

Monetary Policy & Anchored Expectations

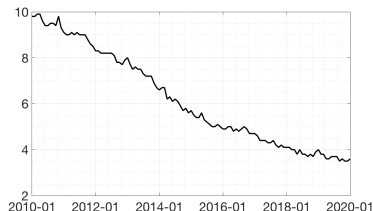
An Endogenous Gain Learning Model

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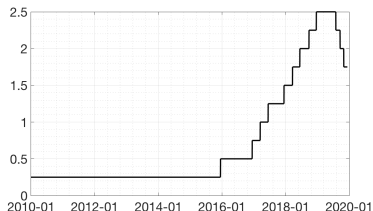
Boston College

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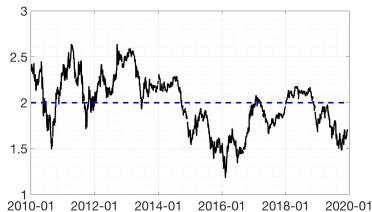
Puzzling Fed behavior fall 2019



(a) Unemployment rate, %



(b) Fed funds rate target, upper limit, %



(c) Market-based inflation expectations, 10 year, % average

This project

Model anchored expectations as an endogenous gain learning scheme

- How to conduct optimal monetary policy in interaction with the anchoring expectation formation?

Preview of results

1. Two layers of new intertemporal tradeoffs
2. Optimal monetary policy time-inconsistent

→ Illustrate analytically in special case: target criterion
3. Not today: short-run costs vs. long-run benefits of anchoring expectations

Related literature

- **Optimal monetary policy in New Keynesian models**

Clarida, Gali & Gertler (1999), Woodford (2003)

- **Econometric learning**

Evans & Honkapohja (2001), Preston (2005), Molnár & Santoro (2014)

- **Anchoring / endogenous gain**

Carvalho et al (2019), Svensson (2015), Hooper et al (2019), Milani (2014)

Structure of talk

1. Model

2. Solving the Ramsey problem

3. Implications

Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j)) dj \right] \quad (1)$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j) + \Pi_t^i(j) dj - T_t - P_t C_t^i \quad (2)$$

► Consumption, price level

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} \mathcal{Q}_{t,T} \left[\Pi_t^j(p_t(j)) \right] \quad (3)$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \quad (4)$$

► Profits, stochastic discount factor

Expectations: $\hat{\mathbb{E}}$ instead of \mathbb{E}

- If use \mathbb{E} (rational expectations, RE)

Model solution

$$s_t = h s_{t-1} + \epsilon_t \quad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \quad (5)$$

$$y_t = g s_t \quad (6)$$

$$s_t \equiv (r_t^n, u_t)' \quad (\text{states})$$

$$y_t \equiv (\pi_t, x_t, i_t)' \quad (\text{jumps})$$

- If use $\hat{\mathbb{E}} \rightarrow$ private sector does not know g
 \rightarrow estimate using observed states & knowledge of (5)
- Households and firms don't know they are identical

Adaptive learning

- Estimate g using recursive least squares (RLS)

→ nonrational expectations:

$$\hat{\mathbb{E}}_t y_{t+1} = \phi_{t-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix} \quad (7)$$

- Note: misspecified

Can write:

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \quad (8)$$

In RE, $a_{t-1} = (0, 0, 0)'$, $b_{t-1} = g h \quad \forall t$

Recursive least squares

Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g h \quad \forall t \quad (9)$$

→ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))}_{\equiv fe_{t|t-1}, \text{ forecast error}} \quad (10)$$

$k_t \in (0, 1)$ gain

b_1 first row of b

► General RLS algorithm

Anchoring mechanism: endogenous gain

Gain in literature usually exogenous:

$$k_t = \begin{cases} \frac{1}{t} & \text{decreasing} \\ k & \text{constant} \end{cases}$$

Here instead

$$k_t = k_{t-1} + \mathbf{g}(fe_{t|t-1}) \quad (11)$$

► Functional forms

Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (12)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (13)$$

► Derivations

- Expectations evolve according to RLS with the endogenous gain given by (11)

→ How should $\{i_t\}$ be set?

Structure of talk

1. Model

2. Solving the Ramsey problem

3. Implications

Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

- \mathbb{E} is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

Special case

- Only inflation intercept learned
- Anchoring function simplified to

$$k_t = \mathbf{g}(fe_{t|t-1}) \quad (14)$$

Target criterion for special case

Result

In the simplified model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa} \left\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_{\pi,t} \right) \right.$$

$$\left. \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\pi,t+j}) \right) \right\}$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ at t , $\prod_{j=0}^0 \equiv 1$ and b_1 is the first row of b .

Two layers of intertemporal tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa} \mathbf{x}_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + f e_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} \mathbf{x}_{t+i} \\ - \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + f e_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} \mathbf{x}_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j} + f e_{t+1+j|t+j} \mathbf{g}_{\tilde{\pi},t+j})$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

Corollary

Optimal policy under adaptive learning is time-inconsistent.

Lemma

The commitment solution of the Ramsey problem does not exist under adaptive learning.

► Why no commitment?

Structure of talk

1. Model

2. Solving the Ramsey problem

3. Implications

Implementation: feedback rule

- Related issue under RE: optimal interest rate sequence implies indeterminate equilibrium

⇒ Reaction function stabilizes expectations

Recall IS-curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n)$$

- E.g. Taylor rule disciplines expectations:

$$\hat{\mathbb{E}}_t i_T = \psi_\pi \hat{\mathbb{E}}_t \pi_T + \psi_x \hat{\mathbb{E}}_t x_T$$

Next steps: form of reaction function

- Model suggests $i_t = \mathbf{f}(\pi_t, k_t, \bar{\pi}_{t-1}; t)$ nonlinear
- Explains deviations from Taylor rule
- However: no commitment makes Taylor rule more viable than under RE as a rough approximation of optimal feedback rule
- If Taylor rule, model prefers being less aggressive on inflation

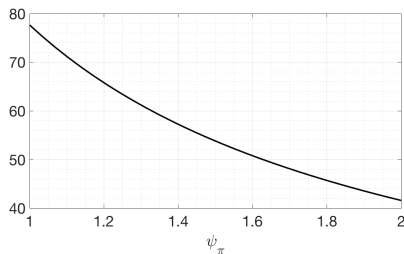
Conclusion

- Interaction between monetary policy and anchoring
- Optimal policy conditions on stance of current and expected future anchoring
 - ↪ determine intertemporal tradeoffs
- Explain departures from the Taylor rule like US, fall 2019
- If Fed acted to anchor expectations, then Missing Deflation and Inflation are not “missing”

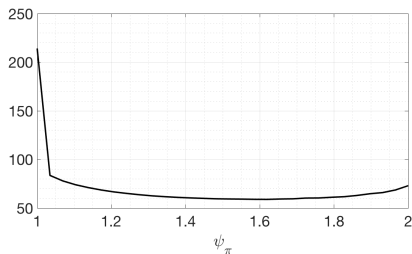
Short-run costs, long-run benefits

Assume Taylor rule and no concern for output gap stabilization

$$i_t = \psi_\pi \pi_t \quad \lambda_x = 0$$



(d) RE



(e) Anchoring

Figure: Central bank loss as a function of ψ_π

Functional forms for \mathbf{g}

- Smooth anchoring function

$$k_t = k_{t-1} - c + dfe_{t|t-1}^2 \quad (15)$$

$$c, d > 0$$

- Kinked anchoring function

$$k_t = \begin{cases} \frac{1}{t} & \text{when } \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \quad (16)$$

θ_t criterion, $\bar{\theta}$ threshold value

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \quad (17)$$

Σ variance-covariance matrix of shocks

$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1} (fe_{t|t-1} fe'_{t|t-1} - \omega_{t-1}) \quad (18)$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1} (fe'_{t|t-1} \omega_t^{-1} fe_{t|t-1} - \theta_{t-1}) \quad (19)$$

ω_t estimated forecast-error variance

Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right) \right)' \quad (20)$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} [1 \quad s_{t-1}] - R_{t-1} \right) \quad (21)$$

Compact notation

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \quad (22)$$

$$s_t = h s_{t-1} + \epsilon_t \quad (23)$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \quad s_t \equiv \begin{pmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{pmatrix} \quad (24)$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \quad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \quad (25)$$

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t) \right. \\ \left. + \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \right\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa}x_t - \varphi_{2,t}(k_t + \mathbf{g}_{\pi}(\pi_t - f_{t-1})) = 0 \quad (26)$$

$$-2\beta\frac{\lambda}{\kappa}x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_f(\pi_{t+1} - f_t)) = 0 \quad (27)$$

Short-run costs from oscillatory dynamics

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \quad (28)$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \quad (29)$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \quad (30)$$

Solve for forecast error $fe_t \equiv \pi_t - f_{t-1}$:

$$fe_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k \rightarrow 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \quad (31)$$

Target criterion system for anchoring function as changes of the gain

$$\begin{aligned} \varphi_{6,t} = & -cfe_{t|t-1}x_{t+1} + \left(1 + \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1}) - fe_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1} \\ & - \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1})\varphi_{6,t+2} \end{aligned} \quad (32)$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{fe_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{fe_{t|t-1}}\varphi_{6,t+1} \quad (33)$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (33) is given by:

$$\varphi_{6,t} = -2\mathbb{E}_t \sum_{i=0}^{\infty} \left(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}\right) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{fe_{t+j|t+j-1}}}{\frac{k_{t+j}}{fe_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \quad (34)$$

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (35)$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \quad (36)$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \quad (37)$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \quad (38)$$

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \quad (39)$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \quad (40)$$

where ‘hats’ denote log-linear approximation and

$$\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}.$$

1. Solve (39) backward to some date t , take expectations at t
 2. Sub in (40)
 3. Aggregate over households i
- Obtain (12)