Materials 13 - Still looking for a version of the model w/o overshooting

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Overview

1 Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

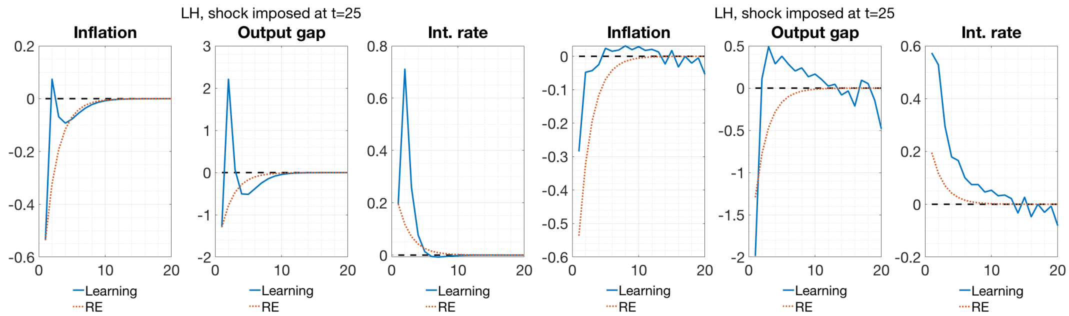
$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \tag{3}$$

$$\hat{\mathbb{E}}_t z_{t+h} = \begin{bmatrix} \bar{\pi}_{t-1} \\ 0 \ (\bar{x}_{t-1}) \\ 0 \ (\bar{i}_{t-1}) \end{bmatrix} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x \ h_x \qquad \text{PLM} \tag{4}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \underbrace{\big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big)}_{\text{fcst error using (4)}} \qquad (b_1 \text{ is the first row of } b) \tag{5}$$

$$k_t = \begin{cases} k_{t-1} + 1 & \text{for decreasing gain learning} \\ \bar{g}^{-1} & \text{for constant gain learning.} \end{cases} \tag{6}$$

Figure 1: Reference: baseline model



(a) Learning constant only (b) Learning slope and constant

2 Ideas

1. Check ψ_π above but close to 1

   $\rightarrow$ works but only quantitatively; qualitatively, the overshooting is still there, likely because this only cancels out one of the two channels through which $\mathbb{E}\,\pi$ affects x_t negatively.

2. Fix shock for simulation

   Indeed the issue was that for learning, I accidentally scaled down the shock by $\sigma_i < 1$, while for RE I had maintained $\sigma_i = 1$.

3. Interest rate smoothing as $i_t = \rho i_{t-1} + (1-\rho)(\psi_\pi \pi_t + \psi_x x_t) + \bar{i}_t$

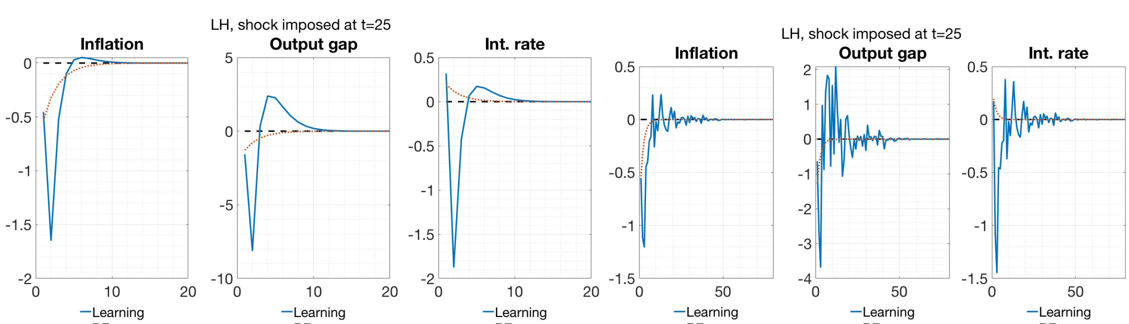
   Doesn't work either - it doesn't change the model except reduces ψ_π.

4. Indexation in NKPC

   Doesn't work either - same model dynamics.

5. Learn h_x

Figure 2: Learning h_x, baseline



(a) Learning constant only (b) Learning slope and constant

Like learning the Taylor rule b/c agents initially don't know if the shock will continue.

6. Central bank's $\mathbb{E}\,\pi_{t+1}$ in TR?

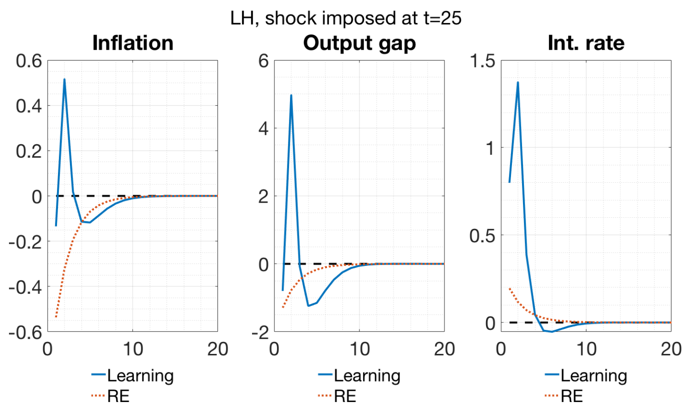
   Done a correction for $\hat{\mathbb{E}}\pi_{t+1}$ in TR, now both are stable, but overshooting is still there in both. Not so dissimilar to baseline except that the periods are shifted.

7. Initialize beliefs away from RE somehow

   Slobodyan & Wouters do this, but in an estimation context, which I think is necessary because you need pre-sample data to condition priors on.

8. Slobodyan & Wouters' "VAR-learning": use lagged observables to learn from, not from states.

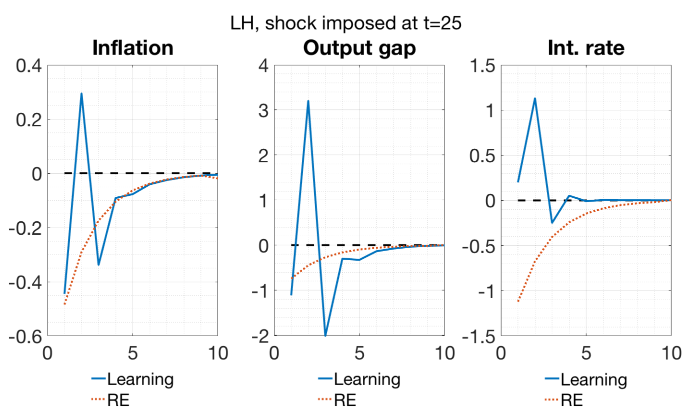
Figure 3: VAR learning, baseline, learning only constant



For learning both slope and constant, not E-stable. Kind of makes sense since I'd think that this amplifies positive feedback.

9. Davig & Leeper-style switching Taylor rule where only long-run Taylor principle holds?

Figure 4: Markov-switching Taylor rule, baseline, learning only constant (slope learning unstable, why?)



10. Some kind of moving average of inflation (or average) in the TR?

A quick question on projection facility: checking `eig(phi)` when ϕ isn't square? Right now I'm splitting up ϕ as `a = phi(:,1), b = phi(:, 2:end)` and then checking `eig(b)` and `eig(diag(a))`.

3 Details on the Markov-switching setup

Model equations remain the same, except the Taylor rule now is:

$$i_t = \psi_\pi(r_t)\pi_t + \psi_x x_t + \bar{i}_t \tag{7}$$

$$r_t = \begin{cases} 1 \quad \text{active regime} \quad \psi_\pi = \psi_1 = 2.19 \\ 2 \quad \text{passive regime} \quad \psi_\pi = \psi_2 = 0.89 \end{cases} \tag{8}$$

$$r_{t+1} = \begin{cases} p_{11}1 + (1-p_{11})2 \quad \text{if} \quad r_t = 1 \\ (1-p_{22})1 + p_{22}2 \quad \text{if} \quad r_t = 2 \end{cases} \qquad \text{where} \quad p_{ji} \equiv Prob(s_{t+1} = j | s_t = i) \tag{9}$$

So I solve the RE model by introducing the new jump variables $\pi_{it}, x_{it}, i_{it},\ i = 1, 2$ and writing the model equations as

$$x_{it} = (p_{1i}\,\mathbb{E}_t\, x_{1t+1} + p_{2i}\,\mathbb{E}_t\, x_{2t+1}) - \sigma(i_{it} - (p_{1i}\,\mathbb{E}_t\, \pi_{1t+1} + p_{2i}\,\mathbb{E}_t\, \pi_{2t+1})) + \sigma r_t^n \tag{10}$$

$$\pi_{it} = \kappa x_{it} + \beta(p_{1i}\,\mathbb{E}_t\, \pi_{1t+1} + p_{2i}\,\mathbb{E}_t\, \pi_{2t+1})) + u_t \tag{11}$$

$$i_{it} = \psi_i \pi_{it} + \psi_x x_{it} + \bar{i}_t \tag{12}$$

Now I unleash the usual method of solving for the observable and state transition matrix g_x, h_x. The only difference will be that since the number of jumps now is double the old number, g_x will be $2n_y \times n_x$. Is it correct to interpret $g_x(1) \equiv$ `gx(1:ny,:)` as pertaining to regime 1, and $g_x(2) \equiv$ `gx(ny+1:end,:)` to regime 2?

Then, generating an exogenous regime sequence r, I compute RE IRFs as usual for the state block, but depending on the state, I use the corresponding block of g_x. With x_0 being the impulse, so that $IR_1^x = x_0$, I do the following:

$$IR_t^y = \begin{cases} g_x(1)x_t \quad \text{if} \quad r_t = 1 \\ g_x(2)x_t \quad \text{if} \quad r_t = 2 \end{cases} \tag{13}$$

$$IR_{t+1}^x = h_x x_t \tag{14}$$

As for the learning model, the compact notation for the model was:

$$z_t = A_a f_a(t) + A_b f_b(t) + A_s s_t \tag{15}$$

where the A-matrices are functions of the parameters, including ψ_π. The LH expectations f_a and f_b are only functions of the learning coefficients a, b and of the state transition matrix h_x. So the only salient difference is that agents react to expectations differently depending on the regime: the A-matrices become state-dependent.

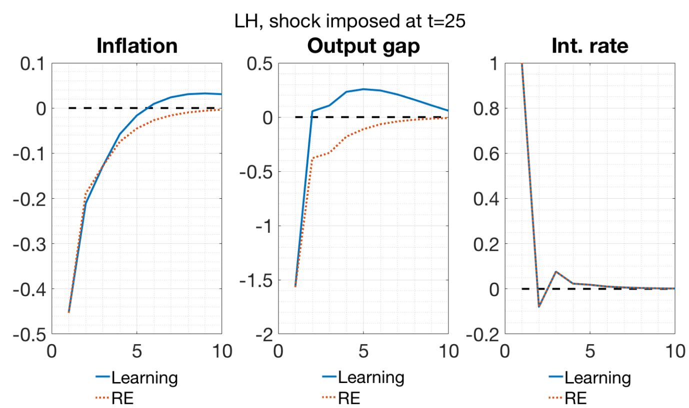
Is it correct then to simulate the model as

$$z_t = A_a(i) f_a(t) + A_b(i) f_b(t) + A_s(i) s_t \quad \text{for} \quad i = 1, 2 \tag{16}$$

?

4 Lagged inflation in the TR *can* work

Figure 5: Lagged inflation in TR, "suboptimal forecasters" info assumption, $\beta = 0.96$ instead of 0.99



Why can this work? Because the entire issue comes from two channels, one of which is the fact that in the NKIS relation, future inflation enters with the coefficient $1 - \beta\psi_\pi$. This is what can be shut off if $\psi_\pi < 1/\beta$. For the lagged inflation extension, this coefficient is modified to $1 - \beta^2\psi_\pi$. Thus the non-overshooting condition becomes $\psi_\pi < 1/\beta^2$. So one can either lower β, or one can have the Taylor rule feature more lags For example, for 2 lags we'd get $1 - \beta^3\psi_\pi$, having the condition modify to $\psi_\pi < 1/\beta^3$.