Materials 7 - IRFs and making sure RE is right

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Overview

1 Model summary with interest rate smoothing ρi_{t-1}

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \rho i_{t-1} + \bar{i}_t \tag{3}$$

$$\hat{\mathbb{E}}_t z_{t+h} = \begin{bmatrix} \bar{\pi}_{t-1} \\ 0 \\ 0 \end{bmatrix} + b\, hx^{h-1} s_t \quad \forall h \geq 1 \qquad b = gx\ hx \qquad \text{PLM} \tag{4}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \underbrace{\big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}) \big)}_{\text{fcst error using (4)}} \qquad (b_1 \text{ is the first row of } b) \tag{5}$$

$$k_t = \mathbb{I} \times (k_{t-1} + 1) + (1 - \mathbb{I}) \times \bar{g}^{-1} \tag{6}$$

$$\mathbb{I} = \begin{cases} 1 & \text{if } \theta_t \leq \bar{\theta} \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

$$\theta_t = |\hat{\mathbb{E}}_{t-1}\pi_t - \mathbb{E}_{t-1}\,\pi_t| / \sigma_s \qquad \text{CEMP criterion for the gain} \tag{8}$$

The alternative criterion for the choice of gain is a recursive variant of the CUSUM-test (Brown, Durbin, Evans 1975):

1. Let FE_t denote the short-run forecast error, and ω_t firms' estimate of the FE variance.

2. Let $\kappa \in (0, 1)$ and $\tilde{\theta}$ be the new threshold value for the criterion.

3. Then for initial (ω_0, θ_0), firms in every period estimate the criterion and the FEV as:

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1} (FE_t^2 - \omega_{t-1}) \tag{9}$$

$$\theta_t = \theta_{t-1} + \kappa k_{t-1}^{-1} (FE_t^2 / \omega_t - \theta_{t-1}) \tag{10}$$

$$k_t = \mathbb{I} \times (k_{t-1} + 1) + (1 - \mathbb{I}) \times \bar{g}^{-1} \tag{11}$$

$$\mathbb{I} = 1 \quad \text{if} \quad \theta_t \leq \tilde{\theta} \tag{12}$$

2 Compact notation - with lagged interest rate term in TR

$$z_t = A_p^{RE}\,\mathbb{E}_t\, z_{t+1} + A_s^{RE} s_t \tag{13}$$

$$z_t = A_a^{LH} f_a(t) + A_b^{LH} f_b(t) + A_s^{LH} s_t \tag{14}$$

$$s_t = P s_{t-1} + \epsilon_t \qquad \rightarrow \quad s'_t = hx\; s'_{t-1} + \epsilon'_t \tag{15}$$

$$\text{where} \quad s'_t \equiv \begin{pmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{pmatrix} \quad hx \equiv \begin{pmatrix} \rho_r & 0 & 0 & 0 \\ 0 & \rho_i & 0 & 0 \\ 0 & 0 & \rho_u & 0 \\ gx_{3,1} & gx_{3,2} & gx_{3,3} & gx_{3,4} \end{pmatrix} \quad \epsilon'_t \equiv \begin{pmatrix} \varepsilon_t^r \\ \varepsilon_t^i \\ \varepsilon_t^u \\ 0 \end{pmatrix} \quad \text{and} \quad \Sigma' = \begin{pmatrix} \sigma_r & 0 & 0 & 0 \\ 0 & \sigma_i & 0 & 0 \\ 0 & 0 & \sigma_u & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{16}$$

i_{t-1} is an endogenous state and breaks the link that previously had $P = hx$; now this is no longer true. In particular, using Matlabby notation, $P = hx(1:3, 1:3)$.

And the A_s^{RE} and A_s^{LH} are given by:

$$A_s^{RE} = \begin{pmatrix} \frac{\kappa\sigma}{w} & -\frac{\kappa\sigma}{w} & 1 - \frac{\kappa\sigma\psi_\pi}{w} & 0 \\ \frac{\sigma}{w} & -\frac{\sigma}{w} & -\frac{\sigma\psi_\pi}{w} & 0 \\ \psi_x(\frac{\sigma}{w}) + \psi_\pi(\frac{\kappa\sigma}{w}) & \psi_x(-\frac{\sigma}{w}) + \psi_\pi(-\frac{\kappa\sigma}{w}) + 1 & \psi_x(-\frac{\sigma\psi_\pi}{w}) + \psi_\pi(1 - \frac{\kappa\sigma\psi_\pi}{w}) & \rho \end{pmatrix} \tag{17}$$

$$A_s^{LH} = \begin{pmatrix} g_{\pi s} \\ g_{xs} \\ \psi_\pi g_{\pi s} + \psi_x g_{xs} + \begin{bmatrix} 0 & 1 & 0 & \rho \end{bmatrix} \end{pmatrix} \tag{18}$$

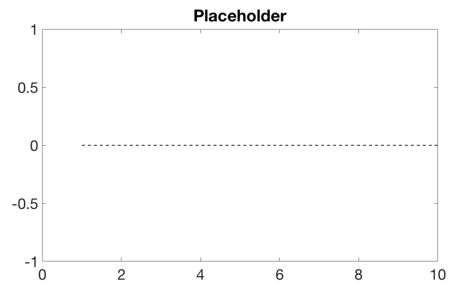
$$g_{\pi s} = (1 - \frac{\kappa\sigma\psi_\pi}{w}) \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} (I_4 - \alpha\beta hx)^{-1} - \frac{\kappa\sigma}{w} \begin{bmatrix} -1 & 1 & 0 & \rho \end{bmatrix} (I_4 - \beta hx)^{-1} \tag{19}$$

$$g_{xs} = \frac{-\sigma\psi_\pi}{w} \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} (I_4 - \alpha\beta hx)^{-1} - \frac{\sigma}{w} \begin{bmatrix} -1 & 1 & 0 & \rho \end{bmatrix} (I_4 - \beta hx)^{-1} \tag{20}$$

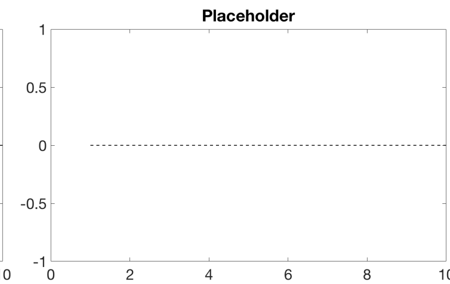
3 IRFs splitting by decreasing, constant and endogenous gain learning

3.1 Natural rate shock

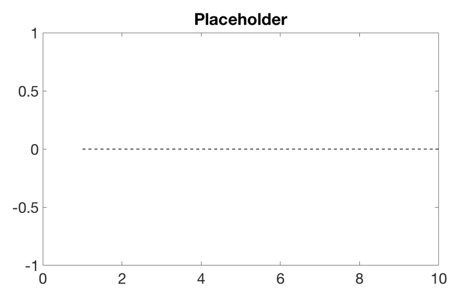
Figure 1: Moving ρ



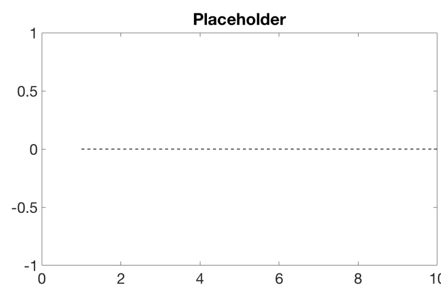
(a) IRFs, $\rho = 0$



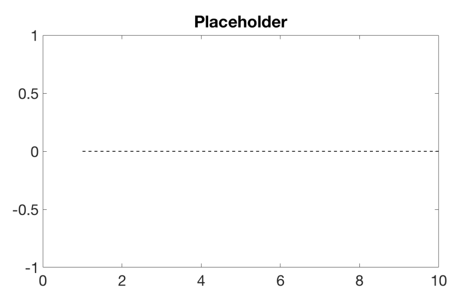
(b) Gain, $\rho = 0$



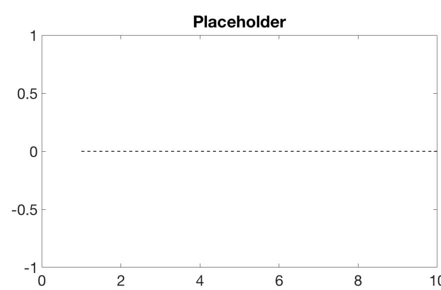
(c) IRFs, $\rho = 0.3$



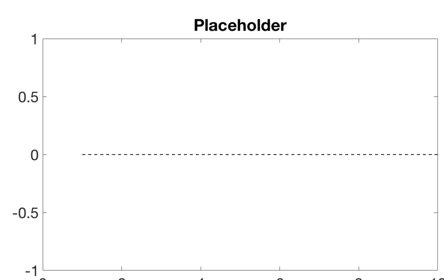
(d) Gain, $\rho = 0.3$



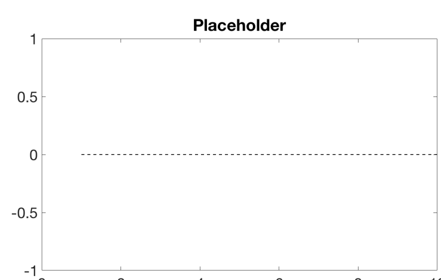
(e) IRFs, $\rho = 0.6$



(f) Gain, $\rho = 0.6$

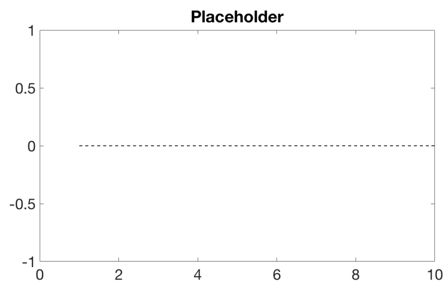


(g) IRFs, $\rho = 0.9$

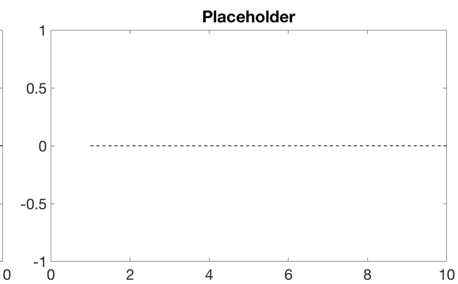


(h) Gain, $\rho = 0.9$

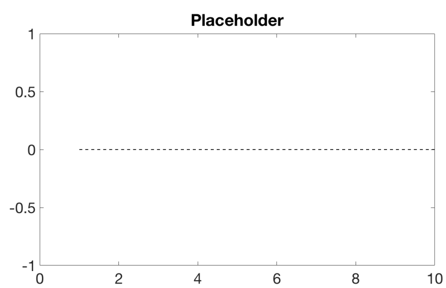
Figure 2: Moving ψ_π



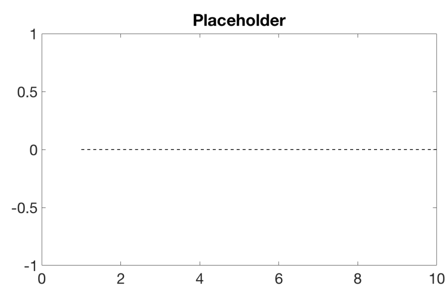
(a) IRFs, $\psi_\pi = 1.1$



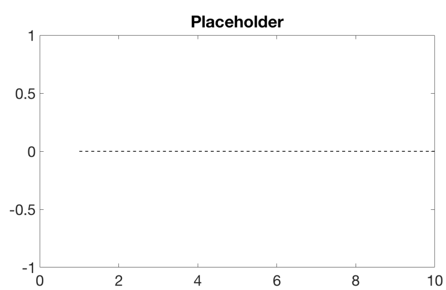
(b) Gain, $\psi_\pi = 1.1$



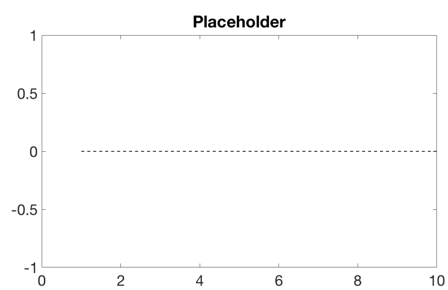
(c) IRFs, $\psi_\pi = 1.5$



(d) Gain, $\psi_\pi = 1.5$



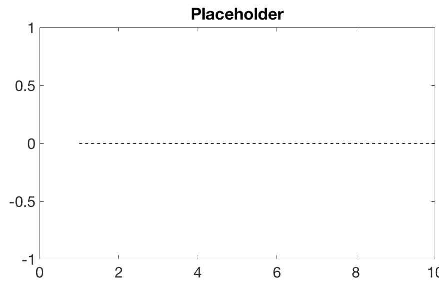
(e) IRFs, $\psi_\pi = 2$



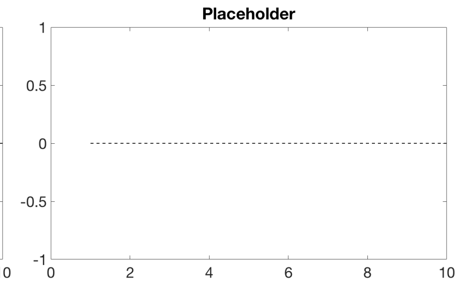
(f) Gain, $\psi_\pi = 2$

3.2 Monetary policy shock

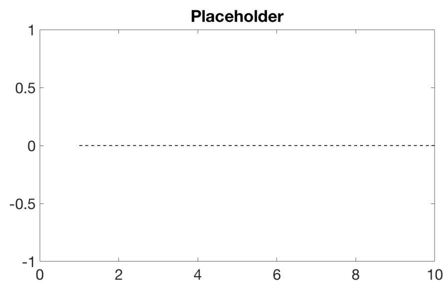
Figure 3: Moving ρ



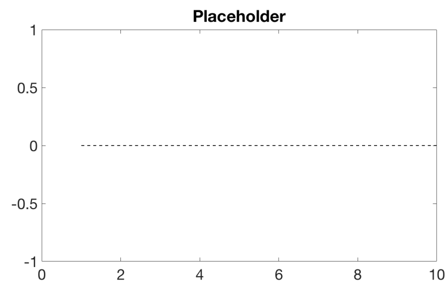
(a) IRFs, $\rho = 0$



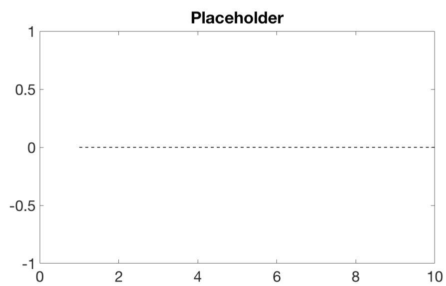
(b) Gain, $\rho = 0$



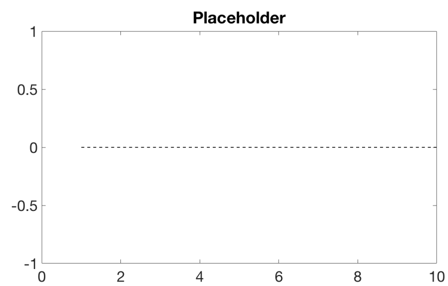
(c) IRFs, $\rho = 0.3$



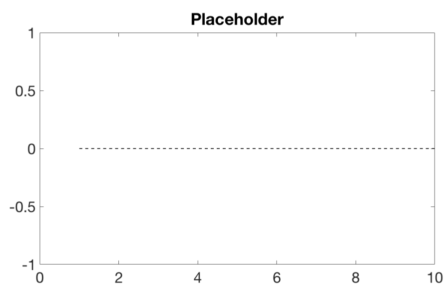
(d) Gain, $\rho = 0.3$



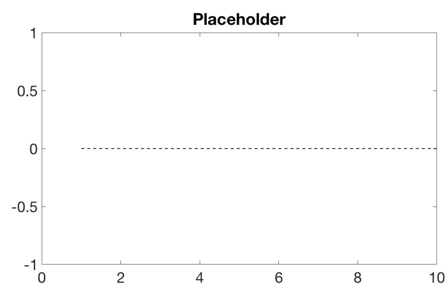
(e) IRFs, $\rho = 0.6$



(f) Gain, $\rho = 0.6$

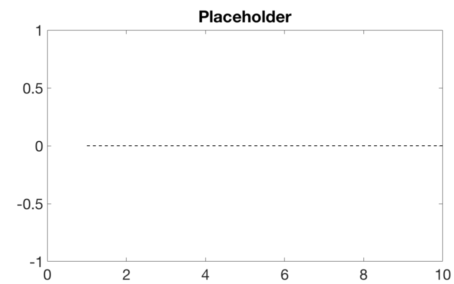


(g) IRFs, $\rho = 0.9$

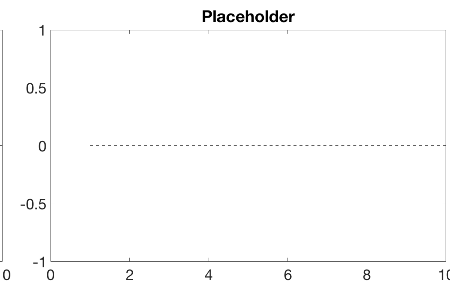


(h) Gain, $\rho = 0.9$

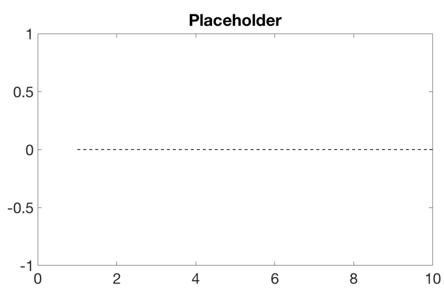
Figure 4: Moving ψ_π



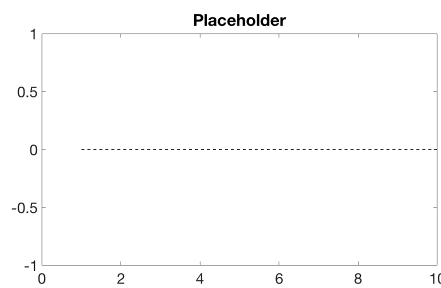
(a) IRFs, $\psi_\pi = 1.1$



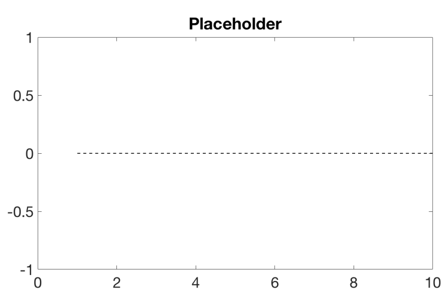
(b) Gain, $\psi_\pi = 1.1$



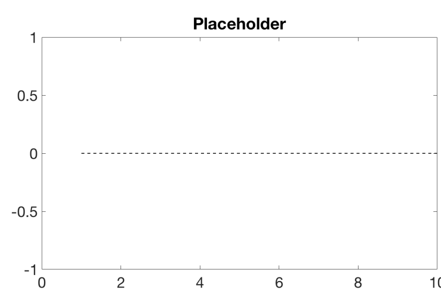
(c) IRFs, $\psi_\pi = 1.5$



(d) Gain, $\psi_\pi = 1.5$



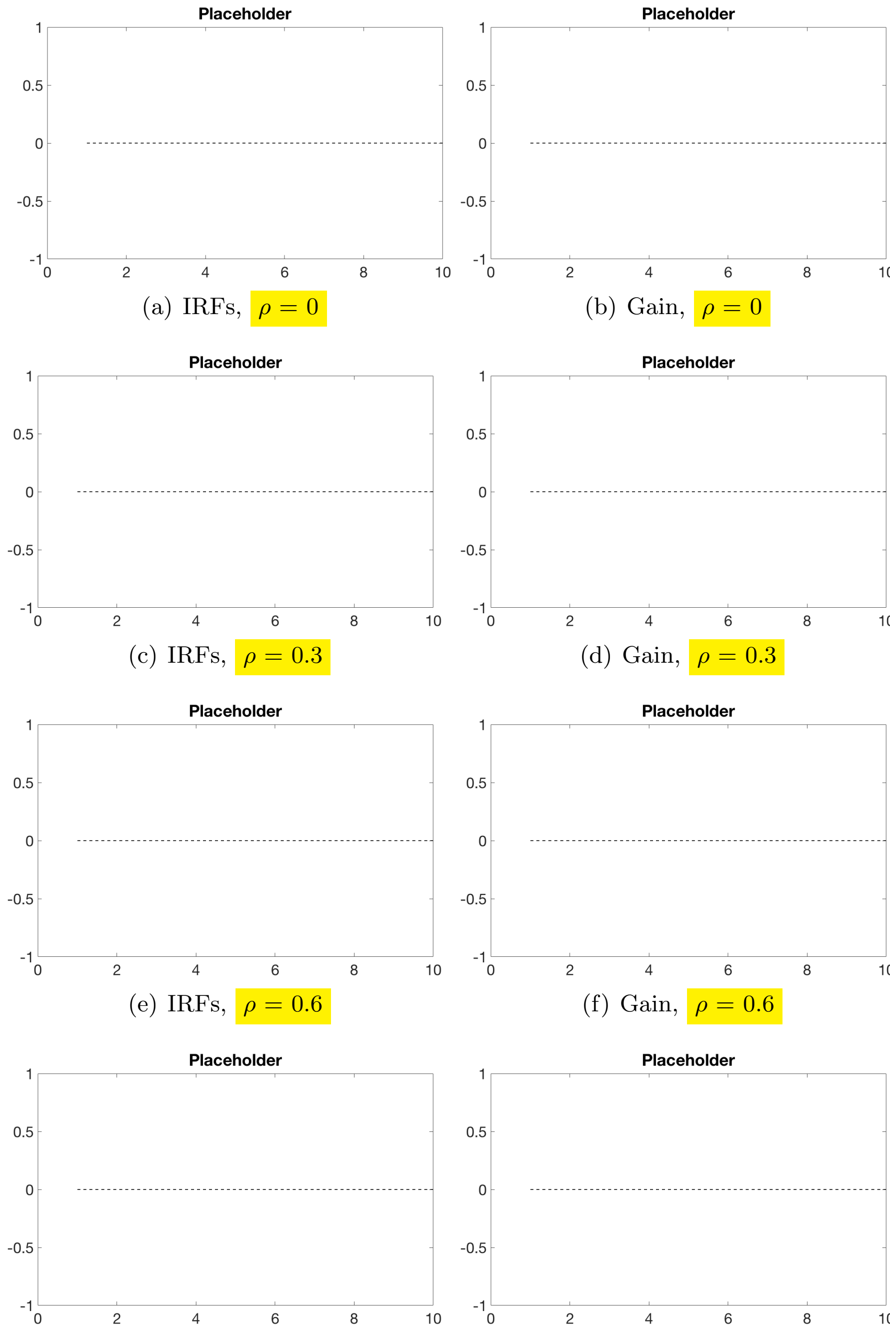
(e) IRFs, $\psi_\pi = 2$



(f) Gain, $\psi_\pi =2$

3.3 Cost-push shock

Figure 5: Moving ρ



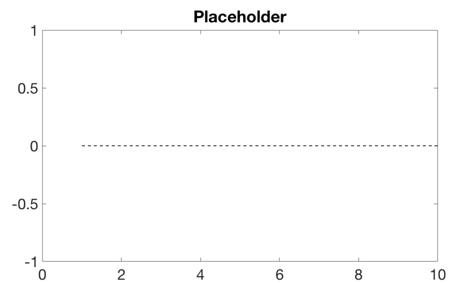
(a) IRFs, $\rho = 0$ (b) Gain, $\rho = 0$

(c) IRFs, $\rho = 0.3$ (d) Gain, $\rho = 0.3$

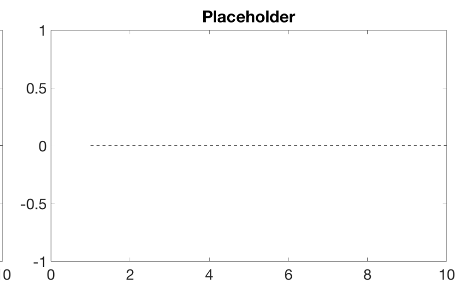
(e) IRFs, $\rho = 0.6$ (f) Gain, $\rho = 0.6$

(g) IRFs, $\rho = 0.9$ (h) Gain, $\rho = 0.9$

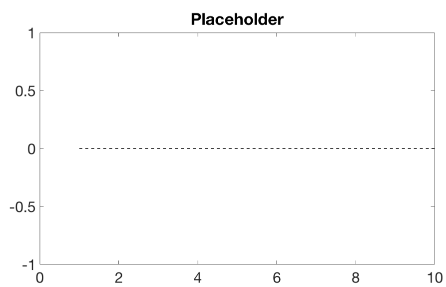
Figure 6: Moving ψ_π



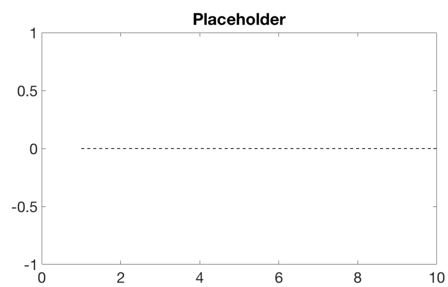
(a) IRFs, $\psi_\pi = 1.1$



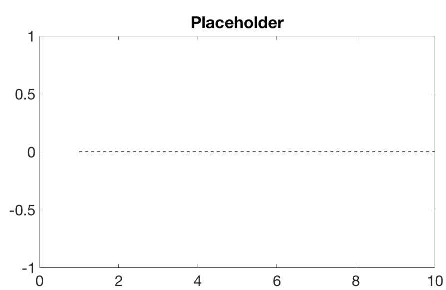
(b) Gain, $\psi_\pi = 1.1$



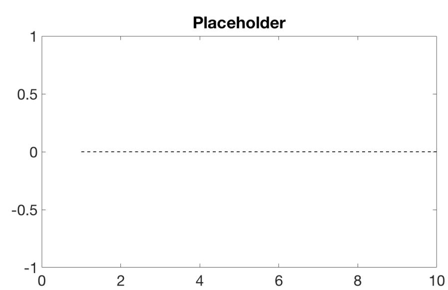
(c) IRFs, $\psi_\pi = 1.5$



(d) Gain, $\psi_\pi = 1.5$



(e) IRFs, $\psi_\pi = 2$



(f) Gain, $\psi_\pi =2$