Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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Inflation that runs below its desired level can lead to an unwelcome fall in ***longer-term inflation expectations****, which, in turn, can pull actual inflation even lower, resulting in an adverse cycle of ever-lower inflation and inflation expectations. [...]* ***Well-anchored inflation expectations*** *are critical[.]*
Jerome Powell, Chairman of the Federal Reserve [1]
(Emphases added.)

[1] "New Economic Challenges and the Fed's Monetary Policy Review," August 27, 2020.

Long-run expectations: capturing responsiveness to short-run conditions

Individual SPF forecasts: for 1991-Q4 onward, estimate rolling regression

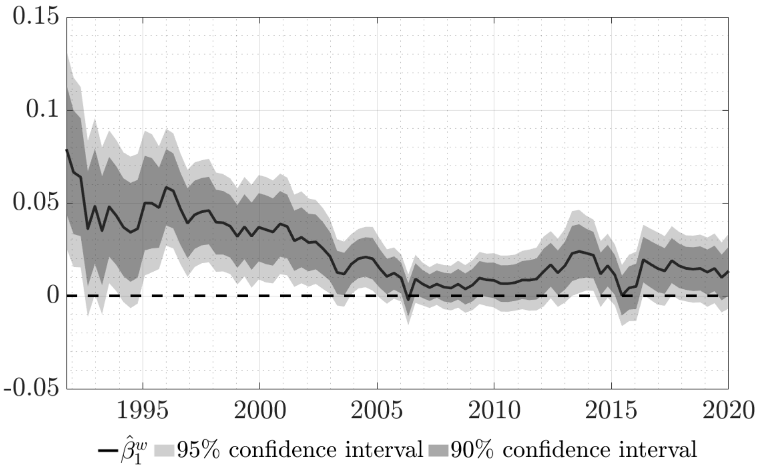
$$\Delta\bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{1}$$

where w indexes windows of 20 quarters length,

$f_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1}\,\pi_t$ individual one-year-ahead forecast error

Time-varying responsiveness

$$\Delta \bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{1}$$



Disagreement measures

This project

- How to conduct monetary policy in interaction with the anchoring expectation formation?

- Model of expectations unanchoring
  $\hookrightarrow$ extension to adaptive learning that captures time-varying responsiveness of long-run expectations

- Estimate how unanchoring takes place in data
  $\hookrightarrow$ quantify novel anchoring channel and use for monetary policy analysis

Preview of results

1. Estimation
    - Larger mistakes unanchor more
    - Overestimating inflation unanchors more than underestimating it (Hebden et al 2020)
    - On average, people discount observations older than 8 quarters
2. Optimal policy
    - Responds aggressively to inflation when unanchored, accommodates inflation when anchored
3. Taylor rule
    - Less aggressive on inflation than under rational expectations

Related literature

- **Optimal monetary policy in the New Keynesian model**

  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Adaptive learning**

  Evans & Honkapohja (2001, 2006), Sargent (1999), Primiceri (2006), Lubik & Matthes (2018), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Mele et al (2019), Eusepi & Preston (2011), Milani (2007, 2014), Marcet & Nicolini (2003)

- **Anchoring and the Phillips curve**

  Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Reis (2020), Hebden et al 2020, Gobbi et al (2019), Carvalho et al (2019)

Structure of talk

1. Model of anchoring expectations
2. Quantification of learning channel
3. Solving the Ramsey problem
4. Implementing optimal policy
5. Approximating optimal policy with a Taylor rule

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Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j))dj \right] \tag{2}$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_t C_t^i \tag{3}$$

▸ Consumption, price level

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \tag{4}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{5}$$

▸ Profits, stochastic discount factor

Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- Model implies mapping between exogenous states s_t and observables $y_t \equiv (\pi, x, i)'$

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- Under rational expectations (RE), private sector knows model $\rightarrow$ knows (6)

$$\mathbb{E}_t \, y_{t+1} = g \, \mathbb{E} \, s_{t+1} \tag{7}$$

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- $\hat{\mathbb{E}}$: agents do not internalize that identical $\rightarrow$ do not know aggregate model $\rightarrow$ do not know (6)

Adaptive learning

- Agents know exogenous evolution of states

$$s_t = hs_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \tag{8}$$

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$a \rightarrow$ concept of long-run expectations in the model

- Estimate a, b using recursive least squares (RLS) using observed states and knowledge of (8)

Recursive least squares (RLS)

Observables are: $(\pi, x, i)'$

Assumption: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\, h \quad \forall t \tag{10}$$

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$\rightarrow$ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\right)}_{\equiv f_{t|t-1}\text{, forecast error}} \tag{11}$$

$k_t \in (0, 1)$ gain
b_1 first row of b

General RLS algorithm

Alternatives for the gain

1. Decreasing gain:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f_{t|t-1} \tag{12}$$

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Model summary

- New Keynesian core: IS and Phillips curves

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big) \tag{15}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \big) \tag{16}$$

▸ Derivations ▸ Actual laws of motion

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→ How should $\{i_t\}$ be set?

Structure of talk

1. Model of anchoring expectations

2. Quantification of learning channel

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Estimating form of gain function

$$\mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}) \tag{19}$$

- $b_i(f_{t|t-1})$ = piecewise linear basis
- γ_i = approximating coefficient at node i

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- Calibrate variances of disturbances to match moments
- Estimate $\hat{\gamma}$ to match moments

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- Moments: autocovariances of inflation, output gap, federal funds rate and 1-year ahead SPF inflation expectations at lags $0, \ldots, 4$

Calibration - parameters from the literature

β	0.98	stochastic discount factor
σ	1	intertemporal elasticity of substitution
α	0.5	Calvo probability of not adjusting prices
κ	0.0842	slope of the Phillips curve
ψ_π	1.5	coefficient of inflation in Taylor rule
$\bar{g}$	0.145	initial value of the gain
λ_x	0.05	weight on the output gap in central bank loss

Main sources: Chari et al 2000, Woodford 2003, Nakamura & Steinsson 2008

Calibration - matching moments

ψ_x	0.3	coefficient of the output gap in Taylor rule
σ_r	0.01	standard deviation, natural rate shock
σ_i	0.01	standard deviation, monetary policy shock
σ_u	0.5	standard deviation, cost-push shock
$\hat{\gamma}_i$	(0.82; 0.61; 0; 0.33; 0.45)	coefficients in anchoring function

Estimated form for $\mathbf{g}(\cdot)$

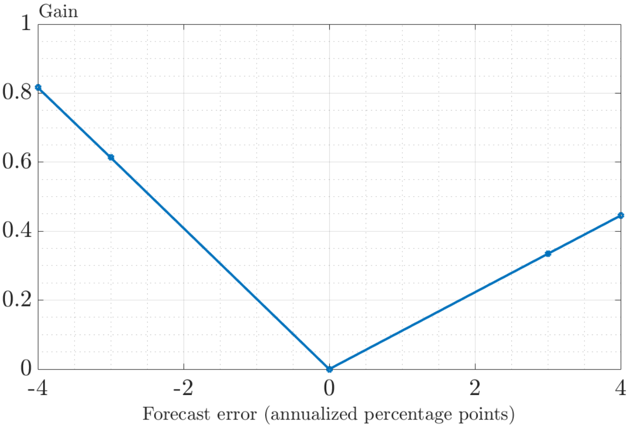
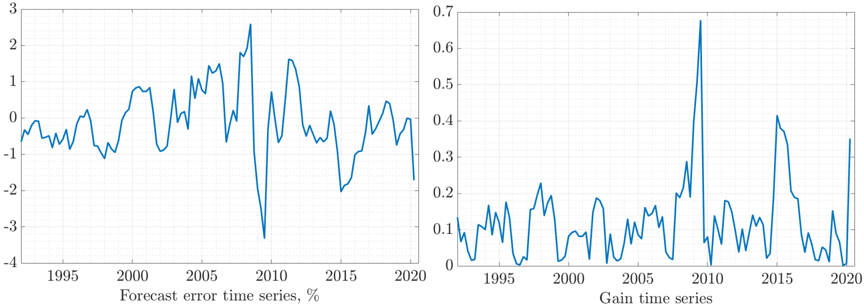


Figure: Gain as a function of forecast errors in inflation

Forecast errors in the data

Figure: Time series of 1-year ahead forecast errors and implied gain in the SPF



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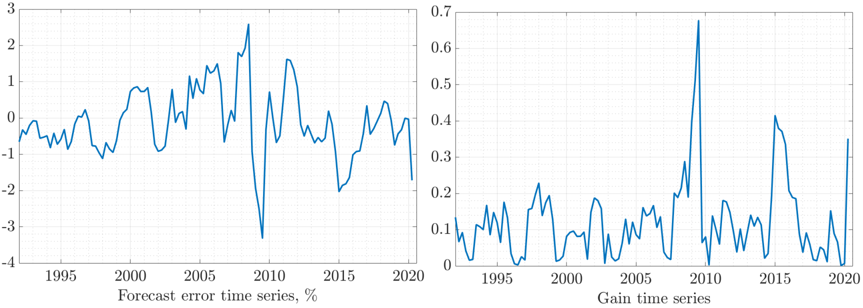

Mean gain ≈ 0.12

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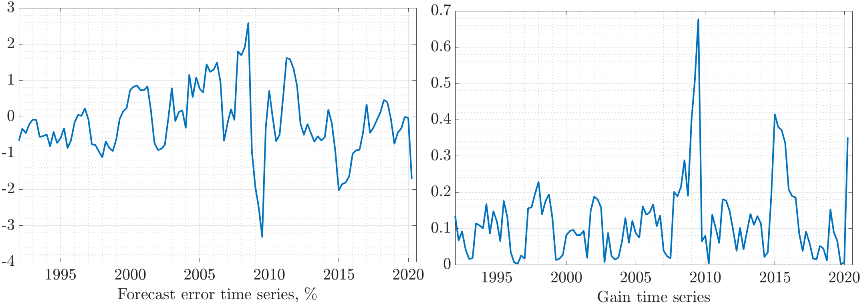

Mean gain ≈ 0.12 $\quad\rightarrow$ discount forecast errors older than 8 quarters

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Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

- $\mathbb{E}$ is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

Target criterion

Proposition

In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t$$
$$+ \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - f_{t+1+j|t+j} \mathbf{g}_{\bar{\pi},\mathbf{t+j}}) \right)$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t*, and* b_1 *is the first row of* b*.*

General case

Responding to cost-push shocks

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - f_{t+1+j|t+j} \mathbf{g}_{\bar{\pi},\mathbf{t+j}}) \right)$$

Intratemporal tradeoffs in RE (discretion)

Postpone current tradeoff to future as long as gain > 0

Extent to which can postpone depends on not unanchoring too much in future

Responding to cost-push shocks

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Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

▸ Why no commitment?

Corollary

Optimal policy under adaptive learning is time-consistent.

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Solution procedure

Solve system of model equations + target criterion

$\hookrightarrow$ solve using parameterized expectations (PEA)

$\hookrightarrow$ obtain a cubic spline approximation to optimal policy function

Optimal policy - responding to unanchoring

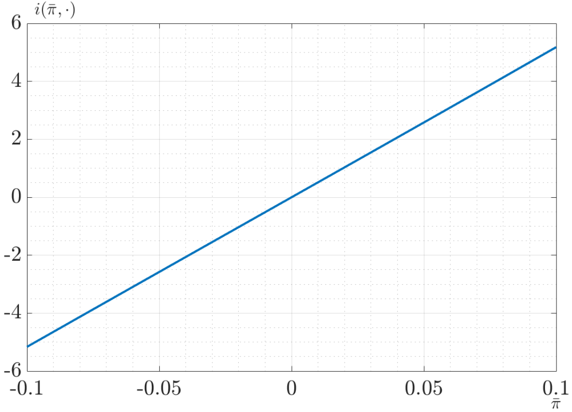

Figure: Policy function: $i(\bar{\pi}$, all other states at their means)

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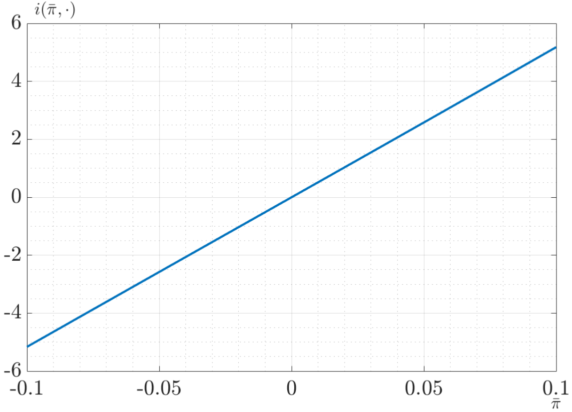

Figure: Policy function: $i(\bar{\pi}$, all other states at their means)

$\rightarrow$ For 5 bp drop in $\bar{\pi}$, lower i by 2.5 pp

Unanchoring causes volatility

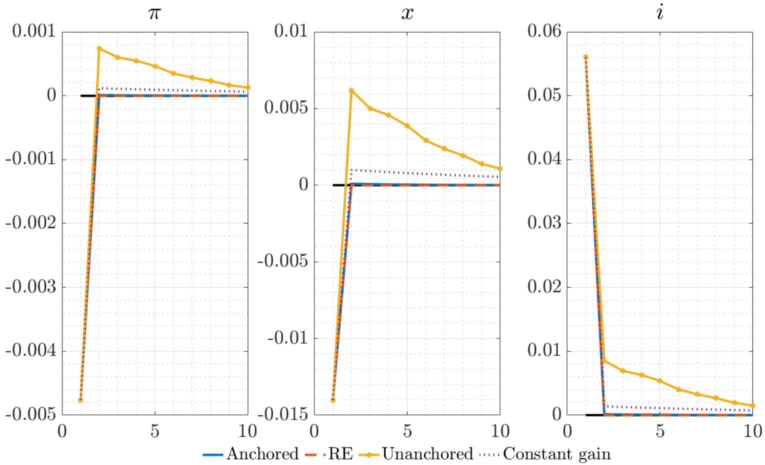

Figure: Impulse responses after a contractionary monetary policy shock when policy follows a Taylor rule

Volatility comes from endogenous gain

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Shocks raise the gain → central bank needs to anchor

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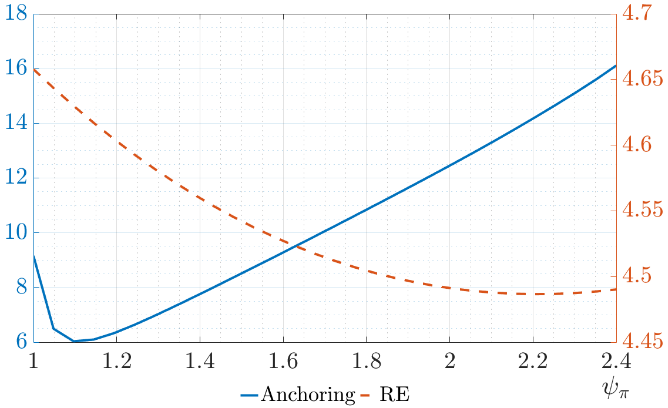
Optimal Taylor-coefficient on inflation

Figure: Central bank loss as a function of ψ_π



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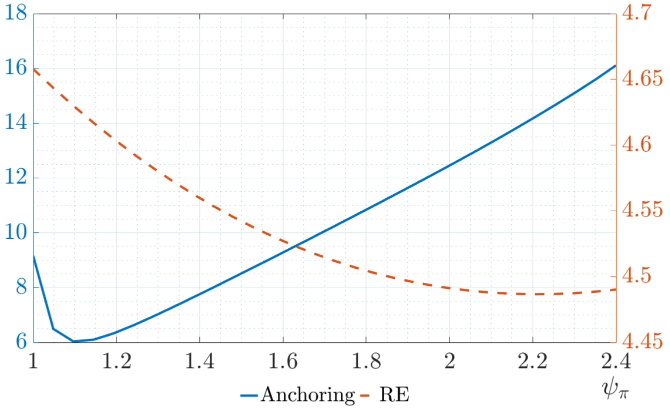

Anchoring-optimal coefficient: $\psi_\pi^A = 1.09$ RE-optimal coefficient: $\psi_\pi^{RE} = 2.21$

Respond but not too much

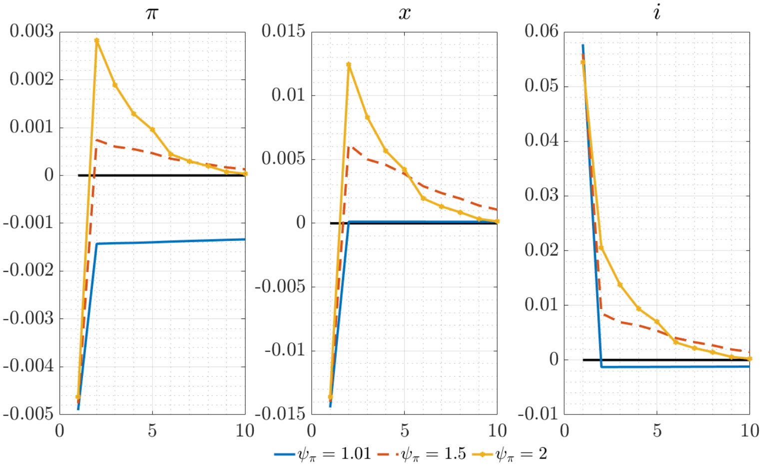

Figure: Impulse responses for unanchored expectations for various values of ψ_π

Why less aggressive? Future interest rate expectations

IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big)$$

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- Current interest rate i_t: one channel of policy

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$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \big)$$

- Current interest rate i_t: one channel of policy
- Expected future interest rates: additional channel of policy

Why less aggressive? Future interest rate expectations

IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\Big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\Big)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\Big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\Big)$$

- Current interest rate i_t: one channel of policy
- Expected future interest rates: additional channel of policy
- Only if policy reaction function internalized

Conclusion

First theory of monetary policy for potentially unanchored expectations

Estimation of novel unanchoring channel

- Large and negative surprises unanchor more
- Estimated gain time series: on average, people only use the last 8 quarters of data

Monetary policy

- Expectations unanchoring makes smoothing shocks over time possible
- Optimal policy aggressive when unanchored, accommodates otherwise
- Taylor rule less aggressive than under rational expectations

Future work

$\hookrightarrow$ How to anchor at zero-lower bound?

$\hookrightarrow$ Other applications: currency crises

Appendix

Breakeven inflation

Figure: Market-based inflation expectations, various horizons, %

Return

Correcting the TIPS from liquidity risk

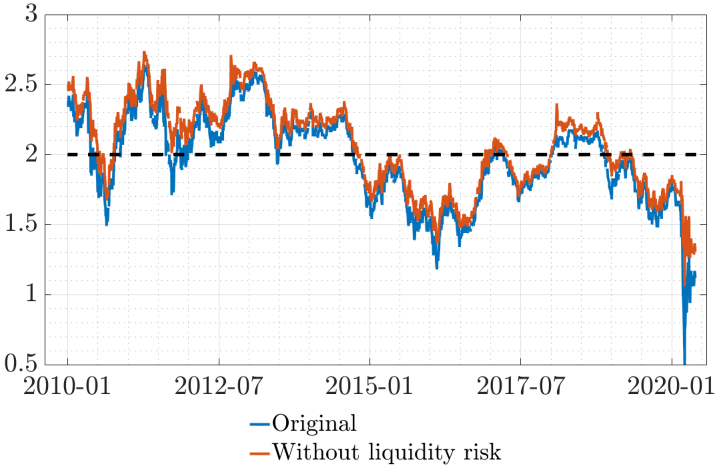

Figure: Market-based inflation expectations, 10 year, %

Return

Further evidence

Figure: Livingston Survey of Firms:
Interquartile range of 10-year ahead inflation expectations

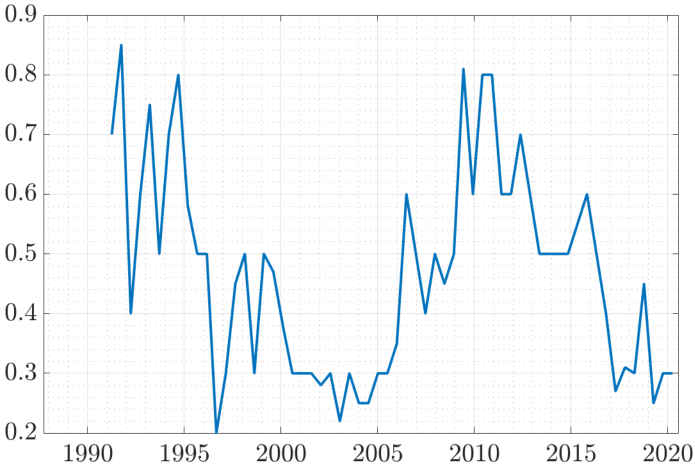

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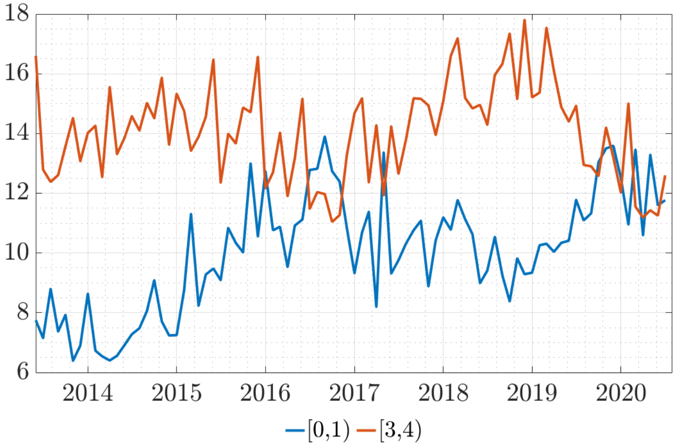
Figure: New York Fed Survey of Consumers:
Percent of respondents indicating 3-year ahead inflation will be in a particular range



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Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{20}$$
$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{21}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1-k^{-1}}{1-k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1-k^{-1}\beta} u_t \tag{22}$$

Solve for forecast error $f_t \equiv \pi_t - f_{t-1}$:

$$f_t = \underbrace{-\frac{1-\beta}{1-k\beta}}_{\lim_{k\to 1}=-1} f_{t-1} + \frac{1}{1-k\beta} u_t \tag{23}$$

Functional forms for **g** in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \tag{24}$$

$p \equiv Prob$(liquidity trap regime)
y_{t-1} output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{25}$$

θ_t criterion, $\bar{\theta}$ threshold value

Return

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \tag{26}$$

Σ variance-covariance matrix of shocks
$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(f_{t|t-1} f'_{t|t-1} - \omega_{t-1}) \tag{27}$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(f'_{t|t-1} \omega_t^{-1} f_{t|t-1} - \theta_{t-1}) \tag{28}$$

ω_t estimated forecast-error variance

Return

Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right)' \right)' \tag{29}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{30}$$

◂ Return

Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{31}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{32}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \tag{33}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{34}$$

Return

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t)$$
$$+ \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \Big\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t + \mathbf{g}_\pi(\pi_t - f_{t-1})) = 0 \tag{35}$$
$$-2\beta\frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_\mathbf{f}(\pi_{t+1} - f_t)) = 0 \tag{36}$$

Return

Target criterion system for anchoring function as changes of the gain

$$\varphi_{6,t} = -cf_{t|t-1}x_{t+1} + \left(1 + \frac{f_{t|t-1}}{f_{t+1|t}}(1 - k_{t+1}) - f_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1}$$
$$- \frac{f_{t|t-1}}{f_{t+1|t}}(1 - k_{t+1})\varphi_{6,t+2} \tag{37}$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{f_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{f_{t|t-1}}\varphi_{6,t+1} \tag{38}$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (38) is given by:

$$\varphi_{6,t} = -2\,\mathbb{E}_t \sum_{i=0}^{\infty}(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{f_{t+j|t+j-1}}}{\frac{k_{t+j}}{f_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \tag{39}$$

Return

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{40}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{41}$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \tag{42}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{43}$$

Return

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{44}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{45}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (44) backward to some date t, take expectations at t
2. Sub in (45)
3. Aggregate over households i

$\rightarrow$ Obtain (15)

Return