Info assumptions 8 Jun 2020 cont'd

Let me give names to the info-assumptions:

The particular question is: supposing agents don't know the NKPC, NKIS, do know the TR, do they know the linking equation between the jump (i_t or π_t) and the endogenous lagged value (il_t or pil_t), and if yes, what do they use to fcst them?

- No: fcst i_t using gx and il_t using hx → "myopic info"
- Yes: fcst i_t using gx and il_t using hx → "schizophrenic"
- Yes: both using gx → "suboptimal forecaster info ass"
- Yes: both using hx → "optimal forecaster info ass"

→ these are the two that at least are internally consistent

The irony is that the old approach to il did the "myopic info", and the corrected MN & PQ methods outlined in materials12f1-3 all do the suboptimal forecaster.

So -lol- 12f is not internally consistent!

A "middle road" was ...intrate_smoothing3.m, b/c it does the MN approach w/ the myopic info ass, which is why it always coincides w/ older approaches.

At least what I'm more & more converging on is the idea that, concerning the big-picture question, agents

- do NOT know NKPC & NKIS → b/c they don't know they are identical
- do know the TR → b/c that doesn't require knowledge that they are identical, so CB can just announce it.

But as for whether they know the linking equation:

- both myopic info & optimal forecaster ass are consistent
- but my hunch is that ass. myopic info at the same time as I ass. that they know the TR is weird b/c it amounts to ass-ing that the CB announces the TR but people don't understand that the i_t on the RHS is i_{t-1}.

So from a consistency-realisticness standpoint the "optimal forecaster info" ass seems to be the best.

But in terms of desirability of model dynamics, which should we prefer? In particular, can some overcome the overshooting?

- materials6 introduced int-rate smoothing, now I know using the "myopic info" ass and it didn't do much to dampen the overshooting
- materials12 introduced learning the slope and found it was desirable
  → "optimal forecaster" ass would undo some of this
  ↳ this suggests that having them know less slows down things

This is confusing b/c the "myopic info" ass endowes them w/ less info than the "optimal forecaster". So that suggests that I need to withdraw more info: make 'em learn h x too.

The reason that makes me anxious is that over-shooting is already happening b/c $E(\cdot)$ moves so much. So if I increase the role of $E(\cdot)$ b/c hx isn't known either, then will that not make more overshooting?

Step back 1 sec: $E(\cdot)$ moving wouldn't necess be a prob if i_t wasn't moving so fast or if the TR wasn't known. → No! Whether the TR is known might not matter actually b/c whether agents know how i_t is set, it just affects stuff.

→ No that's not true either b/c LH fcsts matter and thus fcsts of i_{t+k} matter, and how you do those depends on whether you know the TR.

→ Does that mean that I've been ass-ing the TR is known all along?

I think so b/c otherwise the (*)-conditions wouldn't be valid.

Do you agree w/ the statement 9 Jan 2020

$\hat{E}^i_t(i_{t+1}) = \hat{E}_t(i_{t+1})$?

For this to be true, it has to be that agents first aggregates the same way: $\hat{E}^i_t(y_{t+k}) \overset{!}{=} \hat{E}^j_t(y_{t+k})$ for y being an aggregate variable. Given that agents are identical and observe the same aggregates, this should be true. In fact, it may be true for disagg. variables too, except agents don't realize that either.

The fact that agents know the TR allows me to use the (*)-relations to write $\hat{E}_t(i_{t+k}) = \psi_\pi \hat{E}_t(\pi_{t+k}) + \psi_x \hat{E}_t(x_{t+k})$
$+ \hat{E}_t$ (shocks)

But in a certain sense this doesn't help HHs b/c they don't know $\hat{E}_t(\pi_{t+k})$ nor $\hat{E}_t(x_{t+k})$, and so they still estimate $\hat{E}_t(i_{t+k})$ as they estimate gx. Recall that you could solve the model using the PQ-method w/o using (*); then you get a different soln of course when HHs don't know the

TR. But the MN-method is not valid if HHs don't know the TR b/c then you can't sub in $\hat{E}_t\left[\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \text{shocks}_{T+1}\right]$ into $\hat{E}_t i_{T+1}$.

So, since Preston uses the MN method, it means that he ass-s that agents know the Taylor-rule!

Ok: so big-picture info ass are cleared up:

- agents do <u>not</u> know NKIS, NKPC
- agents <u>do</u> know the TR

Now we just need to solve the in-depth issue: do agents internalize the linking equation?

- I think it's tough to tell what has the most appealing dynamics.
- But intuitively I'd say yes.

⇒ Resolve pil & il using the "optimal forecasters" info ass.

⇒ materials 12g2-3.

pil

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t$$

If agents use hx to fcst π, then $f_\beta(1)$ or $f_\alpha(1)$ will not be used.

PQ:

$$\frac{\sigma}{\beta} + \sigma\beta\left(\beta\psi_\pi \frac{\sigma}{\beta}\right) = \frac{\sigma}{\beta} + \sigma^2\beta\psi_\pi$$

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{cases} [0, 1-\beta, -\sigma\beta] f_\beta + \sigma[1, 0, 0, 1](I_{nx} - \beta h_x)^{-1} s_t - [0\ 0\ 0\ \sigma] s_t \\ [0, \kappa\alpha\beta, 0] f_\alpha + [0, 0, 1, (1-\alpha)\beta](I_{nx} - \alpha\beta h_x)^{-1} s_t - [0\ 0\ 0\ (1-\alpha)\beta] s_t \\ [0, 1, 0, \psi_\pi] s_t \end{cases}$$

$$\sigma \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \pi_{T+1} + r_T^n \right\} = \sigma \sum_{T=t}^{\infty} \beta^{T-t} \hat{E}_t \left\{ pil_{T+2} + r_T^n \right\}$$

$$= \sigma\left[pil_{t+2} + r_t^n + \beta\, pil_{t+3} + \beta r_{t+1}^n + \beta^2 pil_{t+4} + \beta^2 r_{t+2}^n + \dots \right]$$

$$= \sigma\left[\frac{1}{\beta^2}\left(\beta^2 pil_{t+2} + \beta^3 pil_{t+3} + \dots \right) + \left(r_t^n + \beta r_{t+1}^n + \beta^2 r_{t+2}^n + \dots \right)\right]$$

$$= \sigma\left[\frac{1}{\beta^2}\left(pil_t + \beta\, pil_{t+1} + \beta^2 pil_{t+2} + \dots \right) + (-\,\|\,-) - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1} \right]$$

$$= \sigma\left[\sum_{T=t}^{\infty} \beta^{T-t} \left(\frac{1}{\beta^2} pil_T + r_T^n \right) - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1} \right]$$

$$= -\frac{\sigma}{\beta}\pi_t + \sigma \sum_{T=t}^{\infty} \beta^{T-t}\left(\frac{1}{\beta^2} pil_T + r_T^n \right) - \frac{\sigma}{\beta^2}[0, 0, 0, 1] s_t$$

The problem is that $\hat{E}_t\, pil_t = \hat{E}_t \pi_{t-1}$ & $\hat{E}_t\, pil_{t+1} = \hat{E}_t \pi_t$

I actually now think it's fine to write

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\, pil_T = \frac{1}{1-\beta(hx_{\pi})}\, pil_t$$

b/c this is $pil_t \,(= \pi_{t-1}) + \beta\, pil_{t+1} \,(= \beta\, hx_{\pi}\, pil_t = \pi_t)$

$+ \beta^2 pil_{t+2} \,(= \beta \pi_{t+1})$.

With this logic, $\hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ (1-\alpha)\beta\, \pi_{T+1} + u_T \right\}$

$$= \hat{E}_t \left[u_t + \alpha\beta u_{t+1} + (\alpha\beta)^2 u_{t+2} + \dots + pil_{t+2} + (\alpha\beta)^2 pil_{t+3} + \dots \right]$$

$$= \hat{E}_t \left[-||- + \frac{1}{(\alpha\beta)^2} \left[pil_t + \alpha\beta\, pil_{t+1} + (\alpha\beta)^2 pil_{t+2} + \dots \right] - \frac{1}{(\alpha\beta)^2} pil_t - \frac{1}{\alpha\beta} pil_{t+1} \right]$$

$$= \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[0, 0, 1, \frac{1}{(\alpha\beta)^2}\right] s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1] s_t - \frac{1}{\alpha\beta}\pi_t$$

$$= \left[0, 0, 1, \frac{1}{(\alpha\beta)^2}\right] (I_{nx} - \alpha\beta\, hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1] s_t - \frac{1}{\alpha\beta}\pi_t$$

So using

$$-\frac{\sigma}{\beta}\pi_t + \sigma \sum_{T=t}^{\infty} \beta^{T-t}\left(\frac{1}{\beta^2} pil_T + r_T^n\right) - \frac{\sigma}{\beta^2}[0,0,0,1]\, s_t$$

and using

$$[0,0,1,\tfrac{1}{\kappa\beta^2}](I_{nx}-\alpha\beta h_x)^{-1} s_t - \frac{1}{\kappa\beta^2}[0,0,0,1]\, s_t - \frac{1}{\alpha\beta}\pi_t$$

we get

$$\begin{bmatrix} \frac{\sigma}{\beta} & 1 & \sigma \\ (1+\frac{1}{\alpha\beta})\kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [0, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0,\frac{1}{\beta^2}](I_{nx}-\beta h_x)^{-1} s_t - \frac{\sigma}{\beta^2}[0,0,0,1]\, s_t \\ [0, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,\frac{1}{\kappa\beta^2}](I_{nx}-\alpha\beta h_x)^{-1} s_t - \frac{1}{\kappa\beta^2}[0,0,0,1]\, s_t \\ [0,1,0,\psi_\pi]\, s_t \end{bmatrix}$$

$(1+\frac{1}{\alpha\beta})$

And we need to modify (*) as well.

But wait a sec: using the above, can I not, instead of rewriting everything, simply add two linking equations that relate $f_\beta(1)$ and $f_\alpha(1)$ to $s_t(4)$?

Something like: $f_\beta(1) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \pi_{T+1}$

$= \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} pil_{T+2} = \hat{E}_t \left[pil_{t+2} + \beta pil_{t+3} + \dots \right]$

$$= \hat{E}_t \left[\frac{1}{\beta^2} \left(pil_t + \beta\, pil_{t+1} + \beta^2 pil_{t+2} + \dots \right) - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1} \right]$$

$$= \frac{1}{\beta^2} \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} pil_T \quad - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} \pi_t$$

$$= \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t - \frac{1}{\beta} \pi_t$$

L1: $f_\beta(1) = \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t - \frac{1}{\beta} \pi_t$

L2: $f_\alpha(1) = \frac{1}{(\alpha\beta)^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0,0,0,1] s_t - \frac{1}{\alpha\beta} \pi_t$

OR: simply add $\underbrace{\text{stuy} \times \left(\frac{1}{\beta}\right)\pi_t}$ and $\underbrace{\text{stuy} \times \left(\frac{1}{\alpha\beta}\right)\pi_t}$ to the LHS, $\quad P(1,1) = \frac{2}{\beta} \qquad P(2,1) = \left(1 + \frac{(1-\alpha)\beta}{\alpha\beta}\right) = \frac{1}{\alpha}$

L1': $f_\beta(1) = \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t$

L2': $f_\alpha(1) = \frac{1}{(\alpha\beta)^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0,0,0,1] s_t$

and then to deal w/ (*), add stuy $\times\, \psi_\pi \pi_t$ to the LHS and lastly there's a $\beta \psi_\pi f_\beta(1)$ in (*), which from the NKIS relation has a -2β coefficient, so put things together:

$$-2\beta \cdot \beta \psi_\pi f_\beta(1) \Rightarrow 2\beta \psi_\pi \cdot \pi_t \Rightarrow \text{add } -2\beta \psi_\pi \pi_t$$

to the LHS, i.e. to $P(1,1)$

$P(1,1)_{\text{final}} = \left(\frac{2}{\beta} - 2\beta \psi_\pi\right)$, don't change anything else but add L1' & L2'.

MN (the red highlighted stuff is correct, see Mathematica)

$$x_t = -b i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + b\pi_{T+1} - b\beta i_{T+1} + b r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \underline{\pi_{t-1}} + \psi_x x_t + \bar{i}_t$$

$$\rightarrow x_t = -b(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-b\beta\psi_x) x_{T+1} + b\pi_{T+1} - b\beta(\psi_\pi \pi_T + \bar{i}_{T+1}) + b r_T^N \right\}$$

$$(1+b\psi_x) x_t = -b\psi_\pi \pi_{t-1}$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-b\beta\psi_x) x_{T+1} + b\pi_{T+1} - b\beta \psi_\pi \pi_T + b(r_T^N + \bar{i}_T) \right\}$$

$$(1+b\psi_x) x_t =$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-b\beta\psi_x) x_{T+1} + b\pi_{T+1} - b\psi_\pi \, pil_T + b(r_T^N - \bar{i}_T) \right\}$$

$$\begin{bmatrix} 0 & 1+b\psi_x \\ 1 & -\kappa \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [b, 1-\beta-b\beta\psi_x, 0] f_\beta + b[1,-1,0,-\psi_\pi](I_{nx}-\beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx}-\alpha\beta h_x)^{-1} s_t \end{bmatrix}$$

and

$$L1: \; f_\beta(1) = \frac{1}{\beta^2}[0,0,0,1](I_{nx}-\beta h_x)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1] s_t - \frac{1}{\beta}\pi_t$$

$$L2: \; f_\alpha(1) = \frac{1}{(\alpha\beta)^2}[0,0,0,1](I_{nx}-\beta h_x)^{-1} s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1] s_t - \frac{1}{\alpha\beta}\pi_t$$

$$\begin{bmatrix} \frac{b}{\beta} & 1+b\psi_x \\ \frac{1}{\alpha} & -\kappa \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [b, 1-\beta-b\beta\psi_x, 0] f_\beta + b[1,-1,0,-\psi_\pi](I_{nx}-\beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx}-\alpha\beta h_x)^{-1} s_t \end{bmatrix}$$

$$1 + \frac{(1-\alpha)\cancel{\beta}}{\alpha\cancel{\beta}} = \frac{\alpha+1-\alpha}{\alpha} = \frac{1}{\alpha}$$

w/ L1' & L2'.

Another attempt at PQ:

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \underline{\pi_{t-1}} + \psi_x x_t + \bar{i}_t$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0,0](I_{nx} - \beta h x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h x)^{-1} s_t \\ [0, 1, 0, \psi_\pi] s_t \end{bmatrix}$$

$P(1,1)$ $\frac{1}{\alpha}$

L1: $f_\beta(1) = \frac{1}{\beta^2}[0,0,0,1](I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1] s_t - \frac{1}{\beta}\pi_t$

L2: $f_\alpha(1) = \frac{1}{(\alpha\beta)^2}[0,0,0,1](I_{nx} - \beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1] s_t - \frac{1}{\alpha\beta}\pi_t$

$P(2,1)$: $1 - \frac{(1-\alpha)\beta}{\alpha\beta}(-1) = 1 + \frac{1-\alpha}{\alpha} = \frac{\alpha + 1 - \alpha}{\alpha} = \underline{\underline{\frac{1}{\alpha}}}$ $P(2,1)$

$P(1,1)$: $\sigma f_\beta(1)$ | (*) But need to rewrite (*)

$= \sigma(-\frac{1}{\beta})$ | over to LHS $\rightarrow \frac{\sigma}{\beta}$ $P(1,1)$

(*) needs to take the "optimal forecaster" ass into account

$$f_\beta(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [i_{T+1}] = \hat{E}_t \sum_{T=t} \beta^{T-t} \left\{ \psi_\pi \pi_T + \psi_x x_{T+1} + \bar{i}_{T+1} \right\}$$

$$= \psi_x f_\beta(2) + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [0,1,0,\psi_\pi] s_{T+1}$$

$$f_\beta(3) = \psi_x f_\beta(2) + \frac{1}{\beta}[0,1,0,\psi_\pi](I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta}[0,1,0,\psi_\pi] s_t$$

Ok, having pil, let's now do it w/ "optimal precoders."

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}$$

Now the linking equations have to relate $f_a(3)$ & $f_b(3)$ to the errors.

$$f_b(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1}$$ | Don't plug the Taylor-rule b/c that is less info than using hx_i.

$$= \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{t+2} = \hat{E}_t \left[i_{t+2} + \beta i_{t+3} + \dots \right]$$

$$= \frac{1}{\beta^2} \hat{E}_t \left[i_t + \beta i_{t+1} + \dots \right] - \frac{1}{\beta^2} i_t - \frac{1}{\beta} i_{t+1}$$

$\underline{\underline{L1}}$

$$f_b(3) = \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t - \frac{1}{\beta} i_t$$

$\underline{\underline{L2}}$

$$f_a(3) = \frac{1}{(\alpha\beta)^2} [0,0,0,1] (I_{nx} - \alpha\beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0,0,0,1] s_t - \frac{1}{\alpha\beta} i_t$$

I wonder what happens to (*) ...

MN

$$x_t = -\sigma\left(\psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}\right)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta)x_{T+1} + \sigma\pi_{T+1} - \sigma\beta\left(\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \bar{i}_T + \rho i_T\right) \sigma r_T^n\right\}$$

But again, this doesn't make sense b/c you've got sthg better than the TR to fest i:

PQ:

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta)x_{T+1} + \sigma\pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n\right\}$$
$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\left\{\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\right\}$$
$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}$$

$$\begin{bmatrix} 0 & 1 & -\sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix}\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0,0](I_{nx} - \beta hx)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + [0,0,1,0](I_{nx} - \alpha\beta hx)^{-1} s_t \\ [0,1,0,\rho] s_t \end{bmatrix}$$

L1 s.t. L1 instead of (*) [and L2 is obsolete!]

$$f_\beta(3) = \frac{1}{\beta^2}[0,0,0,1](I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1] s_t - \frac{1}{\beta} i_t$$

So in the first equation, coeffs of i are:

$$-\sigma - (-\sigma\beta) f_\beta = -\sigma + \sigma\beta\left(-\frac{1}{\beta}\right) = -2\sigma \Leftarrow P(1,3)$$

I just have to hope that this is right!

I guess it's time to compare results and check work. 10 Jan 2020

But first: implement pil w/ "myopic" info ass!
(which actually in terms of forecasting equals the "schizophrenic" info ass b/c either you're myopic and so you don't realize $pil_{t+1} = \pi_t$, so you fcst pil using hx and π_t using gx; or you do realize this but you're schizophrenic and still forecast them separately).

pil - myopic → materials12h2

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t$$

MN

agents don't realize this is the same var

$$x_t = -\sigma(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta \left[\psi_\pi \pi_T + \bar{i}_{T+1} \right] + \sigma r_T^n \right\}$$

$$(1+\sigma\psi_x) x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma \pi_{T+1} - \sigma \left(\psi_\pi \pi_{T-1} + \bar{i}_T \right] + \sigma r_T^n \right\}$$

$$(1+b\psi_x)x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-b\beta\psi_x)x_{T+1} + b\pi_{T+1} - b\left(\psi_\pi \pi_{T-1} + \bar{i}_T\right] + br_T^n \right\}$$

$$\begin{bmatrix} 0 & 1+b\psi_x \\ 1 & -k \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [b,\ 1-\beta-b\beta\psi_x,\ 0] f_\beta + b[1,-1,0,-\psi_\pi](I_{nx}-\beta hx)^{-1} s_t \\ [(1-\alpha)\beta,\ \overset{k\alpha\beta}{\vee},\ 0] f_\alpha + [0,0,1,0](I_{nx}-\alpha\beta hx)^{-1} s_t \end{bmatrix}$$

PQ

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -k & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [b,\ 1-\beta,\ -b\beta] f_\beta + b[1,0,0,0](I_{nx}-\beta hx)^{-1} s_t \\ [(1-\alpha)\beta,\ \overset{k\alpha\beta}{,}\ 0] f_\alpha + [0,0,1,0](I_{nx}-\alpha\beta hx)^{-1} s_t \\ [0,\ 1,\ 0,\ \psi_\pi] s_t \end{bmatrix}$$

(β)

$$f_\beta(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \psi_\pi pil_{T+1} + \psi_x x_{T+1} + \bar{i}_{T+1}$$

$$= \psi_x f_\beta(2) + \frac{1}{\beta}\left\{ [0,1,0,\psi_\pi](I_{nx}-\beta hx)^{-1} s_t - [0,1,0,\psi_\pi] s_t \right\}$$

Ok, so look at IRFs in materials12.tex, quickly:
(Note: should check all work to make sure they are really correct → much of this will be my task on Mon, Tues & Wed.)

1) Baseline: learning slope & constant seems better b/c since agents don't know $gx(2:end,:)$, the Ball-effect can't pan out.

2) Epi:

constant only → instrument instability

slope & constant → not E-stable (visually)

To me it makes intuitive sense that Epi should exhibit more instability than baseline b/c instability in the baseline also comes from the Ball-effect, $E(\cdot)$ moving a lot and mattering a lot due to fwd-lookingness.
In Epi, they get to matter more b/c now if $E(\cdot)$ are unstable, i becomes unstable too ⇒ that's why you get instr-instability. When agents are learning all of gx, then

an exploding i is not sufficient to keep $E(\cdot)$ at bay, leading agents to "learn the wrong thing" and so they don't converge to RE.

3) pil

"myopic": for both learning PLMs, seems not E-stable

In a sense it makes sense b/c their forecasting is not consistent w/ RE. Gets worse the less they know of gx. Why don't they learn that $gx = hx$ for π? Maybe b/c this assumes that $pil \neq \pi$ in RE.

"suboptimal fcst": both behave nicely, in particular the & "optimal fcst": latter

In the former, learning only constant or slope & constant matters of course b/c you're using gx to fcst.

In the latter, this distinction doesn't matter b/c you're using hx to fcst anyway.

I think the reason this model works so nicely in general is b/c it takes out π_t as a jump (and that's what "myopic" fails to do), and since π_t is gone as a jump, $E(\pi)$ don't misbehave, and so the Ball-type disinflationary boom effect doesn't occur.

4) iL

"myopic": behaves a lot like the baseline, where learning the slope too dampens the Ball-effect.

"suboptimal" vs "optimal fcst": again, optimal fcst is more stable b/c your fcst is closer to RE since you're using hx (which comes from RE). Learning slope too makes things worse $\rightarrow$ it seems that the gx you're learning here is unstable and b/c you use it to fcst x & π in both cases, you diverge.

$\Rightarrow$ The overarching theme seems to be that gx is not E-stable and so the only way to restore stability is to effectively make things known: preferably π (piL) b/c it's more $E(\pi)$ that lead to instability $\rightarrow$ need to check E-stability!

Check thm work 14 Jan 2020

(1) Epi

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma \beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \hat{E}_t \pi_{t+1} + \psi_x x_t + \bar{i}_t$$

MN

$$x_t = -\sigma(\psi_\pi \hat{E}_t \pi_{t+1} + \psi_x x_t + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta(\psi_\pi \hat{E}_T \pi_{T+2} + \psi_x x_{T+1} + \bar{i}_{T+1}) + \sigma r_T^n \right\}$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \hat{E}_t \pi_{t+1}$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta\psi_\pi \hat{E}_T \pi_{T+2} \quad + \sigma[1,-1,0] s_T \right\}$$

$$\hat{E}_t \hat{E}_T \pi_{T+1} = \hat{E}_t \pi_{T+1}$$

I've been using all along that $\hat{E}$, not just $\hat{E}^i$, satisfies LIE

$$(1+\sigma\psi_x) x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma(1-\psi_\pi) \pi_{T+1} + \sigma[1,-1,0] s_T \right\}$$

$$\begin{bmatrix} 0 & 1+\sigma\psi_x \\ 1 & -\kappa \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [\sigma(1-\psi_\pi), 1-\beta-\sigma\beta\psi_x, 0] f_\beta + \sigma[1,-1,0](I_{nx}-\beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] + [0,0,1](I_{nx}-\alpha\beta h_x)^{-1} s_t \end{bmatrix}$$

materials12f1.tex ✓ .nb ✓

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \hat{E}_t \pi_{t+1} + \psi_x x_t + \bar{i}_t$$

$+0 \cdot \hat{E}_t \pi_{t+1}$

<u>PQ</u>

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0](I_{nx} - \beta h x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1](I_{nx} - \alpha\beta h x)^{-1} s_t \\ [0,1,0] s_t + \psi_\pi \cdot \hat{E}_t \pi_{t+1} \end{bmatrix}$$

materials12f1. tex ✓ nb ✓

From now on: only MN and only the x-equation!

② <u>PiL</u> <u>first "myopic"</u> (12h2 - which I apparently never typed up on LaTeX)

$$i_t = \psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t$$

<u>MN</u>

$$x_t = -\sigma(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta(\psi_\pi \pi_T + \psi_x x_{T+1} + \bar{i}_{T+1}) + \sigma r_T^n \right\}$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \pi_{t-1}$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta\sigma\psi_x) x_{T+1} + \sigma\pi_{T+1} - \sigma\beta(\psi_\pi \pi_T) + \sigma[1,-1,0,0] s_T \right\}$$

$-\psi_\pi$

✓

Pil – second "suboptimal fasters" (first both using gx)

MN step 1 is the same:

$$x_t = -b(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \{ (1-\beta) x_{T+1} + b\pi_{T+1} - b\beta(\psi_\pi \pi_T + \psi_x x_{T+1} + \bar{i}_{T+1}) + b r_T^n \}$$

$$(1+b\psi_x) x_t = -b\psi_\pi \pi_{t-1} - b\beta\psi_\pi \pi_t$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \{ (1-\beta-\beta b\psi_\pi) x_{T+1} + \underbrace{b\pi_{T+1} - b\beta^2 \psi_\pi \pi_{T+1}}_{b(1-\beta^2\psi_\pi)} + b[1, -1, 0, 0] \cdot \dots \xi_T \}$$

.tex ✓ .nb ✓

Pil – third "optimal fasters" (first both using hx)

MN step 2 is even the same!

$$(1+b\psi_x) x_t = -b\psi_\pi \pi_{t-1} - b\beta\psi_\pi \pi_t$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \{ (1-\beta-\beta b\psi_\pi) x_{T+1} + b(1-\beta^2\psi_\pi)\pi_{T+1} + b[1, -1, 0, 0] \cdot \dots \xi_T \}$$

The diff is that the $b(1-\beta^2\psi_\pi)\pi_{T+1}$ term now goes into the error.

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} b(1-\beta^2\psi_\pi)\pi_{T+1} = b(1-\beta^2\psi_\pi) \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} pil_{T+2}$$

$$= b(1-\beta^2\psi_\pi) \hat{E}_t (pil_{t+2} + \beta\, pil_{t+3} + \dots)$$

$$= b(1-\beta^2\psi_\pi) \hat{E}_t \frac{1}{\beta^2} (pil_t + \beta\, pil_{t+1} + \beta^2 pil_{t+2} + \beta^3 pil_{t+3} + \dots)$$

$$- \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1}$$

$$= b(1-\beta^2\psi_\pi)\,\hat{E}_t\,\frac{1}{\beta^2}\left(pil_t + \beta\, pil_{t+1} + \beta^2 pil_{t+2} + \beta^3 pil_{t+3} + \dots\right)$$

$$- \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1}$$

$$= b(1-\beta^2\psi_\pi)\,[0,0,0,1](I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2}\pi_{t-1} - \frac{1}{\beta}\pi_t$$

So then

$$(1+b\psi_x)x_t = -b\psi_\pi \pi_{t-1} - b\beta\psi_\pi \pi_t$$

$$+\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta-\beta b\psi_\pi)x_{T+1} + b(1-\beta^2\psi_\pi)\pi_{T+1} + b[1,1,0,0]\dots s_T\right\}$$

becomes

$$(1+b\psi_x)x_t = -(b\psi_\pi + \frac{1}{\beta^2})\pi_{t-1} - (b\beta\psi_\pi + \frac{1}{\beta})\pi_t$$

$$+\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta-\beta b\psi_\pi)x_{T+1} + b[1,1,0,1-\beta^2\psi_\pi]s_T\right\}$$

which is NOT what I have at all! LOL!

And using this, you can also rewrite the NKPC

$$\pi_t - \kappa x_t = \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \kappa\alpha\beta\, x_{T+1} + (1-\alpha)\beta\,\hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} pil_{T+2}$$

$$+ [0,0,1,0](I_{nx} - \alpha\beta hx)^{-1} s_t$$

Note that the linking equations do exactly this.

So if I just write the expectations separately, but impose the linking eqs L1 & L2, do I get the same?

MN, Step 1:

$$x_t = -\sigma(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta(\psi_\pi \pi_T + \psi_x x_{T+1} + \bar{i}_{T+1}) + \sigma r_T^n \right\}$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \pi_{t-1}$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta\sigma\psi_\pi) x_{T+1} \underbrace{+\sigma\pi_{T+1} - \sigma\beta\psi_\pi \pi_T}_{\sigma\pi_{T+1} - \sigma\psi_\pi pid_t \text{ (pulling in } \pi_{t-1})} + \sigma[1,-1,0,0] s_T \right\}$$

$$(1+\sigma\psi_x) x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta\sigma\psi_\pi) x_{T+1} + \sigma\pi_{T+1} + \sigma[1,-1,0,-\psi_\pi] s_T \right\}$$
$$+ L1$$

Yes!

$$1 + \frac{(1-\alpha)\beta}{\alpha\beta} = \frac{\alpha + (1-\alpha)}{\alpha} = \frac{1}{\alpha}$$

okidoki; this is fine too!

(3) ii : here I'll only check subopti & opti fcsters.

First: subopt fcsters

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}$$

PQ only

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0,0](I_{nx} - \beta h x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + [0,0,1,0](I_{nx} - \alpha\beta h x)^{-1} s_t \\ [0,1,0,\rho] s_t \end{bmatrix}$$

This initial setup is true for both info assumptions. The two will only change the (*)-condition which relates $f_\beta(3)$ to $f_\beta(1)$ & $f_\beta(2)$, and in the case of "opti fcsters", to $s(4)$

$$f_\beta(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \bar{i}_{T+1} + \rho i_T$$

$$= \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \hat{E}_t(\bar{i}_{t+1} + \beta \bar{i}_{t+2} + \dots) + \rho \hat{E}_t \sum_{T=t} i_T$$

$$= \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \frac{1}{\beta} \hat{E}_t(\bar{i}_t + \beta \bar{i}_{t+1} + \dots) - \frac{1}{\beta} \bar{i}_t + \rho [i_t + \beta i_{t+1} + \dots]$$

$$= \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \frac{1}{\beta} \hat{E}_t(\bar{i}_t + \beta \bar{i}_{t+1} + \dots) - \frac{1}{\beta} \bar{i}_t + \rho\beta [i_{t+1} + \beta i_{t+2} + \dots] + \rho i_t$$

$f_\beta(3)$

$$= \gamma_\pi f_\beta(1) + \gamma_x f_\beta(2) + \frac{1}{\beta}\hat{E}_t\left(i_t + \beta i_{t+1} + \dots\right) - \frac{1}{\beta} i_t + \rho\beta\Big[\overbrace{i_{t+1} + \beta i_{t+2} + \dots}^{f_\beta(3)}\Big] + \rho i_t$$

$$(1-\rho\beta) f_\beta(3) = -''-$$

$$f_\beta(3) = \frac{1}{1-\rho\beta}\Big(\gamma_\pi f_\beta(1) + \gamma_x f_\beta(2) + \frac{1}{\beta}\left\{[0,1,0,0](I_{nx} - \beta h_x)^{-1} s_t - [0,1,0,0] s_t\right\}$$
$$+ \rho i_t\Big)$$

The (*)-condition is the same.

Use this on the $-b\beta f_\beta(3) = -b\beta \cdot \left(\frac{\rho}{1-\rho\beta} i_t\right)$ is on

the RHS $= \frac{b\beta\rho}{1-\rho\beta}$ → so on the LHS we'll have

$$\Rightarrow b + \frac{b\beta\rho}{1-\rho\beta}$$

→ subopti is correct!

Now: "opti feders"

$$f_\beta(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \hat{E}_t\left[i_{t+1} + \beta i_{t+2} + \beta^2 i_{t+3} + \dots\right]$$

$$= \frac{1}{\beta}\left[i_{t-1} + \beta i_t + \beta^2 i_{t+1} + \dots\right] - \frac{1}{\beta^2} i_{t-1} - \frac{1}{\beta} i_t$$

$$= \frac{1}{\beta^2}\left\{[0,0,0,1](I_{nx} - \beta h_x)^{-1} s_t - [0,0,0,1] s_t\right\} \quad = L1' \checkmark$$

And on the RHS we have $-b\beta f_\beta(3) = b\beta \frac{1}{\beta} i_t = b i_t$

→ so on the LHS we'll have $(b - b) i_t = 0$.

LHS: $b - \left[-b\beta\left(-\frac{1}{\beta}\right)\right] = b - b = 0$. $\Rightarrow -2b$ was wrong!

$P(1,3) = 0$.

Ok so we've checked thru everything - it should be all correct! So now: E-stability & interpretation of plots.

conclusions / questions

1) Epi is never E-stable

I think this is b/c although i could dampen things by reacting to $E(\pi)$, $E(\pi)$ internalizes this → feedback effects

2) Why is myopic the only unstable for pil, why is it the only stable for il?

- I think pil explodes for myopic b/c $E(\pi)$ are governed by gx, so explosive, and E(pil) are governed by hx, so stable, so they don't counterbalance.
- For pil the opposite happens b/c when E(i) & E(il) are governed by the same thing, then ppl internalize that i is unstable, either in an oscillatory way (gx) or in a smooth way (hx). By contrast, when they don't

realize that $i = il$, then they think that i is driven by something stable, so it's balanced out.

$\Rightarrow$ it seems like behind all of this are (at least initially) unstable $E(\cdot)$ that need to be balanced out somehow. When the TR fails to do that, you explode, either smoothly or via oscillations.

Somehow I'm not so hot on deriving E-stab. analytically, maybe I'm just tired. I'm more fixated on seeing where to go from here, and what we learn.

Preston (2005, p. 112) suggests that E-stability requires that the TR itself be not an additional source of instability.

$\rightarrow$ but that is exactly what it seems to be when for il the agents recognize $i = il$, b/c this opens a new feedback

$\rightarrow$ and also for pil when agents don't recognize $\pi = pil$

↳ then it's as if they missed a dampening, "negative" feedback.

il (as opposed to pil) makes the TR recursive which generates instability unless you shut off recursiveness by making agents not realize it! (i.e. when they don't internalize that il = i, then you effectively shut off the recursiveness)
The situ is different for pil b/c it doesn't make the TR recursive. Instead, when agents don't realize that π = pil, then the TR is effectively independent of endog stuff ($\psi_x = 0$) and thus not E-stable, as Preston found.

⇒ does pil, fcsting using hx for both ("opt. fcsters") work when agents are learning hx too?

Overshooting comes from $E(\pi)$ moving around a lot and pushing around $E(i)$ due to knowledge of the TR.
⇒ So actually a cheap way to make 'em not know the

TR is simply not to impose (*)!

So take the baseline for example

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

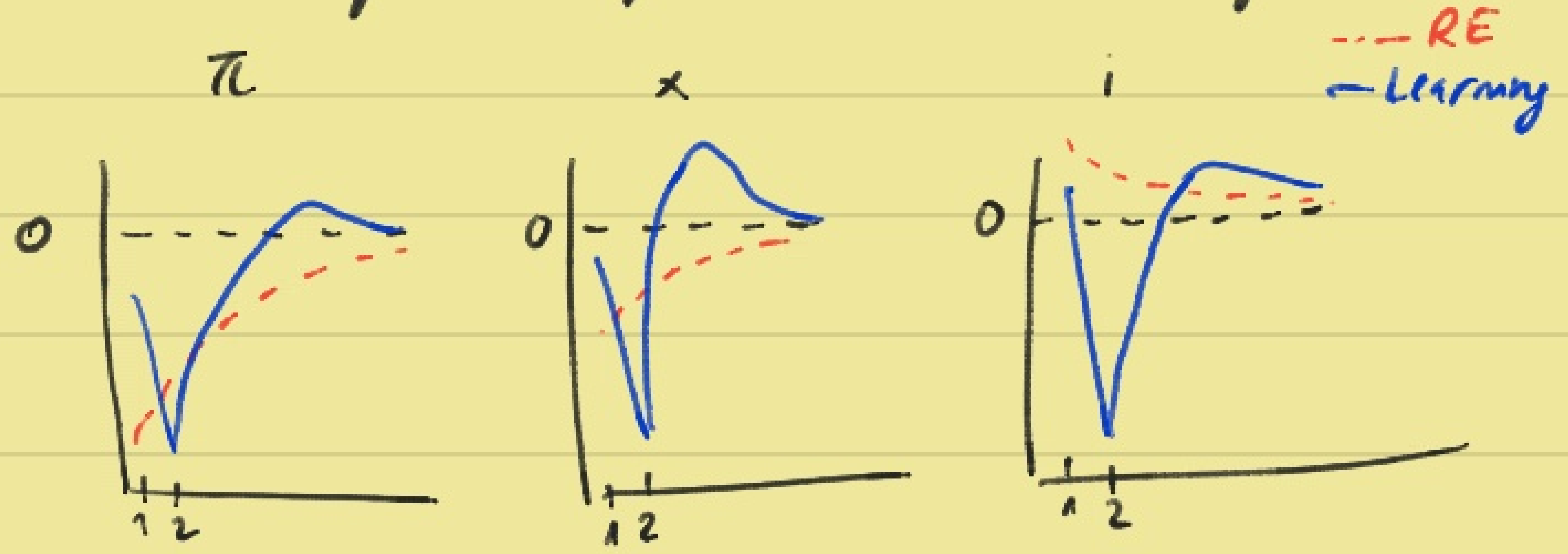
PQ

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma [1,0,0] (I_{nx} - \beta hx)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1] s_t \\ [0,1,0] s_t \end{bmatrix}$$

and no (*) imposed!

→ <u>12i</u>

↳ very interesting IRFs! (constant-only PLM)



→ You can literally see the overshooting be born as they learn the TR!

Ryan meeting 15 Jan 2020

Macro seminars this semester: Last date: Apr 23 or 30?
→ CEMP might be next semester so you're good.

Josephine de Karman: Needs to be received by Jan 31
in one package by snail mail. You'll need to sign
my application document too!

CSWEP due Feb 1 → gonna try Boston Fed
Summer
↳ I'll send the request soon.

Is it an issue that for pil, mon pol shock isn't 1
on impact?
→ looks like a scaled-down version of RE.

Not knowing the TR is like Angeletos: "agents don't
understand GE".

check in data whether EE learning fits data better check Fabian Winkler's work → he'll have dealt w/ these issues!

Work after

Looking at lit w/ estimation of learning and Winkler:

- Winkler 2019: Beautifully oscillating IRFs! p. 12 (Fig 2)

Should try int. rate smoothing as

16 Jan 2020

- $i_t = \rho i_{t-1} + (1-\rho)[\psi_\pi \pi_t + \psi_x x_t] + \varepsilon_t$ (Milani 2007, p. 4 Mac)

- Indexation in NKPC (Milani 2004b p 27 Mac)
- set $k^* =$ optimal k from Kalman filter = relative variances of transitory & permanent shocks (Orphanides & W, 2005 footnote 18, p. 13 Mac)

- Durig & Leeper's generalized Taylor principle suggests that maybe I could have an alternating, state-contingent TR.
- or have initial beliefs different (not RE) like Leeble
  - Add burn-in!

- Milani (2006) finds for LH learning a gain of 0.028
  - also contrasts EE & LH learning, and finds the fit of the former better.

- Slobodyan & Wouters' "VAR learning" may be similar to my "let them learn hx too".
  It seems like VAR learning uses a smaller set of observables in the RLS algorithm than MSV.
  ↳ could also be an alternative to consider
- Erceg & Levin 2003 do "3-horizon learning" and they differentiate them $\tilde{E}$ no problem :)
  But they have a Kalman-filter type of "distinguish temporary from persistent component" learning.
  → not entirely sure about distinction btwn constant gain learning and the Kalman filter.
- Preston et al "Limits": int. rate expectations induce instability in the model ⇒ aggressive policy leads to learning about i which creates instability ⇒ related to what I find!

- Eusepi & Preston (2011, AER) actually do compare LH & EE, finding that EE yields much worse properties (no amplification or propagation, moments are too close to RE) and they also cite another paper to reach similar conclusions (p. 25 Mac)

  But Lubik also got stuff like this: EE learning only matters a lot when initial beliefs are not RE.

- Eusepi & Preston (2018) also has stuff that resembles "Limits" and my instability results. But finds that you should be more aggressive on π.

  But Result 6 (p. 30 Mac) makes clear that it's learning i and not the internalization of $E(\pi) \to E(i)$ that drives this, so when $i_t \uparrow \to$ agents expect LR-i to be higher ($\to$ in my case, lower b/c $E(\pi)\downarrow$)

- Graham: as I remembered, he prefers LH to EE b/c the former converges faster. EE is more volatile than LH.

Peter meeting

16 Jan 2020

- Modify the NKPC
- Davig & Leeper : $\psi_\pi = 1.01$ would that work?
- Learn hx

Somehow add states as "copies" to jump vector

→ some accounting trick to relabel stocks as flows

Like you can rewrite an AR(1) WLOG to AR(2), there might be some trick to rewrite the eq system $\left\{\begin{array}{l}\text{states}\\\text{flows}\end{array}\right.$ as one equation $\begin{bmatrix}\text{states}\\\text{flows}\end{bmatrix}$ and then use the existing learning machinery on that matrix.

- low gain is ok: if high gain produces these counterfactual IRFs, then an MLE would give you a low gain estimate to make sure that IRFs align w/ those in data.
- not spend too much time (more than 1 month)

⇒ You want the best model possible, yes, but don't want to spend too much time finding it!