Materials 6 - More on IRFs

Laura Gáti

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Overview

1 Model summary, adding ρi_{t-1}

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \rho i_{t-1} + \bar{i}_t \tag{3}$$

$$\hat{\mathbb{E}}_t z_{t+h} = \begin{bmatrix} \bar{\pi}_{t-1} \\ 0 \\ 0 \end{bmatrix} + bP^{h-1}s_t \quad \forall h \geq 1 \qquad b = gx\ hx \qquad \text{PLM} \tag{4}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \underbrace{\big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big)}_{\text{fcst error using (4)}} \qquad (b_1 \text{ is the first row of } b) \tag{5}$$

$$k_t = \mathbb{I} \times (k_{t-1} + 1) + (1 - \mathbb{I}) \times \bar{g}^{-1} \tag{6}$$

$$\mathbb{I} = \begin{cases} 1 & \text{if } \theta_t \leq \bar{\theta} \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

$$\theta_t = |\hat{\mathbb{E}}_{t-1}\pi_t - \mathbb{E}_{t-1}\,\pi_t|/\sigma_s \qquad \text{CEMP criterion for the gain} \tag{8}$$

The alternative criterion for the choice of gain is a recursive variant of the CUSUM-test (Brown, Durbin, Evans 1975):

1. Let FE_t denote the short-run forecast error, and ω_t firms' estimate of the FE variance.

2. Let $\kappa \in (0, 1)$ and $\tilde{\theta}$ be the new threshold value for the criterion.

3. Then for initial (ω_0, θ_0), firms in every period estimate the criterion and the FEV as:

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1}(FE_t^2 - \omega_{t-1}) \tag{9}$$

$$\theta_t = \theta_{t-1} + \kappa k_{t-1}^{-1}(FE_t^2/\omega_t - \theta_{t-1}) \tag{10}$$

$$k_t = \mathbb{I} \times (k_{t-1} + 1) + (1 - \mathbb{I}) \times \bar{g}^{-1} \tag{11}$$

$$\mathbb{I} = 1 \quad \text{if} \quad \theta_t \leq \tilde{\theta} \tag{12}$$

2 Compact notation - with lagged interest rate term in TR

$$z_t = A_p^{RE}\,\mathbb{E}_t\,z_{t+1} + A_s^{RE} s_t \tag{13}$$

$$z_t = A_a^{LH} f_a(t) + A_b^{LH} f_b(t) + A_s^{LH} s_t \tag{14}$$

$$s_t = P s_{t-1} + \epsilon_t \tag{15}$$

$$\text{where}\quad s_t \equiv \begin{pmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{pmatrix} \quad P \equiv \begin{pmatrix} \rho_r & 0 & 0 & 0 \\ 0 & \rho_i & 0 & 0 \\ 0 & 0 & \rho_u & 0 \\ 0 & 0 & 0 & \rho \end{pmatrix} \quad \epsilon_t \equiv \begin{pmatrix} \varepsilon_t^r \\ \varepsilon_t^i \\ \varepsilon_t^u \\ 0 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} \sigma_r & 0 & 0 & 0 \\ 0 & \sigma_i & 0 & 0 \\ 0 & 0 & \sigma_u & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{16}$$

Adding i_{t-1} to the state vector fortunately doesn't change A_p^{RE}, A_a^{LH} or A_b^{LH}, but it does change A_s^{RE} and A_s^{LH}. The latter two get an additional column to account for the new state variable. With $g_{i,j}\;\; i = \pi, x,\;\; j = a, b$ unchanged from Materials 4, the new coefficient matrices are given by (new elements highlighted in blue):

$$A_s^{RE} = \begin{pmatrix} \frac{\kappa\sigma}{w} & -\frac{\kappa\sigma}{w} & 1 - \frac{\kappa\sigma\psi_\pi}{w} & 0 \\ \frac{\sigma}{w} & -\frac{\sigma}{w} & -\frac{\sigma\psi_\pi}{w} & 0 \\ \psi_x(\frac{\sigma}{w}) + \psi_\pi(\frac{\kappa\sigma}{w}) & \psi_x(-\frac{\sigma}{w}) + \psi_\pi(-\frac{\kappa\sigma}{w}) + 1 & \psi_x(-\frac{\sigma\psi_\pi}{w}) + \psi_\pi(1 - \frac{\kappa\sigma\psi_\pi}{w}) & \rho \end{pmatrix} \tag{17}$$

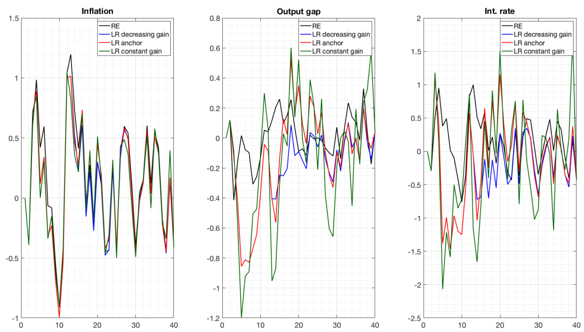
$$A_s^{LR} = \begin{pmatrix} g_{\pi s} \\ g_{xs} \\ \psi_\pi g_{\pi s} + \psi_x g_{xs} + \begin{bmatrix} 0 & 1 & 0 & \rho \end{bmatrix} \end{pmatrix} \tag{18}$$

$$g_{\pi s} = (1 - \frac{\kappa\sigma\psi_\pi}{w}) \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} (I_3 - \alpha\beta P)^{-1} - \frac{\kappa\sigma}{w} \begin{bmatrix} -1 & 1 & 0 & \rho \end{bmatrix} (I_3 - \beta P)^{-1} \tag{19}$$

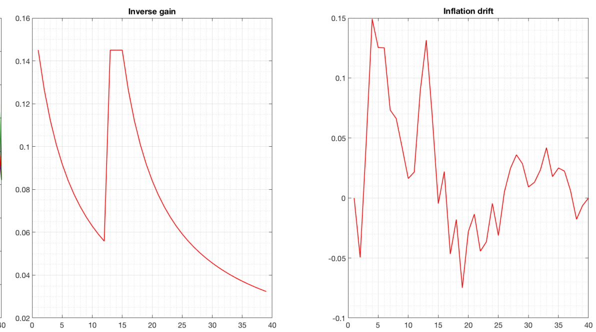
$$g_{xs} = \frac{-\sigma\psi_\pi}{w} \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} (I_3 - \alpha\beta P)^{-1} - \frac{\sigma}{w} \begin{bmatrix} -1 & 1 & 0 & \rho \end{bmatrix} (I_3 - \beta P)^{-1} \tag{20}$$

3 Small deviations in π, large ones in x, overshooting - IRFs to a natural real rate shock

Figure 1: A baseline shock sequence

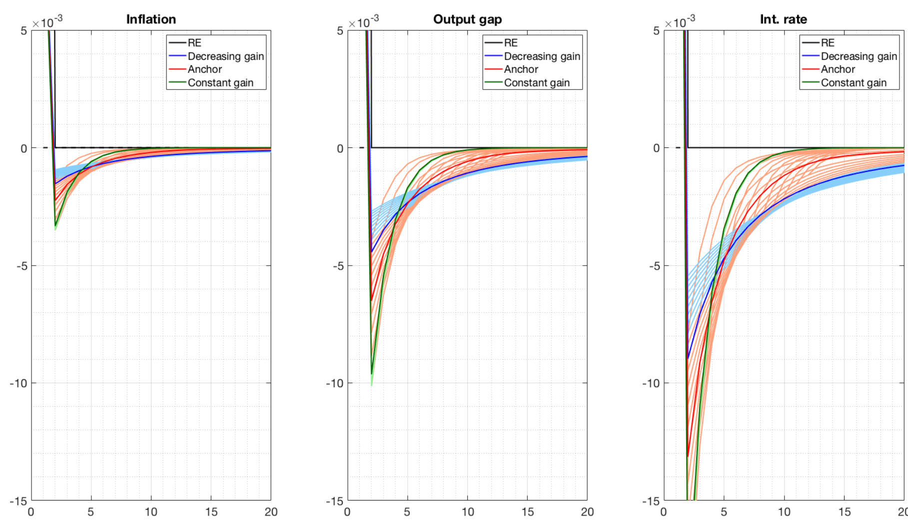


(a) The observables for the specific shock sequence



(b) Inverse gain and drift for the specific shock sequence

Figure 2: IRFs to a natural rate shock (r^n)



Proposition 1. *The degree to which inflation responds to expectational deviations from rational expectations (RE) depends on κ, the slope of the Phillips curve. A lower κ means higher price rigidity and translates to current inflation responding less to expectation gaps.*

Corollary 1. *Whether expectation gaps between subjective and rational expectations show up in inflation or output depends on the value of κ. When nominal rigidities are high (κ is low), money is strongly nonneutral, and thus output is the margin of adjustment. When prices are flexible, money is neutral and inflation is the margin of adjustment.*

Proposition 2. *The fact that firms and households choose between decreasing and constant gains endogenously (anchoring mechanism) introduces a novel tradeoff for monetary policy relative to exogenous gain learning (decreasing or constant). With exogenous gain learning, too strong monetary responsiveness to shocks leads to overshooting due to increased persistence of inflation. With endogenous gain learning, however, monetary policy has to hit a lower bound on responsiveness in order to keep inflation expectations anchored. With endogenous gain learning, monetary policy faces a tradeoff between overshooting and anchoring.*