Finally back to work! To do <u>7 Oct 2019</u>

- generate IRFs from all models } Susanto
- think about about slope, b

✓- change κ (and α) and see if π fluctuates as much as x then } Peter

✗ - $\hat{E}$-operator

- θ_t as SR FE

- small π vs. larger x & i → contrast to rest of Lit to see what mechanism is responsible } Ryan

✓- think about whether ppl know the TR?

✓- read Thomas Lubik: Indeterminacy & Learning (JME)

✓- derive Preston's IS curve → me.

Lubik: has a comment that b/c of anticipated utility (Kreps 1998, Cogley & Sargent 2008), their model can be solved using standard, RE algorithms

→ in Lubik, the CB learns the econ and announces

$i = \psi_\pi \pi_t + \psi_x x_t$ w/ (ψ_π, ψ_x) each period, updated according to its estimates. Depending on $(\psi_{x,t}, \psi_{\pi,t})$ we can land in the determinacy or indeterminacy region. Ryan is right: this amounts to ppl not knowing the TR. And he's doubly right b/c in my setting, they know the Taylor-rule, the only thing they don't know are future things.

Let's first try to derive the IS curve in Preston <u>8 Oct 2015</u>

HH: $\max \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[u(C_T^i; \xi_T) - \int_0^1 v(h_T^i(j), \xi_T)\, dj \right]$ (1)

$M_t^i + B_t^i \leq (1+i_{t-1}^m) M_{t-1}^i + (1+i_{t-1}) B_{t-1}^i + P_t Y_t^i - T_t - P_t C_t^i$ (2)

$$P_t Y_t^i = \int_0^1 \left[W_t(j) h_t^i(j) + \Pi_t(j) \right] dj$$

So the HH chooses $\{C_t^i(j), h_t^i(j), M_t^i, B_t^i\}$ $\forall j \in [0,1]$ so as to $\max (1)$ s.t. (2), taking as given $\{p_T(j), w_T(j), \Pi_T, i_{T-1}, i_{T-1}^m, \xi_T\}$ $\forall T \geq t$.

In the App, Preston defines $W_{t+1}^i = (1+i_t^m) M_t^i + (1+i_t) B_t^i$ as beginning-of-period wealth at time t+1

$\Rightarrow$ (2) $M_t^i + B_t^i \leq W_t^i + P_t Y_t^i - T_t - P_t C_t^i$

Now let $\Delta_t \equiv \dfrac{i_t - i_t^m}{1+i_t}$ recall: $W_{t+1}^i = (1+i_t^m) M_t^i + (1+i_t) B_t^i$

$$\Leftrightarrow \quad P_t C_t^i + \underbrace{M_t^i + B_t^i}_{(1+i_t^m - i_t^m) M_t^i + \frac{1+i_t}{1+i_t} B_t^i} \leq W_t^i + \left[P_t Y_t^i - T_t\right]$$

$$= -i_t^m M_t^i + (1+i_t^m) M_t^i + \frac{(1+i_t)}{1+i_t} B_t^i$$

$$= -i_t^m M_t^i + \frac{(1+i_t)(1+i_t^m) M_t^i}{1+i_t} + \frac{(1+i_t)}{1+i_t} B_t^i$$

$$= \underbrace{-i_t^m M_t^i + \frac{i_t (1+i_t^m)}{1+i_t} M_t^i}_{= \frac{(1+i_t)(-i_t^m) M_t^i + i_t (1+i_t^m) M_t^i}{1+i_t}} + \underbrace{\frac{(1+i_t^m) M_t^i + (1+i_t) B_t^i}{1+i_t}}_{= \frac{1}{1+i_t} W_{t+1}}$$

$$= \frac{(-i_t^m - i_t i_t^m + i_t + i_t i_t^m) M_t^i}{1+i_t} = \frac{i_t - i_t^m}{1+i_t} M_t^i = \Delta_t M_t^i$$

So $\Rightarrow$ (2) $P_t C_t^i + \Delta_t M_t^i + \dfrac{1}{1+i_t} W_{t+1}^i \leq W_t^i + \left[P_t Y_t^i - T_t\right]$ (32)

Now solve this fwd.

$$W_t^i \geq P_t C_t^i + \Delta_t M_t^i - (P_t Y_t^i - T_t) + \frac{1}{1+i_t} W_{t+1}^i$$

$$\to W_t^i \geq P_t C_t^i + \Delta_t M_t^i - (P_t Y_t^i - T_t)$$

$$+ \left(\frac{1}{1+i_t}\right)\left[P_{t+1} C_{t+1}^i + \Delta_{t+1} M_{t+1}^i - (P_{t+1} Y_{t+1}^i - T_{t+1})\right]$$

$$+ \dots$$

$$+ \underbrace{\left(\frac{1}{1+i_t}\right) \cdot \ldots \cdot \left(\frac{1}{1+i_{t+j-1}}\right)}_{=: R_{t,t+j} = \prod_{s=1}^{j} \left(\frac{1}{1+i_{t+s-1}}\right)} \left[P_{t+j} C_{t+j}^i + \Delta_{t+j} M_{t+j}^i - (P_{t+j} Y_{t+j}^i - T_{t+j})\right]$$

$$W_t^i \geq \sum_{j=0}^{\infty} R_{t,t+j} \left[P_{t+j} C_{t+j}^i + \Delta_{t+j} M_{t+j}^i - (P_{t+j} Y_{t+j}^i - T_{t+j})\right]$$

The flow BC solved fwd to get the intertemporal BC, the IBC.

Let's now take the FOCs using the new flow BC, 32

$$\mathcal{L} = \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[u(C_T^i; \xi_T) - \int_0^1 v(h_T^i(j); \xi_T)\, dj\right]$$

$$+ \lambda_t \left(- P_t C_t^i - \Delta_t M_t^i - \frac{1}{1+i_t} W_{t+1}^i + W_t^i + [P_t Y_t^i - T_t]\right)$$

$w_t^j h_t^j$

c 1) $u_c = \lambda_t P_t \to \lambda_t = \frac{u_c}{P_t}$

h^j 2) $v_{h_j} = \lambda_t P_t w_t^j \to \lambda_t = \frac{v_{h_j}}{P_t w_t}$

$$\frac{u_c}{P_t} = \frac{v_h}{P_t w_t} \Rightarrow \frac{v_h}{u_c} = w_t$$ real wage

B (3) $\lambda_t (1+i_t) = \beta \hat{E}_t \lambda_{t+1} \to 1+i_t = \beta \hat{E}_t \frac{\lambda_{t+1}}{\lambda_t} = \beta \hat{E}_t \frac{P_t}{P_{t+1}} \frac{u' c_{t+1}}{u' c_t}$

ok so I *almost* get what Preston gets: (difference)

$$1 + i_t = \beta E_t \frac{P_t}{P_{t+1}} \frac{u_c(c_{t+1})}{u_c(c_t)} \qquad (33)$$

$$\frac{v'_n}{u'_c} = w_t \qquad (34)$$

- A cashless econ implies $i_t = i_t^m$ or $M_t^i = 0$

  since $M^S > 0$, we get $i_t = i_t^m \Rightarrow \Delta_t = 0 \quad \forall t$

- Market clearing implies $y_t(j) = c_t(j) \quad \forall j \Rightarrow C_t = Y_t$

- Zero debt fiscal policy: $\underbrace{B_t = 0 \Rightarrow T_t = (1 + i_{t-1})M_{t-1} - M_t}_{\text{ain't so sure if this is right!}}$

  ↳ thus the IBC becomes

$$W_t^i = \sum_{j=0}^{\infty} R_{t,t+j} \left[P_{t+j} C_{t+j}^i \qquad - (P_{t+j} Y_{t+j}^i - T_{t+j}) \right]$$

$$W_t^i = \sum_{j=0}^{\infty} R_{t,t+j} \left[P_{t+j} C_{t+j}^i - P_{t+j} Y_{t+j}^i - (1 + i_{t+j-1}) M_{t+j-1} + M_{t+j} \right]$$

Maybe the point is that there is no diff btwn bonds & money,

so $W_{t+1}^i = (1+i_t)M_t + (1+i_t)B_t = (1+i_t)\left[M_t + B_t \right]$

so we can ass that ppl only hold bonds, $M_t = 0$

$\rightarrow W_{t+1}^i = (1+i_t)B_t \quad \rightarrow T_t = 0$ (a little non-kosher but ok)

$$W_t^i \geq \sum_{j=0}^{\infty} R_{t,t+j} \left[P_{t+j} C_{t+j}^i - P_{t+j} Y_{t+j}^i \right]$$

Funnily, the App. stops here.

But it is clear that w/ a specific U-fct ass, loglin of the EE (33) gives rise to (3). But does loglin of the IBC $W_t^i \geq \sum_{j=0}^{\infty} R_{t,t+j} \left[P_{t+j} C_{t+j}^i - P_{t+j} Y_{t+j}^i \right]$ give rise to (4), $\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i = \bar{\omega}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T^i$?

What's the loglin of $R_{t,t+j} = \prod_{s=1}^{j} \left(\frac{1}{1+i_{t+s-1}} \right)$?

$$= \sum_{s=1}^{j} \ln\left(\frac{1}{1+i_{t+s-1}} \right) = \sum_{s=1}^{j} -\ln(1+i_{t+s-1})$$

Take the gross interest rate $1+i_t$ as a unit, call it R_t

$$\Rightarrow \sum_{s=1}^{j} -\ln(R_{t+s-1}) \quad \text{Total diff} = \sum_{s=1}^{j} -\frac{dR_{t+s-1}}{R} = -\sum_{s=1}^{j} \hat{R}_{t+s-1}$$

where $R = \beta^{-1}$ so $\bar{i} = i^* = \beta^{-1} - 1$

Supp I write the IBC as

$$W_t^i \geq \sum_{j=0}^{\infty} R_{t,t+j} P_{t+j} \left[C_{t+j}^i - Y_{t+j}^i \right] \quad \Big| : P_t \bar{Y} \qquad \bar{Y} = Y^*$$

$$\bar{\omega}_t^i = \sum_{j=0}^{\infty} R_{t,t+j} \frac{P_{t+j}}{P_t} \left[\frac{C_{t+j}^i}{\bar{Y}} - \frac{Y_{t+j}^i}{\bar{Y}} \right]$$

$$\bar{\omega}_t^i = \sum_{j=0}^{\infty} R_{t,t+j} \Pi_{t,t+j} \left[\hat{C}_{t+j}^i - \hat{Y}_{t+j}^i \right] \quad \text{To get eq (4),}$$

we need the loglin of $R_{t,t+j}\,\Pi_{t,t+j}$ to be β^j

Honestly, I don't think there's a way. Or?

$R_{t,t+j}\,\Pi_{t,t+j}$ is the sdf. It's just like the firms' sdf in Calvo, $\phi_{t+\tau,t} = \beta\dfrac{P_t C_t}{P_{t+\tau}C_{t+\tau}}$, so then, analogously to there, it cancels and we're left w/ β every time.

$\rightarrow$ yes, you can see it from eq (33).

$\Rightarrow$ ok so eq. (4) is good!

The next step is solving (3) backwards.

(3): $\hat{c}_t^i = \hat{E}_t^i \hat{c}_{t+1}^i - \sigma(\hat{i}_t - \hat{E}_t^i \hat{\pi}_{t+1}) + g_t - \hat{E}_t^i g_{t+1}$

$$\hat{E}_t^i \hat{c}_{t+1}^i = \hat{c}_t^i - g_t + \hat{E}_t^i g_{t+1} + \sigma(\hat{i}_t - \hat{E}_t^i \pi_{t+1})$$

$$\hat{E}_t^i \hat{c}_{t+1}^i - \hat{E}_t^i g_{t+1} = \hat{c}_t^i - g_t + \sigma(\hat{i}_t - \hat{E}_t^i \pi_{t+1})$$

Let's call $t+1 = T$

$$\hat{E}_t \hat{c}_T^i - \hat{E}_t g_T = \underbrace{\hat{c}_{T-1}^i - g_{T-1}} + \sigma\underbrace{(\hat{i}_{T-1} - \hat{E}_t^i \pi_T)}$$

$$= \hat{c}_{T-2}^i - g_{T-2} + \sigma(\hat{i}_{T-2} - \hat{E}_t^i \pi_{T-1}) + \cdots$$

$$= \ldots \hat{c}_t^i - g_t + \sigma\sum_{s=t}^{T-1}(\hat{i}_s - \hat{\pi}_{s+1}) \quad \checkmark$$

$$\hat{E}^i_t \hat{C}^i_T = \hat{E}^i_t g_T + \hat{c}^i_t - g_t + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1})$$

$$= \hat{c}^i_t - g_t + \hat{E}_t \left[g_T + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \right]$$

So now sub $\hat{E}^i_t \hat{C}^i_T$ (I guess) into the IBC, eq (4)

$$\hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}^i_T = \tilde{\omega}^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}^i_T$$

$$\Rightarrow \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \hat{c}^i_t - g_t + \hat{E}_t \left[g_T + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \right] \right\}$$

$$= \tilde{\omega}^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}^i_T$$

$$\Leftrightarrow \hat{c}^i_t - g_t = \tilde{\omega}^i_t + \hat{E}^i_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}^i_T - g_T - \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \right]$$

Conjecture: $\sum_{T=t}^{\infty} \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) = \sum_{T=t}^{\infty} (\hat{i}_T - \hat{\pi}_{T+1})$.

Why? B/c $\sum_{T=t}^{\infty} \sum_{s=t}^{T-1} \text{stuff}_s = \sum_{s=t}^{t-1} \text{stuff}_s + \sum_{s=t}^{t} \text{stuff}_s$

So what if I don't sub the (3) solved fwd into (4), but just (3)?

(3) $\hat{c}_t^i = \hat{E}_t^i \hat{c}_{t+1}^i - \sigma(\hat{i}_t - \hat{E}_t^i \hat{\pi}_{t+1}) + g_t - \hat{E}_t^i g_{t+1}$

(4) $\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{c}_T^i = \tilde{\omega}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T^i$

$\rightarrow \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{c}_{T+1}^i - \sigma(\hat{i}_T - \hat{\pi}_{T+1}) + g_T - g_{T+1} \right] = \tilde{\omega}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T^i$

that doesn't seem to do it at all!

Let's pause the issue of eq (5). Supp. we have it.

- Then integrate over i $\rightarrow \tilde{\omega}_t \Rightarrow 0$

$c_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\hat{Y}_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta(g_T - g_{T+1}) \right]$

- $\hat{Y}_t = \hat{c}_t$

$\hat{Y}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\hat{Y}_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta(g_T - g_{T+1}) \right]$

- $x_t := \hat{Y}_t - \hat{Y}_t^n$

$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta(g_T - g_{T+1}) - (1-\beta)\hat{Y}_T^n \right]$

$- \hat{Y}_t^n$ even that doesn't work out! damn!

Ryan meeting

8 Oct 2019

- Invite Eusepi / Preston for macro seminar? Who organizes it?
  - ↳ Ryan, Susanto & Pablo for the spring, Rosen, Fabio & Jaromir for fall

Ryan said that Susanto connects b (slope) w/ Missing Deflation b/c b transmits shocks to SR fcsts

Work after:

8 Oct 2019

So supp my previous conjecture is right. Then

$$\hat{C}_t^i - g_t = \tilde{\omega}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}_T^i - g_T + \sigma(\hat{i}_T - \hat{\pi}_{T+1}) \right]$$

and I still don't get (5).

But if I now do the last 3 steps:

$$\underbrace{\hat{Y}_t - \hat{Y}_t^n}_{} \underbrace{- g_t + \hat{Y}_t^n}_{} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}_T - \hat{Y}_T^n - g_T + \hat{Y}_T^n + \sigma(\hat{i}_T - \hat{\pi}_{T+1}) \right]$$

$$\Rightarrow \quad x_t + r_t^n = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[x_t \right.$$

↳ no, the problem is that that ain't right b/c

$$r_t^n = Y_{t+1}^n - Y_t^n \left[+ (g_t - g_{t+1}) \text{ if we have this preference shock} \right]$$

↳ and for me, $r_t^n = \frac{1}{\sigma}(Y_{t+1}^n - Y_t^n)$, in line w/ Basu, SUM part 2, p.58)

So let's set all $g_t = 0 \ \forall t$ b/c I don't have it.

→ Then the EE is $\hat{C}_t^i = \hat{E}_t^i \hat{C}_{t+1} - \sigma(\hat{i}_t - \hat{E}_t^i \hat{\pi}_{t+1})$

Solve backwd:

$$\hat{E}_t C_{t+1}^i = \hat{C}_t^i + \sigma(\hat{i}_t - \hat{E}_t^i \pi_{t+1})$$

$$\hat{E}_t C_T^i = \hat{C}_{T-1}^i + \sigma(\hat{i}_{T-1} - \hat{E}_t^i \pi_T)$$

$$\hat{E}_t^i \hat{C}_T^i = \sigma(\hat{i}_{T-1} - \hat{E}_t^i \pi_T) + \sigma(\hat{i}_{T-2} - \hat{E}_t^i \pi_{T-1}) + \dots$$
$$+ \sigma(\hat{i}_t - \hat{E}_t \pi_{t+1}) + \hat{c}_t^i$$

$$\hat{E}_t^i \hat{C}_T^i = \hat{c}_t^i + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{E}_t \pi_{s+1}) \quad \left(EE^{\text{simple}}_{\text{solved bwd}}\right)$$

Sub this into (4)

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{C}_t^i + \sigma \sum_{s=t}^{T-1} (\hat{i}_s - \hat{E}_t \pi_{s+1}) \right] = \tilde{\omega}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T^i$$

$$\Leftrightarrow \hat{c}_t^i \sum_{T=t}^{\infty} \beta^{T-t} + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \sigma \sum_{s=0}^{T-1} (\hat{i}_s - \hat{E}_t \pi_{s+1})] = \tilde{\omega}_t^i + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_t^i$$

$$\Leftrightarrow \underset{\checkmark}{\hat{c}_t^i} + \underbrace{(1-\beta) \sigma \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \sum_{s=t}^{T-1} (\hat{i}_s - \hat{E}_t \pi_{s+1})} = \underset{\checkmark}{(1-\beta) \tilde{\omega}_t^i} + \underset{\checkmark}{\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} (1-\beta) \hat{Y}_T^i}$$

now I only need this to equal $\beta\sigma(\hat{i}_T - \hat{\pi}_{T+1})$

But potentially, it should actually be $(1-\beta)(\sigma?)$

Now I'm only missing the $\hat{i}_T - \hat{\pi}_{T+1}$ term <u>9 Oct 2019</u>

in eq (5). So supp. again that I have eq (5), w/o g_s,

and I do the last 3 steps:

· aggregate, set $C_t = Y_t$ and am about to get $x_t = \hat{Y}_t - \hat{Y}_t^n$

$$\hat{Y}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) \hat{Y}_T - \beta\sigma(\hat{i}_T - \pi_{T+1}) \right]$$

$$\hat{Y}_t - \hat{Y}_t^n = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)\hat{Y}_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1})\right] - \hat{Y}_t^n$$

(*) $$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)[\hat{Y}_T - \hat{Y}_T^n] - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + (1-\beta)\hat{Y}_T^n\right] - \hat{Y}_t^n$$

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)(\hat{Y}_{T+1} - \hat{Y}_{T+1}^n) - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + (1-\beta)\hat{Y}_{T+1}^n\right]$$
$$- \hat{Y}_t^n + (1-\beta)\hat{Y}_t$$

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)x_{T+1} - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1})\right]$$
$$+ \underline{x_t - \beta\hat{Y}_t + (1-\beta)\hat{Y}_{t+1}^n + (1-\beta)\beta\hat{Y}_{t+2}^n + (1-\beta)\beta^2\hat{Y}_{t+3}^n} + \dots$$

$$\underbrace{x_t - \beta\hat{Y}_t + Y_{t+1}^n}\ \underbrace{-\beta\hat{Y}_{t+1}^n + \beta\hat{Y}_{t+2}^n}\ \underbrace{-\beta^2\hat{Y}_{t+2}^n + \beta^2\hat{Y}_{t+3}^n}\ \underbrace{-\beta^3\hat{Y}_{t+3}^n} + \dots$$

$$\beta r_{t+1}^n + \beta^2 r_{t+2}^n + \dots$$ cool

$$\hat{Y}_t - \hat{Y}_t^n - \beta\hat{Y}_t + \hat{Y}_{t+1}^n$$
$$= \hat{Y}_t - \beta\hat{Y}_t + r_t^n$$
$$= (1-\beta)\hat{Y}_t + r_t^n$$

grrr.. let's retry, from step (*)

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)[\hat{Y}_T - \hat{Y}_T^n] - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + (1-\beta)\hat{Y}_T^n\right] - \hat{Y}_t^n$$
$$x_t = \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)x_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1})\right]$$
$$- \hat{Y}_t^n + (1-\beta)\hat{Y}_t^n + (1-\beta)\beta\hat{Y}_{t+1}^n + (1-\beta)\beta^2\hat{Y}_{t+2}^n + \dots$$

The last series is $-\hat{Y}_t^n + (1-\beta)\hat{Y}_t^n + (1-\beta)\beta\hat{Y}_{t+1}^n + (1-\beta)\beta^2\hat{Y}_{t+2}^n + \dots$

$$= \underbrace{-\beta\hat{Y}_t^n + \beta\hat{Y}_{t+1}^n}\ \underbrace{-\beta^2\hat{Y}_{t+1}^n + \beta^2\hat{Y}_{t+2}^n}\ -\beta^3\hat{Y}_{t+2}^n + \dots$$

$$\beta r_t^n + \beta^2 r_{t+1}^n + \dots$$

$$= \beta\left[r_t^n + \beta r_{t+1}^n + \beta^2 r_{t+2}^n + \dots\right]$$

$$= \beta\sum_{T=0}^{\infty}\beta^{T-t} r_T^n$$

now this guy is superfluous ...

and we have x_T, not x_{T+1} in the Σ, i.e.

$$x_t = \sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta)x_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta r_T^n\right] \quad \Big| -x_t$$

$$\beta x_t = \sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta)x_{T+1} - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta r_T^n\right]$$

$$x_t = \sum_{T=t}^{\infty}\beta^{T-t}\left[\frac{1-\beta}{\beta}x_{T+1} - \sigma(\hat{i}_T - \hat{\pi}_{T+1}) + r_T^n\right]$$

hát ezt nem hiszem el ... mostmár

minden stimmel csak ez nem ...

(Még az $r_T^n = \sigma r_t^n$ is jó, ha átdefiniálom

$r_t^n = \frac{1}{\sigma}(\hat{Y}_{t+1}^n - \hat{Y}_t^n)$ mint Bosen Sem 2 p. 58)

So at the current point in time I'm inclined to say that if eq(5) is right, then eq.(6) is too, except the coefficient of x_{T+1} should be $\frac{1-\beta}{\beta}$, not $1-\beta$.

So we need to turn back to deriving eq (5) from (3).

$$\hat{c}_t^i = \hat{E}_t c_{t+1}^i - \sigma(i_t - \hat{E}_t^i \hat{\pi}_{t+1}) \qquad (3)$$

Drop i's and hats for simplicity:

$$c_t - E_t c_{t+1} = -\sigma(i_t - E_t \pi_{t+1})$$

$$(1 - L^{-1}) c_t = -\sigma(i_t - E_t \pi_{t+1})$$

$$\text{or} \quad -(1 - L) E_t c_{t+1} = -\sigma(i_t - E_t \pi_{t+1})$$

$$\Leftrightarrow \quad (1 - L) E_t c_\tau = \sigma(i_\tau - E_t \pi_{\tau+1})$$

$$E_t c_\tau = (1 - L)^{-1} \sigma(i_\tau - E_t \pi_{\tau+1})$$

See rotemberg_pricing_peter_ _mynotes.pdf.

not trivial

$$= \sigma \sum_{i=0}^{\infty} 1^i \text{ RHS} = \sigma \sum_{T=t}^{\tau-1} (i_T - E_t \pi_{T+1})$$

I think this is pretty much what I got before, and it's also in accordance w/ Preston's (4.5)

Plugging this into the LHS of (4), we have

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\sigma \sum_{s=t}^{T-1} (i_s - \hat{E}_t \pi_{s+1}) \right]$$

$$= \sigma \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \sum_{s=t}^{T-1} (i_s - \hat{E}_t \pi_{s+1})$$

$$= \sum_{s=t}^{t-1} \text{stuff} + \beta \sum_{s=t}^{t} + \beta^2 \sum_{s=t}^{t+1} + \beta^3 \sum_{s=t}^{t+2}$$

That don't look good.

$$= \delta \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \sum_{s=t}^{T-1} (i_s - \hat{E}_t \pi_{s+1})$$

Let's try to redefine the indices

$$\delta \hat{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{s=0}^{t-1} \text{stuff}_s$$

$$= \delta \cdot [\underset{t=0}{0} + \underset{t=1}{\beta \cdot \sum_{s=0}^{0}} + \underset{t=2}{\beta^2 \cdot \sum_{s=0}^{1}} \quad \dots \quad \text{mmm} \dots \quad :S$$

Also I kinda feel that even if I don't take time t, but time 0 expectations, I can't get around the second sum ...

$$= \delta [\beta \cdot \text{stuff}_0 + \beta^2 \text{stuff}_0 + \beta^2 \text{stuff}_1 + \beta^3 \text{stuff}_0 + \beta^3 \text{stuff}_1 + \beta^3 \text{stuff}_2 + \dots$$

$$= \delta [(\beta + \beta^2 + \dots) \text{stuff}_0 + (\beta^2 + \beta^3 + \dots) \text{stuff}_1 + \text{etc.} \quad | \text{Ignore } \delta$$

$$= \beta(1 + \beta^2 + \beta^3 + \dots) \text{stuff}_0 + \beta^2 (1 + \beta^2 + \beta^3 + \dots) \text{stuff}_1 + \dots$$

$$= (\beta^1 \text{stuff}_0 + \beta^2 \text{stuff}_1 + \beta^3 \text{stuff}_2 + \dots) \frac{1}{1-\beta} \quad (!)$$

$$= (\beta^0 \text{stuff}_0 + \beta^1 \text{stuff}_1 + \beta^2 \text{stuff}_2 + \dots) \frac{\beta}{1-\beta} \quad (!)$$

So the LHS is

$$\delta \hat{E}_t \sum_{T=t}^{\infty} \frac{\beta}{1-\beta} (i_T - \hat{E}_t \pi_{T+1}) \qquad \text{Holy Moly!}$$

So (5) is

$$\frac{1}{1-\beta} c_t + \sigma \hat{E}_t \sum_{T=t}^{\infty} \frac{\beta}{1-\beta} (i_T - \hat{E}_t \pi_{T+1}) = \hat{E}_t \sum_{T=t}^{\infty} \hat{Y}_T + \bar{w}_t$$

$$\Rightarrow \hat{c}_t^i + \sigma \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \beta (i_T - \hat{E}_t \pi_{T+1}) = (1-\beta)\bar{w}_t + \hat{E}_t \sum_{T=t}^{\infty} (1-\beta) \hat{Y}_T$$

So ...

$$\hat{c}_t^i = (1-\beta)\bar{w}_t^i + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\hat{Y}_T - \beta\sigma (i_T - \hat{E}_t \pi_{T+1}) \right]$$

= Preston's (5). Yay!

The last 3 steps again: aggregate and set $\hat{C}_t = \hat{Y}_t$, drop hats.

$$Y_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) Y_T - \beta\sigma (i_T - \pi_{T+1}) \right]$$

$$Y_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)(Y_t - Y_t^n) - \beta\sigma (i_T - \pi_{T+1}) + (1-\beta) Y_t^n \right]$$

$$Y_t = \underbrace{\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \beta\sigma (i_T - \pi_{T+1}) \right]}_{=: CS} + (1-\beta) x_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} (1-\beta) Y_t^n$$

$$x_t = \text{correct-stuff} - Y_t^n + (1-\beta) x_t + (1-\beta) \left[Y_t^n + \beta Y_{t+1}^n + \beta^2 Y_{t+2}^n + \dots \right.$$

$$\beta x_t = CS \cancel{- Y_t^n} \cancel{+ Y_t^n} - \beta Y_t^n + \beta Y_{t+1}^n - \beta^2 Y_{t+1}^n + \beta^2 Y_{t+2}^n - \beta^3 Y_{t+2}^n + \dots \Big]$$

$$\beta x_t = CS + \beta r_t^n + \beta^2 r_{t+1}^2 + \dots$$

$$x_t = \frac{1}{\beta} CS + r_t^n + \beta r_{t+1}^2 + \dots$$

σr_T^N w/ my def.

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\frac{1-\beta}{\beta} x_{T+1} - \sigma (i_T - \pi_{T+1}) + r_T^N \right].$$

Wait... can it be that I did one thing wrong:

when I have $\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_T \right] = (1-\beta) \hat{E}_t \sum_{T=t}^{T-t} x_T$

$$= (1-\beta) x_t + (1-\beta) \hat{E}_t \left[\beta x_{t+1} + \beta^2 x_{t+2} + \dots \right]$$

$$= (1-\beta) x_t + (1-\beta) \hat{E}_t \left[\beta \sum_{T=t}^{\infty} \beta^{T-t} x_{T+1} \right]$$

Yeah!

So that means that my truly correct CS is:

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) \beta x_{T+1} - \beta \sigma (i_T - \pi_{T+1}) \right]$$

so that when I take $\frac{1}{\beta}$ CS I obtain

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \sigma (i_T - \pi_{T+1}) + \sigma r_T^N \right]$$

which is Preston's (6) = (18), and the equation I had in my DW prezi and in materials4. Yay!

Great! We can go on to investigating the expectation-operator, $\hat{E}_t$. Obs. 1. Preston took derivatives thru it no problem, w/o saying anything. So let's go to Evans & Honkapohja (2001).

Evans & Honk (2001) on the nonrational expectation-operator $\hat{E}$

- p. 68-69 Ramsey model: they derive them at no point.
- Minimum-state-variable (MSV) solution (introduced in McCallum (1983)
- heterogeneous learning p. 223-225 (Evans, Honk & Marimon 2000)
- "regular" vs "irregular" models (Farmer 1999, Pesaran 1987)
  - ↳ unique REE (Blanchard-Kahn conditions)
- RE is an eqb concept p. 11
- Expectation is a fct of observables, and so is the updating p. 17-18 → I think it's fair to say that we conjecture $\hat{E}$ to be linear and of the form $\bar{\pi} + bs_t$, and $\bar{\pi}_t = Q(\bar{\pi}_{t-1}, \theta_t)$ nonlinear updating rule and to somehow verify this using undet. coeffs.

Reading the chapter on Nonlinear models   10 Oct 2019

(Chapter 11)

Let the nonlinear univariate model be of the form

$$y_t = F(y_{t+1})^e + v_t \qquad (11.2)$$

$\uparrow$ nonlinear $\quad \uparrow$ shock $v_t \sim iid(0, \cdot)$ If $v_t = 0$

$$= E_t\left[F(y_{t+1})\right] \qquad (11.2) = (11.1)$$

A more general nonlinear model is

$$y_t = H\left(G(y_{t+1}, v_{t+1})^e, v_t\right) \qquad (11.3)$$

Here's an argument (p. 273) for linearity:

1. Supp agents don't know $G(y_{t+1}, v_{t+1})^e$, but have data on its past values $G(y_j, v_j)$ $j = 1, \dots, t$.
2. A natural estimator for $G(y_{t+1}, v_{t+1})^e$ is the sample mean: $\theta_t = \frac{1}{t}\sum_{j=1}^{t} G(y_j, v_j)$ which is then updated using RLS as more data becomes available.
3. the sample mean is a linear operator.

Peter's argument for differentiating them:

- E is an integral over states
- Differentiation proceeds over different variables, not states.

By the way, Evans & Honk derive them $\hat{E}$ all over the place here w/o remarks

- That's all Evans & Honk had to say about $\hat{E}$.
- Liam Graham also simply differentiates thru $\tilde{E}$ (Macp 5)

Moving on: <u>small π, large x</u>

- well, setting $\alpha = 1$ and $\kappa = 1$ still <u>doesn't</u> get π to move as much as x (and thus i). So what's going on?
- now what I did is on top of $(\alpha = 1, \kappa = 1)$, I shut off all shocks except u_t, so now only π is affected. Now the magnitude of the responses is the same (almost)
  ↳ But even then, even then x & i fluctuate more!
- shocks shut off except u, α back to 0.5
  $\Rightarrow$ same as w/ $\alpha = 1$ $\Rightarrow$ α doesn't matter
- –11–, κ back to 0.51.
  $\Rightarrow$ κ matters a <u>lot</u>: this really brings in the gap between π and x.

Let's note a couple of things

1. You'd think that the higher the shock volatilities, the more unanchoring happens. But that is only partly so:
   - If I shut off $\bar{i}$-shocks, c.p., I'm always anchored
   - If I shut off the other shocks, c.p., I'm <u>unanchored</u>

→ What's going on?

3. What matters for the size of gaps in π & x is
   - κ → but this doesn't explain it all
     ↳ Why?
   - size of shocks DOESN'T really matter, α doesn't matter

→ Something else going on?

4. CEMP say about the criterion θ_t that it's a reduced-form way to capture model misspecification tests
   ↳ they simulate a calibrated version where firms instead employ a CUSUM-test of shifting means (Brown, Durbin, Evans 1975) (see CEMP p. 18)
   ↳ get nearly identical results but w/ more parameters to estimate.

Analysis of the issues:

1) κ = slope of NKPC $\quad \pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t$

- If $\kappa = 0$, no rel. btwn π & x $\Rightarrow$ money is strongly non-neutral, x persistently $\neq 0$.
- If $\kappa \to \infty$, then flex prices: changes in x translate immediately and a <u>lot</u> into π $\Rightarrow$ money is neutral

Note that $\frac{\partial \kappa}{\partial \alpha} < 0$ $\Rightarrow$ if firms are stuck w/ a price for longer ($\alpha \uparrow$), then $\kappa \downarrow$ and money becomess less neutral!

- So if $\alpha \uparrow$ (from 0.5 to 1) $\Rightarrow$ we get anchoring where we previously didn't
- if $\alpha \downarrow$ (from 0.5 to 0.1) $\Rightarrow$ we get deanchoring and the gaps in π are indeed huger

$\Rightarrow$ the lower α (higher κ), the more deanchoring we get and the bigger the gaps in π are relatively to x

$\Rightarrow$ I think what is happening is that when α high (κ low), we have a lot of price stickiness ⇒ so inflation cannot respond a lot to shocks, so the margin of adjustment is x

↳ we need sufficiently flex. prices for inflation to move away from its RE path.

⇒ cool, now we understand that fairly well SOLVED

⇒ a RESULT:

the more price stickiness in a learning world w/ anchoring, the more deanchored periods will show up in output gaps instead of inflation b/c x will be the margin of adjustment!

Let's do a survey of lit on learning w/ price stickiness to see if we have anything similar

- CEMP: can't adress b/c no x in model
- Graham: no inflation in model
- Preston: doesn't do simulations

- Eusepi et al, Limits: doesn't get it
- Ferrero (2007): has π & x, but doesn't seem to discuss the relative role of these.
- Orphanides & Williams (2004) is a little difficult to compare b/c the model isn't an NK model
  → there doesn't seem to be a clear mapping btw a parameter and price stickiness (maybe α?)

  They do have a related result: when inflation inertia is low (flex prices) then RLS w/ a constant gain produces very bad fcsts ⇒ i.e. π moves far from π^{RE}.

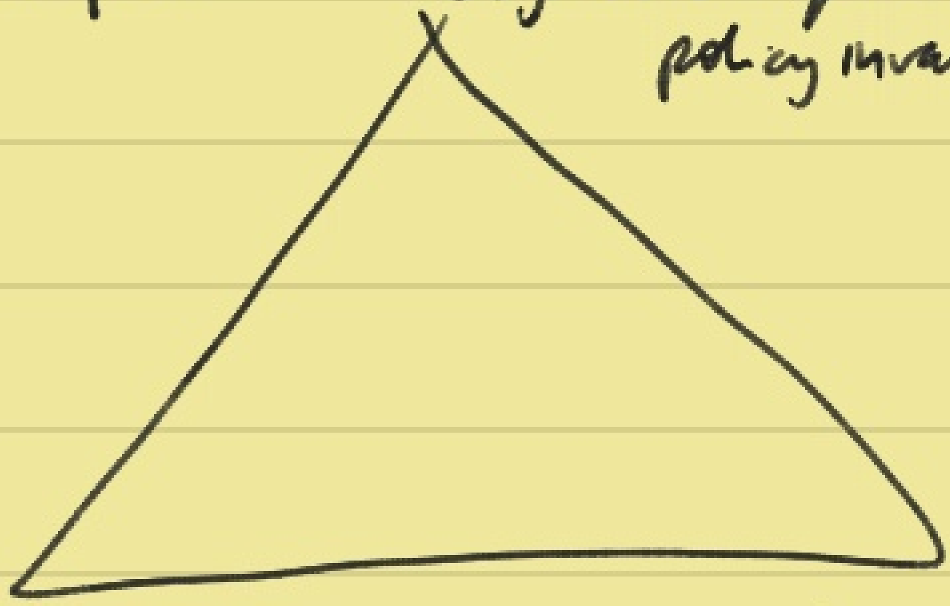
  → it might be useful for me to
  1) write out the ALM, in particular of $\bar{\pi}$ and π
  2) look into the Lit on π-persistence (CEMP allude to it) b/c if the conclusion is that there's a drift in π, well, boom! there's our $\bar{\pi}$.
- Compare my IRFs w/ Orph & Will p. 19 (Mac)

Angeletos

10 Oct 2019

Eusepi Preston

His main point: Dogmatic agents: econ & beliefs are policy invariant



adaptive learning

bayesian learning

1) Why isn't dogmatic beliefs enough? (i.e. why learning?)

→ b/c w/ dogmatic E, E are policy invariant!

Mon. pol. might depart from TR to correct E, but can't influence E.

2) Why adaptive and not Bayesian learning?

It might not matter?

And lastly: what evidence is there in the data for learning?

Send him paper when you have it!

Angeletos Talk

it is the same as NBER SI

- Angeletos & Lian 2018, AER

does "Preston": NK model $\underline{\underline{\text{w/o common knowledge}}}$ (key)

gives rise to LH probs (for IS & NKPC!)

what mon pol does here is to anchor E.

"Anchored Beliefs" = lack of common knowledge

when policy says X,

$E_i[X] = X$ but $E_i\left[\bar{E}[X]\right] = (1-\lambda)\underset{\uparrow \text{prior}}{0} + \lambda E_i[X]$

i.e. I update my beliefs but I strongly believe that $(1-\lambda)$ fraction are "anchored to the prior", zero.

Estimate λ by taking the diff btwn IRFs of beliefs of C and IRFs of C (smart!)

→ Coibion & Goro do this, get $\lambda \sim 0.5$.

(but not for ZLB)

Back to my work

- Eusepi & Preston (2018)'s Fig 4 (p. 23, Mac)

  shows volatilities of π & x $\quad \sigma_\pi^2 + \lambda \cdot \sigma_x^2$

  as a function of λ in learning vs. RE.

  1.) volatility is higher in learning

  2.) as $\lambda \uparrow$, volatility $\uparrow$ i.e. $\sigma_x^2 > \sigma_\pi^2$ (like me!)

- Noah Williams (2003)'s Fig 2.

  under a constant gain, π responds less than x

  (to a shock to r^n, NK model)

  $\rightarrow$ more support for my results!

Now we need to tackle the issue of IRFs in the learning world

11 Oct 2019

Recall that in the linear RE, the impulse response is:

$$IRF = hx^j \cdot \eta \cdot \delta \qquad \left[\text{where } x_t = hx \cdot x_{t-1} + \eta \varepsilon_t\right]$$

But this is a special case of the generalized IR, the GIR (Lect 3 (and 4))

$$GIR(j, \delta, \tilde{X}_{t-1}, \tilde{\varepsilon}_t) = E_t[X_{t+j} | \tilde{X}_{t-1}, \tilde{\varepsilon}_t + \underline{\delta}] - E_t[X_{t+j} | \tilde{X}_{t-1}, \tilde{\varepsilon}_t]$$

j: periods; δ: shock; $\tilde{X}_{t-1}$: econ up to now; $\tilde{\varepsilon}_t$: other shocks

In the linear world, $E_t[X_{t+j} | \tilde{X}_{t-1}, \tilde{\varepsilon}_t] = \sum_{\tau=j}^{\infty} hx^{\tau} \eta \tilde{\varepsilon}_{t+j-\tau}$ (Wold-representation), and thus

$E_t[X_{t+j} | \tilde{X}_{t-1}, \tilde{\varepsilon}_t + \delta] = \sum_{\tau=j}^{\infty} hx^{\tau} \eta \tilde{\varepsilon}_{t+j-\tau} + hx^{j} \eta \delta$, so

$GIR^{linear} = hx^{j} \eta \delta$.

But in the nonlinear world, the Wold representation isn't valid b/c you can't neatly separate shocks.

→ a solution is to integrate out shocks: Ryan's IR:

$RIR = E[GIR]$ → don't need to know <u>where</u> in the state-space the economy is.

- simulate a bunch of histories $\tilde{X}_{t-1}$, calc GIR at all and take an average

Initially I thought that the learning IRFs will be a similar issue, but I don't think so anymore b/c the world is linear (shocks are), but the problem is that gx isn't.

One part that will be constant is the effect on exog states: $hx^j \eta \delta$ will be the same as for RE.
But every time I calculate the effect on the jumps, $gx\ hx^j \eta \delta$, gx will be different.
But what <u>is</u> a "GIR"-issue is that the econ's position in the state-space matters b/c the stance of convergence of learning to RE means that reaction to shocks will be diff. So maybe I <u>do</u> have to do the RIR-thing!

One option: supp. $T=40$. For $t=1,\dots,T$, take a "GIR" for every t and then calc the average (kind of like (I)
→ in fact, I think that might work...

⇒ In particular:

for t = 1 ... T_h (so that IRFs are the same length, h)

[sim_x, sim_y] = sim_learn(same, t)

add shock to innov in that period

for the <u>same</u> shock sequence otherwise!

and then take

sim_x − x_LR_anchor and sim_y − y_LR_anchor

⇒ let these diffs be ir_h_t (h-period ahead IRFs started at time t), or GIR

end

now we have $\overset{\text{GIR}}{\text{ir\_h\_t}}$ which is h × $\overset{\text{ny}}{3}$ × T.

1 approach: take average over the 3rd dim, T.

2nd approach: sort and take median

- Have done IRFs   15 Oct 2019
- and cusum-based alternative criterion for

Now I want to write out the ALM and the criterion at least in Mathematica to investigate the processes for π and $\bar{\pi}$.

→ materials5.nb. See ALM & criterion in materials3, eq (41) & (43).

Mathematica, materials5.nb shows that you can write the LOM of π as:

$$\pi_t = a_1 \cdot \bar{\pi}_{t-1} + a_2 r^n_t + a_3 \cdot \bar{i}_t + a_4 \cdot u_t$$

where a_i, $i=1,\dots,4$ are scalars.

At the same time, the PLM is $\hat{E}_{t-1} z_t = \bar{z}_{t-2} + b s_{t-1}$

which for π reads $\hat{E}_{t-1}\pi_t = \bar{\pi}_{t-2} + b_{11} r^n_{t-1} + b_{12} \bar{i}_{t-1} + b_{13} u_{t-1}$

Then the criterion is $\bar{\pi}_{t-2}$ check timing! → for Ryan's PLM

$$\left|\hat{E}_{t-1}\pi_t - E_{t-1}\pi_t\right| = \left| \pi_{t-1} + b_{11} r^n_{t-1} + b_{12}\bar{i}_{t-1} + b_{13} u_{t-1} - a_1 \bar{\pi}_{t-1} - a_2 \rho_r r^n_{t-1} - a_3 \rho_i \bar{i}_{t-1} - a_4 \rho_u u_{t-1} \right|$$

$\bar{\pi}_{t-2}$ ✓

↳ as usual I'm sidestepping the timing issues for now.

The point is that for LOM(π) of

$$\pi_t = a_1 \cdot \bar{\pi}_{t-1} + a_2 r_t^n + a_3 \cdot \bar{i}_t + a_4 \cdot u_t$$

check timing!

$\bar{\pi}_{t-2}$

the "scalar criterion" θ_t is

$$\theta_t(z_r + z_i + z_u) = \left| (1-a_1)\bar{\pi}_{t-1} + (b_{11} - a_2 \rho_r) r_{t-1}^n + (b_{12} - a_3 \rho_i)\bar{i}_{t-1} + (b_{13} - a_4 \rho_u) u_{t-1} \right|$$

$\bar{\pi}_{t-2}$

which we can summarize as

For Ryan's PLM it's $\bar{\pi}_{t-2}$, for Honky's it's $\bar{\pi}_{t-1}$.

check timing

$\bar{\pi}_{t-2}$

$$\pi_t = a_1 \bar{\pi}_{t-1} + [a_2 \ a_3 \ a_4] \, s_t$$

$$\theta_t(z_r + z_i + z_u) = \left| (1-a_1)\bar{\pi}_{t-1} + [b_{11} - a_2\rho_r, \, b_{12} - a_3 \rho_i, \, b_{13} - a_4 \rho_u] \, s_t \right|$$

$\bar{\pi}_{t-2}$

while we still have, as in CEMP, that

$$\Big[\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \cdot f_{t-1}$$

but the FE f_{t-1} is diff,

in particular it is ALM − PLM, $\pi_{t-1} - \hat{E}_{t-2}\pi_{t-1}$

$$f_t = (a_1 - 1)\bar{\pi}_{t-1} + (a_2 - b_{11}) r_t^n + (a_3 - b_{12})\bar{i}_t + (a_4 - b_{13}) u_t$$

where I've again been sloppy w/ the timing.

$$f_t = (a_1 - 1)\bar{\pi}_{t-1} + [a_2 - b_{11}, \, a_3 - b_{12}, \, a_4 - b_{13}] \, s_t$$

⇒ So supposing the timing is right, I'd get

$$\bar{\pi}_t = \left(1 + k_t^{-1}(a_1 - 1)\right)\bar{\pi}_{t-1} + k_t^{-1} [a_2 - b_{11}, \, a_3 - b_{12}, \, a_4 - b_{13}] \, s_{t-1}$$

check timing!

The only element that is def. true regardless of timing is $\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} f_{t-1}$

$$= \bar{\pi}_{t-2} + k_{t-1}^{-1} f_{t-2} + k_t^{-1} f_{t-1}$$

$$= \bar{\pi}_0 + \sum_{\tau=0}^{t-1} k_{\tau+1}^{-1} f_\tau$$

Obs. 1. The estimate of LR π-mean will be $\neq$ RE (0) when there have been
- a bunch of FE w/ the same sign
- some very large FE.

The difference to CEMP is that there, it is only $\bar{\pi}$ that causes a deviation between PLM-EALM (i.e. in $\hat{E}_{t-1}\pi_t - E_{t-1}\pi_t$) whereas in my case there's an additional wedge that comes from the fact that the RE coefficient b isn't correct as long as we haven't converged to RE!

Look at timing one more time:

1.) $f_{t-1} = \pi_t - E_{t-1}\pi_t$ (index on FE refers to the time the fcst was made)

This is NOT what CEMP have. For them,

$f_{t-1} = \pi_{t-1} - \hat{E}_{t-2}\pi_{t-1}$ (index refers to what you're fcsting)

and $\theta_t = |\hat{E}_{t-1}\pi_t - E_{t-1}\pi_t|$

$= |E_{t-1} f_t|$

The criterion at t is the date t-1 expectation of time t FEs.

I'm really starting to think that Ryan's code isn't a 100% consistent. But so isn't CEMP either!

Let's go thru timing in words.

In t=2: have $\bar{z}_1, s_2, k_1, f_1^{CEMP} = z_1 - \underbrace{E_0 z_1}$

Form $E_2 z_3 = \bar{z}_1 + b s_2$ } z_2 realized. $\underbrace{\bar{z}_{-1} + b s_0}$

Form $\theta_2 = \hat{E}_1 z_2 - E_1 z_2$ $\downarrow$ f_2^{CEMP} realized

CEMP – that cannot be right!

Form $k_2 = f(\theta_2)$

Update $\bar{z}_2 = \bar{z}_1 + k_2^{-1}(f_1)$

Repeat

In $t=2$: have $\bar{z}_1, s_2, k_1, f_1^{CEMP} = z_1 - \underbrace{E_0 z_1}_{\bar{z}_{-1} + b s_0}$

Form $E_2 z_3 = \bar{z}_1 + b s_2$ } z_2 realized

Form $\theta_2 = \hat{E}_1 z_2 - E_1 z_2$ $\Downarrow$ f_2^{CEMP} realized $= f_1^{my}$ realized

Form $k_2 = f(\theta_2)$

Update $\bar{z}_2 = \bar{z}_1 + k_2^{-1}(f_1^{my})$

Issues:

① timing of fcst $\rightarrow$ CEMP cannot be right, so let me use my notation $f_1^{my} = z_2 - E_1 z_2$ (= Ryan's)

$$f_1^{my} = z_2 - (\bar{z}_0 + b s_1)$$

"The fcst made at period 1 of z_2"

$f_2^{assess} = z_2 - (\bar{z}_1 + b s_1)$ "if I'd have to redo yesterday's expectation of z_2 given having updated $\bar{z}$, I'd do this"

② This brings us to our 2nd issue: timing of the PLM

I need to suffer w/ the assessment fcst b/c I'm using

Ryan's PLM: $E_t z_{t+1} = \bar{z}_{t-1} + b s_t$

But if I instead ass that ppl update $\bar{z}$ at the beginning of the period, then: at $t=2$, have $\bar{z}_2, s_2, k_1, f_0^{my}$

Form $E_2 z_3 = \bar{z}_2 + b s_2$ $\}$ z_2 realized, $f_1^{my} = z_2 - E_1 z_2$ realized

Form $\theta_2 = \hat{E}_1 \theta_2 - E_1 \theta_2$

Form $k_2 = f(\theta_2)$

Update $\bar{z}_3 = \bar{z}_2 + k_2^{-1}(f_1^{my})$

$\quad \hookrightarrow = z_2 - (\bar{z}_1 + b s_1)$

Let's write the two approaches for time t next to each other

Ryan:	Honky
At t, have: $\bar{z}_{t-1}, s_t, k_{t-1}, f_{t-2}^{my}$	$\bar{z}_t, s_t, k_{t-1}, f_{t-2}^{my}$
Form: $E_t z_{t+1} = \bar{z}_{t-1} + b s_t \Rightarrow z_t$	$E_t z_{t+1} = \bar{z}_t + b s_t \Rightarrow z_t$
Form $\theta_t = \hat{E}_{t-1} z_t - E_{t-1} z_t$	
Form $k_t = f(\theta_t)$	
Update $\bar{z}_t = \bar{z}_{t-1} + k_t^{-1}(f_{t-1}^{my})$	$\bar{z}_{t+1} = \bar{z}_t + k_t^{-1}(f_{t-1}^{my})$
$z_t - (\bar{z}_{t-2} + b s_{t-1})$	$z_t - (\bar{z}_{t-1} + b s_{t-1})$

$\hookrightarrow$ clearly the two formulations are equivalent, except that for Ryan's code to work out, you need to use an assessment fcst $f_{t-1,eve}^{assess}$ for updating $\bar{z}$ instead of f_{t-1}^{my}.

$= f_{t,morning}^{assess}$ ($\Leftarrow$ before s_t realizes) $\Rightarrow$ Does the assessment fcst matter?

Let's see if the PLM matters for the ALM.

Ryan: $E_t z_{t+1} = \bar{z}_{t-1} + b s_t$

available at t

$f_{a,b} = \text{stuff}\, \bar{z}_{t-1} + \text{stuff} \cdot s_t$

$z_t = \text{stuff}\, \bar{z}_{t-1} + \text{stuff} \cdot s_t$

$\underline{\underline{E_{t-1} z_t = \text{stuff}\, \bar{z}_{t-2}}} + \text{stuff} \cdot \rho s_{t-1}$

hmmm...

$\hat{E}_{t-1} z_t = \underline{\underline{\bar{z}_{t-2}}} + b s_{t-1}$

$E_t z_{t+1} = \bar{z}_t + b s_t$ Harley

available at t

$f_{a,b} = \text{stuff}\, \bar{z}_t + \text{stuff} \cdot s_t$

$z_t = \text{stuff}\, \bar{z}_t + \text{stuff} \cdot s_t$

$E_{t-1} z_t = \text{stuff}\, \underline{\underline{\bar{z}_{t-1}}} + \text{stuff}\, \rho s_{t-1}$

$\hat{E}_{t-1} z_t = \underline{\underline{\bar{z}_{t-1}}} + b s_{t-1}$

Aaah... either way you get θ_t being a function of the $\bar{z}$ at the <u>same time</u>: the point is it's the one that was available <u>last period</u> (whether you denote that $\bar{z}_{t-1}$ or $\bar{z}_{t-2}$ doesn't matter!)

$\theta_t = \theta_t(\bar{z}_{t-2}, s_{t-1})$

$f_t = \text{ALM} - \text{PLM}$

$= z_{t+1} - E_t z_{t+1}$

$= \text{stuff} \cdot \bar{z}_{t-1} + \text{stuff} \cdot s_t$

$- (\bar{z}_{t-1} + b s_t)$ → these work out as well! ⇒ <u>but</u>:

$\theta_t = \theta_t(\bar{z}_{t-1}, s_{t-1})$

$f_t = z_{t+1} - E_t z_{t+1}$

$= \text{stuff}\, \bar{z}_t + \text{stuff} \cdot s_t$

$- (\bar{z}_t + \text{stuff}\, s_t)$

in both cases if I have to update using f_{t-1}, then

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1}\left(z_t - (\bar{z}_{t-2} + bs_{t-1})\right) \quad \Big| \quad \bar{z}_{t+1} = \bar{z}_t + k_t^{-1}\left(z_t - (\bar{z}_{t-1} + bs_{t-1})\right)$$

not the one I used at the beginning of this period

⇒ so I have the assessment fcst trouble no matter what!

i.e. both for Ryan's & Hanley's PLM

So is the assessment fcst the sol. to all my troubles?

It's a new day and today I think <u>16 Oct 2019</u>

the assessment fcst is fine: it's yesterday evening's forecast so it does use the most recent info available yesterday to fcst today's state. (See materials5c.tex)

→ so I'm ok w/ using the assessment fcst to update $\bar{\pi}$ (or $\bar{z}$) (Issue #1 is "solved").

But issue #2 is a bigger problem.

→ tomorrow: let's tackle that and see if Comp's timing can be written like mine!

Before I do that, just one note: 17 Oct 2019

$$\theta_t = \hat{E}_{t-1}(z_t) - E_{t-1}(z_t)$$

We can interpret both expectations as $t-1$ evening!

Then $\theta_t = \bar{z}_{t-1} + b s_{t-1} - (\text{stuff}\ \bar{z}_{t-1} + \text{stuff} \cdot s_{t-1})$

$$= \chi(\bar{z}_{t-1}, s_{t-1})$$

My feeling is that the CEMP formulation (when agents at time $t-1$ fcst time t stuff) works out only if

$$\theta_t = |\hat{E}_{t-2}\pi_{t-1} - E_{t-2}\pi_{t-1}|$$

$$= |\gamma\pi_{t-2} + (1-\gamma)\bar{\pi}_{t-1} + \text{stuff} - (\gamma\pi_{t-2} + (1-\gamma)\rho\bar{\pi}_{t-1} + \text{stuff})|$$

$$= |\text{stuff} \cdot \bar{\pi}_{t-1}|$$

that seems to work. But let's try to change the notation so that $\bar{\pi}$ makes more sense in light of standard learning things.

Suppose: PLM: $\pi_t = \bar{\pi}_{t-2} + \rho\psi_{t-1}$ (ignoring indexing)

This maps perfectly to my PLM.

Then: at time $t-1$

$$\left.\begin{aligned} &\pi_t = \bar{\pi}_{t-2} + \rho y_{t-1} \\ &\rightarrow \bar{\pi}_{t-2} \text{ was formed in } t-2 \text{ evening.} \end{aligned}\right] \text{Morning. } \pi_{t-1} \text{ realized.}$$

Now we wanna update to $\bar{\pi}_{t-1}$

$$\bar{\pi}_{t-1} = \bar{\pi}_{t-2} + k_{t-1}^{-1} \underbrace{(\pi_{t-2} - \hat{E}_{t-3}\pi_{t-2})}_{\text{this is my } FE_{t-2}, \text{ should be } FE_{t-1}}$$

$\pi_{t-1} - \hat{E}_{t-2}\pi_{t-1}$ is what I'd have.

→ even my FE_{t-1} only works out if it's the evening:

$$FE^e_{t-2} = \pi_{t-1} - (\bar{\pi}_{t-2} + \rho y_{t-2}).$$

$$k_{t-1} = f(\theta_{t-1}, k_{t-2})$$

$$\theta_{t-1} = |\hat{E}_{t-2}\pi_{t-1} - E_{t-2}\pi_{t-1}|$$

Ok, if I ass $|\hat{E}^e_{t-2}\pi_{t-1} - E^e_{t-2}\pi_{t-1}|$ I get

$$\bar{\pi}_{t-2} + \rho y_{t-2} - (\text{stuff } \bar{\pi}_{t-2} + \text{stuff} \cdot y_{t-2}) \checkmark$$

- Ok, so changing the notation for $\bar{\pi}_t$ to $\bar{\pi}_{t-2}$
- imposing evening fcsts in $\bar{\pi}$-update and criterion θ
- changing the timing of CEMP's fcst in $\bar{\pi}$-update

→ I get stng that 1) maps to mine 2) makes sense 3) works out.

Honestly, I think I won't even ask Ryan about this b/c the important thing is that I know what I'm assuming. I'll still send him materials5c, but I don't think I'll talk about it.

<u>Ryan meeting</u> <u>17 Oct 2019</u>

- IRFs: present Susanto ones conditional on being anchored or not before the shock hits
- kill χ_x to clarify storytelling
- after: add persistence term to TR → kills overshooting?

ρi_{t-1}

- For Susanto:
  - only show mean lines (not all – at most, confidence bands)
  - have IRFs conditional on being < anchored / unanchored before shock
  - understand why & how the overshooting happen
- as for θ_t: keep both specifications, CEMP's θ_t and the CUSUM one – right now they're tied b/c the CUSUM is closer to

what firms do and isn't technically harder to implement (as long as we're not estimating – which is not clear to him that that's where we're headed ... mmm...)

⇒ trade whether they lead to different results

- as for $\hat{E}$:

Keep this question at the back of your mind and when you get to talk to some of these people, ask them!

→ then you can say, in a job talk or when you're asked this question, gingerly, that "experts in the field told me that we don't worry about it." Then you've shown that you've at least thought about it.

Work after materials6 18 Oct 2019

Adding ρi_{t-1} to TR isn't that simple b/c it changes a bunch of things in the LR models.

i_{t-1} is a new state variable, so should it show up in s_t? $s_t = \begin{bmatrix} r_t^n \\ i_t \\ u_t \\ i_{t-1} \end{bmatrix}$? → that's what I did for the RE model.

In materials6, eqs (1)-(3) are Preston's (18), (19) & (27), NKIS, NKPC & TR. Now that I modified the TR, to get the compact notation I need to redo the steps w/ the new Taylor-rule.

$$x_t = -\sigma(\psi_\pi \pi_t + \psi_x x_t + \rho i_{t-1} + \bar{i}_t) + \Sigma(\cdot)$$

$$(1+\sigma\psi_x)x_t = -\sigma\psi_\pi \pi_t - \sigma\rho i_{t-1} - \sigma\bar{i}_t$$

$$+\sum_{T=t}^{\infty}\beta^{T-t}\left((1-\beta)x_{T+1} + \sigma\pi_{T+1} + \sigma r_T^n - \sigma\beta\left(\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \rho i_T + \bar{i}_{T+1}\right)\right)$$

$$(1+\sigma\psi_x)x_t = -\sigma\psi_\pi \pi_t - \sigma\rho i_{t-1} - \sigma\bar{i}_t$$

$$+\sum_{T=t}^{\infty}\beta^{T-t}\left((1-\beta-\sigma\beta\psi_x)x_{T+1} + (\sigma-\sigma\beta\psi_\pi)\pi_{T+1} + \sigma r_T^n - \sigma\beta\bar{i}_{T+1} - \sigma\beta\rho i_T\right)$$

$$(1+b\psi_x)x_t = -b\psi_\pi \pi_t - b\rho i_{t-1} - b\bar{i}_t$$
$$+\sum_{T=t}^{\infty}\beta^{T-t}\Big((1-\beta-b\beta\psi_x)x_{T+1} + (b-b\beta\psi_\pi)\pi_{T+1} + br_T^N - b\beta\bar{i}_{T+1} - b\beta\rho i_T\Big)$$
$$\Leftrightarrow (1+b\psi_x)x_t = -b\psi_\pi \pi_t$$
$$+\sum_{T=t}^{\infty}\beta^{T-t}\Big((1-\beta-b\beta\psi_x)x_{T+1} + (b-b\beta\psi_\pi)\pi_{T+1} + br_T^N - b\bar{i}_T \quad - b\rho i_{T-1}\Big)$$

The i_{t-1} terms

$$-b\rho i_{t-1} - b\rho\beta i_t - b\rho\beta^2 i_{t+1} + \dots$$
$$-b\rho\left[i_{t-1} + \beta i_t + \beta^2 i_{t+1} + \dots\right]$$
$$= -b\rho \sum_{T=t}^{\infty}\beta^{T-t} i_{T-1}$$ ok! Now plug π_t

$$\Rightarrow (1+b\psi_x)x_t = -b\psi_\pi\left[\kappa x_t + \sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\Big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\Big)\right]$$
$$+\sum_{T=t}^{\infty}\beta^{T-t}\Big((1-\beta-b\beta\psi_x)x_{T+1} + (b-b\beta\psi_\pi)\pi_{T+1} + br_T^N - b\bar{i}_T \quad - b\rho i_{T-1}\Big)$$
$$(\underbrace{1+b\psi_x-\kappa b\psi_\pi}_{=w})x_t = \text{stuff}$$
$$x_t = \frac{-b\psi_\pi}{w}\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\Big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\Big)$$
$$+\frac{1}{w}\sum_{T=t}^{\infty}\beta^{T-t}\Big((1-\beta-b\beta\psi_x)x_{T+1} + (b-b\beta\psi_\pi)\pi_{T+1} + br_T^N - b\bar{i}_T \quad - b\rho i_{T-1}\Big)$$

$$\pi_t = \left(1 - \frac{\kappa b \psi_\pi}{w}\right) \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\right)$$
$$+ \frac{\kappa}{w} \sum_{T=t}^{\infty} \beta^{T-t} \left((1 - \beta - b\beta\psi_x) x_{T+1} + (b - b\beta\psi_\pi)\pi_{T+1} + b r_T^n - b\bar{i}_T - b\rho i_{T-1} \right)$$

Ah yeah, so all I need to do is to expand the s-vector

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = f(fa, fb) + \underbrace{f_\gamma \text{ which is different}}_{\text{but only to the extent that it has}}$$

a 4^{th} row for i_{T-1}, and A_s is different

$$f_\gamma = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} s_T = (I_4 - \beta P)^{-1} s_t \quad w/\ s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{bmatrix}$$

does P include ρ? I think so!

$$P = \begin{bmatrix} \rho_r & & 0 & \\ & \rho_i & & \\ & & \rho_u & \\ 0 & & & \rho \end{bmatrix}$$

And A_s^{LR} looks as follows:

for $x_t \Rightarrow \frac{-b\psi_\pi}{w}[1 - \alpha\beta P]^{-1}[0\ 0\ 1\ 0]\, s_t + \frac{b}{w}[1 - \beta P]^{-1}[1\ {-1}\ 0\ {-\rho}]\, s_t$

for $\pi_t \Rightarrow \left(1 + -\frac{\kappa b\psi_\pi}{w}\right)[1 - \alpha\beta P]^{-1}[0\ 0\ 1\ 0]\, s_t + \frac{\kappa b}{w}[1 - \beta P]^{-1}[1\ {-1}\ 0\ {-\rho}]\, s_t$

and for $i_t \Rightarrow \underbrace{\psi_x \text{ stuff } x + \psi_\pi \text{ stuff } \pi}_{-\shortparallel-} + \rho i_{t-1} + \bar{i}_t$

$+ [0\ 1\ 0\ \rho]\, s_t$

Yeah so I only need to adjust A_S^{LR} & A_S^{RE}. <u>19 Oct 2019</u>

It's now 3×4 instead of 3×3.

I can summarize the previous as: (new in blue)

$$g_{xs} = \left(\frac{-\gamma\psi_\pi}{w}\left[1-\alpha\beta P\right]^{-1}\left[0\ 0\ 1\ 0\right] - \frac{\gamma}{w}\left[1-\beta P\right]^{-1}\left[-1\ 1\ 0\ \rho\right]\right)$$

$$g_{\pi s} = \left(\left(1 + -\frac{\kappa\gamma\psi_\pi}{w}\right)\left[1-\alpha\beta P\right]^{-1}\left[0\ 0\ 1\ 0\right] - \frac{\kappa\gamma}{w}\left[1-\beta P\right]^{-1}\left[-1\ 1\ 0\ \rho\right]\right)$$

and for $i_t \Rightarrow \psi_x g_{xs} + \psi_\pi g_{\pi s} + \left[0\ 1\ 0\ \rho\right]$

Of course it doesn't work, even if I set $\rho = 0$ I don't get what I had before, not even for the RE model.

So:

$$x_t = hx \cdot x_{t-1} + \eta \cdot \varepsilon_t$$

$$y_t = gx \cdot x_t$$

$$\begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{bmatrix} = \begin{bmatrix} \rho_r & & & 0 \\ & \rho_i & & \\ & & \rho_u & \\ 0 & & & \rho \end{bmatrix} \begin{bmatrix} r_{t-1}^n \\ \bar{i}_{t-1} \\ u_{t-1} \\ i_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_r & & & 0 \\ & \sigma_i & & \\ & & \sigma_u & \\ 0 & & & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_r \\ \varepsilon_i \\ \varepsilon_u \\ 0 \end{bmatrix}$$

is this right? I don't think so...

$$\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \underbrace{\begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}}_{3\times 4} \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{bmatrix}$$

} this stuff ain't right!

Everything looks correct. So I'll take a big step back: go back to materials5.m and

✓ 1) implement the learning & IRFs w/ a single code, making sure you keep obtaining the same thing

2) change RE to account for the smoothing in i_t making sure you obtain the same thing as before

Ok, did that, reverted.

→ Let's think this thru one more time:

$$\begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{bmatrix} = \begin{bmatrix} h_x & & & \\ & & & \\ & & & \\ i_r & i_i & i_u & \rho \end{bmatrix} \begin{bmatrix} r_{t-1}^n \\ \bar{i}_{t-1} \\ u_{t-1} \\ i_{t-2} \end{bmatrix} + \begin{bmatrix} \eta & & & \\ & & & \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_r \\ \varepsilon_i \\ \varepsilon_u \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} g_x & & & \\ & & & \\ i_r & i_i & i_u & \rho \end{bmatrix} \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{bmatrix}$$

→ it makes sense that the last row of h_x = that of g_x!

↳ and that does look like this is where rounds part between P and h_x b/c i_{t-1} is an endogenous state!

→ so I don't think I need to redefine P or Σ,
but in fa & fb, agents will use hx instead of P
to test states! Well, Σ maybe...
↳ will need to correct materials6.tex!
↳ still need to think it thru – this ain't done yet!
But maybe the 1st step is to replace P w/ hx in
fafb, fk (with CEMP & CUSUM) and let it adjust
and make sure we still get the same things.
To be sure I'd do new functions for everything!

<u>Analyzing IRFs (all of the above done & works!)</u> <u>20 Oct 2015</u>

1.) CUSUM criterion doesn't respond a whole lot to ρ, to the point of me suspecting that something isn't quite right there.

2) In general, when $\uparrow \rho$, CEMP criterion gives more anchoring
→ makes sense b/c then π isn't moving around as much, as so they aren't making as large errors.

3) Same for $\psi_\pi \uparrow$, except after a while it reverses

Focus for a sec on constant gain learning:

1) overshooting ↑ when ψ_π ↑

2) ρ↑ doesn't really do much to overshooting

3) ρ↑ or ψ_π↑ → anchoring → AM behaves more like decreasing gain learning

Both cgain & dgain have overshooting if ψ_π high enough but cgain has a more "sign-switching convergence"-behavior in the sense that it's initially further from RE and also moving further away, but then it overtakes dgain learning.

➔ It's as if the overshooting was happening b/c the shock interrupts an ongoing learning process

⇒ that's why there are these reversals in π b/c cgain learners change their mind about where π is headed!

$$b = \underset{ny \cdot nx}{gx} \cdot \underset{nx \cdot nx}{hx} \quad \Big\} \quad \begin{matrix} ny \cdot nx \\ 3 \times 4 \end{matrix}$$

20 Oct 2019

M1 is 3×3 M2 is 3×4 $\Rightarrow \underset{3\times4}{M2} \cdot \underset{4\times1}{s} = 3\times1$

Still trying to understand IRFs:

- $\rho\uparrow$ just means that RE starts overshooting too
  the learning models also overshoot more, but the change is way less pronounced than for RE.

It seems like $\rho>0$ in RE is a substitute for learning!

RE responses closest to learning when $\rho \approx 0.3$ (save for monpol shock but I'm ignoring that a bit b/c it's persistent)

But what is quite diff is the int rate, even under $\rho=0.3$

→ it's more quick to react

it switches sign

→ why is i doing this? It's not the case that the impact effect of shocks would be greater under learning, in fact, the impact is always greater in RE on everything

→ in RE, a bigger x is cancelled by a bigger i↑
whereas in learning a smaller x is reversed by a smaller i

So it seems like the same ψ_π has a bigger effect in a learning model, especially if you have constant gain learning → you get overshooting b/c of that and so the CB reverses course to correct it

Why does the same ψ_π have a bigger effect under learning, especially under cgain?

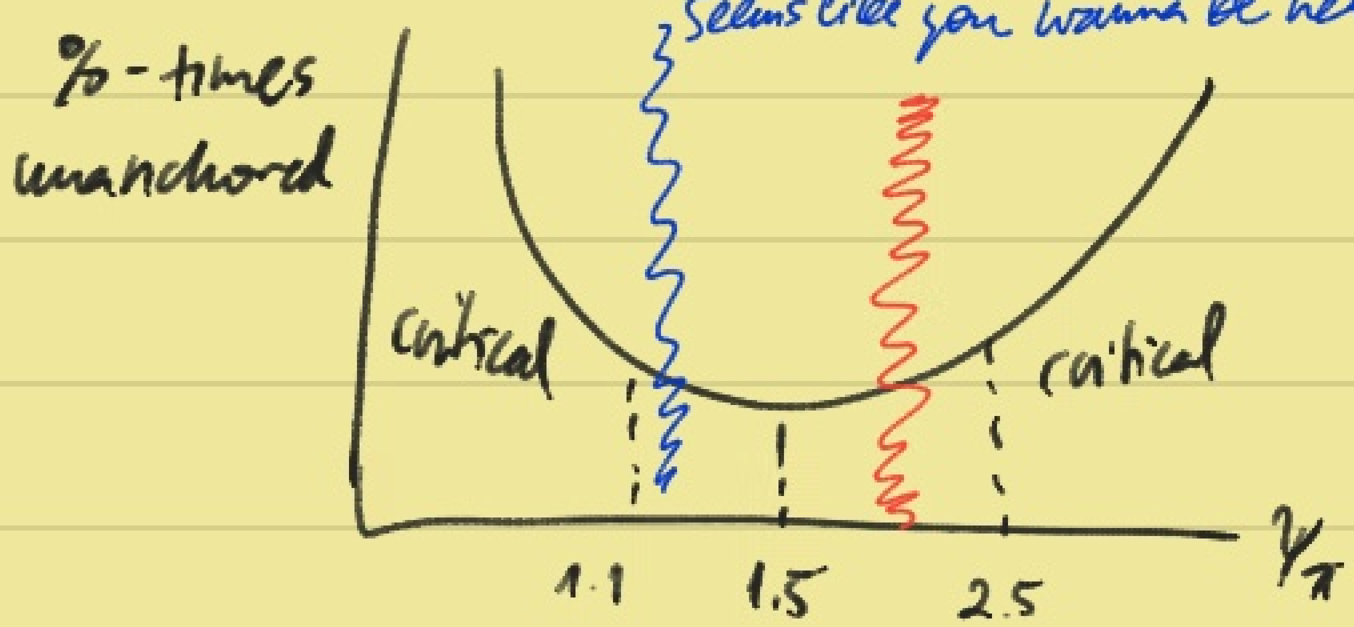
→ b/c agents use it strongly to infer the π^*

⇒ this is the same reason why a too high ψ_π leads to constant gains: it leads to "excess volatility" which suggests to agents that they need to monitor the sitn.

What doesn't make sense is that $\psi_\pi = 1.1$ which seems to do the optimal cancelling, gives you cgains!

Maybe here's what's going on:

- impact effects are smaller in learning b/c E are slow-moving (states feed thru the system slower)
- ψ_π has a bigger effect though somehow (...? :S)
  (maybe jumps feed faster thru the system)
- ψ_π has a U-shaped unanchoring curve



⇒ it seems like you're trading off excess volatility and anchoring

Here: you could ↑ ψ_π to get more anchoring, but then you get more overshooting

Here that tradeoff is gone b/c ψ_π↑ means less anchoring and more overshooting.

⇒ It seems like there's some "interior level" of anchoring that's

optimal in that you both learn fast enough (again)
but it doesn't lead to excess volatility.

But why does learning respond

- slower to states → b/c $E = f(\text{states})$
- faster to jumps? → b/c E doesn't absorb this

Result 1. Under learning, shocks to states propagate slower through the system b/c expectations are functions of states and thus slow-moving.

Cor. 1. Impact effects of structural shocks are smaller than under RE.

Result 2. Under learning, changes to jump variables (such as $\uparrow i_t$ when ψ_π is high) propagate faster through the system b/c expectations take time to respond & absorb them (this is like prices when money isn't neutral).

Cor. 2. the same ψ_π leads to bigger responses (overshooting).

→ Result 1 implies that mon. pol should anchor b/c you can dampen responses.

→ Result 2 implies that mon. pol. shouldn't anchor too much b/c the more anchored, the more overshooting you get. X That's not true.

Monetary policy trades off dampening shocks on impact versus overshooting dynamics.

This tradeoff might disappear if you have sthg that can raise anchoring w/o ψ_π ($\rho\uparrow$ or $\alpha\uparrow$).

Result 3. The $\downarrow\alpha$ ($\kappa\uparrow$), the more is π the margin of adjustment. This is not just b/c in this case, prices are more flexible, but also b/c flex prices lead to more unanchoring as $\bar{a}$ moves more.

I'm a little sceptical about the overshooting result's interpretation b/c if it's the case that we're overshooting b/c E(·) is slow to adapt, then shouldn't Dgain have the biggest overshooting? But currently, clearly cgain does! 22 Oct 2019

cgain learning updates E(·) by much more each period
→ this is why especially long after the shock, dgain converges to RE much slower than cgain does.
⇒ so the overshooting has to have sthg to do w/ learning happening.
↳ from the perspective of overshooting, you want to be anchored b/c that gets you closer to dgain.
But initially (early after impulse), cgain & dgain are close: so why does cgain lead to more overshooting then? B/c these are avg IRFs, so cgain & dgain aren't actually close!

Ryan

24 Oct 2013

- Even in RE $g_{3,4} \neq \rho$

  E.g. if we only have i_{t-1} as a state:

  $i_t = \phi \pi_t + \rho i_{t-1}$

  $\pi_t = g i_{t-1}$

  $\Rightarrow i_t = (\phi g + \rho) i_{t-1}$ The jump picks up effects of the state that are a) direct b) indirect.

- Revisit RE effects on impact of nat. rate shock.

- IRF of expectations & FE $\rightarrow$ plot 'em!

- Check MP shock. fall in i can't happen w/o capital.

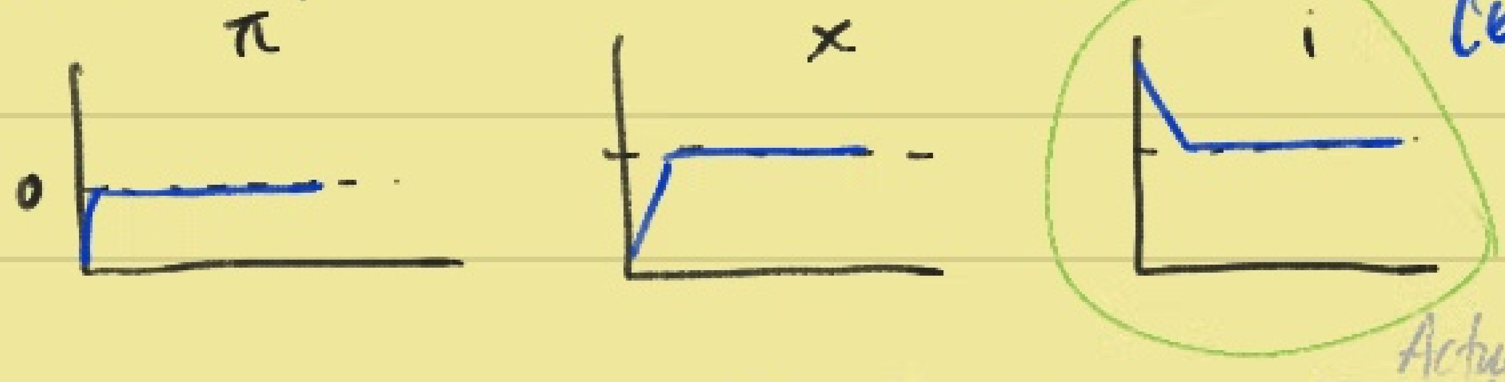
Work after

27 Oct 2019

Let's work out what these shocks should be doing in the RE NK model:

① mon. pol. shock ($\bar{i}_t \uparrow$) ($\rho = 0$)

(by def b/c $\bar{i}\uparrow$, so $i\uparrow$)

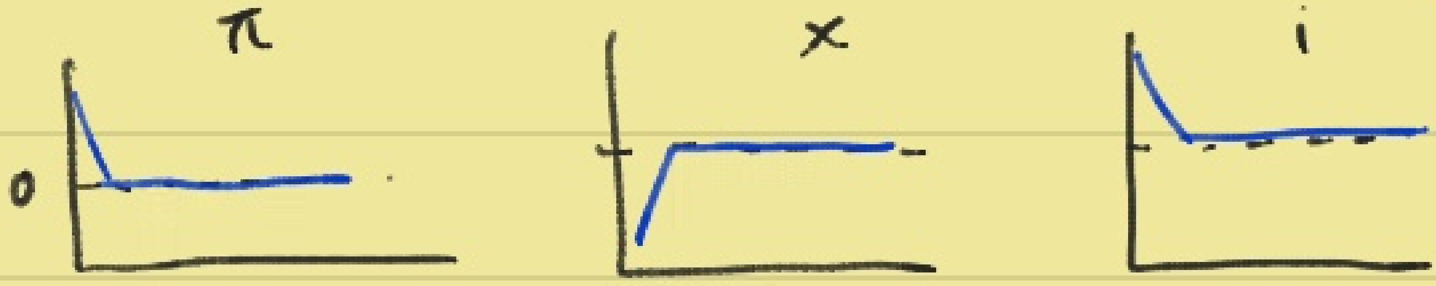


I get the opposite.

Actually, I get this if $\bar{i}$ is iid.

② cost-push shock ($u\uparrow$) ✓ I get this.

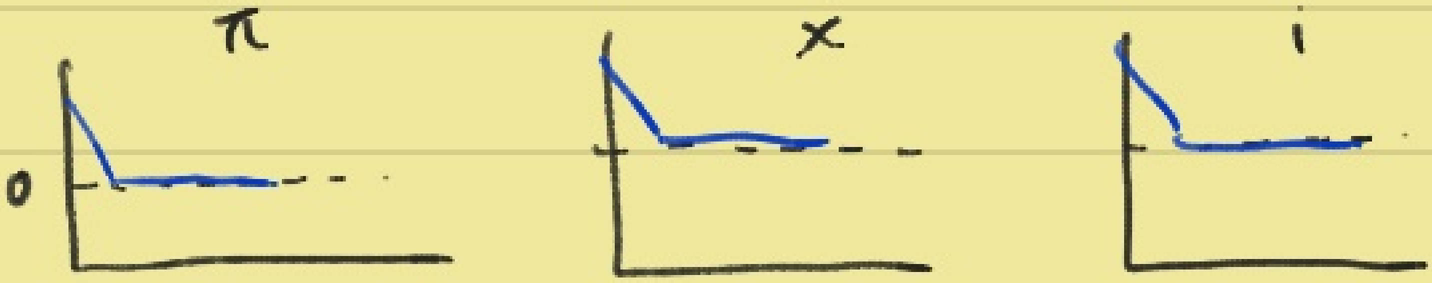
I guess b/c $\psi_x = 0$



This is why Ryan called it a stagflation shock.

③ nat. rate shock (a news shock?) ($r^n \uparrow$)

✓ Which is what I get



It's a stand-in for all kinds of demand shocks

However, all shocks are fine when AR(1) except mon pol.

The nat. rate of interest shock

There are two competing interpretations of this shock

1) $r^n \uparrow$ as a tech shock $z \uparrow$

In Peter Ireland's notation, $r^n = (1-\omega)(1-\rho_a)\hat{a}_t$

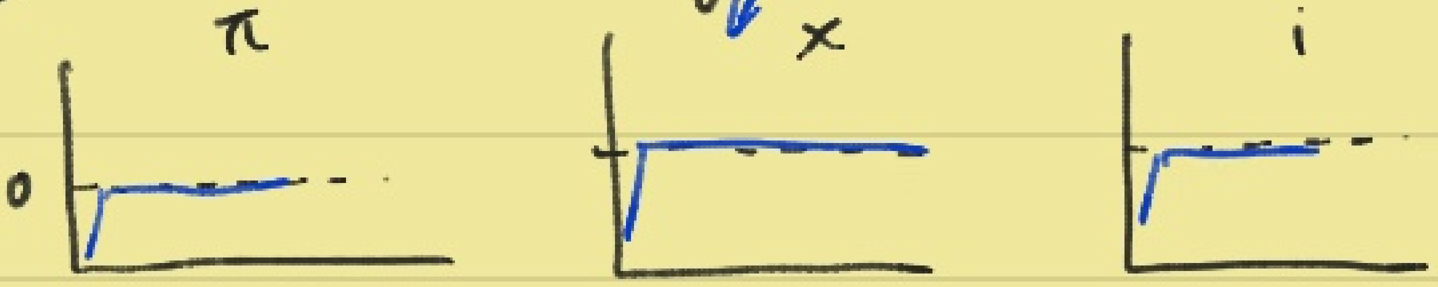
where $\hat{a}_t$ is the tech process

2) $r^n \uparrow$ as a news shock

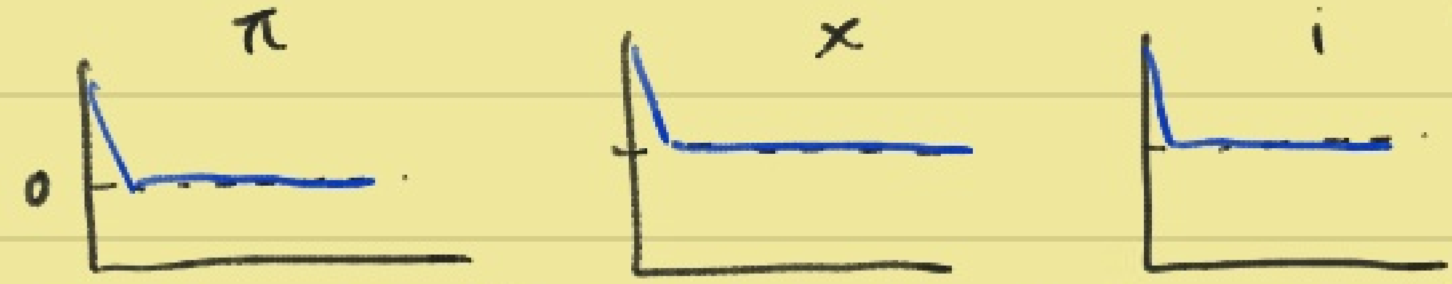
since $r^n = \sigma\left(E(Y^f_{t+1}) - Y^f_t\right)$

Clearly, w/ perfect foresight $E(Y^n_{t+1}) = Y^n_{t+1}$ and r^n is a tech shock, but not today but tomorrow!

(4) Tech shocks $z \uparrow$ today b/c $Y^f_t \uparrow$



(5) Tech shock tomorrow (news shock)



This is what I get

I think a nat. rate shock is best thought of as a news shock, yielding these responses.

Walsh p. 251 (Mac 272) writes

$$x_t = E_t x_{t+1} - \tilde{\sigma}^{-1}\left(\hat{i}_t - E_t \pi_{t+1}\right) + \xi_t \quad \text{(NKIS)}$$

where $\tilde{\sigma}^{-1} = \sigma^{-1}(C/Y)^{ss}$ (used to extend $Y=C$ w/ $+G$)

Then we can interpret the shock in NKIS, ξ_t, as

1.) a gov-spend shock

2.) a preference/taste shock ($\rightarrow$ hits $u'(C)$)

3) expected change in y^t.

$\Rightarrow$ are all these just stand-ins for a demand shock?

I think so!

Shocks in NKIS $\rightarrow$ demand shocks (G, taste, news, nat. rate)

Shock in NKPC $\rightarrow$ supply shock (cost-push)

So why do we get this trouble w/ the monpol shock (only) when it is persistent?

In fact, the impact of the monpol shock is correct up to and including $\rho_i = 0.6$. If $\rho_i > 0.6$, then $i\downarrow$ on impact.

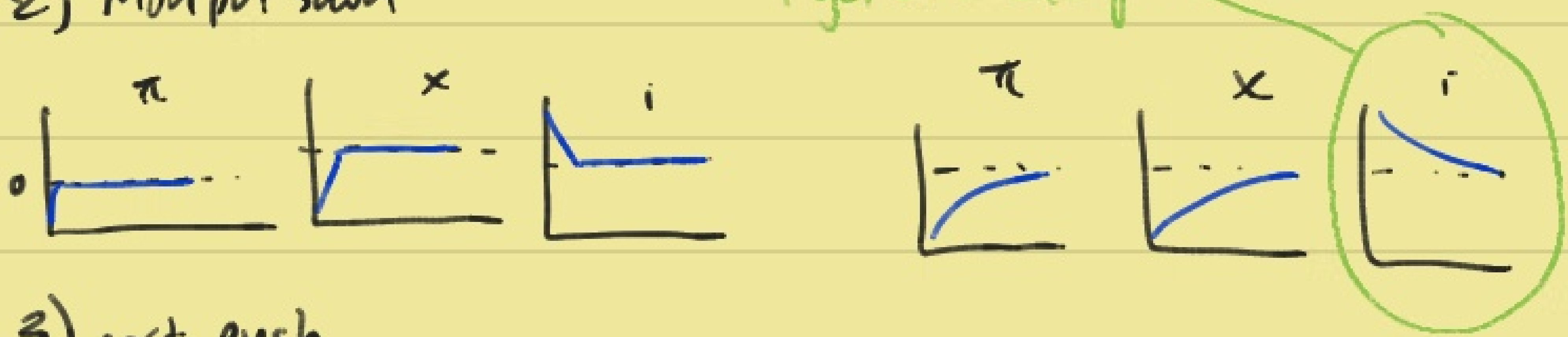
$i_t = \phi_\pi \pi_t + \rho_i \bar{i}_t$ 28 Oct 2019

Why do I get the wrong sign on impact if the shock is persistent but only for the mon pol shock?

1) nat rate shock

2) mon pol shock

I get all except this:



3) cost push

I obtain the same IRFs if I consider w/ or w/o int rate smoothing, so it's not that.

I can reverse the result if $\alpha \uparrow \downarrow \kappa$

Initially, I had $\alpha = 0.5$ ($\rightarrow \kappa = 0.51$)

but you need $\alpha \geq 0.87$ ($\rightarrow \kappa \leq 0.022$) to get $i \uparrow$ on impact!

Let me reconsider IRFs in the linear (RE) world.

Ryan writes (Lect 3, p. 9)

$$X_t = h_x \cdot X_{t-1} + \eta \varepsilon_t$$

$$= \ldots$$

$$X_t = \sum_{\tau=0}^{\infty} h_x^{\tau} \eta \varepsilon_{t-\tau}$$

So on impact that is | on jumps

$X_t = \eta \varepsilon_t$ (impact) | $Y_t = g_x \cdot \eta \varepsilon_t$

$X_{t+1} = h_x \cdot \eta \varepsilon_t$ (1-period out) | $Y_{t+1} = g_x h_x \cdot \eta \varepsilon_t$

$\vdots$ | $\vdots$

So since my h_x only has >0 elements, the problem seems to be g_x

$g_{x_{3,2}} < 0$, so the response of i to $\bar{i} \uparrow$ is $\downarrow$.

(If I set $\alpha \geq 0.87$, $g_{x_{3,2}}$ turns >0.)

$$Y_t = \underbrace{-[f_y\ f_{xp}]^{-1}[f_{yp}\ f_x]}_{\text{top left block}} E_t Y_{t+1}$$

$$\underbrace{-[f_y\ f_{xp}]^{-1}[f_{yp}}_{\text{top right block}}\ f_x]\ X_t$$

A_p^{RE} = top left block, A_s^{RE} = top right block.

: $z_t = A_p^{RE} E_t z_{t+1} + A_s^{RE} s_t$

Let $biggny := -[f_y\ f_{xp}]^{-1}[f_{yp}\ f_x]$

$(nx+ny,\ nx+ny)$

then $A^{RE}p = biggny(1:ny, 1:ny)$ $\quad ny \times ny$

$A^{RE}_s = biggny(1:ny, ny+1:end)$ $\quad ny \times nx$

Yeah, these guys are right!

this biggny is what Ryan in Lect 2 called B

So then he says

$$\begin{bmatrix} Y_t \\ X_{t+1} \end{bmatrix} = B \begin{bmatrix} Y_{t+1} \\ X_t \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} Y_{t+1} \\ X_t \end{bmatrix}$$ (where $A_p = B_{11}$, $A_s = B_{12}$)

Looking at Basu Sun 2, NK w/ capital 29 oct 2019

- For some reason we have a fix cost which means we have IRS I think.
- We also have 2 assets: bonds (B) & capital (K)

Bonds pay $1+r_t$. Capital pays $1+R_t-\delta$ (net of depreciation)

Arbitrage between the two assets requires:

in st st: $R = r + \delta$

outside st st: $\hat{r}_t = \frac{r+\delta}{r} \hat{R}_t$

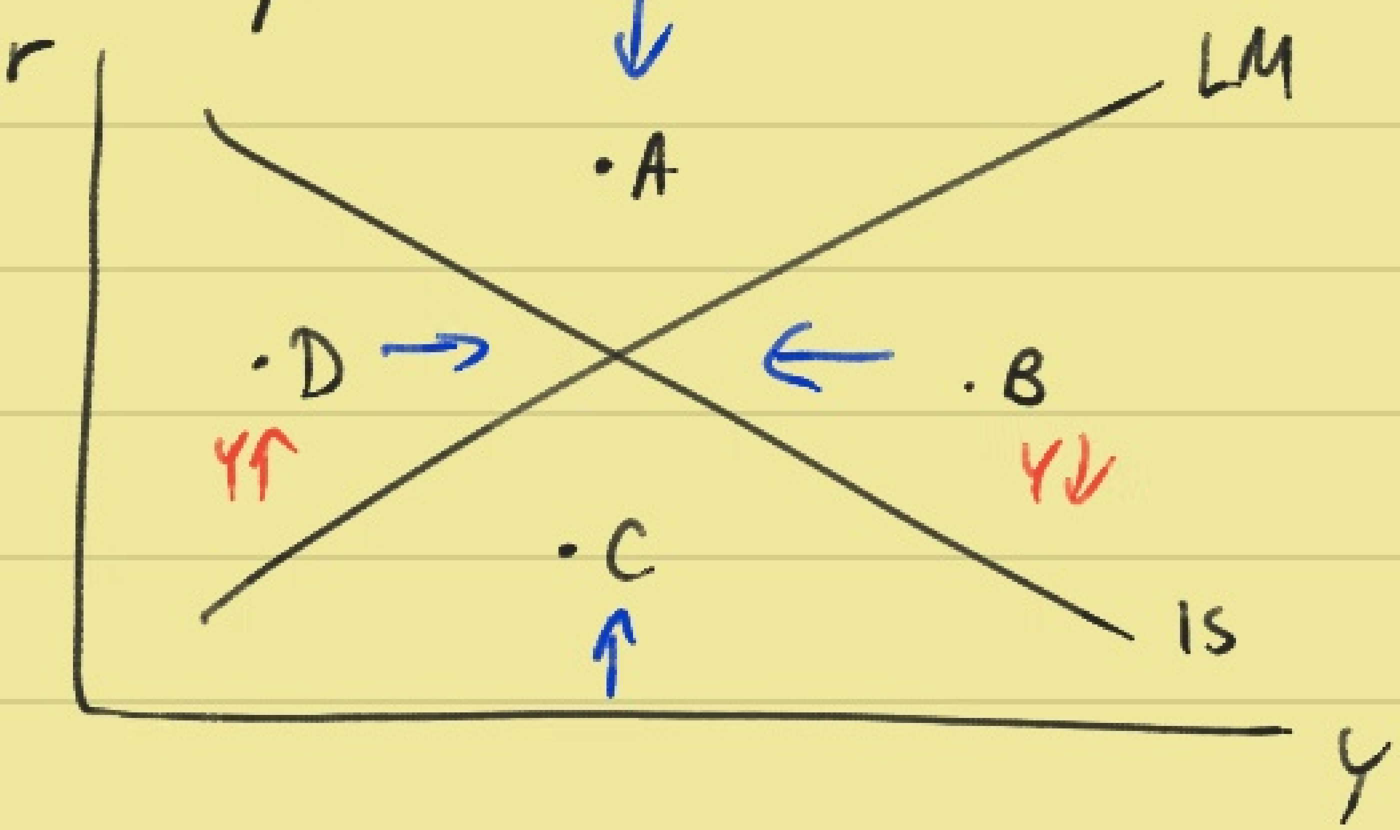
We switch out $\hat{R}_t$ from factor demands to obtain

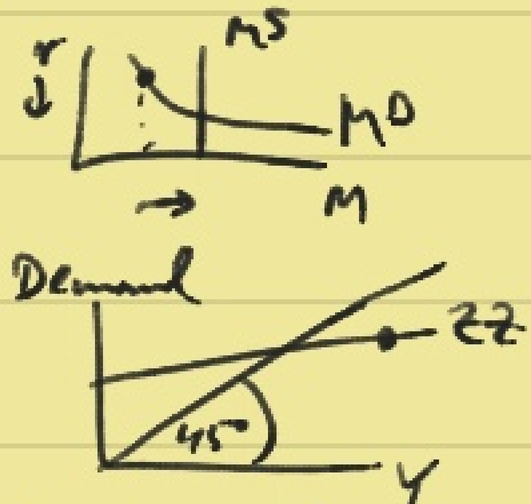
$\hat{r}_t = \frac{r+\delta}{r} \left(\hat{w}_t + \hat{H}_t - \hat{K}_t \right)$ (NRR curve)

instead of the int. rate being pinned down by the NKIS (one EE, here it's two)

I think the point is that this NRR is the "new IS-curve" which is why Basu focuses on the intersection of IS-LM:

He says





A: excess supply of money & goods
$r\downarrow$ from money market
$r\downarrow$ from loanable funds market

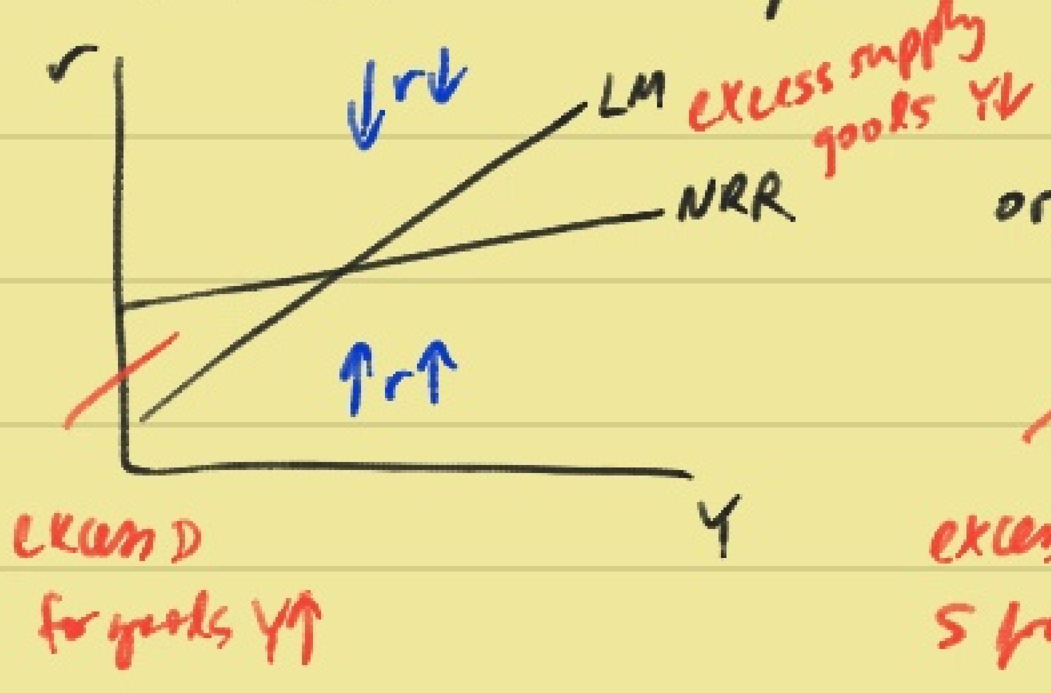
B excess supply of goods $r\downarrow$ } Y (supply?) $\downarrow$
excess demand of money $r\uparrow$ } to equilibrate out

C: excess demand of both $r\uparrow$

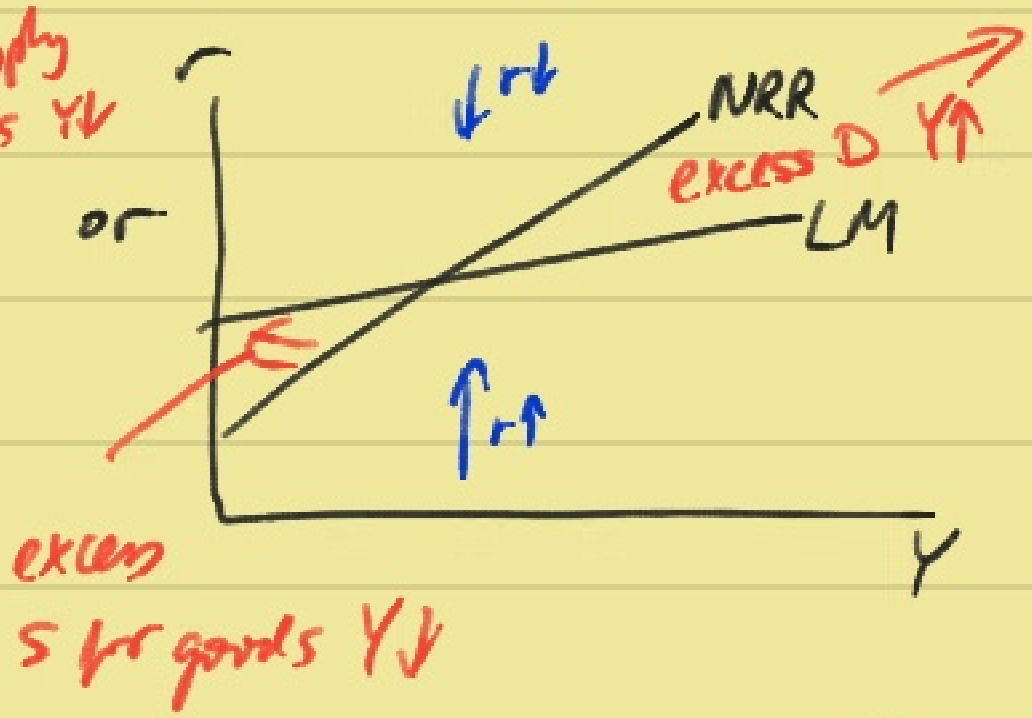
D: excess supply of money $r\downarrow$ } Y (supply?) $\uparrow$
excess demand of goods $r\uparrow$ } to equilibrate out

But unlike IS, NRR is upward-sloping.

So it can do 2 things:



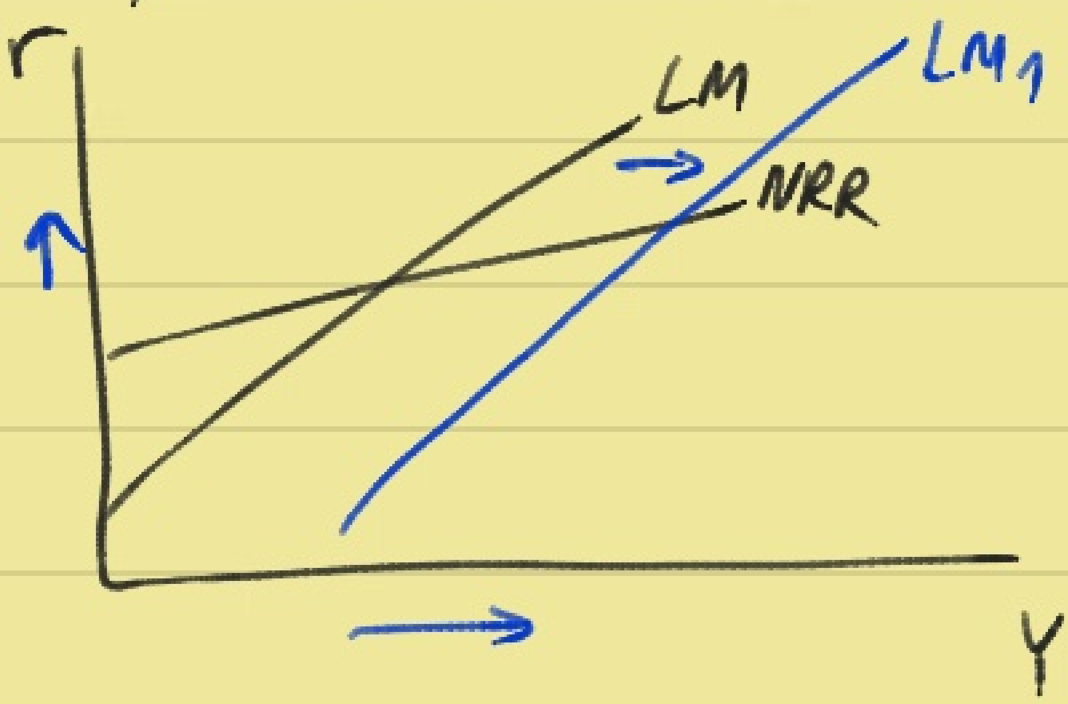
or



unstable!

the channel seems to be SR-supply adjusting!

Ok, and this means: $M\uparrow \rightarrow LM \rightarrow R$



$\Rightarrow$ Y↑ and r↑ (!) Expansionary monpol causes r↑

Why is this happening?

b/c $M\uparrow$ raises $E_t\pi_{t+1}$, so money will be taxed by future inflation $\Rightarrow$ the asset K becomes more valuable relative to money, but via arbitrage, so must bonds! (R & $r\uparrow$)

Ok, so let's turn back to our NK model w/o capital.

So: mon. pol. shock $\bar{i}\uparrow$

Can it be that only the nom int rate $\downarrow$, the real $\uparrow$?

Nah...

So back to thinking: why is $gx_{3,2} < 0$?

In particular, why does $gx_{3,2}$ depend on whether the shock is iid or persistent?

$gx_{3,2} > 0$ if $\rho_i = 0$

$gx_{3,2} < 0$ if $\rho_i > 0$

I mean, it's clear that hx changes when ρ_i changes
In the plain (no-int-rate-smoothing) NK, there are only exog states and so

$$hx = P = \begin{pmatrix} \rho_r & 0 & 0 \\ 0 & \rho_i & 0 \\ 0 & 0 & \rho_u \end{pmatrix}$$

If all shocks iid, then $hx = \mathbf{0}$, which is why IRFs are all zeros after the impact $(= gx \cdot \eta \varepsilon_t)$

So what is gx?

$Y_t = gx\, X_t$, in this case

<0

$$\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} \cdot & g_{12} & \cdot \\ \cdot & gx_{22} & \cdot \\ gx_{31} & gx_{32} & gx_{33} \end{bmatrix} \begin{bmatrix} r^n_t \\ i_t \\ u_t \end{bmatrix}$$

All other $gx_{ij} > 0$. And the point is that gx_{ij} don't ever change sign, except for gx_{32}, which

	$\rho_i = 0$	$\rho_i = 0.5$	$\rho_i = 0.6$	$\rho_i = 0.7$	$\rho_i = 0.9$
gx_{32}	0.72	0.25	0.03	−0.28	−1.32

Setting IES $= \sigma = 1$ (instead of my 0.5) lowers these numbers !

Ok so what about the Frisch? For Basu, the Frisch

is denoted either ε_{HW} (e.g. Lect 11 slide 9 Mac)
or η (SUM 2, p. 48)
and it's set to 1,2 (micro) or 4 (macro).
→ is it missing from my κ?

$$\kappa = \frac{(\eta + \zeta)(1-\omega)(1-\beta\omega)}{\omega} \qquad \zeta = \text{IES}, \ \eta = \text{Frisch}$$

In COMP, $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$ $(\alpha = \omega)$
b/c there ain't no demand side!
In Preston, $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \left(\frac{\omega + \zeta^{-1}}{1+\omega\theta} \right)$

where $\omega > 0$ is the "elasticity of a firm's real marg. cost function wrt its own output, $y_t(i)$." (?)
And θ seems to be the elast. of demand.

4 questions — I think so!
1) Is Preston really using ζ as Basu $\frac{1}{\zeta}$? (if $\zeta = 1$ don't matter)
2) Why does η (Frisch) show up in Basu, not in Preston?
3) Why does θ (price elast of demand) show up in Preston?
4) What's ω in Preston and why is it there?

King & Rebelo 1999, Table 2 Calibration

σ	b	θ	η	γ	α	δ	ρ	σ_ε
1	0.984	3.48	1	1.004	0.667	0.025	0.979	0.0072
IES = CRRA	β (sdf)	disutility of labor?	Frisch	grt rate of tech	K-share	depreciation	tech shock	tech shock

$$u(c_t, L_t) = \log(C_t) + \theta \log(L_t) \Rightarrow (\sigma = 1, \eta = 1)$$

Basu says: $\sigma + \eta$ = GE elasticity of $\frac{W}{P}$ wrt ΔH

In this model, real wage = real MC

$\Rightarrow$ $\sigma + \eta$ = elasticity of real MC wrt output.

Susanto also says somewhere that we need $\kappa \leq 0.1$ or even $\kappa \approx 0.01$ to get reasonable persistence.

(Lect 22, slide 3 Mac)

It also says that w/ $\sigma \in [1,2]$, $\eta \in [1,4]$, $\beta = 0.99$, ω = avg duration $= 6$, you don't get a low enough k.

6 months =

2 quarters.

Wiki says: if $h = \text{Prob}(\text{can reset}) = 1-\alpha$ for Preston.

Then the prob to last i periods is

$$Pr[i] = (1-h)^{i-1} h \qquad \text{(geometric distrib)}$$

Expected duration is h^{-1}

Then if expected duration is 2 quarters, then

$$\text{expected duration} = (1-\alpha)^{-1} = \frac{1}{1-\alpha}$$

$$2 = \frac{1}{1-\alpha} \Rightarrow 1-\alpha = \frac{1}{2} \rightarrow \alpha = 1-\frac{1}{2} = 0.5$$

If it was 6 months (if the model was monthly)

$$6 = \frac{1}{1-\alpha} \Rightarrow 1-\alpha = \frac{1}{6} \rightarrow \alpha = 1-\frac{1}{6} = \frac{5}{6}.$$

New set of default params: $\alpha = 0.5$ (= as before)

$\sigma = 1$ (instead of 0.5, although that would correspond to Basu's 2)

Frisch = 1.

But even w/o worrying about questions (2)-(1),
we have that the correctly specified κ makes
the problem worse!

Let's see Gali: (Monetary Policy & Theory) $Y_t(i) = A_t N_t(i)^{1-\alpha}$

$$\kappa = \underbrace{\frac{(1-\theta)(1-\beta\theta)}{\theta} \underbrace{\left(\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\right)}_{=:\Theta}}_{=\lambda} \qquad \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$$

Collard:
$$\left(\sigma + \frac{\varphi + 1-\alpha}{\alpha}\right)$$
$$\frac{\sigma\alpha + \nu + 1-\alpha}{\alpha}$$

θ = Calvo prob of stuck $\qquad$ σ = IES

Collard's χ is only right if $\sigma = 1$.

ε = price elast. of demand $\qquad$ φ = Frisch

$1-\alpha$ = labor share (in Basu, $\alpha = 0$, w/ that you get $\qquad$ PF: $y_{it} = z_t h_{it}$

$$\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta} \cdot (1) \cdot (\sigma + \varphi) \quad = \text{Basu!}$$

Check Collard (Monitor 4, p. 16 Mac) $\qquad$ PF: $Y_t = A_t h_t^{\alpha}$

$$\text{slope} = \underbrace{\frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}}_{=\kappa} \underbrace{\left(\frac{\alpha}{\alpha+(1-\alpha)\theta}\right)}_{=:\Theta} \underbrace{\left(\frac{1+\nu}{\alpha}\right)}_{=:\chi}$$

In this notation, Basu has $\alpha = 1 \Rightarrow \dfrac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p} \left(\overset{\sigma \,(\log u)}{\overset{\parallel}{1}} + \nu\right)$

So let's use Collard's notes to interpret the parts of the slope of the NKPC:

$$\text{slope}(NKPC) = \frac{(1-\omega)(1-\beta\omega)}{\omega} \quad \frac{\alpha}{\alpha+(1-\alpha)\theta} \quad \frac{\sigma-\alpha\sigma+\eta+\alpha}{\alpha}$$

$$= \kappa \qquad \cdot \qquad \Theta \qquad \cdot \qquad \chi$$

ω = Calvo prob of stuck , β = sdf

α = labor share (cf: $Y = A_t \cdot H_t^{\alpha}$)

θ = price elasticity of demand for varieties

σ = IES/CRRA of utility function

η = inverse of labor supply elasticity

Collard's interpretation:

κ = dynamics of price setting (frequency of adjustment & sdf)

Θ = relates firm's MC to the average MC

χ = relates agg MC to the output gap

More interpretation..

$\underline{\underline{\Theta}}$ • $\alpha\uparrow$, $\theta\uparrow$ $\quad$ α measures returns to scale. Since there's no capital in this model $Y = A \cdot N^{\alpha}$ is a DRS PF. The higher α, the higher returns to scale. In Basu's case, $\alpha = 1$, the PF is CRS.

$$\frac{\partial \theta}{\partial \alpha} = \frac{\partial}{\partial \alpha} \frac{\alpha}{\alpha + (1-\alpha)\theta} = \frac{1}{\alpha + (1-\alpha)\theta} \cdot (-1) \frac{1}{(\alpha + (1-\alpha)\theta)^2} (1-\theta)$$

$\alpha \geq 0$, $\underline{\theta > 1}$ I think b/c $\mu^* = \frac{\theta}{\theta - 1} > 0$

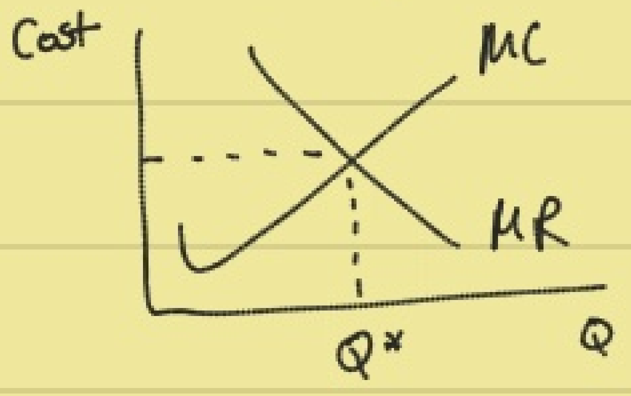
↳ Yes! Basu Sum 1 p. 81 Mac & p. 84.

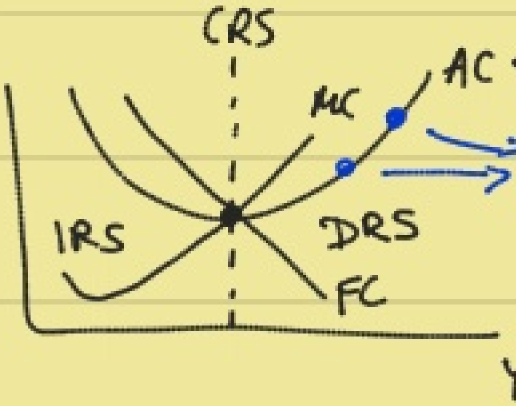
$$\frac{-\overbrace{(1-\theta)}^{>0}}{\underbrace{(\alpha + (1-\alpha)\theta)^3}_{>0}} > 0 \Rightarrow \theta \uparrow \text{ w/ } \alpha \text{ indeed!}$$

So the higher returns to scale, to less flat the NKPC.

The more DRS, the more flat the NKPC.

I think the point is that





the more we're here, the more strongly does AC & MC ↑ when you Y↑.

So when α low, θ low, then $\varepsilon_{MC/y}$ high, in other words, MC fluctuates more (out-of-st.st) when α & θ low.

→ This point isn't quite clear right now! Get back to it later!

- $\Theta \downarrow$ when $\theta \uparrow$

  when closer to perfect comp (θ high), then $\uparrow P$ means demand falls more (flatter D-curve)

  ⇒ so firms won't adjust prices as much, more stickiness, flatter NKPC

$$\underline{\chi} = \frac{\gamma\alpha + \nu + 1 - \alpha}{\alpha} \qquad = \gamma - 1 + \frac{\nu + 1}{\alpha}$$

- $\frac{\partial \chi}{\partial \gamma} = \frac{1-\alpha}{\alpha} > 0$ when IES higher, slope NKPC higher

$$\frac{u'(c)}{u'(c')} = \left(\frac{c_{t+1}}{c_t}\right)^{\gamma} \rightarrow \text{sub more across time}$$

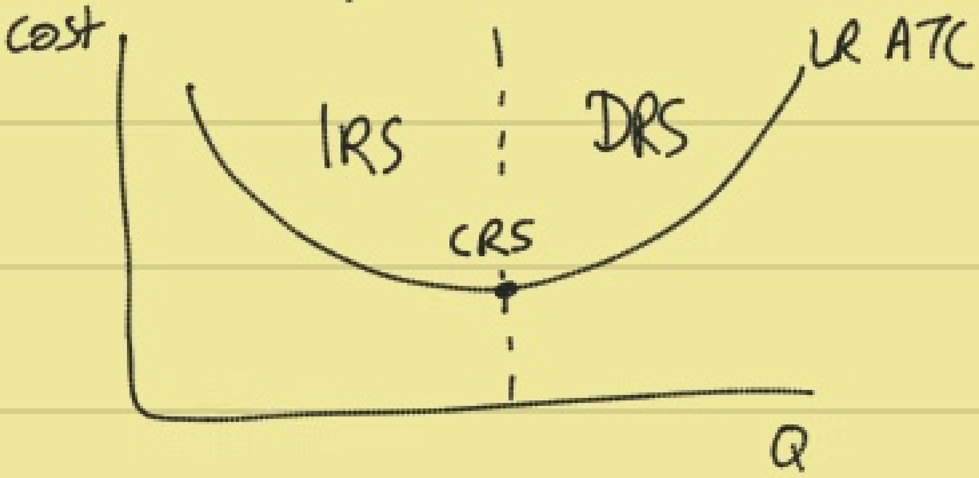
  → firms change prices less

- $\frac{\partial \chi}{\partial \eta} = \frac{1}{\alpha} > 0$ → when η higher, χ higher → NKPC flat when L^s-elasticity low, i.e. when wages need to move a lot to affect L^s. (c.t. & i.t.m. bamez)

- $\frac{\partial \chi}{\partial \alpha} = -\frac{\nu+1}{\alpha^2} < 0$ Now we have the opposite: low α (more DRS) leads to more flexible prices.

Bazinga!

Review from micro:



In Basu: $Y = Z \cdot h \Rightarrow CRS$

OR $= Y = Zh + \Phi \Rightarrow IRS$

$\uparrow$ fix cost

Collard & Gali: $Y = A \cdot h^{\alpha} \Rightarrow DRS$

$\Rightarrow$ seems like a monopoly can be any of these.

Basu Sum P. 1 p. 86 THM: $\gamma = AC/MC$ for a cost-minimizing firm

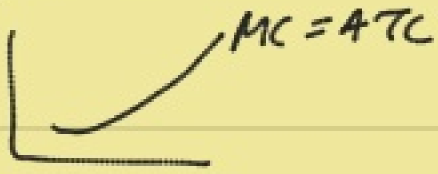
$\gamma \rightarrow$ returns to scale

and $\mu = \gamma \Rightarrow$ st.st. markup = returns to scale.

Jeff just explained this as the following:

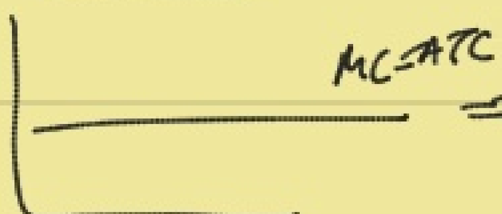
1.) if there's no fix cost (as is the case in NK model)

$MC = AVC = ATC$.



$\Rightarrow$ DRS

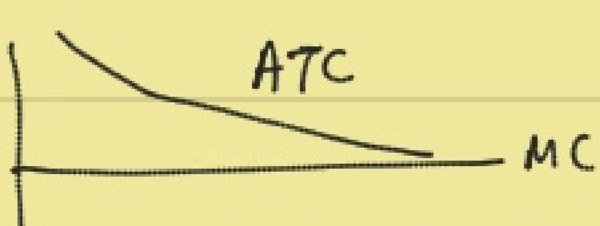
or if MC constant



$\Rightarrow$ CRS

2) If there is fix cost

or



if MC is constant. $\Rightarrow$ IRS

3) Normally we think that MC is upward-sloping, in fact it's convex. The reason is diminishing marg. product

If MPL is   then MC of labor is

b/c to get 1 unit of Y extra, you need to use more & more labor.

Using this, let's check connection btwn returns to scale & MC.

MC

Let $y_{it} = z_t h_{it}^{\alpha}$ $\frac{W_t}{P_t} = \alpha \cdot z_t h_{it}^{\alpha-1}$

If $\alpha = 1$ (CRS), then

$\frac{W_t}{P_t} = z_t \Rightarrow$ constant (given technology)

wrt output or scale of prod.

$\frac{W_t}{P_t} = \alpha \cdot \frac{Y_t}{h_t}$ if $\alpha \neq 1$ MC depends (+) on scale of prod.

I'm not sure I get that b/c if I pay my workers their

MP, and that's diminishing, then MC diminishes too?
What if I call MC the things I need to pay to get another unit of output?

$1 = F'(\cdot)$?

$$1 = \underbrace{\underbrace{\alpha \cdot z_t}_{\text{constant}} \underbrace{h_{it}^{\alpha-1}}_{\downarrow}}_{\text{MP of 1 labor}} \cdot \underbrace{dh_{it}}_{\uparrow}$$

$\Rightarrow$ overall I pay each hour less, but I need a lot of additional hours

$\downarrow$

In the CRS case dh_{it} is always the same b/c MPL is constant, making MC constant.

This means that having CRS $(\alpha = 1)$ let's $\Theta = 1$ meaning that a firm i's MC equal avg MC.

- $\frac{\partial \Theta}{\partial \alpha} > 0$ $\Rightarrow$ DRS means that α falls below 1, so $\Theta < 1$ too, i.e. NKPC flatter $\Rightarrow$ a difference in firm i's MC to the avg MC means a bigger distrib of MCs and prices

$\Rightarrow$ you get more procyclical MC!

$\cdot \frac{\partial \Theta}{\partial \theta} < 0$ More competition means demand reacts more when you change prices, so firms will abstain from doing so if they can.

If CRS, then I don't care about this.

$\chi = \frac{\sigma\alpha + \eta + 1 - \alpha}{\alpha}$

2) Let's set $\alpha = 1 \Rightarrow \sigma + \eta$ Basu's

1) Let's set $\sigma = 1 \Rightarrow \chi = \frac{1+\eta}{\alpha}$ = Collard's

• RTS has opposite effects: $\chi \downarrow$ as $\alpha \uparrow$

The higher RTS, the more MC is scale-dependent

Susanto calls $\sigma + \eta$ = Elasticity of MC to the change in the output gap

↳ comes from changes in M

↳ comes from: $\Delta L^D \to \eta$

$\Delta L^S \to \sigma$

I think it's the other way around

η is the L^S-elasticity

σ is IES $\to$ will show up in $\Delta C \to \Delta Y \to$ firms' demand of L

Θ relates firm i's MC to the average MC

χ relates agg MC to the output gap

Gali says that the slope of NKPC is $\downarrow$ in α and θ

$\rightarrow$ I should check the Woodford book!

I'm still not quite clear on the intuition here but at least I've understood a lot and need to be more careful w/ writing the slope.

<u>Woodford · Interest & Prices</u> <u>31 Oct 2019</u>

$$\pi_t = \kappa\left(\hat{Y}_t - \hat{Y}_t^n\right) + \beta E_t \pi_{t+1} \qquad (2.12)$$

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\,\zeta > 0 \qquad (2.13)$$

the smaller ζ, the larger the strategic complementarity in price setting, the flatter the NKPC (aha!)

He discusses sources of strategic complementarity in Sect 1.4

p. 164

$$\zeta = \frac{\omega + \sigma^{-1}}{1 + \omega\theta}$$ so κ becomes

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{\omega + \sigma^{-1}}{1 + \omega\theta}$$ which is exactly what

Preston has! But how to map this to Basu, Gali & Collard's framework?

Determinants of strategic complementarity (ζ)

Conclusions summed up in Table 3.1. (p.172)

$\sigma^{-1} = 1$ and $\omega = 1.25$ seems reasonable.

- homogeneous factor markets $\rightarrow s_y = 0, \; s_Y = \omega + \sigma^{-1}$

vs. <u>specific factor markets</u> $\rightarrow s_y = \omega, \; s_Y = \sigma^{-1}$

- constant desired markup $\rightarrow \theta \cdot \epsilon_\mu = 0$

vs. <u>variable desired markups</u> $\rightarrow \theta \epsilon_\mu = 1$

- no intermediate inputs ($s_m = 0$) vs. <u>$\mu s_m = 0.6$</u>

Prop 3.3. $$\zeta = \frac{(1-\mu s_m)(s_y + s_Y)}{1 + \theta\left[\epsilon_\mu + (1-\mu s_m) s_y\right]} \qquad (1.43)$$

The underlined $\zeta \downarrow$, leading to more strat. comp.

Supp no intermediate inputs & specific factor markets.

then: $\zeta = \dfrac{\omega + \sigma^{-1}}{1 + \theta\,[\epsilon_\mu + \omega\qquad]}$

To get Preston's thing, then you need to assume constant desired markups: $\epsilon_\mu = 0$.

Why does specific vs homogeneous/common factor markets matter so much, and how?

ϵ_μ = Elasticity of markup function $\mu(x)$ at x (how much do target markups change at different levels of output)

s_y = elasticity of real marginal cost function wrt. $y_t(i)$

s_Y = -//- wrt Y_t ← exog disturbances

Real MC function $s(y, Y, \tilde{\xi})$ is given by (1.10) (p. 149)

Ok, let's walk thru the real MC fct, $s(y, Y, \bar{\xi})$

Peter meeting 31 Oct 2019

He's saying it's b/c $E[\pi]$-effect is stronger when shock persistent
$\rightarrow$ leads $\pi \downarrow$ more so that i drops (on net). But real $i \uparrow$

Easy fix:

- lagged inflation in TR $\rightarrow$ might cause trouble in learning models though, but Peter usually uses this when estimating an NK model
- int. rate smoothing term
- iid shock or low persistence

justify: "in NK model $\exists$ liq. effect & $E[\pi]$ effect. In the SR, the liq. effect is stronger than the $E[\pi]$-effect"

1980s VAR lit: couldn't find liq. effect b/c they used data from 1970s when mon pol shock were persistent and so $E[\pi]$-effect dominated

$\hookrightarrow$ CGG-story: MP raised $i \uparrow$ but not sufficiently to $r \uparrow$, so opened door to $E[\pi] \uparrow$.

Back to Woodford

The real marginal cost function (p. 148)

Variable cost of supplying quantity $y_t(i)$ of good i

is $\quad w_t(i) \; f^{-1}(y_t(i)/A_t) \quad$ where $f^{-1} \approx h$

where PF is $\quad y_t(i) = A_t \, f(h_t(i))$

$\frac{\partial VC}{\partial y_t(i)}$ = nominal MC of supplying good i

$$\Rightarrow S_t(i) = w_t(i) \cdot \frac{1}{f^{-1}(\cdot)} \cdot f' \cdot \frac{1}{A_t} = \frac{w_t(i)}{A_t} \cdot \underbrace{\frac{f'}{f^{-1}(\cdot)}}_{\Psi(y)}$$

$$S_t(i) = \frac{w_t(i)}{A_t} \cdot \underbrace{\Psi(y_t(i)/A_t)}_{\text{"how much do we need to } \uparrow \text{labor"}} \qquad (1.8)$$

wage of that labor (nominal)

$\Rightarrow$ Real MC:

$$s_t(i) = \frac{S_t(i)}{P_t} =: s(y_t(i), Y_t; \tilde{\xi}_t)$$

$$s(y, Y, \bar{\xi}) = \frac{v_h(f^{-1}(y/A); \xi)}{u_c(Y; \xi) A} \, \Psi(y/A) \qquad (1.10)$$

Let's interpret the real marginal cost function:

$$s(y, Y, \bar{\xi}) = \frac{v_h\left(f^{-1}(y/A); \xi\right)}{u_c(Y; \xi)A} \underbrace{\Psi(y/A)}$$

The disutility of $L\uparrow$ for different levels of L and for shocks ξ

↳ Frisch-inverse η

↑ "utility of L" coming from extra consumption

→ IES param σ^{-1}

by how much do I need to increase labor to get ↑ output

→ RTS param α

MPL of labor

Log-linear approx to this gives

$$\hat{s}_t = \omega \hat{y}_t(i) + \sigma^{-1}\hat{Y}_t - (\omega + \sigma^{-1})\hat{Y}_t^n \qquad (1.15)$$

elasticity of $s(y, Y, \bar{\xi})$ wrt y ($\to s_y$)

The above makes clear that this elasticity,

$$s_y = \omega = \omega_w + \omega_p \qquad (1.16)$$

disutility of work (holding u_c fixed)

elasticity of Ψ (how much I need to adjust L to increase prod)

Let's understand specific factor markets by looking at MC:

$$\hat{s}_t = \omega \hat{y}_t(i) + \sigma^{-1} \hat{Y}_t - (\omega + \sigma^{-1}) \hat{Y}_t^n$$

As Woodford details starting p. 164, common economy-wide factor markets means that the MC of supply is equal for all goods i $\forall t$

In English: "1 unit of labor costs the same, no matter in which industry it's used to produce" (see p 166)

And look: Basu, Lect 21, p. 5 Mac:

"Common MC result rests on four important ass's:

1. CRS
2. Common technology
3. Economy-wide factor markets ⟵ there it is!
4. No adj-cost for inputs (i.e labor) "

Woodford says that in this case,

$$s_t = \frac{w_t}{A_t f'(h_t)}$$

b/c this way $w_t(i) = w_t$ $\forall i$ and $h_t(i) = h_t$ $\forall i$.

Woodford goes on to say that in this case,

$$\zeta = \omega + \sigma^{-1} \quad > 0$$

and since in RBC, $\sigma^{-1} = 1$, $\zeta > 1$, i.e. by design, pricing decisions are strategic substitutes!

(sic!)

This implies that $\omega = \eta$: somehow there still is a disconnect between ζ and ω in the sense that "which one is actually the MC elasticity"?

Where is the specific factor markets ass introduced?

it's in the PF: $y_t(i) = A_t f(h_t(i))$

"All producers of good i hire type i-labor, and get that labor from a labor market that's distinct from type j-labor markets." This doesn't mean that firms have mon. power on these specific labor markets b/c they're still a continuum in each industry.

w/ specific factor markets (but no intermediate goods and no variation in desired markups)

$$\xi = \frac{\omega + \sigma^{-1}}{1 + \theta\omega}$$ exactly as in Preston.

Now take Chari et al (2000) assumptions for the values of these params: $\omega = 1.25$, $\sigma = 1$, $\theta = 10$

$$\xi = \frac{2.25}{1 + 12.5} = 0.1\overline{66}$$

Now all I need to do is to interpret 1) why specific factor gives this ξ (lower) 2) what is ω.

1) specific factor markets gives more strat comp in pricing. Maybe it's b/c one way of interpreting specific factor markets is that it's as if factors were "quasi-fixed" → if I can't adjust my quantity, I will find it more important to price as others are.

→ This is why higher strat comp "amplifies price stickiness" b/c if a fraction α of the economy can't adjust, then strat comp makes the other $1-\alpha$ fraction not to adjust as well!

2) What is ω?

The two conflicting things are these:

Basu	Woodford
$\sigma + \eta$ = El of MC wrt output	$\omega = \omega_w + \omega_p$
$= s_Y$ (and $s_y = 0$)	El. of MC wrt. $y(i)$
in Woodford's terms	ω_w = El of real wage demands ($\Rightarrow \eta$) wrt output $y(i)$
$= \zeta$	$\omega_p = (-1) \cdot$ Elasticity of MPL wrt output $y(i)$
$\Rightarrow \omega = \eta$	

w/ CRS, $MPL = 1$, $\omega_p = 0$ $\rightarrow \omega = \omega_w = \eta$

w/ DRS, MPL = decreasing fct of labor, so ω_p = some number

$\Rightarrow \omega = \eta + \omega_p$

I think that Woodford's ω takes out (doesn't include) u_c, which is why σ^{-1} only shows up in s_Y.

⇒ Ok so big conclusion for me:

1.) Need specific labor markets

Use Chari, Kehoe & McGrattan (2000) & Woodford 2003/2011

values of $\theta = 10$, $\sigma = 1$, $\omega = 1.25$ (w/ η inverse Frisch worked into ω I think)

2) I think Woodford is assuming DRS (I'm not sure and I'm not sure it's necessary to make that ass.)

I don't think Preston assumes anything here and again, I don't think it's necessary b/c ω (or s_y & s_Y) will subsume it.

So then

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\zeta \qquad \text{w/ } \zeta = \frac{\omega + \sigma^{-1}}{1+\omega\theta}$$

w/ $\theta = 10$, $\sigma = 1$, $\omega = 1.25$, $\beta = 0.95$, α = whatevs.

But even that isn't sufficient to get $gx_{3,2} > 0$ when $\rho_i \geq 0.8$

But it is if $\rho_i < 0.8$! So I set it to $\rho_i = 0.7$.

Follow Peter: check what happens if we add int rate smoothing!

→ it doesn't do anything b/c the value of $gx(i, \bar{i})$ remains the same!

To do:

- check the interest rate smoothing w/ $\rho > 0$ (idiot!)
- understand how the learning models behave especially when moving between $(\alpha, \rho_i) = (0.75, 0.7)$ & $(\alpha, \rho_i) = (0.5, 0.6)$

⇒ restudy dynamics of learning, together w/ expectations.

<u>Interpretation of IRFs</u> <u>1 Nov 2019</u>

1) RE: • ρ only makes IRFs hump-shaped but does nothing on impact
   • You can move ρ_i up to 0.78 if you ↑ α to 0.75.

2) Differences between learning IRFs in mat8 vs materials7:
   • not much, only anchoring is diff b/c ρ_i ↓ to 0.6 from 0.877

→ less shocks, more anchoring, so anchoring responses resemble decreasing gain ones.

- monpol shock looks diff b/c I decreased ρ_i and so $i \uparrow$ on impact

⇒ but it seems to me that we still get the same conclusions as in materials7:

- overshooting, especially for constant gain learning

Actually, what I find is that taking CI for the gain, all gain sequences are the same. This means that the shock didn't knock the econ out of whatever its gain sequence was. ⇒ Return to this!

- Moving ρ doesn't do a lot in the learning models b/c 1) dynamics was already sluggish
2) since we endog. choose d gain, ρ doesn't help us there

⇒ Need to understand 3 things:

1) What moves the endogenous gain / why don't indiv. shocks move it? → shock IS moving mean gain just not much

→ gain IRFs & equations

2) It seems to be equivalent to compare

a) dgain vs. cgain

b) endog. gain which was anchored vs. unanchored when the shock hit IF the shock doesn't influence the further evolution of the gain. But my feeling is that it should!

3) Overshooting: why is it happening in learning?

Is sthg like this happening:

time 1: shock hits, $\pi \uparrow$, $E_t^m(\pi_{t+1})$ hasn't yet responded

time 2: $E_2^m(\pi_3) \uparrow$, but $\pi \downarrow$

this is the key question

CEMP's criterion: $\theta_t^{CEMP} = |(1-\gamma)(\Gamma-1)\bar{\bar{\pi}}_t|$

mine: $\theta_t = |\text{shuff } \bar{\bar{\pi}}_{t-2} + \text{shuff } s_{t-1}|$

In CEMP: once you've converged, ain't nothing gon' move you out

But mine should be moved out when large enough shocks hit.

$\rightarrow$ and I think they are: the point is: not much.

1) tiny changes 2) they happen early in the sample (makes sense: once $\bar{\bar{\pi}} \rightarrow 0$, the shocks don't have a large chance to hit it out of anchoredness)

3) they seem even smaller b/c we're talking inverse gains: $|1-2| > |1-\frac{1}{2}|$

$\Rightarrow$ so actually, to do conf. bands is quite tough for the gain

My feeling is that overshooting may indeed be coming from a messed-up timing, and here's my hunch:

IRFs: for each t, I shock the econ at $t+1$
so I shouldn't subtract t from t?

✓ To do: plot $\bar{\pi}$ - IRF <u>4 Nov 2019</u>

"what happens to LR-expectations?

✓ figure out why gain IRF changed → it was b/c the shock δ was bigger :)

• timing: morning vs. evening fcsts, timing of IRF

→ 1) I'm doing it correctly: for $t = 2 \dots T-h$

• I impose the shock $\delta = 1$ at time t

• I subtract sim(w/shock) − sim(w/o shock)

for time: $t \dots t+h-1$ (to get a h-horizon IRF)

Now also evening fcsts adjust on impact too, which is correct, and by more than morning ones b/c $\bar{\pi}$ gets updated in the meanwhile

$\Rightarrow$ could it be that some deviation between the morning and evening fcst causes the overshooting?

A shock hits: $i\uparrow \rightarrow s_t \uparrow$

morning fcst $= \bar{\pi}_{t-1} + b \cdot s_t \downarrow$

$\Rightarrow (\pi, x, i)$ realized

$\Rightarrow \bar{\pi}$ updated to $\bar{\pi}_t$

evening fcst $= \bar{\pi}_t + b \cdot s_t \downarrow\downarrow$

But starting from $t=2$, there should be no surprises coming from s_t b/c b incorporates the P-matrix.

The thing is: observables are realized based on morning fcsts, but updating is done based on yesterday's evening fcst

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k^{-1}\Big(\pi_t - \overbrace{(\bar{\pi}_{t-1} + b s_{t-1})}^{f^e_{t-1}}\Big)$$

On impact $(t=1)$

$f^m_{t+1|t} > f^e_{t+1|t}$ b/c π was revised down

At time $t=2$

$f^m_{t+1|t} < f^e_{t+1|t} \Rightarrow \pi$ is being revised up

At time $t=3$ and onward the two are basically the same

But here's another observation:

- evening and morning fcsts 1-period ahead are always negative
- fa & fb for π are always negative

... so why on all earth do (π, x) revert to positive?

$\Rightarrow$ it makes no sense unless A_a^{LH}, A_b^{LH} (or A_s^{LH}) are wrong!

But the fact that this only happens for Cgain indicates that the problem is w/ the expectations, fa, fb and the fcsts ^{e} & ^{m}

It does come from expectations: 5 Nov 2019

when $\psi_\pi = 2$, and cgain yields oscillations, both 1-period ahead fcsts and fa & fb exhibit oscillations

It seems like what is happening is that cgain gets into oscillations after a shock – how long those last depend on ψ_π (the ↑ the longer).

Oscillations arise when morning & evening fcsts criss-cross. What induces differences between morning & evening fcsts? ⟶ the updating of $\bar{\pi}$

?⇅ Observables at t = $fcst^m_t(\downarrow \overline{\bar{\pi}_{t-1}}, \overset{\downarrow}{\overline{s_t}})$ when shock hits

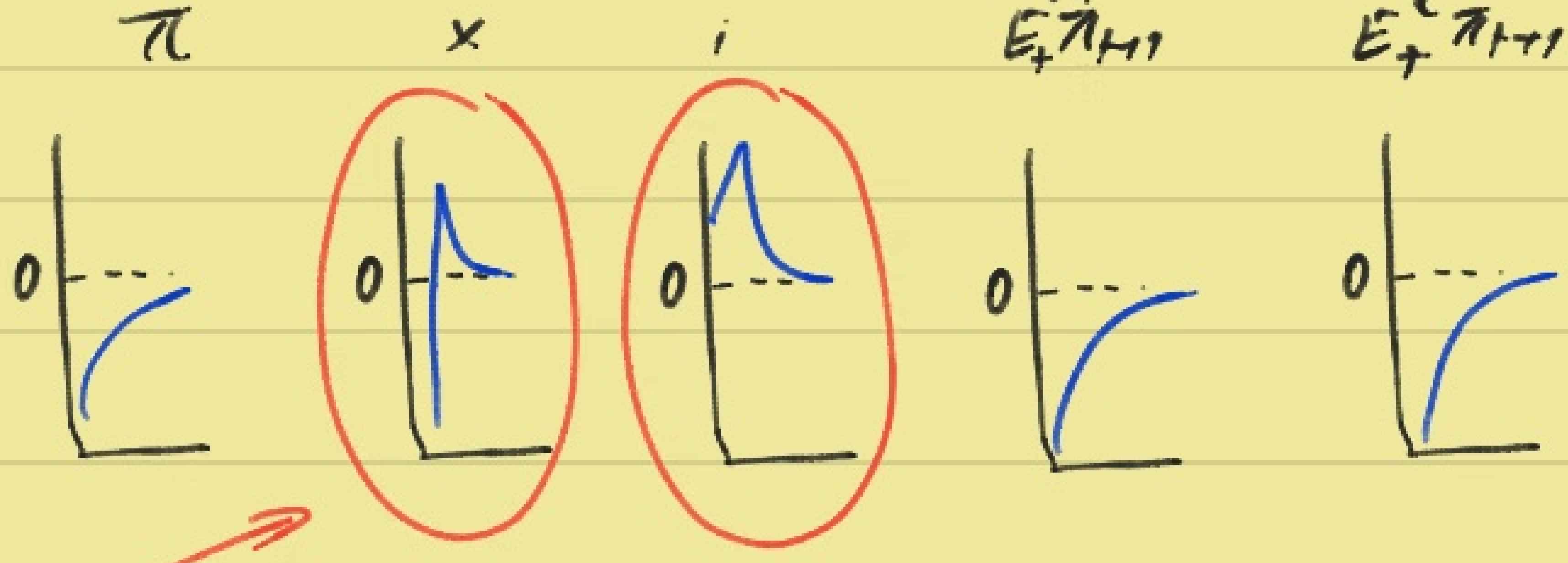
The period after

$\bar{\pi}_t$ = function$\left(fcst^e_{t-1}\left(\overline{\bar{\pi}_{t-1}}, \overline{s_{t-1}}\right)\right) \downarrow$

$fcst^e_t(\downarrow\bar{\pi}_t, \downarrow s_t)$ ↓↓

it's gotta be that at this stage, w/ cgain, your expectation "overadjusted", so you keep readjusting w/ opposite sign ⟹ so the oscillations make sense, but not (π, x) shooting into ⊕ territory!

Wait a sec ... for $\psi_\pi = 1.2$ I get a puzzling response



why is this happening? Why $x\uparrow$ when $i\uparrow$? And why does $i\uparrow$ in period 2 even though $\pi\downarrow$ and $\psi_x = 0$?

You'd think gx & hx aren't right, but this is only happening in the cgain world, so it has to do w/ expectations.

No cost-push shock.

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \right)$$

$$x_t = -\sigma i_t + \sum_{T=t}^{\infty} \beta^{T-t}\left[\underbrace{(1-\beta)\hat{E}_t x_{T+1}} + \sigma \underbrace{\hat{E}_t(\pi_{T+1})} \underbrace{- \sigma\beta \hat{E}_t(i_{T+1})} \right]$$

this needs to $\uparrow$ by more than $\downarrow$ and $\downarrow$

But that ain't happening, and in particular, $f_b \downarrow$ by a <u>lot</u>!

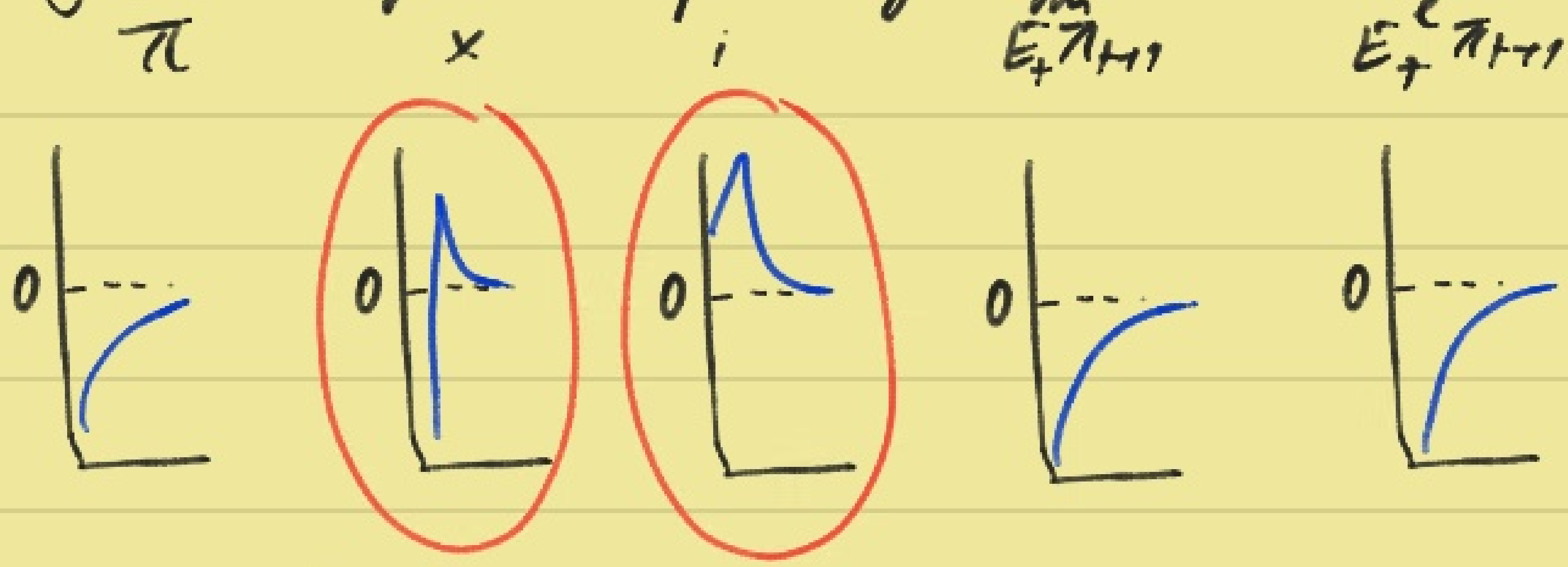
→ I'm starting to suspect that maybe a constant the

IRFs wrong (maybe only for cgain). Maybe a negative sign slips in there somewhere...

→ but that doesn't seem to be the case either!

Now I decreased $\bar{g}$ → even 0.1 decreases the sign-switch, and 0.01 tames things completely.

$\bar{g} = 0.05$ gives the following responses:



⇒ just like $\psi_\pi = 1.2$

Let's analyze A_a, A_b, A_s:

$$A_s = \begin{array}{c} \\ \pi \\ x \\ i_t \end{array} \begin{pmatrix} +\checkmark & -\checkmark & +\checkmark & -\checkmark \\ +\checkmark & -\checkmark & -\checkmark & -\checkmark \\ +\checkmark & +\checkmark & +\checkmark & +\checkmark \end{pmatrix} \begin{array}{c} r^n \\ i \\ u \\ i_{t-1} \end{array}$$

$$\begin{pmatrix} \pi \\ x \\ i \end{pmatrix} = \begin{array}{c} \begin{bmatrix} + & + & - \\ + & + & - \\ + & + & -? \end{bmatrix} \end{array} \begin{array}{l} \alpha\beta \\ \alpha \\ \beta \end{array} \cdot \hat{E}_t \begin{bmatrix} \pi \\ x \\ i \end{bmatrix}$$

*should be from a theoretical perspective

$$A_a = \begin{matrix} + & + & 0 \\ - & - & 0 \\ + & + & 0 \end{matrix}$$

$$A_b = \begin{matrix} - & + & 0 \\ - & + & 0 \\ - & + & 0 \end{matrix}$$

$A_b - A_a$:

$$\begin{matrix} - & - & 0 \\ + & - & 0 \\ - & - & 0 \end{matrix}$$

hmm...

↑ this may be why $i\uparrow$ when $\hat{E}(\pi)\downarrow$

this may be why $x\uparrow$ when $\hat{E}(\pi)\downarrow$

⇒ check g_{xa} (the $-\frac{b\psi_\pi}{w}$ part!)

⇒ check $g_{\pi b}$ (the $b - b\beta\psi_\pi$ part!) and g_{xb} (−||−) } responsible for $Ab(2{:}3, 1) < 0$

Check A_a & A_b (g_{xb} & $g_{\pi b} \to \boxed{z(1-\beta\psi_\pi)}$ are particularly important, and $g_{xa} \to \boxed{\frac{-z\psi_\pi}{w}}$ a little bit)

One troubling thing though is that Peter has exactly these values in preston2005-peter.

Setting all shocks to 0 for simplicity

$$x_t = -zi_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)x_{T+1} - z(\beta i_{T+1} - \pi_{T+1})\right] \quad (1)$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1}\right] \quad (2)$$

$$i_t = \psi_\pi \pi_t + \underbrace{\psi_x x_t}_{\text{set to 0 for simplicity}} \quad (3)$$

$$x_t = -z(\psi_\pi\pi_t) + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)x_{T+1} - z(\beta\psi_\pi\pi_{T+1} - \pi_{T+1})\right]$$

$$x_t = -z\psi_\pi\pi_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)x_{T+1} \boxed{- z(\beta\psi_\pi - 1)\pi_{T+1}}\right]$$

this also means that part of f_b has a ⊖ effect via expected $i\uparrow$

$$x_t = -z\psi_\pi\pi_t + \left[\boxed{z(1-\beta\psi_\pi)},\ (1-\beta),\ 0\right] f_\beta$$

$$\pi_t = \kappa x_t + \left[(1-\alpha)\beta,\ \kappa\alpha\beta,\ 0\right] fa$$

$$\to x_t = -z\psi_\pi\left[\kappa x_t + \left[(1-\alpha)\beta, \kappa\alpha\beta, 0\right] fa\right] + \left[z(1-\beta\psi_\pi), (1-\beta), 0\right] f_\beta$$

$$\underbrace{(1+z\psi_\pi\kappa)}_{=w} x_t = \underbrace{\boxed{-z\psi_\pi}\left[(1-\alpha)\beta, \kappa\alpha\beta, 0\right] fa}_{} + \left[z(1-\beta\psi_\pi), (1-\beta), 0\right] f_\beta$$

this means that x_t depends ⊖ on fa b/c higher fa implies higher i!

$$x_t = \boxed{\frac{-z\psi_\pi}{w}} [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + \frac{1}{w}[\boxed{z(1-\beta\psi_\pi)}, (1-\beta), 0] f_\beta$$

⊖ b/c E[i] ↑ ⊖ b/c E[i] ↑

$$\pi_t = \kappa\left[\frac{-z\psi_\pi}{w} [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + \frac{1}{w}[z(1-\beta\psi_\pi), (1-\beta), 0] f_\beta \right]$$
$$+ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a$$
$$= -\frac{\kappa z\psi_\pi}{w} [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + \frac{\kappa}{w} [z(1-\beta\psi_\pi), (1-\beta), 0] f_\beta$$
$$+ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a$$

$$\pi_t = \left(1 - \frac{\kappa z\psi_\pi}{w}\right) [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + \frac{\kappa}{w} [\boxed{z(1-\beta\psi_\pi)}, (1-\beta), 0] f_\beta$$

⊖ effect of f_b on x mediated here by $\frac{\kappa}{w}$

$z=1$. $w>2 \Rightarrow \max(\kappa z\psi_\pi) = \psi_\pi$
which is usually < 2
In that case this expression > 0
$\Rightarrow$ π depends ⊕ on f_a

However, if ψ_π crosses a threshold value, $\bar{\psi}_\pi$, then
$1 - \frac{\kappa z\psi_\pi}{w} < 0$ and π depends negatively on f_a !

$$1 < \frac{\kappa z\bar{\psi}_\pi}{w} \Rightarrow \frac{w}{\kappa z} < \bar{\psi}_\pi$$

$$\Rightarrow \frac{1 + z\psi_x + \kappa z\bar{\psi}_\pi}{\kappa z} < \bar{\psi}_\pi \qquad | z = 1, \psi_x = 0$$

$$\frac{1 + \kappa\bar{\psi}_\pi}{\kappa} < \bar{\psi}_\pi \Rightarrow 1 + \kappa\bar{\psi}_\pi < \kappa\bar{\psi}_\pi$$ ↯, oh, it can never happen!

Alright so let's recap:

$$x_t = \frac{-\kappa\psi_\pi}{w}[(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + \frac{1}{w}[\kappa(1-\beta\psi_\pi), (1-\beta), 0] f_\beta$$

x ⊖ on $\hat{E}(\pi, x)$ thru $E[i]$ via $\hat{E}[\pi]$ (!)

x ⊖ on $\hat{E}[\pi]$ b/c $E[i]$ but ⊕ on $\hat{E}[x]$

$$\pi_t = \underbrace{\left(1 - \frac{\kappa\kappa\psi_\pi}{w}\right)[(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a}_{\oplus \text{ effect of } \hat{E}(\pi, x) \text{ on } \pi} + \frac{\kappa}{w}[\kappa(1-\beta\psi_\pi), (1-\beta), 0] f_\beta$$

⊖ effect of $\hat{E}[\pi]$ on $\hat{E}[x]$ mediated by NKPC slope

2 notes:

1) $$\frac{\partial \frac{\kappa\kappa\psi_\pi}{w}}{\partial \psi_\pi} = \frac{\partial}{\partial \psi_\pi}(\kappa\kappa\psi_\pi)(1 + \kappa\psi_x + \kappa\kappa\psi_\pi)^{-1}$$

$$= \underbrace{\frac{\kappa\kappa}{1+\kappa\psi_x+\kappa\kappa\psi_\pi}}_{>0} \cdot (-1)\frac{\kappa\kappa}{(1+\kappa\psi_x+\kappa\kappa\psi_\pi)^2} < 0$$

So when $\psi_\pi \uparrow$, $\frac{\kappa\kappa\psi_\pi}{w} \downarrow$, so ⊕ effect of f_a on π stronger!

2) Is $\beta\psi_\pi < 1$ ever? For $\beta = 0.99 \rightarrow \psi_\pi < 1.0101$

So the point is that mon pol would have to be very weak on π for the effect that higher $\pi \rightarrow E[i] \uparrow \rightarrow x \downarrow$ to disappear!

~~An interne zzo:~~ 6 Nov 2019

~~Do materials8b.m to use sim_team.m to simply generate simulations of EE vs LH learning w/ constant & slope learning~~

→ Actually, scrap that – it's all implemented correctly in materials3!

$w = 1 + b\psi_x + kb\psi_\pi$ 7 Nov 2019

Then $\frac{-b\psi_\pi}{w} = -\frac{b\psi_\pi}{1+b\psi_x+kb\psi_\pi}$

$\frac{\partial}{\partial\psi_x}\left[\frac{b\psi_\pi}{1+b\psi_x+kb\psi_\pi}\right] = -\frac{b\psi_\pi}{(1+b\psi_x+kb\psi_\pi)^2} b < 0$

→ so $-\frac{b\psi_\pi}{w}$ ↑ (approaches 0 from below) when ψ_x ↑

$\frac{\partial}{\partial\psi_\pi}\left[\frac{b\psi_\pi}{1+b\psi_x+kb\psi_\pi}\right] = \frac{b}{w} \cdot (-1)\frac{b\psi_\pi}{w^2} kb < 0$

→ same for ψ_π ↑

it doesn't cancel out even when $\psi_x = 0$ b/c π ↑ when $E(x)$ ↑ and thus i ↑

x_t depends $\ominus$ on $\hat{E}_t(\pi_{t+h}, x_{t+h})$ no matter what, and ψ_x ↑ or ψ_π ↑ mitigate this a bit (especially ψ_x) but don't solve it.

$$1 - \frac{k\beta\psi_\pi}{w} = \frac{w - k\beta\psi_\pi}{w} = \frac{1 + \beta\psi_x + k\beta\psi_\pi - k\beta\psi_\pi}{1 + \beta\psi_x + k\beta\psi_\pi}$$

$$= \frac{1 + \beta\psi_x}{1 + \beta\psi_x + k\beta\psi_\pi} = C_{\pi, fa}$$

$$\frac{\partial C_{\pi,fa}}{\partial \psi_x} = \frac{\beta}{w}(-1) \cdot \frac{\beta\psi_x}{w^2}\beta < 0$$

$$\frac{\partial C_{\pi,fa}}{\partial \psi_\pi} = (-1)\frac{1+\beta\psi_x}{w^2} \cdot k\beta < 0$$

$\Rightarrow$ π_t depends $\oplus$ on $\hat{E}_t[\pi_{t+h}, x_{t+h}]$ (direct effect)
but this is $\downarrow$ the higher ψ_π & ψ_x (especially ψ_π).

$f_b \rightarrow$ direct effect of LH-E on x, indirect on π
$f_a \Rightarrow$ direct effect —"— on π, indirect on x

$$x_t = -\beta i_t + [\beta, 1-\beta, -\beta\beta] f_a \qquad + \text{shocks}$$

$$\pi_t = k x_t + [(1-\alpha)\beta, k\alpha\beta, 0] f_b \qquad + \text{shocks}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t \qquad + \text{shocks}$$

if $E[x] \uparrow \Rightarrow \pi_t \uparrow \rightarrow i \uparrow$

Why isn't this happening for dgain?

$-\frac{3\psi_\pi}{w} = -1.2656$ $\qquad (1-\alpha)\beta = 0.495$

$\kappa\alpha\beta = 0.0833$

Ahaa! But the point is that the diff between dgain and cgain is really f_b, not f_a

<u>Peter meeting</u> <u>7 Nov 2019</u>

You usually estimate $\sigma \ll 1$ (micro estimates of 1/4 or 1/5) (asset pricing gets $\sigma = 1/100$)

→ that can decrease the pricing response!

AD doesn't depend on hours if U is separable

Other ways of coping w/ inconsistency w/ balanced growth

e.g. leisure is h.h production

"in RE you can get away w/ ass. log utility but not in learning"

all of HANK research agenda
is motivated by the idea that NK relies
on intertemp subst, whereas there's no
evidence for it in data

Theory ahead of BC measurement
or Kydland & Prescott

→ "You can't just change $E(\cdot)$ and leave
params, you need to rethink
calibration"

<u>Work after</u> <u>7 Nov 2019</u>

- Basu SUM p. 1 p. 20 ⇔ Lect. 5 p. 8.

Equity premium puzzle implies $\sigma \approx 30$
(This σ^{-1} for me, that is $\sigma^{mine} \approx 1/30$.
The higher σ^{Basu} (lower σ^{mine}), the more risk-averse.

CES utility: $u(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \Rightarrow u'(C_t) = C_t^{-\sigma}$

Or in my formulation (Preston-Woodford's)

$$u(C_t) = \frac{C_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} \rightarrow u'(C_t) = C_t^{-\sigma^{-1}}$$

Back to Basu-notation: $u''(C_t) = -\sigma C_t^{1-\sigma} < 0$

$$\underbrace{\frac{\partial u''(C_t)}{\partial \sigma}}_{} = - C_t^{1-\sigma} \cdot \ \ldots\ldots \quad ?$$

call this "stuff"

Use "logarithmic derivative" (Simon & Blume p. 79 Mac)

to compute $\frac{\partial}{\partial \sigma} C_t^{1-\sigma}$

(19) S&B: $u'(x) = (\ln u(x))'\, u(x)$

Where does this come from?

Take $u(x)$ and $u'(x)$. $\mid \ln(\cdot)$

$\ln(u(x))$. $\mid$ Take $\frac{\partial}{\partial x}$

$\frac{1}{u(x)} \cdot u'(x)$ by chain rule. So:

$$\frac{\partial}{\partial x} \ln(u(x)) = \frac{u'(x)}{u(x)}$$

$$\Rightarrow \underline{\underline{u'(x) = \frac{\partial}{\partial x}\left[\ln(u(x))\right] \cdot u(x)}}$$

Derivative of a^x

Logarithmic derivative would say

$$\frac{\partial}{\partial x} a^x = \frac{\partial}{\partial x} \ln(a^x) \cdot a^x = \ln(a) \cdot a^x$$

Here we have $g(z) = -z\, c_+^{1-z}$ $\qquad f(z) = c_+^{1-z}$

$$f'(z) = \ln\left(c_+^{1-z}\right)' \cdot f(z)$$

$$= \left[(1-z)\ln(c_+)\right]' \cdot c_+^{1-z}$$

$$= -\ln(c_+) \cdot c_+^{1-z}$$

from PR

So then stuff $= \left[-\ln(c_+)\, c_+^{1-z}\right]\left[-c_+^{1-z}\right] > 0$

$$g'(z) = \text{stuff} = \ln\left(-z\, c_+^{1-z}\right)' \cdot \left(-z\, c_+^{1-z}\right)$$

$$= \left[\ln(-z) + (1-z)\ln(c_+)\right]' \cdot \left(-z\, c_+^{1-z}\right)$$

$$= \left[\frac{1}{-z}(-1) - \ln(c_+)\right]\left[-z\, c_+^{1-z}\right]$$

M tells me this is right

$$= \left[1 - z\ln(c_+)\right]\left[-c_+^{1-z}\right] > 0$$

I don't know, the two don't look the same, but both are > 0. $z\uparrow$, $u''(c) \to 0$ from below.

I wanted to define the IES and interpret its magnitude, but I'm not getting at it from Basu's SNM.

But there is sthg in Macro HW 1 Corr (1st Basu PS, p. 3)

NB. 2. $\text{sign}\left(\frac{\partial c_t}{\partial r}\right) = \text{sign}\left(\frac{1-\gamma}{\gamma}\right)$ Why γ?

$\Rightarrow$ b/c $r\uparrow$ has 2 effects on c, and γ decides which dominates:

wealth effect / income effect (IE):

since $r\uparrow$, your savings are worth more ($\rightarrow$ richer) $\rightarrow c_t\uparrow$ (and $c_{t+1}\uparrow$)

substitution effect (SE)

since $r\uparrow$, c_{t+1} is worth more $\rightarrow c_t\downarrow$ (and $c_{t+1}\uparrow$)

Wiki also says that IES is a measure for how the grt of cons, $\frac{c_{t+1}}{c_t}$, responds to r

IES $\rightarrow 0$
$\gamma > 1$ When $\gamma \rightarrow \infty$, only IE, $\Delta\frac{c_{t+1}}{c_t} = 0$ (same) b/c $\Delta c_t\uparrow = \Delta c_{t+1}\uparrow$

IES = 1
$\gamma = 1$ IE = SE $\Leftrightarrow \partial c/\partial r = 0$ $\Delta\frac{c_{t+1}}{c_t} > 0$

IES $\rightarrow \infty$
$\gamma < 1$ $\Delta c_t\downarrow$ & $\Delta c_{t+1}\uparrow$ $\Delta\frac{c_{t+1}}{c_t} > 0$ b/c the SE "slows" c_t down.

Let's be precise:

For $u(C_t) = \dfrac{C_t^{1-\gamma}}{1-\gamma}$

$IES = \frac{1}{\gamma}$ (so for me, $\gamma = IES$)

$CRRA = \gamma$ (so for me, $\frac{1}{\gamma} = CRRA$) when CES prefs.

Acemoglu's slides say

$$\varepsilon_u(c(t)) = -\frac{u''(c(t)) \cdot c(t)}{u'(c(t))}$$ is the inverse of IES.

He also writes $$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\varepsilon_u} \cdot (r(t) - \rho)$$

real int rate

discount rate

so finally! if this = IES,

- then the higher this value, the more the grt of cons reacts to $r \uparrow$ (IES $\to \infty$)
- ... the stronger the SE (relative to IE)
- ... the more C_t moves in opposite dir to C_{t+1}

In my (Pretoris's) world, this is $\gamma \to \infty \Rightarrow$ I need to kill the SE!

NKIS:

$$x_t = \text{stuff} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right) + \text{stuff}$$

$$x_t = \quad - \text{IES}\,(r_t) \quad + \text{stuff}$$

⇒ in the NK world, output responds negatively to r thru its effects on borrowing

Actually, reading Hall, 1988, JPE, the first few sentences, is very illuminating

"the IES = response of the change rate in C to changes in expected real int. rate"

"If cons. can be induced to postpone cons. by modest increases in the int. rates, then (1) movements in int rates will make cons decline whenever other components of AD rise"

For this channel to be at work in NK, where I = 0,

$SE \overset{!}{>} IE$ so that C↓ when r↑.

⟹ Yes, so I need a lower IES which lowers the SE.

Check Huo & Angeletos
↓ at Yale
his work (maybe JMP) is the one that solves the ∞ state-space problem of higher-order beliefs using frequency domain.

Ryan meeting 7 Nov 2019

1) Redo shocks & IRF for shock at $t=5$ & $t=25$ for 400 different underlying sequences of shocks → average across cross-section
→ need to see diff btwn cgain & dgain, or, they should be the same early on in sample!

2) wanna see FEs (IRFs)
they should flip sign for cgain for the criss-crossing to happen

3) gain $\bar{g}$ → play w/ it → Ryan has less priors on that
one way to choose is to see where the dgain is after 50 periods.