Materials 34 - Still estimating the anchoring function

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1 Estimation procedure

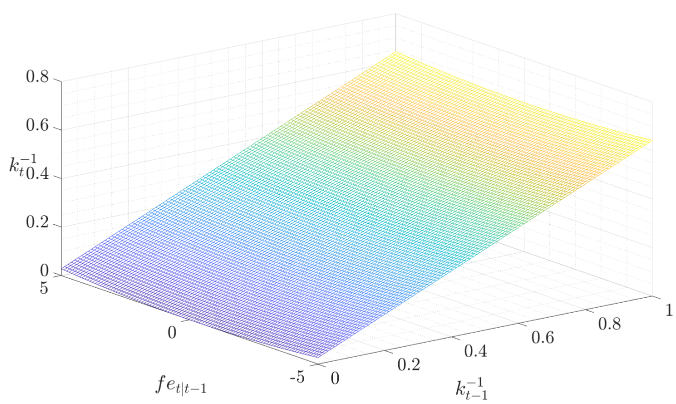
Instead of the AR(1) anchoring function used so far (Equation A.6), I use the following equation

$$k_t^{-1} = \alpha s(X) \tag{1}$$

where $X = (k_{t-1}^{-1}, fe_{t|t-1})$ and I use piecewise linear interpolation. I initialize α_0 by specifying a grid for X, passing the grid through Equation (A.6) to generate k_t^{-1}-values, and approximating by fitting the grid to the k_t^{-1}-values. See Fig. 1.

Then I estimate α using GMM, targeting the autocovariance structure of inflation, the output gap and the nominal interest rate (federal funds rate) in the data.

Figure 1: Initialization via Equation (A.6) implies this functional relationship

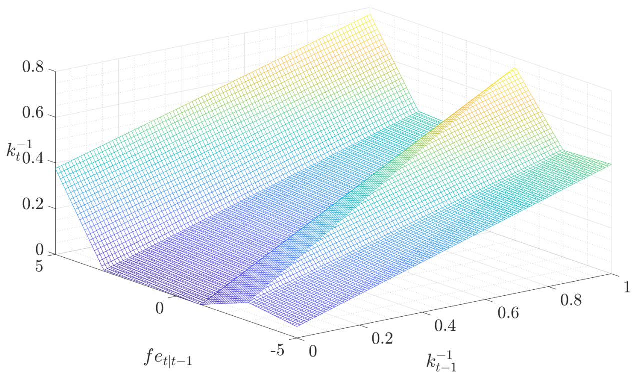


$T = 233$ before BK-filtering, $T = 209$ after BK-filtering. Using the "constant-only, inflation-only" learning PLM. I drop the $ndrop = 5$ initial values. I restrict $\alpha \in (0, 1)$, the support of k^{-1} in the grid. I target the lag $0, \dots, 4$ autocovariance matrices, dropping repeated entries at lag 0, leaving me with 42 moments.

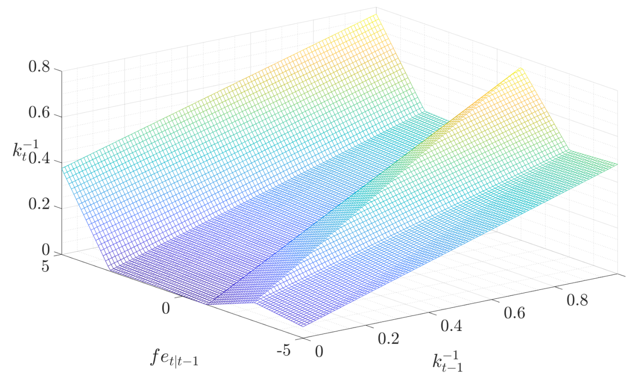
2 Estimating the 2D anchoring function

2.1 Real data, 2D function

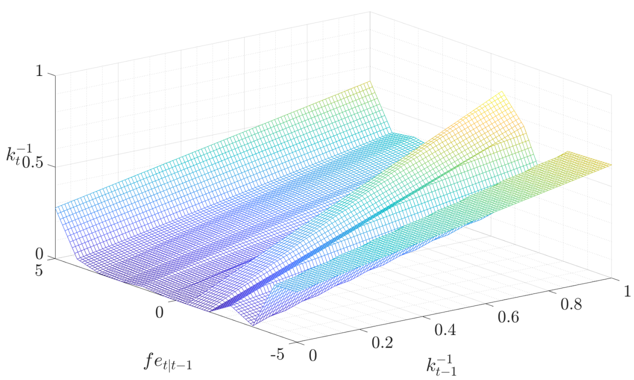
Figure 2: k_t^{-1} as a function of k_{t-1}^{-1} and $fe_{t|t-1}$ given $\hat{\alpha}^{GMM} \in (0,1)$. The dimension of the gain-grid is 2.



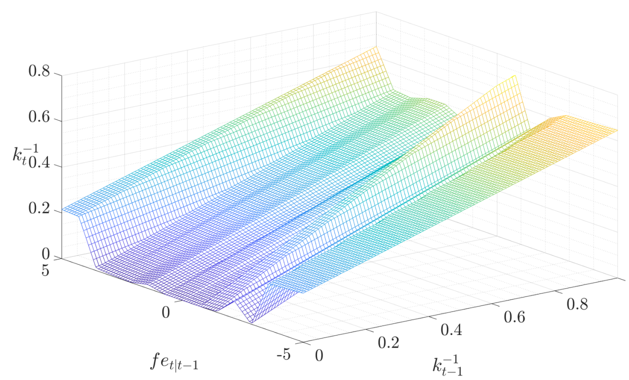
(a) $n_{fe-grid} = 6$



(b) $n_{fe-grid} = 9$



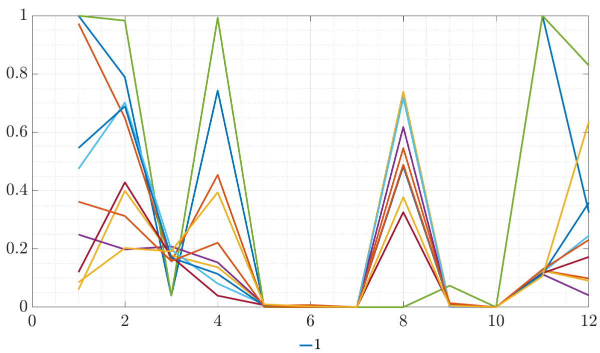
(c) $n_{grid} = 12$



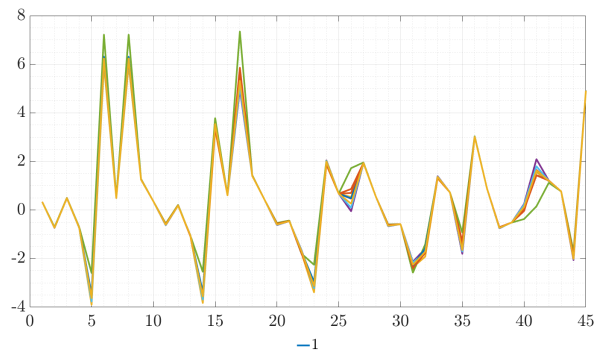
(d) $n_{fe-grid} = 15$

This actually looks quite robust i) to the gridsize, ii) to starting from random points. All I have done at this point is to increase the support of the gain, decrease its gridpoints to 2, increase the forecast error gridpoints, and implement `lsqnonlin`. Increasing the support of the gain also had the feature that now there are no explosions. So all these measures seem to have been beneficial.

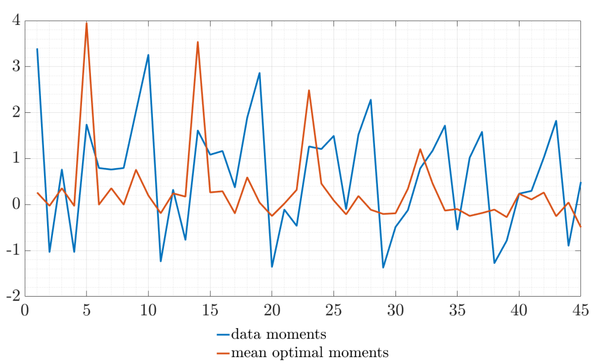
Figure 3: $\hat{\alpha}$ for various random starting points



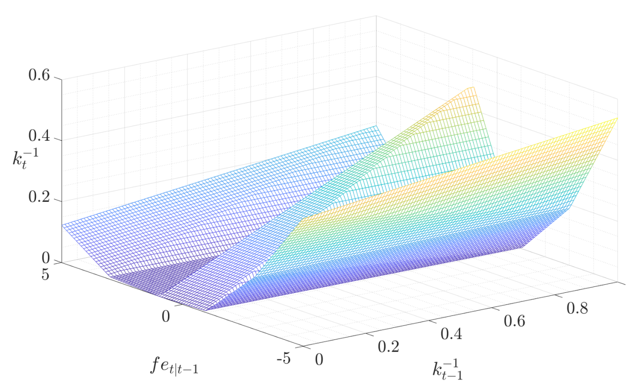
(a) $\hat{\alpha}$



(b) Residuals



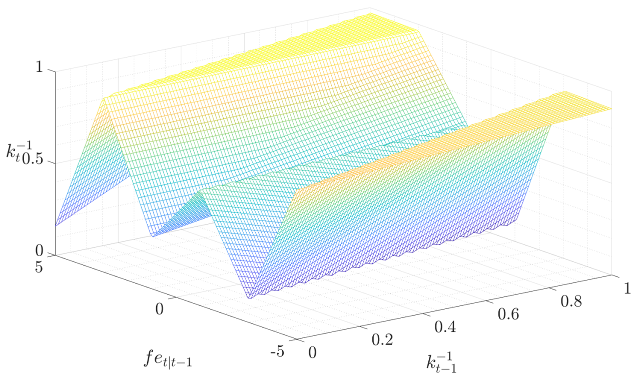
(c) Moments



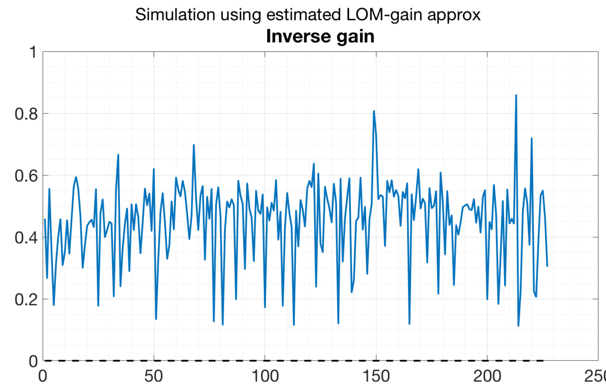
(d) Implied relationship at median estimate

2.2 Simulated data, 2D function

Figure 4: The truth using 2 gridpoints on $[0, 1]$ for the gain, 6 on $[-5, 5]$ for the forecast error

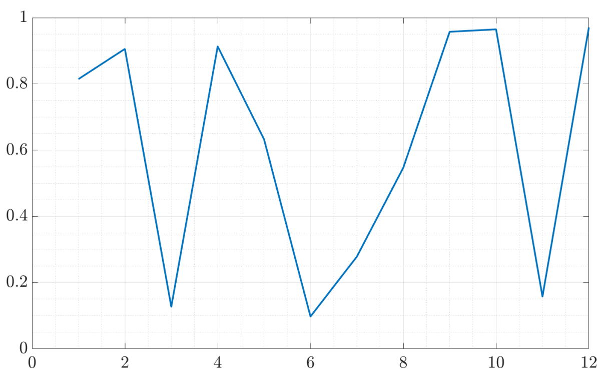


(a) k_t^{-1} as a function of k_{t-1}^{-1} and $fe_{t|t-1}$ given α^{true}

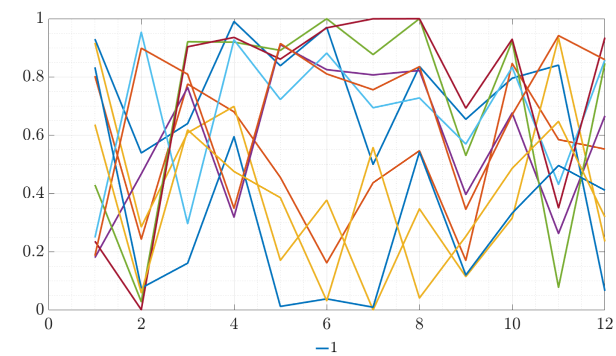


(b) A simulated history of k_t^{-1} given α^{true}

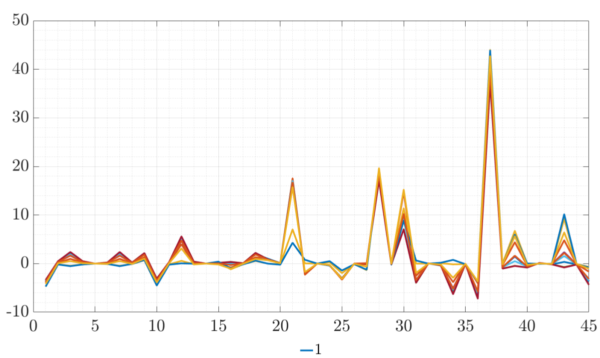
Figure 5: $\hat{\alpha}$ for various random starting points



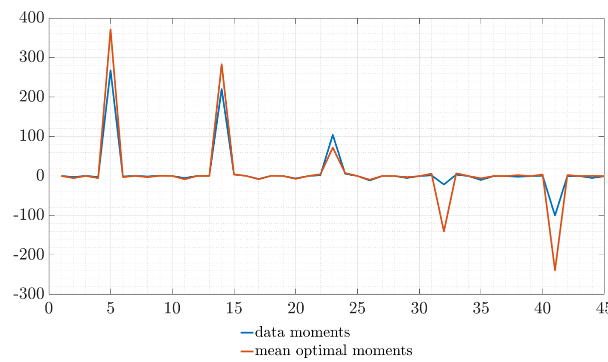
(a) α^{true}



(b) $\hat{\alpha}$



(c) Residuals

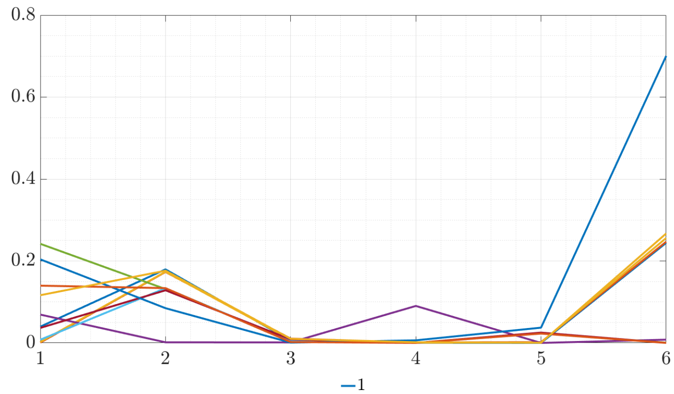


(d) Moments

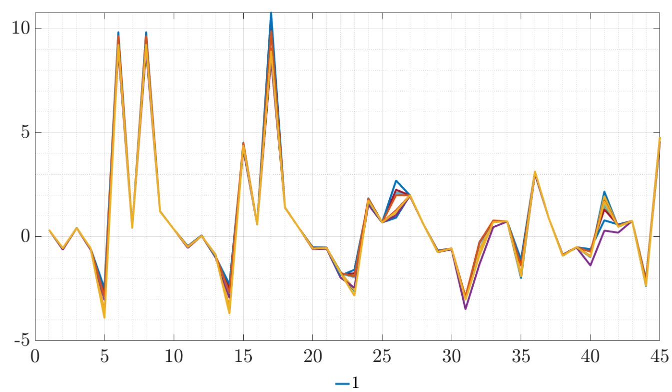
3 Univariate anchoring function

3.1 Real data, 1D function

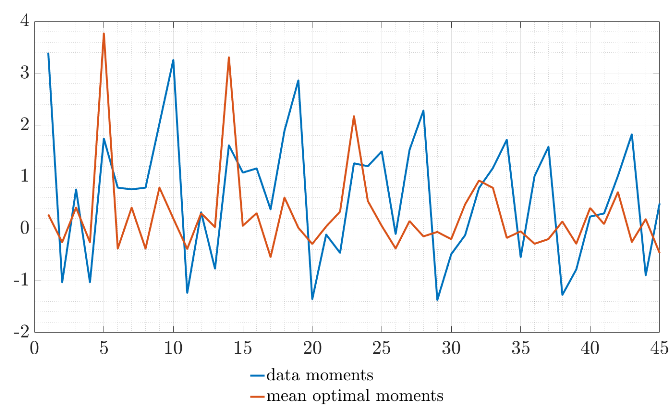
Figure 6: $\hat{\alpha}$ for various random starting points



(a) $\hat{\alpha}$



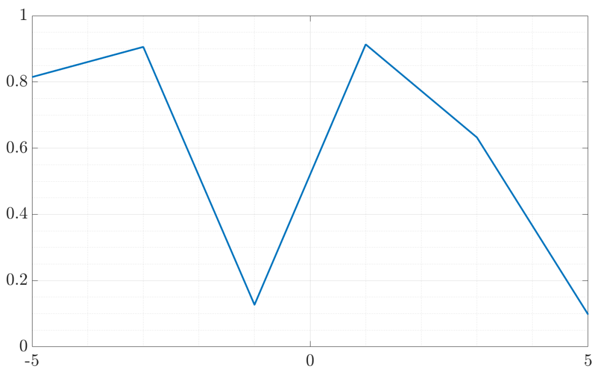
(b) Residuals



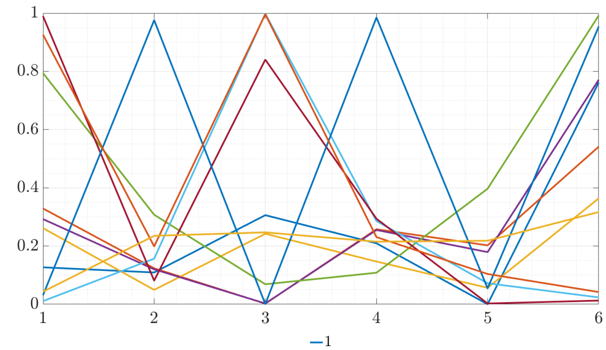
(c) Moments

3.2 Simulated data, 1D function

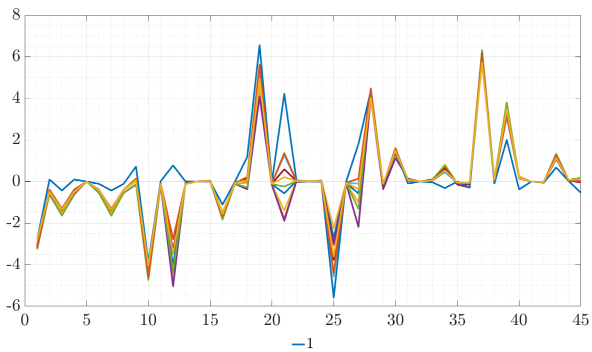
Figure 7: $\hat{\alpha}$ for various random starting points



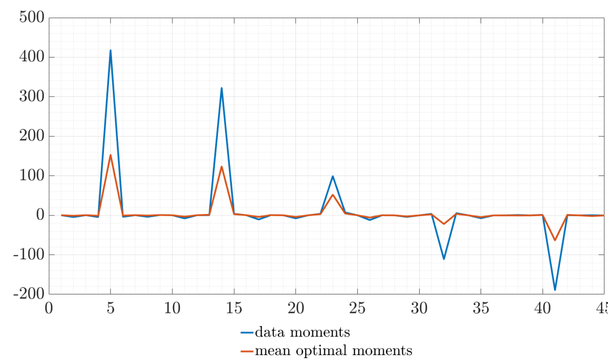
(a) α^{true}



(b) $\hat{\alpha}$



(c) Residuals

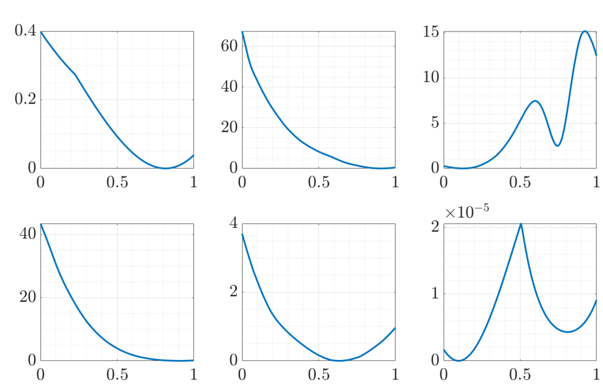


(d) Moments

4 Investigate loss function for 1D case in detail

4.1 Loss changing one parameter at a time

Figure 8: Objective function when I hold all coefficients at α^{true}, move one at a time from 0 to 1



These minima correspond to the true values.

A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{A.1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{A.2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \qquad \text{(if imposed)} \tag{A.3}$$

$$\text{PLM:} \qquad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\, h_x \tag{A.4}$$

$$\text{Updating:} \qquad a_t = a_{t-1} + k_t^{-1}\big(z_t - (a_{t-1} + b s_{t-1})\big) \tag{A.5}$$

$$\text{Anchoring function:} \qquad k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k f e_{t-1}^2 \tag{A.6}$$

$$\text{Forecast error:} \qquad f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \tag{A.7}$$

$$\text{LH expectations:} \qquad f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \qquad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \tag{A.8}$$

This notation captures vector learning (z learned) for intercept only. For scalar learning, $a_t = \begin{pmatrix}\bar{\pi}_t & 0 & 0\end{pmatrix}'$ and b_1 designates the first row of b. The observables (π, x) are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix}\sigma & 1-\beta & -\sigma\beta\end{bmatrix} f_b + \sigma \begin{bmatrix}1 & 0 & 0\end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \tag{A.9}$$

$$\pi_t = \kappa x_t + \begin{bmatrix}(1-\alpha)\beta & \kappa\alpha\beta & 0\end{bmatrix} f_a + \begin{bmatrix}0 & 0 & 1\end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \tag{A.10}$$

B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only, $k_t^{-1} = \mathbf{g}(f e_{t-1})$):

$$\pi_t = -\frac{\lambda_x}{\kappa}\Bigg\{x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_\pi(t)\Big) \\ \Bigg(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi}}(t+j))\Bigg)\Bigg\} \tag{B.1}$$

where I'm using the notation that $\prod_{j=0}^{0} \equiv 1$. For interpretation purposes, let me rewrite this as follows:

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + f e_{t|t-1}^{eve}\mathbf{g}_\pi(t)\Big)\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ -\frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + f e_{t|t-1}^{eve}\mathbf{g}_\pi(t)\Big)\Bigg(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(k_{t+1+j}^{-1} + f e_{t+1+j|t+j}^{eve})\mathbf{g}_{\bar{\pi}}(t+j)\Bigg) \tag{B.2}$$

Interpretation: tradeoffs from discretion in RE + effect of current level and change of the gain on future tradeoffs + effect of future expected levels and changes of the gain on future tradeoffs