seems to be the same paper as woodford1999inertia

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Optimal Interest-Rate Smoothing

MICHAEL WOODFORD
Princeton University

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This paper considers the desirability of the observed tendency of central banks to adjust interest rates only gradually in response to changes in economic conditions. It shows, in the context of a simple model of optimizing private-sector behaviour, that assignment of an interest-rate smoothing objective to the central bank may be desirable, even when reduction of the magnitude of interest-rate changes is not a social objective in itself. This is because a response of policy to "irrelevant" lagged variables may be desirable owing to the way it steers private-sector expectations of future policy.

1. OPTIMAL MONETARY POLICY INERTIA

Many students of central bank behaviour have noted that the level of nominal interest rates in the recent past appears to be an important determinant of where the central bank will set its interest-rate instrument in the present. Changes in observed conditions, such as in the rate of inflation or in the level of economic activity, result in changes in the level of the central bank's operating target for the short-term interest rate that it controls, but these changes typically occur through a series of small adjustments in the same direction, drawn out over a period of months, rather than through an immediate once-and-for-all response to the new development. This type of behaviour is especially noticeable in the case of the Federal Reserve in the U.S., but characterizes many other central banks to at least some extent as well.[1]

Such behaviour may be rationalized on the ground that central banks seek to "smooth" interest rates, in the sense that they seek to minimize the variability of interest-rate changes, in addition to other objectives of policy such as inflation stabilization. Yet it remains unclear why it should be desirable for central banks to pursue such a goal. There are several plausible reasons why policymakers should prefer policies that do not require the *level* of short-term interest rates to be too variable. On the one hand, the zero nominal interest-rate floor (resulting from the availability of cash as a riskless, perfectly liquid zero-return asset) means that rates *cannot* be pushed below zero. This means that a policy consistent with a low average rate of inflation, which implies a low average level of nominal interest rates, cannot involve interest-rate reductions in response to deflationary shocks that are ever too large. And at the same time, high nominal interest rates always imply distortions, as resources are wasted on unnecessary efforts to economize on cash balances. Friedman (1969) stresses that this is a reason to prefer a regime with low average inflation, or even moderate deflation; but it is actually the level of nominal interest rates that directly determines the size of the distortion, and the argument applies as much to short-run variation in nominal interest rates as to their average level. Thus it is also

1. See, *e.g.* Cook and Hahn (1989), Rudebusch (1995), Goodhart (1996) and Sack (1998*a*,*b*). The presence of lagged interest rates in estimated central-bank reaction functions (*e.g.* Clarida, Galí and Gertler (1998, 2000), Judd and Rudebusch (1998) or Sack (1998*b*)) is often interpreted in terms of partial-adjustment dynamics for the gap between the actual level of the interest-rate instrument and a desired level that depends on variables such as current inflation and real activity.

desirable on this ground for policy not to raise nominal interest rates too much in response to inflationary shocks.[2] But while it makes a great deal of sense for a central bank to seek to achieve its other aims in a way consistent with as low as possible a variance of the *level* of short-run nominal rates, this in no way implies a direct concern with the variability of interest-rate *changes*.

Nonetheless, I shall argue that a concern with interest-rate smoothing on the part of a central bank can have desirable consequences. This is because such an objective can result in *history-dependent* central-bank behaviour which, when anticipated by the private sector, can serve the bank's stabilization objectives through the effects upon current outcomes of anticipated future policy.

If the private sector is forward looking, so that the effects of policy depend to an important extent on expectations regarding future policy, it is well known that discretionary minimization of a loss function representing true social objectives will generally lead to a (Markov) equilibrium which is suboptimal from the point of view of those same objectives. The reason is that a central bank that optimizes under discretion neglects at each point in time the effects that anticipations of its current actions have had upon equilibrium determination at earlier dates, as these past expectations can no longer be affected at the time that the bank decides how to act. Yet a different systematic pattern of conduct, justifying different expectations, might have achieved a better outcome in terms of the bank's own objectives.

As a consequence, a better outcome can often be obtained (in the Markov equilibrium associated with discretionary optimization) if the central bank is assigned an objective *different* from the true social objective; the problem of choosing an appropriate objective is sometimes called the problem of "optimal delegation". Famous examples include the proposal by Rogoff (1985) that a central banker should be chosen who is "conservative", in the sense of placing a greater weight on inflation stabilization than does the social loss function, or the proposal by King (1997) that the central bank should aim to stabilize the output gap around the level consistent with achieving its inflation target on average, even when a higher level of output relative to potential would be socially optimal. In both cases, modification of the central bank's objective can eliminate the bias toward higher-than-optimal average inflation resulting from discretionary policy when the central bank seeks to minimize the true social loss function.

In these examples, the central bank's assigned loss function is still a quadratic function of the same *target variables* as is the true social loss function; the assigned *target values* for these variables may be changed (as in the King proposal) or the relative weights on alternative stabilization objectives may be altered (as in the Rogoff proposal), but the variables that one wishes to stabilize are not changed. However, in general, there will also be advantages to introducing *new* target variables into the central bank's assigned loss function. This is the argument given here for assigning a central bank an interest-rate smoothing objective.

In particular, it will often be desirable to assign the central bank a loss function that involves lagged endogenous variables that are irrelevant to the computation of true social losses in a given period, as a way of causing policy to be *history-dependent*.[3] In the case of any loss function that is a function of the same target variables as the true social loss function, and no others, policy must be *purely forward looking* in a Markov equilibrium resulting from discretionary optimization by the bank. This means that at each point in time, policy (and the resulting values of the target variables) depend *only* on those aspects of the state of the world that define the set of feasible

2. Both of these grounds for inclusion of a quadratic stabilization objective for a short-term nominal interest rate in the objective function that monetary policy should be designed to minimize are analysed in detail in Woodford (2003, Chapter 6).

3. Other examples of this general idea, appearing since the first draft of this paper, include Vestin (2002) and Walsh (2002).

paths for the target variables from the present time onward. Yet in general, optimal policy is not purely forward looking (Woodford, 2000). This is shown in the example considered in this paper through explicit computation of the optimal state-contingent evolution of the economy subject to the constraint that policy be purely forward looking, and comparing this with the optimal state-contingent evolution when this constraint is relaxed.

It might seem that familiar "dynamic programming" arguments imply that optimal policy *should* be purely forward looking. But such arguments apply to the optimal control of backward-looking systems of the kind considered in the engineering literature, and not to the control of a forward-looking system of the kind that a central bank is concerned with, as a result of private-sector optimization (under rational expectations). In a case of the latter sort, the evolution of the target variables depends not only on the central bank's current actions, but also upon how the private sector *expects* monetary policy to be conducted in the future. It follows from this that a more desirable outcome may be achieved if it can be arranged for private-sector expectations of future policy actions to adjust in an appropriate way in response to shocks. But if the private sector has rational expectations, it is not possible to arrange for expectations to respond to shocks in a desired way unless *subsequent policy* is affected by those past shocks in the way that one would like the private sector to anticipate. This will generally require that the central bank's behaviour be history dependent—that it *not* depend solely upon current conditions and the bank's current forecast of future conditions, but also upon past conditions, to which it was desirable for the private sector to be able to count upon the central bank's subsequent response.

The essential insight into why interest-rate smoothing by a central bank may be desirable is provided by a suggestion of Goodfriend (1991), also endorsed by Rudebusch (1995). Goodfriend argues that output and prices do not respond to daily fluctuations in the (overnight) federal funds rate, but only variations in *longer-term* interest rates. The Fed can thus achieve its stabilization goals only in so far as its actions affect these longer-term rates. But long rates should be determined by market expectations of future short rates. Hence an effective response by the Fed to inflationary pressures, say, requires that the private sector be able to believe that the entire *future path* of short rates has changed. A policy that maintains interest rates at a higher level for a period of time once they are raised—or even following initial small interest-rate changes by further changes in the same direction, in the absence of a change in conditions that makes this unnecessary—is one that, if understood by the private sector, will allow a moderate adjustment of current short rates to have a significant effect on long rates. Such a policy offers the prospect of significant effects of central-bank policy upon aggregate demand, without requiring excessively volatile short-term interest rates.

This paper offers a formal analysis of the benefits of inertial behaviour along essentially the lines sketched by Goodfriend, in the context of a simple, and now rather standard, forward-looking macro model, with clear foundations in optimizing private-sector behaviour. Section 2 presents the model, poses the problem of optimal monetary policy, and derives the optimal state-contingent responses of endogenous variables, including nominal interest rates, to shocks under an optimal regime.[4] Section 3 highlights the need for policy to be history dependent, by contrasting the fully optimal responses with the optimal responses subject to the constraint that policy be non-inertial. Section 4 then considers the optimal delegation problem, showing that it is desirable for the central bank's loss function to include an interest-rate smoothing objective, even though the true social loss function does not. Section 5 concludes.

4. The analysis of optimal state-contingent policy follows Woodford (1999*a*), which also contrasts optimal state-contingent policy under commitment with the Markov equilibrium associated with discretionary minimization of the true social loss function. The analysis of optimal forward-looking policy is new here, as is the treatment of the optimal delegation problem.

2. OPTIMAL RESPONSES TO FLUCTUATIONS IN THE NATURAL RATE OF INTEREST

In order to illustrate more concretely the themes of the preceding discussion, it is useful to introduce a simple optimizing model of inflation and output determination under alternative monetary policies, where monetary policy is specified in terms of a feedback rule for a short-term nominal interest rate instrument. The model is similar, if not identical, to the small forward-looking models used in a number of recent analyses of monetary policy rules, including Kerr and King (1996), Bernanke and Woodford (1997), Rotemberg and Woodford (1997, 1999), Clarida, Galí and Gertler (1999) and McCallum and Nelson (1999*a*,*b*). As is explained in Woodford (2003, Chapter 4), the model's equations can be derived as log-linear approximations to the equilibrium conditions of a simple intertemporal general equilibrium model with sticky prices. While the model is quite simple, it incorporates forward-looking private-sector behaviour in three respects, each of which is surely of considerable importance in reality, and would therefore also be present in some roughly similar form in any realistic model.

The model's two key equations are an *intertemporal IS equation* (or Euler equation for the intertemporal allocation of private expenditure) of the form

$$x_t = E_t x_{t+1} - \sigma[i_t - E_t\pi_{t+1} - r_t^n], \tag{2.1}$$

and an *aggregate supply equation* of the form

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1}, \tag{2.2}$$

where x_t is the deviation of the log of real output from its natural rate, π_t is the rate of inflation (first difference of the log of the price level), and i_t is the deviation of the short-term nominal interest rate (the central bank's policy instrument) from its steady-state value in the case of zero inflation and steady output growth. These two equations, together with a rule for the central bank's interest-rate policy, determine the equilibrium evolution of the three endogenous variables π_t, x_t, and i_t.

The exogenous disturbance r_t^n corresponds to Wicksell's "natural rate of interest", the interest rate (determined by purely *real* factors) that would represent the equilibrium real rate of return under flexible prices, and that corresponds to the nominal interest rate consistent with an equilibrium with constant prices.[5] In our simple model, disturbances to the natural rate represent a useful summary statistic for *all* non-monetary disturbances that matter for the determination of inflation and the output gap, for no other disturbance term enters either equation (2.2) or (2.1), once they are written in terms of the output gap x_t as opposed to the level of output. Hence if, as we shall suppose, the goals of stabilization policy can be described in terms of the paths of the inflation rate, the output gap, and interest rates alone, then the problem of optimal monetary policy may be formulated as a problem of the optimal response to disturbances to the natural rate of interest.

We shall assume that the objective of monetary policy is to minimize the expected value of a loss criterion of the form

$$E[W] \equiv E\left\{\sum\nolimits_{t=0}^{\infty} \beta^t L_t\right\}, \tag{2.3}$$

where $0 < \beta < 1$ is a discount factor, and the loss each period is given by

$$L_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2, \tag{2.4}$$

for some weights $\lambda_x, \lambda_i > 0$. The expectation is conditional upon the economy's state at the time at which alternative policies are to be evaluated, assumed to be prior to the realization of the exogenous disturbance in period zero. The assumed form of (2.4) is relatively conventional,

5. See Woodford (2003, Chapter 4) for further discussion of the importance of this concept for monetary policy.

except that an interest-rate stabilization objective is included, for either or both of the reasons discussed in the introduction.[6] Note that an interest-rate "smoothing" objective is *not* assumed. The "target values" of each of the target variables are assumed to be those associated with a steady state with zero inflation in the absence of real disturbances. Thus target values are not assumed that result in any bias in the *average* rate of inflation, or in the average values of other state variables, under discretionary policy; the reason for assigning the central bank a loss function other than the social loss function has solely to do with the sub-optimality of the dynamic responses to shocks under discretionary minimization of (2.4).[7]

I begin by characterizing the dynamic responses to shocks that would occur under an optimal commitment, and comparing these to the consequences of discretionary policy when the central bank seeks to minimize the true social loss function. Our problem is to choose stochastic processes π_t, x_t, and i_t—specifying each of these variables as a function of a random state I_t that includes not only the complete history of the exogenous disturbances $(r_t^n, r_{t-1}^n, \ldots, r_0^n)$, but also all public information at date t about the future evolution of the natural rate—in order to minimize the criterion defined by (2.3) and (2.4), subject to the constraint that the processes satisfy equilibrium conditions (2.2) and (2.1) at all dates $t \geq 0$. We shall imagine in this calculation that a policymaker can choose the entire future (state-contingent) evolutions of these variables, once and for all, at date zero. Once this benchmark has been characterized, we can then consider the problem of implementation of such an optimal plan.

2.1. *Characterization of the optimal plan*

This sort of linear-quadratic optimization problem can be treated using methods that are by now familiar.[8] It is useful to write a Lagrangian of the form[9]

$$E\left\{\sum_{t=0}^{\infty} \beta^t \{(1/2)L_t + \phi_{1t}[x_t - x_{t+1} + \sigma(i_t - \pi_{t+1} - r_t^n)] + \phi_{2t}[\pi_t - \kappa x_t - \beta\pi_{t+1}]\}\right\}. \tag{2.5}$$

An optimal plan then must satisfy the first-order conditions

$$\pi_t - \beta^{-1}\sigma\phi_{1t-1} + \phi_{2t} - \phi_{2t-1} = 0, \tag{2.6}$$

$$\lambda_x x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} = 0, \tag{2.7}$$

$$\lambda_i i_t + \sigma\phi_{1t} = 0, \tag{2.8}$$

obtained by differentiating the Lagrangian with respect to π_t, x_t, and i_t respectively. Each of conditions (2.6)–(2.8) must hold at each date $t \geq 1$, and the same conditions also must hold at date $t = 0$, where however one adds the stipulation that

$$\phi_{1,-1} = \phi_{2,-1} = 0. \tag{2.9}$$

We may omit consideration of the transversality conditions, as we shall consider only bounded solutions to these equations, which necessarily satisfy the transversality conditions. A (bounded) optimal plan is then a set of bounded processes $\{\pi_t, x_t, i_t, \phi_{1t}, \phi_{2t}\}$ for dates $t \geq 0$, that satisfy (2.2), (2.1) and (2.6)–(2.8) at all of these dates, consistent with the initial conditions (2.9).

6. A welfare-theoretic justification for this objective function, in the context of the microfoundations of the structural model behind equations (2.1) and (2.2), is presented in Woodford (2003, Chapter 6).

7. Allowing for different target values would affect only the optimal long-run average values of the endogenous variables, and not the nature of optimal responses to shocks. Since our concern here is with stabilization issues, we abstract from any complications that may be involved in bringing about a desirable long-run average state.

8. See, *e.g.* Backus and Driffill (1986) for treatment of a general linear-quadratic problem. See Woodford (1999*b*) and Giannoni and Woodford (2003*a*) for further discussion of the optimal plan for this model.

9. Note that conditional expectations are dropped from the way in which the constraints are written inside the square brackets, because the expectation E at the front of the entire expression makes them redundant.

If the optimal plan is bounded (which is the only case in which our log-linear approximations to the model structural equations and our quadratic approximation to the social welfare function can be expected to accurately characterize it), one can show that this system of equations has a unique bounded solution of the form

$$z_t = G\phi_{t-1} - H\sum\nolimits_{j=0}^{\infty} \tilde{A}^{-(j+1)} a E_t r^n_{t+j}, \tag{2.10}$$

where $z'_t \equiv [\pi_t \ x_t \ i_t]$ is the vector of endogenous variables and $\phi'_t \equiv [\phi_{1t} \ \phi_{2t}]$ is the vector of Lagrange multipliers, for certain matrices of coefficients that depend on the model parameters.[10] The eigenvalues of the matrix $\tilde{A}$ lie outside the unit circle, so that the infinite sum converges in the case of any bounded process for the natural rate. The corresponding solution for the Lagrange multipliers is of the form

$$\phi_t = N\phi_{t-1} - C\sum\nolimits_{j=0}^{\infty} \tilde{A}^{-(j+1)} a E_t r^n_{t+j}, \tag{2.11}$$

where the eigenvalues of the matrix N lie inside the unit circle.[11] This property of the matrix N implies that (2.11) defines a bounded stochastic process for the multipliers ϕ_t, given any bounded process for the natural rate.

It is obvious that such an optimal plan will, in general, not be *time consistent*, in the sense discussed by Kydland and Prescott (1977). For a policymaker that solves a corresponding problem starting at some date $T > 0$ will choose processes for dates $t \geq T$ that satisfy equations (2.10) and (2.11), but starting from initial conditions

$$\phi_{1,T-1} = \phi_{2,T-1} = 0$$

corresponding to (2.9). Yet these last conditions will, in general, not be satisfied by the optimal plan chosen at date zero, according to solution (2.11) for the evolution of the Lagrange multipliers. This is why discretionary optimization leads to a different equilibrium outcome than the one characterized here.

The presence of the lagged Lagrange multipliers in (2.10) and (2.11) is also the reason why optimal policy cannot be implemented through any purely forward-looking procedure. These terms imply that the endogenous variables at date t—and in particular, the central bank's setting of the interest rate at that date—should not depend solely upon current and forecasted future values of the natural rate of interest. They should also depend upon the predetermined state variables ϕ_{t-1}, which represent an additional source of inertia in optimal monetary policy, independent of any inertia that may be present in the exogenous disturbance process r^n_t. The additional terms represent the way in which policy should deviate from what would be judged optimal simply taking into account the current outlook for the economy, in order to follow through upon *commitments* made at an earlier date. It is the desirability of the central bank's being able to credibly commit itself in this way that makes it desirable for monetary policy to be somewhat inertial.

The extent to which these equations imply inertial behaviour of the nominal interest rate can be clarified by writing a law of motion for the interest rate that makes no reference to the Lagrange multipliers. Let us suppose that the relevant information at date t about the future evolution of the natural rate can be summarized by an exogenous state vector s_t, with law of motion

$$s_{t+1} = Ts_t + \varepsilon_{t+1}, \tag{2.12}$$

10. Here G and H are 3×2 matrices, $\tilde{A}$ is 2×2, and a is 2×1.
11. Here N and C are 2×2 matrices.

where ε_{t+1} is a vector of exogenous disturbances unforecastable at t, and let the natural rate be given by some linear function of these states,

$$r_t^n = k' s_t. \tag{2.13}$$

Equation (2.11) can then be written in the form

$$\phi_t = N\phi_{t-1} + n s_t, \tag{2.14}$$

for a certain matrix of coefficients n.

The endogenous variable ϕ_{2t} can then be eliminated from the system of equations (2.14), yielding an equation with instead two lags of ϕ_{1t}. Then using (2.8) to substitute out ϕ_{1t}, we obtain the law of motion

$$Q(L) i_t = R(L) s_t \tag{2.15}$$

for the nominal interest rate, where

$$Q(L) \equiv \det[I - NL], \qquad R(L) \equiv -(\lambda_r \sigma)^{-1}[n_1' + (N_{12} n_2' - N_{22} n_1')L]. \tag{2.16}$$

(In this last expression, n_i' is the i-th row of the matrix n.) The degree of persistence in the *intrinsic* dynamics of the nominal interest rate under the optimal plan, unrelated to any persistence in the fluctuations in the exogenous states s_t is determined by the roots μ_i of the characteristic equation

$$Q(\mu) = 0,$$

which roots are just the eigenvalues of the matrix N. These roots are determined by factors independent of the dynamics of the exogenous disturbances. Thus it may be optimal for nominal interest rates to exhibit a great deal of persistence, regardless of the degree of persistence of the fluctuations in the natural rate.

Finally, we note that first-order conditions (2.6)–(2.8) imply not only that interest-rate fluctuations should exhibit persistence in an optimal equilibrium, but that conditioning current policy on the past level of the nominal interest may be an especially useful type of history dependence, if one's goal is to bring about optimal, or nearly optimal dynamic responses to shocks. We have seen that the optimal equilibrium involves dependence on the lagged Lagrange multipliers, for reasons, and to an extent, that are independent of the nature of the real disturbances and their assumed dynamics. But (2.8) also implies that in an optimal equilibrium, the multiplier ϕ_{1t} will co-vary perfectly with the level of the nominal interest rate, and again this relationship is independent of the nature of the real disturbances and of their dynamics. Hence dependence of policy in period t on i_{t-1} can be a substitute for explicit dependence on the value of the multiplier ϕ_{t-1} (a variable that has no meaning outside the context of the planning problem considered here), and dependence on i_{t-1} with an appropriate coefficient creates the desired type of history dependence not only in the case of the particular type of exogenous variation in the natural rate of interest considered in the numerical examples given below, but much more generally.[12]

12. Giannoni and Woodford (2003*a*) show that it is possible to implement optimal policy in a model of this kind through commitment to an interest-rate feedback rule, and Giannoni and Woodford (2003*b*) similarly show that it is possible to implement the optimal equilibrium through commitment to a history-dependent inflation targeting rule. In both cases, the policy rules that are proposed can be shown to implement an optimal equilibrium in the case of completely arbitrary (additive) real disturbances; the desire for robustness in this sense tightly constrains the type of relation among endogenous variables that represents a suitable policy commitment. And in both cases, the robustly optimal policy rule states a criterion that period t policy should satisfy that involves two lags of the nominal interest rate.

2.2. *A simple limiting case*

The extent to which the equations just derived imply behaviour that might appear to involve interest-rate "smoothing" can be clarified by considering a limiting case, in which a closed-form solution is possible. This is the limiting case in which the value of the parameter κ (the slope of the "short-run Phillips curve") approaches zero. In this limit, variations in output relative to potential cause no change in the level of real marginal cost, and firms accordingly have no reason to change their prices at any time. Hence $\pi_t = 0$ at all times, regardless of monetary policy. We shall assume that the values of all other parameters are unchanged.[13]

In this limiting case, the $\kappa\phi_{2t}$ term in (2.7) can be neglected, so that it becomes possible to solve for the variables x_t, i_t, and ϕ_{1t} using only equations (2.1), (2.7), and (2.8).[14] This system of equations can then be written in the form

$$\begin{bmatrix} E_t x_{t+1} \\ \phi_{1t} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_t \\ \phi_{1,t-1} \end{bmatrix} + \begin{bmatrix} a \\ 0 \end{bmatrix} r_t^n, \tag{2.17}$$

where

$$\begin{aligned} A &= 1 + \sigma^2\lambda_x/\lambda_i, \\ B &= -\sigma^2/(\beta\lambda_i), \\ C &= -\lambda_x, \\ D &= 1/\beta, \\ a &= -\sigma. \end{aligned}$$

The characteristic equation for this system is then

$$\mu^2 - [1 + \beta^{-1} + \sigma^2\lambda_x/\lambda_i]\mu + \beta^{-1} = 0. \tag{2.18}$$

One observes that it necessarily has two real roots, satisfying

$$0 < \mu_1 < 1 < \beta^{-1} < \mu_2,$$

and that $\mu_2 = (\beta\mu_1)^{-1}$. Because exactly one root is inside the unit circle, the system of equations (2.17) has a unique bounded solution. The solution is again of the form (2.10) and (2.11), where now G, N, and $\tilde{A}$ are scalars

$$\begin{aligned} G &= (\beta^{-1} - \mu_1)\lambda_x^{-1} > 0, \\ N &= \mu_1 > 0, \\ \tilde{A} &= \mu_2 > 1. \end{aligned}$$

Since equation (2.11) now involves only $\hat{\phi}_{1t}$, it is possible to use (2.8) to substitute i_t for ϕ_{1t}, and thus directly obtain an equation for the optimal interest-rate dynamics:

$$i_t = \mu_1 i_{t-1} + \sigma^2(\lambda_x/\lambda_i)\sum_{j=0}^{\infty} \mu_2^{-(j+1)} E_t r_{t+j}^n. \tag{2.19}$$

13. This does not necessarily make sense, if the coefficients in the loss function (2.4) are intended to represent an approximation to true social welfare, as suggested above, since in that case there is a theoretical relation between λ_x and the various preference and technology parameters that also determine κ. But even in that case, we may view the present calculation as solving a mathematical problem that is closely similar in form, even if not identical, to one that arises in an optimal monetary policy problem.

14. The intuition for this reduction in the order of the system of equations is simple. We no longer need to solve for π_t, as we set this variable equal to zero. We no longer obtain a first-order condition corresponding to (2.6) by differentiating the Lagrangian with respect to π_t, because it is not possible to vary inflation. Likewise, we no longer obtain a Lagrange multiplier ϕ_{2t} corresponding to constraint (2.2), as this constraint is already guaranteed to be satisfied once we have set π_t equal to zero at all times.

This gives us a law of motion of the form (2.15), but in this limiting case, a representation is possible in which $Q(L)$ is only of first order, and $R(L)$ is a constant (there are no lags at all). In fact, one can easily show that (2.19) is a partial-adjustment equation of the form

$$i_t = \theta i_{t-1} + (1-\theta)\bar{\imath}_t, \tag{2.20}$$

where the inertia coefficient $\theta = \mu_1$, and the time-varying interest-rate "target" is given by[15]

$$\bar{\imath}_t = (1-\mu_2^{-1})\sum\nolimits_{j=0}^{\infty} \mu_2^{-j} E_t r_{t+j}^n. \tag{2.21}$$

Thus the optimal interest-rate dynamics are described by partial adjustment toward a moving average of current and expected future natural rates of interest.

In the case that the natural rate is a simple first-order autoregressive process,

$$r_{t+1}^n = \rho r_t^n + \varepsilon_{t+1} \tag{2.22}$$

for some $0 \leq \rho < 1$, the target rate is just a function of the current natural rate of interest, although (because of expected mean reversion of the natural rate in the future) it varies less than does the natural rate itself. Specifically, we have

$$\bar{\imath}_t = k r_t^n, \tag{2.23}$$

where $k \equiv (\mu_2 - 1)/(\mu_2 - \rho)$, so that $0 < k < 1$. If the fluctuations in the natural rate are largely transitory, the elasticity k may be quite small, though it is any event necessarily greater than $1-\beta$. If the fluctuations in the natural rate are nearly a random walk (ρ is near one), the elasticity k instead approaches one. In this case, interest rates eventually change by nearly as much as the (nearly permanent) change that has occurred in the natural rate; but even in this case, the change in the level of nominal interest rates is delayed. As a result, an innovation in the natural rate is followed by a series of interest rate changes in the same direction, as in the characterizations of actual central-bank behaviour by Rudebusch (1995) and Goodhart (1996).

While this partial-adjustment representation of optimal interest-rate dynamics is only exactly correct in an unrealistic limiting case, it provides considerable insight into the optimal interest-rate responses in more realistic cases. This is shown through numerical analysis of a case with $\kappa > 0$ in the next section. It is also worth noting that in this special case, we have shown that the only form of history dependence required for optimality is dependence of period t policy on the value of $\phi_{1 \cdot t-1}$. But since ϕ_{1t} is always proportional to i_t in an optimal equilibrium (owing to (2.8)), it follows that dependence of policy on the previous period's level of the nominal interest rate introduces precisely the kind of history dependence required in order to implement an optimal equilibrium. An example of how this type of history dependence may be introduced into policy is offered in Section 4.2. Once again, this simple form of history dependence is shown to lead to fully optimal responses to disturbances only in the limiting case considered in this section. But numerical analysis shows that the same kind of history dependence can yield a fairly close approximation to optimal policy even when $\kappa > 0$.

3. THE VALUE OF INTEREST-RATE INERTIA

A central theme of this paper is the desirability of assigning to the central bank an objective which makes lagged nominal interest rates relevant to the bank's evaluation of possible current states. In order to show the need for an objective of that form, it is appropriate to consider first the degree to which responses similar to those associated with the optimal plan can be achieved

15. This representation is possible because (2.18) implies that $\sigma^2 \lambda_x / \lambda_i$ is equal to $(1-\mu_1)(\mu_2 - 1)$.

through choice of a suitable central-bank loss function that does *not* depend on any such lagged variables. To consider this question, we need not consider the Markov equilibria associated with alternative central-bank loss functions at all. Instead, we may simply consider what the best possible equilibrium would be like that can be achieved by *any* purely forward-looking decision procedure. To the extent that that pattern of responses to shocks—the optimal non-inertial plan—remains substantially inferior to the optimal plan in the absence of such a restriction, there are clear benefits to the introduction of history dependence of the proper sort into the central bank's decision procedures. Once this has been established, we may turn to the question of what form of history-dependent central-bank objective is needed.

3.1. *Evaluating alternative stabilization policies*

Before attempting to characterize the welfare gains from history-dependent policy, it is first necessary to discuss the criterion on which we propose to rank alternative policies. The expected discounted loss (2.3) can be decomposed as

$$E[W] = W^{\text{det}} + W^{\text{stab}},$$

where the *deterministic loss* is defined as

$$W^{\text{det}} \equiv \sum_{t=0}^{\infty} \beta^t \{E[\pi_t]^2 + \lambda_x E[x_t]^2 + \lambda_i E[i_t]^2\},$$

and the *stabilization loss* is defined as

$$W^{\text{stab}} \equiv \sum_{t=0}^{\infty} \beta^t \{\text{var}[\pi_t] + \lambda_x \text{var}[x_t] + \lambda_i \text{var}[i_t]\}, \tag{3.1}$$

in which expression the variances are also conditional upon the state at the time that policies are evaluated. Note that the first term depends only on the deterministic component of the economy's equilibrium path—the expected path prior to the realization of any of the disturbances in periods zero and later—while the second term depends only on the equilibrium responses to unexpected shocks in periods zero and later. Because (under our log-linear approximate characterization of the structural equations) there are no restrictions on the possible equilibrium evolution of the economy that connect these two aspects of equilibrium, it is useful to consider separately the contribution to welfare of each of the two aspects of equilibrium, and correspondingly, each of the two aspects of policy. Thus the question of what the long-run average inflation target should be can be considered solely with regard to the consequences of this decision for W^{det}, while the question of how the path of inflation should be affected by random disturbances should be considered from the point of view of minimizing W^{stab}.

Here we shall consider the specification of aspects of policy that affect the equilibrium responses to disturbances—for example, the way in which policy may be affected by the level of interest rates in the recent past—from the point of view of minimization of the stabilization loss W^{stab}. Thus we shall compute the implied value of W^{stab} under alternative policies, and seek to identify those policies that make this quantity as small as possible. We do not consider here the problem of choosing the proper deterministic component of policy, though in Section 4 we consider only policies that imply a long-run average inflation rate of zero and (as a consequence) an average output gap of zero as well.[16]

It is worth noting that this criterion is not universally adopted in the related literature. For example, Levine (1991) chooses an "optimal simple policy rule", from within a finitely

16. This assumption regarding the target values in the loss function assigned to the central bank is not restrictive as far as the achievable value of the stabilization loss is concerned. Grounds for choosing a time-invariant policy rule under which the deterministic component of policy is of this kind are discussed in Woodford (2003, Chapter 7).

parametrized family of linear rules, by computing the total expected loss $E[W]$ in the equilibrium determined by a commitment to follow the rule from period zero onward, conditional on the economy's state at the time that the rule is chosen. The application of this criterion to our problem would yield somewhat different conclusions, for example with regard to the optimal loss function to assign to the central bank from among one of the simple families considered in Section 4. The reason is that when candidate policies are restricted to a simple family of time-invariant rules, parameters that affect the equilibrium responses to disturbances—for example, a coefficient that determines the degree of interest-rate inertia—may also affect the deterministic component of equilibrium, when expectations are evaluated conditional upon initial conditions, such as the initial level of interest rates. Use of this criterion would not only mean that the degree of interest-rate inertia chosen would not be selected solely in order to minimize W^{stab}; it also implies that the "optimal simple policy rule" will differ depending on the initial level of interest rates, so that the choice will not be time-consistent. Instead, choice of a linear relation among fluctuations in endogenous and exogenous variables so as to minimize W^{stab} results in the same selection regardless of initial conditions, and hence a time-consistent choice.

The criterion used by Jensen and McCallum (2002) to evaluate alternative policy rules avoids this sort of dependence on initial conditions by selecting the rule that minimizes an *unconditional* expectation $E[W]$, rather than conditioning on initial conditions at the time of the decision. (Their criterion thus amounts to minimization of the unconditional expectation $E[L]$, making the degree of discounting irrelevant.) But the Jensen–McCallum criterion penalizes policies that create greater volatility of inflation, output or interest rates for the fact that—when one integrates over all possible initial conditions under the stationary distribution implied by the equilibrium dynamics under that policy—the *initial conditions* are assumed to be more volatile under such a policy, and not simply for the greater variability in the responses to subsequent shocks. This is not a fair comparison among the alternative policies that might be pursued from a given date onward, taking as given the economy's initial state at the time of the policy choice; the "optimal" pattern of responses to shocks as judged by Jensen and McCallum differs from the one characterized in Section 2 in the same way as the "golden rule" steady-state capital stock in growth theory differs from the "modified golden rule" capital stock that actually characterizes optimal capital accumulation (with discounting of utility).[17]

An important advantage of the criterion proposed here, apart from its time consistency, is the fact that if the class of policies considered is flexible enough to allow the kind of responses to shocks that occur under the optimal state-contingent evolution characterized in Section 2, then those responses will be selected as optimal. That is, the responses to shocks implied by (2.10) and (2.11) are those that minimize W^{stab}. This avoids the potential embarrassment of using a criterion to select among various constrained families of "simple" policies that, when also used to evaluate the unconstrained optimal policy, could rank one of the other policies as better than the unconstrained optimum.

3.2. *The optimal non-inertial plan*

By a purely forward-looking procedure we mean one that makes the bank's policy decision a function solely of the set of possible equilibrium paths for the economy from the present date onward. In a Markov equilibrium associated with any such procedure, the endogenous variables must be functions *only* of the state vector s_t. Hence we may proceed by optimizing over possible

17. The Jensen–McCallum criterion leads to no different ranking than that used here in the case of purely forward-looking policies of the kind considered in Section 3.2. But it leads to different rankings among alternative forms of history-dependent policies.

state-contingent evolutions of the economy that satisfy this restriction. We call the optimal pattern of responses to disturbances subject to this restriction the *optimal non-inertial plan.*[18]

We shall simplify by here considering only the case in which the natural rate evolves according to (2.22). In this case, non-inertial plans are those in which each endogenous variable y_t is a time-invariant linear function[19] of the current natural rate of interest,

$$y_t = f_y r_t^n. \tag{3.2}$$

Substituting the representation (3.2) for each of the variables $y = \pi, x, i$ into (2.2)–(2.1), we find that feasible non-inertial plans correspond to coefficients f_y that satisfy

$$(1-\beta\rho) f_\pi = \kappa f_x, \tag{3.3}$$

$$(1-\rho) f_x = -\sigma(f_i - 1 - \rho f_\pi). \tag{3.4}$$

Among these plans, we seek the one that minimizes W^{stab}, the stabilization loss defined in (3.1).

Given our restriction to non-inertial plans, minimization of $E[W]$ is equivalent to minimization of $E[L]$, the unconditional expectation of the period loss (2.4). Thus we seek to minimize

$$E[L] = [f_\pi^2 + \lambda_x f_x^2 + \lambda_i f_i^2]\text{var}(r^n), \tag{3.5}$$

subject to the linear constraints (3.3) and (3.4). The first-order conditions for optimal choice of the f_y imply that

$$f_i = \frac{\kappa(1-\beta\rho)^{-1} f_\pi + \lambda_x f_x}{[(1-\rho)\sigma^{-1} - \rho\kappa(1-\beta\rho)^{-1}]\lambda_i}. \tag{3.6}$$

This condition along with (3.3) and (3.4) determines the optimal response coefficients.

3.3. *A numerical example*

To consider what degree of interest-rate inertia might be optimal in practice, it is useful to consider a numerical example, "calibrated" to match certain quantitative features of the Rotemberg and Woodford (1997, 1999) analysis of optimal monetary policy for the U.S. economy.[20] The numerical values that we shall use are given in Table 1. We assume an AR(1) process for the fluctuations in the natural rate of interest as in (2.22), so that we need only calibrate a single parameter ρ. The value of most interest here is unclear, since in reality there are probably different types of disturbances, with differing degrees of persistence.[21] The value considered here,[22] $\rho = 0{\cdot}35$, however, is chosen so as to imply, in the simpler model used here, a degree of concern for reduction of interest-rate variability similar to that obtained by Rotemberg and Woodford in their estimated model.

For the parameter values in Table 1, the matrix N is given by

$$N = \begin{bmatrix} 0{\cdot}4611 & 0{\cdot}0007 \\ -0{\cdot}7743 & 0{\cdot}6538 \end{bmatrix},$$

18. This is essentially what Clarida *et al.* (1999, Section 4.2.1) call (in the context of a simpler model) "the optimum within a simple family of policy rules (that includes the optimal rule under discretion)". For further discussion of the general concept of the optimal non-inertial plan, see Woodford (2003, Chapter 7). For characterization of the optimal non-inertial plan for an extension of the present model that allows for "cost-push" shocks, see Giannoni and Woodford (2003*a*).

19. The omission of a constant term implies that we consider only policies consistent with a long-run average inflation rate of zero, as noted in the previous section. Allowance for non-zero constant terms would not affect the value of the stabilization loss W^{stab}, though the deterministic component of policy is obviously relevant to the other component of our welfare measure, W^{det}.

20. Details of the justification for this calibration are set out in Woodford (1999*b*).

21. The shock process estimated by Rotemberg and Woodford has two innovations per period and complicated dynamics. See Appendix 2 of their NBER working paper.

22. See Woodford (1999*b*) for numerical analysis of cases with values of ρ both smaller and larger than this.

TABLE 1

"Calibrated" parameter values

Structural parameters	
β	0·99
σ^{-1}	0·157
κ	0·024
Shock process	
ρ	0·35
sd(r^n)	3·72
Loss function	
λ_x	0·048
λ_i	0·236

and its eigenvalues are found to be approximately 0·65 and 0·46. Both of these are substantial positive quantities, suggesting that once interest rates are perturbed in response to some shock, it should take several quarters for them to be restored to nearly their normal level, even if the shock is completely transitory.

Figure 1 illustrates this by showing the optimal responses of inflation and the output gap, as well as of the short-term nominal interest rate, to a unit positive innovation ε_t in the natural-rate process. This might be due either to a temporary increase in the autonomous component of spending, $\hat{G}_t$, or to a temporary decrease in the natural rate of output due to an adverse "supply shock". Either type of shock would imply a temporary increase in the equilibrium real rate of interest in a flexible-price model, and, in our model with sticky prices, will increase both the output gap and inflation,[23] in the absence of an offsetting adjustment of monetary policy. The natural rate of interest is made higher by $(0{\cdot}35)^j$ percentage points in quarter $t + j$ by this disturbance. The figure shows the dynamic responses of inflation, output and nominal interest rates in quarters $t + j$, for $j = 0$ through 10, both under the optimal plan and under the optimal non-inertial plan.

Under the optimal non-inertial plan (dash–dot lines in the figure), the nominal interest rate is raised in response to the real disturbance, but only by about two-thirds the amount of the increase in the natural rate. As a result, monetary policy does not fully offset the inflationary pressure created by the disturbance, and both inflation and the output gap increase;[24] this is optimal within the class of non-inertial policies because it involves less interest-rate variability than would be required to completely stabilize inflation and the output gap (by perfectly tracking the variations in the natural rate). Because the policy is non-inertial, inflation, the output gap, and the nominal interest rate all decay back to their long-run average values at exactly the same rate as the real disturbance itself decays, *i.e.* in proportion to $(0{\cdot}35)^j$.

Under the fully optimal plan (solid lines in the figure), instead, the nominal interest rate is raised by less at the time of the shock. But the increase is more *persistent* than is the disturbance to the natural rate of interest, so that policy is expected to be tighter under this policy than under the optimal non-inertial plan from quarter $t + 2$ onward. Thus interest rates are more inertial under the optimal plan, both in the sense that the central bank is slow to raise rates when the

23. Note that an adverse supply shock *lowers* output, but increases the output *gap,* as sticky prices prevent output from falling as much as it would in the case of fully flexible prices.

24. See Woodford (2003, Chapter 4) for discussion of the effects on inflation and output of fluctuations in the natural rate of interest in a model like this one.

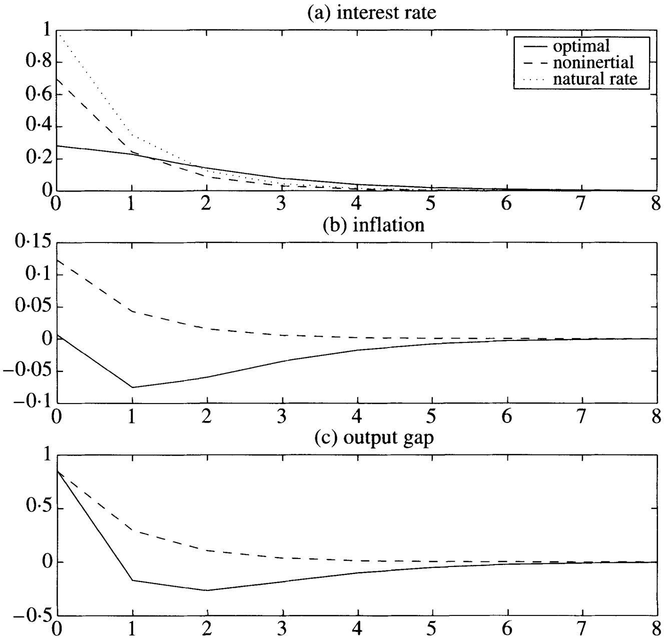

FIGURE 1

Impulse responses under fully optimal policy and under the optimal non-inertial plan

natural rate unexpectedly increases, and also in the sense that it is slow to bring them back down when the natural rate returns to its normal level.

The advantages of more inertial adjustment of the interest rate can be seen in the other two panels of the figure. Despite the gentler immediate interest-rate response, the initial increase in the output gap is no greater under this policy, because spending is restrained by the anticipation of tight policy farther into the future; and the output gap returns to its normal level much more rapidly under this policy, as interest rates are kept relatively high despite the decay of the natural rate back toward its normal level. Because the output stimulus is expected to be short lived (the output gap is actually expected to undershoot its normal level by the quarter after the shock) the increase in inflation resulting from the shock is minimal under the optimal policy. Thus monetary policy is as successful at stabilizing inflation and the output gap under this policy as under the optimal non-inertial plan (actually, somewhat more successful overall), yet the desired result is achieved with much less variability of interest rates, owing to a commitment to adjust them in a smoother way.

Statistics regarding the variability of the various series under the two plans are reported in Table 2. Here independent drawings from the same distribution of shocks ε_t are assumed to occur each period,[25] and infinite-horizon stochastic equilibria are computed under each policy. The measure of variability reported for each variable z_t is (the square root of)

$$V[z] \equiv (1-\beta)\sum_{t=0}^{\infty} \beta^t \operatorname{var}[z_t], \tag{3.7}$$

25. The assumed variance of the shocks ε_t is chosen so as to imply a standard deviation for the natural rate of interest of the size indicated in Table 1. This is irrelevant, of course, for the comparisons of *relative* variances between the two regimes, or comparison of the relative size of expected losses $E[W]$.

TABLE 2

Statistics for alternative policies

Policy	$V[\pi]^{1/2}$	$V[x]^{1/2}$	$V[i]^{1/2}$	W^{stab}
Discretion	0·698	4·79	2·01	2·547
Non-inertial	0·459	3·15	2·59	2·279
Optimal	0·361	3·26	1·39	1·097

where the variances refer to the distribution conditional upon the economy's state prior to the realization of the period zero shocks. Except for the discounting, $V[z]$ corresponds to the unconditional variance of z_t, and in the case of non-inertial plans, it is equal to the unconditional variance regardless of the value of β. In the case of history-dependent plans, the discounted measure is of greater interest, because our stabilization objective W^{stab} is in that case just a weighted sum of squares of the previous three columns. (The variability measure reported is $V[z]^{1/2}$, which bears a similar relation to the unconditional standard deviation of the series in question, so that the units are percentage points of variation.)

Note that the optimal plan minimizes W^{stab} among all possible plans consistent with the structural equations, while the optimal non-inertial plan minimizes W^{stab} among all non-inertial plans; thus there is a necessary ranking of the stabilization loss between the two plans. For purposes of comparison, the table also presents statistics for the Markov equilibrium resulting from discretionary minimization of the true social loss function.[26] Since this is an example of a purely forward-looking policy, there is also a necessary ranking of the values of W^{stab} between this equilibrium and the optimal non-inertial plan as well.

The table shows that for the calibrated parameter values, there is a substantial gain from commitment to an inertial policy, relative to the best possible non-inertial policy. This is primarily due to the lower volatility of nominal interest rates under the optimal plan ($V[i]$ is reduced by more than 70%), although the central bank's other stabilization goals are better served as well ($V[\pi] + \lambda_x V[x]$ is also reduced).

Visual inspection of the optimal interest-rate dynamics in Figure 1 suggests that partial adjustment of the nominal interest rate toward a level determined by the current natural rate of interest, just as in the limiting case analysed in Section 2.2, gives a reasonable approximation to optimal interest-rate dynamics. The reason for this is not hard to see. Note that the element N_{12} of the matrix N is quite small; it is three orders of magnitude smaller than the other elements of the matrix. Hence one would not obtain too poor an approximation to the optimal interest-rate dynamics by setting N_{12} equal to zero in expression (2.15). But in that case, $Q(L)$ and $R(L)$ reduce to

$$Q(L) = (1 - N_{11}L)(1 - N_{22}L),$$
$$R(L) = -(\sigma/\lambda_i)n_1'(1 - N_{22}L),$$

as a result of which (2.15) is equivalent to

$$i_t = N_{11}i_{t-1} - (\sigma/\lambda_i)n_1' s_t. \tag{3.8}$$

This implies partial adjustment of the interest rate, as in equation (2.20), toward a time-varying "target" interest rate equal to

$$\bar{\imath}_t = -(\sigma/\lambda_i)(1 - N_{11})^{-1}n_1' s_t, \tag{3.9}$$

26. See Woodford (1999*a*) for further discussion of the differences between the optimal plan and the outcome of discretionary policy.

with an inertia coefficient of $\theta = N_{11}$. In the case that the natural rate evolves according to (2.22), this time-varying target rate is again described by an equation of the form (2.23). In our numerical example, the target rate is given by $\bar{\imath}_t = 0{\cdot}52 r_t^n$, while the inertia coefficient is equal to $\theta = 0{\cdot}46$, indicating that interest rates should be adjusted only about half of the way toward the current target level (implied by the natural rate) within the quarter.

4. ADVANTAGES OF A CENTRAL-BANK SMOOTHING OBJECTIVE

We turn now to the question of the type of objective that should be assigned to the central bank in order to bring about equilibrium responses to shocks similar to those associated with the optimal plan.[27] We thus wish to address what is sometimes called the problem of *optimal delegation* of authority to conduct monetary policy. In such an analysis, one asks what objective the central bank should be charged with, understanding that the details of the pursuit of the goal on a day-to-day basis should then be left to the bank, and expecting that the bank will then act as a discretionary minimizer of its assigned loss function.[28] Our results in the previous section suggest that equilibrium responses to shocks can be improved if the central bank is assigned an interest-rate *smoothing* objective, leading to partial-adjustment dynamics for the bank's interest-rate instrument, even though the lagged nominal interest rate is irrelevant to both the true social objective function (2.4) and the structural equations of our model.

4.1. *Markov equilibrium with a smoothing objective*

Let us consider the consequences of delegating the conduct of monetary policy to a central banker that is expected to seek to minimize the expected value of a criterion of the form (2.3), where however (2.4) is replaced by a function of the form

$$L_t^{cb} = \pi_t^2 + \hat{\lambda}_x x_t^2 + \hat{\lambda}_i i_t^2 + \lambda_\Delta (i_t - i_{t-1})^2. \tag{4.1}$$

Here we allow the weights $\hat{\lambda}_x$, $\hat{\lambda}_i$ to differ from the weights λ_x, λ_i associated with the true social loss function. We also allow for the existence of a term that penalizes interest-rate *changes*, not present in the true social loss function (2.4).

The time-consistent optimizing plan associated with such a loss function can be derived using familiar methods, expounded for example in Soderlind (1998). Note that the presence of a term involving the lagged interest rate in the period loss function (4.1) means that even in a Markov equilibrium, outcomes will depend on the lagged interest rate. This in turn means that the central bank's expectations at date t about equilibrium in periods $t + 1$ and later are not independent of its choice of i_t. However, in such an equilibrium, the central bank's value function in period t is given by a function $V(i_{t-1}; r_t^n)$, which function is time invariant. (Here we simplify by assuming that the natural rate of interest is itself Markovian, with law of motion (2.22), though we could easily generalize our results to allow for more complicated linear state-space models.)

Standard dynamic programming reasoning implies that the value function must satisfy the Bellman equation[29]

$$V(i_{t-1}; r_t^n) = \min_{(i_t, \pi_t, x_t)} E_t\{\tfrac{1}{2}[\pi_t^2 + \hat{\lambda}_x x_t^2 + \hat{\lambda}_i i_t^2 + \lambda_\Delta (i_t - i_{t-1})^2] + \beta V(i_t; r_{t+1}^n)\}, \tag{4.2}$$

27. This is not, of course, the only way that one might seek to bring about the desired type of equilibrium responses. See Woodford (1999*b*), Giannoni and Woodford (2003*a*,*b*) and Svensson and Woodford (2003) for discussion of alternative approaches in the context of similar models.

28. Alternatively, the question is sometimes framed as asking what *type* of central banker (or monetary policy committee) should be appointed, taking it as given that the central banker will seek to maximize the good as he or she personally conceives it, again optimizing under discretion.

29. Once again, multiplication of the central bank's loss function by a factor 1/2 here eliminates a factor of 2 from subsequent expressions such as (4.6), and is purely a normalization of the value function.

where the minimization is subject to the constraints

$$\pi_t = \kappa x_t + \beta E_t[\pi(i_t; r^n_{t+1})],$$
$$x_t = E_t[x(i_t; r^n_{t+1}) - \sigma(i_t - r^n_t - \pi(i_t; r^n_{t+1}))].$$

Here the functions $\pi(i_t; r^n_{t+1}), x(i_t; r^n_{t+1})$ describe the equilibrium that the central bank *expects* to occur in period $t+1$, conditional upon the exogenous state r^n_{t+1}. This represents the consequence of discretionary policy at that date and later, that the current central banker regards him or herself as unable to change. Similarly, $V(i_t; r^n_{t+1})$ represents the value expected for the central bank's objective as of date $t+1$, in the discretionary equilibrium described by those functions.

Discretionary optimization by the central bank at date t is then defined by the minimization problem on the R.H.S. of (4.2). The solution to this problem, for any given current state $(i_{t-1}; r^n_t)$, defines a set of functions $i(i_{t-1}; r^n_t)$, $\pi(i_{t-1}; r^n_t)$, $x(i_{t-1}; r^n_t)$, indicating the optimal values of i_t, π_t, and x_t, and a function $V(i_{t-1}; r^n_t)$ indicating the minimized value of the R.H.S. of the equation. Consistency of the central bank's expectations then requires that the functions V, π, x used to *define* this minimization problem are identical to the functions obtained as its *solution*.

We shall furthermore restrict attention to solutions of the Bellman equation in which the value function is a *quadratic* function of its arguments, and the solution functions for i, π, and x are each linear functions of their arguments. The solution functions can accordingly be written

$$i(i_{t-1}; r^n_t) = i_i i_{t-1} + i_n r^n_t, \tag{4.3}$$
$$\pi(i_{t-1}; r^n_t) = \pi_i i_{t-1} + \pi_n r^n_t, \tag{4.4}$$
$$x(i_{t-1}; r^n_t) = x_i i_{t-1} + x_n r^n_t, \tag{4.5}$$

where i_i, i_n, and so on are constant coefficients to be determined by solving a fixed-point problem. The first-order conditions for the optimization problem in (4.2) involve the partial derivative of the value function with respect to the lagged interest rate. This too must be a linear function of its arguments. In fact, differentiation of (4.2) using the envelope theorem implies that when the value function is defined, the partial derivative with respect to its first argument must satisfy

$$V_1(i_{t-1}; r^n_t) = \lambda_\Delta[i_{t-1} - i(i_{t-1}; r^n_t)]. \tag{4.6}$$

Thus linearity of the solution function i guarantees the linearity of this function as well.

We turn now to the fixed-point problem for the constant coefficients in the solution functions. First of all, substitution of the assumed linear solution functions into the two constraints following (4.2), and using

$$E_t r^n_{t+1} = \rho r^n_t, \tag{4.7}$$

allows us to solve for x_t and π_t as linear functions of i_t and r^n_t. (Let the coefficients on i_t in the solutions for x_t and π_t be denoted X_i and Π_i respectively. These coefficients are themselves linear combinations of the coefficients x_i and π_i introduced in (4.4) and (4.5).) Requiring the solution functions defined in (4.3)–(4.5) to satisfy these linear restrictions yields a set of four nonlinear restrictions on the coefficients x_i, x_n, and so on.[30]

Substituting these solutions for x_t and π_t into the R.H.S. of (4.2), the expression inside the minimization operator can be written as a function of i_t and r^n_t. This expression is quadratic in i_t, and so it achieves a minimum if and only if it is convex, in which case the optimum is characterized by the first-order condition obtained from differentiation with respect to i_t. Because the function is quadratic, it is globally convex if and only if the second-order condition

30. The restrictions are nonlinear because the coefficients X_r and so on are themselves functions of the coefficients x_r and so on.

is satisfied; thus a solution satisfying the first- and second-order conditions is both necessary and sufficient for optimality.

Substituting (4.6) for the derivative of the value function, the first-order condition may be written as

$$\Pi_i \pi_t + \hat{\lambda}_x X_i x_t + \hat{\lambda}_i i_t + \lambda_\Delta (i_t - i_{t-1}) + \beta \lambda_\Delta (i_t - E_t i_{t+1}) = 0. \tag{4.8}$$

Substituting into this the above solutions for π_t and x_t as functions of i_t and r_t^n, and the assumed solution (4.3) for i_{t+1} as a function of i_t and r_{t+1}^n, we see that the second-order condition may be written as

$$\Omega \equiv \Pi_i^2 + \hat{\lambda}_x X_i^2 + \hat{\lambda}_i + \lambda_\Delta + \beta \lambda_\Delta (1 - i_i) \geq 0. \tag{4.9}$$

Now requiring that the solutions defined in (4.3)–(4.5) always satisfy the linear equation (4.8) gives us another set of two nonlinear restrictions on the constant coefficients of the solution functions. We thus have a set of six nonlinear equations to solve for the six coefficients of equations (4.3)–(4.5). A set of coefficients satisfying these equations, and also satisfying the inequality (4.9), represent a linear Markov equilibrium for the central bank objective (4.1).

We shall as usual be interested solely in the case of a *stationary* equilibrium, so that fluctuations in i_t, π_t, and x_t are bounded if the fluctuations in r_t^n are bounded.[31] Given (4.3)–(4.5) and the assumption of stationary fluctuations in r_t^n, it is clear that π_t and x_t will be stationary processes as long as i_t is. It is also obvious that i_t will be stationary (bounded) if and only if

$$|i_i| < 1. \tag{4.10}$$

Thus we are interested in solutions to the nine nonlinear equations that satisfy both inequalities (4.9) and (4.10).

In the case that $\hat{\lambda}_x, \hat{\lambda}_i, \lambda_\Delta \geq 0$, it will be observed that (4.10) implies condition (4.9), so that we need not concern ourselves with the convexity issue in that case. However, non-negativity of these weights in the central-bank objective is not *necessary* for convexity of the central bank's optimization problem, and it is of some interest to consider delegation to a central banker with a *negative* weight on some term. In particular, we shall see that there are advantages to delegation to a central banker with $\hat{\lambda}_i < 0$, while $\lambda_\Delta > 0$: the central banker dislikes large interest-rate *changes* of either sign, but actually *prefers* for interest rates to deviate farther their steady-state level (which in such a case is not properly speaking a "target" level!). Such preferences need not result in a violation of convexity, though the negative $\hat{\lambda}_i$ term makes it harder for (4.9) to be satisfied. Nonetheless, the condition will still be satisfied as long as the other four terms together outweigh the negative $\hat{\lambda}_i$ term.[32]

We turn now to the question of what loss function the central bank should be assigned to minimize, if a Markov equilibrium of this kind is assumed to result from delegation of such an objective. We first note that setting $\lambda_\Delta > 0$ results in inertial interest-rate responses to fluctuations in the natural rate. If we take the partial derivative of the L.H.S. of (4.8) with respect to i_{t-1}, using the solution functions to express all terms as functions of i_{t-1} and r_t^n, we obtain a coefficient equal to $\Omega i_i - \lambda_\Delta$, where Ω is defined in (4.9). The first-order condition (4.8) thus implies that

$$\Omega i_i = \lambda_\Delta$$

31. Once again, this is the case in which our linear-quadratic approximations are justifiable in terms of a Taylor series approximation to the exact conditions associated with private-sector optimization, in the case of small enough exogenous disturbances.

32. Note that the conditions required for convexity of the bank's objective at date t, when it *takes as given* equilibrium outcomes from date $t+1$ onward, are much weaker than the conditions that would be required for convexity of its objective if it viewed itself as being able to commit itself at date t to any state-contingent plan from that date onward that was consistent with the structural equations (2.1) and (2.2).

in any solution. Then if $\lambda_\Delta > 0$, both Ω and i_i must be non-zero, and of the same sign. The second-order condition (4.9) then implies that in any equilibrium, both quantities must be positive. Combining this result with (4.10), we conclude that in any stationary equilibrium,

$$0 < i_i < 1. \tag{4.11}$$

Given this, the law of motion (4.3) for the nominal interest rate implies partial adjustment toward a time-varying target that is a linear function of the current natural rate of interest. Since this is at least a rough characterization of a way in which the optimal responses to shocks differ from the time-consistent responses when the central bank seeks to minimize true social losses, it is plausible that delegating monetary policy to a central banker who believes it is better to reduce the variability of interest-rate changes can improve social welfare.

One may wonder whether it is possible to choose the weights in the central bank's loss function so as to completely eliminate the distortions associated with discretion, and achieve the same responses as under an optimal commitment. It should be immediately apparent that it is not in general possible to achieve this outcome exactly. For we have shown in Section 2.1 that the optimal interest-rate dynamics have a representation of the form (2.15), where in general $Q(L)$ is of second order and $R(L)$ is of first order; thus they generally do not take a form as simple as (4.3). Nonetheless, we can show that exact implementation of the optimal plan is possible at least in a limiting case. And we can also show that it is possible to achieve a pattern of responses nearly as good as the optimal plan, in the calibrated numerical example of Section 3.2. These points are taken up in succession in the next two subsections.

4.2. *Optimal delegation in a limiting case*

Here we consider again the limiting case with $\kappa = 0$ taken up in Section 2.2. We have shown there that in this special case, the optimal interest-rate and output dynamics do take the form given by (4.3) and (4.5), where the coefficients (neglecting the constant term) are given by

$$i_i = \mu_1, \qquad i_n = (1-\mu_1)\left(\frac{\mu_2 - 1}{\mu_2 - \rho}\right), \tag{4.12}$$

$$x_i = -\frac{\beta^{-1} - \mu_1}{\sigma}\frac{\lambda_i}{\lambda_x}, \qquad x_n = \frac{\sigma}{\mu_2 - \rho}. \tag{4.13}$$

Here μ_1, μ_2 refer to the two roots of (2.18) discussed earlier. The question that we wish to ask, then, is whether it is possible to choose the weights in (4.1) so that the optimal values (4.12) and (4.13) solve the equilibrium conditions just derived. Since the optimal values necessarily satisfy the conditions required for consistency with the structural equation (2.1), it suffices that they also be consistent with conditions (4.8) and (4.9) for time-consistent optimizing behaviour on the part of the central bank.

In this limiting case, the conditions required for consistency of (4.3) and (4.5) with (4.8) simplify to

$$(x_i - \sigma)\hat{\lambda}_x x_i + \hat{\lambda}_i i_i + (1 - \beta i_i)\lambda_\Delta(i_i - 1) = 0,$$
$$(x_i - \sigma)\hat{\lambda}_x x_n + \hat{\lambda}_i i_n + (1 + \beta(1 - i_i - \rho))\lambda_\Delta i_n = 0.$$

These two equations depend only upon the ratios of the weights in the policy objective, $\hat{\lambda}_i/\hat{\lambda}_x$ and $\lambda_\Delta/\hat{\lambda}_x$, rather than upon the absolute size of the three weights. (This is because inflation variations are negligible under any policy regime, so the relative weight on inflation variability no longer matters.) Hence we may, without loss of generality, suppose that $\hat{\lambda}_x = \lambda_x$, the weight in the true social objective function. Using this simplification, and substituting the optimal values

(4.12) and (4.13), the above two linear equations can be solved for the unique values of $\hat{\lambda}_i$ and λ_Δ consistent with the optimal equilibrium responses. These are given by

$$\lambda_\Delta = \lambda_i \frac{\lambda_i(\beta^{-1} - \mu_1) + \lambda_x \sigma^2}{(1 - \beta\rho\mu_1)\beta\lambda_x\sigma^2} > 0, \tag{4.14}$$

$$\hat{\lambda}_i = -(1 - \beta\rho)(1 - \beta\mu_1)\lambda_\Delta < 0. \tag{4.15}$$

While one finds that the kind of partial-adjustment interest-rate dynamics associated with the optimal plan do require $\lambda_\Delta > 0$, as conjectured, one finds that they cannot be exactly matched through delegation to a central banker with discretion unless in addition $\hat{\lambda}_i < 0$. This is another difference between the best objective for the central banker and the true social objective function, since in the latter $\lambda_i > 0$. As noted earlier, a negative value for $\hat{\lambda}_i$ does not necessarily imply violation of the convexity condition (4.9) needed for central-bank optimization. In fact, we have shown above that the convexity condition holds in the case of any solution to the first-order condition with $\lambda_\Delta > 0$ and $i_i > 0$. As (4.12) implies that $i_i > 0$, and (4.14) implies that $\lambda_\Delta > 0$, the above assumed central-bank objective does result in a convex optimization problem for the central bank. Thus the optimal pattern of responses to shocks can in this case be supported as an equilibrium outcome under discretion, as long as the central bank is charged with pursuit of an objective that involves interest-rate smoothing.

4.3. *Optimal delegation in a numerical example*

When $\kappa > 0$, assignment of an objective from the simple class (4.1) does not suffice to implement the precise optimal plan characterized in Section 2. Nonetheless, it is possible to achieve quite a good approximation to the optimal pattern of responses to shocks, in the case of plausible parameter values. We demonstrate this through numerical analysis, using the same calibrated parameter values as earlier.

Specifically, we assume that the parameters of the structural equations and the shock process are as specified in Table 1. But we now assume a central-bank loss function of the form (4.1), and consider how the Markov equilibrium associated with discretionary optimization varies with the assumed weights in that loss function. To begin, we shall assume that $\hat{\lambda}_x = \lambda_x = 0{\cdot}048$ (the value in Table 1), and consider only the consequences of variation in $\hat{\lambda}_i$ and λ_Δ.

We first note that the nonlinear equations referred to above do not always have a unique solution for the coefficients i_i, i_n, and so on. It can be shown that given a value for i_i consistent with these equations, a unique solution can be obtained, generically, for the other coefficients. However, i_i solves a quintic equation, which equation may have as many as five real roots. For example, Figure 2 plots the solutions to this equation, as a function of $\hat{\lambda}_i$, in the case that $\lambda_\Delta = 0$. One solution is always $i_i = 0$, which is the unique Markov equilibrium, since when $\lambda_\Delta = 0$, the lagged nominal interest rate is an irrelevant state variable. However, for small non-zero values of λ_Δ, the graph of the solutions is similar, and all solutions count as Markov equilibria. One observes that there is a unique real root, $i_i = 0$, in the case of any $\hat{\lambda}_i > 0$; but for $\hat{\lambda}_i < 0$, there are multiple solutions, and given the results of the previous sub-section, we are interested in considering loss functions of this kind.

In the figure, solutions that also satisfy conditions (4.9) and (4.10), and so correspond to stationary equilibria, are indicated by solid lines, while additional branches of solutions that do not correspond to stationary equilibria are indicated by dashed lines.[33] We observe that while

33. Technically, in the case shown in the figure, because $\lambda_\Delta = 0$, the second-order condition is (weakly) satisfied even by solutions in which $i_i < 0$. But our real interest is in the set of solutions that exist for small positive values of λ_Δ. The solutions shown in Figure 2 with $i_i < 0$ also correspond to solutions with $i_i < 0$ in the case of small positive λ_Δ,

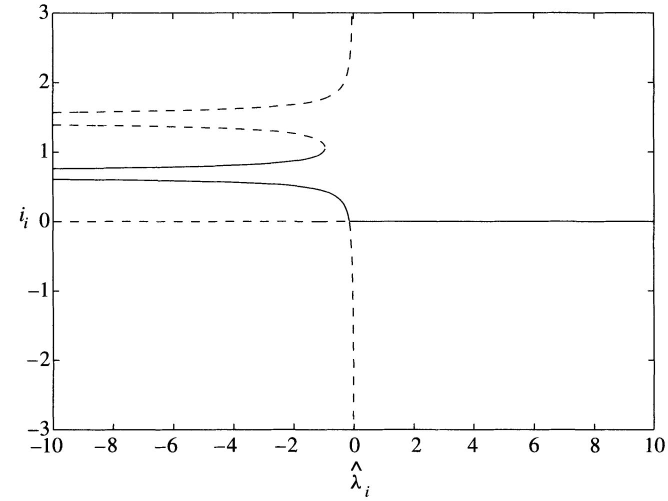

FIGURE 2
Solutions for i_i as a function of the weight on the interest-rate stabilization objective

there exist multiple solutions to the nonlinear equations for all $\hat{\lambda}_i < 0$, there is still a unique stationary equilibrium involving optimization under discretion for all $\hat{\lambda}_i > -1$.[34] Only for even larger negative values do we actually have multiple stationary Markov equilibria. The same turns out to be true for $\lambda_\Delta > 0$ as well, at least in the case of the moderate values of λ_Δ that we shall consider here. (For very high values of $\lambda_\Delta > 0$, not shown in the figures below, there exist multiple equilibria even for higher values of $\hat{\lambda}_i$.) Note that our outcome under discretion is essentially a Nash equilibrium in a game played by successive central bankers, rather than the solution to an optimization problem. Thus there is nothing paradoxical about the possible existence of multiple solutions.

We turn now to a consideration of how the properties of the stationary Markov equilibrium vary with the parameters $\hat{\lambda}_i$ and λ_Δ. In each of Figures 3–5, the white region indicates the set of loss function weights for which there is a unique stationary equilibrium of the linear form characterized above. In this region, the contour lines plot properties of this equilibrium. The grey region indicates weights for which there are multiple stationary equilibria. Here we plot the values associated with the best of these equilibria, the one with the lowest value of W^{stab}. As it turns out, the best equilibrium that is attainable corresponds to weights in the white region, so that we do not have to face the question of whether one should choose weights that are consistent with *one* good equilibrium but also with other bad ones.

Figure 3 shows how the inertia coefficient i_i in representation (4.3) of the equilibrium interest-rate dynamics varies with the loss function weights. As one might expect, the equilibrium inertia coefficient increases as λ_Δ is increased, for any given value of $\hat{\lambda}_i$. At the same time, for any given value of $\lambda_\Delta > 0$, the inertia coefficient also increases if $\hat{\lambda}_i$ is reduced. This continues to be true as $\hat{\lambda}_i$ is made negative. Figure 4 shows the corresponding equilibrium values of the

and under that perturbation these solutions cease to satisfy the second-order condition. Hence we show these branches of solutions with dashed lines in Figure 2.

34. To be more precise, for any small enough value $\lambda_\Delta > 0$, there exists a unique stationary equilibrium for all $\hat{\lambda}_i > -1$. This identifies the boundary of the white region in Figures 3–5, near the horizontal axis. It is interesting to note that for values of $\hat{\lambda}_i$ below a critical value, approximately $-0{\cdot}02$, the unique stationary equilibrium no longer corresponds to the "minimum state variable solution", *i.e.* the solution in which lagged interest rates are irrelevant.

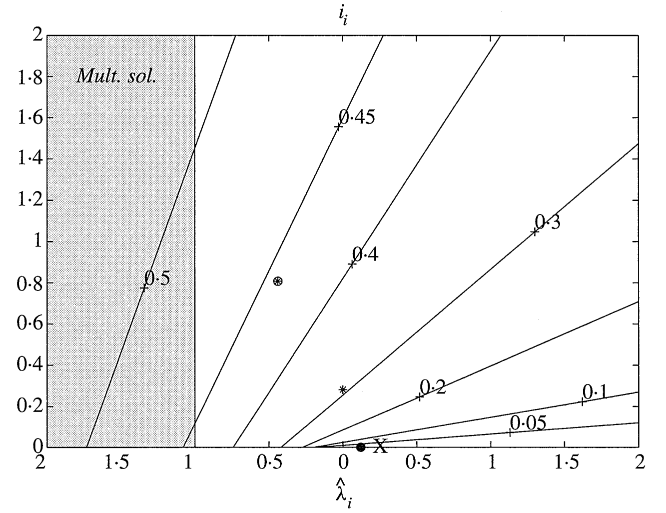

FIGURE 3
The value of i_i for alternative central-bank objectives

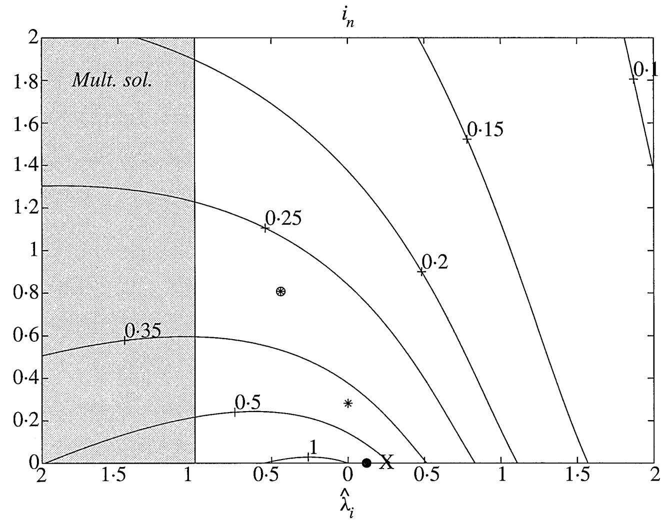

FIGURE 4
The value of i_n for alternative central-bank objectives

coefficient i_n, describing the immediate interest-rate response to an increase in the natural rate of interest. Increasing either $\hat{\lambda}_i$ or λ_Δ lowers this response coefficient, at least in the region where both are positive, though the response is positive.

Figure 5 presents the corresponding contour plot for the stabilization loss W^{stab}.[35] Four sets of policy weights are marked on this figure (as on the others). The X indicates the weights in the true social loss function; but charging a discretionary central bank to minimize this objective

35. Note that this corresponds to the value of the true social loss measure, $E[W]$, rather than the one pursued by the central bank, in the case of initial conditions under which $i_{-1} = r^n_{-1} = 0$.

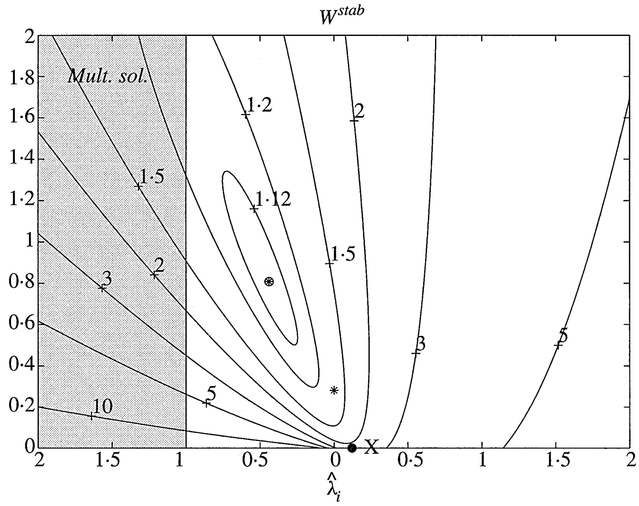

FIGURE 5
The stabilization loss for alternative central-bank objectives

TABLE 3
Stationary Markov equilibria with alternative policy weights

Policy weights				Equilibrium statistics			
$\hat{\lambda}_\pi$	$\hat{\lambda}_x$	$\hat{\lambda}_i$	λ_Δ	$V[\pi]^{1/2}$	$V[x]^{1/2}$	$V[i]^{1/2}$	W^{stab}
1	0·048	0·236	0	0·698	4·79	2·01	2·547
1	0·048	0·120	0	0·459	3·15	2·59	2·279
1	0·048	0	0·282	0·286	3·42	1·71	1·337
1	0·048	−0·439	0·807	0·367	3·24	1·39	1·097
1	0	−0·232	0·204	0·367	3·24	1·39	1·097
20·7	1	−9·09	16·7	0·367	3·24	1·39	1·097
0	1	0	5·51	0·278	3·46	1·63	1·281

does not lead to the best equilibrium, even under this criterion. (These weights, and properties of the resulting Markov equilibrium, are described in the first line of Table 3.) The large black dot instead indicates the weights that lead to the best outcome, when one still restricts attention to central-bank loss functions with no smoothing objective ($\lambda_\Delta = 0$). This corresponds to a weight $\hat{\lambda}_i$ that implements the optimal *non-inertial* plan, characterized in Section 3.[36] It involves a value $\hat{\lambda}_i < \lambda_i$, so that interest rates respond more vigorously to variations in the natural rate of interest than occurs under discretion when the central bank seeks to minimize the true social loss function.

The circled star (or wheel) instead indicates the minimum achievable value of W^{stab}, among time-consistent equilibria of this kind. These weights therefore solve the optimal delegation problem, if we restrict ourselves to central-bank objectives of the form (4.1). As in the limiting case solved explicitly above, the optimal weights involve $\lambda_\Delta > 0$, $\hat{\lambda}_i < 0$. (The optimal weights and the properties of the resulting equilibrium are described on the fourth line of Table 3.)

How good an equilibrium can be achieved through optimal delegation of this kind? The minimum value of W^{stab} shown in Figure 5 is the same, to three significant digits, as that shown

36. Compare the second line of Table 3 with the second line of Table 2.

in Table 2 for the optimal plan under commitment. Thus more than 99·9% of the reduction in expected loss (relative to the outcome under discretionary minimization of the true social loss function) that is possible in principle, through an optimal commitment, can be achieved through an appropriate choice of objective for a discretionary central bank.[37] The exact optimal pattern of responses could presumably be supported as a time-consistent equilibrium if we were to consider more complex central-bank loss functions; but our analysis here suffices to indicate the desirability of assigning the central bank an interest-rate smoothing objective.

It may not be thought possible, in practice, to assign the central bank a smoothing objective that involves a negative weight on one of the "stabilization" objectives. The star without a circle in Figures 3–5 indicates the best Markov equilibrium that can be achieved subject to the constraint that $\hat{\lambda}_i \geq 0$. This point corresponds to a point of tangency between an isoquant of W^{stab} and the vertical axis at $\hat{\lambda}_i = 0$ (though neither curve is drawn in Figure 5). In this case, it is still desirable to direct the central bank to penalize large interest-rate changes, though the optimal λ_Δ is smaller than if it were possible to choose $\hat{\lambda}_i < 0$. (These constrained-optimal weights are shown on the third line of Table 3.)

In this exercise, we have assumed that the relative weight on output gap variability, $\hat{\lambda}_x$, equals the weight in the true social loss function, λ_x, given in Table 1. In fact, consideration of values $\hat{\lambda}_x \neq \lambda_x$ allows us to do no better, either in the case of loss functions with no smoothing objective, or in the case of the fully unconstrained family. For the policy on the second line of Table 3 already implements the optimal non-inertial plan, and the policy on the fourth line already implements the optimal plan of the form given by equations (4.3)–(4.5). This is not because $\hat{\lambda}_x = \lambda_x$ is a uniquely optimal value in either case, but rather because we can find weights that support the optimal plan for an arbitrary value of $\hat{\lambda}_x$, so that the constraint that $\hat{\lambda}_x = \lambda_x$ has no cost. For example, it would also be possible to impose the constraint that $\hat{\lambda}_x = 0$, so that there is no output-gap term in the central-bank loss function at all. The optimal weights in this case are given on the fifth line of Table 3. Note that again $\hat{\lambda}_i < 0$, $\lambda_\Delta > 0$.

This ceases to be true if we impose the constraint that all weights be non-negative. In this case, the constraint that $\hat{\lambda}_i \geq 0$ binds. But if we must set $\hat{\lambda}_i = 0$, the additional degree of freedom allowed by varying $\hat{\lambda}_x$ as well as λ_Δ does allow some improvement of the time-consistent equilibrium, in general. In fact, for the numerical parameter values used above, the optimal $\hat{\lambda}_x$ is infinite; that is, the relative weight on the inflation term is best set to zero. To analyse this case, it is thus convenient to adopt an alternative normalization for the central bank loss function,

$$L_t^{cb} = x_t^2 + \tilde{\lambda}_\pi \pi_t^2 + \tilde{\lambda}_i i_t^2 + \tilde{\lambda}_\Delta (i_t - i_{t-1})^2. \tag{4.16}$$

In terms of this alternative normalization, the loss function described on the fourth line of Table 3 is instead described as on the sixth line of the table. The optimal objective in the family (4.16), when we impose the constraint that $\tilde{\lambda}_i \geq 0$, is instead given by the seventh line. Once again we find that a positive weight on the smoothing objective is desirable, though the constrained-optimal central-bank objective puts no weight on either inflation stabilization or on reducing variation in the level of nominal interest rates.[38]

5. CONCLUSION

Even if there is no intrinsic benefit to minimizing the size of changes in the central bank's interest-rate instrument, it can be desirable for a central bank to seek to minimize a loss function that

37. The equilibrium achieved in this way is also very similar in other respects, such as the other statistics for the optimal plan reported in Table 2.

38. See Woodford (1999*b*) for further discussion of optimal delegation under this constraint.

includes a smoothing objective. For pursuit of such an objective will lead a central bank that optimizes under discretion to adjust interest rates in a more inertial fashion, and interest-rate dynamics of this kind are desirable for the sake of objectives that *are* important for monetary policy—namely, achieving a greater degree of stability of inflation and the output gap, without requiring so much variation in the level of interest rates.

Of course, the assignment to the central bank of an objective different from the true social loss function, in the expectation that it will pursue that objective with discretion, is not the only possible approach to the achievement of a desirable pattern of responses to disturbances. One defect of the "optimal delegation" approach considered here is that it presumes that the stationary Markov equilibrium associated with a particular distorted objective will be realized. Yet there may well be other possible rational expectations equilibria consistent with discretionary optimization by the central bank, "reputational" equilibria in which the bank may do a *better* job of minimizing the objective it has been assigned, but as a consequence bring about a pattern of responses that is *less* desirable from the point of view of the true social objective. It is also unclear how the public ought to monitor the central bank's commitment to its assigned objective under such a scheme; should the bank be judged on its success in pursuing its assigned objective, or its success in achieving the true social objective for the sake of which the delegation scheme has been designed?

An alternative approach that would not raise these difficulties would be a commitment by the central bank to conduct policy according to an interest-rate feedback rule along the lines of the "Taylor rule". Interest-rate rules that would implement the optimal plan in the context of the model considered here are discussed in Woodford (1999*b*) and Giannoni and Woodford (2003*a*). Under this approach as well, an optimal rule makes the current interest rate setting a function of the recent past level of interest rates. A purely contemporaneous rule—one that makes the current nominal interest rate a linear function of the current inflation rate and current output gap only, as proposed by Taylor (1993)—can at best implement only the optimal non-inertial plan. The more inertial interest rate dynamics shown in Section 2 to characterize the optimal plan instead require a feedback rule that responds to lagged endogenous variables; in particular, the rule must specify that the level chosen for the current nominal interest rate will be higher the higher nominal interest rates already are. Thus this feature of estimated central-bank reaction functions can also be justified as a characteristic of optimal policy.

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