

# Materials 36 - Convince that estimation is robust

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## Overview

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# 1 Calibration issues

## 1.1 $\alpha = \text{Prob}(\text{keep same price}) = ?$

So far I've used 0.5.

- Nikolay Hristov notes: expected duration of contract =  $\frac{1}{1-\alpha}$  periods.
- Evidence on average duration of prices:
  - Bils & Klenow (2004): 4.3 months
  - Klenow & Kryvstov (2008): mean 7-9 months, median 4-7 months
  - Nakamura & Steinsson (2008): 7-9 months
  - Klenow & Malin (2010): 6.9 months
  - Eichenbaum, Jaimovitch & Rebelo (2008, published as 2011): 10.6 months

→ On average this gives us 7.56 months, a little more than two quarters. The implied  $\alpha \approx 0.6$ .

Rotemberg & Woodford (1997) calibrate  $\alpha = 0.66$ . To err on the flexible price side, I set  $\alpha = 0.5$ .

## 1.2 The composite parameter $\kappa$ once and for all

I've used  $\kappa = \frac{(1-\alpha\beta)}{\alpha}\zeta$  where I should have used  $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\zeta$ . In Preston (2005), and I think this is also Woodford's favorite specification,  $\zeta = \frac{\omega+\sigma^{-1}}{1+\omega\theta}$ . Let's define terms:

- $\alpha$ : Prob(keep price unchanged)
- $\beta$ : discount factor
- $\zeta$ : measure of strategic complementarity in price setting. The smaller  $\zeta$ , the more complementarity. This depends on a bunch of things:
  - homogenous vs. specific factor markets ( $s_y = 0$  or not)
  - constant vs. variable desired markup ( $\epsilon_\mu = 0$  or not)
  - no vs. intermediate inputs ( $s_m = 0$  or not)

In particular (Prop 3.3 in Woodford 2011, equation 1.43, Chapter 3, p. 171):

$$\zeta = \frac{(1 - \mu s_m)(s_y + s_Y)}{1 + \theta[\epsilon_\mu + (1 - \mu s_m)s_y]} \quad (1)$$

with

- $\theta$ : price elasticity of demand
- $\mu(x)$ : markup function
- $\epsilon_\mu$ : elasticity of markup function (how much do target markups change at different levels of output)
- $s(y, Y, \xi)$ : real marginal cost function
- $s_y$ : elasticity of real marginal cost function wrt firm i's output,  $y_t(i)$
- $s_Y$ : elasticity of real marginal cost function wrt aggregate output,  $y_t$
- $s_m$ : elasticity of real marginal cost function wrt intermediate inputs,  $m_t(i)$

Then expression  $\zeta = \frac{\omega + \sigma^{-1}}{1 + \omega\theta}$  is obtained by assuming no intermediate inputs, constant desired markups wrt. output levels and specific factor markets, so that

$$\zeta = \frac{s_y + s_Y}{1 + s_y\theta} \quad (2)$$

What is  $\omega$ ? It's the derivative of the MC function wrt own output, but this only coincides with  $s_y$  for specific factor markets. Woodford shows that for specific factor markets,  $s_y = \omega$ ,  $s_Y = \sigma^{-1}$ , while for common factor markets  $s_Y = \omega + \sigma^{-1}$  because for the latter, there is no distinction between own and aggregate output for the purpose of wage setting and thus marginal cost. So, more broadly,  $\omega$  is a measure of how marginal cost reacts to some wage-relevant measure of output. Woodford, Chapter 3, (1.16), p. 152:

$$\omega = \underbrace{\omega_w}_{= \eta, \text{ Frisch elasticity of disutility of labor wrt output}} + \underbrace{\omega_p}_{\text{elasticity of MPL wrt output}} \quad (3)$$

Denoting the Frisch elasticity as  $\eta$ , and noting that for Cobb-Douglas,  $\partial MPL / \partial y_t = 0$ ,

$$\omega = \eta \quad (4)$$

- Chari, Kehoe & McGrattan (2000) and Woodford (2011) values:  $\theta = 10, \sigma = 1, \omega = 1.25, \beta = 0.99$   
These values are not controversial. I just want to check that the Frisch elasticity is Kosher, because by setting  $\omega = 1.25$ , we are implicitly setting the Frisch. According to Susanto (compare my summaries Part 1, p. 56 Mac and Part 2 p. 46-47 Mac), the inverse Frisch elasticity,  $\varepsilon_{H,W} = \eta^{-1}$  needs to be 4 to flatten the labor supply curve (Barro-King comovement puzzle), but estimates suggests it's  $< 1$ . Here,  $\omega = 1.25 = \eta$  implies  $\eta^{-1} = 4/5 < 1$ . I guess that's at least in line with estimates.

- The value of  $\kappa$

Susanto suggests (somewhere in my notes) that for a NK model to display reasonable dynamics,  $\kappa$  needs to be below 0.1, but preferably even less than 0.01.

Chari, Kehoe & McGrattan (2000) and Woodford (2011) values with  $\alpha = 0.5 \rightarrow \kappa = 0.0842$ .

Chari, Kehoe & McGrattan (2000) and Woodford (2011) values with  $\alpha = 0.6 \rightarrow \kappa = 0.0451$ .

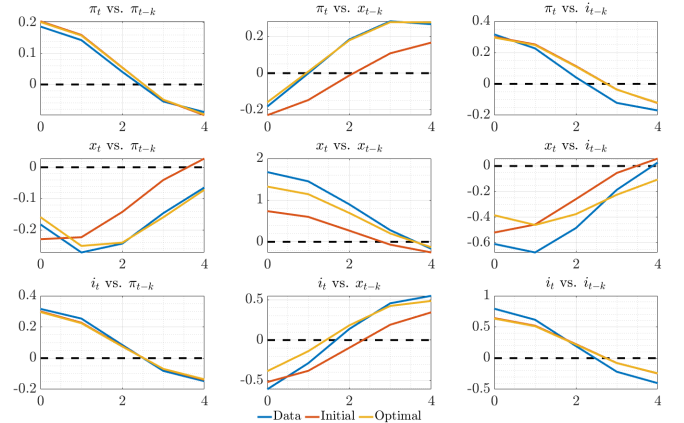
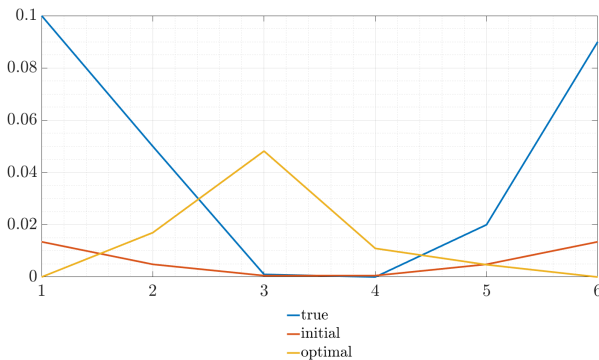
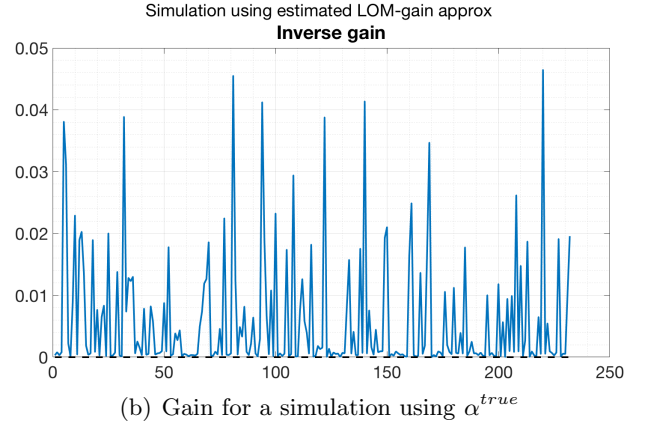
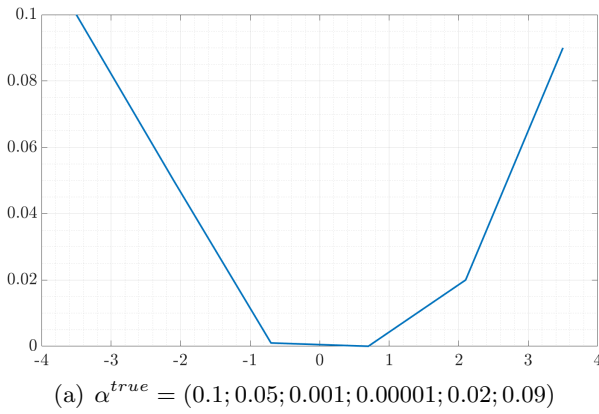
Chari, Kehoe & McGrattan (2000) and Woodford (2011) values with  $\alpha = 0.66 \rightarrow \kappa = 0.0298$ .

Chari, Kehoe & McGrattan (2000) and Woodford (2011) values with  $\alpha = 0.7 \rightarrow \kappa = 0.0219$ .

## 2 Back to estimation

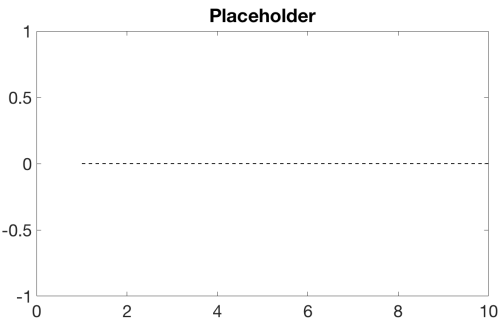
### 2.1 Simulated data with different seed

**Figure 1:** A seed for shocks of `rng(1)` when true data was generated using `rng(0)`

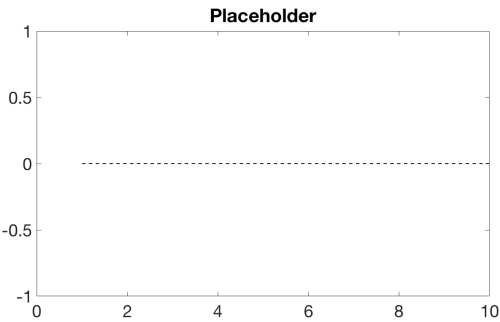


2.2 Autocovariogram for real data

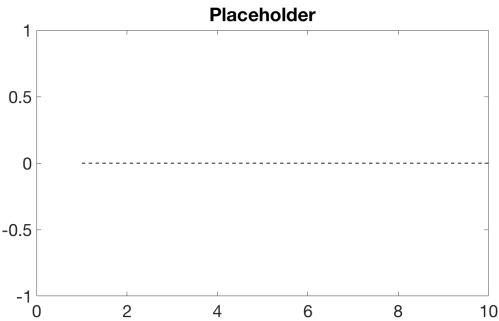
Figure 2: Candidate points



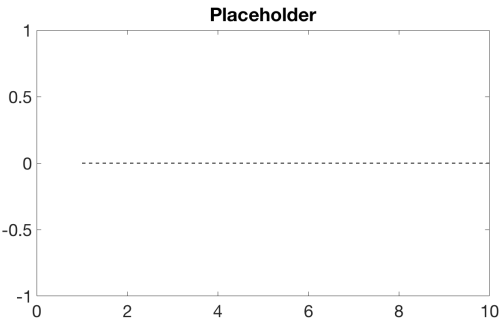
(a)  $\hat{\alpha}$



(b) Autocovariogram



(c)  $\hat{\alpha}$



(d) Autocovariogram

## A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (\text{A.1})$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (\text{A.2})$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \quad (\text{if imposed}) \quad (\text{A.3})$$

$$\text{PLM:} \quad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \quad b = g_x h_x \quad (\text{A.4})$$

$$\text{Updating:} \quad a_t = a_{t-1} + k_t^{-1} (z_t - (a_{t-1} + b s_{t-1})) \quad (\text{A.5})$$

$$\text{Anchoring function:} \quad k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k f e_{t-1}^2 \quad (\text{A.6})$$

$$\text{Forecast error:} \quad f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \quad (\text{A.7})$$

$$\text{LH expectations:} \quad f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \quad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \quad (\text{A.8})$$

This notation captures vector learning ( $z$  learned) for intercept only. For scalar learning,  $a_t = (\bar{\pi}_t \ 0 \ 0)'$  and  $b_1$  designates the first row of  $b$ . The observables  $(\pi, x)$  are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix} \sigma & 1-\beta & -\sigma\beta \end{bmatrix} f_b + \sigma \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \quad (\text{A.9})$$

$$\pi_t = \kappa x_t + \begin{bmatrix} (1-\alpha)\beta & \kappa\alpha\beta & 0 \end{bmatrix} f_a + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \quad (\text{A.10})$$

## B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only,  $k_t^{-1} = \mathbf{g}(f e_{t-1})$ ):

$$\pi_t = -\frac{\lambda_x}{\kappa} \left\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_\pi(t) \right) \right. \\ \left. \left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\bar{\pi}}(t+j)) \right) \right\} \quad (\text{B.1})$$

where I'm using the notation that  $\prod_{j=0}^0 \equiv 1$ . For interpretation purposes, let me rewrite this as follows:

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ - \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j}^{-1} + f e_{t+1+j|t+j}^{eve} \mathbf{g}_{\bar{\pi}}(t+j)) \right) \quad (\text{B.2})$$

Interpretation: **tradeoffs from discretion in RE** + **effect of current level and change of the gain on future tradeoffs** + **effect of future expected levels and changes of the gain on future tradeoffs**