

Work after

9 Nov 2019

Form the FEs correctly:

$$f_t^m = \hat{E}_t(\pi_{t+1} | \mathcal{I}_t^m) = \hat{E}_t(\pi_{t+1} | s_t, \bar{\pi}_{t-1})$$

$$f_t^e = \hat{E}_t(\pi_{t+1} | \mathcal{I}_t^e) = \hat{E}_t(\pi_{t+1} | s_t, \bar{\pi}_t)$$

$$\left. \begin{array}{l} FE_t^m = \pi_{t+1} - f_t^m \\ FE_t^e = \pi_{t+1} - f_t^e \end{array} \right\} \text{so in either case I just need to subtract } F_t \text{ from } y_{sim,t+1} \text{ and note that these } F_t \text{ are realized at } t+1.$$

The problem is that the FEs I construct this way aren't equal to the ones I get from the sim-learn.m code. This is puzzling b/c in principle they come from the same simulated π - same fest of π .

The problem is that the FE coming out of sim-learn

- 1) always changes, despite IRF-ing & averaging
- 2) there are diffs b/wn FE_{shocked} & FE_{unshocked} even before I impose the thresov δ (!)

FEs are solved.

14 Nov 2015

I think that the cross-coupling of fots is well-understood: when (gain), you update your foot too much and so your FE switches sign and oscillates.

At a certain point, your FE is small enough so that no overupdating of expectations happens any more.

→ my bet is you can kill this overupdating /

crosscoupling w/ a sufficiently low gain

i.e., w/ $\bar{g} = 0.1$ (instead of 0.145) you already have dgain & cgain similar at $t=5$

w/ $\bar{g} = 0.0145$ they're identical at $t=25$ too

- But you always get some overshooting, whether it's in the 2nd period (cgain) or later on (dgain)

- Moreover, it's puzzling that $i \uparrow$ as $\pi < 0$ in 2nd period

One way to get perfectly normal, RE-like responses

is to set $\alpha = 1$ b/c then $f_a \approx f_b$. But even $\alpha = 0.99$

gets a quite sig diff b/wn $f_a \& f_b$ & overshooting too!

$$f_A = \frac{a}{1-\alpha\beta} + b (I_{nx} - \alpha\beta h x)^{-1} s$$

What is $\frac{1}{1-\alpha\beta} = 50.2573$ and $\frac{1}{1-\beta} = 100$

for $\alpha = \beta = 0.95$?

$$(F_A - \alpha\beta h x)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2.4275 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.0049 & 2.3267 & 1.9678 & 1 \end{bmatrix}$$

$$\text{and } (F_B - \beta h x)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2.4639 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.0050 & 2.3643 & 1.9776 & 1 \end{bmatrix}$$

ha! The diff in f_A & f_B is most pronounced in the part that comes from the intercept!

I think M_N would change a bit as α moves away from 1 (M_0)

$$\alpha = 0.5, \beta = 0.95$$

$$\frac{1}{1-\alpha\beta} = 1.0802 \quad \text{and} \quad \frac{1}{1-\beta} = 100$$

$$(F_4 - \alpha \beta h x)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1.4225 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.0015 & 0.6817 & 0.7388 & 1 \end{bmatrix}$$

$$\text{and } (F_4 - \beta h x)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2.7639 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.0050 & 2.5643 & 1.9776 & 1 \end{bmatrix}$$

Yes, now the part relating the slope is diff, but the part relating the intercept is even more diff!

\Rightarrow the lower α (the higher k , the less price children)
the more b_0 loads on the intercept both in absolute terms
(it reacts more) and relative terms (vs. the slope)

\Rightarrow This may be driving (some of) the overshooting b/c

for the std param value of $\alpha = 0.5$, f_b is almost 50 times more driven by the intercept than the slope + shocks. \Rightarrow so overreaction in updating the intercept drives f_b , which is what drives x_+ up for gains.

Let's interpret

$$\begin{array}{c} \alpha\beta \cdot h_x \\ \left[\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0.297 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.0025 & 0.48 & 0.7388 & 0 \end{matrix} \right] \end{array} \text{ vs } \begin{array}{c} \beta \cdot h_x \\ \left[\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0.554 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.005 & 0.56 & 1.5776 & 0 \end{matrix} \right] \end{array}$$

r^n \bar{r} u i_{t-1}

2 differences

- 1) effect of shocks on i_t } $\alpha < 1$ mutes these somewhat
 - 2) effect of \bar{r} on s }
- \rightarrow you're just discounting shocks in the future more!

Also, when $\alpha=1$, then $f_a(1)$ doesn't matter
for (x, π) " $E\pi$

The puzzling $i \uparrow$ at $t=2$

$$A_a(3,1) = 0.5928 \rightarrow i \uparrow \text{if } f_a(1) \uparrow$$

$$A_b(3,1) = -0.0978 \rightarrow i \downarrow \text{if } f_b(1) \uparrow$$

\Rightarrow so when f_b moves a lot more than f_a ,
(which is in general not true for avg again), then
 $i \uparrow$ even when $E[\pi]$ is \downarrow (!)

But why?

if $\gamma_x \uparrow$ (now it's 0) then $A_a(3,1) \downarrow$ and
 $A_b(3,1) \downarrow$ too!

But $A_a(3,1)$ never < 0 , not even for $\gamma_x = 5$.

When $\gamma_x = 0$, it's b/c $\pi \downarrow$ when $f_b \uparrow$

\rightarrow it seems like $i \uparrow$ in $t=2$ b/c π is \uparrow from $t=1$ to $t=2$

:S

$$A_a(3,1) = \gamma_{\bar{a}} A_a(1,1) + \gamma_x A_a(2,1)$$

$$A_b(3,1) = \gamma_{\bar{b}} A_b(1,1) \quad "0"$$

these are true!

⇒ ah I see: i^{\uparrow} at $t=2$ b/c it was going up much more at $t=1$ due to the innovation, but since π fell so much, i has depressed a lot.

At $t=2$, since $\pi \uparrow$ (but is still < 0), i is depressed below $0.6 \cdot 1^{\uparrow \delta}$ (i^{\uparrow} 's only ≈ 0.1) but it's not depressed as much

Puzzling i-response

$$i_t = \pi + \text{innovation}(\delta)$$

$\downarrow \quad \uparrow$

Initially $|\delta| > |\pi|$

At $t=2$ $|\pi|$ shrinks so i^{\uparrow}

What remains to be understood is why the overshooting happens regardless, just later:

- maybe what's going on is that $E[\bar{\pi}]$ are pushing stuff up but i^* is pushing them down, and i reacts faster

Check: if $\bar{\pi}$ -shock is iid, overshooting should happen

at $t=2$

\Rightarrow exactly, and it does!

The only thing that isn't a 100% clear is why $\pi_t \neq \bar{\pi}$ reaction to expectations, when RE doesn't have bias?

$$\text{In RE: } x_t = E_t x_{t+1} - \beta E_t(i_t - \pi_{t+1})$$

$$\pi_t = K x_t + \beta E_t \pi_{t+1}$$

$$i_t = \gamma_\pi \pi_t$$

$$x_t = -\beta i_t + E_t x_{t+1} + \beta E_t \pi_{t+1}$$

$$\pi_t = K x_t + \beta E_t \pi_{t+1}$$

$$x_t = -\beta \gamma_\pi \pi_t + E_t x_{t+1} + \beta E_t \pi_{t+1}$$

$$x_t = -2\gamma_\pi [K x_t - \beta E_t \pi_{t+1}] + E_t x_{t+1} + \beta E_t \pi_{t+1}$$

$$X_t = -\gamma \pi_t [kx_t + \beta E_t \pi_{t+1}] + E_t x_{t+1} + \beta E_t \pi_{t+1}$$

$$(1 + \gamma \pi_t k) X_t = -\gamma \pi_t \beta E_t \pi_{t+1} + E_t x_{t+1} + \beta E_t \pi_{t+1}$$

$$X_t = \underbrace{\frac{1}{w} \beta (1 - \gamma \pi_t \beta) E_t \pi_{t+1}}_{< 0 (!)} + \frac{1}{w} E_t x_{t+1}$$

$\Rightarrow RE$ has it too, only

$$\Rightarrow \pi_t = \frac{k}{w} \beta (1 - \gamma \pi_t \beta) E_t \pi_{t+1} + \frac{k}{w} E_t x_{t+1} + \beta E_t \pi_{t+1}$$

[E]

$$\pi_t = \underbrace{\left[\frac{k}{w} \beta (1 - \beta \gamma \pi_t) + \beta \right]}_{\text{likely } > 0} E_t \pi_{t+1} + \frac{k}{w} E_t x_{t+1}$$

more
as much

(for current params = 0.9298)

RE :

$$X_t = \ominus E_t \pi_{t+1} + \oplus E_t x_{t+1}$$

$$\pi_t = \oplus E_t \pi_{t+1} + \ominus E_t x_{t+1}$$

Learning

$$X_t = \ominus E_t \pi_{t+1}^{fa} + \ominus E_t \pi_{t+1}^{fb} + \circlearrowleft \ominus E_t x_{t+1}^{fa} + \oplus E_t x_{t+1}^{fb}$$

$$\pi_t = \oplus E_t \pi_{t+1}^{fa} + \circlearrowleft \ominus E_t \pi_{t+1}^{fb} + \oplus E_t x_{t+1}^{fa} + \ominus E_t x_{t+1}^{fb}$$

$$X_t = \ominus E \pi + \circlearrowleft \ominus E X$$

① *This is not a mistake*

$$\pi_t = \oplus E \pi + \oplus E X$$

②

$$\textcircled{1} \quad -\frac{\beta \gamma_{\pi} k \alpha \beta}{w} + \frac{1}{w} (1-\beta) = \frac{1-\beta - \beta \gamma_{\pi} k \alpha \beta}{w} < 0$$

$$\textcircled{2} \quad \left(1 - \frac{k \beta \gamma_{\pi}}{w}\right)(1-\alpha)\beta + \frac{k}{w} \beta (1-\beta \gamma_{\pi}) > 0$$

\Rightarrow why do we have this diff b/w RE & Learn? 15 Nov 2017

$$\beta \gamma_{\pi} k \alpha \beta < 1 - \beta$$

$$\beta \gamma_{\pi} k \alpha \beta + \beta < 1$$

$$(\beta \gamma_{\pi} k \alpha + 1) \beta < 1 \quad \text{but it's } 1.1150$$

In the RE world, x depends on $E(x)$ only directly
 My conjecture is that $E(\pi)$ in RE will incorporate $E(x)$
 In some way. So π must depend stronger on $E(\pi)$
 in RE than in learning.

$$\text{RE: } \pi \text{ in } E(\pi): \quad \frac{k \beta + \beta}{\beta \gamma_{\pi}} = 0.7722 \text{ under current params}$$

$$\text{Learn: } \left(1 - \frac{k \beta \gamma_{\pi}}{w}\right)(1-\alpha)\beta + \frac{k}{w} \beta (1-\beta \gamma_{\pi}) = 0.33 \quad -11-$$

\rightarrow So yes, this is true

In fact, you can reason that in RE,

$\pi = E(\pi)$ only b/c only via
④

while in learning

$\pi = E(\pi)$ but part of π_{RL} is
④ → from f_1
⑤ → from f_2

Ryan meeting

15 Nov 2019

fix point where find the gain stat with $\text{Var}(\text{FE})$
data is generated by a gain = 0.145 gain,
given this, let an agent set a best gain
→ it must be lower than 0.145 !

Analogy to RE for the gain problem.

→ Pooya Molavi's JMP does RL, using a
Kullback - Leibler distance.

Stage 0: Establish that again learning causes excess volatility:

0.1. Do learning rule where I don't do RE-pred

$$PLM = \bar{\pi}_{t+1} \text{ and that's it.}$$

0.2. My learning the slope

\Rightarrow in those contexts, do I continue to get the hiccups?

2. Didn't quite get to the bottom of RE is.

learning loading on EC.)

\rightarrow connect to those equations

3. These features can become worse if ΓY_π

$$\uparrow Y_\pi$$

Do it in a week. Schedule to talk to Basu after.

Tell him: in learning models, there's this endemic instability. This can become worse if $Y_\pi \uparrow$.

Here's how it works.

Work after

- 15 Nov: did "only-mean" Pm and "slope & constant"
- ✓ check that the latter is correct
 - ✓ print figs w/o cutting them off
 - polish explanation of $E(\cdot) \rightarrow z$
 - do the fixed point thing, use Molari

Reading Molari JMP

16 Nov 2019

I'm not superimposed b/c the "constrained RE eqb" (CREE) is really just saying that give agents a set of models Θ and let them choose (their expectation formation) the subset Θ^* that $\min H(\cdot)$ where $H(\theta, T)$ is the Kullback-Leibler distance between model θ and the ALM T . (Sometimes Θ^* is a singleton, Molari calls this a pure CREE)
He shows (Thm 2 & 3) that Bayesian & adaptive learning coincide w/ a CREE in the LH
 \rightarrow of course b/c the CREE must be = REE if

the REE $\in \Theta$! This is why Molen says that the "id behavior of the econ is independent of ... the learning process" b/c they all converge to REE!
 (Unless they are not E-stable, which is the analog of Molen's concept of $\text{REE} \notin \Theta$, i.e. when agents don't include the REE in the set of models they consider.)

→ ok so trying to solve for \bar{g}^*

$$\bar{g}^* = \arg \min FEV$$

I have 2 ways to construct FEV.

- 1) analytically (don't know if possible)
- 2) numerically in Matlab

↳ here I'm confused whether the FEV is across time or cross-section \leftarrow I suppose time.

$$\text{Var}(x) = E(x^2) - E(x)^2$$

$$\text{If } x = \text{FE}, \text{ then } E(x) = 0, \text{ so } \text{FEV} = E(\text{FE}^2)$$

$$\text{Var}(x) = E \left[\underbrace{(x - E[x])^2}_{\text{FE}} \right] \quad \text{this is why}$$

$\text{FEV}(x) = \text{Var}(x)$ when you initialize!

$$\text{Otherwise } \text{FEV} = E \left[(x_{t+k} - x_{t+k,t}) (x_{t+k} - x_{t+k,t})' \right]$$

\uparrow
fcast

Ok let's clarify one thing:

$$\underbrace{\text{Var}(x) = E \left[(x - E[x])^2 \right]}_{\text{it seems right now that } \text{Var}(x) = \text{FEV}(x)} = E(x^2) - E(x)^2$$

are the same thing?

Leaving that aside for a moment

- My posts are given by the $\text{PLM}(\bar{g})$
 - $\text{FE}_{t+1} = \pi_t - \text{PLM}_{t-1}^{e_t}(\bar{g})$
 - $\text{FEV}_{t-1} = E \left[(\pi_t - \text{PLM}_{t-1}^{e_t}(\bar{g}))^2 \right]$
- Let's take a general case: $\text{PLM}_{t-1}^{e_t}(\bar{g}) = \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}$
- $$\Rightarrow \text{FEV}_{t-1} = E \left[(\pi_t - \phi_{t-1}(\bar{g}) \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix})^2 \right]$$
- $$\phi_{t-1}(\bar{g}) = (\phi_{t-2} + \bar{g} (\pi_t - \phi_{t-2} \begin{bmatrix} 1 \\ s_{t-2} \end{bmatrix}))'$$

I don't think I can solve the problem analytically
b/c it's so recursive: to find \bar{g}_t^* , I need to
have \bar{g}_{t-1}^* etc.

Also I'm not sure if I should

- restrict agents to use the same \bar{g} in every period
- make them optimize over \bar{g} in every period.

Ok - what I have now is FEV across time \rightarrow I make it
min TEV for each history n

\rightarrow This gives me $N \bar{g}^*$'s, which I then average.

So far I got 0.00021076 ($2.1076 \cdot 10^{-4}$) ≈ 0.0002

\Rightarrow Think more on this tomorrow!

RE vs learning: responses to $E(\cdot)$

17 Nov 2019

RE:

$$X_t = -\beta \gamma_{\pi} (K X_t + \beta E_t \pi_{t+1}) + E_t x_{t+1} + \beta E_t \pi_{t+1}$$

$$= -\beta \gamma_{\pi} K X_t - \beta \beta \gamma_{\pi} E_t \pi_{t+1} + E_t X_{t+1} + \beta E_t \pi_{t+1}$$

$$(1 + \beta \gamma_{\pi} K) X_t = \beta (1 - \beta \gamma_{\pi}) E_t \pi_{t+1} + E_t X_{t+1}$$

$$X_t = \frac{\beta (1 - \beta \gamma_{\pi}) E_t \pi_{t+1} + \frac{1}{1 + \beta \gamma_{\pi} K} E_t X_{t+1}}{1 + \beta \gamma_{\pi} K}$$

$$\pi_t = \frac{K \beta (1 - \beta \gamma_{\pi}) E_t \pi_{t+1}}{1 + \beta \gamma_{\pi} K} + \frac{K}{1 + \beta \gamma_{\pi} K} E_t X_{t+1} + \beta E_t \pi_{t+1}$$

$$\pi_t = \left(\frac{K \beta (1 - \beta \gamma_{\pi})}{1 + \beta \gamma_{\pi} K} + \beta \right) E_t \pi_{t+1} + \frac{K}{1 + \beta \gamma_{\pi} K} E_t X_{t+1}$$

Learning

$$X_t = \left(-\frac{\beta \gamma_{\pi} (1 - \alpha) \beta}{w} + \frac{\beta (1 - \beta \gamma_{\pi})}{w} \right) E_t^{\alpha, \beta} \pi_{\infty}$$

$$\left(-\frac{\beta \gamma_{\pi} (\alpha \beta)}{w} + \frac{1 - \beta}{w} \right) E_t^{\alpha, \beta} x_{\infty}$$

$$\pi_t = \left(\left(1 - \frac{K \beta \gamma_{\pi}}{w} \right) (1 - \alpha) \beta + \frac{K \beta (1 - \beta \gamma_{\pi})}{w} \right) E_t^{\alpha, \beta} \pi_{\infty}$$

$$+ \left(\left(1 - \frac{K \beta \gamma_{\pi}}{w} \right) (\alpha \beta) + \frac{K (1 - \beta)}{w} \right) E_t^{\alpha, \beta} x_{\infty}$$

matlab10 \rightarrow parameter values:

$$\cdot \frac{k\beta(1-\beta\gamma_\pi)}{1+\beta\gamma_\pi K} + \beta = \frac{k\beta - k\beta\beta\gamma_\pi + \beta + \beta^2\gamma_\pi K}{1+\beta\gamma_\pi K} > 0$$

$$\cdot -\frac{\beta\gamma_\pi K\alpha\beta}{1+\beta\gamma_\pi K} + \frac{1-\beta}{1+\beta\gamma_\pi K} \propto 1-\beta - \beta\gamma_\pi K\alpha\beta$$

For this to be positive, we need $\beta + \beta\gamma_\pi K\alpha\beta < 1$

For current params, this is $1.155 > 1$.

$$\cdot \left(1 - \frac{k\beta\gamma_\pi}{1+k\beta\gamma_\pi}\right)(1-\alpha)\beta + \frac{k\beta(1-\beta\gamma_\pi)}{1+\beta\gamma_\pi K}$$

$$\propto (1+k\beta\gamma_\pi - k\beta\gamma_\pi)(1-\alpha)\beta + k\beta - k\beta\beta\gamma_\pi$$

$$= (1+\alpha)\beta + k\beta(1-\beta\gamma_\pi) = 1.4034 > 0$$

$$\underbrace{\beta + \alpha\beta}_{\approx 1.5} + \underbrace{k\beta}_{\approx 20} - \underbrace{k\beta\beta\gamma_\pi}_{\approx \gamma_\pi \text{ even if } \gamma_\pi = 5} > 0$$

Ok, so now explain why, if I recursively substitute into the RE system, why do I not get the learning system? Even though you can pull out the next term from the learning system to reduce to RE.

→ it seems that LIE holds for the idiosyncratic expectation $\hat{E}_t^i \hat{E}_{t+1}^i = \hat{E}_t^i$ (in fact, this is anticipated utility!) but not for the average expectation: $\hat{E}_t^i \hat{E}_{t+1}^i \neq \hat{E}_t^i \Rightarrow$ it's a little bit like the distinction b/w PLM & ALM b/c firms act based on $\hat{E}_t^i \hat{E}_{t+1}^i = \hat{E}_t^i$, i.e. knowing that LIE holds, but in the actual law of motion $\hat{E}_t^i \hat{E}_{t+1}^i$ turns out not to equal \hat{E}_t^i since updating happens!

19 Nov 2015

For Susanto, use

- from materials 10: "A more concise rephrasing"
- I think I wanna show IRFs from Dgns & again against RE for std params for the 3 shorts (take iid shorts & except mngd.)

Ryan meeting

(500 years) 20 Nov 2015

↳ do for FER-min $T = 5 \cdot 400$
→ question: maybe this isn't ergodic $\Rightarrow \bar{g}^* \text{ is too large}$
Fabio Milani } have estimated gains
& Preston }

so if I generate a data sequence from RE, and I allow agents to choose gain, optimal is 0.

Ryan conjecture:

"If you do $T=5 \cdot 400 \rightarrow$ will you squeeze the dispersion and shrink the mean? Yes."

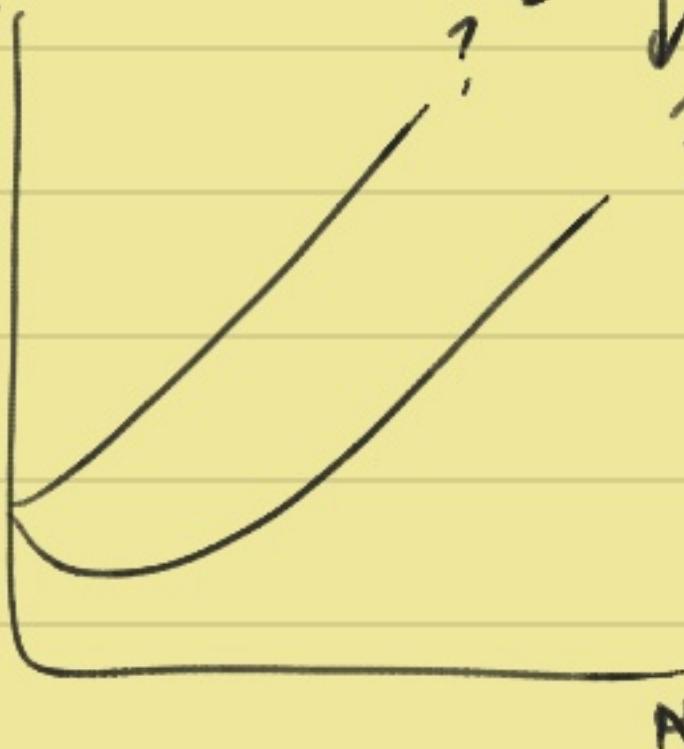
↳ But if this exercise gives $\bar{g}^* = 0$ then it's not the right model-based notion of what the \bar{g} should be)

→ Ryan says something like: our exercise here is to get a model-based notion of what the gain should be. But we do not yet know what the right notion is.

→ maybe b/c that's saying that the learning model is not optimal.
(?)

plot this

MSE



We don't know yet what π_{M3} fit looks like

does it wobble
a lot?

if yes, taking average
is necessary
but Ryan thinks
it won't.

However it could be
that this point is a
local, but
not a global
min!

→ Move slightly neg. α to see if it comes

back up. That is, if the MSE function looks like
this:



If not unique for reasonable K ,
can $\pi - E(\cdot)$ look absurd?

↳ What he means by this is the following:
We have found that the "puzzling IRFs" ("overshooting")

is an endemic feature of learning. It decreases when gains are smaller. So we are trying to find a way to pick a reasonable gain & - either from a model & optimality perspective or from the data. And the question is: supp. we have a reasonable κ , κ^R .

1) Do we get overshooting for that?

The point is that the IRFs of the econ for κ^R are going to be the model's prediction for what unanchored expectations look like.

If overshooting is endemic for κ^R , then for anchoring to be a good model, you'd need the overshooting IRFs to fit data.

⇒ So, as Ryan said, this can put me in a dilemma, or, I could call it a crossroads: maybe the anchored E model isn't a good model of $E(\cdot)$? Maybe they have to learn about x and/or i too, or maybe something entirely different.

work after

20 Nov 2019

let's gather some reasonable numbers for the gain.

Estimates Calibration

0.002

Eusepi & Preston (2011)

$g < 2(1-\beta)$ for beliefs to be
stable. For $\beta = 0.95$,

Eusepi, Gramm, Preston (2019)
Limits

$g < 0.02$.

$\hat{g} = 0.05$

0.145

CAMP

0.0183

Milan (2007)

0.1, 0.05, 0.03

Williams (2003)

0.062

Branch & Evans (2006)

0.075, 0.05, 0.025

Orphanides & Williams (2004)

0.02

Orphanides & Williams (2005)

Avg = 0.05

Avg w/o CAMP & avg estimate w/o CAMP = 0.04

Let's interpret the gain number:

- Euzgi & Preston (2011)

Data T quarters old receives the weight
 $(1 - k)^T$

- Milani (2007)

$\frac{1}{\text{gain}} \approx$ "how many past observations agents use
to form expectations"

[Euzgi & Preston (2011) \rightarrow 500 quarters ($\Rightarrow 125$ years!)
avg w/o COMP \rightarrow 24 quarters ($\Rightarrow 6$ years)
COMP \rightarrow 7 quarters (< 2 years!)]

I think it's reasonable that humans shouldn't use more
than approx 50 years of data on avg (200 quarters)

$$\hookrightarrow \min(\bar{g}) = 0.005$$

They also shouldn't use less than 5 years (20 quarters)

$$\hookrightarrow \max(\bar{g}) = 0.05$$

$\bar{g} \in [0.005, 0.05]$, and in particular $0.02 \rightarrow 12$ years
seems reasonable.

When $b_{\text{min}} = 0$, $T = 400$, $N = 100$ (48 sec)

$$\bar{g}^* = 0.0005 \quad \text{Var}(\bar{g}_n^*) = 1.2602 \cdot 10^{-6}$$

but it varies per run!

$b_{\text{min}} = 50$, $T = 400$, $N = 100$

$$0.00018217 \quad 1.8037e-07$$

and still varies

$b_{\text{min}} = 1600$, $T = 400$, $N = 100$ (48 sec)

$$\bar{g}^* = 0.00033 \quad \text{Var}(\bar{g}_n^*) = 6.0104e-07$$

vary still.

$b_{\text{min}} = 0$, $T = 2000$, $N = 100$ (150 sec)

$$\bar{g}^* = 5.27773e-05 \quad \text{Var}(\bar{g}_n^*) = 1.5801e-08$$

$$\bar{g}^* = 6.4184e-05 \quad \text{Var}(\bar{g}_n^*) = 1.5136e-08$$

$$2.6731e-05 \quad N=1 \quad (2 \text{ sec})$$

$$9.6657e-05$$

$$1.511e-05$$

$$1.0546e-05$$

3 obs: 1) b_{min} doesn't matter (why? b/c doesn't impact FER)

2) longer T decreases \bar{g}^* } Ryan's conjecture was
3) decreases dispersion across n } right.

Moving on to MSE:

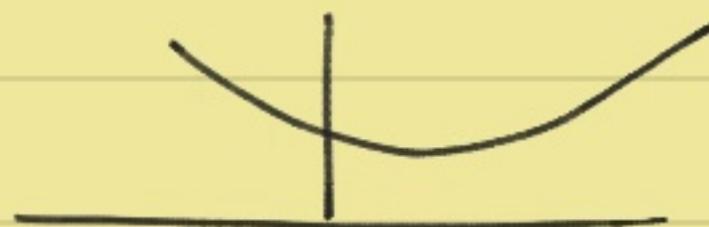
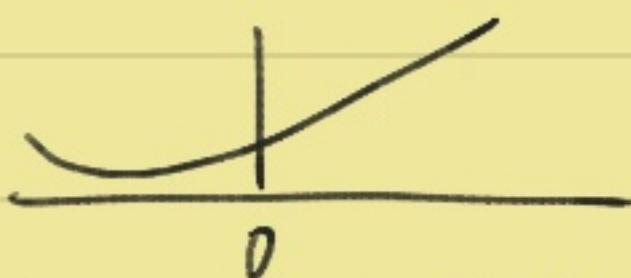
Note $\text{MSE}(y_j) = \frac{1}{T} \sum_{t=1}^T (y_{j,t} - \hat{y}_{j,t})^2$ (Brand & Grams 2006)

$\text{MSE} = \text{FEV}$ b/c $\text{FE} = x_t - x_{t-1}$

$\text{Var(FE)} = E[\text{FE}^2] = \text{mean squared error}$

Plotting this thing has a very clear shape.

it has a 2nd deriv > 0 (convex) and takes a min



either slightly to the
left of 0

or slightly to the right

\Rightarrow but maybe this means that if I increase T,
the min will be pushed to 0 "from both sides"?

Any interesting finding: The more volatile the environment
(the higher $\Sigma = \text{VC}(\text{shocks})$) the lower \bar{g}^* (more negative)

Ok so what Roger had had in mind was something like: sim Y using $\bar{g} = 0.145$.

Now, for different values of \bar{g} , compute forecasts of Y_{t+1} at each t , allowing only $\bar{\pi}$ to update and thus forecasts, but not Y . What's the \bar{g} that mins the FEV?

→ For this I obtain $\bar{g}^* = 1.003e-05$ $\text{Var}(\bar{g}^*) = 3.357e-16$

whereas for my exercise I get $\bar{g} = 2.424e-05$ $1.7218e-8$
($b_{\min} = 0$, $T = 2000$, $N = 100$)

The figs show that ideally you'd get $\approx -1.5 \cdot 10^{-3}$

→ and this is consistent no matter how long T is

(actually that's not too surprising given that here T doesn't make the FER larger b/c the data is created given $\bar{g} = 0.145$)

Why am I getting negative values in both exercises?

Why is a $\bar{g}^* = 0$ not good? What other action to use?

Given the relationship between the gain and the Kalman gain, is there no KF-related notion of optimality we could use?

→ In fact, for Ryan's minFEV, I get $\bar{g}^* < 0$ even when the DGP is RE!

Schenkman & Xiong, 2003 "Overconfidence & Speculative bubbles"
(setup on optimism & pessimism on financial markets)
→ This stuff relates to my confirmation bias idea

Understand why minFEV

23 Nov 2015

Ryan's method: $y = \text{generated by gain} = 0.145$

Now set $K = \bar{g}^* = \arg\min FEV(y(\bar{g} = 0.145))$

My method: $K = \bar{g} = \arg\min FEV(y(\bar{g}))$

1) I don't understand why Ryan said that I didn't go all the way to the fixed point. To me it seems like my method does go to the fixed point, his doesn't!

2.) Why did Ryan call his method "analog to RE"?

Interpretation of Ryan's method:

"Data is what it is. Let me update my beliefs so that they have as little errors given the structure of the data as possible."

→ I think that's why: b/c I wanna choose the optimal gain such that my expectations are close to model-consistent.

Interpretation of my method:

"Give me the gain that makes my beliefs model-consistent, internalizing that my beliefs affect the DGP."

→ to me my way still seems closer to a fixed point.

What would I expect the two methods to yield?

- Ryan expected his method to yield a lower \bar{g}^* than $\bar{g} = 0.145$, the one using which data was generated.

But why should it? This means Ryan expected that FEs are smaller when $\bar{g} < \bar{g}^{\text{DGP}}$. I don't think this is

a general statement; i.e. I think he only expected that compared to $\bar{g}^{\text{COMP}} = 0.145$. Why do I think this?

B/c fcsks are $\hat{E}_t \pi_{t+1} = \bar{\pi}_{t-1} + b_1 s_t$

$$\text{where } \bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1}(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))$$

- So the statement is that even as data g is generated w/ $k_t = \bar{g}^{\text{COMP}}$, that leads to FEs b/c the PLM doesn't coincide w/ the ALM. So if $k_t = \bar{g}^{\text{COMP}}$ and FEs switch sign, that means that by lowering \bar{g} , we could lower FE b/c agents are overpredicting.
- On the other hand, even $k_t = 0$ will not be optimal, b/c then $\bar{\pi}_t = \bar{\pi}_{t-1} = \bar{\pi}$, so $\hat{E}_t \pi_{t+1} = \bar{\pi} + b_1 s_t$ which yields permanent FEs when the data is generated by a gain of $k = \bar{g}^{\text{COMP}}$. If $k_t = 0$, and initialized at RE, $\hat{E}_t \pi_{t+1} = 0 + b_1 s_t = \text{RE first}$. So if the DGP is RE, then \bar{g}^* better be 0, and it's a problem that that's not what I find!

- Also, this shows that for the learning DGP, $b^* = 0$

Cannot be optimal.

So for Ryan's method, we'd expect an integer k^* .

What does it mean if this is < 0 ?

$$\text{fcsks are } \hat{E}[\bar{\pi}_{t+1}] = \bar{\pi}_{t-1} + b_1 s_t$$

$$\text{where } \bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1}(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))$$

If $k < 0$, then if I underestimated π_t ($\text{test} < \pi_t$)

then I lower $\bar{\pi}$, that is, I lower my fcsk even further. So that makes no sense!

\Rightarrow it must be that these negative values are a result of a coding error, which potentially also explains why RE DGP doesn't give me $k^* = 0$.

• So what do I expect my method to give?

Reinterpretation of my method: "give me the gain that gives me the sequence of Y for which the FEV gives that gain is minimized" \rightarrow i.e. "find the gain \bar{g}^* that generates the sequence Y w/ minimal FEV associated w/ it"

→ but this should give you $k^* = 0$ b/c then the $\bar{\pi}_0 = 0$ means that $PLM = ALM = RE$, which again shows you why this isn't a good notion of an optimal gain!

⇒ Ryan's notion is better than mine

→ in fact I think Branch & Evans are doing something similar when estimating the gain, so I should obtain figures that look like these:



But: neither my nor Ryan's should give $k^* < 0$.

So there's still wrong in both!

So now I'm focusing on Ryan's method alone:

1) I'm getting $\approx 70\%$ of $\bar{g}^* < 0$ (!)

2) Increasing T makes 1) $\bar{g}^* \rightarrow 0$ from below 2) shrinks $\text{Var}(\bar{g}_n^*)$

3) Increasing N doesn't do anything

→ in any case, $\bar{g}^* < 0$ about 60-70% of the time!

More observations:

1) If I make the shocks be centered around \pm some number, k^* 's become positive!

2) Changing Σ doesn't really do anything.

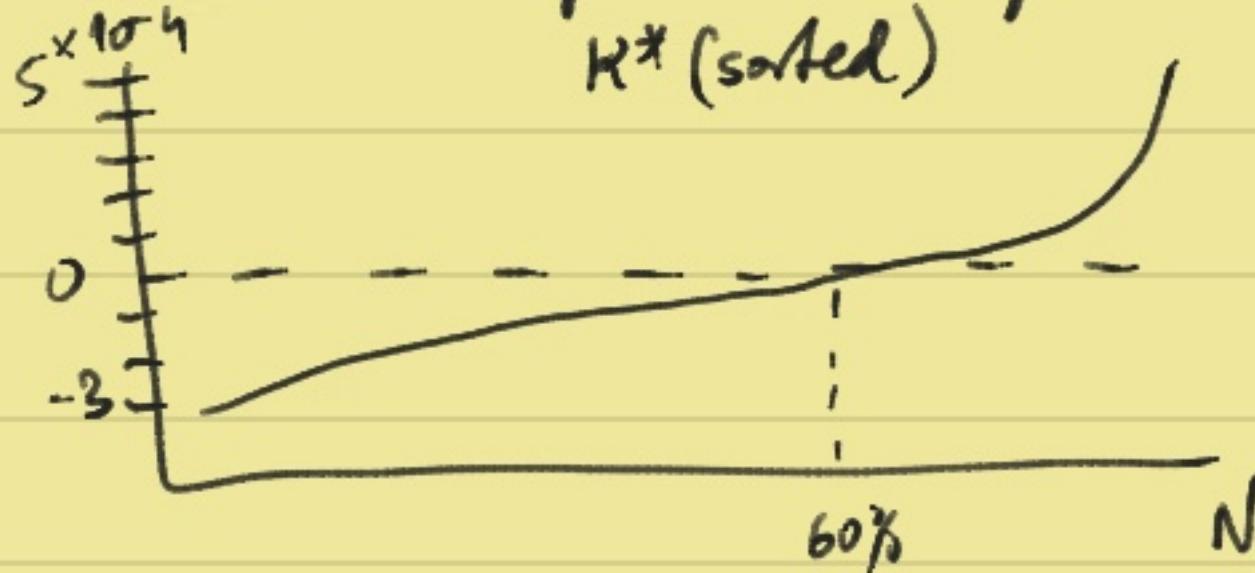
Why is this happening? And why doesn't the sign(mean(shocks)) matter? (\rightarrow i.e. why doesn't it matter whether shocks are centered around 1 or -1?)

\rightarrow Try shutting off all shocks except 1 \rightarrow see how that affects k^* .

If in the objective function, I shut the effect of shocks on FE off, I obtain all $k^* > 0$

If you $T \uparrow$, $k^* \rightarrow 0$ but always from below

and the distnb of k^* always is skewed like this:



Reading Berkoo 2019

2 Dec 2019

I just understood something: $T(\phi)$ is the mapping from the PLM to the ALM. That is, given a belief ϕ , what ALM does the PLM imply, i.e. how would the model expect observables to evolve (abstact shocks) given these beliefs?

$T(\phi) = E(\text{ALM})$ (it just disregards shocks, answering the question: "w/ these beliefs, on average, what will observables do?")

The E-stability condition is the differential equation

$$\frac{d}{dt}(\phi) = T(\phi) - \phi \quad (2.8, \text{EH p. 31})$$

$E(\text{ALM}(\phi)) - \text{PLM}$ (either in both cases w/o, or

In both cases w/o observables)

Camp's criterion $\theta_t = \text{PLM} - E(\text{ALM})$, so it's really

the E-stability diff. equation $\theta_t = \frac{d}{dt}(\phi) \quad (!)$

Susanto meeting

2 Dec 2019

Work after

3 Dec 2019

Ball 1994 AER

Deflation causes recessions as we thought.

But disinflation (changing the growth of money \downarrow)
causes a boom.

I think the idea is this:

$$x_t = m_t - E_t m_{t+1} \quad \text{(roughly)}$$

↑ all future m^s

↓ deflation causes $x_t \downarrow$

but disinflation is $m \downarrow$ so m_t doesn't change,
only m_{t+1} does (here's where credibility comes in:
 $E_t m_{t+1} = m_{t+1}$)

↳ this is exactly the contractionary $E_t \pi_{t+1}$, I get!

$$x_t = -\beta \pi_t \pi_t + \sum_{T=1}^{\infty} \beta^{T-t} \left((1-\beta)x_{T+1} + \underbrace{2(1-\beta \pi_T) \pi_{T+1}}_{< 0} \right)$$

I think that the intuition is that a

credible disinflation means a movement in expectations

only, but no current values.

I think Whelan's intuition is that when $E\pi_{t+1} \downarrow$,

$\hat{q}_t \uparrow$ a bit and so consumers are richer ($\frac{M_t}{P_t} \uparrow$)

$\rightarrow x_t \uparrow$

And this is also what Ball's intuitive proof circles around: firms choose lower prices bc they anticipate future decreases in the increase of m .

↳ So, like for me, it's all about expectations moving and credibility: ppl have to believe mon. pol. (in my case, know & believe the Taylor-rule).

One could also summarize Ball's argument as

A disinflation has two effects: 1) contractionary via the interest rate 2) expansionary via expectations

For the expectations channel to dominate, Ball figured you need to do policy quickly, so "1) doesn't get to move".

In my case, you just need expectations to move strongly enough.

Whelan adds 2 things:

- Since Ball himself said that disinflationary booms don't fit the data well, it must be that CB's have credibility problems.
→ But, though likely during Great Inflation, is it still likely now?
- Something else must be wrong w/ the UK model.

One thing I'm a little worried about is that $E(\cdot)$ moving strengthens the Ball effect, and $E(\cdot)$ move a lot when unanchored (again) which is exactly a measure of the CB not being credible. — ah but that may be fine actually, b/c I think Ball needs credibility for $E(\cdot)$ to move. I can get them to move otherwise.

Instrument instability seems to happen 4 Dec 2019

here in the sense that expectations are the instrument.

In particular, the FE oscillates.

Let's try to write out the first error. It's

$$FE_{t-1} = \pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})$$

$$\text{where } \bar{\pi}_{t-1} = \bar{\pi}_{t-2} + k^{-1} \underbrace{(\pi_{t-1} - (\bar{\pi}_{t-2} + b_1 s_{t-2}))}_{FE_{t-2}}$$

$$FE_{t-1} = \pi_t - \left[\bar{\pi}_{t-2} + k^{-1} FE_{t-2} + b_1 s_{t-1} \right]$$

$$FE_{t-1} = \pi_t - \left[\bar{\pi}_{t-3} + k^{-1} FE_{t-3} + k^{-1} FE_{t-2} + b_1 s_{t-1} \right]$$

$$FE_{t-1} = \pi_t - b_1 s_{t-1} - \bar{\pi}_0 - k^{-1} \sum_{s=0}^{t-2} FE_s \quad | \pm FE_j \\ j=t-2, \dots, 1''$$

$$\Delta FE_{t-1} = \underbrace{\pi_t - b_1 s_{t-1}}_{\text{ignore this}} - k^{-1} \sum_{s=0}^{t-2} \Delta FE_s$$

Ignore this and switch $t-1$ to t for simplicity

$$\Delta FE_t = -k^{-1} \left[\Delta FE_{t-1} + \Delta FE_{t-2} + \dots \right]$$

so the weights are k^{-1} for all and 1 for ΔFE_t

or $k < -1$ for ΔFE_t and 1 for all the rest.

$k \Delta FE_t + \Delta FE_{t-1} + \Delta FE_{t-2} + \dots$ leads to the

characteristic equation

$$kx^n + x^{n-1} + \dots + x + 1 = 0$$

If we only had one lag, this would be

$$kx + 1 = 0$$

$$x = -k^{-1} > -1$$

That would be stable
but oscillating

Two lags

$$kx^2 + x + 1 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 4k}}{2k}$$

For the roots to be real $1 - 4k > 0 \Rightarrow 1 > 4k$

$k < \frac{1}{4} = 0.25$ It usually is, even comp's.

$$\text{If } k = 0.25, \quad x_1 = x_2 = -\frac{1}{2k} = -2$$

\rightarrow that would be unstable.

For $k \in (0, 0.25)$, both roots are < -1 always!

So the system is unstable.

My guess is that the more lags you include,

The closer you will get to stability, but you will have (potentially all) roots < 0 , which is why we get the oscillation.

But what this doesn't account for is the role of γ_{π} in getting the oscillation. What seems to be clear is that the oscillations in FE are driving it.

So in a sense I'm not even sure if it's instrument instability or simply instability.

The connection to γ_{π} must come via the role of i in the rule x .

$$\left. \begin{array}{l} x = -\beta i_t + E(\text{stuff}) \\ \pi_t = \kappa x_t + E(\text{stuff}) \\ i_t = \gamma_{\pi} \pi_t \end{array} \right\} \begin{array}{l} \text{In period 1, a shock hits, } E \\ \text{moves a little, } (x, \pi) \text{ move, } i \\ \text{moves.} \end{array}$$

Evening of period 1: $E(\cdot)$ adjusts. 2 things influence the adjustment: 1) $\kappa = \bar{g}$ the size 2) γ_{π} the size of Ball's disinflationary boom effect (direction of E -adjustment)

$$FE = \pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})$$

↑ ↑
↑ governs the change here

γ_π governs the change in both, and the bigger it is, the more $E(.)$ moves compared to π_t , opening up FEs.

→ it's as if $E(.)$ were the policy instrument?

Chaining ; works partly via its effect on x_t and π_t today, but its main effect is on $E(.)$.

So in that sense it's like instrument instability b/c the instrument becomes unstable, but: it's also not like instr. instab. b/c a too high γ_π makes FEs unstable thereby rendering the objective variables unstable too.

Ryan meeting

4 Dec 2019

- Gerko: send to Ryan
- Data-IRFs oscillated for Ryan early on \rightarrow so it's not quite the case that empirical IRFs never oscillate ...
- L'Hourris estimated $\hat{\gamma}_\pi \approx 1.004$ or sthg
Note: it's Blanchard, L'Huillier, Lourenco, Mers & Nauk, AER 2013
and they get $\hat{\gamma}_\pi = 1.0137$, $\hat{\gamma}_x = 0.005$
- Try $\hat{\gamma}_\pi < 1$ \rightarrow it seems like if for a fwd-looking system $\hat{\gamma}_\pi > 1$ gives stability, for a bw-looking one, $\hat{\gamma}_\pi < 1$ will.
- Consider flex price model

$$r_t = i_t - E_t[\pi_{t+1}] \quad \left. \begin{array}{l} \text{do expectations from} \\ \text{out similarly here?} \end{array} \right\}$$
$$i_t = \phi \pi_t$$

- The big picture question is:

Do we take the model's implications seriously and explore what they imply for policy?

OR: do we change sthg about the model? 2 options

- 1) change E-formation
- 2) change policy (e.g. have $E(\pi)$ in TR instead of π)

If you change policy, then you can make a statement like: "central bankers say they have $E(\pi)$ in TR, and look, indeed it works better than π "

If you change $E(\cdot)$ -formation, you can say: "std learning implies this, but here's a learning that works"

Think of ways we can change policy

$E(\pi)$ instead of π in TR

Think of ways we can change E-formation.

Work after

5 Dec 2019

One thing I've tried is to add interest rate smoothing, $\rho > 0$. It doesn't change much b/c on the net it acts like p_i , the persistence of the non pol shock, except that it has a contractionary effect (stuck w/ initial shock for longer) and an expansionary effect (stuck w/ expansionary policy reaction for longer). In gen, these two seem to balance so we still have the overshooting.

Peter meeting

5 Dec 2019

A big-picture comment:

What you might worry about in learning:

Systematic pattern of FE's

→ similarly corr errors in the same direction

Peter Ireland similar paper to Ball:

Stoppage Inflation, B.g & Small 1997 JMCB

Ric. EA seems to hold today? Chris Sims

30-yr T-bill i-rate is low despite high debt

→ ask if

alternative expectation-formation schemes that

deliver more appealing dynamics for

$E(\cdot)$ and then for observables.

Work after

5 Dec 2019

Reading Townsend (1983): "Forecasting the forecasts of others" which seems to be the first paper to introduce dispersed info!

One claim he makes is that w/ a normal signal extraction problem:

- forecasts $E[v_t | \mathcal{S}_t]$ is correlated w/ $E[v_{t-1} | \mathcal{S}_{t-1}]$ (i.e. first serially corr) b/c both contain v_{t-1}
- FEs $[E(\theta_t | \mathcal{S}_t) - \theta_t]$ will be serially corr b/c it'll be an MA of past random vars v_t and E_t .

Another is that in a simple model of forecasting the forecasts of others, FEs of those forecasting the forecasts of others (market 2) exhibit damped oscillations (!)

→ This may be part of the reason for me too b/c in this learning model agents don't know other people's forecasts but are implicitly passing them when forming aggregates.

To do's:

- ① Add $E(\pi)$ instead of π in TR
- ② Try $\gamma_h < 1$
- ③ The Townsend analysis¹³ suggests that IRFs to EE-learning won't give oscillations b/c these agents aren't testing each others' posts
- ④ Do IRFs to vector learning from the spinoff LH paper
- ⑤ Figure out alternative learning that exhibits no oscillations
• Can Bayesian learning do it?

↳ refs in Ryan's class reading list

Susanto commented on

6 Dec 2015

oscillating IRFs : 1) they can and do arise more frequently than you'd think b/c published IRFs look "nice" b/c a lot of work goes into making them nice
2) check local projections which estimates the MA process directly. Since a VAR estimates an AR and imposes

restrictions on the VC matrix (and the AR-matrix), it's likely to give you more smooth IRFs, while those of local projections are likely to be more choppy.

Check Valerie Ramey's Handbook chapter on IRFs!

It also has a bunch of replication files.

Adding $E(\pi)$ to TR

7 Dec 2015

$$x_t = -\beta (\gamma_{\pi} \hat{E}_t \pi_{t+1} + \gamma_x x_t + \underbrace{\rho i_{t-1} + \bar{i}_t}_{\text{shocks}}) + \Sigma(\cdot)$$

clearly the shocks will

$(1+\beta \gamma_x)x_t = -\beta \gamma_{\pi} \hat{E}_t \pi_{t+1} + \Sigma$ be the same, ignore 'em

$$(1+\beta \gamma_x)x_t = -\beta \gamma_{\pi} \hat{E}_t \pi_{t+1} + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \gamma_{\pi} \hat{E}_{T+1} \pi_{T+2} + \gamma_x x_{T+1} + \text{shocks}]$$

$$(1+\beta \gamma_x)x_t = -\beta \gamma_{\pi} \hat{E}_t \pi_{t+1} + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta - \beta \gamma_{\pi})x_{T+1} + \beta \pi_{T+1} - \beta \gamma_{\pi} \hat{E}_{T+1} \pi_{T+2}]$$

now these two pose difficulties

b/c I don't know if I can use LIE on \hat{E}_t or not.

Let's see if we can shove β it into the expectation instead.

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} \left[(1-\beta)x_{T+1} + \beta \pi_{T+1} \right]$$

$$+ \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} \left[-\beta \beta i_{T+1} \right]$$

$$\Rightarrow x_t = \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} \left[-\beta i_T \right]$$

$$\Rightarrow x_t = \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} \left[(1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta i_t \right]$$

$$= \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} \left[(1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta (\gamma_{\pi} \hat{E}_T \pi_{T+1} - \gamma_x x_T) \right]$$

shocks
un-
changed

$$(1+\beta \gamma_x) x_t = \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} \left[\underbrace{x_{T+1} + \beta \pi_{T+1} - \beta \gamma_{\pi} \hat{E}_T \pi_{T+1}}_{\text{shocks unchanged}} \right]$$

LIE is still a question

but at least this expression is neater.

So $\hat{E}_t^i \hat{E}_{t+1}^i = \hat{E}_t^i$, i.e. the idios \hat{E}^i fulfills LIE b/c that is anticipated utility. There are 2 questions: if LIE holds for \hat{E}^i , why don't HMs use the recursive representation?

→ Preston would argue that b/c then they would ignore their wealth b/c they don't know they're identical. ok fine so they don't use LIE and so we get the full expression when we aggregate.

But it seems to me - again! - from Preston 2005 p 16 that

that \hat{E} doesn't satisfy LIE - not on the aggregate.

So also CEMP define anticipated utility as $\hat{E}_{t-1}^+ \bar{\pi}_T = \bar{\pi}_t$
i.e. as an idiosyncratic statement.

However it seems to me like maybe actually even
the agg $\hat{E}(\cdot)$ satisfied it somehow! so when we
compute expectations I think we make use of it?

In particular, the modeler knows, as CEMP state,
that $\hat{E}_t^+ = \hat{E}_t$, i.e. that agents hold identical beliefs.

So then E_t will fulfill LIE too. So maybe it's really
just the case that agg beliefs just "inheret" the
discounted sums.

Anyway - the usual confusion about LI is there: it
seems like everything fulfills LIE and yet still since
LI forecasts are optimal for agents who don't know
that they're identical, agg \hat{E} "inheret" them.

yes this is the stance of my thinking later on too (8 Jan 2020)

Will do so after that detour:

$$(1+b\gamma_x)x_+ = \hat{E}_1 \sum_{T=1}^{\infty} \beta^{T+1} \left[(1-\beta)x_{T+1} + b\pi_{T+1} - b\gamma_T \hat{E}_T \pi_{T+1} \right] \xrightarrow{\text{+ shocks unchanged}}$$

$$= (b - b\gamma_T) \pi_{T+1} \text{ here}$$

$$(1+b\gamma_x)x_+ = \hat{E}_1 \sum_{T=1}^{\infty} \beta^{T+1} \left[(1-\beta-\beta^2\gamma) x_{T+1} + b(1-\gamma_T)\pi_{T+1} \right]$$

$$= g_{xb}$$

well this will always be < 0.

So then: $x_+ = \frac{1}{1+b\gamma_x} [b(1-\gamma_T), (1-\beta-\beta^2\gamma), 0] f_\beta$

$\underline{\underline{g_{xa} = [0 \ 0 \ 0]}}$

$$\pi_+ = K x_+ + [(1-\alpha)\beta, K\alpha\beta, 0] f_\alpha$$

$$\pi_+ = \frac{K}{1+b\gamma_x} [b(1-\gamma_T), 1-\beta, 0] f_\beta + [(1-\alpha)\beta, K\alpha\beta, 0] f_\alpha$$

$\underline{\underline{= g_{\pi a}}}$

$$x_+ = g_{xb} f_\beta + g_{xa} f_\alpha$$

$\rightarrow [0 \ 0 \ 0]$

$$\pi_+ = K \cdot g_{xb} f_\beta + g_{\pi a} f_\alpha$$

$$i_+ = (\gamma_\pi g_{\pi a} + \gamma_x g_{xa}) f_\alpha + (\gamma_\pi g_{\pi b} + \gamma_x g_{xb}) f_\beta$$

\rightarrow shocks that
are unchanged

actually that's not quite true,

see Mathematica materials 12.m

or Notes 10 Dec 2019.

Stuff got lost here. Kozlowski et al: agents are learning the distribution of shocks, $g(\cdot)$.

For koz et al, this is crucial b/c when a tail event occurs, it leads to agents to estim a shock distrib $\hat{g}(\cdot)$ w/ fat tails (potentially asymmetric?)

They estimate $\hat{g}(\cdot)$ using a normal kernel density estimator, I guess the most std. non-parametric estimator:

$$\hat{g}_t(x) = \frac{1}{n_t} \sum_{s=0}^{n_t-1} \Omega(x - x_{t-s}, \Sigma_t)$$

↑ x_t ? not clear but can check kernel estimators
 multivariate normal density or mean, \bar{x} ?
 VC matrix (aka "bandwidth matrix")
 (see Baum lecture 3)

obs at time t

$X = d \times 1$ shock vector

Σ_t = diagonal matrix, I write like this:

$$\Sigma = \begin{pmatrix} \hat{\sigma}_1 & & \\ & \ddots & \\ & & \hat{\sigma}_d \end{pmatrix} \cdot \left(\frac{4}{(2+d)n_t} \right)^{\frac{1}{(4+d)}}$$

where $\hat{\sigma}_j$ = sample std dev of shock j

So 2 main differences to adaptive learning:

- 1) here agents learn about shocks, not observables
- 2) learning is nonparametric b/c they learn a distrib

↳ (1) is not so interesting for me b/c exog. shocks have no connection to central bank credibility, so that one can no longer talk about anchoring

↳ I expect (2) to be more interesting, yet maybe not too different to my setting (quantitatively) b/c

I can imagine a generalization of my adaptive learning framework in which agents don't know the distrib of the LOM (they don't know that g_x is bivar and normal) \Rightarrow that would be similar

to learning slope and constant I guess except it would be less restrictive on their "prior" if you will.

\rightarrow But one could try it out!

What is not clear to me is what would be the analogue to the gain? The bandwidth matrix?

Read Collins-Dufresne et al (2016)

Honestly, Bayesian learning seems very analogous to adaptive (frequentist) learning except that the updating rule is Bayes' rule, the prior matters AND one difference that might matter is that while in adaptive learning, the anticipated utility as means that in every period, agents treat the PLM as if it was the ALM, Bayesian learning can account for the uncertainty around the correctness of the PLM

(This is referred to as parameter uncertainty)
And it often refers to the case where agents plot using an average of different models
↳ can this mitigate overshooting?

Or is this simply the Bayesian counterpart to the gains/anchoring?

The problem is that this is an wort pötzlig paper and doesn't describe how Bayesian learning works, nor does it give references.

Evans, Honkapohja & Sargent (2013)

(In the book *Macs at the Service of Public Policy* - which I haven't downloaded but I did save the ref.)

Truth market model (6.1)

$$p_t = \mu + \alpha \hat{z}_{t-1}^*, p_t + \delta z_{t-1} + \eta_t; \quad \eta_t \sim \text{WN}(0, \bar{\sigma}_\eta^2)$$

$$z_t = \beta z_{t-1} + w_t \quad w_t \sim \text{iid}(0, \bar{\sigma}_w^2)$$

$$\mu = 0$$

REE model for (6.1) is

$$p_t = \hat{\beta} z_{t-1} + \eta_t \quad \text{w/ } \hat{\beta} = \frac{\delta}{1-\alpha}$$

Now EHS consider a constant parameter learning rule that's estimated using Bayesian techniques.

The PLM is

$$\frac{\hat{\sigma}_\eta^2}{\hat{\sigma}_w^2} \neq \frac{\bar{\sigma}_\eta^2}{\bar{\sigma}_w^2}$$

$$p_t = b_{t-1} z_{t-1} + \eta_t \quad \text{w/ } \eta_t \sim N(0, \hat{\sigma}_\eta^2)$$

Agents' prior distib for β is $\beta \sim N(b_0, V_0)$ and updates

$$\text{are } b_t = b_{t-1} + \frac{V_{t-1} z_{t-1}}{\hat{\sigma}_\eta^2 + V_{t-1} z_{t-1}^2} (p_t - b_{t-1} z_{t-1})$$

$$V_t = V_{t-1} - \frac{z_{t-1}^2 V_{t-1}}{\hat{\sigma}_\eta^2 + V_{t-1} z_{t-1}^2} \quad \text{using the Kalman filter.}$$

Prop. 6.1 Bayesian analog to E-stability

(convergence to RE w/ prob 1 if $\alpha < 1$)

and $V_t = \frac{\beta_n^2}{(t+1)S_1 - 2\beta^2} \rightarrow 0$ irrespective of
whether β_n^2 is corrct!
 \downarrow
WTF is $S_t \dots$?

I'm a little suspicious of this b/c it says that if $V_t \rightarrow 0$
then the Kalman gain goes to zero too and you
don't get that w/ the KF (instead $\Sigma_{+1+} \rightarrow \Sigma$,
the st. st. and so the gain K_{+1+} goes to α, γ
st. st gain) \rightarrow this is normally how the KF is
analogous w/ constant gain learning and it
always keeps a tiny variance around RE.

But maybe they are looking at gain learning
analogues here - that might be what they mean
w/ "constant param learning". In the following
section, they consider time-varying param β .
It is useful to understand how time-varying param (TVP)

models are related to misspecification of test rules.
When agents think that f_p may be time-varying
they entertain the idea that they may be using a
misspecified model. This is intimately connected
to again learning b/c agents prefer to track the process
because they do not know whether a structural
change may render their converged forecasting rule
misspecified.

EHS show w/ a simple example (where agents
aren't sure whether a constant param PLM or
a TVP PLM is the right one) that if feedback
effects from beliefs are sufficiently strong then
agents may select the non-RE model b/c E(.)
induce actual drifts in the ALM that induce the
MSE of the non-RE PLM to be lower than the RE
one. (This is like a self-confirming learning result.)

It's like an endog. again choice !!!

A note: EHS claim on p. 105 that

they guess that for this TVP learning case the stable parameter space will be $-1 < \alpha < 0.5$ b/c under LS learning:

- 1) $0.5 < \alpha < 1$ may yield slow convergence to RE
- 2) " $\alpha < -1$ a possible problem of overshooting can emerge when agents overparametrize the PLM"

⇒ oh really ??

So when the feedback from expectations is negative and sufficiently strong, least-squares learning may yield overshooting ???

You don't say!

so when RLS learning, so agents learn 9 Dec 2019

g_x , then in terms of the NKPC, NKIS & TR, what do they know?

they know h_x

$$x_t = h_x x_{t-1} + \eta \varepsilon_t$$

$$y_t = g_x x_t$$

I think they always know the TR - what they don't seem to know is how shocks affect observables $z_t = \begin{bmatrix} \pi_t \\ x_t \\ i_t \\ s_t \end{bmatrix}$.

I think they must understand contemporaneous relationships between observables \rightarrow so even when they learn the slope also, they know $i_t = \gamma_{\pi} \pi_t$.

What they get wrong are forecasts $\hat{E}_t z_{t+1} = \hat{g}_x h x_s$.

If $\hat{g}_x = \underbrace{a + g_x}_{\text{constant}}$, then they shift expectations up/down.

If $\hat{g}_x = \hat{g}_x \leftarrow \text{constant \& slope}$ \rightarrow so when a persistent shock hits, now agents might not know how that will impact observables in the future

(in particular $i_t \uparrow \rightarrow \pi_t \downarrow \rightarrow i_{t+h} \downarrow$ will be less clear, so expectations move less.)

\Rightarrow I'm not sure that a different updating scheme (KF or PF) would change anything about the speed of updating, or its size.

→ in this sense if they learned b_x as well might help b/c then in the case of the persistent mon pol shock they wouldn't actually be sure that it will persist.

→ It seems like my hunch is that the more they are uncertain about, the less expectations will move.

→ Is learning a distrib. helpful here? I'm not sure. I kinda feel that learning a functional form might... or might not?

⇒ I kinda want them not to know the TR!

↳ what if I put π_{t-1} in the Taylor-rule instead of $\bar{\pi}_t$?
(Or add a π_{t-1} term?)

So if $i_t = \psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t$ 10 Dec 2019
don't ignore shocks again b/c they might not be unchanged!

$$\text{then } x_t = -3\bar{i}_t + \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} [(1-\beta)x_{T-1} + 3\pi_{T+1} - 3\beta i_{T-1} + 3\pi_T]$$

$$(1-3\psi_x) x_t = -3\psi_\pi \pi_{t-1} + \hat{E}_t 2\beta [(1-\beta - 3\beta \psi_x) x_{t+1} + 2\pi_{t+1} - 3\beta \pi_t + 3\pi_{t+1}]$$

Or, if instead starting from \star ,

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} + \beta \pi_{T+1} - \beta i_t + \beta r_t^N \right]$$

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} + \beta \pi_{T+1} - \beta (\gamma_\pi \pi_{T-1} + \gamma_x x_T + i_T) + \beta r_t^N \right]$$

$$(1-2\gamma_x)x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta-2\beta\gamma_x)x_{T+1} + \beta \pi_{T+1} - \beta \gamma_\pi \pi_{T-1} \right] \\ - \beta \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [i_t - r_t^N]$$

$$-2\gamma_\pi \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \pi_{T-1} = -2\gamma_\pi [\pi_{t-1} + \beta \pi_t + \beta^2 \pi_{t+1}]$$

So we get $-2\gamma_\pi (\pi_{t-1} + \beta \pi_t) + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \beta^2 \pi_{t+1}$

So

$$(1+2\gamma_x)x_t = -2\gamma_\pi \pi_{t-1} - 2\gamma_\pi \beta \pi_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta-2\beta\gamma_x)x_t + \beta(1-\beta^2\gamma_\pi)\pi_{T+1} \right]$$

↑
 long term
 novel here

↑
 β's novel here and will
 dampen feedback

↗
 $-2\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [i_t - r_t^N]$

$$(1+2\gamma_x)x_t = -2\gamma_\pi \pi_{t-1} - 2\gamma_\pi \beta \left[kx_t + [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\beta + \hat{E}_t \sum \alpha \beta^{T-t} u_T \right] \\ + \left[\beta(1-\beta^2\gamma_\pi), 1-\beta-2\beta\gamma_x, 0 \right] f_\beta - 2\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [i_t - r_t^N]$$

$$(1+2\gamma_x + k\beta\gamma_\pi \beta) x_t = -2\gamma_\pi \pi_{t-1} - 2\gamma_\pi \beta \left[(1-\alpha)\beta, \kappa\alpha\beta, 0 \right] f_\alpha \\ + \left[\beta(1-\beta^2\gamma_\pi), 1-\beta-2\beta\gamma_x, 0 \right] f_\beta - 2\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [i_t - r_t^N] - 2\gamma_\pi \beta \hat{E}_t \sum \alpha \beta^{T-t} u_T$$

$$\Rightarrow (1+2\gamma_x + k\beta\gamma_\pi \beta) x_t = -2\gamma_\pi \pi_{t-1} - 2\gamma_\pi \beta \left[(1-\alpha)\beta, \kappa\alpha\beta, 0 \right] f_\alpha \\ + \left[\beta(1-\beta^2\gamma_\pi), 1-\beta-2\beta\gamma_x, 0 \right] f_\beta - 2\begin{bmatrix} -1 & 1 & 0 & 0 & 0 \end{bmatrix} (I_5 - \beta h x)^{-1} s_T \\ - 2\gamma_\pi \beta \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} (I_5 - \alpha \beta h x)^{-1} s_T$$

$$S_0 \cdot x_t = \frac{1}{(1 + 2\gamma_x + k\beta\gamma_\pi \beta)} \cdot \left\{ \begin{array}{l} -2\gamma_\pi \pi_{t-1} - 2\gamma_\pi \beta [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha \\ \hookrightarrow -2\gamma_\pi [0 \ 0 \ 0 \ 0 \ 1] s_t \end{array} \right.$$

$$+ [2(1-\beta^2\gamma_\pi), 1-\beta-2\beta\gamma_x, 0] f_\beta - \beta [-1 \ 1 \ 0 \ 0] (I_4 - \beta h x)^{-1} s_t - 2\gamma_\pi \beta [0 \ 0 \ 1 \ 0] (I_4 - \alpha\beta h x)^{-1} s_t \} \\ \uparrow \beta^2$$

$\Rightarrow \text{coeffa} = \text{uncharged } [(1-\alpha)\beta, k\alpha\beta, 0] \quad \text{coeffspil} = [0 \ 0 \ 0 \ 0 \ 1]$

$$\text{coeffb} = [2(1-\beta^2\gamma_\pi), 1-\beta-2\beta\gamma_x, 0]$$

$$\text{coeffa} = \text{uncharged } [0 \ 0 \ 1 \ 0 \ 0] (I_4 - \alpha\beta h x)^{-1}$$

$$\text{coeffb} = \text{uncharged } [-1 \ 1 \ 0 \ 0 \ 0] (I_4 - \beta h x)^{-1}$$

$$\text{coeffs} = \text{uncharged } [0 \ 1 \ 0 \ \rho \ 0] \quad (\text{shorts in TR})$$

$$x_t = \frac{1}{(1 + 2\gamma_x + k\beta\gamma_\pi \beta)} \cdot \left\{ \begin{array}{l} -2\gamma_\pi \pi_{t-1} - 2\gamma_\pi \beta \text{coeffa} \cdot f_\alpha \\ \hookrightarrow -2\gamma_\pi [0 \ 0 \ 0 \ 0 \ 1] s_t \end{array} \right.$$

$$+ \text{coeffb} \cdot f_\beta + [-2 \text{coeffb} - 2\gamma_\pi \beta \text{coeffa}] \cdot s_t \} \quad \boxed{\quad}$$

$$x_t = \frac{-2\gamma_\pi}{(1 + 2\gamma_x + k\beta\gamma_\pi \beta)} \pi_{t-1} - \frac{2\gamma_\pi \beta}{(1 + 2\gamma_x + k\beta\gamma_\pi \beta)} \text{coeffa} \cdot f_\alpha$$

$$+ \frac{1}{(1 + 2\gamma_x + k\beta\gamma_\pi \beta)} \text{coeffb} \cdot f_\beta - \frac{2\gamma_\pi}{1 + 2\gamma_x + k\beta\gamma_\pi \beta} \text{coeffspil} [0 \ 0 \ 0 \ 0 \ 1] s_t$$

$$+ \frac{1}{(1 + 2\gamma_x + k\beta\gamma_\pi \beta)} [-2 \text{coeffb} - 2\gamma_\pi \beta \text{coeffa}] \cdot s_t$$

$$\begin{aligned}
 \pi_t^+ &= \frac{-\beta \gamma_\pi K}{(1 + \beta \gamma_x + K \beta \gamma_\pi \beta)} \cancel{\pi_{t-1}} - \frac{\beta \gamma_\pi \beta K}{(1 + \beta \gamma_x + K \beta \gamma_\pi \beta)} \text{coeff}_\alpha \cdot f_\alpha \\
 &\quad + \frac{K}{(1 + \beta \gamma_x + K \beta \gamma_\pi \beta)} \text{coeff}_\beta \cdot f_\beta - \frac{K \cdot \beta \gamma_\pi}{1 + \beta \gamma_x + K \beta \gamma_\pi \beta} [0 \ 0 \ 0 \ 0 \ 1] s_t \\
 &\quad + \frac{K}{(1 + \beta \gamma_x + K \beta \gamma_\pi \beta)} [-3 \text{coeff}_\beta - \beta \gamma_\pi \beta \text{coeff}_\alpha] \cdot s_t \\
 &\quad + \underbrace{[(1-\alpha)\beta, \kappa \alpha \beta, 0]}_{\text{coeff}_\alpha \cdot f_\alpha} \cdot f_\alpha + \underbrace{[0 \ 0 \ 1 \ 0] [I_4 - \alpha \beta h x]^{-1}}_{\text{coeff}_\alpha \cdot s_t} s_t
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \pi_t^+ &= \frac{-\beta \gamma_\pi K}{(1 + \beta \gamma_x + K \beta \gamma_\pi \beta)} \pi_{t-1} + \left(1 - \frac{\beta \gamma_\pi \beta K}{(1 + \beta \gamma_x + K \beta \gamma_\pi \beta)} \right) \text{coeff}_\alpha \cdot f_\alpha \\
 &\quad + \frac{K}{(1 + \beta \gamma_x + K \beta \gamma_\pi \beta)} \text{coeff}_\beta \cdot f_\beta + \left\{ -\frac{K \beta \gamma_\pi}{1 + \beta \gamma_x + K \beta \gamma_\pi \beta} [0 \ 0 \ 0 \ 0 \ 1] \right. \\
 &\quad \left. + \frac{-K \beta}{1 + \beta \gamma_x + K \beta \gamma_\pi \beta} \text{coeff}_\beta + \left(1 - \frac{K \beta \gamma_\pi \beta}{1 + \beta \gamma_x + K \beta \gamma_\pi \beta} \right) \text{coeff}_\alpha \right\} s_t
 \end{aligned}$$

What is kinda funny is that I forgot that π_{t-1} is a new state, so it's a 5th entry in s_t .

→ the way I'm gonna deal with it is as a zero in s_t

and thus in $\text{coefs}a$ & $\text{coefs}b$, & coefs
b/c π_{t-1} doesn't enter in the expectations

$$z_t = \text{new} \cdot \pi_{t-1} + A_a \cdot f_a + A_b \cdot f_b + A_S \cdot s_t$$

↑ bigger size
b/c h_x is 5×5

Or, one can avoid

having to redesign the learning code and instead
incorporate the effects of π_{t-1} into A_S ! This

is what I'll do!

↳ change in previous pages using red!

Ok done, but there's even more overshooting in IRFs.

Is something wrong?

- Also RE-i oscillates a bit b/c it doesn't move on impact
(THIS is instrument instability: it reacts to inflation w/
a lag)
- But it doesn't make sense that the shock doesn't have
bigger impact effects both for RE and learning

Err - I think that w/ π_{t-1} in TR instead of π_t should have larger IRFs for RE on impact

So I was looking at Basu

11 Dec 2019

lectures and what do I find in Lect 22 (in a part we haven't covered)? \rightarrow disinflationary booms!

slide 36: $\pi_t - \beta E_t \pi_{t+1} = k (\hat{y}_t - \hat{y}_{t+1})$

1) $\pi_t \uparrow \rightarrow x_t \uparrow$ standard

2) $E_t \pi_{t+1} \downarrow$ (anticipated disinflation) $\rightarrow x_t \uparrow$

slide 38: having a π_{t-1} term in NKPC would make disinflation costly

- this is equivalent to imperfect credibility or learning (!)

"we don't believe π_t will fall unless we see π_{t-1} fall"

The intuition of why it's costly is that in the disinflationary boom case, you could get $E_t \pi_{t+1}$ to move w/o π_t moving \Rightarrow w/ a lagged π term, you can't do that, and firms will depress the output gap.

The weird thing is that I get exactly the opposite:
I add π_{t+1} from learning and I still get
disinflationary booms.

→ I think I know why: b/c of what I outline in
materials 10 Susanto: when γ_T sufficiently large
the expectations effect still outweighs the "I demand
to see a fall"-effect.

There's an issue around the concept of credibility
here: Ball: credibility is when $E_t \pi_{t+1}$ moves a lot
for me, that's exactly no credibility b/c agents are
reassessing the target.

But I've gotten a little off track: I was gonna try to
understand RE responses when π_{t+1} in TR.

The only reason the leg- π -TR can yield higher π
is if due to lower movement in expectations have smaller
effects on $X \rightarrow$ and yes: although $X \downarrow$ more on impact
in π_{t+1} -TR, the cumulative fall in X is smaller in leg- π -TR.

What's going on? In the lag- π -TR RE model...

Although the CB won't respond on impact, it will in the 2nd period and more strongly so b/c of the "missed response" on impact. Agents w/ RE internalize this which is why $E(\pi_t)$ don't drop as much; and the cumulative drop in X is thus lower, leading π_T to drop less.

⇒ Ok, so surprising but makes sense!

Ok so to do: understand the learning responses

To do so, work hard to develop a general solution method so that adapting the model is quicker.

Also check from the pit-matrices!

Ok so : The goal is to get (quidly & painlessly) to

$$z_t = A_a f_\alpha + A_b f_\beta + A_s s_t$$

Suppose we start from the std NKIS & NKPC

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \pi_{T+1} + \beta r_T)$$

$$\pi_t = K x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} (K \alpha \beta x_{T+1} + (1-\alpha) \beta \pi_{T+1} + u_T)$$

and the Taylor rule will be something like

$$i_t = \gamma_\pi \pi_t + \gamma_x x_t + \rho i_{t-1} + \bar{i}$$

As a first step, let's isolate the shortes/states

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \pi_{T+1} + \beta r_T)$$

$$\pi_t = K x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} (K \alpha \beta x_{T+1} + (1-\alpha) \beta \pi_{T+1}) + \hat{E}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} u_T$$

$$i_t = \gamma_\pi \pi_t + \gamma_x x_t + [0 \ 1 \ 0 \ \rho] s_t$$

As a 2nd step, introduce LH expectations

$$x_t = -\beta i_t + [0, 1-\beta, -\beta \beta] f_\beta + \beta [1 \ 0 \ 0 \ 0] (I_{nx} - \beta h x)^{-1} s_t$$

$$\pi_t = K x_t + [(1-\alpha)\beta, K \alpha \beta, 0] f_\alpha + [0 \ 0 \ 1 \ 0] (I_{nx} - \alpha \beta h x)^{-1} s_t$$

$$i_t = \gamma_\pi \pi_t + \gamma_x x_t + [0 \ 1 \ 0 \ \rho] s_t$$

Already this should be enough to solve things!

$$x_+ = -\beta i_t + [0, 1-\beta, -\beta\beta] f_B + \beta [1 \ 0 \ 0 \ 0] (\mathbf{I}_{nx} - \beta h x)^{-1} s_+$$

$$\pi_+ = K x_+ + [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0 \ 0 \ 1 \ 0] (\mathbf{I}_{nx} - \alpha\beta h x)^{-1} s_+$$

$$i_+ = \gamma_{\pi} \pi_+ + \gamma_x x_+ + [0 \ 1 \ 0 \ p] s_+$$

\Rightarrow

$$x_+ = -\beta i_t + c_{xb} \cdot f_B + c_{xs} \cdot s_+$$

$$\pi_+ = K x_+ + c_{\pi a} \cdot f_\alpha + c_{\pi s} \cdot s_+$$

$$i_+ = \gamma_{\pi} \pi_+ + \gamma_x x_+ + c_{is} \cdot s_+$$

where c_{xy} stands for "coefficient of x on y " and are:

$$c_{xb} = [0, 1-\beta, -\beta\beta] \quad (1 \times ny)$$

$$c_{xs} = \beta [1 \ 0 \ 0 \ 0] (\mathbf{I}_{nx} - \beta h x)^{-1} \quad (1 \times nx)$$

$$c_{\pi a} = [(1-\alpha)\beta, \kappa\alpha\beta, 0] \quad (1 \times ny)$$

$$c_{\pi s} = [0 \ 0 \ 1 \ 0] (\mathbf{I}_{nx} - \alpha\beta h x)^{-1} \quad (1 \times nx)$$

$$c_{is} = [0 \ 1 \ 0 \ p] \quad (1 \times nx)$$

Then Mathematica can solve this (or you can)

and it should be easier to manipulate the model

by tweaking the C -matrices.

$$\left. \begin{aligned} x_+ &= -\gamma_{\pi} \pi_+ + c_{xb} \cdot f_{\beta} + c_{xs} \cdot s_+ \\ \pi_+ &= K x_+ + c_{\pi a} \cdot f_{\alpha} + c_{\pi s} \cdot s_+ \\ i_+ &= \gamma_{\pi} \pi_+ + \gamma_x x_+ + c_{is} \cdot s_+ \end{aligned} \right\}$$

$$x_+ = -2(\gamma_{\pi} \pi_+ + \gamma_x x_+ + c_{is} \cdot s_+) + c_{xb} \cdot f_{\beta} + c_{xs} \cdot s_+$$

$$\pi_+ = K x_+ + c_{\pi a} \cdot f_{\alpha} + c_{\pi s} \cdot s_+$$

$$x_+ = \frac{1}{1+2\gamma_x} [-2\gamma_{\pi} \pi_+ - 2c_{is} \cdot s_+ + c_{xb} \cdot f_{\beta} + c_{xs} \cdot s_+]$$

$$x_+ = \frac{1}{1+2\gamma_x} \left[-2\gamma_{\pi} (K x_+ + c_{\pi a} \cdot f_{\alpha} + c_{\pi s} \cdot s_+) - 2c_{is} \cdot s_+ + c_{xb} \cdot f_{\beta} + c_{xs} \cdot s_+ \right]$$

$$= \frac{1}{1+2\gamma_x} \left[-2\gamma_{\pi} K x_+ - 2\gamma_{\pi} c_{\pi a} f_{\alpha} - 2\gamma_{\pi} c_{\pi s} s_+ - 2c_{is} s_+ + c_{xb} f_{\beta} + c_{xs} s_+ \right]$$

$$= \frac{-2\gamma_{\pi} K}{1+2\gamma_x} x_+ - \frac{2\gamma_{\pi}}{1+2\gamma_x} c_{\pi a} f_{\alpha} + \frac{1}{1+2\gamma_x} [-2\gamma_{\pi} c_{\pi s} - 2c_{is} + c_{xs}] s_+ + \frac{1}{1+2\gamma_x} \cdot c_{xb} f_{\beta}$$

$$\underbrace{\left(1 + \frac{2\gamma_{\pi} K}{1+2\gamma_x}\right)}_{\frac{1+2\gamma_x+2\gamma_{\pi} K}{1+2\gamma_x}} x_+ = -\frac{2\gamma_{\pi}}{1+2\gamma_x} c_{\pi a} f_{\alpha} + \frac{1}{1+2\gamma_x} c_{xb} f_{\beta}$$

$$+ \frac{1}{1+2\gamma_x} [-2\gamma_{\pi} c_{\pi s} - 2c_{is} + c_{xs}] s_+$$

$$\left(\frac{1+2\gamma_x + 2\gamma_{\pi} k}{1+2\gamma_x} \right) s_t = - \frac{2\gamma_{\pi}}{1+2\gamma_x} C_{\pi a} f_{\alpha} + \frac{1}{1+2\gamma_x} C_{x b} f_{\beta}$$

$$+ \frac{1}{1+2\gamma_x} [-2\gamma_{\pi} C_{\pi s} - 2C_{is} + C_{xs}] s_t$$

$$s_t = - \frac{2\gamma_{\pi}}{1+2\gamma_x + 2\gamma_{\pi} k} C_{\pi a} f_{\alpha} + \frac{1}{1+2\gamma_x + 2\gamma_{\pi} k} C_{x b} f_{\beta}$$

$$+ \frac{1}{1+2\gamma_x + 2\gamma_{\pi} k} [-2\gamma_{\pi} C_{\pi s} - 2C_{is} + C_{xs}] s_t$$

Now check this against bottom of Mathematica,
materials 12.nb. It's correct (solGEN)

Of course it doesn't equal my old solution.
I wonder if it's b/c I'm somehow not adequately
dealing w/ the monpol shock; $\bar{i} \rightarrow$ it should
maybe enter the $E(\cdot)$ in the NKPC? But then
again - one could argue the same about cost prob
shocks? It certainly seems like only the MP

short has different loadings (2nd column of A_S)
 and in terms of $\mathbb{E}[\cdot]$, $x \& i$ have different
 ones (1st & 3rd column of A_b)

Need to understand why the two

12 Dec 2019

solutions for the A-matrices differ.

$$x_t = -\beta i_t + \hat{\mathbb{E}}_t \sum_{T=1}^{\infty} \beta^{T-t} [(+\beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta i_{T+1} + \beta r_T^n] \quad (1)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=1}^{\infty} (\kappa \beta)^{T-t} [\kappa \alpha \beta x_{T+1} + (1-\alpha) \beta \pi_{T+1} + u_T] \quad (2)$$

$$i_t = \gamma_\pi \pi_t + \gamma_x x_t + \rho i_{t-1} + \bar{i}_t \quad (3)$$

The old sol was:

$$x_t = -\beta [\gamma_\pi \pi_t + \gamma_x x_t + \rho i_{t-1} + \bar{i}_t]$$

$$+ \hat{\mathbb{E}}_t \sum_{T=1}^{\infty} \beta^{T-t} [(1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta [\gamma_\pi \pi_{T+1} + \gamma_x x_{T+1} + \rho i_T + \bar{i}_{T+1}] + \beta r_T^n]$$

$$(1+\beta \gamma_x)x_t = -\beta \gamma_\pi \pi_t - \beta [\rho i_{t-1} + \bar{i}_t]$$

$$+ \hat{\mathbb{E}}_t \sum_{T=1}^{\infty} \beta^{T-t} [(1-\beta - \beta \beta \gamma_x)x_{T+1} + \beta(1-\beta \gamma_\pi)\pi_{T+1} - \beta \beta [\rho i_T + \bar{i}_{T+1}] + \beta r_T^n]$$

$$(1+\beta \gamma_x)x_t = -\beta \gamma_\pi \pi_t$$

$$+ \hat{\mathbb{E}}_t \sum_{T=1}^{\infty} \beta^{T-t} [(1-\beta - \beta \beta \gamma_x)x_{T+1} + \beta(1-\beta \gamma_\pi)\pi_{T+1} - \beta [\rho i_{T-1} + \bar{i}_T] + \beta r_T^n]$$

So we can write (1) & (2) as

$$(1+b\gamma_x)x_+ = -b\gamma_\pi \pi_+$$

$$+ \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} \left[(1-\beta-b\beta\gamma_x)x_{T+1} + b(1-\beta\gamma_\pi)\pi_{T+1} - b[\rho i_{T-1} + \bar{i}_T] + b r_T^n \right]$$

$$\pi_+ = Kx_+ + \hat{E}_t \sum_{T=1}^{\infty} (\beta P)^{T-t} [K\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T]$$

\Leftrightarrow

$$x_+ = \frac{-b\gamma_\pi}{1+b\gamma_x} \pi_+ + \frac{1}{1+b\gamma_x} [b(1-\beta\gamma_\pi), (1-\beta-b\beta\gamma_x), 0] f_\beta$$

$$+ \frac{1}{1+b\gamma_x} (-b) [-1 \ 1 \ 0 \ \rho] (I_{hx} - \beta h x)^{-1} s_+$$

(*)

$$\pi_+ = Kx_+ + [(1-\alpha)\beta, K\alpha\beta, 0] f_K + [0 \ 0 \ 1 \ 0] (I_{hx} - \alpha\beta h x)^{-1} s_+$$

Hm. Anyway, so then

$$(1+b\gamma_x)x_+ =$$

$$-b\gamma_\pi \left\{ Kx_+ + [(1-\alpha)\beta, K\alpha\beta, 0] f_K + [0 \ 0 \ 1 \ 0] (I_{hx} - \alpha\beta h x)^{-1} s_+ \right\}$$

$$+ [b(1-\beta\gamma_\pi), (1-\beta-b\beta\gamma_x), 0] f_\beta + (-b) [-1 \ 1 \ 0 \ \rho] (I_{hx} - \beta h x)^{-1} s_+$$

$$(1+b\gamma_x + bK\gamma_\pi)x_+ =$$

$$-b\gamma_\pi [(1-\alpha)\beta, K\alpha\beta, 0] f_K - b\gamma_\pi [0 \ 0 \ 1 \ 0] (I_{hx} - \alpha\beta h x)^{-1} s_+$$

$$+ [b(1-\beta\gamma_\pi), (1-\beta-b\beta\gamma_x), 0] f_\beta + \frac{(-b)}{W} [-1 \ 1 \ 0 \ \rho] (I_{hx} - \beta h x)^{-1} s_+$$

$$K_+ = \frac{-b\gamma_\pi}{W} [(1-\alpha)\beta, K\alpha\beta, 0] f_K + \frac{1}{W} [b(1-\beta\gamma_\pi), (1-\beta-b\beta\gamma_x), 0] f_\beta$$

$$+ \left[-\frac{b\gamma_\pi}{W} [0 \ 0 \ 1 \ 0] (I_{hx} - \alpha\beta h x)^{-1} + \frac{(-b)}{W} [-1 \ 1 \ 0 \ \rho] (I_{hx} - \beta h x)^{-1} \right] s_+$$

x^* , old method

Now let's solve using the new method (for x only)

$$\left. \begin{aligned} x_+ &= -\gamma_{it} + [3, 1-\beta, -2\beta] f_\beta + 2[1 \ 0 \ 0 \ 0] (\mathcal{I}_{hx} - \beta h_x)^{-1} s_1 \\ \pi_+ &= Kx_+ + [(1-\alpha)\beta, \alpha\beta, 0] f_\alpha + [0 \ 0 \ 1 \ 0] (\mathcal{I}_{hx} - \alpha\beta h_x)^{-1} s_4 \\ i_t &= \gamma_\pi \pi_+ + \gamma_x x_+ + [0 \ 1 \ 0 \ \rho] s_4 \end{aligned} \right\}$$

So if I solve for x , I get

$$\begin{aligned} x_+ &= -2[\gamma_\pi \pi_+ + \gamma_x x_+ + [0 \ 1 \ 0 \ \rho] s_4] \\ &\quad + [3, 1-\beta, -2\beta] f_\beta + 2[1 \ 0 \ 0 \ 0] (\mathcal{I}_{hx} - \beta h_x)^{-1} s_1 \end{aligned}$$

$$\begin{aligned} (1+2\gamma_x) x_+ &= -2\gamma_\pi \pi_+ - 2[0 \ 1 \ 0 \ \rho] s_4 \\ &\quad + [3, 1-\beta, -2\beta] f_\beta + 2[1 \ 0 \ 0 \ 0] (\mathcal{I}_{hx} - \beta h_x)^{-1} s_1 \end{aligned}$$

$$\begin{aligned} (1+2\gamma_x) x_+ &= -2[0 \ 1 \ 0 \ \rho] s_4 \\ &\quad - 2\gamma_\pi \left\{ Kx_+ + [(1-\alpha)\beta, \alpha\beta, 0] f_\alpha + [0 \ 0 \ 1 \ 0] (\mathcal{I}_{hx} - \alpha\beta h_x)^{-1} s_4 \right\} \\ &\quad + [3, 1-\beta, -2\beta] f_\beta + 2[1 \ 0 \ 0 \ 0] (\mathcal{I}_{hx} - \beta h_x)^{-1} s_1 \end{aligned}$$

$$\begin{aligned} (1+2\gamma_x) x_+ &= -2[0 \ 1 \ 0 \ \rho] s_4 - 2\gamma_\pi K x_+ \\ &\quad - 2\gamma_\pi [(1-\alpha)\beta, \alpha\beta, 0] f_\alpha - 2\gamma_\pi [0 \ 0 \ 1 \ 0] (\mathcal{I}_{hx} - \alpha\beta h_x)^{-1} s_4 \\ &\quad + [3, 1-\beta, -2\beta] f_\beta + 2[1 \ 0 \ 0 \ 0] (\mathcal{I}_{hx} - \beta h_x)^{-1} s_1 \end{aligned}$$

$$(1+2\gamma_x + 2\gamma_\pi K) x_+ = \text{RHS}$$

$$x_t = -\frac{3}{w} [0 \ 1 \ 0 \ \rho] s_t$$

$$-\frac{2\gamma_n}{w} [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha - \frac{b\gamma_n}{w} [0 \ 0 \ 1 \ 0] (I_{hx} - \alpha\beta h_x)^{-1} s_t \\ + \frac{1}{w} [3, 1-\beta, -2\beta] f_\beta + \frac{b}{w} [1 \ 0 \ 0 \ 0] (I_{hx} - \beta h_x)^{-1} s_t$$

$$x_t = -\frac{2\gamma_n}{w} [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + \frac{1}{w} [3, 1-\beta, -2\beta] f_\beta$$

$$+ \left\{ -\frac{b\gamma_n}{w} [0 \ 0 \ 1 \ 0] (I_{hx} - \alpha\beta h_x)^{-1} + \frac{b}{w} [1 \ 0 \ 0 \ 0] (I_{hx} - \beta h_x)^{-1} \right.$$

$$\left. -\frac{3}{w} [0 \ 1 \ 0 \ \rho] \right\} s_t$$

x^* using new method

Let's compare old and new:

$$x_t = -\frac{2\gamma_n}{w} [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + \frac{1}{w} [3(1-\beta\gamma_n), (1-\beta-2\beta\gamma_n), 0] f_\beta$$

$$+ \left[-\frac{2\gamma_n}{w} [0 \ 0 \ 1 \ 0] (I_{hx} - \alpha\beta h_x)^{-1} + \left(-\frac{b}{w} \right) [-1 \ 1 \ 0 \ \rho] (I_{hx} - \beta h_x)^{-1} \right] s_t$$

the second thing is this
matrix on shores

clearly one diff is
the loadings on f_β

$$x_t = -\frac{2\gamma_n}{w} [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + \frac{1}{w} [3, 1-\beta, -2\beta] f_\beta$$

$$+ \left\{ -\frac{b\gamma_n}{w} [0 \ 0 \ 1 \ 0] (I_{hx} - \alpha\beta h_x)^{-1} + \frac{b}{w} [1 \ 0 \ 0 \ 0] (I_{hx} - \beta h_x)^{-1} \right.$$

$$\left. -\frac{3}{w} [0 \ 1 \ 0 \ \rho] \right\} s_t$$

I'm thinking right now that the two systems should be different but give rise to the same dynamics.

Another way to summarize the two methods is:

Old method: sub in i_T to reduce the system to

$$M \begin{bmatrix} \pi_T \\ x_T \end{bmatrix} = N \quad (M \text{ } 2 \times 2, N \text{ } 2 \times 1)$$

New method: don't sub in i_T , just write

$$P \begin{bmatrix} \pi_T \\ x_T \\ i_T \end{bmatrix} = Q \quad (P \text{ } 3 \times 3, Q \text{ } 3 \times 1)$$

I think maybe this way of writing makes clear that the two approaches might treat shocks and expectations differently, for which reason the old method may still be preferable.

If this is so, then I should still be able to write a general solution procedure using M and N.

general method based on the old method
(M-N method)

Plugging in the TR-rule, from System (t) we have:

$$(1 + \beta \gamma_x) x_t = -\beta \gamma_n \pi_t + [2(1-\beta \gamma_n), (1-\beta - 2\beta \gamma_x), 0] f_\beta \\ - \beta [-1 \ 1 \ 0 \ \rho] (I_{hx} - \beta h_x)^{-1} s_t$$

$$\pi_t = K x_t + [(1-\alpha)\beta, \alpha\beta, 0] f_\alpha + [0 \ 0 \ 1 \ 0] (I_{hx} - \alpha\beta h_x)^{-1} s_t$$

④

$$\begin{bmatrix} \gamma_n & 1 + \beta \gamma_x \\ 1 & -K \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} \underbrace{[2(1-\beta \gamma_n), (1-\beta - 2\beta \gamma_x), 0]}_{CXB} f_\beta & \underbrace{-\beta [-1 \ 1 \ 0 \ \rho] (I_{hx} - \beta h_x)^{-1} s_t}_{CXS} \\ \underbrace{[(1-\alpha)\beta, \alpha\beta, 0]}_{CPA} f_\alpha + \underbrace{[0 \ 0 \ 1 \ 0] (I_{hx} - \alpha\beta h_x)^{-1} s_t}_{CPS} \end{bmatrix}$$

Or, more compactly,

$$\underbrace{\begin{bmatrix} \gamma_n & 1 + \beta \gamma_x \\ 1 & -K \end{bmatrix}}_M \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \underbrace{\begin{bmatrix} CXB f_\beta + CXS \cdot s_t \\ CPA f_\alpha + CPS \cdot s_t \end{bmatrix}}_N$$

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = M^{-1} N$$

Mathematica
mathematica12.nb
"Resist12Dec2015"

$$\rightarrow i_t = \gamma_n \cdot \pi^* + \gamma_x \cdot x^* + \underbrace{[0 \ 1 \ 0 \ \rho]}_{CIS} s_t$$

\rightarrow get g_{xy} as coeff of x on y for $x, y = \pi, x, i$

Alright, then if we have π_{t-1} in the TR, then using method M-N,

$$i_t = \gamma_\pi \pi_{t-1} + \gamma_x x_t + \rho i_{t-1} + \bar{i}_t$$

$$i_t = \gamma_x x_t + \underbrace{\begin{bmatrix} 0 & 1 & 0 & \rho & \gamma_\pi \end{bmatrix}}_{\text{new } C_{is}} s_t \quad \downarrow s \times 1$$

$$x_t = -2(\gamma_x x_t + C_{is}^{\text{new}} s_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} + 3\pi_{T+1} - 3\beta i_{T+1} + 3r_T^n \right]$$

$$\text{while } \pi_t = k x_t + \underbrace{\begin{bmatrix} (1-\alpha)\beta & \alpha\beta & 0 \end{bmatrix}}_{C_{pa} (\text{undamaged})} f_\alpha + \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}}_{\text{new } C_{ps}} (I_{nx} - \alpha\beta h_x)^{-1} s_t$$

$$\text{So } (1+3\gamma_x)x_t = \rho i_{t-1} + \bar{i}_t + \gamma_\pi \pi_{t-1}$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} + 3\pi_{T+1} - 3\beta \left[\gamma_x k_{T+1} + \bar{i}_{T+1} + \gamma_\pi \pi_{T+1} \right] + 3r_T^n \right]$$

$$(1+3\gamma_x)x_t = \hat{E} \sum_{T=1}^{\infty} \beta^{T-t} \left[(1-\beta-3\beta\gamma_x) x_{T+1} + 3\pi_{T+1} - 3 \left[\bar{i}_T + \rho i_{T-1} + \gamma_\pi \pi_{T-1} - r_T^n \right] \right]$$

$$= \underbrace{\begin{bmatrix} 3 & 1-\beta-3\beta\gamma_x & 0 \end{bmatrix}}_{\text{new } C_{xb}} f_\beta - \underbrace{3[-1, 1, 0, \rho, \gamma_\pi]}_{\text{new } C_{xs}} (I_{nx} - \beta h_x)^{-1} s_t$$

$$(1+3\gamma_x)x_t = C_{xb} f_\beta + C_{xs} \cdot s_t$$

$$\pi_t = k x_t + C_{pa} \cdot f_\alpha + C_{ps} \cdot s_t$$

$$(1-\beta\gamma_x)x_t = c_{xb} f_\beta + c_{xs} \cdot s_t$$

$$\pi_t = Kx_t + c_{pa} \cdot f_\alpha + c_{ps} \cdot s_t$$

$$\Rightarrow \underbrace{\begin{bmatrix} 0 & 1-\beta\gamma_x \\ 1 & -K \end{bmatrix}}_M \underbrace{\begin{bmatrix} \pi_t \\ x_t \end{bmatrix}}_{(\text{new})} = \underbrace{\begin{bmatrix} c_{xb} f_\beta + c_{xs} \cdot s_t \\ c_{pa} \cdot f_\alpha + c_{ps} \cdot s_t \end{bmatrix}}_N \quad (\text{new})$$

$$\text{And also new is } i_t = \gamma_x \cdot x_t^* + c_{is} \cdot s_t$$

Erm - IRFs are explosive... but at the same time, when I look at the dd sol, that sure don't look good!

↳ but it has a grain of truth... b/c at a certain point the lagged π becomes sthg we need to expect (here) - Let's reconsider...

$$i_t = \gamma_\pi \pi_{t-1} + \gamma_x x_t + \rho i_{t-1} + \bar{i}_t$$

$$\pi_t = Kx_t + \underbrace{[(1-\alpha)\beta, K\alpha\beta, 0]}_{c_{pa}(\text{undamaged})} f_\alpha + \underbrace{[00100]}_{\text{new } c_{ps}} (I_w - \alpha\beta Kx)^{-1} s_t$$

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} + \beta \pi_{T+1} - \beta \rho i_{T+1} + \beta r_T^n \right]$$

$$x_t = -\beta(\gamma_\pi \pi_{t-1} + \gamma_x x_t + \rho i_{t-1} + \bar{i}_t) \\ + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(\gamma - \beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta i_{T+1} + \beta r_T^n \right]$$

$$(1+\beta \gamma_x) x_t = -\beta(\gamma_\pi \pi_{t-1} + \rho i_{t-1} + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(\gamma - \beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta [\gamma_\pi \pi_T + \gamma_x x_{T+1} + \rho i_T + \bar{i}_{T+1}] + \beta r_T^n \right]$$

④

$$(1+\beta \gamma_x) x_t = -\beta(\gamma_\pi \pi_{t-1} + \rho i_{t-1} + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(\gamma - \beta - \beta \gamma_x) x_{T+1} + \beta \pi_{T+1} - \beta \beta [\gamma_\pi \pi_T + \rho i_T + \bar{i}_{T+1}] + \beta r_T^n \right]$$

⑤

$$(1+\beta \gamma_x) x_t = \hat{E} \sum_{T=t}^{\infty} \beta^{T-t} \left[(\gamma - \beta - \beta \gamma_x) x_{T+1} + \beta \pi_{T+1} - \beta [\underbrace{\gamma_\pi \pi_{T-1} + \rho i_{T-1} + \bar{i}_T + r_T^n}_{\text{known}}] \right]$$

here the questions
emerge, and they
emerge for i_{T-1} as well!

$$\hat{E} \sum_{T=t}^{\infty} \beta^{T-t} [\gamma_\pi \pi_{T-1} + \rho i_{T-1}]$$

$$= \underbrace{(\gamma_\pi \pi_{t-1} + \rho i_{t-1})}_{\text{known}} + \beta (\gamma_\pi \pi_t + \rho i_t) + \underbrace{\beta^2 (\gamma_\pi \pi_{t+1} + \rho i_{t+1})}_{\text{unknown}} \dots$$

$$= \text{known stuff} + \beta^2 \left[\hat{E} \sum_{T=t}^{\infty} \beta^{T-t} (\gamma_\pi \pi_{T+1} + \rho i_{T+1}) \right]$$

This is the problem!

In fact it hints at the fact that even the model w/
lagged- i is wrong b/c it doesn't take into account
that i is an endogenous state \rightarrow agents only
know the LOM of exog states. I think! -right?
agents should only know exog states? or? But only
knowing exog states amounts to ... not knowing a
subset of hx ... omg... but I think that's correct
b/c looking at the lag- i model, the bottom part of
 hx (the row pertaining to i_{t-1}) is just a combo
of elements of gx ... it wouldn't be consistent
w/ learning gx if agents actually knew it in
 hx ...

Look: materials6, p.2 (eq 16): $hx(4;:) = gx(4,:)$.
 \Rightarrow so we need indeed to account for $hx(4,:)$
not known!

It shouldn't really matter for results so far b/c
we've in general set $p=0$, shutting off i_{t-1} . But need to
correct!

So taking another step back ... model with lagged i...

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^T \left\{ (\alpha - \beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta i_{T+1} + \beta r_T^N \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \left\{ \kappa \alpha \beta x_{T+1} + (1-\alpha) \beta \pi_{T+1} + u_T \right\}$$

$$i_t = \gamma_\pi \pi_t + \gamma_x x_t + \rho i_{t-1} + \bar{i}_t$$

$$(1 + \beta \gamma_x) x_t = -\beta \left[\gamma_\pi \pi_t + \rho i_{t-1} + \bar{i}_t \right]$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (\alpha - \beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta [\gamma_\pi \pi_{T+1} + \gamma_x x_{T+1} + \rho i_T + \bar{i}_{T-1}] + r_T^N \right\}$$

⇒

$$(1 + \beta \gamma_x) x_t = -\beta \left[\gamma_\pi \pi_t + \rho i_{t-1} + \bar{i}_t \right]$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (\alpha - \beta - \beta \gamma_\pi) x_{T+1} + \beta (1 - \beta \gamma_\pi) \pi_{T+1} - \beta \beta [\rho i_T + \bar{i}_{T-1}] + r_T^N \right\}$$

$$(1 + \beta \gamma_x) x_t = -\beta \gamma_\pi \pi_t - \beta \rho i_{t-1}$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (\alpha - \beta - \beta \gamma_\pi) x_{T+1} + \beta (1 - \beta \gamma_\pi) \pi_{T+1} - \beta \beta \rho i_T - \beta [-r_T^N + \bar{i}_T] \right\}$$

↑ now I'm worried
that this gives us
an endless
recursion ...

But: as the simulation code

makes clear [e.g. sim_teamH.m], when $\hat{E}_t z_{t+s}$
is formed, z_t is not realized yet - does that help us?

Not really, b/c x_t, π_t aren't either. No - but no prob, just
pull it out!

$$(1-\beta Y_X)X_t = -\beta Y_{t+1} \pi_t - \beta p_{t+1}$$

$$+ \hat{E} \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta - 2\beta Y_X) X_{T+1} + \beta(1-\beta Y_T) \pi_{T+1} - \beta p_{T+1} - \beta [-r_T^N + i_T] \right\}$$

$$\Rightarrow (1-\beta Y_X)X_t = -\beta Y_{t+1} \pi_t - \beta p_{t+1} - \beta \beta p_{t+1}$$

$$+ \hat{E} \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta - 2\beta Y_X) X_{T+1} + \beta(1-\beta Y_T) \pi_{T+1} - \beta^2 p_{T+1} - \beta [-r_T^N + i_T] \right\}$$

$$\text{b/c } E \sum \beta^{T-t} \beta p_{t+1} = \beta p_{t+1} + \beta^2 i_{t+1} + \beta^3 i_{t+2} + \dots \\ = \beta p_{t+1} + \beta^2 \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1}$$

Now the problem is that by the same token that not
subbing i out of f_β before was wrong, it still should
be wrong...

What to do now?

I think that I need to return to the

29 Dec 2019

simplest model and the question of M-N method vs PQ-
method (old vs. new) and how these relate to ass's
about what is known and what is learned.

A generic representation of the model is the state-space
form:

$$X_t = h_x \cdot X_{t-1} + \eta_t E_t$$

$$Y_t = g_x \cdot X_t \quad (\text{caps denote vectors})$$

As long as all X exog, agents think

$$X_t = h_x \cdot X_{t-1} + \eta \varepsilon_t \quad (\text{const})$$

$$Y_t = \hat{a} + \hat{g} \cdot X_t \Leftrightarrow Y_t = \hat{p} X_t$$

This representation is the solution to

$$E_t(f_x X_t + f_y Y_t + f_{xp} X_{t+1} + f_{yp} Y_{t+1}) = 0$$

(possibly w/ many signs somewhere.)

So if the agents know f_x, f_y, f_{xp}, f_{yp} , which they should given that these are loglin FOCs, then why can't they just do LU-decomp to solve the model & find g_x ? Although right now I'm not sure they know the f_s b/c even though they obtain the same FOCs they don't know that others did so too.

But that's what I don't get: do they then know the NKPC, the NKIS, the TR? More generally, do they know the relationships between the elements of Y ?

Right now I think no - when I work w/ PC, IS, TR, I'm working w/ agg rules that govern dynamics, but that

doesn't mean that agents know those rules.
 But that should imply that the PQ - method (the new)
 should be the same ... unless it embodies an
 expectational assumption somewhere that I don't see yet.

Ignore shocks for a moment:

The most basic system is:

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=1}^{\infty} \beta^T \left\{ (1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \pi_{T+1} \right\}$$

$$\pi_t = kx_t + \hat{E}_t \sum_{T=1}^{\infty} (\alpha \beta)^{T-t} \left\{ \kappa \alpha \beta x_{T+1} + (1-\alpha) \beta \pi_{T+1} \right\}$$

$$i_t = \gamma_{\pi} \pi_t + \gamma_x x_t$$

The only way there can be a difference between
 the two methods is if the computer doesn't understand
 that $\hat{E}_t i_{T+1} = \hat{E}_t (\gamma_{\pi} \pi_T + \gamma_x x_{T+1})$

$$x_t + \beta i_t = [2 - 1 - \beta - \beta] f_{\beta}$$

30 Dec 2015

$$\pi_t - kx_t = [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_{\alpha}$$

$$-\gamma_{\pi} \pi_t - \gamma_x x_t + i_t = 0$$

$$\begin{bmatrix} 0 & 1 & \beta \\ 1 & -k & 0 \\ -\gamma_{\pi} & -\gamma_x & 0 \end{bmatrix}_{3 \times 3} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix}_{3 \times 1} = \begin{bmatrix} [2, 1 - \beta, -\beta] f_{\beta} \\ [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_{\alpha} \\ 0 \end{bmatrix}_{3 \times 1}$$

vs old method

$$X_+ = -\beta (\gamma_{\pi} \pi_+ + \gamma_x x_+)$$

$$+ \hat{E}_+ \sum_{T=1}^{\infty} \beta^T \left\{ (1-\beta) X_{T+1} + \beta \pi_{T+1} - \beta \beta (\gamma_{\pi} \pi_{T+1} + \gamma_x x_{T+1}) \right\}$$

$$\pi_+ - k x_+ = [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha$$

$$2\gamma_{\pi} \pi_+ + (1+2\gamma_x) x_+ = \hat{E} \left\{ (1-\beta - 2\beta\gamma_x) X_{T+1} + (2 - 2\beta\gamma_{\pi}) \pi_{T+1} \right\}$$

So

$$2\gamma_{\pi} \pi_+ + (1+2\gamma_x) x_+ = [2(1-\beta\gamma_{\pi}), (1-\beta-2\beta\gamma_x), 0] f_\beta$$

$$\pi_+ - k x_+ = [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha$$

$$\begin{bmatrix} 2\gamma_{\pi}, & 1+2\gamma_x \\ 1 & -k \end{bmatrix} \begin{bmatrix} \pi_+ \\ x_+ \end{bmatrix} = \begin{bmatrix} [2(1-\beta\gamma_{\pi}), (1-\beta-2\beta\gamma_x), 0] f_\beta \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha \end{bmatrix}$$

So

$$\begin{bmatrix} \pi_+ \\ x_+ \end{bmatrix} = \begin{bmatrix} 2\gamma_{\pi} & 1+2\gamma_x \\ 1 & -k \end{bmatrix}^{-1} \begin{bmatrix} [2(1-\beta\gamma_{\pi}), (1-\beta-2\beta\gamma_x), 0] f_\beta \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha \end{bmatrix} (M-N)$$

vs.

$$\begin{bmatrix} \pi_+ \\ x_+ \\ i_+ \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -k & 0 \\ -\gamma_{\pi} & -\gamma_x & 0 \end{bmatrix}^{-1} \begin{bmatrix} [2, 1-\beta, -2\beta] f_\beta \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha \\ 0 \end{bmatrix} (P-Q)$$

In Mathematica (matroids12.nb) $\gamma = \gamma_\pi$ and $\delta = \delta_\pi$

$$\begin{bmatrix} \pi_+ \\ x_+ \\ i_+ \end{bmatrix} = \underbrace{\begin{bmatrix} 2\gamma & 1+\beta\delta \\ 1 & -k \end{bmatrix}}_{=: M}^{-1} \underbrace{\begin{bmatrix} [2(1-\beta\gamma), (1-\beta-\beta\delta), 0] f_\beta \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha \\ 0 \end{bmatrix}}_{=: N} (M-N)$$

vs.

$$\begin{bmatrix} \pi_+ \\ x_+ \\ i_+ \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & \gamma \\ 1 & -k & 0 \\ -\gamma & -\delta & 0 \end{bmatrix}}_{=: P}^{-1} \underbrace{\begin{bmatrix} [2, 1-\beta, -\beta\delta] f_\beta \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha \\ 0 \end{bmatrix}}_{=: Q} (P-Q)$$

②

$$\begin{bmatrix} \pi_+ \\ x_+ \\ i_+ \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{k}{1+\beta\delta+k\beta\gamma} & \frac{1+\beta\delta}{1+\beta\delta+k\beta\gamma} \\ \frac{1}{1+\beta\delta+k\beta\gamma} & \frac{-\beta\gamma}{1+\beta\delta+k\beta\gamma} \end{bmatrix}}_{=: M}^{-1} \underbrace{\begin{bmatrix} [2(1-\beta\gamma), (1-\beta-\beta\delta), 0] f_\beta \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha \\ 0 \end{bmatrix}}_{=: N} (M-N)$$

vs.

$$\begin{bmatrix} \pi_+ \\ x_+ \\ i_+ \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{-\beta\delta}{-\delta\beta-k\beta\gamma} & \frac{k\beta}{-\delta\beta-k\beta\gamma} \\ 0 & \frac{\beta\gamma}{-\delta\beta-k\beta\gamma} & \frac{\beta}{-\delta\beta-k\beta\gamma} \\ \frac{-\beta-k\gamma}{-\delta\beta-k\beta\gamma} & \frac{-\gamma}{-\delta\beta-k\beta\gamma} & \frac{-1}{-\delta\beta-k\beta\gamma} \end{bmatrix}}_{=: P}^{-1} \underbrace{\begin{bmatrix} [2, 1-\beta, -\beta\delta] f_\beta \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha \\ 0 \end{bmatrix}}_{=: Q} (P-Q)$$

Let's solve the following, very stupid system:

$$x_+ = -3\pi_+$$

$$\pi_+ = kx_+$$

$$\pi_+ = 4\pi_+ + \delta x_+$$

M-N method: $x_+ = -2(4\pi_+ + \delta x_+) = -34\pi_+ - 2\delta x_+$

$$\pi_+ = kx_+$$

Hmm. Maybe the problem w/ the P-Q method is that
the TR has a 0 RHS. Look:

$$34\pi_+ + (1+2\delta)x_+ = 0$$

$$\pi_+ - kx_+ = 0$$

$$\begin{bmatrix} 34, & 1+2\delta \\ 1 & -k \end{bmatrix} \begin{bmatrix} \pi_+ \\ x_+ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \pi_+ \\ x_+ \end{bmatrix} = \begin{bmatrix} -k & -(1+2\delta) \\ -1 & 34 \end{bmatrix} \begin{bmatrix} 1 \\ -2\pi k - (1+2\delta) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This is a homogeneous system! (RHS vector is the null)

A homo system has 1 sol or ∞ , the latter if # vars >
of eqs. So the fa & fb act like a nonhomogen part!

consider the following system:

X

$$x_+ = -3\pi_+ + a$$

$$\pi_+ = Kx_+ + b$$

$$i_+ = \gamma\pi_+ + \delta x_+ + c$$

Now I bet M-N & P-Q yield the same sol:

$$x_+ = -3\gamma\pi_+ - 3\delta x_+ - 3c + a$$

$$\pi_+ = Kx_+ + b$$

$$\Rightarrow 2\gamma\pi_+ + (1+3\delta)x_+ = -3c + a$$

$$\pi_+ - Kx_+ = b$$

$$\begin{bmatrix} 2\gamma & 1+3\delta \\ 1 & -K \end{bmatrix} \begin{bmatrix} \pi_+ \\ x_+ \end{bmatrix} = \begin{bmatrix} -3c + a \\ b \end{bmatrix}$$

$$\begin{bmatrix} \pi_+ \\ x_+ \end{bmatrix} = \begin{bmatrix} 2\gamma & 1+3\delta \\ 1 & -K \end{bmatrix}^{-1} \begin{bmatrix} -3c + a \\ b \end{bmatrix} \quad (M-N)$$

$$\begin{aligned} x_+ &= -3\pi_+ + a \\ \pi_+ &= kx_+ + b \\ i_+ &= \varphi \pi_+ + \delta x_+ + c \end{aligned} \Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 1 & -k & 0 \\ -\varphi & -\delta & 1 \end{bmatrix} \begin{bmatrix} \pi_+ \\ x_+ \\ i_+ \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} \pi_+ \\ x_+ \\ i_+ \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -k & 0 \\ -\varphi & -\delta & 1 \end{bmatrix}^{-1} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad (P-Q)$$

You can see in Mathematica that the two sols are the same! (I've checked for π & x , for i it's not so obvious.)

If you set $c=0$, you still get $MN = PQ$.

So this homo is non-homo thing isn't the only.

Plus w/ shadows the TR is also non-homo anyway.

So it has to do w/ expectations

31 Dec 2019

The most basic system again: supp μ_3 doesn't ex

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=1}^{t+1} \beta^T \left\{ (1-\beta)x_{T+1} + \beta \pi_{T+1} - \cancel{\beta i_{T+1}} \right\}$$

$$\pi_t = kx_t + \hat{E}_t \sum_{T=1}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} \right\}$$

$$i_t = \gamma_{\pi} \pi_t + \gamma_x x_t$$

Now I bet that the two methods give the same sol

$$x_t = -\beta(\gamma_{\pi} \pi_t + \gamma_x x_t) + [\beta, 1-\beta, 0] f \beta$$

$$\pi_t = kx_t + [(1-\alpha)\beta, \kappa\alpha\beta, 0] f \alpha$$

$$\begin{bmatrix} \gamma_{\pi} & (1-\gamma_x) \\ 1 & -k \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [\beta, 1-\beta, 0] f \beta \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f \alpha \end{bmatrix} \quad (M-N)$$

$$\begin{bmatrix} 0 & 1 & \beta \\ 1 & -k & 0 \\ -\gamma_{\pi} & -\gamma_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\beta, 1-\beta, 0] f \beta \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f \alpha \\ 0 \end{bmatrix} \quad (P-Q)$$

Note that this corresponds to the RHS of $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ w/ $c=0$

and we've already seen that in that case the sols

$(M-N)$ & $(P-Q)$ correspond.

I think this causes a problem b/c it's as if you changed the independent RHS in $AB = C$.

What this suggests tho is that

$$\hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \gamma \pi_{T+1} - \gamma \beta i_{T+1} \right\} \neq$$

$$\hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \gamma \pi_{T+1} - \gamma \beta (\gamma_\pi \pi_{T+1} + \gamma_x x_{T+1}) \right\}$$

⇒

$$[b, 1-\beta, -\gamma \beta] f_\beta \neq [b - \gamma \beta \gamma_\pi, 1-\beta - \gamma \beta \gamma_x, 0] f_\beta$$

$$\Leftrightarrow [b, 1-\beta, 0] f_\beta + [0, 0, -\gamma \beta] f_\beta \neq$$

$$[b, 1-\beta, 0] f_\beta + [-\gamma \beta \gamma_\pi, -\gamma \beta \gamma_x, 0] f_\beta$$

$$\Leftrightarrow [0, 0, -\gamma \beta] f_\beta \neq [-\gamma \beta \gamma_\pi, -\gamma \beta \gamma_x, 0] f_\beta \quad | : (-\gamma \beta)$$

$$\Leftrightarrow [0, 0, 1] f_\beta \neq [\gamma_\pi, \gamma_x, 0] f_\beta$$

which is weird!

No wait - it makes perfect sense that the computer doesn't realize this and this is why I get different things!

$$f_\beta(3,1) = \gamma_\pi f_\beta(1,1) + \gamma_x f_\beta(2,1)$$

this is what the computer doesn't internalize.

To do: 1) show on Mathematica that w/ this piece of info,

The M-N & P-Q sets coincide \rightarrow Yes! materials 12.nb

2) Correct general method

3) Check why on Matlab the two still don't give

the same dynamics. \rightarrow b/c shocks weren't treated correctly
1 min (condition (ii) wasn't correct yet)

2) Now take the errors back in:

2 Jan 2019

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta i_{T+1} + \beta r_T^N \right\}$$

$$\pi_t = k x_t + \hat{E}_t \sum_{T=t}^{\infty} (\beta)^{T-t} \left\{ \kappa \beta x_{T+1} + (1-\alpha) \beta \pi_{T+1} + u_T \right\}$$

$$i_t = \gamma_\pi \pi_t + \gamma_x x_t + \bar{i}_t$$

(no i -rate smoothing)

M-N method: Sub out i_t :

$$x_t = -\beta (\gamma_\pi \pi_t + \gamma_x x_t + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta (\gamma_\pi \pi_{T+1} + \gamma_x x_{T+1} + \bar{i}_{T+1}) + \beta r_T^N \right\}$$

$$(1+\beta \gamma_x) x_t = -\beta \gamma_\pi \pi_t$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \beta(1-\beta \gamma_\pi) \pi_{T+1} + (1-\beta-\beta \gamma_x) x_{T+1} - \beta \bar{i}_T + \beta r_T^N \right\}$$

$$2\gamma_\pi \pi_t + (1+\beta \gamma_x) x_t = [b(1-\beta \gamma_\pi), (1-\beta-\beta \gamma_x), 0] f_\beta - 2[-1, 1, 0] (I_{hx} - \beta h_x)^{-1} s_t$$

$$\pi_t - k x_t = [(1-\alpha)\beta, \kappa \beta, 0] f_\alpha + [0 \ 0 \ 1] (I_{hx} - \alpha \beta h_x)^{-1} s_t$$

$$\underbrace{\begin{bmatrix} 2\gamma_\pi & 1-\beta\gamma_x \\ 1 & -k \end{bmatrix}}_M \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \underbrace{\begin{bmatrix} [b(1-\beta\gamma_\pi), (1-\beta-2\beta\gamma_x), 0] f_\beta + [(-1, 1, 0)] (I_{hx} - \beta h x)^{-1} s_t \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + [0 \ 0 \ 1] (I_{hx} - \alpha\beta h x)^{-1} s_t \end{bmatrix}}_N$$

= d1 = f_{\beta c} = f_{\beta b}
= d2 = f_{\alpha a} = f_{\alpha a}

and then $i_t = \gamma_\pi \pi_t^* + \gamma_x x_t^* + \underbrace{[0 \ 1 \ 0] s_t}_{= d3}$

P-Q method

$$x_t + bi_t = [0, 1-\beta, -2\beta] f_\beta + 3[1 \ 0 \ 0] (I_{hx} - \beta h x)^{-1} s_t$$

$$\pi_t - kx_t = [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + [0 \ 0 \ 1] (I_{hx} - \alpha\beta h x)^{-1} s_t$$

$$-\gamma_\pi \pi_t - \gamma_x x_t + i_t = [0 \ 1 \ 0] s_t$$

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & -k & 0 \\ -\gamma_\pi - \gamma_x & 1 \end{bmatrix}}_P \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \underbrace{\begin{bmatrix} [0, 1-\beta, -2\beta] f_\beta + 3[1 \ 0 \ 0] (I_{hx} - \beta h x)^{-1} s_t \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + [0 \ 0 \ 1] (I_{hx} - \alpha\beta h x)^{-1} s_t \\ [0 \ 1 \ 0] s_t \end{bmatrix}}_Q$$

= d4 = f_{\beta c}
= d5 = f_{\alpha a}
= d6 = f_{\beta b}

w/ the extra condition that:

$$f_\beta(3, 1) = \gamma_\pi f_\beta(1, 1) + \gamma_x f_\beta(2, 1) + \underbrace{[0 \ 1 \ 0] (I_{hx} - \beta h x)^{-1} s_t}_{= d7}$$



The new term that's supp to account for the error term

Mathematica has some trouble here so there are 2 ways I can check whether the two sets coincide:

$$1) i_{NM}^* - i_{PQ}^* = -d5 + (12 - d4)\gamma \\ + (d1 - d3 - d7\beta)(\delta + \kappa\gamma) \\ + d6(1 + \delta\beta + \kappa\beta\gamma)$$

2) Let Mathematica give you M^{-1} and P^{-1} and calculate $i_{NM}^* - i_{PQ}^*$ analytically from these.

$$\begin{aligned} & 1) -[0 \ 0 \ 1] (I_{nx} - \alpha\beta h_x)^{-1} s_+ \\ & + \left\{ [0 \ 0 \ 1] (I_{nx} - \alpha\beta h_x)^{-1} s_+ - 2[1 \ 0 \ 0] (I_{nx} - \beta h_x)^{-1} s_+ \right\} \gamma \\ & + (\delta + \kappa\gamma) \left\{ -2[-1, 1, 0] (I_{nx} - \beta h_x)^{-1} s_+ - [0 \ 1 \ 0] s_+ \right. \\ & \quad \left. + [0 \ 1 \ 0] (I_{nx} - \beta h_x)^{-1} s_+ \right\} \\ & + (1 + \delta\beta + \kappa\beta\gamma) [0 \ 1 \ 0] s_+ \\ & = \left\{ -[0 \ 0 \ 1] (I_{nx} - \alpha\beta h_x)^{-1} - (1 + \delta\beta + \kappa\beta\gamma) [0 \ 1 \ 0] \right\} s_+ \\ & + \gamma \left\{ [0 \ 0 \ 1] (I_{nx} - \alpha\beta h_x)^{-1} - 2[1 \ 0 \ 0] (I_{nx} - \beta h_x)^{-1} \right\} s_+ \\ & + (\delta + \kappa\gamma) \left\{ [0, 1 + \beta, 0] (I_{nx} - \beta h_x)^{-1} - [0 \ 1 \ 0] \right\} s_+ \end{aligned}$$

$$\begin{aligned}
&= \left\{ -[0 \ 0 \ 1] (I_{nx} - \alpha \beta h x)^{-1} + (1 + \delta b + k b^2) [0 \ 1 \ 0] \right\} s_t \\
&+ \gamma \left\{ [0 \ 0 \ 1] (I_{nx} - \alpha \beta h x)^{-1} - b [1 \ 0 \ 0] (I_{nx} - \beta h x)^{-1} \right\} s_t \\
&+ (\delta + k \gamma) \left\{ [b, 1+b, 0] (I_{nx} - \beta h x)^{-1} - [0 \ 1 \ 0] \right\} s_t \\
&= \left\{ \frac{[0 \ 0 \ 1] (I_{nx} - \alpha \beta h x)^{-1} + [0, 1 + \delta b + k b^2, 0]}{\underline{[0 \ 0 \ \gamma]}} \right. \\
&\quad \left. + \frac{[0 \ 0 \ \gamma] (I_{nx} - \alpha \beta h x)^{-1} + [-b \ 0 \ 0] (I_{nx} - \beta h x)^{-1}}{\underline{[b(1+k\gamma), (b+k\gamma)(1+b), 0]}} \right. \\
&\quad \left. + [0, -\delta - k \gamma, 0] \right\} s_t \\
&= \left\{ [0 \ 0 \ \gamma-1] (I_{nx} - \alpha \beta h x)^{-1} \right. \\
&\quad \left. + [b + b(b+k\gamma), (b+k\gamma)(1+b), 0] (I_{nx} - \beta h x)^{-1} \right. \\
&\quad \left. + [0, 1 + \delta b + k b^2 - \delta - k \gamma, 0] \right\} s_t
\end{aligned}$$

Honestly that doesn't look like it was = 0.

$$2) NM: \begin{bmatrix} n_1 \\ x_1 \end{bmatrix}_{2 \times 2} = M^{-1} N_{2 \times 1} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$n_1 = m_1 \cdot n_1 + m_2 \cdot n_2 \quad x_1 = m_3 \cdot n_1 + m_4 \cdot n_2$$

$$i_{mn}^* = \gamma_n (m_1 \cdot n_1 + m_2 \cdot n_2) + \gamma_x (m_3 \cdot n_1 + m_4 \cdot n_2) + [0 \ 1 \ 0] s_t$$

$$M^{-1} = \begin{bmatrix} \frac{k}{w} & \frac{1+\delta b}{w} \\ \frac{1}{w} & \frac{-b\gamma}{w} \end{bmatrix}$$

$$PQ = \begin{bmatrix} \bar{x}_+ \\ x_+ \\ i_+ \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}$$

$$i_{PR}^* = p_{31} \cdot Q_1 + p_{32} \cdot Q_2 + p_{33} \cdot Q_3$$

$$P^{-1} = \begin{bmatrix} \frac{k}{w} & \frac{1+\delta b}{w} & -\frac{k b}{w} \\ \frac{1}{w} & \frac{-2b}{w} & -\frac{b}{w} \\ \frac{\delta+k\gamma}{w} & \frac{\gamma}{w} & \frac{1}{w} \end{bmatrix} \quad (= M^{-1})$$

MN:

$$\begin{bmatrix} \bar{x}_+ \\ x_+ \\ i_+ \end{bmatrix} = \begin{bmatrix} \frac{k}{w} & \frac{1+\delta b}{w} \\ \frac{1}{w} & \frac{-2b}{w} \end{bmatrix} \begin{bmatrix} [b(1-\beta\gamma_\pi), (1-\beta-2\beta\gamma_\pi), 0] f_\beta - 2[-1, 1, 0] (I_{hx} - \beta h_x)^{-1} S_+ \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + [0 \ 0 \ 1] (I_{hx} - \alpha\beta h_x)^{-1} S_+ \end{bmatrix}$$

$$\begin{aligned} \bar{x}_+ &= \frac{k}{w} [b(1-\beta\gamma_\pi), (1-\beta-2\beta\gamma_\pi), 0] f_\beta - \frac{kb}{w} [-1, 1, 0] (I_{hx} - \beta h_x)^{-1} S_+ \\ &\quad + \frac{1+\delta b}{w} [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + \frac{1+\delta b}{w} [0 \ 0 \ 1] (I_{hx} - \alpha\beta h_x)^{-1} S_+ \end{aligned}$$

PQ:

$$\begin{bmatrix} \pi_4 \\ x_+ \\ i_+ \end{bmatrix} = \begin{bmatrix} \frac{k}{w} & \frac{1+\delta b}{w} & -\frac{k b}{w} \\ \frac{1}{w} & -\frac{2b}{w} & -\frac{b}{w} \\ \frac{s+k\gamma}{w} & \frac{\gamma}{w} & \frac{1}{w} \end{bmatrix} \begin{bmatrix} [0, 1-\beta, -b\beta] f_\beta + 3[1, 0, 0] (I_{nx} - \beta h x)^{-1} s_+ \\ [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_\alpha + [0, 0, 1] (I_{nx} - \alpha \beta h x)^{-1} s_+ \\ [0, 1, 0] s_+ \end{bmatrix}$$

$$\begin{aligned} \pi_4^{PQ} &= \frac{k}{w} [0, 1-\beta, -b\beta] f_\beta + \frac{k b}{w} [1, 0, 0] (I_{nx} - \beta h x)^{-1} s_+ \\ &\quad + \frac{1+\delta b}{w} [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_\alpha + \frac{1+\delta b}{w} [0, 0, 1] (I_{nx} - \alpha \beta h x)^{-1} s_+ \\ &\quad - \frac{k b}{w} [0, 1, 0] s_+ \end{aligned}$$

Compare w/ π_{MN} :

$$\begin{aligned} \pi_4^{MN} &= \frac{k}{w} [0(1-\beta \gamma_\alpha), (1-\beta-b\beta \gamma_\lambda), 0] f_\beta - \frac{k b}{w} [-1, 1, 0] (I_{nx} - \beta h x)^{-1} s_+ \\ &\quad + \frac{1+\delta b}{w} [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_\alpha + \frac{1+\delta b}{w} [0, 0, 1] (I_{nx} - \alpha \beta h x)^{-1} s_+ \end{aligned}$$

$$\begin{aligned} \pi_4^{PQ} - \pi_4^{MN} &= \frac{k}{w} [0, 1-\beta, -b\beta] f_\beta + \frac{k b}{w} [1, 0, 0] (I_{nx} - \beta h x)^{-1} s_+ \\ &\quad - \frac{k b}{w} [0, 1, 0] s_+ \end{aligned}$$

$$\begin{aligned} &\quad - \left\{ \frac{k}{w} [0(1-\beta \gamma_\alpha), (1-\beta-b\beta \gamma_\lambda), 0] f_\beta - \frac{k b}{w} [-1, 1, 0] (I_{nx} - \beta h x)^{-1} s_+ \right\} \\ &= \frac{k}{w} [0, 1-\beta, -b\beta] f_\beta + \frac{k b}{w} [1, 0, 0] (I_{nx} - \beta h x)^{-1} s_+ - \frac{k b}{w} [0, 1, 0] s_+ \\ &\quad - \left\{ \frac{k}{w} [0, 1-\beta, 0] f_\beta + \frac{k}{w} [-3\beta \gamma_\alpha, -3\beta \gamma_\lambda, 0] f_\beta + \frac{k b}{w} [1, 0, 0] (I_{nx} - \beta h x)^{-1} s_+ \right. \\ &\quad \left. + \frac{k b}{w} [0, 1, 0] (I_{nx} - \beta h x)^{-1} s_+ \right\} \end{aligned}$$

$$\begin{aligned}
&= \cancel{\frac{k}{w} [0, 1, \beta, 0] f_B} + \frac{k\beta}{w} [1, 0, 0] (\mathbb{I}_{nx} - \beta h x)^{-1} s_+ - \frac{k\beta}{w} [0, 1, 0] s_+ \\
&\quad + \frac{k}{w} [0, 0, -2\beta] f_B \\
&\quad - \left\{ \cancel{\frac{k}{w} [0, 1 - \beta, 0] f_B} + \frac{k}{w} [-2\beta \gamma_n, -2\beta \gamma_x, 0] f_B + \cancel{\frac{k\beta}{w} [1, 0, 0] (\mathbb{I}_{nx} - \beta h x)^{-1} s_+} \right. \\
&\quad \left. - [0, -1, 0] (\mathbb{I}_{nx} - \beta h x)^{-1} s_+ \right\} \\
&= -\frac{k\beta}{w} [0, 1, 0] s_+ + \cancel{\frac{k}{w} [0, 0, -2\beta] f_B} - \left\{ \cancel{\frac{k}{w} [-2\beta \gamma_n, -2\beta \gamma_x, 0] f_B} \right. \\
&\quad \left. + \frac{k\beta}{w} [0, -1, 0] (\mathbb{I}_{nx} - \beta h x)^{-1} s_+ \right\} \\
&= -\frac{k\beta}{w} [0, 1, 0] s_+ - \left\{ + \frac{k\beta}{w} [0, -1, 0] (\mathbb{I}_{nx} - \beta h x)^{-1} s_+ \right\} \\
&\quad + \frac{k}{w} (-2\beta) [0, 1, 0] (\mathbb{I}_{nx} - \beta h x)^{-1} s_+
\end{aligned}$$

Recall that from (X)

$$f_B(3,1) = \gamma_n f_B(1,1) + \gamma_x f_B(2,1) + [0, 1, 0] (\mathbb{I}_{nx} - \beta h x)^{-1} s_+$$

which is where the last term comes from.

Take the last line, divide by $\frac{k\beta}{w}$

$$-\tilde{i}_t + \frac{1}{1 - \beta p_i} \tilde{i}_t - \frac{\beta}{1 - \beta p_i} \tilde{i}_t = \frac{[-(1 - \beta p_i) + 1 - \beta]}{1 - \beta p_i} \tilde{i}_t$$

$$= \frac{\beta p_i - \beta}{1 - \beta p_i} \tilde{i}_t = \frac{\beta(p_i - 1)}{1 - \beta p_i} \tilde{i}_t \quad \text{that should ain't 0.}$$

Let's compare the 2 systems:

MN

$$\begin{bmatrix} \pi_+ \\ x_+ \\ z_+ \end{bmatrix} = \begin{bmatrix} \frac{k}{w} & \frac{1+\beta b}{w} & -\frac{k^2}{w} \\ \frac{1}{w} & -\frac{\alpha b}{w} & -\frac{b}{w} \end{bmatrix} \begin{bmatrix} [b(1-\beta Y_\pi), (1-\beta-\alpha b)Y_\pi, 0] f_\beta + 3[1, 1, 0] (I_{hx} - \beta h_x)^{-1} s_+ \\ [(1-\alpha)\beta, \alpha b \beta, 0] f_\alpha + [0, 0, 1] (I_{hx} - \alpha b h_x)^{-1} s_+ \end{bmatrix}$$

PQ

$$\begin{bmatrix} \pi_+ \\ x_+ \\ z_+ \end{bmatrix} = \begin{bmatrix} \frac{k}{w} & \frac{1+\beta b}{w} & -\frac{k^2}{w} \\ \frac{1}{w} & -\frac{\alpha b}{w} & -\frac{b}{w} \\ \frac{s_+ k Y}{w} & \frac{w}{w} & \frac{1}{w} \end{bmatrix} \begin{bmatrix} [b, 1-\beta, -\beta b] f_\beta + 3[1, 0, 0] (I_{nx} - \beta h_x)^{-1} s_+ \\ [(1-\alpha)\beta, \alpha b \beta, 0] f_\alpha + [0, 0, 1] (I_{nx} - \alpha b h_x)^{-1} s_+ \\ [0, 1, 0] s_+ \end{bmatrix}$$

(copy w) changes: (show only stuff for π & x that are different)

$$\begin{bmatrix} \pi_+ \\ x_+ \end{bmatrix} = \begin{bmatrix} \frac{k}{w} & \cdot \\ \cdot & \cdot \end{bmatrix} \left[\begin{array}{l} [b, 1-\beta, 0] f_\beta + [-\beta b Y_\pi, -\beta b Y_\pi, 0] f_\beta + 3[1, 0, 0] (I_{nx} - \beta h_x)^{-1} s_+ \\ + 3[0, -1, 0] (I_{nx} - \beta h_x)^{-1} s_+ \end{array} \right]$$

$$\begin{bmatrix} \pi_+ \\ x_+ \\ z_+ \end{bmatrix} = \begin{bmatrix} \frac{k}{w} & \cdot & -\frac{k^2}{w} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \left[\begin{array}{l} [b, 1-\beta, 0] f_\beta + [0, 0, -\beta b] f_\beta + 3[1, 0, 0] (I_{nx} - \beta h_x)^{-1} s_+ \\ \cdot \\ [0, 1, 0] s_+ \end{array} \right]$$

So what we really need is

$$-\frac{k b \beta}{w} [Y_\pi, Y_\pi, 0] f_\beta + \frac{k^2}{w} [0, -1, 0] (I_{nx} - \beta h_x)^{-1} s_+$$

$$\stackrel{!}{=} -\frac{k b \beta}{w} [0, 0, 1] f_\beta + \frac{k^2}{w} [0, -1, 0] s_+$$

which is exactly what I obtained before.

Can we rewrite this using tricks on the sums such that it works?

$$\begin{aligned}
 -\bar{i}_t + \frac{1}{1-\beta\rho_i} \bar{i}_t - \beta \bar{i}_{t+1} &= -\bar{i}_t + \sum_{T=t}^{\infty} (\beta\rho_i)^{T-t} \bar{i}_T + \sum_{T=t}^{\infty} (\beta\rho_i)^{T-t} \beta \bar{i}_{T+1} \\
 &= \sum_{T=t}^{\infty} \left[(\beta\rho_i)^{T-t} \bar{i}_T \right] - \bar{i}_t + \sum_{T=t+1}^{\infty} \left[\beta^{T-t+1} \rho_i^{T-t} \bar{i}_T \right] \\
 &= \bar{i}_t + \beta\rho_i \bar{i}_{t+1} + (\beta\rho_i)^2 \bar{i}_{t+2} + \dots \\
 &\quad - \bar{i}_t \\
 &\quad + \beta \bar{i}_t + \beta^2 \rho_i \bar{i}_{t+1} + \beta^3 \rho_i^2 \bar{i}_{t+2} + \dots
 \end{aligned}$$

Ah wait... can it be that the $f_p(3,1)$ -term only is i_{t+1} onward while the last element of Q has the t -term?

Exactly! Look: Preston 2005, p. 40 (mac)

$$f_{\beta} := \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} z_{T+1} \text{ which is } \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \bar{i}_{T+1}$$

And like her, I see that that works

3 Jan 2019

except that somehow I have a β too many!

unless $(*)$ doesn't treat the errors quite right!

If $(*)$ had to divide errors by β , will be fine.

(*) was

$$f_{\beta}(3,1) = \gamma_n f_{\beta}(1,1) + \gamma_x f_{\beta}(2,1) + [0 \ 1 \ 0] (I_{nx} - \beta h_x)^{-1} s_+$$

But here the errors are $\sum_{T=1}^{\infty} (\beta \rho_i)^{T-t} \bar{i}_t$

while they should be $\sum_{T=t+1}^{\infty} \beta^{T-t} \rho_i^{T-t+1} \bar{i}_{T+1}$ since they are future expectations

$$= \frac{1}{\beta} \sum_{T=1}^{\infty} (\beta \rho_i^{T-t+1}) \bar{i}_{T+1}$$

$$= \frac{1}{\beta} \left[\beta \rho_i i_{t+1} + (\beta \rho_i)^2 i_{t+2} + \dots \right]$$

$$- \frac{1}{\beta} \left[\underbrace{[0 \ 1 \ 0] (I_{nx} - \beta h_x)^{-1} s_+ - [0 \ 1 \ 0] s_+}_{\text{The full sum minus the first term}} \right]$$

The full sum minus the first term

$y(x)$ is

$$f_{\beta}(3,1) = \gamma_n f_{\beta}(1,1) + \gamma_x f_{\beta}(2,1)$$

$$+ \frac{1}{\beta} \left[[0 \ 1 \ 0] (I_{nx} - \beta h_x)^{-1} s_+ - [0 \ 1 \ 0] s_+ \right]$$

then what we need becomes:

$$- \frac{k_1 b \beta}{w} [\gamma_n, \gamma_x, 0] f_{\beta} + \frac{k_2}{w} [0, -1, 0] (I_{nx} - \beta h_x)^{-1} s_+$$

$$\stackrel{!}{=} - \frac{k_2 \beta}{w} [0, 0, 1] f_{\beta} + \frac{k_2}{w} [0, -1, 0] s_+$$

which is

$$\begin{aligned}
 & -\frac{k_B}{w} [Y_1, Y_2, 0] f_p + \frac{k_B}{w} [0, -1, 0] (I_{nx} - \beta h_x)^{-1} s_+ \\
 & = + \frac{k_B}{w} [0, -1, 0] s_+ \\
 & - \frac{k_B p}{w} [Y_1 f_p(1,1) + Y_2 f_p(2,1)] \\
 & - \frac{k_B p}{w} \frac{1}{\beta} [[0, 1, 0] (I_{nx} - \beta h_x)^{-1} s_+ - [0, 1, 0] s_+]
 \end{aligned}$$

Yeah!!! So indeed the condition in PQ, (*) needs to be:

$$\begin{aligned}
 f_p(3,1) = & Y_1 f_p(1,1) + Y_2 f_p(2,1) \\
 & + \frac{1}{\beta} [[0, 1, 0] (I_{nx} - \beta h_x)^{-1} s_+ - [0, 1, 0] s_+]
 \end{aligned}$$

Ok, so I've found the equality between the MN-PQ methods even when shocks are present. The problem is that they are too intensive for Mathematica. So either I find a way to simplify them or I just do the MN-method. The thing is that the PQ condition (*) may change when changing the model and therefore I think that the MN-method is more robust.

So what Mathematica (materials1d.nb) now does is it calculates M^{-1} to give

$$\begin{bmatrix} \pi_+ \\ x_+ \end{bmatrix} = \begin{bmatrix} \frac{k}{\omega} & \frac{1+\beta^2}{\omega} \\ \frac{1}{\omega} & -\frac{\alpha\beta}{\omega} \end{bmatrix} \begin{bmatrix} [b(1-\beta\gamma_\pi), (1-\beta-\beta^2\gamma_\pi), 0] f_\beta + 2[1, 1, 0] (I_{hx} - \mu\omega)^{-1} s_+ \\ [(1-\alpha)\beta, \alpha\beta, 0] f_\alpha + [0, 0, 1] (I_{hx} - \alpha\beta\omega)^{-1} s_+ \end{bmatrix}$$

$= d1$
 $= d2$

which it solves for (π_+, x_+) and for i_+ as

$$i_+ = 4\pi_+^* + f \cdot x_+^* + \underbrace{[0 \ 1 \ 0]}_{= d3} s_+$$

Then it collects the g_i s and assemble

$$A_a = \begin{pmatrix} gp_1 \\ gx_1 \\ gis \end{pmatrix} \quad A_b = \begin{pmatrix} gpb \\ gx_1 \\ gib \end{pmatrix}$$

$$A_s = \begin{pmatrix} gps \\ gxs \\ gis \end{pmatrix}$$

$$\text{Check that } gis = 4gps + 5gxs - d6 \quad \checkmark$$

And checked these A -matrices against the old 4 Jan 2015 ones in materialsb.m, they are identical.

matrices - A - int rate smoothing 2.m is WRONG

matrices - A - int rate smoothing 3.m is correct as long as $\rho = 0$

To do next: check what happens to the system once we have interest rate smoothing, and then eventually $E[\pi]$ in TR or π_{t-1} in TR.

So: adding ρi_{t-1}

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta i_{T+1} + \beta r_T^n \right\}$$

$$\pi_t = k x_t + \hat{E}_t \sum_{T=t}^{\infty} (\beta)^{T-t} \left\{ \kappa \alpha \beta x_{T+1} + (1-\alpha) \beta \pi_{T+1} + u_T \right\}$$

$$i_t = \gamma_\pi \pi_t + \gamma_x x_t + \bar{i}_t + \rho i_{t-1}$$

☞

$$y_t = -\beta(\gamma_\pi \pi_t + \gamma_x x_t + \bar{i}_t + \rho i_{t-1})$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta - \beta \gamma_x) x_{T+1} + \beta(1-\beta \gamma_\pi) \pi_{T+1} - \beta \bar{i}_{T+1} - \underline{\beta \rho i_T} + \beta r_T^n \right\}$$

NKPC unchanged

A problem b/c even if

you sub the TR in again, i_{t-1} will show up and since there are ∞ terms, expectations of $\underline{i_{t+1}}$ will show up.

\Rightarrow it looks to me as if this forced us to use the PQ method, either w/ an augmented (\hat{x}) or w/ an additional linking eq.

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta i_{T+1} + \beta r_T \right\}$$

$$\pi_t = k x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \left\{ \kappa \alpha \beta x_{T+1} + (1-\alpha) \beta \pi_{T+1} + u_T \right\}$$

$$i_t = \gamma_{\pi} \pi_t + \gamma_x x_t + \hat{i}_t + \rho i_{t-1}$$

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -\kappa & 0 \\ -\gamma_{\pi} & -\gamma_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [3, 1-\beta, -\beta] f_{\beta} + b [1 \ 0 \ 0 \ 0] (I_{4x4} - \beta L_X)^{-1} S_+ \\ [(\alpha-\kappa)\beta, \kappa \alpha \beta, 0] + [0 \ 0 \ 1 \ 0] (I_{4x4} - \alpha \beta L_X)^{-1} S_+ \\ [0 \ 1 \ 0 \ \underline{\rho}] S_+ \end{bmatrix}$$

so far this is identical to the PQ method, except S_+ is 4×1 .

(\hat{x}) used to be:

$$f_{\beta}(3,1) = \gamma_{\pi} f_{\beta}(1,1) + \gamma_x f_{\beta}(2,1)$$

$$+ \frac{1}{\beta} [[0 \ 1 \ 0] (I_{4x4} - \beta L_X)^{-1} S_+ - [0 \ 1 \ 0] S_+]$$

An easy (but likely not quite correct) modification would be:

$$f_{\beta}(3,1) = \gamma_{\pi} f_{\beta}(1,1) + \gamma_x f_{\beta}(2,1)$$

$$+ \frac{1}{\beta} [[0 \ 1 \ 0 \ \underline{\rho}] (I_{4x4} - \beta L_X)^{-1} S_+ - [0 \ 1 \ 0 \ \underline{\rho}] S_+]$$

1. This takes into account expectation of future i_{t-1} ✓
2. but it doesn't take into account that $i_{t-1} = i_t$ ✗

→ I'm scared that this treats " i_t " as an exog. state,

$$\text{ignoring that } \hat{E}_t[i_{t+1}] = \hat{E}_t[i_{t+2}] = \hat{E}_t[i'_t]$$

This worry is not done if hx accounts for this
which - oh wonder - it does! Recall that

$$hx_{it} = gx_i \quad (\text{more or less...})$$

So the "easy fix" of $(*)$ may be correct.

The worry then is that the "il-row" of hx may reveal
parts of gx ?

B/c now I'm saying that putting il into the exog.
state block is ok for expectations of il as long
as hx encompasses the structural relationship
between i_t and i_{t-1} , which I think it does.

→ Ryan 24 Oct 2015 suggests that the "il-row" of hx ≠ row of gx
b/c of indirect & direct effects → check on Matlab!

Bad news:

6 Jan 2020

$hx(4, :) = gx(4, :)$ in Matlab

and $gx_{3,4} = hx_{3,4} \neq p$ b/c since i_{t-1} is an endog. state, it doesn't depend on s_t as $[0 \ 0 \ 0 \ p]$, instead it depends on s_t as it does, from the effects of other shocks.

⇒ Then we do have the problem that hx reveals a part of gx . Which necessitates a learning where

- either only the exog part of hx is known
- none of hx is known ...

But in any case, the first step is to figure out the PQ method in Mathematica.

→ materials12e.nb ✓

Type it up in Latex materials12e.tex ✓

And compare in materialsb.m too - it works ✓

"

So now w/ the PQ method working for the baseline model, let's try to tackle extensions where the Taylor-rule is modified:

- 1) interest smoothing: $\rho i_{t-1} m$ or $\rho \bar{i}_t$ w/ $\bar{i}_t = i_{t-1}$
- 2) pil: π_{t-1} in TR m
- 3) Epi: $\hat{E}_t[\pi_{t+1}]$ in TR m

In each case the NKPC is unchanged, so I just need to see how the NKIS and TR change and how (*) has to change to take them into account.

Epi may not be so diff b/c it doesn't introduce a new, endog. state, but pil will.

$$1) x_t = -\beta i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta i_{T+1} + \beta r_T^n \right\}$$

$$\bar{i}_t = \gamma_\pi \pi_t + \gamma_x x_t + \bar{i}_t + \rho i_{t-1}$$

$$2) \bar{i}_t = \gamma_\pi \pi_{t-1} + \gamma_x x_t + \bar{i}_t$$

$$3) i_t = \gamma_x \hat{E}_t \pi_{t+1} + \gamma_x x_t + \bar{i}_t$$

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -\alpha & 0 \\ -\gamma_{\pi} & \gamma_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [3, 1-\beta, -b\beta] f_{\beta} + b[1 \ 0 \ 0 \ 0] (I_{Mx} - \beta h_x)^{-1} s_2 \\ [(1-\alpha)\beta, k\alpha\beta, 0] + [0 \ 0 \ 1 \ 0] (I_{Mx} - \alpha\beta h_x)^{-1} s_1 \\ [0 \ 1 \ 0 \ \rho] s_x \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -\alpha & 0 \\ 0 & -\gamma_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [3, 1-\beta, -b\beta] f_{\beta} + b[1 \ 0 \ 0 \ 0] (I_{Mx} - \beta h_x)^{-1} s_2 \\ [(1-\alpha)\beta, k\alpha\beta, 0] + [0 \ 0 \ 1 \ 0] (I_{Mx} - \alpha\beta h_x)^{-1} s_1 \\ [0 \ 1 \ 0 \ \gamma_{\pi}] s_x \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -\alpha & 0 \\ 0 & -\gamma_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [3, 1-\beta, -b\beta] f_{\beta} + b[1 \ 0 \ 0 \ 0] (I_{Mx} - \beta h_x)^{-1} s_2 \\ [(1-\alpha)\beta, k\alpha\beta, 0] + [0 \ 0 \ 1 \ 0] (I_{Mx} - \alpha\beta h_x)^{-1} s_1 \\ [0 \ 1 \ 0 \] s_x + \gamma_{\pi} \hat{E}_t [\pi_{t+1}] \end{bmatrix}$$

\hat{E}_t → need to be

evaluated as a 1-period-ahead

first in learning rule

and $(\star - \text{Epi})$ is

$$f_B(3,1) = \cancel{\gamma_n f_p(1,1)} + \gamma_x f_B(2,1) \\ + \frac{1}{\beta} [[0 \ 1 \ 0] (I_{nx} - \beta h_x)^{-1} s_+ - [0 \ 1 \ 0] s_+] \\ + \hat{E}_+ \bar{u}_{t+1} \quad \leftarrow \text{this ain't right! Miss (7 Jan 2020)}$$

→ so the Epi-model can be done directly w/ those green modifications, and the learning rule needs to spit out $\hat{E}_+ \bar{u}_{t+1}$, and the compound notation needs to be $z_+ = A_a f_a + A_b f_b + A_s s_+ + A_c \hat{E}_+ \bar{u}_{t+1}$

→ As for il and pil, the PQ-method can deal w/ the general solutions, and I suggest that the modified (\star)

$$f_B(3,1) = \cancel{\gamma_n f_p(1,1)} + \gamma_x f_B(2,1) \quad (\star - \text{il}) \\ + \frac{1}{\beta} [[0 \ 1 \ 0 \ \underline{\rho}] (I_{nx} - \beta h_x)^{-1} s_+ - [0 \ 1 \ 0 \ \underline{\rho}] s_+]$$

and the implied for pil

$$\cancel{f_B(3,1) = \cancel{\gamma_n f_p(1,1)} + \gamma_x f_B(2,1)} \quad (\star - \text{pil}) \\ + \frac{1}{\beta} [[0 \ 1 \ 0 \ \underline{\psi}] (I_{nx} - \beta h_x)^{-1} s_+ - [0 \ 1 \ 0 \ \underline{\psi}] s_+]$$

might also work. The uncertainty though is h_x !

For the pil-model, MN would look like:

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta i_{T+1} + \beta r_T^n \right\}$$

$$i_t = \gamma_x \pi_{t-1} + \gamma_x x_t + i_t$$

$$\Rightarrow x_t = -\beta (\gamma_x \pi_{t-1} + \gamma_x x_t + i_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta (\gamma_x \pi_T + \gamma_x x_{T-1} + i_{T-1}) + \beta r_T^n \right\}$$

$$(1+\beta \gamma_x) x_t = -\beta \gamma_x \pi_{t-1}$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta \beta \gamma_x) x_{T+1} + \underbrace{\beta \pi_{T+1} - \beta \beta \gamma_x \pi_T}_{+ \beta [1, -1, 0, 0] s_T} + \beta [1, -1, 0, 0] s_T \right\}$$

This is the prob.

$$\Rightarrow -\beta \beta \gamma_x \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \pi_T = -\beta \beta \gamma_x \left[\pi_t + \beta \pi_{t+1} + \beta^2 \pi_{t+2} + \dots \right]$$

$$= -\beta \beta \gamma_x \pi_t + (-\beta \beta \gamma_x) \beta \left[\pi_{t+1} + \beta \pi_{t+2} + \dots \right]$$

So I can write the NKIS as:

$$(1+\beta \gamma_x) x_t = -\beta \gamma_x \pi_{t-1} - \beta \beta \gamma_x \pi_t$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta \beta \gamma_x) x_{T+1} + \underbrace{\beta \pi_{T+1} - \beta \beta \gamma_x \pi_{T+1}}_{+ \beta [1, -1, 0, 0] s_T} + \beta [1, -1, 0, 0] s_T \right\}$$

$$\beta \beta \gamma_x \pi_t + (1+\beta \gamma_x) x_t = -\beta \gamma_x \pi_{t-1}$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta \beta \gamma_x) x_{T+1} + \underbrace{\beta (1-\beta^2 \gamma_x) \pi_{T+1}}_{+ \beta [1, -1, 0, 0] s_T} + \beta [1, -1, 0, 0] s_T \right\}$$

\Rightarrow So I think the MN-method could deal w/ this . . . ?

It's a new day and I no longer think 7 Jan 2020

that it's necessarily a problem that parts of hx equal parts of gx . It can be a problem but it doesn't have to be if we assume that agents

don't know that $gx(4, :) = hx(4, :)$. Then hx doesn't reveal anything about gx . And I think that implicitly I've been making this assumption.

But: if I continue making this assumption then I can solve all 3 extensions: il (PQ-method), pil and Epi (both MN and PQ methods).

Since il is actually the hardest (MN sol doesn't exist)

I'll reverse the order:

1) Epi: materials12f1 .nb / m / .tex

matrices - A-12f1.m

2) pil: materials12f2 .nb / m / .tex

matrices - A-12f2.m

3) il: materials12f3 .nb / m / .tex

matrices - A-12f3.m

① Epi

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta i_{T+1} + \beta r_T^H \right\}$$

$$i_t = \gamma_x \hat{E}_t \pi_{t+1} + \gamma_x x_t - \bar{i}_t$$

$$\Rightarrow PQ: x_t + \beta i_t = [0, 1-\beta, -\beta \beta] f_\beta + \beta [1, 0, 0] (I_{nx} - \beta h_x)^{-1} s_t$$

$$\pi_t - K x_t = [(1-\alpha)\beta, \alpha\beta, 0] f_\alpha + [0, 0, 1] (I_{nx} - \alpha \beta h_x)^{-1} s_t$$

$$-\gamma_x x_t + i_t = \gamma_x \hat{E}_t \pi_{t+1} + [0 \ 1 \ 0] s_t$$

$$\begin{bmatrix} 0 & 1 & \beta \\ 1 & -k & 0 \\ 0 & -\gamma_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [0, 1-\beta, -\beta \beta] f_\beta + (x_s \cdot s_t) \\ [(1-\alpha)\beta, \alpha\beta, 0] f_\alpha + (c_{ps} \cdot s_t) \\ c_{is} \cdot s_t + \gamma_x \hat{E}_t \pi_{t+1} \end{bmatrix}$$

and (*-Epi) is

$$f_p(3) = \gamma_x f_\beta(2)$$

$$+ \frac{1}{\beta} [[0 \ 1 \ 0] (I_{nx} - \beta h_x)^{-1} s_t - [0 \ 1 \ 0] s_t]$$

$$+ \gamma_x \hat{E}_t \sum_{T=t+1}^{\infty} \pi_{T+2} \quad \text{now what?}$$

b/c

$$\hat{E}_t i_{t+1} = \gamma_x \hat{E}_t x_{t+1} + c_{is} \hat{E}_t s_{t+1} + \gamma_x \hat{E}_t \hat{E}_{t+1} \pi_{t+2}$$

$$\hat{E}_t i_{t+2} = \gamma_x \beta \hat{E}_t x_{t+2} + c_{is} \hat{E}_t s_{t+2} + \gamma_x \hat{E}_t \pi_{t+3}$$

$$\Rightarrow Mx = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta)x_{T+1} + b\pi_{T+1} - 2\beta \left(k_T \hat{E}_T \pi_{T+1} x_{T+1} - s_T \right) + b\pi_T \right\}$$

$$x_t = -b\gamma_x Epi - b\gamma_x x_t - b\bar{s}_t$$

$$(1+b\gamma_x)x_t = -b\gamma_T \hat{E}_T \pi_{T+1} + 2[1-\alpha] (I_{nx} - \beta h_x)^{-1} s_T$$

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-2b\gamma_x)x_{T+1} + b\pi_{T+1} - 2\beta \gamma_T \hat{E}_T \pi_{T+2} \right\}$$

Assume LIE holds for \hat{E} (which I think is true by anticipated utility)

$$\Rightarrow (1+b\gamma_x)x_t = -b\gamma_T \hat{E}_T \pi_{T+1} + 2[1-\alpha] (I_{nx} - \beta h_x)^{-1} s_T$$

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-2b\gamma_x)x_{T+1} + b\pi_{T+1} - 2\beta \gamma_T \pi_{T+2} \right\}$$

$$\Rightarrow (1+b\gamma_x)x_t = -b\gamma_T \hat{E}_T \pi_{T+1} + 2[1-\alpha] (I_{nx} - \beta h_x)^{-1} s_T$$

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-2b\gamma_x)x_{T+1} + (2-2\gamma_T) \pi_{T+1} \right\}$$

$$\begin{bmatrix} 0 & (1+b\gamma_x) \\ 1 & -k \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [b(1-\gamma_T), 1-\beta-2b\gamma_x, 0] f_p + k x_s s_t \\ [(1-\alpha)\beta, k \alpha \beta, 0] f_n + d p_s s_t \end{bmatrix}$$

and in both cases the compact notation is

$$z_t = A_a \cdot f_a + A_b \cdot f_b + A_c \cdot \underbrace{E_t \pi_{t+1}}_{Epi}$$

Try $(\star - \text{Epi})$ again:

$$f_\beta(3) = \hat{E} \sum_{T=1}^{\infty} \beta^{T+1} i_{T+1}$$

$$= \hat{E} \sum_{T=1}^{\infty} \beta^{T+1} \left[\gamma_x x_{T+1} + c_{1S} s_{T+1} + \gamma_\pi \hat{E} \pi_{T+2} \right]$$

$$= \underbrace{\gamma_x f_\beta(2) + c_{1S} [ps_+ + \beta p s_{1+} \dots]}_{\gamma_\pi \hat{E} \sum_{T=1}^{\infty} \beta^{T+1} \hat{E}_+ \pi_{T+2}}$$

$$\hookrightarrow \gamma_\pi \left[\pi_{1+2} + \beta \pi_{1+3} + \dots \right]$$

$$= \frac{1}{\beta} \gamma_\pi \left[\pi_{1+1} + \beta \pi_{1+2} + \beta^2 \pi_{1+3} + \dots - \pi_{1+1} \right]$$

$$= \frac{1}{\beta} \gamma_\pi \left[f_\beta(1) - \hat{E}_+ \pi_{1+1} \right]$$

So $(\star - \text{Epi})$ is:

$$\left[f_\beta(3) = \gamma_x f_\beta(2) + \underline{\gamma_\pi / \beta} f_\beta(1) - \underline{\gamma_\pi / \beta \hat{E}_+ \pi_{1+1}} \right. \\ \left. + \frac{1}{\beta} c_{1S} [(I_{1X} - \beta I_{1X})^{-1} s_+ - s_+] \right]$$

This is the correct $(\star - \text{Epi})$!

\Rightarrow ok, the two are equal in Mathematica! ☺

→ need to adapt the learning code

Sim-learn L11 - 12f1.m

I haven't changed the EE-learning b/c we don't care so much about it.

② pil-Extension

MN

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta i_{T+1} + \beta r_T^N \right\}$$

$$i_t = \gamma_\pi \pi_{t-1} + \gamma_x x_t + i_t$$

$$\Rightarrow x_t = -\beta (\gamma_\pi \pi_{t-1} + \gamma_x x_t + i_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta)x_{T+1} + \beta \pi_{T+1} - \beta \beta (\gamma_\pi \pi_T + \gamma_x x_{T+1} + i_{T+1}) + \beta r_T^N \right\}$$

$$(1+\beta \gamma_x) x_t = -\beta \gamma_\pi \pi_{t-1}$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta \gamma_x) x_{T+1} + \beta \pi_{T+1} - \beta \beta \gamma_\pi \pi_T - \beta i_T + \beta r_T^N \right\}$$

$$\Leftrightarrow (1+\beta \gamma_x) x_t = -\beta \gamma_\pi \pi_{t-1} - \beta \gamma_\pi \beta \pi_t$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta \gamma_x) x_{T+1} + \beta \pi_{T+1} - \beta \beta \gamma_\pi \pi_{T+1} - \beta i_T + \beta r_T^N \right\}$$

$$\beta \gamma_\pi \pi_t + (1+\beta \gamma_x) x_t = -\beta \gamma_\pi \pi_{t-1}$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta \gamma_x) x_{T+1} + \beta (1-\beta^2 \gamma_\pi) \pi_{T+1} - \beta i_T + \beta r_T^N \right\}$$

$$b\psi_{\pi} \bar{x}_+ + (-b\psi_x)x_+ = -b\psi_{\pi} \bar{\pi}_{T+1}$$

$$+ \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} \left\{ (1-\beta - \beta b\psi_x) x_{T+1} + b(1-\beta^2 \psi_{\pi}) \bar{\pi}_{T+1} - \bar{\pi}_T + b\gamma^N \right\}$$

$$\rightarrow \begin{bmatrix} b\psi_{\pi} \beta & 1+b\psi_x \\ 1 & -k \end{bmatrix} \begin{bmatrix} \bar{\pi}_t \\ x_t \end{bmatrix} = \begin{bmatrix} [2(1-\beta^2 \psi_{\pi}), (1-\beta - \beta b\psi_x), 0] f_{\beta} + d_{xS} s_t + [0 \dots 0 - b\psi_{\pi}] s_t \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_{\alpha} + d_{\pi S} s_t \end{bmatrix}$$

PK

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -k & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \bar{\pi}_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [b, 1-\beta, -b\beta] f_{\beta} + b[1, 0, 0, 0] (I_{nx} - \beta I_{nx})^{-1} s_t \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_{\alpha} + c_{ps} s_t \\ [0, 1, 0, \psi_{\pi}] s_t \end{bmatrix}$$

c_{ps} is new

And (*-part)

$$\begin{aligned} f_{\beta}(s) &= \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} i_{T+1} = \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} \{ \psi_{\pi} \bar{\pi}_T + \psi_x x_{T+1} + \bar{i}_{T+1} \} \\ &= \psi_x f_{\beta}(z) + \frac{1}{\beta} \{ [0, 1, 0, 0] (I_{nx} - \beta I_{nx})^{-1} s_t - [0, 1, 0, 0] s_t \} \\ &\quad + \underbrace{\psi_{\pi} [\bar{\pi}_t + \beta \hat{E}_t \bar{\pi}_{t+1} + \beta^2 \hat{E}_t \bar{\pi}_{t+2} + \dots]}_{\psi_{\pi} \bar{\pi}_t + \beta [\hat{E}_t \bar{\pi}_{t+1} + \beta \hat{E}_t \bar{\pi}_{t+2} + \dots]} \\ &= \psi_{\pi} \bar{\pi}_t + \beta \psi_{\pi} f_{\beta}(1) \end{aligned}$$

$$\begin{aligned} f_{\beta}(s) &= \underline{\beta \psi_{\pi} f_{\beta}(1)} + \psi_x f_{\beta}(z) - \underline{\psi_{\pi} \bar{\pi}_t} \\ &\quad + \frac{1}{\beta} \{ [0, 1, 0, 0] (I_{nx} - \beta I_{nx})^{-1} s_t - [0, 1, 0, 0] s_t \} \end{aligned}$$

now what?

what if we took RHS for $\underline{\beta \psi_{\pi} f_{\beta}(1)}$ over to the LHS as $2\beta \psi_{\pi} \bar{\pi}_t$ and cancelled it in (*)?

(\hookrightarrow works! \cup)

Ok, so pil also implemented in Matlab!

- 1) Tomorrow do "il"-extension
- 2) Do something about command IRF_moving.m
so that one can switch between the TRs easily.
- 3) Now it's the end of the day and I'm thinking that agents knowing hx might still cause trouble b/c no matter what I ass, I need to be consistent.
If they know hx but don't realize that $T_{t-1} = pil$,
then they should be able to find all of pil, even
 $pil_{t+k} \quad k \geq 2$! If they aren't able to find pil_{t+k},
then they realize the linking equation but then
don't know a part of hx.
- 4) In materials 12, I really need to spell out the \hat{f} problem,
and also the problem of whether they know relationships
between jumps. Do they know model equations??

③ il-extension

8 Jan 2020

MN doesn't work here, we know that.

$$PQ \quad \frac{3+2\beta\rho}{1-\rho\beta}$$

$$\begin{bmatrix} 0 & 1 & 3 \\ 1 & -k & 0 \\ -\gamma_x & -\gamma_x & 1 \end{bmatrix} \begin{bmatrix} \pi_4 \\ x_4 \\ i_4 \end{bmatrix} = \begin{bmatrix} [0, 1-\beta, -2\beta] f_p + 3[1, 0, 0, 0] (\mathbf{I}_{nx} - \beta h x)^{-1} s_+ \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_A + [0, 0, 1, 0] (\mathbf{I}_{nx} - \alpha\beta h x)^{-1} s_+ \\ [0, 1, 0, 1] s_+ \end{bmatrix}$$

$(x - i)$ is

$$\begin{aligned} f_p(3) &= \hat{\mathbb{E}}_{T=1}^{\infty} \beta^{T-t} i_{T+1} = \hat{\mathbb{E}}_{T=1}^{\infty} \beta^{T-t} [\gamma_x \pi_{T+1} + \gamma_x x_{T+1} + i_{T+1} + \rho i_T] \\ &= \gamma_x f_p(1) + \gamma_x f_p(2) + \underbrace{\frac{1}{\beta} \left\{ [0, 1, 0, 0] (\mathbf{I}_{nx} - \beta h x)^{-1} s_+ - [0, 1, 0, 0] s_+ \right\}}_{+ \underbrace{\rho \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} i_T}_{\rho [i_t + \beta i_{t+1} + \beta^2 i_{t+2} + \dots]} \\ &= \rho i_t + \rho \beta [i_{t+1} + \beta i_{t+2} + \dots] \\ &= \rho i_t + \rho \beta f_p(3) \end{aligned}$$

$$(1 - \rho\beta) f_p(3) = \gamma_x f_p(1) + \gamma_x f_p(2) + \underbrace{\frac{1}{\beta} \left\{ [0, 1, 0, 0] (\mathbf{I}_{nx} - \beta h x)^{-1} s_+ - [0, 1, 0, 0] s_+ \right\}}_{+ \rho i_t}$$

$$f_p(3) = \frac{1}{1-\rho\beta} \left[\gamma_x f_p(1) + \gamma_x f_p(2) + \frac{1}{\beta} \left\{ [0, 1, 0, 0] (\mathbf{I}_{nx} - \beta h x)^{-1} s_+ - [0, 1, 0, 0] s_+ \right\} \right]$$

$$+ \frac{\rho}{1-\rho\beta} i_t$$

again, like in p1, we subtract $\frac{-2\beta\rho}{1-\rho\beta}$ it in 1st row of P.

Interesting: I get the same A-matrices I got w/
1) original intake smoothing
2) intake-smoothing 3 (which used the MN method,
treating it as an exogenous state)

This breaks down if $\rho \neq 0$, in which case the two
old codes still coincide. Why?

- 1) b/c ... intake-smoothing.m = intake-smoothing 3.m
→ both use the MN method, except 3 is more
explicit about it
- 2) ... 12f3.m however uses the PD-method so actually
it's great news that it equals the other two when
 $\rho=0$ b/c it means that it's correct, and it should
also not equal the other two when $\rho \neq 0$ b/c
of the different info assumption!

⇒ Info ass's:

MN: Since you put in it and treat all future ils
as exogenous, the info ass you make is that agents

don't realize that $i_t = i_{t+1}$, but they know (all of) hx , so they fast the shock, but not the jump.

PQ: here the info ans you're making is that agents realize that $i_t = i_{t+1}$, and so they are fishing i_t as i_t , not realizing that they can use hx .
Neither is really kosher!

⇒ the difference is really what they are using to fast an endog. state which equals a lagged jump, gx , which they don't know, or hx , which they know.