Materials 11 - trying to get a reasonable gain $\bar{g}$

Laura Gáti

December 4, 2019

Overview

1 Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \rho i_{t-1} + \bar{i}_t \tag{3}$$

$$\hat{\mathbb{E}}_t z_{t+h} = \begin{bmatrix} \bar{\pi}_{t-1} \\ 0 \\ 0 \end{bmatrix} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\, h_x \qquad \text{PLM} \tag{4}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \underbrace{\big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big)}_{\text{fcst error using (4)}} \qquad (b_1 \text{ is the first row of } b) \tag{5}$$

$$k_t = \begin{cases} k_{t-1} + 1 & \text{for decreasing gain learning} \\ \bar{g}^{-1} & \text{for constant gain learning.} \end{cases} \tag{6}$$

2 Recap of timing

Define some objects: *(I usually let t denote the time in which the variable is formed.)*

$$FE_{t-1} = z_t - \hat{\mathbb{E}}_{t-1}(z_t) \quad \text{one-period-ahead forecast error realized at time } t \tag{7}$$

$$= ALM(t) - PLM(t-1) \tag{8}$$

$$\theta_t = \hat{\mathbb{E}}_{t-1}(z_t) - \mathbb{E}_{t-1}(z_t) \quad \text{CEMP's criterion} \tag{9}$$

$$= PLM(t-1) - \mathbb{E}_{t-1}\, ALM(t) \tag{10}$$

$PLM(t) : \hat{\mathbb{E}}_t z_{t+1} = \bar{z}_{t-1} + b s_t$

Morning: morning of time t available: $\mathcal{I}_t^m = \{\bar{z}_{t-1}, s_t, k_{t-1}, FE_{t-2}\}$

1. Form all future expectations using $PLM(t)$(morning forecast) $\rightarrow z_t$ realized, $\rightarrow FE_{t-1}$ realized

2. Form $\theta_t \rightarrow k_t$ realized

3. **Evening**: Update $\bar{z}_t = \bar{z}_{t-1} + k_t^{-1}(z_t - (\bar{z}_{t-1} + b s_{t-1}))$

   $\rightarrow$ evening of time t available: $\mathcal{I}_t^e = \{\bar{z}_t, s_t, k_t, FE_{t-1}\}$

3 Trying to get an optimal gain

$T = 2000, N = 100$. Note: MSE as a function of gain is convex. Stats:

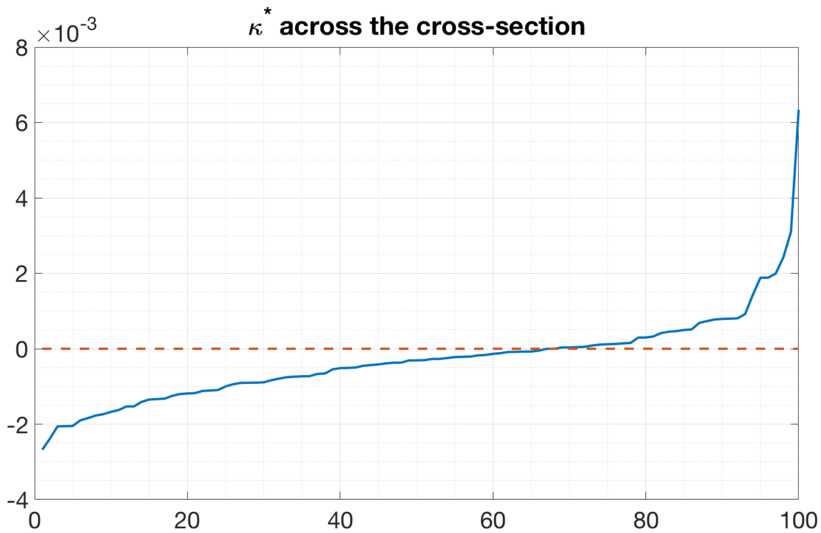
DGP is learning with constant gain $\bar{g} = 0.1450$.

- $Mean(\bar{g}_n^*) = -0.00028929$
- $Var(\bar{g}_n^*) = 1.4561e - 06$
- % of $\bar{g}_n^* < 0$ is 68%.

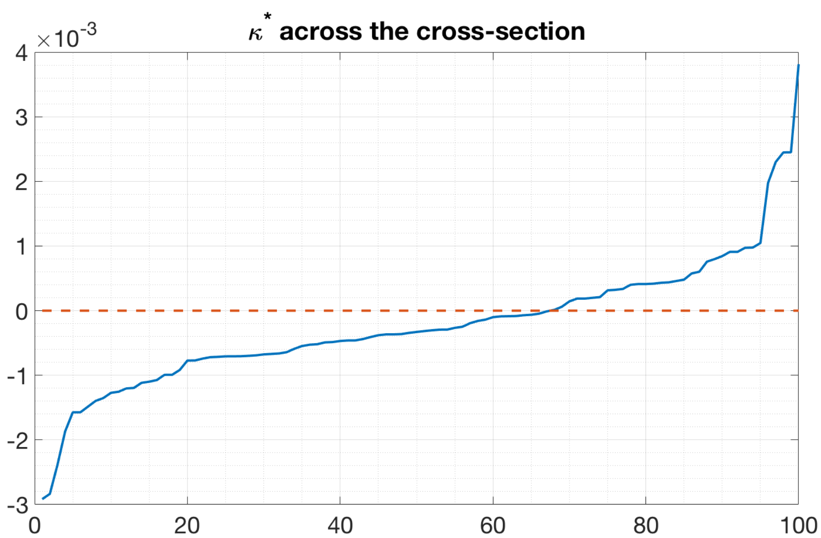
DGP is RE.

- $Mean(\bar{g}_n^*) = -0.00021913$
- $Var(\bar{g}_n^*) = 1.0134e - 06$
- % of $\bar{g}_n^* < 0$ is 67%.

Figure 1: Optimal gains across the histories



(a) Learning



(b) RE

4 Susanto's points and big picture

- L. Ball: credible disinflation causes booms in NK model (1994)
- instrument instability (Holbrook 1972, Sims 1974, Lane 1984)