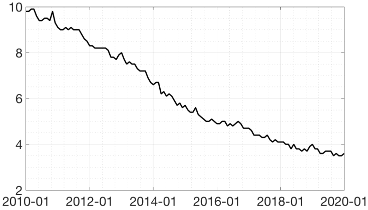
Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

Laura Gáti

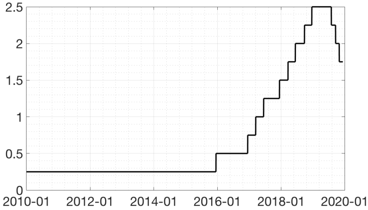
Boston College

April 15, 2020

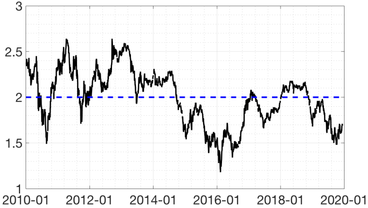
Puzzling Fed behavior fall 2019



(a) Unemployment rate, %



(b) Fed funds rate target, upper limit, %



(c) Market-based inflation expectations, 10 year, % average

This project

Model anchored expectations as an endogenous gain learning scheme

$\rightarrow$ How to conduct optimal monetary policy in interaction with the anchoring expectation formation?

Preview of results

1. Two layers of new intertemporal tradeoffs

2. optimal monetary policy time-inconsistent

$\rightarrow$ illustrate analytically in special case: target criterion

3. Not today: short-run costs vs. long-run benefits of anchoring expectations

Related literature

- **Optimal monetary policy in New Keynesian models**
  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Econometric learning**
  Evans & Honkapohja (2001), Preston (2005), Molnár & Santoro (2014)

- **Anchoring / endogenous gain**
  Carvalho et al (2019), Svensson (2015), Hooper et al (2019), Milani (2014)

Structure of talk

1. Model

2. Solving the Ramsey problem

3. Implications

Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 \upsilon(h_T^i(j))dj \right] \tag{1}$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j) + \Pi_t^i(j)dj - T_t - P_tC_t^i \tag{2}$$

▸ Consumption, price level

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \tag{3}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{4}$$

▸ Profits, stochastic discount factor

Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- If use $\mathbb{E}$ (rational expectations, RE)

  Model solution

$$s_t = hs_{t-1} + \epsilon_t \tag{5}$$
$$y_t = gs_t \tag{6}$$

  $s_t \equiv (r_t^n, u_t)'$ (states)
  $y_t \equiv (\pi_t, x_t, i_t)'$ (jumps)

- If use $\hat{\mathbb{E}} \rightarrow$ don't know g
  $\rightarrow$ estimate using observed states & knowledge of (5)

Adaptive learning

- Estimate g using recursive least squares (RLS)

  $\rightarrow$ nonrational expectations:

$$\hat{\mathbb{E}}_t y_{t+1} = \phi_{t-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix} \tag{7}$$

- Note: misspecified

  Can write:

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{8}$$

  In RE, $a_{t-1} = (0, 0, 0)', b_{t-1} = g\, h \quad \forall t$

Recursive least squares

Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\,h \quad \forall t \tag{9}$$

$\rightarrow$ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\right)}_{\equiv fe_{t|t-1},\ \text{forecast error}} \tag{10}$$

k_t gain
b_1 first row of b

▸ General RLS algorithm

Anchoring mechanism: endogenous gain

Gain in literature usually exogenous:

$$k_t = \begin{cases} \frac{1}{t} & \text{decreasing} \\ k & \text{constant} \end{cases}$$

Here instead

$$k_t = k_{t-1} + \mathbf{g}(fe_{t|t-1}) \tag{11}$$

▸ Functional forms

Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{12}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{13}$$

▸ Derivations

- Expectations evolve according to RLS with the endogenous gain given by (11)

→ How should $\{i_t\}$ be set?

Structure of talk

1. Model

2. Solving the Ramsey problem

3. Implications

Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$
$$\text{s.t. model equations}$$

- $\mathbb{E}$ is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

Special case

- Only inflation intercept learned
- Anchoring function simplified to

$$k_t = \mathbf{g}(fe_{t|t-1}) \tag{14}$$

Target criterion for special case

Result

In the simplified model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa}\Bigg\{x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\Bigg(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_{\pi,t}\Bigg)$$

$$\Bigg(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi},\mathbf{t+j}})\Bigg)\Bigg\}$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t, $\prod_{j=0}^{0} \equiv 1$ *and* b_1 *is the first row of* b.

General case

Two layers of intertemporal tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i}$$
$$- \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j} + fe_{t+1+j|t+j}) \mathbf{g}_{\bar{\pi},\mathbf{t+j}}$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff from current level and change of the gain

Intertemporal tradeoff from future expected levels and changes of the gain

Lemma

The commitment solution of the Ramsey problem does not exist under adaptive learning.

Corollary

Optimal policy under adaptive learning is time-inconsistent.

Why no commitment?

Structure of talk

1. Model

2. Solving the Ramsey problem

3. Implications

How to implement?

- Related issue under RE: optimal interest rate sequence implies indeterminate equilibrium

$\Rightarrow$ Reaction function stabilizes expectations

Recall IS-curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big)$$

- E.g. Taylor rule disciplines expectations:

$$\hat{\mathbb{E}}_t i_T = \psi_\pi \hat{\mathbb{E}}_t \pi_T + \psi_x \hat{\mathbb{E}}_t x_T$$

Next steps: form of reaction function

- Model suggests $i_t = \mathbf{f}(\pi_t, k_t, \bar{\pi}_{t-1}; t)$ nonlinear
- Would explain deviations from Taylor rule
- However: no commitment makes Taylor rule more viable than under RE as a rough approximation of optimal feedback rule

Conclusion

- Interaction between monetary policy and anchoring

- Optimal policy conditions on stance of current and expected future anchoring

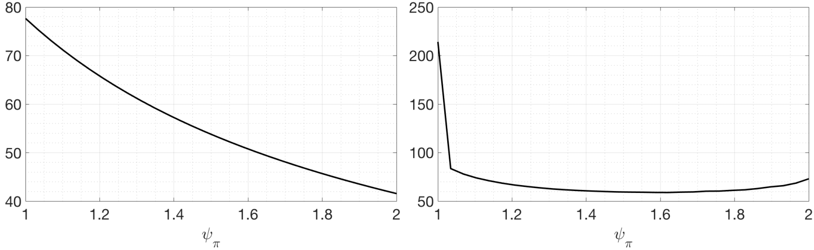
  $\hookrightarrow$ determine intertemporal tradeoffs

- Explain departures from the Taylor rule like US, fall 2019

Short-run costs, long-run benefits

Assume Taylor rule and no concern for output gap stabilization

$$i_t = \psi_\pi \pi_t \qquad \lambda_x = 0$$



(d) RE (e) Anchoring

Figure: Central bank loss as a function of ψ_π

▸ Short-run costs

Functional forms for **g**

- Smooth anchoring function

$$k_t = k_{t-1} - c + dfe^2_{t|t-1} \tag{15}$$

$c, d > 0$

- Kinked anchoring function

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{16}$$

θ_t criterion, $\bar{\theta}$ threshold value

Return

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \tag{17}$$

Σ variance-covariance matrix of shocks
$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(fe_{t|t-1} fe'_{t|t-1} - \omega_{t-1}) \tag{18}$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(fe'_{t|t-1} \omega_t^{-1} fe_{t|t-1} - \theta_{t-1}) \tag{19}$$

ω_t estimated forecast-error variance

◂ Return

Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right)' \right)' \tag{20}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{21}$$

◂ Return

Compact notation

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{22}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{23}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{pmatrix} \tag{24}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{25}$$

◂ Return

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t)$$
$$+ \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \Big\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t + \mathbf{g}_\pi(\pi_t - f_{t-1})) = 0 \qquad (26)$$

$$-2\beta\frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_\mathbf{f}(\pi_{t+1} - f_t)) = 0 \qquad (27)$$

◂ Return

Short-run costs from oscillatory dynamics

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{28}$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{29}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1-k^{-1}}{1-k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1-k^{-1}\beta} u_t \tag{30}$$

Solve for forecast error $fe_t \equiv \pi_t - f_{t-1}$:

$$fe_t = \underbrace{-\frac{1-\beta}{1-k\beta}}_{\lim_{k\to 1} = -1} f_{t-1} + \frac{1}{1-k\beta} u_t \tag{31}$$

◂ Return

Target criterion system for anchoring function as changes of the gain

$$\varphi_{6,t} = -cfe_{t|t-1}x_{t+1} + \left(1 + \frac{fe_{t|t-1}}{fe_{t+1|t}}(1-k_{t+1}) - fe_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1}$$
$$- \frac{fe_{t|t-1}}{fe_{t+1|t}}(1-k_{t+1})\varphi_{6,t+2} \quad (32)$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{fe_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{fe_{t|t-1}}\varphi_{6,t+1} \quad (33)$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (33) is given by:

$$\varphi_{6,t} = -2\,\mathbb{E}_t \sum_{i=0}^{\infty}(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{fe_{t+j|t+j-1}}}{\frac{k_{t+j}}{fe_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \quad (34)$$

◂ Return

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{35}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{36}$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \tag{37}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{38}$$

◂ Return

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{39}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{40}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (39) backward to some date t, take expectations at t
2. Sub in (40)
3. Aggregate over households i

$\rightarrow$ Obtain (12)

◂ Return