

You know, I don't know about that.

16 March 2020

- I wanna pause on for a sec b/c I can't seem to get it to work and I'm confused about how to get it time-varying anyway. So let's turn to estimation.

Estimation of the anchoring function

The issue is that we wanna estimate the anchoring function together w/ the model. On the top of my head I can think of 3 ways of doing that:

- 1) IR-matching
- 2) likelihood-based (either MLE or Bayesian)
- 3) VAR-representation \Rightarrow exist? est. that!

\hookrightarrow it would be a time-varying one.

- \hookrightarrow I'm leaning toward #2 b/c 1) it's sexier
2) it's more general than conditional on shocks
3) a TV-VAR sounds challenging

The thing is:

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need to derive the log-likelihood of my model

Take the midsample model w/ no TR

In materials 21, this is eq. (5)-(10) + TR

$$\pi_t = kx_t - (1-\alpha)\beta f_{\alpha}(t) + [-\kappa\alpha\beta(f_3 - \alpha\beta h_x)^{-1} - e_3(f_3 - \alpha\beta h_x)^{-1}]s_t$$

$$x_t = -b\bar{\pi}_t - b f_b(t) + [-(1-\beta)b_2(f_3 - \beta h_x)^{-1} + 2\beta b_3(f_3 - \beta h_x)^{-1} - 2e_1(f_3 - \beta h_x)^{-1}]s_t$$

$$f_{\alpha}(t) = \frac{1}{1-\alpha\beta} \bar{\pi}_{t-1} - b_1(f_3 - \alpha\beta h_x)^{-1}s_t$$

$$f_b(t) = \frac{1}{1-\beta} \bar{\pi}_{t-1} - b_1(f_3 - \beta h_x)^{-1}s_t$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} (\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))$$

$$k_t^{-1} = k_{t-1}^{-1} + d(\bar{\pi}_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})) + c$$

$$i_t = \gamma_{\pi}\pi_t + \gamma_x x_t + \bar{i}_t$$

this is a state-space model (believe it or not)

and I'm gonna eliminate some variables

$$\begin{aligned} \pi_t &= kx_t - \frac{(1-\alpha)\beta}{1-\alpha\beta} \bar{\pi}_{t-1} - (1-\alpha)\beta b_1(f_3 - \alpha\beta h_x)^{-1}s_t \\ &\quad + [-\kappa\alpha\beta(f_3 - \alpha\beta h_x)^{-1} - e_3(f_3 - \alpha\beta h_x)^{-1}]s_t \end{aligned}$$

$$\begin{aligned} x_t &= -b\gamma_{\pi}\bar{\pi}_t - b\gamma_x x_t - b\bar{i}_t - \frac{b}{1-\beta} \bar{\pi}_{t-1} - b b_1(f_3 - \beta h_x)^{-1}s_t \\ &\quad + [-(1-\beta)b_2(f_3 - \beta h_x)^{-1} + 2\beta b_3(f_3 - \beta h_x)^{-1} - 2e_1(f_3 - \beta h_x)^{-1}]s_t \end{aligned}$$

$$\pi_4 = \kappa x_+ - \frac{(1-\alpha)\beta}{1-\alpha\beta} \bar{\pi}_{t-1}$$

$$+ [-(1-\alpha)\beta b_1(I_3 - \alpha\beta h_x)^{-1} s_1 - \kappa\alpha\beta(f_3 - \alpha\beta h_x)^{-1} - e_3(f_3 - \alpha\beta h_x)^{-1}] s_t$$

$$x_+ = -b\gamma_\pi \pi_4 - b\gamma_x x_+ - bi_r - \frac{b}{1-\beta} \bar{\pi}_{t-1} - b b_1(I_3 - \beta h_x)^{-1} s_t$$

$$+ [-(1-\beta)b_2(f_3 - \beta h_x)^{-1} - (1-\beta)b_3(f_3 - \beta h_x)^{-1} + 2e_1(f_3 - \beta h_x)^{-1}] s_t$$

$$(1+b\gamma_x)x_+ = -b\gamma_\pi \pi_4 - \frac{b}{1-\beta} \bar{\pi}_{t-1}$$

$$[-2e_2 - 2b_1(I_3 - \beta h_x)^{-1} - (1-\beta)b_2(f_3 - \beta h_x)^{-1} + 2\beta b_3(f_3 - \beta h_x)^{-1} - 2e_1(f_3 - \beta h_x)^{-1}] s_t$$

$$\Rightarrow x_+ = -\frac{b\gamma_\pi}{1+b\gamma_x} \pi_4 - \frac{1}{1+b\gamma_x} \frac{b}{1-\beta} \bar{\pi}_{t-1}$$

$$+ \frac{1}{1+b\gamma_x} [-2e_2 - 2b_1(I_3 - \beta h_x)^{-1} - (1-\beta)b_2(f_3 - \beta h_x)^{-1} + 2\beta b_3(f_3 - \beta h_x)^{-1} - 2e_1(f_3 - \beta h_x)^{-1}] s_t$$

Can even sub x_+ out!

$$\pi_4 = -\frac{\kappa b\gamma_\pi}{1+b\gamma_x} \pi_4 - \frac{\kappa}{1+b\gamma_x} \frac{b}{(1-\beta)} \bar{\pi}_{t-1}$$

$$+ \frac{\kappa}{1+b\gamma_x} [-2e_2 - 2b_1(I_3 - \beta h_x)^{-1} - (1-\beta)b_2(f_3 - \beta h_x)^{-1} + 2\beta b_3(f_3 - \beta h_x)^{-1} - 2e_1(f_3 - \beta h_x)^{-1}] s_t$$

$$- \frac{(1-\alpha)\beta}{1-\alpha\beta} \bar{\pi}_{t-1}$$

$$+ [-(1-\alpha)\beta b_1(I_3 - \alpha\beta h_x)^{-1} s_1 - \kappa\alpha\beta(f_3 - \alpha\beta h_x)^{-1} - e_3(f_3 - \alpha\beta h_x)^{-1}] s_t$$

$$\Rightarrow \left(1 + \frac{\kappa b\gamma_\pi}{1+b\gamma_x}\right) \pi_4 = -\left(\frac{\kappa b}{(1+b\gamma_x)(1-\beta)} + \frac{(1-\alpha)\beta}{(1-\alpha\beta)}\right) \bar{\pi}_{t-1} + \text{stuff} \cdot s_t$$

$$\left(1 + \frac{k_2 Y_\pi}{1+bY_x}\right) \bar{\pi}_+ = - \left(\frac{k_2}{1+bY_x(1-\beta)} + \frac{(1-\alpha)\beta}{(1-\alpha\beta)} \right) \bar{\pi}_{t-1} +$$

$$+ \left\{ \frac{k}{1+bY_x} \left[-2e_2 - 3b_1(f_3 - \beta h_x)^{-1} - (1-\beta)b_2(f_3 - \beta h_x)^{-1} + 3\beta b_3(f_3 - \beta h_x)^{-1} - 2e_1(f_3 - \beta h_x)^{-1} \right] \right.$$

$$\left. + \left[-(1-\alpha)\beta b_1(f_3 - \alpha\beta h_x)^{-1} s_1 - K\alpha\beta(f_3 - \alpha\beta h_x)^{-1} - e_3(f_3 - \alpha\beta h_x)^{-1} \right] \right\} s_t$$

$$\frac{1+bY_x+k_2 Y_\pi}{1+bY_x} \bar{\pi}_+ = \text{same}$$

$$\Rightarrow \bar{\pi}_+ = - \frac{1+bY_x}{1+bY_x+k_2 Y_\pi} \left(\frac{\frac{k_2(1-\alpha\beta)}{(1+bY_x)(1-\beta)} + (1-\alpha)\beta(1-\beta)(1+bY_x)}{(1-\alpha\beta)} \right) \bar{\pi}_{t-1}$$

$$+ \left\{ \frac{k}{1+bY_x+k_2 Y_\pi} \left[-2e_2 - 3b_1(f_3 - \beta h_x)^{-1} - (1-\beta)b_2(f_3 - \beta h_x)^{-1} + 3\beta b_3(f_3 - \beta h_x)^{-1} - 2e_1(f_3 - \beta h_x)^{-1} \right] \right.$$

$$\left. + \frac{1+bY_x}{1+bY_x+k_2 Y_\pi} \left[-(1-\alpha)\beta b_1(f_3 - \alpha\beta h_x)^{-1} s_1 - K\alpha\beta(f_3 - \alpha\beta h_x)^{-1} - e_3(f_3 - \alpha\beta h_x)^{-1} \right] \right\}$$

$$\Rightarrow \bar{\pi}_+ = - \frac{\frac{k_2(1-\alpha\beta)}{(1+bY_x+k_2 Y_\pi)} + \beta(1-\alpha)(1-\beta)(1+bY_x)}{(1+bY_x+k_2 Y_\pi)(1-\beta)(1-\alpha\beta)} \bar{\pi}_{t-1} +$$

$$\left\{ \frac{k}{1+bY_x+k_2 Y_\pi} \left[-2e_2 - 3b_1(f_3 - \beta h_x)^{-1} - (1-\beta)b_2(f_3 - \beta h_x)^{-1} + 3\beta b_3(f_3 - \beta h_x)^{-1} - 2e_1(f_3 - \beta h_x)^{-1} \right] \right.$$

$$\left. + \frac{1+bY_x}{1+bY_x+k_2 Y_\pi} \left[-(1-\alpha)\beta b_1(f_3 - \alpha\beta h_x)^{-1} s_1 - K\alpha\beta(f_3 - \alpha\beta h_x)^{-1} - e_3(f_3 - \alpha\beta h_x)^{-1} \right] \right\} s_t$$

Damn damn! So we have \xrightarrow{EA}

$$\bar{\pi}_t = - \frac{k_2(\alpha-\alpha\beta) + \beta(1-\alpha)(1-\beta)(1+bY_X)}{(1+bY_X+k_2Y_H)(1-\beta)(1-\alpha\beta)} \bar{\pi}_{t-1} +$$

$$\left[\frac{k}{1+bY_X+k_2Y_H} \left[-2e_2 - 3b_1(f_3 - \beta h_x)^{-1} - (1-\beta)b_2(f_3 - \beta h_x)^{-1} + 3\beta b_3(f_3 - \beta h_x)^{-1} - 2e_1(f_3 - \beta h_x)^{-1} \right] \right.$$

$$\left. + \frac{1+bY_X}{1+bY_X+k_2Y_H} \left[-(1-\alpha)\beta b_1(f_3 - \alpha\beta h_x)^{-1} s_1 - \kappa\alpha\beta(f_3 - \alpha\beta h_x)^{-1} - e_3(f_3 - \alpha\beta h_x)^{-1} \right] \right] s_t \xrightarrow{EB}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} (\bar{\pi}_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))$$

$$k_t^{-1} = k_{t-1}^{-1} + d (\bar{\pi}_t - (\bar{\pi}_{t-1} - b_1 s_{t-1})) + c$$

\hookrightarrow 1 jump ($\bar{\pi}_t$), 3 exogenous states ($s_t = \begin{bmatrix} r_t \\ i_t \\ h_t \end{bmatrix}$) and 2

endogenous states $\xi_t = \begin{bmatrix} \bar{\pi}_t \\ k_t^{-1} \end{bmatrix}$ ($\propto \begin{bmatrix} \bar{\pi}_{t-1} \\ k_t^{-1} \end{bmatrix}$) so $X_t = \begin{bmatrix} \xi_t \\ s_t \end{bmatrix}$ (states)

$$\pi_t = A \bar{\pi}_{t-1} + B s_t = [A \ B] \begin{bmatrix} \xi_t \\ s_t \end{bmatrix}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} (\bar{\pi}_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))$$

$$k_t^{-1} = k_{t-1}^{-1} + d (\bar{\pi}_t - (\bar{\pi}_{t-1} - b_1 s_{t-1})) + c$$

call this the state (?) f_{t-1}

$$\pi_t = A \bar{\pi}_{t-1} + B s_t = [A \ B] \begin{bmatrix} \bar{s}_t \\ s_t \end{bmatrix}$$

$$Y_t = g x \cdot X_t$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} f c_{t-1}$$

$$X_{t+1} = h x X_t + \eta \epsilon_t$$

$$k_t^{-1} = k_{t-1}^{-1} + d \cdot f c_{t-1} + c$$

$$f c_{t-1} = \pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}$$

Several issues y'all:

- 1) fc state or jump? depends on π_t (a jump)
 → there's gotta be some trick (like π_t is LEMP)
 to make it a pure state

- 2.) $\bar{\pi}_t$ nonlinear Lom!



ok - that's troubling. But let's pause it for a sec & let's read what Litterman has to say about MLE & log-likelihoods.

Supp. our VAR(p) looks like

$$y_{t-p} = A_1(y_{t-1} - \mu) + \dots + A_p(y_{t-p} - \mu) + u_t \quad (3.3.1)$$

then if the VAR(p) is Gaussian, that is

$$u \equiv \text{vec}(U) = \begin{bmatrix} u_1 \\ \vdots \\ u_T \end{bmatrix} \sim N(0, I_T \otimes \frac{1}{T} I_n) , \text{ which}$$

p.75

equivalently means that the prob. density of u is

$$f_u(u) = \frac{1}{(2\pi)^{kT/2}} \left| I_T \otimes \Sigma_u \right|^{-1/2} \exp \left[-\frac{1}{2} u' (I_T \otimes \Sigma_u^{-1}) u \right]$$

then we can use the fact that $u = y - \mu^* - (x' \otimes I_K) \alpha$

where $\alpha := \text{vec}(A)$, $A := (A_1, \dots, A_p)$ $k \times kp$

$$k^2 p \times 1$$

$$Y^0 := (y_1 - \mu, \dots, y_T - \mu) \quad k \times T$$

$$Y_t^0 := \begin{bmatrix} y_t - \mu \\ y_{t-p+1} - \mu \end{bmatrix} \quad (k_p \times 1)$$

$$X := (Y_0^0, \dots, Y_{T-1}^0),$$

to write

$$\begin{aligned} f_y(y) &= \left| \frac{\partial y}{\partial u} \right| f_u(u) \\ &= \frac{1}{(2\pi)^{kT/2}} \left| I_T \otimes \Sigma_u \right|^{-1/2} \exp \left[-\frac{1}{2} (y - \mu^* - (x' \otimes I_K) \alpha)' (I_T \otimes \Sigma_u^{-1}) \right. \\ &\quad \left. \cdot (y - \mu^* - (x' \otimes I_K) \alpha) \right] \quad (3.4.4) \end{aligned}$$

and

$$\ln L(\mu, \alpha, \Sigma_u) = -\frac{kT}{2} \ln(2\pi) - \frac{T}{2} \ln |\Sigma_u| - \frac{1}{2} \text{tr} (Y^0 - Ax)' \Sigma_u^{-1} (Y^0 - Ax)$$

is the log-likelihood

(3.4.5)

Given that the updating w/ endog. gain 18 March 2020 introduces non-linearities, I'm afraid that even a "simple" & quick estimation has to involve some form of particle filter. But let's see whether Litterpoli has anything interesting to say about 1) state-space models 2) non-linearities

Litterpoli, State-space models (Ch. 13, p. 415 ff.)

$$\begin{aligned} z_{t+1} &= B_t z_t + F_t x_t + w_t \xrightarrow{\text{noise}} t=0,1,2, \dots \quad (13.2.1_1) \\ y_t &= H_t z_t + G_t x_t + v_t \quad t=1,2, \dots \quad (13.2.2) \end{aligned}$$

transition
matrix input
measurement
matrix inputs/
measurements/
policy vars measurement error

$$\text{and } \begin{bmatrix} w_t \\ v_t \end{bmatrix} \sim WN\left[0, V_L\right] \quad V_L = \begin{bmatrix} \frac{1}{2} w_t^2 & \frac{1}{2} w_t v_t \\ \frac{1}{2} v_t w_t & \frac{1}{2} v_t^2 \end{bmatrix}$$

Nonlinear state-space models

$$\begin{aligned} z_{t+1} &= b_t(z_t, x_t, w_t, \delta_1) \\ y_t &= h_t(z_t, x_t, v_t, \delta_2) \end{aligned}$$

vectors of params

Example of nonlinear state-space is the "bilinear" model:

$$y_t = \alpha y_{t-1} + u_t + \beta y_{t-1} u_{t-1} \quad p. 427 w/ Refs.$$

$$\hookrightarrow z_{t+1} = B z_t + w_t + C \text{vec}(z_t z_t') \quad (13.2.33)$$

$$y_t = [I_k \ 0 \dots 0] z_t \quad (13.2.34)$$

\Rightarrow Bounds of refs on bilinear systems, univariate & multivariate on p. 427 bottom.

MLE of state-space models p. 434

Gather the time-invariant params from $B, F, H_1, G_1, \Sigma_w, \Sigma_0$ and $\Sigma_0 \& \mu_0$ in δ as

$$\delta = \begin{bmatrix} \text{vec}[v, A_1, \dots, A_p] \\ \text{vech}(\Sigma_0) \end{bmatrix}$$

where $\text{vech} = \text{"half-vectorization"}$, $\text{vech}\begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{bmatrix} a \\ b \\ d \end{bmatrix}$

The log-likelihood for the Gaussian state-space model is:

$$\ln L(\delta | y_1, \dots, y_T) = -\frac{kT}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T \ln |\Sigma_y(t|t-1)| - \frac{1}{2} \sum_{t=1}^T (y_t - y_{t|t-1})' \Sigma_y(t|t-1)^{-1} (y_t - y_{t|t-1}) \quad (13.4.1)$$

Denoting the first error $e_t(\delta) := y_t - \hat{y}_{t|t-1}$

and $\hat{\epsilon}_t(\delta) := \hat{E}_{y_t}(t|t-1)$, we can rewrite this as

$$\ln l(\delta) = -\frac{kT}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T [\ln |\hat{\epsilon}_t(\delta)| + e_t'(\delta) \hat{\epsilon}_t(\delta)^T e_t(\delta)] \quad (13.4.3)$$

which makes explicit

- 1) the dependence of $\ln l$ on δ ,
- 2) that all quantities in this $\ln l$ are functions of δ and can (most) be computed using the Kalman filter.

locally identified: when in a subspace of the param space, δ is uniquely determined.

v. globally identified: when δ is uniquely determined in the entire param space.

↪ identification: we need a min of $-\ln l$, so we need some sort of Hessian = pos. def. \Rightarrow the information matrix, $= E[\text{Hessian}] = E \left[\frac{\partial^2 (-\ln l)}{\partial \delta \partial \delta} \right]_{\delta_0}$

A quick note on DSEMs (dynamic simultaneous equations models | a.k.a. "linear systems") p. 323

essentially, these are linear VARMAX(p, s, q) models

$$A_0 y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + B_0 x_t + B_1 x_{t-1} + \dots + B_s x_{t-s} + w_t \quad (10.1.1)$$

- VARMAX(p, s, q) if $w_t \sim WN$
- VARX(p, s) if $w_t \sim WN$.

VAR(p) models w/ time-varying coefficients p. 891ff.

periodic VARs \rightarrow e.g. w/ seasonal dummies

intervention models \rightarrow DGP₁ is replaced by DGP₂ at time T .

$$y_t = v_t + A_{1,t} y_{t-1} + \dots + A_{p,t} y_{t-p} + u_t \quad (12.2.1)$$

$\hookrightarrow WN(0, \Sigma_t)$

also time-varying
(not identically distib.)

Rewrite the VAR(p) as a VAR(1)

$$Y_t = V_t + A_t Y_{t-1} + U_t \quad (12.2.2)$$

$$Y_t := \begin{bmatrix} y_t \\ \vdots \\ y_{t-p+1} \end{bmatrix}_{kp \times 1}, \quad v_t := \begin{bmatrix} v_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{kp \times 1}, \quad A_t := \begin{bmatrix} A_{1,1} & \dots & A_{p-1,1} & A_{p,1} \\ I_k & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & I_k & 0 \end{bmatrix}_{kp \times kp}, \quad u_t := \begin{bmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{kp \times 1}$$

And by recursive subst we get

$$Y_t = \left[\prod_{j=0}^{h-1} A_{t-j} \right] Y_{t-h} + \sum_{i=0}^{h-1} \left[\prod_{j=0}^{i-1} A_{t-j} \right] v_{t-i} + \sum_{i=0}^{h-1} \left[\prod_{j=0}^{i-1} A_{t-j} \right] u_{t-i} \quad (12.2.3)$$

Defining $J := [I_k \ 0]$ such that $y_t = J Y_t$, we can premultiply (12.2.3) by J , define

$$\bar{\Phi}_{it} := J \left[\prod_{j=0}^{i-1} A_{t-j} \right] J' \quad \text{to get}$$

$$y_t = \mu_t + \sum_{i=0}^{\infty} \bar{\Phi}_{it} u_{t-i} \quad (12.2.4)$$

where $\mu_t = E[y_t]$

\Rightarrow Then the MSE (or FEV of the FE $y_{t+h} - \hat{y}_t(h)$) is

$$\bar{\Phi}_t(h) := \sum_{i=0}^{h-1} \bar{\Phi}_{i,t+h} \bar{\Phi}_{t+h-i} \bar{\Phi}_{i,t+h}' \quad (12.2.10)$$

MLE of TV-VAR

p. 394

$$\text{Write (12.2.1) as } y_t = B_t z_{t-1} + u_t \quad (12.2.11)$$

$$\text{where } B_t := [r_t, A_{1t}, \dots, A_{pt}], \quad z_{t-1} := (1, y_{t-1}')'$$

B_t depend on the vector γ of time-invariant params.

z_{t-1} depend on β of fixed params.

$y_t \sim N(0, \Sigma_t)$, the log-likelihood is

$$\ln l(\gamma, \beta) = -\frac{kT}{2} \ln 2\pi - \frac{1}{2} \sum_{t=1}^T \ln |\Sigma_t| - \frac{1}{2} \sum_{t=1}^T u_t' \Sigma_t^{-1} u_t \quad (12.2.12)$$

where initial conditions have been ignored.

You can also derive an info-matrix.

So it seems like you just min -ln $l(\gamma, \beta)$!

It also seems like for certain special cases you can even derive the estimators in closed-form!

Ryan meeting

18 March 2010

from

not yet convinced that the procedure is sensible
→ the only procedure that can work is

$f_{2,t}$ and more RHS-variables: $k^{-1}, \bar{\pi}$

$$z_t = \bar{z}_t + f_{2,t} z_{t-1} + f_{4,t} h_t$$

values in $f_{2,t}$ will be 0 or simply
described by the LOMs

It may be too restrictive

A computationally intensive exercise is:

- long path for the int. rate (e.g.)
 - solve the model for next int. rate
 - check the target criterion, compute the resid
 - fmincon to min that resid
- ⇒ find simulated optimal plan.

- Simulate the Ramsey model
- Simulate the model w/ Taylor rule

→ see how close you can get

It might then be that most results won't
be pencil & paper.

Comments:

- Concerning w/ linearity of (10)
 - "negative surprises cause me to be unanchored" shouldn't be "big mistakes" → so take the square
 - ↳ like smooth one better than jumpy
- Estim: filter the data → App paper w/ Robert
 match the params to moments of data
 → would give you results

They est an NK model: HP-filter both data and
model, compute moments and try to match
those.

Work after

It's the primal economy paper w/ Robert and Ryan meant. \rightarrow ChauhanWorld.pdf in "literature."

\Rightarrow they¹⁾ take data on real per-capita output, inflation, nominal interest rates & per-capita hours, ^(BK) 2) high-pass filter them, 3) use GMM to estimate the process $\tilde{\tau}_t$ that best-matches the autocorrelation structure of the data.

App. D. p. 46 (Mac).

Wedges are an MA(14):

$$\tau_t = \Phi_\varepsilon(L) \varepsilon_t + \Phi_u(L) u_t$$

$$\varepsilon_t \sim WN(0, \sigma_\varepsilon^2) \quad u_t \sim WN(0, \sigma_u^2)$$

\Rightarrow params to be estimated are

$$\gamma_{ma} = (\Phi_\varepsilon, \Phi_u, \sigma_\varepsilon^2)$$

I think that the following is the target: $t=8$

$$\tilde{\Sigma}_{\tau, T} = \text{rech} \left\{ \text{Var} [\tilde{q}_t^{\text{data}}, \dots, \tilde{q}_{t+k}^{\text{data}}] \right\} \quad q = y_t, \pi_t, b_t, w_t$$

Filting: Baxter-King filter w/ truncation horizon 32, lag-length 12

$\rightarrow \tilde{q}_t = BK_{32}(q_t)$, so \tilde{q}_t is filtered data.

then, they do a trick in converting γ_{ma} into γ_{ar} to finally estimate $\hat{\gamma}$ as:

$$\hat{\gamma}_{\text{ar}} = \underset{\gamma_{\text{ar}}}{\operatorname{argmin}} (\tilde{\Sigma}_T - \tilde{\Sigma}(\gamma_{\text{ar}}))' W^{-1} (\tilde{\Sigma}_T - \tilde{\Sigma}(\gamma_{\text{ar}}))$$

- W is a diagonal matrix w/ the bootstrapped variances of $\tilde{\Sigma}_T$ along the main diagonal.
- The model analogue $\tilde{\Sigma}(\gamma_{\text{ar}})$ is computed after the model data has been similarly filtered as the data.

↳ Hold it there, now I know what I need to do.

↳ That and

- the numerical implementation of the target criterion \Rightarrow both are things to do once I have the big screen. So now give a last try to do the time-varying one.

The ori - a first shot

Ryan wrote:

$$z_t = \bar{z}_t + f_{z,t} z_{t-1} + f_{u,t} u_t$$

I think he meant $z_t = \bar{z}_t + h_{z,t} z_{t-1} + f_{z,t} u_t$

let me ignore r_t^n b/c it just blows things up and I just wanna see if it works. (ignore \bar{z} too.)

$$\begin{aligned}
 & h_{\pi,t} \pi_{t-1} + f_{\pi,t} u_t - \kappa(h_{x,t} x_{t-1} + f_{x,t} u_t) \\
 & - (1-\alpha)\beta(h_{fa,t} f_a(t-1) + f_{fa,t} u_t) + exog_1 \cdot u_t = D \quad (9) \\
 & h_{x,t} x_{t-1} + f_{x,t} u_t + \beta(h_{i,t} i_{t-1} + f_{i,t} u_t) \\
 & - \beta(h_{fb,t} f_b(t-1) + f_{fb,t} u_t) + exog_2 \cdot u_t = D \quad (10)
 \end{aligned}$$

↪

$$\begin{aligned}
 & h_{\pi,t} \pi_{t-1} - \kappa h_{x,t} x_{t-1} - (1-\alpha)\beta h_{fa,t} f_a(t-1) \\
 & + \underbrace{(f_{\pi,t} - \kappa f_{x,t} - (1-\alpha)\beta f_{fa,t} + exog_1)}_{u_t} = 0 \quad (10)
 \end{aligned}$$

This is exactly what I have in materials 21

$$\begin{aligned}
 & h_{x,t} x_{t-1} + \beta h_{i,t} i_{t-1} - \beta h_{fb,t} f_b(t-1) \\
 & + (f_{x,t} + \beta f_{i,t} - \beta f_{fb,t} + exog_2) u_t = 0
 \end{aligned}$$

$$z_t = \bar{z}_t + h_{\bar{z},t} z_{t-1} + f_{\bar{z},t} u_t$$

$$h_{fat,t} f_{at}(t-1) + f_{fat,t} u_t - \frac{1}{1-\alpha\beta} \left(h_{\bar{\pi},t-1} \bar{\pi}_{t-2} + f_{\bar{\pi},t-1} u_{t-1} \right) + exog_3 u_t$$

$$h_{fat,t} f_{at}(t-1) - \frac{1}{1-\alpha\beta} h_{\bar{\pi},t-1} \bar{\pi}_{t-2} + \underbrace{(f_{fat,t} + exog_3) u_t - \frac{1}{1-\alpha\beta} f_{\bar{\pi},t-1} u_{t-1}}_{\text{same as in materials 21}} = 0 \quad (9)$$

$$h_{fb,t} f_b(t-1) - \frac{1}{1-\beta} h_{\bar{\pi},t-1} \bar{\pi}_{t-2} + \underbrace{(f_{fb,t} + exog_3) u_t - \frac{1}{1-\beta} f_{\bar{\pi},t-1} u_{t-1}}_{(10)} = 0$$

Honestly, I don't even think I will complete this b/c

we can see that if, in say (10), $h_{fb,t} = 0$ and $h_{\bar{\pi},t-1} = 0$

then we're back to exactly the system I had in mat21.

So suppose $h_{fb,t} \neq 0$. But then in each iteration we have an earlier iteration of itself. Now I might argue

$h_{\bar{\pi}} = h_x = h_i = 0$, but if I ass that for $h_{fat,t} = h_{fb,t} = 0$ then again the link between (9)-(10) and (5)-(6) is broken. Then (10) would give an additional constraint

$$h_{fb,t} f_b(t-1) = \frac{1}{1-\beta} h_{\bar{\pi},t-1} \bar{\pi}_{t-2}$$

In two unknowns. Mm... You'd get a proliferation of unknowns...! I'll stop here, this wasn't for endog states, merde!

① GMM of midsimple

19 March 2020

② Numerical implementation of target criterion

① GMM of midsimple

sim_learnM.m w/ PLM = constant-only

→ Need to:

1) add smooth function as a third criterion

2) To have "midsimple", we need that x (and i)

aren't learned, so input $a = \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix}$, $b = b^{\text{RE}}$

(\Rightarrow a_i , I've learned one thing

(comparing w/ IRFs of materials 9)

⇒ when only π is learned, IRFs are less

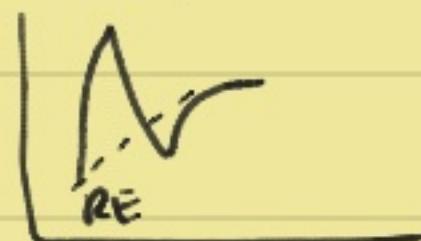
oscillatory in the sense that they overshoot

only once:

when $(\pi, x, i)^T$ learned



vs.



→ did I ever conclude this? I thought so, but now it doesn't seem like it. In materials? I just state that vector vs. scalar learning is pretty much the same

... And it almost is, just not quite.
⇒ so mental note!

Learning only $\text{Lom}(\alpha)$ dampens the oscillations b/c $E(\cdot)$ in NKIS & NKPC are moving less, and therefore $E(i+k)$ is moving less. I think more importantly since the bulk of action is in x , the fact that $\text{FE}(x)$ is now cut out gets rid of the FE that was most oscillatory.

Ok I just wasted an hour trying to customize buys in Matlab ...

Back to 1) the anchoring function:

$$k_t^{-1} - k_{t-1}^{-1} = c + d(FE)$$

has the interpretation that the gain decreases when $FE < 0$. Ryan is right: the easiest thing that makes Δk_t^{-1} big when FE is big in absolute value is FE^2

$$\rightarrow k_t^{-1} - k_{t-1}^{-1} = c + d FE_t^2$$

↑ does this even make sense?

> Not really: it's saying that the gain always changes, even if the $FE^2 = 0$.

↳ So then gather data and implement

- HP
- BK
- Hamilton filters

But there is a problem: for this current form

$$k_t^{-1} - k_{t-1}^{-1} = (c + d \cdot FE^2),$$

it always explodes, even more so if $c=0$.

And it's b/c $k \rightarrow 0$, so I guess k^{-1} explodes.

↳ Need to be smart about this!

Normally, a gain would be

20 March 2020

$$k_{t+1} = k_t + 1$$

$$\text{so I could just do } k_t = k_{t-1} + \frac{1}{FE^2}$$

so that if in the limit $FE^2 \rightarrow \infty$, $k_t = k_{t-1}$ (gain)

then one could have

$d < 1$

$$k_t = k_{t-1} + d \frac{1}{FE^2} \quad \text{in this case I guess } d \text{ small}$$

$$\text{or } k_t = k_{t-1} + \frac{1}{d FE^2} \quad \text{in this case I guess } d \text{ big}$$

$d > 1$

↳ both of them work and exactly the way I hoped

$$\text{or even } k_t = k_{t-1} + \left(\frac{1}{d FE}\right)^2 = k_{t-1} + (d FE)^{-2}$$

works.

HP-filter

A time series y_t (in logs) is

$$y_t = g_t + c_t$$

\uparrow \uparrow
 growth component cyclical component

Obtain g_t as

$$\min_{\{g_t\}_{t=1}^T} \left\{ \sum_{t=1}^T c_t^2 + \lambda \sum_{t=1}^T [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \right\}$$

where $\lambda = 1600$.

Hamilton provides a closed-form solution to this problem as $g^* = (H'H + \lambda Q'Q)^{-1} H'y$ (2)

where $\tilde{T} := T+2$

$$y = (y_T, y_{T-1}, \dots, y_1)' \quad q = (g_T, g_{T-1}, \dots, g_0, g_{-1})'$$

$T \times 1$ $\tilde{T} \times 1$

$$\text{and } H = \begin{bmatrix} I_T & 0 \\ 0 & I_{\tilde{T}} \end{bmatrix} \quad \text{and } Q = \begin{bmatrix} 1 & -2 & 1 & 0 & \dots & 0 \\ 0 & 1 & -2 & 1 & 0 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & 0 & \dots & 0 & 1 & -2 & 1 \end{bmatrix}$$

Hamilton filter

21 March 2020

Idea: $c = y_{t+h} - E[y_{t+h} | y_t]$

choose $h=8$ for quarterly data.

How to calc $E[y_{t+h} | y_t]$?

just regress y_{t+h} on $y_t, y_{t-1}, y_{t-2}, y_{t-3}$
and take the fitted value from that reg.

In fact, Hamilton's eq (21) gives us c directly:

$$c = \hat{v}_{t+h} = y_{t+h} - \hat{\beta}_0 - \hat{\beta}_1 y_t - \hat{\beta}_2 y_{t-1} - \hat{\beta}_3 y_{t-2} - \hat{\beta}_4 y_{t-3}$$

So: $Y = y_{t+h}$, $X = \begin{bmatrix} y_t \\ y_{t-1} \\ y_{t-2} \\ y_{t-3} \end{bmatrix}^{4 \times 1} \Rightarrow \beta = (X'X)^{-1} X' Y$

and $\hat{v}_{t+h} = y_{t+h} - \beta X$

Baxter & King filter (BK filter, bandpass filter)

Def: business cycle = cyclical components of a ts that last between 6 and 32 quarters in duration

Notation: $\text{BP}_k(p, q) =$ filter that processes cycles between $q=32$ and $p=6$ cycle length, and is truncated at leads/lags k . (\rightarrow recommend $k=12$)

\hookrightarrow Such a filter is a MA(k).

So we're interested in constructing the filtered series

$$y_t^* = \sum_{k=-N}^K a_k y_{t-k} \quad (1)$$

and the question is how to obtain a_k ?

(where the MA is symmetric, i.e. $a_k = a_{-k}$)

Eq. (10) gives the answer:

$$a_h^{BP} = (\bar{b}_h - b_h) + (\bar{\theta} - \underline{\theta})$$

where the bars denote things pertaining to low-pass filters that only allow stuff below the frequency $\bar{\omega}$ & $\underline{\omega}$,

i.e periodicities \bar{p} & \underline{p} ,

where $p = \frac{2\pi}{\omega}$

θ is a normalizing constant (coming from truncation)

$$\theta = \left(1 - \sum_{h=-K}^K b_h\right) \frac{1}{2K+1}$$

and $b_0 = \frac{\omega}{\pi}$ and $b_h = \frac{\sin(h\omega)}{h\pi}$ for $h=1, 2, \dots$ (7)

Ok, so we have $\underline{p} = 32$, $\bar{p} = 6$

$$\Rightarrow \underline{\omega} = \frac{2\pi}{\underline{p}} = \frac{2\pi}{32} = \frac{\pi}{16}, \quad \bar{\omega} = \frac{2\pi}{\bar{p}} = \frac{\pi}{3}$$

Autocorrelations of VARs (Lütkepohl, p. 21)

For a stationary VAR(1)

$$y_t = v + A_1 y_{t-1} + u_t \quad \hookrightarrow N(0, \Sigma_u)$$

define $\Gamma_y(h) \equiv E[(y_t - \mu)(y_{t+h} - \mu)']$

where $\mu := E[y_t]$. Then if you know $\Gamma_y(0)$,
you can compute

$$\Gamma_y(h) = A_1 \Gamma_y(h-1) \text{ recursively! } (2.1.31)$$

If you know A_1 and Σ_u , you can compute

$$\text{vec} \Gamma_y(0) = (I_k^2 - A_1 \otimes A_1)^{-1} \text{vec} \Sigma_u \quad (2.1.32)$$

But I don't want to est a VAR... I just want
the empirical auto-covariance matrix of the data
and of the simulated data.

So, the 6MM of the anchoring fit works

23 March 2020

so far. Issues.

1) I'm not sure if I should model the raw data w/ a time series. So e.g. if they were a VAR(1) then I could estimate a RF-VAR, I could bootstrap using the residuals and I could estimate the anterior structure as in (2.1.32) & (2.1.31) in *filterpost*.

This issue shows up for $W = \begin{bmatrix} \hat{\sigma}_{act_1}^2 & & \\ & \ddots & \\ & & \hat{\sigma}_{act_{15}}^2 \end{bmatrix}$
b/c $\hat{\sigma}_{act_i}^2$ are super small, thus W^{-1} is huge,
and therefore the estimation does not move.

2) Filters may not be working?

→ I think now they are!

↳ check bootstrap! ✓ It's fine.

Write up materials 22 - peter ✓

Read "Stochastic" On it!

I'm not sure what I'm learning from 24 March 2020
the fMM so far, so until I talk to Ryan, I'll
postpone it and turn to questions of implementation
of the target condition.

Peter meeting

24 March 2020

- Ask Clough

- Intro: CBs always talk about anchoring,
e.g. (2) formulates that!

- Where to go #1:

- Suggestions

- (1): A common theme: does it make sense
to have $b_t = k_t^{-1}$ and just track the
evolution of that.

(2) : Eq. (2)

If you work w/ nonlinear diff eqs, they

are harder but not impossible to solve

→ mit-shock business is all about that

w/ perfect foresight

- take FOCs

- keep 'em in nonlinear form

- ass. the econ is hit by 1-time shock

- trace out transition to new st. st.

A precise way of putting it:

any dynamic econ: all vars $\rightarrow X_t$

$$F(X_t, X_{t-1}, \varepsilon_t) = 0$$

If F has a lin form, $A X_t + B X_{t-1} + C \varepsilon_t = 0$

then we can solve it immediately quite
easily.

If F is nonlinear, but nonstochastic:

$$F(x_t, x_{t+1}) = 0$$

is harder, but doable.

↳ Sect 2. point (1)

"Can we analytic methods to describe eqs
in opt policy & key variables"

& simulate vars under those policies.

⇒ Now you do answer

- 1) How close does a TR come to opt. policy.
- 2) Opt TR will try to do the same thing
as opt policy and you'll see what they
both are trying to do.

Suggestion overall:

- simulate numerically the model under
some kind of optimal pol that's more flex
than a TR.

2nd suggestion w/ nonlinearity:

Analogy: stochastic Ramsey model
(neoclassical growth model)

no closed-form sol once depreciation isn't
 $= 1$ and not $\log U$. So to solve this:

- loglin

- discretize the state-space

- TFP follows not an AR(1) but
one of 3 values

- Then decision rules are also 3-form

tophi (continues working w/ full-blown non-linear
model but replace the stock, light sources

$$u = \begin{cases} u_{\text{high}} \\ u_{\text{low}} \end{cases} \quad r^n = \begin{cases} r^{\text{high}} \\ r^{\text{low}} \end{cases}$$

\Rightarrow 4 states of the world

\hookrightarrow calculating $E(\cdot)$ of a future term in (1)
may be easier

2 option: take (1)

$$f_t = f_{t-1} + h_t (\pi_t - f_{t-1})$$

and login

Another way to solve a DSGE model

login can's that decision rules are lin
functions of the states

→ well you can say that they are
quadratic function of the states

↳ so instead of Taylor-approx, maybe the
right thing is a cubic or spline

In eq (3) I try to approx something of
unknown form w/ a class

Simulate the model under some optimal targeting rule & compare w/ behavior under an optimized TR economy.

Draft for end of semester
→ she'll read it then
and afterwards too.

↳ talk to Ryan now:
what is the most promising route?

Tomorrow noon → send Peter email for major
summer