Journal of Macroeconomics 61 (2019) 103070

Contents lists available at ScienceDirect

Journal of Macroeconomics

journal homepage: www.elsevier.com/locate/jmacro

Monetary policy, de-anchoring of inflation expectations, and the "new normal"

Lucio Gobbi[a], Ronny Mazzocchi[b], Roberto Tamborini[c,*]

[a] Doctoral Programme in Economics and Management, School of Social Sciences, University of Trento, Trento, Italy
[b] European Parliament, Brussels, Belgium
[c] Department of Economics and Management, University of Trento, Trento, Italy

A R T I C L E I N F O

Keywords:
Monetary policy
Zero lower bound
New normal
Inflation expectations

JEL codes:
E50
E52
E58

A B S T R A C T

Persistently low inflation rates, followed by declining inflation expectations, in advanced economies after the Great Recession have raised the question whether central banks are still able to credibly anchor inflation to their medium-term targets. The purpose of this paper is twofold. First, we investigate why agents' expectations that over the business cycle inflation will remain in line with the target begin to falter. Our hypothesis is that agents form expectations in terms of their probabilistic belief that the economy may switch from a normal to a depression state (permanently low output and inflation), which is updated upon observing the actual state of the economy. Second, we study how the de-anchoring of expectations interacts with monetary policy to determine whether the central bank is still able to achieve its target - and hence re-anchor inflation expectations - or whether the system drifts towards depressed states. We obtain two main findings. The first is that, facing unfavourable shocks, if inflation expectations ``fall faster" than the policy rate, and the zero lower bound is reached without correcting the shock, the system converges to a new steady state - the "new normal" - with permanent negative gaps. The second is that a more aggressive monetary policy is ineffective both at the zero lower bound and *above* it, when the shock is large and/or when the reactivity of inflation expectations is sufficiently high. This latter finding seems to support the necessity, in those conditions, to abandon conventional monetary policy and to switch to an aggressive reflationary policy package that prevents the entrenchment of deflationary expectations.

"Inflation expectations are well
anchored until they are not"

(Orphanides, 2015)

1. Introduction

In the aftermath of the Great Recession inflation dynamics in advanced economies has surprised many economists. On the one hand, just after the outbreak of the crisis there was a period of *missing disinflation*: based on historical data and given the

We would like to thank, without any implications, the participants in the XXX 'Villa Mondragone' International Economic Seminar (Rome, June 25–27, 2018), Giancarlo Altieri, Andrea Baldini, Dirk Ehnts, Dario Paternoster, and two anonymous referees of this Journal for valuable comments and suggestions. The views expressed in this article are personal and should not be attributed to the European Parliament or its services.

* Corresponding author.
E-mail address: roberto.tamborini@unitn.it (R. Tamborini).

https://doi.org/10.1016/j.jmacro.2018.10.006
Received 5 May 2018; Received in revised form 18 October 2018; Accepted 29 October 2018
Available online 31 October 2018

depth of the recession, inflation should have declined much more than it actually did (IMF, 2013; Murphy, 2014; Blanchard et al., 2015). On the other hand, subsequently inflation has surprised for the opposite reason. Three bodies of evidence are relevant here.

First, since the end of 2011, the Euro Area, the United Kingdom and the United States have been experiencing a *downward drift of inflation* away from the central bank's target. This has become more pronounced since 2013. Even greater surprise has been provoked by the fact that this process has been taking place in parallel with an unprecedented easing of monetary conditions to the limit of the zero lower bound (ZLB) of policy interest rates. Second, this concomitance has challenged the consensus on the worldwide "flattening" of the Phillips Curves pointing to their "steepening" (for the Euro Area see e.g. Riggi and Venditti, 2014; Bank of Ireland, 2014; Oinonen and Palovitta, 2014). Third, direct investigation of expectation forecasts has shown both their downward drift and a significant under-estimation of actual inflation (Riggi and Venditti, 2014; Miccoli and Neri, 2015).

Overall, though episodes of strict deflation (negative inflation rate) have been limited (e.g. December 2014–March 2015, and February–May 2016 in the Euro Area) these phenomena have raised concern among central bankers about their ability to govern the dynamics of the price level by credibly anchoring expectations to the medium-term inflation target pursued by central banks. (Iakova, 2007; Kuttner and Robinson, 2010; ECB, 2014). In the words of the President of the European Central Bank (ECB) Mario Draghi,

"[...] The most fundamental question facing all major central banks today is this: can our price stability mandate still be delivered? Across advanced economies inflation is low and has been low for some time. And in several of those economies, long-term inflation expectations, based on market prices, remain below our numerical definitions of price stability. That has led some to question whether it makes sense for central banks to pursue expansionary policies to meet their inflation objectives. Are they fighting a futile battle against forces beyond their control?" (Draghi, 2016, p. 1).

This question has come to be known as the *de-anchoring problem* of inflation expectations, and this problem plays a key role in the communication strategy of today's conventional and non-conventional monetary policies (Draghi, 2014a, b, 2016; Yellen, 2014; Eggertsson and Woodford, 2004).

Independent empirical analyses support concern with the de-anchoring problem in various countries. Though common across advanced economies, the problem seems particularly pronounced in the Euro Area, confirming Ehrmann et al. (2015) claim that de-anchoring is more likely the more persistent is low inflation. In the empirical literature, the more common criterion used to test the anchoring of inflation expectations is based on the idea that firmly anchored expectations should be insensitive to the announcement of macroeconomic news (Gürkaynak et al., 2010; Fracasso and Probo, 2017). Survey-based analyses suggest that in recent years inflation expectations have shown some signs of de-anchoring, moving away from the ECB target even at the longer horizons.[1] Lyziak and Paloviita (2016) report evidence of increased sensitivity of longer-term inflation forecasts to shorter-term forecasts and to actual HICP inflation, and find that the role of ECB inflation targets for those expectations has diminished in recent years. Buono and Formai (2016) show that the short-term inflation expectations of the professional forecasters interviewed by Consensus Economics started falling in 2012 and that its reduction implies a reduction by 0.3 percentage points of long-run expectations.

Empirical evidence of deanchoring

Market-based analyses[2] largely confirm these results. By applying the multiple endogenous break point tests of Bai (1997), Nautz et al. (2017) find that expectations in the Euro Area became less well anchored after September 2011. Using a news-regression approach to assess the sensitivity (or lack thereof) of expectations to the release of unexpected macroeconomic news, Fracasso and Probo (2017) find evidence that the de-anchoring of expectations started in December 2011 and never reversed. Symptoms have also been detected by Natoli and Sigalotti (2018): studying the pass-through of inflation expectations, they find that, since mid-2014, negative tail events affecting short-term inflation expectations have been increasingly channelled into long-term ones, and that this phenomenon has generated both downward revisions in expectations and upward shifts in uncertainty.[3]

This evidence of de-anchoring of inflation expectations challenges standard macro-theory of conventional monetary policy based on agents' long-run rational expectation (RE) that actual inflation will remain anchored to the central bank's target up to short-run deviations (Woodford, 2003, ch. 2).[4] In parallel, it supports the concern of central banks about their ability to fulfil their inflation target once expectations are de-anchored.

Benhabib et al. (2001) first warned that a feedback rule *à la* Taylor may generate multiple RE equilibria, one of which at the ZLB

[1] Various European institutions regularly conduct surveys involving both consumers and professional forecasters. For example, in the European Commission's Consumer Survey on Inflation Expectations, consumers are asked monthly to indicate their expectations in terms of the direction of inflation development over the next year. The ECB Survey of Professional Forecasters (SPF) is conducted quarterly and includes a panel of more than 70 forecast experts.

[2] Market-based measures of inflation expectations avoid many problems of measures taken from inflation surveys. Firstly, data frequency is not an issue since market data are easily available on a daily basis. In contrast to survey-based measures, market analyses are determined by market activities and actual trading behaviour. Therefore, market data can be considered to represent the market's actual inflation expectations and should not be distorted by psychological factors (ECB, 2006). For an overview of survey-based and market-based measures, see Grothe and Meyler (2015).

[3] This result is in line with the high sensitivity of medium-to-long term expectations to inflation surprises observed in Miccoli and Neri (2015).

[4] Indeed, today's theory of conventional monetary policy is a modernized version of Wicksell's theory put forward in Interest and Prices (1898), where the problem of anchoring inflation expectations was identified as a consequence of the central bank controlling (or seeking to control) the nominal interest rate instead of the quantity of money (Woodford, 2003; Mazzocchi et al., 2014).

with a perpetual "liquidity trap" with low inflation (below target) and output (below potential).[5] Woodford (2003, Ch. 2, Section 4) discusses "self-fulfilling inflations and deflations". He also finds that under an interest-rate feedback rule a multiplicity of equilibria may arise and that the RE equilibrium that fulfils the central bank's inflation target is locally stable, but not globally unique. Moreover he highlights that the drift towards self-fulfilling inflations and deflations has to be driven by de-anchored inflation expectations that, in the case of deflations, *combine with the ZLB of the interest rate*. In simple words, expected inflation should "fall faster" than the interest rate.[6]

In light of this account, the de-anchoring problem consists of *two phenomena* that we investigate in this paper. The first is why agents' expectations that inflation will remain in line with the central bank's target begin to falter. The second is how the de-anchoring of expectations interacts with monetary policy determining whether the central bank is still able to achieve the inflation target, and hence re-anchor inflation expectations, or whether the system drifts towards negative inflation and output gaps.

Insights into the de-anchoring of inflation expectations can be found in different strands of literature that drop the RE hypothesis. In the first place, one may find studies which question the validity of the RE hypothesis empirically. For the Euro Area, using the European Commission survey, Forsells and Kenny (2004) find that inflation expectations appear unbiased, but they reject the hypothesis of orthogonality with respect to the available information. Several studies provide evidence in favour of adaptive learning in inflation expectations Pfajfar and Santoro, 2010; Slobodyan and Wouters, 2012; Molnar and Ormeno, 2015; Carvalho et al., 2017). A more theoretical literature investigates processes of expectation formation and whether they converge to the unique RE equilibrium. Generally, this literature posits agents lacking full information or knowledge about the data generating process, and examines some form of learning mechanism thereof. Multiple equilibria typically emerge, among which self-fulfilling liquidity traps in conjunction with the ZLB on the nominal interest rate. Some authors (e.g. Bullard and Mitra, 2002; Evans et al., 2008; Evans and McGough, 2018) show that, under least-square learning introduced by Marcet and Sargent (1989) and Evans and Honkapohja (2001), the liquidity trap exists, but it fails to be stochastically stable. Others, who employ different learning mechanisms, instead show that it can be stable (Arifovic et al., 2012; 2017; Busetti et al., 2014; De Grauwe and Ji, 2016).

In this paper we present a model consisting of the standard New-Keynesian three equations used in this literature (e.g. De Grauwe and Ji, 2016; Arifovic et al., 2017),[7] augmented with an inflation-expectation mechanism which endogenises the de-anchoring process. It is both consistent with a broad notion of "rational beliefs" (e.g. Kurz, 2011), and amenable to simple parameterisation and control for comparative simulations. We draw on the key idea of "regime switch" in Arifovic et al. (2017). Forming their expectation of the future path of inflation, agents elaborate a probabilistic belief that, whenever the economy has negative inflation and output gaps, it may shift from the "normal regime", where gaps will always return to zero, to a "depression regime" where the gaps will be permanent.[8] The regime-switch probability $p \in [0, 1]$ held by agents affects their inflation expectations. In fact, the belief-consistent expectation for each point in time is the (p; $1 - p$) mean value of the inflation (gaps) in the two regimes: as long as agents believe in the normal regime ($p = 0$), their expected inflation gap is zero at all times; as p rises, the expectation is tilted towards the existing inflation gap. Hence, $p > 0$ can be regarded as an indicator of de-anchoring.

The key element in our model is that the regime-switch probability is formed and updated upon observing the actual state of the economy, namely it increases *vis-à-vis* worse output and inflation conditions, in accordance with the empirical methodology that identifies de-anchoring with the reactivity of expectations to macroeconomic news (see above). As we shall see, this makes the regime switch – the transition from a negative episode in the normal regime to the depression regime – dependent on agents' beliefs themselves. In this sense, we are also in the field of self-fulfilling beliefs.

With respect to the largest part of the literature, we extend our analysis beyond the single issue of the existence and stability of depressed states of the economy at the ZLB. Starting from the zero-gaps equilibrium, we introduce a shock to the natural rate of interest and, by means of simulations, we investigate the global dynamic behaviour of the system. Key to whether the system tends to return to the zero-gaps steady state, to settle down into a negative-gaps steady state or to display global instability is the interaction

[5] Of course, the problem is not low inflation *per se*, but rather that the economy settles down below potential output. From this point of view, the use of the Keynesian term "liquidity trap" to denote this equilibrium - apparently reintroduced by Krugman (1998) - seems appropriate. However, it should also be recalled that in Keynes' *General Theory*, and in the Keynesian tradition, the liquidity trap had different causes: i.e. the inability of monetary expansion to lower the nominal interest rate, *even above zero*, owing to *asset holders' expectations* of falling *asset prices* leading to infinitely elastic supply of bonds in exchange for money. This original notion of liquidity trap is more relevant to what is now called unconventional monetary policy.

[6] "Along such a path, interest rates are constantly being lowered in response to decline in inflation, but because *expected* future inflation falls at the same time, *real* interest rates are not reduced, and continue to be high enough to restrain demand despite the falling prices" (Woodford, 2003, p.126).

[7] On the foundations of the New-Keynesian three-equation model see e.g. Carlin and Soskice (2005).

[8] The term "regime" broadly defines a configuration of the economy inclusive of the central bank policy conduct, which support agents' determined beliefs regarding its realisations. Kurz (2011) illustrates the class of rational beliefs concerning state transitions of the economy. Accordingly, our model can also be viewed as an approach to the more general problem of the central bank's commitment and credibility (we owe this point to an anonymous referee of this journal). Credibility can easily be interpreted as the complement $(1-p)$ to the regime-switch probability p. In our model agents need a reason to believe in the central bank's commitment, and this reason should be based on the observed states of economy. Though faithfully committed to the declared inflation target under the guidance of the Taylor Rule, the central bank may lose credibility owing to the adverse, endogenous, mechanisms that will be treated in the paper. See also fn. 18.

between the inflation-expectation mechanism, conditioned by agents' belief about a regime switch, and monetary policy, together with contour conditions given by structural parameters and the extent of the initial shock. Therefore, our study also offers some insights for both empirical analysis and policy conduct.

More in detail, Section 2 introduces the model. The New Keynesian block of the model consists of the standard equations of the output gap relative to potential, of the inflation gap relative to the central bank's inflation target, and of the nominal interest rate (policy rate for short) set by the central bank following a Taylor Rule. Then we introduce a fourth equation which, under suitable conditions that will be discussed, determines the regime-shift probability p elaborated by agents as a function of the observed state of the economy. The single key parameter of this function is its reactivity (σ) to output gaps: larger σ induces larger increase of p for a given output gap. This conditions their inflation expectations as described above, which in turn affect the actual output and inflation gaps and hence the monetary policy response, normally a cut in the interest rate aimed at closing the output and inflation gaps. This induces a revision of agent's beliefs and inflation expectations, and so on and so forth. Hence, we have precise identification and full control of the conditions that determine the extent of de-anchoring of inflation expectations, and whether they "fall faster" than the interest rate.

In Section 3 we analyse the properties of the system and present simulations of its long-run evolution. In the normal regime ($p = 0$), the system displays the usual RE equilibrium with zero gaps. With p endogenous, however, the system becomes nonlinear, and dependent on the values of six parameters plus the initial natural-rate shock. Hence, we examine its global dynamics by means of simulations. These are organised in a baseline case with parameter values consistent with the empirical literature, and then pairwise interactions of the reactivity parameter σ with three other ones: the extent of the initial shock, the output gap elasticity of inflation gaps (κ) and the inflation gap parameter (γ_π) in the central bank's rule.

Our main findings, summarised and discussed in Section 4, are the following. Depending on the parameter values and the initial shock, the system's dynamic behaviour is more complex and richer than in similar studies on the de-anchoring problem. The system displays *three* possible regions: (1) *convergence to the zero-gaps steady state and* $p = 0$, i.e. the system remains in the normal regime; (2) convergence to a steady state with *negative gaps and* $p > 0$, that we call a "new normal", i.e. a depressed state of the economy to which there corresponds a permanent, though finite in size, de-anchoring of inflation expectations from the central bank's target; (3) *global instability*, i.e. divergence from any steady state.

Key to the destiny of the economy is the interaction between the magnitude of the shock and of the sensitivity parameter σ. The economy remains in the first region as long as the policy rate can accommodate the shock above the ZLB; yet this basin of attraction shrinks as σ increases. In the second and third region we have a clear representation of the phenomenon of inflation expectations that "fall faster" than the policy rate. The economy is pushed from the second to the third region for combinations of high σ and a large shock.

Controlling for structural price stickiness (κ), simulations show that, *ceteris paribus,* as κ increases (prices are more flexible and the Phillips Curve is steeper) the convergence regions shrink, the reason being that a steeper Phillips Curve amplifies the output gaps. With regard to the inflation parameter in the central banks' policy rule (γ_π), simulations show that a more aggressive (conventional) monetary policy, i.e. with larger γ_π, is obviously ineffective at the ZLB, but it is also ineffective above the ZLB when adverse conditions push the system into the divergence region. This finding seems to support the necessity, in those conditions, to abandon conventional monetary policy and to switch to an aggressive reflationary policy that prevents the entrenchment of deflationary expectations.

2. The model

2.1. The standard new Keynesian three equations

We draw on the standard New Keynesian three equations of the so-called output gap, the inflation gap or Phillips Curve and a Taylor Rule that determines the policy rate controlled by the central bank.

The output gap equation (OG) yields the logarithmic difference y between the current output and the potential output given by technology and endowment of factors.[9] The OG equation, for any period t, is:

$$y_t = E_t y_{t+1} - \alpha(i_t - E_t \pi_{t+1} - r_t) \quad (1)$$

where E_t is the usual expected value operator conditional on information at time t, i_t is the nominal interest rate controlled by the central bank (policy rate for short), π_t is the inflation rate, r_t* is the "natural" (real) interest rate, i.e. the interest rate corresponding to the general equilibrium of the economy at potential output.[10]

It is assumed that the natural rate remains constant (r*) unless, at any time t, it is unexpectedly changed by a permanent amount ρ_t. Hence the term in parentheses can be read as the "interest-rate gap" $\hat{i}_t$, i.e. the deviation of the policy rate from its target given by the natural rate augmented with the expected inflation, with α measuring the (constant) elasticity of substitution of aggregate spending.

[9] The potential output can be measured as the market clearing, Pareto efficient, level of output, or alternatively as the maximal level of output obtainable in the presence of distortionary "real rigidities" (Woodford, 2003; Trautwein and Zouache, 2009). This difference is immaterial here.

[10] In the standard New Keynesian model, the natural interest rate equates the rate of time preference of households with marginal return to capital.

Inflation is driven by its own expectations and the output gap according to the so-called Phillips Curve (PC):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t \tag{2}$$

where $\beta < 1$ is the time discount factor and $\kappa > 0$ is a parameter reflecting the degree of price stickiness in the goods market (κ is decreasing in price stickiness).[11]

Finally, the model is closed by the equation determining the policy rate, which is typically stated in the format of a Taylor Rule (TR), namely

$$i_t = (r_\tau^* + \pi^*) + \gamma_\pi(\pi_\tau - \pi^*) + \gamma_y y_\tau \ \geq 0 \tag{3}$$

where $\pi^* \geq 0$ is the inflation target, and τ is a time index that can be determined according to various specifications, e.g. "real time" $\tau = t$, forward looking $\tau = t + n(n = 1, \ldots)$, lagged $\tau = t + n(n = 1, \ldots)$. For reasons that will be explained below, we adopt the lagged specification with $n = 1$.

The three equations OG-PC-TR form a consistent system that determines the equilibrium values of the three endogenous variables, corresponding to the steady-state values of the OG and PC equations. The steady-state inflation implied by the PC is the key input to the central bank's determination of the inflation target, which in turn provides the anchor for inflation expectations. The steady-state inflation is $\bar{\pi} = [\kappa/(1-\beta)]\bar{y}$ hence it is nil when the output gap is nil, so that $\bar{\pi} = 0$ is the central bank's inflation target,[12] and the PC is also the inflation-gap equation. The equilibrium of the economy is ($\bar{y} = 0, \bar{\pi} = 0, \bar{i} = r^*$). Clarida et al. (1999, 2000), Woodford (2003, Ch. 2), and others have also shown that under suitable conditions in the TR equation (crucially $\gamma_\pi > 1$), the system also ensures convergence to the steady state. Thus, in the presence of random shocks to the system, the RE of inflation and output should be consistent with, and remain "anchored" to, the zero-gaps (stochastic) equilibrium of the economy (being also called "long-run expectations", e.g. Woodford 2003, Ch. 2).[13]

2.2. De-anchoring inflation expectations

We now address the issue of the formation of long-run inflation expectations more closely. Given the PC equation, we have jumped to the steady-state solution, but it is also necessary for us to work out the single steps, and we do so by forward iteration in accordance with the model-consistency principle (also Woodford, 2003, pp. 91-ff.). Therefore, we carry Eq. (2) one period forward, so that

$$E_t \pi_{t+1} = \beta E_{t+2} \pi_{t+2} + \kappa E_t y_{t+1}$$

Substituting back into 2 we obtain

$$\pi_t = \beta^2 E_t \pi_{t+2} + \beta\kappa E_t y_{t+1} + \kappa y_t$$

After N iterations, the equation of the current inflation – the inflation gap for the central bank – is

$$\pi_t = \beta^N E_t \pi_{t+N} + \kappa\left(y_t + \sum_{n=1}^{N} \beta^n E_t y_{t+n}\right) \tag{4}$$

LH-expectations

In the second term, we have to track the series $E_t y_{t+n}$ for $n = 1, \ldots, N$. What is the rational expectation for this series? There is no unambiguous answer because the evolution of the output gaps depends on the central bank's ability to control the system (Woodford 2003, pp. 91-ff). Indeed, if agents derive the expected output gap consistently with the full model solution, then $E_t y_{t+n} = 0$ at all times as explained above. Yet this solution *presumes* that the central bank exerts control on the system up to random deviations, and this in turn *presumes* the RE hypothesis. In other words, the central bank succeeds in anchoring the expectations if the agents expect it to succeed, and vice versa. Hence, we face the classic circularity of models with the RE hypothesis (e.g. Farmer, 1993), which in our context means that the problem of anchoring the expectations is resolved by assumption.

well explained

To deal with this issue, we follow a different route, which starts from the premise that the central bank succeeds in anchoring agents' expectations insofar as *they have reason to believe in the central bank's success* – we may call this a *rational belief* (Kurz, 2011). Minimal requirements of a rational belief are that (1) it is logically consistent (e.g. in accordance with probabilistic metrics), (2) it is consistent with the data generating process (though not necessarily the "true" process), (3) it is corroborated by experience (Bayesian updating is an example). Hence, it should not be systematically falsified, and it should be verified with some nonzero probability. On

[11] According to the Calvo pricing mechanism embedded in the standard New Keynesian Phillips Curve, $\kappa = (1-\phi)(1-\phi\beta)\phi - 1$ where ϕ is the probability of prices being unchanged (the fraction of firms not changing their price) after a change in aggregate demand. Clearly, $\phi = 1$, $\kappa = 0$, represent the Old Keynesian fixed-price economy where the Phillips Curve is horizontal, and the steady-state inflation is zero, whereas $\phi = 0$, $\kappa \to \infty$, represents the New Classical flex-price economy where the Phillips Curve is vertical, and the steady-state inflation is undetermined.

[12] The central bank may set a nonzero inflation target if, for welfare reasons, it also targets $\bar{y} > 0$, insofar as the potential output with distortions falls short of the Pareto efficient potential output (Woodford 2003, Ch. 2).

[13] In common DSGE applications of the New Keynesian model, the output and inflation expectational terms refer to one-period ahead values ("short-run expectations") because shocks are assumed to be autoregressive. On the distinction between short-run and long-run expectations in this class of models see also Preston (2005) and Mazzocchi et al. (2014).

the other hand, a nontrivial treatment of rational beliefs is that a chance exists that they *are* falsified.

We draw on Arifovic et al. (2017) idea of regime switch. That is to say, observing the current state of the economy $(y_t, \pi_t) < 0$, the agents elaborate the series of future expected output gaps according to the probabilistic belief that, at each point in time, the economy may switch from the "normal regime", where it will return to the zero-gaps equilibrium, to a "depression regime" where the gaps will remain unchanged. If agents believe that $p \in [0, 1]$ is the probability of regime switch, *vis-à-vis* the alternative hypothesis (i.e. the confidence in the normal regime with zero gaps) with probability $(1 - p)$, a consistent rational belief about the future series of y_{t+n} is the $(p; 1 - p)$ mean value of the output gaps in the two regimes, i.e. $E_t y_{t+1} = p y_t$, $E_t y_{t+2} = p E_t y_{t+1}$, …, $E_t y_{t+n} = p E_t y_{t+n-1}$, … where $p > 0$ can be interpreted as an indicator of the de-anchoring of expectations, and is conditional on the information available in t. This opens the model to the possibility that agents change their belief as information (e.g. the realised state of the economy) changes over time as will be seen in the next section.

Given the series of expected output gaps, Eq. (4) becomes

$$\pi_t = \beta^N E_t \pi_{t+N} + \kappa y_t \left(1 + \sum_{n=1}^{N} (\beta p)^n\right) \tag{5}$$

Since $(\beta, p) \in [0, 1]$, then $\lim_{N\to\infty} \beta^N E_t \pi_{t+N} = 0$, and $\lim_{N\to\infty} \sum_{n=1}^{N} (\beta p)^n = \frac{\beta p}{(1-\beta p)}$. Therefore,

$$\pi_t = \omega \kappa y_t \tag{6}$$

with $\omega \equiv 1 + \beta p/(1 - \beta p)$. Consequently, the PC turns out to be a function of the current output gap, and its slope has two determinants: one is the structural parameter κ determined by price stickiness; the other is the weight ω of de-anchored inflation expectations, which increases above 1 as the regime-switch probability p rises above 0.

We thus gain a first insight into the effect of the de-anchoring of expectations. The current negative output gap is translated into a *larger* inflation gap, which is *amplified* by the implicit projection of the current state of the economy into the future. At the same time, the economy becomes more difficult to control, which may corroborate the belief in the regime switch. This loop also sheds light on the puzzling evidence of sustained, larger than expected, fall of inflation after the Great Recession recalled in Section 1.[14]

We can now reformulate also the OG equation according to our hypothesis of expectation formation. We have seen that the one-step expectation of the output gap is given by $E_t y_{t+1} = p y$, whereas the concomitant inflation gap implied by the forward iteration procedure is $E_t \pi_{t+1} = \omega \kappa E_t y_{t+1}$ (which is indeed the expectation of (6)). Substituting these values into (1) we obtain

$$y_t = -\theta \alpha \hat{i}_t \tag{7}$$

where $\theta \equiv (1 - \beta p)[\beta p^2 - (1 + \alpha\kappa + \beta)p + 1)]^{-1}$.

As in the standard New Keynesian model, negative output gaps are function of excess interest-rate gaps. The important content of this OG equation is that $\partial\theta/\partial p > 0$, i.e. the impact of the interest-rate gap, in absolute terms, is larger when the de-anchoring of expectations is larger.[15] Hence de-anchoring has a *compound amplification effect*, both on the inflation gap, for a given output gap, and on the output gap, for a given interest-rate shock.

2.3. Endogenising the de-anchoring of expectations

Having highlighted the effect of the de-anchoring of expectations, we now want to endogenise it in a tractable and controllable way in view of our analysis of the global behaviour of this economy. Pursuing the suggested idea of rational beliefs, i.e. beliefs driven by experience, we want the regime-switch probability p to be driven by the observed state of the system, in particular the output gap, given its role in the expectation formation process.[16] Thus, we posit a relationship between p and y, that we call "de-anchoring function" (DA), with the following properties. First, p is updated with a lag, i.e. $p_t = h(y_{t-1})$, which seems a realistic feature once account is taken of the time necessary to collect and process information. Second, updating is such that p is changed proportionally to the absolute value of y. Third, we allow, and want to control, for different degrees of reactivity in updating (i.e. for a given output gap, p may be changed more or less largely). A function that embodies these properties is the following logistic

$$h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \tag{8}$$

logistic de-anchoring function (DA)

[14] According to Riggi and Venditti (2014), these phenomena can be explained by the evidence of a "steepening" of the PCs in various countries, whereas previous estimates where based on the "flattening" of the PC (e.g. Blanchard et. al, 2015). In fact, as our equation (6) shows, $\omega > 1$ makes the PC steeper. However, it also shows that the empirical slope of the PC may be the result of (permanent) structural factors (κ) as well as (transient) expectational factors (ω). The steepening of the PC when falling inflation expectations take hold (ω rises as y falls) does not contradict the structural flattening of the PC (lower κ). Likewise, given κ, if monetary policy restores confidence in the return towards the normal regime (ω falls as y rises) the PC appears flatter than during the deflationary period.

[15] For $p = 0$, in the normal regime, $\theta = 1$, i.e. a positive interest-rate gap generates a negative output gap proportional to the elasticity of aggregate spending.

[16] Central banks communicate their target and assess their policy stance in terms of output, inflation, or both, depending on their mandate (and communication style). Here we are concerned with the expectation formation of the private sector, and as shown by Eq. (6), this hinges on the forecast of future output gaps. Anyway, the inflation and output gaps are univocally related by way of Eq. (6).

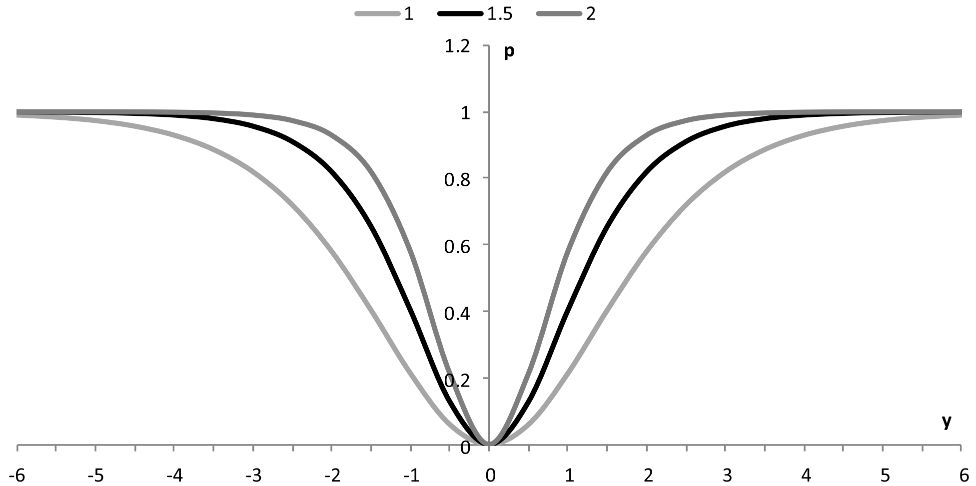

Fig. 1. The DA function with increasing values of reactivity σ.

In order to determine the values of the parameters (A, B, C, D), we impose the following conditions

- $h(0) = 0$ <—- 0 at 0 condition
- $\lim \cdot h_{y\to+/-\infty} = 1$
- $h'(0) = 0$
- $h''(D) = 0, \text{ with } D > 0$

0 at 0

The first condition states that with zero output gap agents believe in the normal regime with certainty (de-anchoring is nil). The second states that as the output gap grows unboundedly, p tends to 1 (de-anchoring is maximal). The third condition ensures that p is bounded at 0 when the output gap is zero. The fourth condition controls for reactivity of p. The solution of the system given by the four conditions yields the following parameter values, $A = 1,\ B = -4,\ C = 1,\ D \equiv \sigma$ Therefore we can write

I didn't exactly get where sigma entered the picture.

$$p_t = 1 - \frac{4e^{-\sigma y_{t-1}}}{(e^{-\sigma y_{t-1}} + 1)^2} \tag{9}$$

Fig. 1 shows the plots of function (9) for increasing values of reactivity σ; as can be seen, higher values of σ determine larger revision of p for a given y.[17] The factors that determine the magnitude of σ fall outside the scope of this paper, but we shall explore the effects of different values of this parameter.[18]

We now have a system consisting of the four equations OG (7), PC (6) TR (3) and DA (9). For symmetry with the private sector, we assume that the central bank, too, adjusts its policy rate with one lag after observing the output and inflation gaps, i.e. $\tau = t - 1$. The reduced form of the four equations form a dynamic first-order nonlinear system governed by the parameters (α, β, κ, γ_π, γ_y, σ), where each endogenous variable v_t^i in the vector $\mathbf{v}_t = [y_t, \pi_t, \hat{i}_t, p_t]$ is determined by a specific function of general form $v^i(y_{t-1}, \pi_{t-1}, \hat{i}_{t-1}, p_{t-1}; y_0, \pi_0, \hat{i}_0, p_0)$, i.e. the lagged values and the initial state (or shock) of the endogenous variables themselves. Having four equations for four endogenous, the system admits a determined steady-state solution vector $\bar{v}$ for any given vector of initial states. The question is under what conditions $\bar{v} = 0$, i.e. the system returns to a state with *zero gaps and zero probability of regime switch*. These conditions would be as many as the number of possible combinations of parameter values and initial states. The examination of analytical solutions would be both awkward and little informative.

In the first place, we only consider initial states where all the endogenous have zero value, except the interest-rate gap which

[17] Note that this function is behaviorally consistent with Bayesian inference. Our p is in fact analogous to the posterior belief $p(S|yt-1)$ where S is the hypothesis of regime switch conditional on the observed nonzero output gap $yt-1$. Let the prior $p(S) = 0$. Then, according to Bayes' principle, the posterior belief is driven by the likelihood $L(yt-1|S)/\phi(yt-1)$ to observe $yt-1$ if S holds relative to its unconditional occurrence. Our DA function implies that this likelihood is increased proportionally to $yt-1$. An alternative characterization of the function might hinge on the *speed* of revision based on the *frequency* of negative output gaps (i.e. after how many negative output gaps agents revise their probability of regime switch). However, whether the dimension or the frequency of events is more relevant is a long-standing open issue, and our characterization is flexible enough for the purpose of simulation analyses. For instance, a low value of σ implies that sequences of positive and negative output gaps around zero, or even a sequence of small negative output gaps, have negligible effect on p.

[18] The reactivity σ might be determined by history and conceived as changing over time, for instance in relation to the performance, or reputation, of the central bank in pursuing its target. For instance, a newly created central bank, or one with low reputation may generate high reactivity. This may also improve or deteriorate for a variety of reasons, such as a financial crisis, a crisis of confidence in the governor, or a political conflict between the monetary authority and the government (Bordo and Siklos, 2014). Notably, high reactivity makes the central bank's task harder.

Table 1
Empirical parameter values.

$r^* = 2\%$, $\alpha = 0.2$	Empirical estimates of the natural interest rate, and the interest-rate elasticity of private demand in the EU and US (e.g. Garnier and Wilhelmsen, 2005; Laubach and Williams, 2003)
$\beta = 0.98$	Standard model value of the time discount factor[a]
$\kappa = 0.09$	Structural elasticity of the PC, implied by the Calvo equation, given β and 75% of non-adjusted prices (e.g. Smets and Wouters 2003; Luk and Vines 2015)
$\gamma_\pi = 1$, $\gamma_y = 0.5$	Standard model values of the Taylor Rule
$\sigma = 1$	Our own reference value of the reactivity of the p function (see Fig. 1).[b]

[a] In the standard New Keynesian model, $r = \beta^{-1} - 1 \approx 2\%$.
[b] We do not have direct evidence for σ. The empirical work closest to our model, the already mentioned Gürkaynak et al. (2010), finds significant reactivity of long-term inflation expectations to various macroeconomic news in US, UK and Sweden. Following news about real GDP, the estimated reactivity varies between 0.3 in Sweden and 1.8 in the US. In our model, the correspondent relationship is $E_t\pi_{t+1} = \omega\kappa p y_t$. For one point of output gap, the range of values of σ consistent with the above estimates is (approximately) between 2.8 and 4.8. As can be seen from Fig. 1, such order of magnitude can be regarded as quite reactive, possibly too reactive if the system initial state is in equilibrium (and the central bank enjoys a good reputation). Hence we have chosen the more conservative value of $\sigma = 1$ which nonetheless is sufficient for the system to display the phenomena of interest.

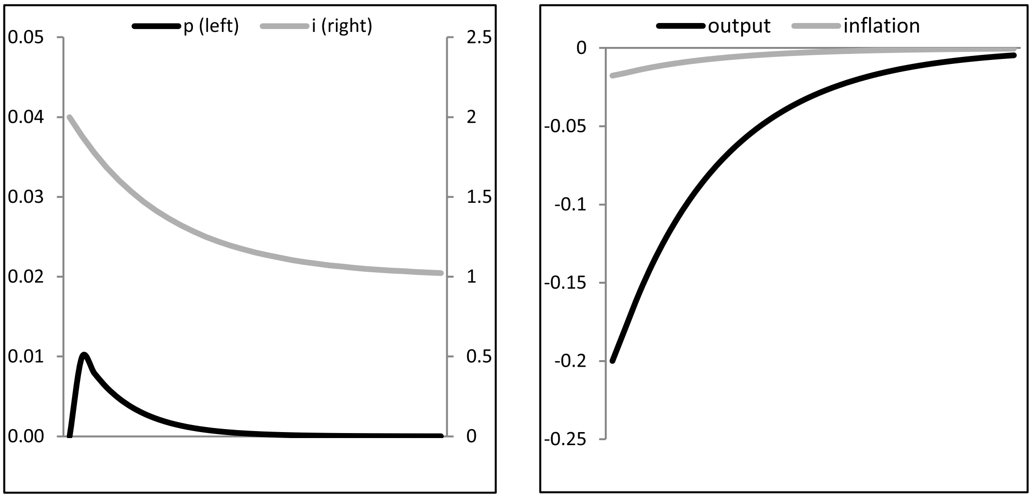

Fig. 2. Baseline case. Effects on the endogenous variables of − 1% shock to the natural interest rate.

results from a negative shock $\rho_0 < 0$ to the natural rate, i.e. $\hat{i}_0 = i_0 - (r^* + \rho_0)$.[19] In the second place, after a shock, the trajectory taken by the system, and hence its dynamic evolution, depend on the values of the parameters. Since the parameters are several, we study the dynamic behaviour of the system by means of simulations within the space defined by couples of parameters, while keeping the others constant.

3. Simulations

3.1. A baseline case

To begin with, we present a baseline case, given the following values of the parameters in Table 1 drawn from the relevant empirical literature and employed in standard quantitative policy models. The baseline exercise has a pure illustrative purpose of the functioning of the system, whereas in the subsequent simulations we shall explore the system's dynamic behaviour in a wide range of values of parameters.

As said above, the initial state of the system is a zero-gaps steady state for all the endogenous variables except a positive interest-rate gap, namely ($y_0 = 0$, $\pi_0 = 0$, $\hat{i}_0 > 0$, $p_0 = 0$). The latter results from a negative shock to the natural rate such that

[19] This is common to the relevant literature in the field. It was also Wicksell's original concern with interest-rate-based monetary policy, in particular because he regarded the natural interest rate as a non-observable variable subject to various sources of modification (e.g. Boianovsky and Trautwein, 2004; Mazzocchi et al., 2014). The fall of the natural rate also plays a prominent role in the debate on the so-called "secular stagnation", in particular because it may entail a permanent interest-rate gap as the policy rate hits the zero lower bound (e.g. Teulings and Baldwin, 2014)

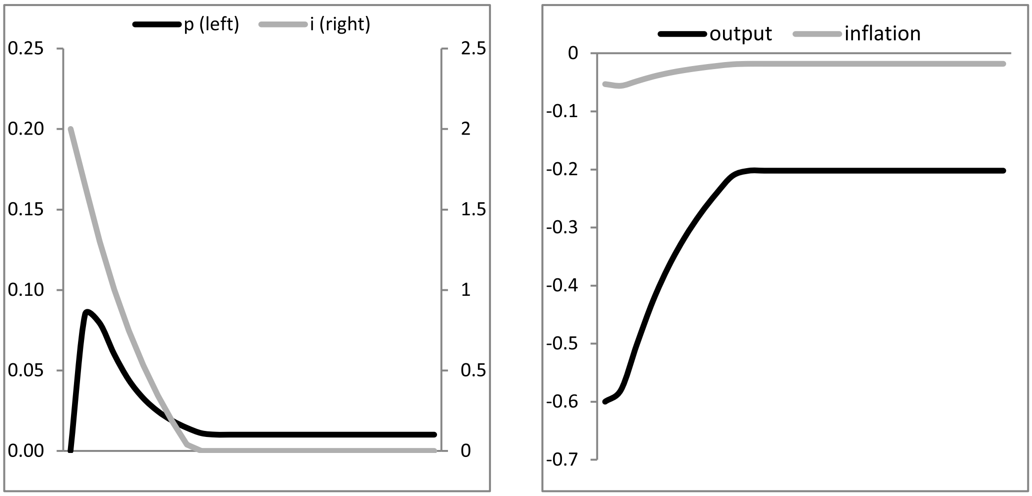

Fig. 3. The economy at the ZLB (parameters with baseline values).

$\hat{i}_0 = i_0 - (r^* + \rho_0)$. We consider three types of shocks, small ($\rho_0 = -1\%$), medium ($\rho_0 = -2\%$), large ($\rho_0 = -3\%$). Since $i_0 = r^* = 2\%$, each type of shock gives rise, respectively, to a small ($\hat{i}_0 = -1\%$), medium ($\hat{i}_0 = -2\%$), large ($\hat{i}_0 = -3\%$) interest-rate gap. Note that the small and medium shocks can be accommodated by letting the policy rate fall to zero, whereas the large shock cannot: as the policy rate falls to zero, a permanent interest-rate gap of 1% remains. Virtual time is scaled in quarters.

Fig. 2 plots the four endogenous variables in percent points (except *p* which is measured in decimal points) over 30 periods after a small shock.

The system converges monotonically to a new zero-gaps steady state, after an initial depressionary phase. It should be noted that the policy rate correctly converges to the new value of 1% given by the new natural rate at zero inflation, while the regime-switch probability is virtually unaffected (to be precise, it rises to 1% in the first period and then falls back). The output gap is larger and more persistent than the inflation one, but this is almost entirely due to the role of the New-Keynesian structural parameters, which generate a rather flat PC combined with a Taylor Rule less reactive to output than to inflation gaps. As *p* rises above zero, the structural parameter of the PC κ is increased by the factor $\omega > 1$, which however remains close to unity. Overall, the system shows substantial resilience. Empirically, it could hardly be distinguishable from an "anchored" system.

We can observe the same pattern in the case of a medium shock (not reported here), with, of course, larger and prolonged effects which are eventually reabsorbed as the policy rate falls to zero.

3.2. The zero lower bound, and the "new normal"

If a large shock occurs, we know that the adjustment of the policy rate is limited by the ZLB, which entails a permanent interest-rate excess of 1%. The simulation of this case has been run by freezing the policy rate when hitting the ZLB. The plots are drawn in Fig. 3.

First, the system fails to reach a new zero-gaps steady state. Output and inflation, however, do converge to a steady state with *permanent* negative gaps ($-$ 0.2 and $-$ 0.02 respectively). This result is in line with the literature on the "liquidity traps" at the ZLB recalled above. Our simulation also shows that, in parallel, the regime-switch probability *p* remains above zero, albeit slightly so (about 1%). This indicates a *permanent* de-anchoring of expectations, or loss of confidence in the return to the normal regime. This fact deserves further considerations.

This state of the economy arises from the interplay between the OG and DA functions: as *y* stops falling, so *p* stops rising (see below), one being mutually consistent with the other. Our suggested interpretation is that this kind of equilibrium represents a state of affairs commonly dubbed the "new normal". Put otherwise, while agents, at the ZLB, have no reason to believe that the central bank can do anything else to correct the non-zero gaps (*p* does not fall to zero), *ceteris paribus* nothing can drive the system into an even worse condition (*p* does not escalate to one). To modify this equilibrium, a structural change is necessary, where the structure of the economy includes the policy rule(s) (e.g. a change in the monetary policy conduct or a shift from monetary to fiscal policy). Then the existing degree of de-anchoring may be regarded as the hurdle to be overcome in order to restore full confidence in the normal regime (indeed, the agents have reason to believe that the farther (the closer) are *y* and *p* from (to) zero, the less (more) likely is the restoration of the normal regime).

To better understand the properties of the system at the ZLB, and to see to what extent the result of the simulation can be of general value, we can go back to the four equations. At the ZLB, the Taylor Rule stops working leaving a permanent interest-rate gap, as if it were exogenously fixed, in the OG equation. From that point onwards the system is driven by the three equations OG, PC, DA. Since the inflation gap is monotonically determined by the output gap, we can study the system formed by just the two equations OG, DA. If a steady state exists, it is a fixed point of the two variables (*y, p*) along the OG and DA functions. Recalling the relationship

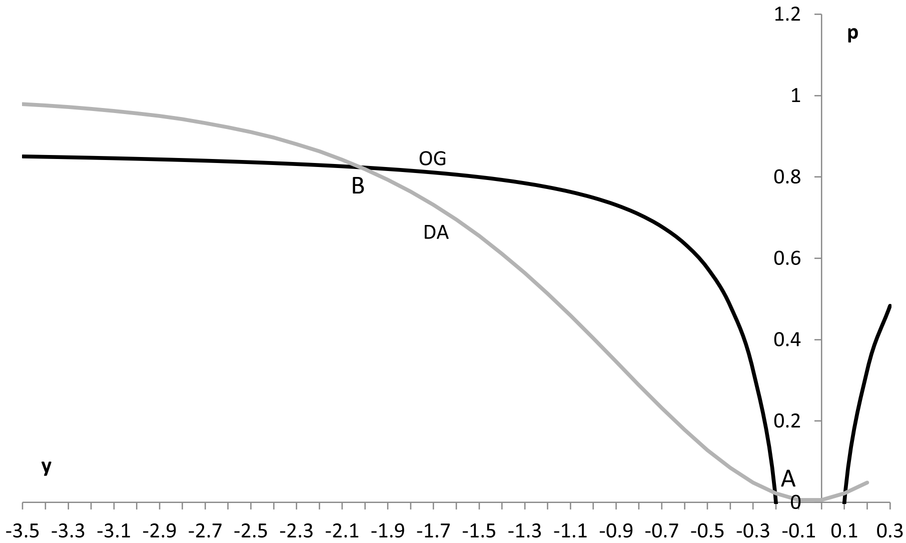

Fig. 4. The OG and DA functions at the ZLB (baseline values of parameters).

between y and p in the two functions, we can infer that, in general, one steady state *may* indeed exist *conditional* upon the value of the parameters and the interest-rate gap. Given the baseline parameter values and the fixed interest-rate gap -1%, it can be shown that two steady states exist.

Let us consider the functions DA and (inverted) OG in the Cartesian plan (y, p) in Fig. 4. The intersection A between the functions corresponds to the steady-state values in Fig. 3.

According to the DA function, as y tends to large negative values, the asymptotic value of p is 1, whereas it is less than 1 according to the OG function (0.873). This necessarily implies that a second steady state exists (B with, approximately, $\bar{y} = -2.1$ and $\bar{p} = 0.825$). Observing the relative slope of the two functions in A and B, it follows that A is stable whereas B is not. Hence, approaching A from the right, the economy settles down at A as shown by the simulation.

From this analysis we can also say that either two steady states or none exist depending on combinations of parameters and shocks. In fact, for larger shocks, *cet. par.* the OG function shifts to the left to the effect that also A moves along the DA function, determining "new normal" equilibria with worse output and inflation gaps and de-anchoring of expectations. There may be shocks large enough that displace OG to the far left of DA where no steady state exists and the system becomes globally unstable. On the other hand, larger values of the sensitivity σ of the DA function make it steeper. Now, *cet. par.* point A moves along the OG function, determining again worse "new normal" equilibria. For values of σ large enough, DA may entirely lay to the right of OG preventing the existence of any steady state and generating global instability.

The conclusion of our analysis of the system at the ZLB is that either the economy settles down in a "new normal", depressed equilibrium or, in the presence of unfavourable combinations of large shocks and/or high sensitivity of agents' inflation expectations, the economy may be bound to global instability.

3.3. Exploring the space of the parameter values

So far we have examined the properties of the economy represented by our four equations for a given set of parameters with values taken from the empirical literature. We have seen that this economy can safely accommodate natural-rate shocks up to the ZLB of the policy rate. This may be representative of the Great Moderation era. At the ZLB, with the baseline configuration of parameters the economy reaches a "new normal" equilibrium with (moderately) low output and inflation and de-anchoring of expectations. However, we have also seen that the economy is bound to global instability in the presence of combinations of large shocks and high reactivity of expectations. Therefore, now we explore the dynamic behaviour of the economy in the space of parameter values beyond their baseline values.

We have selected three parameters of interest, the sensitivity of the DA function (σ), the structural slope of the PC (κ), and the inflation parameter of the TR (γ_π). The first simulation tests the resilience of the system when the parameters (κ, γ_π) are kept constant at their baseline value, while σ and the natural-rate shock ρ_0 are allowed to vary within a determined range. The subsequent simulations focus on σ *vis-à-vis*, respectively, γ_π and κ, while keeping the other constant at the baseline value, for each of the three types of shocks ρ_0, small (-1%), medium (-2%), large (-3%). The number of iterations (30 "quarters") has been chosen to mimic the typical medium-term horizon of business cycles. However, the number of iterations is inevitably arbitrary; hence we have set the following criteria for evaluation of the system's behaviour. First, the benchmark variable is the level of the output gap at the last iteration. Then, we have identified four areas that gauge the extent of divergence (convergence) of the system relative to the zero-gaps steady state, indexed from 0 to 3:

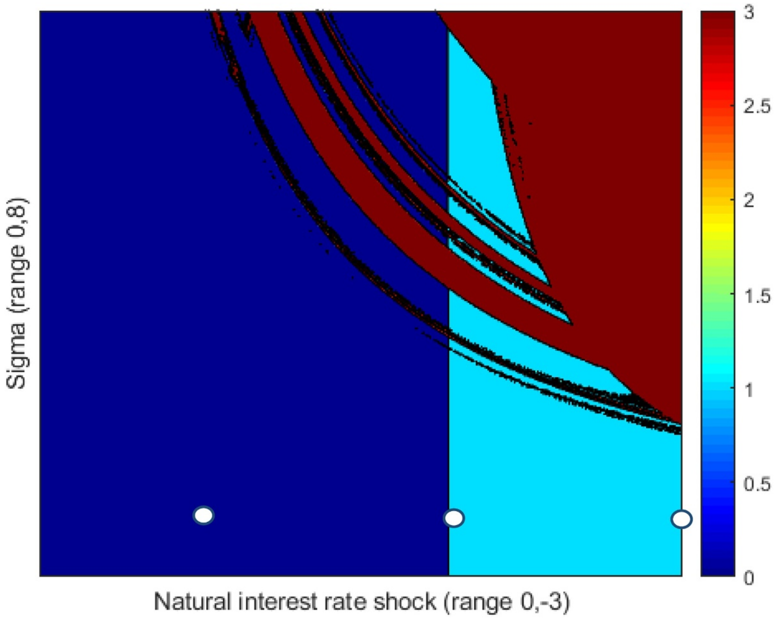

Fig. 5. The σ-ρ interaction.

- 0 (blue) = *convergence*, the residual output gap is less than 10% of the initial shock[20]
- 1 (light blue/green) = *tendential convergence*, the residual output gap is between 10% and 50% of the initial shock
- 2 (yellow) = *tendential divergence*, the residual output gap is between 50% and 100% of the initial shock
- 3 (red) = *divergence*, the residual output gap is larger than the initial shock.

3.3.1. The σ-ρ interaction

Two questions are particularly relevant. How do greater values of σ affect the system given the baseline values of γ_π and κ? How does the magnitude of the shock modify the picture? The answer is given by the simulation results depicted in Fig. 5 in the σ-ρ space. The range of values of σ has been set $\sigma \in [0, 8]$ and the natural-rate shock in the full range $\rho_0 \in [0, -3\%]$. Recall that for $\rho_0 < -2\%$, the ZLB is binding. The white dots indicate the positions of the system in the baseline cases with $\sigma = 1$, $\rho_0 = [-1, -2, -3]$.

The first notable feature is that the regions of convergence or tendential convergence shrink with greater values of σ and/or ρ_0 as argued above in Sections 3.1 and 3.2. A small shock ($\rho_0 \geq -1\%$) allows the central bank to control the system up to relatively large values of σ (about 6). With a medium shock (up to −2%), the central bank is still able to control the system but only up to a value of σ of about 4. Interestingly, a frontier of the convergence region emerges as a convex relationship between σ and ρ.

The second notable feature is the tendential convergence region (light blue) for large shocks, i.e. under the ZLB regime. This region embeds the states of the system that we have named "new normal". The system stabilises itself "close" to, but below, the zero-gaps steady state. As shown above in 3.2, in these states the regime-switch probability remains "close" to, but above, zero. It is also confirmed that these states are attainable only for values of σ not too large (approximately less than 3).

3.3.2. The σ-γπ interaction

In light of the previous results, the next relevant question is whether an aggressive Taylor Rule (larger γ_π) can counteract the effects of increasing σ. The ranges of the two parameters have been set at $\sigma \in [0, 8]$ and $\gamma_\pi \in [0, 4]$. Fig. 6 displays the graded divergence/convergence regions of the system in the space (σ-γ_π) after the three types of shocks to the natural rate.

Simulations are in line with the previous ones. They show that with a small shock (−1%), a standard value of γ_π above 1 allows the central bank to control the system up to relatively large values of σ. Above this threshold, the system enters regions of tendential or outright divergence, typically with oscillatory dynamics. The extent of the shock systematically reduces the regions of convergence, so that the magnitude of σ becomes more binding. With the large shock (−3%), under the ZLB, the system again settles down in the ""new normal" (the light grblue region) provided that σ is sufficiently small (less than 2.5 approximately). Notably, the answer to the above question is that in the divergence regions, *a more aggressive (conventional) monetary policy with larger γ_π is ineffective, and it is obviously ineffective at the ZLB.*

[20] To discard casual convergence of the very last observation, we have also checked that from the 25th period onwards the values of the output gap do not exceed the value of the initial shock.

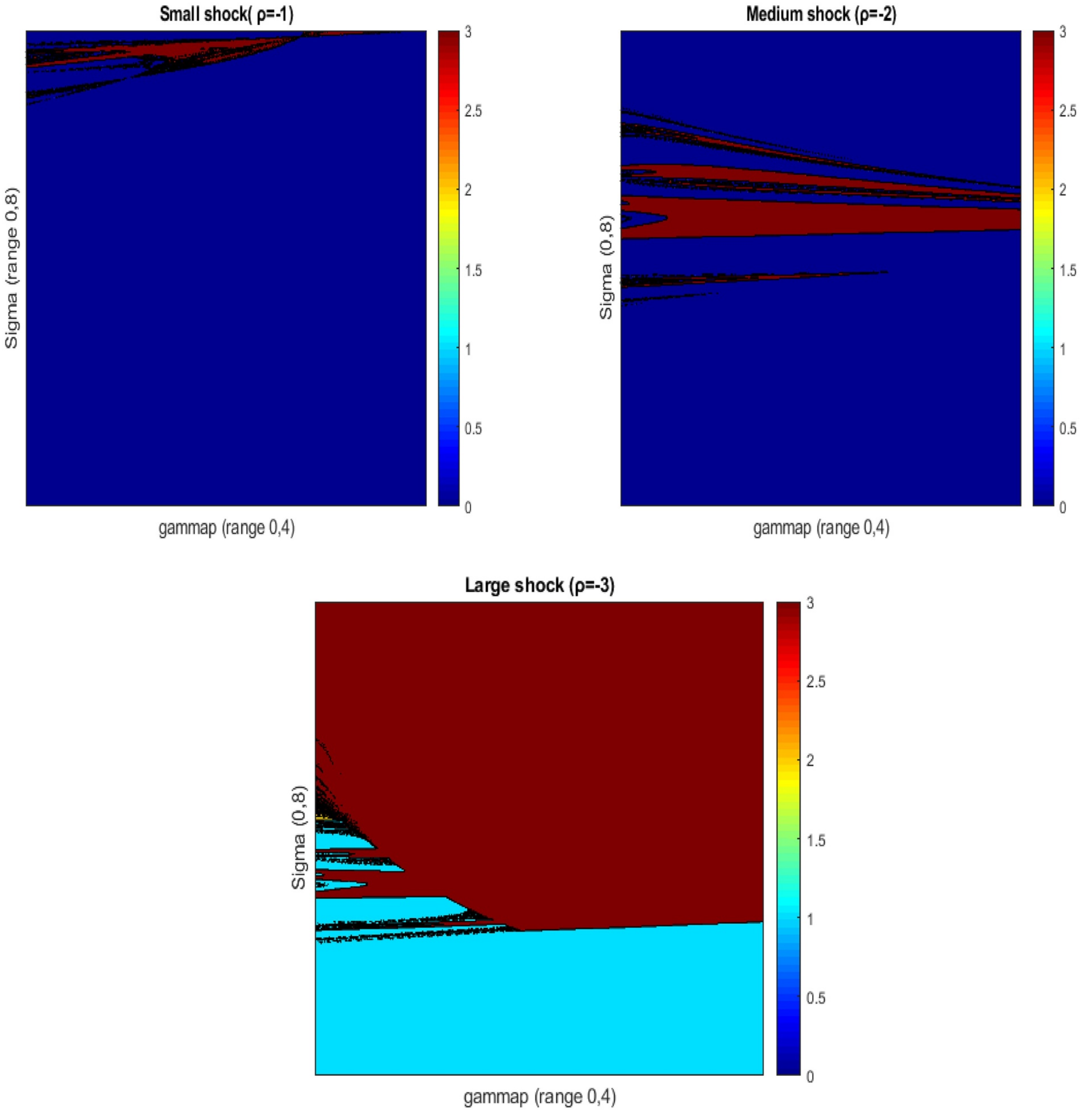

Fig. 6. The σ-γ_π interaction.

3.3.3. The σ-κ interaction

Price stickiness is a key component of the New Keynesian framework in which our simulations are set. As said above, the usual estimations or calibration of the parameter κ that measures the slope of the PC yield very small values. On the other hand, we have shown that the de-anchoring of expectations steepens the PC. The interaction between σ and κ is therefore an interesting issue to address and is shown in Fig. 7. The range of values of the two parameters have been set at $\sigma \in [0, 8]$ and $\kappa \in [0, 4]$.[21]

We have already seen that for standard values of κ and γ_π, the economy enjoys a large region of convergence up to relatively high values of σ. The noteworthy feature of the parameter κ is that if it increases, the region of convergence shrinks, although this effect is relatively moderate. Medium and large shocks magnify this phenomenon, especially at the ZLB. In other words, as prices become more flexible, the central bank *loses control over the system and is more exposed to de-anchoring.* This may seem a counterintuitive result,

[21] According to the Calvo equation, $\kappa = 4$ corresponds (approximately) to less than 20% of firms not changing their price, i.e. an economy with almost full price flexibility.

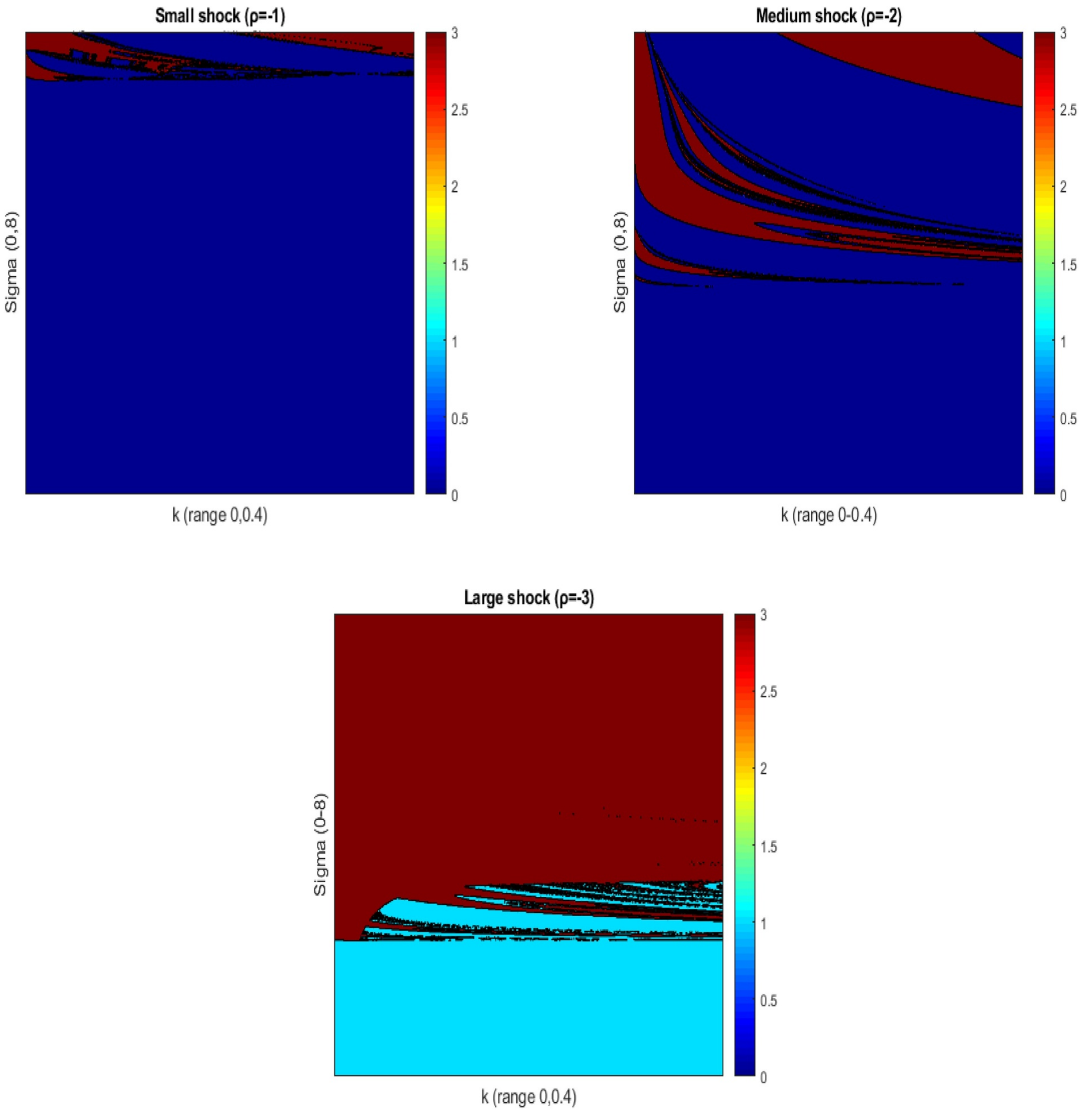

Fig. 7. The σ-κ interaction.

since greater price flexibility is, generally, regarded as an unconditional benefit for the economy. However, the problem that price flexibility would undermine monetary policy based on the interest rate control was raised by Wicksell himself (Mazzocchi, 2013; Mazzocchi et al., 2014).

4. Conclusions

The Great Recession has severely challenged monetary policy. Since 2012 inflation rates, and inflation forecasts, have been diminishing rapidly despite extraordinarily expansive monetary conditions. These phenomena have raised concern about central banks' ability to deliver their medium-term inflation targets according to the current doctrine of anchoring expectations.

In the light of the study presented in this paper, the first critical factor is the interaction between the reactivity of inflation expectations to the current state of the economy and the dimension of the shocks. The normal regime with anchored expectations and inflation on track is resilient as long as the policy rate can accommodate shocks above the ZLB, and reactivity is sufficiently small. In other words, deviations from targeted inflation are self-amplifying and make the system more difficult to manage. If the reactivity/

shock combination is such that expectations "fall faster" than the policy rate, which reaches the ZLB without correcting the shock, the economy *may* converge to a new steady state with permanent negative gaps. This state of affairs represents the "new normal" in the sense that it reduces the confidence in the resilience of the system by a finite amount without triggering a complete fall of confidence. Otherwise, the economy may enter in regions of global instability. Controlling for structural price stickiness, simulations show that, *ceteris paribus,* as stickiness decreases the instability regions widen, the reason being that a steeper Phillips Curve amplifies the output gaps.

With regard to the empirical literature, our study shows that the three main stylised facts – sustained deviation of inflation from the target, de-anchoring of expectations and steepening of the Phillips Curve – are mutually consistent. In fact, the best suited empirical identification strategy of de-anchoring is that expectations are sensitive to shocks to inflation and/or output gaps and are revised procyclically. In this connection, a relevant implication for empirical analysis of the inflation-output relationship is that the standard estimation equation with the expectational term (either forward or backward looking) assumed to be independent of the output gap may be misleading. The steepening of the Phillips Curve may be due to the amplifying effect of de-anchored expectations rather than to pure structural factors.

I think the point is that shocks may be too large for a TR

Our study also has policy implications. More aggressive (conventional) monetary policy, i.e. with larger inflation coefficient, is obviously ineffective at the ZLB, but it is also ineffective above the ZLB when adverse conditions (large shocks and/or expectation reactivity) push the system into the instability regions. This finding seems to support the necessity, in those conditions, to abandon conventional monetary policy and to switch to an aggressive reflationary policy that prevents the entrenchment of deflationary expectations.

Two main questions require further research. One regards the DA function. Another reasonable characterization of reactivity may be the *speed* of updating, i.e. after how many periods of observed output gaps agents update *p*. This gives salience to the persistence of output gaps, rather than amplitude, as a determinant of the regime-switch probability. Another is whether and how non-conventional monetary policies (e.g. quantitative easing) may succeed in reverting the de-anchoring of inflation expectations when the conventional one fails. To this end, the model should be modified in order to accommodate money demand and supply equations suitably specified to capture the channels through which asset purchases by the central bank can affect output and inflation (e.g. Curdia and Woodford, 2011; Saraceno and Tamborini, 2015; Bletzinger and Von Tadden, 2018).

References

Arifovic, J., Bullard, J., Kostyshyna, O., 2012. Social learning and monetary policy rules. Econ. J. 123, 38–76.
Arifovic, J., Schmitt-Grohé, S., Uribe, M., 2017. In: Learning to Live in a Liquidity Trap, *NBER Working Paper Series*, n. 23725.
Bai, J., 1997. Estimating multiple breaks one at a time. Econometr. Theory 13 (3), 315–352.
Bank of Ireland, 2014. Examining the sensitivity of inflation to the output gap. Quart. Bull April.
Benhabib, J., Schmitt-Grohé, S., Uribe, M., 2001. The perils of Taylor rules. J. Econ. Theory 96, 40–69.
Blanchard, O.J., Cerutti, E., Summers, L., 2015. In: Inflation and Activity. Two Explorations and their Monetary Policy Implications, IMF Working Paper, n. 15/230.
Bletzinger, T., von Tadden, L., 2018. In: Designing QE in a Fiscally Sound Monetary Union, ECB Working Paper Series, n. 2156.
Boianovsky, M., Trautwein, H.M., 2004. Wicksell after Woodford. In: Paper Presented at the History of Economics Society meeting, Toronto, June.
Bordo, M.D., Siklos, P., 2014. In: Central Bank Credibility, Reputation and Inflation Targeting in Historical Perspective, *NBER Working Papers Series*, n. 20693.
Bullard, J., Mitra, K., 2002. Learning about monetary policy rules. J. Monet. Econ. 49, 1105–1129.
Buono, I., Formai, S., 2016. In: The Evolution of the Anchoring of Inflation Expectations. Bank of Italy. Occasional Papers, n. 321.
Busetti, F., Ferrero, G., Gerali, A., Locarno, A., 2014. In: Deflationary Shocks and De-anchoring of Inflation Expectations. Bank of Italy. Occasional Papers, n. 252.
Carlin, W., Soskice, D., 2005. The 3-equation new keynesian model. A graphical exposition. Contrib. Macroecon. 5, 13.
Carvalho, C., Eusepi, S., Moench, E., Preston, B., 2017. Anchored Inflation Expectations. mimeo. Available at SSRN: https://ssrn.com/abstract=3018198.
Clarida, R., Galì, J., Gertler, M., 1999. The science of monetary policy: a new Keynesian perspective. J. Econ. Lit. 37, 1661–1707.
Clarida, R., Galì, J., Gertler, M., 2000. Monetary policy rules and macroeconomic stability: evidence and some theory. Quart. J. Econ. 115, 147–180.
Curdia, V., Woodford, M., 2011. The central-bank balance sheet as an instrument of monetary policy. J. Monet. Policy 58, 58–79.
De Grauwe, P., Ji, Y., 2016. In: Inflation Targets and the Zero Lower Bound in a Behavioral Macroeconomic Model, *CEPR, Discussion Paper Series*, n.11320.
Draghi, M., 2014a. Monetary policy communication in turbulent times. In: Speech at the Conference 'De Nederlandsche Bank 200 Years: Central Banking in the Next Two Decades', Amsterdam, April, URL: http://www.ecb.europa.eu/press/key/date/2014/html/sp140424.en.html.
Draghi, M., 2014b. Monetary policy in the euro area. In: Opening Keynote Speech by Mario Draghi, President of the ECB at the Frankfurt European Banking Congress, Frankfurt am Main, November, URL: http://www.ecb.europa.eu/press/key/date/2014/html/sp141121.en.html.
Draghi, M., 2016. How central banks meet the challenge of low inflation. In: Marjolin Lecture, SUERF Conference, Deutsche Bundesbank, Frankfurt, February, URL: http://www.ecb.europa.eu/press/key/date/2016/html/sp160204.en.html.
Eggertsson, G.B., Woodford, M., 2004. Policy options in a liquidity trap. Papers and Proceedings of the American Economic Association. Am. Econ. Rev. 94, 76–79.
Ehrmann, M, Eijffinger, S., Fratzscher, M., 2015. The role of central bank transparency for guiding private sector forecasts. Scand. J. Econ. 114, 1018–1052.
European Central Bank, 2006. Measures of inflation expectations in the Euro area. Monthly Bull. 59–68 July.
European Central Bank, 2014. The Phillips curve relationship in the Euro Area. Monthly Bull. 99–114 July.
Evans, G.W., Guse, E., Honkapohja, S., 2008. Liquidity traps, learning and stagnation. Eur. Econ. Rev. 52, 1438–1463.
Evans, G.W., Honkapohja, S., 2001. Learning and Expectations in Macroeconomics. Princeton University Press, Princeton, NJ.
Evans, G.W., McGough, B., 2018. Interest rate pegs in new Keynesian models. J. Money Credit Bank. 50, 939–965.
Farmer, R.A.E., 1993. The Macroeconomics of Self-fulfilling Prophecies. MIT Press, Cambridge Mass.
Foresell, M., Kenny, G., 2004. Survey expectations, rationality and the dynamics of Euro area inflation. J. Bus. Cycle Measur. Anal. 1, 13–41.
Fracasso, A., Probo, R., 2017. When did inflation expectations in the Euro area de-anchor? Appl. Econ. Lett. 24, 1481–1485.
Garnier, J., Wihelmsen, B., 2005. In: The Natural Real Interest and the Output Gap in the Euro Area. A Joint Estimation, ECB Working Paper Series, n. 546.
Grothe, M., Meyler, A., 2015. In: Inflation Forecasts: Are Market-Based and Survey-Based Measures Informative? ECB Working Paper Series, n. 1865.
Gürkaynak, R.S., Swanson, E., Levin, A., 2010. Does inflation targeting anchor long-run inflation expectations? Evidence from the U.S., UK, and Sweden. J. Eur. Econ. Assoc. 8, 1208–1242.
Iakova, D., 2007. In: Flattening of the Phillips Curve: Implication for Monetary Policy, *IMF Working Paper*, n. 07/76.
International Monetary Fund, 2013. The dog that didn't bark: has inflation been muzzled or was it just sleeping? World Economic Outlook. International Monetary Fund, Washington, DC chapter 3.
Krugman, P., 1998. It's Baaaack! Japan's slump and the return of liquidity trap. Brook. Pap. Econ. Act. 2, 137–205.
Kurz, M., 2011. Symposium: on the role of market beliefs in economic dynamics, an introduction. Econ. Theory 47, 189–204.

Kuttner, K., Robinson, T., 2010. Understanding the flattening of the phillips curve. North Am. J. Econ. Finance 21, 110–125.
Laubach, T., Williams, J.C., 2003. Measuring the natural rate of interest. Rev. Econ. Stat. 85, 1063–1070.
Luk, P., Vines, D., 2015. In: Optimal Monetary and Fiscal Policy in an Economy with Endogenous Public Debt, CEPR, Discussion Paper Series, n. 10580.
Lyziak, T., Paloviita, M., 2016. In: Anchoring of Inflation Expectations in the Euro Area: Recent Evidence Based on Survey Data, ECB Working Paper Series, n. 1945.
Marcet, A., Sargent, T., 1989. Convergence of least square learning mechanisms in self-referential linear stochastic models. J. Econ. Theory 48, 337–368.
Mazzocchi, R., 2013. In: Intertemporal Coordination Failure and Monetary Policy,, Department of Economics and Management, University of Trento, DEM Working Papers, n. 15.
Mazzocchi, R., Tamborini, R., Trautwein, H.M., 2014. Wicksell, Keynes, and the new neoclassical synthesis: what can we learn for monetary policy? Econ. Notes 43, 79–114.
Miccoli, M., Neri, S., 2015. In: Inflation Surprises and Inflation Expectations in the Euro Area, Occasional Papers, Bank of Italy, n. 265.
Molnar, K., Ormeno, A., 2015. Using survey data of inflation expectations in the estimation of learning and rational expectations models. J. Money, Credit Bank. 47, 673–699.
Murphy, R., 2014. Explaining inflation in the aftermath of the great recession. J. Macroecon. 40, 228–244.
Natoli, F., Sigalotti, L., 2018. Tail comovement in inflation expectations as an indicator of anchoring. Int. J. Central Bank. 14, 35–71.
Nautz, D., Pagenhardt, L., Strohsal, T., 2017. The (De-)anchoring of inflation expectations: new evidence from the Euro area. North Am. J. Econ. Finance 40, 103–115.
Oinonen, S., Palovitta, M., 2014. In: Updating the Euro Area Phillips Curve: The Slope Has Increased, Bank of Finland Research, Discussion Papers, n. 31.
Orphanides, A., 2015. Fear of lift off: uncertainty, rules, and discretion in monetary policy normalization. Fed. Res. Bank St. Louis Rev. 97 (3), 173–196.
Pfajfar, D., Santoro, E., 2010. Heterogeneity, learning and information stickiness in inflation expectations. J. Econ. Behav. Organ. 75 (3), 426–444.
Preston, B., 2005. Learning about monetary policy rules when long horizon expectations matter. Int. J. Central Bank. 1, 81–126.
Riggi, M., Venditti, F., 2014. In: Surprise! Euro Area Inflation Has Fallen, Bank of Italy, Occasional Papers, n. 237.
Saraceno, F., Tamborini, R., 2015. In: How Can it Work? The Impact of quantitative Easing in the Eurozone. Department of Economics and Management, University of Trento DEM Working Papers, n.3.
Slobodyan, S., Wouters, R., 2012. Learning in a medium-scale DSGE model with expectations based on small forecasting models. Am. Econ. J. 4, 65–101.
Smets, F., Wouters, R., 2003. An estimated dynamic stochastic general equilibrium model of the Euro area. J. Eur. Econ. Assoc. 5, 1123–1175.
Teulings, C., Baldwin, R., 2014. Secular Stagnation: Facts, Causes and Cures. CEPR, London.
Trautwein, H.M., Zouache, A., 2009. Natural rates in the new synthesis: same old trouble? Eur. J. Econ. Econ. Policies 6 (2), 207–225.
Wicksell, K., 1898. Geldzins und Güterpreise. Eine Untersuchung über die den Tauschwert des Geldes bestimmenden Ursachen. Jena, Gustav Fischer; Eng. tr.: Interest and Prices. A Study of the Causes Regulating the Value of Money. Macmillan, London 1936.
Woodford, M., 2003. Interest and Prices. Princeton University press, Princeton.
Yellen, J., 2014. House Financial Services Committee Hearing on Federal Reserve Semi-Annual Monetary Policy Report to Congress. Senate Hearing, 11 February 2014. .