Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

Preliminary and Incomplete

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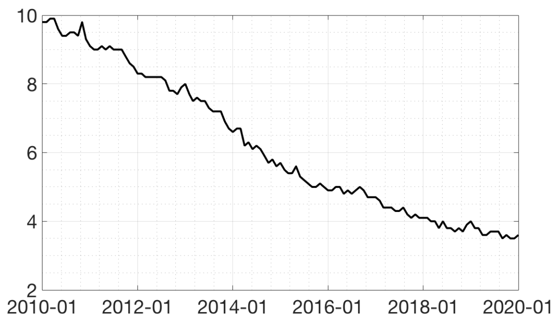
Abstract

This paper analyzes optimal monetary policy in a behavioral model where expectation formation is characterized by potential anchoring of expectations. Expectations are anchored when in an adaptive learning setting, the private sector endogenously chooses a decreasing learning gain. Within the context of an otherwise standard macro model with nominal rigidities and natural-rate and cost-push shocks, I find that the anchoring mechanism augments the intratemporal tradeoff between inflation and the output gap with two layers of intertemporal tradeoffs. Optimal policy is characterized by a targeting rule that spells out these novel tradeoffs conditional on the extent to which expectations are anchored. The anchoring expectation formation alleviates the intratemporal tradeoff because expectations only adjust slowly over time. Moreover, while concerns for stabilization lead the central bank to seek to anchor expectations in the long run, getting them anchored is costlier the more current expectations are unanchored. The monetary authority thus also trades off the short-run costs with the long-run benefits of anchoring expectations.

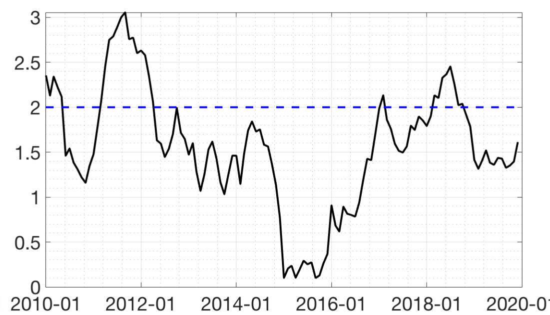
1 Introduction

The Missing Deflation during the Great Recession and the Missing Inflation in the subsequent recovery have puzzled economists in academia and at central banks alike. Figure 1 focuses on the recovery period. As seen from panels (a) and (b), the historically low unemployment level has not resulted in rising inflation, contrary to the implications of state-of-the-art macro models. Instead, personal consumption expenditures (PCE) inflation has persistently undershot the Federal Reserve's 2% target (panel (b)). Moreover, as panel (c) of the figure depicts, the Fed, initially raising interest rates in response to the prolonged expansion, reversed course in fall 2019 and turned expansionary, thus deviating from the policy response implied by a feedback rule à la Taylor (1993).

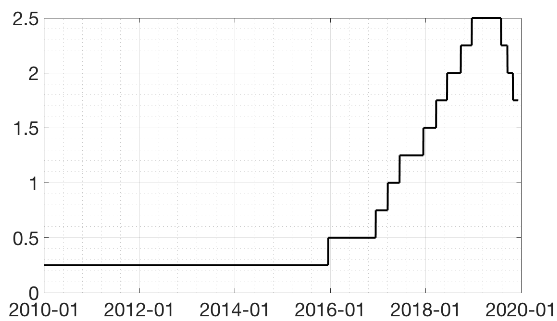
Figure 1



(a) Unemployment rate, %



(b) PCE inflation, % change from 12 months ago



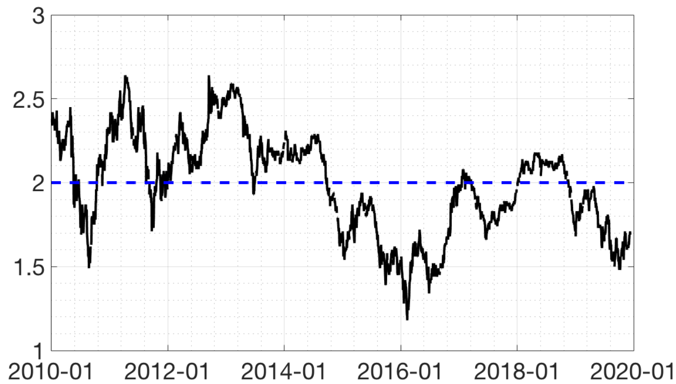
(c) Fed funds rate target, upper limit, %

In this paper I argue that the key to understanding both the puzzling behavior of inflation and the Fed's response is the time series of expected mean inflation.[1] As Fig. 2 shows, long-run inflation expectations of the public, averaging a little above the 2% target prior to 2015, display a marked downward drift since 2015. This indicates that the public has doubts whether the Fed is able to restore inflation to the target. Confronted with what it sees as a changing environment, the public revises its predictions about the future course of the economy.

Modeling the public sector's expectation formation as rational forecloses any role for long-run expectations because it does not allow expectations to fluctuate away from the rational expectations

[1] By "expected mean inflation" I mean the private sector's expectation of where inflation will be on average over a longer period in the future. Therefore I also refer to this as "long-run inflation expectations."

Figure 2



(a) Market-based inflation expectations, 10 year, average, %

equilibrium of the endogenous variables. Macroeconomic theory that seeks to understand this phenomenon thus needs to account for expectation formation that a) departs from rational expectations, b) features a margin along which the public updates its forecasting behavior. I propose a behavioral model in the adaptive learning tradition where the public sector's choice of learning gain is endogenous, as in Carvalho et al. (2019). This captures the idea that in normal times, when firms and households observe economic data that confirms their previous predictions, agents choose a decreasing gain and thus do not change their forecasting rules by much. By contrast, when incoming data suggests that the current forecasting rule is incorrect, agents switch to a constant gain, updating their forecasting rule strongly. I refer to the former case as *anchored* and to the latter as *unanchored expectations*.

The contribution of this paper is to investigate how the anchoring expectation formation affects the optimal conduct of monetary policy. The main result is that optimal monetary policy under anchoring takes the stance of expectations explicitly into account. In particular, optimal policy follows a targeting rule that spells out the configuration of intra- and intertemporal tradeoffs determined by the current and expected future stance of anchoring. It is not unambiguously desirable to anchor expectations. The central bank prefers to have some extent of learning happening on the part of the private sector because this allows the bank to postpone some of its current intratemporal tradeoffs to the future. Unanchored expectations however imply faster learning and thus faster convergence. Moreover, unanchored expectations also induce undesired volatility to the economy. Anchoring expectations is therefore desirable from a long-run volatility standpoint, yet it comes at a short-run cost of heightened volatility and an amplified inflation-output gap stabilization tradeoff. Thus the monetary authority trades off the short-run costs with the long-run benefits of anchoring expectations.

My second result is that, unlike time-zero optimal commitment under rational expectations, optimal policy will be time consistent. But not only that. I also find that optimal policy in the anchoring model is not history-dependent: the commitment and discretion solutions of the Ramsey problem are indistinguishable and resemble discretion under rational expectations. This is due to the fact that backward-looking expectation formation cannot incorporate promises on the part of the policymaker regarding the course of future policy.

A third result characterizes optimal response coefficients to inflation and the output gap when monetary policy is conducted using a Taylor rule. It turns out that monetary policy is optimally less aggressive on inflation than under rational expectations. This is because the relationship between the volatility of observables and the central bank's aggressiveness is non-monotonic. This relates to the intertemporal tradeoff between the cost and benefits of anchoring. By being aggressive on inflation, the central bank introduces negative feedback into a system with positive feedback, and can thus anchor expectations. However, when the interest rate responds strongly to realized inflation, unanchored expectations induce high fluctuations in forecasts and observables, making it costly to choose large coefficients.

Overall, my work uncovers that in the presence of expectations that can become unanchored, the monetary authority needs to monitor the evolution of long-run expectations and adjust its policy instrument in accordance with their stance. This insight invites an interpretation of the Fed's fall 2019 decision to lower interest rates despite a booming economy as an attempt to anchor expectations.

The model I use to study the interaction between monetary policy and anchoring is a behavioral version of standard New Keynesian (NK) model of the type widely used for monetary policy analysis. Monetary policy in the rational expectations version of this model has been studied extensively, for example in Clarida et al. (1999) or Woodford (2011), whose exposition I follow. I rely on Svensson (1999)'s formulation of a target criterion to implement robustly optimal policy.

The behavioral part of the model is the departure from a rational expectation formation on the part of the public sector. Instead, I allow the public sector to form expectations via an adaptive learning scheme, where the learning gain - the parameter governing the extent to which forecasting rules are updated in the direction of the most recent forecast error - is endogenous. The learning framework belongs to the statistical learning literature advocated in the book by Evans and Honkapohja (2001). This literature replaces the rational expectations assumption by postulating an ad-hoc forecasting rule, the perceived law of motion (PLM), as the expectation-formation process. Agents use the PLM to form expectations and update it in every period using recursive estimation techniques.

There is a small number of studies that reevaluate optimal monetary policy from the lens of a learning model. Some examples are Orphanides and Williams (2005), Gaspar et al. (2006), Evans and Honkapohja (2006), Ferrero (2007), Preston (2008), Molnár and Santoro (2014), Eusepi and Preston (2018) and Eusepi et al. (2018). While there is no consensus on how learning affects optimal monetary policy when modeled using a Taylor rule, the conventional wisdom is that the central bank should react more strongly to inflation in order to dampen the positive feedback effects that learning introduces. In the case of the New Keynesian model, for example, Eusepi and Preston (2018) and Molnár and Santoro (2014) conclude that optimal monetary policy is more aggressive on inflation than under rational expectations, yet Eusepi et al. (2018) find the exact opposite. My finding that the relationship between the central bank's aggressiveness and observable volatility is non-monotic reconciles these findings as it emphasizes the temporal dimension of optimal policy. It also coincides with the insight of Lubik and Matthes (2016) that Taylor-rule coefficients are time-varying, but in my model the reason for this is that the intertemporal tradeoff induces monetary policy to be time consistent.

Lastly, I contribute to the tiny literature that writes down the Ramsey problem for adaptive learning models. The main references here are Molnár and Santoro (2014) and Mele et al. (2019); summaries of their results can be found in Eusepi and Preston (2018) and Gaspar et al. (2010). I add to these treatments by considering the effect of an endogenous gain on the optimal Ramsey plan.

To the best of my knowledge, my work is the first to investigate optimal monetary policy in a learning model with an anchoring mechanism. Only few papers study models with an endogenous gain: Marcet and Nicolini (2003), Milani (2014), and Carvalho et al. (2019).[2] Milani (2014) estimates an endogenous gain model developed in Marcet and Nicolini (2003). He shows that endogenous gain formation can account for time-varying volatility in macroeconomic aggregates, even when the model is only subject to exogenous disturbances with constant volatility. Carvalho et al. (2019) are the first to propose an endogenous gain specification as a model of anchored expectations. They interpret anchored expectations as a metric for the public's confidence in the central bank's inflation target. Their estimate the evolution of the endogenous gain reinterprets the Great Inflation and the Great Moderation as a successful anchoring of initially unanchored expectations. I go further to investigate how a concern for the anchoring of expectations affects the conduct of monetary policy.

The paper is structured as follows. Section 2 introduces the model. Section 3 describes the learning framework and spells out the anchoring mechanism. Section 4 presents analytical results in two simplified versions of the model. Section 5 simulates the model to develop intuition about the results. Section 6 concludes.

2 The model

Apart from expectation formation, the model is a standard New Keynesian model with nominal frictions à la Calvo (1983). The advantage of having a standard NK backbone to the model is that one can neatly isolate the way the anchoring mechanism alters the behavior of the model. Since the mechanics of the rational expectations version of this model are well understood, I only lay out the model briefly and pinpoint the places where the assumption of nonrational expectations matters.[3]

2.1 Households

The representative household is infinitely-lived and maximizes expected discounted lifetime utility from consumption net of the disutility of supplying labor hours:

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j))dj \right] \tag{1}$$

$U(\cdot)$ and $v(\cdot)$ denote the utility of consumption and disutility of labor respectively and β is the discount factor of the household. $h_t^i(j)$ denotes the supply of labor hours of household i at time t to the production

[2] My notion of anchoring as an endogenous gain learning model is distinct from the idea of "anchoring and adjustment" advocated originally by Tversky and Kahneman (1974) and taken up in the macro learning context for example by Anufriev and Hommes (2012). Such rules represent a simple expectation-formation heuristic that is a convex combination of a default value (the anchor) and a learned or observed value that the expectation is adjusted toward.

[3] For the specifics of the NK model the reader is referred to Woodford (2011).

of good j and the household participates in the production of all goods j. Similarly, household i's consumption bundle at time t, C_t^i, is a Dixit-Stiglitz composite of all goods in the economy:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{2}$$

$\theta > 1$ is the elasticity of substitution between the varieties of consumption goods. Denoting by $p_t(j)$ the time-t price of good j, the aggregate price level in the economy can then be written as

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{3}$$

The budget constraint of household i is given by

$$B_t^i \leq (1 + i_{t-1}) B_{t-1}^i + \int_0^1 w_t(j) h_t^i(j) + \Pi_t^i(j) dj - T_t - P_t C_t^i \tag{4}$$

where $\Pi_t^i(j)$ denotes profits from firm j remitted to household i, T_t taxes, and B_t^i the riskless bond purchases at time t.[4]

The only difference to the standard New Keynesian model thus far is the expectation operator, $\hat{\mathbb{E}}$. This is the subjective expectation operator that differs from its rational expectations counterpart, $\mathbb{E}$, in that it does not encompass knowledge of the model. In particular, households have no knowledge of the fact that they are identical and by extension they also do not internalize that they hold identical beliefs about the evolution of the economy. As we will see in Section 2.3, this has implications for their forecasting behavior and will result in decision rules that differ from those of the rational expectations version of the model.

2.2 Firms

Firms are monopolistically competitive producers of the differentiated varieties $y_t(j)$. The production technology of firm j is $y_t(j) = A_t f(h_t(j))$, whose inverse, $f^{-1}(\cdot)$, signifies the amount of labor input. Noting that A_t is the level of technology and that $w_t(j)$ is the wage per labor hour, firm j profits at time t can be written as

$$\Pi_t^j = p_t(j) y_t(j) - w_t(j) f^{-1}(y_t(j)/A_t) \tag{5}$$

Firm j's problem then is to set the price of the variety it produces, $p_t(j)$, to maximize the present discounted value of profit streams

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \tag{6}$$

[4] For ease of exposition I have suppressed potential money assets here. This has no bearing on the model implications since it represents the cashless limit of an economy with explicit money balances.

subject to the downward-sloping demand curve

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{7}$$

where

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{8}$$

is the stochastic discount factor from households. Nominal frictions enter the model through the parameter α in Equation (6). This is the Calvo probability that firm j is not able to adjust its price in a given period.

Analogously to households, the setup of the production side of the economy is standard up to the expectation operator. Also here the rational expectations operator $\mathbb{E}$ has been replaced by the subjective expectations operator $\hat{\mathbb{E}}$. This implies that firms, like households, do not know the model equations and fail to internalize that they are identical. Thus their decision rules, just like those of the households, will be distinct from their rational expectations counterparts.

2.3 Aggregate laws of motion

The model solution procedure entails deriving first order conditions, taking a loglinear approximation around the nonstochastic steady state and imposing market clearing conditions to reduce the system to two equations, the New Keynesian Phillips curve (NKPC) and IS curve (NKIS). The presence of subjective expectations, however, implies that firms and households are not aware of the fact that they are identical. Thus, as Preston (2005) points out, imposing market clearing conditions in the expectations of agents is inconsistent with the assumed information structure.[5]

Instead, I prevent firms and households from internalizing market clearing conditions. As Preston (2005) demonstrates, this leads to long-horizon forecasts showing up in firms' and households' first order conditions. As a consequence, instead of the familiar expressions, the NKIS and NKPC take the following form:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big) \tag{9}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \big) \tag{10}$$

Here x_t, π_t and i_t are the log-deviations of the output gap, inflation and the nominal interest rate from their steady state values, and σ is the intertemporal elasticity of substitution. The variables r_t^n and u_t

[5] The target of Preston (2005)'s critique is the Euler-equation approach as exemplified for example by Bullard and Mitra (2002). This approach involves writing down the loglinearized first order conditions of the model, and simply replacing the rational expectations operators with subjective ones. In a separate paper, I demonstrate that the Euler-equation approach is not only inconsistent on conceptual grounds as Preston (2005) shows, but also delivers substantially different quantitative dynamics in a simulated New Keynesian model. Thus relying on the Euler-equation approach when investigating the role of learning is not only incorrect in terms of microfoundations, but also leads to mistaken quantitative inferences. In the context of this model, the problem becomes more acute when expectations are unanchored.

are exogenous disturbances representing a natural rate shock and a cost-push shock respectively.

The laws of motion (9) and (10) are obtained by deriving individual firms' and households' decision rules, which involve long-horizon expectations, and aggregating across the cross-section. Importantly, agents in the economy have no knowledge of these relations since they don't know that they are identical and thus are not able to impose market clearing conditions required to arrive at (9) and (10). Thus, although the evolution of the observables (π, x) is governed by the exogenous state variables (r^n, u) and long-horizon expectations via these two equations, agents in the economy are unaware of this. As I will spell out more formally in Section 3, it is indeed the equilibrium mapping between states and jump variables the agents are attempting to learn.[6]

To simplify notation, I gather the exogenous state variables in the vector s_t and observables in the vector z_t as

$$s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{bmatrix} \qquad z_t = \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} \tag{11}$$

which allows me to denote long-horizon expectations by

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} z_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} z_{T+1} \tag{12}$$

As detailed in App. A, one can use this notation to reformulate the laws of motion of jump variables (Equations (9), (10) and (25)) compactly as

$$z_t = A_a f_{a,t} + A_b f_{b,t} + A_s s_t \tag{13}$$

where the matrices $A_i, \ i = \{a, b, s\}$ gather coefficients and are given in App. A. Assuming that exogenous variables evolve according to independent AR(1) processes, I write the state transition matrix equation as

$$s_t = h s_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \tag{14}$$

where h gathers the autoregressive coefficients ρ_j, ϵ_t the Gaussian innovations ε_t^j, and η the standard deviations σ_t^j, for $j = \{r, i, u\}$. $\Sigma = \eta\eta'$ is the variance-covariance matrix of disturbances.[7]

3 Learning with an anchoring mechanism

The informational assumption of the model is that agents have no knowledge of the equilibrium mapping between states and jumps in the model. Therefore they are not able to form rational expectations forecasts.[8] Instead, agents postulate an ad-hoc forecasting relationship between states and jumps and

[6] The learning of (9) and (10) is complicated by the fact that the current stance of expectations figures into the equations, resulting in the well-known positive feedback effects of learning.

[7] For the sake of conciseness, I have suppressed the expressions for these in the main text. See App. A.

[8] To see this, observe that an agent with rational expectations would internalize the rational expectations state-space system (see (**??**) - (**??**) in App. A) and would therefore forecast future jumps as $\mathbb{E}_t\, z_{t+h} = g^{RE}\, \mathbb{E}_t\, s_{t+h} = g^{RE} h^h s_t$, where

seek to refine it in light of incoming data.

3.1 Perceived law of motion

I assume agents consider a forecasting model for jumps of the form

$$\hat{\mathbb{E}}_t z_{t+1} = a_{t-1} + b_{t-1} s_t \tag{15}$$

where a and b are estimated coefficients of dimensions 3×1 and 3×3 respectively. This perceived law of motion (PLM) reflects the assumption that agents forecast jumps using a linear function of current states and a constant, with last period's estimated coefficients. I also assume that

$$\hat{\mathbb{E}}_t \phi_{t+h} = \phi_t \quad \forall\, h \geq 0 \tag{16}$$

This assumption, known in the learning literature as anticipated utility, means that agents fail to internalize that they will update the forecasting rule in the future.[9] This is the most behavioral element in the expectation-formation process since it postulates that agents think differently about their own behavior than how they actually act. Clearly, this poses a higher level of irrationality than not knowing the model and using statistical techniques to attempt to learn it.

Assuming that agents know the evolution of states, that is they have knowledge of Equation (**??**)[10], the PLM together with anticipated utility implies that h-period ahead forecasts are constructed as

$$\hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b_{t-1} h^{h-1} s_t \quad \forall h \geq 1 \tag{17}$$

Summarizing the estimated coefficients as $\phi_{t-1} \equiv \begin{bmatrix} a_{t-1} & b_{t-1} \end{bmatrix}$, here 3×4, I can rewrite Equation (15) as

$$\hat{\mathbb{E}}_t z_{t+1} = \phi_{t-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix} \tag{18}$$

The timing assumptions of the model are as follows. In the beginning of period t, the current state s_t is realized. Agents then form expectations according to (15) using last period's estimate ϕ_{t-1} and the current state s_t. Given exogenous states and expectations, today's jump vector z_t is realized. This allows agents to evaluate the most recent forecast error $fe_{t|t-1} \equiv z_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}$ to update their

g^{RE} is the mapping between states and jumps under rational expectations. Agents in the learning model however don't know g^{RE} and are thus indeed unable to form the rational expectations forecast.

[9] This is a conventional assumption in the learning literature and serves to simplify the algebra. As Sargent (1999) shows, similar results obtain upon relaxing anticipated utility.

[10] This is another common simplifying assumption in studies of adaptive learning. In an extension, I relax this assumption and find that it has similar implications as having agents learn the Taylor rule: initial responses to shocks lack intertemporal expectation effects, but these reemerge as the evolution of state variables is learned.

forecasting rule. The estimate is updated according to the following recursive least-squares algorithm:

$$\phi_t = \left(\phi'_{t-1} + k_t^{-1} R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(z_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}\right)'\right)' \tag{19}$$

$$R_t = R_{t-1} + k_t^{-1} \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{20}$$

where R_t is the 4×4 variance-covariance matrix of the regressors and k_t^{-1} is the learning gain, specifying to what extent the updated estimate loads on the forecast error. Clearly, a high gain implies high loadings and thus strong changes in the estimated coefficients ϕ. A low gain, by contrast, means that the current forecast error only has a small effect on ϕ_t.

3.2 Endogenous gain as anchoring mechanism

The vast majority of the learning literature specifies the gain either as a constant, $\bar{g}$, or decreasing with time so that $k_t^{-1} = (k_{t-1} + 1)^{-1}$. Instead, in the spirit of Carvalho et al. (2019), I allow firms and households in the model to choose whether to use a constant or a decreasing gain. I use the following endogenous gain specification: let $fe_{t|t-1}$ denote the forecast error of time t variables given information at $t - 1$. Then the gain evolves as

$$k_t^{-1} = k_{t-1}^{-1} + \mathbf{g}(fe_{t|t-1}) \tag{21}$$

where $\mathbf{g}(\cdot)$ is a smooth, increasing function in the forecast error that I refer to as the anchoring function. App. C compares alternative specifications for $\mathbf{g}(\cdot)$.

Having an endogenous gain has the interpretation of agents being able to adapt their forecasting behavior to the volatility of their environment. If they observe small forecast errors, $\mathbf{g}(\cdot) < 0$. Firms and households thus use decreasing gains, reflecting their belief that the underlying data-generating process (DGP) has not changed compared to their earlier held beliefs. I refer to this case as *anchored expectations* because it captures the notion that the private sector's estimate of g, g^l, is not updated strongly. In other words, the previously maintained PLM is seen as close enough to the true DGP.

By contrast, observing large forecast errors leads to increasing gains, which corresponds to assigning a higher weight to more recent observations than old ones. Such a forecasting scheme outperforms the decreasing gain scheme when the environment is volatile, reflecting a possible regime switch. If the previous DGP has been replaced by a new one, having high gains allows agents to discount old observations generated by the previous DGP that is no longer in effect, and rely more on the newest observations that come from the current DGP. In this way, agents can learn the new DGP faster, correcting their previously held PLM. This is the case of *unanchored expectations*.

It is intuitive why the central bank might care whether expectations are anchored or not. When expectations are unanchored at time t, the private sector believes that the true DGP involves a different mapping g between states and jumps than g_{t-1}^l, so thatg_t^l is updated strongly. Private sector forecasts will thus drift in the direction of the update in g^l, implying that the observables will also shift in the same direction owing to the law of motion (13). From the perspective of the central bank, stabilization of the observables therefore implies stabilization of expectations. However, it is not obvious that the central bank prefers to anchor expectations at all points in time because regime shifts in model parameters might warrant letting the private sector learn the new DGP fast. Indeed, the contribution of this paper is to characterize formally the nature of the monetary policy problem when expectation formation is characterized by the anchoring mechanism.

3.3 Actual law of motion

To complete the model, I now use the specifics of the anchoring expectation formation to characterize the evolution of the jump variables under learning. Using the PLM from Equation (15), I write the long-horizon expectations in (12) as

$$f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b_{t-1}(I_3 - \alpha\beta h)^{-1} s_t \qquad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b_{t-1}(I_3 - \beta h)^{-1} s_t \tag{22}$$

Substituting these into the law of motion of observables (Equation (13)) yields the actual law of motion (ALM):

$$z_t = g^l_{t-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix} \tag{23}$$

where g^l is a 3×4 matrix given in App. B. Thus, instead of the state-space solution of the RE version of the model (Equations (14) and (B.1)), the state-space solution for the learning model is characterized by the pair of equations (14) and (23).

4 Monetary policy and anchoring

In this section I use the model developed above to analyze the interaction between monetary policy and the anchoring mechanism. I begin with an analytical characterization of optimal monetary policy. As a first step, I analyze the Ramsey problem of determining optimal paths for the endogenous variables that policy seeks to bring about. While the anchoring mechanism introduces substantial nonlinearity into the model, it is possible to derive an optimal targeting criterion for the policymaker to follow. As we shall see, the optimal rule prescribes for monetary policy to act conditionally on the stance of expectations. In particular, whether expectations are anchored or not matters for the extent to which there is a tradeoff between inflation and output gap stabilization, alleviating this tradeoff when expectations are anchored.

I then turn to question of how to implement optimal policy. Since history-dependence is not a feature of the optimal solution, purely forward-looking Taylor rules are no longer excluded from the class of rules that can implement the Ramsey solution. In Section 4.3, I therefore restrict attention to Taylor-type feedback rules for the interest rate. I solve for the optimal Taylor-rule coefficients numerically and investigate how the choice of Taylor-rule coefficients affects the anchoring mechanism. In contrast to conventional wisdom in learning models, optimal monetary policy is *less* aggressive on inflation (and on the output gap) in the anchoring model than under rational expectations. As I explain in detail below, the reason is that while having anchored expectations is beneficial in order to lower economic volatility in the long run, in the short run it is costly to get expectations anchored.

4.1 The Ramsey problem

I assume the monetary authority seeks to maximize welfare of the representative household under commitment. As shown in Woodford (2011), a second-oder Taylor approximation of household utility delivers a central bank loss function of the form

$$L^{CB} = \mathbb{E}_t \sum_{T=t}^{\infty} \{\pi_T^2 + \lambda_x (x_T - x^*)^2 + \lambda_i (i_T - i^*)\} \tag{24}$$

where λ_j $j = \{x, i\}$ is the weight the central bank assigns to stabilizing variable j and j^* is its target value. The central bank's problem, then, is to determine paths for inflation, the output gap and the interest rate that minimize the loss in Equation (24), subject to the model equations (9) and (10), as well as the evolution of long-horizon expectations, spelled out in Section 3. A second question is how to implement the optimal allocation; that is, to find a response function for the policy instrument i_t that implements the optimal sequences of the observables.

While for most of the paper I consider a general specification for monetary policy, in Section 4.3, I will restrict attention to a standard Taylor rule:

$$i_t = \psi_\pi (\pi_t - \bar{\pi}) + \psi_x (x_t - \bar{x}) + \bar{i}_t \tag{25}$$

where ψ_π and ψ_x represent the responsiveness of monetary policy to inflation and the output gap respectively, $\bar{\pi}$ and $\bar{x}$ are the central bank's targets. Lastly, $\bar{i}_t$ is a monetary policy shock. I also assume that when the Taylor rule is in effect, the central bank publicly announces this. Thus Equation (25) is common knowledge and is therefore not the object of learning.[11]

4.2 Ramsey policy under anchoring

4.2.1 Optimal Ramsey policy as a target criterion

Appendix D lays out the policy problem for a simplified version of the baseline model. In particular, I make three simplifying assumptions compared to the baseline model. First, I assume that only the inflation process is learned; expectations about the output gap and the interest rate are rational evaluations of the infinite sum of future expectations. Second, I assume that only the constant of the inflation process is learned. Third, I consider the following simplification of the anchoring mechanism:[12]

$$k_t = \mathbf{g}(fe_{t|t-1}) \tag{26}$$

[11] In an extension I consider the case where the Taylor rule is not known (or not believed) by the public and therefore is learned together with the relations (9) and (10). This dampens intertemporal expectation effects as long as the Taylor rule is not learned; afterwards, the model dynamics are identical to those of the baseline.

[12] These assumptions are made for algebraic convenience only and do not alter the qualitative implications of the model. For a more general version, see App. E.

The solution of the Ramsey problem under these assumptions is stated in the following result.

Result 1. *Target criterion in the anchoring model*
The targeting rule in the simplified learning model with anchoring is given by

$$\pi_t = -\frac{\lambda_x}{\kappa}\left\{x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_{\pi,t}\right)\right.$$
$$\left.\left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi},\mathbf{t+j}})\right)\right\} \tag{27}$$

For the derivation, see Appendix D. For a general target criterion without assumption (26), see Appendix E.

The interpretation of Equation (27) is that the *intra*temporal tradeoff between inflation and output gaps due to cost-push shocks is complemented by two *intertemporal* tradeoffs due to the anchoring mechanism. The first is the current stance of anchoring, captured by the gain k_t^{-1} and the derivative of the anchoring function, $\mathbf{g}_{\pi,t}$, in the second term on the right-hand side.

One part of this effect, namely the current level of the gain, is related to the result of Molnár and Santoro (2014). In the context of a decreasing or constant gain Euler-equation learning model, they show that the presence of learning introduces a novel intertemporal tradeoff between inflation and output gap stabilization. That effect is thus present here too since a nonzero gain indicates that the learning process has not yet converged. The presence of anchoring amplifies this intertemporal tradeoff because now the degree and direction in which the gain changes matters too.

However, there is a second intertemporal tradeoff due to the anchoring mechanism. It is manifest in the multiplication of $(1 - k_{t+j}^{-1}(\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})$ in the second bracket on the right-hand side. This says that additionally to the intertemporal tradeoff coming from a not-yet converged learning mechanism, there is another novel intertemporal tradeoff coming from all future expected levels of the gain given the expected future changes of the gain. In other words, the central bank needs to consider whether its chosen interest rate sequence contributes to anchoring expectations in future periods, or whether it actually serves to unanchor them.

Let me investigate these channels in isolation. To see exactly what the role of anchoring is in the target criterion, consider first the special case of exogenous gain adaptive learning, for simplicity with a constant gain specification.[13] In this case the anchoring function and the forecast error are irrelevant (since $\mathbf{g_z} = 0, z = \pi, \bar{\pi}$) and (27) boils down to

$$\pi_t = -\frac{\lambda_x}{\kappa}\left\{x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}k^{-1}\left(\sum_{i=1}^{\infty} x_{t+i}(1-k^{-1})^i\right)\right\} \tag{28}$$

which now is the analogue of Gaspar et al. (2010)'s Equation (24).[14] This result, due to Molnár and

[13] The intuition is identical if the public were using a decreasing gain specification but the math would not convey that same intuition as cleanly.

[14] In their Handbook chapter, Gaspar et al. (2010) provide a parsimonious treatment of Molnár and Santoro (2014). I am referring to their expression for the target criterion because Molnár and Santoro (2014) do not provide one explicitly.

Santoro (2014), suggests that already the presence of learning introduces an intertemporal tradeoff between inflation and output gap stabilization. However, the fact that the central bank now has future output gaps as a margin of adjustment means that it does not have to face the full tradeoff in the current period. Learning allows the central bank to improve the current output gap without sacrificing inflation stability today; however, this results in a worsened tradeoff in the future. In other words, adaptive learning by itself allows the central bank to postpone the current tradeoff to later periods.

Intuitively, this happens because adaptive expectations are slow in converging to rational expectations. In the transition, the private sector's expectations do not adjust to fully internalize the intratemporal tradeoff. This gives the monetary authority room to transfer the tradeoff to the future.

Contrasting Equations (28) and (27) highlights the role of anchoring. With anchoring, the extent to which policy can transfer the intratemporal tradeoff to future periods depends not only on the stance of the learning process, as in (28), but also on whether expectations are anchored or not and in which direction they are moving. In fact, not only the current stance and change of anchoring matters, but also all future levels and changes.

Anchoring, however, complicates the possibility of transfering today's tradeoff to the future. If the central bank is able to achieve zero forecast errors in all periods, then we're back to the decreasing gain specification. This is not likely, however, given that the noise in the evolution of exogenous states gives rise to forecast errors at all periods, both positive and negative. And while only the size of squared forecast errors matters for the sign of the derivative of the anchoring function, the sign of the forecast errors proper matters for the sign of the two large parentheses on the right-hand side.

Let me therefore rewrite Equation (D.8) from App. D, the first of the three-equation system obtained by solving the Ramsey problem:

$$2\pi_t = -2\frac{\lambda}{\kappa}x_t + \varphi_{5,t}k_t^{-1} + \varphi_{6,t}\mathbf{g}_{\pi,t} \tag{29}$$

The Lagrange multipliers $\varphi_5 \geq 0$ and $\varphi_6 \geq 0$ are the multipliers of the RLS updating and the anchoring equation respectively. This equation, upon substitution of the solutions for the two multipliers, yields the target criterion and is thus ideal for interpretation. First, since $\varphi_{5,t}k_t^{-1} > 0$, one immediately obtains the above-discussed conclusion that as long as the adaptive learning equation is a constraint to the policymaker ($\varphi_{5,t} > 0$), the central bank has more room to transfer the contemporaneous tradeoff between inflation and the output gap to the future.[15]

However, whether the anchoring equation alleviates or exacerbates the inflation-output gap tradeoff depends on the sign of $\mathbf{g_{\pi,t}}$. If the derivative is positive, the effect is the same as above, and the central bank has more leeway to postpone the tradeoff to the future. By contrast, if the derivative is negative,

[15] Strictly speaking, φ_5 and φ_6 are never zero in this model. The reason is that the anchoring model is a convex combination of decreasing and constant gain learning. The former has the RE solution as a limit, while the latter fluctuates around the RE solution with bounded variance. In the anchoring setting, if expectations are anchored and there are no inflation surprises, the gain converges to zero and the decreasing gain limit of discretionary RE obtains. However, exogenous disturbances induce unforecastable variation, producing forecast errors that will unanchor expectations, restarting the learning process. This is in stark contrast with Carvalho et al. (2019), where the anchoring function is the map between the PLM and the expected ALM and thus only depends on the endogenous component of the forecast error. Therefore, in their model absent regime switches, expectations can never become unanchored once learning has converged.

that is expectations are becoming anchored, the intratemporal tradeoff is worsened.

Why do unanchored expectations give the central bank the possibility to postpone its current inflation-output gap tradeoff? The reason is that when expectations become unanchored, the learning process is restarted. A not-yet converged learning process implies, as I discussed above, that postponing the tradeoff is possible. Restarting the convergence process thus unlocks this possibility.

This does not imply that the central bank should prefer to have unanchored expectations. In fact, the answer to whether expectations should be anchored from the viewpoint of the central bank depends on the current stance of learning. Clearly, the central bank prefers to face a learning process that on the one hand has not yet converged, on the other is converging only slowly. A high gain under unanchored expectations implies both a sizable distance from convergence as well as faster learning and thus faster convergence. Therefore, ideally the central bank would have expectations anchored but the gain far from zero. Once the gain approaches zero, only unanchored expectations can raise it again to as to restart the learning process.

Unanchoring expectations, however, is costly. As I demonstrate in Section 5, unanchored expectations lead to more volatility in the observables due to more volatile forecast errors in the wake of a higher gain. Therefore heightened inflation- and output gap volatility may be too high a price to pay for an ameliorated inflation-output gap tradeoff.

4.2.2 No commitment under adaptive learning

Now simplify the target criterion further, assuming that learning has converged, $k_t^{-1} = \mathbf{g}_\pi = 0$. We're left with

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t \tag{30}$$

which corresponds to the optimal discretionary solution for rational expectations in Clarida et al. (1999). This is formalized in the following result.

Result 2. *Coincidence of commitment and discretion under adaptive learning*
In an adaptive learning model with exogenous or endogenous gain, the optimal Ramsey policies under commitment and discretion coincide. The optimal Ramsey plan is more akin to discretion than to commitment as it does not involve making promises about future policy actions and is time consistent.

To illustrate this result in a parsimonious manner, consider a simplified version of the model. The planner chooses $\{\pi_t, x_t, f_t, k_t^{-1}\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t) \\ + \varphi_{2,t}(f_t - f_{t-1} - k_t^{-1}(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t^{-1} - \mathbf{g}(\pi_t - f_{t-1})) \Big\}$$

where the IS-curve, $x_t = \mathbb{E}_t\, x_{t+1} + \sigma f_t - \sigma i_t + \sigma r_t^n$, is a non-binding constraint, therefore excluded from the problem, and $\mathbb{E}_t\, x_{t+1}$ is rational. In this simplified setting, f_t is a stand-in variable capturing inflation expectations that evolves according to a recursive least squares algorithm. The anchoring function $\mathbf{g}$ specifies how the gain k_t^{-1} changes as a function of the current forecast error according to assumption (26).[16] Note that the problem involves commitment because the monetary authority internalizes the effects of its actions both on the evolution of expectations and on that of the gain.

After some manipulation, first order conditions reduce to:

$$2\pi_t + 2\frac{\lambda}{\kappa}x_t - \varphi_{2,t}(k_t^{-1} + \mathbf{g}_\pi(\pi_t - f_{t-1})) = 0 \tag{31}$$

$$-2\beta\frac{\lambda}{\kappa}x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1}^{-1} - \mathbf{g}_\mathbf{f}(\pi_{t+1} - f_t)) = 0 \tag{32}$$

Inspection of this system reveals that unlike the rational expectations case ($f_t = \mathbb{E}_t\,\pi_{t+1}$), the optimal solution does not involve lagged multipliers.[17] This implies that the monetary authority cannot condition the optimal time path of inflation and the output gap on the past; optimal policy is not history-dependent.

The intuition for this result is easy to see if one compares rational expectations and learning in an infinitely repeated game setting. Under rational expectations, the lagged multiplier appears in the solution because expected inflation is a jump variable. This reflects that expectations fulfill a form of optimality, rendering the private sector a strategic player. The adjustment of expectations under rational expectations enables the central bank to make promises about future policy that are incorporated into expectations.

Not so for adaptive expectations. Learning agents look exclusively to past data to form expectations. Their expectation formation thus cannot incorporate the policymaker's promises about the future course of policy. In fact, a private sector with adaptive expectations has a pre-specified, non-strategic expectation formation.[18] Therefore, households and firms act as an automaton, leaving the central bank unable to make promises that have any effect on expectations. [19]

4.2.3 Implementation without a feedback rule

Is the target criterion (27) implementable without recourse to an interest rate rule, but by virtue of announcing it to the public? Even in the case of "sophisticated central banking"[20] in which the monetary authority observes the public's expectation and learning process fully, evaluating the expectation in the last term on the right-hand side can be challenging. Such a calculation requires the central bank to form expectations of not just future exogenous disturbances, but also how gains, the expected mean inflation $\bar{\pi}$ and inflation will evolve as a function of disturbances and of the endogenous model mechanisms.

Moreover, communicating the rule to the public poses further challenges. In light of the maintained

[16] Here I maintain assumption (26) for presentational purposes. It has no bearing on the results.

[17] This echoes the findings of Molnár and Santoro (2014) and Gaspar et al. (2010).

[18] The literature on central bank reputation seeks to address this issue by reintroducing some form of optimality to expectation formation. See Cho and Matsui (1995) and Ireland (2000). Another possible remedy is to retain a sufficient degree of forward-looking expectations as in the finite-horizon planning approach advocated by Woodford (2019).

[19] Mele et al. (2019) report a similar finding in a decreasing gain learning model.Their terminology of contrasting "inflation-targeting" with "price-level targeting" policy rules renders the connection to commitment less immediate, but it is helpful to recall that in the rational expectations NK model, optimal discretionary monetary policy involves inflation stabilization, while optimal commitment entails full price-level stabilization.

[20] Gaspar et al. (2006)'s term.

assumption of anticipated utility, in which the public fails to internalize its own future updating of its forecasting rule, communicating a rule that incorporates such updating is a daunting task. For implementation purposes, I therefore suggest a simplified targeting rule that replaces expected future gains by the current, observed gain, and ignores expected future forecast errors:

$$\pi_t = -\frac{\lambda_x}{\kappa}\left\{x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_{\pi,t}\right)\left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i}(1-k_t^{-1})^i\right)\right\} \tag{33}$$

While still a long expression, the interpretation of (33) is straightforward. It instructs the policymaker to weigh the inflation-output gap tradeoff by the current output gap and an additional term that captures the effects of learning and anchoring. The learning-and-anchoring term corresponds exactly to expected future output gaps, discounted by future gains, assuming those to be equal to current ones, where the entire expression is weighted by the current gain and the extent of the change in the current gain times the forecast error.

To the public, such a rule could be communicated in simple terms along the lines of: "current inflation will target the negative of today's output gap, plus the present discounted value of all expected future output gaps, discounted by the current gain, times the sum of the current gain and the forecast error times the change in the current gain." The advantage of such a targeting rule is that it is simple to communicate and to compute, and, given that it is more aligned with the long-run expectation of learning agents with anticipated utility, it can also reduce forecast errors and thus aid the anchoring of expectations.

4.3 Optimal Taylor rule with anchoring

The optimal commitment rational expectations version of the targeting rule (27) would take the form of (30) with an additional x_{t-1} term capturing the benefits of commitment. Therefore, due to its purely forward-looking nature, a Taylor-type interest rate rule is not fully optimal in the rational expectations NK model. Since Result 2 tells us that in the anchoring model commitment is inexistent, there is a renewed interest in formulating monetary policy as a Taylor rule, as these types of rules are no longer excluded from the class of fully optimal rules. In particular, the targeting rule (27) is also purely forward-looking. One may then wonder whether Taylor-rule coefficients (ψ_π, ψ_x) exist that can implement the optimal plan. Therefore I now consider the restricted set of purely forward-looking policy rules and ask what values of the Taylor-rule coefficients are optimal in the case of the anchoring model.

Table 1: Calibrated parameters

β	0.99	stochastic discount factor
σ	1	intertemporal elasticity of substitution
α	0.5	Calvo probability of not adjusting prices
ψ_π	1.5	coefficient of inflation in Taylor rule
ψ_x	0	coefficient of the output gap in Taylor rule
$\bar{g}$	0.145	value of the constant gain
$\tilde{\theta}$	2.5	threshold value for criterion of endogenous gain choice
$\tilde{\kappa}$	0.2	scaling parameter of gain for forecast error variance estimation
ρ_r	0	persistence of natural rate shock
ρ_i	0.6	persistence of monetary policy shock
ρ_u	0	persistence of cost-push shock
σ_i	1	standard deviation of natural rate shock
σ_r	1	standard deviation of monetary policy shock
σ_u	1	standard deviation of cost-push shock
λ_x	0	weight on the output gap in central bank loss
λ_i	0	weight on the interest rate in central bank loss

5 Simulations

5.0.1 Calibration

In this section I simulate the rational expectations and learning versions of the model and compute the optimal Taylor rule coefficients numerically.[21] Table 1 summarizes the calibrated parameter values. For most of the parameters, I assign values commonly used in the macroeconomic literature. In particular, I follow Woodford (2011)'s calibration. For this section, I shut off the monetary policy parameters λ_i, λ_x and ψ_x in order to focus on the role of inflation in the central bank's problem and thus on the optimal choice of inflation aggressiveness, ψ_π.

For the simulations in this section, I use the CUSUM-inspired criterion outlined in App. C. The learning parameters $\bar{g}, \tilde{\theta}$ and $\tilde{\kappa}$ require some discussion. While the choice of $\tilde{\kappa}$ only matters for the smoothness of the endogenous gain decision and thus can be set relatively freely, the threshold $\tilde{\theta}$ has more bearing on the behavior of the model. Intuitively, the higher $\tilde{\theta}$, the more forecast error volatility agents in the economy are willing to tolerate before switching to a constant gain. Experimenting with different values reveals that once $\tilde{\theta}$ is higher than a particular threshold, expectations are anchored for any value of ψ_π. Analogously if $\tilde{\theta}$ is below a lower threshold, expectations are always unanchored regardless of the value of ψ_π. My choice of $\tilde{\theta} = 2.5$ is thus motivated by assigning a value for which the comparative static of anchoring with respect to ψ_π is meaningful.

The choice of $\bar{g}$ is far from innocent as it has considerable implications for model dynamics. In particular, as periodically noted in the adaptive learning literature, constant gain learning models have

[21]To concentrate on the intuition, in this section I implement the learning algorithm such that only the constant is learned. The general formulation has qualitatively similar features but is more impacted by small-sample concerns prevalent in simulations.

a tendency to produce impulse responses that exhibit damped oscillations.[22] The reason is that under an adaptive learning framework, forecast errors following an impulse are oscillatory. In fact, the higher the learning gain, the higher the amplitude of forecast error oscillations. If the gain is high enough, the oscillations become explosive.

Unfortunately, the model gives no guidance on the appropriate value for $\bar{g}$.[23] I thus turn to the thin literature on estimating learning gains. I assign the value 0.145, obtained by Carvalho et al. (2019), to my knowledge the only study to estimate the value of the constant gain for an endogenous gain model. It has to be observed, however, that this is a significantly higher value than what was found in the literature estimating gains for constant gain learning. Branch and Evans (2006)'s number of 0.062 is quite close to Eusepi et al. (2018)'s estimate of 0.05, while Milani (2007) finds an even lower number of 0.0183. Studies that use calibrated gains such as Williams (2003) or Orphanides and Williams (2005) tend to experiment with a range of values in the $[0.01, 0.1]$ interval. The value of 0.05 seems to have attained particular prominence, but also much lower numbers have been used, such as 0.002 in Eusepi and Preston (2011). I speculate that Carvalho et al. (2019)'s estimate is so large relative to other estimates because they estimate a switching-gain model, while the rest of the estimates come from constant gain specifications. In an endogenous gain specification, data needs to assign a higher value to the constant gain parameter to rationalize the same average gain in the time series. With these caveats in mind, I adopt Carvalho et al. (2019)'s value.

5.0.2 Simulated dynamics

Having thus assigned values to the parameters, I turn to the model's behavior. Table 2 presents an overview of the optimal Taylor rule coefficient ψ_π, obtained via grid-search, for the rational expectations and anchoring models. The table also compares the baseline parameterization with several alternatives. One notices that if the central bank has no concern to stabilize the output gap ($\lambda_x = 0$) or the nominal interest rate ($\lambda_i = 0$), ψ_π^{RE} is infinity. As discussed in Section 4, this is because if the central bank suffers no loss upon output variation, then the fact that the divine coincidence doesn't hold does not pose a problem. Similarly, if the monetary authority is willing to allow the nominal interest rate to fluctuate vastly in order to stabilize inflation, this also allows the central bank to be infinitely aggressive on inflation.

Table 2: Optimal coefficient on inflation, RE against learning for alternative parameters

	$\psi_\pi^{*,RE}$	$\psi_\pi^{*,learn}$
Baseline	∞	1.6243
$\lambda_x = 1$	2.1042	1.0571
$\lambda_i = 1$	1.1	1.0978

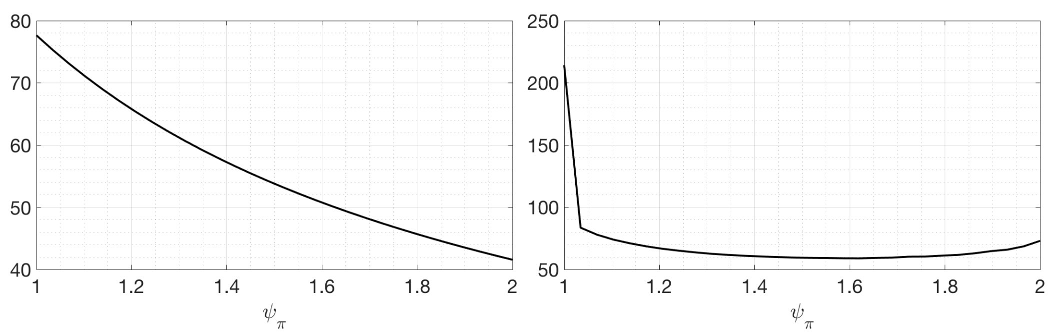
[22] Authors making explicit note of this phenomenon include Evans and Honkapohja (2001), Evans et al. (2013) and Anufriev and Hommes (2012). App. F presents a simple illustration for why this is the case.

[23] The analogy of the Kalman gain from the Kalman filter does not prove helpful either because it requires a steady state forecast error variance matrix which is not available in a learning context.

The main observation however is that ψ_π is always lower for the anchoring model than for the RE model. This is reinforced in Fig. 3 which plots the central bank's loss in the RE and learning models for various values of ψ_π but otherwise the baseline specification. The message is clear: while for rational expectations, the loss is strictly decreasing in ψ_π, this is not the case for the anchoring model.

This echoes the analytical results of Section 4: the anchoring mechanism introduces a novel tradeoff for the central bank. On the one hand, as we will see shortly in Fig. 4, having unanchored expectations increases the volatility of the observables. This results in the central bank wishing to anchor expectations. As Fig. 5 makes clear, this requires raising ψ_π. But in an environment where agents know the Taylor rule, a higher coefficient on inflation will lead to initially higher volatility due to the agents anticipating the endogenous responses of the nominal interest rate far in the future.

Figure 3: Central bank loss function as a function of ψ_π



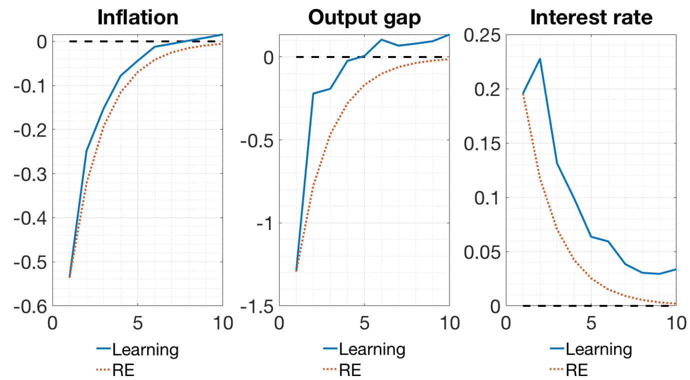
(a) RE (b) Learning

To understand what's going on in the model in detail, consider Fig. 4, portraying the impulse responses of the model after a contractionary monetary policy shock. The red dashed lines show the responses of the observables in the rational expectations version of the model. The blue lines show the responses in the learning model, on panel (a) conditional on expectations being anchored when the shock hits, on panel (b) being unanchored upon the arrival of the shock.

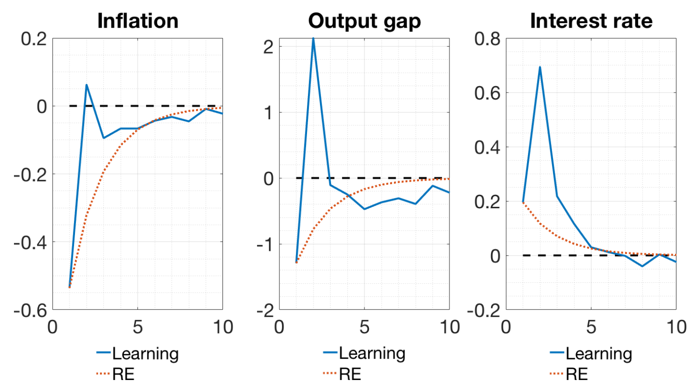
Not only do the impulse responses show the usual behavior of learning models - dampened responses and increased persistence. More importantly, responses differ strongly depending on whether expectations are anchored or not when the shock hits. In particular, if expectations are anchored, responses are closer to rational expectations than when expectations are unanchored. Moreover, when expectations are unanchored, the endogenous responses of the observables become much more volatile, indeed, oscillatory. This makes intuitive sense: expectations being unanchored reflects the fact that firms and households are confronted with an environment that does not line up with their currently held perceived law of motion. They thus believe that a structural change has occurred and are therefore revising their expectations. Expectations are therefore fluctuating strongly, and as they feed back to the observables, the latter inherit their volatility.

Therefore, to avoid the volatility that results from unanchored expectations, the central bank wishes to anchor expectations. What choice of ψ_π will do the job? Fig. 5 provides the answer. The figure

Figure 4: Impulse responses after a contractionary monetary policy shock



(a) RE against learning, expectations anchored



(b) RE against learning, expectations unanchored

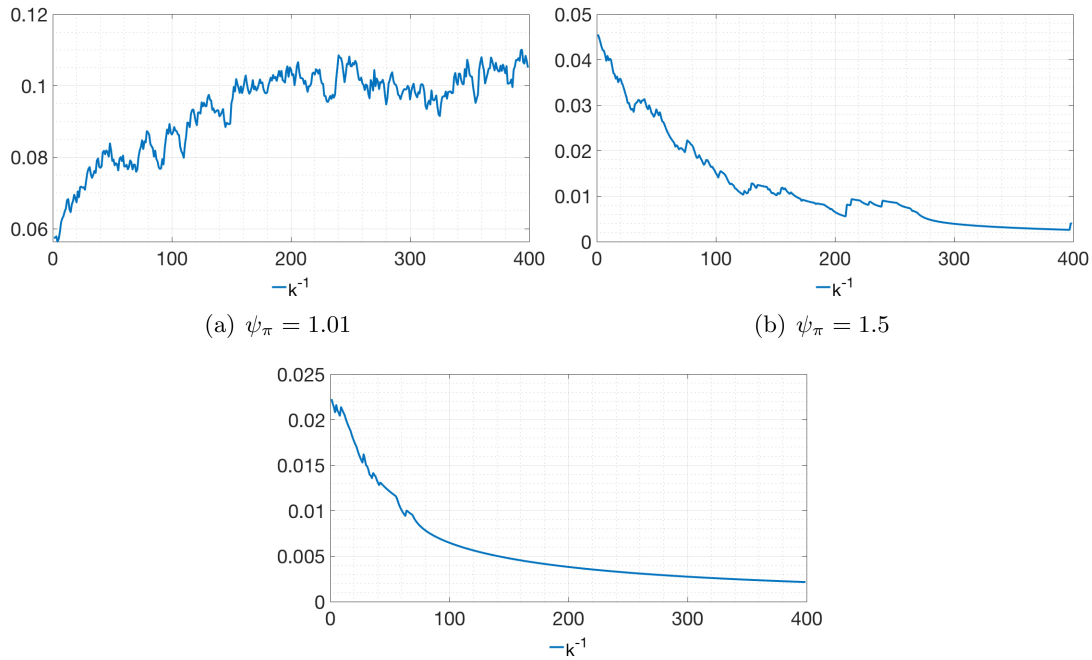
Shock imposed at $t = 25$ of a sample length of $T = 400$ (with 100 initial burn-in periods), cross-sectional average with a cross-section size of $N = 100$. For the rest of the paper, I keep these simulation values unless otherwise stated. For the learning model, the remark refers to whether expectations are anchored at the time the shock hits.

shows the cross-sectional average of inverse gains that result when ψ_π takes on different values. Clearly, a higher ψ_π results in lower and decreasing gains.[24] Thus a central bank aiming to anchor expectations needs to employ a high ψ_π. This is also intuitive. A more aggressive central bank can signal to the public that it is determined to achieve the announced inflation target. Thus agents can rest assured that their believed inflation target is indeed the one the central bank can and will implement. Whenever ψ_π is low, however, the central bank is willing to tolerate bigger deviations from the target. This opens the door to speculation about whether the central bank is really committed to the target. In this case, deviations from the believed target of the same magnitude can cast doubt on the central bank's commitment, so that agents decide to monitor recent data closely to learn the seemingly shifting average value of inflation.

But if unanchored expectations cause heightened volatility, and being aggressive on inflation is able to anchor expectations, why does the monetary authority optimally choose a lower value for ψ_π than under rational expectations? The reason is that conditional on being unanchored, a higher ψ_π actually

[24] As I remark in Section 3, if one uses the anchoring criterion of Carvalho et al. (2019), this conclusion is overturned.

Figure 5: Cross-sectional average inverse gains for various values of ψ_π



(a) $\psi_\pi = 1.01$

(b) $\psi_\pi = 1.5$

(c) $\psi_\pi = 2$

causes higher volatility than a lower one. This can be seen on Fig. 6 which depicts the same impulse responses to a contractionary monetary policy shock as Fig. 4, focusing however only on responses conditional on expectations being unanchored upon the shock. It shows these responses for three different values of ψ_π.

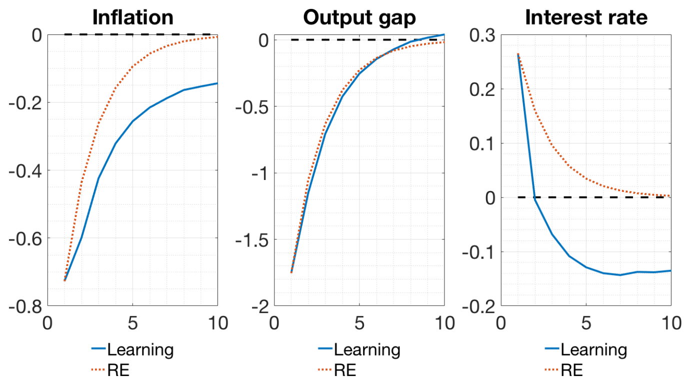
As the figure shows, a high ψ_π leads to more volatility than a low one does. The intuition is a little subtle. Since expectations are unanchored, they are also volatile. This implies that inflation far ahead in the future is expected to fluctuate strongly. Since agents know the Taylor rule, this also means that they expect the nominal interest rate far in the future to respond. The more aggressive the central bank, the stronger an interest rate response will the agents expect. This however feeds back into current output gaps and thus inflation. Higher overall volatility is the result.

Unexpectedly, the model dynamics here echo the predictions of Ball (1994). But the underlying channels are quite different. Ball (1994) observes that, contrary to conventional wisdom, rational expectations New Keynesian models imply expansionary disinflations. To reconcile this model feature with data pointing to the costliness of disinflations, he concludes that central bank announcements must suffer from credibility issues.

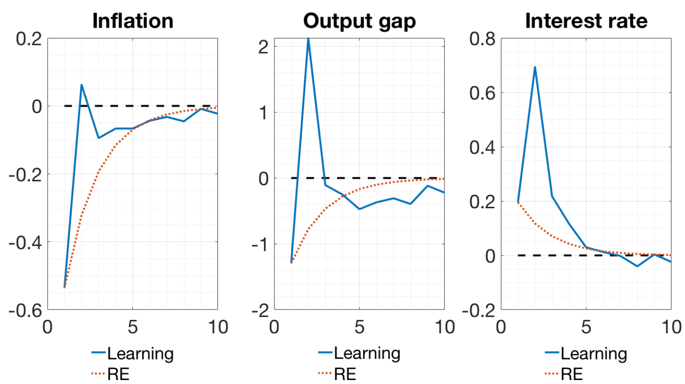
Note that in the present context, when expectations are anchored (panel (a) of Fig. 4), impulse responses do not exhibit this feature. However, when expectations are unanchored (panel (b) of Fig. 4), impulse responses look exactly as Ball (1994) predicts: we obtain an expansionary disinflation.

The reason this is happening is that when agents know the Taylor rule, long-horizon expectations

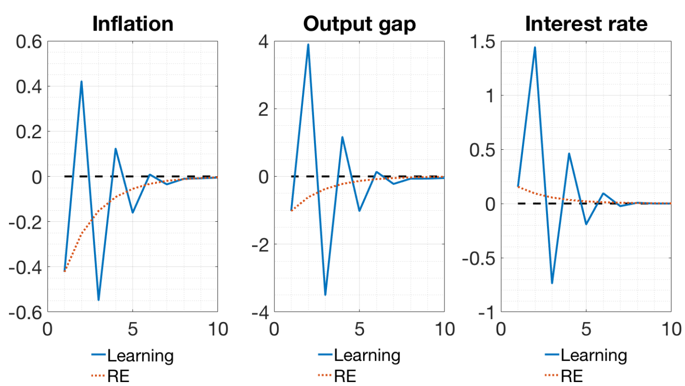
Figure 6: Impulse responses for unanchored expectations for various values of ψ_π



(a) $\psi_\pi = 1.01$



(b) $\psi_\pi = 1.5$



(c) $\psi_\pi = 2$

of the interest rate move in tandem with the same expectations of inflation in the far future. A current disinflation lowers long-horizon inflation expectations, leading the public to expect low interest rates far out in the future. Through the NKIS-curve (Equation 9), this stimulates current output.[25] But the

[25]The extension in which the public has to learn the Taylor rule is interesting in this regard. As expected, the Ball-type disinflationary boom does not initially show up in impulses responses obtained in that extension. However, as the agents are learning the Taylor rule, the expansionary disinflation slowly reemerges in the impulse responses.

absence of the "Ball-effect" from the anchored expectations impulse responses indicates that the channel is only operational when expectations are moving sufficiently. Thus I arrive at a different conclusion than Ball (1994); instead of credibility issues, it is anchored expectations that are responsible for the absence of expansionary disinflations of the type seen on Fig 4, panel (b).

Thus the simulations bear out the analytical results in illustrating how the presence of an expectation formation that allows for the anchoring and unanchoring of expectations interacts with monetary policy in the New Keynesian model. Unsurprisingly, it is desirable for the central bank to anchor expectations. It is also intuitive that being aggressive on inflation helps to anchor expectations. However, less intuitive is the fact that the optimal degree of aggressiveness on inflation is lower than under rational expectations. This has to do with the heightened volatility of the expectations process when ψ_π is high. A higher ψ_π increases the response of future nominal interest rate expectations, thus raising the feedback from expectations to current observables. Thus the central bank faces an intertemporal tradeoff: to reduce volatility in the long-run, it seeks to anchor expectations. However, the price the bank has to pay in order to get expectations anchored is higher short-run volatility. Thus, the monetary authority trades off the short-run cost with the long-run benefit of anchoring expectations.

5.1 Discussion

I have thus established both analytically and by way of simulations that optimal monetary policy in the anchoring model is characterized by the following three features: 1) conditioning on whether expectations are anchored, 2) a non-monotonic relationship between the inflation response coefficient in the Taylor rule and the central bank's loss function, and 3) history-independence.

These characteristics are all related to Result 2, which states that no commitment device exists in models with fully backward-looking expectation formation. It is immediate that if the monetary authority is allowed to reoptimize at a particular date τ, it will choose Taylor-rule coefficients depending on whether expectations are anchored or not. Or, to put it differently, policy adhering to the targeting rule (27) will face different tradeoffs between inflation and output gap stabilization depending on the current value of the gain. As the gain varies over time, it follows that the central bank will choose different Taylor-rule coefficients; optimal policy will be time-inconsistent.

As emphasized at the end of Section 2, this result bears resemblance to that of Lubik and Matthes (2016) who show that central bank learning and data misperceptions lead to time-varying Taylor-rule coefficients. But it is important to point out that this result would not arise in a setting where the public sector is learning with a constant or a decreasing gain specification. The reason is that in such a model, the monetary authority would not find it optimal to reset its coefficient at different points in time because this choice would have no impact on the learning process. With constant gain learning, the central bank would just have to accept heightened volatility and accordingly set a lower ψ_π than under rational expectations. With decreasing gain learning, the bank would have to accept higher volatility initially as the learning process converges to RE.

Either case can be thought of as simply scaling the path of volatility, but with the central bank having no influence on the slope of the path. Under anchoring, the authority can exert direct influence on the volatility path, and its options of doing so are more or less costly depending on whether expectations are currently anchored or not. In this manner, the anchoring mechanism not only introduces a novel tradeoff for policy, but also a new mechanism underlying the time-inconsistency of optimal policy.

One can thus use the model to provide fresh interpretations of the current, 2019-2020 monetary episode in the US. The model suggests that the downward drift of the public's long-horizon inflation expectations is a sign of expectations threatening to become unanchored. The Fed seems to have internalized that anchoring expectations will be much more costly once they are truly unanchored, and thus moved swiftly to lower interest rates in the fall of 2019. In so doing, it communicated to the public its commitment to the 2% target. Through the lens of my model, then, we can interpret this counterintuitive easing at the height of an expansion as an effort on the Fed's part to keep expectations anchored.

6 Conclusion

Central bankers frequently voice a concern to anchor expectations. The fact that rational expectations New Keynesian models have nothing to say about this aspect of monetary is a gap in the macroeconomic literature. Absent a behavioral theory of anchored expectations, it is difficult for macroeconomists to understand periods where central banks are clearly off the Taylor rule. The fall 2019 stance of US monetary policy is an example of such an episode: the business cycle calls for monetary tightening, yet inflation lags below the target and if anything, the Fed is expansionary. My work suggests that the Fed reads the downward drift of long-run inflation expectations as a threat that expectations may become unanchored. In order to prevent that from happening, the Fed therefore is anxious to signal that it is determined to achieve its 2% inflation target. Its expansionary actions are thus not intended to stimulate an already tight labor market; instead the Fed's present objective is to keep expectations anchored.

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A Compact model notation

The A-matrices are given by

$$A_a = \begin{pmatrix} g_{\pi a} \\ g_{xa} \\ \psi_\pi g_{\pi a} + \psi_x g_{xa} \end{pmatrix} \quad A_b = \begin{pmatrix} g_{\pi b} \\ g_{xb} \\ \psi_\pi g_{\pi b} + \psi_x g_{xb} \end{pmatrix} \quad A_s = \begin{pmatrix} g_{\pi s} \\ g_{xs} \\ \psi_\pi g_{\pi s} + \psi_x g_{xs} + \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \end{pmatrix} \tag{A.1}$$

$$g_{\pi a} = (1 - \frac{\kappa\sigma\psi_\pi}{w}) \left[(1-\alpha)\beta, \kappa\alpha\beta, 0\right] \tag{A.2}$$

$$g_{xa} = \frac{-\sigma\psi_\pi}{w} \left[(1-\alpha)\beta, \kappa\alpha\beta, 0\right] \tag{A.3}$$

$$g_{\pi b} = \frac{\kappa}{w} \left[\sigma(1-\beta\psi_\pi), (1-\beta-\beta\sigma\psi_x, 0\right] \tag{A.4}$$

$$g_{xb} = \frac{1}{w} \left[\sigma(1-\beta\psi_\pi), (1-\beta-\beta\sigma\psi_x, 0\right] \tag{A.5}$$

$$g_{\pi s} = (1 - \frac{\kappa\sigma\psi_\pi}{w}) \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (I_3 - \alpha\beta P)^{-1} - \frac{\kappa\sigma}{w} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} (I_3 - \beta P)^{-1} \tag{A.6}$$

$$g_{xs} = \frac{-\sigma\psi_\pi}{w} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (I_3 - \alpha\beta P)^{-1} - \frac{\sigma}{w} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} (I_3 - \beta P)^{-1} \tag{A.7}$$

$$w = 1 + \sigma\psi_x + \kappa\sigma\psi_\pi \tag{A.8}$$

The matrices of the state transition equation (14) are

$$h \equiv \begin{pmatrix} \rho_r & 0 & 0 \\ 0 & \rho_i & 0 \\ 0 & 0 & \rho_u \end{pmatrix} \quad \epsilon_t \equiv \begin{pmatrix} \varepsilon_t^r \\ \varepsilon_t^i \\ \varepsilon_t^u \end{pmatrix} \quad \text{and} \quad \eta \equiv \begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_i & 0 \\ 0 & 0 & \sigma_u \end{pmatrix} \tag{A.9}$$

Note that this is the formulation for the case where a Taylor rule is in effect and is known by the private sector. It is straightforward to remove any of these two assumptions.

B The observation matrix for learning

Instead of the rational expectations observation equation

$$z_t = g s_t \tag{B.1}$$

$$g_{t-1}^l = \begin{bmatrix} F_{t-1} & G_{t-1} \end{bmatrix} \tag{B.2}$$

with

$$F_{t-1} = \left(A_a \frac{1}{1-\alpha\beta} + A_b \frac{1}{1-\beta} \right) a_{t-1} \tag{B.3}$$

$$G_{t-1} = A_a b_{t-1} \left(I_3 - \alpha\beta h \right)^{-1} + A_b b_{t-1} \left(I_3 - \beta h \right)^{-1} + A_s \tag{B.4}$$

C Alternative specifications for the anchoring function

Specifications for **g**. TO DO.
The only other paper to consider an endogenous gain as a model for anchored expectations is Carvalho et al. (2019). In their model, the anchoring function is a discrete choice function as follows. Let θ_t be a criterion to be defined. Then, for a threshold value $\tilde{\theta}$, the gain evolves according to

$$k_t = \begin{cases} k_{t-1} + 1 & \text{if} \quad \theta_t < \tilde{\theta} \\ \bar{g}^{-1} & \text{otherwise.} \end{cases} \tag{C.1}$$

In other words, agents choose a decreasing gain when the criterion θ_t is lower than the threshold $\tilde{\theta}$; otherwise they choose a constant gain. The criterion employed by Carvalho et al. (2019) is computed as the absolute difference between subjective and model-consistent expectations, scaled by the variance of shocks:

$$\theta_t = \max |\Sigma^{-1}(\phi_{t-1} - \begin{bmatrix} F_{t-1} & G_{t-1} \end{bmatrix})| \tag{C.2}$$

where Σ is the VC matrix of shocks, ϕ is the estimated matrix, $[F, G]$ is the ALM (see App. B).

As a robustness check, Carvalho et al. (2019) also compute an alternative criterion.[26] Let ω_t denote agents' time t estimate of the forecast error variance and θ_t be a statistic evaluated by agents in every period as

$$\omega_t = \omega_{t-1} + \tilde{\kappa} k_{t-1}^{-1}(fe_{t|t-1} fe'_{t|t-1} - \omega_{t-1}) \tag{C.3}$$

$$\theta_t = \theta_{t-1} + \tilde{\kappa} k_{t-1}^{-1}(fe'_{t|t-1} \omega_t^{-1} fe_{t|t-1} - \theta_{t-1}) \tag{C.4}$$

where $\tilde{\kappa}$ is a parameter that allows agents to scale the gain compared to the previous estimation and $fe_{t|t-1}$ is the most recent forecast error, realized at time t. Indeed, this is a multivariate time series version of the squared CUSUM test.[27]

It is worthwhile to compare the two criteria. On the one hand, the first specification requires the private sector to evaluate model-consistent expectations, which runs counter to the maintained informational assumptions. It is more consistent with the present model, then, to assume that firms and households employ a statistical test of structural change. Therefore the CUSUM-based criterion is more appealing on conceptual grounds.

On the other hand, simulation of the model using Carvalho et al. (2019)'s preferred criterion reveals that it leads to the opposite comparative statics of anchoring with respect to monetary policy aggressiveness. In particular, while policy that is more aggressive on inflation (a higher ψ_π in the Taylor rule) leads to more anchoring in a model with the CUSUM-inspired criterion, if one uses Carvalho et al. (2019)'s criterion, the same comparative static involves *less* anchoring. This comes from the fact that Carvalho et al. (2019)'s criterion endows the public sector with capabilities to disentangle volatility due

[26] Note that for both criteria, I present the matrix generalizations of the scalar versions considered by Carvalho et al. (2019).
[27] See Brown et al. (1975) and Lütkepohl (2013) for details.

to the learning mechanism from that owing to exogenous disturbances. Thus agents in the Carvalho et al. (2019) model are able to make more advanced inferences about the performance of their forecasting rule and understand that a higher ψ_π causes more learning-induced volatility. This is however not possible for agents who process data in real time without knowledge of the model. Therefore the CUSUM-inspired criterion is preferable both on conceptual and quantitative grounds.

D The policy problem in the simplified baseline model

Denote by $\mathbf{g}_{i,t} \in (0,1),\ i = \pi, \bar{\pi}$, the potentially time-varying derivatives of the anchoring function $\mathbf{g}_t$. In this simplified setting, $\bar{\pi}_t = e_1 a_t$, the estimated constant for the inflation process. e_i is a selector vector, selecting row i of the subsequent matrix. I also use the notation $b_i \equiv e_i b$. The planner chooses $\{\pi_t, x_t, f_{a,t}, f_{b,t}, \bar{\pi}_t, k_t^{-1}\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ (\pi_t^2 + \lambda x_t^2) \tag{D.1}$$

$$+ \varphi_{1,t}\Big(\pi_t - \kappa x_t - (1-\alpha)\beta f_a(t) - \kappa\alpha\beta b_2(I_3 - \alpha\beta h_x)^{-1} s_t - e_3(I_3 - \alpha\beta h_x)^{-1} s_t\Big) \tag{D.2}$$

$$+ \varphi_{2,t}\Big(x_t + \sigma i_t - \sigma f_b(t) - (1-\beta) b_2(I_3 - \beta h_x)^{-1} s_t + \sigma\beta b_3(I_3 - \beta h_x)^{-1} s_t - \sigma e_1(I_3 - \beta h_x)^{-1} s_t\Big) \tag{D.3}$$

$$+ \varphi_{3,t}\Big(f_a(t) - \frac{1}{1-\alpha\beta}\bar{\pi}_{t-1} - b_1(I_3 - \alpha\beta h_x)^{-1} s_t\Big) \tag{D.4}$$

$$+ \varphi_{4,t}\Big(f_b(t) - \frac{1}{1-\beta}\bar{\pi}_{t-1} - b_1(I_3 - \beta h_x)^{-1} s_t\Big) \tag{D.5}$$

$$+ \varphi_{5,t}\Big(\bar{\pi}_t - \bar{\pi}_{t-1} - k_t^{-1}\big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big)\Big) \tag{D.6}$$

$$+ \varphi_{6,t}\Big(k_t^{-1} - \mathbf{g}(\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})\Big)\Big\} \tag{D.7}$$

After a little bit of simplifying, the first-order conditions boil down to the following three equations:

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{5,t} k_t^{-1} - \varphi_{6,t}\mathbf{g}_{\pi,t} = 0 \tag{D.8}$$

$$-\frac{2(1-\alpha)\beta}{1-\alpha\beta}\frac{\lambda}{\kappa} x_{t+1} + \varphi_{5,t} - (1 - k_t^{-1})\varphi_{5,t+1} + \mathbf{g}_{\bar{\pi},t}\varphi_{6,t+1} = 0 \tag{D.9}$$

$$\varphi_{6,t} = (\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})\varphi_{5,t} \tag{D.10}$$

Note that Equation (D.8) is the analogue of Gaspar et al. (2010)'s Equation (22) (or, equivalently, of Molnár and Santoro (2014)'s (16)), except that there's an additional multiplier, φ_6. This multiplier reflects the fact that in addition to the constraint coming from the expectation process itself, with shadow value φ_5, learning involves the gain equation as a constraint as well. One can also clearly read off Result 2: when the learning process has converged such that neither expectations nor the gain process are constraints ($\varphi_5 = \varphi_6 = 0$), the discretionary inflation-output gap tradeoff familiar from Clarida et al. (1999) obtains. Combining the above three equations and solving for $\varphi_{5,t}$, using the notation that $\prod_{j=1}^{0} = 1$, one obtains the target criterion (27).

The system of first order conditions (31)-(32) and model equations for this simplified system also reveal how the endogenous gain introduces nonlinearity to the equation system. In particular, notice how in equations (31)-(32) the gain k_t^{-1} shows up multiplicatively with the Lagrange multiplier, $\varphi_{2,t}$. In fact, the origin of the problem is the recursive least squares learning equation

$$f_t = f_{t-1} + k_t^{-1}(\pi_t - f_{t-1}) \tag{D.11}$$

where the first interaction terms between the gain and other endogenous variables show up. This results in an equation system of nonlinear difference equations that does not admit an analytical solution.

Considering equation (D.11) is instructive to see how it is indeed the endogeneity of the gain that causes these troubles. Were we to specify a constant gain setup, k_t^{-1} would merely equal the constant $\bar{g}$ and the anchoring function $\mathbf{g}$ would trivially reduce to $\bar{g}$ as well. In such a case, all interaction terms would reduce to multiplication between endogenous variables and parameters; linearity would be restored and a solution for the optimal time paths of endogenous variables would be obtainable. Similarly, a decreasing gain specification would also be manageable since for all t, the gain would simply be given by t^{-1}, and the anchoring function would also be deterministic and exogenous.

Although a full optimal time path for the endogenous variables is thus not available for the anchoring model, it is still possible to characterize optimal monetary policy in terms of a target criterion. That is, one can express a relationship between inflation and output gaps from the first order conditions that characterizes the optimal plan that the monetary authority can rely on to implement and to communicate its policy.[28]

E A target criterion for an anchoring mechanism specified in terms of gain changes

Consider the general anchoring mechanism of equation (21):

$$k_t = k_{t-1} + \mathbf{g}(fe_{t|t-1}) \tag{E.1}$$

With this assumption, the FOCs of the Ramsey problem are

$$2\pi_t + 2\frac{\lambda}{\kappa}x_t - k_t^{-1}\varphi_{5,t} - \mathbf{g}_{\pi,t}\varphi_{6,t} = 0 \tag{E.2}$$

$$cx_{t+1} + \varphi_{5,t} - (1 - k_t^{-1})\varphi_{5,t+1} + \mathbf{g}_{\bar{\pi},t}\varphi_{6,t+1} = 0 \tag{E.3}$$

$$\varphi_{6,t} + \varphi_{6,t+1} = fe_{t|t-1}\varphi_{5,t} \tag{E.4}$$

[28] See Woodford (2011) for a discussion of the desirability of target criteria for robustly optimal policy as well as ease of communication with the public.

where the red multiplier is the new element vis-à-vis the case where the anchoring function is specified in levels ($k_t^{-1} = \mathbf{g}(fe_{t|t-1})$), and I'm using the shorthand notation

$$c = -\frac{2(1-\alpha)\beta}{1-\alpha\beta}\frac{\lambda}{\kappa} \tag{E.5}$$

$$fe_{t|t-1} = \pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1} \tag{E.6}$$

(D.8) says that in anchoring, the discretion tradeoff is complemented with tradeoffs coming from learning ($\varphi_{5,t}$), which are more binding when expectations are unanchored (k_t^{-1} high). Moreover, the change in the anchoring of expectations imposes an additional constraint ($\varphi_{6,t}$), which is more strongly binding if the gain responds strongly to inflation ($\mathbf{g}_{\pi,t}$). One can simplify this three-equation-system to:

$$\varphi_{6,t} = -cfe_{t|t-1}x_{t+1} + \left(1 + \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1}^{-1}) - fe_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1} - \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1}^{-1})\varphi_{6,t+2} \tag{E.7}$$

$$0 = 2\pi_t + 2\frac{\lambda}{\kappa}x_t - \left(\frac{k_t^{-1}}{fe_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t^{-1}}{fe_{t|t-1}}\varphi_{6,t+1} \tag{E.8}$$

Thus a central bank that follows the target criterion has to compute $\varphi_{6,t}$ as the solution to (E.8), and then evaluate (E.7) as a target criterion. The solution to (E.8) is given by:

$$\varphi_{6,t} = -2\,\mathbb{E}_t \sum_{i=0}^{\infty}(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}^{-1}}{fe_{t+j|t}}}{\frac{k_{t+j}^{-1}}{fe_{t+j|t}} + \mathbf{g}_{\pi,t+j}} \tag{E.9}$$

Interpretation: the anchoring constraint is not binding ($\varphi_{6,t} = 0$) if the central bank always hits the target ($\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i} = 0,\ \forall i$); or expectations are always anchored ($k_{t+j}^{-1} = 0,\ \forall j$).

F Oscillatory dynamics in adaptive learning models

Here I present an illustration for why adaptive learning models produce oscillatory impulse responses if the gain is high enough. Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{F.1}$$

$$f_t = f_{t-1} + k^{-1}(\pi_t - f_{t-1}) \tag{F.2}$$

The reader can recognize in (F.1) a simplified Phillips curve in which I am abstracting from output gaps to keep the presentation as clear as possible. Like in the simple model of Section 4.2 in the main text, f_t represents the one-period inflation expectation $\hat{\mathbb{E}}_t\pi_{t+1}$. (F.2), then, represents the simplest possible recursive updating of the expectations f_t. My notation of the gain as k^{-1} indicates a constant gain specification, but the intuition remains unchanged for decreasing (or endogenous) gains.

Combining the two equations allows one to solve for the time series of expectations

$$f_t = \frac{1 - k^{-1}}{1 - k^{-1}\beta} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \tag{F.3}$$

which, for β close but smaller than 1, is a near-unit-root process. (In fact, if the gain were to go to zero, this would be a unit root process.) Defining the forecast error as $fe_{t|t-1} \equiv \pi_t - f_{t-1}$, one obtains

$$fe_{t|t-1} = -\frac{1 - \beta}{1 - k^{-1}\beta} f_{t-1} + \frac{1}{1 - k^{-1}\beta} u_t \tag{F.4}$$

Equation (F.4) shows that in this simple model, the forecast error loads on a near-unit-root process with a coefficient that is negative and less than one in absolute value. Damped oscillations are the result.

Note that even if the gain would converge to zero, the coefficient on f_{t-1} would be negative and less than one in absolute value. Thus even for decreasing gain learning, one obtains oscillations, but the lower the gain, the more damped the oscillations become. This corroborates my findings in the impulse responses of Fig. 4. But importantly, the opposite extreme, when $k^{-1} \to 1$, results in a coefficient of exactly -1, giving perpetual oscillations. This clearly illustrates how the oscillatory behavior of impulse responses comes from the oscillations in the forecast error that obtain when the gain is sufficiently large.