# Monetary Policy & Anchored Expectations
# An Endogenous Gain Learning Model

Laura Gáti

Boston College

TO DO April 15, 2020

*Inflation that runs below its desired level can lead to an unwelcome fall in* ***longer-term inflation expectations****, which, in turn, can pull actual inflation even lower, resulting in an adverse cycle of ever-lower inflation and inflation expectations. [...]* ***Well-anchored inflation expectations*** *are critical[.]*
*Jerome Powell, Chairman of the Federal Reserve* [1]
*(Emphases added.)*

[1] "New Economic Challenges and the Fed's Monetary Policy Review," August 27, 2020.

# Anchored expectations

$$\mathbb{E}_t \, \pi_{t+1} = \bar{\pi} + f(shocks) \tag{1}$$

$\bar{\pi}$ = long-run expectations

# Anchored expectations

$$\mathbb{E}_t\, \pi_{t+1} = \bar{\pi} + f(\mathit{shocks}) \tag{1}$$

$\bar{\pi} =$ long-run expectations

*Anchored expectations*: long-run expectations at Fed's target ($\pi^*$)

# Anchored expectations

$$\mathbb{E}_t\, \pi_{t+1} = \bar{\pi} + f(shocks) \tag{1}$$

$\bar{\pi}$ = long-run expectations

*Anchored expectations*: long-run expectations at Fed's target ($\pi^*$)

$\rightarrow$ short-run expectations "anchored" to stable mean:

$$\mathbb{E}_t\, \pi_{t+1} = \pi^* + f(shocks) \tag{2}$$

# Anchored expectations

$$\mathbb{E}_t\, \pi_{t+1} = \bar{\pi} + f(shocks) \tag{1}$$

$\bar{\pi}$ = long-run expectations

*Anchored expectations*: long-run expectations at Fed's target ($\pi^*$)

$\rightarrow$ short-run expectations "anchored" to stable mean:

$$\mathbb{E}_t\, \pi_{t+1} = \pi^* + f(shocks) \tag{2}$$

*Unanchored expectations*: long-run expectations deviate systematically from the target

$$\mathbb{E}_t\, \pi_{t+1} = \bar{\pi}_{t-1}(shocks) + f(shocks) \tag{3}$$

# Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \, \mathbb{E}_t \, \pi_{t+1} + u_t \tag{4}$$

(5)

# Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \, \mathbb{E}_t \, \pi_{t+1} + u_t \tag{4}$$

$$\pi_t = \kappa x_t + \beta (2\% + f(shocks)) + u_t \tag{5}$$

# Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta\, \mathbb{E}_t\, \pi_{t+1} + u_t \tag{4}$$

$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t \tag{5}$$

The Fed's uneasiness: public thinks inflation averages only 1%

$$\pi_t = \kappa x_t + \beta(\underset{\downarrow}{1\%} + f(shocks)) + u_t \tag{6}$$

# Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta\,\mathbb{E}_t\,\pi_{t+1} + u_t \tag{4}$$

$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t \tag{5}$$

The Fed's uneasiness: public thinks inflation averages only 1%

$$\underset{\downarrow}{\pi_t} = \kappa x_t + \beta(\underset{\downarrow}{1\%} + f(shocks)) + u_t \tag{6}$$

# Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta\,\mathbb{E}_t\,\pi_{t+1} + u_t \tag{4}$$

$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t \tag{5}$$

The Fed's uneasiness: public thinks inflation averages only 1%

$$\underset{\downarrow}{\pi_t} = \kappa x_t + \beta(\underset{\downarrow\downarrow}{1\%} + f(shocks)) + u_t \tag{6}$$

# Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \, \mathbb{E}_t \, \pi_{t+1} + u_t \tag{4}$$

$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t \tag{5}$$

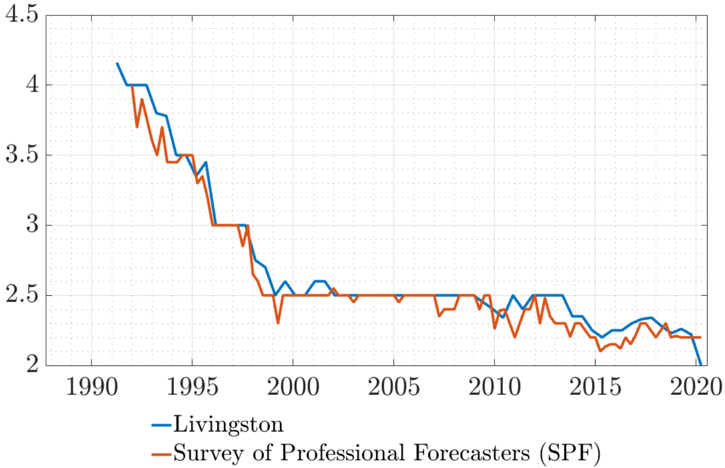
The Fed's uneasiness: public thinks inflation averages only 1%

$$\underset{\downarrow}{\pi_t} = \kappa x_t + \beta(\underset{\downarrow\downarrow}{1\%} + f(shocks)) + u_t \tag{6}$$

→ unanchored expectations can cause a deflationary spiral

# Long-run expectations drifting down?

Figure: Expectations of average inflation over 10 years ($\bar{\pi}$ in data)




▸ TIPS

# Long-run expectations moving systematically?

Individual SPF forecasts: for 1991-Q4 onward, estimate rolling regression

$$\Delta\bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{7}$$

where $w$ indexes windows of 19 quarters length

# Long-run expectations moving systematically?

Individual SPF forecasts: for 1991-Q4 onward, estimate rolling regression

$$\Delta \bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{7}$$

where $w$ indexes windows of 19 quarters length

Table: Rolling regression coefficients

| Window (start quarter) | $\hat{\beta}_1^w$ |
|---|---|
| 1991-Q4 | $0.0687^{**}$ |
| 1996-Q3 | $0.0464^{***}$ |
| 2001-Q2 | 0.0116 |
| 2006-Q1 | 0.0061 |
| 2010-Q4 | 0.0135 |
| 2015-Q3 | $0.0496^{***}$ |

**** significance at 1% ** significance at 5%*

▸ Further evidence

# This project

- How to conduct monetary policy in interaction with the anchoring expectation formation?

- Model of anchoring expectation formation as an extension to adaptive learning
  $\hookrightarrow$ twist: systematic fluctuations in long-run expectations

- Estimation of the anchoring function: when do expectations become unanchored?

# Preview of results

- Optimal policy aggressive when expectations unanchor, accommodates when anchored

- Taylor rule policy less aggressive on inflation than under rational expectations

$\hookrightarrow$ Anchoring-optimal Taylor rule eliminates 84% of loss from volatility

# Related literature

- **Optimal monetary policy in the New Keynesian model**

  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Adaptive learning**

  Evans & Honkapohja (2001, 2006), Sargent (1999), Primiceri (2006), Lubik & Matthes (2018), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Mele et al (2019), Eusepi & Preston (2011), Milani (2007, 2014)

- **Anchoring and the Phillips curve**

  Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Reis (2020), Gobbi et al (2019), Carvalho et al (2019)

- **Reputation**

  Barro (1986), Cho & Matsui (1995)

# Structure of talk

1. Model of anchoring expectations
2. Solving the Ramsey problem
3. Implementing optimal policy
4. Approximating optimal policy with a Taylor rule

# Structure of talk

1. Model of anchoring expectations

2. Solving the Ramsey problem

3. Implementing optimal policy

4. Approximating optimal policy with a Taylor rule

# Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ U(C_T^i) - \int_0^1 v(h_T^i(j))dj \right] \tag{8}$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_t C_t^i \tag{9}$$

▸ Consumption, price level

# Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[ \Pi_t^j(p_t(j)) \right] \tag{10}$$

subject to demand

$$y_t(j) = Y_t \left( \frac{p_t(j)}{P_t} \right)^{-\theta} \tag{11}$$

▸ Profits, stochastic discount factor

# Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- If use $\mathbb{E}$ (rational expectations, RE)

  Model solution

$$s_t = h s_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \tag{12}$$

$$y_t = g s_t \tag{13}$$

$s_t \equiv$ states
$y_t \equiv$ jumps
$\epsilon_t \equiv$ disturbances

# Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- If use $\mathbb{E}$ (rational expectations, RE)

  Model solution

$$s_t = hs_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \tag{12}$$
$$y_t = gs_t \tag{13}$$

  $s_t \equiv$ states
  $y_t \equiv$ jumps
  $\epsilon_t \equiv$ disturbances

- If use $\hat{\mathbb{E}}$ $\rightarrow$ private sector does not know (13)

  $\hookrightarrow$ estimate using observed states & knowledge of (12)

# Adaptive learning

- Postulate linear functional relationship instead of (13):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{14}$$

# Adaptive learning

- Postulate linear functional relationship instead of (13):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{14}$$

$a \rightarrow$ concept of long-run expectations in the model

# Adaptive learning

- Postulate linear functional relationship instead of (13):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{14}$$

$a \rightarrow$ concept of long-run expectations in the model

In RE, $a_{t-1} = \mathbf{0}, \quad b_{t-1} = g\,h \quad \forall t$

# Adaptive learning

- Postulate linear functional relationship instead of (13):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{14}$$

$a \rightarrow$ concept of long-run expectations in the model

In RE, $a_{t-1} = \mathbf{0}, \quad b_{t-1} = g\, h \quad \forall t$

$\rightarrow$ In RE, expectations anchored by assumption

# Adaptive learning

- Postulate linear functional relationship instead of (13):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{14}$$

$a \rightarrow$ concept of long-run expectations in the model

In RE, $a_{t-1} = \mathbf{0}, \quad b_{t-1} = g\, h \quad \forall t$

$\rightarrow$ In RE, expectations anchored by assumption

- Estimate $a, b$ using recursive least squares (RLS)

# Recursive least squares

Jumps are: $(\pi, x, i)'$

# Recursive least squares

Jumps are: $(\pi, x, i)'$

Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\, h \quad \forall t \tag{15}$$

# Recursive least squares

Jumps are: $(\pi, x, i)'$

Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\, h \quad \forall t \tag{15}$$

$\bar{\pi}_{t-1}$: long-run inflation expectations $\rightarrow$ anchoring

# Recursive least squares

Jumps are: $(\pi, x, i)'$

Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\, h \quad \forall t \tag{15}$$

$\bar{\pi}_{t-1}$: long-run inflation expectations $\rightarrow$ anchoring

$\rightarrow$ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\right)}_{\equiv f_{t|t-1}, \text{ forecast error}} \tag{16}$$

$k_t \in (0, 1)$ gain
$b_1$ first row of $b$

General RLS algorithm

# Decreasing versus constant gain

Decreasing gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f_{t|t-1} \tag{17}$$

$\rightarrow$ consider sample mean of full sample of forecast errors

# Decreasing versus constant gain

Decreasing gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f_{t|t-1} \quad (17)$$

$\rightarrow$ consider sample mean of full sample of forecast errors

Constant gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k\, f_{t|t-1} \quad (18)$$

$\rightarrow$ consider sample mean of most recent observations only

# Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big) \tag{19}$$

$k_t = \mathbf{g}(f_{t|t-1})$: anchoring function

# Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big) \tag{19}$$

$k_t = \mathbf{g}(f_{t|t-1})$: anchoring function

$$\mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}) \tag{20}$$

$b_i(f_{t|t-1}) =$ basis, here: second order spline (piecewise linear)

$\gamma_i =$ approximating coefficients
$\hookrightarrow$ estimate $\hat{\gamma}$ via simulated method of moments (SMM)

Target autocovariances of inflation, output gap, federal funds rate and 1-year ahead SPF inflation expectations at lags $0, \ldots, 4$

▸ Functional forms in literature

# Anchoring function in the data

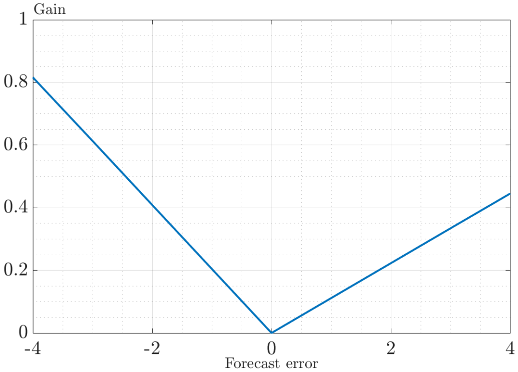
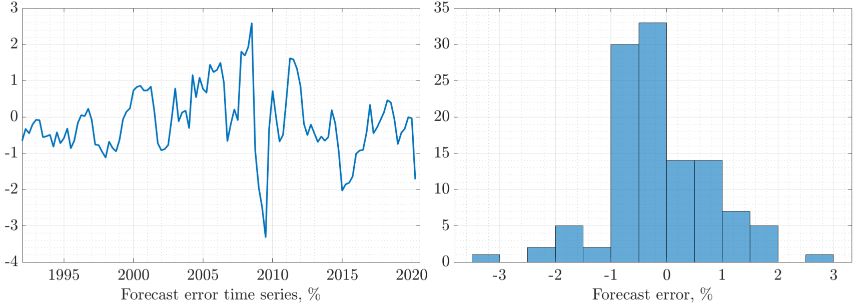



Figure: Learning gain as a function of forecast errors in inflation in %

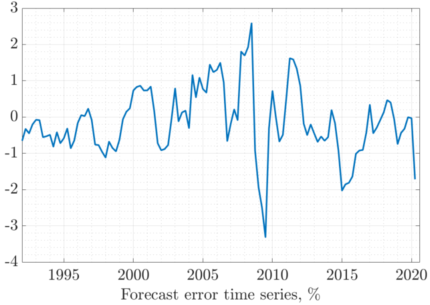
# Forecast errors in the data

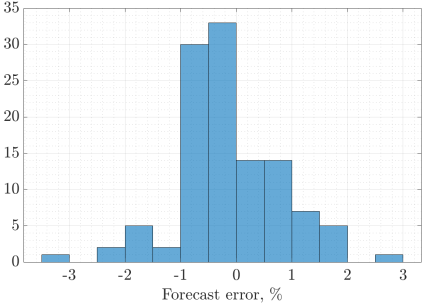
Figure: Time series and distribution of 1-year ahead forecast errors in the SPF



# Forecast errors in the data

Figure: Time series and distribution of 1-year ahead forecast errors in the SPF

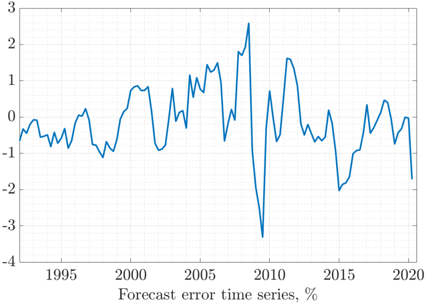


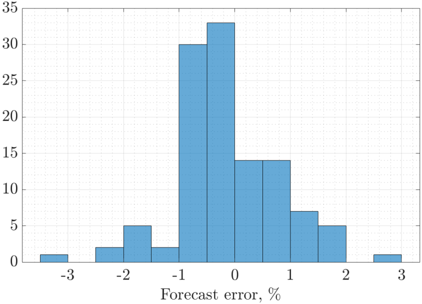




Typical forecast error $\approx -0.5$ pp

# Forecast errors in the data

Figure: Time series and distribution of 1-year ahead forecast errors in the SPF

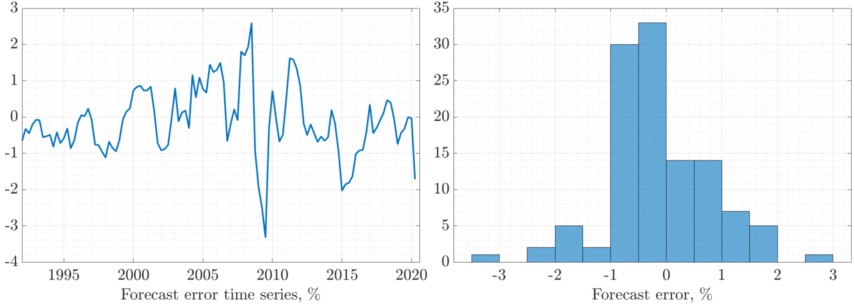



Typical forecast error $\approx -0.5$ pp $\quad \rightarrow$ gain of 0.1

# Forecast errors in the data

Figure: Time series and distribution of 1-year ahead forecast errors in the SPF




Typical forecast error $\approx -0.5$ pp $\quad \rightarrow$ gain of 0.1

$\Rightarrow$ 5 bp shift down in long-run expectations

# Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big( (1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big) \tag{21}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \big( \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \big) \tag{22}$$

▸ Derivations ▸ Actual laws of motion

- Expectations evolve according to RLS with the endogenous gain given by (20)

$\rightarrow$ How should $\{i_t\}$ be set?

# Structure of talk

1. Model of anchoring expectations

2. Solving the Ramsey problem

3. Implementing optimal policy

4. Approximating optimal policy with a Taylor rule

# Ramsey problem

$$\min_{\{y_t,\bar{\pi}_{t-1},k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0}(\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

- $\mathbb{E}$ is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

# Target criterion

## Proposition

*In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap*

$$\pi_t = -\frac{\lambda_x}{\kappa}x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_{\pi,t}\right)$$

$$\left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi},\mathbf{t+j}})\right)$$

*where* $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* $t$, $\prod_{j=0}^{0} \equiv 1$ *and* $b_1$ *is the first row of* $b$.

General case

# Two layers of intertemporal stabilization tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i}$$

$$- \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j} + f_{t+1+j|t+j} \mathbf{g}_{\bar{\pi},\mathbf{t+j}})$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

## Lemma

*The discrion and commitment solutions of the Ramsey problem coincide.*

▸ Why no commitment?

## Corollary

*Optimal policy under adaptive learning is time-consistent.*

$\hookrightarrow$ Foreshadow: optimal policy aggressiveness time-varying

# Structure of talk

1. Model of anchoring expectations

2. Solving the Ramsey problem

3. Implementing optimal policy

4. Approximating optimal policy with a Taylor rule

# Solution procedure

Solve system of model equations + target criterion

$\hookrightarrow$ solve using parameterized expectations (PEA)

$\hookrightarrow$ obtain a cubic spline approximation to optimal policy function

# Calibration - parameters from the literature

| | | |
|---|---|---|
| $\beta$ | 0.98 | stochastic discount factor |
| $\sigma$ | 1 | intertemporal elasticity of substitution |
| $\alpha$ | 0.5 | Calvo probability of not adjusting prices |
| $\kappa$ | 0.0842 | slope of the Phillips curve |
| $\psi_\pi$ | 1.5 | coefficient of inflation in Taylor rule* |
| $\bar{g}$ | 0.145 | initial value of the gain |
| $\lambda_x$ | 0.05 | weight on the output gap in central bank loss |
| $\rho_r$ | 0 | persistence of natural rate shock |
| $\rho_i$ | 0 | persistence of monetary policy shock* |
| $\rho_u$ | 0 | persistence of cost-push shock |

* pertains to sections where Taylor rule is in effect

# Calibration - matching moments

| | | |
|---|---|---|
| $\psi_x$ | 0.3 | coefficient of the output gap in Taylor rule* |
| $\sigma_r$ | 0.01 | standard deviation, natural rate shock |
| $\sigma_i$ | 0.01 | standard deviation, monetary policy shock* |
| $\sigma_u$ | 0.5 | standard deviation, cost-push shock |
| $\hat{\gamma}_i$ | (0.82; 0.61; 0; 0.33; 0.45) | coefficients in anchoring function |

Calibrated ($\sigma_j,\ j = r, i, u$) or estimated ($\hat{\gamma}_i$) to match the autocovariances of inflation, output gap, interest rate and one-period ahead inflation expectations for lags $0, \ldots, 4$.

* pertains to sections where Taylor rule is in effect

# Optimal policy - responding to unanchoring

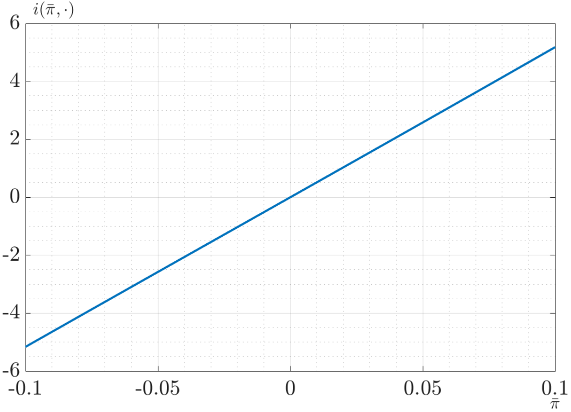



Figure: Policy function: $i(\bar{\pi}$, all other states at their means)

# Optimal policy - responding to unanchoring

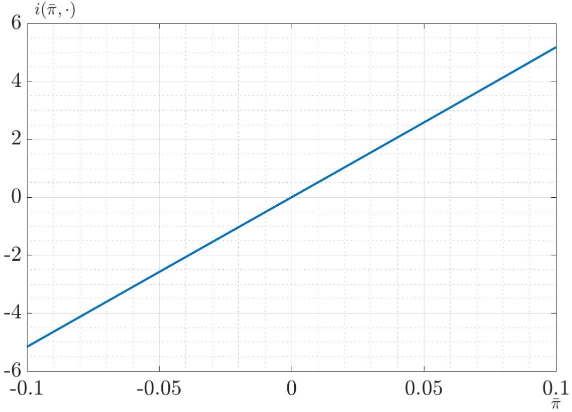
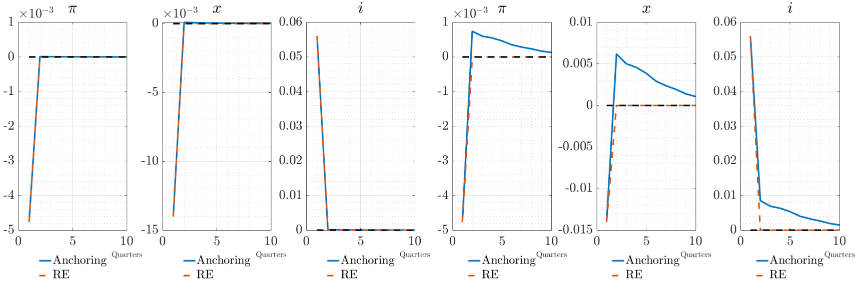



Figure: Policy function: $i(\bar{\pi},$ all other states at their means)

$\rightarrow$ For 5 bp drop in $\bar{\pi}$, lower $i$ by 2.5 pp

# Unanchoring causes volatility




(a) Expectations anchored (b) Expectations unanchored

Figure: Impulse responses after a contractionary monetary policy shock when policy follows a Taylor rule

# Why so volatile? Term structure of expectations

IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big( (1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \big( \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \big)$$

# Why oscillatory? Intertemporal anticipation effects

IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big)$$

# Why oscillatory? Intertemporal anticipation effects

IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big)$$

- Additional channel of policy

# Why oscillatory? Intertemporal anticipation effects

IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r^n_T\big)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big)$$

- Additional channel of policy
- Only if policy reaction function internalized

# Structure of talk

1. Model of anchoring expectations

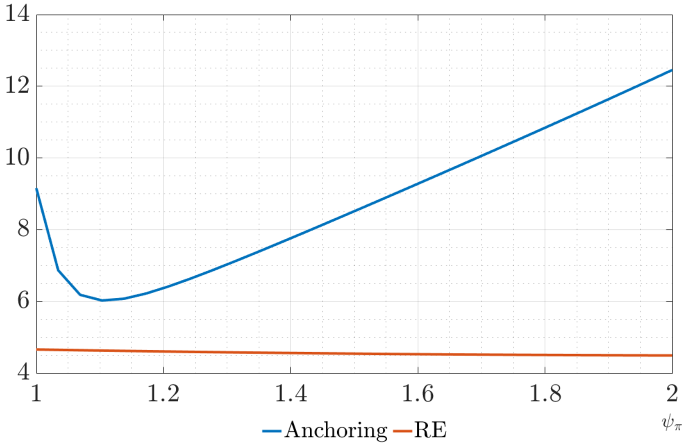
2. Solving the Ramsey problem

3. Implementing optimal policy

4. Approximating optimal policy with a Taylor rule

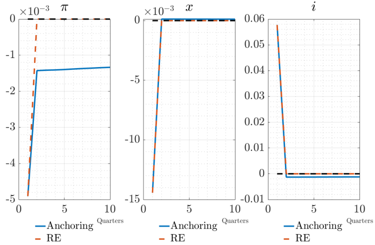
# Optimal Taylor-coefficient on inflation

Figure: Central bank loss as a function of $\psi_\pi$

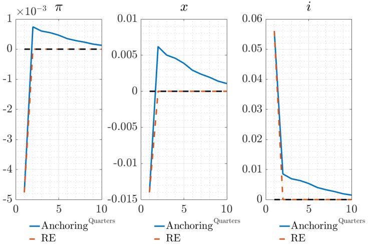



Anchoring-optimal coefficient: $\psi_\pi^A = 1.09$ RE-optimal coefficient: $\psi_\pi^{RE} = 2.21$

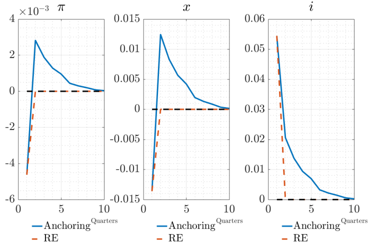
# Respond but not too much



(a) $\psi_\pi = 1.01$



(b) $\psi_\pi = 1.5$



(c) $\psi_\pi = 2$

# Losses for optimal Taylor-rule coefficient on inflation

Table: Loss for RE and anchoring models for choice of RE- or anchoring-optimal $\psi_\pi$

| Anchoring, $\psi_\pi^{RE}$ | Anchoring, $\psi_\pi^{A}$ | RE, $\psi_\pi^{RE}$ | Optimal policy |
|---|---|---|---|
| 14.2633 | 6.0228 | 4.4866 | 6.0985 |

# Losses for optimal Taylor-rule coefficient on inflation

Table: Loss for RE and anchoring models for choice of RE- or anchoring-optimal $\psi_\pi$

| Anchoring, $\psi_\pi^{RE}$ | Anchoring, $\psi_\pi^{A}$ | RE, $\psi_\pi^{RE}$ | Optimal policy |
|---|---|---|---|
| 14.2633 | 6.0228 | 4.4866 | 6.0985 |

$\rightarrow$ Optimal policy or anchoring-optimal $\psi_\pi^{A}$ gets 84% of the distance to RE-optimal $\psi_\pi^{RE}$ under RE

# Losses for optimal Taylor-rule coefficient on inflation

Table: Loss for RE and anchoring models for choice of RE- or anchoring-optimal $\psi_\pi$

| Anchoring, $\psi_\pi^{RE}$ | Anchoring, $\psi_\pi^{A}$ | RE, $\psi_\pi^{RE}$ | Optimal policy |
|---|---|---|---|
| 14.2633 | 6.0228 | 4.4866 | 6.0985 |

$\rightarrow$ Optimal policy or anchoring-optimal $\psi_\pi^{A}$ gets 84% of the distance to RE-optimal $\psi_\pi^{RE}$ under RE

Optimal policy vs. Taylor rule? $\rightarrow$ is reaction function internalized or not

# Conclusion

- First theory of monetary policy for potentially unanchored expectations

- Optimal policy frontloads aggressive interest rate response to suppress potential unanchoring

- Matters: already anchoring-optimal Taylor rule reduces losses by 84%

- Future work
  - $\hookrightarrow$ Better approximation to optimal policy than Taylor rule?
  - $\hookrightarrow$ How to anchor at zero-lower bound?

# Appendix

# Breakeven inflation


Figure: Market-based inflation expectations, various horizons, %

Return

# Correcting the TIPS from liquidity risk

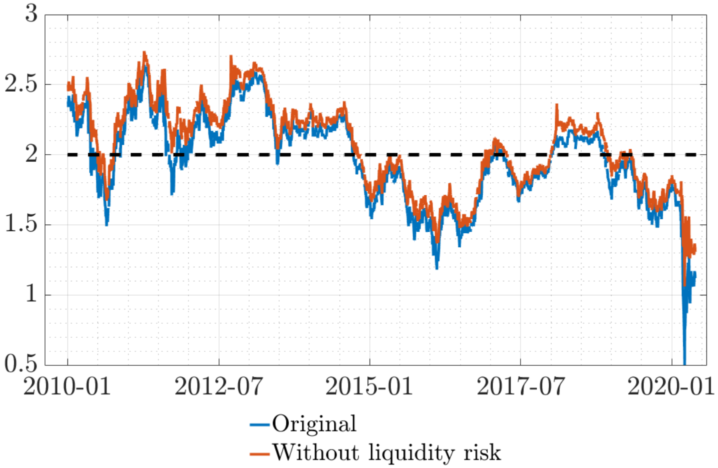
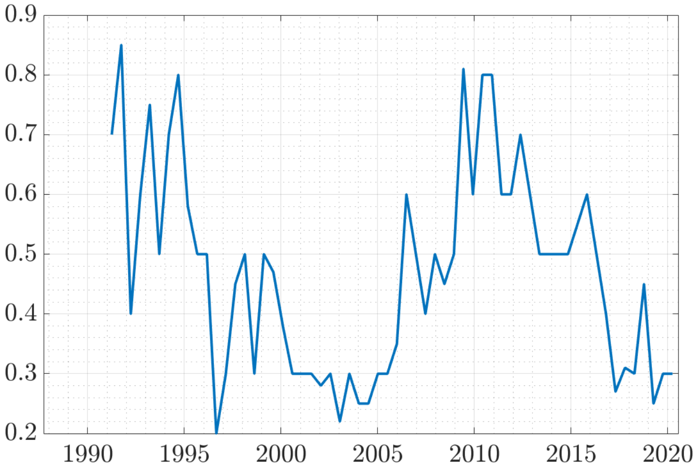



Figure: Market-based inflation expectations, 10 year, %

Return

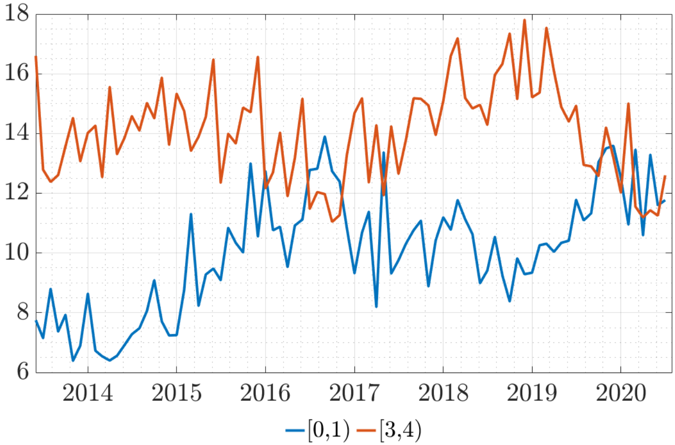
# Further evidence

Figure: Livingston Survey of Firms:
Interquartile range of 10-year ahead inflation expectations




Return

Figure: New York Fed Survey of Consumers:
Percent of respondents indicating 3-year ahead inflation will be in a particular range




Return

# Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{23}$$
$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{24}$$

Solve for the time series of expectations $f_t$

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \tag{25}$$

Solve for forecast error $f_t \equiv \pi_t - f_{t-1}$:

$$f_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k \to 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \tag{26}$$

# Functional forms for **g** in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \tag{27}$$

$p \equiv Prob(\text{liquidity trap regime})$
$y_{t-1}$ output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{28}$$

$\theta_t$ criterion, $\bar{\theta}$ threshold value

Return

# Choices for criterion $\theta_t$

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \tag{29}$$

$\Sigma$ variance-covariance matrix of shocks
$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(f_{t|t-1} f'_{t|t-1} - \omega_{t-1}) \tag{30}$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(f'_{t|t-1} \omega_t^{-1} f_{t|t-1} - \theta_{t-1}) \tag{31}$$

$\omega_t$ estimated forecast-error variance

Return

# Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}\right)'\right)' \tag{32}$$

$$R_t = R_{t-1} + k_t \left( \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{33}$$

Return

# Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{34}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{35}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \tag{36}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{37}$$

Return

# No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t)$$

$$+ \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \Big\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t + \mathbf{g}_\pi(\pi_t - f_{t-1})) = 0 \tag{38}$$

$$-2\beta\frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g_f}(\pi_{t+1} - f_t)) = 0 \tag{39}$$

Return

# Target criterion system for anchoring function as changes of the gain

$$\varphi_{6,t} = -cf_{t|t-1}x_{t+1} + \left(1 + \frac{f_{t|t-1}}{f_{t+1|t}}(1 - k_{t+1}) - f_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1}$$
$$- \frac{f_{t|t-1}}{f_{t+1|t}}(1 - k_{t+1})\varphi_{6,t+2} \tag{40}$$
$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{f_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{f_{t|t-1}}\varphi_{6,t+1} \tag{41}$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (41) is given by:

$$\varphi_{6,t} = -2\,\mathbb{E}_t \sum_{i=0}^{\infty}(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{f_{t+j|t+j-1}}}{\frac{k_{t+j}}{f_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \tag{42}$$

Return

# Details on households and firms

Consumption:

$$C_t^i = \left[ \int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{43}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[ \int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{44}$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \tag{45}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{46}$$

Return

# Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{47}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{48}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (47) backward to some date $t$, take expectations at $t$
2. Sub in (48)
3. Aggregate over households $i$

$\rightarrow$ Obtain (21)

Return