Finally back to work! To do 7 Oct 2019

- generate IRFs from all models } Susanto
- think about about slope, b

✓ - change κ (and α) and see if π fluctuates as much as x then } Peter

✗ - $\hat{E}$-operator

- θ_t as SR FE

- small π vs. larger x & i → contrast to rest of Lit to see what mechanism is responsible } Ryan

✓ - think about whether ppl know the TR?

✓ - read Thomas Lubik: Indeterminacy & Learning (JME)

✓ - derive Preston's IS curve → me.

Lubik: has a comment that b/c of anticipated utility (Kreps 1998, Cogley & Sargent 2008), their model can be solved using standard, RE algorithms

→ in Lubik, the CB learns the econ and announces

$i = \psi_\pi \pi_t + \psi_x x_t$ w/ (ψ_π, ψ_x) each period, updated according to its estimates. Depending on $(\psi_{x,t}, \psi_{\pi,t})$ we can land in the determinacy or indeterminacy region. Ryan is right: this amounts to ppl not knowing the TR. And he's doubly right b/c in my setting, they know the Taylor-rule, the only thing they don't know are future things.

Let's first try to derive the IS curve in Preston <u>8 Oct 2015</u>

HH: $\max \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[u(C_T^i; \xi_T) - \int_0^1 v(h_T^i(j), \xi_T)\, dj \right]$ (1)

$M_t^i + B_t^i \leq (1+i_{t-1}^m) M_{t-1}^i + (1+i_{t-1}) B_{t-1}^i + P_t Y_t^i - T_t - P_t C_t^i$ (2)

$\uparrow$

$\int_0^1 \left[W_t(j) h_t^i(j) + \Pi_t(j) \right] dj$

So the HH chooses $\{c_t^i(j), h_t^i(j), M_t^i, B_t^i\}$ $\forall j \in [0,1]$ so as to $\max$ (1) s.t. (2), taking as given $\{p_T(j), w_T(j), \Pi_T, i_{T-1}, i_{T-1}^m, \xi_T\}$ $\forall T \geq t$.

In the App, Preston defines $W_{t+1}^i = (1+i_t^m) M_t^i + (1+i_t) B_t^i$ as beginning-of-period wealth at time t+1

$\Rightarrow$ (2) $M_t^i + B_t^i \leq W_t^i + P_t Y_t^i - T_t - P_t C_t^i$

Now let $\Delta_t \equiv \dfrac{i_t - i_t^m}{1+i_t}$ recall: $W_{t+1}^i = (1+i_t^m) M_t^i + (1+i_t) B_t^i$

$$\Leftrightarrow \quad P_t C_t^i + \underbrace{M_t^i + B_t^i}_{(1+i_t^m - i_t^m) M_t^i + \frac{1+i_t}{1+i_t} B_t^i} \leq W_t^i + [P_t Y_t^i - T_t]$$

$$= -i_t^m M_t^i + (1+i_t^m) M_t^i + \frac{(1+i_t)}{1+i_t} B_t^i$$

$$= -i_t^m M_t^i + \frac{(1+i_t)(1+i_t^m) M_t^i}{1+i_t} + \frac{(1+i_t)}{1+i_t} B_t^i$$

$$= \underbrace{-i_t^m M_t^i + \frac{i_t (1+i_t^m)}{1+i_t} M_t^i}_{= \frac{(1+i_t)(-i_t^m) M_t^i + i_t (1+i_t^m) M_t^i}{1+i_t}} + \underbrace{\frac{(1+i_t^m) M_t^i + (1+i_t) B_t^i}{1+i_t}}_{= \frac{1}{1+i_t} W_{t+1}}$$

$$= \frac{(-i_t^m - i_t i_t^m + i_t + i_t i_t^m) M_t^i}{1+i_t} = \frac{i_t - i_t^m}{1+i_t} M_t^i = \Delta_t M_t^i$$

So $\Rightarrow$ (2) $P_t C_t^i + \Delta_t M_t^i + \dfrac{1}{1+i_t} W_{t+1}^i \leq W_t^i + [P_t Y_t^i - T_t]$ (32)

Now solve this fwd.

$$W_t^i \geq P_t C_t^i + \Delta_t M_t^i - (P_t Y_t^i - T_t) + \frac{1}{1+i_t} W_{t+1}^i$$

$$\rightarrow W_t^i \geq P_t C_t^i + \Delta_t M_t^i - (P_t Y_t^i - T_t)$$

$$+ \left(\frac{1}{1+i_t}\right)\left[P_{t+1} C_{t+1}^i + \Delta_{t+1} M_{t+1}^i - (P_{t+1} Y_{t+1}^i - T_{t+1})\right]$$

$$+ \ldots$$

$$+ \underbrace{\left(\frac{1}{1+i_t}\right) \cdot \ldots \cdot \left(\frac{1}{1+i_{t+j-1}}\right)}_{=: R_{t,t+j} = \prod_{s=1}^{j}\left(\frac{1}{1+i_{t+s-1}}\right)}\left[P_{t+j} C_{t+j}^i + \Delta_{t+j} M_{t+j}^i - (P_{t+j} Y_{t+j}^i - T_{t+j})\right]$$

$$W_t^i \geq \sum_{j=0}^{\infty} R_{t,t+j}\left[P_{t+j} C_{t+j}^i + \Delta_{t+j} M_{t+j}^i - (P_{t+j} Y_{t+j}^i - T_{t+j})\right]$$

The flow BC solved fwd to get the intertemporal BC, the IBC.

Let's now take the FOCs using the new flow BC, 32

$$\mathcal{L} = \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t}\left[u(C_T^i; \xi_T) - \int_0^1 v(h_T^i(j); \xi_T)\, dj\right]$$

$$+ \lambda_t \left(-P_t C_t^i - \Delta_t M_t^i - \frac{1}{1+i_t} W_{t+1}^i + W_t^i + \underbrace{[P_t Y_t^i - T_t]}_{w_t^j h_t^j}\right)$$

c 1) $u_c = \lambda_t P_t \rightarrow \lambda_t = \frac{u_c}{P_t}$

h^j 2) $v_{h^j} = \lambda_t P_t w_t^j \rightarrow \lambda_t = \frac{v_{h^j}}{P_t w_t}$

$$\left.\right\} \frac{u_c}{P_t} = \frac{v_h}{P_t w_t} \Rightarrow \frac{v_h}{u_c} = w_t$$

real wage

B (3) $\lambda_t (1+i_t) = \beta \hat{E}_t \lambda_{t+1} \rightarrow 1+i_t = \beta \hat{E}_t \frac{\lambda_{t+1}}{\lambda_t} = \beta \hat{E}_t \frac{P_t}{P_{t+1}} \frac{u' C_{t+1}}{u' C_t}$

ok so I *almost* get what Preston gets: (difference)

$$\Delta_t i_t = \beta E_t \frac{P_t}{P_{t+1}} \frac{u_c(C_{t+1})}{u_c(C_t)} \qquad (33)$$

$$\frac{v'_n}{u'_c} = w_t \qquad (34)$$

- A cashless econ implies $i_t = i_t^m$ or $M_t^i = 0$

  since $M^S > 0$, we get $i_t = i_t^m \Rightarrow \Delta_t = 0 \ \forall t$
- Market clearing implies $y_t(j) = c_t(j) \ \forall j \Rightarrow C_t = Y_t$
- Zero debt fiscal policy: $\underbrace{B_t = 0 \Rightarrow T_t = (1+i_{t-1})M_{t-1} - M_t}_{\text{ain't so sure if this is right!}}$

  ↳ thus the IBC becomes

$$W_t^i = \sum_{j=0}^{\infty} R_{t,t+j}\left[P_{t+j}C_{t+j}^i \qquad - (P_{t+j}Y_{t+j}^i - T_{t+j})\right]$$

$$W_t^i = \sum_{j=0}^{\infty} R_{t,t+j}\left[P_{t+j}C_{t+j}^i - P_{t+j}Y_{t+j}^i - (1+i_{t+j-1})M_{t+j-1} + M_{t+j}\right]$$

Maybe the point is that there is no diff btwn bonds & money,

so $W_{t+1}^i = (1+i_t)M_t + (1+i_t)B_t = (1+i_t)[M_t + B_t]$

so we can ass that ppl only hold bonds, $M_t = 0$

$\rightarrow W_{t+1}^i = (1+i_t)B_t \quad \rightarrow \quad T_t = 0$ (a little non-kosher but ok)

$$W_t^i \geq \sum_{j=0}^{\infty} R_{t,t+j}\left[P_{t+j}C_{t+j}^i - P_{t+j}Y_{t+j}^i\right]$$

Funnily, the App. stops here.

But it is clear that w/ a specific U-fct ass, loglin of the EE (33) gives rise to (3). But does loglin of the IBC $W_t^i \geq \sum_{j=0}^{\infty} R_{t,t+j}\left[P_{t+j}C_{t+j}^i - P_{t+j}Y_{t+j}^i\right]$ give rise to (4), $\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i = \bar{\omega}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T^i$?

What's the loglin of $R_{t,t+j} = \prod_{s=1}^{j} \left(\frac{1}{1+i_{t+s-1}}\right)$?

$$= \sum_{s=1}^{j} \ln\left(\frac{1}{1+i_{t+s-1}}\right) = \sum_{s=1}^{j} -\ln(1+i_{t+s-1})$$

Take the gross interest rate $1+i_t$ as a unit, call it R_t

$$\Rightarrow \sum_{s=1}^{j} -\ln(R_{t+s-1}) \text{ Total diff } = \sum_{s=1}^{j} -\frac{dR_{t+s-1}}{R} = -\sum_{s=1}^{j} \hat{R}_{t+s-1}$$

where $R = \beta^{-1}$ so $\bar{i} = i^* = \beta^{-1} - 1$

Supp I write the IBC as

$$W_t^i \geq \sum_{j=0}^{\infty} R_{t,t+j} P_{t+j} \left[C_{t+j}^i - Y_{t+j}^i\right] \quad \Big| : P_t\bar{Y} \qquad \bar{Y} = Y^*$$

$$\bar{\omega}_t^i = \sum_{j=0}^{\infty} R_{t,t+j} \frac{P_{t+j}}{P_t} \left[\frac{C_{t+j}^i}{\bar{Y}} - \frac{Y_{t+j}^i}{\bar{Y}}\right]$$

$$\bar{\omega}_t^i = \sum_{j=0}^{\infty} R_{t,t+j} \Pi_{t,t+j} \left[\hat{C}_{t+j}^i - \hat{Y}_{t+j}^i\right] \quad \text{To get eq (4),}$$

we need the loglin of $R_{t,t+j}\,\pi_{t,t+j}$ to be β^{j}

Honestly, I don't think there's a way. Or?

$R_{t,t+j}\,\pi_{t,t+j}$ is the sdf. It's just like the firms' sdf in Calvo, $\phi_{t+\tau,t} = \beta \dfrac{P_t C_t}{P_{t+\tau} C_{t+\tau}}$, so then, analogously to here, it cancels and we're left w/ β every time.

→ yes, you can see it from eq (33).

⇒ ok so eq. (4) is good!

The next step is solving (3) backwards.

(3): $\hat{c}_t^i = \hat{E}_t^i \hat{c}_{t+1}^i - \sigma(\hat{i}_t - \hat{E}_t^i \hat{\pi}_{t+1}) + g_t - \hat{E}_t^i g_{t+1}$

$\hat{E}_t^i \hat{c}_{t+1}^i = \hat{c}_t^i - g_t + \hat{E}_t^i g_{t+1} + \sigma(\hat{i}_t - \hat{E}_t^i \pi_{t+1})$

$\hat{E}_t^i \hat{c}_{t+1}^i - \hat{E}_t^i g_{t+1} = \hat{c}_t^i - g_t + \sigma(\hat{i}_t - \hat{E}_t^i \pi_{t+1})$

Let's call $t+1 = T$

$$
\begin{aligned}
\hat{E}_t^i \hat{C}_T^i - \hat{E}_t g_T &= \underbrace{\hat{c}_{T-1}^i - g_{T-1}} + \sigma\underbrace{(\hat{i}_{T-1} - \hat{E}_t^i \pi_T)} \\
&= \hat{c}_{T-2}^i - g_{T-2} + \sigma(\hat{i}_{T-2} - \hat{E}_t^i \pi_{T-1}) + \cdots \\
&= \ldots \hat{c}_t^i - g_t + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \quad \checkmark
\end{aligned}
$$

$$\hat{E}^i_t \hat{C}^i_T = \hat{E}^i_t g_T + \hat{c}^i_t - g_t + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1})$$
$$= \hat{c}^i_t - g_t + \hat{E}_t \left[g_T + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \right]$$

So now sub $\hat{E}^i_t \hat{C}^i_T$ (I guess) into the IBC, eq (4)

$$\hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}^i_T = \varpi^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}^i_T$$

$$\Rightarrow \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \hat{c}^i_t - g_t + \hat{E}_t \left[g_T + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \right] \right\}$$

$$= \tilde{\omega}^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}^i_T$$
$$\Leftrightarrow \hat{c}^i_t - g_t = \tilde{\omega}^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}^i_T - g_T - \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \right]$$

Conjecture: $\sum_{T=t}^{\infty} \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) = \sum_{T=t}^{\infty} (\hat{i}_T - \hat{\pi}_{T+1})$.

Why? B/c $\sum_{T=t}^{\infty} \sum_{s=t}^{T-1} \text{stuff}_s = \sum_{s=t}^{t-1} \text{stuff}_s + \sum_{s=t}^{t} \text{stuff}_s$

So what if I don't sub the (3) solved fwd into (4), but just (3)?

(3) $\hat{C}^i_t = \hat{E}^i_t \hat{C}^i_{t+1} - \sigma(\hat{i}_t - \hat{E}^i_t \hat{\pi}_{t+1}) + g_t - \hat{E}^i_t g_{t+1}$

(4) $\hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}^i_T = \tilde{\omega}^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}^i_T$

$\rightarrow \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{C}^i_{T+1} - \sigma(\hat{i}_T - \hat{\pi}_{T+1}) + g_T - g_{T+1} \right] = \tilde{\omega}^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}^i_T$

that doesn't seem to do it at all!

Let's pause the issue of eq (5). Supp. we have it.

- Then integrate over i $\rightarrow$ $\tilde{\omega}_t \Rightarrow 0$

$C_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\hat{Y}_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta(g_T - g_{T+1}) \right]$

- $\hat{Y}_t = \hat{C}_t$

$\hat{Y}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\hat{Y}_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta(g_T - g_{T+1}) \right]$

- $x_t := \hat{Y}_t - \hat{Y}^n_t$

$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta(g_T - g_{T+1}) - (1-\beta)\hat{Y}^n_T \right]$

$- \hat{Y}^n_t$ even that doesn't work out! damn!

Ryan meeting

8 Oct 2019

- Invite Eusepi / Preston for macro seminar? Who organizes it?

  ↳ Ryan, Susanto & Pablo for the spring, Rosen, Fabio & Jaromir for fall

Ryan said that Susanto connects b (slope) w/ Missing Deflation b/c b transmits shocks to SR fcsts

Work after:

8 Oct 2019

So supp my previous conjecture is right. Then

$$\hat{c}_t^i - g_t = \tilde{\omega}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}_T^i - g_T + \sigma(\hat{i}_T - \hat{\pi}_{T+1}) \right]$$

and I still don't get (5).

But if I now do the last 3 steps:

$$\underbrace{\hat{Y}_t - \hat{Y}_t^n}_{} \underbrace{- g_t + \hat{Y}_t^n}_{} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}_T - \hat{Y}_T^n - g_T + \hat{Y}_T^n + \sigma(\hat{i}_T - \hat{\pi}_{T+1}) \right]$$

$$\Rightarrow \quad x_t + r_t^n = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[x_t \right.$$

↳ no, the problem is that that ain't right b/c

$r_t^n = Y_{t+1}^n - Y_t^n$ [$+ (g_t - g_{t+1})$ if we have this preference shock]

↳ and for me, $r_t^n = \frac{1}{\sigma}(Y_{t+1}^n - Y_t^n)$, in line w/ Basu, SUM part 2, p.58)

So let's set all $g_t = 0 \ \forall t$ b/c I don't have it.

→ Then the EE is $\hat{c}_t^i = \hat{E}_t^i \hat{c}_{t+1} - \sigma(\hat{i}_t - \hat{E}_t^i \hat{\pi}_{t+1})$

Solve back: $\hat{E}_t c_{t+1}^i = \hat{c}_t^i + \sigma(\hat{i}_t - \hat{E}_t^i \pi_{t+1})$

$$\hat{E}_t c_T^i = \hat{c}_{T-1}^i + \sigma(\hat{i}_{T-1} - \hat{E}_t^i \pi_T)$$

$$\hat E_t \hat C_T^i = \sigma(\hat i_{T-1} - \hat E_t^i \pi_T) + \sigma(\hat i_{T-2} - \hat E_t^i \pi_{T-1}) + \dots$$
$$+ \sigma(\hat i_t - \hat E_t \pi_{t+1}) + \hat c_t^i$$

$$\hat E_t^i \hat C_T^i = \hat c_t^i + \sigma \sum_{s=t}^{T-1} \left(\hat i_s - \hat E_t \pi_{s+1}\right)$$ (EE simple solved fwd)

Sub this into (4)

$$\hat E_t^i \sum_{T=t}^{\infty} \beta^{T-t}\left[\hat c_t^i + \sigma \sum_{s=t}^{T-1}\left(\hat i_s - \hat E_t \pi_{s+1}\right)\right] = \tilde\omega_t^i + \hat E_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat Y_T^i$$

$$\Leftrightarrow \hat c_t^i \sum_{T=t}^{\infty} \beta^{T-t} + \hat E_t^i \sum_{T=t}^{\infty} \beta^{T-t} \sigma \sum_{s=0}^{T-1}\left(\hat i_s - \hat E_t \pi_{s+1}\right)\Big] = \tilde\omega_t^i + \hat E_t \sum_{T=t}^{\infty} \beta^{T-t} \hat Y_t^i$$

$$\Leftrightarrow \hat c_t^i + \underbrace{(1-\beta)\sigma \hat E_t^i \sum_{T=t}^{\infty} \beta^{T-t} \sum_{s=t}^{T-1}\left(\hat i_s - \hat E_t \pi_{s+1}\right)} = (1-\beta)\tilde\omega_t^i + \hat E_t \sum_{T=t}^{\infty} \beta^{T-t}(1-\beta)\hat Y_T^i$$

now I only need this to equal $\beta\sigma(\hat i_T - \hat\pi_{T+1})$

But potentially, it should actually be $(1-\beta)(\sigma?)$

Now I'm only missing the $\hat i_T - \hat \pi_{T+1}$ term <u>9 Oct 2019</u>

in eq (5). So supp. again that I have eq (5), w/o g_s,

and I do the last 3 steps:

- aggregate, set $C_t = Y_t$ and am about to get $x_t = \hat Y_t - \hat Y_t^n$

$$\hat Y_t = \hat E_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)\hat Y_T - \beta\sigma(\hat i_T - \pi_{T+1})\right]$$

$$\hat{Y}_t - \hat{Y}_t^n = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)\hat{Y}_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1})\right] - \hat{Y}_t^n$$

(*) $$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)[\hat{Y}_T - \hat{Y}_T^n] - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + (1-\beta)\hat{Y}_T^n\right] - \hat{Y}_t^n$$

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)(\hat{Y}_{T+1} - \hat{Y}_{T+1}^n) - \beta\sigma(\hat{i}_t - \hat{\pi}_{T+1}) + (1-\beta)\hat{Y}_{T+1}^n\right]$$
$$- \hat{Y}_t^n + (1-\beta)\hat{Y}_t$$

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)x_{T+1} - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1})\right]$$
$$+ \underline{x_t - \beta\hat{Y}_t + (1-\beta)\hat{Y}_{t+1}^n + (1-\beta)\beta\hat{Y}_{t+2}^n + (1-\beta)\beta^2\hat{Y}_{t+3}^n} + \dots$$

$$\underbrace{x_t - \beta\hat{Y}_t + Y_{t+1}^n}\ \underbrace{-\beta\hat{Y}_{t+1}^n + \beta\hat{Y}_{t+2}^n}_{\beta r_{t+1}^n}\ \underbrace{-\beta^2\hat{Y}_{t+2}^n + \beta^2\hat{Y}_{t+3}^n}_{+\ \beta^2 r_{t+2}^n}\ \underbrace{-\beta^3\hat{Y}_{t+3}^n}_{+\ \dots\ \text{cool}} + \dots$$

$$\hat{Y}_t - \hat{Y}_t^n - \beta\hat{Y}_t + \hat{Y}_{t+1}^n$$
$$= \hat{Y}_t - \beta\hat{Y}_t + r_t^n$$
$$= \underline{(1-\beta)\hat{Y}_t} + r_t^n$$

grrr.. let's retry, from step (*)

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)[\hat{Y}_T - \hat{Y}_T^n] - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + (1-\beta)\hat{Y}_T^n\right] - \hat{Y}_t^n$$
$$x_t = \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)x_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1})\right]$$
$$- \hat{Y}_t^n + (1-\beta)\hat{Y}_t^n + (1-\beta)\beta\hat{Y}_{t+1}^n + (1-\beta)\beta^2\hat{Y}_{t+2}^n + \dots$$

The last series is $-\hat{Y}_t^n + (1-\beta)\hat{Y}_t^n + (1-\beta)\beta\hat{Y}_{t+1}^n + (1-\beta)\beta^2\hat{Y}_{t+2}^n + \dots$

$$= \underbrace{-\beta\hat{Y}_t^n + \beta\hat{Y}_{t+1}^n}_{\beta r_t^n}\ \underbrace{-\beta^2\hat{Y}_{t+1}^n + \beta^2\hat{Y}_{t+2}^n}_{+\ \beta^2 r_{t+1}^n\ +\ \dots} - \beta^3\hat{Y}_{t+2}^n + \dots$$

$$\beta r_t^n + \beta^2 r_{t+1}^n + \dots$$

$$= \beta\left[r_t^n + \beta r_{t+1}^n + \beta^2 r_{t+2}^n + \dots\right]$$

$$= \beta\sum_{T=0}^{\infty}\beta^{T-t} r_T^n$$

now this guy is superfluous ...

and we have x_T, not x_{T+1} in the Σ, i.e.

$$x_t = \sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta)x_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta r_T^n\right] \quad \Big|\ -x_t$$

$$\beta x_t = \sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta)x_{T+1} - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta r_T^n\right]$$

$$x_t = \sum_{T=t}^{\infty}\beta^{T-t}\left[\frac{1-\beta}{\beta}x_{T+1} - \sigma(\hat{i}_T - \hat{\pi}_{T+1}) + r_T^n\right]$$

hát ezt nem hiszem el ... mostmár minden stimmel csak ez nem ...

(Még az $r_T^n = \sigma r_t^n$ is jó, ha átdefiniálom $r_t^n = \frac{1}{\sigma}(\hat{Y}_{t+1}^n - \hat{Y}_t^n)$ mint Basu Sem 2 p. 58)

So at the current point in time I'm inclined to say that if eq.(5) is right, then eq.(6) is too, <u>except</u> the coefficient of x_{T+1} should be $\frac{1-\beta}{\beta}$, <u>not</u> $1-\beta$.

So we need to turn back to deriving eq (5) from (3).

$$\hat{c}_t^i = \hat{E}_t c_{t+1}^i - \sigma(i_t - \hat{E}_t^i \hat{\pi}_{t+1}) \qquad (3)$$

Drop i's and hats for simplicity:

$$c_t - E_t c_{t+1} = -\sigma(i_t - E_t \pi_{t+1})$$

$$(1 - L^{-1}) c_t = -\sigma(i_t - E_t \pi_{t+1})$$

or $-(1 - L) E_t c_{t+1} = -\sigma(i_t - E_t \pi_{t+1})$

$\Leftrightarrow \quad (1 - L) E_t c_\tau = \sigma(i_\tau - E_t \pi_{\tau+1})$ | See rotemberg_pricing_peter_

$E_t c_\tau = (1 - L)^{-1} \sigma(i_\tau - E_t \pi_{\tau+1})$ | _mynotes.pdf.

$$= \sigma \sum_{i=0}^{\infty} 1^i \text{ RHS } = \sigma \sum_{T=t}^{\tau-1} (i_T - E_t \pi_{T+1})$$

not trivial

I think this is pretty much what I got before, and it's also in accordance w/ Preston's (4.5)

Plugging this into the LHS of (4), we have

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\sigma \sum_{s=t}^{T-1} (i_s - \hat{E}_t \pi_{s+1}) \right]$$

$$= \sigma \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \sum_{s=t}^{T-1} (i_s - \hat{E}_t \pi_{s+1})$$

$$= \sum_{s=t}^{t-1} \text{stuff} + \beta \sum_{s=t}^{t} + \beta^2 \sum_{s=t}^{t+1} + \beta^3 \sum_{s=t}^{t+2}$$

That don't look good.

$$= \mathcal{B}\,\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \sum_{s=t}^{T-1} (i_s - \hat{E}_t \pi_{s+1})$$

let's try to redefine the indices

$$\mathcal{B}\,\hat{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{s=0}^{t-1} \text{stuff}_s$$

$$= \mathcal{B} \cdot [\underset{t=0}{0} + \underset{t=1}{\beta \cdot \sum_{s=0}^{0}} + \underset{t=2}{\beta^2 \cdot \sum_{s=0}^{1}} \quad \ldots$$ mmm... :S

Also I kinda feel that even if I don't take time t, but time 0 expectations, I can't get around the second sum ...

$$= \mathcal{B}\, [\beta \cdot \text{stuff}_0 + \beta^2 \text{stuff}_0 + \beta^2 \text{stuff}_1 + \beta^3 \text{stuff}_0 + \beta^3 \text{stuff}_1 + \beta^3 \text{stuff}_2 + \ldots$$

$$= \mathcal{B}\, [(\beta + \beta^2 + \ldots) \text{stuff}_0 + (\beta^2 + \beta^3 + \ldots) \text{stuff}_1 + \text{etc.}$$ | Ignore $\mathcal{B}$

$$= \beta (1 + \beta^2 + \beta^3 + \ldots) \text{stuff}_0 + \beta^2 (1 + \beta^2 + \beta^3 + \ldots) \text{stuff}_1 + \ldots$$

$$= (\beta^1 \text{stuff}_0 + \beta^2 \text{stuff}_1 + \beta^3 \text{stuff}_2 + \ldots) \frac{1}{1-\beta} \quad (!)$$

$$= (\beta^0 \text{stuff}_0 + \beta^1 \text{stuff}_1 + \beta^2 \text{stuff}_2 + \ldots) \frac{\beta}{1-\beta} \quad (!)$$

So the LHS is

$$\mathcal{B}\,\hat{E}_t \sum_{T=t}^{\infty} \frac{\beta}{1-\beta} (i_T - \hat{E}_t \pi_{T+1})$$ Holy Moly!

So (5) is

$$\frac{1}{1-\beta} c_t + \sigma \hat{E}_t \sum_{T=t}^{\infty} \frac{\beta}{1-\beta} (i_T - \hat{E}_t \pi_{T+1}) = \hat{E}_t \sum_{T=t}^{\infty} \hat{Y}_T + \bar{w}_t$$

$$\Rightarrow \hat{c}_t^i + \sigma \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \beta (i_T - \hat{E}_t \pi_{T+1}) = (1-\beta) \bar{w}_t + \hat{E}_t \sum_{T=t}^{\infty} (1-\beta) \hat{Y}_T$$

So ...

$$\hat{c}_t^i = (1-\beta) \bar{w}_t^i + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) \hat{Y}_T - \beta \sigma (i_T - \hat{E}_t \pi_{T+1}) \right]$$

= Preston's (5). Yay!

The last 3 steps again: aggregate and set $\hat{C}_t = \hat{Y}_t$, drop hats.

$$Y_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) Y_T - \beta \sigma (i_T - \pi_{T+1}) \right]$$

$$Y_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)(Y_T - Y_T^n) - \beta \sigma (i_T - \pi_{T+1}) + (1-\beta) Y_T^n \right]$$

$$Y_t = \underbrace{\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \beta \sigma (i_T - \pi_{T+1}) \right]}_{=: CS} + (1-\beta) x_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} (1-\beta) Y_T^n$$

$$x_t = \text{correct-stuff} - Y_t^n + (1-\beta) x_t + (1-\beta) \left[Y_t^n + \beta Y_{t+1}^n + \beta^2 Y_{t+2}^n + \dots \right.$$

$$\beta x_t = CS \cancel{- Y_t^n} \cancel{+ Y_t^n} - \beta Y_t^n + \beta Y_{t+1}^n - \beta^2 Y_{t+1}^n + \beta^2 Y_{t+2}^n - \beta^3 Y_{t+2}^n + \dots \left. \right]$$

$$\beta x_t = CS + \beta r_t^n + \beta^2 r_{t+1}^2 + \dots$$

$$x_t = \frac{1}{\beta} CS + r_t^n + \beta r_{t+1}^2 + \dots$$

σr_T^N w/ my def.

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\frac{1-\beta}{\beta} x_{T+1} - \sigma (i_T - \pi_{T+1}) + r_T^N \right] .$$

Wait... can it be that I did one thing wrong:

when I have $\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta) x_T] = (1-\beta) \hat{E}_t \sum_{T=t}^{T-t} x_T$

$$= (1-\beta) x_t + (1-\beta) \hat{E}_t [\beta x_{t+1} + \beta^2 x_{t+2} + \dots]$$

$$= (1-\beta) x_t + (1-\beta) \hat{E}_t [\beta \sum_{T=t}^{\infty} \beta^{T-t} x_{T+1}]$$

Yeah!

So that means that my truly correct CS is:

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)\beta x_{T+1} - \beta\sigma(i_T - \pi_{T+1})]$$

so that when I take $\frac{1}{\beta}$ CS I obtain

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta) x_{T+1} - \sigma(i_T - \pi_{T+1}) + \sigma r_T^N]$$

which is Preston's (6) = (18), and the equation I had in my DW prezi and in materials 4. Yay!

Great! We can go on to investigating the expectation-operator, $\hat{E}_t$. Obs. 1. Preston took derivatives thru it no problem, w/o saying anything. So let's go to Evans & Honkapohja (2001).

Evans & Honk (2001) on the nonrational expectation-operator $\hat{E}$

- p. 68-69 Ramsey model: they derive them at no point.
- Minimum-state-variable (MSV) solution (introduced in McCallum (1983)
- heterogeneous learning p. 223-225 (Evans, Honk & Marimon 2000)
- "regular" vs "irregular" models (Farmer 1999, Pesaran 1987)
  ↳ unique REE (Blanchard-Kahn conditions)
- RE is an eqb concept p. 11
- Expectation is a fct of observables, and so is the updating p. 17-18 → I think it's fair to say that we conjecture $\hat{E}$ to be linear and of the form $\bar{\pi} + bs_t$, and $\bar{\pi}_t = Q(\bar{\pi}_{t-1}, \theta_t)$ nonlinear updating rule and to somehow verify this using undet. coeffs.

Reading the chapter on Nonlinear models — 10 Oct 2019

(Chapter 11)

Let the nonlinear univariate model be of the form

$$y_t = F(y_{t+1})^e + v_t \qquad (11.2)$$

nonlinear $\quad$ shock $v_t \sim iid(0, \cdot)$ if $v_t = 0$

$$= E_t\left[F(y_{t+1})\right] \qquad (11.2) = (11.1)$$

A more general nonlinear model is

$$y_t = H\left(G(y_{t+1}, v_{t+1})^e, v_t\right) \qquad (11.3)$$

[Here's an argument (p. 273) for linearity:

1. Supp agents don't know $G(y_{t+1}, v_{t+1})^e$, but have data on its past values $G(y_j, v_j)$ $j = 1, \dots, t$.
2. A natural estimator for $G(y_{t+1}, v_{t+1})^e$ is the sample mean: $\theta_t = \frac{1}{t}\sum_{j=1}^{t} G(y_j, v_j)$ which is then updated using RLS as more data becomes available.
3. The sample mean is a linear operator.

[Peter's argument for differentiating them:

- E is an integral over states
- Differentiation proceeds over different variables, not states.

By the way, Evans & Honk derive them $\hat{E}$ all over the place here w/o remarks

- That's all Evans & Honk had to say about $\hat{E}$.
- Liam Graham also simply differentiates thru $\tilde{E}$ (Mac p 5)

Moving on: <u>small π, large x</u>

- well, setting $\alpha = 1$ and $\kappa = 1$ still <u>doesn't</u> get π to move as much as x (and thus i). So what's going on?
- now what I did is on top of ($\alpha = 1, \kappa = 1$), I shut off all shocks except u_t, so now only π is affected. Now the magnitude of the responses is the same (almost)
  ↳ But even then, even then x & i fluctuate more!
- shocks shut off except u, α back to 0.5
  $\Rightarrow$ same as w/ $\alpha = 1$ $\Rightarrow$ α doesn't matter
- –11–, κ back to 0.51.
  $\rightarrow$ κ matters a <u>lot</u>: this really brings in the gap between π and x.

Let's note a couple of things

1. You'd think that the higher the shock volatilities, the more unanchoring happens. But that is only partly so:
   - If I shut off $\bar{\imath}$-shocks, c.p., I'm always anchored
   - If I shut off the other shocks, c.p., I'm unanchored

→ What's going on?

3. What matters for the size of gaps in π & x is
   - κ → but this doesn't explain it all
     ↳ Why?
   - size of shocks DOESN'T really matter, α doesn't matter

→ Something else going on?

4. CEMP say about the criterion θ_t that it's a reduced-form way to capture model misspecification tests
   ↳ they simulate a calibrated version where firms instead employ a CUSUM-test of shifting means (Brown, Durbin, Evans 1975) (see CEMP p. 18)
   ↳ get nearly identical results but w/ more parameters to estimate.

Analysis of the issues:

1) κ = slope of NKPC $\quad \pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t$

- If $\kappa = 0$, no rel. btwn π & x $\Rightarrow$ money is strongly non-neutral, x persistently $\neq 0$.
- If $\kappa \to \infty$, then flex prices: changes in x translate immediately and a <u>lot</u> into π $\Rightarrow$ money is neutral

Note that $\frac{\partial \kappa}{\partial \alpha} < 0$ $\Rightarrow$ if firms are stuck w/ a price for longer ($\alpha \uparrow$), then $\kappa \downarrow$ and money becomess less neutral!

- So if $\alpha \uparrow$ (from 0.5 to 1) $\Rightarrow$ we get anchoring where we previously didn't
- If $\alpha \downarrow$ (from 0.5 to 0.1) $\Rightarrow$ we get deanchoring and the gaps in π are indeed huger

$\Rightarrow$ the lower α (higher κ), the more deanchoring we get and the bigger the gaps in π are relatively to x

$\Rightarrow$ I think what is happening is that when α high (κ low), we have a lot of price stickiness ⇒ so inflation cannot respond a lot to shocks, so the margin of adjustment is x

↳ we need sufficiently flex. prices for inflation to move away from its RE path.

⇒ cool, now we understand that fairly well SOLVED

⇒ a RESULT:

the more price stickiness in a learning world w/ anchoring, the more deanchored periods will show up in output gaps instead of inflation b/c x will be the margin of adjustment!

Let's do a survey of lit on learning w/ price stickiness to see if we have anything similar

- CEMP: can't adress b/c no x in model
- Graham: no inflation in model
- Preston: doesn't do simulations

- Eusepi et al, Limits: doesn't get it
- Ferrero (2007): has π & x, but doesn't seem to discuss the relative role of these.
- Orphanides & Williams (2004) is a little difficult to compare b/c the model isn't an NK model

→ there doesn't seem to be a clear mapping btwn a parameter and price stickiness (maybe α?)

They do have a related result: when inflation inertia is low (→ flex prices) then RLS w/ a constant gain produces very bad fcsts ⇒ i.e. π moves far from π^{RE}.

⇒ it might be useful for me to

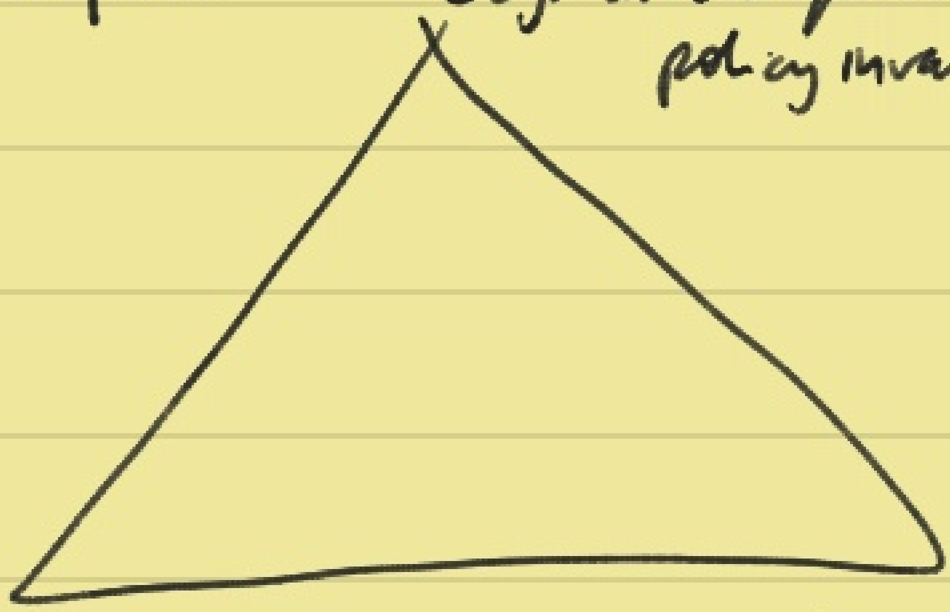
1) write out the ALM, in particular of $\bar{\pi}$ and π
2) look into the Lit on π-persistence (CEMP allude to it) b/c if the conclusion is that there's a drift in π, well, boom! there's our $\bar{\pi}$.

- Compare my IRFs w/ Orph & Will p. 19 (Mac)

Angeletos 10 Oct 2019

Eusepi Preston

His main point: Dogmatic agents: econ & beliefs are policy invariant



adaptive learning

bayesian learning

1) Why isn't dogmatic beliefs enough? (i.e. why learning?)

→ b/c w/ dogmatic E, E are policy invariant!

Mon. pol. might depart from TR to correct E, but can't influence E.

2) why adaptive and not Bayesian learning?

It might not matter?

And lastly: what evidence is there in the data for learning?

Send him paper when you have it!

Angeletos Talk

it is the same as NBER SI

- Angeletos & Lian 2018, AER

does "Preston": NK model <u>w/o common knowledge</u> ← key

gives rise to LH probs (for IS & NKPC!)

what mon pol does here is to anchor E.

"Anchored Beliefs" = lack of common knowledge

when policy says X,

$E_i[X] = X$ but $E_i[\bar{E}[X]] = (1-\lambda)0 + \lambda E_i[X]$

↑ prior

i.e. I update my beliefs but I wrongly believe that $(1-\lambda)$ fraction are "anchored to the prior", zero.

Estimate λ by taking the diff btwn IRFs of beliefs of C and IRFs of C (smart!)

→ Coibion & Goro do this, get $\lambda \sim 0.5$

(but not for ZLB)

Back to my work

- Eusepi & Preston (2018)'s Fig 4 (p. 23, Mac)
  shows volatilities of π & x $\quad \sigma_\pi^2 + \lambda \cdot \sigma_x^2$
  as a function of λ in learning vs. RE.
  1.) volatility is higher in learning
  2.) as $\lambda \uparrow$, volatility $\uparrow$ i.e. $\sigma_x^2 > \sigma_\pi^2$ (like me!)
- Noah Williams (2003)'s Fig 2.
  under a constant gain, π responds less than x
  (to a shock to r^n, NK model)
  $\to$ more support for my results!

Now we need to tackle the issue of IRFs <u>11 Oct 2019</u>
<u>in the learning world</u>

Recall that in the linear RE, the impulse response is:

$IRF = hx^j \cdot \eta \cdot \delta \qquad [\text{where } x_t = hx \cdot x_{t-1} + \eta \epsilon_t]$

But this is a special case of the generalized IR, the GIR (Lect 3 (and 4))

$$GIR(j, \delta, \tilde{X}_{t-1}, \tilde{\varepsilon}_t) = E_t[X_{t+j} | \tilde{X}_{t-1}, \tilde{\varepsilon}_t + \underline{\delta}] - E_t[X_{t+j} | \tilde{X}_{t-1}, \tilde{\varepsilon}_t]$$

j: periods; δ: shock; $\tilde{X}_{t-1}$: econ up to now; $\tilde{\varepsilon}_t$: other shocks

In the linear world, $E_t[X_{t+j} | \tilde{X}_{t-1}, \tilde{\varepsilon}_t] = \sum_{\tau=j}^{\infty} hx^{\tau} \eta \tilde{\varepsilon}_{t+j-\tau}$ (Wold-representation), and thus

$E_t[X_{t+j} | \tilde{X}_{t-1}, \tilde{\varepsilon}_t + \delta] = \sum_{\tau=j}^{\infty} hx^{\tau} \eta \tilde{\varepsilon}_{t+j-\tau} + hx^{j} \eta \delta$, so

$GIR^{linear} = hx^{j} \eta \delta$.

But in the nonlinear world, the Wold representation isn't valid b/c you can't neatly separate shocks.

→ a solution is to integrate out shocks: Ryan's IR:

RIR = E[GIR] → don't need to know <u>where</u> in the state-space the economy is.

- simulate a bunch of histories $\tilde{X}_{t-1}$, calc GIR at all and take an average

Initially I thought that the learning IRFs will be a similar issue, but I don't think so anymore b/c the world is linear (shocks are), but the problem is that gx isn't.

One part that will be constant is the effect on exog states: $hx^j \eta \delta$ will be the same as for RE. But every time I calculate the effect on the jumps, $gx\, hx^j \eta \delta$, gx will be different.

But what is a "GIR"-issue is that the econ's position in the state-space matters b/c the stance of convergence of learning to RE means that reaction to shocks will be diff. So maybe I do have to do the RIR-thing!

One option: supp. $T=40$. For $t=1,\ldots,T$, take a "GIR" for every t and then calc the average (kind of like (I)

→ in fact, I think that might work ...

⇒ In particular:

for t = 1 ... T_h (so that IRFs are the same length, h)

[sim_x, sim_y] = sim_learn (same, t)

add shock to innov in that period

for the <u>same</u> shock sequence otherwise!

and then take

sim_x - x_LR_anchor and sim_y - y_LR_anchor

⇒ let these diffs be ir_h_t (h-period ahead IRFs started at time t), or GIR

end

now we have ir_h_t (GIR) which is h × 3 (ny) × T.

1 approach: take average over the 3rd dim, T.

2nd approach: sort and take median

- Have done IRFs 15 Oct 2019
- and cusum-based alternative criterion for

Now I want to write out the ALM and the criterion at least in Mathematica to investigate the processes for π and $\bar{\pi}$.

→ materials5.nb See ALM & criterion in materials3, eq (41) & (43).

Mathematica, materials5.nb shows that you can write the LOM of π as:

$$\pi_t = a_1 \cdot \bar{\pi}_{t-1} + a_2 r_t^n + a_3 \cdot \bar{i}_t + a_4 \cdot u_t$$

where a_i, $i = 1, \dots, 4$ are scalars.

At the same time, the PLM is $\hat{E}_{t-1} z_t = \bar{z}_{t-2} + b s_{t-1}$

which for π reads $\hat{E}_{t-1} \pi_t = \bar{\pi}_{t-2} + b_{11} r_{t-1}^n + b_{12} \bar{i}_{t-1} + b_{13} u_{t-1}$

Then the criterion is $\bar{\pi}_{t-2}$ check timing! → for Ryan's PLM

$$\left| \hat{E}_{t-1} \pi_t - E_{t-1} \pi_t \right| = \left| \bar{\pi}_{t-1} + b_{11} r_{t-1}^n + b_{12} \bar{i}_{t-1} + b_{13} u_{t-1} \right.$$

$$\left. - a_1 \bar{\pi}_{t-1} - a_2 \rho_r r_{t-1}^n - a_3 \rho_i \bar{i}_1 - a_4 \rho_u u_{t-1} \right|$$ ✓ $\bar{\pi}_{t-2}$

↳ as usual I'm sidestepping the timing issues for now.

The point is that for LOM(π) of

$$\pi_t = a_1 \cdot \bar{\pi}_{t-1} + a_2 r_t^n + a_3 \cdot \bar{i}_t + a_4 \cdot u_t$$

check timing!

$\bar{\pi}_{t-2}$

the "scalar criterion" θ_t is

$$\theta_t(\gamma_r + \gamma_i + \gamma_u) = \left| (1-a_1)\bar{\pi}_{t-1} + (b_{11} - a_2 \rho_r) r_{t-1}^n + (b_{12} - a_3 \rho_i)\bar{i}_{t-1} + (b_{13} - a_4 \rho_u) u_{t-1} \right|$$

$\bar{\pi}_{t-2}$

which we can summarize as

check timing

For Ryan's PLM it's $\bar{\pi}_{t-2}$, for Honky's it's $\bar{\pi}_{t-1}$.

$\bar{\pi}_{t-2}$

$$\pi_t = a_1 \bar{\pi}_{t-1} + [a_2 \; a_3 \; a_4] \, s_t$$

$$\theta_t(\gamma_r + \gamma_i + \gamma_u) = \left| (1-a_1)\bar{\pi}_{t-1} + [b_{11} - a_2\rho_r, \; b_{12} - a_3\rho_i, \; b_{13} - a_4\rho_u] \, s_t \right|$$

$\bar{\pi}_{t-2}$

while we still have, as in CEMP, that

$\left[\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \cdot f_{t-1} \right.$ but the FE f_{t-1} is diff,

in particular it is ALM − PLM, $\pi_{t-1} - \hat{E}_{t-2}\pi_{t-1}$

$$f_t = (a_1 - 1)\bar{\pi}_{t-1} + (a_2 - b_{11}) r_t^n + (a_3 - b_{12})\bar{i}_t + (a_4 - b_{13}) u_t$$

where I've again been sloppy w/ the timing.

$$f_t = (a_1 - 1)\bar{\pi}_{t-1} + [a_2 - b_{11}, \; a_3 - b_{12}, \; a_4 - b_{13}] \, s_t$$

$\Rightarrow$ So supposing the timing is right, I'd get

$$\bar{\pi}_t = \left(1 + k_t^{-1}(a_1 - 1)\right)\bar{\pi}_{t-1} + k_t^{-1} [a_2 - b_{11}, \; a_3 - b_{12}, \; a_4 - b_{13}] \, s_{t-1}$$

check timing!

The only element that is def. true regardless of timing is $\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} f_{t-1}$

$$= \bar{\pi}_{t-2} + k_{t-1}^{-1} f_{t-2} + k_t^{-1} f_{t-1}$$

$$= \bar{\pi}_0 + \sum_{\tau=0}^{t-1} k_{\tau+1}^{-1} f_\tau$$

Obs. 1. The estimate of LR π-mean will be $\neq$ RE (0) when there have been
- a bunch of FE w/ the same sign
- some very large FE.

The difference to CEMP is that there, it is only $\bar{\pi}$ that causes a deviation between PLM-EALM (i.e. in $\hat{E}_{t-1}\pi_t - E_{t-1}\pi_t$) whereas in my case there's an additional wedge that comes from the fact that the RE coefficient b isn't correct as long as we haven't converged to RE!

Look at timing one more time:

1.) $f_{t-1} = \pi_t - E_{t-1}\pi_t$ (index on FE refers to the time the fcst was made)

This is NOT what CEMP have. For them,

$f_{t-1} = \pi_{t-1} - \hat{E}_{t-2}\pi_{t-1}$ (index refers to what you're fcsting)

and $\theta_t = |\hat{E}_{t-1}\hat{\pi}_t - E_{t-1}\pi_t|$

$= |E_{t-1} f_t|$

The criterion at t is the date $t-1$ expectation of time t FEs.

I'm really starting to think that Ryan's code isn't a 100% consistent. But so isn't CEMP either!

Let's go thru timing in words.

In $t=2$: have $\bar{z}_1, s_2, k_1, f_1^{CEMP} = z_1 - \underbrace{E_0 z_1}_{\bar{z}_{-1} + b s_0}$

Form $E_2 z_3 = \bar{z}_1 + b s_2$ } z_2 realized. $\underbrace{\bar{z}_{-1} + b s_0}$

CEMP – that cannot be right!

Form $\theta_2 = \hat{E}_1 z_2 - E_1 z_2$ ↓ f_2^{CEMP} realized

Form $k_2 = f(\theta_2)$

Update $\bar{z}_2 = \bar{z}_1 + k_2^{-1}(f_1)$

Repeat

In $t=2$: have $\bar{z}_1$, S_2, k_1, $f_1^{CEMP} = z_1 - \underbrace{E_0 z_1}$

Form $E_2 z_3 = \bar{z}_1 + bS_2$ } z_2 realized $\bar{z}_{-1} + bS_0$

Form $\theta_2 = \hat{E}_1 z_2 - E_1 z_2$ $\Downarrow$ f_2^{CEMP} realized $= f_1^{my}$ realized

Form $k_2 = f(\theta_2)$

Update $\bar{z}_2 = \bar{z}_1 + k_2^{-1}(f_1^{my})$

Issues:

① timing of fcst $\rightarrow$ CEMP cannot be right, so let me use my notation $f_1^{my} = z_2 - E_1 z_2$ (= Ryan's)

$$f_1^{my} = z_2 - (\bar{z}_0 + bS_1)$$

"The fcst made at period 1 of z_2"

$f_2^{assess} = z_2 - (\bar{z}_1 + bS_1)$ "If I'd have to redo yesterday's expectation of z_2 given having updated $\bar{z}$, I'd do this"

② This brings us to our 2nd issue: timing of the PLM

I need to suffer w/ the assessment fcst b/c I'm using Ryan's PLM: $E_t z_{t+1} = \bar{z}_{t-1} + bS_t$

But if I instead ass that ppl update $\bar{z}$ at the beginning of the period, Then: at $t=2$, have $\bar{z}_2, s_2, k_1, f_0^{my}$

Form $E_2 z_3 = \bar{z}_2 + b s_2$ } z_2 realized, $f_1^{my} = z_2 - E_1 z_2$ realized

Form $\theta_2 = \hat{E}_1 \theta_2 - E_1 \theta_2$

Form $k_2 = f(\theta_2)$

Update $\bar{z}_3 = \bar{z}_2 + k_2^{-1}(f_1^{my})$

$\hookrightarrow = z_2 - (\bar{z}_1 + b s_1)$

Let's write the two approaches for time t next to each other

Ryan:	Honky
At t, have: $\bar{z}_{t-1}, s_t, k_{t-1}, f_{t-2}^{my}$	$\bar{z}_t, s_t, k_{t-1}, f_{t-2}^{my}$
Form: $E_t z_{t+1} = \bar{z}_{t-1} + b s_t \Rightarrow z_t$	$E_t z_{t+1} = \bar{z}_t + b s_t \Rightarrow z_t$
Form $\theta_t = \hat{E}_{t-1} z_t - E_{t-1} z_t$	
Form $k_t = f(\theta_t)$	
Update $\bar{z}_t = \bar{z}_{t-1} + k_t^{-1}(f_{t-1}^{my})$	$\bar{z}_{t+1} = \bar{z}_t + k_t^{-1}(f_{t-1}^{my})$
$z_t - (\bar{z}_{t-2} + b s_{t-1})$	$z_t - (\bar{z}_{t-1} + b s_{t-1})$

$\hookrightarrow$ clearly the two formulations are equivalent, except that for Ryan's code to work out, you need to use an assessment fcst $f_{t-1,eve}^{assess}$ for updating $\bar{z}$ instead of f_{t-1}^{my}.

$= f_{t,morning}^{assess}$ ($\Leftarrow$ before s_t realizes) $\Rightarrow$ Does the assessment fcst matter?

Let's see if the PLM matters for the ALM.

Ryan: $E_t z_{t+1} = \bar{z}_{t-1} + b s_t$

$\uparrow$ available at t

$f_{a,b} = \text{stuff}\, \bar{z}_{t-1} + \text{stuff} \cdot s_t$

$z_t = \text{stuff}\, \bar{z}_{t-1} + \text{stuff} \cdot s_t$

$\underline{\underline{E_{t-1} z_t = \text{stuff}\, \bar{z}_{t-2}}} + \text{stuff} \cdot \rho s_{t-1}$

hmmm...

$\hat{E}_{t-1} z_t = \underline{\underline{\bar{z}_{t-2}}} + b s_{t-1}$

$E_t z_{t+1} = \bar{z}_t + b s_t$ Hanky

$\uparrow$ available at t

$f_{a,b} = \text{stuff}\, \bar{z}_t + \text{stuff} \cdot s_t$

$z_t = \text{stuff}\, \bar{z}_t + \text{stuff} \cdot s_t$

$E_{t-1} z_t = \text{stuff}\, \underline{\underline{\bar{z}_{t-1}}} + \text{stuff}\, \rho s_{t-1}$

$\hat{E}_{t-1} z_t = \underline{\underline{\bar{z}_{t-1}}} + b s_{t-1}$

Aaah... either way you get θ_t being a function of the $\bar{z}$ at the <u>same time</u>: the point is it's the one that was available <u>last period</u> (whether you denote that $\bar{z}_{t-1}$ or $\bar{z}_{t-2}$ doesn't matter!)

$\theta_t = \theta_t(\bar{z}_{t-2}, s_{t-1})$

$f_t = \text{ALM} - \text{PLM}$

$= z_{t+1} - E_t z_{t+1}$

$= \text{stuff} \cdot \bar{z}_{t-1} + \text{stuff} \cdot s_t$

$- (\bar{z}_{t-1} + b s_t)$

$\theta_t = \theta_t(\bar{z}_{t-1}, s_{t-1})$

$f_t = z_{t+1} - E_t z_{t+1}$

$= \text{stuff}\, \bar{z}_t + \text{stuff} \cdot s_t$

$- (\bar{z}_t + \text{stuff}\, s_t)$

$\rightarrow$ these work out as well! $\Rightarrow$ <u>but</u>:

in both cases if I have to update using f_{t-1}, then

$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1}\left(z_t - (\bar{z}_{t-2} + bS_{t-1})\right)$ | $\bar{z}_{t+1} = \bar{z}_t + k_t^{-1}\left(z_t - (\bar{z}_{t-1} + bS_{t-1})\right)$

not the one I used at the beginning of this period

$\Rightarrow$ so I have the assessment fcst trouble no matter what!

i.e. both for Ryan's & Hanley's PLM

So is the assessment fcst the sol. to all my troubles?

It's a new day and today I think — 16 Oct 2019

the assessment fcst is fine: it's yesterday evening's forecast so it does use the most recent info available yesterday to fcst today's state. (See materials5c.tex)

$\Rightarrow$ so I'm ok w/ using the assessment fcst to update $\bar{\pi}$ (or $\bar{z}$) (Issue #1 is "solved").

But issue #2 is a bigger problem.

$\rightarrow$ tomorrow: let's tackle that and see if Comp's timing can be written like mine!

Before I do that, just one note: <u>17 Oct 2019</u>

$\theta_t = \hat{E}_{t-1}(z_t) - E_{t-1}(z_t)$

We can interpret both expectations as $t-1$ evening!

Then $\theta_t = \bar{z}_{t-1} + bs_{t-1} - (\text{stuff } \bar{z}_{t-1} + \text{stuff} \cdot s_{t-1})$

$\quad = \chi(\bar{z}_{t-1}, s_{t-1})$

My feeling is that the CEMP formulation (when agents at time $t-1$ fcst time t stuff) works out only if

$\theta_t = |\hat{E}_{t-2}\pi_{t-1} - E_{t-2}\pi_{t-1}|$

$\quad = |\gamma\pi_{t-2} + (1-\gamma)\bar{\pi}_{t-1} + \text{stuff} - (\gamma\pi_{t-2} + (1-\gamma)\rho\bar{\pi}_{t-1} + \text{stuff})$

$\quad = |\text{stuff} \cdot \bar{\pi}_{t-1}|$

that seems to work. But let's try to change the notation so that $\bar{\pi}$ makes more sense in light of standard learning things.

Suppose: PLM: $\pi_t = \bar{\pi}_{t-2} + \rho y_{t-1}$ (ignoring indexing)

This maps perfectly to my PLM.

Then: at time $t-1$

$$\pi_t = \bar{\pi}_{t-2} + \rho y_{t-1}$$] Morning. π_{t-1} realized.

$\rightarrow \bar{\pi}_{t-2}$ was formed in $t-2$ evening.

Now we wanna update to $\bar{\pi}_{t-1}$

$$\bar{\pi}_{t-1} = \bar{\pi}_{t-2} + k_{t-1}^{-1} \underbrace{(\pi_{t-2} - \hat{E}_{t-3}\pi_{t-2})}$$

this is my FE_{t-2}, should be FE_{t-1}

$\pi_{t-1} - \hat{E}_{t-2}\pi_{t-1}$ is what I'd have.

$\rightarrow$ even my FE_{t-1} only works out if it's the evening:

$$FE^e_{t-2} = \pi_{t-1} - (\bar{\pi}_{t-2} + \rho y_{t-2}).$$

$$k_{t-1} = f(\theta_{t-1}, k_{t-2})$$

$$\theta_{t-1} = |\hat{E}_{t-2}\pi_{t-1} - E_{t-2}\pi_{t-1}|$$

Ok, if I use $|\hat{E}^e_{t-2}\pi_{t-1} - E^e_{t-2}\pi_{t-1}|$ I get

$$\bar{\pi}_{t-2} + \rho y_{t-2} - (\text{stuff}\ \bar{\pi}_{t-2} + \text{stuff}\cdot y_{t-2})$$ ✓

- Ok, so changing the notation for $\bar{\pi}_t$ to $\bar{\pi}_{t-2}$
- imposing evening fcsts in $\bar{\pi}$-update and criterion θ
- changing the timing of CEMP's fcst in $\bar{\pi}$-update

$\rightarrow$ I get sthg that 1) maps to mine 2) makes sense 3) works out.

Honestly, I think I won't even ask Ryan about this b/c the important thing is that I know what I'm assuming. I'll still send him materials5c, but I don't think I'll talk about it.

<u>Ryan meeting</u> <u>17 Oct 2019</u>

- IRFs: present Susanto ones conditional on being anchored or not before the shock hits
- kill γ_x to clarify storytelling
- after: add persistence term to TR → kills overshooting?

  ρi_{t-1}
- For Susanto:
  - only show mean lines (not all – at most, confidence bands)
  - have IRFs conditional on being anchored / unanchored before shock
  - understand why & how the overshooting happen
- as for θ_t: keep both specifications, CEMP's θ_t and the CUSUM one – right now they're tied b/c the CUSUM is closer to

what firms do and isn't technically harder to implement (as long as we're not estimating — which is not clear to him that that's where we're headed ... mmm...)

⇒ trade whether they lead to different results

· as for $\hat{E}$:

Keep this question at the back of your mind and when you get to talk to some of these people, ask them!

→ then you can say, in a job talk or when you're asked this question, gingerly, that "experts in the field told me that we don't worry about it." Then you've shown that you've at least thought about it.

Work after materials6 18 Oct 2019

Adding ρi_{t-1} to TR isn't that simple b/c it changes a bunch of things in the LR models.

i_{t-1} is a new state variable, so should it show up in s_t? $s_t = \begin{bmatrix} r_t^n \\ i_t \\ u_t \\ i_{t-1} \end{bmatrix}$? → that's what I did for the RE world.

In materials6, eqs (1)-(3) are Preston's (18), (19) & (27), NKIS, NKPC & TR. Now that I modified the TR, to get the compact notation I need to redo the steps w/ the new Taylor-rule.

$$x_t = -\sigma(\psi_\pi \pi_t + \psi_x x_t + \rho i_{t-1} + \bar{i}_t) + \Sigma(\cdot)$$

$$(1+\sigma\psi_x)x_t = -\sigma\psi_\pi \pi_t - \sigma\rho i_{t-1} - \sigma\bar{i}_t$$

$$+\sum_{T=t}^{\infty} \beta^{T-t}\Big((1-\beta)x_{T+1} + \sigma\pi_{T+1} + \sigma r_T^n - \sigma\beta\big(\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \rho i_T + \bar{i}_{T+1}\big)\Big)$$

$$(1+\sigma\psi_x)x_t = -\sigma\psi_\pi \pi_t - \sigma\rho i_{t-1} - \sigma\bar{i}_t$$

$$+\sum_{T=t}^{\infty} \beta^{T-t}\Big((1-\beta-\sigma\beta\psi_x)x_{T+1} + (\sigma-\sigma\beta\psi_\pi)\pi_{T+1} + \sigma r_T^n - \sigma\beta\bar{i}_{T+1} - \sigma\beta\rho i_T\Big)$$

$$(1+b\psi_x)x_t = -b\psi_\pi \pi_t - b\rho\, i_{t-1} - b\bar{i}_t$$
$$+\sum_{T=t}^{\infty}\beta^{T-t}\Big((1-\beta-b\beta\psi_x)x_{T+1} + (b-b\beta\psi_\pi)\pi_{T+1} + b r_T^N - b\beta\bar{i}_{T+1} - b\beta\rho\, i_T\Big)$$
$$\Leftrightarrow (1+b\psi_x)x_t = -b\psi_\pi \pi_t$$
$$+\sum_{T=t}^{\infty}\beta^{T-t}\Big((1-\beta-b\beta\psi_x)x_{T+1} + (b-b\beta\psi_\pi)\pi_{T+1} + b r_T^N - b\bar{i}_T - b\rho\, i_{T-1}\Big)$$

The i_{t-1} terms

$$-b\rho\, i_{t-1} - b\rho\beta\, i_t - b\rho\beta^2 i_{t+1} + \dots$$
$$-b\rho\left[i_{t-1} + \beta i_t + \beta^2 i_{t+1} + \dots\right]$$
$$= -b\rho \sum_{T=t}^{\infty}\beta^{T-t} i_{T-1}$$

ok! Now plug π_t

$$\Rightarrow (1+b\psi_x)x_t = -b\psi_\pi\left[\kappa x_t + \sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\Big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\Big)\right]$$
$$+\sum_{T=t}^{\infty}\beta^{T-t}\Big((1-\beta-b\beta\psi_x)x_{T+1} + (b-b\beta\psi_\pi)\pi_{T+1} + b r_T^N - b\bar{i}_T - b\rho\, i_{T-1}\Big)$$
$$(\underbrace{1+b\psi_x-\kappa b\psi_\pi}_{=w})x_t = \text{stuff}$$
$$x_t = \frac{-b\psi_\pi}{w}\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\Big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\Big)$$
$$+\frac{1}{w}\sum_{T=t}^{\infty}\beta^{T-t}\Big((1-\beta-b\beta\psi_x)x_{T+1} + (b-b\beta\psi_\pi)\pi_{T+1} + b r_T^N - b\bar{i}_T - b\rho\, i_{T-1}\Big)$$

$$\pi_t = \left(1 - \frac{\kappa b \psi_\pi}{w}\right) \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\right)$$
$$+ \frac{\kappa}{w} \sum_{T=t}^{\infty} \beta^{T-t} \left((1 - \beta - b\beta\psi_x) x_{T+1} + (b - b\beta\psi_\pi)\pi_{T+1} + b r_T^n - b\bar{i}_T - b\rho\, i_{T-1} \right)$$

Ah yeah, so all I need to do is to expand the s-vector

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = f(f_a, f_b) + \underbrace{f_f \text{ which is different}}_{\text{but only to the extent that it has}}$$

a 4^{th} row for i_{T-1}, and A_s is different

$$f_f = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} s_T = (I_4 - \beta P)^{-1} s_t \quad w/\ s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{bmatrix}$$

does P include ρ? I think so!

$$P = \begin{bmatrix} \rho_r & & & 0 \\ & \rho_i & & \\ & & \rho_u & \\ 0 & & & \rho \end{bmatrix}$$

And A_s^{LR} looks as follows:

for $x_t \Rightarrow \frac{-b\psi_\pi}{w}[1 - \alpha\beta P]^{-1}[0\ 0\ 1\ 0]\, s_t + \frac{b}{w}[1-\beta P]^{-1}[1\ {-1}\ 0\ {-\rho}]\, s_t$

for $\pi_t \Rightarrow \left(1 - \frac{\kappa b\psi_\pi}{w}\right)[1 - \alpha\beta P]^{-1}[0\ 0\ 1\ 0]\, s_t + \frac{\kappa b}{w}[1-\beta P]^{-1}[1\ {-1}\ 0\ {-\rho}]\, s_t$

and for $i_t \Rightarrow \underbrace{\psi_x \text{ stuff } x + \psi_\pi \text{ stuff } \pi}_{-11-} + \rho\, i_{t-1} + \bar{i}_t$

$$+ [0\ 1\ 0\ \rho]\, s_t$$

Yeah so I only need to adjust A_S^{LR} & A_S^{RE}. 19 Oct 2019

It's now 3×4 instead of 3×3.

I can summarize the previous as: (new in blue)

$$g_{xs} = \left(\frac{-\partial \psi_\pi}{w} [1-\alpha\beta P]^{-1} [0\ 0\ 1\ 0] \quad - \frac{\partial}{w} [1-\beta P]^{-1} [-1\ 1\ 0\ \rho] \right)$$

$$g_{\pi s} = \left(\left(1 + -\frac{\kappa \partial \psi_\pi}{w}\right) [1-\alpha\beta P]^{-1} [0\ 0\ 1\ 0] \quad - \frac{\kappa \partial}{w} [1-\beta P]^{-1} [-1\ 1\ 0\ \rho] \right)$$

and for $i_t \Rightarrow \psi_x g_{xs} + \psi_\pi g_{\pi s} + [0\ 1\ 0\ \rho]$

Of course it doesn't work, even if I set $\rho = 0$ I don't get what I had before, not even for the RE model.

So:

$$x_t = hx \cdot x_{t-1} + \eta \cdot \varepsilon_t$$

$$y_t = gx \cdot x_t$$

is this right? I don't think so...

$$\begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{bmatrix} = \begin{bmatrix} \rho_r & & & 0 \\ & \rho_i & & \\ & & \rho_u & \\ 0 & & & \rho \end{bmatrix} \begin{bmatrix} r_{t-1}^n \\ \bar{i}_{t-1} \\ u_{t-1} \\ i_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_r & & & 0 \\ & \sigma_i & & \\ & & \sigma_u & \\ 0 & & & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_r \\ \varepsilon_i \\ \varepsilon_u \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \underbrace{\begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}}_{3\times 4} \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{bmatrix}$$

} this stuff ain't right!

Everything looks correct. So I'll take a big step back:
go back to materials5.m and

✓ 1) implement the learning & IRFs w/ a single code, making sure you keep obtaining the same thing

2) change RE to account for the smoothing in i_t making sure you obtain the same thing as before

ok, did that, reverted.

→ Let's think this thru one more time:

$$\begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{bmatrix} = \begin{bmatrix} hx & & & \\ & & & \\ & & & \\ i_r & i_i & i_u & \rho \end{bmatrix} \begin{bmatrix} r_{t-1}^n \\ \bar{i}_{t-1} \\ u_{t-1} \\ i_{t-2} \end{bmatrix} + \begin{bmatrix} \eta & & & \\ & & & \\ & & & \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_r \\ \varepsilon_i \\ \varepsilon_u \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} gx & & & \\ & & & \\ i_r & i_i & i_u & \rho \end{bmatrix} \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{bmatrix}$$

→ it makes sense that the last row of hx = that of gx!

↳ and that does look like this is where roads part between P and hx b/c i_{t-1} is an endogenous state!

→ so I don't think I need to redefine P or Σ,

but in fa & fb, agents will use hx instead of P

to test states! Well, Σ maybe...

↳ will need to correct materials6.tex!

↳ still need to think it thru — this ain't done yet!

But maybe the 1st step is to replace P w/ hx in

fafb, fk (with CEMP & CUSUM) and let it adjust

and make sure we still get the same things.

To be sure I'd do new functions for everything!

<u>Analyzing IRFs (all of the above done & works!)</u> <u>20 Oct 2019</u>

1.) CUSUM criterion doesn't respond a whole lot to ρ, to the point of me suspecting that something isn't quite right there.

2) In general, when $\uparrow \rho$, CEMP criterion gives more anchoring

→ makes sense b/c then π isn't moving around as much, as so they aren't making as large errors.

3) Same for $\psi_\pi \uparrow$, except after a while it reverses

Focus for a sec on constant gain learning:

1) overshooting ↑ when ψ_π ↑
2) ρ↑ doesn't really do much to overshooting
3) ρ↑ or ψ_π↑ $\rightarrow$ anchoring $\rightarrow$ AM behaves more like decreasing gain learning

Both cgain & dgain have overshooting if ψ_π high enough but cgain has a more "sign-switching convergence"-behavior in the sense that it's initially further from RE and also moving further away, but then it overtakes dgain learning.

➜ It's as if the overshooting was happening b/c the shock interrupts an ongoing learning process

$\Rightarrow$ that's why there are these reversals in π b/c cgain learners change their mind about where π is headed!

$$b = \underset{ny \cdot nx}{gx} \cdot \underset{nx \cdot nx}{hx} \quad \Big\} \quad \begin{matrix} ny \cdot nx \\ 3 \times 4 \end{matrix}$$

<u>20 Oct 2019</u>

M1 is 3×3 M2 is 3×4 $\Rightarrow$ $\underset{3\times 4}{M2} \cdot \underset{4\times 1}{s} = 3 \times 1$

Still trying to understand IRFs:

- $\rho \uparrow$ just means that RE starts overshooting too

  the learning models also overshoot more, but the change

  is way less pronounced than for RE.

It seems like $\rho > 0$ in RE is a substitute for learning!

RE responses closest to learning when $\rho \approx 0.3$ (save for monpol shocks but I'm ignoring that a bit b/c it's persistent)

But what is quite diff is the int rate, even under $\rho = 0.3$

$\rightarrow$ it's more quick to react

it switches sign

$\rightarrow$ why is i doing this? It's not the case that the impact effect of shocks would be greater under learning, in fact, the impact is always greater in RE on everything

→ in RE, a bigger X is cancelled by a bigger i↑

whereas in learning a smaller X is reversed by a smaller i

So it seems like the same ψ_π has a bigger effect in a learning model, especially if you have constant gain learning → you get overshooting b/c of that and so the CB reverses course to correct it

Why does the same ψ_π have a bigger effect under learning, especially under cgain?

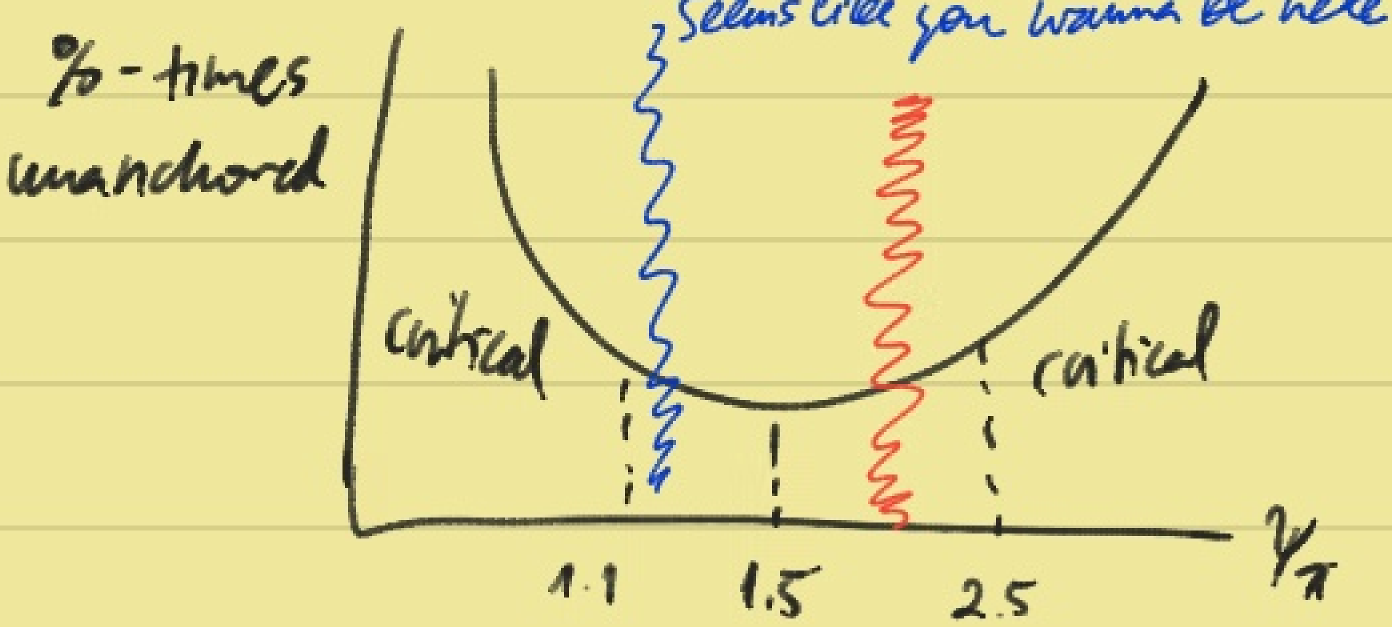
→ b/c agents use it strongly to infer the π^*

⇒ this is the same reason why a too high ψ_π leads to constant gains: it leads to "excess volatility" which suggests to agents that they need to monitor the situ.

What doesn't make sense is that $\psi_\pi = 1.1$ which seems to do the optimal cancelling, gives you cgains!

Maybe here's what's going on:

- impact effects are smaller in learning b/c E are slow-moving (states feed thru the system slower)
- ψ_π has a bigger effect though somehow (...? :S)
  (maybe jumps feed faster thru the system)
- ψ_π has a U-shaped unanchoring curve



$\Rightarrow$ it seems like you're trading off excess volatility and anchoring

Here: you could $\uparrow \psi_\pi$ to get more anchoring, but then you get more overshooting

Here that tradeoff is gone b/c $\psi_\pi \uparrow$ means less anchoring and more overshooting.

$\Rightarrow$ It seems like there's some "interior level" of anchoring that's

optimal in that you both learn fast enough (again)
but it doesn't lead to excess volatility.

But why does learning respond
- slower to states → b/c $E = f(\text{states})$
- faster to jumps? → b/c E doesn't absorb this

Result 1. Under learning, shocks to states propagate slower through the system b/c expectations are functions of states and thus slow-moving.

Cor. 1. Impact effects of structural shocks are smaller than under RE.

Result 2. Under learning, changes to jump variables (such as $\uparrow i_t$ when ψ_π is high) propagate faster through the system b/c expectations take time to respond & absorb them (this is like prices when money isn't neutral).

Cor. 2. the same ψ_π leads to bigger responses (overshooting).

→ Result 1 implies that mon. pol should anchor b/c you can dampen responses.

→ Result 2 implies that mon. pol. shouldn't anchor too much b/c the more anchored, the more overshooting you get. X That's not true.

Monetary policy trades off dampening shocks on impact versus overshooting dynamics.

This tradeoff might disappear if you have sthg that can raise anchoring w/o ψ_π ($\rho\uparrow$ or $\alpha\uparrow$).

Result 3. The $\downarrow\alpha$ ($\alpha\uparrow$), the more is π the margin of adjustment. This is not just b/c in this case, prices are more flexible, but also b/c flex prices lead to more unanchoring as $\bar{\pi}$ moves more.

I'm a little sceptical about the overshooting result's interpretation b/c if it's the case that we're overshooting b/c E(·) is slow to adapt, then shouldn't Dgain have the biggest overshooting? But currently, clearly cgain does! 22 Oct 2019

cgain learning updates E(·) by much more each period
→ this is why especially long after the shock, dgain converges to RE much slower than cgain does.
⇒ so the overshooting has to have sthg to do w/ learning happening
↳ from the perspective of overshooting, you want to be anchored b/c that gets you closer to dgain.
But initially (early after impulse), cgain & dgain are close: so why does cgain lead to more overshooting then? B/c these are avg IRFs, so cgain & dgain aren't actually close!

Ryan

24 Oct 2017

- Even in RE $g_{x_{3,4}} \neq \rho$

  E.g. if we only have i_{t-1} as a state:

  $i_t = \phi \pi_t + \rho i_{t-1}$

  $\pi_t = g i_{t-1}$

  $\Rightarrow i_t = (\phi g + \rho) i_{t-1}$ The jump picks up effects of the state that are a) direct b) indirect.

- Revisit RE effects on impact of nat. rate shock.

- IRF of expectations & FE → plot 'em!

- Check MP shock. fall in i can't happen w/o capital.