15 Identification Based on Extraneous Data

Yet another approach to identifying structural VAR shocks is to rely on additional data not included among the VAR model variables. This chapter discusses two such approaches. The first approach relies on information from high-frequency futures prices, whereas the second approach relies on external exogenous instruments such as measures of exogenous OPEC oil supply shocks or the narrative measures of exogenous monetary policy shocks and exogenous fiscal policy shocks already discussed in Chapter 7.

15.1 Identification Based on High-Frequency Futures Prices

Discomfort with semistructural VAR models of monetary policy, in which the monetary policy shock is identified based on a recursive ordering of the model variables, has stimulated the development of yet another approach to identification that relies on high-frequency futures market data to identify monetary policy shocks. One motivation for this approach is that the sequences of policy shocks identified by recursive structural VAR models do not always correspond to common perceptions of when policy shocks occurred and indeed vary widely across models (see Rudebusch 1998).[1]

The other motivation is that the reduced-form interest rate forecasts of conventional VAR models of monetary policy are difficult to reconcile with financial market measures of interest rate expectations. For example, Rudebusch (1998) documents a low correlation between the quarterly reduced-form prediction errors for the federal funds rate implied by conventional VAR

[1] Sims (1998) stresses that this finding is to be expected in a simultaneous equations model. For example, if we compare estimates of a model of demand and supply using alternative exogenous supply shifters (say, weather and insect density), then there is no reason for these instruments to generate the same sequence of supply shocks. This fact does not preclude them from accurately estimating the slope of the demand curve, however. Indeed, as Sims observes, models that imply very different sequences of monetary policy shocks still tend to agree on the response of the economy to a given monetary policy shock.

models of monetary policy and the quarterly changes in the expected federal funds rate implied by the prices of federal funds futures contracts, defined as

$$\widehat{u}_t^{\Delta FFF} \equiv \big(FFR_i + FFR_{i+1} + FFR_{i+2} - FFF_{i-1}^1 - FFF_{i-1}^2 - FFF_{i-1}^3\big)/3,$$

where quarter t contains months i, $i+1$, and $i+2$, FFR_i is the actual federal funds rate in month i, and FFF_i^j denotes the j-months ahead expected federal funds rate at the end of month i, as measured by the price of the federal funds futures contract. The financial market shocks statistically explain only between 10% and 25% of the variation in the quarterly VAR prediction errors, $\widehat{u}_t$, based on data since 1988 when federal funds futures contracts were introduced.[2] Rudebusch interprets this evidence as suggesting that the reduced-form representation of VAR models of monetary policy is inherently misspecified, perhaps reflecting an informational deficiency of these models (see also Chapter 16).

This finding suggests that conventional low-dimensional VAR models of monetary policy are unable to capture the market's interest rate expectations. It also suggests that the timing of quarterly VAR models may be too coarse to separate monetary policy shocks from policy responses to contemporaneous events. These two concerns have spurred the development of an alternative approach to identifying monetary policy shocks in monthly VAR models based on high-frequency futures prices.

Whereas Rudebusch (1998) defines the monetary policy shock as the difference between the realized federal funds rate target and the expected federal funds rate derived from federal funds futures, more recent studies consider monetary policy shocks measured in this way potentially misleading for technical reasons related to the measurement of the realized federal funds rate (see Piazzesi and Swanson 2008). More importantly, these shocks will also be contaminated by risk premia in the futures market. As discussed in Chapter 7, the alternative of measuring monetary policy shocks based on the change in the daily federal funds futures rate around Federal Open Market Committee (FOMC) announcements, as first suggested by Kuttner (2001), tends to be more robust to the presence of risk premia in these contracts. This approach uses daily (or even intradaily) federal funds futures data to narrow the time interval around the FOMC announcement and assumes that risk premia do not change over such a small interval.

Initially, researchers constructed responses to such policy shocks from distributed-lag models, as reviewed in Chapter 7 (see, e.g., Cochrane and Piazzesi 2002). The idea of incorporating such measures of policy shocks into

[2] In addition, Rudebusch shows that the forecast errors from unrestricted VAR models are more variable than the futures price shocks. The latter problem may be overcome with the use of suitable priors when estimating the model (see Robertson and Tallman 2001).

structural VAR models was pioneered by Faust, Rogers, Swanson, and Wright (2003) and Faust, Swanson, and Wright (2004). This approach allows the user not only to incorporate information from financial markets into the VAR model but also to dispense with the exclusion restrictions used in recursively identified VAR models of monetary policy. A potential drawback of this approach is that the structural impulse responses are not longer point-identified, but only set identified.

15.1.1 A Set-Identified Approach

Faust, Swanson, and Wright (2004) consider a standard closed-economy VAR model of U.S. monetary policy using a benchmark model from Christiano, Eichenbaum, and Evans (1996, 1999). The model includes monthly observations for U.S. industrial production, the CPI, the smoothed change in an index of commodity prices, the federal funds rate, nonborrowed reserves, and total reserves. Unlike Christiano et al., they also incorporate information from the federal funds futures market.

Measuring the Policy Shock. Faust et al. assume that the change in the federal funds target rate on the days of Federal Open Market Committee (FOMC) meetings that was not anticipated by the federal funds futures market represents a monetary policy shock. Specifically, they treat the change in the futures rate on the day, on which a change in the Fed's target federal funds rate is announced, as a measure of the change in market expectations. This interpretation requires the risk premium to remain unchanged. Faust et al. further postulate that this change in expectations is due to the policy shock only. In other words, no other news move the market on that day and the policy announcement itself does not reveal information about other structural shocks.

Faust et al.'s procedure involves two key steps: In the first step, the surprise change in the target rate is measured by the change in the closing price of the federal funds contract at 3:00PM Eastern Standard Time, suitably scaled by the ratio

$$\frac{\text{days in month}}{\text{days left in month}}$$

to take account of how far into the current month the surprise occurs. Near the end of the month, this scaling factor is quite large.

Faust et al. then regress for all FOMC meeting dates the expected future change in the monthly federal funds rate on the monthly surprise change in the target rate. These regressions yield an estimate of the response of the monthly federal funds rate to a policy shock that can be scaled appropriately. In the second step, when estimating the VAR model, Faust et al. impose that the response of the monthly federal funds rate to a monetary policy shock in the monthly

VAR model must match the impulse response of the federal funds rate already estimated from the high-frequency data.

While these two steps are conceptually straightforward, the information from the futures market only set-identifies the responses in the structural VAR model, as explained shortly. This means that one must give up on point estimation of the structural responses and focus on confidence intervals instead, similar to classical inference in sign-identified VAR models.

In closely related work, Faust, Rogers, Swanson, and Wright (2003) extend this algorithm to allow for restrictions on the impulse responses of several model variables. They consider open economy VAR models for selected countries, for which quotes from futures markets are available at daily frequency for the domestic interest rate and the U.S. interest rate as well as the bilateral U.S. dollar exchange rate.

Estimation and Inference. Consider a K-dimensional reduced-form VAR model

$$A(L)y_t = u_t.$$

The corresponding structural MA representation of this model can be written as

$$y_t = \Phi(L)B_0^{-1}w_t,$$

where $u_t = B_0^{-1}w_t$. One of the structural shocks in w_t is assumed to be the monetary policy shock of interest. Without loss of generality, Faust et al. assume that this shock is the first shock and let δ denote the first column of B_0^{-1}. Then the set of response functions of the model variables to the policy shock is

$$\Phi(L)\delta = \sum_{j=0}^{\infty} \Phi_j \delta L^j.$$

This expression is a $K \times 1$ vector of lag polynomials. The coefficients of the k^{th} element of this vector trace out the responses of the k^{th} variable to the policy shock. Given that Φ_j can be computed from the reduced-form model, identifying the structural impulse responses only requires restricting the K elements of δ. There are two sets of restrictions. First, it is essential to normalize the impact response of the federal funds rate to a monetary policy shock. Faust et al. choose 25 basis points as this normalization. Second, in addition, sign and magnitude restrictions on the other elements of δ may be imposed based on a priori economic reasoning, as discussed later in this chapter.

The final requirement is that the response of the federal funds rate to the policy shock must match the corresponding response estimated from the futures market data. Denote the latter response at horizon $t+h$ by r_h^{FFF},

$h = 0, 1, \ldots, K-1$, where FFF denotes the federal funds forecast obtained from the futures market. By construction this means that

$$\phi_h^{FF}\delta = r_h^{FFF}, \quad h = 0, 1, \ldots, K-1,$$

where ϕ_h^{FF} denotes the row of Φ_h pertaining to the federal funds rate response. For example, if the federal funds rate is ordered last in the vector of observed variables, y_t, then ϕ_h^{FF} is the last row of Φ_h. After stacking these K equations, we obtain the set of restrictions

$$R\delta = r^{FFF}. \tag{15.1.1}$$

Here $R' = [\phi_0^{FF\prime}, \ldots, \phi_{K-1}^{FF\prime}]$ so that R is $K \times K$ and $r^{FFF} = (r_0^{FFF}, \ldots, r_{K-1}^{FFF})'$ is $K \times 1$. If R is of rank K, then there is a unique solution to this equation and $\delta = R^{-1}r^{FFF}$. A failure of this rank condition can be tested, allowing for the fact that R depends on the reduced-form estimate of the VAR model and r^{FFF} must be estimated from the futures price data. If R is rank deficient, as is typically the case, then there are many solutions for δ and the structural impulse responses are only set-identified by the restriction that $R\delta = r^{FFF}$, which greatly complicates inference, as we have already seen in Chapter 13.

Testing the Rank Condition. Following Faust, Swanson, and Wright (2004), consider testing $\mathbb{H}_0 : \text{rank}(R) = l$ against $\mathbb{H}_1 : \text{rank}(R) > l$. In other words, the null hypothesis is that the model is rank deficient. Let $\boldsymbol{\alpha}$ denote the vector of reduced-form VAR parameters. Provided that $\sqrt{T}(\widehat{\boldsymbol{\alpha}} - \boldsymbol{\alpha}) \xrightarrow{d} \mathcal{N}(0, \Sigma_{\boldsymbol{\alpha}})$, we can write R as a nonlinear function of $\boldsymbol{\alpha}$ with consistent plug-in estimator $\widehat{R}$. By the delta method, $\sqrt{T}(\text{vec}(\widehat{R}) - \text{vec}(R)) \xrightarrow{d} \mathcal{N}(0, \Sigma_R)$, where

$$\Sigma_R = \frac{\partial \text{vec}(R)}{\partial \boldsymbol{\alpha}'} \Sigma_{\boldsymbol{\alpha}} \frac{\partial \text{vec}(R)'}{\partial \boldsymbol{\alpha}}.$$

Hence, the test statistic is

$$T \min_{S \in \pi(l)} \left(\text{vec}(\widehat{R}) - \text{vec}(S)\right)' \widehat{\Sigma}_R^{-1} \left(\text{vec}(\widehat{R}) - \text{vec}(S)\right),$$

where

$$\widehat{\Sigma}_R = \frac{\partial \text{vec}(\widehat{R})}{\partial \boldsymbol{\alpha}'} \widehat{\Sigma}_{\boldsymbol{\alpha}} \frac{\partial \text{vec}(\widehat{R})'}{\partial \boldsymbol{\alpha}}$$

is an estimator of Σ_R and

$$\pi(l) = \{S |\, \text{rank}(S) = l, S\delta = r^{FFF}\}$$

is the space of all conformable matrices S of rank l. Assuming that Σ_R has full rank, the test statistic by Theorem 1 in Cragg and Donald (1997) has a χ^2 null distribution. In practice, it is common to test for successively higher ranks l sequentially until some null cannot be rejected (or until all rank deficient models have been rejected).

In the empirical application of Faust et al. there are six futures contracts and six variables in the VAR model, so if $\text{rank}(R) = 6$, the model would be exactly identified. The normalization of the impact response of the federal funds rate to a policy shock means that R has at least rank 1. Faust et al. proceed by testing for successively higher ranks $l = 1, 2, 3, 4, 5, 6$. While ranks of 1 and 2 are clearly rejected, the null hypothesis of a rank 3 is not rejected, so the model is considered set identified.

Impulse Response Confidence Intervals under Set Identification. The construction of confidence intervals for the structural impulse responses involves two steps. First, we need to construct a confidence set D for the vector δ. Second, we need to construct the implied confidence interval for each structural impulse response coefficient for any fixed $\delta \in D$. Third, we need to construct a Bonferroni bound for these intervals across all $\delta \in D$.

Let $\mathcal{D}$ denote the parameter space for δ. Assume that the restrictions $R\delta = r^{FFF}$ hold, that R is consistently estimated by $\widehat{R}$, that R is rank deficient, that $\sqrt{T}(\text{vec}(\widehat{R}) - \text{vec}(R)) \xrightarrow{d} \mathcal{N}(0, \Sigma_R)$ and that $\sqrt{T}(\widehat{r}^{FFF} - r^{FFF}) \xrightarrow{d} \mathcal{N}(0, \Sigma_{r^{FFF}})$. Consider the objective function

$$S(\delta) = T\big(\widehat{R}\delta - r^{FFF}\big)'\big[(\delta' \otimes I_K)\widehat{\Sigma}_R(\delta \otimes I_K) + \widehat{\Sigma}_{r^{FFF}}\big]^{-1}\big(\widehat{R}\delta - r^{FFF}\big).$$

The estimator $\widehat{\delta}$ that minimizes this objective function is not consistent for the true δ because of the rank deficiency of R. However, $S(\delta_0)$ has a χ^2 null distribution regardless of the rank of R, where δ_0 denotes the true value of the vector δ. Hence,

$$D = \big\{\delta \in \mathcal{D} : S(\delta) \leq \chi^2_{0.95}(K)\big\}$$

is a confidence set for δ with asymptotic coverage probability of 0.95, regardless of the rank of R, where $\chi^2_{0.95}$ denotes the 95th percentile of a χ^2 distribution with degrees of freedom equal to K, the number of elements in r^{FFF}.

In practice, Faust et al. recommend constructing the confidence set D by drawing 10 million candidate solutions for δ at random uniformly from the parameter space $\mathcal{D}$ and retaining the draws that satisfy the definition of D. In Faust, Swanson, and Wright (2004), δ is a 6×1 parameter vector. The parameter space $\mathcal{D}$ they draw from is restricted as follows. After normalizing the coefficient on the federal funds rate to 0.25 (so a unit monetary policy shock raises the federal funds rate by 25 basis points), Faust et al. propose restricting the impact response of industrial production and of the CPI to this policy shock to lie between 0 and -0.1 and the other impact coefficients to lie between 0 and -0.25. The magnitude restrictions are somewhat ad hoc, but they are clearly different from the exclusion restrictions traditionally used in recursive structural VAR models of monetary policy.

Having simulated the $(1-\gamma_1)100\%$ confidence set D, for any fixed $\delta \in D$, one can use conventional bootstrap methods to construct a $(1-\gamma_2)100\%$ confidence interval for each structural impulse response coefficient conditional on δ. Denote this confidence interval by $\left[\underline{\theta}(\delta), \overline{\theta}(\delta)\right]$. Finally, we form the outer envelope of all these intervals across $\delta \in D$ as $\left[\inf_{\delta\in D}\underline{\theta}(\delta), \sup_{\delta\in D}\overline{\theta}(\delta)\right]$. The latter pointwise interval has at least $(1-\gamma_1-\gamma_2)100\%$ asymptotic coverage by the Bonferroni inequality.

Faust et al. report both confidence intervals conditional on the original reduced-form VAR estimate and allowing for sampling uncertainty in the reduced-from parameters. Given the absence of point estimates for the impulse responses, these confidence bands may be difficult to interpret in practice, but the empirical results in Faust et al. are nevertheless informative and allow us to reject hypotheses of economic interest. Most importantly, the usual recursive identification of monetary policy shocks is clearly rejected, as is any identification that insists on a monetary policy shock having no effect on prices contemporaneously. This confirms the earlier concerns with recursive structural monetary policy VAR models. Faust et al.'s identification also eliminates the price puzzle – the finding in the benchmark recursive identification that the impulse response of prices to an unexpected monetary policy tightening first rises significantly, before falling. Faust, Swanson, and Wright (2004) nevertheless find that only a small fraction of the variance of output can be attributed to monetary policy shocks.

15.1.2 A Point-Identified Approach

D'Amico and Farka's (2011) analysis of the interaction of the stock market and the interest rate presents an alternative approach to using high-frequency financial data for the identification of structural VAR models. They propose a methodology for estimating simultaneously the response of stock returns to policy decisions and the Federal Reserve's contemporaneous reaction to the stock market. Their methodology has broad applicability when modeling asset prices.

Overview. D'Amico and Farka's approach involves two steps. In the first step, the response of the stock market to policy shocks is estimated outside the VAR model by measuring changes in intraday S&P500 futures prices immediately before and after policy announcements. The monthly policy shock is obtained by summing the intraday shocks over the course of a given month. In the second step, D'Amico and Farka impose that external estimate when estimating the response of the federal funds rate to stock returns in a monthly VAR model.

D'Amico and Farka consider a 7-variable monthly VAR model of U.S. monetary policy. They propose to partition the structural impact multiplier matrix into two blocks. One block includes the macroeconomic variables (industrial production, the consumer price index, a smoothed index of commodity prices, nonfarm payrolls, and the ISM manufacturing index) and the other block includes the financial variables (federal funds rate, S&P500 stock returns). Their methodology is based on five key identifying assumptions.

1. The structural impact multiplier matrix is recursive except for the VAR variables within the financial block.
2. The relationship between reduced-form and structural errors is the same at the intraday and at the monthly frequency.
3. The monetary policy shock is the only shock at the time of the FOMC announcement.
4. Intraday changes in spot month federal funds futures prices around FOMC announcements reflect the effects of policy shocks.
5. Intraday changes in S&P500 futures prices around FOMC announcements provide a good measure of unexpected changes in stock prices caused by monetary policy shocks.

The assumption of a recursive structure outside of the financial block is innocuous as long as only the simultaneity of stock prices and policy rates is of interest. As in Faust et al., the analysis treats risk premia as approximately constant around FOMC announcements.

Estimation and Inference. Suppose that the economy is described by the K-dimensional structural VAR model

$$B_0 y_t = \widetilde{B}(L) y_t + w_t,$$

where $\widetilde{B}(L) = B_1 L + \cdots + B_p L^p$ and $u_t = B_0^{-1} w_t$ is the vector of reduced-form errors. Suppose that there are two financial variables (y_t^f) and $(K-2)$ macroeconomic variables (y_t^m). The matrix B_0 is partitioned as

$$B_0 = \begin{bmatrix} B_{0,M}^m & 0_{(K-2)\times 2} \\ B_{0,M}^f & B_{0,F}^f \end{bmatrix},$$

where $B_{0,M}^m$ is a $(K-2)\times(K-2)$ lower-triangular matrix, while the $2 \times (K-2)$ matrix $B_{0,M}^f$ and the 2×2 matrix $B_{0,F}^f$ govern the contemporaneous interaction of the macroeconomic variables and financial variables. Note that the model differs from a conventional semistructural model of monetary policy in that it allows for simultaneous interaction between the financial variables. Because the diagonal elements of B_0 can be normalized to 1,

$$B_{0,F}^f = \begin{bmatrix} 1 & -a \\ -b & 1 \end{bmatrix}$$

and we can partition B_0^{-1} such that

$$B_0^{-1} = \begin{bmatrix} \left(B_0^{-1}\right)_M^m & 0 \\ \left(B_0^{-1}\right)_M^f & \left(B_0^{-1}\right)_F^f \end{bmatrix},$$

where

$$\left(B_0^{-1}\right)_F^f = (B_{0,F}^f)^{-1} = \frac{1}{1-ab}\begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix},$$

so $a/(1-ab)$ denotes the contemporaneous response of stock prices to a monetary policy shock, if the federal funds rate is ordered last in the VAR model. In conventional recursive structural VAR models of monetary policy, identification is achieved by imposing either $a = 0$ or $b = 0$. Neither assumption is compelling as discussed earlier. D'Amico and Farka dispense with that assumption. This means that they require one more identifying restriction for exact identification. D'Amico and Farka achieve identification by imposing an extraneous estimate of a obtained by regressing monthly stock returns on measures of monetary policy shocks constructed from intraday changes in the federal funds rate near FOMC announcements.

Denote this intraday change by $w_{t,d}$. Then the monthly policy shock is constructed by cumulating the intradaily policy shocks over the course of a given month t,

$$w_t^{FF} = \sum_{d=1}^{D} w_{t,d},$$

where d indexes the subintervals within the month. In practice, D'Amico and Farka (2011) rely on 20-minute intervals. Likewise, it is assumed that the reduced-form errors may be decomposed as

$$u_t^{FF} = \sum_{d=1}^{D} u_{t,d},$$

and it is assumed that $u_{t,d} = B_0^{-1} w_{t,d}$, where B_0^{-1} is the same matrix as in the monthly data. This is, of course, a very strong assumption.

The premise of the analysis is that, in a short interval around an FOMC announcement, the only new information is the monetary policy news. It is therefore reasonable to assume that all elements but the last element of $w_{t,d}$ are zero within that interval. Thus, if u_t^{FF} and u_t^{ret} denote the reduced-form errors for the equations of the federal funds rate and for S&P500 stock returns, in a narrow window around the policy announcement, we have

$$u_{t,d}^{FF} = \frac{1}{1-ab} w_{t,d}^{policy},$$
$$u_{t,d}^{ret} = \frac{a}{1-ab} w_{t,d}^{policy} = a u_{t,d}^{FF}.$$

The estimation proceeds in two steps. First, following Kuttner (2001), it is assumed that changes in the federal funds futures price around the time of FOMC announcements can be used to measure the difference between the announced funds rate and the ex-ante expectation, $u_{t,d}^{FF}$. Likewise, changes in the S&P500 futures prices within the same window of time are assumed to be a good measure of unexpected changes in stock prices, $u_{t,d}^{ret}$. An OLS regression of changes in S&P500 futures and changes in the federal funds rate for all FOMC announcement dates yields an estimate $\widehat{a}$ of a.

Second, one substitutes this estimate for a in the monthly structural VAR model

$$\begin{bmatrix} 1 & -\widehat{a} \\ -b & 1 \end{bmatrix} \begin{pmatrix} ret_t \\ FF_t \end{pmatrix} = -B_{0,M}^{f} y_t^{m} + \widetilde{B}^{f}(L) y_t + \begin{pmatrix} w_t^{ret} \\ w_t^{policy} \end{pmatrix},$$

where $\widetilde{B}^f(L)$ denotes the last two rows of $\widetilde{B}(L)$. Let $\widehat{w}_t^{ret}$ denote the residual from the first equation. Then the second equation can be estimated by regressing the federal funds rate on contemporaneous and lagged values of all variables, using the residuals $\widehat{w}_t^{ret}$ as an instrument for ret_t. Because the model proposed by D'Amico and Farka (2011) is point-identified, inference is standard and may be implemented by conventional bootstrap methods.

D'Amico and Farka (2011) conclude that a surprise 25 basis points tightening in policy rates is associated with a decline in stock prices by 1.25% . The response of monetary policy to stock prices is positive and statistically significant. Policy rates increase by about 2 basis points in response to a 1% unanticipated increase in stock prices. The use of intraday data allows D'Amico and Farka to address both the endogeneity issue (because there is no simultaneous reaction within the short window around a policy announcement) and the omitted variable problem (by reducing the likelihood that new information is released during the short window). One drawback of their analysis is that accurate information about the timing of U.S. monetary policy announcements is available only after 1994.

This approach may be extended to other forms of interventions and other financial variables including exchange rates, commodity prices, and other financial instruments to the extent that the precise timing of these interventions is known and to the extent that corresponding intraday futures price data are available. Of course, the premise that the risk premium remains constant during policy interventions must be evaluated on a case-by-case basis.

15.1.3 Discussion

Identifying structural VAR models based on high-frequency futures data offers several advantages over conventional exclusion restrictions, but it also has several limitations. One limitation is that this approach is designed for modeling monetary policy decisions and does not easily generalize to other economic shocks. Another concern is that this approach requires high-frequency futures prices for extended periods, which limits its applicability. A third concern is that this approach is only as good as the information we have about the timing of the underlying policy decisions, which limits its applicability to other countries.

There is yet another and more fundamental concern, however, and that is how to measure policy shocks at monthly frequency that are actually observed at daily or intradaily frequency. Indeed, the studies we reviewed made different choices in this regard. As discussed in Chapter 2, temporal aggregation of variables has a strong impact on the dynamic structure of the DGP. If the actual reaction of markets occurs within a very short period following the policy shock, then ideally one would have to consider the daily or intradaily process generating the set of variables under consideration. The effects on the corresponding monthly variables would have to be assessed by analyzing the properly aggregated process. No in-depth investigation of the aggregation implications on the stochastic properties driving the whole system of variables has been conducted thus far.

An alternative approach to allowing higher frequency data in a monthly model would be to consider explicitly models for mixed frequency data. In other words, the DGP of a system of variables being observed at different frequency is modeled explicitly. This approach formally allows one to take into account the changes in the dynamic structure resulting from time aggregation. The approach is used by Foroni and Marcellino (2016). They find that taking into account the mixed frequency nature of the data explicitly can make a substantial difference to the impulse responses. Thus, simply aggregating the shocks and making the assumptions on the aggregated effects that are used by the research reviewed in this chapter so far is problematic and can lead to conclusions that do not reflect the true responses of the variables in the underlying economic system.

15.2 Identification Based on External Instruments

In some situations, researchers have access to direct measures of exogenous shocks obtained from information not contained in the VAR model. Examples of such exogenous variables are monetary policy and fiscal policy shocks obtained from narrative evidence as in Romer and Romer (2004) or Ramey and Shapiro (1998), the exogenous OPEC oil supply shock measures of Hamilton

(2003) and Kilian (2008b), or the oil supply news shock measure proposed in Arezki, Ramey, and Sheng (2017) (see Chapter 7).

Consider measures of exogenous OPEC oil supply shocks, for example. One way of incorporating this exogenous information is to augment the baseline structural VAR model of Kilian (2009) for the growth in global oil production ($\Delta prod_t$), global real economic activity (rea_t), and the real price of oil ($rpoil_t$) to include the exogenous variable in question, x_t. If this variable is serially uncorrelated, we may treat it both as an observable and as a structural shock. For example, Kilian (2006) incorporates the exogenous OPEC oil supply shock series developed in Kilian (2008b) into a restricted monthly structural VAR(24) model

$$B_0 y_t = B_1 y_{t-1} + \cdots + B_{24} y_{t-24} + w_t,$$

where $y_t = (x_t, \Delta prod_t, rea_t, rpoil_t)'$ and the first row of B_i, $i = 1, \ldots, 24$, has been restricted to zero, reflecting the exogeneity of x_t and its lack of serial correlation. The vector w_t denotes the serially and mutually uncorrelated structural shocks. By construction, in this VARX model of the global oil market $y_{1t} = x_t = u_t^x = b_0^{11} w_t^{\text{political OPEC oil supply}}$. The model is estimated by restricted GLS as discussed in Chapter 2. As in Kilian (2009), the identification relies on a recursive structure, but now with the exogenous OPEC oil supply shock ordered first, allowing global oil production to respond endogenously to politically motivated OPEC supply shocks:

$$\begin{pmatrix} u_t^x \\ u_t^{\Delta prod} \\ u_t^{rea} \\ u_t^{rpoil} \end{pmatrix} = \begin{bmatrix} b_0^{11} & 0 & 0 & 0 \\ b_0^{21} & b_0^{22} & 0 & 0 \\ b_0^{31} & b_0^{32} & b_0^{33} & 0 \\ b_0^{41} & b_0^{42} & b_0^{43} & b_0^{44} \end{bmatrix} \begin{pmatrix} w_t^{\text{political OPEC oil supply}} \\ w_t^{\text{other oil supply}} \\ w_t^{\text{aggregate demand}} \\ w_t^{\text{oil}-\text{specific demand}} \end{pmatrix}.$$

Because the distinction between politically motivated OPEC oil supply shocks and other oil supply shocks turns out to be unimportant in practice, Kilian (2009) therefore switched to a simpler version of this model that excluded the OPEC oil supply shock and focused on shocks to aggregate oil production, allowing the model to be estimated by unrestricted least squares.

Similar approaches have also been used in fiscal policy VAR models. For example, Auerbach and Gorodnichenko (2012) augment a (nonlinear) quarterly structural VAR model of fiscal spending to include a measure of news about fiscal spending, defined as the forecast errors implied by professional survey forecasts or other expert forecasts and denoted by $news_t$. The model variables are $y_t = (news_t, gov_t, tax_t, gdp_t)'$, where gov_t is the log of government purchases, tax_t is the log of government receipts, and gdp_t is the log of real GDP. The model imposes the identifying assumption that government spending does not respond to changes in fiscal and macroeconomic conditions in the short run. The news variable is ordered first, allowing an autonomous shock to fiscal spending to be identified as the second structural shock, with the third and fourth structural shocks remaining unidentified from an economic point of view such that

$$\begin{pmatrix} u_t^{news} \\ u_t^{gov} \\ u_t^{tax} \\ u_t^{gdp} \end{pmatrix} = \begin{bmatrix} b_0^{11} & 0 & 0 & 0 \\ b_0^{21} & b_0^{22} & 0 & 0 \\ b_0^{31} & b_0^{32} & b_0^{33} & 0 \\ b_0^{41} & b_0^{42} & b_0^{43} & b_0^{44} \end{bmatrix} \begin{pmatrix} w_t^{\text{fiscal news}} \\ w_t^{\text{fiscal spending}} \\ w_{3t} \\ w_{4t} \end{pmatrix}.$$

Unlike in Kilian (2006), the model is estimated without imposing the assumption that the news shock variable is serially uncorrelated and strictly exogenous. Another difference from the analysis in Kilian (2006) is that Auerbach and Gorodnichenko (2012) use their exogenous variable to deal with the informational deficiencies of their baseline structural VAR model. A closely related approach was also suggested by Favero and Giavazzi (2012).

More recently, a number of authors have suggested to use such exogenous variables instead as instruments for the structural shock of interest (see, e.g., Stock and Watson 2012; Mertens and Ravn 2012, 2014; Montiel Olea, Stock, and Watson 2015b; Piffer and Podstawski 2016). For example, direct measures of monetary policy shocks, as discussed in Chapter 6, may be used as an external instrument (see Stock and Watson 2012). This approach is also sometimes referred to as using a proxy VAR model. One key advantage of this model is that it allows the central bank to respond to asset prices without restricting the feedback from policy shocks to asset prices. As in the approach discussed in Section 15.1, however, the high-frequency proxy shocks must be scaled to monthly frequency.

Likewise, direct measures for exogenous fiscal policy shocks may be used as instruments in modeling fiscal aggregates. For example, Mertens and Ravn (2014) propose to interpret exogenous tax changes identified by the narrative approach to identification as proxy measures of latent structural tax shocks. The key identifying assumption is that the narrative measure is correlated with the tax shocks in the structural VAR model, but orthogonal to other structural shocks in this model. Similarly, Montiel Olea, Stock, and Watson (2015b) propose using external estimates of exogenous OPEC oil supply shocks as instruments for the reduced-form error of world oil production in a VAR model of the impact of oil price shocks on the U.S. economy.

15.2.1 Estimation and Inference

A comprehensive discussion of this approach is provided in Montiel Olea, Stock, and Watson (2015b). Recall the covariance stationary reduced-form

K-dimensional VAR(p) model

$$y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t,$$

where $A_i = B_0^{-1} B_i$, $i = 1, \ldots, p$, and $u_t = B_0^{-1} w_t$.

Instrumental Variable Estimation. We are interested in estimating the responses of the K model variables to structural shock j, h periods ahead,

$$\theta_{j,h} = \Phi_h B_0^{-1} e_j,$$

where e_j denotes the j^{th} column of I_K, $B_0^{-1} e_j$ is the j^{th} column of B_0^{-1}, and Φ_h refers to the $K \times K$ matrix of h-step ahead reduced-form impulse responses, as defined in Chapter 4. Since $B_0^{-1} e_j = \theta_{j,0}$, $\theta_{j,h} = \Phi_h \theta_{j,0}$. Without loss of generality, let the structural shock of interest be w_{1t}. Thus, we focus on the estimation of $\theta_{1,h}$. Suppose that there is only one external instrument, z_t.[3]

The key assumption for estimating $\theta_{1,0}$ is that the external instrument, z_t, is correlated with w_{1t} and uncorrelated with all other structural shocks, i.e., $\mathbb{E}(z_t w_{1t}) = a \neq 0$ and $\mathbb{E}(z_t w_{kt}) = 0$ for $k = 2, \ldots, K$. Except for the fact that w_{1t} is unobserved, these are the standard conditions required for conventional IV analysis. Using $u_t = B_0^{-1} w_t$, it follows that

$$\mathbb{E}(z_t u_t) = \mathbb{E}(B_0^{-1} z_t w_t) = a\theta_{1,0}.$$

Montiel Olea et al. impose the normalization that the $(1, 1)$ element of the structural impact multiplier matrix B_0^{-1} is unity. Under these assumptions the scalar structural parameter a and the $K \times 1$ vector of structural impact responses $\theta_{1,0} = B_0^{-1} e_1$ are identified by the moment condition

$$\mathbb{E}(z_t u_t) = a\theta_{1,0}$$

or, equivalently, by

$$\mathbb{E}\left[z_t(y_t - A_1 y_{t-1} - \cdots - A_p y_{t-p}) - a\theta_{1,0}\right] = 0.$$

In addition, the VAR slope parameters $\boldsymbol{\alpha} = \text{vec}(A_1, \ldots, A_p)$ satisfy the moment conditions

$$\mathbb{E}\left[\left(y'_{t-1}, \ldots, y'_{t-p}\right)' \otimes \left(y_t - A_1 y_{t-1} - \cdots - A_p y_{t-p}\right)\right] = 0.$$

[3] Montiel Olea, Stock, and Watson (2015b) show that their approach may be generalized to a situation in which up to K structural shocks are identified by as many external instruments.

The GMM estimator corresponding to these two sets of moment conditions is given by

$$\widehat{\boldsymbol{\alpha}} = \left[\left(\frac{1}{T} \sum_{t=1}^{T} Y_{t-1} Y_{t-1}' \right)^{-1} \otimes I_K \right] \frac{1}{T} \sum_{t=1}^{T} (Y_{t-1} \otimes y_t),$$

$$\widehat{a} = \frac{1}{T} \sum_{t=1}^{T} z_t \widehat{u}_{1t}, \tag{15.2.1}$$

$$\widehat{\theta}_{1,0} = \frac{1}{\widehat{a}} \frac{1}{T} \sum_{t=1}^{T} z_t \widehat{u}_t, \tag{15.2.2}$$

where $Y_{t-1} \equiv (y_{t-1}', \dots, y_{t-p}')'$ and $\widehat{u}_t = y_t - \left(Y_{t-1}' \otimes I_K\right) \widehat{\boldsymbol{\alpha}}$, such that the K-dimensional vector of structural responses to shock 1 at horizon h is

$$\widehat{\theta}_{1,h} = \widehat{\Phi}_h \widehat{\theta}_{1,0},$$

where $\widehat{\Phi}_h$ is obtained from $\widehat{\boldsymbol{\alpha}}$, as discussed in Chapter 4. The estimator is $\sqrt{T}$-consistent and asymptotically normal. Analogous results may be constructed for the structural forecast error variance decomposition with respect to the first shock. Pointwise inference on the structural impulse responses and forecast error variance decompositions may be based on the plug-in estimators of the asymptotic variance derived in Montiel Olea, Stock, and Watson (2015b).

Weak Instrument Diagnostics. An important concern in applied work is that many external instruments are weak in the econometric sense. Conventional GMM procedures are not robust to the use of weak external instruments. The concern is that the correlation between the instrument, z_t, and w_{1t} is low, calling into question the identifying assumption that $\mathbb{E}(z_t w_{1t}) = a \neq 0$. Under the normalizing assumption that the $(1, 1)$ element of the structural impact matrix B_0^{-1} is unity, a low correlation between z_t and the reduced-form residual $\widehat{u}_{1t}$ resulting in a value of $\widehat{a}$ close to zero in expression (15.2.1), renders the estimator $\widehat{\theta}_{1,0}$ in expression (15.2.2) unreliable.

Montiel Olea et al. show that the asymptotic distribution of the GMM Wald test for the scalar structural impulse response $\widehat{\theta}_{i1,h}$ under weak-instrument asymptotics is equivalent to the asymptotic distribution of a Wald test statistic for the coefficient of an endogenous regressor in a conventional IV regression model given by

$$e_i' \widehat{\Phi}_h \widehat{u}_t = \kappa_0 \widehat{u}_{1t} + v_{2t}, \tag{15.2.3}$$

$$\widehat{u}_{1t} = a z_t + v_{1t}, \tag{15.2.4}$$

where κ_0 is the value of $\theta_{i1,h}$ under the null hypothesis, $e_i' \widehat{\Phi}_h \widehat{u}_t$ is the dependent variable in the second-stage regression (15.2.3), $\widehat{u}_{1t}$ is the endogenous regressor in the second-stage regression, and z_t is the external instrument used in the first-stage regression (15.2.4). This alternative representation illustrates that one can assess the strength of the instrument z_t based on the correlation between $\widehat{u}_{1t}$ and z_t. In practice, an F-statistic for $\mathbb{H}_0 : a = 0$ below 10 in the first-stage regression is usually interpreted as an indication of a weak instrument problem.

Weak instrument problems are common in IV regression analysis. For example, Kilian (2008a, 2008b) provides evidence that measures of exogenous OPEC oil supply shocks of the type proposed by Hamilton (2003) and Kilian (2008b) do not have much predictive power for changes in the real price of oil and, hence, must be considered a weak instrument for the real price of oil. In the context of a VAR model identified by external instruments, Stock and Watson (2012) show that a similar weak instrument problem arises when regressing the VAR prediction error for world oil production, $\widehat{u}_{1t}$, on exogenous OPEC oil supply shocks, z_t.

Robust Inference. The key result in Montiel Olea et al. is that inference about structural impulse responses may be conducted in a way that is robust to the strength of the instrument and remains valid even when using a weak instrument. This involves an alternative asymptotic thought experiment than that underlying the derivation of the conventional GMM estimator discussed earlier. In the context of the IV model (15.2.3) and (15.2.4), this thought experiment amounts to postulating that the population parameter a in the first-stage regression of the IV model is not a constant, but is modeled as local to zero in the sense that $a = 0 + \lambda/\sqrt{T}$, as $T \to \infty$, where λ is a constant. As in the discussion of local-to-unity asymptotics in Chapters 3 and 12, these local asymptotics are merely a device for approximating the finite-sample distribution of the estimator.

Montiel Olea et al. propose an adjusted version of the Anderson-Rubin test statistic for the IV model. This test statistic rejects $\mathbb{H}_0 : \theta_{i1,h} = \kappa_0$ if

$$AR(\kappa_0) = \left(\frac{1}{\sqrt{T}} \sum_{t=1}^{T} z_t \left(e_i' \widehat{\Phi}_h \widehat{u}_t - \kappa_0 \widehat{u}_{1t} \right) \right)^2 \Bigg/ \widehat{\sigma}_T^2(\kappa_0) > \chi^2_{1-\gamma}(1).$$

The statistic $\widehat{\sigma}_T^2(\kappa_0)$ is a consistent estimator of the asymptotic variance of

$$\frac{1}{\sqrt{T}} \sum_{t=1}^{T} z_t \left(e_i' \widehat{\Phi}_h \widehat{u}_t - \kappa_0 \widehat{u}_{1t} \right).$$

Montiel Olea, Stock, and Watson (2015b) derive a formula for $\widehat{\sigma}_T^2(\kappa_0)$ based on the delta method.

In the just-identified case, this test statistic has three desirable properties. First, the test is asymptotically valid whether the instruments are weak or strong. Second, the test achieves the same local power as the GMM test under strong asymptotics. Third, the test is asymptotically efficient within the class of tests that are asymptotically valid in the presence of weak instruments.

Robust $(1-\gamma)100\%$ confidence sets for the structural impulse responses may be obtained by inverting this test statistic such that

$$CS \equiv \left\{\kappa_0 \in \mathbb{R} \,|\, AR(\kappa_0) \leq \chi^2_{1-\gamma}(1)\right\}.$$

Implementation of this robust interval does not require the user to evaluate the $AR(\kappa_0)$ statistic over a grid of κ_0. Montiel Olea, Stock, and Watson (2015b) show that the confidence set for $\theta_{i1,h}$ may also be computed from the roots that solve

$$\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t \left(e_i'\widehat{\Phi}_h\widehat{u}_t - \kappa_0\widehat{u}_{1t}\right)\right)^2 - \widehat{\sigma}^2_T(\kappa_0)\chi^2_{1-\gamma}(1) = 0,$$

which can be shown to be a quadratic equation in κ_0.

15.2.2 Discussion

The literature on the use of external instruments is evolving at a rapid pace. For example, researchers recently have been exploring the use of external shock measures based on futures prices, as discussed in Section 15.1 (see, e.g., Gertler and Karadi 2015; Rogers, Scotti, and Wright 2015). One context in which this approach may prove particularly useful is the identification of monetary policy shocks in factor-augmented VAR (FAVAR) models, because recursive orderings are not easy to defend in this context (see Chapter 16).

At the same time, the econometric methodology continues to be refined. For example, Lunsford (2015a) proposes a test of the strength of external instruments. In related work, Lunsford (2015b) develops a residual-based block bootstrap method that allows for conditional heteroskedasticity in structural VAR models identified by external instruments, building on Brüggemann, Jentsch, and Trenkler (2016).