

# Materials 2

Laura Gáti

September 5, 2019

## Overview

<b>1 A CEMP-Preston mix</b>	<b>1</b>
1.1 Deriving the ALM	2
1.2 SR forecast error and the criterion	3
1.3 Model summary	4

## 1 A CEMP-Preston mix

Suppose we have a NK model with LR forecasts being relevant, as in Preston (2005):

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (\text{Preston, eq. (18)})$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (\text{Preston, eq. (19)})$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \quad (\text{Preston, eq. (27)})$$

where I've 1) added  $\sigma$  in front of  $r_T^n$ , reflecting the derivation of the shock on the NKIS; 2) added  $u_T$ , a cost-push shock to the NKPC.

I'm assuming that the innovations can be summarized as:

$$s_t = P s_{t-1} + \epsilon_t \quad (1)$$

$$\text{where } s_t \equiv \begin{pmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{pmatrix} \quad P \equiv \begin{pmatrix} \rho_r & 0 & 0 \\ 0 & \rho_i & 0 \\ 0 & 0 & \rho_u \end{pmatrix} \quad \text{and} \quad \epsilon_t \equiv \begin{pmatrix} \varepsilon_t^r \\ \varepsilon_t^i \\ \varepsilon_t^u \end{pmatrix} \quad (2)$$

Let  $z_t$  summarize the endogenous variables as

$$z_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \quad (3)$$

This is where the CEMP bit comes in: let agents form forecasts according to the relation

$$\bar{\mathbb{E}}_t z_{t+1} = \bar{z}_t + s_t + e_{t+1} \quad (\text{PLM})$$

where  $\bar{z}_t$  is the LR expectation of all endogenous variables. CEMP would love if we called this the “drift” in beliefs. Let this drift evolve according to CEMP’s criterion as:

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} f_{t-1} \quad (4)$$

$$f_{t-1} = z_{t-1} - \hat{\mathbb{E}}_{t-2} z_{t-1} \quad (\text{short-run forecast error}) \quad (5)$$

$$k_t = \mathbb{I}(k_{t-1} + 1) + (1 - \mathbb{I})\bar{g}^{-1} \quad (6)$$

$$\mathbb{I} = \begin{cases} 1 & \text{if } \theta_t \leq \bar{\theta} \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

$$\text{where } \theta_t = |\hat{\mathbb{E}}_{t-1} z_t - \mathbb{E}_{t-1} z_t| / (\sigma_r + \sigma_i + \sigma_u) \quad (\text{subjective - objective forecast}) \quad (8)$$

## 1.1 Deriving the ALM

Let the discounted infinite sums of expectations be given by

$$f_a \equiv \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} z_{T+1} \quad (9)$$

$$f_b \equiv \sum_{T=t}^{\infty} (\beta)^{T-t} z_{T+1} \quad (10)$$

Given these, and matrices  $A_1, A_2, A_3$ , we can write the reduced form LOM of the system as

$$z_t = A_1 f_a + A_2 f_b + A_3 s_t \quad (\text{RF})$$

where

$$A_1 = \begin{pmatrix} g_{\pi a} \\ g_{xa} \\ \psi_\pi g_{\pi a} + \psi_x g_{xa} \end{pmatrix} \quad A_2 = \begin{pmatrix} g_{\pi b} \\ g_{xb} \\ \psi_\pi g_{\pi b} + \psi_x g_{xb} \end{pmatrix} \quad A_3 = \begin{pmatrix} g_{\pi s} \\ g_{xs} \\ \psi_\pi g_{\pi s} + \psi_x g_{xs} + \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \end{pmatrix} \quad (11)$$

$$g_{\pi a} = \left(1 - \frac{\kappa\sigma\psi_\pi}{w}\right) \left[(1 - \alpha)\beta, \kappa\alpha\beta, 0\right] \quad (12)$$

$$g_{xa} = \frac{-\sigma\psi_\pi}{w} \left[(1 - \alpha)\beta, \kappa\alpha\beta, 0\right] \quad (13)$$

$$g_{\pi b} = \frac{\kappa}{w} \left[\sigma(1 - \beta\psi_\pi), (1 - \beta - \beta\sigma\psi_x), 0\right] \quad (14)$$

$$g_{xb} = \frac{1}{w} \left[\sigma(1 - \beta\psi_\pi), (1 - \beta - \beta\sigma\psi_x), 0\right] \quad (15)$$

$$g_{\pi s} = \left(1 - \frac{\kappa\sigma\psi_\pi}{w}\right) \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (I_3 - \alpha\beta P)^{-1} - \frac{\kappa\sigma}{w} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} (I_3 - \beta P)^{-1} \quad (16)$$

$$g_{xs} = \frac{-\sigma\psi_\pi}{w} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (I_3 - \alpha\beta P)^{-1} - \frac{\sigma}{w} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} (I_3 - \beta P)^{-1} \quad (17)$$

$$w = 1 + \sigma\psi_x + \kappa\sigma\psi_\pi \quad (18)$$

To get the ALM, we need to write the expectations  $f_a, f_b$  based on the PLM. Subbing in the PLM and using the anticipated utility assumption, I get

$$f_a = \frac{1}{1 - \alpha\beta} \bar{z}_t + (I_3 - \alpha\beta P)^{-1} s_t \quad (19)$$

$$f_b = \frac{1}{1 - \beta} \bar{z}_t + (I_3 - \beta P)^{-1} s_t \quad (20)$$

Then the ALM is the reduced-form expression RF, with expectations evaluated using these two expressions:

$$z_t = \left(A_1 \frac{1}{1 - \alpha\beta} + A_2 \frac{1}{1 - \beta}\right) \bar{z}_t + \left(A_1 (I_3 - \alpha\beta P)^{-1} + A_2 (I_3 - \beta P)^{-1} + A_3\right) s_t \quad (\text{ALM})$$

## 1.2 SR forecast error and the criterion

$$f_{t-1} = z_{t-1} - \hat{\mathbb{E}}_{t-2} z_{t-1} \quad (\text{short-run forecast error: ALM - PLM})$$

$$\theta_t = |\hat{\mathbb{E}}_{t-1} z_t - \mathbb{E}_{t-1} z_t| / (\sigma_r + \sigma_i + \sigma_u) \quad (\text{criterion: PLM - } \mathbb{E}_{t-1} \text{ALM})$$

Evaluating PLM, ALM, and  $\mathbb{E}_{t-1}$ ALM

$$f_{t-1} = \left( A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta-I_3} \right) \bar{z}_{t-1} + \left( A_1(I_3 - \alpha\beta P)^{-1} + A_2(I_3 - \beta P)^{-1} + A_3 - I_3 \right) s_{t-1} \quad (21)$$

$$(\sigma_r + \sigma_i + \sigma_u)\theta_t = \left( I_3 - A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} \right) \bar{z}_{t-1} + \left( I_3 - (A_1(I_3 - \alpha\beta P)^{-1} + A_2(I_3 - \beta P)^{-1} + A_3)P \right) s_{t-1} \quad (22)$$

### 1.3 Model summary

$$z_t = \left( A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} \right) \bar{z}_t + \left( A_1(I_3 - \alpha\beta P)^{-1} + A_2(I_3 - \beta P)^{-1} + A_3 \right) s_t \quad (\text{ALM})$$

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} f_{t-1} \quad (\text{Drift LOM})$$

$$k_t = \mathbf{f}_{\mathbf{k}}(\bar{z}_{t-1}, k_{t-1}, s_{t-1}) \quad \text{where } \mathbf{f}_{\mathbf{k}} \text{ evaluates the criterion } \theta_t \quad (\text{Gain LOM})$$

$$f_{t-1} = \left( A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} - I_3 \right) \bar{z}_{t-1} + \left( A_1(I_3 - \alpha\beta P)^{-1} + A_2(I_3 - \beta P)^{-1} + A_3 - I_3 \right) s_{t-1} \quad (23)$$

$$s_t = P s_{t-1} + \epsilon_t \quad (\text{exog. process})$$

More compactly

$$z_t = \left( A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} \right) \bar{z}_t + \left( A_1(I_3 - \alpha\beta P)^{-1} + A_2(I_3 - \beta P)^{-1} + A_3 \right) s_t$$

$$\bar{z}_t = \left( I_3 + k_t^{-1} \left( A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} - I_3 \right) \right) \bar{z}_{t-1} + k_t^{-1} \left( A_1(I_3 - \alpha\beta P)^{-1} + A_2(I_3 - \beta P)^{-1} + A_3 - I_3 \right) s_{t-1}$$

$$k_t = \mathbf{f}_{\mathbf{k}}(\bar{z}_{t-1}, k_{t-1}, s_{t-1})$$

$$s_t = P s_{t-1} + \epsilon_t$$