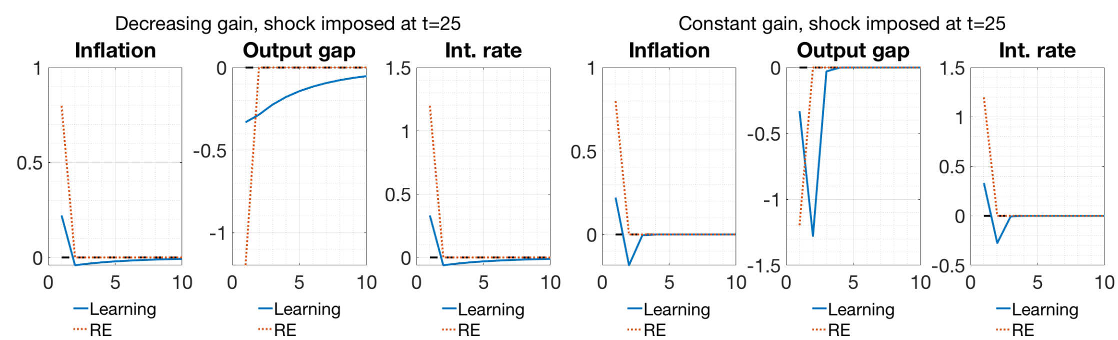
(a) Decreasing gain, $t = 25$ (b) Constant gain, $t = 25$

Figure 2: Monetary policy shock



(a) Decreasing gain, $t = 25$ (b) Constant gain, $t = 25$

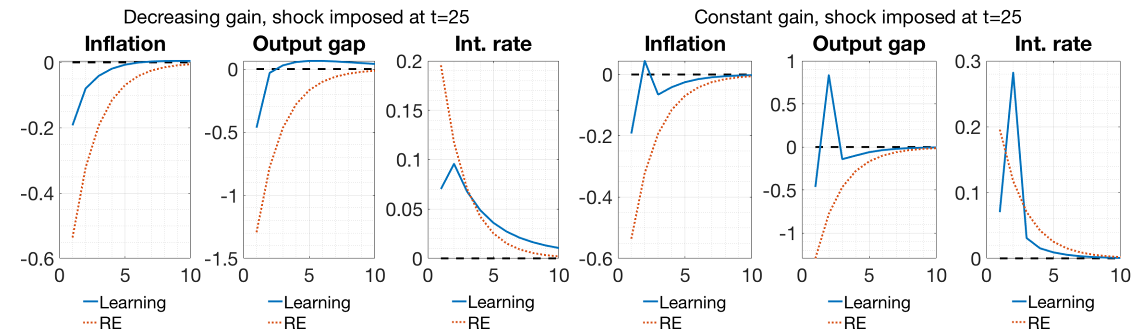
Figure 3: Cost-push shock



(a) Decreasing gain, $t = 25$ (b) Constant gain, $t = 25$

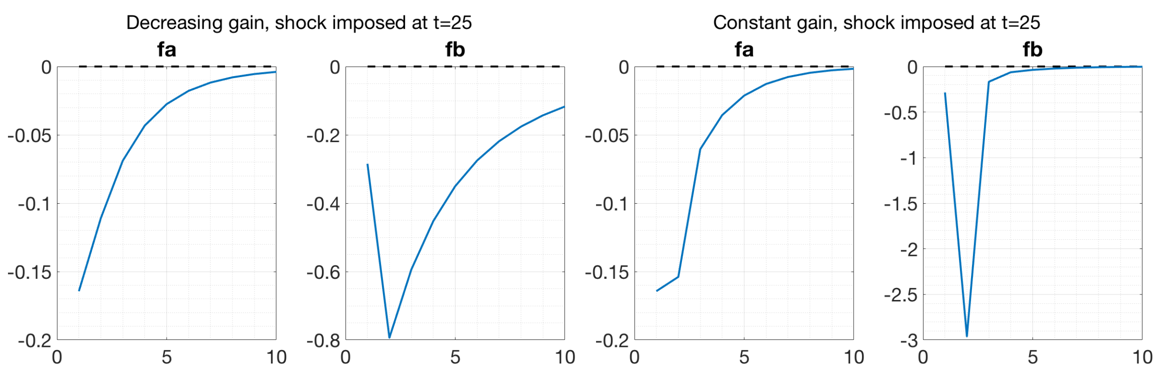
3 Observables and expectations for monetary policy shock - overshooting happens because of overadjustment of expectations

Figure 4: Monetary policy shock - observables



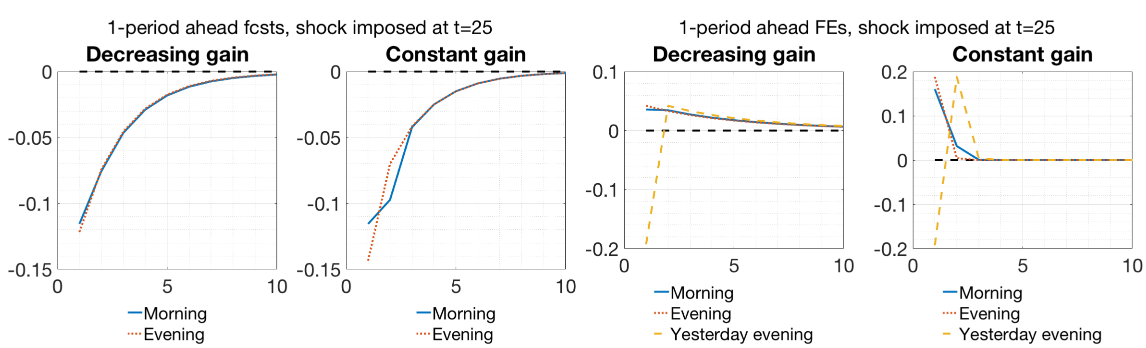
(a) Decreasing gain, $t = 25$ (b) Constant gain, $t = 25$

Figure 5: Monetary policy shock - long-horizon expectations



(a) Decreasing gain, $t = 25$ (b) Constant gain, $t = 25$

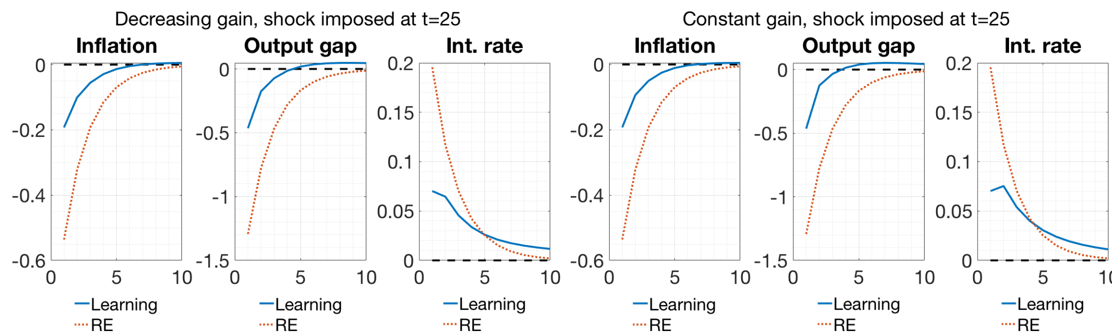
Figure 6: Monetary policy shock - 1-period ahead forecasts and forecast errors



(a) Decreasing gain, $t = 25$ (b) Constant gain, $t = 25$

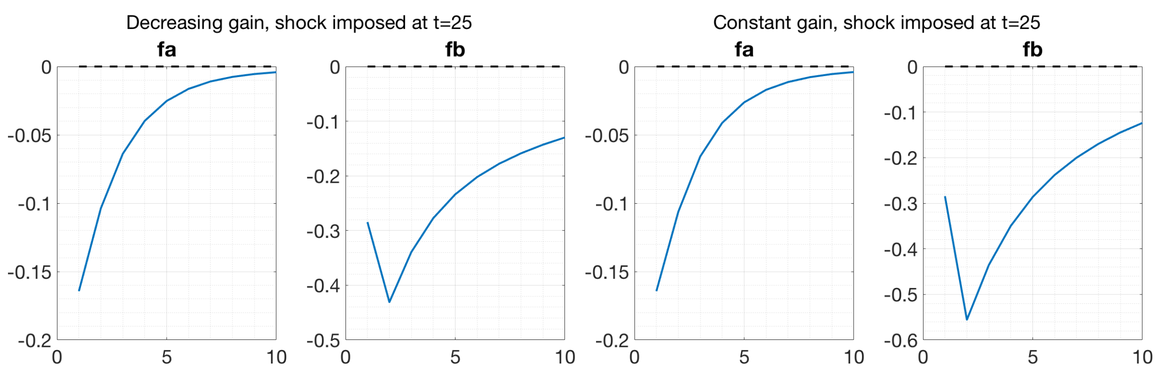
4 Observables and expectations for monetary policy shock - a lower gain ($\bar{g} = 0.02$) mitigates the overadjustment

Figure 7: Monetary policy shock - observables



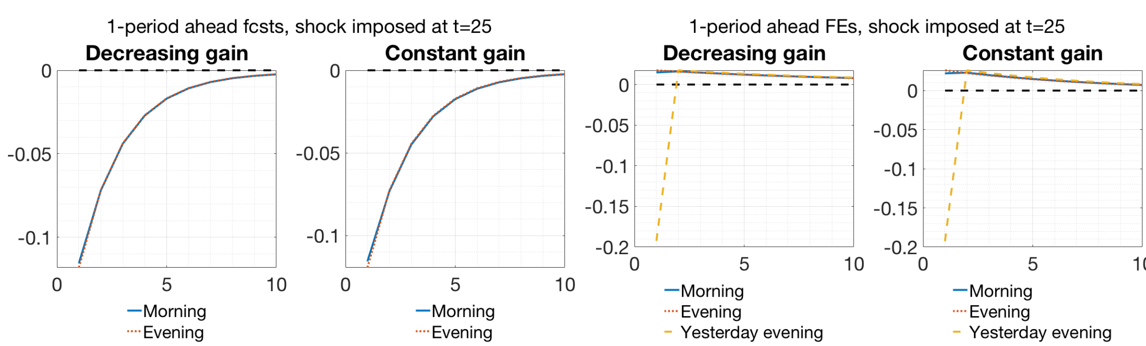
(a) Decreasing gain, $t = 25$ (b) Constant gain, $t = 25$

Figure 8: Monetary policy shock - long-horizon expectations



(a) Decreasing gain, $t = 25$ (b) Constant gain, $t = 25$

Figure 9: Monetary policy shock - 1-period ahead forecasts and forecast errors



(a) Decreasing gain, $t = 25$ (b) Constant gain, $t = 25$

5 Why is overshooting happening for learning and not for RE?

Ignoring shocks and setting $\psi_x = 0$, so the Taylor rule is just $i_t = \psi_\pi \pi_t$, the two systems are (throughout I'm using blue to denote negative values).

$$\begin{aligned}
&RE\\
&x_t = -\sigma\psi_\pi \pi_t + \mathbb{E}_t\, x_{t+1} + \sigma\, \mathbb{E}_t\, \pi_{t+1}\\
&\pi_t = \kappa x_t + \beta\, \mathbb{E}_t\, \pi_{t+1}
\end{aligned}$$

$$\begin{aligned}
&Learning\\
&x_t = -\sigma\psi_\pi \pi_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} + \sigma(1-\beta\psi_\pi)\pi_{T+1}\big)\\
&\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1}\big)
\end{aligned}$$

Expressing x, π as functions of expectations alone, this gives:

$$\begin{aligned}
&RE\\
&x_t = \frac{\sigma(1-\beta\psi_\pi)}{1+\sigma\psi_\pi\kappa}\, \mathbb{E}_t\, \pi_{t+1} + \frac{1}{1+\sigma\psi_\pi\kappa}\, \mathbb{E}_t\, x_{t+1}\\
&\pi_t = \overbrace{\left(\frac{\kappa\sigma(1-\beta\psi_\pi)}{1+\sigma\psi_\pi\kappa} + \beta\right)}^{+} \mathbb{E}_t\, \pi_{t+1} + \frac{\kappa}{1+\sigma\psi_\pi\kappa}\, \mathbb{E}_t\, x_{t+1}
\end{aligned}$$

$$\begin{aligned}
&Learning\\
&x_t = \frac{-\sigma\psi_\pi}{w}\begin{bmatrix}(1-\alpha)\beta & \kappa\alpha\beta & 0\end{bmatrix} f_a + \frac{1}{w}\begin{bmatrix}\sigma(1-\beta\psi_\pi) & 1-\beta & 0\end{bmatrix} f_b\\
&\pi_t = (1 - \frac{\kappa\sigma\psi_\pi}{w})\begin{bmatrix}(1-\alpha)\beta & \kappa\alpha\beta & 0\end{bmatrix} f_a + \frac{\kappa}{w}\begin{bmatrix}\sigma(1-\beta\psi_\pi) & 1-\beta & 0\end{bmatrix} f_b
\end{aligned}$$

This yields the stylized representation of how endogenous variables respond to expectations in the two formulations:

$$\begin{aligned}
&RE\\
&x_t = \overset{-}{\mathbb{E}(\pi)} + \overset{+}{\mathbb{E}(x)}\\
&\pi_t = \overset{+}{\mathbb{E}(\pi)} + \overset{+}{\mathbb{E}(x)}
\end{aligned}$$

$$\begin{aligned}
&Learning\\
&x_t = \overset{-}{\mathbb{E}_a(\pi)} + \overset{-}{\mathbb{E}_b(\pi)} + \overbrace{\overset{-}{\mathbb{E}_a(x)} + \overset{+}{\mathbb{E}_b(x)}}^{+\text{ since } f_a<f_b}\\
&\pi_t = \underbrace{\overset{+}{\mathbb{E}_a(\pi)} + \overset{-}{\mathbb{E}_b(\pi)}}_{+\text{ since } \kappa \text{ tiny}} + \overset{+}{\mathbb{E}_a(x)} + \overset{+}{\mathbb{E}_b(x)}
\end{aligned}$$

Where f_a and f_b denote long-horizon expectations and are given by

$$f_a(t) \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \begin{bmatrix} \pi_{T+1} \\ x_{T+1} \\ i_{T+1} \end{bmatrix} \qquad f_b(t) \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} \begin{bmatrix} \pi_{T+1} \\ x_{T+1} \\ i_{T+1} \end{bmatrix} \tag{7}$$

$$f_a(t) = \frac{1}{1-\alpha\beta} \begin{bmatrix} \bar{\pi}_t \\ 0 \\ 0 \end{bmatrix} + b(I_4 - \alpha\beta h_x)^{-1} s_t \qquad f_b(t) = \frac{1}{1-\beta} \begin{bmatrix} \bar{\pi}_t \\ 0 \\ 0 \end{bmatrix} + b(I_4 - \beta h_x)^{-1} s_t \tag{8}$$

(And $b = g_x h_x$, where h_x is the state transition matrix and g_x is the observation matrix from the RE model solution.)

6 Coefficient on inflation in the Taylor rule (ψ_π)

Current variables depend negatively on expected future ones through $1 - \beta\psi_\pi$ and $-\sigma\psi_\pi$.

$-\sigma\psi_\pi$ is always negative, though smaller in absolute value as $\psi_\pi \downarrow$.

$1 - \beta\psi_\pi > 0$ if $\quad \psi_\pi < 1/\beta$. For $\beta = 0.99 \rightarrow \psi_\pi < 1.0101$, or for $\beta = 0.98 \rightarrow \psi_\pi < 1.0204$.
$\rightarrow$ Monetary policy needs to be more passive on inflation in order to limit overshooting.