Materials 42 - Reverting to a first-pass calibration

Laura Gáti

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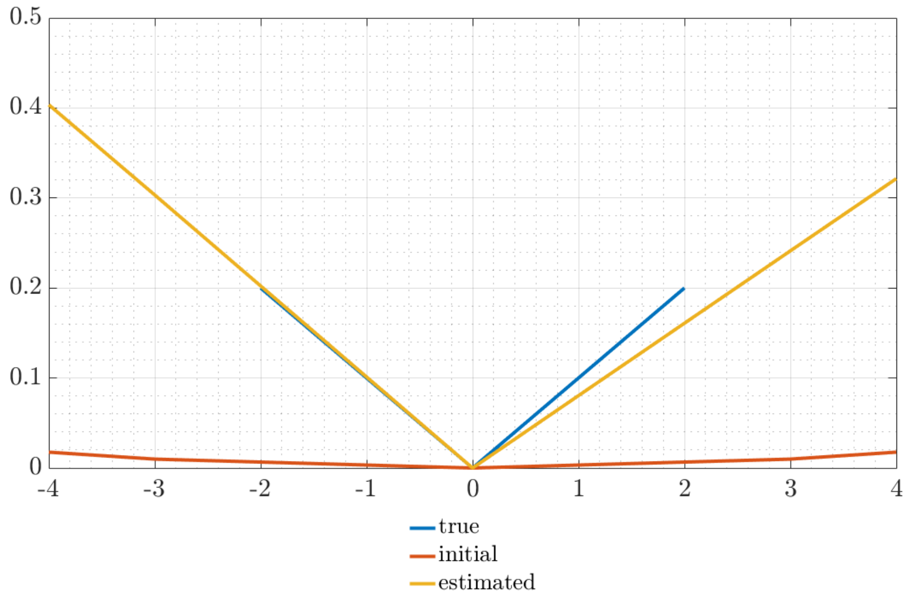
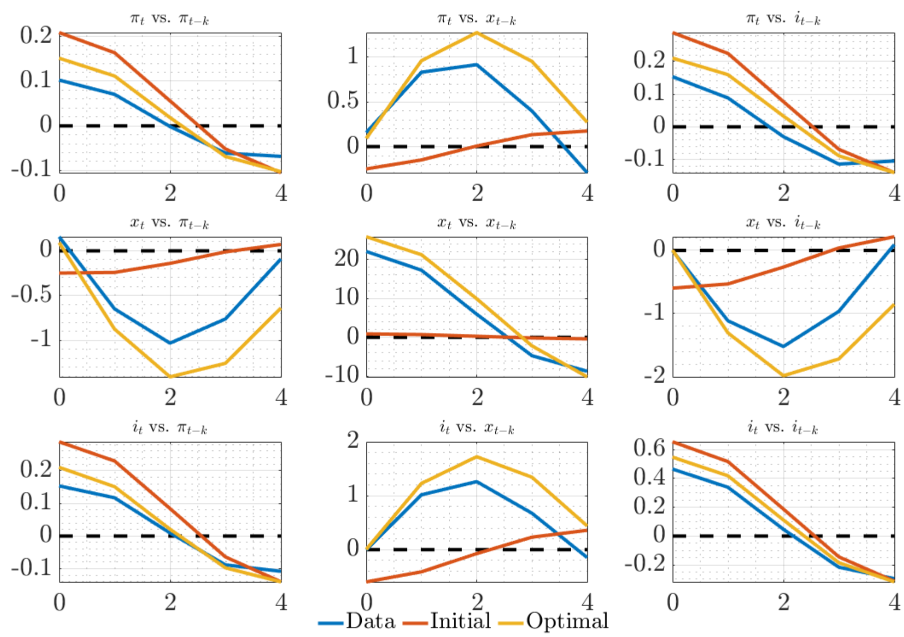
1 Reference figures: best estimation so far

Figure 1: Reference: Figure 2 from Materials 41. Not using 1-step ahead forecasts of inflation, estimate mean moments once, imposing convexity with weight 100K, with 0 at 0 imposed with weight 1000, taking square root of elements of W, w/o measurement error, gridpoints at $fe = [-4, -3, 0, 3, 4]$ true parameters (0.2; 0.1; 0; 0.1; 0.2) at fe (-2,-1,0,1,2)



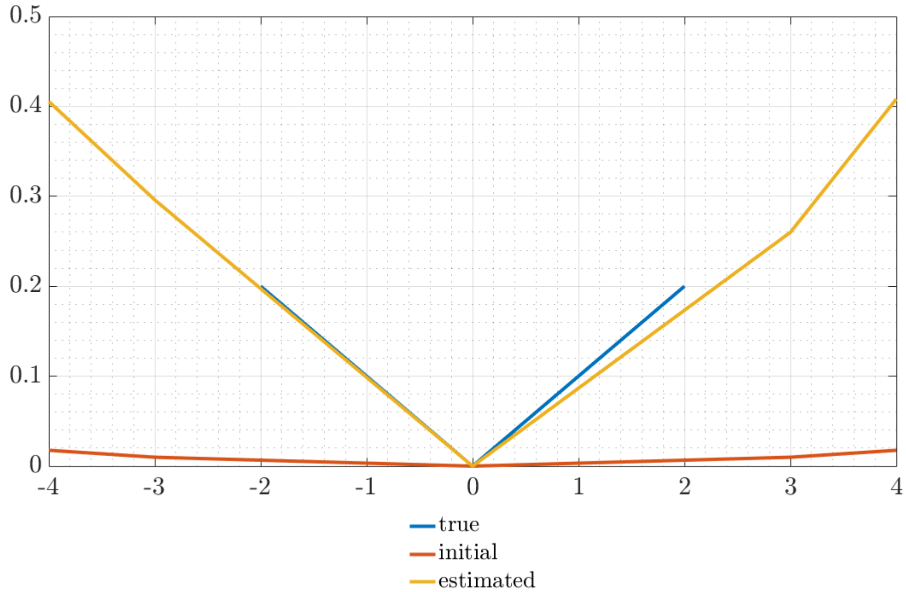
(a) Estimated parameters



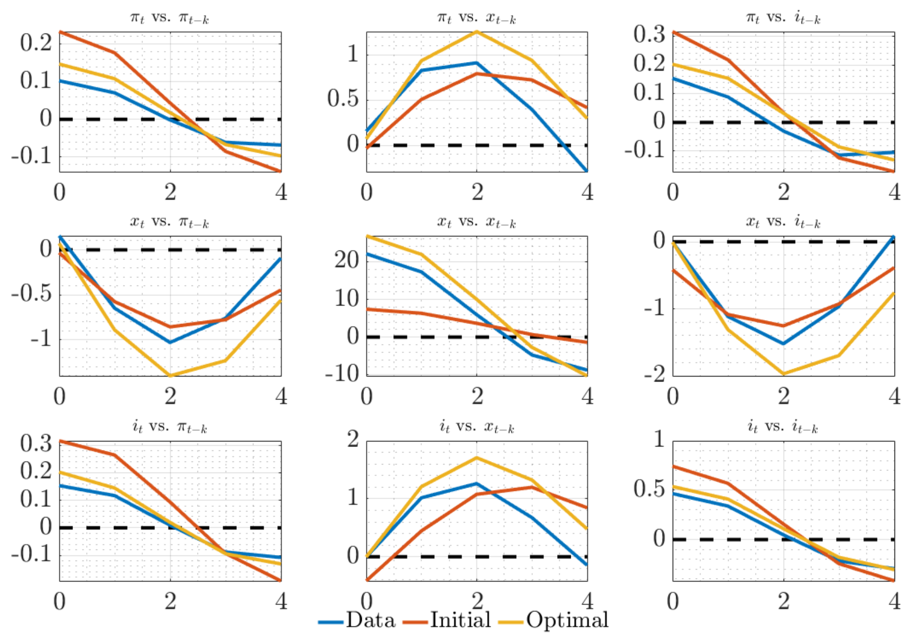
(b) Autocovariogram

2 A correction for the loss: treating explosive or negative gain paths

Figure 2: Setting $|fe| > 5$ to 5, $k^{-1} < 0$ to 0. Not using 1-step ahead forecasts of inflation, estimate mean moments once, imposing convexity with weight 100K, with 0 at 0 imposed with weight 1000, taking square root of elements of W, w/o measurement error, gridpoints at $fe = [-4, -3, 0, 3, 4]$ true parameters (0.2; 0.1; 0; 0.1; 0.2) at fe (-2,-1,0,1,2)



(a) Estimated parameters



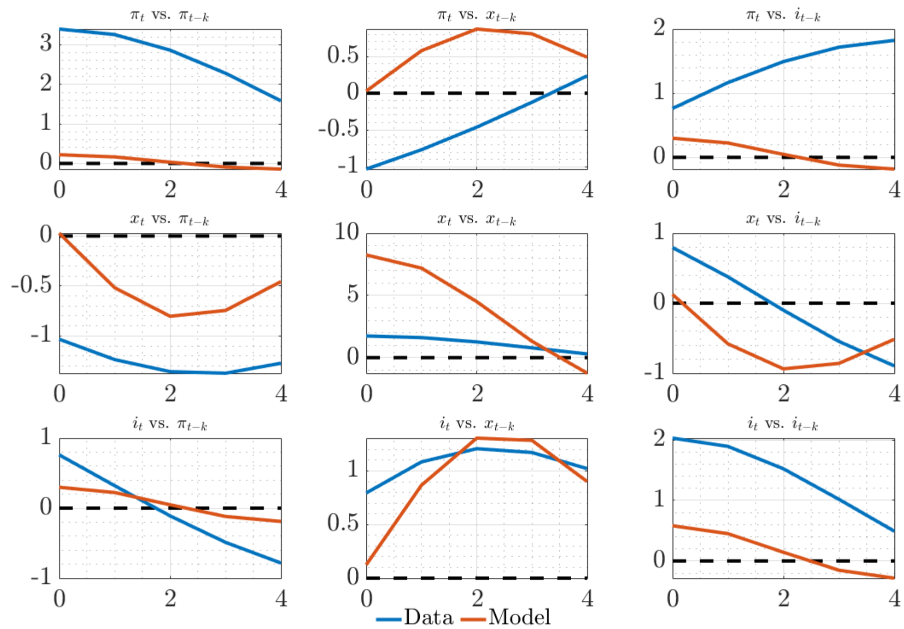
(b) Autocovariogram

Experimented with a fe-threshold of 8 or 10, but those are more off ($\hat{\alpha}$ lower than α^{true} at the edges). Same holds for a threshold of 3. The threshold seems to be a balance-act between allowing volatile histories to happen, but not allowing explosions to screw up the moments.

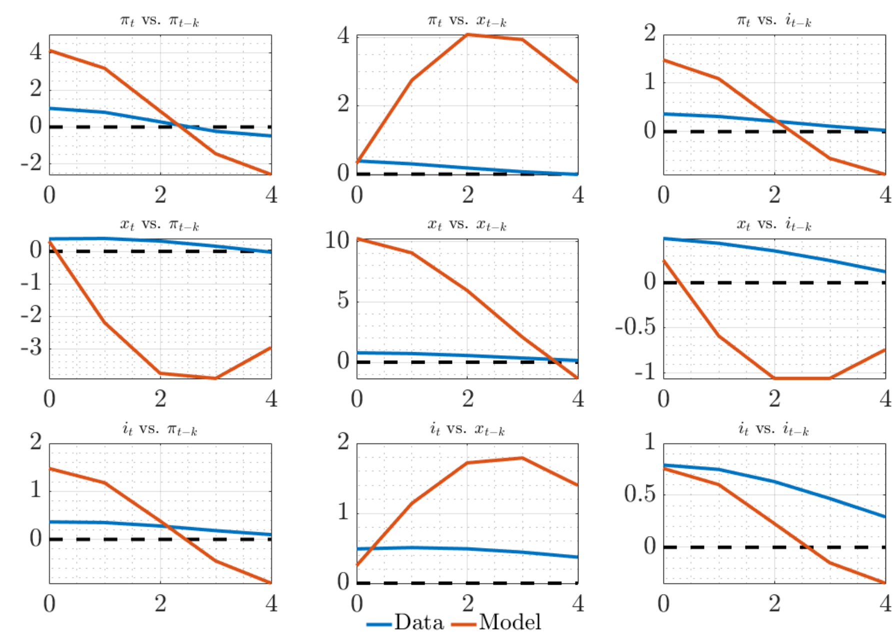
3 Calibrating shock volatilities to match autocovariances of π, x, i

Keep α fix at (0.2,0.1,0,0.1,0.2)

Figure 3



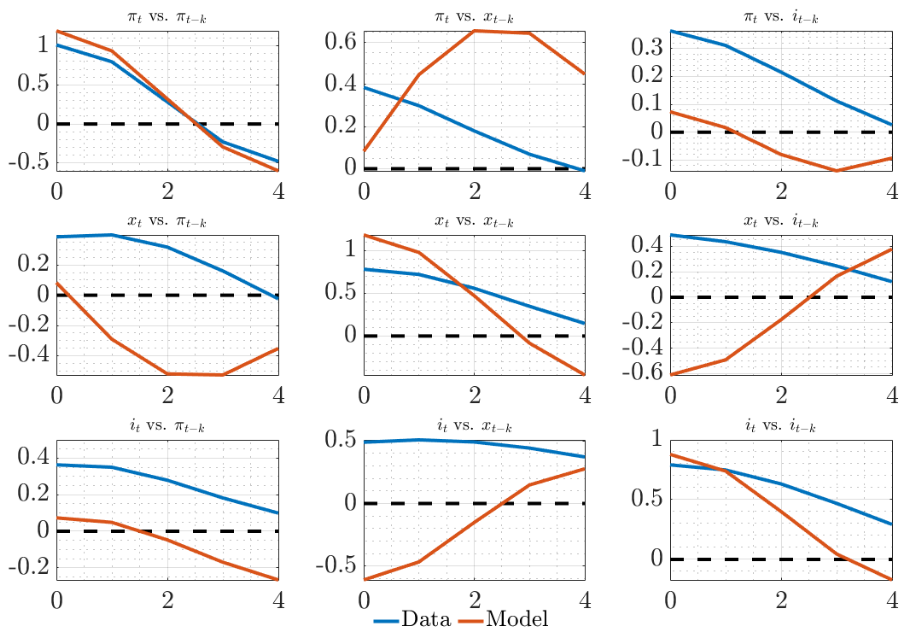
(a) $(\sigma_r, \sigma_i, \sigma_u) = (1, 1, 1)$



(b) $(\sigma_r, \sigma_i, \sigma_u) = (1, 1, 1)$, Using annualized quarterly percent changes for inflation and inflation expectations

Don't understand why autocovariance of x and i is changing. Can it be because I have fewer periods in the real data (I think it's only 2 fewer)? Or has there been data revision in the 3 months since I downloaded the data last?

Figure 4

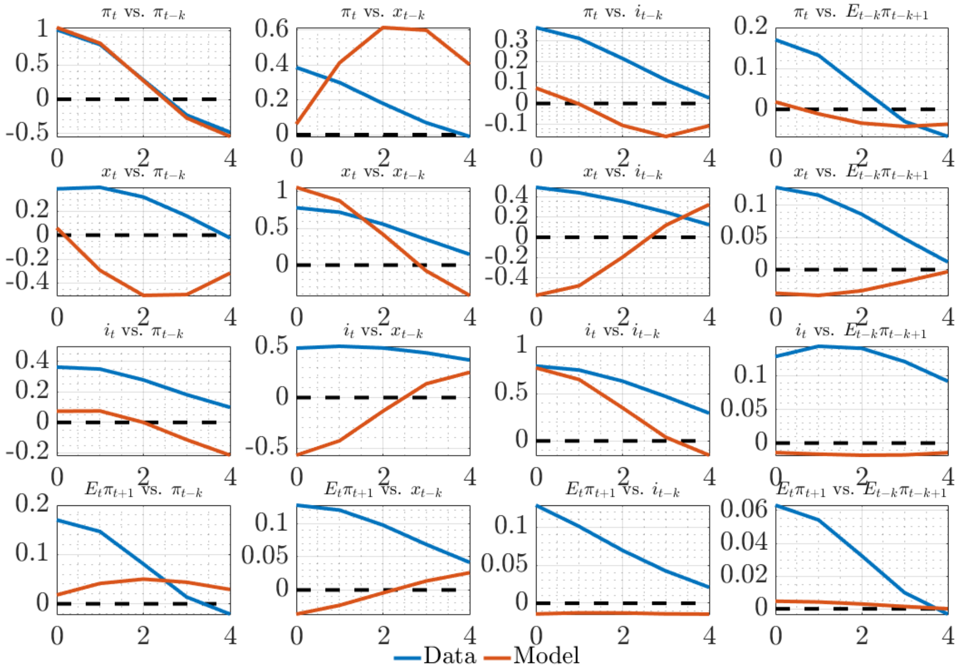


(a) $(\sigma_r, \sigma_i, \sigma_u) = (0.01, 2, 0.5)$

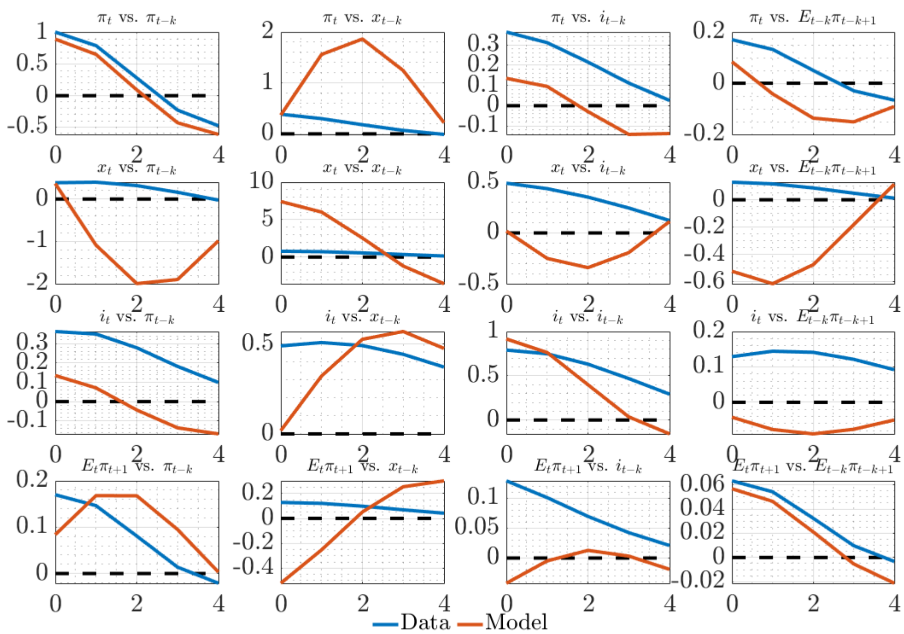
4 Calibrating αs to match autocovariances of $\pi, \mathbb{E}(\pi)$

Keep $\sigma_r, \sigma_i, \sigma_u$ fix at (0.01, 2, 0.5).

Figure 5: Using expectations (estimating VAR using ridge regression with ridge parameter 0.001 if unstable)



(a) $\alpha = (0.2, 0.1, 0, 0.1, 0.2)$



(b) $\alpha = (1, 0.5, 0, 0.5, 1)$

A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{A.1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{A.2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \qquad \text{(if imposed)} \tag{A.3}$$

$$\text{PLM:} \qquad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\, h_x \tag{A.4}$$

$$\text{Updating:} \qquad a_t = a_{t-1} + k_t^{-1}\big(z_t - (a_{t-1} + b s_{t-1})\big) \tag{A.5}$$

$$\text{Anchoring function:} \qquad k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k fe_{t-1}^2 \tag{A.6}$$

$$\text{Forecast error:} \qquad fe_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \tag{A.7}$$

$$\text{LH expectations:} \qquad f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \qquad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \tag{A.8}$$

This notation captures vector learning (z learned) for intercept only. For scalar learning, $a_t = \begin{pmatrix} \bar{\pi}_t & 0 & 0 \end{pmatrix}'$ and b_1 designates the first row of b. The observables (π, x) are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix} \sigma & 1-\beta & -\sigma\beta \end{bmatrix} f_b + \sigma \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \tag{A.9}$$

$$\pi_t = \kappa x_t + \begin{bmatrix} (1-\alpha)\beta & \kappa\alpha\beta & 0 \end{bmatrix} f_a + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \tag{A.10}$$

B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only, $k_t^{-1} = \mathbf{g}(fe_{t-1})$):

$$\begin{aligned} \pi_t = -\frac{\lambda_x}{\kappa}\Bigg\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_\pi(t)\Big) \\ \Big(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi}}(t+j))\Big)\Bigg\} \end{aligned} \tag{B.1}$$

where I'm using the notation that $\prod_{j=0}^{0} \equiv 1$. For interpretation purposes, let me rewrite this as follows:

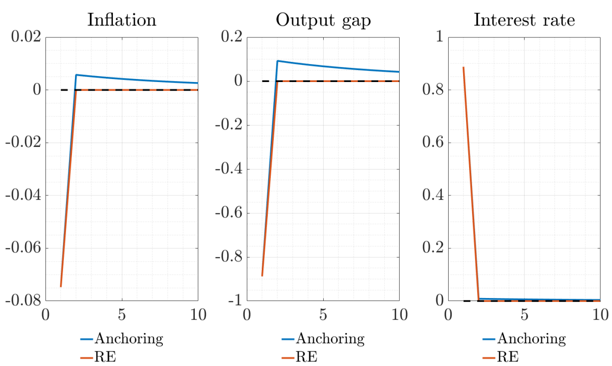
$$\begin{aligned} \pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + fe_{t|t-1}^{eve}\mathbf{g}_\pi(t)\Big)\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ -\frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + fe_{t|t-1}^{eve}\mathbf{g}_\pi(t)\Big)\Big(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j}^{-1} + fe_{t+1+j|t+j}^{eve})\mathbf{g}_{\bar{\pi}}(t+j)\Big) \end{aligned} \tag{B.2}$$

Interpretation: tradeoffs from discretion in RE + effect of current level and change of the gain on future tradeoffs + effect of future expected levels and changes of the gain on future tradeoffs

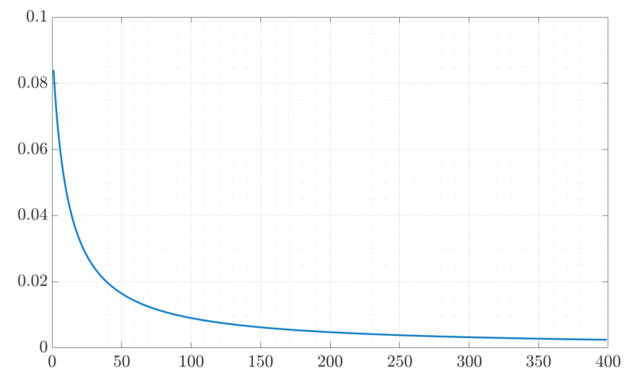
C Impulse responses to iid monpol shocks across a wide range of learning models

$T = 400, N = 100, n_{drop} = 5$, shock imposed at $t = 25$, calibration as above, Taylor rule assumed to be known, PLM = learn constant only, of inflation only.

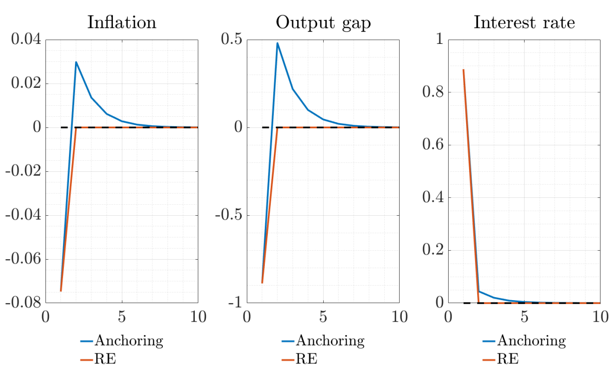
Figure 6: IRFs and gain history (sample means)



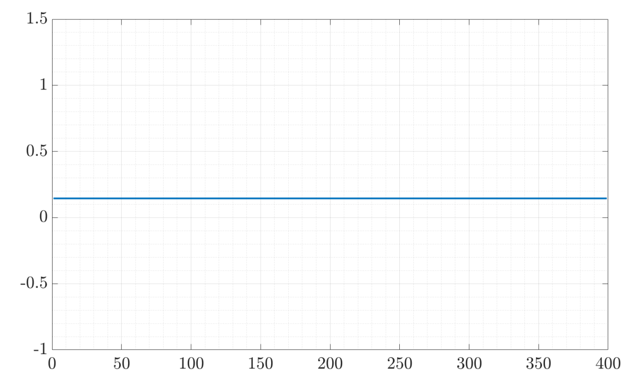
(a) Decreasing gain learning



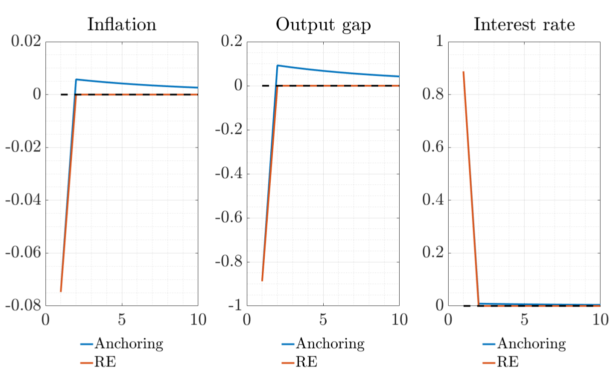
(b) Mean gain



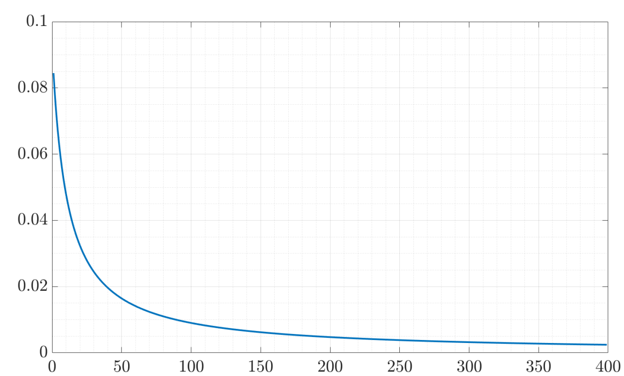
(c) Constant gain learning



(d) Mean gain

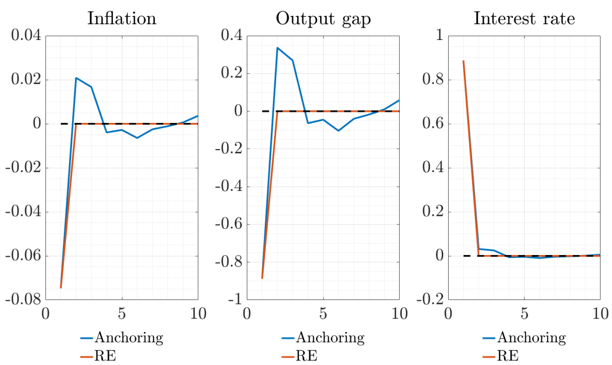


(e) CEMP criterion (vector)

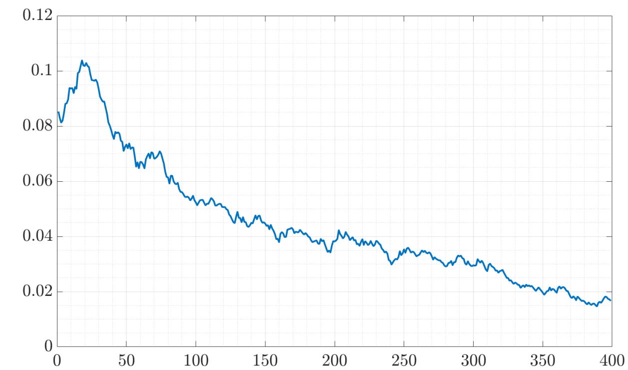


(f) Mean gain

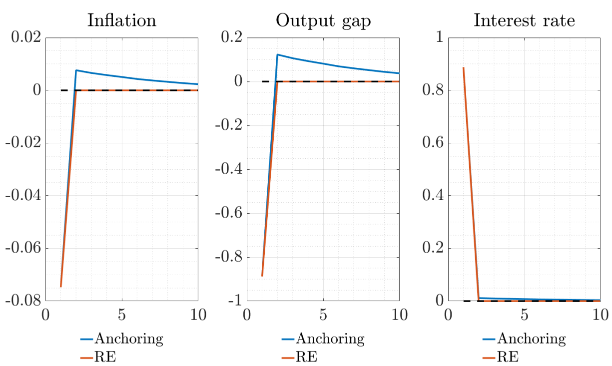
Figure 7: IRFs and gain history (sample means), continued



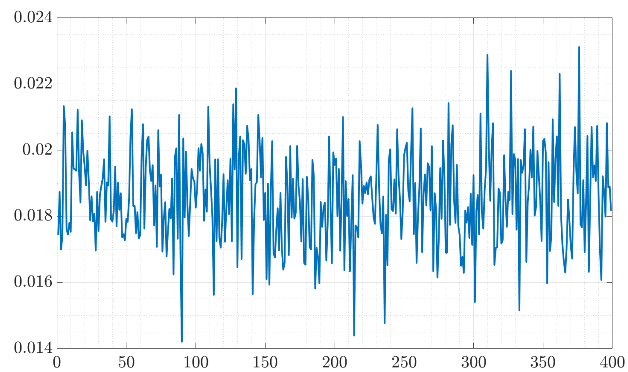
(a) CUSUM criterion (vector)



(b) Mean gain



(c) Smooth criterion, approximated, using $\alpha^{true} = (0.05; 0.025; 0; 0.025; 0.05)$, on $fe \in (-2, 2)$.



(d) Mean gain