NEXT

26 Aug 2019

See Experimentation Notes 21 August 2019 Ryan Meeting

Benhabib, Schmitt-Grohe & Uribe (1999)

- challenges the conventional notion that active pol. (mon. pol that responds more than 1:1 to π) is stabilizing → it is only stable (unique) in a "very" local neighborhood of the st. st.

→ π fluctuates around its "stable" value for a while before converging to the passive st. st.
↳ makes emerging liq. traps hard to detect!

Needs ZLB.

↳ Is this a feature of learning models?

I said in the meeting that "ZLB episode should never have happened if beliefs were anchored"

→ I meant that $\pi = E(\pi)$, (kind of) and if $E(\pi) = 3\%$, π could never have gone down so low as to warrant $i = 0\%$.

→ Ryan said something like what if we were in the low-π period just τ periods too short, so that if it had persisted τ more periods, expectations had become unanchored and we'd never have gotten out?

2 options for Benhabib et al (1999):

- either learning models rationalize multiplicity for active policy
- or learning models offer a different explanation for the slide into liq. traps.

Davig & Leeper (2007)

Markov-process for Taylor-rule parameters

expectations-effects: even if you're in an active regime, if ppl. expect that you may switch, macro volatility ↑

You can get indeterminacy if passive regime is a) sufficiently permanent b) or sufficiently passive

↳ This is in spirit like unanchored E(·).

⇒ Regime-switching <u>increases</u> the local determinacy region b/c you can "store up on" hawkishness so you have more allowed "dovishness credit"

↳ regime-change on the policy-side

vs. CEMP: regime-change on the learning-side

Interpretations of US mon. history: 27 Aug 2019

- Benhabib et al (1999):

  It's not actually active, but it's spiralling into a liq. trap (:S)

- Davig & Leeper (2007)

  The 70's wasn't indeterminate b/c the LRTP (LR Taylor principle) holds, but there were large shocks that were amplified by policy.

- CEMP

  The 70's was due to large gains.

  In "Limits to Mon. Pol" C&P & Giannoni also argue that the gain was large mainly due to loose policy in the 70's (less due to shocks)

Inflation targeting countries:

New Zealand (1990)

Canada (1991)

UK (1992) (RPI of 2.5% → CPI of 2% since 2003)

SWE (1993 announced, applied 1995)

ECB 2%

US (2012) (2% PCE)

→ in the data, what I see is that CPI inflation came ≈5 yrs after the intro of π targeting.

Let's add that Svensson is essentially saying that the "LR-PC becomes non-vertical when infl. E(·) are anchored" i.e. he's saying that the u-π tradeoff survives into the LR then

→ money non-neutrality in the LR.

CEMP-view of π-development:

$$\pi_t = E^{LR}(\bar{\pi}) + \underset{\uparrow MC \ \nwarrow \text{mon. pol}}{\text{shocks}}$$

Svensson is saying

$$\pi_t = \overline{E^{LR}(\bar{\pi})} + \underset{\uparrow \text{MP didn't do enough!}}{\text{shocks}} < \text{what it should be}$$

$\Rightarrow$ so when $E(\cdot)$ anchored, MP very strong in terms of output gap / unemployment control.

$$\pi_t = E^{LR}(\pi_t - \pi^{Target}) + \text{shocks}$$

when unanchored, E^{LR} responds a lot to missing the target $\rightarrow$ overshooting gets amplified (& vice versa)

$\Rightarrow$ self-referentiality

$\Rightarrow$ self-referentiality seems to make money neutral faster (the LR arrives quicker / earlier)

$\Rightarrow$ unanchored makes MP weaker: bigger interventions are needed.

→ the "Svensson scenario" doesn't describe current <u>US monpol well</u>, though:

• $\pi < \bar{\pi}$ while $u < u^n$

Or does it? Can we think of a story in which

$$\pi_t = E^{LR}(\pi_t - \bar{\pi}) + \text{shocks} \qquad \text{(unanchored)}$$

② ↑ ↑ ④ ③ ↑ ① ↑

vs

$$\pi_t = E^{LR}(\bar{\pi}) + \text{shocks} \qquad \text{(anchored)}$$

② ↑ ① ↑

→ π doesn't increase as much while labor market effects are large!

But this is where the "where are they anchored?" comes in:

$$1.5\% = \underbrace{E^{LR}(1.5\% - 2\%)}_{<0} + \text{shocks}$$

<0 but maybe not large or persistent enough for expectations to adjust. → this story is harder

to tell if expectations are anchored at 3%,
a much higher level.

⇒ the question though also is "whose expectation"?

→ the 3% may be lower if the agg. econ's expectations overshoot less than those of HHs.

But ok, at least I can rationalize Svensson's story
– and maybe past errors need to be very big or very persistent for expectations to become unanchored.

⇒ and I've also rationalized why the ECB was scared of unanchoring of expectations during the crisis:

- spiral down
- loss of control

- ZLB

I agreed w/ myself that much larger MP shocks are necessary to move π if $E(\cdot)$ are <u>not</u> anchored.

→ it's poss. to get into ZLB w/ anchored beliefs by being unlucky:

$\pi_t = E^{LR}(\bar{\pi}) + \text{shocks}\downarrow$

→ and getting out should be a lot harder (require bigger MP shocks) if beliefs become unanchored b/c as long as $\pi_t < \bar{\pi}$

$\Rightarrow E^{LR} < 0$ which pushes $\pi \downarrow$

What if they get anchored again at a lower $\pi = 1\%$? Well then it's easy to get to $\pi = 2\%$ one-time, but it will take a persistent series of MP shocks to maintain that level unless you "unanchor" beliefs in order to shift the anchor to the correct place → this seems to be an "overshoot-risky" thing.

Divine coincidence

no tradeoff btwn output gap - & π stabilization

In CONP there's no demand side

↳ well now after the Peter meeting it feels like the DC doesn't hold: Fed trades off π-exp(·) ⇒ π vs. output in the SR!

Peter meeting 27 Aug. 2019

· diss-fells. 2 Prezi oct 1.

· Preston

· new dir: a learning model take on std issues:

i) infl targeting ii) credibility iii) effectiveness of MP

iv) "anomalies of Taylor-rules

Benhabib, Schmitt-Grohe & Uribe (1999)

global stability of TR: 2 eqb < active / passive

→ fall into Liq. traps

Davig & Leeper (2007)

LR-TP: expanded determinacy region when

monpol can be thought of regime-switching.

Preston

· Try a simul w/ only 1 source of randomness

or no randomness at all ($\bar{i}$ or r^n)

· − instead of a +.

To Benhabib et al:

Fed changes TR only around ZLB

→ we follow a TR but when i = 0%

we keep it at 0%.

→ nonlinearity in TR (Benhabib et al)

vs. we switch to sthg else at ZLB
(regime-switching)

Benhabib:

In RE, the bad eqb is the attractor: how much of that depends on RE ass? Would it be worse/better w/ learning?

Beware: global analysis w/ learning might be tough

Another: CEMP-exp.

Compared to the case of RE → the CB may be forced in some sense to deviate from a TR or adopt a diff TR w/ diff coefficients just to keep expectations anchored / maintain credibility

↳ tradeoff btwn π-management & credibility

→ sweep in Leeper

std TR

diff regime when unanchored

⇒ could be done using linear methods and simulation

"here's a MP rule that preserves credibility and here's a switching regime that gets the best of both worlds"

↳ and data (estimate using the gain-result from CEMP so you don't have to estimate the beast)

Erceg & Levin JME (2003)

Volcker-disinflation not explainable using RE

dynamics of (π, Y) cannot be explained

w/o a signal extraction on CB's target

Marvin Goodfriend (1993) "Interest rate policy

and the Infl. Scare Problem"

Credibility of Fed was called into question

→ so when FOMC took actions that

weren't justified by TR it's b/c they

wanna preserve credibility.

+ "whatever it takes" (Draghi)

"the Fed listens" Chicago conference

idea here (Powell & John Williams & Clarida)

"we don't wanna get into low E( )"

"better act now than later"

Leeper, Preston, Margaret Jacobson
"Recovery of 1933"
what ended the Deflation of Great Recession
FDR took over and I'm in charge of the Fed"
→ under those circumstances it's ok

<u>Work after</u>

<u>Erceg & Levin:</u> a DSGE model in which agents
try to disentangle permanent vs transitory shocks
to the inflation target

↳ Evans & Wachtel (1993): show using survey
data that persistent fcst errors aren't irrational,
instead they reflect uncertainty on the regime

<u>Goodfriend:</u> inflation scare: when market-based π-
expectations jump up (here: the LR int rate), indicating
low credibility of the Fed → Fed has to raise the FFR
to indicate its commitment to low π & maintain credibility

The main takeaway seems to be:

a diff take on US mon history (echoing Goodfriend's idea of an "inflation scare") is that under learning, there's a tradeoff btwn mon. objective and credibility → this can explain US mon history as well as the recent int. rate cut (July 2019) as signalling commitment to the 2% target.

→ Could demonstrate in an NK model that

- when anchored, a TR does fine
- when unanchored, a new rule does better

⇒ a hybrid rule that is regime-switching gets the "best of both worlds"

→ and this kind of behavior is what policy-makers are talking about.

⇒ would also shed light on the "flat NKPC" issue:

- when unanchored, not flat, but you fight to get anchoring
- when anchored, flat b/c $E(\pi)$ don't respond.

Let's return to the question 28 Aug. 2019

of <u>when monpol is powerful under learning</u>:

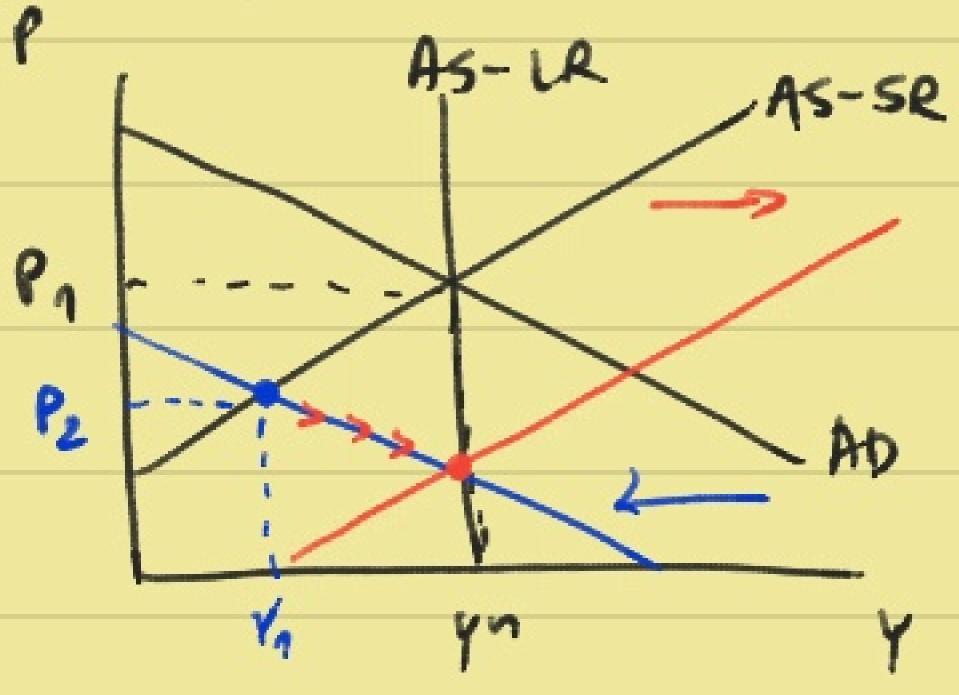
$$\pi_t = E^{LR}(\pi_t - \bar{\pi}) + \text{shocks}$$

anchored: can't move expectations

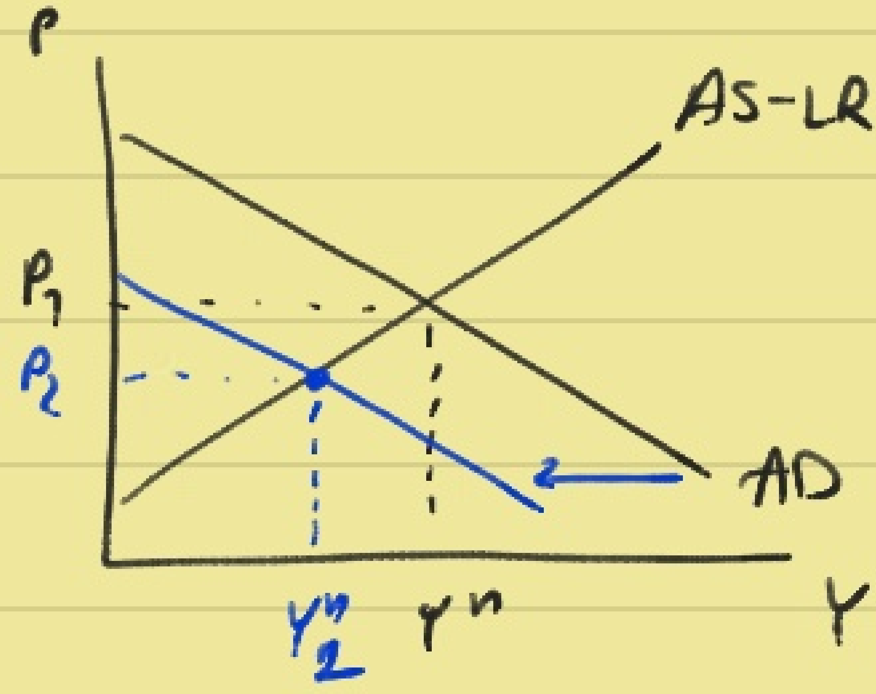
need bigger shocks to move π

(can be a blessing & a curse)

Unanchored



Anchored



$P_2 < E(P) \Rightarrow E(P)\downarrow$

$\Rightarrow$ AS-SR shifts R

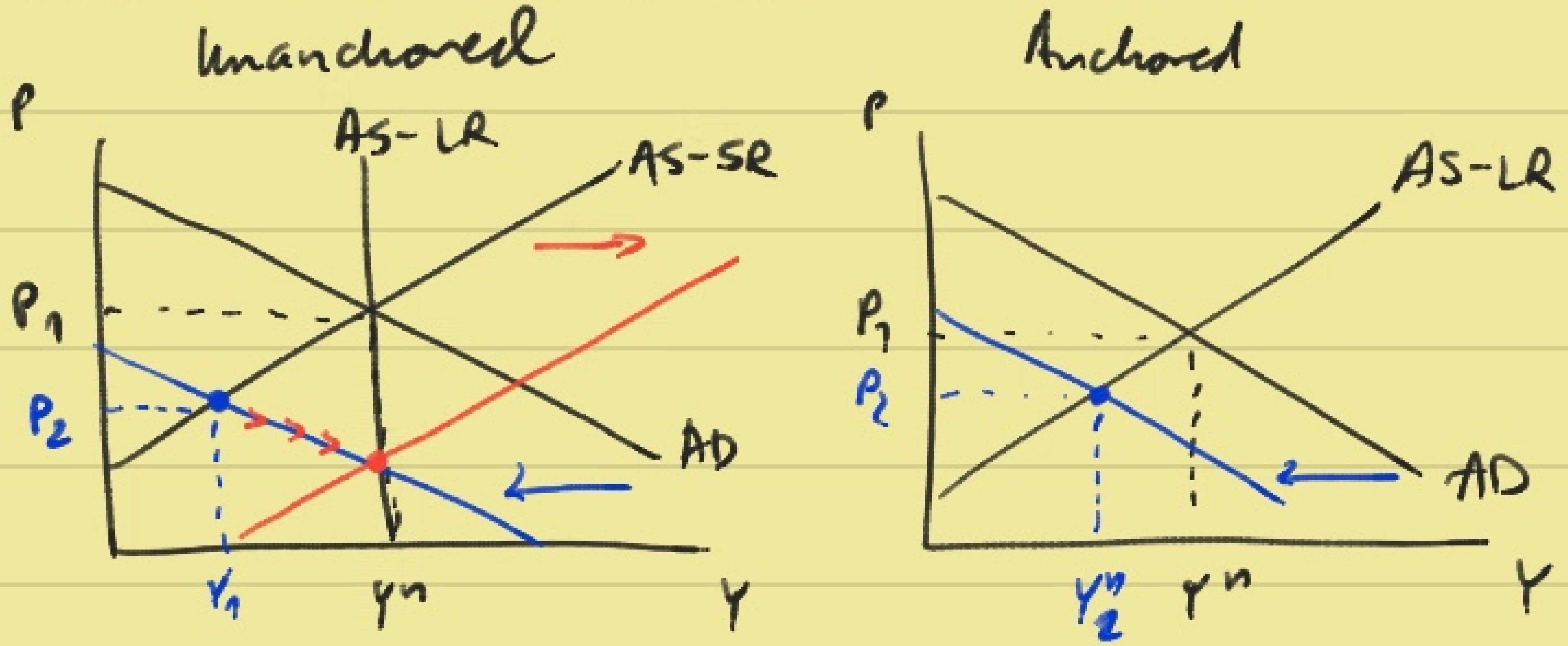
Technically, if one had ∞ shocks one could keep the output gap open forever

$\rightarrow$ but then they deanchor!

$\Rightarrow$ so <u>anchoring makes MP powerful</u> b/c you can

- keep the output gap positive for a longer time
- shift AD back to smooth a crisis before the extra deflation occurs.

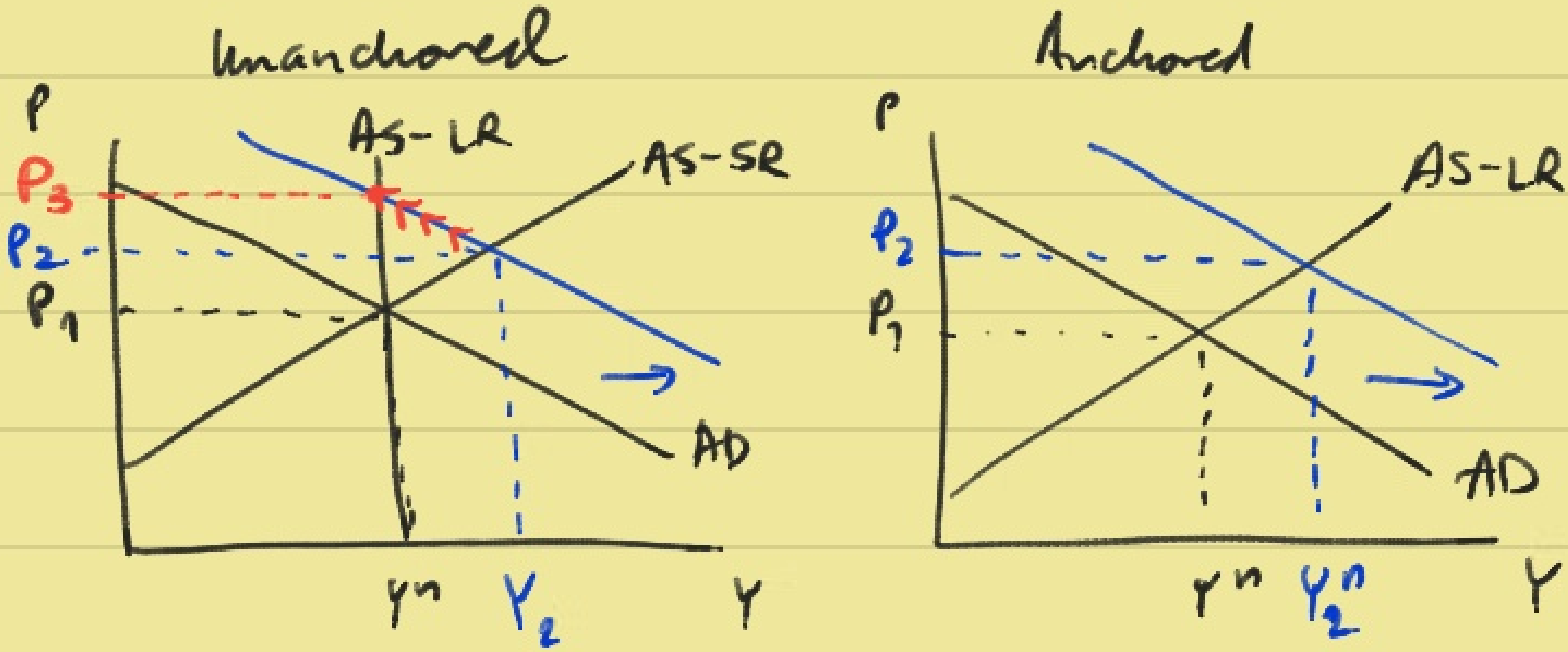
Let's look at this situ more:



Already here being anchored is two-sided:

- Curse: in principle the unanchored econ would go back to a higher Y^n, lower u^n situ by itself.
- Blessing: prices stay higher. And given that mon pol needs to intervene to get back to target π^* (and implicitly u^*, Y^* too), it's better that prices haven't fallen as much. Also, if unanchored, adjustment is costlier (you need bigger shocks to move AD by the same amount)

Let's look at the reverse situ: an inflation scare:



→ mon pol, when E(·) anchored, could "persistently" push up output (open the output gap). The problem is that there is a threshold surprise, or a threshold length of time of consecutive surprises such that E(·) deanchor → you set off inflation (red)

⇒ it is to keep that from happening that you raise int. rates and thus dampen momentum although you might not even have hit the bound → credibility

Ryan meeting 28 Aug 2019

Pol: • Inflation-targeting countries (some mixed evidence)

• Benhabib global → diff.

↕ opposition

• Davig & Leeper related though to Evans & H (2003)

A synthesis: investigate the tradeoff btwn stabilization & credibility ⇒ propose a modified TR under learning

Addresses: – current US policy

– current talk of anchoring

– flat NKPC

– ZLB a bit → a case for unconventional pol.

Doesn't address:

– divine coincidence (I don't know!)

– why E(·) didn't become unanchored in SWE / Riksbank undershooting

A confusion: anchoring doesn't mean in CEMP that SR-Exp. don't move!

The actual mechanism:

CB min $\mathcal{L}$ s.t. credibility stock

Try to do this:

- think through how a cost push shock works in an NK model
- do the same in an NK model w/ learning where the cost push shock affects π and thus also $E(\pi)$ → what's diff?

Note: a cost push shock is a shock to desired markups → it increases the wedge between perfect competition and mon. comp, and then the flex. price isn't efficient (b/c distortions). So we don't want to stabilize to flex prices b/c that outcome isn't optimal.

Work after

what are cost push shocks?

Ryan said: shocks to the desired level of the markup. (μ)

$$\mu^* = \frac{\theta}{\theta - 1} \quad \text{where } \theta = \text{el of subst between varieties.}$$

Peter in his 2004 NBER WP "Tech Shocks..." indeed defs it as

$$\ln(\theta_t) = (1 - \rho_\theta)\ln(\theta) + \rho_\theta \ln(\theta_{t-1}) + \varepsilon_{\theta,t}$$

→ i.e. ψ_t process of CEMP.

And this is what makes sense to me!

But Peter refers to Clarida, Galí & Gertler (1999), who define it as:

NKPC: $\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t$ (2.2)

which "captures anything else [than excess demand, which in turn is captured by the output gap, x_t] that might affect expected marginal costs."

or: u_t is the deviations from the condition $mc_t = \kappa x_t$.

Why are markups endog. in a sticky prices world?

And let's see the condition $mc_t = \kappa x_t$

- $\hat{Y}_t \uparrow \rightarrow W \& R \uparrow \rightarrow \hat{\varphi}_t\ (mc) \uparrow \Rightarrow \hat{\mu}_t \downarrow$
(b/c prices sticky)

- and Barro Sims 2, p. 47 & 49, we have

$$\hat{\varphi}_t = \hat{mc}_t = (\eta + b)\, \hat{x}_t$$

Further on p. 49:

target markup $= \frac{\theta}{\theta - 1}$ (always constant)

vs. actual markup $\hat{\mu}_t = -\hat{\varphi}_t$

25 Aug 2019

at the same location, I find the following.

"From PS we see that sticky prices, that is variable MC or markups, means that $\hat{P}^*_t \neq \hat{P}_t \quad \forall t$ (!)"

and the wedge between those is MC:

$$\hat{P}^*_t - \hat{P}_t = \hat{\varphi}_t \qquad (PS).$$

I think the argument is sthg like this:

w/ mon. competition, the PS is:

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta - 1} \varphi_t \qquad \text{(PS, Basu summ Part 2, p. 47)}$$

(already ass-ing symmetry btw i & j)

Loglin gives:

$$\hat{P}_t^* - \hat{P}_t = \hat{\varphi}_t$$

With flex prices, $\frac{P_t^*}{P_t} = 1$ ⇒ all firms set a price that's equal to the agg. price ⇒ $\hat{P}_t^* - \hat{P}_t = 0$ ⇒ MC constant!

2 reasons for confusion

- In Rotemberg, isn't $P_{it}^* = P_{jt}^* = P_t$?
- And it's fine that $\hat{P}_t^* - \hat{P}_t \neq 0$, but why does that change my <u>cost structure</u>?

↳ answer this might be ("CalvoOnceForAll.pdf")

$$\hat{s}_{it} = \hat{s}_t + \frac{1-\alpha}{\alpha}(\hat{y}_{it} - \hat{y}_t)$$

MC

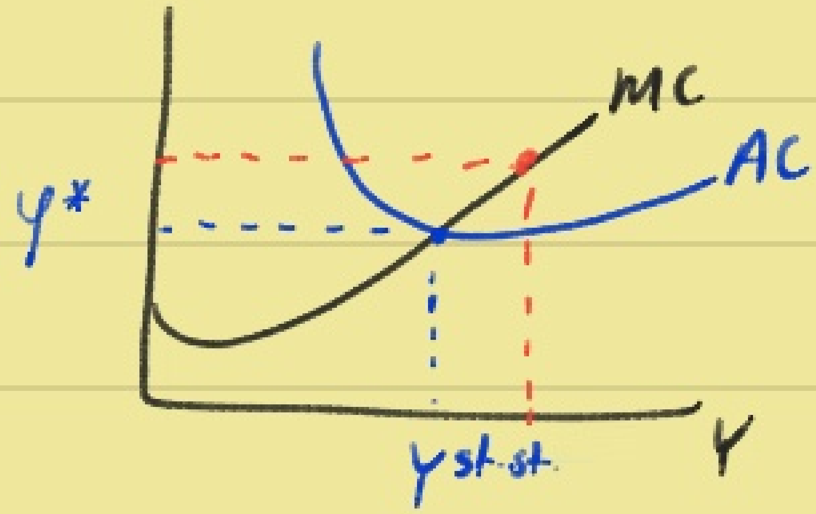
"my MC is that of everyone plus a demand term"

the demand term tho is: $\hat{y}_{it} - \hat{y}_t = -\theta(\hat{P}_{it}^* - \hat{P}_t)$

My demand is
- how much cheaper my good is $(\hat{P}_{it}^* - \hat{P}_t)$
- how much different the good is (θ)

↳ but that brings me back to the same question: Why does this change my cost?

→ I think I see actually: Recall from micro that MC is upward-sloping:



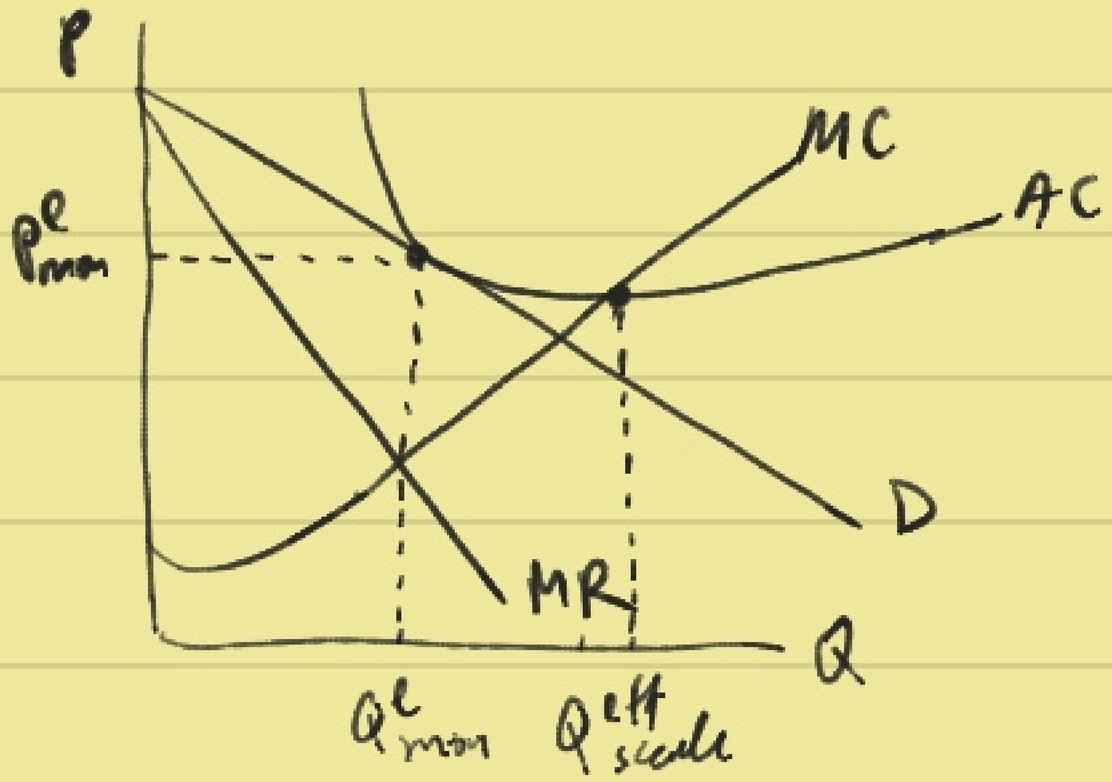
⟹ the higher the prod, the higher MC.

And there's a constant st. st. level.

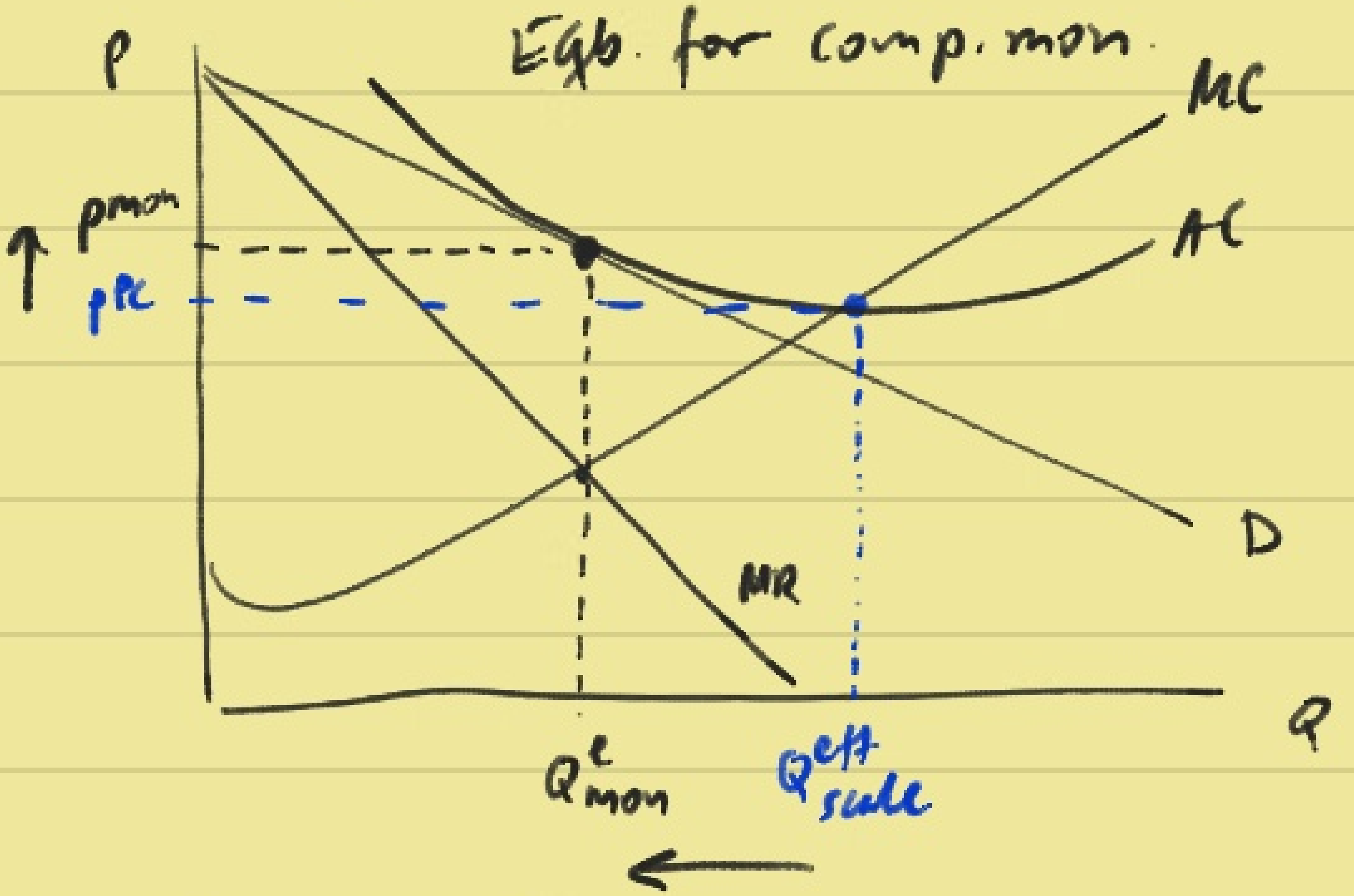
But when $\hat{P}^*_{it} \neq \hat{P}_t$, as is the case in Calvo, this causes $Y_{it} \neq Y_t$, i.e. your demand isn't = to the agg, and so your level of production isn't either.

If your prod. level exceeds the agg, so will your MC.

Let's go back to micro principles: competitive mon.

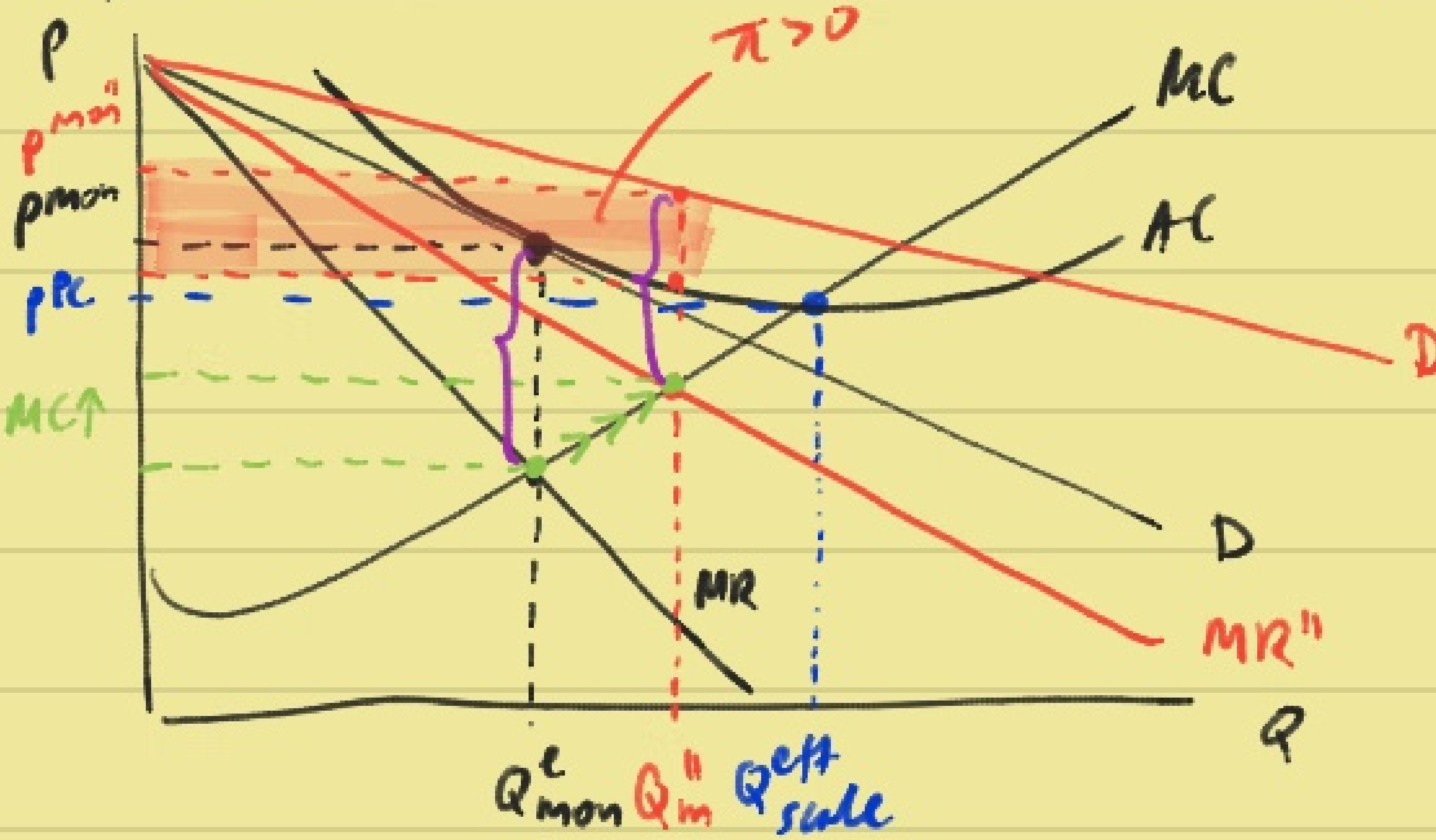


Let's compare that w/ perfect comp.



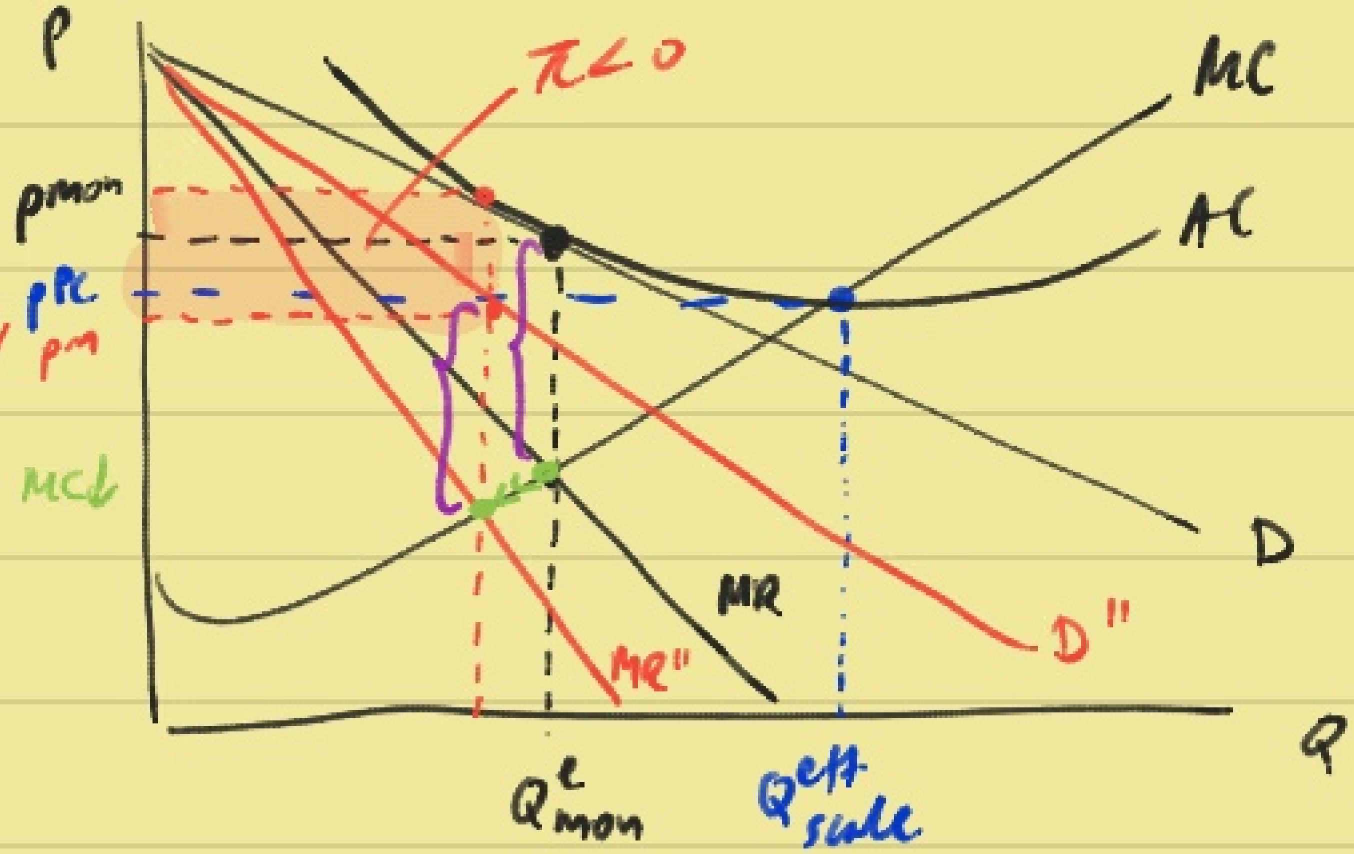
(p^{mon}, Q^{mon}) is the st. st. value.

I think Susanto's logic is that there are idio movements in Y_{it}, i.e. demand: here: $D\uparrow \rightarrow MC\uparrow \rightarrow \mu\downarrow$



Isn't this a feature of any imperfect comp. model?
→ yes, but w/ flex prices, eqb is restored immediately, whereas sticky prices keep the adjustment from happening right away.

Let's do a ⊖ demand shock now for fun:



D↓ → MC↓ → μ↑

⇒ ok so we know now why sticky prices introduce endog. time-varying markups / mc!

Both Eric Sims & Collard notes indicate more or less explicitly that in Rotemberg:

$$P_{it} = P_{jt} = P_t$$

Maybe the point is that this is an eqb condition, but out of eqb, it doesn't hold b/c indi. demands may be different, so the analysis of the previous pages carries through!

Ok so we've settled those 2 problems. So let's get back to the cost push shock.

We were kinda saying that 2 things can move mc:

- demand → moves "out-of-st-st" mc
- θ (el. of subst.) → moves st. st. mc

I think cost push refers to the latter based on Peter's def, and based on Clarida, Gali & Gertler's statement that it's "anything else than demand" that moves mc.

Clarida, Gali, Gertler (1999), Result 1:

with cost push shocks, there's a SR tradeoff btwn inflation and output variability.

→ so I think the divine coincidence doesn't hold!

And look at this: Basu Sum 2, p. 57

Divine coincidence in DNK model:

NKPC: $\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t$

if $\pi_t = 0$, $\Rightarrow \hat{x}_t = 0$!

Divine coincidence breaks down however if the NKPC has a shock...

... which is exactly how Clarida, Gali & Gertler define the cost push shock! It's

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + u_t$$

and I betcha it comes from the El. of substi!

and you know what... look at the fucking NKPC of CEMP:

$$\pi_t = \underset{\uparrow \text{ hybrid}}{\gamma \pi_{t-1}} + \underset{\text{expectation term}}{(1-\gamma)\Gamma \bar{\pi}_t} + \underbrace{\rho \varphi_{t-1} + \eta_t}_{\text{what's this?}}$$

$\Rightarrow$ it's a shock in the NKPC!!

Specifically, $\eta_t = \frac{\xi_p}{1-\beta\rho} \underset{\text{mc-shock}}{\varepsilon_t} + \mu_t$

$\mu_t = -\xi_p \frac{\hat{\psi}_t}{\bar{\psi} - 1}$

where ψ_t = el. substi of demand!

$\rightarrow$ η_t IS a cost-push shock! And this is what they mean by saying that their model doesn't distinguish between cost-push (μ_t) and mc-shocks (ε_t)!

So let's look at Clarida, Gali & Gertler in detail:

CB loss: $\max -\frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\alpha x_{t+i}^2 + \pi_{t+i}^2 \right] \right\}$ (2.7)

CB's problem:

max (2.7) s.t. (2.1) (NKIS) and (2.2) (NKPC).

Result 1 referred to optimal mon. pol under discretion.

Result 2 as well:

"Optimal mon. pol is a kind of "LR inflation targeting" in which π converges to the target only over time.

"Extreme inflation targeting", in which you reach the target immediately, is only optimal if

1) No cost-push shocks, or

2.) $\alpha = 0$ (weight on output gap)

Result 3 = Taylor-Principle

Result 4 The optimal policy has the int. rate move so as to:

1. offset demand shocks ($AD\uparrow \Rightarrow i_t \uparrow$)

$\Rightarrow$ b/c $i_t\uparrow$ pushes both $x_t\downarrow$ and $\pi_t\downarrow$

(no SR tradeoff here)

2. accomodate shocks to potential output ($\rightarrow \bar{i}_t$)

b/c $Y_t^n\uparrow$ but $Y_t\uparrow$ 1:1 (PIH) $\rightarrow x_t = 0$ still.

Result 5: If under discretion the CB wishes to have $Y_t > Y_t^n$, then we get higher inflation w/o a gain in output (inflationary bias)

A side note: in Basu Sum Part 2, p. 85:

"To get results qualitatively diff. from an RBC, we need suboptimal mon. pol b/c the Taylor-rule is too good!" → maybe that can change under learning?

Ok so let's think thru a cost-push shock in the DNK model. Would it look like the $\hat{\mu}_t \downarrow$-shock in Basu Sum 2, p. 7?

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + u_t \qquad \text{(NKPC)}$$

$$\hat{x}_t = \hat{E}_t x_{t+1} - \frac{1}{\sigma}\left[i_t - E_t \pi_{t+1}\right] + \frac{1}{\sigma} r_t^n \qquad \text{(NKIS)}$$

$$\hat{i}_t = \delta_\pi \pi_t + \delta_x \hat{x}_t \qquad \text{(TR)}$$

$$\kappa = \frac{(\eta+\sigma)(1-\omega)(1-\beta\omega)}{\omega}$$

ω = Calvo param.

η = Frisch

σ = IES param in $U(\cdot)$ fct.

θ = El. of subst. in demand

$$\rightarrow \mu = \frac{\theta}{\theta-1} \qquad \text{(I think } \theta > 1\text{)}$$

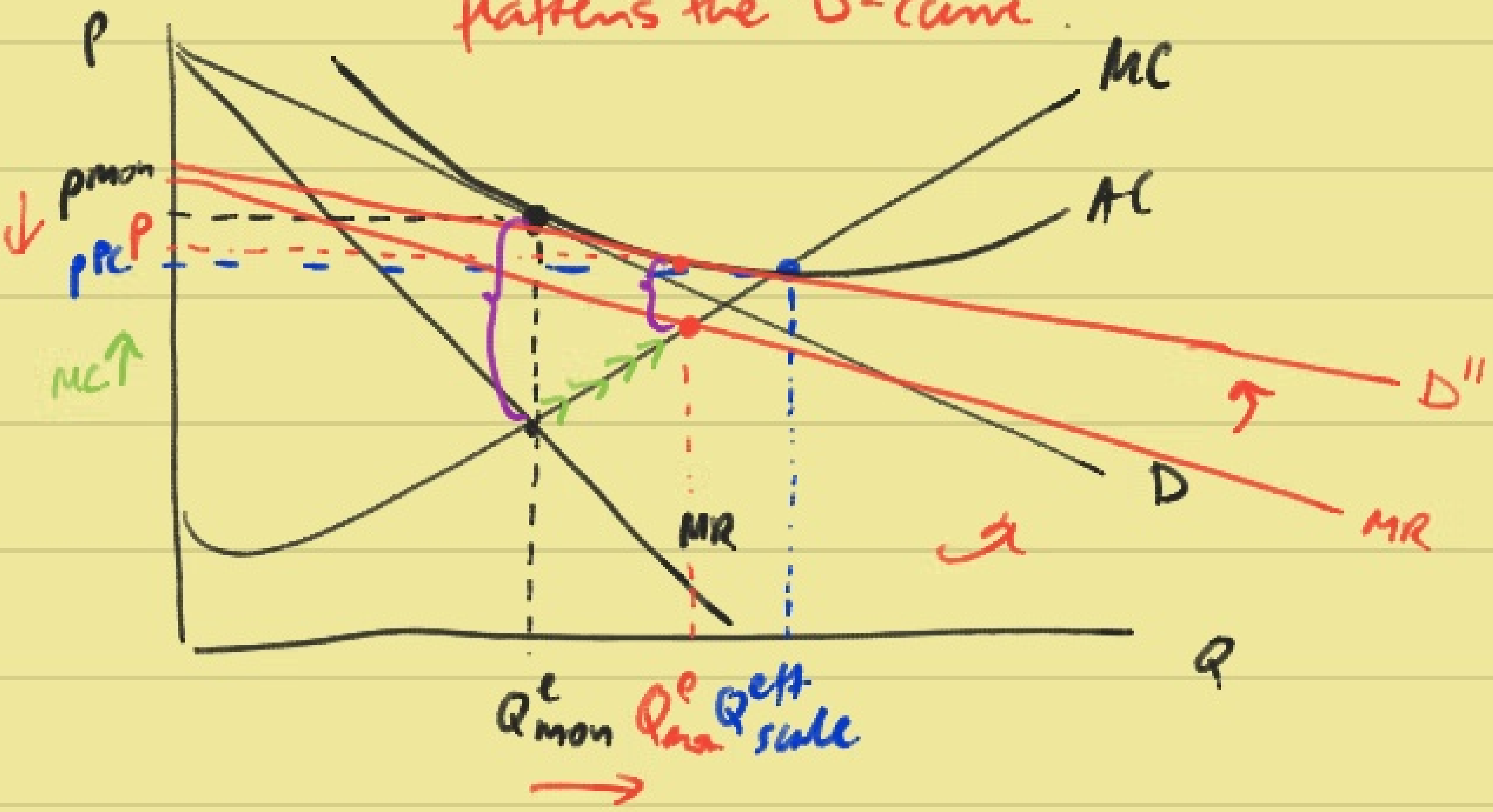
$$\frac{\partial \mu}{\partial \theta} = \underbrace{\frac{1}{\theta-1}}_{>0} + (-1)\frac{\theta}{(\theta-1)^2} = \frac{\theta-1-\theta}{(\theta-1)^2} = -\frac{1}{(\theta-1)^2} < 0$$

So when $\theta\uparrow \rightarrow \mu\downarrow$

which makes sense btw. b/c as $\theta \to \infty$, the goods b/c perfect substitutes and so you lose mon. power.

In our micro principles class, a $\theta\uparrow$ / $\mu\downarrow$ shock:

flattens the D-curve.



$\Rightarrow \mu\downarrow$

Now in the NK model:

$$\uparrow\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t\downarrow + u_t \uparrow \qquad \text{(NKPC)}$$

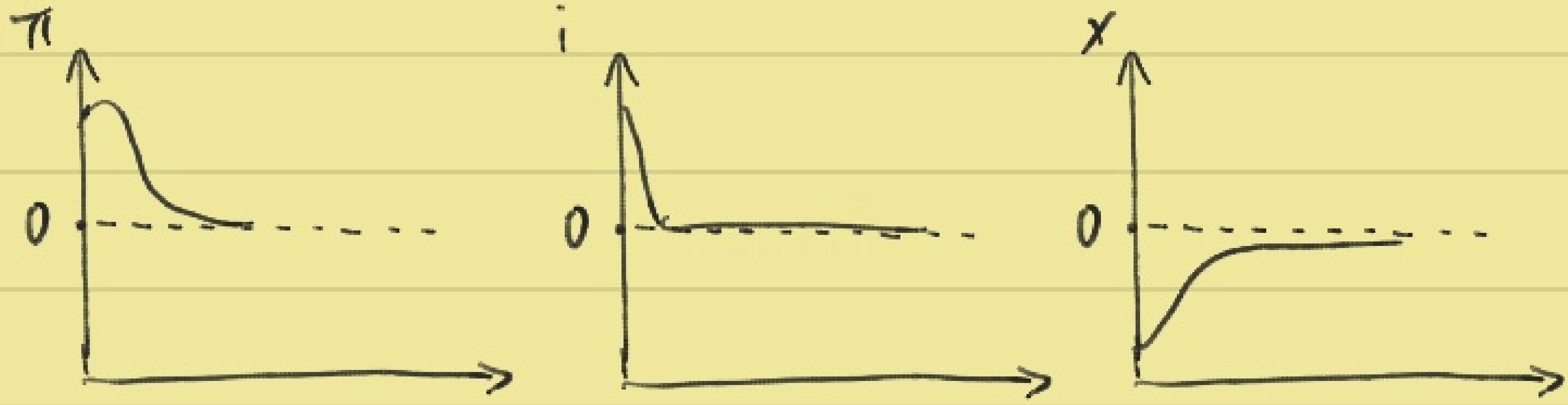
$$\downarrow\hat{x}_t = \hat{E}_t x_{t+1} - \frac{1}{\sigma}\left[i_t\uparrow - E_t \pi_{t+1}\right] + \frac{1}{\sigma} r_t^n \qquad \text{(NKIC)}$$

$$\uparrow\hat{i}_t = \delta_\pi \pi_t\uparrow + \delta_x \hat{x}_t \qquad \text{(TR)}$$

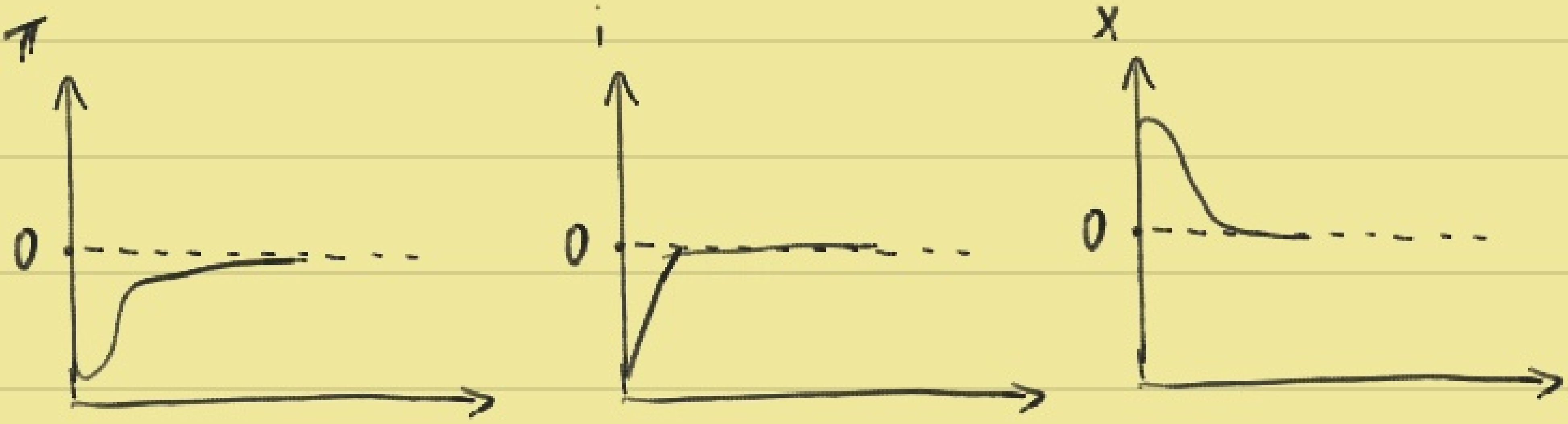
and stop.

(All shocks are one-time shocks here!)

So it would look like this in IRFs: $\theta\downarrow / \mu\uparrow$



$\theta\uparrow / \mu\downarrow$ (closer to perfect comp.)



It doesn't look like the μ-shock b/c here mon. pol. responds

But if there was no TR, we'd transition to a new eqb. w/ lower prices and higher output, so in that sense the θ/μ-shock does resemble the one on Basu p.7 in that it resembles a tech shock.

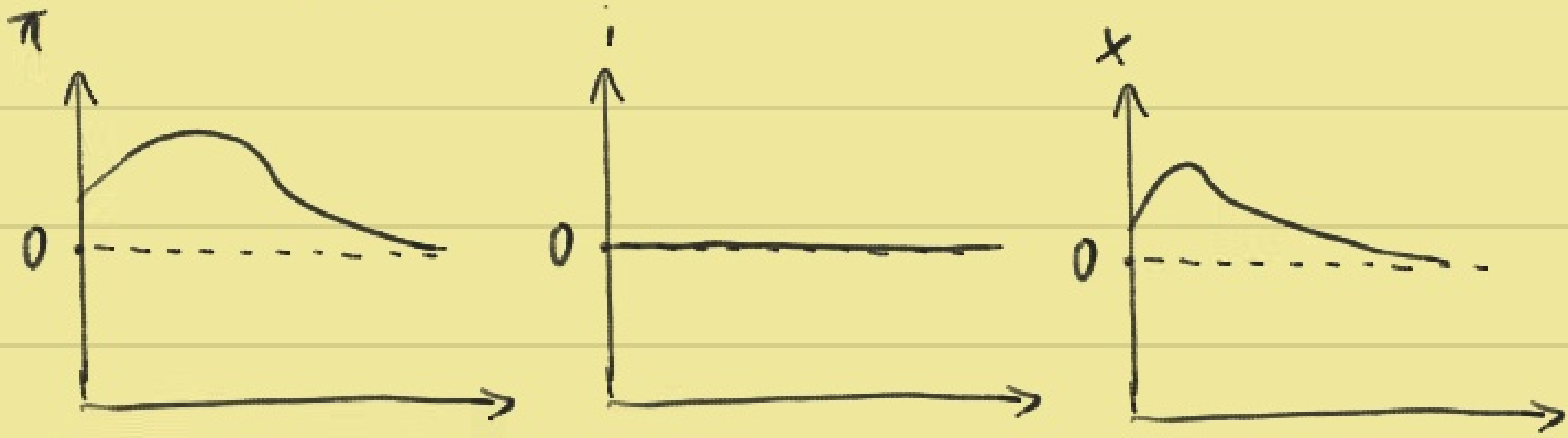
→ you can also see the SR nature of the (π, x)-tradeoff:

If mon. pol. doesn't respond:

$$\uparrow\uparrow \pi_t = \beta E_t \pi_{t+1} \uparrow + \kappa \hat{x}_t \uparrow + u_t \uparrow \qquad \text{(NKPC)}$$

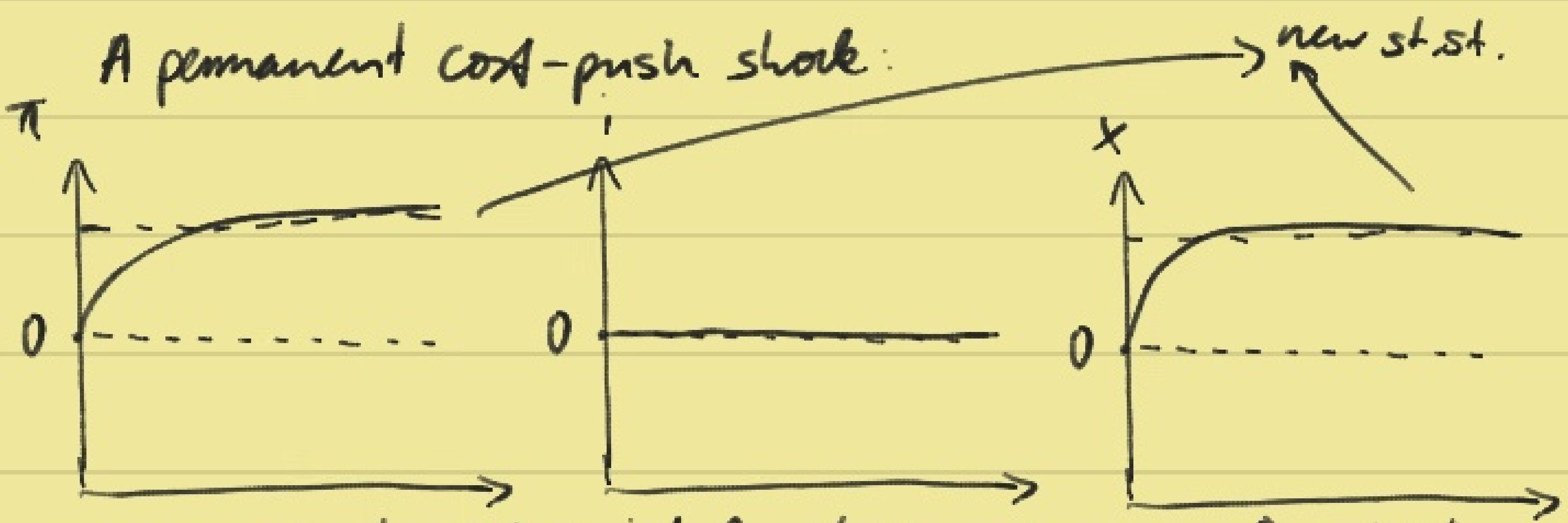
$$\uparrow \hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma}\left[i_t - E_t \pi_{t+1} \uparrow\right] + \frac{1}{\sigma} r_t^n \qquad \text{(NKIS)}$$

$$\hat{i}_t = \delta_\pi \pi_t + \delta_x \hat{x}_t \qquad \text{(TR)}$$



→ permanently higher price-level.

A permanent cost-push shock:



⇒ you don't want to interfere if the cost-push is "positive"

in the sense that it moves towards perfect competition.

However if it's a negative cost-push ($\theta\downarrow/\mu\uparrow$), the Fed can react

(and this is what Peter gets in his paper!)

the problem I'm now noticing is that here, I'm analyzing the SR ($\pi - x$)-tradeoff that happens when $\exists$ cost-push shocks in NK models, i.e. a failure of divine coincidence GIVEN that we want to close the output gap.

Ryan said however that we don't want to close the output gap in this case b/c the flex price is inefficient.

→ Eric Sims says that in the NK model $\exists$ 2 distortions

- SR distortion due to nominal frictions
- LR distortion due to imperfect comp.

"We ass the CB is concerned w/ the SR one and that the LR one has been taken care of w/ some Pigouvian tax".

→ and I think that's the std. way of business:
also in Basu we simply wanna close the output gap.
Also in absence of cost-push shocks I argue that y^{flex} is suboptimal (too low) in the NK model.
→ but you can't stabilize to it b/c the Phillips curve breaks down: the neutrality of money happens.

Justiniano & Primiceri, National Bank of Beligium.
Slides of Presi, 22 March 2010 "presentatic_..._pdf"

<u>Efficient output</u> := Y under perfect comp. (Y^{PC})

<u>Potential output</u> := flex price Y under imperfect comp (Y^{P}) and constant markups

<u>Natural output</u> := flex price Y under imperfect comp (Y^{n}) but time-varying markups

$Y^{PC} - Y^{n}$ → importance of nom. rigidities

$Y^{P} - Y^{n}$ → importance of exog. markup variation → cost-push! ($\pi - Y$) tradeoff for monpol!

If markup shocks (i.e. cost push) is interpreted as tech shocks:

$Y^p = Y^n$ (!)

Then prezi observes that while Y^p is very smooth (and close to actual observed Y), Y^n is crazy volatile, implying implausibly big markup shocks.

$\Rightarrow$ JP argue that much of this is meas. error, and show that a model w/ meas. error obtains $Y^p \approx Y^n$, and a much better fit to data.

(Since they consider wage markups, the idea of meas. error is plausible b/c wage series aren't great.)

If Y^p is smooth, it means that tech shocks aren't very volatile

If Y^n is volatile, it means that BCs are big $\Leftrightarrow$ markups fluctuate a lot. But IMO that doesn't need to come from cost-push exclusively: it can also come from demand!

My point is: mon pol cannot stabilize to Y^{PC} if there's imperfect comp. → If in addition there are time-varying markups, then Y^P is the only thing we can stabilize to. And to the extent that there are also nominal rigidities, we have time-varying markups.

What the output gap is depends on what we're stabilizing to:

E.g. if $X := Y^{PC} - Y_t$ then cost-push shocks don't affect Y^{PC} and we thus get a substantial output gap.

but if $X := Y^n - Y_t$ then since cost-push shocks affect Y^n, X doesn't open at all.

→ so we can really only stabilize to Y^P in a NK world.

Ok so I think I understand not too badly how the cost-push shocks work in the NK model

→ since they move markups / mc, they open up the output gap

Peter's results in the Tech Shocks in NK models paper bear out that cost-push shocks behave this way, and he also shows that they've played an important role (especially in inflation) fluctuations (Var. decomp).

⇒ So the task is: how does the model react differently under learning? CEMP:

$$\left.\begin{array}{l} k_t = f_k \\ \bar{\pi}_t = f_{\bar{\pi}} + f_k^{-1} \eta_{t-1} \end{array}\right\} \text{learning block}$$

Cost-push

$$\xi_t = f_\xi + A_\xi \xi_{t-1} + S_\xi \begin{pmatrix} \varepsilon_t \\ \mu_t \end{pmatrix} \left\{ \begin{array}{l} \text{exog sum of shocks } \eta_t \\ \text{exog shock: } y_t \\ \text{NKPC: } \pi_t \end{array}\right.$$

Right now there's no mon. pol (but effect of mon pol is on Γ in AS)

I want to approach the CEMP-version in three ways 30 Aug 2019

1.) Intuition 2) w/o mon. pol 3.) w/ mon. pol in the model

1) Intuition: there's a $\theta\downarrow / \mu\uparrow$ (less comp) cost-push shock

$$\uparrow\uparrow \pi_t = \beta E_t \pi_{t+1} \uparrow + \kappa \hat{x}_t \uparrow + u_t \uparrow \qquad \text{(NKPC)}$$

$$\uparrow \hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma}\left[i_t - E_t \pi_{t+1} \uparrow\right] + \frac{1}{\sigma} r_t^n \qquad \text{(NKIS)}$$

$$\hat{i}_t = \delta_\pi \pi_t + \delta_x \hat{x}_t \qquad \text{(TR)}$$

Note by the way that $\hat{x}_t > 0$ here b/c Y_t^n moved down, so it's actually not clear that we wanna close this gap!

But supp CB ignores that, & just closes.

Time-out: I think I haven't yet understood the tradeoff well:

CGG: discretion: $\max -\frac{1}{2}\left[\alpha x_t^2 + \pi_t^2\right] + F_t$

$$\text{s.t.} \quad \pi_t = \lambda x_t + f_t$$

$$\Rightarrow \max_x -\frac{1}{2}\left[\alpha x_t^2 + (\lambda x_t + f_t)^2\right] + F_t$$

$$\Rightarrow \max_x -\frac{1}{2}\left[\alpha x_t^2 + \lambda^2 x_t^2 + 2\lambda f_t x_t + f_t^2\right] + F_t$$

FOC: $-\frac{1}{2}\left[2\alpha x_t + 2\lambda^2 x_t + 2\lambda f_t\right] = 0$

$$(\alpha + \lambda^2) x_t \overset{!}{=} -\lambda f_t$$

$$x_t = \frac{-\lambda}{\alpha + \lambda^2} f_t$$

Most comp shock

$$\Rightarrow \quad x_t = -\frac{\lambda}{\alpha + \lambda^2}\left(\beta E_t \pi_{t+1} + u_t\right)$$

$$\Rightarrow \quad \pi_t = \lambda x_t + f_t = -\frac{\lambda^2}{\alpha + \lambda^2} f_t + f_t = \left(1 - \frac{\lambda^2}{\alpha + \lambda^2}\right) f_t$$

$$\pi_t = \frac{\alpha}{\alpha + \lambda^2} f_t$$

$$\rightarrow \quad \pi_t = \frac{\alpha}{\alpha + \lambda^2}\left(\beta E_t \pi_{t+1} + u_t\right)$$

$\Rightarrow$ a cost-push shock looks like a supply shock

- x & π move in opposite directions following a cost-push shock: if $u_t \downarrow \rightarrow x_t \uparrow$ & $\pi_t \downarrow$; if $u_t \uparrow \rightarrow x_t \downarrow$ & $\pi_t \uparrow$

  $\Rightarrow$ stabilizing π requires: $i_t \downarrow \Rightarrow x_t \uparrow\uparrow$ and $i_t \uparrow \Rightarrow x_t \downarrow\downarrow$

  IF expectations don't move: a tradeoff.

- Note: that $E(\cdot)$ moving makes this worse.

- In the LR I guess the tradeoff dies out as $E_t \pi_{t+1} \rightarrow 0$.

→ intuition: a mon. pol that can stop $E(\pi)$ from moving needs to use "less ammunition" to fight the cost-push shock than one that doesn't manage expectations.

Problem: how to manage $E(\pi)$?

- bigger cuts in i_t today → but then you're back to higher costs, except you pay them today instead of tomorrow (→ intertemporal tradeoff)
  - → this still is preferable though if $var(\pi)$ is costly per se!
- abandon TR: use communication to keep $E(\pi)$ "anchored"

⇒ if you have a "credibility stock" (low gain) you don't have to resort to any of these measures as long as you don't allow π to deviate from its st. st. value by too much or for too long.

It would just be nice to see the tradeoff w/ commitment too, that is, simply in the 3-eq DNK:

$$\downarrow\downarrow \pi_t = \beta E_t \pi_{t+1}^{\downarrow} + \kappa \hat{x}_t + u_t \downarrow \qquad \text{(NKPC)}$$

$$\uparrow\downarrow \hat{x}_t = \hat{E}_t x_{t+1} - \frac{1}{\sigma}\left[i_t^{\downarrow} - E_t \pi_{t+1}^{\downarrow}\right] + \frac{1}{\sigma} r_t^n \downarrow \qquad \text{(NKIC)}$$

$$\downarrow \hat{i}_t = \delta_\pi \pi_t \downarrow + \delta_x \hat{x}_t \qquad \text{(TR)}$$

Maybe the natural rate is affected? $r_t^n = \sigma E_t\left[y_{t+1}^f - y_t^f\right]$ $\downarrow$ $\uparrow$

b/c closer to P.C.

grrr....

The only thing that's clear is that policy $i_t \downarrow \rightarrow x_t \uparrow$ & $\pi_t \uparrow$

But we should have $\hat{x}_t \uparrow$ as a response to the shock.

Ok, let's just supp. that a TR is in place

- then, by virtue of propagating through the system of the economy (and only then) does a cost-push shock look like a supply shock

- Movement in EC) aggravates the problem and the tradeoff.

2) Now to CEMP w/o mon. pol:

$$\left.\begin{aligned} k_t &= f_k \\ \bar{\pi}_t &= f_{\bar{\pi}} + f_k^{-1}\eta_{t-1} \end{aligned}\right\} \text{learning block}$$

Cost-push

$$\xi_t = f_\xi + A_\xi \xi_{t-1} + S_\xi \begin{pmatrix} \varepsilon_t \\ \mu_t \end{pmatrix} \left\{ \begin{array}{l} \text{exog sum of shocks } \eta_t \\ \text{exog shock: } y_t \\ \text{NKPC: } \pi_t \end{array} \right.$$

Right now there's no mon. pol (but effect of mon pol is on Γ in AS)

Supp $\mu_t \downarrow$ ⇒ closer to P.C. cost-push shock.

⇒ $\eta_t \downarrow$ → $\pi_t(\eta_t) \downarrow$

→ 1.) anchored ($f_k^{-1} \to 0$): $E(\pi)$ don't move and that's it.

2) unanchored: $\bar{\pi}_{t+1} \downarrow$ ⇒ again,

the same intuition is straightforward to see

⇒ $E(\cdot)$ moving makes the stabilization costlier.

3) CEMP w/ mon. pol. "appended"

$$\left.\begin{aligned} k_t &= f_k \\ \bar{\pi}_t &= f_{\bar{\pi}} + f_k^{-1}\,\eta_{t-1} \end{aligned}\right\}$$ learning block

Cost-push $\rightarrow$ η_t

$$\xi_t = f_\xi + A_\xi \xi_{t-1} + S_\xi \begin{pmatrix} \varepsilon_t \\ \mu_t \end{pmatrix}$$

} exog sum of shocks η_t; exog shock: y_t; NKPC: π_t

$\downarrow \pi_t$ = stuff + $\uparrow$ demand (new term $\rightarrow x_t$)

+ $\uparrow x_t$ = stuff $-(i_t - E_t\pi_{t+1})$

+ TR: $\downarrow i_t = \delta_\pi \pi_t + \delta_x x_t$

} mon pol appended

Supp $\mu_t \downarrow$ $\Rightarrow$ closer to P.C. cost-push shock.

$\rightarrow \eta_t \downarrow \rightarrow \pi_t \downarrow$

$\rightarrow i_t \downarrow \rightarrow x_t \uparrow$ (mechanisms same as when mon. pol. isn't explicitly specified)

$\rightarrow$ Movement in $\bar{\pi}$ amplifies $\pi_t \downarrow$ & $x_t \uparrow$

$\Rightarrow$ again in such a case a higher int. rate drop is necessary, worsening the tradeoff

The point is: what we know about the effect 31 Aug 2019
of learning on mon. pol seems to be captured by
Eusepi & Preston (2018, JEL) & "On the Limits of Mon. Policy"
Conclusion 1: In the presence of LR expectations on real rates,
the divine coincidence doesn't hold.
Conclusion 2: Opt. mon. pol is either more aggressive (EP 2018,
Molnar-Santoro, Ferrero 2007) or less aggressive ("Limits")
on infl. than the RE counterpart.

This seems to be the research frontier. (I'm ignoring all the
technical results like E-stability (Evans & H, 2003) or
Taylor-principle for E-stab. (Preston 2005).)

Economically interesting:

- anchoring as credibility stock → maybe this can break the
divine coincidence.
- ZLB: anchoring gives you leeway when ZLB hits!

Ok – there's no way around it: generalize CEMP w/ a HH (that is demand-side) so we can specify a TR w/ ZLB

As a first step, investigate different (exog?) gain sequences to see what they mean for policy and welfare. → write out the social welfare fct as an approx to HH utility.

Would be neat: show that larger gains are c.p. welfare-decreasing due to
- larger volatility in π
- larger i-movements & thus volatility in x

In CEMP App. there is a IS-curve, but it doesn't feature LR-expectations, so I don't actually think it's the relevant one.

→ I think they use it in CEMP in order to avoid complication for the NKPC?

So "adapt" the Preston NKPC (eq 19) and NKIS (eq 18) for our purposes:

CEMP - AS:

$$(2)\quad \pi_t - \gamma \pi_{t-1} = \mu_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\xi_p s_T + (1-\alpha)\beta(\pi_{T+1} - \gamma \pi_T) \right]$$

Preston NKPC: eq (19)

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta \cdot x_{T+1} + (1-\alpha)\beta \pi_{T+1} \right]$$

⇒ The "hybrid NKPC" would be (my guess)

$$\pi_t - \gamma \pi_{t-1} = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta(\pi_{T+1} - \gamma \pi_T) \right]$$

$+u_t$ ← cost-push shock (Guess-NKPC)

CEMP demand side was just the s_t-process

Preston demand side is

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + r_T^n \right] \quad (18)$$

Let the TR be given by Preston's (27)

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

Now I just have to manipulate the shock processes so as to get an ALM that resembles CEMP's (eq. 6).

p. 144. Experimentation Notes 1.

And p. 166

The crux in CEMP is the expectation $\hat{E}_t(\pi_{T+1} - \gamma \pi_T)$

the PLM: $\pi_t = \gamma \pi_{t-1} + (1-\gamma)\bar{\pi} + \rho \varphi_{t-1} + e_t$

So $\hat{E}_t \pi_{T+1} = \gamma \pi_T + (1-\gamma)\bar{\pi} + \rho \varphi_T$

$$\Rightarrow \hat{E}_t(\pi_{T+1} - \gamma \pi_T) = \hat{E}_t\left[\cancel{\gamma \pi_T} + (1-\gamma)\bar{\pi} + \rho \varphi_T \cancel{- \gamma \pi_T}\right]$$

this is the crux which makes the eval. of this sum sooo much easier.

This way the ugly sum $(1-\alpha)\beta \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \hat{E}_t(\pi_{T+1} - \gamma \pi_T)$

simply becomes $(1-\alpha)\beta \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \hat{E}_t\left[(1-\gamma)\bar{\pi} + \rho \varphi_T\right]$

$$= \frac{(1-\alpha)\beta(1-\gamma)\bar{\pi}}{1-\alpha\beta} + (1-\alpha)\beta \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \rho \hat{E}_t \varphi_T$$

$\uparrow$ RE $= \rho^{T-t}\varphi_t$

$$= \frac{(1-\alpha)\beta(1-\gamma)\bar{\pi}}{1-\alpha\beta} + \frac{(1-\alpha)\beta}{1-\alpha\beta\rho}\varphi_{t-1}$$

So I'll need to design PLM and exog. processes such that I can do a trick like this. Otherwise I have to do the f_α, f_β-thing.

An alternative way is to take the Preston system (18), (19) & (27) but change the PLM to CEMP's (or sthg like it) (except I wanna allow for a cost-push shock in (19).)

So if the model is: 1 Sep 2019

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + r_T^N \right] \quad (18)$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta \, x_{T+1} + (1-\alpha)\beta \, \pi_{T+1} + u_T \right] \quad (19)$$

cost-push shock

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \quad (27)$$

$$\underbrace{}_{} = z_t = \text{function}(f\alpha, f\beta, f\sigma, \bar{i}_t) \quad (\text{RF LOM of econ})$$

then PLM needs to be sthg like CEMP's eq (5):

$$\pi_t = \gamma \pi_{t-1} + (1-\gamma)\bar{\pi}_t + \rho y_{t-1} + e_t \quad (5)$$

i.e. like: $z_t = \Gamma z_{t-1} + (I_3 - \Gamma)\bar{z}_t + \rho s_{t-1} + e_t \quad (\text{PLM}_1)$

where $s_t = \begin{bmatrix} r_t^N \\ \bar{i}_t \\ u_t \end{bmatrix}$ and $\rho = \begin{bmatrix} \rho_r & 0 & 0 \\ 0 & \rho_{\bar{i}} & 0 \\ 0 & 0 & \rho_u \end{bmatrix}$

with $\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} \cdot f_{t-1}$

$f_{t-1} = z_{t-1} - \hat{E}_{t-2} z_{t-1} \quad (3\times 1)$

So let's derive the ALM here, which is going to be identical to Peter's ALM in Preston, except for the presence of the cost-push shock (r_t): Plug i_t:

$$x_t = -\sigma(\psi_\pi \pi_t + \psi_x x_t + \bar{i}_t)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)x_{T+1} - \sigma\left(\beta(\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \bar{i}_{T+1}) - \pi_{T+1}\right) + \overset{\sigma}{r_T^N}\right]$$

$$(1+\sigma\psi_x)x_t = -\sigma\psi_\pi \pi_t - \sigma\bar{i}_t$$
$$+ \hat{E}_t \sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta-\sigma\beta\psi_x)x_{T+1} - \sigma[\beta\psi_\pi - 1]\pi_{T+1} - \sigma\beta\bar{i}_{T+1} + \overset{\sigma}{r_T^N}\right]$$

One realization: in the Bayes SUM, we saw that on the NKIS, whatever is the coeff. on i_t, r_t^N gets the same coeff w/ the opposite sign! So likely Preston wrote the IS-curve wrong, but this explains why the coeffs of r_t are $-\sigma$ and σ!

$$(1+\sigma\psi_x)x_t = -\sigma\psi_\pi\left[\kappa x_t + \hat{E}_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[\kappa\alpha\beta\, x_{T+1} + (1-\alpha)\beta\,\pi_{T+1} + u_T\right]\right]$$
$$+ \hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta-\beta\sigma\psi_x)x_{T+1} + (\sigma - \sigma\beta\psi_\pi)\pi_{T+1} - \sigma\bar{i}_T + \sigma r_T^N\right]$$

$$\underbrace{(1+\sigma\psi_x - \sigma\psi_\pi\kappa)}_{=:W}x_t = -\sigma\psi_\pi\left[(1-\alpha)\beta,\ \kappa\alpha\beta,\ 0\right]f_\alpha - \sigma\psi_\pi\sum(\alpha\beta)^{T-t}u_T$$
$$+\left[\sigma(1-\beta\psi_\pi),\ (1-\beta-\beta\sigma\psi_x),\ 0\right]f_\beta \qquad - \sigma\hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}(\bar{i}_T + r_T^N)$$

$$x_t = -\frac{\sigma\psi_\pi}{W}\left[(1-\alpha)\beta,\ \kappa\alpha\beta,\ 0\right]f_\alpha + \frac{1}{W}\left[\sigma(1-\beta\psi_\pi),\ (1-\beta-\beta\sigma\psi_x),\ 0\right]f_\beta$$
$$-\frac{\sigma\psi_\pi}{W}\sum(\alpha\beta)^{T-t}\hat{E}_t u_T - \frac{\sigma}{W}\sum\beta^{T-t}\hat{E}_t(\bar{i}_T - r_T^N)$$

✓

$$\pi_t = K\Big\{-\frac{b\psi_\pi}{w}\left[(1-\alpha)\beta,\ \kappa\alpha\beta,\ 0\right]f\alpha + \frac{1}{w}\left[b(1-\beta\psi_\pi),\ (1-\beta-\beta b\psi_x),\ 0\right]f_\beta$$

$$-\frac{b\psi_\pi}{w}\sum(\alpha\beta)^{T-t}\hat{E}_t u_T - \frac{b}{w}\sum\beta^{T-t}\hat{E}_t(\bar{i}_T + r_T^N)\Big\}$$

$$+\hat{E}_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[\kappa\alpha\beta\cdot x_{T+1} + (1-\alpha)\beta\,\pi_{T+1} + u_T\right]$$

$$= -\frac{Kb\psi_\pi}{w}\left[(1-\alpha)\beta,\ \kappa\alpha\beta,\ 0\right]f\alpha + \frac{K}{w}\left[b-b\beta\psi_\pi,\ (1-\beta-\beta b\psi_x),\ 0\right]f_\beta$$

$$-\frac{b\psi_\pi K}{w}\sum(\alpha\beta)^{T-t}\hat{E}_t u_T - \frac{Kb}{w}\sum\beta^{T-t}\hat{E}_t(\bar{i}_T + r_T^N)$$

$$+\left[(1-\alpha)\beta,\ \kappa\alpha\beta,\ 0\right]f\alpha + \hat{E}_t\sum(\alpha\beta)^{T-t}u_T$$

$$\pi_t = \left(1-\frac{Kb\psi_\pi}{w}\right)\left[(1-\alpha)\beta,\ \kappa\alpha\beta,\ 0\right]f\alpha + \frac{K}{w}\left[b-b\beta\psi_x,\ (1-\beta-\beta b\psi_x),\ 0\right]f_\beta$$

$$+\left(1-\frac{Kb\psi_\pi}{w}\right)\sum(\alpha\beta)^{T-t}\hat{E}_t u_T - \frac{Kb}{w}\sum\beta^{T-t}\hat{E}_t(\bar{i}_T \cancel{+} r_T^N) \quad \checkmark$$

The error terms:

$$\sum(\alpha\beta)^{T-t}\hat{E}_t u_T = \sum(\alpha\beta)^{T-t}[0\ 0\ 1]\hat{E}_t S_T = \sum(\alpha\beta)^{T-t}[0\ 0\ 1]\rho^{T-t}S_t$$

I hope this is allowed $= \sum(\alpha\beta\rho)^{T-t}[0\ 0\ 1]S_t = \cancel{[I_3 - \alpha\beta\rho]^{-1}[0\ 0\ 1]S_t}$

should be $[0\ 0\ 1][I_3 - \alpha\beta P]^{-1}S_t$

$$\sum\beta^{T-t}\hat{E}_t(\bar{i}_T + r_t^N) = \sum\beta^{T}\hat{E}_t[1\ 1\ 0]S_T = \sum(\beta\rho)^{T-t}[1\ 1\ 0]S_t$$

$$= [I_3 - \beta\rho]^{-1}[1\ 1\ 0]S_t$$

$[-1\ 1\ 0][I_3 - \beta P]^{-1}S_t$

So then

$$x_t = -\frac{b\psi_\pi}{w}\left[(1-\alpha)\beta,\ \kappa\alpha\beta,\ 0\right] f_\alpha + \frac{1}{w}\left[b(1-\beta\psi_\pi),\ (1-\beta-\beta b\psi_x),\ 0\right] f_\beta$$
$$\quad -\frac{b\psi_\pi}{w}\left[I_3-\alpha\beta P\right]^{-1}\left[0\ 0\ 1\right] s_t - \frac{b}{w}\left[I_3-\beta P\right]^{-1}\left[1\ 1\ 0\right] s_t$$

$$\pi_t = \left(1-\frac{\kappa b\psi_\pi}{w}\right)\left[(1-\alpha)\beta,\ \kappa\alpha\beta,\ 0\right] f_\alpha + \frac{\kappa}{w}\left[b-b\beta\psi_\pi,\ (1-\beta-\beta b\psi_x),\ 0\right] f_\beta$$
$$+\left(1-\frac{\kappa b\psi_\pi}{w}\right)\left[I_3-\alpha\beta P\right]^{-1}\left[0\ 0\ 1\right] s_t - \frac{\kappa b}{w}\left[I_3-\beta P\right]^{-1}\left[1\ 1\ 0\right] s_t$$

$\Rightarrow$

$$x_t = \underbrace{-\frac{b\psi_\pi}{w}\left[(1-\alpha)\beta,\ \kappa\alpha\beta,\ 0\right]}_{=:g_{xa}\ ✓} f_\alpha + \underbrace{\frac{1}{w}\left[b(1-\beta\psi_\pi),\ (1-\beta-\beta b\psi_x),\ 0\right]}_{=:g_{xb}\ ✓} f_\beta$$
$$+\underbrace{\left[-\frac{b\psi_\pi}{w}\left[I_3-\alpha\beta P\right]^{-1}\left[0\ 0\ 1\right] - \frac{b}{w}\left[I_3-\beta P\right]^{-1}\left[1\ 1\ 0\right]\right]}_{=:g_{xs}} s_t$$

$$\left(-\frac{b\psi_\pi}{w}\left[0\ 0\ 1\right]\left(I_3-\alpha\beta P\right)^{-1} - \frac{b}{w}\left[-1\ 1\ 0\right]\left(I_3-\beta P\right)^{-1}\right) s_t \quad ✓$$

$$\pi_t = \left(1-\frac{\kappa b\psi_\pi}{w}\right)\left[(1-\alpha)\beta,\ \kappa\alpha\beta,\ 0\right] f_\alpha + \frac{\kappa}{w}\left[b-b\beta\psi_\pi,\ (1-\beta-\beta b\psi_x),\ 0\right] f_\beta$$
$$+\left[\left(1-\frac{\kappa b\psi_\pi}{w}\right)\left[I_3-\alpha\beta P\right]^{-1}\left[0\ 0\ 1\right] - \frac{\kappa b}{w}\left[I_3-\beta P\right]^{-1}\left[1\ 1\ 0\right]\right] s_t$$

$$\left(\left(1-\frac{\kappa b\psi_\pi}{w}\right)\left[0\ 0\ 1\right]\left(I_3-\alpha\beta P\right)^{-1} - \frac{\kappa b}{w}\left[-1\ 1\ 0\right]\left(I_3-\beta P\right)^{-1}\right) s_t \quad ✓$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \left[0\ 1\ 0\right] s_t$$

$$x_t = g_{xa}\cdot f_\alpha + g_{xb}\cdot f_\beta + g_{xs}\cdot s_t \quad ✓$$

$$\pi_t = g_{\pi a}\cdot f_\alpha + g_{\pi b}\cdot f_\beta + g_{\pi s}\cdot s_t \quad ✓$$

$$i_t = \left(\psi_\pi g_{\pi a} + \psi_x g_{xa}\right) f_\alpha + \left(\psi_\pi g_{\pi b} + \psi_x g_{xb}\right) f_\beta$$
$$+ \left(\psi_\pi g_{\pi s} + \psi_x g_{xs} + \left[0\ 1\ 0\right]\right) s_t \quad ✓$$

So $x_t = g_{xa} \cdot f_\alpha + g_{xb} \cdot f_\beta + g_{xs} \cdot s_t$

$\pi_t = g_{\pi a} \cdot f_\alpha + g_{\pi b} \cdot f_\beta + g_{\pi s} \cdot s_t$

$i_t = (\psi_\pi g_{\pi a} + \psi_x g_{xa}) f_\alpha + (\psi_\pi g_{\pi b} + \psi_x g_{xb}) f_\beta$

$+ (\psi_\pi g_{\pi s} + \psi_x g_{xs} + [0\ 1\ 0]) s_t$

$$\Rightarrow z_t = A_1 f_\alpha + A_2 f_\beta + A_3 s_t \qquad \text{(ALM)}$$

where

$$A_1 = \begin{bmatrix} g_{\pi a} \\ g_{xa} \\ \psi_\pi g_{\pi a} + \psi_x g_{xa} \end{bmatrix}$$

all A_i should be 3×3.

$$A_2 = \begin{bmatrix} g_{\pi b} \\ g_{xb} \\ \psi_\pi g_{\pi b} + \psi_x g_{xb} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} g_{\pi s} \\ g_{xs} \\ \psi_\pi g_{\pi s} + \psi_x g_{xs} + [0\ 1\ 0] \end{bmatrix}$$

But we still need to redefine f_α & f_β in light of the new PLM!

If the PLM is $z_t = \Gamma z_{t-1} + (I_3 - \Gamma)\bar{z}_t + \rho s_{t-1} + e_t$ (PLM$_1$)

and $f_\alpha = \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \hat{E}_t \underset{z_{T+1}}{\underline{\underline{z_T}}}$; $f_\beta = \sum_{T=t}^{\infty} \beta^{T-t} \hat{E}_t z_T$

then

$\swarrow$ some generic df.

$$"f" = \sum_{T=t} \delta^{T-t} \Big[\underbrace{\Gamma z_{T-1}}_{} + (I_3 - \Gamma)\bar{z}_T + \rho s_{T-1} \Big]$$

this guy is causing trouble → shut it off

→ simplify the PLM to $z_t = \bar{z}_t + \rho s_{t-1} + e_t$ (PLM$_2$)

$\uparrow$ drift

Then   Actually: PLM$_3$: $z_t = \bar{z}_t + s_t$

$$"f" = \sum_{T=t} \delta^{T-t} \Big[\overset{\bar{z}_t}{\bar{z}_T} + \overset{+ s_T}{\rho s_{T-1}} \Big] \Big| \text{ anticipated utility ass.}$$

$$= \sum_{T=t} \delta^{T-t} \bar{z}_t + \sum_{T=t} \delta^{T-t} \rho s_{T-1}$$

$$= \frac{1}{1-\delta} \bar{z}_t + \sum_{T=t} (\delta\rho)^{T-t} s_t$$

$$= \frac{1}{1-\delta} \bar{z}_t + \frac{1}{1-\delta\rho} s_t$$

So

$$\boxed{\begin{aligned} f_\alpha &= \frac{1}{1-\alpha\beta} \bar{z}_t + (I_3 - \alpha\beta\rho)^{-1} s_t \quad \checkmark \\ f_\beta &= \frac{1}{1-\beta} \bar{z}_t + (I_3 - \beta\rho)^{-1} s_t \quad \checkmark \end{aligned}}$$

So then the model is:

$$z_t = A_1 f_\alpha + A_2 f_\beta + A_3 s_t \qquad (RF)$$

$$f_\alpha = \frac{1}{1-\alpha\beta} \bar{z}_t + \frac{1}{1-\alpha\beta\rho} s_t \qquad (LR\text{-}E_1)$$

$$f_\beta = \frac{1}{1-\beta} \bar{z}_t + \frac{1}{1-\beta\rho} s_t \qquad (LR\text{-}E_2)$$

↳ A_i are given by p. 55, g_{ij} on p. 54.

$$s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{bmatrix} = \underbrace{\begin{bmatrix} \rho_r & 0 & 0 \\ 0 & \rho_i & 0 \\ 0 & 0 & \rho_u \end{bmatrix}}_{=: P \text{ (capital } \rho)} s_{t-1} + \begin{bmatrix} \varepsilon_r \\ \varepsilon_i \\ \varepsilon_u \end{bmatrix} \qquad \text{(Exog. process)}$$

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} \overbrace{\left(z_{t-1} - \hat{E}_{t-2} z_{t-1} \right)}^{\text{fcst error } f_{t-1}} \qquad (LR\text{-beliefs})$$

$$k_t = \mathbb{I}(k_{t-1} + 1) + (1 - \mathbb{I}) \bar{g}^{-1}$$

$$\mathbb{I} = \begin{cases} 1 & \text{if } \theta_t \leq \bar{\theta} \\ 0 & \text{else} \end{cases}$$

w/ $\theta_t = \left| \hat{E}_{t-1} z_t - E_{t-1} z_t \right|$ / (variance of noise)

E_{t-1}: objective expectations

(which might be $= \sigma_r + \sigma_i + \sigma_u$ here.)

What is f_{t-1}? I guess it's ALM − PLM, so

$$A_1 f_\alpha + A_2 f_\beta + A_3 s_{t-1} - \Big(\bar{z}_{t-1} + \overbrace{\cancel{\rho s_{t-2}}}^{= s_{t-1} \text{ w/ PLM}_3}\Big)$$

$$= A_1 \left[\frac{1}{1-\alpha\beta} \bar{z}_{t-1} + \frac{1}{1-\alpha\beta\rho} s_{t-1}\right] + A_2 \left[\frac{1}{1-\beta} \bar{z}_{t-1} + \frac{1}{1-\beta\rho} s_{t-1}\right]$$

$$+ A_3 s_{t-1} - \bar{z}_{t-1} - \cancel{\rho s_{t-2}} \quad - s_{t-1}$$

$$= \left[A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} - I_3\right] \bar{z}_{t-1}$$

$$+ \underbrace{\left[A_1 \frac{1}{1-\alpha\beta\rho} + A_2 \frac{1}{1-\beta\rho} + A_3\right]}_{A_1(1-\alpha\beta\rho)^{-1} + A_2(1-\beta\rho)^{-1} + A_3 - I_3} s_{t-1} \quad \overbrace{- \cancel{\rho s_{t-2}}}^{- s_{t-1}}$$

need to make sure s_{t-2} doesn't enter here.

$$\theta_t = \left[A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} - I_3\right] \bar{z}_{t-1}$$

diff btw ALM − PLM absent shocks ...

(... it seems to me but need to check this further!)

I think w/ PLM$_3$ that's ok now.

On Tuesday:

1. Check these things, correct s_{t-1}, s_{t-2} & θ_t
2. Type up (→ materials1) & simulate!

So 3 Sep 2019

$$f_{t-1} = \left[A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} - I_3\right] \bar{z}_{t-1}$$
$$\left[A_1 (I_3 - \alpha\beta P)^{-1} + A_2 (I_3 - \beta P)^{-1} + A_3 - I_3\right] s_{t-1}$$

Next: check θ_t !

I checked in Experimentation Notes 1, p. 39 4 Sep 2019

that $f_t = (6) - (5) = ALM - PLM$

There it also says

$$\theta_t = \frac{|\underbrace{\hat{E}_{t-1}\pi_t}_{(5)} - \underbrace{E_{t-1}\pi_t}_{E[(6)]}|}{\bar{\theta}_n}$$

which makes sense b/c $E[ALM]$ is the objective expectation that one should have given the model.

(now) $\theta_t = |PLM - E[ALM]|$

ALM is evaluating fa & fb in RF

RF: $z_t = A_1 f_a + A_2 f_b + A_3 s_t$

$$z_t = A_1 \frac{1}{1-\alpha\beta} \bar{z}_t + A_1 (I_3 - \alpha\beta P)^{-1} s_t + A_2 \frac{1}{1-\beta} \bar{z}_t + A_2 (I_3 - \beta P)^{-1} s_t$$
$$+ A_3 s_t$$

$$\underline{\underline{z_t = \left(A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta}\right) \bar{z}_t + \left(A_1 (I_3 - \alpha\beta P)^{-1} + A_2 (I_3 - \beta P)^{-1} + A_3\right) s_t}} \quad \text{(ALM)}$$

So $\theta_t = \left| \hat{E}_{t-1} z_t - E_{t-1} z_t \right| \cdot$ (noise)

$$E_{t-1} z_t = E_{t-1} [\text{ALM}]$$
$$= \left(A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta}\right) \bar{z}_{t-1}$$
$$+ \left((I_3 - \alpha\beta P)^{-1} + (I_3 - \beta P)^{-1} + A_3\right) P s_{t-1}$$

$$\hat{E}_{t-1} z_t = \text{PLM}(t-1) = \bar{z}_{t-1} + s_{t-1}$$

So (noise) $\theta_t = \left(I_3 - A_1 \frac{1}{1-\alpha\beta} - A_2 \frac{1}{1-\beta} \right) \bar{z}_{t-1}$

$$\underline{\underline{+ \left(I_3 - \left(A_1 (I_3 - \alpha\beta P)^{-1} + A_2 (I_3 - \beta P)^{-1} + A_3\right) P \right) s_{t-1}}}$$

1.) For CEMP, s_{t-1} drops out b/c it enters PLM & ALM the same way

2.) Need to be careful about timing. I think it's correct:

$k_t = f_k(\bar{z}_{t-1}) \rightarrow \theta_t$ is a fct of $\underline{\underline{\bar{z}_{t-1}}}$ & $\underline{\underline{s_{t-1}}}$.

Let's try to put together model equations like in CEMP App:

$$(\text{ALM}):\quad z_t = \left(A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta}\right)\bar{z}_t + \left(A_1(I_3-\alpha\beta P)^{-1} + A_2(I_3-\beta P)^{-1} + A_3\right)s_t$$

$$(\bar{z}):\quad \bar{z}_t = \bar{z}_{t-1} + k_t^{-1} f_{t-1}$$

$$(k_t):\quad k_t = f_k(\bar{z}_{t-1}, k_{t-1}, s_{t-1}) \quad \text{(the rule involving } \theta_t\text{)}$$

$$(f_{t-1}):\quad f_{t-1} = \left[A_1 \frac{1}{1-\alpha\beta} + A_2 \frac{1}{1-\beta} - I_3\right]\bar{z}_{t-1}$$
$$\left[A_1 (I_3-\alpha\beta P)^{-1} + A_2 (I_3-\beta P)^{-1} + A_3 - I_3\right]s_{t-1}$$

$$(s_t):\quad s_t = P s_{t-1} + \epsilon_t$$

<u>Ryan meeting</u> <u>4 Sep 2019</u>

Idea that CB can change $E(r)^{LR}$ (= LR growth prospects) ⇒ he finds that not quite credible!

The story of more aggressive optimal mon. pol. is more credible!

Peter meeting 5 Sep 2019

- loss fct & parametric family of TRs
  you can see how changing params on TR affects the loss (→ grid-search over params of TR)
- loss fct : quadratic on π & x
- Extension: do a welfare-theoretic exercise
- In FIRE, the mon. pol. can offset 1:1 real rate shocks
  → would be interesting to see whether this holds for learning.

Work after 10 Sep 2019

Goal: get s_t out of f_k !

1) f_{t-1} is wrong. It's

$$\underbrace{z_{t-1}}_{ALM_{t-1}} - \hat{E}_{t-2} z_{t-1}$$

$$ALM_{t-1} - PLM_{t-1|t-2}$$

$$B_1 \bar{z}_{t-1} + B_2 s_{t-1} - (\bar{z}_{t-2} + s_{t-2})$$

$$f_{t-1} = B_1 \bar{z}_{t-1} - \bar{z}_{t-2} + B_2 s_{t-1} - s_{t-2}$$ VAR w/ 2 lags...

noise $\theta_t = \left| \hat{E}_{t-1} z_t - E_{t-1} z_t \right|$

$$= \left| PLM_{t|t-1} - E_{t-1}(ALM_t) \right|$$

$$= \left| \bar{z}_{t-1} + s_{t-1} - \left(B_1 \bar{z}_{t-1} + B_2 P s_{t-1} \right) \right|$$

$$= \left| (I_3 - B_1) \bar{z}_{t-1} + \underbrace{(I_3 - B_2 P)}_{} s_{t-1} \right|$$

for this to cancel, we need $B_2 P \overset{!}{=} I_3$

How does CEMP do it?

$$PLM^{CEMP}_{t|t-1} = \gamma \pi_{t-1} + (1-\gamma) \bar{\pi}_t + \rho \varphi_{t-1}$$

$$E_{t-1} ALM_t = \gamma \pi_{t-1} + (1-\gamma) \Gamma \bar{\pi}_t + \rho \varphi_{t-1} + \overset{=0}{E(\eta_t)}$$

Right now, my PLM is:

$$PLM_3 = \hat{E}_t z_{t+1} = \bar{z}_t + s_t + e_{t+1}$$

$$z_t = \bar{z}_{t-1} + s_{t-1} + e_t$$

Try

$$z_t = \bar{z}_{t-1} + \overset{P s_{t-1}}{C s_{t-1}} + e_t \qquad (PLM_4)$$

$\uparrow$ not yet determined.

$$f_a = \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} z_{T+1} = \frac{1}{1-\alpha\beta} \bar{z}_t + \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \overset{P s_T}{C s_T}$$

$$= \frac{1}{1-\alpha\beta} \bar{z}_t + C \sum_{T=t}^{\infty} (\alpha\beta P)^{T-t} s_t$$

$$f_a = \frac{1}{1-\alpha\beta}\bar{z}_t + {}^{P}C(1-\alpha\beta P)^{-1} s_t$$

$$f_b = \frac{1}{1-\beta}\bar{z}_t + {}^{P}C(1-\beta P)^{-1} s_t$$

So B_1 will always be the same in the ALM.

B_2 modifies to

$$B_2 = A_1 C(1-\alpha\beta P)^{-1} + A_2 C(I_3-\beta P)^{-1} + A_3$$

What if we try a PLM$_4$

$$PLM_4 = \quad z_t = \bar{z}_{t-1} + P s_{t-1} + e_t$$

→ then P takes the place of C, which is

1) not super helpful 2) but reasonable

$$z_t = B_1 \bar{z}_t + B_2 s_t \quad (ALM) \rightarrow B_1 \bar{z}_t + B_2 P s_{t-1} + B_2 e_t$$

$$ALM_{t|t-1} = B_1 \bar{z}_{t-1} + B_2 P s_{t-1}$$

If shocks are iid, then P = zeros

$$(\text{noise})\,\theta_t = (I_3 - B_1)\bar{z}_{t-1}$$

Ok, so following this logic, keep PLM 4, and let C solve the equation $B_2 P \stackrel{!}{=} I_3$

$\Leftrightarrow \left(A_1 C (1-\alpha\beta P)^{-1} + A_2 C (I_3 - \beta P)^{-1} + A_3\right) P = I_3$

→ do it on mathematica!

It's running, takes a while... (started 3.45 pm)

Took almost 10 min.

Even to show more takes 5 min!

Show all (started 4.01 pm) - crashed :S

$f_{t-1} = z_{t-1} - \hat{E}_{t-2}\, z_{t-1}$

$\quad = (B_1 \bar{z}_{t-1} + B_2 s_{t-1}) - (\bar{z}_{t-2} + C s_{t-2})$

$f_{t-1} = B_1 \bar{z}_{t-1} - \bar{z}_{t-2} + B_2 s_{t-1} - C s_{t-2}$

One thing I tried is to artificially shut down s_{t-1} in θ_t.

→ doesn't change the fact that $\bar{\theta}$ needs to be large!

So I think I can leave θ_t as a fct of s, it's ok.

⟹ Try to write f_{t-1} simpler!

B_1, B_2 same every time! Correct? → Yes!

Robert Ulbricht meeting

10 Sep 2019

- tell about his/my research
- can I come to you for job market advice?
- can you look turn / give feedback on proposal / & job market documents
- external letter writer – valuable?

He said about info project: try the setting where the CB can only comm about today, but E(tomorrow) matter as well → what's the optimal comm level there? Do we see the same tradeoff?

11 Sep 2019

$$f_{t-1} = z_{t-1} - \hat{E}_{t-2} z_{t-1}$$

In CEMP: $\gamma \pi_{t-2} + (1-\gamma)\Gamma \bar{\pi}_{t-1} + \rho \psi_{t-2} + \eta_{t-1}$

$$-\left(\gamma \pi_{t-2} + (1-\gamma)\bar{\pi}_{\underline{\underline{t-1}}} + \rho \psi_{t-2}\right)$$

$$f_{t-1} = (1-\gamma)(\Gamma - 1)\bar{\pi}_{t-1} + \eta_{t+1}$$

Key: if z_t is the fcsted thing, the PLM needs to feature $\bar{z}_t$ and s_{t-1}

PLM_5 :

$$\begin{cases} z_t = \bar{z}_t + P s_{t-1} + e_t \\ \hat{E}_t z_{t+1} = \bar{z}_{t+1} + P s_t \\ \hat{E}_{t-1} z_t = \bar{z}_t + P s_{t-1} \end{cases}$$

Then

$$f_a = \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \hat{E}_t z_{T+1} = \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \bar{z}_{t+1} + \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} P s_T$$

$$= \frac{1}{1-\alpha\beta} \bar{z}_{t+1} + P \sum_{T=t}^{\infty} (\alpha\beta P)^{T-t} s_t$$

$$= \frac{1}{1-\alpha\beta} \bar{z}_{t+1} + P (I_n - \alpha\beta P)^{-1} s_t$$

B_1 unchanged, B_2 unchanged except $C = P$.

$\Rightarrow$ ALM is $z_t = B_1 \bar{z}_{t+1} + B_2 s_t$

So ALM_{t-1} is $z_{t-1} = B_1 \bar{z}_t + B_2 s_{t-1}$

$PLM_{t-1|t-2} = \hat{E}_{t-2} z_{t-1} = \bar{z}_{t-1} + P s_{t-2}$

At least you can say $f_{t-1} = z_{t-1} - \hat{E}_{t-2} z_{t-1}$

$$= B_1 \bar{z}_t + B_2 P s_{t-2} + B_2 e_{t-1} - \bar{z}_{t-1} - P s_{t-2}$$

$$= B_1 \bar{z}_t - \bar{z}_{t-1} + (B_2 - I_3) P s_{t-2} + B_2 e_{t-1}$$

but you can do that in the current formulation too, and $\bar{z}$ still ain't cool.

Let's redo the current PLM calculation (PLM$_4$)

$$\hat{E}_t z_{t+1} = \bar{z}_t + C s_t + e_{t+1}$$

$$\Leftrightarrow \quad z_t = \bar{z}_{t-1} + C s_{t-1} + e_t \qquad (= \text{PLM}_4)$$

The difference between PLM$_4$ & PLM$_5$ is CEMP's ass. (5.5): the "shifting end-point model"

$$\hat{E}_{t-1} \bar{\pi}_T = \bar{\pi}_t \quad \forall T \geq t \quad (\to \text{PLM}_5)$$

vs. PLM$_4$ implicitly assumes $\hat{E}_{t-1} \bar{\pi}_T = \bar{\pi}_{t-1} \; \forall T \geq t$

The diff isn't substantial I don't think: it's notational.

To be precise, CEMP's ass (5.5) means that

$$\bar{\pi}_t = \bar{\pi}_{t|t-1} \qquad (\text{that is } \bar{\pi}_{t+1} = \bar{\pi}_{t+1|t})$$

which is what PLM$_5$ captures.

I'm going to use the modified (5.5) of CEMP:

$$\hat{E}_{t-1} \bar{z}_T = \bar{z}_{t-1} \quad \forall T \geq t \qquad \text{or} \qquad \hat{E}_t \bar{z}_T = \bar{z}_t \quad \forall T > t$$

in accordance w/ PLM$_4$ (which is used in materials 2)

which also means $\bar{z}_t = \bar{z}_{T|t} \quad \forall T > t$

Then, for the current PLM, PLM4,

$$f_a = \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} z_{T+1} = \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \big(\underbrace{\bar{z}_t}_{= \bar{z}_{T+1|t}} + c s_{T|t} \big)$$

$$= \frac{1}{1-\alpha\beta} \bar{z}_t + c \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} P^{T-t} s_t$$

$$= \frac{1}{1-\alpha\beta} \bar{z}_t + c \left(I_3 - \alpha\beta P \right)^{-1} s_t \qquad = \text{eq (22) of materials 2.}$$

and so the ALM will equal eq. (24).

and so $f_{t-1} = z_{t-1} - \hat{E}_{t-2} z_{t-1}$

$$= B_1 \bar{z}_{t-1} + B_2 s_{t-1} - \left(\bar{z}_{t-2} + c s_{t-2} \right)$$

which is indeed what I have in materials 2.

Start to formulate a loss function

- initially, just check how for the same simulation, loss responds to changing the params of the Taylor-rule
- then find optimal (ψ_π^*, ψ_x^*).

Clarida, Gali & Gertler (1999)'s loss (eq. 2.7)

$$\max -\frac{1}{2} E_t \left[\sum_{i=0}^{\infty} \beta^i \left[\alpha x_{t+i}^2 + \pi_{t+i}^2 \right] \right]$$

$$\mathcal{L} = -E_t\left[\sum_{i=0}^{\infty}\beta^i\left[\alpha x_{t+i}^2 + \pi_{t+i}^2\right]\right]$$

$$= -E_t\sum_{i=0}^{\infty}\beta^i\begin{bmatrix}1 & \alpha & 0\end{bmatrix}\underbrace{\begin{bmatrix}z(1)^2\\ z(2)^2\\ z(3)^2\end{bmatrix}}_{\text{elementwise product: } z \cdot z}$$

Turns out that this kind of operation is known in math as the Hadamard product or the Schur product:

$$A \circ B = \underset{m\times n}{A} \odot \underset{m\times n}{B} = a_{ij}\cdot b_{ij} \quad \forall i,j$$

In Latex, $\circ$ is \circ, and $\odot$ is \odot

So

$$\mathcal{L} = -E_t\sum_{i=0}^{\infty}\beta^i\begin{bmatrix}1 & \alpha & 0\end{bmatrix}(z \odot z)$$

weight on output gap
(have to denote it w/ sthg else, so maybe γ?)

Ryan meeting 11 Sep 2019

One idea: have only one thing (π) that they can learn about:
LR level of x and i fixed.

- $\forall$ knows that $x = 0$ in LR
- —— || —————— $i = \pi$ in LR

The C-issue:
If P strictly positive

$$P' I_3 = B_2(C)$$

↑
vec-operator

- maybe it's a result that w/ these TR-coeffs, it's hard to get anchored $E(\cdot)$.

- Solve special cases: would be valuable to solve
  e.g. Preston → solve the simulation
- Recursive representation of the economy (BM)
  if it's easier as a start → can see what's happening

Simul w/ decreasing gain

RE BM LR-exp

run expectation, adding N expectation terms at a time

⇒ can tease out these errors are b/c you should eventually get the same thing.

- projection facility → it might solve some of the explosion issues.
Liam Graham paper.

Yes - he is willing to look at my application documents yay! But: it'd be good to have some results to point to!

By the way, he also said TR coeffs between 1 to 5 were plausible.

Work after 12 Sep 2015

First, a summary:

① To the model as it now is:

Ⓘ Have only 1 thing for agents to learn about

Ⓘ Maybe the explosions are a result: harder to get anchoring w/ these TR coeffs than it would be otherwise

Ⓘ Maybe the projection facility can get around explosions (Liam Graham paper)

Ⓘ If you restrict P to be strictly positive, then you can try to use vec(·) to see if you can solve the problem.

↳ I think it suffices to have $P = \begin{pmatrix} \rho_r & 0 & 0 \\ 0 & \rho_i & 0 \\ 0 & 0 & \rho_u \end{pmatrix}$.

② Special cases (and I think this is what we should focus on for materials 3)

- Simulate RE model
- Simulate EE model (BM)
- Simulate LR model (Preston)

in the ΣExp, add one term at a time to see diffs between EE & LR

- Simulate LR model w/ anchoring (CEMP-Preston mix, call it LRA)

In the RE-EE case, plug TR into i:

$$x_t = E_t x_{t+1} - \sigma\left(\bar{i}_t + \psi_\pi \pi_t + \psi_x x_t - E_t \pi_{t+1}\right) + \sigma r_t^N$$

$$= E_t x_{t+1} + \sigma E_t \pi_{t+1} - \sigma\psi_\pi \pi_t - \sigma\psi_x x_t - \sigma\bar{i}_t + \sigma r_t^N$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \pi_t + E_t x_{t+1} + \sigma E_t \pi_{t+1} - \sigma \bar{i}_t + \sigma r_t^N$$

$$= -\sigma\psi_\pi\left[\kappa x_t + \beta E_t \pi_{t+1} + u_t\right] + \text{rest}$$

$$\underbrace{(1+\sigma\psi_x - \kappa\sigma\psi_\pi)}_{=:w} x_t = E_t x_{t+1} + (\sigma - \sigma\psi_\pi\beta) E_t \pi_{t+1} - \sigma\psi_\pi u_t - \sigma \bar{i}_t + \sigma r_t^N$$

$$x_t = \frac{(\sigma-\sigma\psi_\pi\beta)}{w} E_t \pi_{t+1} + \frac{1}{w} E_t x_{t+1} + \frac{\sigma}{w} r_t^N - \frac{\sigma}{w} \bar{i}_t - \frac{\sigma\psi_\pi}{w} u_t$$

$$\rightarrow \underline{\underline{x_t = \left[\frac{(\sigma-\sigma\psi_\pi\beta)}{w}, \frac{1}{w}, 0\right] E_t z_{t+1} + \left[\frac{\sigma}{w}, -\frac{\sigma}{w}, -\frac{\sigma\psi_\pi}{w}\right] s_t}}$$

$$\pi_t = \kappa\left[\frac{(\sigma-\sigma\psi_\pi\beta)}{w}, \frac{1}{w}, 0\right] E_t z_{t+1} + \kappa\left[\frac{\sigma}{w}, -\frac{\sigma}{w}, -\frac{\sigma\psi_\pi}{w}\right] s_t$$

$$+ \beta E_t \pi_{t+1} + u_t$$

$$\underline{\underline{\pi_t = \left[\left(\beta + \frac{\kappa(\sigma-\sigma\psi_\pi\beta)}{w}\right), \frac{\kappa}{w}, 0\right] E_t z_{t+1} + \left[\frac{\kappa\sigma}{w}, -\frac{\kappa\sigma}{w}, \left(1 - \frac{\kappa\sigma\psi_\pi}{w}\right)\right] s_t}}$$

$$i_t = \psi_\pi\left(\left[\left(\beta + \frac{\kappa(\sigma-\sigma\psi_\pi\beta)}{w}\right), \frac{\kappa}{w}, 0\right] E_t z_{t+1} + \left[\frac{\kappa\sigma}{w}, -\frac{\kappa\sigma}{w}, \left(1 - \frac{\kappa\sigma\psi_\pi}{w}\right)\right] s_t\right)$$

$$+ \psi_x\left(\left[\frac{(\sigma-\sigma\psi_\pi\beta)}{w}, \frac{1}{w}, 0\right] E_t z_{t+1} + \left[\frac{\sigma}{w}, -\frac{\sigma}{w}, -\frac{\sigma\psi_\pi}{w}\right] s_t\right)$$

$$+ \bar{i}_t$$

$$i_t = \psi_\pi \left(\left[\left(\beta + \frac{\kappa(b - 3\psi_\pi \beta)}{w} \right), \frac{\kappa}{w}, 0 \right] E_t z_{t+1} + \left[\frac{\kappa 3}{w}, -\frac{\kappa 3}{w}, \left(1 - \frac{\kappa 3 \psi_\pi}{w} \right) \right] s_t \right)$$
$$+ \psi_x \left(\left[\frac{(b - 3\psi_\pi \beta)}{w}, \frac{1}{w}, 0 \right] E_t z_{t+1} + \left[\frac{3}{w}, -\frac{3}{w}, -\frac{3\psi_\pi}{w} \right] s_t \right)$$
$$+ \bar{i}_t$$

$$i_t = \left[\psi_\pi \left(\beta + \frac{\kappa(b - 3\psi_\pi \beta)}{w} \right) + \psi_x \left(\frac{(b - 3\psi_\pi \beta)}{w} \right), \psi_\pi \frac{\kappa}{w} + \psi_x \frac{1}{w}, 0 \right] E_t z_{t+1}$$
$$+ \left[\psi_\pi \frac{\kappa 3}{w} + \psi_x \frac{3}{w}, \psi_\pi \left(-\frac{\kappa 3}{w} \right) + \psi_x \left(-\frac{3}{w} \right) + 1, \psi_\pi \left(1 - \frac{\kappa 3 \psi_\pi}{w} \right) + \psi_x \left(-\frac{3\psi_\pi}{w} \right) \right] s_t$$

So the RE model's LOM is:

$$z_t = A_p^{RE} E_t z_{t+1} + A_s^{RE} s_t$$

and the EE model's LOM is:

$$z_t = A_p^{RE} \hat{E}_t z_{t+1} + A_s^{RE} s_t$$

with

$$A_p^{RE} = \begin{bmatrix} \left[\left(\beta + \frac{\kappa(b - 3\psi_\pi \beta)}{w} \right), \frac{\kappa}{w}, 0 \right] \\ \left[\frac{(b - 3\psi_\pi \beta)}{w}, \frac{1}{w}, 0 \right] \\ \left[\psi_\pi \left(\beta + \frac{\kappa(b - 3\psi_\pi \beta)}{w} \right) + \psi_x \left(\frac{(b - 3\psi_\pi \beta)}{w} \right), \psi_\pi \frac{\kappa}{w} + \psi_x \frac{1}{w}, 0 \right] \end{bmatrix}$$

$= BM_1$ (materials8, Experimentation)

$$A_s^{RE} = \begin{bmatrix} \left[\frac{\kappa 3}{w}, -\frac{\kappa 3}{w}, \left(1 - \frac{\kappa 3 \psi_\pi}{w} \right) \right] \\ \left[\frac{3}{w}, -\frac{3}{w}, -\frac{3\psi_\pi}{w} \right] \\ \left[\psi_\pi \frac{\kappa 3}{w} + \psi_x \frac{3}{w}, \psi_\pi \left(-\frac{\kappa 3}{w} \right) + \psi_x \left(-\frac{3}{w} \right) + 1, \psi_\pi \left(1 - \frac{\kappa 3 \psi_\pi}{w} \right) + \psi_x \left(-\frac{3\psi_\pi}{w} \right) \right] \end{bmatrix}$$

$\neq BM_2$

BM_2 is wrong.

ALM of EE:

$$\hat{E}_t z_{t+1} = \phi_t + \rho s_t$$

So ALM is:

$$z_t = A_p^{RE}\left(\phi_t + \rho s_t\right) + A_s^{RE} s_t$$

$$z_t = A_p^{RE} \phi_t + \left(A_p^{RE}\rho + A_s^{RE}\right) s_t$$

ALM of LR

$$f_a = \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} z_{T+1} = \hat{E}_t \left(z_{t+1} + \alpha\beta\left(z_{t+2}\right) + \ldots\right)$$

$$= \hat{E}_t z_{t+1} + \alpha\beta \hat{E}_t z_{t+2} + \ldots$$

$$= \phi_t + \rho s_t + \alpha\beta\left(\phi_t + \rho^2 s_t\right) + \ldots$$

$$= \sum_{n=0}^{\infty} (\alpha\beta)^n \phi_t + \rho s_t + \alpha\beta\rho^2 s_t + (\alpha\beta)^2 \rho^3 s_t + \ldots$$

$$= \frac{1}{1-\alpha\beta}\phi_t + \rho\left(s_t + \alpha\beta\rho s_t + (\alpha\beta\rho)^2 s_t + \ldots\right)$$

$$= \frac{1}{1-\alpha\beta}\phi_t + \rho\left(I_3 + \alpha\beta\rho + (\alpha\beta\rho)^2 + \ldots\right) s_t$$

$$= \frac{1}{1-\alpha\beta}\phi_t + \rho\left(I_3 - \alpha\beta\rho\right)^{-1} s_t$$

So the ALM is

$$z_t = A_a f_a + A_b f_b + A_s s_t$$
$$= A_a \left(\frac{1}{1-\alpha\beta} \phi_t + P \left(I_3 - \alpha\beta P \right)^{-1} s_t \right)$$
$$+ A_b \left(\frac{1}{1-\beta} \phi_t + P \left(I_3 - \beta P \right)^{-1} s_t \right)$$
$$+ A_s s_t$$

$$z_t = \left(A_a \frac{1}{1-\alpha\beta} + A_b \frac{1}{1-\beta} \right) \phi_t$$
$$+ \left(A_a P \left(I_3 - \alpha\beta P \right)^{-1} + A_b P \left(I_3 - \beta P \right)^{-1} + A_s \right) s_t$$

Reduce LR to EE $(T=t)$ 14 Sep 2015

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left((1-\beta) x_{T+1} - \sigma i_{T+1} + \sigma \pi_{T+1} + \sigma r_T^N \right)$$

$T=t$ only: $x_t = \underline{(1-\beta)} \hat{E}_t x_{t+1} - \sigma \hat{E}_t (i_{t+1} - \pi_{t+1}) + \sigma r_t^N$

$$\pi_t = \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left(\kappa x_T + (1-\alpha)\beta \pi_{T+1} + u_T \right)$$

$T=t$ only: $\pi_t = \kappa x_t + \underline{(1-\alpha)\beta \hat{E}_t \pi_{t+1}} + u_t$

Let's leave this issue for a moment.

Let's continue to state that in this RE model:

$$y_t = z_t = \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} = \text{jumps} \qquad X_t = s_t = \begin{pmatrix} r_t^n \\ i_t \\ u_t \end{pmatrix} \text{ states}$$

and all states are exogenous here!

So in Ryan's sim_learn.m, agents are learning gx^l which encompasses a constant and a slope.

$$\underset{3\times 1}{y_t} = \underset{3\times 3}{gx} \cdot \underset{3\times 1}{X_t} \qquad \text{and } y_t = \underset{3\times 4}{g^l} \cdot \underset{4\times 1}{(1 \ x_t')'}$$

$$\text{where } g^l = \begin{bmatrix} 1 & \\ 1 & \underset{3\times 3}{gx \cdot hx} \\ 1 & \end{bmatrix}$$

If agents only learn about the constant, then $g^l = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

And $E_t y_{t+1} = gx\, hx \cdot x_t$ in general. (Ryan Lec. 2)

I'm confused why agents would kind of assume that gx is I_3. ($hx = P$ in my case).

→ this implies that a PLM in which agents at least use their knowledge of hx, and assume $gx = I_3$ is:

$$z_t = \phi_{t-1} + P s_{t-1} + \varepsilon_t \qquad (PLM_6)$$

and then I can map more directly to Ryan's code

b/c in each t, $g^l = [\phi \;\; P]$

[where this is updated.

1.) Figure out timing 2.) Rework h-horizon fcsts and ALM w/ PLM$_6$.

1) sim_learn.m

<u>Initialize:</u> $g^l = \begin{bmatrix} 0 & \\ 0 & gx\,hx \\ 0 & \end{bmatrix} = \underset{3\times4}{[\phi_0, P]}$ $\quad R_0 = \underset{4\times4}{\begin{bmatrix} 1 & & & 0 \\ & b_r & & \\ & & b & \\ 0 & & & b_u \end{bmatrix}}$

$t = 1$ $\quad xsim_1 = ysim_1 = 0$.

$ysim_t = gx \cdot xsim_t \quad (=0)$

$xesim_t = hx \cdot xsim_t \quad (=0) \quad = E_t X_{t+1}$

$xsim_{t+1} = xesim_t + \eta \cdot shock_{t+1}$ (where shock = innovation)

$t = 2 \dots T-1$

$yp\_e = \underset{3\times4}{gl} \underset{4\times1}{\begin{bmatrix} 1 \\ xsim_t \end{bmatrix}}$ $\quad \left(E_t y_{t+1} = g^l x_t\right)$ (gl_{t-1})

$yx = [f_y \;\; f_{xp}]^{-1} [f_{yp} \;\; f_x] \begin{bmatrix} E_t Y_{t+1} \\ X_t \end{bmatrix} = \begin{bmatrix} Y_t \\ E_t X_{t+1} \end{bmatrix}$

→ $ysim_t = Y_t$ from yx

$xesim_t = E_t X_{t+1}$ from yx

$xsim_{t+1} = xesim_t + \eta \cdot shock_{t+1}$

Update using RLS $\quad g^l_t = g^l_{t-1}$ etc.

$t = T$

$ysim_t = gx \cdot xsim_t$ (b/c you ran out of shocks)

Note: Y_t from gx can be written

$$Y_t = [f_y \; f_{xp}]^{-1} f_{yp} E_t Y_{t+1} + [f_y \; f_{xp}]^{-1} f_x X_t$$

which Ryan writes as (Lect 2. p. 8)

$$Y_t = B_{11} Y_{t+1} + B_{12} X_t$$ and which corresponds to

$$z_t = A_p^{RE} \hat{E}_t z_{t+1} + A_s^{RE} s_t$$, my eq. (6) in materials3

but this kinda suggests that

$$A_p^{RE} = \underbrace{[f_y \; f_{xp}]^{-1} f_{yp}}_{= (-\text{ the top half})} \quad \text{and} \quad A_s^{RE} = \underbrace{[f_y \; f_{xp}]^{-1} f_x}_{= (-1)(\text{top half})}$$

and indeed ...

Moreover, the RE expectations are just

$$E_t X_{t+1} = hx \cdot X_t \quad \text{and} \quad E_t Y_{t+1} = gx \cdot hx \cdot X_t \quad \ldots$$

So we could simulate the RE model using the sim_learn.m-framework, just subbing in $gx \cdot hx \, X_t$ for yp_e every time!

So when Ryan extracts $ysim_t$ from gx, he is implicitly evaluating my ALM for the EE learning case!

PLM$_6$: $z_t = \bar{z}_{t-1} + \rho s_{t-1} + \varepsilon_t = \underset{3\times 4}{\phi_{t-1}} \cdot \underset{4\times 1}{\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}} + \varepsilon_t$

$$\hat{E}_t z_{t+1} = \bar{z}_t + \rho s_t = \phi_t \begin{bmatrix} 1 \\ s_t \end{bmatrix}$$

$$\hat{E}_t z_{t+2} = \bar{z}_t + \rho^2 s_t \neq$$

$$\hat{E}_t z_{t+h} = \bar{z}_t + \rho^h s_t$$

$$f_b = \hat{E}_t \sum_{s=0}^{\infty} \beta^s z_{t+1+s} = \hat{E}_t z_{t+1} + \beta \hat{E}_t z_{t+2} + \beta^2 \hat{E}_t z_{t+3}$$

$$= \bar{z}_t + \rho s_t + \beta \bar{z}_t + \beta \rho^2 s_t + \beta^2 \bar{z}_t + \beta^2 \rho^3 s_t + \dots$$

$$= \frac{1}{1-\beta} \bar{z}_t + \rho (s_t + \beta \rho s_t + \beta^2 \rho^2 s_t + \dots)$$

$$= \frac{1}{1-\beta} \bar{z}_t + \rho (I_3 - \beta \rho)^{-1} s_t$$

So my f_a & f_b are correct, and so are the ALMs.

PLM$_7$: $z_t = \bar{z}_{t-2} + \rho s_{t-1} + \varepsilon_t = \phi_{t-2} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}$

NOW I think timing is consistent everywhere.

$\longrightarrow$ to do: solve ALMs for the case of learning about both constant and slope → that way you'll be able to compare expectations and ALM w/ Ryan's code.

15 Sep 2019

Supp that the PLM is (as in materials 3)

$$z_t = a_{t-2} + b_{t-2}\, s_{t-1} + \varepsilon_t$$
$$= \phi_{t-2} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \quad \text{where} \quad \phi_{t-2} = (\overset{3\times 1}{a_{t-2}}, \overset{3\times 3}{b_{t-2}})$$

$$\Rightarrow \hat{E}_t z_{t+1} = \phi_{t-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix}$$

(and I think this means that agents are learning $hx = P$ too, not just gx.)

$$\Rightarrow \hat{E}_t z_{t+h} = \phi_{t-1}^h \begin{bmatrix} 1 \\ s_t \end{bmatrix}$$

So then the ALM of EE is:

$$z_t = A_p^{RE}\, \underbrace{\hat{E}_t z_{t+1}}_{\phi_{t-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix}} + A_s^{RE}\, s_t$$

Ok cool sim_learn_EE.m & sim_learn_EE_check.m obtain the same simulated states, the first doing Ryan's approach, the second the explicit ALM.

Now we have 2 things to do:

1) Compare EE w/ LR when agents learn the constant and the slope

2) Obtain a representation for when agents only learn the constant.

fa ff_general

If $\hat{E}_t z_{t+h} = \phi_{t-1} \begin{bmatrix} 1 \\ \rho^{h-1} s_t \end{bmatrix}$

$$f_b = \sum_{h=1}^{\infty} \beta^{h-1} \phi_{t-1} \begin{bmatrix} 1 \\ \rho^{h-1} s_t \end{bmatrix} = \phi_{t-1} \sum_{h=1}^{\infty} \beta^{h-1} \begin{bmatrix} 1 \\ \rho^{h-1} s_t \end{bmatrix}$$

$f_b = \sum_{s=0}^{\infty} \beta^s \phi_{t-1} \begin{bmatrix} 1 \\ \rho^s s_t \end{bmatrix}$ but to pull it apart nicely it makes sense to take out the constant

$$f_b = \sum_{s=0}^{\infty} \beta^s \left[a_{t-1} + b_{t-1} \rho^s s_t \right]$$

$$= a_{t-1} \frac{1}{1-\beta} + b_{t-1} \left(I_3 - \beta P \right)^{-1} s_t$$

Right now, keep to this formulation, the one w/o P.

Now f_a^{anal} & f_a^{trunc} coincide, but f_b^{anal} & f_b^{trunc} don't.

In EE learning, right now 1-period ahead fcsts <u>16 Sep 2019</u>

are generated using

$$\hat{E}_t z_{t+1} = a_{t-1} + b_{t-1} \cdot s_t$$

which means that agents don't forecast anything exog. here, they just load on the most recent obs., s_t.

But the situ changes when

$$\hat{E}_t z_{t+2} = a_{t-1} + b_{t-1} E_t s_{t+1}$$

$$= a_{t-1} + b_{t-1} P s_t$$

$$\rightarrow \hat{E}_t z_{t+h} = a_{t-1} + b_{t-1} P^{h-1} s_t$$

$$f_b = \hat{E}_t \sum_{h=1}^{\infty} \beta^{h-1} z_{t+h} = \hat{E}_t \sum_{s=0}^{\infty} \beta^s z_{t+1+s}$$

$$= \sum_{s=0}^{\infty} \beta^s \hat{E}_t z_{t+1+s} = \sum_{s=0}^{\infty} \beta^s \left[a_{t-1} + b_{t-1} P^s s_t \right]$$

$$= \underline{\underline{\frac{1}{1-\beta} a_{t-1} + b_{t-1} (I_3 - \beta P)^{-1} s_t}}$$

$$= \hat{E}_t z_{t+1} + \beta \hat{E}_t z_{t+2} + \beta^2 \hat{E}_t z_{t+3} + \dots$$

Again: it seems to me like

1.) fa, fb were correct (and that fb takes longer to converge b/c it's discounted by $\beta > \alpha\beta$) (both anal & trunc)

2.) the LR-simulation seems correct too

... and guess what? It is! After 2 million periods ($T = 2$ million), LR almost (but not quite) coincides w/ RE! Yay!

So the question now is:

implement learning algorithm such that only the drift is learned.

That is: $\phi_{t-1} = \bar{z}_{t-1}$

h-period ahead fcsts are given by (24) in materials 3 and I need to figure out a good way to rewrite the RLS algorithm in (25)-(26).

$$\underset{3\times 1}{\bar{z}_t} = \underset{3\times 1}{\bar{z}_{t-1}} + t^{-1} R_t^{-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix} f_{t-1}$$

$$f_{t-1} = z_{t-1} - \hat{E}_{t-2} z_{t-1} = z_{t-1} - (\bar{z}_{t-2} + s_{t-1})$$

$= z_{t-1}$ – or is this actually $\hat{E}_{t-1} z_{t-1}$, a weird "backward-looking" application of anticipated utility?

$$f_{t-1} = z_{t-1} - \hat{E}_{t-1} z_{t-1}$$

$$= z_{t-1} - (\bar{z}_{t-1} + s_{t-1})$$

So ok

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$$\bar{z}_t = \bar{z}_{t-1} + t^{-1} R_t^{-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix} f_{t-1}^{assess}$$

$$f_{t-1}^{assess} = z_{t-1} - \hat{E}_{t-1} z_{t-1} = z_{t-1} - \phi_{t-1} s_{t-1}$$

(the "assessment fcst error" – only used to assess ϕ)

$$f_{t-1}^{assess} = z_{t-1} - (\underbrace{\bar{z}_{t-1} + s_{t-1}}_{\text{fcst w/o learning the slope}})$$

$$\Rightarrow \bar{z}_t = \bar{z}_{t-1} + t^{-1} R_t^{-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix} (z_{t-1} - (\bar{z}_{t-1} + s_{t-1}))$$

R_t is the matrix of 2nd moments for $\begin{bmatrix} 1 \\ s_t \end{bmatrix}$, i.e. it seems to be an estimate of $\begin{bmatrix} 1 \\ s_t \end{bmatrix} [1 \;\, s_t]$. That's why it gets updated in every step as

$$\underset{4\times4}{R_t} = \underset{4\times4}{R_{t-1}} + t^{-1}\left(\underset{4\times1}{\begin{bmatrix}1\\ s_{t-1}\end{bmatrix}}\underset{1\times4}{\begin{bmatrix}1 & s_{t-1}\end{bmatrix}} - \underset{4\times4}{R_{t-1}}\right)$$

So this isn't impacted when agents only learn the constant.

And it's <u>not</u> $\eta\eta'$! It's Σ_x, which is a fct of $\eta\eta'$

So:

$$\cancel{\underset{3\times1}{\bar{z}_t} = \underset{3\times1}{\bar{z}_{t-1}} + \underbrace{\underset{1\times1}{t^{-1}}\,\underset{4\times4}{R_t^{-1}}\underset{4\times1}{\begin{bmatrix}1\\ s_t\end{bmatrix}}}_{4\times1}\underset{3\times1}{\left(z_{t-1} - (\bar{z}_{t-1} + s_{t-1}\right)}}$$

Wait: when agents only learn the constant, then the regressor is not technically $\begin{bmatrix}1\\ s_t\end{bmatrix}$; it's 1!

That's why R disappears for CEMP, b/c it would be a scalar: $R_t = R_{t-1} + t^{-1}(1\cdot 1 - R_{t-1})$

And then $\begin{bmatrix}1\\ s_t\end{bmatrix}$ is $= 1$, so $R_t^{-1}\begin{bmatrix}1\\ s_t\end{bmatrix}$ becomes

$1^{-1}\cdot 1 = 1$ for CEMP! That's why it disappears!

Ok, so R is out!

$$\underset{3\times1}{\bar{z}_t} = \underset{3\times1}{\bar{z}_{t-1}} + t^{-1}\left(\underset{3\times1}{z_{t-1}} - (\underset{3\times1}{\bar{z}_{t-1}} + \underset{3\times1}{s_{t-1}})\right)$$

Here, EE & LR don't converge to RE, not even after 2 million periods. Which leads me to problematize the notion of E-stability again: is it the property that the estimates converge to their RE counterpart w/ prob = 1? Evans & Honkapohja (2001) define E-stability as the condition under which an RE eqb (REE) "is stable under least-squares learning". That is, ϕ^{RE} is stable if a deviation from ϕ^{RE} returns to ϕ^{RE} under the learning rule (PLM).

→ yes: E-stability governs to which extent a learning model's learning coefficients converge to the RE coefficients.

In CEMP, E-stability is why $\bar{\pi} \to 0$, b/c $\bar{\pi}$ is the constant and the RE eqb implies a 0 constant!

→ so also for me the constant should converge to 0.

↳ Maybe I need to change the learning rule such that

b = constant = $gx \cdot hx$ or sthg ... so that the PLM nests the REE.

→ Yay! That works!

Ok so anchoring

$$\theta_t \cdot \text{noise} = \left| \hat{E}_{t-1} z_t - E_{t-1} z_t \right|$$

So $\hat{E}_{t-1} z_t = \bar{z}_{t-2} + b s_{t-1}$

$$E_{t-1} z_t = E_{t-1} \left[A_a^{LR} fa_t + A_b^{LR} fb_t + A_s^{LR} s_t \right]$$

$$= E_{t-1} \Big[A_a^{LR} \frac{1}{1-\alpha\beta} \bar{z}_{t-1} + A_a^{LR} b (I_3 - \alpha\beta P)^{-1} s_t$$

$$+ A_b^{LR} \frac{1}{1-\beta} \bar{z}_{t-1} + A_b^{LR} b \left(I_3 - \beta P \right)^{-1} s_t + A_s^{LR} s_t \Big]$$

$$= A_a^{LR} \frac{1}{1-\alpha\beta} \bar{z}_{t-2} + A_a^{LR} b (I_3 - \alpha\beta P)^{-1} P s_{t-1}$$

$$+ A_b^{LR} \frac{1}{1-\beta} \underbrace{\bar{z}_{t-2}}_{\text{anticipated } u \text{ at } t-1} + A_b^{LR} b \left(I_3 - \beta P \right)^{-1} \underbrace{P s_{t-1}}_{E_{t-1}[s_t]} + A_s P s_{t-1}$$

$$= \left[A_a \frac{1}{1-\alpha\beta} + A_b \frac{1}{1-\beta} \right] \bar{z}_{t-2}$$

$$+ \left[A_a b (I_3 - \alpha\beta P)^{-1} + A_b b (I_3 - \beta P)^{-1} + A_s \right] P s_{t-1}$$

Which means that θ_t · noise =

$$= \left| \left[I_3 - A_a \frac{1}{1-\alpha\beta} - A_b \frac{1}{1-\beta} \right] \bar{z}_{t-2} + \left[b - A_a b (I_3 - \alpha\beta P)^{-1} P - A_b \, b (I_3 - \beta P)^{-1} P - A_s P \right] s_{t-1} \right|$$

(this is still materials3, eq. (42).)

ok: simulation stands, but why are gains always equal for all variables? → Typo, lol!

→ and to do: the Ryan-type simplification!

Timing issue: 18 Sept 2019

$\bar{z}_{t-2}$: it'd be better if this was $\bar{z}_{t-1}$.

For that we need

$$\hat{E}_{t-1} z_t = \bar{z}_{t-1} + b s_{t-1} \quad \rightarrow \hat{E}_t z_{t+h} = \bar{z}_t + b P^{h-1} s_t$$

$$E_{t-1} z_t = A_a f_{a_t} + A_b f_{b_t} + A_s s_t$$

Or I can use as a criterion the "assessment fcts"

$$\left| \hat{E}_t z_t - E_t z_t \right| / 3^{noise}$$

But let's try to get away from the "assessment fwd"
b/c conceptually, I think it's wrong

EH: $\phi_t = \phi_{t-1} + t^{-1} R_t^{-1} x_{t-1} \underbrace{(y_t - \phi_{t-1}' x_{t-1})}_{FE_t}$

$\Rightarrow E_{t-1} y_t = \underline{\underline{\phi_{t-1}'}}\ \underline{\underline{x_{t-1}}}$

In Preston, this is not what is happening, as

$$\hat{E}_t z_T = a_{t-1} + b_{t-1} \rho^{T-t} r_t^n$$

$\Rightarrow E_{t-1} z_t = \underline{\underline{\phi_{t-2}'}}\ \underline{\underline{x_{t-1}}}$

(although when introducing the PLM, he is even more sloppy b/c he says $z_t = a_t + b_t s_t + \varepsilon_t$, implying that $E_t z_t = \phi_t s_t$, and not z_t (FI!))

CEMP: $\pi_t = \gamma \pi_{t-1} + (1-\gamma)\bar{\pi}_t + \rho y_{t-1} + e_t$

$$\hat{E}_{t-1} \pi_t = \gamma \pi_{t-1} + (1-\gamma)\bar{\pi}_t + \rho y_{t-1} + e_t$$
$$= \underline{\underline{\phi_t'}}\ \underline{\underline{x_{t-1}}}$$

↳ a third approach!

kind of implying that firms update ϕ' first, and use that, giving them access to ϕ_t in $t-1$.

To get $\theta_t = (\bar{z}_{t-1}, s_{t-1})$, we need

1.) EH's PLM: $\hat{E}_{t-1} z_t = \bar{z}_{t-1} + s_{t-1}$

2.) fa_t, fb_t to be functions of z_{t-1}

But $fb_t = \hat{E}_t \sum_{h=0}^{\infty} \beta^h z_{t+1+h}$

$$= \underbrace{\frac{1}{1-\beta} \bar{z}_t + \text{stuff} \cdot s_t}$$

That can be fine if E_{t-1} (of this) $= \underbrace{\text{stuff}\ \bar{z}_{t-1}}_{\text{does this part hold?}} + \text{stuff}\ s_{t-1}$ ✓

Let's go back to CEMP:

$\hat{E}_{t-1} \pi_t = \gamma \pi_{t-1} + (1-\gamma) \bar{\pi}_t + \rho y_{t-1}$

$E_{t-1} \pi_t = \gamma \pi_{t-1} + \underbrace{(1-\gamma) \Gamma \bar{\pi}_t}_{\text{here it \underline{doesn't} seem to hold!}} + \rho y_{t-1}$ (E_{t-1} ALM)

CEMP is implicitly saying, $E_{t-1} \phi_t = \phi_t$.

One option is to modify CEMP's criterion so that things work out:

1) $\hat{E}_{t-1} z_t - E_{t-1} \hat{E}_{t-1} z_t$

What I'm also thinking is that

$E_{t-1}\bar{z}_t$ may actually be $\bar{z}_{t-1}$ b/c how should a non-FI rational thing know something that's not decided yet at $t-1$ (at least, not in the beginning of the period ...)?

So I'm thinking that for COMP, $E_{t-1}\bar{\pi}_t = \bar{\pi}_t$ b/c of the different assumption on when $\bar{\pi}_t$ is known (already at beginning of $t-1$).

Yes, so I think the ass. $E_{t-1}\bar{z}_t = \bar{z}_{t-1}$ is ok!

<u>Learning about LR inflation only</u>

If agents know that $x=0$ in LR then they also know (from TR): $i_t^{LR} = \psi_\pi \pi_t^{LR}$

I'm not sure at what point in time they think the long-run hit, so <u>let's attack from the other direction</u>: <u>Option 1</u>

what they learn is $\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1}(\pi_t - (\bar{\pi}_{t-1} + b \cdot s_{t-1}))$

So now, given this $\bar{\pi}$, we can evaluate 1-period-ahead expectations of all observables as

$$z_t = \bar{\pi}_{t-1} + b s_{t-1} + \varepsilon_t$$

but w/o further adjustment, I've only said that there's a common intercept in the learning rule.

I need to tie this into f_a & f_b

But I can also just sub this PLM into f_a & f_b, to get

$$f_{a,t} = \underbrace{\frac{1}{1-\alpha\beta}\bar{\pi}_t}_{} + (I_3 - \alpha\beta P)^{-1} s_t \quad f_{b,t} = \underbrace{\frac{1}{1-\alpha\beta}}_{}\bar{\pi}_t + (I_3 - \beta P)^{-1} s_t$$

where these are now scalars

The statement is "I think the inflation drift is where all my variables, π_t, x_t, i_t, will be in the LR"

and that preserves the matrices A_a, A_b and A_s.

<u>Another option</u> (option 2) is to rewrite the model, claiming that agents don't need to fcst $E_t z_{t+h}$ $\forall h$, but they only need that path for π. I like that

less b/c

1) it means obtaining different A_a, A_b & A_s

2) the ass that they don't care about these feats is more ad-hoc than to say that they know that there isn't a drift in x or in i

$\Rightarrow$ ok, so I buy Option 1. What is not clear is whether the PLM should be $E_t z_{t+1} =$

$$\begin{bmatrix} \bar{\pi}_t \\ 0 \\ \bar{i}_t \end{bmatrix} + b s_t \quad \text{or} \quad \begin{bmatrix} \bar{\pi}_t \\ 0 \\ 0 \end{bmatrix} + b s_t$$

or $\psi_\pi \bar{\pi}$

i.e. what should agents ad-hocly assume about the drift in i_t: that it equals that of inflation, or that it's 0?

Ryan meeting 20 Sept 2019

- $E_{t-1}\bar{z}_t = \bar{z}_{t-1}$? (p. 14, eq 92)
- projection facility when learn only constant?
- reconsider timing: PLM$_t$ fct of z_{t-1}.
- show results for different mon. pol. rules

Ryan was saying for $E_{t-1}\bar{z}_t$ that "I wouldn't have to evaluate this expectation b/c $\bar{z}_{stuff}$ would be stuff that's decided at $t-1$"

Ryan is saying what CEMP was saying:

$E_{t-1}\phi_t = \phi_t$ (or $E_{t-1}\bar{\pi}_t = \bar{\pi}_t$) b/c $\bar{\pi}_t$ is decided in $t-1$ so it's known, RE-guy doesn't need to evaluate E_{t-1}.

So I do think that this issue is really a notation issue: the way I have it in materials 3, $\bar{z}_x$, x denotes the time of use, not of forming; This coefficient was formed in t-1.

If we write what Ryan suggested,

$$z_t = f(\phi_{t-1}, s_t)$$

then we have the opposite notation w/ ϕ_x, x designating the <u>time of formation, not of use.</u>

With Ryan's notation, one would have to modify the PLM: $\hat{E}_{t-1} z_t = \bar{z}_{t-2} + b s_{t-1}$

$$\left(\text{i.e. } \hat{E}_t z_{t+1} = \bar{z}_{t-1} + b\, s_t\right)$$

→ so that $f_a(t)$ & $f_b(t)$ = stuff $\cdot\ \bar{z}_{t-1}$ + stuff $\cdot\ s_t$

And Ryan's notation is neater, after all, so let's do a materials 4 that prepares for the DW presentation and uses that notation, and only lays out the LR model w/ & w/o scalar anchoring.

As for projection, he wouldn't know how to do it for just the constant being learned. → talk more on it later! Could it be that it's only an issue if slope is learned?

But the notation issue still isn't quite resolved
b/c w/ Ryan's notation, the learning update
seems to wreak havoc.
b/c there, the notation seems to refer more to
the period in which it was used (RHS),
but the period in which the update happens (LHS)
Eq (1) & (2) in DW_prezi!