

# Materials 23

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## Overview

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## 1 Implementation of target criterion

Problem: Model behaves as if Taylor rule was specified as just a function of exogenous process, i.e. is not E-stable. Need to trick it somehow.

## A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (\text{A.1})$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (\text{A.2})$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \quad (\text{if imposed}) \quad (\text{A.3})$$

$$\text{PLM:} \quad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \quad b = g_x h_x \quad (\text{A.4})$$

$$\text{Updating:} \quad a_t = a_{t-1} + k_t^{-1} (z_t - (a_{t-1} + b s_{t-1})) \quad (\text{A.5})$$

$$\text{Anchoring function:} \quad k_t = k_{t-1} + \mathbf{g}(f e_{t-1}^2) \quad (\text{A.6})$$

$$\text{Forecast error:} \quad f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \quad (\text{A.7})$$

This notation captures vector learning ( $z$  learned) for intercept only. For scalar learning,  $a_t = (\bar{\pi}_t \ 0 \ 0)'$  and  $b_1$  designates the first row of  $b$ . The learning setup allows me to evaluate the long-horizon expectations  $f_a, f_b$ , and then the observables  $(\pi, x)$  are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix} \sigma & 1-\beta & -\sigma\beta \end{bmatrix} f_b + \sigma \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \quad (\text{A.8})$$

$$\pi_t = \kappa x_t + \begin{bmatrix} (1-\alpha)\beta & \kappa\alpha\beta & 0 \end{bmatrix} f_a + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \quad (\text{A.9})$$

## B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only,  $k_t^{-1} = \mathbf{g}(f e_{t-1})$ ):

$$\begin{aligned} \pi_t = & -\frac{\lambda_x}{\kappa} \left\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_\pi(t) \right) \right. \\ & \left. \left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\bar{\pi}}(t+j)) \right) \right\} \end{aligned} \quad (\text{B.1})$$

where I'm using the notation that  $\prod_{j=0}^0 \equiv 1$ . For interpretation purposes, let me rewrite this as follows:

$$\begin{aligned} \pi_t = & -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x (1-\alpha)\beta}{\kappa (1-\alpha\beta)} \left( k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ & - \frac{\lambda_x (1-\alpha)\beta}{\kappa (1-\alpha\beta)} \left( k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j}^{-1} + f e_{t+1+j|t+j}^{eve} \mathbf{g}_{\bar{\pi}}(t+j)) \right) \end{aligned} \quad (\text{B.2})$$

Interpretation: **tradeoffs from discretion in RE** + **effect of current level and change of the gain on future tradeoffs** + **effect of future expected levels and changes of the gain on future tradeoffs**

## C Tried to derive a target criterion for the case when the anchoring function is specified in terms of changes in the gain

$$k_t = k_{t-1} + \mathbf{g}(fe_{t|t-1}) \quad (\text{C.1})$$

Turns out the  $k_{t-1}$  adds one  $\varphi_{6,t+1}$  too many which makes the target criterion unwieldy. The FOCs of the Ramsey problem are

$$2\pi_t + 2\frac{\lambda}{\kappa}x_t - k_t^{-1}\varphi_{5,t} - \mathbf{g}_\pi(t)\varphi_{6,t} = 0 \quad (\text{C.2})$$

$$cx_{t+1} + \varphi_{5,t} - (1 - k_t^{-1})\varphi_{5,t+1} + \mathbf{g}_{\bar{\pi}}(t)\varphi_{6,t+1} = 0 \quad (\text{C.3})$$

$$\varphi_{6,t} + \varphi_{6,t+1} = fe_t\varphi_{5,t} \quad (\text{C.4})$$

where the red multiplier is the new element vis-a-vis the case where the anchoring function is specified in levels ( $k_t^{-1} = \mathbf{g}(fe_{t-1})$ ), as in App. B), and I'm using the shorthand notation

$$c = -\frac{2(1-\alpha)\beta}{1-\alpha\beta}\frac{\lambda}{\kappa} \quad (\text{C.5})$$

$$fe_t = \pi_t - \bar{\pi}_{t-1} - bs_{t-1} \quad (\text{C.6})$$

(C.2) says that in anchoring, the discretion tradeoff is complemented with tradeoffs coming from learning ( $\varphi_{5,t}$ ), which are more binding when expectations are unanchored ( $k_t^{-1}$  high). Moreover, the change in the anchoring of expectations imposes an additional constraint ( $\varphi_{6,t}$ ), which is more strongly binding if the gain responds strongly to inflation ( $\mathbf{g}_\pi(t)$ ). One can simplify this three-equation-system to:

$$\varphi_{6,t} = -cfe_tx_{t+1} + \left(1 + \frac{fe_t}{fe_{t+1}}(1 - k_{t+1}^{-1}) - fe_t\mathbf{g}_{\bar{\pi}}(t)\right)\varphi_{6,t+1} - \frac{fe_t}{fe_{t+1}}(1 - k_{t+1}^{-1})\varphi_{6,t+2} \quad (\text{C.7})$$

$$0 = 2\pi_t + 2\frac{\lambda}{\kappa}x_t - \left(\frac{k_t^{-1}}{fe_t} + \mathbf{g}_\pi(t)\right)\varphi_{6,t} + \frac{k_t^{-1}}{fe_t}\varphi_{6,t+1} \quad (\text{C.8})$$

Unfortunately, I haven't been able to solve (C.7) for  $\varphi_{6,t}$  and therefore I can't express the target criterion so nicely as before. The only thing I can say is to direct the targeting rule-following central bank to compute  $\varphi_{6,t}$  as the solution to (C.8), and then evaluate (C.7) as a target criterion. The solution to (C.8) is given by:

$$\varphi_{6,t} = -2\mathbb{E}_t \sum_{i=0}^{\infty} \left(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}\right) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}^{-1}}{fe_{t+j}}}{\frac{k_{t+j}^{-1}}{fe_{t+j}} + \mathbf{g}_\pi(t+j)} \quad (\text{C.9})$$

Interpretation: the anchoring constraint is not binding ( $\varphi_{6,t} = 0$ ) if the CB always hits the target ( $\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i} = 0 \quad \forall i$ ); or expectations are always anchored ( $k_{t+j}^{-1} = 0 \quad \forall j$ ).