Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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Inflation that runs below its desired level can lead to an unwelcome fall in ***longer-term inflation expectations****, which, in turn, can pull actual inflation even lower, resulting in an adverse cycle of ever-lower inflation and inflation expectations. [...]* ***Well-anchored inflation expectations*** *are critical[.]*
Jerome Powell, Chairman of the Federal Reserve [1]
(Emphases added.)

[1] "New Economic Challenges and the Fed's Monetary Policy Review," August 27, 2020.

Long-run expectations: capturing responsiveness to short-run conditions

Individual SPF forecasts: for 1991-Q4 onward, estimate rolling regression

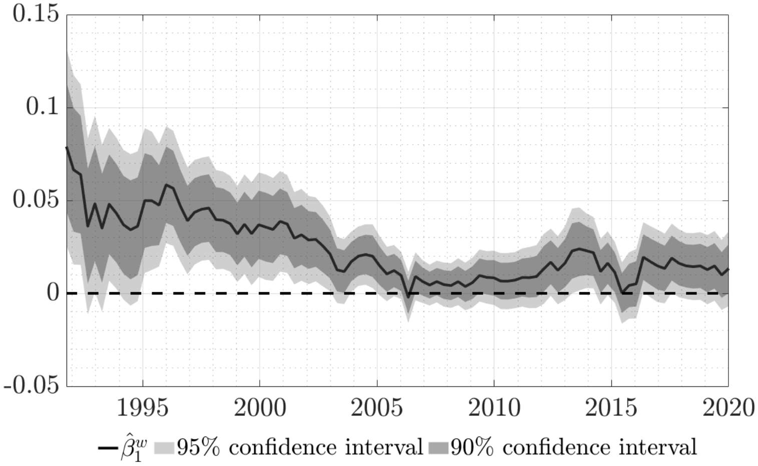
$$\Delta\bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{1}$$

where w indexes windows of 20 quarters length,

$f_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1}(\pi_t)$ individual one-year-ahead forecast error

Time-varying responsiveness

$$\Delta \bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{1}$$



▸ Disagreement measures

This project

- How to conduct monetary policy in interaction with the anchoring expectation formation?

- Model of expectations unanchoring
  $\hookrightarrow$ extension to adaptive learning that captures time-varying responsiveness of long-run expectations

- Estimate how unanchoring takes place in data
  $\hookrightarrow$ quantify novel anchoring channel and use for monetary policy analysis

Preview of results

1. Estimation
    - Larger mistakes unanchor more
    - Overestimating inflation unanchors more than underestimating it (Hebden et al 2020)
    - On average, people discount observations older than 8 quarters
2. Optimal policy
    - Responds aggressively to inflation when unanchored, accommodates inflation when anchored
3. Taylor rule
    - Less aggressive on inflation than under rational expectations

Related literature

- **Optimal monetary policy in the New Keynesian model**

  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Adaptive learning**

  Evans & Honkapohja (2001, 2006), Sargent (1999), Primiceri (2006), Lubik & Matthes (2018), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Mele et al (2019), Eusepi & Preston (2011), Milani (2007, 2014)

- **Anchoring and the Phillips curve**

  Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Reis (2020), Hebden et al 2020, Gobbi et al (2019), Carvalho et al (2019)

- **Reputation**

  Barro (1986), Cho & Matsui (1995)

Structure of talk

1. Model of anchoring expectations
2. Quantification of learning channel
3. Solving the Ramsey problem
4. Implementing optimal policy
5. Approximating optimal policy with a Taylor rule

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Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j))dj \right] \tag{2}$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_tC_t^i \tag{3}$$

▸ Consumption, price level

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \tag{4}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{5}$$

▸ Profits, stochastic discount factor

Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- Model implies mapping between exogenous states (s_t) and observables ($y_t \equiv (\pi, x, i)'$)

$$y_t = g s_t \tag{6}$$

- Under rational expectations (RE), private sector knows model $\rightarrow$ knows (6)

$$\mathbb{E}_t(y_{t+1}) = g\,\mathbb{E}(s_{t+1}) \tag{7}$$

- $\hat{\mathbb{E}}$: agents do not internalize that identical $\rightarrow$ do not know aggregate model $\rightarrow$ do not know (6)

Adaptive learning

- Agents know exogenous evolution of states

$$s_t = hs_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \tag{8}$$

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- Postulate linear functional relationship instead of (6):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{9}$$

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$a \rightarrow$ concept of long-run expectations in the model

- Estimate a, b using recursive least squares (RLS) using observed states and knowledge of (8)

Recursive least squares

Jumps are: $(\pi, x, i)'$

Assumption: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\, h \quad \forall t \tag{10}$$

Recursive least squares

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$\bar{\pi}_{t-1}$: long-run inflation expectations $\rightarrow$ anchoring

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$\bar{\pi}_{t-1}$: long-run inflation expectations $\rightarrow$ anchoring

$\rightarrow$ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\right)}_{\equiv f_{t|t-1}, \text{ forecast error}} \tag{11}$$

$k_t \in (0,1)$ gain
b_1 first row of b

General RLS algorithm

Decreasing versus constant gain

Decreasing gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f_{t|t-1} \tag{12}$$

→ consider sample mean of full sample of forecast errors

Decreasing versus constant gain

Decreasing gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f_{t|t-1} \tag{12}$$

$\rightarrow$ consider sample mean of full sample of forecast errors

Constant gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k\, f_{t|t-1} \tag{13}$$

$\rightarrow$ consider sample mean of most recent observations only

Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t\big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big) \tag{14}$$

$k_t = \mathbf{g}(f_{t|t-1})$: anchoring function

Anchoring mechanism: endogenous gain

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$k_t = \mathbf{g}(f_{t|t-1})$: anchoring function

$$\mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}) \tag{15}$$

$b_i(f_{t|t-1})$ = basis, here: second order spline (piecewise linear)

γ_i = approximating coefficients
$\hookrightarrow$ estimate $\hat{\gamma}$ via simulated method of moments (SMM)

Target autocovariances of inflation, output gap, federal funds rate and 1-year ahead SPF inflation expectations at lags $0, \ldots, 4$

▸ Functional forms in literature

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Anchoring function in the data

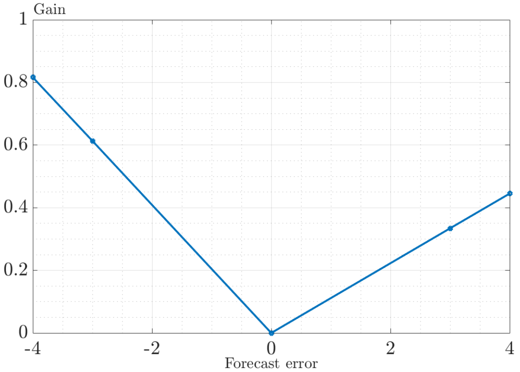
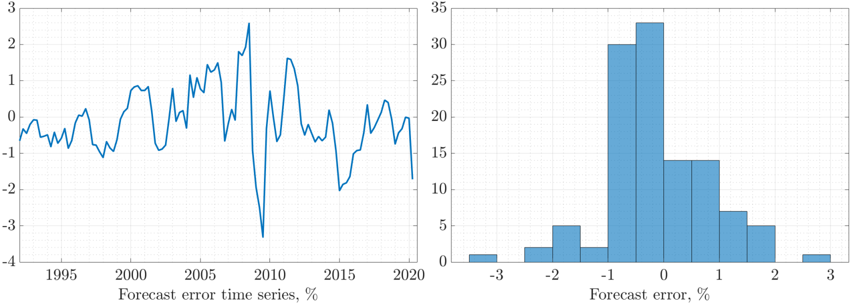


Figure: Learning gain as a function of forecast errors in inflation in %

Forecast errors in the data

Figure: Time series and distribution of 1-year ahead forecast errors in the SPF



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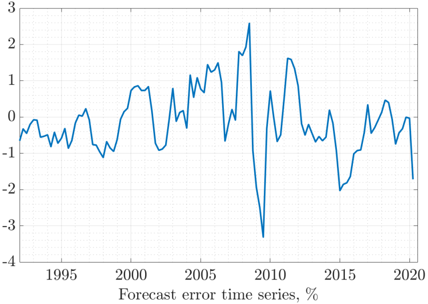

Typical forecast error ≈ -0.5 pp

Forecast errors in the data

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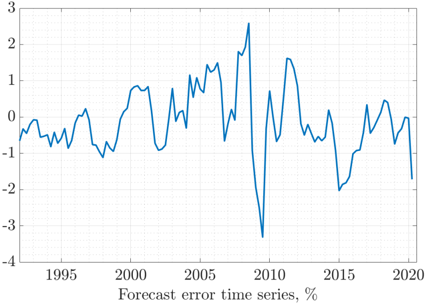

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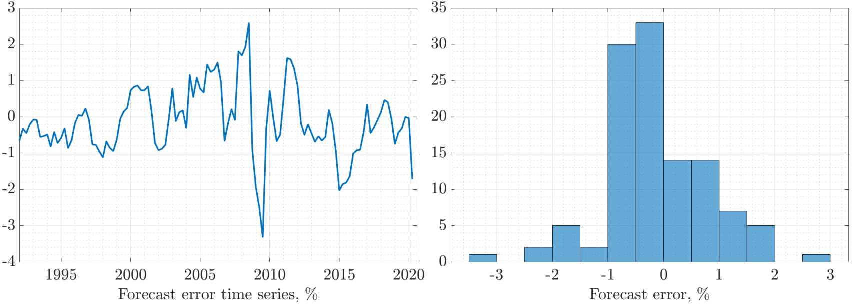

Typical forecast error ≈ -0.5 pp $\rightarrow$ gain of 0.1

$\Rightarrow$ 5 bp shift down in long-run expectations

Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \Big) \tag{16}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \Big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \Big) \tag{17}$$

Derivations Actual laws of motion

- Expectations evolve according to RLS with the endogenous gain given by (15)

→ How should $\{i_t\}$ be set?

Structure of talk

Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

- $\mathbb{E}$ is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

Target criterion

Proposition

In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa}x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_{\pi,t}\right)$$

$$\left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi},\mathbf{t+j}})\right)$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t, $\prod_{j=0}^{0} \equiv 1$ *and* b_1 *is the first row of* b.

General case

Two layers of intertemporal stabilization tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i}$$
$$- \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} \left(k_{t+1+j} + f_{t+1+j|t+j} \mathbf{g}_{\bar{\pi},\mathbf{t+j}} \right)$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

▸ Why no commitment?

Corollary

Optimal policy under adaptive learning is time-consistent.

$\hookrightarrow$ Foreshadow: optimal policy aggressiveness time-varying

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Solution procedure

Solve system of model equations + target criterion

$\hookrightarrow$ solve using parameterized expectations (PEA)

$\hookrightarrow$ obtain a cubic spline approximation to optimal policy function

Calibration - parameters from the literature

β	0.98	stochastic discount factor
σ	1	intertemporal elasticity of substitution
α	0.5	Calvo probability of not adjusting prices
κ	0.0842	slope of the Phillips curve
ψ_π	1.5	coefficient of inflation in Taylor rule*
$\bar{g}$	0.145	initial value of the gain
λ_x	0.05	weight on the output gap in central bank loss
ρ_r	0	persistence of natural rate shock
ρ_i	0	persistence of monetary policy shock*
ρ_u	0	persistence of cost-push shock

* pertains to sections where Taylor rule is in effect

Calibration - matching moments

ψ_x	0.3	coefficient of the output gap in Taylor rule*
σ_r	0.01	standard deviation, natural rate shock
σ_i	0.01	standard deviation, monetary policy shock*
σ_u	0.5	standard deviation, cost-push shock
$\hat{\gamma}_i$	(0.82; 0.61; 0; 0.33; 0.45)	coefficients in anchoring function

Calibrated ($\sigma_j,\ j = r, i, u$) or estimated ($\hat{\gamma}_i$) to match the autocovariances of inflation, output gap, interest rate and one-period ahead inflation expectations for lags $0, \ldots, 4$.

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Optimal policy - responding to unanchoring

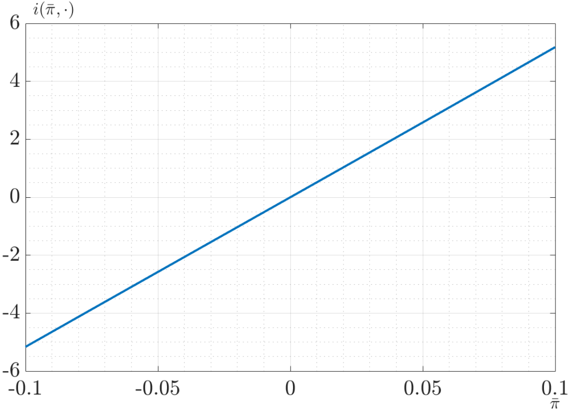

Figure: Policy function: $i(\bar{\pi}$, all other states at their means)

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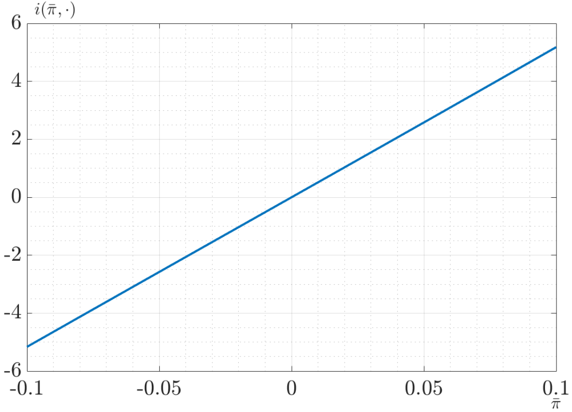
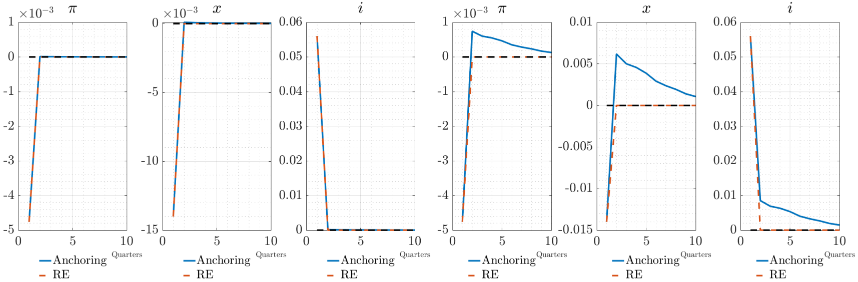


Figure: Policy function: $i(\bar{\pi}$, all other states at their means)

$\rightarrow$ For 5 bp drop in $\bar{\pi}$, lower i by 2.5 pp

Unanchoring causes volatility



(a) Expectations anchored

(b) Expectations unanchored

Figure: Impulse responses after a contractionary monetary policy shock when policy follows a Taylor rule

Why so volatile? Term structure of expectations

IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big)$$

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Why oscillatory? Intertemporal anticipation effects

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- Additional channel of policy

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- Additional channel of policy
- Only if policy reaction function internalized

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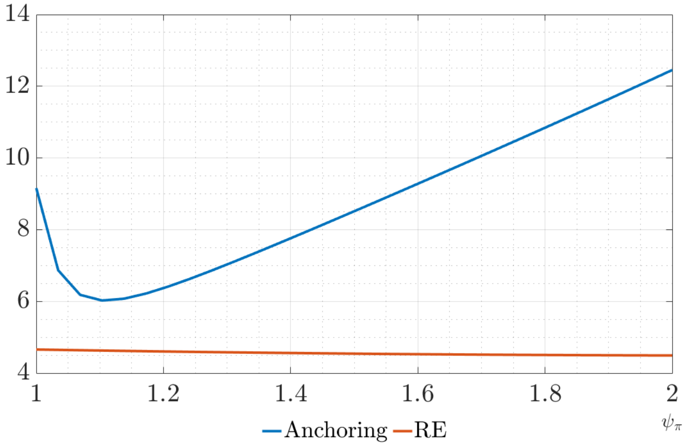
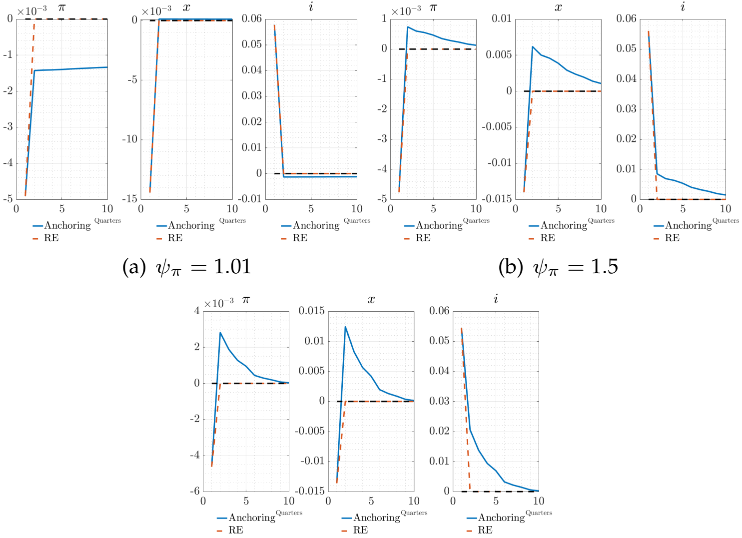
Optimal Taylor-coefficient on inflation

Figure: Central bank loss as a function of ψ_π



Anchoring-optimal coefficient: $\psi_\pi^A = 1.09$ RE-optimal coefficient: $\psi_\pi^{RE} = 2.21$

Respond but not too much



(a) $\psi_\pi = 1.01$

(b) $\psi_\pi = 1.5$

(c) $\psi_\pi = 2$

Losses for optimal Taylor-rule coefficient on inflation

Table: Loss for RE and anchoring models for choice of RE- or anchoring-optimal ψ_π

Anchoring, ψ_π^{RE}	Anchoring, ψ_π^{A}	RE, ψ_π^{RE}	Optimal policy
14.2633	6.0228	4.4866	6.0985

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Optimal policy vs. Taylor rule? $\rightarrow$ is reaction function internalized or not

Conclusion

- First theory of monetary policy for potentially unanchored expectations
- Optimal policy frontloads aggressive interest rate response to suppress potential unanchoring
- Matters: already anchoring-optimal Taylor rule reduces losses by 84%
- Future work
  - $\hookrightarrow$ Better approximation to optimal policy than Taylor rule?
  - $\hookrightarrow$ How to anchor at zero-lower bound?

Appendix

Breakeven inflation

Figure: Market-based inflation expectations, various horizons, %

Return

Correcting the TIPS from liquidity risk

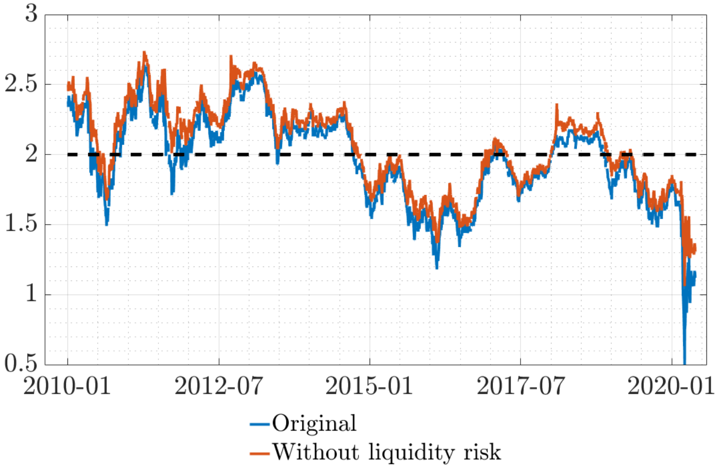

Figure: Market-based inflation expectations, 10 year, %

Return

Further evidence

Figure: Livingston Survey of Firms:
Interquartile range of 10-year ahead inflation expectations

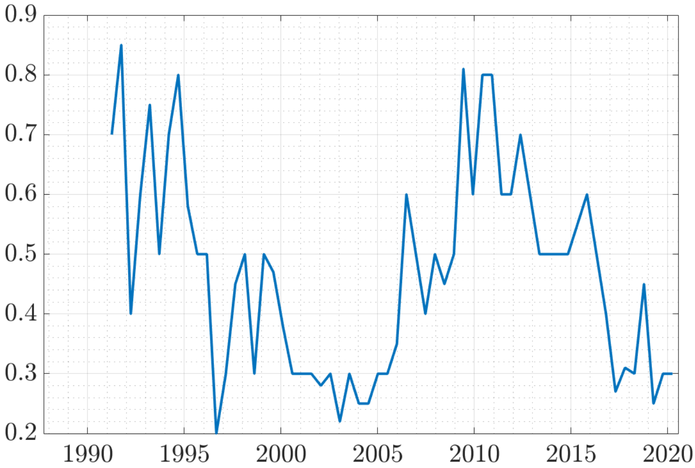

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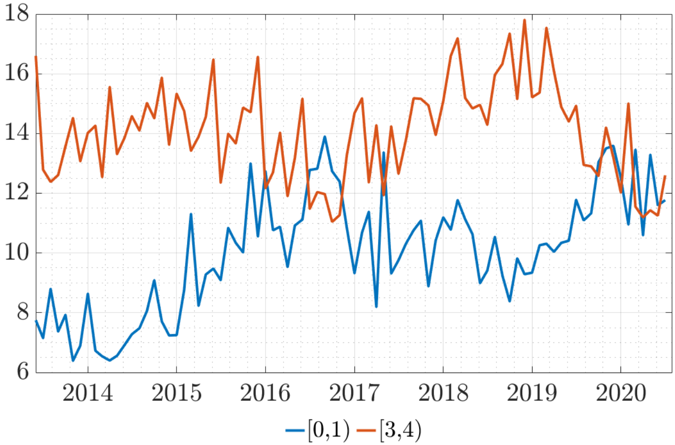
Figure: New York Fed Survey of Consumers:
Percent of respondents indicating 3-year ahead inflation will be in a particular range



Return

Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{18}$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{19}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \tag{20}$$

Solve for forecast error $f_t \equiv \pi_t - f_{t-1}$:

$$f_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k \to 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \tag{21}$$

Functional forms for **g** in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \tag{22}$$

$p \equiv Prob(\text{liquidity trap regime})$
y_{t-1} output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{23}$$

θ_t criterion, $\bar{\theta}$ threshold value

Return

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \tag{24}$$

Σ variance-covariance matrix of shocks
$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(f_{t|t-1}f'_{t|t-1} - \omega_{t-1}) \tag{25}$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(f'_{t|t-1}\omega_t^{-1}f_{t|t-1} - \theta_{t-1}) \tag{26}$$

ω_t estimated forecast-error variance

Return

Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}\right)'\right)' \tag{27}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{28}$$

Return

Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{29}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{30}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \tag{31}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{32}$$

Return

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t)$$

$$+ \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \Big\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t + \mathbf{g}_\pi(\pi_t - f_{t-1})) = 0 \tag{33}$$

$$-2\beta\frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_\mathbf{f}(\pi_{t+1} - f_t)) = 0 \tag{34}$$

Return

Target criterion system for anchoring function as changes of the gain

$$\varphi_{6,t} = -cf_{t|t-1}x_{t+1} + \left(1 + \frac{f_{t|t-1}}{f_{t+1|t}}(1-k_{t+1}) - f_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1} - \frac{f_{t|t-1}}{f_{t+1|t}}(1-k_{t+1})\varphi_{6,t+2} \tag{35}$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{f_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{f_{t|t-1}}\varphi_{6,t+1} \tag{36}$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (36) is given by:

$$\varphi_{6,t} = -2\,\mathbb{E}_t\sum_{i=0}^{\infty}(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i})\prod_{j=0}^{i-1}\frac{\frac{k_{t+j}}{f_{t+j|t+j-1}}}{\frac{k_{t+j}}{f_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \tag{37}$$

Return

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{38}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{39}$$

Profits:

$$\Pi_t^j = p_t(j) y_t(j) - w_t(j) f^{-1}(y_t(j)/A_t) \tag{40}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{41}$$

Return

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{42}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{43}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (42) backward to some date t, take expectations at t
2. Sub in (43)
3. Aggregate over households i

$\rightarrow$ Obtain (16)

Return