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This paper develops a model of the term structures of nominal and real interest rates driven by state variables representing the short-term real interest rate, expected inflation, inflation's central tendency, and four volatility factors that follow GARCH processes. We derive analytical solutions for nominal bond yields, yields on inflation-indexed bonds that have an indexation lag, and the term structure of expected inflation. Unlike prior studies, the model's parameters are estimated using data on inflation swap rates, as well as nominal yields and survey forecasts of inflation. The volatility state variables fully determine bonds' time-varying risk premia and allow for stochastic volatility and correlation between bond yields, yet they have small effects on the cross section of nominal yields. Allowing for time-varying volatility is particularly important for real interest rate and expected inflation processes, but long-horizon real and inflation risk premia are relatively stable. Comparing our model prices of inflation-indexed bonds to those of Treasury Inflation Protected Securities (TIPS) suggests that TIPS were significantly underpriced prior to 2004 and again during the 2008-2009 financial crisis.

Key words: Term structure of interest rates, inflation expectations, asset pricing.

JEL code: E43, G12, E52.

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1 Introduction

Policymakers and finance professionals often use the term structure of Treasury yields to infer expectations of inflation and real interest rates. Inflation expectations can gauge the credibility of a government's fiscal and monetary policies while real interest rates measure the economic cost of financing investments and the tightness of monetary policy. The goal of this paper is to provide an improved methodology for extracting information on these components of Treasury yields. The paper has two main innovations relative to the existing literature. First, it develops a new model of the nominal and real terms structures that provides a convenient, yet realistic, framework for estimating the dynamics of real and inflation-related factors. Second, the paper introduces a new data source for estimating models of real and nominal terms structures, namely, zero-coupon inflation swaps. We present evidence that inflation swaps can provide more reliable information on real yields than inflation-indexed bonds.

Our model of the nominal and real term structures is in the completely affine family, a class which prior empirical work has concluded cannot match both the time series and cross section characteristics of nominal Treasury yields. The completely affine models considered in Dai and Singleton (2000) fail to incorporate the stylized fact that risk premia (expected excess returns) on longer-maturity bonds are highest when the yield curve is steep (Fama and Bliss (1987) and Campbell and Shiller (1991)). Duffee (2002) and Dai and Singleton (2002) show that an "essentially affine" generalization of the completely affine class, which permits more flexible time-varying risk premia, can produce the positive correlation between bond risk premia and the slope of the yield curve, but only when the model's state variables are Gaussian. However, Gaussian models cannot capture the equally well-established time-variation in yields' volatilities that tends to be positively related to their levels (Ait-Sahalia (1996), Brenner, Harjes, and Kroner (1996), and Gallant and Tauchen (1998)). A wealth of empirical work surveyed in Dai and Singleton (2003) and Piazzesi (2005) confirm across a variety of data sets the difficulties in matching both first and second moments of yield changes.

Thus, it may be surprising that this paper's model displays the positive relationship between bond risk premia and the term structure's slope, yet it also incorporates changing yield volatility that increases with yield levels. While the model is completely affine, it does not fall into the typical affine classification of Dai and Singleton (2000) and has yet to be empirically examined. Our model differs because it has four stochastic drivers (sources of risk) yet seven state variables. Three of the state variables in our model are the short-term real interest rate, expected inflation, and inflation's "central tendency," and they have a large influence on the cross-section of bond yields. However, they play no direct role in determining bonds' risk premia. Rather, bond risk premia are completely determined by four volatility state variables whose dynamics are mixtures of normal and chi squared innovations that derive from changes in the aforementioned three state variables plus realized inflation.[1] This de-coupling of the state variables that largely determine the cross-section of yields versus the state variables that solely determine risk premia allows for time-varying risk premia that can even change sign.[2] As a result, the model's ability to fit the cross-section and time-series of nominal yields exceeds that of traditional affine models. Because our model better matches the empirical properties of nominal yields, one may have greater confidence that it provides the correct starting point for decomposing nominal yields into their real yield, expected inflation, and risk premia parts.

The paper's second main contribution is its use of data on inflation swap rates, in addition to nominal Treasury yields and survey forecasts of inflation, to estimate the model's parameters. Note that identifying the parameters of a joint model of nominal and real term structures requires more than just information on nominal yields. Previous work using U.S. data typically employs nominal Treasury yields along with data on Treasury Inflation Protected Securities (TIPS) (D'Amico, Kim, and Wei (2008), Chen, Liu, and Cheng (2010), Christensen, Lopez, and Rudebusch (2010)) or data on survey forecasts of inflation (Pennacchi (1991), Chernov and Mueller (2008)).[3][4] Unlike most studies, we use *three* different sources of data to estimate our model's parameters: nominal Treasury yields, survey forecasts of inflation, and inflation swap rates. As will be shown, real yields on inflation-indexed bonds can be derived as the difference between equivalent maturity nominal yields and inflation swap rates, and these derived real yields are less prone to uncertain changes in liquidity than TIPS yields. For this reason, inflation swaps can be a more reliable indicator of real yields. While also using survey forecasts of inflation creates

[1] The chi squared innovations lead to yield changes whose volatilities are stochastic, displaying both skewness and kurtosis.

[2] In this regard, our model is quite different from the usual completely affine model specifications. We have neither constant market prices of risk, as in Vasicek (1977) specifications, nor specifications proportional to the square root of variables that directly affect the levels of yields, as in Cox, Ingersoll, and Ross (1985) square root specifications.

[3] Similarly, Barr and Campbell (1997) and Evans (1998) use UK data on yields from nominal and indexed-linked gilts.

[4] Ang, Bekaert, and Wei (2008) develop a regime-switch model that is estimated using data on nominal yields and only actual inflation. Essentially they achieve identification, by inferring expected inflation from the actual inflation process along with imposing other parameter restrictions. Buraschi and Jiltsov (2005) develop a structural monetary model of an economy that allows them to estimate the essentially affine model's nominal and real processes using nominal yields and the processes for actual inflation and the M2 money supply.

extra demands on our model's ability to match all observations, it allows us to better identify physical expectations of inflation (as reflected in survey forecasts) from inflation risk premia (which are present in nominal yields).[5]

Based on our model's parameter estimates, we are able to compute term structures of inflation expectations and inflation-indexed (real) yields over our entire 1982 to 2010 sample period. Comparing our real yields to those of TIPS beginning in 1999, we confirm the results of prior studies that found massive underpricing of TIPS during their early years, followed by fair pricing from 2004 to 2008. Enormous underpricing of TIPS returns during the 2008-2009 financial crisis years.

We obtain several other noteworthy results. First, we find that the short term real interest rate is typically the most volatile component of the yield curve, and it is especially important to allow its volatility to be stochastic. Real rates were negative for much of the 2002 to 2005 period, which may have helped inflate a credit bubble. Second, we find that expected inflation over short horizons is also volatile, displays significant stochastic behavior, and has high negative correlation with real rates, likely an artifact of the Federal Reserve's practice of pegging the short term nominal interest rate. Moreover both real rates and expected inflation display rather strong mean reversion. Third, over our 1982 to 2010 sample period, inflation's central tendency, which can be viewed as investors' expectation of longer-term inflation, declined substantially, consistent with greater investor credibility of the Federal Reserve's desire to maintain low inflation. Fourth, we find a real interest rate risk premium that is substantial, varying between 87 and 121 basis points for a ten-year bond during our sample period. The inflation risk premium is more moderate, varying between 23 and 55 basis points for the 10-year maturity.

The paper proceeds as follows. Section 2 introduces a model of real interest rates and inflation that is used to derive the term structures of nominal bonds, inflation forecasts, inflation-indexed bonds, and inflation swap rates. Section 3 develops the analytical solutions. Section 4 describes the data used and explains the estimation technique. Section 5 describes the results and Section 6 concludes.

[5]Expectations of inflation are independent of risk premia while nominal and real yields incorporate risk premia. Employing both inflation swaps and survey forecasts permits better identification of physical versus risk-neutral processes.

2 A Model of Nominal and Real Term Structures

Consider a discrete time environment with multiple periods, each of length Δt measured in years. Let M_t be the nominal pricing kernel with dynamics

$$\frac{M_{t+\Delta t}}{M_t} = e^{-i_t \Delta t - \frac{1}{2}\sum_{j=1}^{4} \phi_j^2 h_{j,t}^2 \Delta t - \sum_{j=1}^{4} \phi_j h_{j,t} \sqrt{\Delta t} \epsilon_{j,t+\Delta t}} \tag{1}$$

Here $\epsilon_{j,t+\Delta t}, j = 1, 2, .., 4$ are independent standard normal random variables and $\phi_j h_{j,t}$, $j = 1, 2, .., 4$ are market prices of risk associated with these four sources of uncertainty. The ϕ_j are constants while the $h_{j,t}$ are volatility state variables whose dynamics will be specified shortly. i_t is the annualized, one-period nominal interest rate.

Denote the consumer price index (dollar value of the consumption basket) at date t as I_t. Its dynamics are

$$\frac{I_{t+\Delta t}}{I_t} = e^{\pi_t \Delta t - \frac{1}{2} h_{1,t}^2 \Delta t + h_{1,t} \sqrt{\Delta t} \epsilon_{1,t+\Delta t}} \tag{2}$$

where the variable $\pi_t = \frac{1}{\Delta t} \ln \left(E_t \left[I_{t+\Delta t} / I_t \right] \right)$ is the rate of expected inflation for the period from t to $t + \Delta t$.

Given the processes for the nominal pricing kernel and the price index, the process for the real (inflation-indexed) pricing kernel, m_t, is

$$\begin{aligned} \frac{m_{t+\Delta t}}{m_t} &= \frac{M_{t+\Delta t}}{M_t} \frac{I_{t+\Delta t}}{I_t} \\ &= e^{\left(\pi_t - i_t - \frac{1}{2} h_{1,t}^2\right)\Delta t - \frac{1}{2}\sum_{j=1}^{4} \phi_j^2 h_{j,t}^2 \Delta t - \sum_{j=1}^{4} \phi_j h_{j,t} \sqrt{\Delta t} \epsilon_{j,t+\Delta t} + h_{1,t} \sqrt{\Delta t} \epsilon_{1,t+\Delta t}} \end{aligned} \tag{3}$$

Taking expectations of the left-hand-side of (3) defines r_t, the one period real rate:

$$E_t \left[\frac{m_{t+\Delta t}}{m_t} \right] = e^{-r_t \Delta t} \tag{4}$$

Taking expectations on the right-hand-side of (3) and equating it to (4) implies

$$i_t = \pi_t + r_t - \phi_1 h_{1,t}^2. \tag{5}$$

To complete the model, the dynamics of the state variables are specified as

$$
\begin{aligned}
\pi_{t+\Delta t} - \pi_t &= [\alpha_t + a_1 r_t + a_2 \pi_t]\,\Delta t + \sqrt{\Delta t}\Sigma_{j=1}^{2}\beta_j h_{j,t}\epsilon_{j,t+\Delta t} \\
r_{t+\Delta t} - r_t &= [b_0 + b_1 r_t + b_2 \pi_t]\,\Delta t + \sqrt{\Delta t}\Sigma_{j=1}^{3}\gamma_j h_{j,t}\epsilon_{j,t+\Delta t} \\
\alpha_{t+\Delta t} - \alpha_t &= [c_0 + c_1 \alpha_t]\,\Delta t + \sqrt{\Delta t}\Sigma_{j=1}^{4}\rho_j h_{j,t}\epsilon_{j,t+\Delta t} \\
h^2_{j,t+\Delta t} - h^2_{j,t} &= \left[d_{j0} + d_{j1} h^2_{j,t}\right]\Delta t + d_{j2}\Delta t(\epsilon_{j,t+\Delta t} - d_{j3} h_{j,t})^2, \quad j = 1, ..., 4
\end{aligned}
\tag{6}
$$

where α_t is an additional state variable that shifts the future path of π_t. Subject to parameter stationarity conditions, the unconditional means (steady state levels) of expected inflation and the real interest rate are

$$
\overline{\pi} = -\frac{a_1 b_0 c_1 + b_1 c_0}{(a_1 b_2 - a_2 b_1)\, c_1} \tag{7}
$$

$$
\overline{r} = \frac{a_2 b_0 c_1 + b_2 c_0}{(a_1 b_2 - a_2 b_1)\, c_1} \tag{8}
$$

The unconditional mean of α_t is $-c_0/c_1 = -(a_1\overline{r} + a_2\overline{\pi})$. If a constant is added to α_t such that $\widehat{\alpha}_t \equiv \alpha_t + a_1\overline{r} + (1 + a_2)\,\overline{\pi}$, then the unconditional mean of $\widehat{\alpha}_t$ equals $\overline{\pi}$, and $\widehat{\alpha}_t$ is commonly referred to as the 'central tendency' of the rate of expected inflation.[6] For simplicity, we refer to α_t as the central tendency, but it should be understood that it differs from the true central tendency, $\widehat{\alpha}_t$, by a constant.

The $h_{j,t}$ are volatility state variables that satisfy the Nonlinear Asymmetric GARCH model of Engle and Ng (1993). Subject to stationarity conditions, their steady-state levels are

$$
\overline{h}_j^2 = -\frac{d_{j0} + d_{j2}}{d_{j1} + d_{j2} d_{j3}^2}\ , \qquad j = 1, ..., 4 \tag{9}
$$

The equations in (2) and (6) specify that actual inflation, expected inflation, the real interest rate, and inflation's central tendency follow imperfectly correlated, stochastic volatility processes. Their correlations depend on the β_j, γ_j, and ρ_j coefficients multiplying the four orthogonal shocks, $h_{j,t}\epsilon_{j,t+\Delta t}$, $j = 1, ..., 4$, but without loss of generality, we can restrict $\beta_2 = \gamma_3 = \rho_4 = 1$.[7] If the four volatility state variables are shut down, the model

[6] Prior research supports a time-varying central tendency for inflation in order to adequately fit the term structure (Balduzzi, Das, and Foresi (1998), Jegadeesh and Pennacchi (1996)). Kozicki and Tinsley (2001) show that a "shifting endpoint" for the short term interest rate process captures historical changes in market perceptions of the policy target for inflation and significantly improves long-horizon forecasts of short term interest rates. Gurkaynak, Sack, and Swanson (2005) also find that modeling a short rate with a central tendency is required to match the high-frequency response of long term bond yields to macroeconomic announcements.

[7] These restrictions permit identification of the levels of the volatility factors $h_{2,t}$, $h_{3,t}$, and $h_{4,t}$.

becomes Gaussian in the three state variables, π_t, r_t, and α_t.[8]

The model's state variables can be written as a 7×1 vector $x_t \equiv (\pi_t \; r_t \; \alpha_t \; h_{1t}^2 \; h_{2t}^2 \; h_{3t}^2 \; h_{4t}^2)'$ while the market prices of risk associated with each of the four shocks $h_{j,t}\epsilon_{j,t+\Delta t}$, $j = 1, ..., 4$ is the 4×1 vector $\Lambda_t \equiv (\phi_1 h_{1t} \; \phi_2 h_{2t} \; \phi_3 h_{3t} \; \phi_4 h_{4t})'$. The compensation for risk depends on the levels of the square roots of the $h_{j,t}^2$ state variables, but not the other state variables. Furthermore, because the processes for the $h_{j,t}^2$ state variables depend on both the levels of the innovations, $\epsilon_{j,t}$, and their squares, $\epsilon_{j,t}^2$, the date $t + \Delta t$ distribution of the state vector $x_{t+\Delta t}$ conditional on x_t is not multivariate normal but a mixture of normals and chi squared distributions. Since bond yields are shown to be affine in x_t, yield changes will display the skewness and kurtosis derived from x_t.

As $\Delta t \to 0$ our model can be made to converge to many possible diffusive limits. The proposition below describes one possible case.

Proposition 1

If we define $d_{j0} = (\kappa_j \theta_j - \frac{v_j^2}{4})$, $d_{j1} = -\frac{1}{\Delta t}$, $d_{j2} = \frac{v_j^2}{4}$, *and* $d_{j3} = \frac{2(1-\kappa_j \Delta t/2)}{v_j \sqrt{\Delta t}}$, *then the limiting dynamics of (6) is*

$$\begin{aligned} d\pi_t &= (\alpha_t + a_1 r_t + a_2 \pi_t)dt + \Sigma_{j=1}^2 \beta_j h_{j,t} dW_j(t) \\ dr_t &= (b_0 + b_1 r_t + b_2 \pi_t)dt + \Sigma_{j=1}^3 \gamma_j h_{j,t} dW_j(t) \\ d\alpha_t &= (c_0 + c_1 \alpha_t)dt + \Sigma_{j=1}^4 \rho_j h_{j,t} dW_j(t) \\ dh_{j,t}^2 &= \kappa_j(\theta_j - h_{j,t}^2)dt - v_j \sqrt{h_{j,t}^2} dW_j(t), \quad j = 1, ..., 4 \end{aligned} \tag{10}$$

where dW_{jt}, $j = 1, ..., 4$ *are independent Wiener processes.*

Proof: See Appendix

One can view our discrete time model as an approximation to the above continuous time, stochastic volatility model.[9] Our empirical work assumes a discrete time period of $\Delta t = 1/12$ year; that is, one month.

2.1 Nominal and Inflation-Indexed Bonds

The model is estimated using data on nominal Treasury yields, inflation swap rates, and survey forecasts of inflation. This section derives model prices for nominal and inflation-

[8]This no-GARCH case occurs when $d_{j1} = -1/\Delta t$ and $d_{j2} = d_{j3} = 0$, so that $h_{j,t}^2 = \overline{h}_j^2$ $\forall t$. It corresponds to a multivariate Vasicek (1977) model as developed in Langetieg (1980).

[9]Duan, Ritchken, and Sun (2006) provide examples of convergence of discrete time GARCH models to continuous time stochastic volatility models.

indexed (real) bonds, which also determine inflation swap rates. The following section gives model formula for inflation forecasts.

Let $P_{N,t}^{(n)}$ and $y_{N,t}^{(n)}$ be the date t price and continuously-compounded yield, respectively, of a nominal bond that pays \$1 at date $t + n\Delta t$. Similarly, $P_{R,t}^{(n)}$ and $y_{R,t}^{(n)}$ are the date t real price and yield of a bond that pays one unit of the consumption basket at date $t + n\Delta t$. In practice, an inflation-indexed bond typically pays semi-annual coupons, but since its payments are a portfolio of zero-coupon payments, it is sufficient to value a zero-coupon inflation-indexed bond. Moreover, inflation-indexed payments are not fully protected against inflation. For example, the inflation index for a TIPS payment is based on the Consumer Price Index (CPI) recorded three months prior to the payment date.[10]

Since this indexation lag can be important, define $V_{t,t_s}^{(n,d)}$ to be the date t nominal price of a zero-coupon TIPS that has n periods remaining to its payment at date of $t_p = t+n\Delta t$. Let t_0 be this TIPS bond's initiation (issue) date, and let d be the indexation lag in periods. Following actual practice, the payment made at date t_p is based on accumulated inflation over the period from $t_s \equiv t_0 - d\Delta t$ to $t_e \equiv t_p - d\Delta t$. Thus, the payment at date t_p equals accumulated inflation, I_{t_e}/I_{t_s}, over the life of the bond but lagged d periods. For TIPS, $d\Delta t = 3 \times 1/12 = \frac{1}{4}$ year or 3 months. Now note that at date t_e the value of the payment to be made d periods later is

$$V_{t_e,t_s}^{(d,d)} = \frac{I_{t_e}}{I_{t_s}} P_{N,t_e}^{(d)} \ , \tag{11}$$

and at date t, with n periods to go to date t_p, we have

$$V_{t,t_s}^{(n,d)} = E_t \left[\frac{M_{t+\Delta t}}{M_t} V_{t+\Delta t,t_s}^{(n-1,d)} \right] . \tag{12}$$

Since $V_{t,t_s}^{(n,d)}$ is the date t nominal price of receiving I_{t_e}/I_{t_s} dollars at date t_p, $V_{t,t_s}^{(n,d)} I_{t_s}$ is the nominal price of receiving I_{t_e} dollars at date t_p. If $P_{R,t}^{(n,d)}$ is defined to be the date t real price of receiving I_{t_e} dollars at date t_p, then

$$P_{R,t}^{(n,d)} = V_{t,t_s}^{(n,d)} \frac{I_{t_s}}{I_t} \ . \tag{13}$$

With no indexation delay ($d = 0$), $P_{R,t}^{(n,0)} = P_{R,t}^{(n)}$ represents the real price of a claim that pays one unit of the consumption basket in n periods.

[10] A reason for this delay is that the CPI is not reported immediately at the date for which it is recorded, but with a lag. Most valuation models of inflation-indexed bonds ignore this indexation lag feature. An exception is Risa (2001) which is a multifactor, essentially affine, Gaussian model.

Inflation Swaps

Derivative securities known as 'zero coupon inflation swaps' are the most liquid inflation derivative contracts that trade in the over-the-counter market. They are quoted with maturities ranging from 1 to 30 years. Together with nominal Treasuries, they provide a measure of real yields that is an alternative to TIPS yields.

A zero coupon inflation swap is a forward contract whereby the inflation buyer pays a predetermined fixed nominal rate and in return receives from the seller an inflation-linked payment. Denote the inflation swap's initiation date as t_0 and its maturity (payment) date as t_p. Similar to TIPS, the inflation-linked payment made at date t_p equals I_{t_e}/I_{t_s} where, as before, $t_s = t_0 - d\Delta t$, $t_e \equiv t_p - d\Delta t$, and $d\Delta t = \frac{1}{4}$ years. In return for receiving I_{t_e}/I_{t_s}, the inflation buyer makes a predetermined fixed payment of $e^{k(t_e - t_s)}$ where k is the continuously-compounded inflation swap rate.[11] Thus, the net fixed for inflation swap payment is $e^{k(t_e-t_s)} - I_{t_e}/I_{t_s}$.

Viewed from date t, the value of the fixed (nominal) leg is simply

$$V_{fix}(t) = P_{N,t}^{(n)} e^{k(t_e - t_s)}. \tag{14}$$

The value of the inflation leg, $V_{inf}(t)$ say, equals the value of a zero coupon TIPS with payouts at date t_p linked to the index values at dates t_s and t_e:

$$V_{inf}(t) = V_{t,t_s}^{(n,d)} = P_{R,t}^{(n,d)} \frac{I_t}{I_{t_s}}. \tag{15}$$

At the initiation date, t_0, the fair inflation swap rate is the value k that equates $V_{fix}(t_0)$ with $V_{inf}(t_0)$:

$$k^*(t_0; t_s, t_e) = y_{N,t_0}^{(n)} - y_{R,t_0}^{(n,d)} = be_{t_0}^{(n,d)} \tag{16}$$

where $y_{R,t_0}^{(n,d)}$ is defined as:

$$y_{R,t_0}^{(n,d)} = -\frac{1}{n\Delta t} \ln V_{t_0,t_s}^{(n,d)} = -\frac{1}{n\Delta t} \ln \left(P_{R,t_0}^{(n,d)} I_{t_0}/I_{t_s} \right) \tag{17}$$

and $be_t^{(n,d)}$ is the break-even inflation rate for a maturity of $n\Delta t$ years. The above discussion shows that once we have a valuation equation for a TIPS, we also have a valuation equation for a fair inflation swap rate. Moreover, $y_{N,t_0}^{(n)} - k^*(t_0; t_s, t_e) = y_{R,t_0}^{(n,d)}$ is a measure of an n-period maturity real yield that is an alternative to a TIPS yield.

[11] In practice, inflation swap rates are quoted as annually-compounded rates, say k_a, where $k_a = \ln(k) - 1$. Our analysis translates these rates to continuously-compounded ones.

The following proposition provides the recursive equation for the values of both the nominal and the real bonds which, in turn, can be used to value TIPS and inflation swaps.

Proposition 2

Under the above dynamics, nominal and real bond prices are given by the following recursive equations:

$$P_{N,t}^{(n)} = e^{-K_n - A_n\pi_t - B_n r_t - C_n\alpha_t - \sum_{j=1}^{4} D_{j,n}h_{j,t}^2} \quad \textit{for } n \geq 1 \tag{18}$$

$$P_{R,t}^{(n,d)} = e^{-\tilde{K}_n - \tilde{A}_n\pi_t - \tilde{B}_n r_t - \tilde{C}_n\alpha_t - \sum_{j=1}^{4} \tilde{D}_{j,n}h_{j,t}^2} \quad \textit{for } n \geq d, \tag{19}$$

where $K_1 = 0$, $A_1 = \Delta t$, $B_1 = \Delta t$, $C_1 = 0$, $D_{1,1} = -\phi_1\Delta t$, $D_{j,1} = 0$ *for* $j = 2, 3, 4$, *and* $\tilde{K}_d = K_d$, $\tilde{A}_d = A_d$, $\tilde{B}_d = B_d$, $\tilde{C}_d = C_d$, $\tilde{D}_{j,d} = D_{j,d}$ *for* $j = 1, 2, 3, 4$ *and the recursive equations are contained in the Appendix.*

Proof: See the Appendix

2.2 Expected Inflation Rates

Our model's parameters are estimated with data that includes survey forecasts of an inflation rate that begins and ends at two future dates. If the current date is t while $t + n\Delta t$ and $t + (n + m)\Delta t$ are the dates when the inflation rate starts and ends, then the forecast of this continuously compounded inflation rate is[12]

$$E_t\left[\frac{1}{m\Delta t}\ln\left(\frac{I_{t+(n+m)\Delta t}}{I_{t+n\Delta t}}\right)\right] = \frac{1}{m\Delta t}\left(E_t\left[\ln\left(\frac{I_{t+(n+m)\Delta t}}{I_t}\right)\right] - E_t\left[\ln\left(\frac{I_{t+n\Delta t}}{I_t}\right)\right]\right) \tag{20}$$

which is the difference between expectations of an inflation rate over two different horizons. Proposition 3 provides the formula for such an expected rate of inflation.

Proposition 3

The date t expectation of the inflation rate for a horizon of n periods is

$$E_t[\ln(I_{t+n\Delta t}/I_t)] = K_n^* + A_n^*\pi_t + B_n^* r_t + C_n^*\alpha_t + \Sigma_{j=1}^4 D_{j,n}^* h_{j,t}^2 \quad \textit{for } n \geq 1 \tag{21}$$

where $K_1^* = 0$, $A_1^* = \Delta t$, $B_1^* = 0$, $C_1^* = 0$, $D_{1,1}^* = -\frac{1}{2}\Delta t$, *and* $D_{j,1}^* = 0$, *for* $j = 2, 3, 4$ *and where the recursions are provided in the Appendix.*

Proof: See the Appendix.

As detailed in the Appendix, the market price of risk parameters, ϕ_j, $j = 1, .., 4$,

[12]For example, $m = 3$ months when the forecasted inflation rate is for a future quarter of a year.

appear in the formulas for nominal yields and inflation swap rates (as well as inflation-indexed yields), but are absent from the above formula for a forecasted inflation rate. By combining data that reflect risk premia as well as data that do not, we can better identify parameters that determine expectations of the state variables versus those that characterize risk premia.[13]

3 Bond Risk Premia and Yield Volatility

Since our empirical work examines our model's risk premia (expected excess returns) for nominal and real bonds as well as the variances and covariances of nominal and real yields, this section derives formulae for these quantities. Rates of return on n-period bonds are given by

$$r_{j,t+\Delta t}^{(n)}\Delta t = \ln\left(P_{j,t+\Delta t}^{(n-1)}/P_{j,t}^{(n)}\right) = y_{j,t}^{(n)}(n\Delta t) - y_{j,t+\Delta t}^{(n-1)}(n-1)\Delta t, \text{ for } j = N, R. \quad (22)$$

where if $j = N$ ($j = R$) equation (22) denotes the nominal (*real*) rate of return on a nominal (*real*) bond. Since $y_{N,t}^{(1)} = i_t$ and $y_{R,t}^{(1)} = r_t$, equation (22) implies that the corresponding expected excess returns (risk premia) equal

$$\begin{aligned} \pi_{j,t}^{(n)} &= E_t\left[r_{j,t+\Delta t}^{(n)}\right] - y_{j,t}^{(1)}, \text{ for } j = N, R \\ &= \left(y_{j,t}^{(n)} - E_t\left[y_{j,t+\Delta t)}^{(n-1)}\right]\right)(n-1) + s_{j,t}^{(n)}, \end{aligned} \quad (23)$$

where $s_{j,t}^{(n)} = y_{j,t}^{(n)} - y_{j,t}^{(1)}$ is the slope of the yield curve. The following proposition gives our model's expressions for these risk premia.

Proposition 4

For zero coupon bonds of maturity n periods, the expected excess nominal return on a nominal bond and the expected excess real return on a real bond are

$$\pi_{N,t}^{(n)} = \frac{1}{\Delta t}\Sigma_{j=1}^{4}\left(\nu_{j0}^{(n)} + \nu_{j1}^{(n)}h_{j,t}^2\right) \quad (24)$$

$$\pi_{R,t}^{(n)} = \frac{1}{\Delta t}\Sigma_{j=1}^{4}\left(\nu_{j0}^{*(n)} + \nu_{j1}^{*(n)}h_{j,t}^2\right) \quad (25)$$

[13] Dai and Singleton (2000) discuss identification restrictions for affine models. Chernov and Mueller (2008) estimate a nominal and real term structure model based on Ang and Piazzesi (2003) that uses only Treasury yields and survey forecasts of inflation. To cope with their difficulty in estimating risk premia, they add to their likelihood function a term that penalizes large risk premia estimates. By also using inflation swap rates, we estimate reasonable risk premia without modifying the likelihood function.

where

$$\begin{aligned}
\nu_{j0}^{(n)} &= \frac{1}{2}\ln(1+2D_{j,n-1}d_{j2}\Delta t) - d_{j2}D_{j,n-1}\Delta t \\
\nu_{j1}^{(n)} &= \frac{1}{2}\phi_j^2\Delta t - \frac{Q_{j,n-1}^2}{2(1+2D_{j,n-1}d_{j2}\Delta t)} \\
\nu_{j0}^{*(n)} &= \frac{1}{2}\ln(1+2\tilde{D}_{j,n-1}d_{j2}\Delta t) - d_{j2}\tilde{D}_{j,n-1}\Delta t \\
\nu_{j1}^{*(n)} &= (\frac{1}{2}-\phi_1)I_1\Delta t + \frac{1}{2}\phi_j^2\Delta t - \frac{\tilde{Q}_{j,n-1}^2}{2(1+2\tilde{D}_{j,n-1}d_{j2}\Delta t)}
\end{aligned}$$

where I_1 is an indicator variable equal to 1 if $j = 1$ and 0 otherwise and where $Q_{j,n}$ and $\tilde{Q}_{j,n}$ are defined in the Appendix.

The next proposition gives expressions for the covariances of changes in yields.[14]

Proposition 5

The covariance of the changes in yields for m-period and k-period maturity nominal and real bonds of are given by

$$Cov_t\left(y_{N,t+\Delta t}^{(m)} - y_{N,t}^{(m)}, y_{N,t+\Delta t}^{(k)} - y_{N,t}^{(k)}\right) = \frac{\Sigma_{j=1}^4\left(a_{jm}a_{jk}h_{jt}^2\Delta t + 2D_{jm}D_{jk}d_{j2}^2\Delta t^2\right)}{mk\Delta t^2} \quad (26)$$

$$Cov_t\left(y_{R,t+\Delta t}^{(m)} - y_{R,t}^{(m)}, y_{R,t+\Delta t}^{(k)} - y_{R,t}^{(k)}\right) = \frac{\Sigma_{j=1}^4\left(a_{jm}^*a_{jk}^*h_{jt}^2\Delta t + 2\tilde{D}_{jm}\tilde{D}_{jk}d_{j2}^2\Delta t^2\right)}{mk\Delta t^2} \quad (27)$$

where

$$\begin{aligned}
a_{1n} &= \beta_1 A_n + \gamma_1 B_n + \rho_1 C_n - 2d_{12}d_{13}\sqrt{\Delta t}D_{1n} \\
a_{2n} &= \beta_2 A_n + \gamma_2 B_n + \rho_2 C_n - 2d_{22}d_{23}\sqrt{\Delta t}D_{2n} \\
a_{3n} &= \gamma_3 B_n + \rho_3 C_n - 2d_{32}d_{33}\sqrt{\Delta t}D_{3n} \\
a_{4n} &= \rho_4 C_n - 2d_{42}d_{43}\sqrt{\Delta t}D_{4n}
\end{aligned}$$

and the a_{jn}^* values have the same form as a_{jn} values except the A_n, B_n, C_n, and $D_{j,n}$ values are replaced by $\tilde{A}_n$, $\tilde{B}_n$, $\tilde{C}_n$ and $\tilde{D}_{j,n}$ values, respectively.

[14]The variance of a change in yield is the case when $m = k$ in Proposition 5.

4 Data and Estimation Method

4.1 Data Description

Estimation of our model uses monthly data on U.S. Treasury yields, survey forecasts of inflation, rates of actual (realized) inflation, and inflation swap rates. Most data series are available from January 1982 to May 2010, though data on inflation swap rates starts only in April 2003. Nominal Treasury yields are obtained from two sources. First, we obtain zero coupon yields of 1, 2, 3, 5, 7, 10, and 15 years to maturity from daily off-the-run Treasury yield curves constructed by Gurkaynak, Sack, and Wright (2007).[15] Second, daily secondary market yields for 1-month, 3-month, and 6-month Treasury bills are taken from the Federal Reserve System's H.15 Release.[16] All of the Treasury yields are observed as of the first trading day of each month.

Survey forecasts of CPI inflation come from two sources. First, a monthly series beginning in 1982 is obtained from Blue Chip Economic Indicators (BCEI) which surveys approximately 50 economists employed by financial institutions, non-financial corporations, and research organizations. At the beginning of each month, participants forecast future CPI inflation for quarterly time periods, starting from the current calendar quarter and going out to at most 8 quarters (2 years) in the future. For January, February, and March, inflation rate forecasts for 8 future quarters are made. For April, May, and June, forecasts for 7 future quarters are made. For July, August, and September, forecasts for 6 future quarters are made, while for October, November, and December, forecasts for 5 future quarters are made. We use BCEI's reported 'consensus' forecast which is the average of the participants' forecasts.

Second, we use the median forecast of CPI inflation over the next ten years made by the approximately 40 participants of the Survey of Professional Forecasters (SPF), conducted by the Federal Reserve Bank of Philadelphia. This 10-year forecast is at a quarterly frequency starting in December of 1991.[17] Keane and Runkle (1990) find that SPF forecasts appear to be rational expectations of inflation that incorporate public information. Ang, Bekaert, and Wei (2007) find that SPF forecasts significantly outperform a variety of

[15]Their daily Treasury yield curves are available from 1961 to the present at http://www.federalreserve.gov/econresdata/researchdata.htm.

[16]The Federal Reserve's H.15 Release provides one-month (4-week) Treasury bill yields begining in August of 2001. Prior to this date, one-month Treasury bill yields are from the Center for Research in Security Prices (CRSP).

[17]SPF participants make forecasts at approximately, the middle of February, May, August, and November of each year. To align this survey with our other data, we presume these forecasts come at the start of the next month.

other methods for predicting inflation. Since the participants in the BCEI survey have qualifications similar to those of the SPF participants, it is likely that the BCEI forecasts also possess these attractive features. Along with both sets of survey forecasts of inflation, we also constructed a monthly time series of actual CPI inflation.[18]

In addition, we obtained bid and ask quotes of inflation swap rates for the first trading day of each month from Bloomberg for annual maturities from 2 to 10 years, as well as 12-, 15-, 20-, and 30-year maturities. The 2- to 10-year swap maturities start in April of 2003, the 12-, 15-, and 20-year inflation swap rates start in November 2003, and the 30-year inflation swap rates start in March 2004.

While not used in the estimation of our model, we will compare our model's implied yields for inflation-indexed bonds to the actual yields on TIPS. Data on zero-coupon TIPS yields are obtained from Gurkaynak, Sack, and Wright (2008) who derive them from TIPS coupon bond yields.[19]

Table 1 provides summary statistics on our data. The first panel describes the levels and standard deviation of changes of nominal Treasury yields and survey inflation forecasts over the 1982 to 2010 period. As might be expected, the term structure of average nominal yields is upward sloping. The standard deviation of yield changes declines with maturity, consistent with mean reversion in short-term yields. BCEI forecasts of inflation averaged somewhat over 3% from 1982 to 2010 while the SPF forecasts during the shorter 1991 to 2010 period averaged 2.73%. The standard deviation of changes in forecasts mostly declined with maturity. The one-month forecast was highly volatile, perhaps reflecting survey participants knowledge of how recent wholesale and commodity price changes would soon affect next month's consumer prices.

The second panel of Table 1 gives statistics on the levels (midpoint of bid-ask quotes) and changes of inflation swap rates during the 2003 to 2010 period. The standard deviations of monthly changes generally decline with maturity, and correlations decline as the gap between maturities increase. The average levels of rates increased with maturity, consistent with a positive inflation risk premium. Recall from equation (16) that in a frictionless market, inflation swap rates should equal the difference between equivalent-maturity, zero-coupon nominal Treasury and TIPS yields; that is, the TIPS breakeven inflation rate.

[18]Survey participants are asked to forecast the seasonally-adjusted CPI inflation rate, so our actual monthly CPI time series is also seasonally-adjusted. However, TIPS and zero-coupon inflation swaps are indexed to the seasonally-unadjusted CPI. This difference is unlikely to have much impact on TIPS yields and swap rates, except perhaps for those with very short times to maturity. The variation in the CPI due to seasonal adjustments is likely to be small compared to other sources of CPI variation, particularly for medium- and longer- term horizons.

[19]Their dataset is available at http://www.federalreserve.gov/econresdata/researchdata.htm.

Because we are unaware of any prior studies that have used inflation swap rates in term structure estimation, we detail in Figure 1 how swap rates compare to the TIPS breakeven rate. The top two panels in Figure 1 plot the inflation swap rates and TIPS breakeven rates for five- and ten-year maturities over the April 2003 to June 2010 period.

As can be seen, the rates display significant variation. Typically, the difference between the inflation swap rate and the breakeven rate remained fairly stable, perhaps reflecting the cost of replication. However, during the financial crisis when replication became difficult, this stable relationship became distorted.[20] The solid curve in the bottom right panel of Figure 1 shows the gap (in basis points) between the inflation swap rate and the TIPS breakeven rate. This gap remained fairly flat until the Lehman Brothers bankruptcy in September 2008, after which it increased dramatically by about 60 basis points.

What accounted for this break in historical relationships? The bottom left panel of Figure 1 compares the bid-ask spread of 10-year inflation swap rates with the bid-ask spread of the 10-year TIPS, both series obtained from Bloomberg. Since mid-2005, the inflation swap spread ranged mostly from 6 to 10 basis points, except for a short period in September 2007 when oil prices surged and for very brief periods in 2009. In contrast, the spread on the 10-year TIPS increased from a small base of 0.5 basis points to over 10 basis points during the crisis, before settling down to around 4 basis points. Thus TIPS sustained a relatively larger rise in its bid-ask spread during the crisis, suggesting that it experienced a relatively large, sustained rise in illiquidity. TIPS's illiquidity appears to explain the huge gap between breakeven inflation rates extracted from these two markets. The bottom right panel in Figure 1 shows that the difference between the inflation swap rate and TIPS breakeven rate is highly correlated with TIPS's (scaled) bid-ask spread. This evidence is consistent with Hu and Worah (2009) who attribute the spike in TIPS yields following Lehman Brothers' bankruptcy to Lehman's use of substantial amounts of TIPS to collateralize their repo borrowings and derivative positions. Lehman's bankruptcy led to creditors releasing a flood of TIPS into the market at a time when there were few willing buyers.[21] In contrast, the effect of a liquidity crisis on prices of *derivatives* is theoretically unclear, but, as evidenced by the relatively flat bid-ask spread of inflation

[20] Fleckenstein, Longstaff, and Lustig (2010) find that Treasury supply-related factors affect the difference between inflation swap rates and TIPS breakeven rates. The difference narrows when the U.S. Treasury auctions either nominal Treasuries or TIPS, but it widens when dealers have difficulties obtaining Treasury securities, such as during a period of increased repo failures.

[21] Many hedge funds that had previously bought TIPS also were forced to sell to meet withdrawals by clients. As TIPS yields rose, breakeven inflation rates fell to unreasonable levels that under normal market conditions would trigger arbitrage trades given the much higher inflation swap rates. But during the crisis, institutions abandoned relative value trades to seek safety in nominal Treasuries.

swaps during the crisis, the effect was minimal.[22]

One aspect of TIPS that our analysis has ignored is the embedded put option that protects TIPS investors against deflation on the bond's principal (but not coupon) payment. Since this put option has a non-negative value, its presence increases a TIPS's price, and hence decreases its yield, relative to an inflation-indexed bond that lacks this option. Note that zero-coupon inflation swap contracts do not contain this option. Therefore, all else equal, breakeven inflation based on a TIPS principal strip should be higher than the equivalent-maturity inflation swap rate. Moreover, if the financial crisis raised fears of deflation, the difference between the inflation swap rate and the TIPS breakeven rate should have declined (become more negative). Yet, Figure 1 shows exactly the opposite occurred, implying that TIPS's rising illiquidity dominated any increase in the deflation put option that would have lowered TIPS yields.

Use of data on TIPS yields is problematic not only for the recent financial crisis. Studies by Sack and Elsasser (2004), Shen (2006), and D'Amico, Kim, and Wei (2008) reveal that the TIPS breakeven inflation rate consistently fell below survey measures of inflation expectations and that TIPS yields contain a liquidity premium that, in the time period prior to 2004, was unreasonably large and difficult to account for in any rational pricing framework. Shen (2006) finds evidence of a drop in the liquidity premium on TIPS around 2004 that he attributes to the U.S. Treasury's greater issuance of TIPS around that time, as well as to the beginning of exchange traded funds that purchased TIPS.

This accumulated evidence on the distortions to TIPS yields led us to employ inflation swap rates and survey inflation forecasts as a more reliable reflection of real yields and expected inflation.[23] In addition, by not using TIPS yields to estimate our model, we can compare our model-implied yields on inflation-indexed bonds to actual TIPS yields in order to evaluate earlier studies' conclusions regarding the systematic mispricing of TIPS.

[22]Inflation swap rates may have been less affected because swap dealers abandoned the hedging of their positions by trades in nominal bonds and TIPS. During the crisis, dealers may have acted merely as brokers, so that swap rates adjusted to equate the aggregate demand and supply for inflation protection, irrespective of the market prices of nominal Treasuries and TIPS. Campbell, Shiller, and Viceira (2009) conclude that during the crisis, the inflation swap market provided a more accurate assessment of inflation rates than the underlying TIPS breakeven rates.

[23]An alternative is to use TIPS data only during the period when their yields appear undistorted by large liquidity premia. This approach is taken by Christensen, Lopez, and Rudebusch (2010). They intentionally estimate a model of nominal and real term structures using TIPS yields only from January 2003 to March 2008 since they acknowledge that illiquidity was high before and after this period.

4.2 Estimation Technique

Since our data is monthly, the model's period is taken to be $\Delta t = 1/12^{\text{th}}$ of a year. Thus, the nominal short rate, i_t, is the one-month Treasury bill rate, π_t is the rate of inflation expected over the next month, and r_t is the one-month real interest rate. Our estimation imposes model restrictions on both the cross-sectional and time-series of bond yields, inflation forecasts, and inflation swap rates which, in general, are assumed to be measured with the independent errors $\omega_{t,i} \sim N\left(0, w^2\right)$, $\upsilon_{t,i} \sim N\left(0, v^2\right)$, and $\mu_{t,i} \sim N\left(0, u^2\right)$, respectively, where the subscript i denotes a particular bond, inflation forecast, or inflation swap rate maturity.

While most bond yields and inflation forecasts are assumed to be observed with error, we need to assume perfect observation of the one-month nominal rate, $i_t = \pi_t + r_t - \phi_1 h_{1,t}^2$ and the survey inflation forecast at the one-month horizon, π_t. These assumptions allow us to recover the exact one period real rate, $r_t = i_t - \pi_t + \phi_1 h_{1,t}^2$, given that $h_{1,t}$ is observed. However to update the volatility factors $h_{i,t}$, $i = 1, ..., 4$, we also need to observe the central tendency, α_t, which can be done if another particular bond yield is measured without error, say one of maturity n_x periods:

$$\alpha_t = \left(n_x \Delta t y_{N,t}^{(n_x)} - K_{n_x} - A_{n_x}\pi_t - B_{n_x} r_t - \Sigma_{j=1}^4 D_{j,n_x} h_{j,t}^2\right)/C_{n_x} \tag{28}$$

Since α_t largely determines the slope of the term structure, our estimation assumes this bond is the one with a five-year maturity ($n_x = 60$).

These assumptions allow us to observe π_t, r_t, and α_t and recover the $\epsilon_{j,t+\Delta t}$, $j = 1, ..., 4$ in equations (2) and (6). In turn, this allows us to update each of the volatility factors, $h_{j,t}$, $j = 1, ..., 4$. Given the state variables $(\pi_t, r_t, \alpha_t, h_{j,t}^2,\ j = 1, .., 4)$ at date t, all of the theoretical bond yields, inflation forecasts, and inflation swap rates can be computed. The difference between these theoretical quantities and their actual counterparts determine the measurement errors for bond yields, inflation forecasts, and inflation swap rates.

Let n_1^b, ..., n_B^b be the maturities of the B different bonds, let n_1^f, ..., n_F^f be the horizons of the F different inflation rate forecasts, and let n_1^s, ..., n_S^s be the maturities of the S

different swap rates.[24] Then the month t vector of observed variables is

$$Y_t = \left(\ln(I_{t+\Delta t}/I_t), \pi_{t+\Delta t}, r_{t+\Delta t}, \alpha_{t+\Delta t}, \quad y_{N,t}^{(n_1^b)} ... y_{N,t}^{(n_B^b)}, \quad s_t^{(n_1^f)} ... s_t^{(n_F^f)}, \; k_t^{(n_1^s)} .... k_t^{(n_S^s)} \right)' \tag{29}$$

and can be written as a linear function of the state variables:

$$Y_t = A_t + M_t x_t + \Upsilon_t \tag{30}$$

where $x_t = \left(\pi_t \; r_t \; \alpha_t \; h_{1,t}^2 \; h_{2,t}^2 \; h_{3,t}^2 \; h_{4,t}^2\right)'$ are the seven state variables, and A_t and M_t are appropriately defined vectors and matrices of the model parameters based on equations (5), (6), (18), (19), and (21). Also based on these equations, the vector Υ_t is a function of the four stochastic drivers, $\epsilon_{j,t+\Delta t}$, $j = 1, ..., 4$ and the measurement errors $\omega_{t,i}$, $\upsilon_{t,i}$, and $\mu_{t,i}$. Given the assumed distribution of the $\epsilon_{j,t+\Delta t}$ and measurement errors, the model parameters are estimated by maximum likelihood by recursively calculating the likelihood function composed of equation (30) for each date.

In principle, the model's 36 parameters can be estimated in one step using equation (30). However, the first element of Y_t is the log inflation process $\ln(I_{t+\Delta t}/I_t) = \pi_t \Delta t - \frac{1}{2}\Delta t h_{1,t}^2 + \sqrt{\Delta t} h_{1,t} \epsilon_{1,t+\Delta t}$. By estimating this equation alone using data only on I_t and π_t, we can recover estimates of the four parameters of the $h_{1,t}$ GARCH process, namely d_{10} (equivalently, $\overline{h}_1$), d_{11}, d_{12}, and d_{13}. Therefore, to make overall parameter estimation more manageable, a two-step procedure is implemented where we first estimate the parameters of the $h_{1,t}$ process using data on only I_t and π_t and the 32 other parameters are estimated in a second step using equation (30) but with the parameters of the $h_{1,t}$ process fixed at those estimated in the first step.[25]

[24] At each monthly observation date the bond yield maturities measured with error are the same, equal to 3, 6, 12, 24, 36, 84, 120, and 180 months, but due to the nature of the inflation survey data, the number of inflation forecasts, F, and their horizons vary over different observation months. Similarly, the number of inflation swap rates, S, (but not their horizons) vary over different observation months.

[25] This procedure is equivalent to a one step weighted maximum likelihood procedure where the observations on $\ln(I_{t+\Delta t}/I_t)$ are given much larger weights relative to those of the other observations. Since the monthly time series of inflation is directly observed over our 28-year sample period, it is reasonable that we heavily weight these observations when estimating the four inflation-specific parameters.

5 Empirical Results

5.1 Parameter Estimates and State Variable Dynamics

Table 2 reports the first step estimates of the parameters of the inflation volatility process, $h_{1,t}$, using data on the CPI (I_t) and the one-month forecast of inflation (π_t) derived from BCEI surveys. The annualized, conditional standard deviation for inflation over a one-month horizon has a steady-state value of $\overline{h}_1 = 88$ basis points.[26] The volatility of inflation displays GARCH effects since the coefficient on a shock to inflation in the GARCH updating, d_{12}, is significantly positive.[27] However, since d_{13} is insignificantly different from zero, there is no evidence that inflation's volatility responds asymmetrically to innovations.

Table 3 reports estimates of the model's other parameters. To gauge the statistical significance of permitting GARCH behavior, we estimated the unrestricted model as well as restricted models that assume some of the volatilities are constant; that is, $h_{j,t} = \overline{h}_j$. The first column of Table 3 reports estimates assuming no GARCH behavior ($h_{j,t} = \overline{h}_j$, for $j = 2$, 3, and 4) while the second, third, and fourth columns assume GARCH behavior only for $h_{2,t}$ or $h_{3,t}$ or $h_{4,t}$, respectively. Finally, the last column of Table 3 is the unrestricted model that permits GARCH behavior for $h_{2,t}$, $h_{3,t}$, and $h_{4,t}$.

Inspection of the log likelihood values for the different models at the bottom of Table 3 indicates that one can reject at the 1% level of significance the hypothesis of no GARCH behavior for each of the less restricted cases. Relative to the model with no GARCH behavior, the largest increase in likelihood value from permitting GARCH behavior for any single volatility process occurs with $h_{3,t}$, the volatility process for the independent component of the real interest rate, r_t. The second largest increase occurs when $h_{2,t}$ is able to display GARCH, which is the independent volatility component for expected inflation, π_t. Allowing for GARCH effects appears to be least important for $h_{4,t}$, the independent volatility component of the central tendency. As indicated in the last column of Table 3, for the fully unrestricted where $h_{2,t}$, $h_{3,t}$, and $h_{4,t}$ all follow GARCH processes, the GARCH volatility parameters for all processes (d_{22}, d_{32}, and d_{42}) are significantly positive. Based on these unrestricted model estimates and those for the inflation GARCH process in Table 2, measures of persistence for $h^2_{j,t}$, $j = 1, ..., 4$ can be computed. The half-life for a shock in $h^2_{j,t}$ to revert to its steady-state of $\overline{h}^2_j$ is 3.4 months, 0.7 months, 1.6 months, and 4.6

[26] Jarrow and Yildirim (2003) obtain a comparable inflation volatility estimate of 87 basis points.

[27] Furthermore, the process displays mean-reversion since the estimate of d_{11} is significantly different from the random walk value of $-1/\Delta t = -12$.

months for $j = 1$, 2, 3, and 4, respectively.[28]

5.2 Levels of State Variables

We obtain reasonable estimates for the unconditional means of inflation and the real interest rate. The unrestricted model gives an estimate for $\overline{\pi}$ of 2.99%, just below the sample average BCEI one-month inflation forecast of 3.05% shown in Table 1. This estimate along with the estimated steady-state one-month real rate of $\overline{r} = 1.76\%$ and the estimated steady-state risk premium of $\phi_1 \overline{h}_1^2 = 0.47\%$ imply from equation (5) that the steady-state one-month nominal interest rate is $\overline{i} = \overline{r} + \overline{\pi} - \phi_1 \overline{h}_1^2 = 4.28\%$, somewhat below the sample average one-month Treasury bill rate of 4.75% given in Table 1. Table 3 also shows that permitting a central tendency for inflation is important since the mean reversion parameter c_1 is estimated as -0.056 with a small standard error that makes it statistically different from both zero and the no central tendency case (α_t constant) of $c_1 = -1/\Delta t = -12$.

Figure 2 plots the model-implied levels and volatilities of the state variables from 1982 to 2010. The top left panel indicates that the rate of expected inflation over one month, π_t, trended downward since the early 1980s. At the beginning, the central tendency for inflation was above this expected inflation rate as investors apparently thought longer term inflation was likely to remain high. However, the Federal Reserve appears to have gained credibility in lowering inflation, since the central tendency later declined to approximately the average of expected inflation. Early in 2008, there was a significant rise in expected inflation followed by a sharp plunge at mid-year as the financial crisis worsened.

The top right panel in Figure 2 displays the one-month real interest rate, r_t. There was an unusually long period from mid-2002 to 2005 when it was negative. This finding supports the belief that a credit bubble may have been inflated by a policy of maintaining interest rates too low for too long. This panel also shows that at the beginning of 2008 the short run real interest rate was negative and then rose dramatically, consistent with the opposite movement in expected inflation as the nominal interest rate remained near zero during this time. At the end of 2009, the short-term real rate became negative again, consistent with the Federal Reserve's pegging of short-term nominal rates near zero.

Our model estimates of these state variables' processes indicate relatively strong mean-reversion for expected inflation and real rates, but high persistence for inflation's central tendency. The half-lives for the variables to return to their steady states following a de-

[28]Note from (6) that $E_t \left[h_{j,t+\Delta t}^2 \right] = \varrho h_{j,t}^2 + (d_{j0} + d_{j2}) \Delta t$, where $\varrho \equiv 1 + \left(d_{j1} + d_{j2} d_{j3}^2 \right) \Delta t$. Thus, the half-life in periods of length Δt (months) is $\ln \left(\frac{1}{2} \right) / \ln (\varrho)$.

viation are 0.26, 0.65, and 12.38 years for π_t, r_t, and α_t, respectively. The very weak mean-reversion for inflation's central tendency suggests that investors' expectations of longer-term inflation are not well-anchored.

5.3 State Variable Volatilities and Correlations

Based on the unrestricted model's parameter estimates, Table 4 reports statistics for the implied standard deviations and correlations for inflation, expected inflation, the real rate, and the central tendency. The first column calculates the state variables' annualized standard deviations and correlations over a one month horizon assuming that each of the GARCH processes begin at their steady state values: $h_{j,t} = \overline{h}_j$, $j = 1, ..., 4$. The real interest rate, r_t, and expected inflation, π_t, have the highest unconditional standard deviations of 3.26% and 3.14%, respectively. Conditional on its mean of π_t, the steady state one-month standard deviation of log inflation is 0.88% while the steady state standard deviation of the central tendency is 1.09%. One also sees that an innovation in actual inflation ($I_{t+\Delta t}$) has a 0.35 correlation with an innovation in expected inflation ($\pi_{t+\Delta t}$) and a 0.13 correlation with an innovation in the central tendency (α_t). This suggests that when investors experience a positive inflation surprise, their one-month expectation of inflation is partially updated and, to a lesser degree, so is their longer-horizon expectation of inflation via the central tendency.

We also see that when starting from the steady state, the one-month expected inflation and real rate are strongly negatively correlated at -0.87. This finding is consistent with Benninga and Protopapadakis (1983), Summers (1983), and Pennacchi (1991) and is a likely consequence of Federal Reserve policy that keeps short-maturity nominal interest rates stable by pegging the federal funds rate. Controlling the short run nominal interest rate implies that any change in short run inflation expectations must lead to an offsetting change in the short run real interest rate. Evidence by Ang, Bekaert, and Wei (2008) confirms that the short term real rate is quite variable.

Of course, due to GARCH behavior, the state variables' standard deviations and correlations are not constant. Columns two, three, and four of Table 4 calculate the model-implied average, minimum, and maximum of the standard deviations and correlations over the sample period. The sample averages for standard deviations and correlations tend to be relatively close to their steady-state values. However, based on the minimum and maximum values, we see that standard deviations and correlations varied significantly. The central tendency's correlation with real rates and expected inflation even changed signs.

To illustrate this variation, the bottom two panels of Figure 2 display the time series of the standard deviations of $\ln(I_{t+\Delta t}/I_t)$, π_t, r_t, and α_t. The standard deviations of expected inflation and the real interest rate were especially high during the early 1980s when the Federal Reserve was battling to lower inflation expectations and also during the late 2000s when commodity price volatility picked up and the financial crisis hit.

5.4 The Model's Fit to the Data

Our paper is largely concerned with decomposing the nominal yield curve into components including real rates, expected inflation, and inflation risk premia. To have confidence in these term structures, we first investigate how our model's implied nominal yields, inflation swap rates, and inflation forecasts fit the data.

5.4.1 Nominal Yields

The top left panel of Figure 3 shows a box plot of measurement errors in basis points for various maturity nominal yields. For any given maturity, the average bias is of the order of 2 basis points. The top right panel indicates that over the 1982-2010 sample period the average nominal yield measurement errors (difference between the observed data yields and the model-implied yields) across all maturities is less than 3 basis points, with the largest errors occurring early in the sample period. Indeed, the mean error from 1990-2010 is less than one basis point.

As reported in the last column of Table 3, our unconstrained model estimates a standard deviation of measurement errors for nominal yields of $w = 36$ basis points, close to the average standard deviation of errors across all maturities over our sample period of 33.5 basis points. The ten-year maturity has a standard deviation of 27 basis points. The largest standard deviation of errors is for the 3 month rate (42 basis points).

Compared to other studies that attempt to fit both nominal and real term structures, our results are satisfactory. For example, Chen, Liu, and Cheng (2010) estimate a multi-factor Cox, Ingersoll, and Ross (1985) (CIR) model using both nominal Treasury and TIPS data and for nominal yields obtain an average measurement error of 24 basis points and an average measurement error standard deviation of 74 basis points. Christensen, Lopez, and Rudebusch (2010) fit a multi-factor Gaussian model to nominal and TIPS yields, finding measurement errors for 10-year nominal yields to average 10 basis points and have a standard deviation of 11 basis points.[29]

[29]Of course studies that use only nominal data can obtain more precise fits of nominal yield curves. For

5.4.2 Inflation Swap Rates

The middle left panel of Figure 3 shows a box plot of errors in basis points for inflation swaps of each maturity. The average error is close to zero, with the possible exception of the 15-year inflation swap which has a bias of 11 basis points. These box plots indicate a standard deviation of errors similar to the $u = 28$ basis points estimated by our model in the last column of Table 3.

The middle right panel of Figure 3 shows the time series of the average monthly errors. The errors stay within a band of 50 basis points from zero except during November 2008 when the errors exceed 100 basis points. Thus, during the financial crisis our model over-predicted actual inflation swap rates, but recall from Figure 1 that break-even inflation rates derived from TIPS were even smaller than inflation swap rates at this time. Thus, if we had used TIPS, rather than swaps, in our model estimation, measurement errors would very likely have been much larger. Indeed, in fitting their model to TIPS, Christensen, Lopez, and Rudebusch (2010) obtained huge measurement errors during this period.[30]

5.4.3 Survey Forecasts of Inflation

The bottom left panel of Figure 3 shows the box plot of errors in basis points for the survey forecasts of inflation. The first seven are the two to eight quarter BCEI inflation forecasts and the last is the SPF 10-year inflation forecast. On average, the model over-predicts two and three quarter inflation by less than 7 basis points but under-predicts seven and eight quarter inflation by around 6 and 9 basis points, respectively. The bias for the 10-year forecast is less than 1 basis point. The sample standard deviations of measurement errors across maturities range from 48 to 36 basis points, consistent with the $v = 40$ basis point standard deviation estimate reported in the last column of Table 3.

The bottom right panel of Figure 3 shows the average measurement error across all forecast maturities during the sample period. The model tended to under-predict the survey forecasts during the late 1980s and over-predict during the late 1990s. During most of the recent financial crisis, the model under-predicted expected inflation relative to the survey forecasts. Recall from the middle right panel of Figure 3 that during this same time

example, Duffie and Singleton (1997) use interest rate swap data, and their average standard deviation of measurement errors across the maturity spectrum is 11 basis points. However, comparison of standard errors should be done with considerable caution since the number of factors, the data, and the sample periods are often very different. Most of our errors are concentrated in the early 1980s, a period of extraordinary interest rate volatility.

[30]Due to their recognition of such large TIPS pricing errors in the latter part of 2008, they ended the baseline estimation of their nominal and real term structure model in March of 2008.

the model was over-predicting inflation swap rates. Hence, during the crisis the model was making a compromise between the relatively high survey expectations of inflation and the relatively low inflation expectations reflected in swap rates.

5.5 Nominal Bond Yields, Risk Premia, and Volatility

This section investigates the model-implied term structure of nominal yields, nominal bonds' risk premia, and the volatility of nominal bond yields. Figure 4 shows the model's nominal yield curve (solid line) when all variables equals their steady states. This term structure appears reasonable, even for maturities out to 30 years, a horizon where no Treasury yield data was used in the model's estimation. The slopes of this steady-state nominal yield curve (difference between yields and the steady-state one-month nominal rate of $i = 4.28\%$) equal 114, 177, 236, and 257 basis points at the 5-, 10-, 20-, and 30-year maturities, respectively. Moving from this steady-state yield curve, let us now consider the time series properties of yields.

Time Series Properties: Campbell-Shiller Tests and Bond Risk Premia

Recall from equation (24) that our model's risk premia for nominal bonds, $\pi_{N,t}^{(n)}$, depend only on the four volatility state variables, $h_{j,t}^2$, $j = 1,..., 4$. Figure 4 shows that if these state variables equal their steady-states ($h_{j,t}^2 = \overline{h}_j^2$), then nominal risk premia are a concave function of maturity (dotted line). Many empirical studies, notably Fama and Bliss (1987) and Campbell and Shiller (1991), provide overwhelming evidence of significant variation in bond risk premia over time. Thus, let us examine whether our model fits the patterns documented by prior research. Note that equation (23) can be re-arranged as

$$E_t(y_{j,t+\Delta t}^{(n-1)}) - y_{j,t}^{(n)} = \frac{s_{j,t}^{(n)}}{n-1} - \frac{\pi_{j,t}^{(n)}}{n-1}, \quad j = N, R. \tag{31}$$

Based on this equation, consider the following regression:

$$y_{j,t+\Delta t}^{(n-1)} - y_{j,t}^{(n)} = \beta_{j,0}^{(n)} + \beta_{j,1}^{(n)} \frac{s_{j,t}^{(n)}}{n-1} + \beta_{j,2}^{(n)} \frac{\pi_{j,t}^{(n)}}{n-1} + \epsilon_{j,t+\Delta t}^{(n)}, \quad j = N, R. \tag{32}$$

For nominal yields ($j = N$), Campbell and Shiller (1991) consider a special case of this regression equation with the risk premium coefficient $\beta_{N,2}^{(n)} = 0$. Under the "Expectations Hypothesis," $\beta_{N,1}^{(n)}$ would then equal one for any maturity, n. Their empirical tests not only reject this hypothesis but their estimates for $\beta_{N,1}^{(n)}$ become increasingly negative as maturity increases. Dai and Singleton (2002) consider whether affine models are consistent with the more general form of the regression specification (32) with $\beta_{N,1}^{(n)} = 1$ and $\beta_{N,2}^{(n)} = -1$. They find that a three-factor Gaussian model with the "essentially affine" factor-dependent risk premium structure of Duffee (2002) generates the Campbell-Shiller regression results ($\beta_{N,1}^{(n)}$ becomes increasingly negative with n) when $\beta_{N,2}^{(n)}$ is constrained to be zero. However, for the regression (32) where $\beta_{N,2}^{(n)}$ is unconstrained so that risk premia can be time varying, this model generates results where the hypothesis that $\beta_{N,1}^{(n)} = 1$ and $\beta_{N,2}^{(n)} = -1$ is not rejected. Hence, such a model appears consistent with the theoretical generalization of the Expectation Hypothesis that allows for time-varying risk premia.

The benefits of this Gaussian model in capturing the dynamics of yields comes at the expense of producing yields with volatilities independent of their levels and correlations that are constant across maturities. Both of these properties are strongly rejected empirically. Unfortunately, when volatilities are permitted to be solely level dependent, as in CIR specifications, then the desirable time series properties are lost with the resulting yield curves not reproducing the Campbell-Shiller regression tests.

As discussed in Dai and Singleton (2002), this tension is an inherent property of standard affine models. Our affine model has the potential to avoid this conflict by introducing factors ($h_{j,t}^2$) that allow for both changing volatility and risk premia. While these factors' GARCH processes are generated by the same sources of uncertainty driving changes in the model's other three state variables (π_t, r_t, α_t), they are not tightly linked to their levels. At the same time, these three state variables are permitted to have a relatively general, time-varying correlation structure. Thus, it is important to examine whether our stochastic volatility model also is consistent with the generalized Campbell-Shiller regression (32).

Using actual monthly nominal Treasury yield data for our 1982 to 2010 sample period, the estimate of the slope coefficient $\beta_{N,1}^{(n)}$ for Campbell-Shiller regressions ($\beta_{N,2}^{(n)}$ constrained to equal zero) are shown in the first column of Table 5. As in Campbell and Shiller (1991), the null hypothesis that the slope coefficient equals 1 is rejected for all maturities at the 10% level of significance, and estimates become more negative as the maturity increases. The second column of the table reports the regression results of equation (32) with our model-implied risk premia, $\pi_{N,t}^{(n)}$, computed at each date. After including these risk premia, one sees that the joint hypothesis that the slope is 1 and the excess return coefficient is -1 cannot be rejected at the 10% level of significance except for the 10 and 15 year maturities. In this regression, the dependent variable and slope are from actual Treasury yields. If these are replaced by the model-implied yields for each date over the 1982-2010 period, similar results are obtained as shown in the rightmost panel of Table 5.

Thus, our model generates variation in risk premia that, for most maturities, are not inconsistent with a generalized Expectations Hypothesis. The variation can be substantial. For example, the risk premium on a 10-year bond during our 1982-2010 sample period sometimes becomes negative, though it is positive over 90% of the time.

While our model's risk premia are entirely determined by the volatility factors ($h^2_{j,t}$, j = 1,...,4), Figure 5 shows that they play a minor role in determining the cross section of yields. The figure illustrates the contribution of each of the seven state variables to the 6 month-, 2 year-, 5 year-, and 10-year yields. Since the four volatility state variables add minimally to the yields, their collective contribution is shown. Nominal yields are largely determined by expected inflation, π_t, the real rate, r_t, and by the central tendency, α_t, with the importance of the last state variable increasing as maturity increases. Only occasionally, do the volatility state variables play a significant role, and then only for the shorter maturities. This result of our model is similar to Duffee (2011) who constructs a multifactor Gaussian model where a subset of factors describe bond yields and a mutually exclusive subset of "hidden factors" determine bond risk premia.

Volatility Effects

Unlike standard affine models, our model's ability to fit a cross-section of yields and to forecast returns in the context of regression (32) does not require the volatility or correlation of yields to be constant. Equation (26) implies that when the volatility state variables are at their steady states, the volatility of nominal yields (annualized standard deviation of monthly changes) is at a minimum at the two-year maturity of just under 1%, rises to a maximum of 1.2% at the eight-year maturity, and then falls again to less than 1% at the 20-year maturity. Allowing the volatility states to vary, the top left panel of Figure 6 shows the term structure of nominal yield volatilities over our sample period. One can see that yield volatilities can change substantially, especially at the short end during the early 1980s and early 1990s. The bottom left panel is a scatterplot of the volatility of the five-year maturity nominal yield against the yield's level. Unlike Gaussian models, our model is capable of inducing a level dependence in volatility. As the yield increases, the volatility generally will increase. The slope of the regression line is statistically different from zero at the 1% significance level.

Also in contrast to Gaussian models, our model's yield changes display excess kurtosis and skewness, with changes in yields of one-year maturity and less tending to display negative skewness while longer-maturity yield changes are positively skewed.[31] Correlations between yields are also time-varying. For example, over our sample period the

[31]For example, average sample skewness for a one-year change in the 3-month and 10-year yields are -0.43 and 0.46, respectively.

model-implied correlation (from equation (26)) between one-month changes in the one-year and 10-year yields averages 0.37, but reaches a maximum and minium of 0.67 and -0.05, respectively.

Collectively, then, while our model is in the completely affine family, it captures the time series properties of nominal yields reasonably well and produces yields with stochastic volatilities that are weakly linked to yield levels. Given the model's reasonable performance for describing nominal yields, we next consider their decomposition into real and inflation components.

5.6 Real Bond Yields, Risk Premia, and Volatility

Figure 4 also shows the model-implied inflation-indexed (real) yield curve when all variables equal their steady states (long dashed line). The slopes of this real yield curve (difference between yields and the steady-state one-month real rate of $\overline{r} = 1.76\%$) equal 52, 96, 149, and 177 basis points at the 5-, 10-, 20-, and 30-year maturities, respectively. Hence, similar to the steady state nominal yield curve, the steady state real yield curve is concave and upward sloping but relatively less steep. There is, however, much time series variation in real yields. The top panel in Figure 7 shows the term structures of inflation indexed yields from 1982 to 2010. For example, the slope of the real yield curve as measured by the five-year yield minus the one month yield fluctuates quite a bit. Specifically the interquartile range of this slope extends from -25 basis points to 172 basis points, and the slope is negative approximately 25% of the time.

Recall from equation (25) that the real risk premium on an inflation-indexed bond, $\pi_{R,t}^{(n)}$, depends on the four volatility state variables. Figure 4 also shows that the term structure of $\pi_{R,t}^{(n)}$ is increasing and concave when these variables are at their steady states ($h_{j,t}^2 = \overline{h}_j^2$). More generally, since equation (31) holds both in real and nominal terms, the generalized Campbell-Shiller regression (32) relating yields to term structure slopes and risk premia should also hold for real yields with $\beta_{R,1}^{(n)} = 1$ and $\beta_{R,2}^{(n)} = -1$. We are not aware of any prior research examining regression (32) for real rates, so we carried out such a test of our model over two different sample periods. First, during the 2003 to 2010 period for which inflation swap data is available, we used equation (16) to obtain "actual" zero coupon real yields, $y_{R,t}^{(n)}$, by subtracting the continuously compounded inflation swap rate from the equivalent maturity nominal Treasury yield. From these real yields, slope variables, $s_{R,t}^{(n)}$, were calculated. Second, we computed model-implied real term structures over the entire 1982 to 2010 period, which are the same term structures shown in Figure

7, and also calculated the slopes $s_{R,t}^{(n)}$ from these model-implied real yields. For both the first and second samples, the model-implied real risk premia, $\pi_{R,t}^{(n)}$, were calculated based on the time series of the volatility state variables.

Table 6 reports results of the Campbell-Shiller regression equation (32) both with the excess return coefficient $\beta_{R,2}^{(n)}$ restricted to zero and unrestricted. The left panel reports results for the "actual" real yields over the 2003 to 2010 period while the right panel shows results for the model-implied real yields from 1982 to 2010. Interestingly, there are some qualitative differences compared to the nominal results of Table 5. Using the "actual" real rates from 2003 to 2010, the regressions that include only the slope term do not reject the hypothesis $\beta_{R,1}^{(n)} = 1$. Unlike the nominal results, the estimates $\beta_{R,1}^{(n)}$ become more *positive* as maturity increases. When the real risk premium, $\pi_{R,t}^{(n)}$, is included in the regression, rejection of the joint hypothesis of $\beta_{R,1}^{(n)} = 1$ and $\beta_{R,2}^{(n)} = -1$ at a 10% level of significance occurs only for the 10-year maturity, but this happens because $\beta_{R,1}^{(n)}$ becomes too positive while $\beta_{R,2}^{(n)}$ becomes too negative, just the opposite of the nominal Campbell-Shiller regressions where rejection occurred because $\beta_{R,1}^{(n)}$ become negative while $\beta_{R,2}^{(n)}$ became positive.

These findings might be partly explained by the short 2003 to 2010 sample period. When in the right panel of Table 6 we use the model-implied real yields over the 1982 to 2010 sample period, the estimate of $\beta_{R,1}^{(n)}$ does decline when the slope, alone, is included in the regression. However, except for the 15-year maturity, this slope coefficient is estimated to be positive and the hypothesis that $\beta_{R,1}^{(n)} = 1$ cannot be rejected for any maturity. Notably, when the real risk premium, $\pi_{R,t}^{(n)}$, is included, the model fits well at the longer maturities but is rejected at the shorter ones. Again, however, rejection at the short maturities occurs because, different from the nominal results, $\beta_{R,1}^{(n)}$ exceeds 1 while $\beta_{R,2}^{(n)}$ is lower than -1.

Volatility Effects

It is apparent from the top panel of Figure 7 that short-maturity real yields tend to be more volatile than longer-maturity ones, consistent with the top right panel of Figure 6 that shows the term structures of real yield volatilities over our sample period. Volatility of short term real yields is also relatively high when all state variables are at their steady states. The steady state one-year real volatility (annualized standard deviation of monthly changes) is 112 basis points, falling to 68 basis points at the four year maturity, rising to 72 basis points at the eight-year maturity, and then gradually falling to 61 basis points at the 20-year maturity. Like the volatility of nominal yields, the volatility of real yields is weakly correlated with yield levels. The bottom right panel in Figure 6 shows a scatter diagram of the levels of five-year real yields versus their volatilities over our sample period. The slope of the regression line in the diagram is statistically different from zero at the 1% level of significance.

The high volatility of short-term real yields is consistent with Figure 2's reported high standard deviation of the one-month real rate, r_t. Since the Federal Reserve pegs short-term nominal rates, changes in short-run inflation expectations induce almost opposite changes in short-run real yields.

5.7 Expected Inflation and Inflation Risk Premia

The bottom panel of Figure 7 shows our model's implied term structures of expected inflation for each month from 1982 to 2010. Consistent with the evidence in Figure 2 of a falling central tendency, inflation expectations generally declined at all maturities. However, the term structure was often upward sloping during the mid-1980s, indicating that investors were not yet convinced that inflation would remain low in the longer run. Expected inflation can be volatile at short maturities, but it changes more smoothly at longer horizons.

Another component of nominal yields that interests policymakers and academics is the term structure of inflation risk premia. There are at least two reasons for wanting to know this quantity. First, saving the cost of an inflation risk premium has been used to justify a government's issuance of inflation-indexed bonds. Second, one needs to subtract an inflation risk premium from an inflation swap or TIPS breakeven inflation rate in order to construct a measure of inflation expectations.

We quantify the term structure of inflation risk premia, as well as the term structure of real interest rate risk premia, in the following manner.[32] First, we compute nominal and real yield curves under the assumption that all of the market prices of risk equal zero; that is, $\phi \equiv (\phi_1\ \phi_2\ \phi_3\ \phi_4) = \mathbf{0}$. Recall that the yields on nominal and inflation-indexed bonds maturing in n periods are denoted as $y_{j,t}^{(n)}$, $j = N,\ R$, respectively, so let their zero-risk premium counterparts be $y_{j,t}^{(n)}\,(\phi = \mathbf{0})$, $j = N,\ R$. Second, define the date t nominal and

[32] Our approach is similar to Chen, Liu, and Cheng (2010) but differs from Christensen, Lopez, and Rudebusch (2010) who define an inflation risk premium as the residual from subtracting the rate of expected inflation from the breakeven inflation rate: $y_{N,t}^{(n)} - y_{R,t}^{(n)} - \frac{1}{n\Delta t}\ln\left(E\left[I_{t+n\Delta t}/I_t\right]\right)$. See their equations (27)-(29). Because the rate of expected inflation exceeds the expected rate of inflation due to Jensen's Inequality, $\frac{1}{n\Delta t}\ln\left(E\left[I_{t+n\Delta t}/I_t\right]\right) \geq E\left[\frac{1}{n\Delta t}\ln\left(I_{t+n\Delta t}/I_t\right)\right]$, their computation creates a downward bias in the inflation risk premium. The bias tends to be greater as inflation uncertainty lengthens with maturity.

real risk premia, $\Phi_{j,t}^{(n)}$, $j = N,\, R$, respectively, for bonds maturing in n periods be:

$$\Phi_{j,t}^{(n)} = y_{j,t}^{(n)} - y_{j,t}^{(n)}\left(\phi = \mathbf{0}\right),\; j = N,\, R \tag{33}$$

Finally, the inflation risk premium, $\Phi_{inf,t}^{(n)}$ is defined as the difference between the nominal risk premium and the real risk premium for the same maturity:

$$\Phi_{inf,t}^{(n)} = \Phi_{N,t}^{(n)} - \Phi_{R,t}^{(n)} \tag{34}$$

Thus, the inflation risk premium is equal to the difference between the actual breakeven inflation rate (which includes the inflation risk premium) and what would be breakeven inflation rate in the absence of risk premia (which excludes the inflation risk premium).

The term structures of nominal, real, and inflation risk premia when all of the state variables are initially at their steady states are plotted in the top panel of Figure 8. One sees that the real risk premia equal 54, 102, 170, and 214 basis points at the 5-, 10-, 20-, and 30-year maturities, respectively. The inflation risk premia equal 17, 45, 80, and 100 basis points at the 5-, 10-, 20-, and 30-year maturities, respectively. Note that the steady-state inflation risk premia are negative for very short maturities, reaching a minimum of -20 basis points at a seven-month maturity before becoming positive at a 29-month maturity. One explanation for this negative premium may be that our model is capturing the relative high liquidity or "moneyness" of shorter maturity Treasuries, particularly Treasury bills. This seems plausible since our longer-maturity yield data was constructed by Gurkaynak, Sack, and Wright (2007) from off-the-run Treasury notes and bonds. In contrast, our shorter-maturity yield data are actual yields of Treasury bills that trade in more liquid money markets. A negative inflation premium is the model's way of adjusting for the greater liquidity at the short end of the nominal yield curve.[33]

We can also examine how these risk premia varied over time during our sample period. The bottom panel of Figure 8 plots expected inflation, the real risk premium, and the inflation risk premium for a 10-year maturity during the 1982 to 2010 period. Interestingly, while inflation expected over 10 years varied substantially, real and inflation risk premia changed more moderately. The real risk premium for a 10-year maturity bond varied from

[33] Other reseach has recognized that Treasury bill yields are reduced by their high liquidity. For example, Campbell, Shiller, and Viceira (2009) incorporate an additional parameter in their model estimation to "capture the liquidity effects which lower the yields on Treasury bills relative to the longer-term real yield curve." Also, note that the top panel of Figure 8 appears to indicate a negative real risk premia at very short maturities. However, this negative premium is due to our real yield curve truly being a yield curve for inflation-indexed bonds that have a three-month indexation lag. Since these bonds lack inflation protection during their last three months, their model yields reflect a small inflation risk premium.

87 to 121 basis points, averaging 102 basis points. This real risk premium is consistent with the significant slope of the real yield curve discussed earlier. The inflation risk premium for a 10-year maturity bond varied from 23 to 55 basis points and averaged 42 basis points.

These inflation risk premia estimates fall within the range of those estimated by other studies. For example, Ang, Bekaert, and Wei (2008) report a 5-year inflation risk premium that averages 114 basis points and ranges from 15 to 204 basis points over their 1955 to 2004 sample period. Buraschi and Jiltsov (2005) estimate a 10-year inflation risk premium averaging 70 basis points and ranging from 20 to 140 basis points over their 1960 - 2000 sample period. Studies that use TIPS yield data generally report lower inflation risk premia. Chen, Liu, and Cheng (2010) find a 10-year inflation risk premium that averaged 56 basis points over their 1997 to 2007 sample period, while over the 2003 to 2008 period Christensen, Lopez, and Rudebusch (2010) and D'Amico, Kim, and Wei (2008) estimate average 10-year inflation risk premia of -5 and 64 basis points, respectively. Using TIPS yields from 2000 to 2007, Grishchenko and zhi Huang (2008) estimate an average 10-year inflation risk premium between 11 and 22 basis points depending on the proxy used for expected inflation.

5.8 Comparison to TIPS Yields

In the spirit of an out-of-sample test, we relate our model's implied yields for inflation-indexed bonds to the actual yields of TIPS. We use zero coupon TIPS yields from Gurkaynak, Sack, and Wright (2008), which are available for the period January 1999 to June 2010. Taking their 5- and 10-year zero coupon TIPS yields, we compare them to our model's implied 5- and 10-year zero coupon yields for inflation-indexed bonds. The results are in Figure 9. It shows that our model's yields are significantly below TIPS yields from 1999 to 2004 and are very close to TIPS yields from 2004 to mid-2008. Starting in mid-2008, the model-implied yields again fall below the yields on TIPS until they converge again in late 2009. Hence at the beginning of our sample and during the height of the financial crisis, our model overprices inflation-indexed bonds relative to TIPS. One interpretation of this comparison is that our model performs poorly in valuing inflation-indexed bonds except during 2004 to mid-2008 and again starting at the end of 2009.

However, recall that prior studies, such as Sack and Elsasser (2004), Shen (2006), and D'Amico, Kim, and Wei (2008), conclude that TIPS were significantly undervalued prior to 2004, and more recently Campbell, Shiller, and Viceira (2009), Hu and Worah (2009), and Christensen, Lopez, and Rudebusch (2010) argue that there was panic selling of TIPS in the latter half of 2008 that drove their yields above, and TIPS breakeven inflation below, reasonable levels. Therefore, an arguably more reasonable interpretation of our model's comparison to TIPS is that the difference in the two curves in Figure 9 is confirming the erratic liquidity premium in TIPS identified by prior studies, and highlighted in our Figure 1. The model's close fit to TIPS beginning in 2004 might also be partly attributed to the initiation of a U.S. inflation swap market in 2003 that, in normal times, allowed dealers to arbitrage significant underpricing of TIPS (Fleckenstein, Longstaff, and Lustig (2010)). In summary, the overall evidence supports our earlier arguments for using data on inflation swaps and survey forecasts of inflation, rather than TIPS, to estimate a nominal and real term structure model. It provides credibility to our model-implied term structures of inflation-indexed yield curves and inflation expectations illustrated in Figure 7.

6 Conclusion

This paper presents an affine model of the term structures of nominal and real yields driven by four sources of uncertainty but having seven state variables. The model's factors include the short term real interest rate, the short term rate of expected inflation, and inflation's central tendency. These factors largely determine the cross-section of bond yields, and their innovations, along with innovation in actual inflation, drive four additional volatility state variables. These volatility state variables follow the nonlinear asymmetric GARCH model of Engle and Ng (1993) and exclusively determine bond risk premia that vary over time and can even change signs.

Unlike most models that belong to the completely affine family, our model permits state variables to have a general correlation structure with stochastic volatilities. It shows reasonable performance in fitting the time series and cross-section of nominal and real yields while permitting stochastic volatilities. Although our conditional distribution of state variables is mixtures of normal and chi squared innovations, we can still obtain analytical solutions for the prices of nominal bonds and inflation-indexed bonds that have an indexation lag, such as TIPS. Closed-form solutions for expected inflation rates, inflation swaps, and all risk premia also can be derived. It may be possible to construct a joint nominal and real term structure model that possesses similar properties, such as one where state variables follow Wishart processes (Buraschi, Cieslak, and Trojani (2008)). Evaluation of this alternative may be a fruitful avenue of research.

We found that allowing for GARCH effects is particularly important for real interest rate and expected inflation processes, but that long-horizon real and inflation risk premia are relatively stable. Our estimate for the 10-year inflation risk premium averaged 42 basis points and varied between 23 and 55 basis points during the 1982 to 2010 sample period. We also find a significant real interest rate risk premium at the 10-year maturity, averaging 102 basis points and varying between 87 and 121 basis points.

Comparing our model's implied yields for inflation-indexed bonds to those of TIPS suggests that TIPS bore a large liquidity premium prior to 2004. Perhaps due to the 2003 introduction of inflation derivatives, such as zero coupon inflation swaps, arbitrage possibilities may have eliminated the mispricing of TIPS until the middle of 2008. However, following the Lehman Brothers bankruptcy in September 2008, the link between nominal Treasuries, TIPS, and inflation swaps was broken, and a very large liquidity premium for TIPS reappeared.

Appendix

Proof of Proposition 1

The dynamics for π_t, r_t and α_t are straight forward. The interesting diffusion limits are for the $h^2_{j,t}$ state variables. Substitute the expressions for $d_{j,i}$, $i = 0,..,3$ given in Proposition 1 into the last line of equation (6). Upon simplification, this leads to:

$$\Delta h_{j,t} = (\kappa_j\theta_j + \frac{1}{4}\upsilon_j^2)\Delta t + \frac{1}{4}\upsilon_j^2(\epsilon^2_{j,t+\Delta t} - 1)\Delta t + h^2_{j,t}\kappa_j\Delta t(\frac{\kappa_j\Delta t}{4} - 1) - \upsilon_j(1 - \frac{\kappa_j\upsilon_j}{2}\Delta t)\sqrt{\Delta t}h_{j,t}\epsilon_{j,t+\Delta t} \tag{A.1}$$

Taking limit of this expression, and noting that the second term converges to zero, leads to the result.

Lemma 1

Let X be a standard normal random variable. Then for $Q_2 > -\frac{1}{2}$,

$$E\left[e^{Q_1X - Q_2X^2}\right] = e^{\frac{Q_1^2}{2(1+2Q_2)} - \frac{1}{2}\ln(1+2Q_2)} \tag{A.2}$$

Proof:

The expectation can be written as:

$$E\left[e^{Q_1X - Q_2X^2}\right] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{Q_1x - Q_2x^2 - \frac{1}{2}x^2}dx \tag{A.3}$$

The result follows after completing the square and using properties of the normal density function.

Proof of Proposition 2

The proof of the first result follows by substituting the nominal pricing kernel into the bond pricing equation, $P^{(n)}_{N,t} = E_t[\frac{M_{t+\Delta t}}{M_t}P^{(n-1)}_{N,t+\Delta t}]$, to obtain:

$$P^{(n)}_{N,t} = E_t\left[e^{-(\pi_t + r_t - \phi_1 h_t^2)\Delta t - \frac{1}{2}\sum_{j=1}^4 \phi_j^2 h^2_{j,t}\Delta t - \sum_{j=1}^4 \phi_j h_{j,t}\sqrt{\Delta t}\epsilon_{j,t+\Delta t}}P^{(n-1)}_{N,t+\Delta t}\right] \tag{A.4}$$

Now assume the bond price has the form

$$P^{(n)}_{N,t} = e^{-(K_n + A_n\pi_t + B_n r_t + C_n\alpha_t + \sum_{j=1}^4 D_{j,n}h^2_{j,t})} \tag{A.5}$$

and substitute equation (A.5) into the left- and right-hand sides of equation (A.4). Substituting in for the state variables at date $t + \Delta t$ using equation (6), collecting all coefficients of the random variables of the same type together, and then taking expectations using Lemma 1 leads to the resulting recursive equations for the coefficients. The initial boundary conditions come from considering the case when $n = 1$. The recursive equations are:

$$\begin{aligned}
K_{n+1} &= K_n + (b_0 B_n + c_0 C_n + \Sigma_{j=1}^4 d_{j0} D_{j,n})\Delta t + \frac{1}{2}\Sigma_{j=1}^4 \ln(1 + 2D_{j,n} d_{j2}\Delta t) \\
A_{n+1} &= \Delta t + (1 + a_2\Delta t) A_n + b_2 B_n \Delta t \\
B_{n+1} &= \Delta t + a_1 \Delta t A_n + (1 + b_1 \Delta t) B_n \\
C_{n+1} &= A_n \Delta t + (1 + c_1\Delta t) C_n \\
D_{j,n+1} &= \left(-\phi_1 I_{\{j=1\}} + \frac{\phi_j^2}{2}\right)\Delta t + D_{j,n}\left(1 + \left(d_{j1} + d_{j2} d_{j3}^2\right)\Delta t\right) - \frac{Q_{j,n}^2}{2(1 + 2D_{j,n} d_{j2}\Delta t)}
\end{aligned} \tag{A.6}$$

where $I_{\{j=1\}}$ is the indicator function equal to 1 if $j = 1$ and 0 otherwise, and

$$\begin{aligned}
Q_{1,n} &= (\phi_1 + A_n\beta_1 + B_n\gamma_1 + C_n\rho_1 - D_{1,n} 2d_{12}d_{13}\sqrt{\Delta t})\sqrt{\Delta t} \\
Q_{2,n} &= (\phi_2 + A_n\beta_2 + B_n\gamma_2 + C_n\rho_2 - D_{2,n} 2d_{22}d_{23}\sqrt{\Delta t})\sqrt{\Delta t} \\
Q_{3,n} &= (\phi_3 + B_n\gamma_3 + C_n\rho_3 - D_{3,n} 2d_{32}d_{33}\sqrt{\Delta t})\sqrt{\Delta t} \\
Q_{4,n} &= (\phi_4 + C_n\rho_4 - D_{4,n} 2d_{42}d_{43}\sqrt{\Delta t})\sqrt{\Delta t}
\end{aligned} \tag{A.7}$$

Next consider the pricing of TIPS. Let t be the current date, $t_e = t + (n - d)\Delta t$ and $t_p = t_e + d\Delta t$. We will compute $V_{t,t_s}^{(n,d)} = P_{R,t}^{(n,d)} I_t / I_{t_s}$. Now, assume the structure in equation (19) of Proposition 2:

$$V_{t,t_s}^{(n,d)} = \frac{I_t}{I_{t_s}} e^{-\tilde{K}_n - \tilde{A}_n\pi_t - \tilde{B}_n r_t - \tilde{C}_n\alpha_t - \sum_{j=1}^4 \tilde{D}_{j,n} h_{j,t}^2} \tag{A.8}$$

Now

$$V_{t,t_s}^{(n,d)} = E_t\left[\frac{M_{t+\Delta t}}{M_t} V_{t+\Delta t,t_s}^{(n-1,d)}\right] \tag{A.9}$$

Substituting in the structure for $V_{t,t_s}^{(n-1,d)}$ leads to:

$$V_{t,t_s}^{(n,d)} = E_t\left[\frac{I_{t+\Delta t}}{I_{t_s}}\frac{M_{t+\Delta t}}{M_t} e^{-\tilde{K}_{n-1} - \tilde{A}_{n-1}\pi_{t+\Delta t} - \tilde{B}_{n-1} r_{t+\Delta t} - \tilde{C}_{n-1}\alpha_{t+\Delta t} - \sum_{j=1}^4 \tilde{D}_{j,n-1} h_{j,t+\Delta t}^2}\right] \tag{A.10}$$

This can be rewritten as:

$$V_{t,t_s}^{(n,d)} = \frac{I_t}{I_{t_s}} E_t \left[\frac{I_{t+\Delta t}}{I_t} \frac{M_{t+\Delta t}}{M_t} e^{-\tilde{K}_{n-1} - \tilde{A}_{n-1}\pi_{t+\Delta t} - \tilde{B}_{n-1} r_{t+\Delta t} - \tilde{C}_{n-1}\alpha_{t+\Delta t} - \sum_{j=1}^{4} \tilde{D}_{j,n-1} h_{j,t+\Delta t}^2} \right] \tag{A.11}$$

Substituting in for the nominal pricing kernel and the inflation process using equations (1) and (2), as well as for the state variables at date $t + \Delta t$ using equation (6), collecting all coefficients of the random variables of the same type together, taking expectations using Lemma 1, and using equation (13) leads to the resulting recursive equations for the coefficients.

The boundary conditions are obtained by recognizing that at date $t + (n - d)\Delta t$, the final payment is known, but is deferred by d periods. So the boundary conditions with d periods to go are given by the known payment multiplied by the d-period nominal bond price. The recursive equations for real bonds are

$$\begin{aligned}
\tilde{K}_{n+1} &= \tilde{K}_n + (b_0 \tilde{B}_n + c_0 \tilde{C}_n + \Sigma_{j=1}^4 d_{j0} \tilde{D}_{j,n})\Delta t + \frac{1}{2}\Sigma_{j=1}^4 \ln(1 + 2d_{j2}\tilde{D}_{j,n}\Delta t) \\
\tilde{A}_{n+1} &= (1 + a_2\Delta t)\tilde{A}_n + b_2 \Delta t \tilde{B}_n \\
\tilde{B}_{n+1} &= (1 + a_1 \tilde{A}_n)\Delta t + (1 + b_1 \Delta t)\tilde{B}_n \\
\tilde{C}_{n+1} &= \Delta t \tilde{A}_n + (1 + c_1 \Delta t)\tilde{C}_n \\
\tilde{D}_{j,n+1} &= (\frac{1}{2} - \phi_1) I_{\{j=1\}} \Delta t + \frac{1}{2}\phi_j^2 \Delta t + \tilde{D}_{j,n}(1 + \left(d_{j1} + d_{j2} d_{j3}^2\right)\Delta t) - \frac{\tilde{Q}_{j,n}^2}{2(1 + 2\tilde{D}_{j,n} d_{j2}\Delta t)}
\end{aligned} \tag{A.12}$$

where $I_{\{j=1\}}$ is the indicator function equal to 1 only if $j = 1$ and

$$\begin{aligned}
\tilde{Q}_{1,n} &= (1 - \phi_1 - \tilde{A}_n\beta_1 - \tilde{B}_n\gamma_1 - \tilde{C}_n\rho_1 + \tilde{D}_{1,n} 2d_{12}d_{13}\sqrt{\Delta t})\sqrt{\Delta t} \\
\tilde{Q}_{2,n} &= (-\phi_2 - \tilde{A}_n\beta_2 - \tilde{B}_n\gamma_2 - \tilde{C}_n\rho_2 + \tilde{D}_{2,n} 2d_{22}d_{23}\sqrt{\Delta t})\sqrt{\Delta t} \\
\tilde{Q}_{3,n} &= (-\phi_3 - \tilde{B}_n\gamma_3 - \tilde{C}_n\rho_3 + \tilde{D}_{3,n} 2d_{32}d_{33}\sqrt{\Delta t})\sqrt{\Delta t} \\
\tilde{Q}_{4,n} &= (-\phi_4 - \tilde{C}_n\rho_4 + \tilde{D}_{4,n} 2d_{42}d_{43}\sqrt{\Delta t})\sqrt{\Delta t}.
\end{aligned} \tag{A.13}$$

Proof of Proposition 3

We need to compute $E_t[\ln(I_{t+n\Delta t}/I_t)]$. This expectation can be written as $E_t[\ln(I_{t+\Delta t}/I_t) + \ln(I_{t+n\Delta t}/I_{t+\Delta t})]$. Assuming the exponential affine structure in Proposition 3 with $(n-1)$

periods to go, then:

$$
\begin{aligned}
E_t\left[\ln\left(\frac{I_{t+n\Delta t}}{I_t}\right)\right] &= E_t\left[\ln\left(I_{t+\Delta t}/I_t\right) + K^*_{n-1} + A^*_{n-1}\pi_{t+\Delta t} + B^*_{n-1}r_{t+\Delta t}\right. \\
&\quad \left. + C^*_{n-1}\alpha_{t+\Delta t} + \Sigma_{j=1}^4 D^*_{j,n} h^2_{j,t+\Delta t}\right]
\end{aligned} \tag{A.14}
$$

Substitute in the dynamics for inflation from equation (2) and compute the resulting expectation. The recursive equations turn out to be:

$$
\begin{aligned}
K^*_{n+1} &= K^*_n + (b_0 B^*_n + c_0 C^*_n + \Sigma_{j=1}^4 \left(d_{j0} + d_{j2}\right) D^*_{j,n})\Delta t \\
A^*_{n+1} &= \Delta t + \left(1 + a_2 \Delta t\right) A^*_n + b_2 \Delta t B^*_n \\
B^*_{n+1} &= a_1 \Delta t A^*_n + (1 + b_1 \Delta t) B^*_n \\
C^*_{n+1} &= \Delta t A^*_n + (1 + c_1 \Delta t) C^*_n \\
D^*_{j,n+1} &= D^*_{j,n}\left[1 + \left(d_{j1} + d_{j2} d^2_{j3}\right)\Delta t\right] - 1_{\{j=1\}}\frac{1}{2}\Delta t,
\end{aligned} \tag{A.15}
$$

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Table 1: Summary Statistics

Nominal Yields

Maturity	Average	Minimum	Maximum	Annualized Std. Dev. Changes
1 month	0.0475	0.0003	0.1347	0.0243
3 months	0.0507	0.0006	0.1431	0.0134
6 months	0.0517	0.0015	0.1457	0.0122
1 year	0.0546	0.0029	0.1437	0.0120
2 years	0.0578	0.0065	0.1440	0.0125
3 years	0.0602	0.0090	0.1424	0.0126
5 years	0.0639	0.0171	0.1398	0.0123
7 years	0.0666	0.0236	0.1390	0.0119
10 years	0.0696	0.0309	0.1393	0.0116
15 years	0.0723	0.0349	0.1402	0.0110

Survey Inflation Forecasts

Maturity	Average	Minimum	Maximum	Annualized Std. Dev. Changes
1 month	0.0305	-0.0540	0.0821	0.0331
2 quarters	0.0314	-0.0030	0.0684	0.0095
4 quarters	0.0330	0.0150	0.0733	0.0050
6 quarters	0.0342	0.0170	0.0704	0.0043
8 quarters	0.0348	0.0190	0.0684	0.0057
10 years	0.0273	0.0223	0.0392	0.0020

Zero-Coupon Inflation Swaps

Correlation of Monthly Changes								Annualized Std. Dev. Changes	Average	Min.	Max.
Maturity	2	3	5	7	10	20	30				
2	1.00	0.89	0.84	0.78	0.59	0.46	0.44	0.0170	0.0205	-0.0240	0.0337
3		1.00	0.91	0.88	0.74	0.63	0.64	0.0138	0.0218	-0.0192	0.0332
5			1.00	0.97	0.88	0.69	0.64	0.0097	0.0238	-0.0006	0.0331
7				1.00	0.94	0.77	0.71	0.0084	0.0250	0.0049	0.0319
10					1.00	0.87	0.79	0.0068	0.0264	0.0130	0.0314
20						1.00	0.92	0.0065	0.0287	0.0147	0.0331
30							1.00	0.0070	0.0298	0.0149	0.0343

Note: The Treasury yields and the 1 month to 8 quarters Blue Chip Economic Indicator survey inflation forecasts are for the period January 1982 to May 2010. The 10-year maturity survey inflation forecast is from the Survey of Professional Forecasters and is for the period December 1991 to March 2010. The zero-coupon inflation swaps are for the period April 2003 to May 2010.

Table 2: Inflation Process Parameter Estimates

$$\frac{I_{t+\Delta t}}{I_t} = e^{\pi_t \Delta t - \frac{1}{2} h_{1,t}^2 \Delta t + h_{1,t} \sqrt{\Delta t} \varepsilon_{1,t+\Delta t}}$$

$$h_{1,t+\Delta t}^2 - h_{1,t}^2 = \left[d_{10} + d_{11} h_{1,t}^2 + d_{12} \left(\varepsilon_{1,t+\Delta t} - d_{13} h_{1,t} \right)^2 \right] \Delta t$$

$$\overline{h}_1^2 = -\frac{d_{10} + d_{12}}{d_{11} + d_{12} d_{13}^2}$$

Parameter	**Estimate**	***t*-Statistic**	***p*-Value**
$\overline{h}_1$	0.0088	6.12	0.000
d_{11}	-2.2034	-2.72	0.007
d_{12}	1.88×10^{-4}	3.67	0.000
d_{13}	2.54	0.159	0.874
Observations: 341			

Note: Estimation uses monthly data on inflation and the Blue Chip Economic Indicator one-month survey forecast of inflation from January 1982 to June 2010.

Table 3: Nominal and Real Term Structure Parameter Estimates

$$\pi_{t+\Delta t} - \pi_t = \left[\alpha_t + a_1 r_t + a_2 \pi_t\right]\Delta t + \sqrt{\Delta t}\ \sum_{j=1}^{2} \beta_j h_{j,t} \varepsilon_{j,t+\Delta t}$$

$$r_{t+\Delta t} - r_t = \left[b_0 + b_1 r_t + b_2 \pi_t\right]\Delta t + \sqrt{\Delta t}\ \sum_{j=1}^{3} \gamma_j h_{j,t} \varepsilon_{j,t+\Delta t}$$

$$\alpha_{t+\Delta t} - \alpha_t = \left[c_0 + c_1 \alpha_t\right]\Delta t + \sqrt{\Delta t}\ \sum_{j=1}^{4} \rho_j h_{j,t} \varepsilon_{j,t+\Delta t}$$

$$h_{j,t+\Delta t}^2 - h_{j,t}^2 = \left[d_{j0} + d_{j1} h_{j,t}^2 + d_{j2}\left(\varepsilon_{j,t+\Delta t} - d_{j3} h_{j,t}\right)^2\right]\Delta t$$

$$\overline{\pi} = -\frac{a_1 b_0 c_1 + b_1 c_0}{c_1\left(a_1 b_2 - a_2 b_1\right)},\quad \overline{r} = -\frac{a_2 b_0 c_1 + b_2 c_0}{c_1\left(a_1 b_2 - a_2 b_1\right)}$$

$$\overline{h}_j^2 = -\frac{d_{j0} + d_{j2}}{d_{j1} + d_{j2} d_{j3}^2},\quad \text{Risk premia: } \phi_j h_{j,t},\quad j = 1,2,3,4$$

Parameter	No GARCH	h_2 GARCH	h_3 GARCH	h_4 GARCH	h_2, h_3, h_4 GARCH
$\overline{\pi}$	0.0302***	0.0292***	0.0403***	0.0181***	0.0299***
a_1	0.6160***	0.5860***	0.5770***	0.5680***	0.6131***
a_2	-3.0914***	-3.0636***	-2.5964***	-3.2317***	-2.6702***
β_1	1.1578***	1.1240***	1.2272***	1.0610***	1.2591***
b_1	-1.4722***	-1.4490***	-1.2792***	-1.4653***	-1.3370***
b_2	2.2994***	2.2677***	1.9623***	2.2113***	2.0219***
γ_1	-0.4931***	-0.4653***	-0.9876***	-0.2520	-0.6599***
γ_2	-0.9987***	-0.9888***	-0.9640***	-0.9962***	-0.9624***
$\overline{r}$	0.0274***	0.0264***	0.0187***	0.0226***	0.0176***
c_1	-0.0599***	-0.0560***	-0.0560***	-0.0623***	-0.0558***
ρ_1	0.1615**	0.1639**	0.1021	0.5647***	0.1683***
ρ_2	0.0205	0.0076	-0.0013	0.0601***	-0.0237
ρ_3	-0.1103***	-0.1123***	-0.1815***	0.0500***	-0.1907***
$\overline{h}_2$	0.0280***	0.0291***	0.0278***	0.0284***	0.0294***
d_{21}		-7.9610***			-7.4680***
d_{22}		0.0047***			0.0048***
d_{23}		-0.35			-1.60
$\overline{h}_3$	0.0256***	0.0253***	0.0167***	0.0280***	0.0151***
d_{31}			-11.8793***		-5.7017***
d_{32}			0.0014***		0.0012***
d_{33}			49.76***		35.72***
$\overline{h}_4$	0.0132***	0.0132***	0.0111***	0.0101***	0.0104***
d_{41}				-7.3717***	-4.0296***
d_{42}				0.00003***	0.00004***
d_{43}				-451.1***	-249.05***
ϕ_1	56.99***	57.20***	38.92***	66.29***	60.50***
ϕ_2	-21.36***	-23.55***	-3.52**	- 30.72***	-20.57***
ϕ_3	5.77	0.34	-77.05***	22.09***	-50.75***
ϕ_4	-17.60***	-20.31***	-37.24***	-54.82***	-44.67***
w	0.0039	0.0039	0.0036	0.0039	0.0036
v	0.0039	0.0039	0.0041	0.0038	0.0040
u	0.0029	0.0029	0.0029	0.0029	0.0028
Ln Likelihood	35848	35891	36058	35886	36199
Reject No GARCH?		Yes	Yes	Yes	Yes

Note: ***, **, and * denotes statistical significance at the 1%, 5%, and 10% level. w, v, and u are the standard deviations of the measurement errors for nominal Treasury yields, survey inflation rate forecasts, and inflation swap rates, respectively. For each set of estimates, the parameters of the GARCH process for inflation (h_1) are fixed at the point estimates reported in Table 2.

Table 4: State Variable Standard Deviations and Correlations

	Steady State	1982 – 2010 Sample Period Average	Minimum	Maximum
Standard Deviations				
$\ln(I_{t+\Delta t}/I_t)$	0.0088	0.0083	0.0039	0.0210
$\pi_{t+\Delta t}$	0.0314	0.0294	0.0166	0.0919
$r_{t+\Delta t}$	0.0326	0.0344	0.0173	0.0886
$\alpha_{t+\Delta t}$	0.0109	0.0113	0.0077	0.0170
Correlations				
$\ln(I_{t+\Delta t}/I_t)$, $\pi_{t+\Delta t}$	0.354	0.371	0.133	0.729
$\ln(I_{t+\Delta t}/I_t)$, $r_{t+\Delta t}$	-0.179	-0.167	-0.406	-0.051
$\ln(I_{t+\Delta t}/I_t)$, $\alpha_{t+\Delta t}$	0.136	0.126	0.050	0.289
$\pi_{t+\Delta t}$, $r_{t+\Delta t}$	-0.874	-0.773	-0.990	-0.285
$\pi_{t+\Delta t}$, $\alpha_{t+\Delta t}$	-0.011	-0.005	-0.167	0.142
$r_{t+\Delta t}$, $\alpha_{t+\Delta t}$	-0.092	-0.187	-0.687	0.148

Note: The annualized standard deviations and correlations are for one-month horizons and based on parameter estimates from the unrestricted model.

Table 5: Campbell-Shiller Regressions for Nominal Yields

The left panel shows the results for Campbell–Shiller regressions of the monthly changes in actual nominal yields against the adjusted slope (slope divided by maturity in months minus 1) over the period January 1982 to May 2010.

$$y_{N,t+\Delta t}^{(n-1)} - y_{N,t}^{(n)} = \beta_{N,0}^{(n)} + \beta_{N,1}^{(n)} \frac{s_{N,t}^{(n)}}{n-1} + \varepsilon_{N,t+\Delta t}^{(n)}$$

For each maturity, the coefficients of the adjusted slope are shown together with the standard error and the *t* statistic for the null hypothesis that $\beta_{N,1}^{(n)} = 1$. The column next to the Slope column reports the *p* value for the test $\beta_{N,1}^{(n)} = 1$.

The next two columns report the results for regressions that include the model-implied time varying risk premium:

$$y_{N,t+\Delta t}^{(n-1)} - y_{N,t}^{(n)} = \beta_{N,0}^{(n)} + \beta_{N,1}^{(n)} \frac{s_{N,t}^{(n)}}{n-1} + \beta_{N,2}^{(n)} \frac{\pi_{N,t}^{(n)}}{n-1} + \varepsilon_{N,t+\Delta t}^{(n)}$$

For each maturity, the coefficients of the adjusted Slope are shown together with the standard error and the *t* statistic for the null hypothesis that $\beta_{N,1}^{(n)} = 1$. The coefficients of the adjusted Risk Premium are shown together with the standard error and the *t* statistic for the null hypothesis that $\beta_{N,2}^{(n)} = -1$. The *p* value is now for the joint test that $\beta_{N,1}^{(n)} = 1$ and $\beta_{N,2}^{(n)} = -1$. The right panel repeats the tests using the model-implied nominal yields.

	Actual					Model-Implied				
	Expectations Hypothesis		Expectations Hypothesis with Model Risk Premium			Expectations Hypothesis		Expectations Hypothesis with Model Risk Premium		
Maturity	Slope	*p* Value	Slope	Risk Premium	Joint Test *p* Value	Slope	*p* Value	Slope	Risk Premium	Joint Test *p* Value
	0.02		0.77	-0.86		0.07		2.53	-1.56	
1	(0.28)	0.00	(0.31)	(0.18)	0.67	(0.41)	0.02	(0.55)	(0.25)	0.02
	-3.54		-0.72	0.81		-2.28		2.77	-2.24	
	-0.15		0.90	-1.12		0.04		1.71	-1.48	
2	(0.51)	0.03	(0.58)	(0.30)	0.84	(0.52)	0.06	(0.60)	(0.29)	0.24
	-2.24		-0.17	-0.39		-1.86		1.19	-1.65	
	-0.45		0.70	-1.38		-0.34		0.93	-1.51	
3	(0.71)	0.04	(0.80)	(0.47)	0.73	(0.66)	0.05	(0.75)	(0.44)	0.39
	-2.04		-0.38	-0.81		-2.01		-0.09	-1.16	
	-1.21		-0.12	-1.69		-1.41		-0.64	-1.35	
5	(1.04)	0.03	(1.11)	(0.97)	0.24	(1.06)	0.02	(1.22)	(1.05)	0.21
	-2.12		-1.01	-0.71		-2.27		-1.35	-0.34	
	-1.93		-1.32	-0.48		-2.48		-2.34	-0.30	
7	(1.35)	0.03	(1.32)	(1.22)	0.20	(1.50)	0.02	(1.64)	(1.46)	0.12
	-2.17		-1.76	0.42		-2.32		-2.03	0.48	
	-2.80		-2.03	0.54		-3.82		-3.86	0.08	
10	(1.80)	0.04	(1.59)	(1.03)	0.08	(2.15)	0.03	(2.21)	(1.30)	0.09
	-2.11		-1.91	1.50		-2.25		-2.19	0.84	
	-3.71		-2.22	0.75		-5.30		-5.28	-0.05	
15	(2.52)	0.06	(2.08)	0.86	0.05	(3.12)	0.04	(3.16)	(1.13)	0.12
	-1.87		-1.55	-2.05		-2.02		-1.99	0.84	

Table 6: Campbell-Shiller Regressions for Real Yields

The left panel shows the results for Campbell–Shiller regressions of the monthly changes in actual real yields against the adjusted slope (slope divided by maturity in months minus 1) over the period January 2003 to May 2010.

$$y_{R,t+\Delta t}^{(n-1)} - y_{R,t}^{(n)} = \beta_{R,0}^{(n)} + \beta_{R,1}^{(n)} \frac{s_{R,t}^{(n)}}{n-1} + \varepsilon_{R,t+\Delta t}^{(n)}$$

Real yields in this panel were derived as the difference between the equivalent maturity nominal Treasury yield and the inflation swap rate. For each maturity, the coefficients of the adjusted slope are shown together with the standard error and the *t* statistic for the null hypothesis that $\beta_{R,1}^{(n)} = 1$. The column next to the Slope column reports the *p* value for the test $\beta_{R,1}^{(n)} = 1$.

The next two columns report the results for regressions that include the model-implied time varying risk premium:

$$y_{R,t+\Delta t}^{(n-1)} - y_{R,t}^{(n)} = \beta_{R,0}^{(n)} + \beta_{R,1}^{(n)} \frac{s_{R,t}^{(n)}}{n-1} + \beta_{R,2}^{(n)} \frac{\pi_{R,t}^{(n)}}{n-1} + \varepsilon_{R,t+\Delta t}^{(n)}$$

For each maturity, the coefficients of the adjusted Slope are shown together with the standard error and the *t* statistic for the null hypothesis that $\beta_{R,1}^{(n)} = 1$. The coefficients of the adjusted Risk Premium are shown together with the standard error and the *t* statistic for the null hypothesis that $\beta_{R,2}^{(n)} = -1$. The *p* value is now for the joint test that $\beta_{R,1}^{(n)} = 1$ and $\beta_{R,2}^{(n)} = -1$. The right panel repeats the tests using the model-implied real yields.

	Actual					Model-Implied				
	Expectations Hypothesis		Expectations Hypothesis with Model Risk Premium			Expectations Hypothesis		Expectations Hypothesis with Model Risk Premium		
Maturity	Slope	*p* Value	Slope	Risk Premium	Joint Test *p* Value	Slope	*p* Value	Slope	Risk Premium	Joint Test *p* Value
	0.21		0.15	0.31		1.91		2.64	-1.26	
2	(0.77)	0.31	(0.81)	(1.19)	0.41	(0.35)	0.01	(0.38)	(0.28)	0.00
	-1.03		-1.05	1.10		2.59		4.32	-0.92	
	0.58		0.66	-0.38		1.60		2.28	-1.42	
3	(0.88)	0.64	(0.93)	(1.46)	0.89	(0.37)	0.10	(0.39)	(0.33)	0.00
	-0.47		-0.37	0.42		1.65		3.30	-1.26	
	1.00		1.55	-2.19		1.14		1.96	-2.04	
5	(1.05)	1.00	(1.17)	(2.03)	0.82	(0.48)	0.78	(0.53)	(0.59)	0.11
	0.00		0.47	-0.59		0.28		1.80	-1.79	
	1.64		2.99	-4.36		0.78		1.80	-2.76	
7	(1.36)	0.64	(1.58)	(2.63)	0.35	(0.65)	0.73	(0.74)	(0.97)	0.19
	0.47		1.26	-1.28		-0.34		1.08	-1.82	
	2.09		4.70	-6.95		0.36		1.19	-2.35	
10	(1.72)	0.53	(1.98)	(2.87)	0.09	(0.91)	0.48	(1.01)	(1.24)	0.53
	0.63		1.86	-2.07		-0.70		0.19	-1.09	
	2.28		5.58	-7.18		-0.10		0.37	-1.29	
15	(2.48)	0.61	(2.86)	(3.31)	0.14	(1.32)	0.40	(1.38)	(1.15)	0.83
	0.51		1.60	-1.87		-0.84		-0.46	-0.25	

Figure 1: Breakeven Inflation Rates and Inflation Swap Rates

The top two panels compare continuously-compounded breakeven inflation rates (zero-coupon Treasury yield minus zero-coupon TIPS yield) with continuously-compounded zero-coupon inflation swap rates over the period from January 2003 to June 2010. The top left panel is for a five-year maturity while the top right panel is for a ten-year maturity. The lower left panel shows the bid-offer yield spread in basis points for the ten-year TIPS and for the ten-year inflation swap rate. The lower right panel compares the difference between the ten-year inflation swap rate and the ten-year breakeven inflation rate (nominal Treasury yield minus TIPS yield) with the bid-offer spread of the ten-year TIPS scaled up by 10.

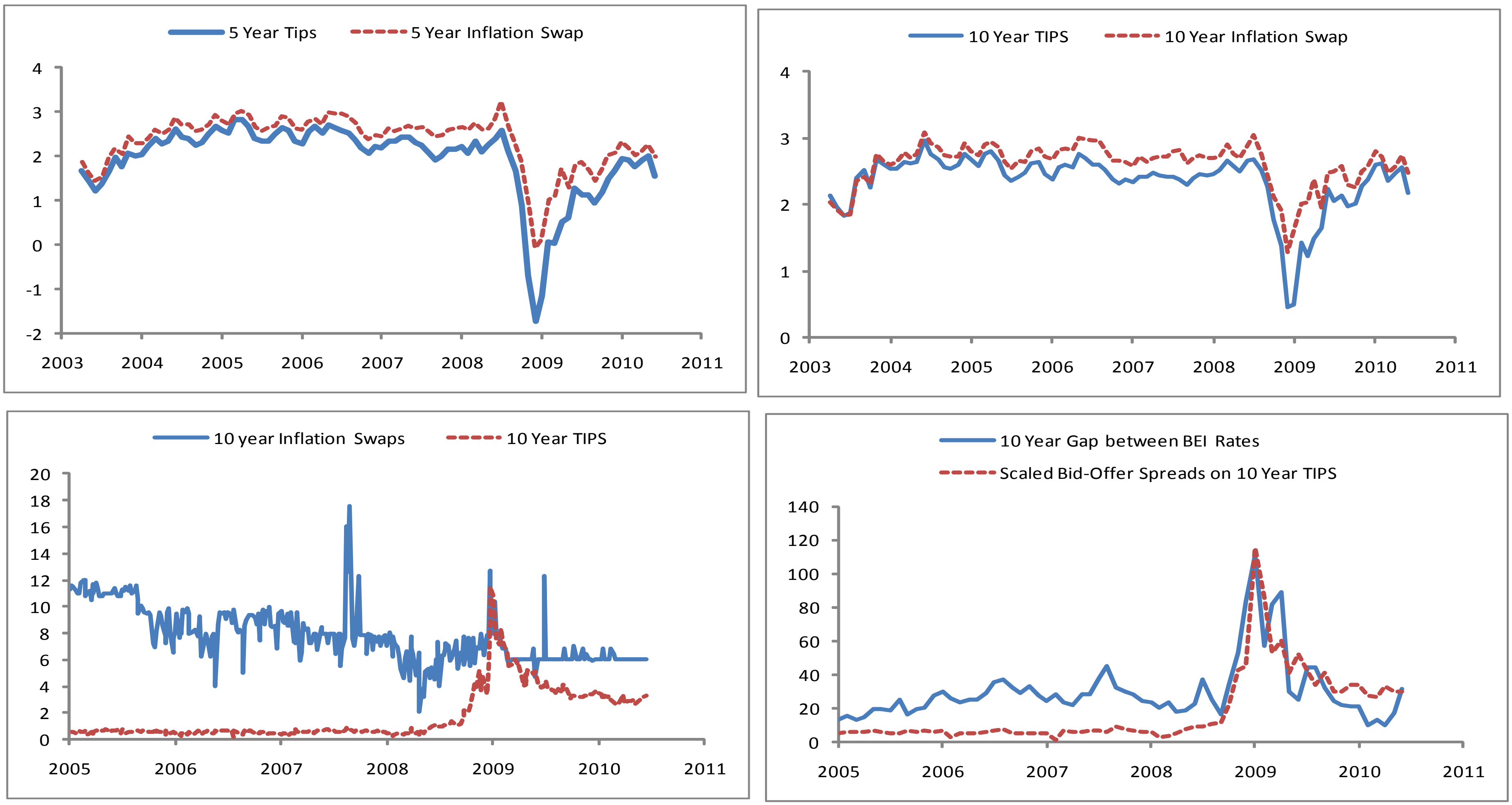


Figure 2: Levels and Standard Deviations of State Variables

The top panel gives the values of one-month expected inflation, inflation's central tendency, and the one-month real interest rate of the 1982 to 2010 sample period. The bottom panel gives the one-month standard deviations of the unexpected component of inflation, the change in expected inflation, the change in the real rate, and the change in the central tendency.

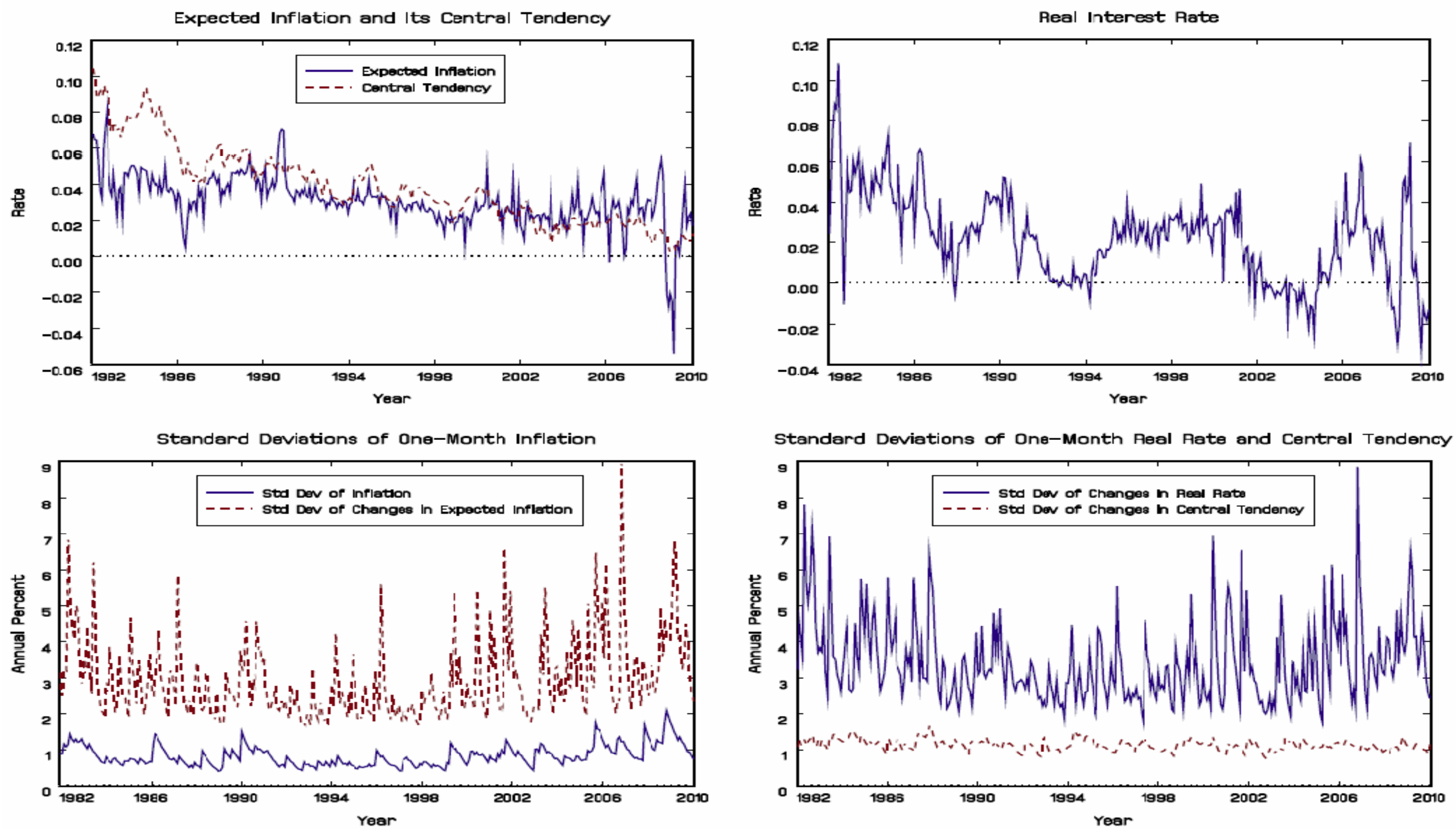
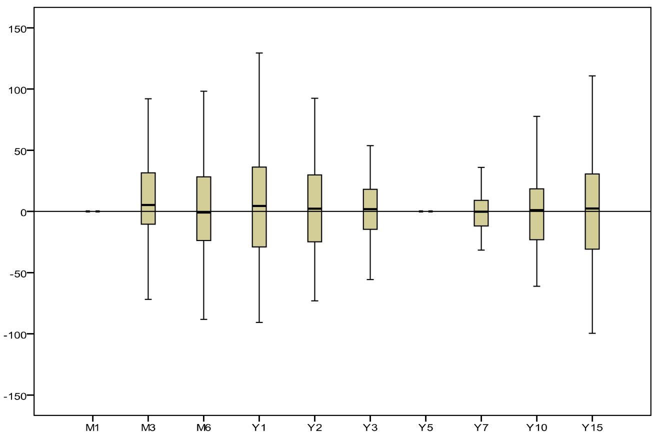
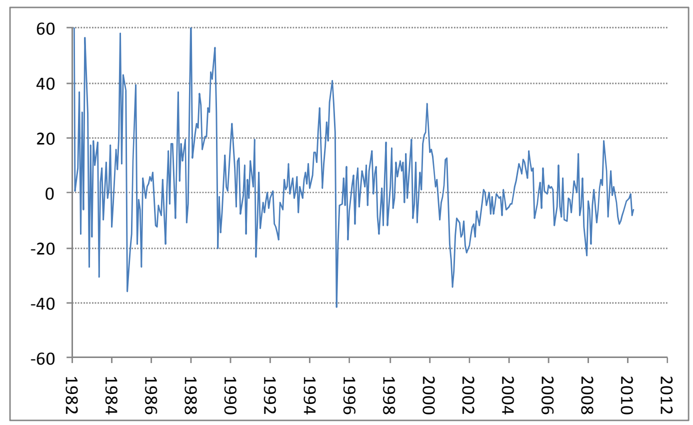


Figure 3: Model Fit of Nominal Yields, Inflation Swap Rates, and Survey Forecasts of Inflation

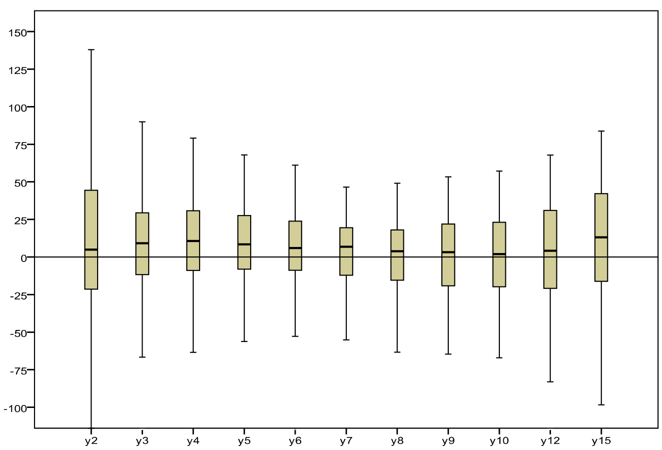
For each row of panels, the left panel shows the box plot of measurement errors (actual minus model-implied) in basis points for various maturities while the right panel is the time series of average measurement errors across all maturities. The top, middle, and bottom panels are for nominal yields, inflation swap rates, and survey forecasts of inflation rates, respectively.

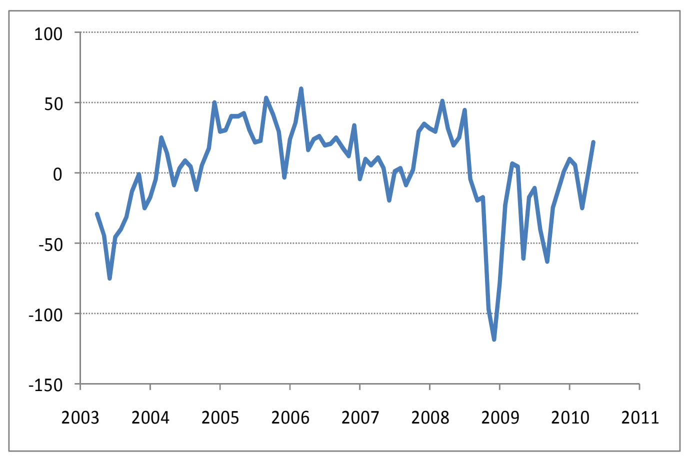
Nominal Yields



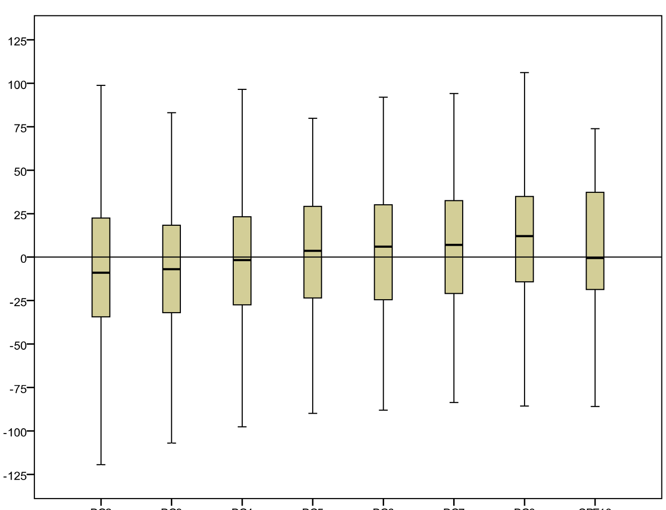


Inflation Swap Rates





Survey Forecasts of Inflation Rates



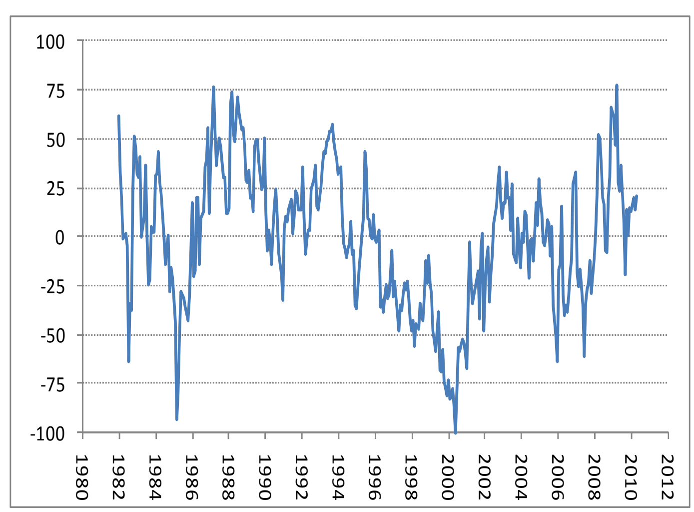


Figure 4: Steady State Yield Curves and Expected Excess Returns

The graph shows the nominal yield curve and inflation-indexed (real) yield curve when all state variables are initially at their steady states. Also shown are the expected excess nominal returns on nominal bonds and the expected excess real returns on real bonds (expected returns relative to one-month maturity).

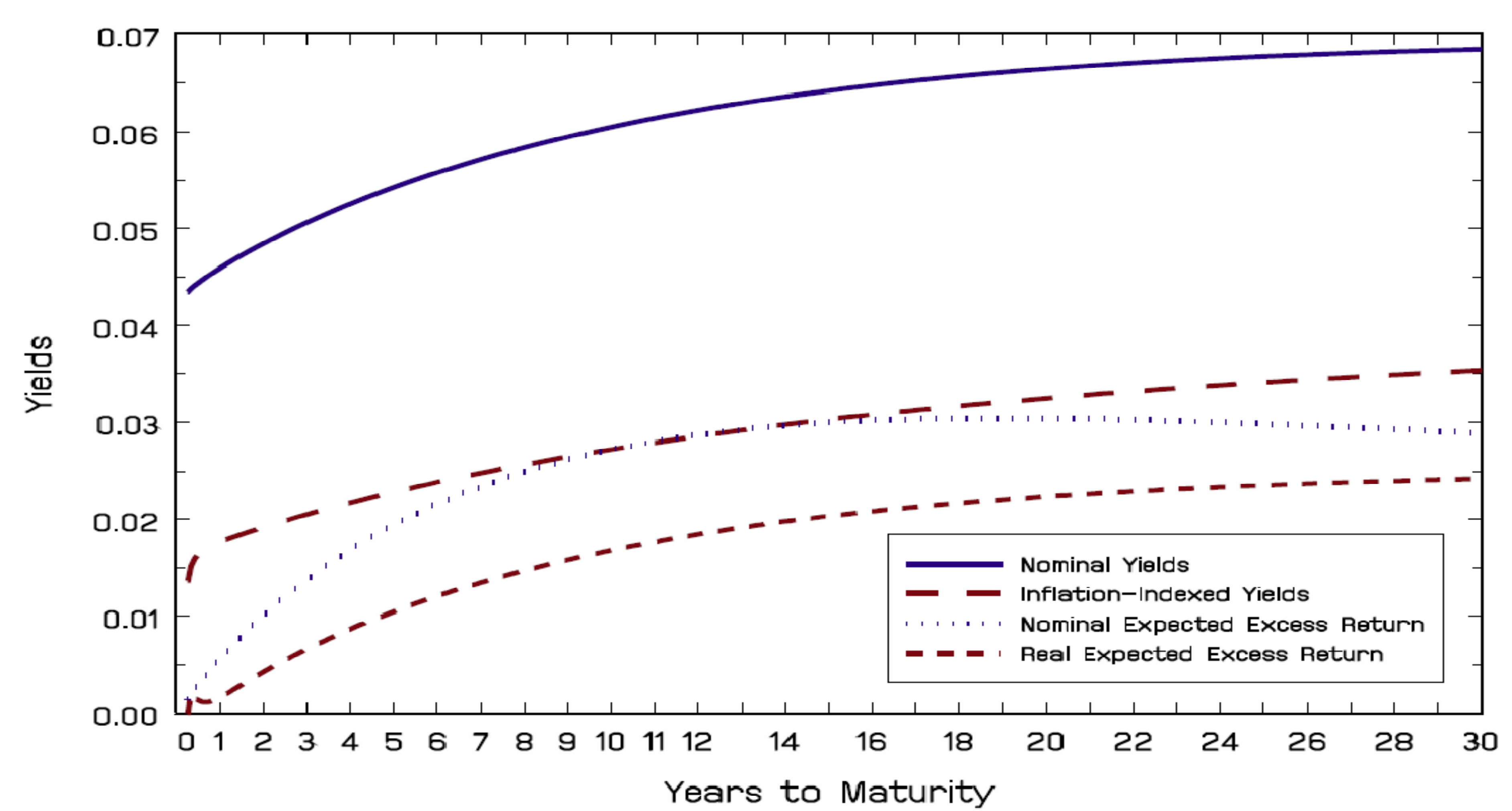


Figure 5: Decomposition of Nominal Interest Rates

The panels show the contribution of one-month expected inflation (π), the one-month real rate (r), the central tendency of inflation (α), and the volatility state variables (h) to the six-month, two-year, five-year and ten-year maturity nominal yields.

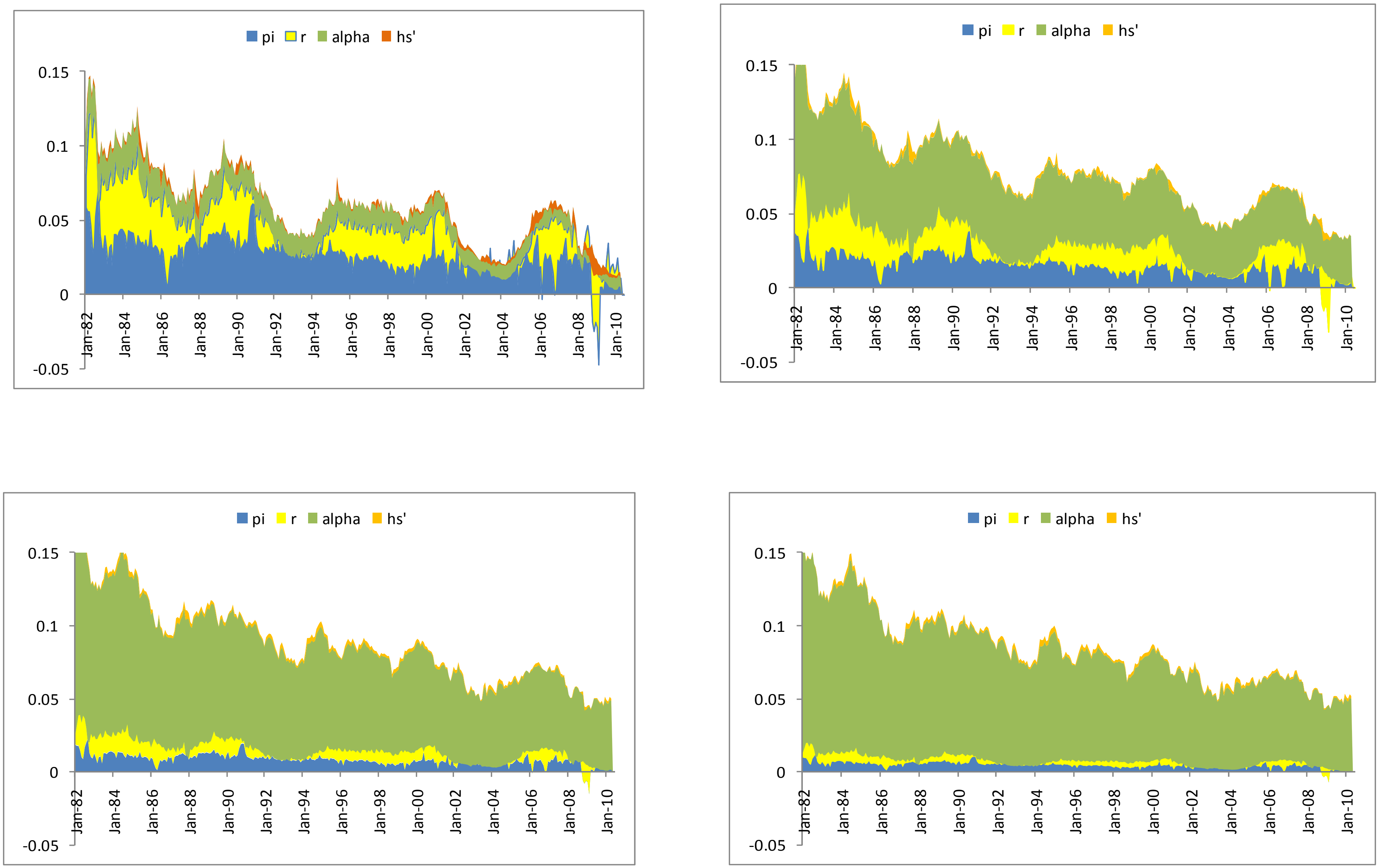
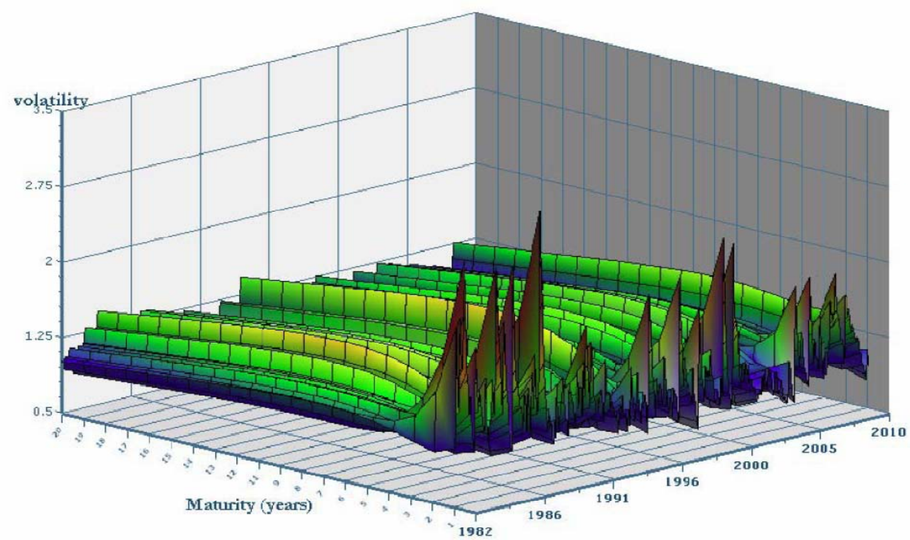
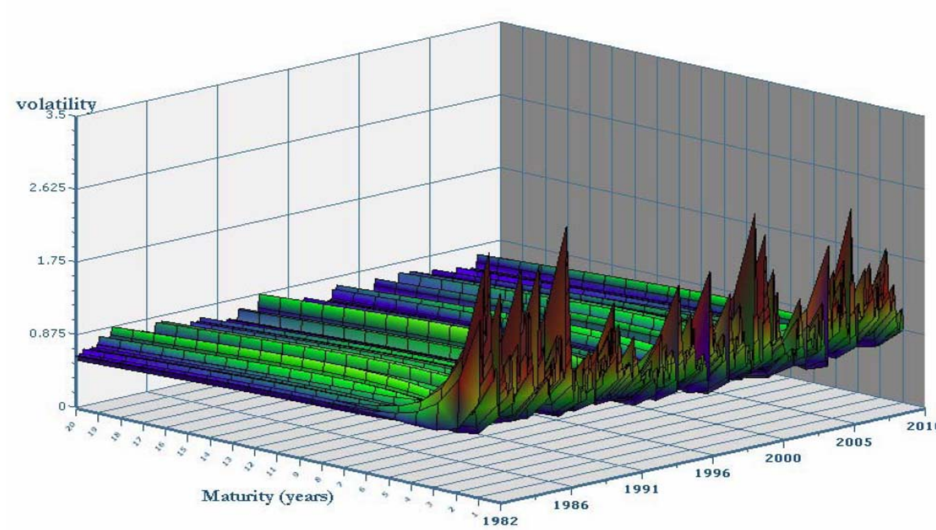


Figure 6: Nominal and Real Yield Volatilities

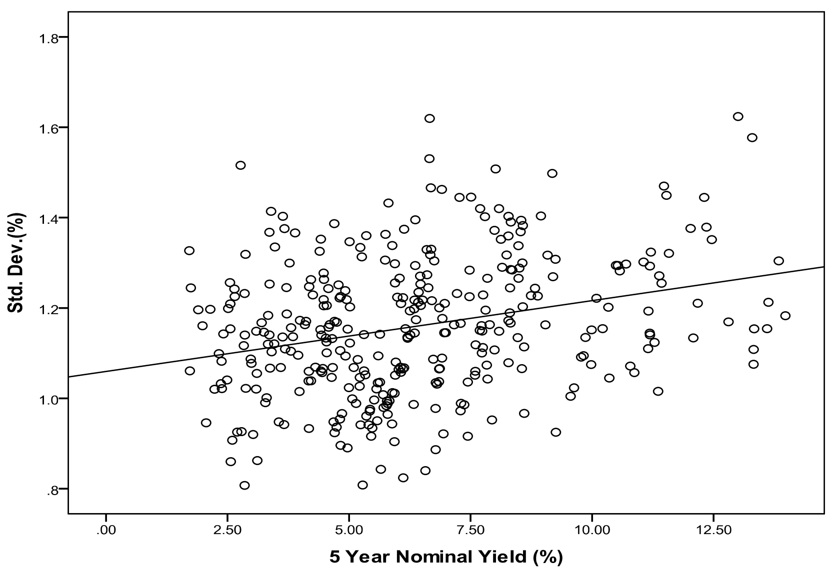
Nominal Yield Volatility



Real Yield Volatility



Monthly Standard Deviation versus Level: Five-Year Nominal Yield



Monthly Standard Deviation versus Level: Five-Year Real Yield

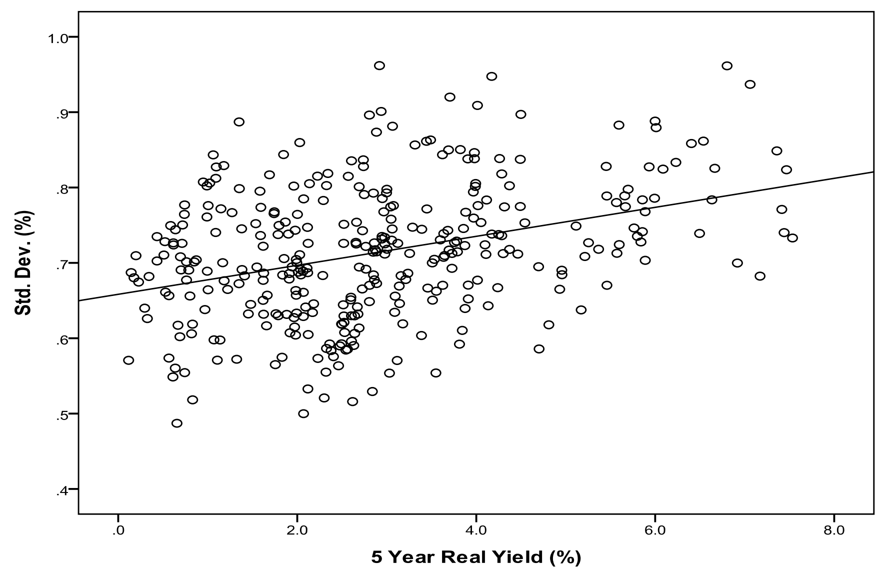
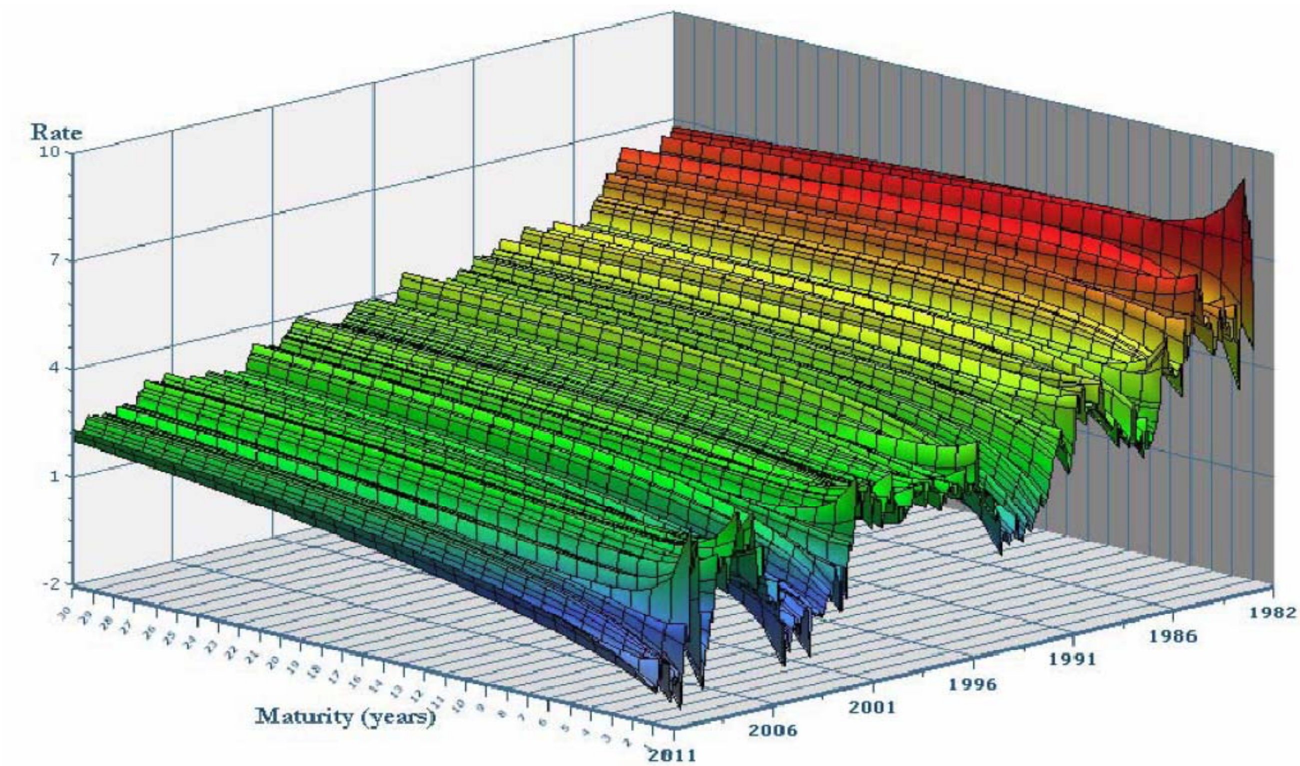


Figure 7: Term Structures of Real Yields and Inflation Expectations

Inflation-Indexed Yield Curves



Term Structures of Expected Inflation

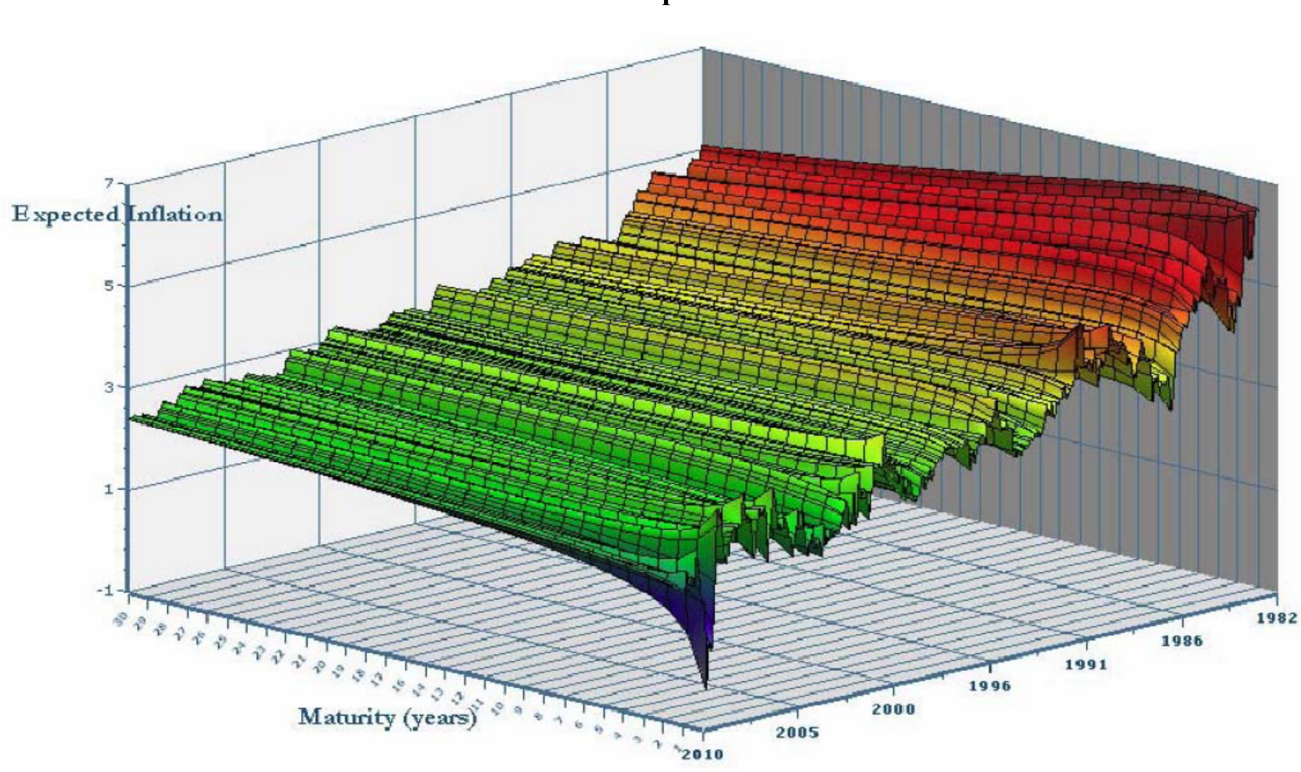
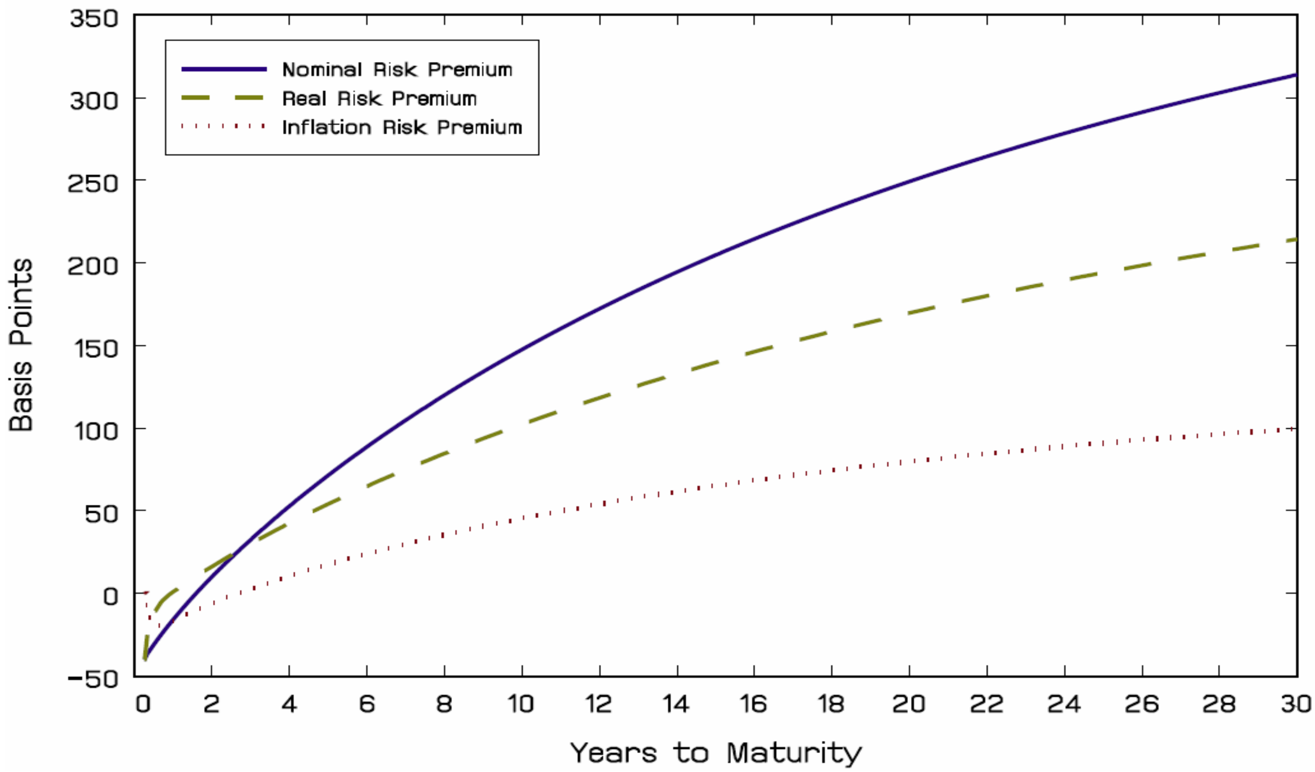


Figure 8 Real and Inflation Risk Premia for Steady State and Ten-Year Maturity

Nominal, Real, and Inflation Risk Premia when All Variables are at Their Steady States



Real and Inflation Risk Premia and Inflation Expectations for Ten-Year Maturity

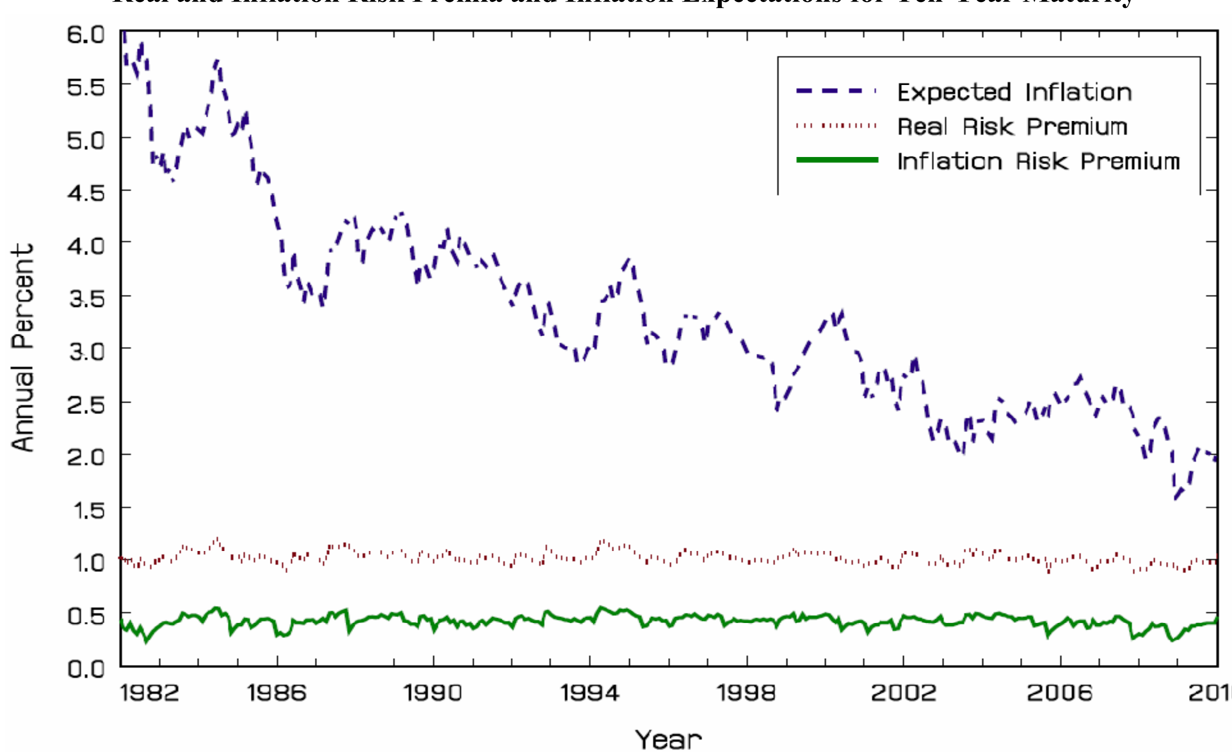


Figure 9: TIPS Yields versus Model-Implied Inflation-Indexed Yields