

NEXT

26 Aug 2019

see Experimentation Notes 21 August 2019 Ryan Meeting

Benhabib, Schmitt-Grohé & Uribe (1999)

- challenges the conventional notion that active pol. (non-pol that responds more than 1:1 to π) is stabilizing \rightarrow it is only stable (unique) in a "very" local neighbourhood of the st. st.
 $\rightarrow \pi$ fluctuates around its "stable" value for a while before converging to the passive st. st.
 \hookrightarrow makes emerging lq. traps hard to detect!

Needs AB.

\hookrightarrow Is this a feature of learning models?

I said in the meeting that "2B episode should never have happened if beliefs were anchored"

→ I meant that $\pi = E(\pi)$, (kind of) and if $E(\pi) = 3\%$, π could never have gone down so low as to warrant $i=0\%$.

→ Ryan said something like what if we were in the low- π period just T periods too short, so that if it had persisted T more periods, expectations had become unanchored and we'd never have gotten out?

2 options for Bernheim et al (1997):

either learning models rationalize multiplicity for active policy

or learning models offer a different explanation for the slide into big traps

Davig & Keepr (2007)

Markov-process for Taylor-rule parameters

expectations-effects: even if you're in an active regime, if ppl. expect that you may switch, macro volatility ↑

You can get indeterminacy if passive regime is

- a) sufficiently permanent
- b) or sufficiently passive

(This is in spirit like unanchored E(·)).

⇒ Regime-switching increases the local determinacy region

b/c you can "store up on" hawtiness so you have more allowed "dovishness credit"

↳ regime-change on the policy-side

vs. CEMP: regime-change on the learning-side

Interpretations of US mon. history:

27 Aug 2019

- Bernanke et al (1995):

It's not actually active, but it's spiraling into
a lig. trap (:-)

- Davig & Keefer (2007)

The 70's wasn't indeterminate b/c the LRTP
(LR-Taylor principle) holds, but there were
large shocks that were amplified by policy.

- CAMP

The 70's was due to large gains.

In "Limits to Mon. Pol" C & P & Giannoni

also argue that the gain was large mainly due to
loose policy in the 70's (less due to shocks)

Inflation targeting countries:

New Zealand (1990)

Canada (1991)

UK (1992) ($RPI \approx 2.5\% \rightarrow CPI \approx 2\% \text{ since 2003}$)

SWE (1993 announced, applied 1995)

ECB 2%

US (2012) (2% PCE)

→ in the data, what I see is that CPI inflation came
≈ 5 yrs after the intro of a targeting.

Let's add that Svensson is essentially saying
that the "LR-PC becomes non-vertical when $\text{infl} \cdot E(\cdot)$
are anchored" i.e. he's saying that the $U - \pi$
tradeoff smirks into the LR then
→ money non-neutrality in the LR.

CEMP-view of π -development:

$$\pi_t = E^{LR}(\bar{\pi}) + \text{shocks}$$

$\uparrow_{MC \uparrow \text{mon. pol}}$

Svensson is saying

$$\pi_t = \frac{E^{LR}(\bar{\pi})}{\uparrow_{MP \text{ didn't do enough!}}}$$

+ shocks < what it should be

→ so when $E(\cdot)$ anchored, MP very strong in terms of output gap / unemployment control.

$$T_t = E^{LR}(\pi_t - \pi^{\text{target}}) + \text{shocks}$$

when unanchored, E^{LR} responds a lot to missing the target → overshooting gets amplified (butterfly)

⇒ self-referentiality

⇒ self-referentiality seems to make money neutral faster (the LR arrives quicker / earlier)

⇒ unanchored makes MP weaker: bigger interventions are needed.

→ the "Swanson scenario" doesn't describe actual US monpol well, though:

$$\cdot \underline{\pi < \bar{\pi} \text{ while } u < u^*}$$

Or does it? Can we think of a story in which

$$\pi_t = E^{LR}(\pi_t - \bar{\pi}) + \text{shocks} \quad (\text{unanchored})$$

$\textcircled{2}\uparrow\textcircled{1}\textcircled{5}$ $\textcircled{3}\uparrow$ $\textcircled{1}\uparrow$

is

$$\pi_t = E^{LR}(\bar{\pi}) + \text{shocks} \quad (\text{anchored})$$

$\textcircled{2}\uparrow$ $\textcircled{1}\uparrow$

→ π doesn't increase as much while labor market effects are huge!

But this is where the "where are they anchored?" comes in:

$$1.5\% = \underbrace{E^{LR}(1.5\% - 2\%)}_{<0} + \text{shocks}$$

<0 but maybe not large or persistent enough for

expectations to adjust. → This story is harder

to tell if expectations are anchored at 3%,
a much higher level.

⇒ the question though also is "whose
expectation"?

→ the 3% may be lower if the app. econ's
expectations overshoot less than those of HMs.

But ok, at least I can rationalize Svensson's story
- and maybe just errors need to be very big
or very persistent for expectations to become
unanchored.

⇒ and I've also rationalized why the ECB
was scared of unanchoring of expectations
during the crisis:

- spiral down
- loss of control

2LB

I agreed w/ myself that much larger MP shocks are necessary to move π_L if $E(\cdot)$ are not anchored.

→ it's poss. to get into 2LB w/ anchored beliefs by being unlucky:

$$\pi_t = E^L(\bar{\pi}) + \text{shocks} \downarrow$$

→ and getting out should be a lot harder (require bigger MP shocks) if beliefs become unanchored b/c as long as $\pi_t < \bar{\pi}$

⇒ $E^L < 0$ which pushes $\pi \downarrow$

What if they get anchored again at a lower $\pi = \bar{\pi}_0$?

Well then it's easy to get to $\pi = \bar{\pi}_0$ one-time, but it will take a persistent series of MP shocks to maintain that level unless you "unanchor"

beliefs in order to shift the anchor to the correct place → this seems to be an "overshoot-risky" thing.

Dinhe coincidence

no tradeoff b/wm output gap - & π stabilization

In CAMP there's no demand side

↳ well now after the Peter meeting it feels like the DC doesn't hold: Fed trades off π -exp(.) $\Rightarrow \pi$ vs. output in the SR!

Peter meeting

27 Aug. 2015

- diss-folls. 8 prez Oct 1.
- Preston
- new dir: a learning model take on std issues:
 - i) MFT targeting
 - ii) credibility
 - iii) effectiveness of MFT
 - iv) "anomalies" of Taylor rules

Benhabib, Schmidt-Grohé & Uribe (1995)

global stability of TR: 2 cgb $\begin{cases} \text{active} \\ \text{passive} \end{cases}$
→ fall into Liq. traps

Dang & Sleeper (2007)

LR-TP: expanded determinacy region when
monopol can be thought of regime-switching.

Preston

- Try a siml w/ only 1 source of randomness or no randomness at all (\bar{i} or r^n)
- - instead of a +

To Benhabib et al:

fed changes TR only around T_B

→ we follow a TR but when $i = 0\%$
we keep it at 0%.

↗ nonlinearity in TR (Benhabib et al)

vs. we switch to steady state at T_B
(regime-switching)

Blanchard:

In RE, the bad eqb is the attractor: how much
of that depends on RE ass? Would it be
worse/better w/ learning?

Beware: global analysis w/ learning might
be tough

Another: Comp-exp.

Compared to the case of RE → the CB may be
forced in some sense to deviate from a TR or

adopt a diff TR w/ diff components just to keep expectations anchored / maintain credibility

↳ tradeoff b/w tx-management & credibility

→ sweep in Leeper

std TR

↳ diff regime when unanchored

⇒ could be done using linear methods and simulation

"here's a MP rule that preserves credibility and here's a switching regime that gets the best of both worlds"

↳ and data (estimate using the gain-results from CEMP or you don't have to estimate the beast)

Ereng & Levin JME (2003)

Voldar-dissipation not explainable using RE
dynamics of (π, Y) cannot be explained
w/o a signal extraction on CB's target

Martin Goodfellow (1993) "Interest rate policy
and the Infl. Scare Problem"

Credibility of Fed was called into question

→ so when FOMC took actions that
weren't justified by TR it's b/c they
wanted to preserve credibility.

+ "whatever it takes" (Draghi)

"the Fed listens" Chicago conference

idea here (Powell & John Williams & Clarida)

"we don't wanna get into low $E(\cdot)$ "

"better act now than later"

Leeper, Preston, Margaret Jacobson

"Recovery of 1933"

what ended the depression of Great Depression
FDR took over and "I'm in charge of the Fed"
→ under those circumstances it's ok

Work after

Errey & Levin: a DSGE model in which agents try to disentangle permanent vs temporary shocks to the inflation target

↳ Evans & Wachtel (1993) : show using survey data that persistent fast errors aren't irrational, instead they reflect uncertainty on the regime

Goodfriend: inflation scare: when market-based TC-expectations jump up (here: the LR int rate), indicating low credibility of the Fed → Fed has to raise the FFR to indicate its commitment to low π & maintain credibility.

The main takeaway seems to be:

a diff take on US mon history (echoing Goodfriend's idea of an "inflation scare") is that under learning, there's a tradeoff b/w mon. objective and credibility → this can explain US mon history as well as the recent int. rate cut (July 2015) as signalling commitment to the 2% target.

→ Could demonstrate in an NK model that

- when anchored, a TR does fine
- when unanchored, a new rule does better

⇒ a hybrid rule that is regime-switching gets the "best of both worlds"

→ and this kind of behavior is what policy-makers are talking about.

⇒ would also shed light on the "flat NKPC" issue:

- when unanchored, not flat, but you fight to get anchoring
- when anchored, flat b/c $E(\pi)$ don't respond.

Let's return to the question

18 Aug 2019

of when monopol is powerful under learning:

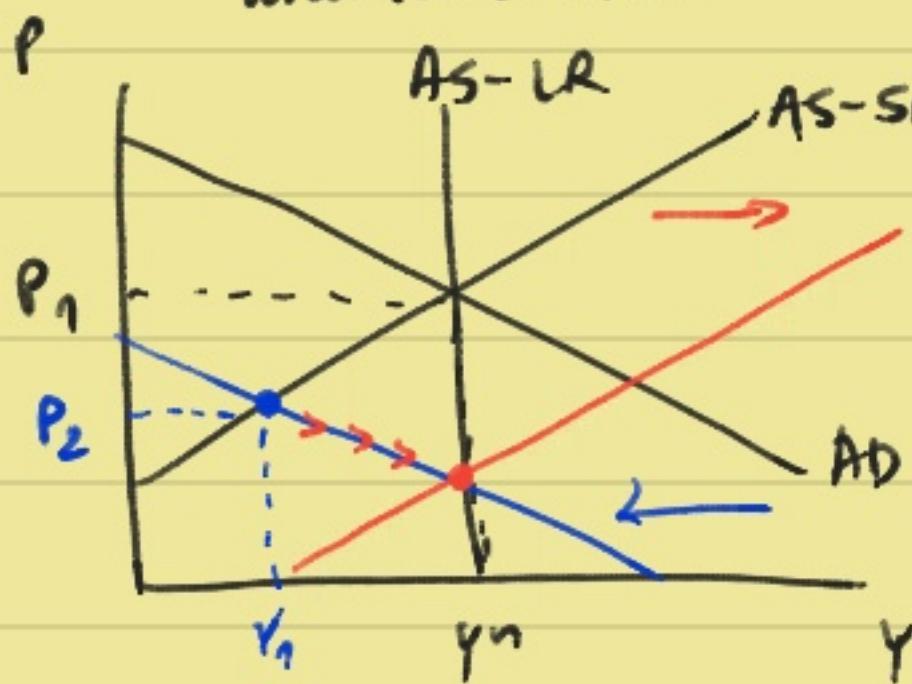
$$\pi_t = E^R(\pi_t - \bar{\pi}) + \text{shocks}$$

anchored: can't move expectations

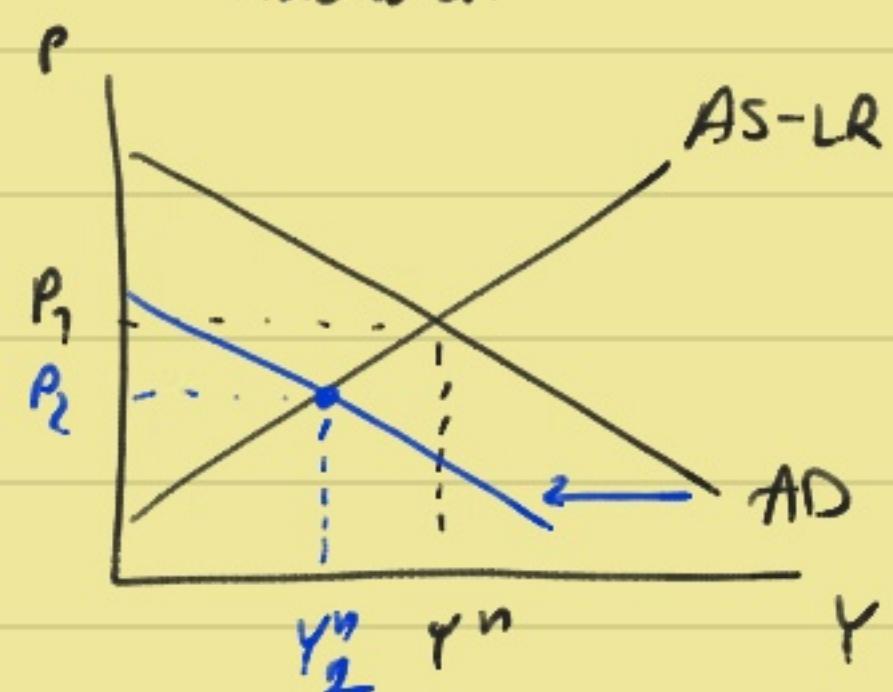
need bigger shocks to move π

(can be a blessing & a curse)

unanchored



Anchored



$$P_2 < E(P) \Rightarrow E(P) \downarrow$$

\Rightarrow AS-SR shifts R

Technically, if one had ∞ shocks one could keep the output gap open forever

\rightarrow but then they deanchor!

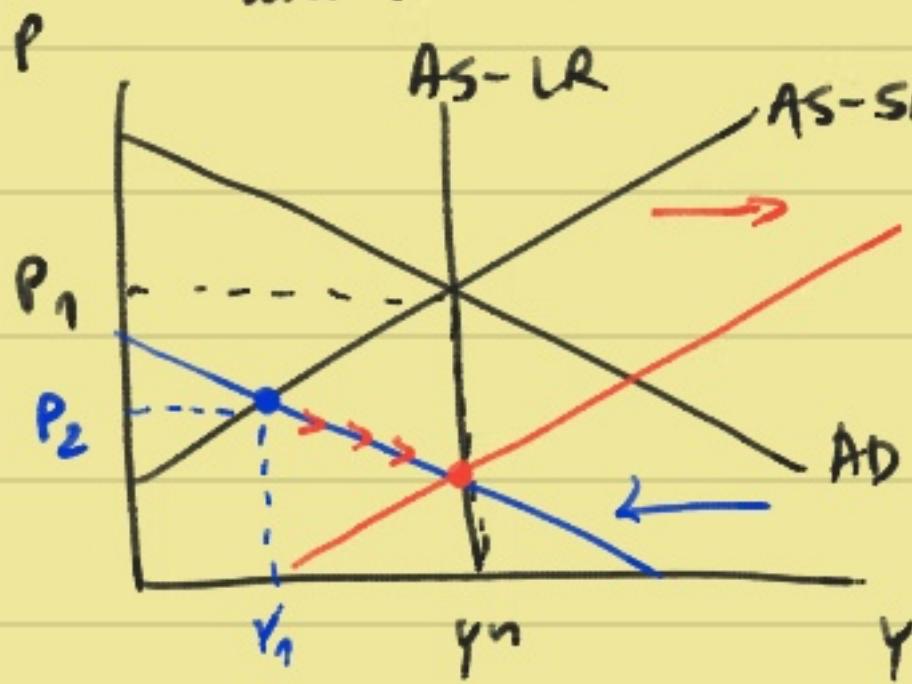
\Rightarrow so anchoring makes MP powerful b/c you can

- keep the output gap positive for a longer time

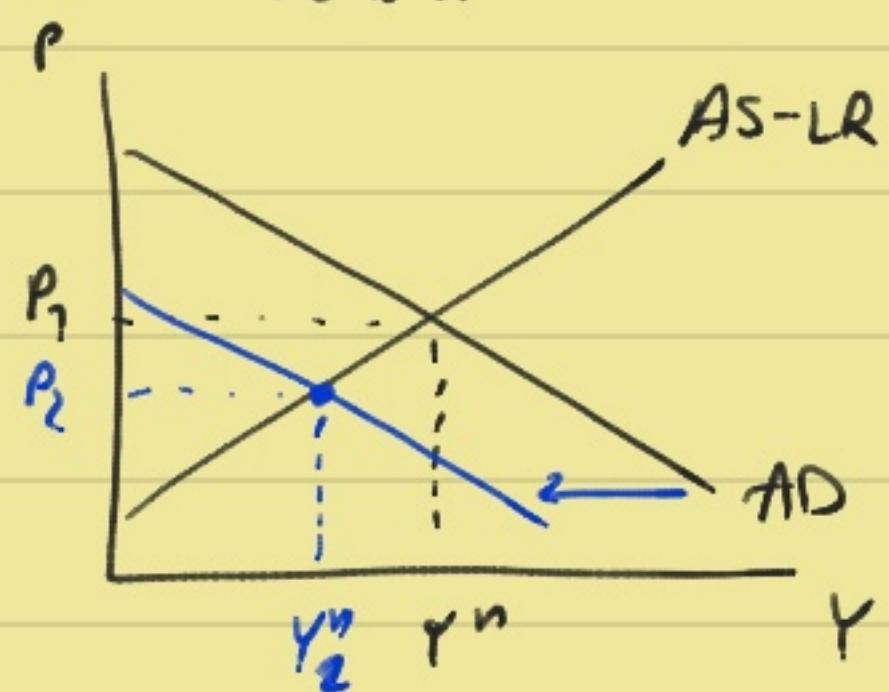
- shift AD back to smooth a crisis before the extra deflation occurs.

Let's look at this situation:

unanchored



Anchored

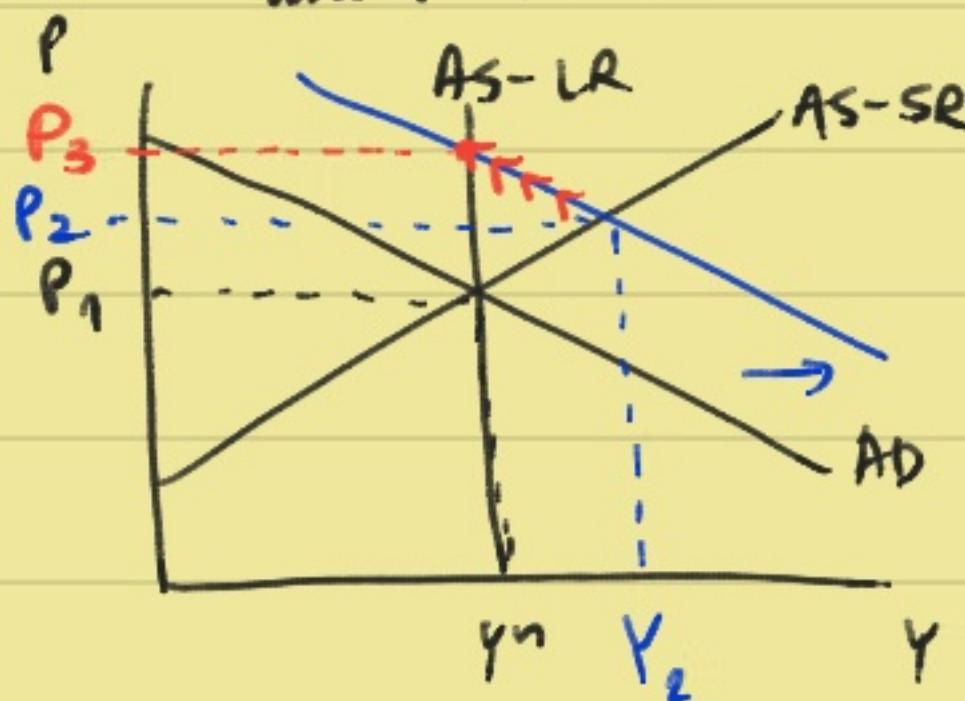


Already here being anchored is two-sided:

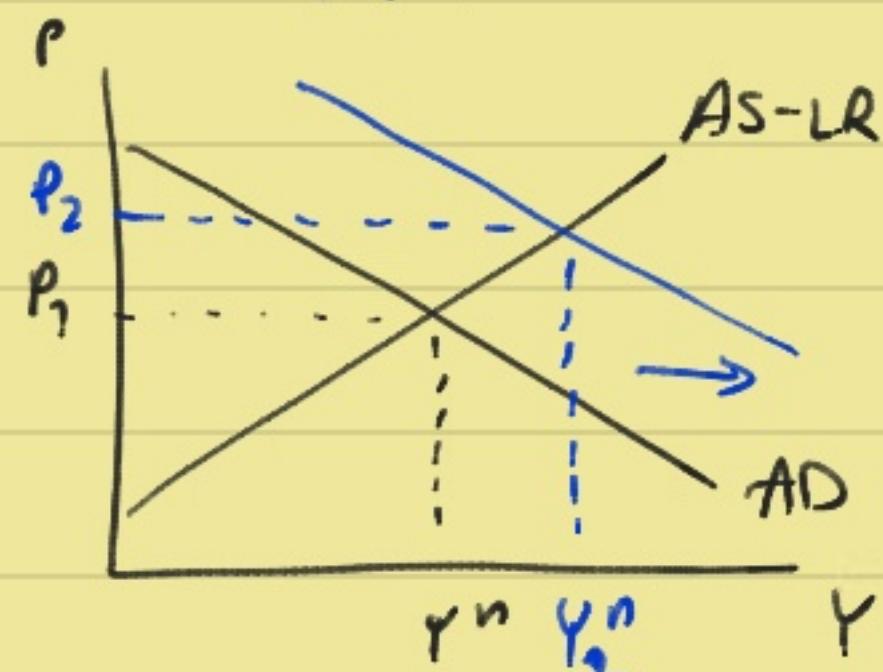
- Curse: in principle the unanchored econ would go back to a higher Y^* , lower u^* situation by itself.
- Blessing: prices stay higher. And given that now π^* needs to intervene to get back to target π^* (and implicitly u^*, Y^* too), it's better that prices haven't fallen as much. Also, if unanchored, adjustment is costlier (you need bigger shocks to move AD by the same amount)

Let's look at the reverse side: an inflation scare:

unanchored



Anchored



→ mon pol, when EC(.) anchored, would "persistently" push up output (open the output gap). The problem is that there is a threshold surprise, or a threshold length of time of consecutive surprises such that EC(.) deanchor \rightarrow you set off inflation (red)

\Rightarrow it is to keep that from happening that you raise int. rates and thus dampen momentum although you might not even have hit the bound \rightarrow credibility

Ryan meeting

28 Aug 2015

Ques: • Inflation-targeting countries (some mixed evidence)

• Bernanke's global \rightarrow diff.

↑ opposition

• Davig & Leeper related through to Evans & H (2003)

A synthesis: investigate the tradeoff between stabilisation & credibility \Rightarrow propose a modified TR under learning

Addresses:

- current US policy

- current talk of anchoring

- flat NKPC

- ZLB a bit \rightarrow a case for unconventional pol.

Doesn't address:

- divine coincidence (I don't know!)

- why $E()$ didn't become unanchored
in SWE / Riksbank undershooting

A confusion: anchoring doesn't mean in CAMP that
SR-Exp. don't move!

The actual meeting:

CB min & s.t. credibility stock

Try to do this:

- think through how a cost push shock works in an NK model
- do the same in an NK model w/ learning where the cost push shock affect π and thus also $E(\pi) \rightarrow$ what's diff?

Note: a cost push shock is a shock to derived markups
→ it increases the wedge between perfect competition and mon. comp., and then the flex. price isn't efficient (HC distortions). So we don't want to stabilize to fix prices b/c that outcome isn't optimal.

Work after

what are cost push shocks?

Ryan said: shocks to the desired level of the markup (μ)

$$\mu^* = \frac{b}{b-1} \quad \text{where } b = \text{el of switch between varieties}$$

Peter in his 2004 NBER WP "Tech Shocks..." indeed defn it as

$$\ln(b_t) = (1 - p_0) \ln(b) + p_0 \ln(b_{t-1}) + \varepsilon_{b,t}$$

→ i.e. π_t process of CEMP.

And this is what makes sense to me!

But Peter refers to Clarida, Gali & Gertler (1999), who define it as:

$$\text{NKPC: } \pi_t = \alpha x_t + \beta E_t \pi_{t+1} + u_t \quad (2.2)$$

which "captures anything else [than excess demand, which in turn is captured by the output gap, x_t] that might affect expected marginal costs."

or: u_t is the deviations from the condition $m_{cf} = Kx_t$.

Why are markups endogenous in a sticky prices world?

And let's see the condition $MC_t = \alpha x_t$

- $\hat{Y}_t \uparrow \rightarrow W&R \uparrow \rightarrow \hat{\varphi}_t (mc) \uparrow \Rightarrow \hat{\mu}_t \downarrow$
(blk prices today)

- and from Simon 2, p. 47 849, we have

$$\hat{\varphi}_t = \hat{MC}_t = (\gamma + b) \hat{x}_t$$

Further on p. 49:

target markup = $\frac{\theta}{\theta-1}$ (always constant)

vs. actual markup $\hat{\mu}_t = -\hat{\varphi}_t$

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at the same location, I find the following.

"From PS we see that sticky prices, that is variable MC or markups, means that $\hat{P}_t^* \neq \hat{P}_t$ at (!)"

and the wedge between those is MC:

$$\hat{P}_t^* - \hat{P}_t = \hat{Y}_t \quad (\text{PS})$$

I think the argument is saying like this:

w/ mon. competition, the PS is:

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta-1} y_t \quad (\text{PS, Basin sum Part 2, p. 47})$$

(already ass-ing symmetry b/w i & j)

Loglin gives:

$$\hat{P}_t^* - \hat{P}_t = \hat{Y}_t$$

With fix prices, $\frac{P_t^*}{P_t} = 1 \Rightarrow$ all firms get a price that's equal to the agg. price. $\Rightarrow \hat{P}_t^* - \hat{P}_t = 0 \Rightarrow \text{MC const!}$
2 reasons for confusion

↳ In Rotemberg, isn't $P_{it}^* = P_{jt}^* = P_t^*$?

↳ And it's fine that $\hat{P}_t^* - \hat{P}_t \neq 0$, but why does that change my cost structure?

↳ answer this might be ("Latro Once for All.pdf")

$$\xrightarrow{\text{MC}} \hat{s}_{it} = \hat{s}_t + \frac{1-\alpha}{\alpha} (\hat{y}_{it} - \hat{y}_t)$$

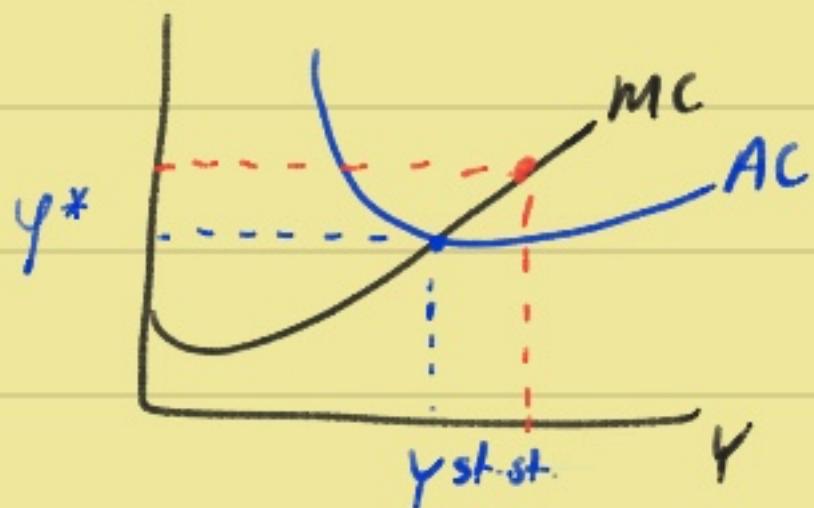
"my MC is that of everyone plus a demand term"

$$\text{the demand term tho is: } \hat{y}_{it} - \hat{y}_t = -\theta(\hat{P}_{it}^* - \hat{P}_t)$$

My demand is $\begin{cases} \text{how much cheaper my good is } (\hat{P}_{it}^* - \hat{P}_t) \\ \text{how much different the good is } (\theta) \end{cases}$

↳ but that brings me back to the same question: Why does this change my cost?

→ I think I see actually: Recall from micro that MC is upward-sloping:

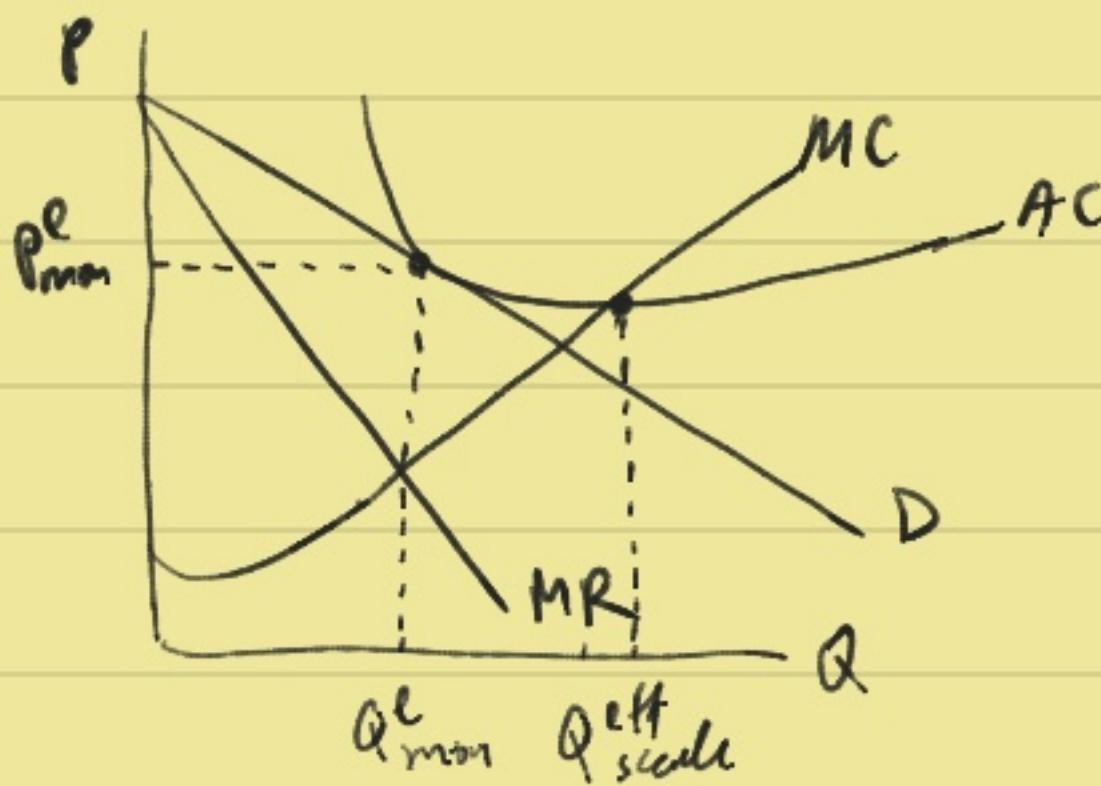


⇒ the higher the prod, the higher MC.

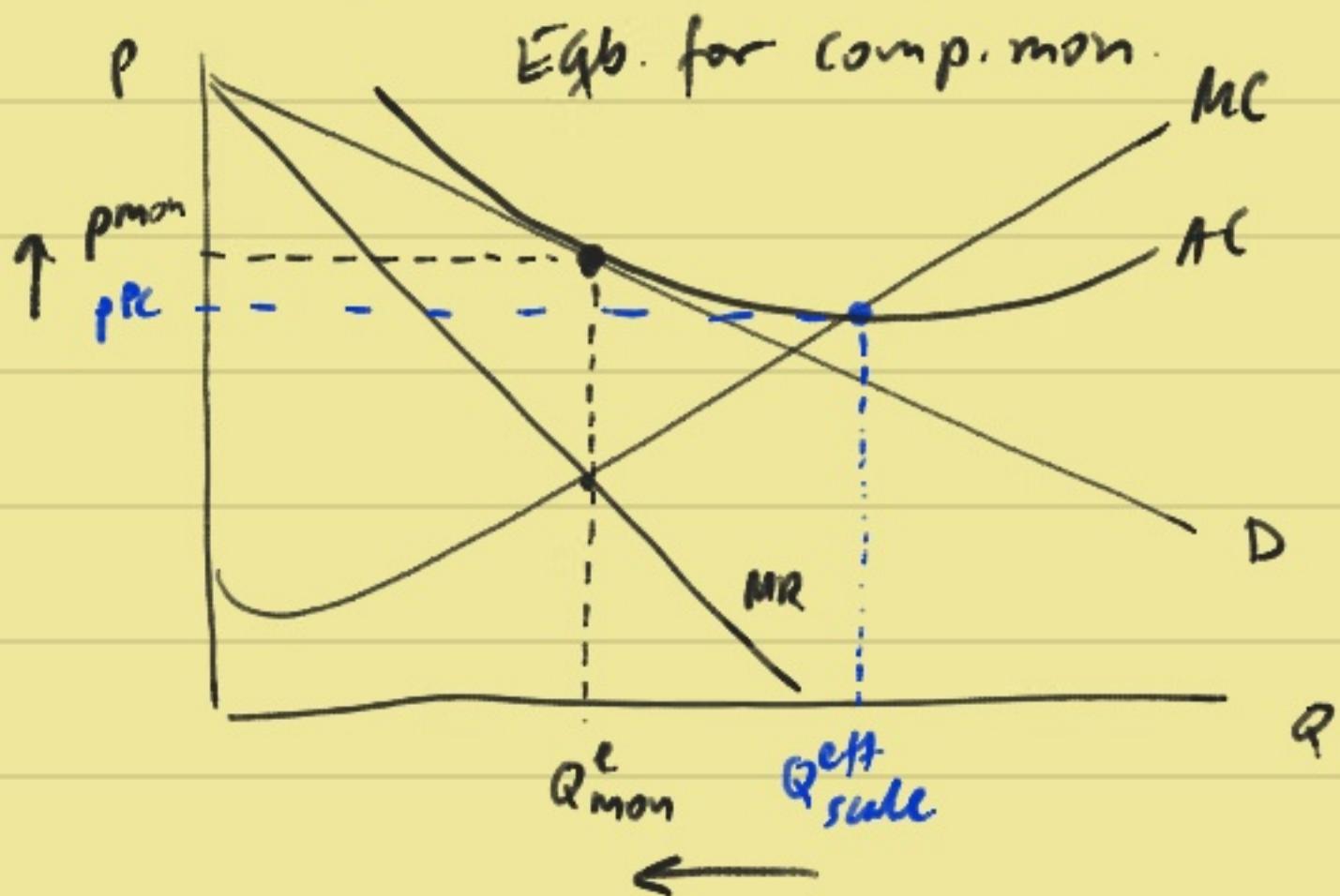
And that's a constant st. st. level.

But when $\hat{P}_{it}^* \neq \hat{P}_t$, as is the case in labor, this causes $Y_t \neq Y_t^*$, i.e. your demand isn't = to the agg., and so your level of production isn't either.
If your prod. level exceeds the agg., so will your MC.

Let's go back to micro principles: competitive mcr.



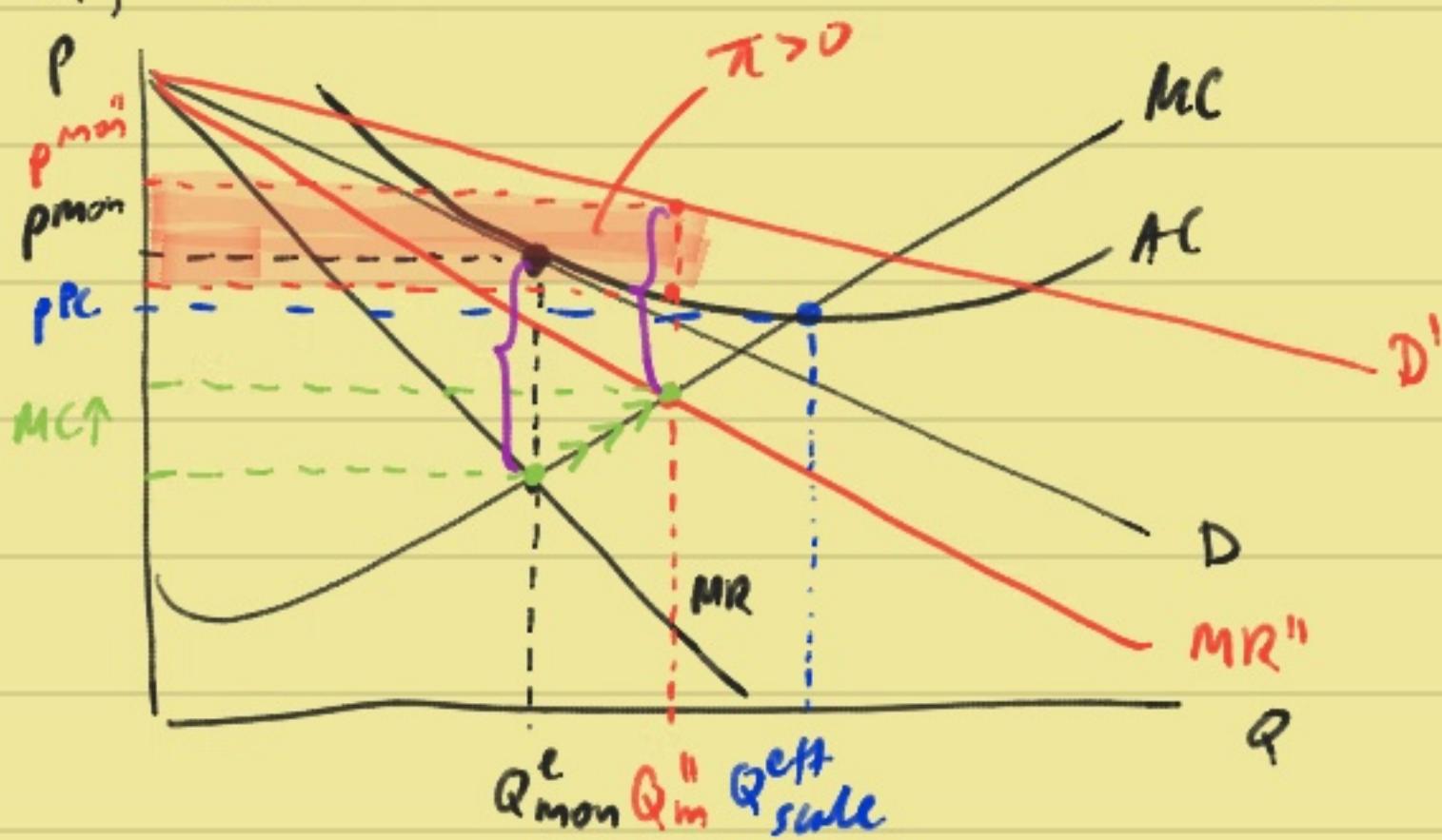
Let's compare that w/ perfect comp.



$(p^{\text{mon}}, Q^{\text{mon}})$ is the st. st. value.

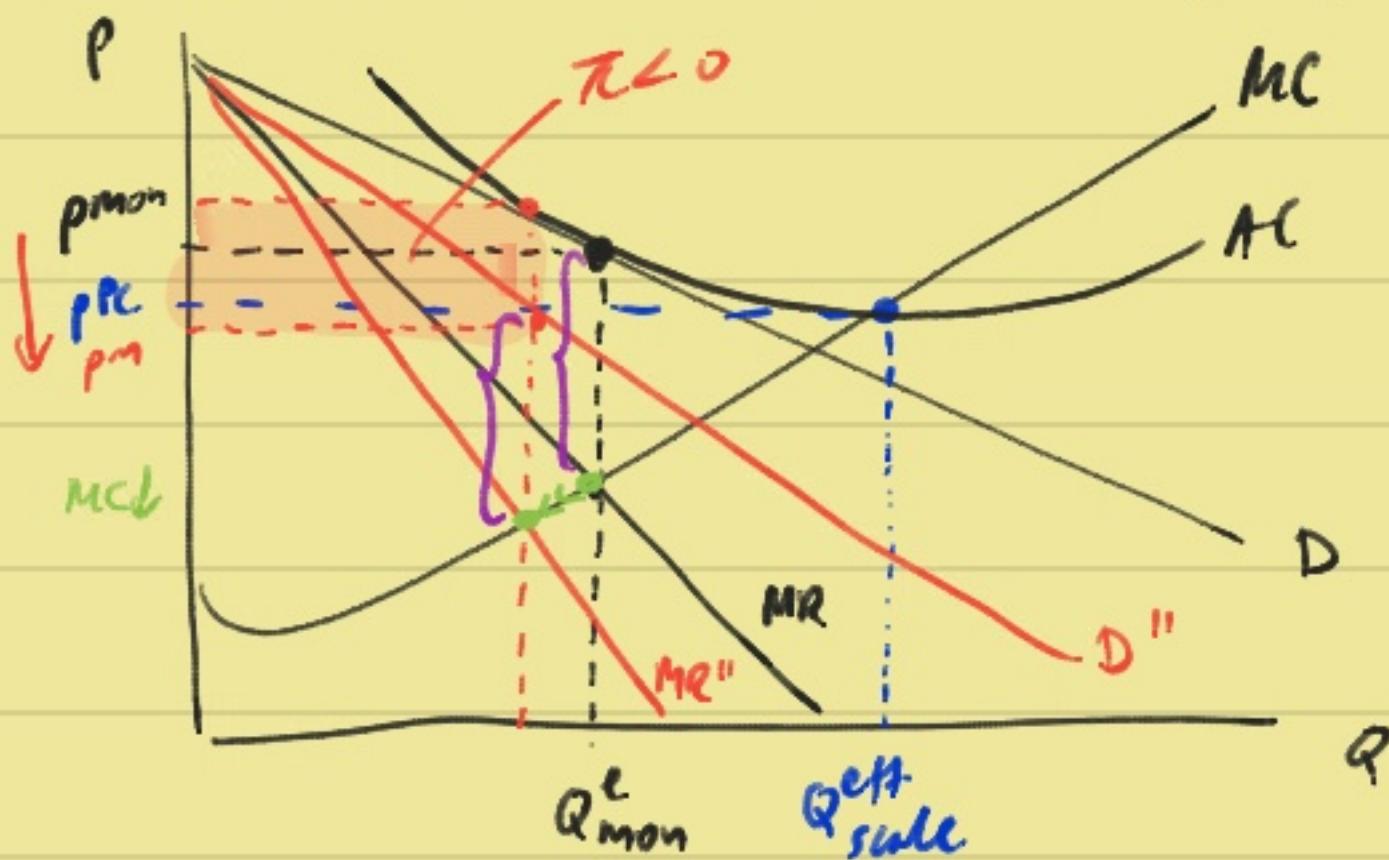
I think Sugawara's logic is that there are also movements in

Yit, i.e. demand: Here: $D \uparrow \rightarrow MC \uparrow \rightarrow \mu \downarrow$



Isn't this a feature of any imperfect comp. model?
 → yes, but w/ flex prices, eqb is restored
 immediately, whereas sticky prices keep the
 adjustment from happening right away.

Let's do a Θ demand & work hor for fun:



$$D \downarrow \rightarrow MC \downarrow \rightarrow p \uparrow$$

\Rightarrow ok so we know now why sticky prices introduce endog. time-varying markups/mc!

Both Eric Sims & Collard notes indicate more or less explicitly that in Rotemberg:

$$P_{it} = P_{jt} = P_t$$

Maybe the point is that this is an egb condition, but out of egb, it doesn't hold b/c indi. demands may be different, so the analysis of the previous pages carries through!

Ok so we've settled those 2 problems. So let's get back to the cost push shock.

We were kinda saying that 2 things can move mc:

↙ demand → moves "out-of-st-st" mc
↙ & (d. of substi) → moves st.st. mc

I think cost push refers to the latter based on Peter's def, and based on Claudio, Bali & Gertler's statement that it's "anything else than demand" that moves mc.

Alvarez, Gali, Gertler (1999), Result 1:

with cost push shocks, there's a SR tradeoff
btwn inflation and output variability.

→ so I think the divine coincidence doesn't hold!

And look at this: Basu Sum 2, p. 57

Divine coincidence in DKK model:

$$NKPC: \pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t$$

$$\text{if } \pi_t = 0, \rightarrow \hat{x}_t = 0 !$$

Divine coincidence breaks down however if the NKPC has a shock...

... which is exactly how Alvarez, Gali & Gertler define the cost push shock! It's

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + u_t$$

and I betcha it comes from the El. of subst!

and you know what... look at the fucking NKPC of GEMP

$$\pi_t = \gamma \pi_{t-1} + (1-\gamma) \pi_t^e + \rho \varphi_{t-1} + \eta_t$$

\uparrow hybrid $\xrightarrow{\text{expectation}}$
 term term

what's new?

\Rightarrow it's a shock in the NKPC!!

$$\text{Specifically, } \eta_t = \frac{\beta\rho}{1-\beta\rho} \varepsilon_t - \mu_t$$

\uparrow \uparrow
 mc-shock $= -\beta\rho \frac{\varphi_t}{\varphi_t - 1}$

where $\varphi_t = \text{el. subst. of demand!}$

$\rightarrow \eta_t$ is a cost-push shock! And this is what they mean by saying that their model doesn't distinguish between cost-push (μ_t) and mc-shocks (ε_t)!

So let's look at Clarida, Gali & Gertler in detail:

$$\text{CB loss: } \max -\frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i [\alpha x_{t+i}^2 + \pi_{t+i}^2] \right\} \quad (2.7)$$

CB's problem:

$$\max (2.7) \text{ st. (2.1) (NKIS) and (2.2) (NKPC).}$$

Result 1 referred to optimal mon. pol under discretion.

Result 2 as well:

"Optimal mon. pol is a kind of "LR inflation targeting"

in which π converges to the target only over time.

"Extreme inflation targeting", in which you reach the target immediately, is only optimal if

- 1) No cost-push shocks, or
- 2.) $\alpha=0$ (weight on output gap)

Result 3 = Taylor-Principle

Result 4 The optimal policy has the int. rate move so as to:

1. offset demand shocks ($AD \uparrow \rightarrow i \uparrow$)

$\Rightarrow b/c$ $i \uparrow$ pushes both $x_t \downarrow$ and $\pi_t \downarrow$
(no SR tradeoff here)

2. accommodates shocks to potential output ($\rightarrow \bar{i}$)

b/c $Y_t^p \uparrow$ but $Y_t \uparrow$ 1:1 (PIH) $\rightarrow x_t = 0$ still.

Result 5: If under discretion the CB wishes to have $Y_t > Y_t^p$,

then we get higher inflation w/o a gain in output (inflationary bias)

A side note: in Baum-Sum Part 2, p. 85:

"To get results qualitatively diff. from an RBC,

we need suboptimal mon. pol. b/c the Taylor rule

is too good!" → maybe that can change under learning?

Ok so let's think from a cost-push shock in the DKN model. Would it look like the $\pi_t \downarrow$ -shock in Baum-Sum 2, p. 72?

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + k \hat{x}_t + u_t \quad (\text{NKPC})$$

$$\hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\delta} [i_t - E_t \hat{\pi}_{t+1}] + \frac{1}{\delta} r_t^n \quad (\text{NKIC})$$

$$\hat{i}_t = \delta_\pi \hat{\pi}_t + \delta_x \hat{x}_t \quad (\text{TR})$$

$$k = \frac{(\eta + \delta)(1 - \omega)}{\omega}$$

ω = calvo param.

η = Frisch.

θ = El. of switch in demand

β = IES param in $U(\cdot)$ fct.

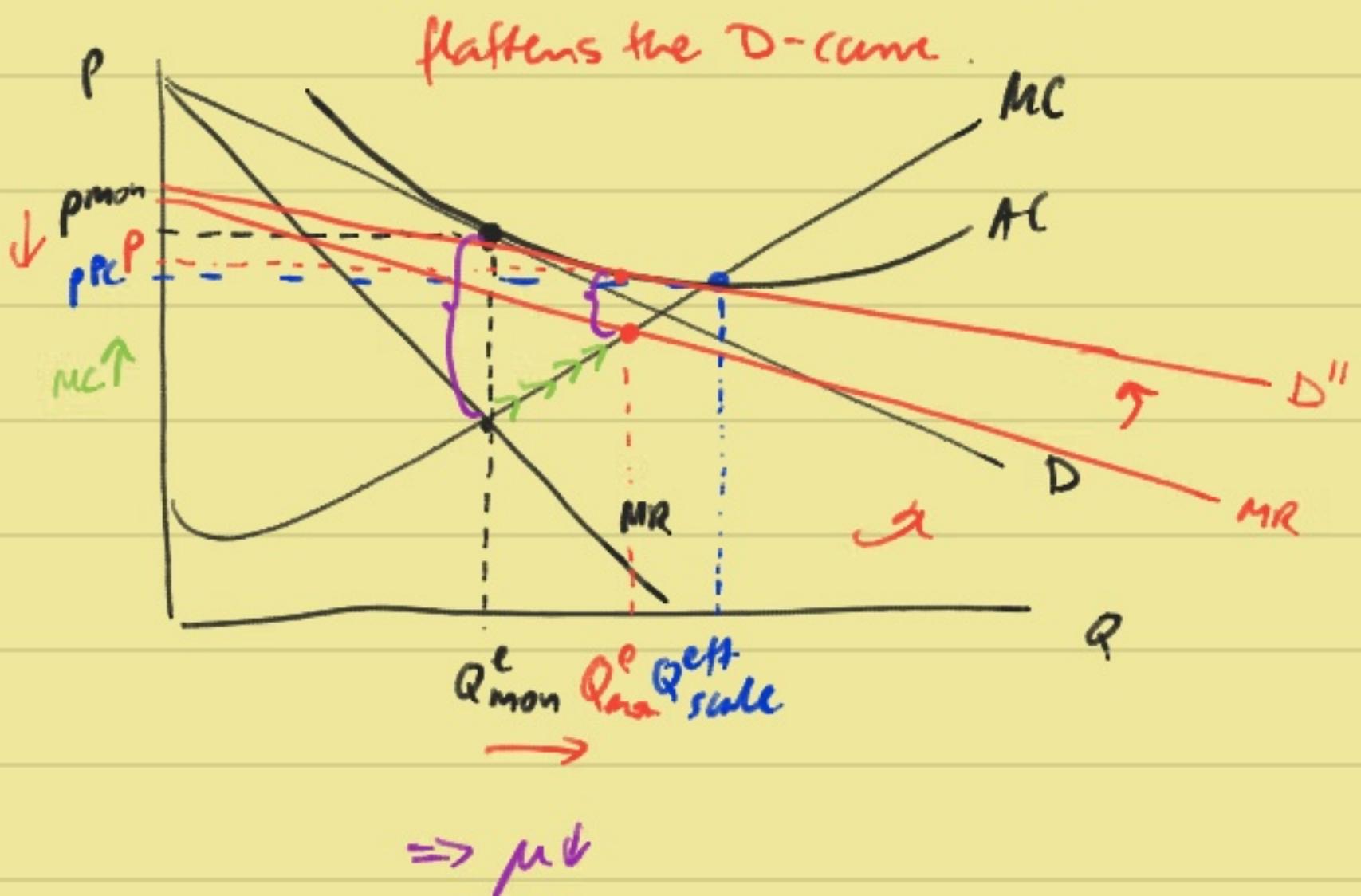
$$\rightarrow \mu = \frac{\theta}{\theta - 1} \quad (\text{I think } \theta > 1)$$

$$\frac{\partial \mu}{\partial \theta} = \frac{1}{\theta - 1} + (-1) \frac{\theta}{(\theta - 1)^2} = \frac{\theta - 1 - \theta}{(\theta - 1)^2} = -\frac{1}{(\theta - 1)^2} < 0$$

\gg

So when $\theta \uparrow \rightarrow \mu \downarrow$

which makes sense b/c as $Q \rightarrow \infty$, the goods b/c perfect substitutes and so you lose mon. power.
In our micro principles class, a $\theta \uparrow / \mu \downarrow$ shock:



Now in the NK model:

$$\uparrow \pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t \downarrow + u_t \quad (\text{NKPC})$$

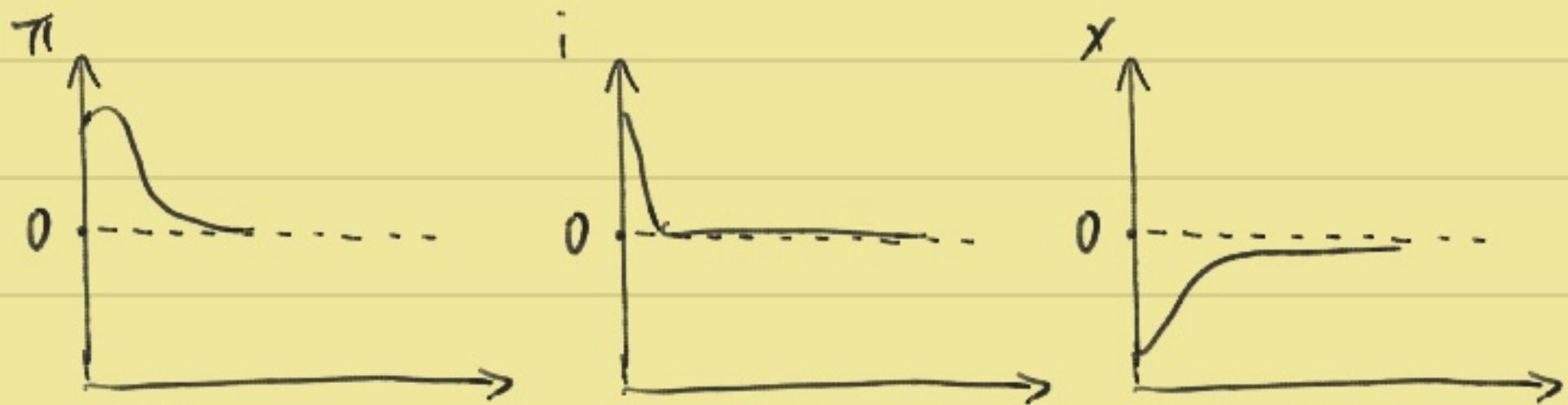
$$\downarrow \hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\delta} [i_t \uparrow - E_t \pi_{t+1}] + \frac{1}{\delta} r_t^n \quad (\text{NKIC})$$

$$\uparrow i_t = \delta_\pi \pi_t \uparrow + \delta_x \hat{x}_t \quad (\text{TR})$$

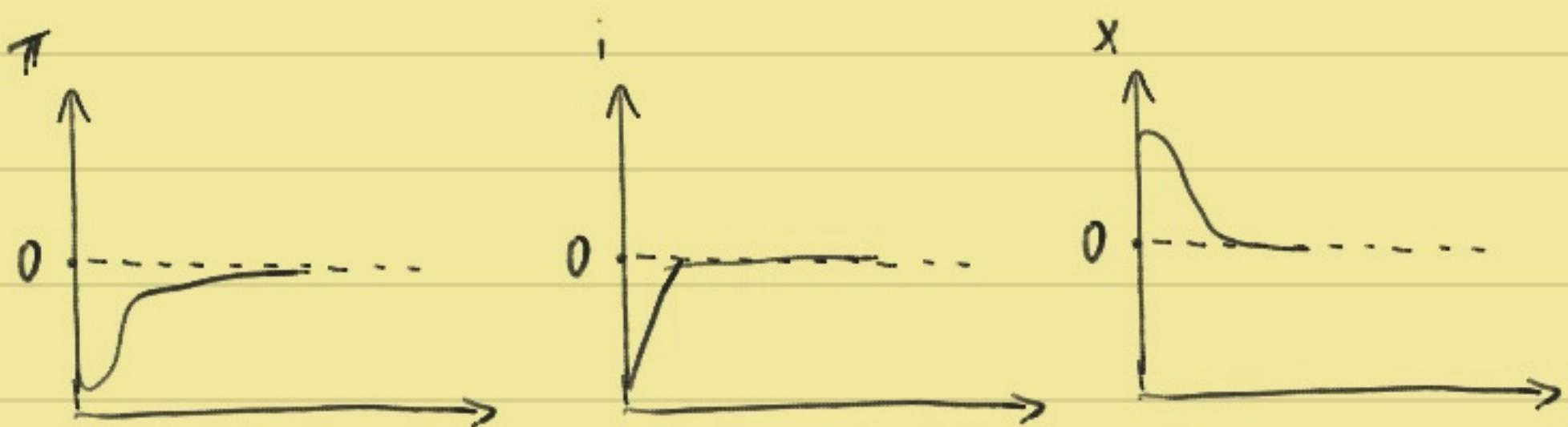
and stop.

(All shocks are one-time shocks here!)

So it would look like this in IRFs: $\theta\uparrow/\mu\uparrow$



$\theta\uparrow/\mu\downarrow$ (closer to perfect comp.)



It doesn't look like the μ -shock b/c here mon. pol. responds

But if there was no TR, we'd transition to a new eqb.

w/ lower prices and higher output, so in that sense
the θ/μ -shock does resemble the one on Basu

p.7 in that it resembles a tech shock.

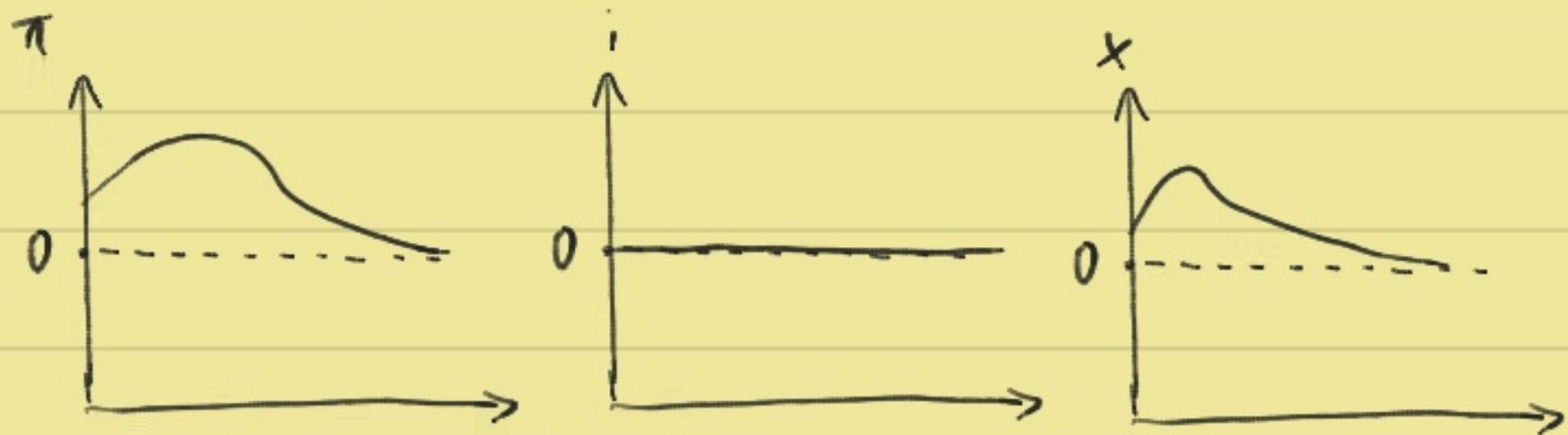
→ you can also see the SR nature of the (π, x) -tradeoff:

If mon. pol. doesn't respond:

$$\uparrow \uparrow \pi_t = \beta E_t \pi_{t+1} + k \hat{x}_t + u_t \quad (\text{NKPC})$$

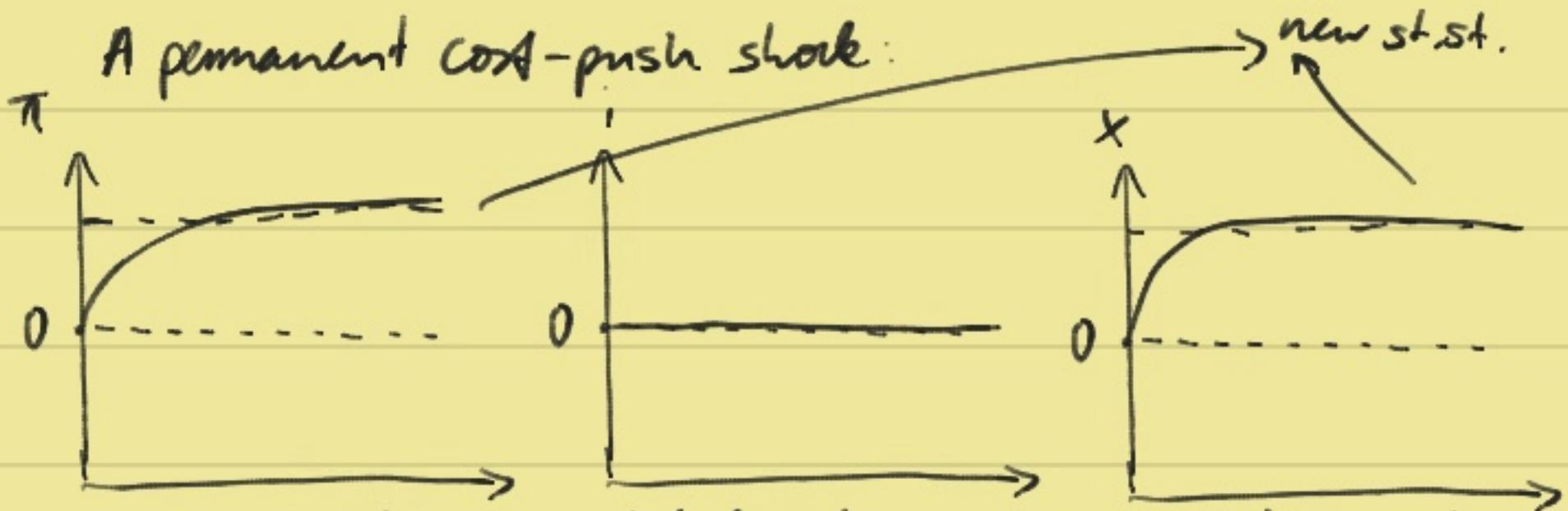
$$\uparrow \hat{x}_t = E_t x_{t+1} - \frac{1}{\delta} [i_t - E_t \pi_{t+1}] + \frac{1}{\delta} r_t^n \quad (\text{NKIC})$$

$$\uparrow i_t = \delta_\pi \pi_t + \delta_x \hat{x}_t \quad (\text{TR})$$



→ permanently higher price-level.

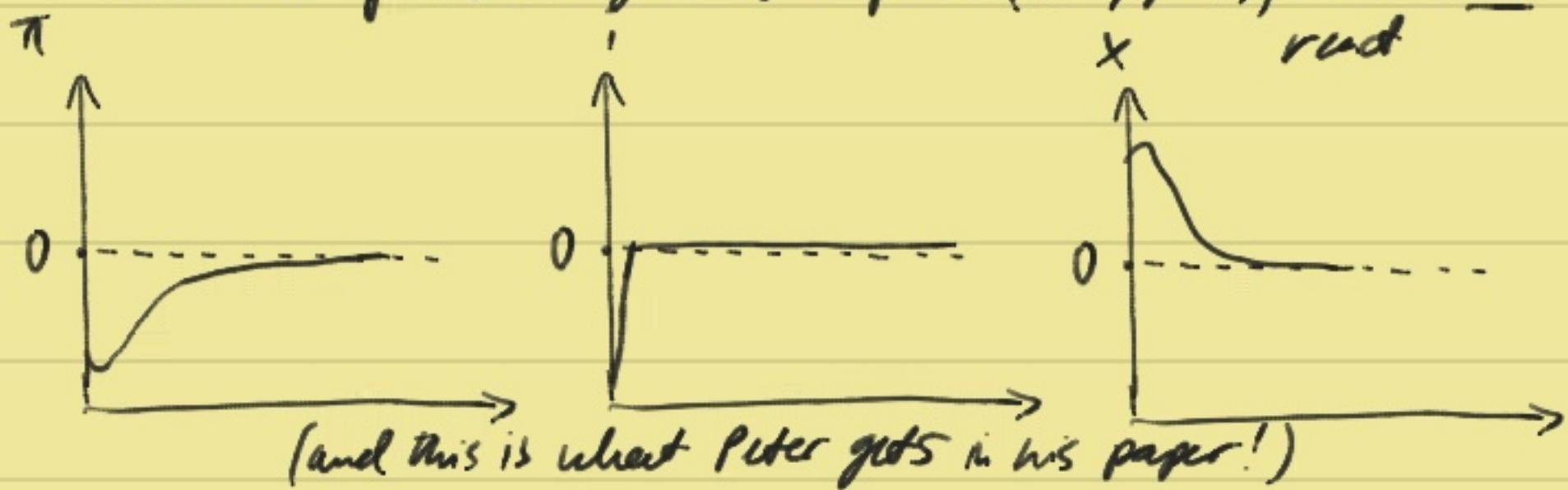
A permanent cost-push shock:



⇒ you don't want to interfere if the cost-push is "positive"

In the sense that it moves towards perfect competition.

However if it's a negative cost-push ($\theta \downarrow / \mu \uparrow$), the Fed can raise



the problem I'm now noticing is that here, I'm analysing the SR ($\pi - x$) - tradeoff that happens when \exists cost-push shocks in NK models, i.e. a failure of divine coincidence GIVEN that we want to close the output gap.

Ryan said however that we don't want to close the output gap in this case b/c the flex price is inflexible.

→ Eric Sims says that in the NK model $\exists 2$ distortions

- SR distortion due to nominal frictions
- LR distortion due to imperfect comp.

"We ask the CB is concerned w/ the SR one and that the LR one has been taken care of w/ some Pigouvian tax".

→ and I think that's the std. way of business:
also in Basu we simply wanna close the output gap.
Also in absence of cost-push shocks I argue that
 γ^{flex} is suboptimal (too low) in the NK model.
→ but you can't stabilize to it b/c the Phillips curve
breaks down: the neutrality of money happens.

Juselius & Pannier, National Bank of Belgium.

Slides of Presi, 22 March 2010 "presentatiepdf"

Efficient output := Y under perfect comp. (γ^{pc})

Potential output := flex price Y under imperfect comp (γ^p)
and constant markups

Natural output := flex price Y under imperfect comp (γ^n)
but time-varying markups

$\gamma^{\text{pc}} - \gamma^n \rightarrow$ importance of non rigidities

$\gamma^p - \gamma^n \rightarrow$ importance of exog. markup variation \rightarrow cost-push!
($\pi - Y$) tradeoff for monopol!

If markup shocks (i.e. cost push) is interpreted as tech shocks.

$$\gamma^P = \gamma^n \quad (?)$$

Their prior observes that while γ^P is very smooth (and close to actual observed γ), γ^n is crazy volatile, implying implausibly big markup shocks.

\Rightarrow JP argue that much of this is meas. error, and show that a model w/ meas. error obtains

$\gamma^P \approx \gamma^n$, and a much better fit to data.

(Since they consider wage markups, the idea of meas. error is plausible b/c wage series aren't great.)

If γ^P is smooth, it means that tech shocks aren't very volatile

If γ^n is volatile, it means that BCs are big \Leftrightarrow markups fluctuate a lot. But IMO that doesn't need to come from cost-push exclusively: it can also come from demand!

My point is: non pol cannot stabilize to Y^P if there's imperfect comp. \rightarrow If in addition there are time-varying markups, then Y^P is the only thing we can stabilize to. And to the extent that there are also nominal rigidities, we have time-varying markups.

What the output gap is depends on what we're stabilizing to:

E.g. if $X := Y^P - Y_t$ then cost-push shocks don't affect Y^P and we thus get a substantial output gap.

but if $X := Y^n - Y_t$ then since cost-push shocks affect Y^n , X doesn't open at all.

\rightarrow so we can really only stabilize to Y^P in a NK world.

OK so I think I understand not too badly how

the cost-push shocks work in the NK model

→ since they move markups/mc, they open up
the output gap

Peter's results in the Tech Shocks in NK models paper

bear out that cost-push shocks behave this way,

and he also shows that they've played an important
role (especially in inflation) fluctuations (Var. decom).

⇒ So the task is: how does the model react differently
under learning? CEMP:

$$k_t = f_k$$

$$\bar{\pi}_t = f_{\bar{\pi}} + f_k^{-1} \eta_{t-1}$$

$$y_t = f_y + A_y y_{t-1} + S_y \left(\begin{matrix} e_t \\ \mu_t \end{matrix} \right)$$

} Learning block

Cost-push

} exog sum of shocks η_t

} exog shock: y_t

NKPC: π_t

Right now there's no mon. pol (but effect of mon. pol is on Γ in AS)

I want to approach the CEMP-version

30 Aug 2015

in three ways

- 1.) Intuition
- 2.) w/o mon. pol
- 3.) w/ mon. pol in the model

1) Intuition: Here's a $\theta \downarrow / \mu \uparrow$ (less comp) cost-push shock

$$\uparrow \uparrow \pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + \textcircled{u_t} \quad (\text{NKPC})$$

$$\uparrow \hat{x}_t = E_t x_{t+1} - \frac{1}{\delta} [i_t - E_t \pi_{t+1}] + \frac{1}{\delta} r_t^n \quad (\text{NKIC})$$

$$\uparrow i_t = \delta_\pi \pi_t + \delta_x \hat{x}_t \quad (\text{TR})$$

Note by the way that $\hat{x}_t > 0$ here b/c y_t^n moved down, so it's actually not clear that we wanna close this gap!

[But supp CB ignores that, & just closes]

Time-out: I think I haven't yet understood the tradeoff well.

$$\text{Cba: } \text{discision: } \max -\frac{1}{2} [\alpha x_t^2 + \pi_t^2] + F_t$$

$$\text{s.t. } \pi_t = \lambda x_t + f_t$$

$$\Rightarrow \max_x -\frac{1}{2} [\alpha x_t^2 + (\lambda x_t + f_t)^2] + F_t$$

$$\Rightarrow \max_x -\frac{1}{2} [\alpha x_t^2 + \lambda^2 x_t^2 + 2\lambda f_t x_t + f_t^2] + F_t$$

$$FDC : -\frac{1}{2} [2\alpha x_+ + 2\gamma^2 x_+ + 2\gamma f_+] = 0$$

$$(\alpha + \gamma^2) x_+ \stackrel{!}{=} -\gamma f_+$$

$$x_+ = \frac{-\gamma}{\alpha + \gamma^2} f_+$$

More comp. shock

$$\Rightarrow x_+ = -\frac{\gamma}{\alpha + \gamma^2} (\beta E \pi_{t+1} + u_t)$$

$$\Rightarrow \pi_t = \alpha x_+ + f_+ = -\frac{\gamma^2}{\alpha + \gamma^2} f_+ + f_+ = \left(1 - \frac{\gamma^2}{\alpha + \gamma^2}\right) f_+$$

$$\pi_t = \frac{\alpha}{\alpha + \gamma^2} f_+$$

$$\rightarrow \pi_+ = \frac{\alpha}{\alpha + \gamma^2} (\beta E \pi_{t+1} + u_t)$$

\Rightarrow a cost-push shock looks like a supply shock

- x & π move in opposite directions following a cost-push

shock: if $u_t \downarrow \rightarrow x_+ \uparrow$ & $\pi_+ \downarrow$; if $u_t \uparrow \rightarrow x_+ \downarrow$ & $\pi_+ \uparrow$

\Rightarrow stabilizing π requires: $u_t \downarrow \Rightarrow x_+ \uparrow$ and $u_t \uparrow \Rightarrow x_+ \downarrow$

IF expectations don't move: a tradeoff.

- Note: that $E(\cdot)$ moving makes this worse.

- In the LR I guess the tradeoff does out as $E\pi_{t+1} \rightarrow 0$.

→ Intuition: a mon. pol that can stop $E(\pi)$ from moving needs to use "less ammunition" to fight the cost-push shock than one that doesn't manage expectations.

Problem: how to manage $E(\pi)$?

- bigger cuts in π_t today → but then you're back to higher costs, except you pay them today instead of tomorrow (\rightarrow intertemporal tradeoff)
 - this still is preferable though if $\text{var}(\pi)$ is costly per se!
- abandon TR: use communication to keep $E(\pi)$ "anchored"
- ⇒ if you have a "credibility stock" (low gain) you don't have to resort to any of these measures as long as you don't allow π to deviate from its st. st. value by too much or for too long.

It would just be nice to see the tradeoff w/ commitment to, that is, simply in the 3-Cg DPK:

$$\downarrow \pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + u_t \quad (\text{NKPC})$$

$$\uparrow \hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\delta} [i_t - E_t \pi_{t+1}] + \frac{1}{\delta} r_t^n \quad (\text{NKIC})$$

$$\downarrow i_t^* = \delta \pi_t + \delta x_t \hat{k}_t \quad (\text{TR})$$

Maybe the natural rate is affected? $r_t^n = \beta E_t [Y_{t+1}^f - Y_t^+]$

$\downarrow \qquad \uparrow$
w/c closer
to P.C.
gov....

The only thing that's clear is that policy $i_t \downarrow \rightarrow x_t^* \uparrow \& \pi_t \uparrow$

But we should have $\hat{x}_t \uparrow$ as a response to the shock.

Ok, let's just supp. that a TR is in place

- Then, by nature of propagating through the system of the economy (and only then) does a cost-push shock look like a supply shock

- Movement in EC) aggravates the problem and the tradeoff.

2) Now to CEMP w/o mon. pol:

$$k_+ = f_k$$

$$\bar{\pi}_+ = f_{\bar{\pi}} + f_k^{-1} \eta_{t+1}$$

} learning block

Cost-push
↓

$$\xi_+ = f_\xi + A_\xi \xi_{t-1} + S_\xi \left(\frac{E_t}{\mu_t} \right)$$

} exog sum of shocks η_t
exog shock: y_t
NKPC: π_t

Right now there's no mon. pol (but effect of mon. pol is on E in AS)

Supp $\mu_t \downarrow \Rightarrow$ closer to P.C. cost-push shock.

$$\rightarrow \eta_t \downarrow \quad \xrightarrow{\hspace{2cm}} \pi_+(\eta_t) \downarrow$$

$\xrightarrow{\hspace{2cm}} 1.)$ anchored ($f_k^{-1} \rightarrow 0$): $E(\pi)$ don't move and that's it.

2) unanchored: $\bar{\pi}_{t+1} \downarrow \Rightarrow$ again,

the same intuition is straightforward to see

$\Rightarrow E(\cdot)$ moving makes the stabilization worse.

3) Comp w/ mon. pol. "appended"

$$k_t = f_k$$

} learning block

$$\bar{\pi}_t = f_{\bar{\pi}} + f_k^{-1} \eta_{t-1}$$

Cost-push
↓

$$\left[\begin{array}{l} \xi_t \\ \end{array} \right] = f_\xi + A_\xi \xi_{t-1} + S_\xi \left(\begin{array}{l} E_t \\ \mu_t \end{array} \right)$$

} exog sum of shocks $\circledcirc \eta_t$

$$\checkmark \left[\begin{array}{l} \pi_t \\ \tau_t \end{array} \right] = \text{stuff} + \checkmark \text{demand new term} \rightarrow x_t$$

} exog shock: y_t
NKPC: π_t

$$+ \uparrow X_t = \text{stuff} - (i_t - E_t \tau_{t-1})$$

} mon. pol. appended

$$+ TR: \downarrow i_t = S_\pi \pi_t + S_X X_t$$

Supp $\mu_t \downarrow \Rightarrow$ closer to P.C. cost-push shock.

$$\rightarrow \eta_t \downarrow \rightarrow \pi_t \downarrow$$

$\rightarrow i_t \downarrow \rightarrow x_t \uparrow$ (mechanisms same as
when mon. pol. isn't explicitly specified)

\rightarrow Movement in $\bar{\pi}$ amplifies $\pi_t \downarrow \& x_t \uparrow$

\Rightarrow again in such a case a higher int. rate

drop is necessary, worsening the tradeoff

The point is: what we know about the effect 31 Aug 2019

of learning on mon. pol. seems to be captured by

Eusepi & Preston (2018, JEL) & "On the Limits of Mon. Policy"

Conclusion 1: In the presence of LR expectations on real rates,
the divine coincidence doesn't hold.

Conclusion 2: Opt. mon. pol. is either more aggressive (F 2018,
Molnar-Sartori, Ferraro 2007) or less aggressive ("limits")
on infl. than the RE counterpart.

This seems to be the research frontier. (I'm ignoring all the
technical results like E-stability (Evans & H, 2003) or
Taylor-principle for E-stab. (Preston, 2005).)

Economically interesting:

- anchoring as credibility stroke \rightarrow maybe this can break the
divine coincidence.
- ZLB: anchoring gives you leeway when ZLB hits!

Ok - there's no way around it: generalize CEM^P w/ a $\Pi\Pi$
(that is demand-side) so we can specify a TR w/ Z_{LB}

As a first step, investigate different (exog?) gain
sequences to see what they mean for policy and
welfare. \rightarrow write out the social welfare fn as an
approx to $\Pi\Pi$ utility.

Would be neat: show that larger gains are CP
welfare-decreasing due to $\begin{cases} \text{larger volatility in } \pi \\ \text{larger } i\text{-movements} \\ \text{larger volatility in } x \end{cases}$

In CEM^P App. there is a IS-curve, but it doesn't feature
LR-expectations, so I don't actually think it's the
relevant one.

\rightarrow I think they use it in CEM^P in order to avoid
complication for the NKPC?

So "adapt" the Preston NKPC (eq 15) and NKIS (eq 18)
for our purposes:

(COMP - AS:

$$(2) \pi_t - \delta \pi_{t-1} = \mu_t + \hat{E}_t \sum_{T=1}^{\infty} (\alpha \beta)^{T-t} \left[\xi_T s_T + (1-\alpha) \beta (\pi_{T+1} - \delta \bar{\pi}_T) \right]$$

Preston NKPC: eq(15)

$$\pi_t = k x_t + \hat{E}_t \sum_{T=1}^{\infty} (\alpha \beta)^{T-t} \left[k \alpha \beta x_{T+1} + (1-\alpha) \beta \pi_{T+1} \right]$$

→ The "hybrid NKPC" would be (my guess)

$$\pi_t - \delta \pi_{t-1} = k x_t + \hat{E}_t \sum_{T=1}^{\infty} (\alpha \beta)^{T-t} \left[k \alpha \beta x_{T+1} + (1-\alpha) \beta (\pi_{T+1} - \delta \bar{\pi}_T) \right]$$

$+ u_t$ ← cost-push shock (Guerr-NKPC)

(COMP demand side was just the s_t -process

Preston demand side is

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \beta (\beta i_{T+1} - \bar{\pi}_{T+1}) + r_T \right] \quad (18)$$

Let the TR be given by Preston's (27)

$$i_t = \psi_{\pi} \pi_t + \psi_x x_t + \bar{i}_t$$

Now I just have to manipulate the shock processes so as to get an ALM that resembles COMP's (eq. 6).

p. 144. Experimentation Notes 1.

And p. 166

The crux in (EMP) is the expectation $\hat{E}_t(\bar{\pi}_{T+1} - \gamma \bar{\pi}_T)$

the PLM: $\bar{\pi}_t = \delta \bar{\pi}_{t-1} + (1-\delta)\bar{\pi} + \rho \psi_{t-1} + e_t$

$\Rightarrow \hat{E}_t \bar{\pi}_{T+1} = \delta \bar{\pi}_T + (1-\delta)\bar{\pi} + \rho \psi_T$

$$\Rightarrow \hat{E}_t(\bar{\pi}_{T+1} - \gamma \bar{\pi}_T) = \hat{E}_t \left[\cancel{\delta \bar{\pi}_T} + (1-\delta)\bar{\pi} + \rho \psi_T - \cancel{\gamma \bar{\pi}_T} \right]$$

This is the crux which makes the eval. of this sum sooo much easier.

This way the ugly sum $(1-\alpha)\beta \sum_{T=1}^{\infty} (\alpha\beta)^{T-1} \hat{E}_T(\bar{\pi}_{T+1} - \gamma \bar{\pi}_T)$

simply becomes $(1-\alpha)\beta \sum_{T=1}^{\infty} (\alpha\beta)^{T-1} \hat{E}_T[(1-\gamma)\bar{\pi} + \rho \psi_T]$

$$= \frac{(1-\alpha)\beta(1-\gamma)\bar{\pi}}{1-\alpha\beta} + (1-\alpha)\beta \sum_{T=1}^{\infty} (\alpha\beta)^{T-1} \rho \hat{E}_T \psi_T \quad \uparrow \text{RE} = \rho^{T-1} \psi_T$$

$$= \frac{(1-\alpha)\beta(1-\gamma)\bar{\pi}}{1-\alpha\beta} + \frac{(1-\alpha)\beta}{1-\alpha\beta\rho} \psi_{T-1}$$

So I'll need to design PLM and eval. processes such that I can do a trick like this. Otherwise I have to do the $f\alpha, f\beta$ -thing.

An alternative way is to take the Proton system (18), (15) & (27)
but change the PLM to COMP's (or stay like it)
(except I wanna allow for a cost-push shock in (19).)

So if the model is:

1 Sep 2019

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} \left[(1-\beta)x_{T+1} - \beta(\beta i_{T+1} - \pi_{T+1}) + r_T^N \right] \quad (18)$$

$$\pi_t = kx_t + \hat{E}_t \sum_{T=1}^{\infty} (\alpha\beta)^{T-t} \left[\alpha\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_t \right] \quad (15)$$

$$i_t = \gamma_\pi \pi_t + \gamma_x x_t + \bar{i}_t \quad (27)$$

↑
cost-push
shock

$$\bar{x}_t = \text{function}(f\alpha, f\beta, f\gamma, \bar{i}_t) \quad (\text{RF form of econ})$$

then PLM needs to be stay like COMP's eq(5):

$$\pi_t = \gamma \pi_{t-1} + (1-\gamma) \bar{x}_t + \rho p_{t-1} + e_t \quad (5)$$

i.e like: $\bar{z}_t = \Gamma \bar{z}_{t-1} + (I_3 - \Gamma) \bar{x}_t + \rho s_{t-1} + e_t \quad (\text{PLM}_1)$

where $s_t = \begin{bmatrix} r_t^N \\ i_t \\ u_t \end{bmatrix}$ and $\rho = \begin{bmatrix} \rho_r & 0 & 0 \\ 0 & \rho_i & 0 \\ 0 & 0 & \rho_u \end{bmatrix}$

with $\bar{x}_t = \bar{z}_{t-1} + k_t^{-1} \cdot f_{t-1}$

$$f_{t-1} = z_{t-1} - \hat{E}_{t-2} z_{t-1} \quad (3 \times 1)$$

So let's derive the ALM here, which is going to be identical to Pötscher's ALM in Preston, except for the presence of the last-period shock (i_T): Plug it:

$$x_t = -\beta(\psi_\pi \pi_t + \psi_x x_{t+1} + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} [(1-\beta)x_{T+1} - \beta(\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \bar{i}_{T+1})] e^{r_T^N}$$

$$(1+\beta\psi_x) x_t = -\beta\psi_\pi \pi_t - \beta\bar{i}_t$$

$$+ \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} [(1-\beta-\beta\psi_x)x_{T+1} - \beta[\beta\psi_T - 1]\pi_{T+1} - \beta\bar{i}_{T+1}] e^{r_T^N}$$

One realization: in the Basin SUM, we saw that on the NKIS, whatever is the coeff. on i_t , r_t^N gets the same coeff w/ the opposite sign! So Preston made the IS-curve wrong, but this explains why the coeffs of i_t are $-\beta$ and β !

$$(1+\beta\psi_x) x_t = -\beta\psi_\pi \left[kx_t + \hat{E}_t \sum_{T=1}^{\infty} (\alpha\beta)^{T-t} [\alpha\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T] \right]$$

$$+ \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} [(1-\beta-\beta\psi_x)x_{T+1} + (1-\beta-\beta\psi_\pi)\pi_{T+1} - \beta\bar{i}_t + \underline{\beta g^N}]$$

$$\underbrace{(1+\beta\psi_x - \beta\psi_\pi k)}_{=w} x_t = -\beta\psi_\pi \left[(1-\alpha)\beta, \alpha\alpha\beta, 0 \right] f_\alpha - \beta\psi_\pi \sum (\alpha\beta)^{T-t} u_T$$

$$+ \left[\beta(1-\beta\psi_\pi), (1-\beta-\beta\psi_x), 0 \right] f_\beta - \beta \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} (\bar{i}_t + g^N)$$

$$x_t = -\frac{\beta\psi_\pi}{w} \left[(1-\alpha)\beta, \alpha\alpha\beta, 0 \right] f_\alpha + \frac{1}{w} \left[\beta(1-\beta\psi_\pi), (1-\beta-\beta\psi_x), 0 \right] f_\beta$$

$$- \frac{\beta\psi_\pi}{w} \sum (\alpha\beta)^{T-t} \hat{E}_t u_T - \frac{\beta}{w} \sum \beta^{T-t} \hat{E}_t (\bar{i}_t + g^N)$$

$$\begin{aligned}
\pi_t &= K \left\{ -\frac{\alpha \psi_{\pi}}{w} [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_{\alpha} + \frac{1}{w} [b(1-\beta \psi_{\pi}), (1-\beta-\beta b \psi_x), 0] f_{\beta} \right. \\
&\quad \left. - \frac{\alpha \psi_{\pi}}{w} \sum (\alpha \beta)^{T-t} \hat{E}_t u_T - \frac{b}{w} \sum \beta^{T-t} \hat{E}_t (\bar{i}_T + r_T^N) \right\} \\
&\quad + \hat{E}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} [\alpha \alpha \beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T] \\
&= -\frac{K \alpha \psi_{\pi}}{w} [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_{\alpha} + \frac{K}{w} [b - b \beta \psi_{\pi}, (1-\beta-\beta b \psi_x), 0] f_{\beta} \\
&\quad - \frac{\alpha \psi_{\pi} K}{w} \sum (\alpha \beta)^{T-t} \hat{E}_t u_T - \frac{K b}{w} \sum \beta^{T-t} \hat{E}_t (\bar{i}_T + r_T^N) \\
&\quad + [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_{\alpha} + \hat{E}_t \sum (\alpha \beta)^{T-t} u_T \\
\pi_t &= \left(1 - \frac{K \alpha \psi_{\pi}}{w}\right) [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_{\alpha} + \frac{K}{w} [b - b \beta \psi_{\pi}, (1-\beta-\beta b \psi_x), 0] f_{\beta} \\
&\quad + \left(1 - \frac{K b \psi_{\pi}}{w}\right) \sum (\alpha \beta)^{T-t} \hat{E}_t u_T - \frac{K b}{w} \sum \beta^{T-t} \hat{E}_t (\bar{i}_T + r_T^N) \quad \checkmark
\end{aligned}$$

The error terms:

$$\sum (\alpha \beta)^{T-t} \hat{E}_t u_T = \sum (\alpha \beta)^{T-t} [0 \ 0 \ 1] \hat{E}_t S_T = \sum (\alpha \beta)^{T-t} [0 \ 0 \ 1] \rho^{T-t} s_T$$

$$\begin{aligned}
\text{I hope } &= \sum (\alpha \beta \rho)^{T-t} [0 \ 0 \ 1] S_T = [I_3 - \alpha \beta \rho]^{-1} [0 \ 0 \ 1] S_T \\
\text{this is allowed!} & \quad \text{should be } [0 \ 0 \ 1] [I_3 - \alpha \beta P]^{-1} S_T
\end{aligned}$$

$$\begin{aligned}
\sum \beta^{T-t} \hat{E}_t (\bar{i}_T + r_T^N) &= \sum \beta^{T-t} \hat{E}_t [1 \ 1 \ 0] S_T = \sum (\beta \rho)^{T-t} [1 \ 1 \ 0] S_T \\
&= [I_3 - \beta \rho]^{-1} [1 \ 1 \ 0] S_T \\
&\quad [-1 \ 1 \ 0] [I_3 - \beta P]^{-1} S_T
\end{aligned}$$

So then

$$x_4 = -\frac{2\pi}{\omega} [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + \frac{1}{\omega} [2(1-\beta\gamma_\pi), (1-\beta-\beta\beta\gamma_x), 0] f_\beta \\ - \frac{2\pi}{\omega} [I_3 - \alpha\beta P]^{-1} [0 \ 0 \ 1] s_+ - \frac{2}{\omega} [I_3 - \beta P]^{-1} [1 \ 1 \ 0] s_+$$

$$\pi_+ = \left(1 - \frac{\kappa\beta\gamma_\pi}{\omega}\right) [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + \frac{\kappa}{\omega} [2 - \beta\beta\gamma_\pi, (1-\beta-\beta\beta\gamma_x), 0] f_\beta \\ + \left(1 - \frac{\kappa\beta\gamma_\pi}{\omega}\right) [I_3 - \alpha\beta P]^{-1} [0 \ 0 \ 1] s_+ - \frac{\kappa\beta}{\omega} [I_3 - \beta P]^{-1} [1 \ 1 \ 0] s_+$$

$$\Rightarrow x_4 = \underbrace{-\frac{2\pi}{\omega} [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + \frac{1}{\omega} [2(1-\beta\gamma_\pi), (1-\beta-\beta\beta\gamma_x), 0] f_\beta}_{=: g_{xa}} \\ + \underbrace{\left[-\frac{2\pi}{\omega} [I_3 - \alpha\beta P]^{-1} [0 \ 0 \ 1] - \frac{2}{\omega} [I_3 - \beta P]^{-1} [1 \ 1 \ 0] \right] s_+}_{=: g_{xs}}$$

$$\left(-\frac{2\pi}{\omega} [0 \ 0 \ 1] (I_3 - \alpha\beta P)^{-1} - \frac{2}{\omega} [-1 \ 1 \ 0] (I_3 - \beta P)^{-1} \right) s_+ \checkmark$$

$$\pi_+ = \left(1 - \frac{\kappa\beta\gamma_\pi}{\omega}\right) [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + \frac{\kappa}{\omega} [2 - \beta\beta\gamma_\pi, (1-\beta-\beta\beta\gamma_x), 0] f_\beta \\ + \left[\left(1 - \frac{\kappa\beta\gamma_\pi}{\omega}\right) [I_3 - \alpha\beta P]^{-1} [0 \ 0 \ 1] - \frac{\kappa\beta}{\omega} [I_3 - \beta P]^{-1} [1 \ 1 \ 0] \right] s_+$$

$$\left(\left(1 - \frac{\kappa\beta\gamma_\pi}{\omega}\right) [0 \ 0 \ 1] (I_3 - \alpha\beta P)^{-1} - \frac{\kappa\beta}{\omega} [-1 \ 1 \ 0] (I_3 - \beta P)^{-1} \right) s_+ \checkmark$$

$$i_4 = \gamma_\pi \pi_+ + \gamma_x x_4 + [0 \ 1 \ 0] s_+$$

$$x_+ = g_{xa} \cdot f_\alpha + g_{xb} \cdot f_\beta + g_{xs} \cdot s_+ \checkmark$$

$$\pi_+ = g_{\pi a} \cdot f_\alpha + g_{\pi b} \cdot f_\beta + g_{\pi s} \cdot s_+ \checkmark$$

$$i_4 = (\gamma_\pi g_{\pi a} + \gamma_x g_{xa}) f_\alpha + (\gamma_\pi g_{\pi b} + \gamma_x g_{xb}) f_\beta \\ + (\gamma_\pi g_{\pi s} + \gamma_x g_{xs} + [0 \ 1 \ 0]) s_+ \checkmark$$

$$\text{So } x_t = g_{xa} \cdot f_\alpha + g_{xb} \cdot f_\beta + g_{xs} \cdot s_t$$

$$\pi_t = g_{\pi a} \cdot f_\alpha + g_{\pi b} \cdot f_\beta + g_{\pi s} \cdot s_t$$

$$\begin{aligned} i_t &= (\gamma_\pi g_{\pi a} + \gamma_x g_{xa}) f_\alpha + (\gamma_\pi g_{\pi b} + \gamma_x g_{xb}) f_\beta \\ &\quad + (\gamma_\pi g_{\pi s} + \gamma_x g_{xs} + [0 \ 1 \ 0]) s_t \end{aligned}$$

$$\Rightarrow z_t = \underbrace{A_1 f_\alpha + A_2 f_\beta + A_3 s_t}_{\text{(ALM)}}$$

where

$$A_1 = \begin{bmatrix} g_{\pi a} \\ g_{xa} \\ \gamma_\pi g_{\pi a} + \gamma_x g_{xa} \end{bmatrix} \quad \text{all } A_i \text{ should be } 3 \times 3.$$

$$A_2 = \begin{bmatrix} g_{\pi b} \\ g_{xb} \\ \gamma_\pi g_{\pi b} + \gamma_x g_{xb} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} g_{\pi s} \\ g_{xs} \\ \gamma_\pi g_{\pi s} + \gamma_x g_{xs} + [0 \ 1 \ 0] \end{bmatrix}$$

But we still need to redefine f_α & f_β in light of the new PLM!

$$\text{If the PLM is } z_t = \Gamma z_{t-1} + (I_3 - \Gamma) \bar{z}_t + \rho s_{t-1} + e_t \quad (\text{PLM}_1)$$

$$\text{and } f_\alpha = \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \hat{E}_t \underline{z_T} \quad ; \quad f_\beta = \sum_{T=t}^{\infty} \beta^{T-t} \hat{E}_t z_T$$

then some generic df.

$$"f" = \sum_{T=t}^{\infty} \delta^{T-t} \left[\underbrace{\Gamma z_{T-1}}_{\bar{z}_T} + (I_3 - \Gamma) \bar{z}_T + \rho s_{T-1} \right]$$

this guy is causing trouble \rightarrow shut it off

$$\rightarrow \text{simplify the PLM to } z_t = \bar{z}_t + \rho s_{t-1} + e_t \quad (\text{PLM}_2)$$

then Actually: $\text{PLM}_3: z_t = \bar{z}_t + s_t$

drift

$$"f" = \sum_{T=t}^{\infty} \delta^{T-t} \left[\bar{z}_t + s_t + \rho s_{T-1} \right] \quad | \text{ anticipated whlty ass.}$$

$$= \sum_{T=t}^{\infty} \delta^{T-t} \bar{z}_t + \sum_{T=t}^{\infty} \delta^{T-t} \rho s_{T-1}$$

$$= \frac{1}{1-\delta} \bar{z}_t + \sum_{T=t}^{\infty} (\delta \rho)^{T-t} s_t$$

$$= \frac{1}{1-\delta} \bar{z}_t + \frac{1}{1-\delta \rho} s_t$$

$$\text{So } f_\alpha = \frac{1}{1-\alpha \beta} \bar{z}_t + (I_3 - \alpha \beta \rho)^{-1} s_t \quad \checkmark$$

$$f_\beta = \frac{1}{1-\beta} \bar{z}_t + (I_3 - \beta \rho)^{-1} s_t \quad \checkmark$$

So then the model is:

$$z_t = A_1 f_\alpha + A_2 f_\beta + A_3 s_t \quad (\text{RF})$$

$$f_\alpha = \frac{1}{1-\alpha\beta} \bar{z}_t + \frac{1}{1-\alpha\beta\rho} s_t \quad (\text{LR-}\epsilon_1)$$

$$f_\beta = \frac{1}{1-\beta} \bar{z}_t + \frac{1}{1-\beta\rho} s_t \quad (\text{LR-}\epsilon_2)$$

↳ A_i are given by p. 55, g_{ij} on p. 54.

$$s_t = \begin{bmatrix} r_t \\ i_t \\ u_t \end{bmatrix} = \underbrace{\begin{bmatrix} p_r & 0 & 0 \\ 0 & p_i & 0 \\ 0 & 0 & p_u \end{bmatrix}}_{=: P \text{ (capital)} \rho} s_{t-1} + \begin{bmatrix} \epsilon_r \\ \epsilon_i \\ \epsilon_u \end{bmatrix} \quad (\text{Exog. process})$$

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} (z_{t-1} - \hat{E}_{t-2} z_{t-1}) \quad (\text{LR-beliefs})$$

$$k_t = \Pi(k_{t-1} + 1) + (1 - \Pi) \bar{g}^{-1}$$

$$\Pi = \begin{cases} 1 & \text{if } \theta_t \leq \bar{\theta} \\ 0 & \text{else} \end{cases}$$

$$w/ \theta_L = \left| \hat{E}_{t-1} z_t - E_{t-1} z_t \right| / (\text{variance of noise})$$

↑
objective
expectations

(which might be
 $= \beta_r + \beta_i + \beta_u$
here.)

What is f_{t-1} ? I guess it's ALM - PLM, so

$$A_1 f_\alpha + A_2 f_\beta + A_3 S_{t-1} - (\bar{z}_{t-1} + \rho S_{t-2}) = S_{t-1} \text{ w/ PLM}_3$$

$$= A_1 \left[\frac{1}{1-\alpha\beta} \bar{z}_{t-1} + \frac{1}{1-\alpha\beta\rho} S_{t-1} \right] + A_2 \left[\frac{1}{1-\beta} \bar{z}_{t-1} + \frac{1}{1-\beta\rho} S_{t-1} \right]$$

$$+ A_3 S_{t-1} - \bar{z}_{t-1} - \rho S_{t-2} = S_{t-1}$$

$$= \left[A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} - I_3 \right] \bar{z}_{t-1}$$

$$+ \underbrace{\left[A_1 \frac{1}{1-\alpha\beta\rho} + A_2 \frac{1}{1-\beta\rho} + A_3 \right]}_{A_1(1-\alpha\beta\rho)^{-1} + A_2(1-\beta\rho)^{-1} + A_3 - I_3} S_{t-1} - \rho S_{t-2}$$

↑ need to make
sure S_{t-2}
doesn't enter here.

$$\theta_t = \left[A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} - I_3 \right] \bar{z}_{t-1}$$

diff from ALM - PLM about shocks ...

(... it seems to me but need to check this further!)

On Tuesday:

think w/ PLM₃ that's
ok near.

1. Check these things, correct S_{t-1}, S_{t-2} & θ_t
2. Type up (\rightarrow materials1) & simulate!

So

3 Sep 2015

$$f_{t+1} = \left[A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} - I_3 \right] \bar{\epsilon}_{t+1}$$

$$\left[A_1 (I_3 - \alpha\beta P)^{-1} + A_2 (I_3 - \beta P)^{-1} + A_3 - \bar{I}_3 \right] s_{t+1}$$

Next: check θ_t !

I checked in Experimentation Notes 1, p. 3,

4 Sep 2015

that $f_t = (6) - (5) = ALM - PLM$

There it also says $E[(6)]$

$$\theta_t = \frac{| \hat{E}_{t+1}^{(5)} \pi_t - \hat{E}_{t+1}^{(6)} \pi_t |}{\beta \eta}$$

which makes sense b/c $E[ALM]$ is the objective expectation that one should have given the model.

$$(\text{noise}) \theta_t = | PLM - E[ALM] |$$

ALM is evaluating f_a & f_b in RF

$$RF: z_t = A_1 f_a + A_2 f_b + A_3 s_t$$

$$z_t = A_1 \frac{1}{1-\alpha\beta} \bar{z}_{t-1} + A_1 (\mathbb{I}_3 - \alpha\beta P)^{-1} s_t + A_2 \frac{1}{1-\beta} \bar{s}_{t-1} + A_2 (\mathbb{I}_3 - \beta P)^{-1} s_t + A_3 s_t$$

$$\underline{z_t = \left(A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} \right) \bar{z}_{t-1} + \left(A_1 (\mathbb{I}_3 - \alpha\beta P)^{-1} + A_2 (\mathbb{I}_3 - \beta P)^{-1} + A_3 \right) s_t } \quad (ALM)$$

$$\therefore \theta_t = \left| \hat{E}_{t-1} z_t - E_{t-1} z_t \right| \cdot \text{(noise)}$$

$$E_{t-1} z_t = E_{t-1} [ALM]$$

$$= \left(A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} \right) \bar{z}_{t-1} + \left((\mathbb{I}_3 - \alpha\beta P)^{-1} + (\mathbb{I}_3 - \beta P)^{-1} + A_3 \right) P s_{t-1}$$

$$\hat{E}_{t-1} z_t = PLM(t-1) = \bar{z}_{t-1} + s_{t-1}$$

$$\begin{aligned} \text{So (noise)} \cdot \theta_t = & \left(\mathbb{I}_3 - A_1 \frac{1}{1-\alpha\beta} - A_2 \frac{1}{1-\beta} \right) \bar{z}_{t-1} \\ & + \underbrace{\left(\mathbb{I}_3 - \left(A_1 (\mathbb{I}_3 - \alpha\beta P)^{-1} + A_2 (\mathbb{I}_3 - \beta P)^{-1} + A_3 \right) P \right) s_{t-1}} \end{aligned}$$

1.) For COMP, s_{t-1} drops out b/c it enters PLM & ALM
the same way

2.) Need to be careful about timing. I think it's correct:

$$k_t = f_k(\bar{z}_{t-1}) \rightarrow \theta_t \text{ is a fit of } \bar{z}_{t-1} \text{ & } s_{t-1}$$

Let's try to put together model equations like in CEMP

App:

$$(ALM): \bar{z}_t = \left(A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} \right) \bar{z}_{t-1} + \left(A_1 (I_3 - \alpha\beta P)^{-1} + A_2 (I_3 - \beta P)^{-1} + A_3 \right) s_t$$

$$(\bar{z}): \bar{z}_t = \bar{z}_{t-1} + k_t^{-1} f_{t-1}$$

$$(k): k_t = f_k (\bar{z}_{t-1}, k_{t-1}, s_{t-1}) \quad (\text{the rule involving } \theta_t)$$

$$(f_{t-1}): f_{t-1} = \left[A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} - I_3 \right] \bar{z}_{t-1}$$

$$\left[A_1 (I_3 - \alpha\beta P)^{-1} + A_2 (I_3 - \beta P)^{-1} + A_3 - I_3 \right] s_{t-1}$$

$$(s): s_t = P s_{t-1} + \epsilon_t$$

Ryan meeting

4 Sep 2019

Idea that CB can change $E(r)^{LR}$ ($= LR$ growth prospects) \rightarrow he finds that not quite credible!

The story of more aggressive optimal mon. pol. is more credible!

Peter Neelings

5 Sep 2019

- loss fit & parametric family of TRs
you can see how changing params on TR affects the loss (\rightarrow grid-search over params of TR)
- loss fit : quadratic on π & x
- Extension: do a welfare-theoretic exercise
In FIRE, the monpol. can offset 1:1 real rate shocks
 \rightarrow would be interesting to see whether this holds for learning.

Work after

10 Sep 2019

Goal: get s_t out of f_t !

1) f_{t-1} is wrong. It's

$$\bar{z}_{t-1} - \hat{E}_{t-2} \bar{z}_{t-1}$$

or

$$A_t M_{t-1} - P_t M_{t-1} I_{t-2}$$

$$B_1 \bar{z}_{t-1} + B_2 s_{t-1} - (\bar{z}_{t-2} + s_{t-2})$$

$$f_{t-1} = B_1 \bar{z}_{t-1} - \bar{z}_{t-2} + B_2 s_{t-1} - s_{t-2} \quad \text{VAR w/ 2 lags.}$$

$$\begin{aligned}
 \text{noise} \cdot \theta_t &= |\hat{E}_{t-1} \bar{z}_t - E_{t-1} \bar{z}_t| \\
 &= |PLM_{t+1-t} - E_{t-1} (ALM_t)| \\
 &= |\bar{z}_{t-1} + s_{t-1} - (B_1 \bar{z}_{t-1} + B_2 P s_{t-1})| \\
 &= |(I_3 - B_1) \bar{z}_{t-1} + \underbrace{(I_3 - B_2 P)}_{\text{for this to cancel, we}} s_{t-1}|
 \end{aligned}$$

need $B_2 P = I_3$

Now does LEMP do it?

$$\begin{aligned}
 PLM_{t+1-t}^{\text{COMP}} &= \gamma \pi_{t-1} + (1-\gamma) \bar{\pi}_t + \rho \psi_{t-1} \\
 E_{t-1} ALM_t &= \gamma \pi_{t-1} + (1-\gamma) \Gamma \bar{\pi}_t + \rho \psi_{t-1} + E(\eta_t) = 0
 \end{aligned}$$

Right now, my PLM is:

$$PLM_3 = E_t \bar{z}_{t+1} = \bar{z}_t + s_t + e_{t+1}$$

$$\bar{z}_t = \bar{z}_{t-1} + s_{t-1} + e_t$$

$$\text{try } \bar{z}_t = \bar{z}_{t-1} + C s_{t-1} + e_t \quad (PLM_4)$$

\uparrow not yet determined.

$$\begin{aligned}
 f_a &= \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} z_{T+1} = \frac{1}{1-\alpha \beta} \bar{z}_t + \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} C s_T \\
 &= \frac{1}{1-\alpha \beta} \bar{z}_t + C \sum_{T=t}^{\infty} (\alpha \beta P)^{T-t} s_t
 \end{aligned}$$

$$f_a = \frac{1}{1-\alpha\beta} \bar{z}_t + C(1-\alpha\beta\rho)^{-1} s_t$$

$$f_b = \frac{1}{1-\beta} \bar{z}_t + C(1-\beta\rho)^{-1} s_t$$

So B_1 will always be the same in the ALM.

B_2 modifies to

$$B_2 = A_1 C (1-\alpha\beta\rho)^{-1} + A_2 C (I_3 - \beta\rho)^{-1} + A_3$$

What if we try a PLM₄

$$\text{PLM}_4 = z_t = \bar{z}_{t-1} + P s_{t-1} + e_t$$

→ then P takes the place of C , which is

1) not super helpful 2) but reasonable

$$z_t = B_1 \bar{z}_t + B_2 s_t \quad (\text{ALM}) \rightarrow B_1 \bar{z}_t + B_2 P s_{t-1} + B_2 e_t$$

$$\text{ALM}_{t|t-1} = B_1 \bar{z}_{t-1} + B_2 P s_{t-1}$$

If shocks are iid, then $P = \text{zeros}$

$$(\text{not } x) \theta_t = (I_3 - B_1) \bar{z}_{t-1}$$

Ok, so following this logic, keep PLM_4 , and let C

solve the equation $B_2 P \stackrel{!}{=} I_3$

$$\Leftrightarrow (A_1 C (I - \alpha \beta P)^{-1} + A_2 C (I_3 - \beta P)^{-1} + A_3) P = I_3$$

→ do it on mathematica!

It's running, takes a while... (started 3.45 pm)

Took almost 10 min.

Even to show more takes 5 min!

Show all (started 4.01 pm) - crashed :S

$$\begin{aligned} f_{t-1} &= z_{t-1} - \tilde{e}_{t-2} z_{t-1} \\ &= (B_1 \tilde{z}_{t-1} + B_2 s_{t-1}) - (\tilde{e}_{t-2} + C s_{t-2}) \end{aligned}$$

$$f_{t-1} = B_1 \tilde{z}_{t-1} - \tilde{e}_{t-2} + B_2 s_{t-1} - C s_{t-2}$$

One thing I did is to artificially shut down s_{t-1} in θ_t .

→ doesn't change the fact that θ needs to be large!

So I think I can leave θ_t as a function of s , it's ok.

→ Try to write f_{t-1} simpler!

B_1, B_2 same every time! Correct? → Yes!

Robert Wright meeting

10 Sep 2019

tell about his/my research

can I come to you for job market advice?

can you look them / give feedback on proposal /

& job market documents

external letter writer - valuable?

He said about info project: try the setting where B can only comment about today, but E(tomorrow) matter as well → what's the optimal comment lead there? Do we see the same tradeoff?

$$f_{t-1} = z_{t-1} - \hat{E}_{t-2} z_{t-1}$$

11 Sep 2019

$$\text{In CEMP: } \gamma \pi_{t-2} + (1-\gamma) \bar{\pi} \bar{\pi}_{t-1} + \rho \psi_{t-2} + \eta_{t-1}$$

$$- (\gamma \pi_{t-2} + (1-\gamma) \bar{\pi} \underline{\pi}_{t-1} + \rho \psi_{t-2})$$

$$f_{t-1} = (1-\gamma)(\bar{\pi}-1) \bar{\pi}_{t-1} + \eta_{t-1}$$

key: if z_t is the fastest thing, the PLM needs to feature \bar{z}_t and s_{t-1}

PLM₅:

$$z_t = \bar{z}_t + p s_{t-1} + e_t$$

$$\hat{E}_t z_{t+1} = \bar{z}_{t+1} + p s_t$$

$$\hat{E}_{t-1} z_t = \bar{z}_t + p s_{t-1}$$

Then

$$\begin{aligned} f_t &= \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \hat{E}_T z_{T+1} = \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \bar{z}_{t+1} + \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} p s_T \\ &= \frac{1}{1-\alpha\beta} \bar{z}_{t+1} + p \sum_{T=t}^{\infty} (\alpha\beta p)^{T-t} s_T \\ &= \frac{1}{1-\alpha\beta} \bar{z}_{t+1} + p (I_n - \alpha\beta p)^{-1} s_t \end{aligned}$$

B_1 unchanged, B_2 unchanged except $C = P$.

$$\Rightarrow ALM \text{ is } z_t = B_1 \bar{z}_{t+1} + B_2 s_t$$

$$\text{So } ALM_{t-1} \text{ is } z_{t-1} = B_1 \bar{z}_t + B_2 s_{t-1}$$

$$PLM_{t-1, t-2} = \hat{E}_{t-2} z_{t-1} = \bar{z}_{t-1} + p s_{t-2}$$

$$\text{At least you can say } f_{t-1} = z_{t-1} - \hat{E}_{t-2} z_{t-1}$$

$$= B_1 \bar{z}_t + B_2 p s_{t-2} + B_2 e_{t-1} - \bar{z}_{t-1} - p s_{t-2}$$

$$= B_1 \bar{z}_t - \bar{z}_{t-1} + (B_2 - I_3) p s_{t-2} + B_2 e_{t-1}$$

but you can do that in the current formulation too, and \bar{z} stay a^{n+1} col.

Let's redo the current PLM calculation (PLM_4)

$$\hat{E}_t \bar{z}_{t+1} = \bar{z}_t + CS_t + \epsilon_{t+1}$$

$$\bullet \quad \bar{z}_t = \bar{z}_{t-1} + CS_{t-1} + e_t \quad (= PLM_4)$$

The difference between PLM_4 & PLM_5 is CEMD's ass. (5.5): the "shifting end-point model"

$$\hat{E}_{t-1} \bar{\pi}_T = \bar{\pi}_t \quad \forall T \geq t \quad (\rightarrow PLM_5)$$

B. PLM_4 implicitly assumes $\hat{E}_{t-1} \bar{\pi}_T = \bar{\pi}_{t-1}, \forall T \geq t$

The diff isn't substantial I don't think: it's notational.

To be precise, CEMD's ass (5.5) means that

$$\bar{\pi}_t = \bar{\pi}_{t|t-1} \quad (\text{that is } \bar{\pi}_{t+1} = \bar{\pi}_{t+1|t})$$

which is what PLM_5 captures.

I'm going to use the modified (5.5) of CEMD:

$$\hat{E}_{t-1} \bar{z}_T = \bar{z}_{t-1} \quad \forall T \geq t \quad \text{or} \quad \hat{E}_t \bar{z}_T = \bar{z}_t \quad \forall T > t$$

in accordance w/ PLM_4 (which is used in materials 2)

which also means $\bar{z}_t = \bar{z}_{T|t} \quad \forall T > t$

Then, for the current PLM, PLM_4 ,

$$f_t = \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} z_{T+1} = \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left(\bar{z}_t + (s_{T+1}) \right) \\ = \bar{z}_{t+1|t}$$

$$= \frac{1}{1-\alpha\beta} \bar{z}_t + \left(\sum_{T=1}^{\infty} (\alpha\beta)^{T-t} p^{T-t} s_t \right)$$

$$= \frac{1}{1-\alpha\beta} \bar{z}_t + C (I_3 - \alpha\beta P)^{-1} s_t \quad = \text{eq (22) of materials?}$$

and so the ALM will equal eq. (24).

$$\text{and so } f_{t-1} = z_{t-1} - \hat{E}_{t-2} z_{t-1}$$

$$= B_1 \bar{z}_{t-1} + B_2 s_{t-1} - \left(\bar{z}_{t-2} + (s_{t-2}) \right)$$

which is indeed what I have in materials 2.

Start to formulate a loss function

- initially, just check how for the same simulation, loss responds to changing the params of the Taylor rule
- then find optimal $(\gamma_\pi^*, \gamma_x^*)$

(Landa, Gallo & Göttsche (1999))'s loss (eq. 2.7)

$$\max -\frac{1}{2} E_t \left[\sum_{i=0}^{\infty} \beta^i \left[\alpha x_{t+i}^2 + \pi_{t+i}^2 \right] \right]$$

$$\begin{aligned} \mathcal{L} &= -E_t \left[\sum_{i=0}^{\infty} \beta^i [\alpha x_{t+i}^2 + \pi_{t+i}^2] \right] \\ &= -E_t \sum_{i=0}^{\infty} \beta^i [1 \circ \sigma] \begin{bmatrix} z(1)^2 \\ z(2)^2 \\ z(3)^2 \end{bmatrix} \end{aligned}$$

elementwise product: \circ

TURNS OUT THAT THIS KIND OF OPERATION IS KNOWN IN MATH AS THE HADAMARD PRODUCT OR THE SCHUR PRODUCT.

$$A \circ B = A \odot B = \underset{m \times n}{a_{ij}} \cdot \underset{m \times n}{b_{ij}} \quad \forall i, j$$

In LaTeX, \circ is \circ, and \odot is \odot

so

$$\mathcal{L} = -E_t \sum_{i=0}^{\infty} \beta^i [1 \circ \sigma](z \odot z)$$

\uparrow
Weight on output gap
(have to denote it w/some else,
so maybe γ ?)

Ryan meeting

11 Sep 2019

One idea: have only one thing (π_i) that they can learn about:
LR level of x and i fixed.

- If knows not $x=0$ in LR

- $\pi_i = \text{vec}(\pi)$ in LR

The C-issue:

If P strictly positive

$$P^T I_3 = B_2(C)$$



vec-operator

- maybe it's a result that w/ these ^{TR-} corps, it's hard to get anchored $E(\cdot)$.

- Solve special cases: would be valuable to solve
e.g. Preston \rightarrow solve the simulation
- Recursive representation of the economy (BM)
if it's easier as a start \rightarrow can see what's happening

Simul w/ decreasing gain

RE

BM

LR-exp

run expectation, adding N expectation terms
at a time

⇒ can tease out where errors are b/c
you should eventually get the same thing.

- projection facility → it might solve some of the
explosion issues.
Liam Graham paper

Yes - he is willing to look at my application
documents, yay! But: it'd be good to have
some results to point to!

By the way, he also said TR corrs between 1 to 5
were plausible.

Work after

12 Sep 2015

First, a summary:

① To the model as it now is:

- ① Have only 1 thing for agents to learn about
- ② Maybe the explorations are a result: harder to get anchoring w/ these TR coeffs than it would be otherwise
- ③ Maybe the projection facility can get around explorations
(Liam Graham paper)
- ④ If you restrict P to be strictly positive, then you can try to use $\text{rec}(\cdot)$ to see if you can solve the problem.

↳ I think it suffices to have $P = \begin{pmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{pmatrix}$

② Special cases (and I think this is what we should focus on for materials 3)

- Simulate RE model
- Simulate EE model (BM) ↪ in the $\sum \text{EXP}$, add one term at a time to see diff's between EE & LR
- Simulate LR model (Preston)
- Simulate LR model w/ anchoring (EMP-Preston mix, call id LAR)

In the RE-EE case, plug TR into i:

$$x_+ = E_+ x_{+-1} - \beta (i_+ + \gamma_\pi \pi_+ + \gamma_x x_+ - E_+ \pi_{+-1}) + \beta r_+^N \\ = E_+ x_{+-1} + \beta E_+ \pi_{+-1} - \beta \gamma_\pi \pi_+ - \beta \gamma_x x_+ - \beta i_+ + \beta r_+^N$$

$$(1 + \beta \gamma_x) x_+ = -\beta \gamma_\pi \pi_+ + E_+ x_{+-1} + \beta E_+ \pi_{+-1} - \beta i_+ + \beta r_+^N \\ = -\beta \gamma_\pi [K x_+ + \beta E_+ \pi_{+-1} + u_+] + \text{rest}$$

$$\underbrace{(1 + \beta \gamma_x - K \beta \gamma_\pi)}_{= w} x_+ = E_+ x_{+-1} + (b - \beta \gamma_\pi / \beta) E_+ \pi_{+-1} - \beta \gamma_\pi u_+ - \beta i_+ + \beta r_+^N \\ \rightarrow x_+ = \underbrace{\left[\frac{(b - \beta \gamma_\pi / \beta)}{w}, \frac{1}{w}, 0 \right] E_+ z_{+-1} + \left[\frac{b}{w}, -\frac{3}{w}, -\frac{\beta \gamma_\pi}{w} \right] s_+}_{= \underline{\underline{s}_+}}$$

$$\pi_+ = K \left[\left(\frac{b - \beta \gamma_\pi / \beta}{w}, \frac{1}{w}, 0 \right] E_+ z_{+-1} + K \left[\frac{b}{w}, -\frac{3}{w}, -\frac{\beta \gamma_\pi}{w} \right] s_+ \right. \\ \left. + \beta E_+ \pi_{+-1} + u_+ \right]$$

$$\pi_+ = \underbrace{\left[\beta + K \left(\frac{b - \beta \gamma_\pi / \beta}{w} \right), \frac{K}{w}, 0 \right] E_+ z_{+-1} + \left[\frac{Kb}{w}, -\frac{Kb}{w}, \left(1 - \frac{K \beta \gamma_\pi}{w} \right) \right] s_+}_{= \underline{\underline{s}_+}}$$

$$i_+ = \gamma_\pi \left(\left[\left(\beta + K \left(\frac{b - \beta \gamma_\pi / \beta}{w} \right), \frac{K}{w}, 0 \right] E_+ z_{+-1} + \left[\frac{Kb}{w}, -\frac{Kb}{w}, \left(1 - \frac{K \beta \gamma_\pi}{w} \right) \right] s_+ \right) \right. \\ \left. + \gamma_x \left(\left[\left(\frac{b - \beta \gamma_\pi / \beta}{w}, \frac{1}{w}, 0 \right] E_+ z_{+-1} + \left[\frac{b}{w}, -\frac{3}{w}, -\frac{\beta \gamma_\pi}{w} \right] s_+ \right) \right. \right. \\ \left. \left. + b_+ \right) \right)$$

$$i_t = \gamma_\pi \left(\left[\left(\beta + \frac{K(b - 3\gamma_\pi \beta)}{w} \right), \frac{K}{w}, 0 \right] E_t z_{t+1} + \left[\frac{K^2}{w}, -\frac{K^2}{w}, \left(1 - \frac{K^2 \gamma_\pi}{w} \right) \right] s_t \right)$$

$$+ \gamma_x \left(\left[\frac{(b - 3\gamma_\pi \beta)}{w}, \frac{1}{w}, 0 \right] E_t z_{t+1} + \left[\frac{3}{w}, -\frac{3}{w}, -\frac{3\gamma_\pi}{w} \right] s_t \right)$$

$$+ b_t$$

$$\underline{i_t = \left[\gamma_\pi \left(\beta + \frac{K(b - 3\gamma_\pi \beta)}{w} \right) + \gamma_x \left(\frac{(b - 3\gamma_\pi \beta)}{w} \right), \gamma_\pi \frac{K}{w} + \gamma_x \frac{1}{w}, 0 \right] E_t z_{t+1} - \left[\gamma_\pi \frac{K^2}{w} + \gamma_x \frac{3}{w}, \gamma_\pi \left(-\frac{K^2}{w} \right) + \gamma_x \left(-\frac{3}{w} \right) + 1, \gamma_\pi \left(1 - \frac{K^2 \gamma_\pi}{w} \right) + \gamma_x \left(-\frac{3\gamma_\pi}{w} \right) \right] s_t}$$

So the RE model's LOM is: | and the EE model's LOM is:

$$z_t = A_p^{RE} E_t z_{t+1} + A_s^{RE} s_t | \quad \hat{z}_t = A_p^{RE} \hat{E}_t \hat{z}_{t+1} + A_s^{RE} s_t$$

with

$$A_p^{RE} = \begin{cases} \left[\left(\beta + \frac{K(b - 3\gamma_\pi \beta)}{w} \right), \frac{K}{w}, 0 \right] \\ \left[\frac{(b - 3\gamma_\pi \beta)}{w}, \frac{1}{w}, 0 \right] \\ \left[\gamma_\pi \left(\beta + \frac{K(b - 3\gamma_\pi \beta)}{w} \right) + \gamma_x \left(\frac{(b - 3\gamma_\pi \beta)}{w} \right), \gamma_\pi \frac{K}{w} + \gamma_x \frac{1}{w}, 0 \right] \end{cases}$$

(material 8, Experimentation)

$$A_s^{RE} = \begin{cases} \left[\frac{K^2}{w}, -\frac{K^2}{w}, \left(1 - \frac{K^2 \gamma_\pi}{w} \right) \right] \\ \left[\frac{3}{w}, -\frac{3}{w}, -\frac{3\gamma_\pi}{w} \right] \\ \left[\gamma_\pi \frac{K^2}{w} + \gamma_x \frac{3}{w}, \gamma_\pi \left(-\frac{K^2}{w} \right) + \gamma_x \left(-\frac{3}{w} \right) + 1, \gamma_\pi \left(1 - \frac{K^2 \gamma_\pi}{w} \right) + \gamma_x \left(-\frac{3\gamma_\pi}{w} \right) \right] \end{cases}$$

is wrong.

ALM of EE:

$$\hat{E}_t z_{t+1} = \phi_t + P s_t$$

So ALM is:

$$z_t = A_p^{RE} (\phi_t + P s_t) + A_s^{RE} s_t$$

$$z_t = A_p^{RE} \phi_t + (A_p^{RE} P + A_s^{RE}) s_t$$

ALM of LR

$$\begin{aligned} f_a &= \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} z_{T+1} = \hat{E}_t (z_{t+1} + \alpha\beta(z_{t+2}) + \dots) \\ &= \hat{E}_t z_{t+1} + \alpha\beta \hat{E}_t z_{t+2} + \dots \\ &= \phi_t + P s_t + \alpha\beta(\phi_t + P^2 s_t) + \dots \\ &= \sum_{n=0}^{\infty} (\alpha\beta)^n \phi_t + P s_t + \alpha\beta P^2 s_t + (\alpha\beta)^2 P^3 s_t + \dots \\ &= \frac{1}{1-\alpha\beta} \phi_t + P(s_t + \alpha\beta P s_t + (\alpha\beta P)^2 s_t + \dots) \\ &= \frac{1}{1-\alpha\beta} \phi_t + P(I_3 + \alpha\beta P + (\alpha\beta P)^2 + \dots) s_t \\ &= \frac{1}{1-\alpha\beta} \phi_t + P(I_3 - \alpha\beta P)^{-1} s_t \end{aligned}$$

So the ALM is

$$\begin{aligned} z_t &= A_a f_a + A_b f_b + A_s s_t \\ &= A_a \left(\frac{1}{1-\alpha\beta} \phi_t + P (\mathbb{I}_3 - \alpha\beta P)^{-1} s_t \right) \\ &\quad + A_b \left(\frac{1}{1-\beta} \phi_t + P (\mathbb{I}_3 - \beta P)^{-1} s_t \right) \\ &\quad + A_s s_t \end{aligned}$$

$$\begin{aligned} z_t &= \left(A_a \frac{1}{1-\alpha\beta} + A_b \frac{1}{1-\beta} \right) \phi_t \\ &\quad + \left(A_a P (\mathbb{I}_3 - \alpha\beta P)^{-1} + A_b P (\mathbb{I}_3 - \beta P)^{-1} + A_s \right) s_t \end{aligned}$$