Materials 35 - ...And still estimating the anchoring function

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Overview

1 Estimation procedure

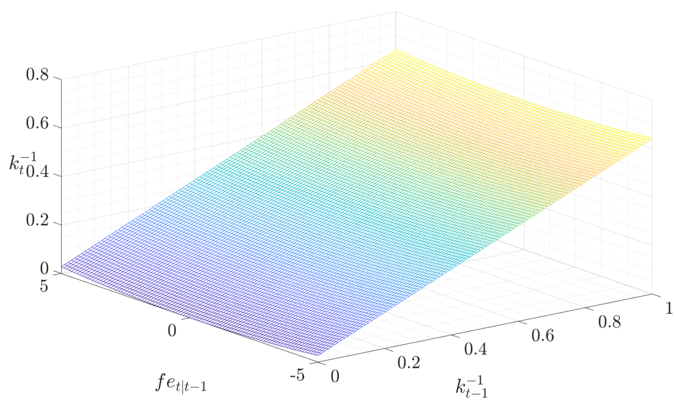
Instead of the AR(1) anchoring function used so far (Equation A.6), I use the following equation

$$k_t^{-1} = \alpha s(X) \tag{1}$$

where $X = (k_{t-1}^{-1}, fe_{t|t-1})$ and I use piecewise linear interpolation. I initialize α_0 by specifying a grid for X, passing the grid through Equation (A.6) to generate k_t^{-1}-values, and approximating by fitting the grid to the k_t^{-1}-values. See Fig. 1.

Then I estimate α using GMM, targeting the autocovariance structure of inflation, the output gap and the nominal interest rate (federal funds rate) in the data.

Figure 1: Initialization via Equation (A.6) implies this functional relationship

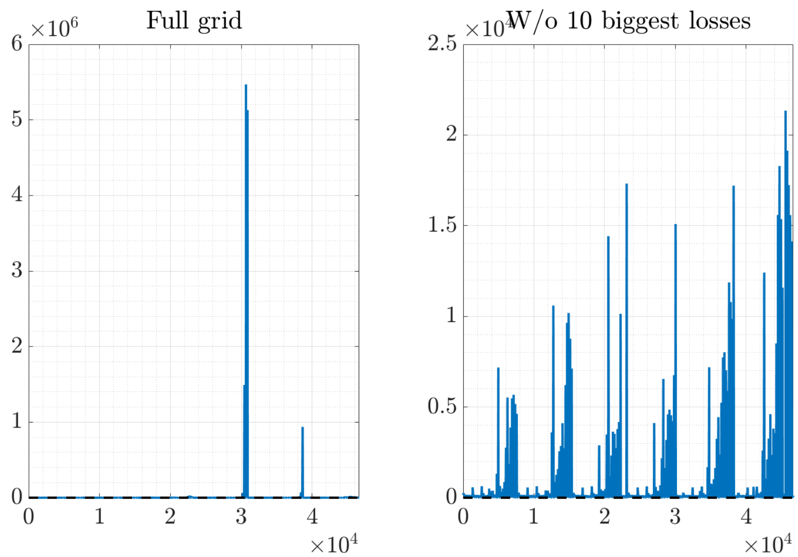


$T = 233$ before BK-filtering, $T = 209$ after BK-filtering. Using the "constant-only, inflation-only" learning PLM. I drop the $ndrop = 5$ initial values. I restrict $\alpha \in (0, 1)$, the support of k^{-1} in the grid. I target the lag $0, \ldots, 4$ autocovariance matrices, dropping repeated entries at lag 0, leaving me with 42 moments.

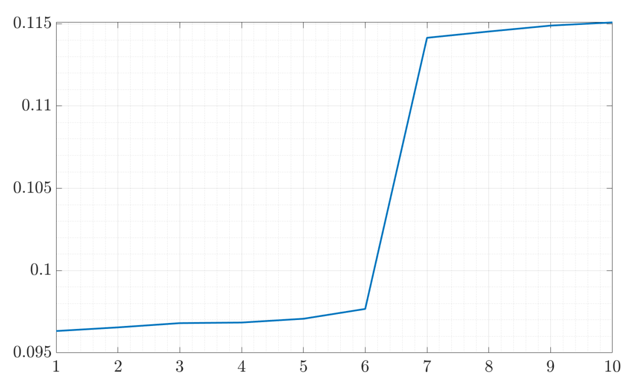
2 1D function

2.1 Simulated data, evaluate loss on a $6^6 = 46656$ grid

Figure 2: Objective function values on a grid with values $(0; 0.2; 0.4; 0.6; 0.8; 1)$

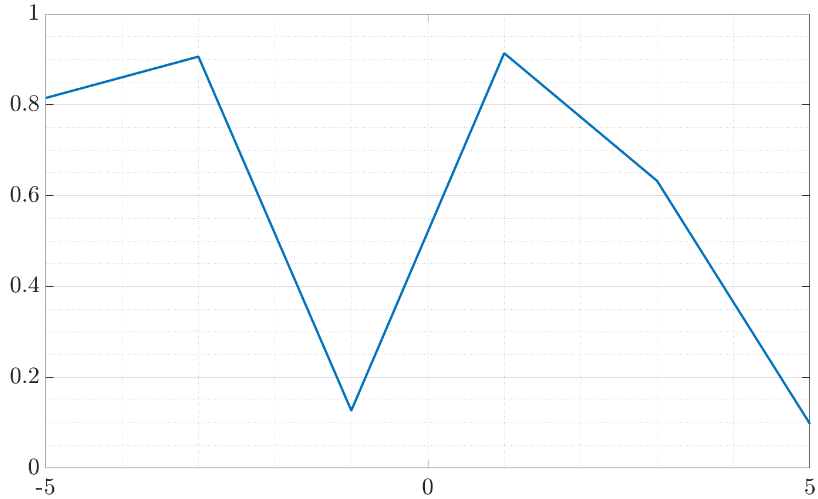


(a) Full grid and excluding the highest 10

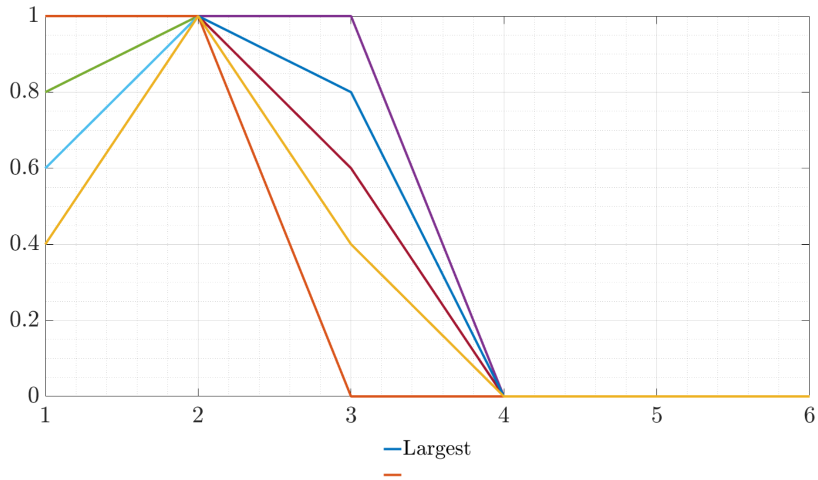


(b) The lowest 10 losses

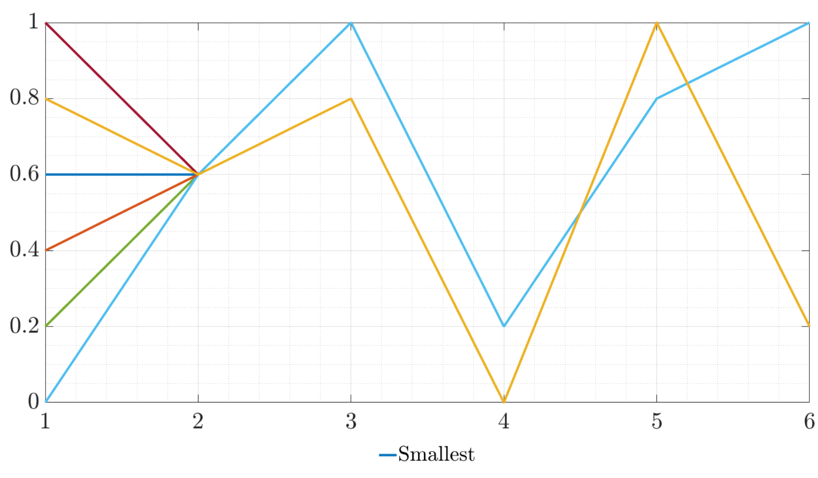
Figure 3: The αs with highest and lowest objective function values



(a) True coefficients



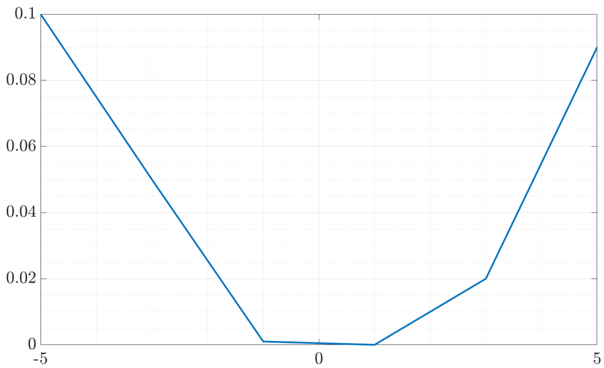
(b) Highest loss



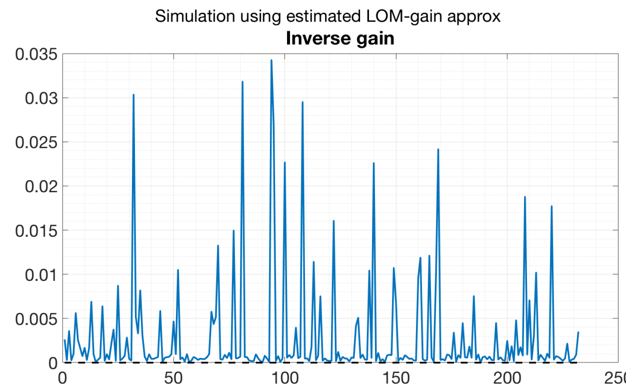
(c) Lowest loss

2.2 Simulated data, $\alpha^{true} \in (0, 0.1)$ and convex

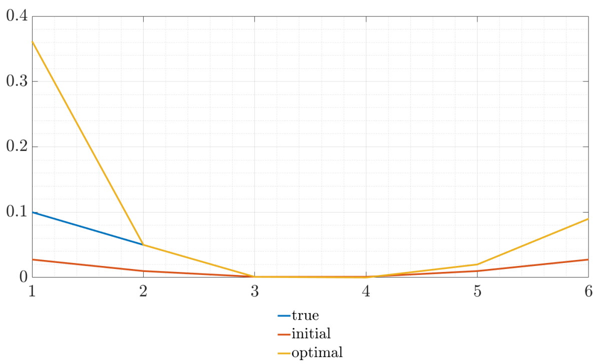
Figure 4: Effects of making the truth i) convex ii) between 0 and 0.1



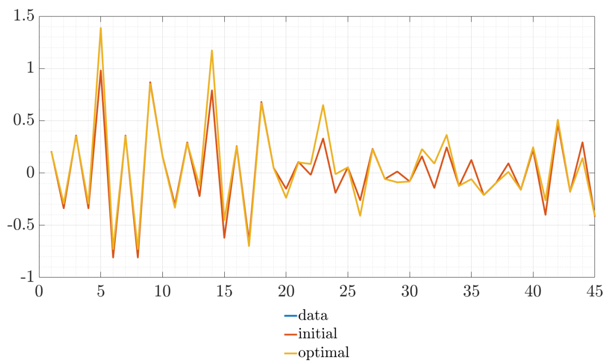
(a) $\alpha^{true} = (0.1; 0.05; 0.001; 0.00001; 0.02; 0.09)$



(b) Gain for a simulation using α^{true}



(c) $\alpha^{true}, \alpha_0, \hat{\alpha}$

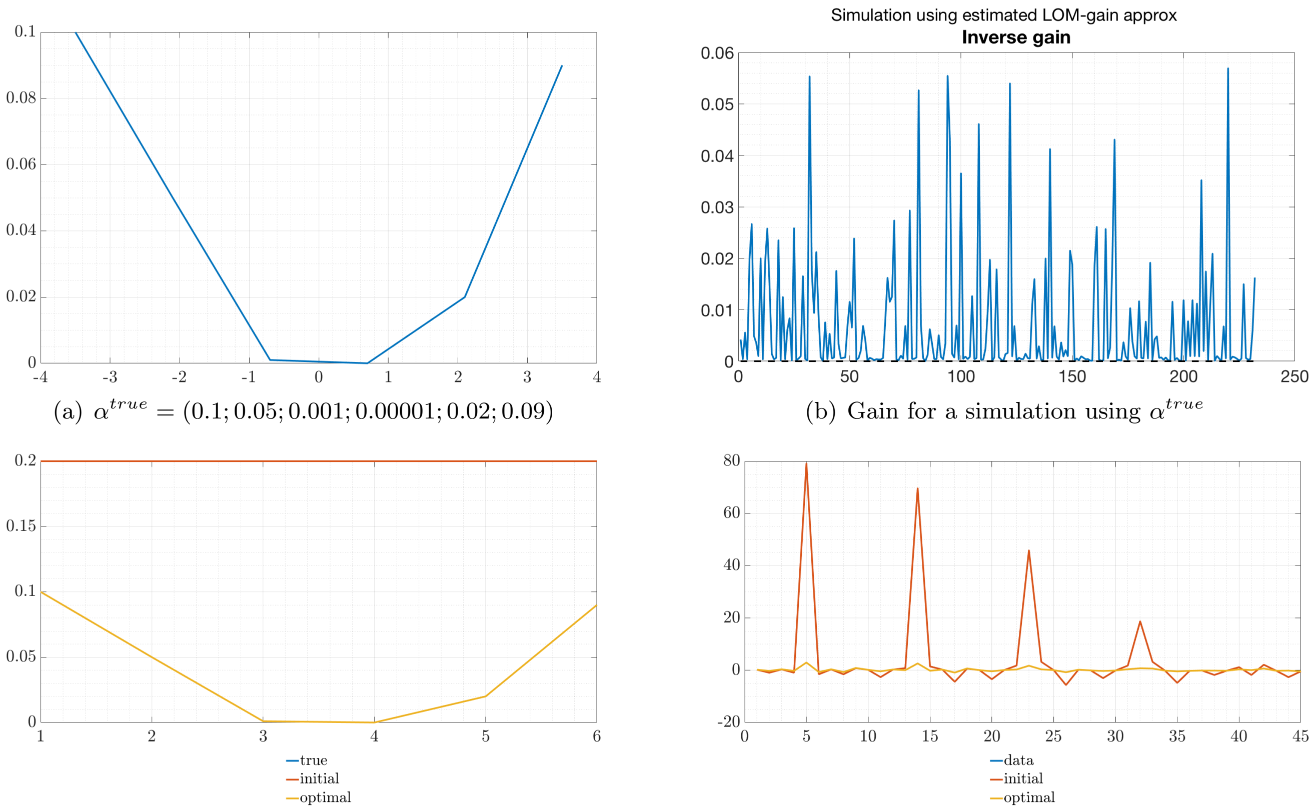


(d) Moments

Actually, this improves the solver's ability to get close (up to $\alpha(fe = -5)$) dramatically! But this indicates the finite-element issue: there might not be any data in the -5 range for the forecast error. If that's so, then redoing this spiel with the true data involving a smaller forecast error support should do the trick.

BAM! That did it! PTO.

Figure 5: Effects of making the truth i) convex ii) between 0 and 0.1 iii) shrinking the true forecast error support to $(-3.5, 3.5)$



(a) $\alpha^{true} = (0.1; 0.05; 0.001; 0.00001; 0.02; 0.09)$

(b) Gain for a simulation using α^{true}

(c) $\alpha^{true}, \alpha_0, \hat{\alpha}$

(d) Moments

2.3 100000 starting points, top 10 minima

Didn't improve on 1000, besides the nonconvex truth was too much of a challenge, so not pursing this.

2.4 Add moments

2.4.1 Strict priors: anchoring function should be convex

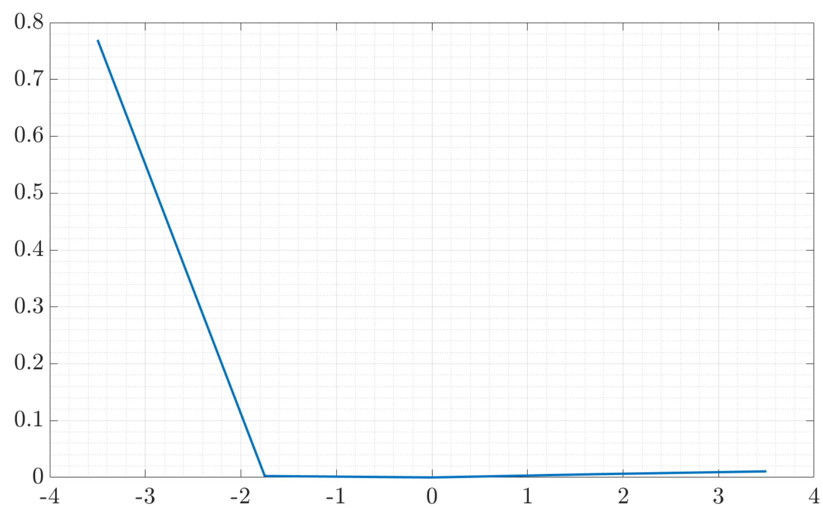
2.4.2 Calibrated moments: e.g. average gain in simulation should be 0.05

Starting with simulated data, and then turning to the real data, I've found that

- without (and also with!) additional moments, data is able identify 5 parameters;
- the convexity moment forces the solution to be convex (otherwise it is often not convex);
- the mean moment helps pinning down the solution when the convexity moment is in place.

Such a solution seems quite robust to starting points (there are some starting points that lead the solver not to converge). The solution looks like this:

Figure 6: The candidate solution

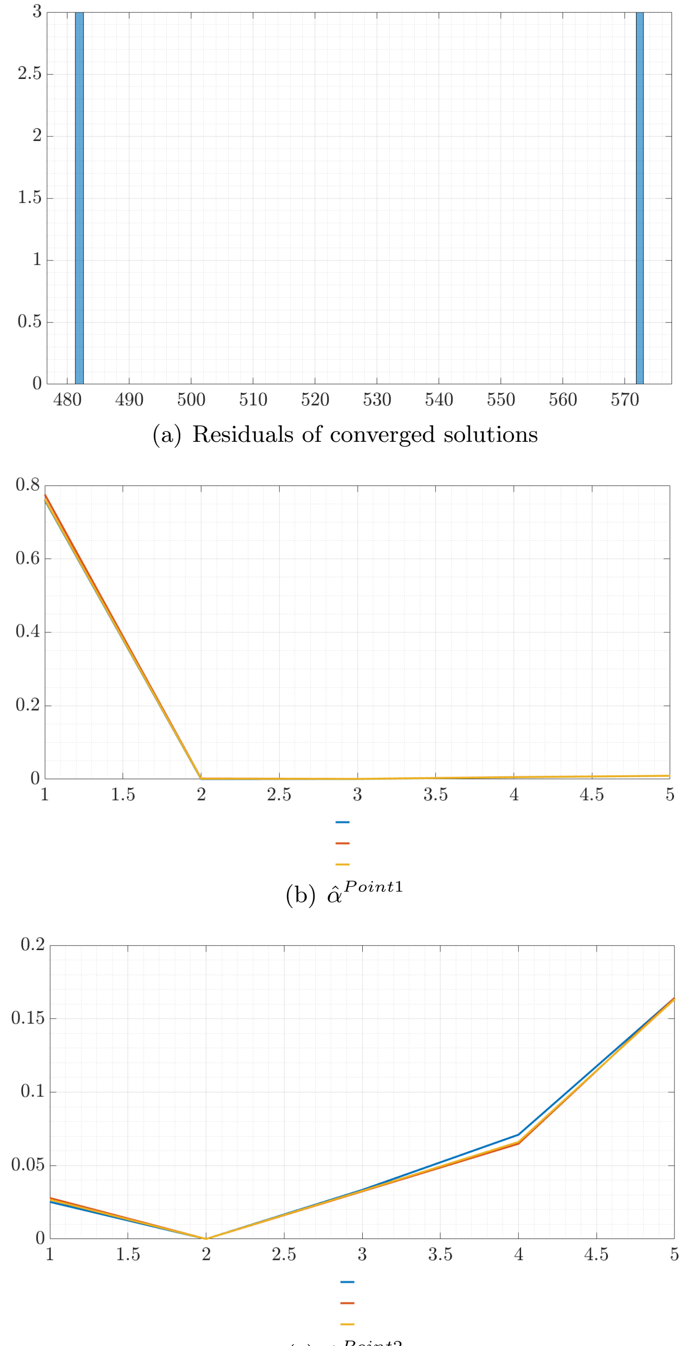


$n_\alpha = 5, fe \in (-3.5, 3.5), \alpha \in (0, 1)$, convexity and mean moment imposed, starting point is given by the AR(1) gain function on this grid. $\hat{\alpha} = (0.7696; 0.0026; 0; 0.0058; 0.0107)$

2.5 But is it robust?

Initialization for robustness check: random straight lines between (0,1). I've also done completely random sets of points and I get similar, but less clean results because initializing at non-convex points impairs the solver's behavior (gets stuck at more local minima).

Figure 7: Converged solutions for 20 runs (6 converged)



(a) Residuals of converged solutions

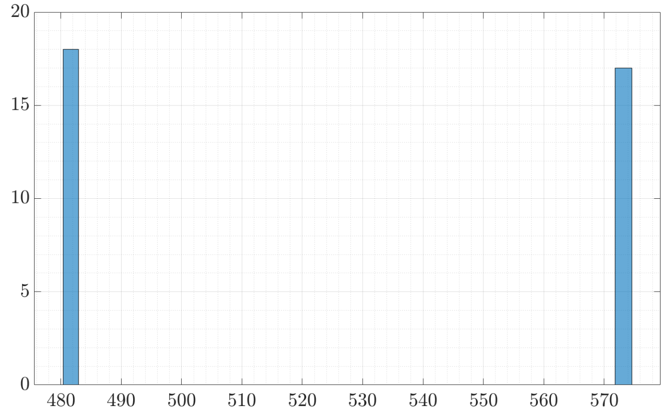
(b) $\hat{\alpha}^{Point1}$

(c) $\hat{\alpha}^{Point2}$

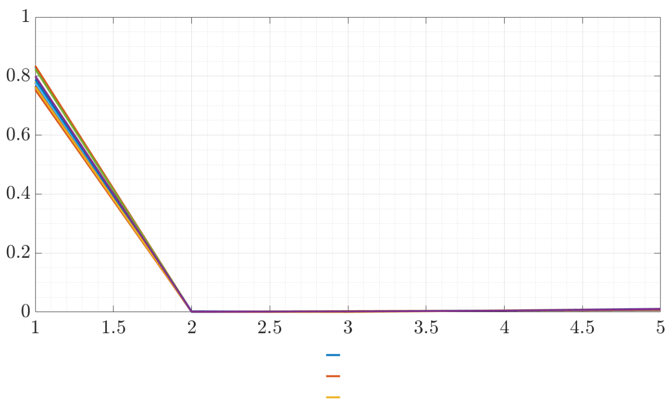
$n_\alpha = 5, fe \in (-3.5, 3.5), \alpha \in (0, 1)$, convexity and mean moment imposed, from different straight lines between (0,1) as initial points.

- Mean coefficients in Point 1 $= (0.7663; 0.0011; 0.0007; 0.0054; 0.0092)$
- Mean coefficients in Point 2 $= (0.0267; 0; 0.0329; 0.0673; 0.1639)$

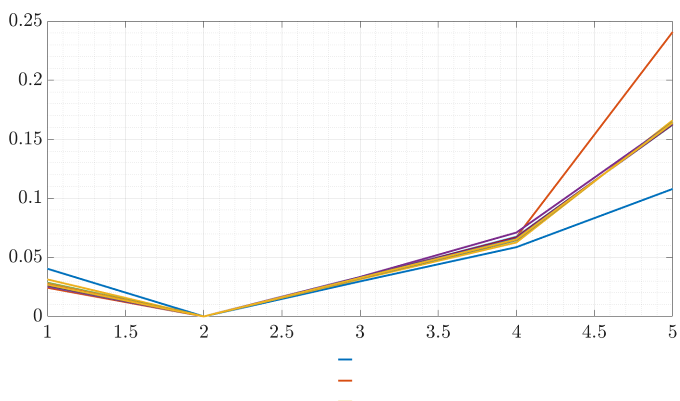
Figure 8: Converged solutions for 100 runs (35 converged)



(a) Residuals of converged solutions



(b) $\hat{\alpha}^{Point1}$



(c) $\hat{\alpha}^{Point2}$

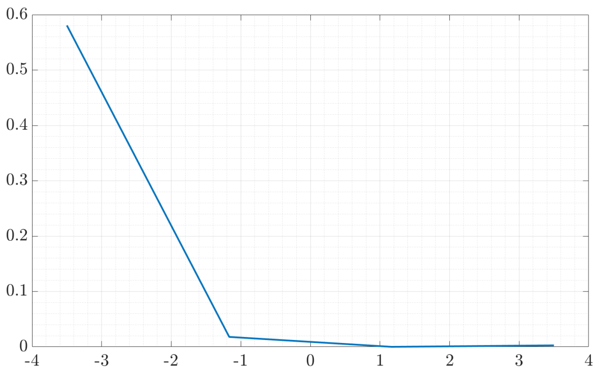
$n_\alpha = 5, fe \in (-3.5, 3.5), \alpha \in (0,1)$, convexity and mean moment imposed, from different straight lines between (0,1) as initial points.

- Mean coefficients in Point 1 = $(0.7816; 0.0014; 0.0012; 0.0054; 0.0091)$
- Mean coefficients in Point 2 = $(0.0279; 0; 0.0324; 0.0654; 0.1652)$

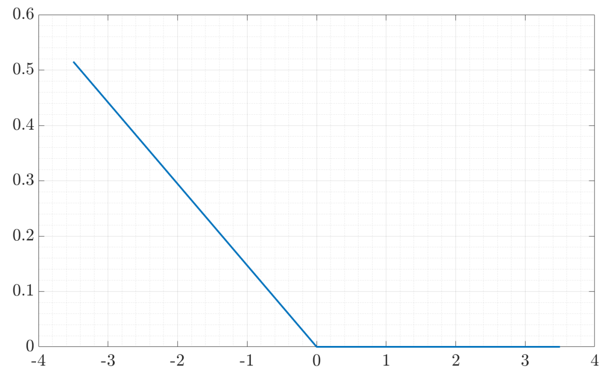
2.6 Is the L-shape a residue of too many coefficients?

No:

Figure 9: The same configurations as above, except less parameters



(a) $n_\alpha = 4$



(b) $n_\alpha = 3$

A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{A.1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{A.2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \qquad \text{(if imposed)} \tag{A.3}$$

$$\text{PLM:} \qquad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\, h_x \tag{A.4}$$

$$\text{Updating:} \qquad a_t = a_{t-1} + k_t^{-1}\big(z_t - (a_{t-1} + b s_{t-1})\big) \tag{A.5}$$

$$\text{Anchoring function:} \qquad k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k f e_{t-1}^2 \tag{A.6}$$

$$\text{Forecast error:} \qquad f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \tag{A.7}$$

$$\text{LH expectations:} \qquad f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \qquad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \tag{A.8}$$

This notation captures vector learning (z learned) for intercept only. For scalar learning, $a_t = \begin{pmatrix}\bar{\pi}_t & 0 & 0\end{pmatrix}'$ and b_1 designates the first row of b. The observables (π, x) are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix}\sigma & 1-\beta & -\sigma\beta\end{bmatrix} f_b + \sigma \begin{bmatrix}1 & 0 & 0\end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \tag{A.9}$$

$$\pi_t = \kappa x_t + \begin{bmatrix}(1-\alpha)\beta & \kappa\alpha\beta & 0\end{bmatrix} f_a + \begin{bmatrix}0 & 0 & 1\end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \tag{A.10}$$

B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only, $k_t^{-1} = \mathbf{g}(f e_{t-1})$):

$$\pi_t = -\frac{\lambda_x}{\kappa}\Bigg\{x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_\pi(t)\Big)$$
$$\Big(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi}}(t+j))\Big)\Bigg\} \tag{B.1}$$

where I'm using the notation that $\prod_{j=0}^{0} \equiv 1$. For interpretation purposes, let me rewrite this as follows:

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + f e^{eve}_{t|t-1}\mathbf{g}_\pi(t)\Big)\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i}$$
$$-\frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + f e^{eve}_{t|t-1}\mathbf{g}_\pi(t)\Big)\Big(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(k_{t+1+j}^{-1} + f e^{eve}_{t+1+j|t+j})\mathbf{g}_{\bar{\pi}}(t+j)\Big) \tag{B.2}$$

Interpretation: tradeoffs from discretion in RE + effect of current level and change of the gain on future tradeoffs + effect of future expected levels and changes of the gain on future tradeoffs