Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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*Inflation that runs below its desired level can lead to an unwelcome fall in **longer-term inflation expectations**, which, in turn, can pull actual inflation even lower, resulting in an adverse cycle of ever-lower inflation and inflation expectations. [...] **Well-anchored inflation expectations** are critical[.]*
Jerome Powell, Chairman of the Federal Reserve [1]
(Emphases added.)

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[1] "New Economic Challenges and the Fed's Monetary Policy Review," August 27, 2020.

Anchored expectations

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$\bar{\pi} =$ long-run expectations

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$\rightarrow$ short-run expectations "anchored" to stable mean:

$$\mathbb{E}_t \, \pi_{t+1} = \pi^* + f(shocks) \tag{2}$$

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$$\mathbb{E}_t \, \pi_{t+1} = \pi^* + f(shocks) \tag{2}$$

Unanchored expectations: long-run expectations deviate systematically from the target

$$\mathbb{E}_t \, \pi_{t+1} = \bar{\pi}_{t-1}(shocks) + f(shocks) \tag{3}$$

Unwelcome fall in longer-term inflation expectations

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \, \mathbb{E}_t \, \pi_{t+1} + u_t$$

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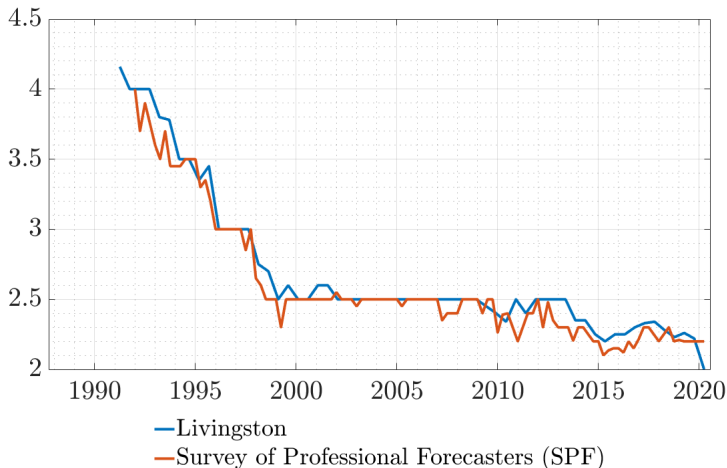$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t$$

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$\rightarrow$ unanchored expectations can cause a deflationary spiral

Long-run expectations drifting down?

Figure: Expectations of average inflation over 10 years ($\bar{\pi}$ in data)

Legend:
— Livingston
— Survey of Professional Forecasters (SPF)

▶ TIPS

Long-run expectations moving systematically?

Individual SPF forecasts: for 1991-Q4 onward, estimate rolling regression

$$\Delta\bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{4}$$

where w indexes windows of 19 quarters length and $f_{t|t-1} = \pi_t - \mathbb{E}_{t-1}\,\pi_t$

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Table: Rolling regression coefficients

Window (start quarter)	$\hat{\beta}_1^w$
1991-Q4	0.0687**
1996-Q3	0.0464***
2001-Q2	0.0116
2006-Q1	0.0061
2010-Q4	0.0135
2015-Q3	0.0496***

*** *significance at 1% ** significance at 5%*

▶ Further evidence

This project

- How to conduct monetary policy in interaction with the anchoring expectation formation?

- Model of anchoring expectation formation as an extension to adaptive learning

  $\hookrightarrow$ twist: fluctuations in long-run expectations responsive to economic conditions

- Estimation of the anchoring function: when do expectations become unanchored?

Preview of results

- Optimal policy aggressive when expectations unanchor, accommodates when anchored

- Taylor rule policy less aggressive on inflation than under rational expectations

$\hookrightarrow$ Anchoring-optimal Taylor rule eliminates 84% of loss from volatility

Related literature

- **Optimal monetary policy in the rational expectations New Keynesian model**
  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Adaptive learning**
  Evans & Honkapohja (2001, 2006), Sargent (1999), Primiceri (2006), Lubik & Matthes (2018), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Mele et al (2019), Eusepi & Preston (2011), Milani (2007, 2014)

- **Anchoring and the Phillips curve**
  Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Reis (2020), Gobbi et al (2019), Carvalho et al (2019)

- **Alternatives to rational expectations**
  Coibion & Gorodnichenko (2015), Mankiw & Reis (2002), Sims (2003), Angeletos & Pavan (2009), Gabaix (2014), Bordalo et al (2018)

- **Reputation**
  Barro (1986), Cho & Matsui (1995)

Structure of talk

1. Model of anchoring expectations

2. Solving the Ramsey problem

3. Implementing optimal policy

4. Approximating optimal policy with a Taylor rule

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Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j)) dj \right] \tag{5}$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1}) B_{t-1}^i + \int_0^1 w_t(j) h_t^i(j) dj + \int_0^1 \Pi_t^i(j) dj - T_t - P_t C_t^i \tag{6}$$

▶ Consumption, price level

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \tag{7}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{8}$$

▶ Profits, stochastic discount factor

Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- If use $\mathbb{E}$ (rational expectations, RE)

  Model solution

$$s_t = hs_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \tag{9}$$

$$y_t = gs_t \tag{10}$$

  $s_t \equiv$ states
  $y_t \equiv$ jumps
  $\epsilon_t \equiv$ disturbances

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  $s_t \equiv$ states
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- If use $\hat{\mathbb{E}} \rightarrow$ private sector does not know (11)

  $\hookrightarrow$ estimate using observed states & knowledge of (10)

Adaptive learning

- Postulate linear functional relationship instead of (11):

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- Estimate a, b using recursive least squares (RLS)

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Special case: learn only intercept of inflation:

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$\bar{\pi}_{t-1}$: long-run inflation expectations $\rightarrow$ anchoring

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$\bar{\pi}_{t-1}$: long-run inflation expectations $\rightarrow$ anchoring

$\rightarrow$ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\right)}_{\equiv\, f_{t|t-1},\ \text{forecast error}} \tag{13}$$

$k_t \in (0, 1)$ gain
b_1 first row of b

Decreasing versus constant gain

Decreasing gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f_{t|t-1} \tag{14}$$

$\rightarrow$ consider sample mean of full sample of forecast errors

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Constant gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k f_{t|t-1} \tag{15}$$

$\rightarrow$ consider sample mean of most recent observations only

Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t\big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big) \tag{16}$$

$k_t = \mathbf{g}(f_{t|t-1})$: anchoring function

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$k_t = \mathbf{g}(f_{t|t-1})$: anchoring function

$$\mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}) \tag{17}$$

$b_i(f_{t|t-1}) =$ basis, here: second order spline (piecewise linear)

$\gamma_i =$ approximating coefficients

$\hookrightarrow$ estimate $\hat{\gamma}$ via simulated method of moments (SMM)

Calibration - parameters from the literature

β	0.98	stochastic discount factor
σ	1	intertemporal elasticity of substitution
α	0.5	Calvo probability of not adjusting prices
κ	0.0842	slope of the Phillips curve
λ_x	0.05	weight on the output gap in central bank loss
$\bar{g}$	0.145	initial value of the gain
ρ_r	0	persistence of natural rate shock
ρ_u	0	persistence of cost-push shock
ρ_i	0	persistence of monetary policy shock*
ψ_π	1.5	coefficient of inflation in Taylor rule*

* pertains to sections where Taylor rule is in effect

Calibration - matching moments

ψ_x	0.3	coefficient of the output gap in Taylor rule*
σ_r	0.01	standard deviation, natural rate shock
σ_u	0.5	standard deviation, cost-push shock
σ_i	0.01	standard deviation, monetary policy shock*
$\hat{\gamma}_i$	(0.82; 0.61; 0; 0.33; 0.45)	coefficients in anchoring function

- Calibrate σ_j, $j = r, i, u$ to match the autocovariances of inflation, output gap, interest rate and one-period ahead inflation expectations for lags $0, \ldots, 4$

- Estimate $\hat{\gamma}_i$ to match the same moments

Anchoring function in the data
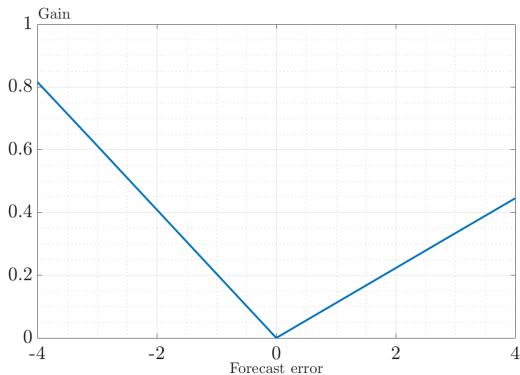
Figure: Learning gain as a function of forecast errors in inflation in %

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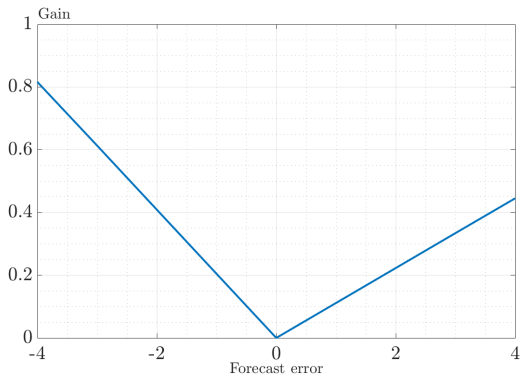
Figure: Learning gain as a function of forecast errors in inflation in %

- Estimates of the gain suggest ≈ 0.05
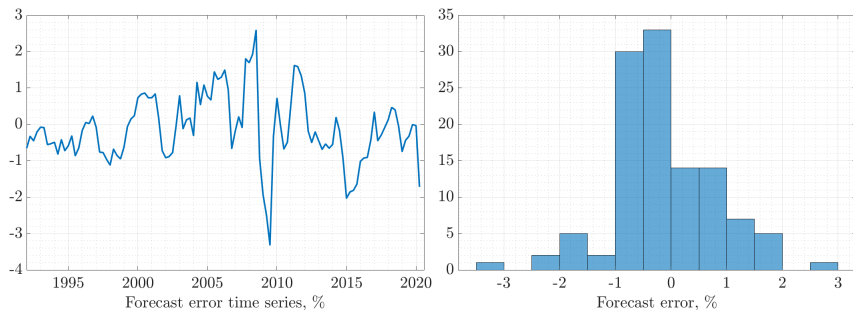
- Carvalho et al (2019)'s gain time series is max 0.145

Forecast errors in the data

Figure: Time series and distribution of 1-year ahead forecast errors in the SPF
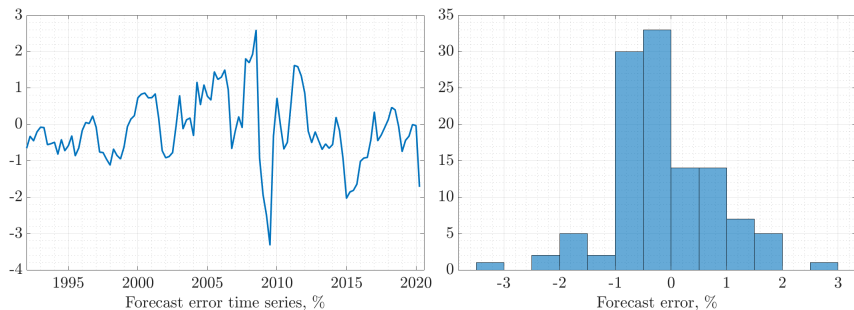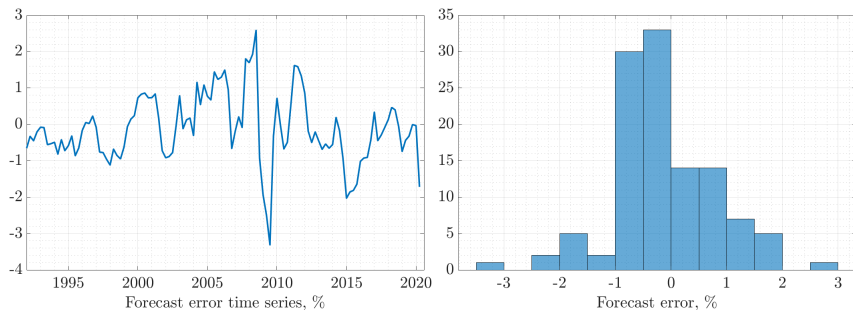
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Typical forecast error ≈ -0.5 pp
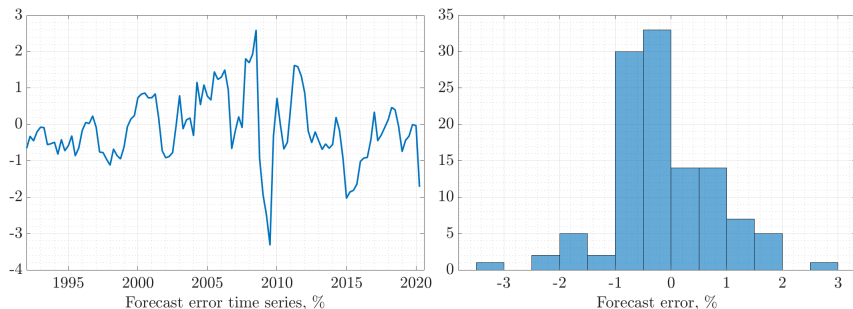
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Typical forecast error ≈ -0.5 pp $\quad \rightarrow$ gain of 0.1

Forecast errors in the data

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Typical forecast error ≈ -0.5 pp → gain of 0.1

$\Rightarrow$ 5 bp shift down in long-run expectations

Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \quad (18)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \quad (19)$$

▸ Derivations ▸ Actual laws of motion

- Expectations evolve according to RLS with the endogenous gain given by (18)

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1} \quad (16)$$

$$k_t = \mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}) \quad (20)$$

$\rightarrow$ How should $\{i_t\}$ be set?

Structure of talk

Ramsey problem

$$\min_{\{y_t, \pi_{t-1}, k_t\}_{i=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0}(\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

- $\mathbb{E}$ is the central bank's (CB) expectation

- Assumption: CB observes private expectations and knows the model

Target criterion

Proposition

In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_{\pi,t} \right)$$

$$\left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\bar{\pi},\mathbf{t+j}}) \right)$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ at t, $\prod_{j=0}^{0} \equiv 1$ and b_1 is the first row of b.

Two layers of intertemporal stabilization tradeoffs

$$
\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i}
$$

$$
- \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j} + f_{t+1+j|t+j} \mathbf{g}_{\bar{\pi},\mathbf{t+j}})
$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

Corollary

Optimal policy under adaptive learning is time-consistent.

↪ Foreshadow: optimal policy aggressiveness time-varying

Structure of talk

1. Model of anchoring expectations

2. Solving the Ramsey problem

3. Implementing optimal policy

4. Approximating optimal policy with a Taylor rule

Solution procedure

Solve system of model equations + target criterion

$\hookrightarrow$ solve using parameterized expectations (PEA)

$\hookrightarrow$ obtain a cubic spline approximation to optimal policy function
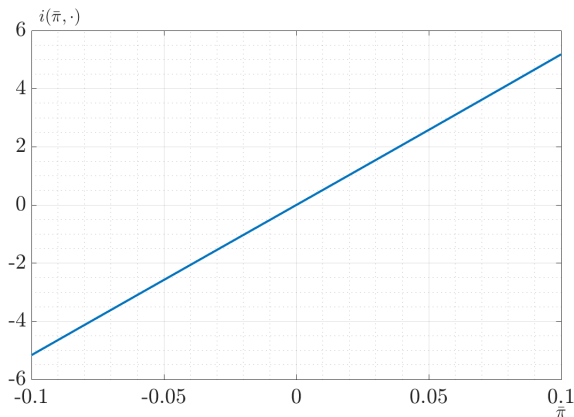
Optimal policy - responding to unanchoring

Figure: Policy function: $i(\bar{\pi}, \text{all other states at their means})$

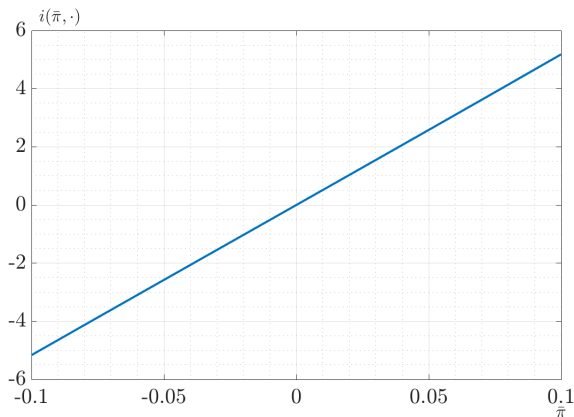# Optimal policy - responding to unanchoring

Figure: Policy function: $i(\bar{\pi}, \text{all other states at their means})$

$\rightarrow$ For 5 bp drop in $\bar{\pi}$, lower i by 2.5 pp

Unanchoring causes volatility

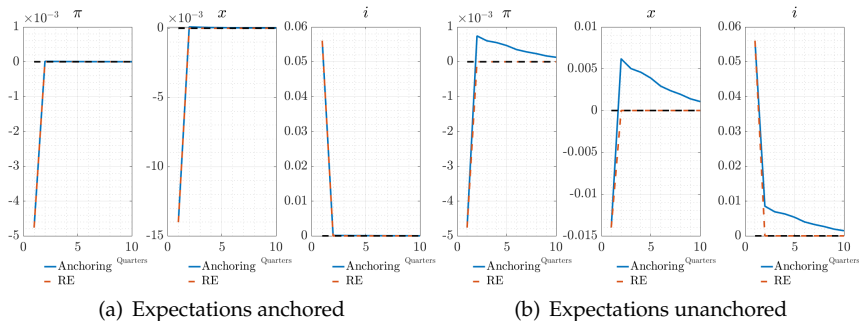(a) Expectations anchored   (b) Expectations unanchored

Figure: Impulse responses after a contractionary monetary policy shock when policy follows a Taylor rule

Why so volatile? Term structure of expectations

When very unanchored, gain is large:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1} \tag{16}$$

$\rightarrow \bar{\pi}$ moves a lot

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$$\hat{\mathbb{E}}_t \pi_{t+1} = \bar{\pi}_{t-1} + b_1^{RE} s_t$$

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Positive feedback: IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big)$$

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Why oscillatory? Intertemporal anticipation effects

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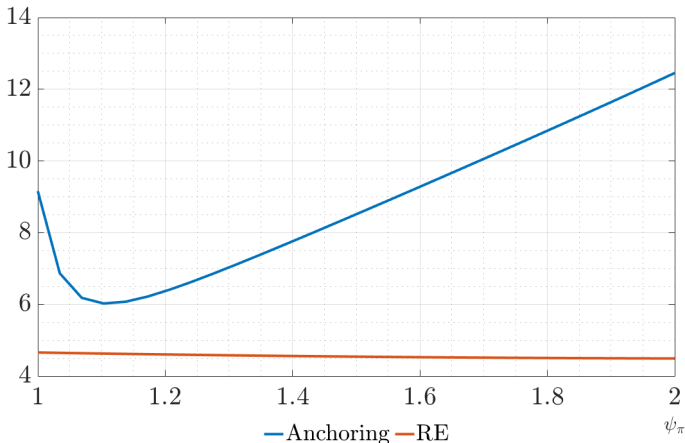
- Additional channel of policy

- Only if policy reaction function internalized

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Optimal Taylor-coefficient on inflation

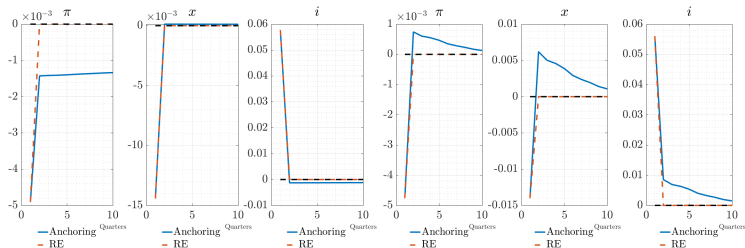Figure: Central bank loss as a function of ψ_π

Anchoring-optimal coefficient: $\psi_\pi^A = 1.09$ RE-optimal coefficient: $\psi_\pi^{RE} = 2.21$
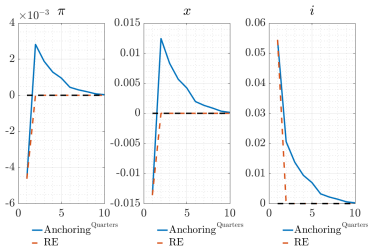
Respond but not too much

(a) $\psi_\pi = 1.01$

(b) $\psi_\pi = 1.5$

(c) $\psi_\pi = 2$

Losses for optimal Taylor-rule coefficient on inflation

Table: Loss for RE and anchoring models for choice of RE- or anchoring-optimal ψ_π

Anchoring, ψ_π^{RE}	Anchoring, ψ_π^{A}	RE, ψ_π^{RE}
14.2633	6.0228	4.4866

Losses for optimal Taylor-rule coefficient on inflation

Table: Loss for RE and anchoring models for choice of RE- or anchoring-optimal ψ_π

Anchoring, ψ_π^{RE}	Anchoring, ψ_π^A	RE, ψ_π^{RE}
14.2633	6.0228	4.4866

Anchoring-optimal ψ_π^A gets 84% of the distance to RE-optimal ψ_π^{RE} under RE

Conclusion

- First theory of monetary policy for potentially unanchored expectations

- Optimal policy frontloads aggressive interest rate response to suppress potential unanchoring

- Matters: already anchoring-optimal Taylor rule reduces losses by 84%

- Future work
  - ↪ Better approximation to optimal policy than Taylor rule?

  - ↪ How to anchor at zero-lower bound?

Appendix

Breakeven inflation

Figure: Market-based inflation expectations, various horizons, %
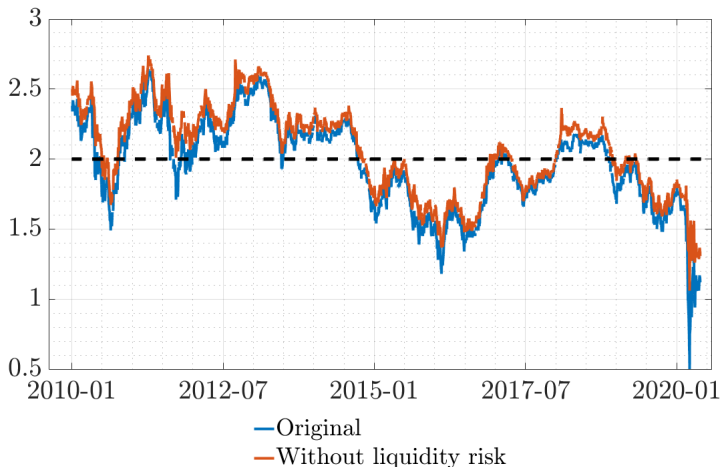
Correcting the TIPS from liquidity risk

Figure: Market-based inflation expectations, 10 year, %

Further evidence

Figure: Livingston Survey of Firms:
Interquartile range of 10-year ahead inflation expectations
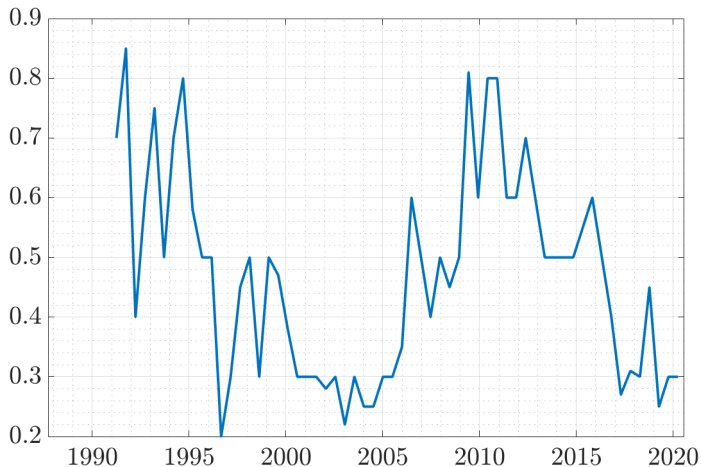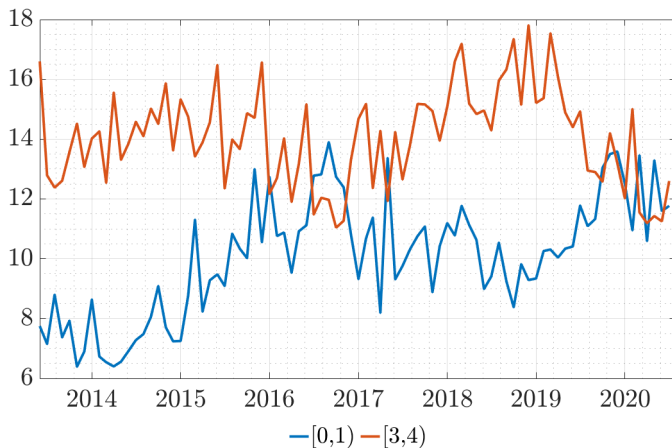

◂ Return

Figure: New York Fed Survey of Consumers:
Percent of respondents indicating 3-year ahead inflation will be in a particular range

Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{20}$$
$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{21}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \tag{22}$$

Solve for forecast error $f_t \equiv \pi_t - f_{t-1}$:

$$f_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k \to 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \tag{23}$$

Functional forms for **g** in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \tag{24}$$

$p \equiv Prob(\text{liquidity trap regime})$
y_{t-1} output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{25}$$

θ_t criterion, $\bar{\theta}$ threshold value

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \tag{26}$$

  Σ variance-covariance matrix of shocks
  $T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(f_{t|t-1}f'_{t|t-1} - \omega_{t-1}) \tag{27}$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(f'_{t|t-1}\omega_t^{-1}f_{t|t-1} - \theta_{t-1}) \tag{28}$$

  ω_t estimated forecast-error variance

Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right)' \right)' \tag{29}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{30}$$

Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{31}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{32}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \tag{33}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{34}$$

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \bigg\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t)$$

$$+ \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \bigg\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa}x_t - \varphi_{2,t}(k_t + \mathbf{g}_\pi(\pi_t - f_{t-1})) = 0 \tag{35}$$

$$-2\beta\frac{\lambda}{\kappa}x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_\mathbf{f}(\pi_{t+1} - f_t)) = 0 \tag{36}$$

Target criterion system for anchoring function as changes of the gain

$$\varphi_{6,t} = -c f_{t|t-1} x_{t+1} + \left(1 + \frac{f_{t|t-1}}{f_{t+1|t}}(1 - k_{t+1}) - f_{t|t-1} \mathbf{g}_{\bar{\pi},t}\right) \varphi_{6,t+1}$$
$$- \frac{f_{t|t-1}}{f_{t+1|t}}(1 - k_{t+1}) \varphi_{6,t+2} \tag{37}$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa} x_t - \left(\frac{k_t}{f_{t|t-1}} + \mathbf{g}_{\pi,t}\right) \varphi_{6,t} + \frac{k_t}{f_{t|t-1}} \varphi_{6,t+1} \tag{38}$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (39) is given by:

$$\varphi_{6,t} = -2 \, \mathbb{E}_t \sum_{i=0}^{\infty} (\pi_{t+i} + \frac{\lambda_x}{\kappa} x_{t+i}) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{f_{t+j|t+j-1}}}{\frac{k_{t+j}}{f_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \tag{39}$$

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{40}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{41}$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \tag{42}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{43}$$

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{44}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{45}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (45) backward to some date t, take expectations at t
2. Sub in (46)
3. Aggregate over households i
$\rightarrow$ Obtain (19)