Materials 15 - More on the CEMP vs. CUSUM criteria

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Overview

1 Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \tag{3}$$

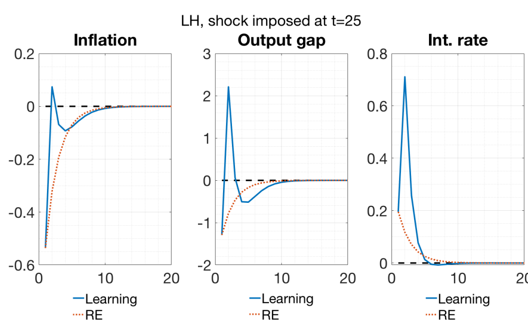
$$\hat{\mathbb{E}}_t z_{t+h} = \bar{z}_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\, h_x \qquad \text{PLM} \tag{4}$$

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} \underbrace{\big(z_t - (\bar{z}_{t-1} + b s_{t-1})\big)}_{\text{fcst error using (4)}} \tag{5}$$

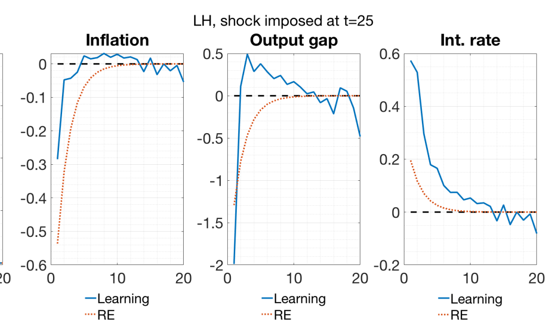
(Vector learning. For scalar learning, $\bar{z} = \begin{pmatrix} \bar{\pi} & 0 & 0 \end{pmatrix}'$. I'm also not writing the case where the slope b is also learned.)

$$k_t = \begin{cases} k_{t-1} + 1 & \text{for decreasing gain learning} \\ \bar{g}^{-1} & \text{for constant gain learning.} \end{cases} \tag{6}$$

Figure 1: Reference: baseline model



(a) Learning constant only



(b) Learning slope and constant

2 The CEMP vs. the CUSUM criterion

CEMP's criterion

$$\theta_t = |\hat{\mathbb{E}}_{t-1}\pi_t - \mathbb{E}_{t-1}\,\pi_t|/(\mathrm{Var}(\mathrm{shocks})) \tag{7}$$

$$\text{i.e.} \quad \text{PLM- E[ALM], scaled by shocks} \tag{8}$$

For my version of CEMP's criterion, I rewrite the ALM

$$z_t = A_a f_a + A_b f_b + A_s s_t \tag{9}$$

$$\text{as} \quad z_t = F + G s_t \tag{10}$$

$$\Leftrightarrow \quad z_t = \begin{bmatrix} F & G \end{bmatrix} \begin{bmatrix} 1 \\ s_t \end{bmatrix} \tag{11}$$

Then, since the PLM is $z_t = \phi \begin{bmatrix} 1 \\ s_t \end{bmatrix}$, the generalized CEMP criterion becomes

$$\theta_t = \max |\Sigma^{-1}(\phi - \begin{bmatrix} F & G \end{bmatrix})| \tag{12}$$

where Σ is the VC matrix of shocks. As for the CUSUM criterion, what I did in Materials 5 was

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1}(FE_t^2 - \omega_{t-1}) \tag{13}$$

$$\theta_t = \theta_{t-1} + \kappa k_{t-1}^{-1}(FE_t^2/\omega_t - \theta_{t-1}) \tag{14}$$

where FE_t is the most recent short-run forecast error ($ny \times 1$), and ω_t is the agents' estimate of the forecast error variance ($ny \times ny$). To take into account that these are now matrices, I now write

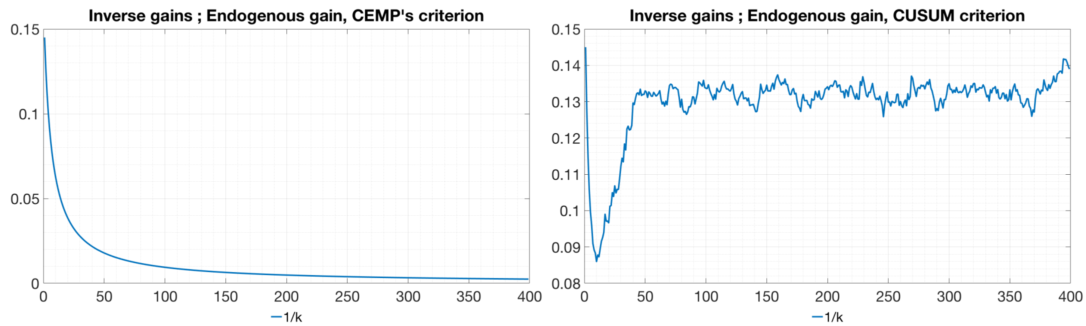
$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1}(FE_t FE_t' - \omega_{t-1}) \tag{15}$$

$$\theta_t = \theta_{t-1} + \kappa k_{t-1}^{-1}\mathrm{mean}((\omega_t^{-1} FE_t FE_t' - \theta_{t-1})) \tag{16}$$

3 Investigating the behavior of CEMP and CUSUM criteria

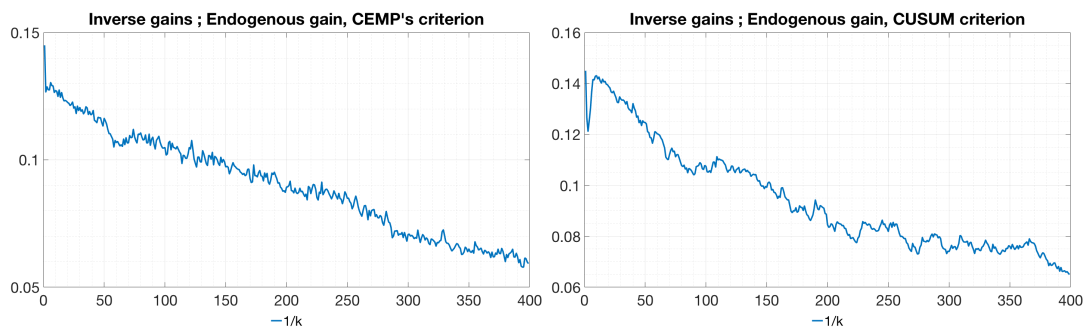
3.1 Anchoring as a function of ψ_π, fixing $\psi_x = 0, \bar{\theta} = 4, \tilde{\theta} = 0.2$

Figure 2: Inverse gains, $\psi_\pi = 1.01$



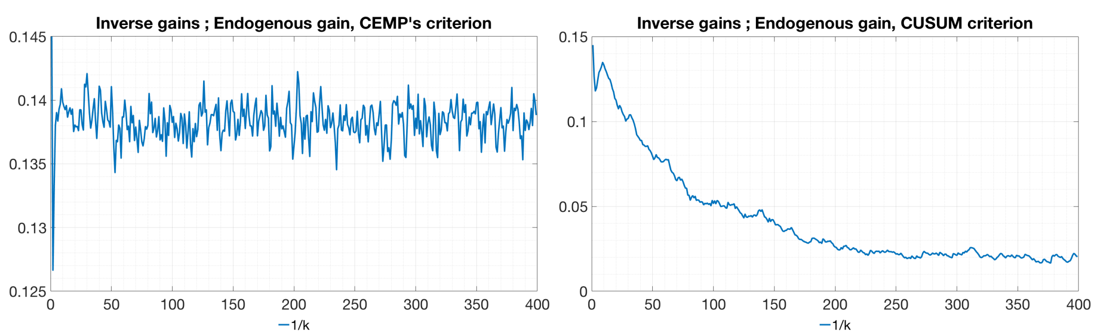
(a) CEMP's criterion (b) CUSUM criterion

Figure 3: Inverse gains, $\psi_\pi = 1.5$



(a) CEMP's criterion (b) CUSUM criterion

Figure 4: Inverse gains, $\psi_\pi = 2$



(a) CEMP's criterion (b) CUSUM criterion

3.2 Why do the two criteria behave opposite ways?

A rough restatement of the two criteria: you get unanchored expectations if:

$$\theta_t^{CEMP} = |(\phi - \begin{bmatrix} F & G \end{bmatrix})| > \bar{\theta} \qquad \text{vs.} \qquad \theta_t^{CUSUM} = f'\omega^{-1} f > \tilde{\theta} \tag{17}$$

where ϕ is the agents' estimated matrix, F, G are the ALM matrices that incorporate long-horizon expectations, f is the one-period ahead forecast error and ω is the estimated forecast error variance matrix. (Note: I'm using Lütkepohl's *Introduction to Multiple Time Series Analysis*, p. 160 to reformulate the CUSUM criterion as a statistic that has a χ^2 distribution.)

Here's the key difference between the two criteria:

- F, G incorporate LH expectations. Thus when ψ_π is large, F, G move a lot, opening up the gap between ϕ and itself, leading to unanchored expectations.
- f doesn't incorporate long-horizon expectations and thus doesn't move as much. In fact, when ψ_π is large, current inflation responds less, and thus one-period ahead forecast errors are *smaller*; you get more anchoring.