Materials 20 - Optimal Taylor rule coefficients

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Overview

1 Procedure

1. obtain the optimal noninertial plan for the endogenous variables,
2. perform coefficient-comparison on the Taylor rule.

Details:

Given optimal noninertial paths of the form $z_t = \bar{z} + f_z u_t + g_z r_t^n$, $z = \pi, x, i$, the Taylor rule

$$i_t = \psi_\pi(\pi_t - \bar{\pi}) + \psi_x(x_t - \bar{x}) + \bar{i}_t \tag{1}$$

can be written as

$$i_t = \bar{i} + \psi_\pi(f_\pi u_t + g_\pi r_t^n) + \psi_x(f_x u_t + g_x r_t^n) \tag{2}$$

which has to satisfy

$$i_t = \bar{i} + f_i u_t + g_i r_t^n \tag{3}$$

allowing one to solve for (ψ_π^*, ψ_x^*) as the solution to

$$f_i = \psi_\pi f_\pi + \psi_x f_x \tag{4}$$

$$g_i = \psi_\pi g_\pi + \psi_x g_x \tag{5}$$

Details on obtaining the coefficients f_z, g_z of the optimal noninertial plan $z_t = \bar{z} + f_z u_t + g_z r_t^n$ for the anchoring model

1. Conjecture $z_t = \bar{z} + f_z u_t + g_z r_t^n$ where $z = \{\pi, x, i, f_a, f_b, \bar{\pi}, k^{-1}\}$

2. Plug conjecture into model equations (6) - (11) (the simplified version of the baseline model):

$$\pi_t - \kappa x_t - (1-\alpha)\beta f_a(t) - \kappa\alpha\beta b_2(I_3 - \alpha\beta h_x)^{-1} s_t - e_3(I_3 - \alpha\beta h_x)^{-1} s_t = 0 \tag{6}$$

$$x_t + \sigma i_t - \sigma f_b(t) - (1-\beta) b_2(I_3 - \beta h_x)^{-1} s_t + \sigma\beta b_3(I_3 - \beta h_x)^{-1} s_t - \sigma e_1(I_3 - \beta h_x)^{-1} s_t\big) = 0 \tag{7}$$

$$f_a(t) - \frac{1}{1-\alpha\beta}\bar{\pi}_{t-1} - b_1(I_3 - \alpha\beta h_x)^{-1} s_t = 0 \tag{8}$$

$$f_b(t) - \frac{1}{1-\beta}\bar{\pi}_{t-1} - b_1(I_3 - \beta h_x)^{-1} s_t = 0 \tag{9}$$

$$\bar{\pi}_t - \bar{\pi}_{t-1} - k_t^{-1}\big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big) = 0 \tag{10}$$

$$k_t^{-1} - f(\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}) = 0 \tag{11}$$

3. Note that there are n_y deterministic components $\bar{z}$, n_y f_z-terms and n_y g_z-terms. Imposing that the conjecture fulfills the model equations (6) - (11) yields $n_y - 1$ constraints *separately* for $\bar{z}$, f_z and g_z. That is, I have three non-interacting equation systems, each consisting of $n_y - 1$ equations in n_y variables.

4. Solve 3 minimizations:

$$\bar{z} = \arg\min L^{det} \quad s.t \quad \text{the 1st set of } \ n_y - 1 \text{ constraints from step 3.} \tag{12}$$

$$f_z = \arg\min L^{stab,u} \quad s.t \quad \text{the 2nd set of } \ n_y - 1 \text{ constraints from step 3.} \tag{13}$$

$$g_z = \arg\min L^{stab,r} \quad s.t \quad \text{the 3rd set of } \ n_y - 1 \text{ constraints from step 3.} \tag{14}$$

   where I'm using the following decomposition of the central bank's loss: The central bank's loss function

$$L^{CB} = \mathbb{E}_t \sum_{T=t}^{\infty} \{\pi_T^2 + \lambda_x(x_T - x^*)^2 + \lambda_i(i_T - i^*)\} \tag{15}$$

   can be decomposed into a component coming from the long-run means, and a component from fluctuations of the endogenous variables:

$$L^{det} = \sum_{T=t}^{\infty} \beta^{T-t} \{\mathbb{E}_t\, \pi_T^2 + \lambda_x (\mathbb{E}_t\, x_T - x^*)^2 + \lambda_i (\mathbb{E}_t\, i_T - i^*)^2\} \tag{16}$$

$$L^{stab} = \sum_{T=t}^{\infty} \beta^{T-t} \{\mathrm{var}_t(\pi_T) + \lambda_x \mathrm{var}_t(x_T) + \lambda_i \mathrm{var}_t(i_T)\} \tag{17}$$

$$L^{stab,u} \propto f_\pi^2 + \lambda_x f_x^2 + \lambda_i f_i^2 \tag{18}$$

$$L^{stab,r} \propto g_\pi^2 + \lambda_x g_x^2 + \lambda_i g_i^2 \tag{19}$$

5. Solve the three decoupled $n_y \times n_y$ equation systems for $\bar{z}$, f_z and g_z.

(I didn't solve for $\bar{z}$ because the TR-coefficients only depend on f_z and g_z.)

2 Remarks on the noninertial plan for the anchoring model

1. Treat long-run expectations f_a, f_b, the gain k^{-1} and expected mean inflation $\bar{\pi}$ as endogenous variables (part of the vector z).

2. Postulate a gain function

$$k_t^{-1} - k_{t-1}^{-1} = c + d(\pi_t - \bar{\pi}_{t-1} - b_{11} r_t^n - b_{13} u_t) \tag{20}$$

3. Step 2 yields interaction terms between $f_k, f_{\bar{\pi}}$ and $g_k, g_{\bar{\pi}}$, as well as between the shocks (of the form $u_t r_t^n, u_t^2, (r_t^n)^2$) and these coefficients loading on lagged shocks u_{t-1}, r_{t-1}^n (which also show up in interaction terms).

   - To keep the solution linear, I therefore impose $f_k = g_k = f_{\bar{\pi}} = g_{\bar{\pi}} = 0$.

     Interpretation: in the optimal plan, the planner wants the gain and expected mean inflation not to fluctuate in response to shocks.

   - A direct consequence of this is:

$$f_{f_a} = \frac{b_{13}}{1 - \alpha\beta\rho_u} \qquad f_{f_b} = \frac{b_{13}}{1 - \beta\rho_u} \qquad g_{f_a} = \frac{b_{11}}{1 - \alpha\beta\rho_r} \qquad g_{f_b} = \frac{b_{11}}{1 - \beta\rho_r} \tag{21}$$

     i.e. long-run expectations are just the discounted sums of the rational expectation of disturbances.

3 Optimal Taylor-rule coefficients

$$\psi_\pi^{anchor} = \frac{\kappa\sigma}{\lambda_i} \tag{22}$$

$$\psi_x^{anchor} = \frac{\lambda_x\sigma}{\lambda_i} \tag{23}$$

For the rational expectations version of the model with the assumption $\rho \equiv \rho_u = \rho_r$, the coefficients are

$$\psi_\pi^{RE} = \frac{\kappa\sigma}{\lambda_i(\rho-1)(\beta\rho-1) - \kappa\lambda_i\rho\sigma} \tag{24}$$

$$\psi_x^{RE} = \frac{\lambda_x\sigma(1-\beta\rho)}{\lambda_i(\rho-1)(\beta\rho-1) - \kappa\lambda_i\rho\sigma} \tag{25}$$