Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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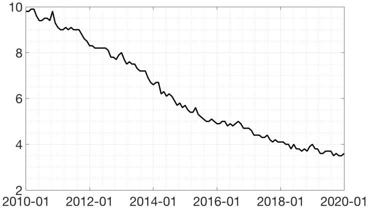
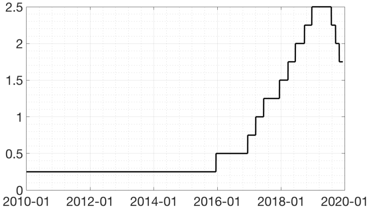
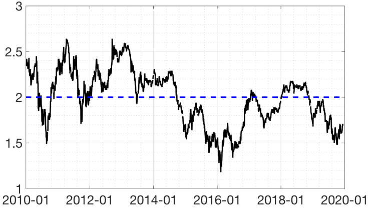
Puzzling Fed behavior fall 2019



(a) Unemployment rate, %



(b) Fed funds rate target, upper limit, %



(c) Market-based inflation expectations, 10 year, % average

This project

Model anchored expectations as an endogenous gain learning scheme

$\rightarrow$ How to conduct optimal monetary policy in interaction with the anchoring expectation formation?

Preview of results

1. Two layers of new intertemporal tradeoffs

2. optimal monetary policy time-inconsistent

$\rightarrow$ illustrate analytically in special case: target criterion

3. Not today: short-run costs vs. long-run benefits of anchoring expectations

Related literature

- **Optimal monetary policy in New Keynesian models**
  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Econometric learning**
  Evans & Honkapohja (2001), Preston (2005), Molnár & Santoro (2014)

- **Anchoring / endogenous gain**
  Carvalho et al (2019), Svensson (2015), Hooper et al (2019), Milani (2014)

Structure of talk

1. Model

2. Solving the Ramsey problem

3. Implications

Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j)) dj \right] \tag{1}$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1}) B_{t-1}^i + \int_0^1 w_t(j) h_t^i(j) + \Pi_t^i(j) dj - T_t - P_t C_t^i \tag{2}$$

▸ Consumption, price level

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \tag{3}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{4}$$

▸ Profits, stochastic discount factor

Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- If use $\mathbb{E}$ (rational expectations, RE)

  Model solution

$$s_t = hs_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \tag{5}$$

$$y_t = gs_t \tag{6}$$

  $s_t \equiv (r_t^n, u_t)'$ (states)
  $y_t \equiv (\pi_t, x_t, i_t)'$ (jumps)

- If use $\hat{\mathbb{E}} \rightarrow$ don't know g
  $\rightarrow$ estimate using observed states & knowledge of (5)

Adaptive learning

- Estimate g using recursive least squares (RLS)

  $\rightarrow$ nonrational expectations:

$$\hat{\mathbb{E}}_t y_{t+1} = \phi_{t-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix} \tag{7}$$

- Note: misspecified

  Can write:

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{8}$$

  In RE, $a_{t-1} = (0, 0, 0)', b_{t-1} = g\, h \quad \forall t$

Recursive least squares

Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\, h \quad \forall t \tag{9}$$

$\rightarrow$ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\right)}_{\equiv fe_{t|t-1},\ \text{forecast error}} \tag{10}$$

$k_t \in (0, 1)$ gain
b_1 first row of b

▸ General RLS algorithm

Anchoring mechanism: endogenous gain

Gain in literature usually exogenous:

$$k_t = \begin{cases} \frac{1}{t} & \text{decreasing} \\ k & \text{constant} \end{cases}$$

Here instead

$$k_t = k_{t-1} + \mathbf{g}(fe_{t|t-1}) \tag{11}$$

▸ Functional forms

Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{12}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{13}$$

▸ Derivations

- Expectations evolve according to RLS with the endogenous gain given by (11)

→ How should $\{i_t\}$ be set?

Structure of talk

1. Model

2. Solving the Ramsey problem

3. Implications

Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

- $\mathbb{E}$ is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

Special case

- Only inflation intercept learned
- Anchoring function simplified to

$$k_t = \mathbf{g}(fe_{t|t-1}) \tag{14}$$

Target criterion for special case

Result

In the simplified model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa}\Bigg\{x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\Bigg(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_{\pi,t}\Bigg)$$

$$\Bigg(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi},\mathbf{t+j}})\Bigg)\Bigg\}$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t, $\prod_{j=0}^{0} \equiv 1$ *and* b_1 *is the first row of* b.

General case

Two layers of intertemporal tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i}$$
$$-\frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j} + fe_{t+1+j|t+j} \mathbf{g}_{\bar{\pi},\mathbf{t+j}})$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

Lemma

The commitment solution of the Ramsey problem does not exist under adaptive learning.

Corollary

Optimal policy under adaptive learning is time-inconsistent.

▸ Why no commitment?

Structure of talk

1. Model

2. Solving the Ramsey problem

3. Implications

How to implement?

- Related issue under RE: optimal interest rate sequence implies indeterminate equilibrium

$\Rightarrow$ Reaction function stabilizes expectations

Recall IS-curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big)$$

- E.g. Taylor rule disciplines expectations:

$$\hat{\mathbb{E}}_t i_T = \psi_\pi \hat{\mathbb{E}}_t \pi_T + \psi_x \hat{\mathbb{E}}_t x_T$$

Next steps: form of reaction function

- Model suggests $i_t = \mathbf{f}(\pi_t, k_t, \bar{\pi}_{t-1}; t)$ nonlinear

- Explains deviations from Taylor rule

- However: no commitment makes Taylor rule more viable than under RE as a rough approximation of optimal feedback rule

- If Taylor rule, model prefers being less aggressive on inflation

Conclusion

- Interaction between monetary policy and anchoring

- Optimal policy conditions on stance of current and expected future anchoring

  $\hookrightarrow$ determine intertemporal tradeoffs

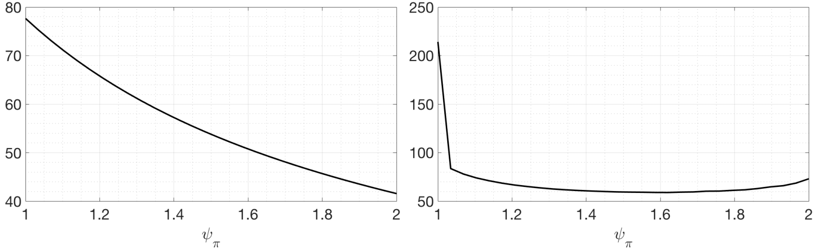
- Explain departures from the Taylor rule like US, fall 2019

- If Fed acted to anchor expectations, then Missing Deflation and Inflation are not "missing"

Short-run costs, long-run benefits

Assume Taylor rule and no concern for output gap stabilization

$$i_t = \psi_\pi \pi_t \qquad \lambda_x = 0$$



(d) RE

(e) Anchoring

Figure: Central bank loss as a function of ψ_π

▸ Short-run costs

Functional forms for $\mathbf{g}$

- Smooth anchoring function

$$k_t = k_{t-1} - c + dfe^2_{t|t-1} \tag{15}$$

$c, d > 0$

- Kinked anchoring function

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{16}$$

θ_t criterion, $\bar{\theta}$ threshold value

◂ Return

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \tag{17}$$

Σ variance-covariance matrix of shocks
$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(fe_{t|t-1} fe'_{t|t-1} - \omega_{t-1}) \tag{18}$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(fe'_{t|t-1} \omega_t^{-1} fe_{t|t-1} - \theta_{t-1}) \tag{19}$$

ω_t estimated forecast-error variance

◂ Return

Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}\right)'\right)' \tag{20}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{21}$$

◂ Return

Compact notation

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{22}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{23}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{pmatrix} \tag{24}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{25}$$

◂ Return

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t) + \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \Big\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t + \mathbf{g}_\pi(\pi_t - f_{t-1})) = 0 \tag{26}$$

$$-2\beta\frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_\mathbf{f}(\pi_{t+1} - f_t)) = 0 \tag{27}$$

◂ Return

Short-run costs from oscillatory dynamics

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{28}$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{29}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1-k^{-1}}{1-k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1-k^{-1}\beta} u_t \tag{30}$$

Solve for forecast error $fe_t \equiv \pi_t - f_{t-1}$:

$$fe_t = \underbrace{-\frac{1-\beta}{1-k\beta}}_{\lim_{k\to 1}=-1} f_{t-1} + \frac{1}{1-k\beta} u_t \tag{31}$$

◂ Return

Target criterion system for anchoring function as changes of the gain

$$\varphi_{6,t} = -cfe_{t|t-1}x_{t+1} + \left(1 + \frac{fe_{t|t-1}}{fe_{t+1|t}}(1-k_{t+1}) - fe_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1} - \frac{fe_{t|t-1}}{fe_{t+1|t}}(1-k_{t+1})\varphi_{6,t+2} \quad (32)$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{fe_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{fe_{t|t-1}}\varphi_{6,t+1} \quad (33)$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (33) is given by:

$$\varphi_{6,t} = -2\,\mathbb{E}_t\sum_{i=0}^{\infty}(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i})\prod_{j=0}^{i-1}\frac{\frac{k_{t+j}}{fe_{t+j|t+j-1}}}{\frac{k_{t+j}}{fe_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \quad (34)$$

Return

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{35}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{36}$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \tag{37}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{38}$$

Return

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{39}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{40}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (39) backward to some date t, take expectations at t
2. Sub in (40)
3. Aggregate over households i

$\rightarrow$ Obtain (12)

◂ Return