## Materials 44 - I think I've got the estimation

### Laura Gáti

#### September 16, 2020

## Overview

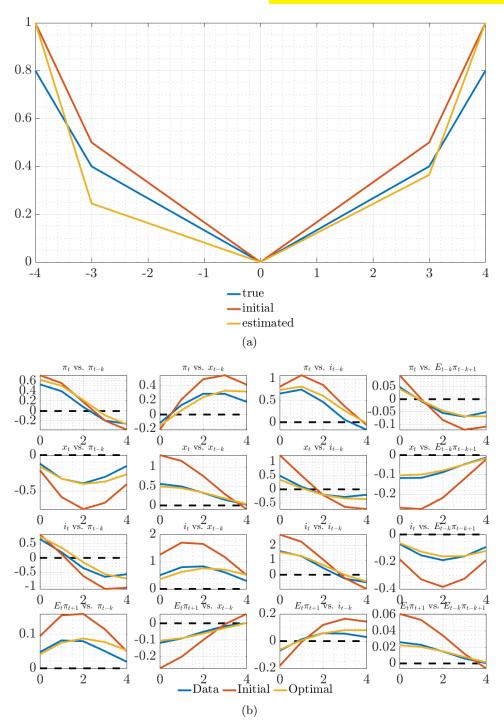
1	Inconsistency was: didn't annualize expectations in true data					
	1.1 Check with same seed as true data and $N=1$	4				
2	Real data	5				
3	Policy isn't a function of $k_t^{-1}$	11				
A	Model summary	12				
В	Target criterion	12				

## 1 Inconsistency was: didn't annualize expectations in true data

0.8 0.70.6 0.50.40.30.2 0.10 -4 -3 -2 -1 0 1 2 3  $--{\rm true}$ —initial —estimated (a)  $\pi_t$  vs.  $x_{t-k}$  $\pi_t$  vs.  $i_{t-k}$ 0.1 1 0.40.5 0.20.5 0 0 0 0 -0.1  $\overline{x_t}$  vs.  $\pi_{t-k}$  $\underset{x_{t} \text{ vs. } x_{t-k}}{2}$  $x_t \text{ vs. } i_{t-k}$  $0 \quad \underset{x_t \text{ vs. } E_{t-k}\pi_{t-k+1}}{\underbrace{2}}$ 0 0 1 0 0 0.5-0.2 0.5 -0.1 -0.4 0 -0.5 -0.6 -0.2 0  $\sum_{i_t \text{ vs.}}^2 \pi_{t-k}$  $\underset{i_t \text{ vs.}}{2} x_{t-k}$  $i_t \text{ vs. } i_{t-k}$ 4 4 0 0  $i_t \text{ vs. } E_{t-k}^2 \pi_{t-k}$ 0 1 -0.1 0 0.50 -0.2 0 -1  $E_t \pi_{t+1}$  vs.  $\pi_{t-k}$  $E_t \pi_{t+1}$  vs.  $x_{t-k}$  $E_t \pi_{t+1}$ vs.  $i_{t-k}$ 0  $0_{E_t\pi_{t+1}} \text{ vs. } 2_{E_{t-k}\pi_{t-k+1}} 4$ 0.10  $0.03 \\ 0.02 \\ 0.01$ 0.1-0.1 0 0.05-0.2 -0.1 Data  $\begin{array}{c}
4 & 0 \\
\text{Initial} & 0 \\
\end{array} \text{Optimal}$ 0 2 2 4 0 (b)

Figure 1: Calibration C, use expectations, ridge tuning = 0.01, initialize at truth

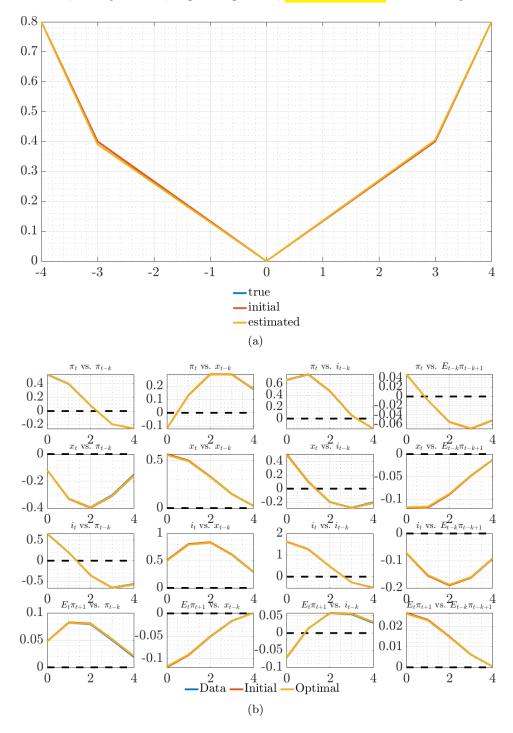
Figure 2: Calibration C, use expectations, ridge tuning = 0.01, initialize above truth, annualize expectations in true data



I think that's success!

#### 1.1 Check with same seed as true data and N=1

Figure 3: Calibration C, use expectations, ridge tuning = 0.01, initialize at truth, annualize expectations in true data



Yes!

## 2 Real data

Figure 4: Settings from Fig 2,  $\frac{10}{10}$  different starting points, showing top 3 (blue is best), N = 1000.2 0.150.1 0.05 0 -2 -4 0 2 4 Forecast error (a)  $\pi_t$  vs.  $\pi_{t-k}$  $\pi_t$  vs.  $x_{t-k}$  $\pi_t$  vs.  $i_{t-k}$  $\pi_t$  vs.  $E_{t-k}\pi_{t-k+1}$ 0.5 0.4 0.2 0.50.1 0.2 0 0 0  $x_t \text{ vs. } \pi_{t-k}$  $\underset{x_{t} \text{ vs. } x_{t-k}}{2}$  $x_t$  vs.  $i_{t-k}$ vs.  $\sum_{t-k}^{2} \pi_{t-k+1}$ 0 4 0.4 0.4 0.1 0.5 0.2 0.2 0.05 0 0 0  $i_t \operatorname{vs.}^2 \pi_{t-k}$  $\sum_{i_t \text{ vs.} x_{t-k}} 2$  $\sum_{i_t \text{ vs. } i_{t-k}}^{2}$  $i_t \text{ vs. } E_{t-k}^2 \pi_{t-k+1}$ 4 0.4 0.40.60.1 0.4 0.2 0 0.2 0.20.050  $E_t \pi_{t+1}$   $\frac{1}{2}$  vs.  $x_{t-k}$  $0_{E_{t}\pi_{t+1} \text{ vs.} 2_{E_{t-k}\pi_{t-k+1}}} 4$  $E_t \pi_{t+1}$  vs.  $\pi_{t-k}$  $E_t \pi_{t+1}$ Vs.  $i_{t-k}$ 4 4 0.2 0.060.10.1 0.04 0.10.050.050.020 0 0 0 2 2 2 0 4 2 -Data -Best 4 (b)

Darn. It doesn't hit the moments I want it to. So let's manually re-weight the own-autocovariances.

Figure 5: Settings from Fig 2, 5 different starting points, showing top 3 (blue is best), N = 100, manually putting more weight on own autocovariances

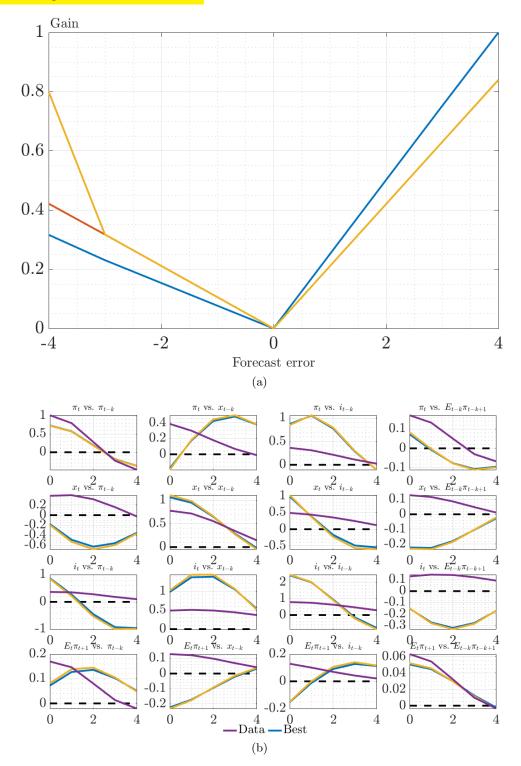
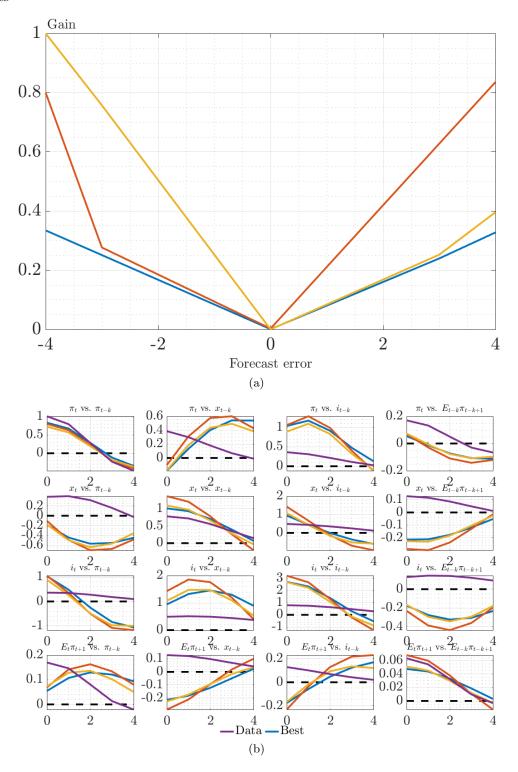


Figure 6: Settings from Fig 2, 5 different starting points, showing top 3 (blue is best), N = 500, manually putting more weight on own autocovariances



This took 5h:12min.

 $\hat{\alpha}_i = (0.3346; 0.2513; 0.001; 0.2399; 0.3277)$ 

Figure 7: Same as the previous, just the best candidate alone

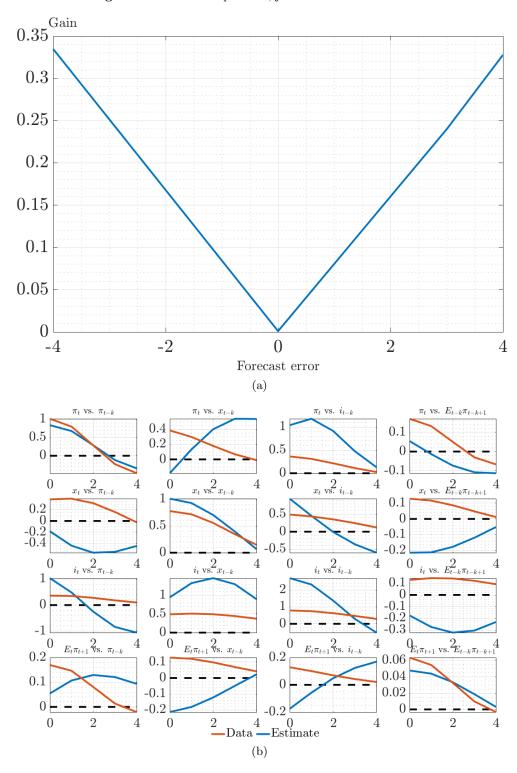
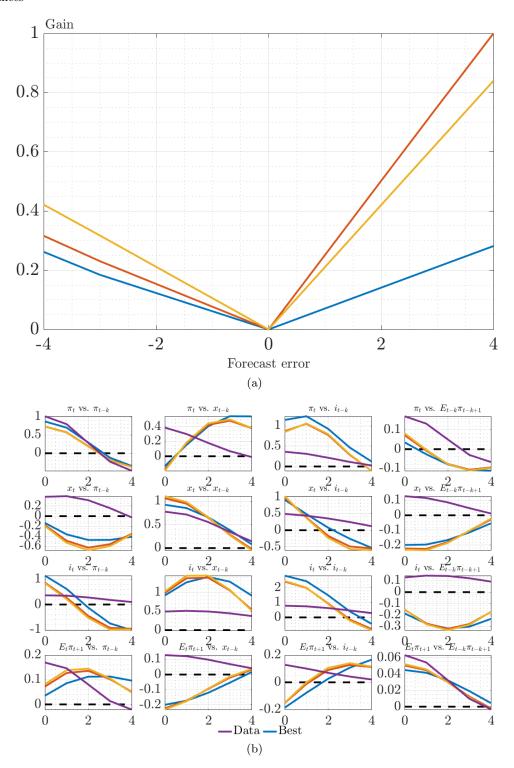
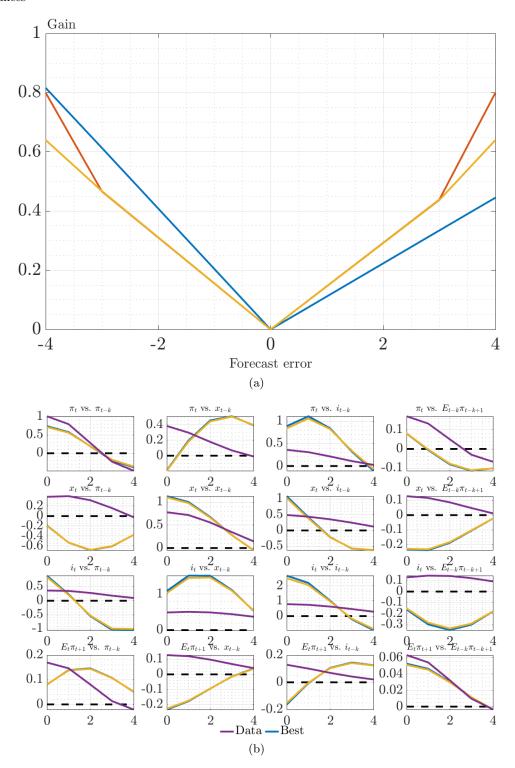


Figure 8: Settings from Fig 2,  $\frac{20}{100}$  different starting points, showing top 3 (blue is best),  $\frac{N}{N} = 100$ , manually putting more weight on own autocovariances



 $\hat{\alpha}_i = (0.2621; 0.1847; 0; 0.2115; 0.2817)$ 

Figure 9: Settings from Fig 2, 10 different starting points, showing top 3 (blue is best), N = 1000, manually putting more weight on own autocovariances



 $\hat{\alpha}_i = (0.8161; 0.61330; 0.3342; 0.4452)$ 

(Saved as estim\_LOMgain\_outputs\_univariate\_coax15\_Sep\_2020\_16\_14\_00.mat)

# 3 Policy isn't a function of $k_t^{-1}$

The anchoring function is (A.6):  $k_t^{-1} = \sum_i \alpha_i b_i (f e_{t|t-1})$ . This essentially eliminates k as a state variable.

#### A Model summary

$$x_{t} = -\sigma i_{t} + \hat{\mathbb{E}}_{t} \sum_{T=t}^{\infty} \beta^{T-t} \left( (1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_{T}^{n} \right)$$
(A.1)

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \left( \kappa \alpha \beta x_{T+1} + (1-\alpha) \beta \pi_{T+1} + u_T \right)$$
(A.2)

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$
 (if imposed) (A.3)

PLM: 
$$\hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \ge 1 \qquad b = g_x h_x$$
 (A.4)

Updating: 
$$a_t = a_{t-1} + k_t^{-1} (z_t - (a_{t-1} + bs_{t-1}))$$
 (A.5)

Anchoring function: 
$$k_t^{-1} = \sum_i \alpha_i b_i (f e_{t|t-1})$$
 (A.6)

Forecast error: 
$$fe_{t-1} = z_t - (a_{t-1} + bs_{t-1})$$
 (A.7)

LH expectations: 
$$f_a(t) = \frac{1}{1 - \alpha \beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha \beta h)^{-1} s_t$$
  $f_b(t) = \frac{1}{1 - \beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t$  (A.8)

This notation captures vector learning (z learned) for intercept only. For scalar learning,  $a_t = \begin{pmatrix} \bar{\pi}_t & 0 & 0 \end{pmatrix}'$  and  $b_1$  designates the first row of b. The observables  $(\pi, x)$  are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix} \sigma & 1 - \beta & -\sigma \beta \end{bmatrix} f_b + \sigma \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t$$
(A.9)

$$\pi_t = \kappa x_t + \begin{bmatrix} (1 - \alpha)\beta & \kappa \alpha \beta & 0 \end{bmatrix} f_a + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (\mathbb{I}_{nx} - \alpha \beta h_x)^{-1} s_t$$
 (A.10)

#### B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only,  $k_t^{-1} = \mathbf{g}(fe_{t-1})$ ):

$$\pi_{t} = -\frac{\lambda_{x}}{\kappa} \left\{ x_{t} - \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_{t}^{-1} + ((\pi_{t} - \bar{\pi}_{t-1} - b_{1}s_{t-1})) \mathbf{g}_{\pi}(t) \right) \right\}$$

$$\left( \mathbb{E}_{t} \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_{1}s_{t+j}) \mathbf{g}_{\bar{\pi}}(t+j)) \right) \right\}$$
(B.1)

where I'm using the notation that  $\prod_{j=0}^{0} \equiv 1$ . For interpretation purposes, let me rewrite this as follows:

$$\pi_{t} = -\frac{\lambda_{x}}{\kappa} x_{t} + \frac{\lambda_{x}}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_{t}^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_{\pi}(t) \right) \mathbb{E}_{t} \sum_{i=1}^{\infty} x_{t+i}$$

$$-\frac{\lambda_{x}}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_{t}^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_{\pi}(t) \right) \left( \mathbb{E}_{t} \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j}^{-1} + f e_{t+1+j|t+j}^{eve}) \mathbf{g}_{\pi}(t+j) \right)$$
(B.2)

Interpretation: tradeoffs from discretion in RE + effect of current level and change of the gain on future tradeoffs + effect of future expected levels and changes of the gain on future tradeoffs