

Materials 12f - “il”-extension of baseline model - Interest rate smoothing, hopefully done right

See Notes 6 Jan 2020

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Compare Mathematica (`materials12f3.nb`).

Blue stuff are changes compared to the baseline model.

1 Model equations

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (1)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (2)$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \textcolor{blue}{\rho i_{t-1}} \quad (3)$$

Compact notation

$$z_t = \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = A_a f_a + A_b f_b + A_s s_t \quad \text{with} \quad s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{bmatrix} \quad (4)$$

2 MN matrices

$$\underbrace{\begin{bmatrix} \sigma\psi_\pi & 1 + \sigma\psi_x \\ 1 & -\kappa \end{bmatrix}}_{\equiv M} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \underbrace{\begin{bmatrix} \left[\sigma(1-\beta\psi_\pi), & 1-\beta-\sigma\beta\psi_x, & 0 \right] f_b + d_{x,s} s_t \\ \left[(1-\alpha)\beta, & \kappa\alpha\beta, & 0 \right] f_a + d_{\pi,s} s_t \end{bmatrix}}_{\equiv N} \quad (5)$$

where

$$d_{x,s} = -\sigma \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} \text{InxBhx} \quad \text{InxBhx} \equiv (I_{nx} - \beta h_x)^{-1} \quad (6)$$

$$d_{\pi,s} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \text{InxABhx} \quad \text{InxABhx} \equiv (I_{nx} - \alpha \beta h_x)^{-1} \quad (7)$$

$$d_{i,s} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \quad (8)$$

3 PQ matrices

$$\underbrace{\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix}}_{\equiv P} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \underbrace{\begin{bmatrix} \begin{bmatrix} \sigma, & 1 - \beta, & \beta(-\sigma) \end{bmatrix} f_b + c_{x,s} s_t \\ \begin{bmatrix} (1 - \alpha)\beta, & \alpha\beta\kappa, & 0 \end{bmatrix} f_a + c_{\pi,s} s_t \\ c_{i,s} s_t \end{bmatrix}}_{\equiv Q} \quad (9)$$

where

$$c_{x,s} = \sigma \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \cdot \text{InxBhx}; \quad (10)$$

$$c_{\pi,s} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \cdot \text{InxABhx} \quad (11)$$

$$c_{i,s} = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} = d_{i,s} \quad (12)$$

where InxABhx and InxBhx are the same as before. The (*)-relation is

$$f_b(3) = \psi_\pi f_b(1) + \psi_x f_b(2) + \frac{1}{\beta} \{ \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} (I_{nx} - \beta h_x)^{-1} s_t - \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} s_t \} \quad (*)$$

The Matlab code that uses this is `matrices_A_12f3.m`.