

Materials 39 - Combing through the code for bugs

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Overview

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1 Look into measurement error

1.1 Autocovariograms w/ and w/o measurement error, not using expectations data

Figure 1: $N = 100$, w/o expectations data, estimate moments N times (N estimations), imposing convexity with weight 100K, truth with $nfe = 5$, $fe \in (-2, 2)$

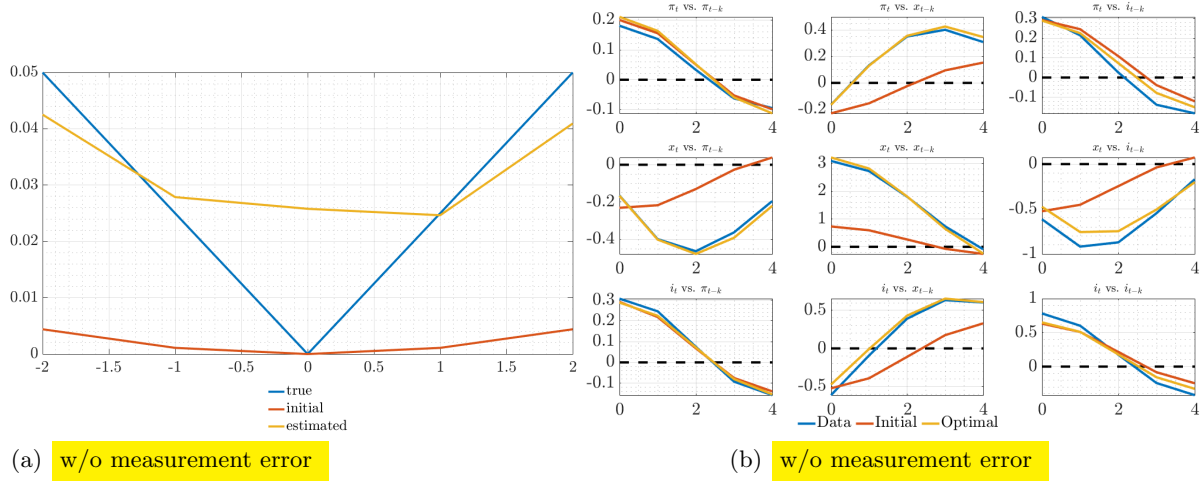
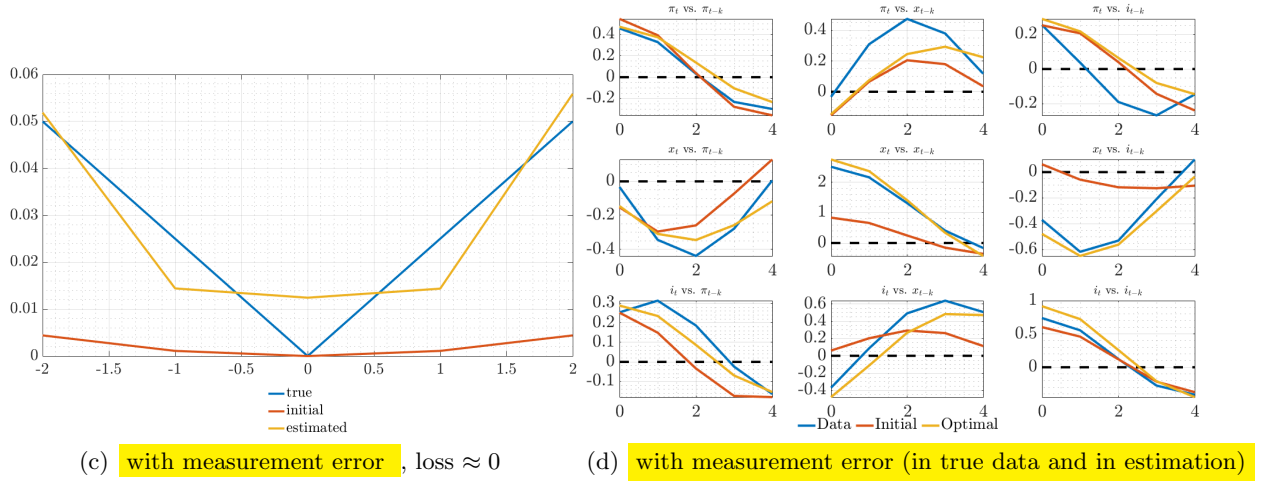


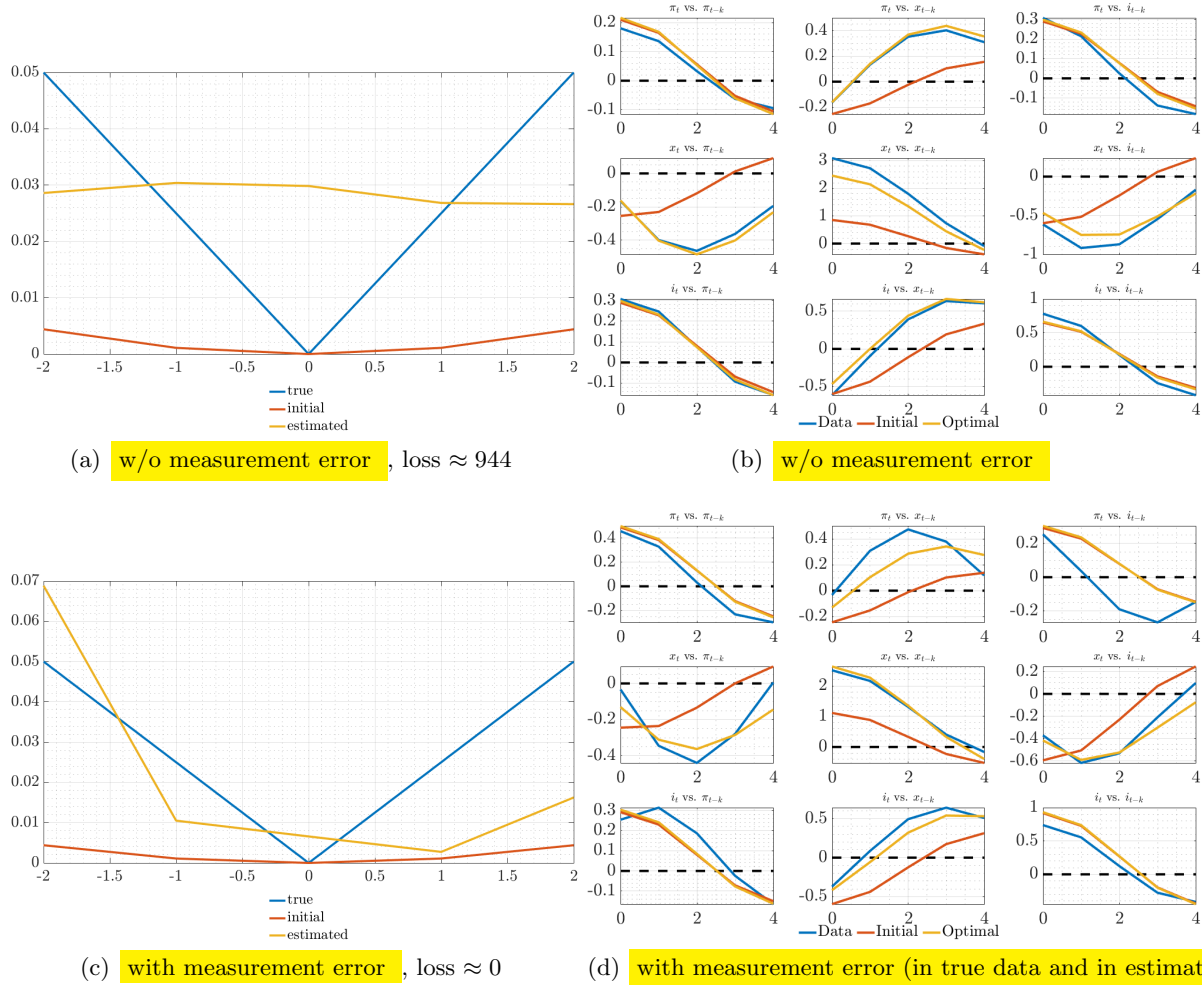
Figure 1(a) nearly = Fig. 1 of Materials 37, loss ≈ 787



The measurement error definitely shows up.

- It should change the true moments, b/c those are just one simulation.
- For N estimations, it should change the initial moments, and it does.

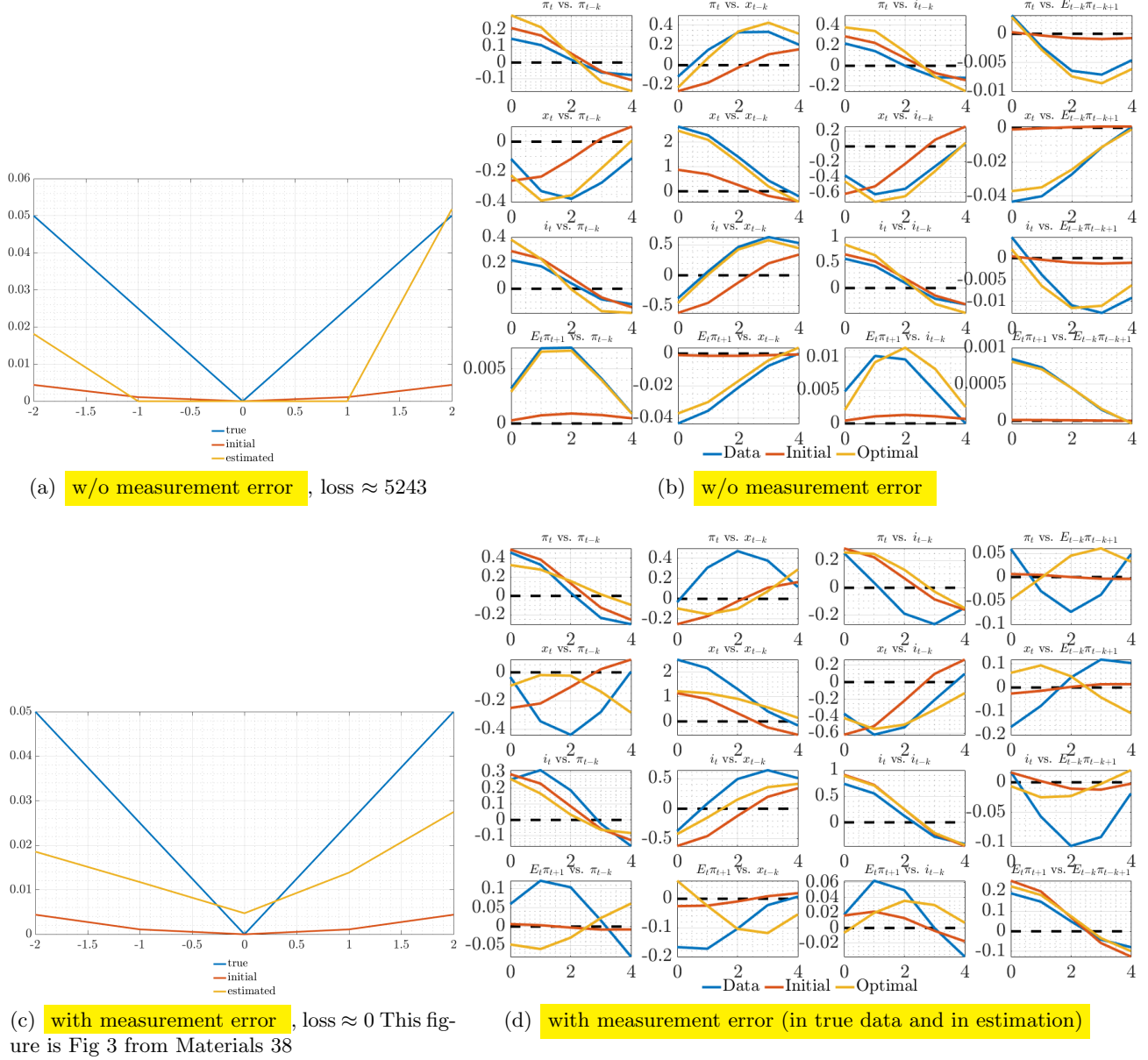
Figure 2: $N = 100$, w/o expectations data, **estimate mean moments once (N simulations)**, imposing convexity with weight 100K, truth with $nfe = 5$, $fe \in (-2, 2)$



- For N simulations, it shouldn't change the initial moments, and it doesn't either.
- But it shouldn't affect the procedure's ability to match the moments. It does though.
- A phenomenon I also can't explain is why losses become orders of magnitude smaller just by using the data that was generated with measurement error in it.

1.1.1 Put expectations back in: w/ measurement error, or w/o

Figure 3: $N = 100$, w/ expectations data, **estimate mean moments once (N simulations)**, imposing convexity with weight 100K, truth with $nfe = 5$, $fe \in (-2, 2)$

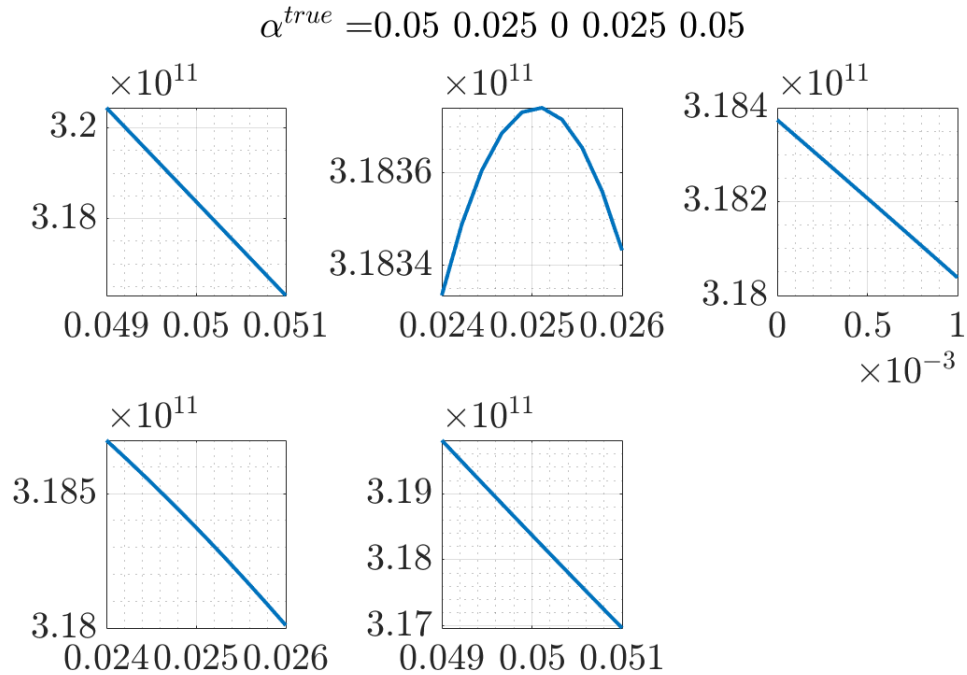
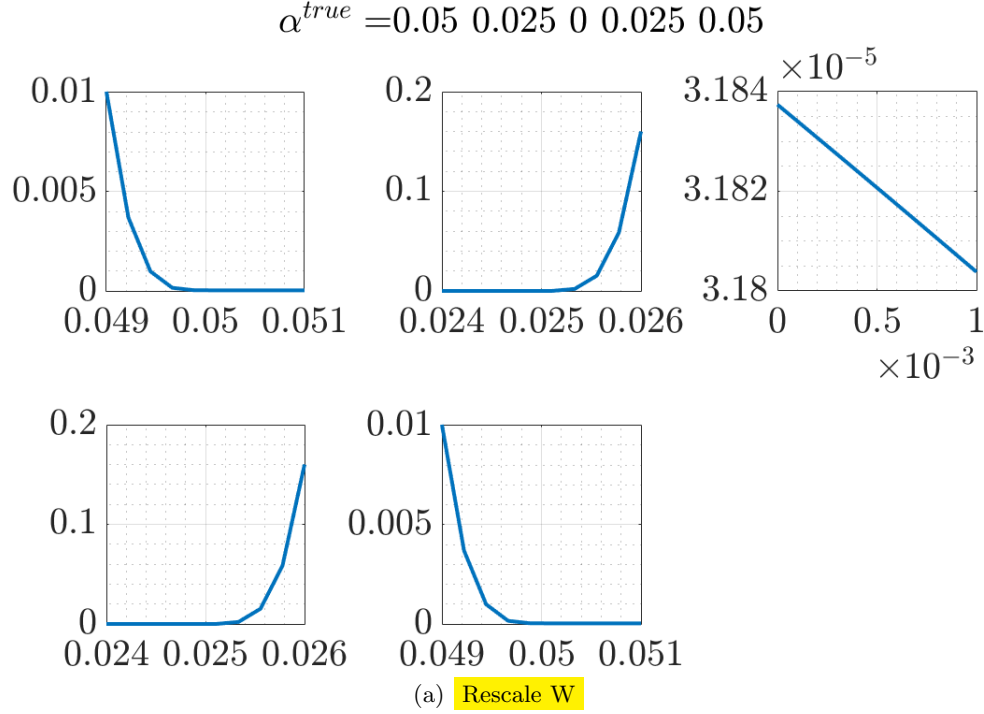


Clearly the measurement error is responsible for the misses!

1.2 Loss for holding α s at true values, and varying one at a time

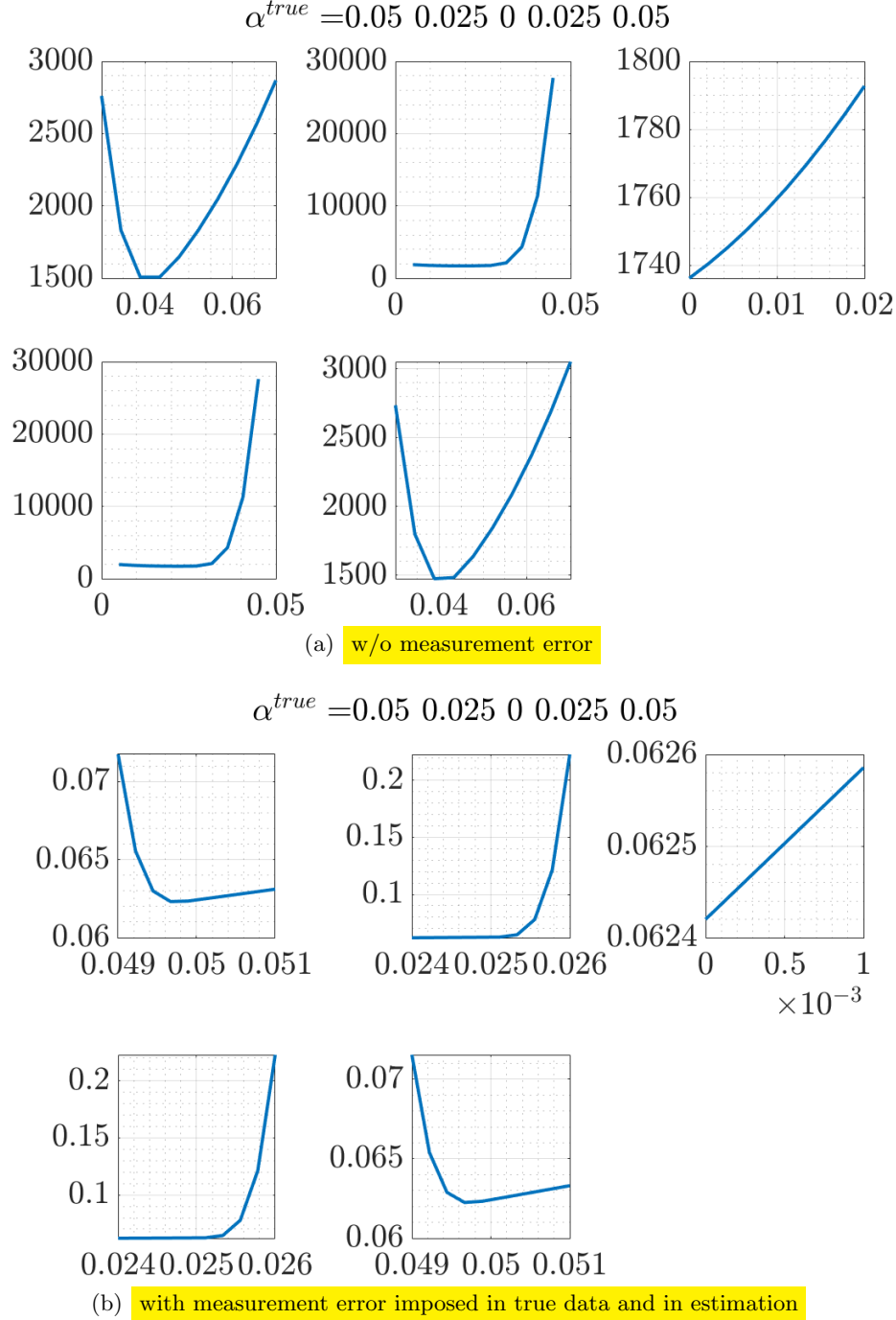
1.2.1 Reference: materials 38

Figure 4: Loss for $N = 100$, incl. 1-step ahead forecasts of inflation, **estimate mean moments once (N simulations)**, imposing convexity with weight 100K, with measurement error imposed, truth with $nfe = 5, fe \in (-2, 2)$



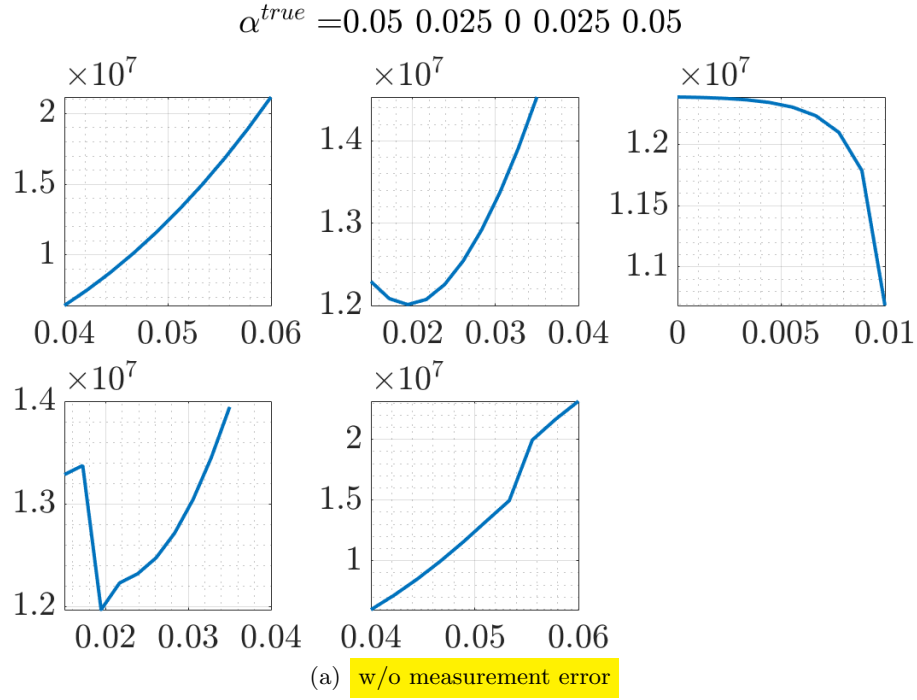
1.2.2 Loss when not using expectations data, all else same as Materials 38

Figure 5: Loss for $N = 100$, **NOT** using 1-step ahead forecasts of inflation, estimate mean moments once, imposing convexity with weight 100K, , truth with $nfe = 5$, $fe \in (-2, 2)$



Note: If I don't use expectations variables, the tiny W issue disappears. So those are the moments that cause the rescaling issue in the first place. Therefore I'm not rescaling here.

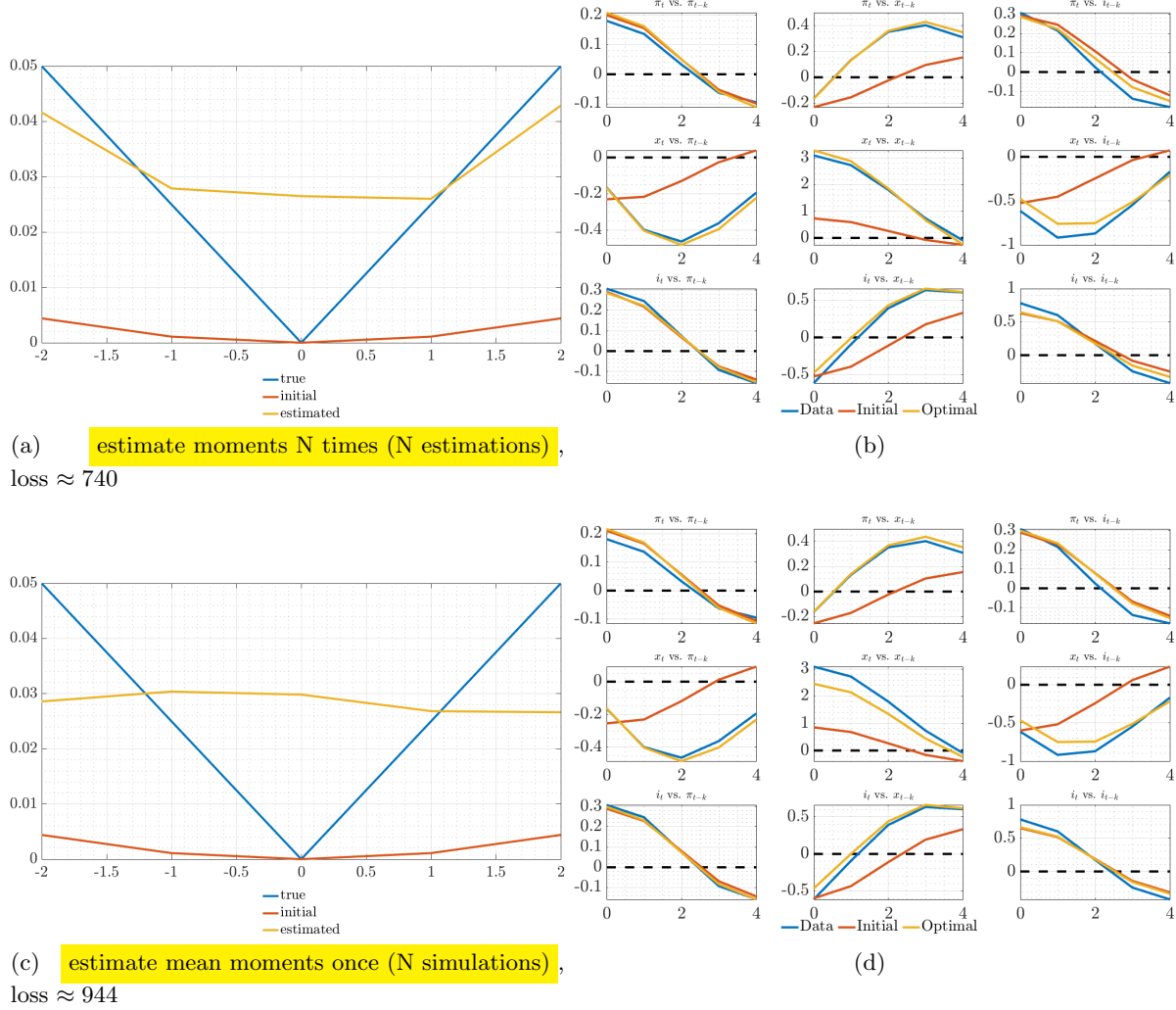
Figure 6: Loss for $N = 100$, using 1-step ahead forecasts of inflation, estimate mean moments once, imposing convexity with weight 100K, , truth with $nfe = 5, fe \in (-2, 2)$



- the 0-forecast-error α was reversed
- also reversed: now middle α s identified, although a little too low, but the sides don't seem so

1.2.3 Get rid of k^{-1} in learning code, just use k (“uninvert k ”)

Figure 7: $N = 100$, w/o expectations data, w/o measurement error, imposing convexity with weight 100K, truth with $nfe = 5$, $fe \in (-2, 2)$



A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (\text{A.1})$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (\text{A.2})$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \quad (\text{if imposed}) \quad (\text{A.3})$$

$$\text{PLM:} \quad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \quad b = g_x h_x \quad (\text{A.4})$$

$$\text{Updating:} \quad a_t = a_{t-1} + k_t^{-1} (z_t - (a_{t-1} + b s_{t-1})) \quad (\text{A.5})$$

$$\text{Anchoring function:} \quad k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k f e_{t-1}^2 \quad (\text{A.6})$$

$$\text{Forecast error:} \quad f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \quad (\text{A.7})$$

$$\text{LH expectations:} \quad f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \quad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \quad (\text{A.8})$$

This notation captures vector learning (z learned) for intercept only. For scalar learning, $a_t = (\bar{\pi}_t \ 0 \ 0)'$ and b_1 designates the first row of b . The observables (π, x) are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix} \sigma & 1-\beta & -\sigma\beta \end{bmatrix} f_b + \sigma \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \quad (\text{A.9})$$

$$\pi_t = \kappa x_t + \begin{bmatrix} (1-\alpha)\beta & \kappa\alpha\beta & 0 \end{bmatrix} f_a + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \quad (\text{A.10})$$

B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only, $k_t^{-1} = \mathbf{g}(f e_{t-1})$):

$$\pi_t = -\frac{\lambda_x}{\kappa} \left\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_\pi(t) \right) \right. \\ \left. \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\bar{\pi}}(t+j)) \right) \right\} \quad (\text{B.1})$$

where I'm using the notation that $\prod_{j=0}^0 \equiv 1$. For interpretation purposes, let me rewrite this as follows:

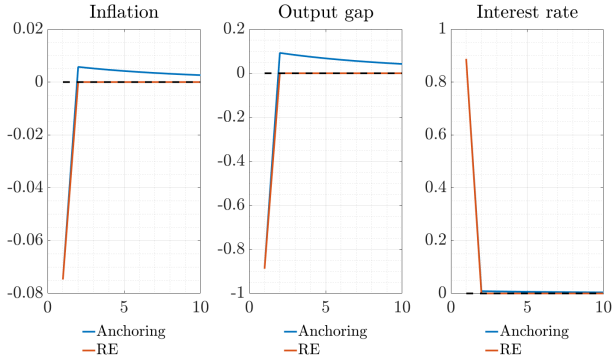
$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ - \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j}^{-1} + f e_{t+1+j|t+j}^{eve} \mathbf{g}_{\bar{\pi}}(t+j)) \right) \quad (\text{B.2})$$

Interpretation: **tradeoffs from discretion in RE** + **effect of current level and change of the gain on future tradeoffs** + **effect of future expected levels and changes of the gain on future tradeoffs**

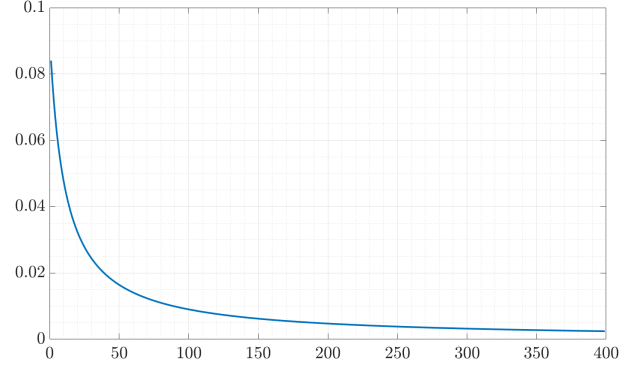
C Impulse responses to iid monpol shocks across a wide range of learning models

$T = 400, N = 100, n_{drop} = 5$, shock imposed at $t = 25$, calibration as above, Taylor rule assumed to be known, PLM = learn constant only, of inflation only.

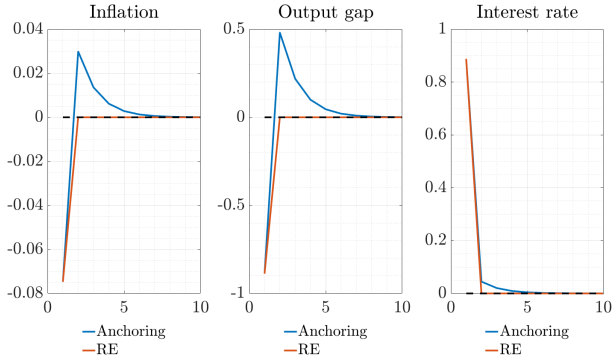
Figure 8: IRFs and gain history (sample means)



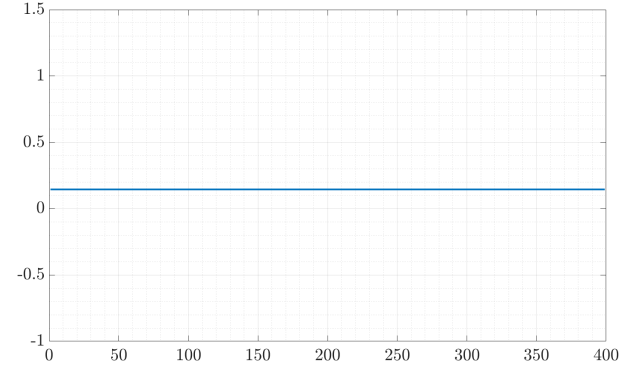
(a) Decreasing gain learning



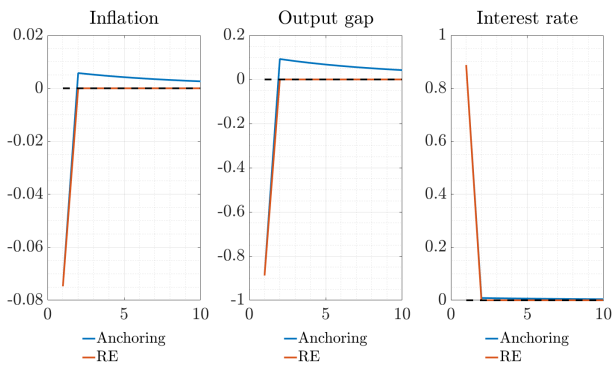
(b) Mean gain



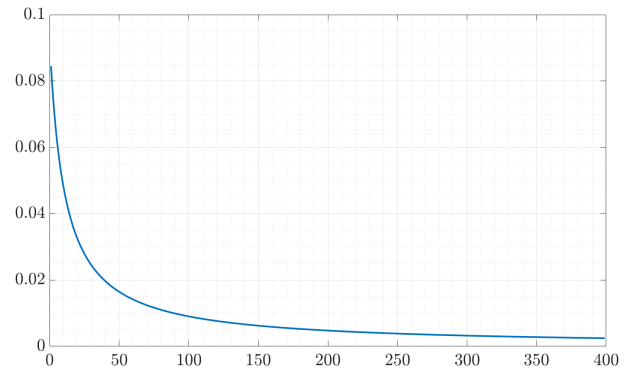
(c) Constant gain learning



(d) Mean gain

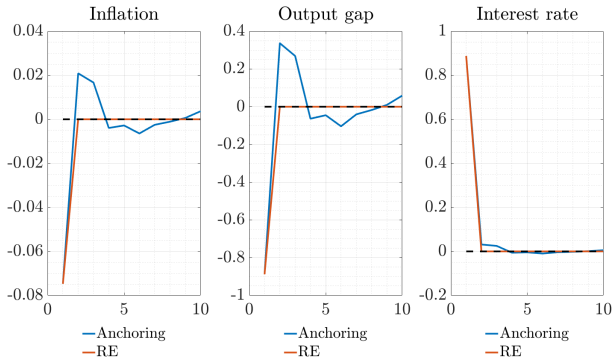


(e) CEMP criterion (vector)

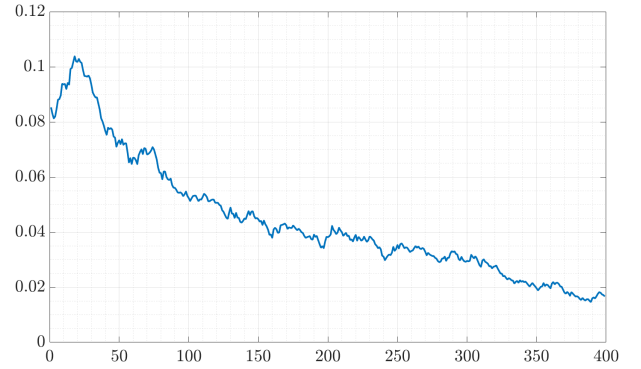


(f) Mean gain

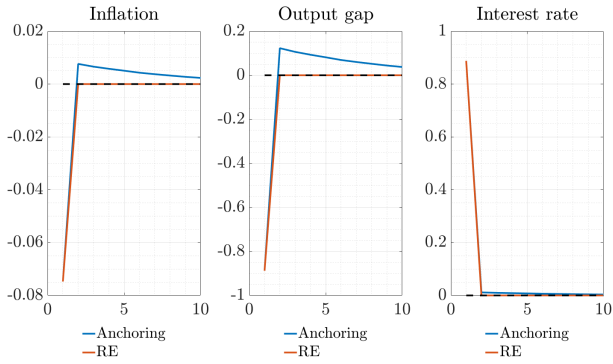
Figure 9: IRFs and gain history (sample means), continued



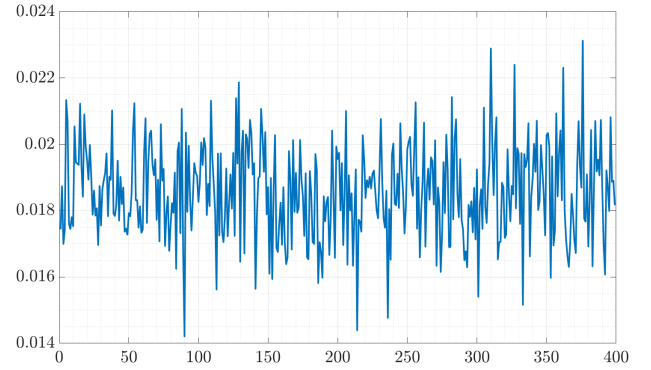
(a) CUSUM criterion (vector)



(b) Mean gain



(c) Smooth criterion, approximated, using $\alpha^{true} = (0.05; 0.025; 0; 0.025; 0.05)$, on $fe \in (-2, 2)$.



(d) Mean gain