

Materials 11 - trying to get a reasonable gain \bar{g}

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1 Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (1)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (2)$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \rho i_{t-1} + \bar{i}_t \quad (3)$$

$$\hat{\mathbb{E}}_t z_{t+h} = \begin{bmatrix} \bar{\pi}_{t-1} \\ 0 \\ 0 \end{bmatrix} + b h_x^{h-1} s_t \quad \forall h \geq 1 \quad b = g_x h_x \quad \text{PLM} \quad (4)$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \underbrace{(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))}_{\text{fcst error using (4)}} \quad (b_1 \text{ is the first row of } b) \quad (5)$$

$$k_t = \begin{cases} k_{t-1} + 1 & \text{for decreasing gain learning} \\ \bar{g}^{-1} & \text{for constant gain learning.} \end{cases} \quad (6)$$

2 Recap of timing

Define some objects: (*I usually let t denote the time in which the variable is formed.*)

$$FE_{t-1} = z_t - \hat{\mathbb{E}}_{t-1}(z_t) \quad \text{one-period-ahead forecast error realized at time } t \quad (7)$$

$$= ALM(t) - PLM(t-1) \quad (8)$$

$$\theta_t = \hat{\mathbb{E}}_{t-1}(z_t) - \mathbb{E}_{t-1}(z_t) \quad \text{CEMP's criterion} \quad (9)$$

$$= PLM(t-1) - \mathbb{E}_{t-1} ALM(t) \quad (10)$$

$$PLM(t) : \hat{\mathbb{E}}_t z_{t+1} = \bar{z}_{t-1} + b s_t$$

Morning: morning of time t available: $\mathcal{I}_t^m = \{\bar{z}_{t-1}, s_t, k_{t-1}, FE_{t-2}\}$

1. Form all future expectations using $PLM(t)$ (morning forecast) $\rightarrow z_t$ realized, $\rightarrow FE_{t-1}$ realized
2. Form $\theta_t \rightarrow k_t$ realized
3. **Evening:** Update $\bar{z}_t = \bar{z}_{t-1} + k_t^{-1}(z_t - (\bar{z}_{t-1} + b s_{t-1}))$

\rightarrow evening of time t available: $\mathcal{I}_t^e = \{\bar{z}_t, s_t, k_t, FE_{t-1}\}$

3 Trying to get an optimal gain