Work after 9 Nov 2019

From the FEs correctly:

$$f_t^m = \hat{E}_t(\pi_{t+1} \mid \mathcal{I}_t^m) = \hat{E}_t(\pi_{t+1} \mid s_t, \bar{\pi}_{t-1})$$

$$f_t^e = \hat{E}_t(\pi_{t+1} \mid \mathcal{I}_t^e) = \hat{E}_t(\pi_{t+1} \mid s_t, \bar{\pi}_t)$$

$$FE_t^m = \pi_{t+1} - f_t^m$$

$$FE_t^e = \pi_{t+1} - f_t^e$$

} so in either case I just need to subtract F_t from $y_{sim_{t+1}}$.

and note that these FE_t are realized at $t+1$.

The problem is that the FEs I construct this way aren't equal to the ones I get from the sim_learn.m code. This is puzzling b/c in principle they come from the same simulated π – same test of π.

The problem is that the FE coming out of sim_learn

1) always changes, despite IRF-ing & averaging
2) There are diffs btwn $FE^{shocked}$ & $FE^{unshocked}$ even before I impose the shock δ (!)

FEs are solved. 14 Nov 2019

I think that the cross-crossing of posts is well-understood: when cgain, you update your post too much and so your FE switches sign and voilà. At a certain point, your FE is small enough so that no overupdating of expectations happens any more.

→ my bet is you can kill this overupdating / crisscrossing w/ a sufficiently low gain

Yes, w/ $\bar{g} = 0.1$ (instead of 0.145) you already have dgain & cgain similar at $t=5$

w/ $\bar{g} = 0.0145$ they're identical at $t=25$ too

- But you always get some overshooting, whether it's in the 2nd period (cgain) or later on (dgain)
- Moreover, it's puzzling that i↑ as $\pi<0$ in 2nd period

One way to get perfectly normal, RE-like responses is to set $\alpha = 1$ b/c then $f_a \approx f_b$. But even $\alpha = 0.99$ gets a quite sig diff btwn f_a & f_b & overshooting too!

$$f_a = \frac{a}{1-\alpha\beta} + b\left(I_{nx} - \alpha\beta hx\right)^{-1} s$$

What is $\frac{1}{1-\alpha\beta} = 50.2513$ and $\frac{1}{1-\beta} = 100$

for $\alpha = \beta = 0.99$?

$$\left(I_4 - \alpha\beta hx\right)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2.4275 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.0049 & 2.3063 & 1.4678 & 1 \end{bmatrix}$$

and $\left(I_4 - \beta hx\right)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2.4631 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.0050 & 2.3643 & 1.4776 & 1 \end{bmatrix}$

ha! The diff in f_a & f_b is most pronounced in the part that comes from the intercept!

While this would change a bit as α moves away from 1 (ITS)

$\alpha = 0.5$, $\beta = 0.99$

$\frac{1}{1-\alpha\beta} = 1.9802$ and $\frac{1}{1-\beta} = 100$

$$(I_4 - \alpha\beta hx)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1.4925 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.0015 & 0.6217 & 0.7588 & 1 \end{bmatrix}$$

and $(I_4 - \beta hx)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2.4631 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.0050 & 2.3643 & 1.9776 & 1 \end{bmatrix}$

Yes, now the part relating the slope is diff, but the part relating the intercept is even more diff!

⇒ The lower α (the higher k, the less price stickiness) the more fp loads on the intercept both in absolute terms (fp reacts more) and relative terms (vs. the slope)

⇒ This may be drowning (some of) the overshooting b/c

for the std param value of $\alpha = 0.5$, f_b is almost 50 times more driven by the intercept than the slope + shocks. ⇒ so overreaction in updating the intercept drives f_b, which is what drives x_t up for again.

Let's interpret

$$\underset{\alpha\beta \cdot h_x}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.297 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.0025 & 0.48 & 0.7388 & 0 \end{bmatrix}} \quad r^n \quad \begin{matrix} \bar{i} \\ u \\ i_{t-1} \end{matrix} \quad \text{vs} \quad \underset{\beta \cdot h_x}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.554 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.005 & 0.56 & 1.4776 & 0 \end{bmatrix}}$$

2 differences

1) effect of shocks on i_t } $\alpha < 1$ mutes these somewhat
2) effect of $\bar{i}$ on s }

→ you're just discounting shocks in the future more!

Also, when $\alpha = 1$, then $f_a(1)$ doesn't matter for (x, π) "$E\pi$"

The puzzling i↑ at $t=2$ "$E\pi$"

$A_a(3,1) = 0.5928 \rightarrow$ i↑ if $f_a(1)$ ↑

$A_b(3,1) = -0.0978 \rightarrow$ i↓ if $f_b(1)$ ↑

⇒ so when f_b moves a lot more than f_a, (which is in general not true for avg dgain), then i↑ even when $E[\pi]$ is ↓ (!)

But why?

if ψ_x ↑ (now it's 0) then $A_a(3,1)$↓ and $A_b(3,1)$↓ too!

But $A_a(3,1)$ never <0, not even for $\psi_x = 5$.

when $\psi_x = 0$, it's b/c π↓ when f_b↑

→ it seems like i↑ in $t=2$ b/c π is ↑ from $t=1$ to $t=2$

:S

$A_A(3,1) = \psi_\pi A_A(1,1) + \psi_x A_A(2,1)$

$A_D(3,1) = \psi_\pi A_D(1,1)$ "D

these are true!

⇒ ah I see: i↑ at t=2 b/c it was going up much more at t=1 due to the innovation, but since π fell so much, this depressed i a lot. At t=2, since π↑ (but is still <0), i is depressed below $0.6 \cdot 1^{"\delta}$ (it's only ≈0.1) but it's not depressed as much

Puzzling i-response

$i_t = \pi + \text{innovation } (\delta)$

↓ ↑

Initially $|\delta| > |\pi|$

At t=2 $|\pi|$ shrinks so i↑

What remains to be understood is why the overshooting happens regardless, just later:

- maybe what's going on is that $E[\pi]$ are pushing things up but i is pushing them down, and i recedes faster

Check: if $\bar{i}$-shock is iid, overshooting should happen at $t=2$

⇒ exactly, and it does!

The only thing that isn't a 100% clear is why the ⊖ reaction to expectations, when RE doesn't have this?

In RE: $x_t = E_t x_{t+1} - \sigma E_t(i_t - \pi_{t+1})$

$\pi_t = \kappa x_t + \beta E_t \pi_{t+1}$

$i_t = \psi_\pi \pi_t$

$x_t = -\sigma i_t + E_t x_{t+1} + \sigma E_t \pi_{t+1}$

$\pi_t = \kappa x_t + \beta E_t \pi_{t+1}$

$x_t = -\sigma \psi_\pi \pi_t + E_t x_{t+1} + \sigma E_t \pi_{t+1}$

$x_t = -\sigma \psi_\pi [\kappa x_t + \beta E_t \pi_{t+1}] + E_t x_{t+1} + \sigma E_t \pi_{t+1}$

$$X_t = -b\psi_\pi \left[k X_t + \beta E_t \pi_{t+1} \right] + E_t X_{t+1} + b E_t \pi_{t+1}$$

$$(1 + b\psi_\pi k) X_t = -b\psi_\pi \beta E_t \pi_{t+1} + E_t X_{t+1} + b E_t \pi_{t+1}$$

$$X_t = \underbrace{\frac{1}{w} b(1 - \psi_\pi \beta)}_{<0\,(!)} E_t \pi_{t+1} + \frac{1}{w} E_t X_{t+1}$$

$\Rightarrow$ RE has it too, only

$$\Rightarrow \pi_t = \frac{k}{w} b(1 - \psi_\pi \beta) E_t \pi_{t+1} + \frac{k}{w} E_t X_{t+1} + \beta E_t \pi_{t+1}$$

$E[\,]$ lost more as such

$$\pi_t = \underbrace{\left[\frac{k}{w} b(1 - \beta\psi_\pi) + \beta \right]}_{\text{likely } > 0} E_t \pi_{t+1} + \frac{k}{w} E_t X_{t+1}$$

(for current params = 0.9248)

RE:

$$X_t = \ominus\, E_t \pi_{t+1} + \oplus\, E_t X_{t+1}$$

$$\pi_t = \oplus\, E_t \pi_{t+1} + \oplus\, E_t X_{t+1}$$

Learning

$$X_t = \ominus\, E_t^{fa} \pi_{t+1} + \ominus\, E_t^{fb} \pi_{t+1} + \ominus\, E_t^{fa} X_{t+1} + \oplus\, E_t^{fb} X_{t+1}$$

$$\pi_t = \oplus\, E_t^{fa} \pi_{t+1} + \ominus\, E_t^{fb} \pi_{t+1} + \oplus\, E_t^{fa} X_{t+1} + \oplus\, E_t^{fb} X_{t+1}$$

$$X_t = \ominus\, E\pi + \ominus\, EX \quad ①$$

this is not a mistake

$$\pi_t = \oplus\, E\pi + \oplus\, EX \quad ②$$

① $-\frac{b\psi_\pi}{w} k\alpha\beta + \frac{1}{w}(1-\beta) = \frac{1-\beta-b\psi_\pi k\alpha\beta}{w} < 0$

② $\left(1-\frac{kb\psi_\pi}{w}\right)(1-\alpha)\beta + \frac{k}{w} b(1-\beta\psi_\pi) > 0$

⇒ why do we have this diff btwn RE & Learn? 15 Nov 2019

$b\psi_\pi k\alpha\beta \overset{!}{<} 1-\beta$

$b\psi_\pi k\alpha\beta + \beta < 1$

$(b\psi_\pi k\alpha + 1)\beta < 1$ but it's 1.1150.

In the RE world, x depends on $E(x)$ only directly

My conjecture is that $E(\pi)$ in RE will incorporate $E(x)$ in some way. So π must depend stronger on $E(\pi)$ in RE than in learning.

RE: π on $E(\pi)$: $\frac{kb+\beta}{b\psi_\pi} = 0.7722$ under current params

Learn: $\left(1-\frac{kb\psi_\pi}{w}\right)(1-\alpha)\beta + \frac{k}{w} b(1-\beta\psi_\pi) = 0.33$ —11—

→ So yes, this is true

In fact, you can reason that in RE,

$\pi = E(\pi_i)$ only b/c only via
⊕

while in learning

$\pi = E(\pi)$ but part of this is ⊕ → from f_A
⊕ ⊖ → from f_B

Ryan meeting 15 Nov 2019

fix point where find the gain that min Var(FE)
data is generated by a gain = 0.145 gain,
given this, let an agent set a best gain
→ it must be lower than 0,145 !
Analog to RE for the gain param.

↳ Pooya Molavi's JMP does this, using a
Kullback - Leibler distance.

Stage 0: Establish that again learning causes excess volatility:

0.1° Do learning rule where I don't do RE-pad PLM = $\bar{\pi}_{t+-1}$ and treat's it.

0.2° Try learning the slope.

=> in those contexts, do I continue to get the flipflops?

2. Didn't quite get to the bottom of REs. learning loading on E(·)
→ connect to those equations

3. These features can become worse if 1 $\uparrow \psi_\pi$.

Do it in a week. Schedule to talk to Basu after.

Tell him: in learning models, there's this endemic instability. This can become worse if $\psi_\pi \uparrow$.

Here's how it works.