Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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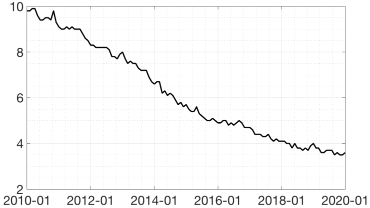
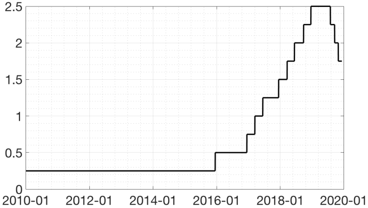
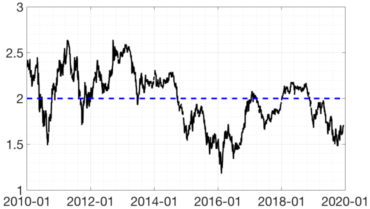
Puzzling US business cycle fall 2019



(a) Unemployment rate



(b) Fed funds rate target, upper limit



(c) Market-based inflation expectations, 10 year, average

This project

Model anchored expectations as an endogenous gain learning scheme

$\rightarrow$ How to conduct optimal monetary policy in interaction with the anchoring expectation formation?

Preview of results

1. intertemporal tradeoff: short-run costs vs. long-run benefits of anchoring expectations

2. optimal monetary policy time-inconsistent

$\rightarrow$ illustrate in special case: target criterion

Related literature

- **Optimal monetary policy in New Keynesian models**
  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Econometric learning**
  Evans & Honkapohja (2001), Preston (2005), Molnár & Santoro (2014)

- **Anchoring / endogenous gain**
  Carvalho et al (2019), Svensson (2015), Hooper et al (2019), Milani (2014)

Structure of talk

1. Model
2. Ramsey problem for special case

Households - standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j))dj \right] \tag{1}$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j) + \Pi_t^i(j)dj - T_t - P_tC_t^i \tag{2}$$

▸ Consumption, price level

Firms - standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \tag{3}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{4}$$

▸ Profits, stochastic discount factor

Expectations - $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- If use $\mathbb{E}$ (rational expectations, RE)

  Model solution

$$s_t = hs_{t-1} + \epsilon_t \tag{5}$$
$$y_t = gs_t \tag{6}$$

  $s_t \equiv (r_t^n, u_t)'$ (states)
  $y_t \equiv (\pi_t, x_t, i_t)'$ (jumps)

- If use $\hat{\mathbb{E}}$ $\rightarrow$ don't know g
  $\rightarrow$ estimate using observed states & knowledge of (5)

- Estimate g using recursive least squares (RLS)

  $\rightarrow$ nonrational expectations:

$$\hat{\mathbb{E}}_t y_{t+1} = \phi_{t-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix} \tag{7}$$

- Note: misspecified

  Can write:

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{8}$$

  In RE, $a_{t-1} = (0, 0, 0)', b_{t-1} = g \quad \forall t$

Anchoring - endogenous gain

Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', b_{t-1} = g \quad \forall \tag{9}$$

$\rightarrow$ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + bs_{t-1})\right)}_{\equiv fe_{t|t-1}, \text{forecast error}} \tag{10}$$

▸ General RLS algorithm

Gain in literature usually exogenous:

$$k_t = \begin{cases} \frac{1}{t} & \text{decreasing} \\ k & \text{constant} \end{cases}$$

Here instead

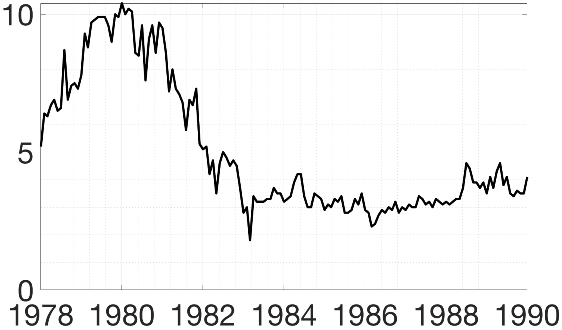
$$k_t = k_{t-1} + \mathbf{g}(fe_{t|t-1}) \tag{11}$$

▸ Functional forms

Anchoring function - interpretation

$$k_t = k_{t-1} + \mathbf{g}(fe_{t|t-1})$$

Figure: U Michigan inflation expectations (%)



- If gain nondecreasing, $\bar{\pi}$ changes $\rightarrow$ *unanchored expectations*
- If gain decreasing, $\bar{\pi}$ stable $\rightarrow$ *anchored expectations*

Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\left((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\right) \tag{12}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\left(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\right) \tag{13}$$

▸ Derivations

- Expectations evolve according to RLS with the endogenous gain given by (11)

→ How should $\{i_t\}$ be set?

Structure of talk

1. Model
2. Ramsey problem for special case

Ramsey problem

$$\max_{\{y_t, \phi_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

- $\mathbb{E}$ is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

Special case

- Only inflation intercept learned
- Anchoring function simplified to

$$k_t = \mathbf{g}(fe_{t|t-1}) \tag{14}$$

Target criterion for special case

Result

In the simplified model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa}\Bigg\{x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\Bigg(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_{\pi,t}\Bigg)$$

$$\Bigg(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi},\mathbf{t+j}})\Bigg)\Bigg\}$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at t,* $\prod_{j=0}^{0} \equiv 1$ *and* b_1 *is the first row of b.*

Interpretation - intertemporal tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t + fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\mathbb{E}_t\sum_{i=1}^{\infty} x_{t+i}$$
$$-\frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t + fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\mathbb{E}_t\sum_{i=1}^{\infty} x_{t+i}\prod_{j=0}^{i-1}(k_{t+1+j} + fe_{t+1+j|t+j})\mathbf{g}_{\bar{\pi},\mathbf{t+j}}$$

tradeoffs from discretion in RE

+ effect of current level and change of the gain on future tradeoffs

+ effect of future expected levels and changes of the gain on future tradeoffs

Lemma

The commitment solution of the Ramsey problem does not exist.

Let $\quad k_t \to 0, \quad \mathbf{g_{z,t}} \to 0.$

Target criterion becomes

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t \tag{15}$$

$=$ target criterion under RE discretion

Corollary

Optimal policy is time-inconsistent.

Already true for exogenous gain learning!

Constant gain specification:

- $k_t = k$
- $g_{z,t} = 0$ (still)

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} k \left(\sum_{i=1}^{\infty} x_{t+i} (1-k)^i \right) \tag{16}$$

$\rightarrow$ A first intertemporal tradeoff

Anchoring as a Second Intertemporal Tradeoff

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t + fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\mathbb{E}_t\sum_{i=1}^{\infty} x_{t+i}$$

$$-\frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t + fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\mathbb{E}_t\sum_{i=1}^{\infty} x_{t+i}\prod_{j=0}^{i-1}(k_{t+1+j} + fe_{t+1+j|t+j})\mathbf{g}_{\bar{\pi},\mathbf{t+j}}$$

+ first intertemporal tradeoff from stance of learning

+ second intertemporal tradeoff from stance of anchoring

How to implement?

- Need reaction function to anchor beliefs

  Recall IS-curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \right)$$

- E.g. Taylor rule disciplines $\hat{\mathbb{E}}_t i_T = \psi_\pi \hat{\mathbb{E}}_t \pi_T + \psi_x \hat{\mathbb{E}}_t x_T$

- Or: need to announce target criterion
- Problem 1: private sector needs to back out reaction function
- Problem 2: private sector cannot compute target criterion

$$\pi_t = -\frac{\lambda_x}{\kappa}\Bigg\{x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\Bigg(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_{\pi,t}\Bigg)$$

$$\Bigg(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi},\mathbf{t+j}})\Bigg)\Bigg\}$$

Form of reaction function?

- Model suggests $\quad i_t = \mathbf{f}(\pi_t, k_t, \bar{\pi}_{t-1})$
- Preliminary results prefer the simple Taylor rule

$$i_t = \psi_\pi \pi_t$$

over

$$i_t = \psi_\pi \pi_t + \psi_k k_t + \psi_{\bar{\pi}} \bar{\pi}_{t-1}$$

- Taylor rule better approximation of optimal policy than under RE?

Conclusion

- Interaction between monetary policy and anchoring
- Optimal policy conditions on
  - stance of expectations
  - stance of anchoring and expected future anchoring
- Optimal policy trades off short-run costs with future benefits of anchoring expectations
- Can explain departures from the Taylor rule like US, fall 2019

Thank you!

FUNCTIONAL FORMS FOR $\mathbf{g}$

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right)' \right)' \tag{17}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{18}$$

◂ Return

Recursive least squares algorithm

$$\phi_t = \left(\phi_{t-1}' + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}\right)'\right)' \tag{19}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{20}$$

◂ Return

Compact notation

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{21}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{22}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{pmatrix} \tag{23}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{24}$$

Return

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{25}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{26}$$

Profits:

$$\Pi_t^j = p_t(j) y_t(j) - w_t(j) f^{-1}(y_t(j)/A_t) \tag{27}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{28}$$

Return

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{29}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{30}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (27) backward to some date *t*, take expectations at *t*
2. Sub in (28)
3. Aggregate over households *i*

$\rightarrow$ Obtain (12)

◂ Return