Info assumptions

8 Jun 2020 cont'd

Let me give names to the info-assumptions:

The particular question is: supposing agents don't know the NKPC, NKIS, do know the TR, do they know the linking equation between the jump (i_t or π_t) and the endogenous lagged value (il_t or pil_t), and if yes, what do they use to fcst them?

- No: fcst i_t using gx and il_t using hx → "myopic info"
- Yes: fcst i_t using gx and il_t using hx → "schizophrenic"
- Yes: both using gx → "suboptimal forecaster info ass"
- Yes: both using hx → "optimal forecaster info ass"

→ these are the two that at least are internally consistent

The irony is that the old approach to il did the "myopic info", and the corrected MN & PQ methods outlined in materials12f1-3 all do the suboptimal forecaster.

So -lol- 12f is not internally consistent!

A "middle road" was ...intrate_smoothing3.m , b/c
it does the MN approach w/ the myopic info ass,
which is why it always coincides w/ older approaches.

At least what I'm more & more converging on is the
idea that, concerning the big-picture question, agents

- do NOT know NKPC & NKIS → b/c they don't know
  they are identical
- <u>do</u> know the TR → b/c that doesn't require knowledge
  that they are identical, so CB can just announce it.

But as for whether they know the linking equation:

- both myopic info & optimal forecaster ass are consistent
- but my hunch is that ass. myopic info at the same time
  as I ass. that they know the TR is weird b/c it amounts
  to ass-ing that the CB announces the TR but people don't
  understand that the i_t on the RHS is i_{t-1}.

So from a consistency-realisticness standpoint the "optimal forecaster info" ass seems to be the best.

But in terms of desirability of model dynamics, which should we prefer? In particular, can some overcome the overshooting?

- materials6 introduced int-rate smoothing, now I know using the "myopic info" ass and it didn't do much to dampen the overshooting
- materials12 introduced learning the slope and found it was desirable
  - → "optimal forecaster" ass would undo some of this
  - ↳ this suggests that having them know less slows down things

This is confusing b/c the "myopic info" ass endowes them w/ less info than the "optimal forecaster". So that suggests that I need to withdraw more info: make 'em learn h_x too.

The reason that makes me anxious is that over-shooting is already happening b/c $E(\cdot)$ moves so much. So if I increase the role of $E(\cdot)$ b/c hx isn't known either, then will that not make more overshooting?

Step back 1 sec: $E(\cdot)$ moving wouldn't neces be a prob if it wasn't moving so fast or if the TR wasn't known. → No! Whether the TR is known might not matter actually b/c whether agents know how i_t is set, i_t just affects stuff.

→ No that's not true either b/c LH fcsts matter and thus fcsts of i_{t+k} matter, and how you do those depends on whether you know the TR.

→ Does that mean that I've been ass-ing the TR is known all along?

I think so b/c otherwise the $(*)$-conditions wouldn't be valid.

Do you agree w/ the statement 9 Jan 2020

$\hat{E}^i_t(i_{t+1}) = \hat{E}_t(i_{t+1})$?

For this to be true, it has to be that agents first aggregates the same way: $\hat{E}^i_t(y_{t+k}) \overset{!}{=} \hat{E}^j_t(y_{t+k})$ for y being an aggregate variable. Given that agents are identical and observe the same aggregates, this should be true. In fact, it may be true for disagg. variables too, except agents don't realize that either.

The fact that agents know the TR allows me to use the (*)-relations to write $\hat{E}_t(i_{t+k}) = \psi_\pi \hat{E}_t(\pi_{t+k}) + \psi_x \hat{E}_t(x_{t+k})$
$+ \hat{E}_t(\text{shocks})$

But in a certain sense this doesn't help HHs b/c they don't know $\hat{E}_t(\pi_{t+k})$ nor $\hat{E}_t(x_{t+k})$, and so they still estimate $\hat{E}_t(i_{t+k})$ as they estimate gx. Recall that you could solve the model using the PQ-method w/o using (*); then you get a different soln of course where HHs don't know the

TR. But the MN-method is not valid if HHs don't know the TR b/c then you can't sub in $\hat{E}_t[\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \text{shocks}_{T+1}]$ into $\hat{E}_t i_{T+1}$.
So, since Preston uses the MN method, it means that he ass-s that agents know the Taylor-rule!

Ok: so big-picture info ass are cleared up:
- agents do <u>not</u> know NKIS, NKPC
- agents <u>do</u> know the TR

Now we just need to solve the in-depth issue: do agents internalize the linking equation?
- I think it's tough to tell what has the most appealing dynamics.
- But intuitively I'd say yes.

⇒ Resolve pil & il using the "optimal forecasters" info ass.
⇒ materials 12g2-3.

pil

$$x_t = -bi_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + b\pi_{T+1} - b\beta i_{T+1} + b r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t$$

If agents use hx to fcst π, then $f_\beta(1)$ or $f_\alpha(1)$ will not be used.

PQ:

$$\frac{b}{\beta} + b\beta\left(\beta\psi_\pi \frac{b}{\beta}\right) = \frac{b}{\beta} + b^2\beta\psi_\pi$$

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -\kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{cases} [0, 1-\beta, -b\beta] f_\beta + b[1, 0, 0, 1](I_{nx} - \beta h_x)^{-1} s_t - [0\ 0\ 0\ b] s_t \\ [0, \kappa\alpha\beta, 0] f_\alpha + [0, 0, 1, (1-\alpha)\beta](I_{nx} - \alpha\beta h_x)^{-1} s_t - [0\ 0\ 0\ (1-\alpha)\beta] s_t \\ [0, 1, 0, \psi_\pi] s_t \end{cases}$$

$$b\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \pi_{T+1} + r_T^n \right\} = b \sum_{T=t}^{\infty} \beta^{T-t} \hat{E}_t \left\{ pil_{T+2} + r_T^n \right\}$$

$$= b\left[pil_{t+2} + r_t^n + \beta pil_{t+3} + \beta r_{t+1}^n + \beta^2 pil_{t+4} + \beta^2 r_{t+2}^n + \dots \right]$$

$$= b\left[\frac{1}{\beta^2}\left(\beta^2 pil_{t+2} + \beta^3 pil_{t+3} + \dots \right) + \left(r_t^n + \beta r_{t+1}^n + \beta^2 r_{t+2}^n + \dots \right) \right]$$

$$= b\left[\frac{1}{\beta^2}\left(pil_t + \beta pil_{t+1} + \beta^2 pil_{t+2} + \dots \right) + \left(- \| - \right) - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1} \right]$$

$$= b\left[\sum_{T=t}^{\infty} \beta^{T-t} \left(\frac{1}{\beta^2} pil_T + r_T^n \right) - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1} \right]$$

$$= -\frac{b}{\beta}\pi_t + b \sum_{T=t}^{\infty} \beta^{T-t} \left(\frac{1}{\beta^2} pil_T + r_T^n \right) - \frac{b}{\beta^2}[0, 0, 0, 1] s_t$$

The problem is that $\hat{E}_t pil_t = \hat{E}_t \pi_{t-1}$ & $\hat{E}_t pil_{t+1} = \hat{E}_t \pi_t$

I actually now think it's fine to write

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \, pil_T = \frac{1}{1-\beta(hx_{\pi})} pil_t$$

b/c this is $pil_t (= \pi_{t-1}) + \beta \, pil_{t+1} (= \beta hx_{\pi} pil_t = \pi_t)$

$+ \beta^2 pil_{t+2} \quad (= \beta \pi_{t+1})$.

With this logic, $\hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ (1-\alpha)\beta \pi_{T+1} + u_T \right\}$

$= \hat{E}_t \left[u_t + \alpha\beta u_{t+1} + (\alpha\beta)^2 u_{t+2} + \dots + pil_{t+2} + (\alpha\beta)^2 pil_{t+3} + \dots \right]$

$= \hat{E}_t \left[-\text{''}- + \frac{1}{(\alpha\beta)^2} \left[pil_t + \alpha\beta pil_{t+1} + (\alpha\beta)^2 pil_{t+2} + \dots \right] - \frac{1}{(\alpha\beta)^2} pil_t - \frac{1}{\alpha\beta} pil_{t+1} \right]$

$= \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[0, 0, 1, \frac{1}{(\alpha\beta)^2} \right] s_t - \frac{1}{(\alpha\beta)^2} [0, 0, 0, 1] s_t - \frac{1}{\alpha\beta} \pi_t$

$$= \left[0, 0, 1, \frac{1}{(\alpha\beta)^2} \right] (I_{nx} - \alpha\beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0,0,0,1] s_t - \frac{1}{\alpha\beta} \pi_t$$

So using

$$-\frac{\sigma}{\beta}\pi_t + \sigma\sum_{T=t}^{\infty}\beta^{T-t}\left(\frac{1}{\beta^2}pil_T + r_T^n\right) - \frac{\sigma}{\beta^2}[0,0,0,1]s_t$$

and using

$$[0,0,1,\frac{1}{\kappa\beta^2}](I_{nx}-\alpha\beta h_x)^{-1}s_t - \frac{1}{\alpha\beta^2}[0,0,0,1]s_t - \frac{1}{\alpha\beta}\pi_t$$

we get

$$\begin{bmatrix} \frac{\sigma}{\beta} & 1 & \sigma \\ (1+\frac{1}{\alpha\beta})\kappa & 0 \\ 0 & -\Psi_x & 1 \end{bmatrix}\begin{bmatrix}\pi_t \\ x_t \\ i_t\end{bmatrix} = \begin{bmatrix} [0, 1-\beta, -\sigma\beta]f_\beta + \sigma[1,0,0,\frac{1}{\beta^2}](I_{nx}-\beta h_x)^{-1}s_t - \frac{\sigma}{\beta^2}[0,0,0,1]s_t \\ [0, \kappa\alpha\beta, 0]f_\alpha + [0,0,1,\frac{1}{\kappa\beta^2}](I_{nx}-\alpha\beta h_x)^{-1}s_t - \frac{1}{\alpha\beta^2}[0,0,0,1]s_t \\ [0,1,0,\Psi_\pi]s_t \end{bmatrix}$$

$(1+\frac{1}{\alpha\beta})$

And we need to modify (*) as well.

But wait a sec: using the above, can I not, instead of rewriting everything, simply add two linking equations that relate $f_\beta(1)$ and $f_\alpha(1)$ to $s_t(4)$?

Something like: $f_\beta(1) = \hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\pi_{T+1}$

$= \hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}pil_{T+2} = \hat{E}_t\left[pil_{t+2} + \beta pil_{t+3} + \ldots\right]$

$$= \hat{E_t}\left[\frac{1}{\beta^2}\left(pil_t + \beta\, pil_{t+1} + \beta^2 pil_{t+2} + \ldots \right) - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1} \right]$$

$$= \frac{1}{\beta^2} \hat{E_t} \sum_{T=t}^{\infty} \beta^{T-t} pil_T \quad - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} \pi_t$$

$$= \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t - \frac{1}{\beta} \pi_t$$

L1: $f_\beta(1) = \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t - \frac{1}{\beta} \pi_t$

L2: $f_\alpha(1) = \frac{1}{(\alpha\beta)^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0,0,0,1] s_t - \frac{1}{\alpha\beta} \pi_t$

OR: simply add $\underbrace{\text{sth} \times \left(\frac{1}{\beta}\right)\pi_t}$ and $\underbrace{\text{sth} \times \left(\frac{1}{\alpha\beta}\right)\pi_t}$ to the LHS, $P(1,1) = \frac{2}{\beta}$ $\quad P(2,1) = \left(1 + \frac{(1-\alpha)\beta}{\alpha\beta}\right) = \frac{1}{\alpha}$

L1': $f_\beta(1) = \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t$

L2': $f_\alpha(1) = \frac{1}{(\alpha\beta)^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0,0,0,1] s_t$

and then to deal w/ (*), add sth $\times\, \psi_\pi \pi_t$ to the LHS

and lastly there's a $\beta \psi_\pi f_\beta(1)$ in (*), which from the NKIS relation has a -2β coefficient, so put things together:

$$-2\beta \cdot \beta\psi_\pi f_\beta(1) \Rightarrow 2\beta\,\psi_\pi \cdot \pi_t \Rightarrow \text{add } -2\beta\psi_\pi \pi_t$$

to the LHS, i.e. to $P(1,1)$

$\underset{\text{final}}{P(1,1)} = \left(\frac{2}{\beta} - 2\beta\psi_\pi\right)$, don't change anything else but add L1' & L2'.

MN (the red highlighted stuff is correct, see Mathematica)

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \underline{\pi_{t-1}} + \psi_x x_t + \bar{i}_t$$

$$\rightarrow x_t = -\sigma(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta(\psi_\pi \pi_T + \bar{i}_{T+1}) + \sigma r_T^N \right\}$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \pi_{t-1}$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta \psi_\pi \pi_T + \sigma(r_T^N + \bar{i}_T) \right\}$$

$$(1+\sigma\psi_x) x_t =$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma \pi_{T+1} - \sigma\psi_\pi \, pil_T + \sigma(r_T^N - \bar{i}_T) \right\}$$

$$\begin{bmatrix} 0 & 1+\sigma\psi_x \\ 1 & -\kappa \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta-\sigma\beta\psi_x, 0] f_\beta + \sigma[1,-1,0,-\psi_\pi](I_{nx}-\beta h x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx}-\alpha\beta h x)^{-1} s_t \end{bmatrix}$$

and

$$L1: \quad f_\beta(1) = \frac{1}{\beta^2}[0,0,0,1](I_{nx}-\beta h x)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1] s_t - \frac{1}{\beta}\pi_t$$

$$L2: \quad f_\alpha(1) = \frac{1}{(\alpha\beta)^2}[0,0,0,1](I_{nx}-\beta h x)^{-1} s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1] s_t - \frac{1}{\alpha\beta}\pi_t$$

$$\begin{bmatrix} \frac{\sigma}{\beta} & 1+\sigma\psi_x \\ \frac{1}{\alpha} & -\kappa \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta-\sigma\beta\psi_x, 0] f_\beta + \sigma[1,-1,0,-\psi_\pi](I_{nx}-\beta h x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx}-\alpha\beta h x)^{-1} s_t \end{bmatrix}$$

$$1 + \frac{(1-\alpha)\cancel{\beta}}{\alpha\cancel{\beta}} = \frac{\alpha+1-\alpha}{\alpha} = \frac{1}{\alpha}$$

w/ L1' & L2'!

Another attempt at PQ:

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \underline{\pi_{t-1}} + \psi_x x_t + \bar{i}_t$$

$P(1,1)$

$$\Rightarrow \begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0,0](I_{nx} - \beta h x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h x)^{-1} s_t \\ [0,1,0,\psi_\pi] s_t \end{bmatrix}$$

$\frac{1}{\alpha}$

$$L1: \quad f_\beta(1) = \frac{1}{\beta^2}[0,0,0,1](I_{nx} - \beta h x)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1] s_t - \frac{1}{\beta}\pi_t$$

$$L2: \quad f_\alpha(1) = \frac{1}{(\alpha\beta)^2}[0,0,0,1](I_{nx} - \beta h x)^{-1} s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1] s_t - \frac{1}{\alpha\beta}\pi_t$$

$$P(2,1): \quad 1 - \frac{(1-\alpha)\beta}{\alpha\beta}(-1) = 1 + \frac{1-\alpha}{\alpha} = \frac{\alpha + 1 - \alpha}{\alpha} = \underline{\underline{\frac{1}{\alpha}}} \qquad P(2,1)$$

$$P(1,1): \quad \sigma f_\beta(1) \qquad | (*) \text{ But need to rewrite } (*)$$

$$= \sigma\left(-\frac{1}{\beta}\right) \quad | \text{ over to LHS} \rightarrow \frac{\sigma}{\beta} \qquad P(1,1)$$

$(*)$ needs to take the "optimal forecaster" ass into account

$$f_\beta(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [i_{T+1}] = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \psi_\pi \pi_T + \psi_x x_{T+1} + \bar{i}_{T+1} \right\}$$

$$= \psi_x f_\beta(2) + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [0,1,0,\psi_\pi] s_{T+1}$$

$$f_\beta(3) = \psi_x f_\beta(2) + \frac{1}{\beta}[0,1,0,\psi_\pi](I_{nx} - \beta h x)^{-1} s_t - \frac{1}{\beta}[0,1,0,\psi_\pi] s_t$$

Ok, having pil, let's now do it w/ "optimal forecasters."

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}$$

Now the linking equations have to relate $f_a(3)$ & $f_b(3)$ to the errors.

$$f_b(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1}$$ | Don't plug the Taylor-rule b/c that is less info than using h_x.

$$= \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{t+2} = \hat{E}_t \left[i_{t+2} + \beta i_{t+3} + \dots \right]$$

$$= \frac{1}{\beta^2} \hat{E}_t \left[i_t + \beta i_{t+1} + \dots \right] - \frac{1}{\beta^2} i_t - \frac{1}{\beta} i_{t+1}$$

$\stackrel{L1}{=}$

$$f_b(3) = \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t - \frac{1}{\beta} i_t$$

$\stackrel{L2}{=}$

$$f_a(3) = \frac{1}{(\alpha\beta)^2} [0,0,0,1] (I_{nx} - \alpha\beta h_x)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0,0,0,1] s_t - \frac{1}{\alpha\beta} i_t$$

I wonder what happens to $(*)$ ...

MN

$$x_t = -\sigma\left(\psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}\right)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta)x_{T+1} + \sigma\pi_{T+1} - \sigma\beta\left(\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \bar{i}_T + \rho i_T\right) \sigma r_T^n\right\}$$

But again, this doesn't make sense b/c you've got stg better than the TR to fcst i:

PQ:

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta)x_{T+1} + \sigma\pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n\right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\left\{\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T\right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}$$

$$\begin{bmatrix} 0 & 1 & -\sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix}\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0,0](I_{nx} - \beta hx)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + [0,0,1,0](I_{nx} - \alpha\beta hx)^{-1} s_t \\ [0,1,0,\rho] s_t \end{bmatrix}$$

L1 s.t. L1 instead of (*) [and L2 is obsolete!]

$$f_\beta(3) = \frac{1}{\beta^2}[0,0,0,1](I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1] s_t - \frac{1}{\beta} i_t$$

So in the first equation, coeffs of i are:

$$-\sigma - (-\sigma\beta) f_\beta = -\sigma + \sigma\beta\left(-\frac{1}{\beta}\right) = -2\sigma \leftarrow P(1,3)$$

I just have to hope that this is right!

I guess it's time to compare results and check work. 10 Jan 2020

But first: implement pil w/ "myopic" info ass!
(which actually in terms of forecasting equals the "schizophrenic" info ass b/c either you're myopic and so you don't realize $pil_{t+1} = \pi_t$, so you fcst pil using hx and π_t using gx; or you do realize this but you're schizophrenic and still forecast them separately).

pil - myopic → materials12h2

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta)x_{T+1} + \sigma\pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n\right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\left\{\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\right\}$$

$$i_t = \psi_\pi \underline{\pi_{t-1}} + \psi_x x_t + \bar{i}_t$$

MN

$$x_t = -\sigma(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$

agents don't realize this is the same var

$$+\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta-\sigma\beta\psi_x)x_{T+1} + \sigma\pi_{T+1} - \sigma\beta\left[\psi_\pi\pi_T + \bar{i}_{T+1}\right] + \sigma r_T^n\right\}$$

$$(1+\sigma\psi_x)x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta-\sigma\beta\psi_x)x_{T+1} + \sigma\pi_{T+1} - \sigma\left(\psi_\pi\pi_{T-1} + \bar{i}_T\right] + \sigma r_T^n\right\}$$

$$(1+\sigma\psi_x)x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta-\sigma\beta\psi_x)x_{T+1} + \sigma\pi_{T+1} - \sigma\left(\psi_\pi \pi_{T-1} + \bar{i}_T\right] + \sigma r_T^n\right\}$$

$$\begin{bmatrix} 0 & 1+\sigma\psi_x \\ 1 & -k \end{bmatrix}\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta-\sigma\beta\psi_x, 0] f_\beta + \sigma[1,-1,0,-\psi_\pi](I_{nx}-\beta hx)^{-1}s_t \\ [(1-\alpha)\beta, \overset{k\alpha\beta}{\vee}, 0] f_\alpha + [0,0,1,0](I_{nx}-\alpha\beta hx)^{-1}s_t \end{bmatrix}$$

PR

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -k & 0 \\ 0 & -\psi_x & 1 \end{bmatrix}\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0,0](I_{nx}-\beta hx)^{-1}s_t \\ [(1-\alpha)\beta, \overset{k\alpha\beta}{,} 0] f_\alpha + [0,0,1,0](I_{nx}-\alpha\beta hx)^{-1}s_t \\ [0,1,0,\psi_\pi] s_t \end{bmatrix}$$

$(\cancel{x})$

$$f_\beta(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \psi_\pi pil_{T+1} + \psi_x x_{T+1} + \bar{i}_{T+1}$$

$$= \psi_x f_\beta(2) + \frac{1}{\beta}\left\{[0,1,0,\psi_\pi](I_{nx}-\beta hx)^{-1}s_t - [0,1,0,\psi_\pi]s_t\right\}$$

Ok, so look at IRFs in materials12.tex, quickly:
(Note: should check all work to make sure they are really correct → much of this will be my task on Mon, Tues & Wed.)

1) Baseline: learning slope & constant seems better b/c since agents don't know gx(2:end,:), the Ball-effect can't pan out.

2) Epi:

constant only → instrument instability

slope & constant → not E-stable (visually)

To me it makes intuitive sense that Epi should exhibit more instability than baseline b/c instability in the baseline also comes from the Ball-effect, E(.) moving a lot and mattering a lot due to fwd-lookingness.
In Epi, they get to matter more b/c now if E(.) are unstable, i becomes unstable too ⇒ that's why you get instr-instability. When agents are learning all of gx, then

an exploding i is not sufficient to keep $E(\cdot)$ at bay, leading agents to "learn the wrong thing" and so they don't converge to RE.

3) pil

"myopic": for both learning PLMs, seems not E-stable

In a sense it makes sense b/c their forecasting is not consistent w/ RE. Gets worse the less they know of gx. Why don't they learn that $gx = hx$ for π? Maybe b/c this assumes that pil $\neq \pi$ in RE.

"suboptimal fcst": both behave nicely, in particular the

& "optimal fcst": latter

In the former, learning only constant or slope & constant matters of course b/c you're using gx to fcst. In the latter, this distinction doesn't matter b/c you're using hx to fcst anyway.

I think the reason this model works so nicely in general is b/c it takes out π_t as a jump (and that's what "myopic" fails to do), and since π_t is gone as a jump, $E(\pi)$ don't misbehave, and so the Ball-type disinflationary boom effect doesn't occur.

4) ii

"myopic": behaves a lot like the baseline, where learning the slope too dampens the Ball-effect.

"suboptimal" vs "optimal fcst": again, optimal fcst is more stable b/c your fcst is closer to RE since you're using hx (which comes from RE). Learning slope too makes things worse → it seems that the gx you're learning here is unstable and b/c you use it to fcst x & π in both cases, you diverge.

⇒ The overarching theme seems to be that gx is not E-stable and so the only way to restore stability is to effectively make things known: preferably π (pil) b/c it's more E(π) that lead to instability → need to check E-stability!

Check firm work 14 Jan 2020

(1) EPi

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \hat{E}_t \pi_{t+1} + \psi_x x_t + \bar{i}_t$$

MN

$$x_t = -\sigma(\psi_\pi \hat{E}_t \pi_{t+1} + \psi_x x_t + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta (\psi_\pi \hat{E}_T \pi_{T+2} + \psi_x x_{T+1} + \bar{i}_{T+1}) + \sigma r_T^n \right\}$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \hat{E}_t \pi_{t+1}$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta\psi_\pi \hat{E}_T \pi_{T+2} \qquad + \sigma [1,-1,0] s_T \right\}$$

$$\hat{E}_t \hat{E}_T \pi_{T+1} = \hat{E}_t \pi_{T+1}$$

I've been using all along that $\hat{E}$, not just $\hat{E}^i$, satisfies LIE

$$(1+\sigma\psi_x) x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma(1-\psi_\pi) \pi_{T+1} + \sigma [1,-1,0] s_T \right\}$$

$$\begin{bmatrix} 0 & 1+\sigma\psi_x \\ 1 & -\kappa \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [\sigma(1-\psi_\pi), 1-\beta-\sigma\beta\psi_x, 0] f_\beta + \sigma[1,-1,0](I_{nx}-\beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] + [0,0,1](I_{nx}-\alpha\beta h_x)^{-1} s_t \end{bmatrix}$$

materials12f1. tex ✓ .nb ✓

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \hat{E}_t \pi_{t+1} + \psi_x x_t + \bar{i}_t$$

<u>PQ</u>

$$+0 \cdot \hat{E}_t \pi_{t+1}$$

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma [1,0,0](I_{nx} - \beta hx)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1](I_{nx} - \alpha\beta hx)^{-1} s_t \\ [0,1,0] s_t + \psi_\pi \cdot \hat{E}_t \pi_{t+1} \end{bmatrix}$$

materials12f1. tex ✓ nb ✓

From now on: only MN and only the x-equation!

② PiL first "myopic" (12h2 - which I apparently never typped up on LaTeX)

$$i_t = \psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t$$

<u>MN</u>

$$x_t = -\sigma(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta(\psi_\pi \pi_T + \psi_x x_{T+1} + \bar{i}_{T+1}) + \sigma r_T^n \right\}$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \pi_{t-1}$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta\sigma\psi_x) x_{T+1} + \sigma\pi_{T+1} - \sigma\beta(\psi_\pi \pi_T) + \sigma[1,-1,0,\underset{-\psi_\pi}{\boxed{0}}] s_T \right\}$$

✓

Pil – second "suboptimal fasters" (fest both using gx)

MN Step 1 is the same:

$$x_t = -b(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \{ (1-\beta) x_{T+1} + b\pi_{T+1} - b\beta(\psi_\pi \pi_T + \psi_x x_{T+1} + \bar{i}_{T+1}) + b r_T^n \}$$

$$(1+b\psi_x) x_t = -b\psi_\pi \pi_{t-1} - b\beta\psi_\pi \pi_t$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \{ (1-\beta-\beta b\psi_x) x_{T+1} + \underbrace{b\pi_{T+1} - b\beta^2 \psi_\pi \pi_{T+1}}_{b(1-\beta^2\psi_\pi)} + b[1,-1,0,0] \cdot z_T \}$$

tex ✓ nb ✓

Pil – third "opti fasters" (fest both using hx)

MN Step 2 is even the same!

$$(1+b\psi_x) x_t = -b\psi_\pi \pi_{t-1} - b\beta\psi_\pi \pi_t$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \{ (1-\beta-\beta b\psi_x) x_{T+1} + b(1-\beta^2\psi_\pi)\pi_{T+1} + b[1,-1,0,0] \cdot z_T \}$$

The diff is that the $b(1-\beta^2\psi_\pi)\pi_{T+1}$ term now goes into the error.

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} b(1-\beta^2\psi_\pi)\pi_{T+1} = b(1-\beta^2\psi_\pi) \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} pil_{T+2}$$

$$= b(1-\beta^2\psi_\pi) \hat{E}_t (pil_{t+2} + \beta\, pil_{t+3} + \dots)$$

$$= b(1-\beta^2\psi_\pi) \hat{E}_t \frac{1}{\beta^2} (pil_t + \beta\, pil_{t+1} + \beta^2 pil_{t+2} + \beta^3 pil_{t+3} + \dots)$$

$$- \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1}$$

$$= b(1-\beta^2\psi_\pi)\,\hat{E}_t\,\frac{1}{\beta^2}\left(pil_t + \beta\, pil_{t+1} + \beta^2 pil_{t+2} + \beta^3 pil_{t+3} + \ldots\right)$$

$$-\frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1}$$

$$= b(1-\beta^2\psi_\pi)\,[0,0,0,1]\,(I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2}\pi_{t-1} - \frac{1}{\beta}\pi_t$$

So then

$$(1+b\psi_x)x_t = -b\psi_\pi \pi_{t-1} - b\beta\psi_\pi \pi_t$$

$$+\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta-\beta b\psi_\pi)x_{T+1} + b(1-\beta^2\psi_\pi)\pi_{T+1} + b[1,1,0,0]\ldots s_T\right\}$$

becomes

$$(1+b\psi_x)x_t = -(b\psi_\pi + \frac{1}{\beta^2})\pi_{t-1} - (b\beta\psi_\pi + \frac{1}{\beta})\pi_t$$

$$+\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta-\beta b\psi_\pi)x_{T+1} + b[1,1,0,1-\beta^2\psi_\pi]\,s_T\right\}$$

which is NOT what I have at all! LOL!

And using this, you can also rewrite the NKPC

$$\pi_t - \kappa x_t = \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\kappa\alpha\beta\, x_{T+1} + (1-\alpha)\beta\,\hat{E}_t \sum_{T=t}^{\infty}(\alpha\beta)^{T-t} pil_{T+2}$$

$$+ [0,0,1,0]\,(I_{nx} - \alpha\beta hx)^{-1} s_t$$

Note that the linking equations do exactly this.

So if I just write the expectations separately, but impose the linking eqs L1 & L2, do I get the same?

MN, Step 1:

$$x_t = -\sigma(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta(\psi_\pi \pi_T + \psi_x x_{T+1} + \bar{i}_{T+1}) + \sigma r_T^n \}$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \pi_{t-1}$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \{ (1-\beta-\beta\sigma\psi_\pi) x_{T+1} \underbrace{+ \sigma\pi_{T+1} - \sigma\beta\psi_\pi \pi_T}_{\sigma\pi_{T+1} - \sigma\psi_\pi pid_t \text{ (pulling in } \pi_{t-1})} + \sigma[1,-1,0,0] s_T \}$$

$$(1+\sigma\psi_x) x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \{ (1-\beta-\beta\sigma\psi_\pi) x_{T+1} + \sigma\pi_{T+1} + \sigma[1,-1,0,-\psi_\pi] s_T \}$$

$$+ L1$$

Yes!

$$1 + \frac{(1-\alpha)\beta}{\alpha\beta} = \frac{\alpha + (1-\alpha)}{\alpha} = \frac{1}{\alpha}$$

okidoki; this is fine too!

③ iii : here I'll only check subopti & opti fcsters.

First: subopt fcsters

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}$$

PQ only

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0,0](I_{nx} - \beta h x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h x)^{-1} s_t \\ [0,1,0,\rho] s_t \end{bmatrix}$$

This initial setup is true for both info assumptions. The two will only change the (*)-condition which relates $f_\beta(3)$ to $f_\beta(1)$ & $f_\beta(2)$, and in the case of "opti fcsters", to $s(4)$

$$f_\beta(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \bar{i}_{T+1} + \rho i_T$$

$$= \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \hat{E}_t(\bar{i}_{t+1} + \beta \bar{i}_{t+2} + \dots) + \rho \hat{E}_t \sum_{T=t}^{\infty} i_T$$

$$= \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \frac{1}{\beta} \hat{E}_t(\bar{i}_t + \beta \bar{i}_{t+1} + \dots) - \frac{1}{\beta} \bar{i}_t + \rho [i_t + \beta i_{t+1} + \dots]$$

$$= \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \frac{1}{\beta} \hat{E}_t(\bar{i}_t + \beta \bar{i}_{t+1} + \dots) - \frac{1}{\beta} \bar{i}_t + \rho\beta [i_{t+1} + \beta i_{t+2} + \dots] + \rho i_t$$

$f_\beta(3)$

$$= \gamma_\pi f_\beta(1) + \gamma_x f_\beta(2) + \frac{1}{\beta}\hat{E}_t\left(i_t + \beta i_{t+1} + \dots\right) - \frac{1}{\beta} i_t + \rho\beta\big[\underbrace{i_{t+1} + \beta i_{t+2} + \dots}^{f_\beta(3)}\big] + \rho i_t$$

$(1-\rho\beta) f_\beta(3) = -\text{''}-$

$$f_\beta(3) = \frac{1}{1-\rho\beta}\Big(\gamma_\pi f_\beta(1) + \gamma_x f_\beta(2) + \frac{1}{\beta}\Big\{[0,1,0,0](I_{nx} - \beta h_x)^{-1} s_t - [0,1,0,0] s_t\Big\}$$
$$+ \rho i_t\Big)$$

The (*)-condition is the same.

Use this on the $-\gamma\beta f_\beta(3) = -\gamma\beta \cdot \left(\frac{\rho}{1-\rho\beta} i_t\right)$ is on

the RHS $= \frac{\gamma\beta\rho}{1-\rho\beta}$ → so on the LHS we'll have

$$\Rightarrow \gamma + \frac{\gamma\beta\rho}{1-\rho\beta}$$

→ subopt. is correct!

Now: "opti fcsters"

$$f_\beta(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \hat{E}_t\left[i_{t+1} + \beta i_{t+2} + \beta^2 i_{t+3} + \dots\right]$$

$$= \frac{1}{\beta}\left[i_{t-1} + \beta i_t + \beta^2 i_{t+1} + \dots\right] - \frac{1}{\beta^2} i_{t-1} - \frac{1}{\beta} i_t$$

$$= \frac{1}{\beta^2}\Big\{[0,0,0,1](I_{nx} - \beta h_x)^{-1} s_t - [0,0,0,1] s_t\Big\} \qquad = L1' \checkmark$$

And on the RHS we have $-\gamma\beta f_\beta(3) = \gamma\beta \frac{1}{\beta} i_t = \gamma i_t$

→ so on the LHS we'll have $(\gamma - \gamma) i_t = 0$.

LHS: $\gamma - \left[-\gamma\beta\left(-\frac{1}{\beta}\right)\right] = \gamma - \gamma = 0$. ⇒ -2γ was wrong!

$P(1,3) = 0$.

Ok so we've checked thru everything – it should be all correct! So now: E-stability & interpretation of plots.

conclusions / questions

1) Epi is never E-stable

I think this is b/c although i could dampen things by reacting to $E(\pi)$, $E(\pi)$ internalizes this → feedback effects

2) Why is myopic the only unstable for pil, why is it the only stable for il?

- I think pil explodes for myopic b/c $E(\pi)$ are governed by gx, so explosive, and E(pil) are governed by hx, so stable, so they don't counterbalance.
- For pil the opposite happens b/c when E(i) & E(il) are governed by the same thing, then ppl internalize that i is unstable, either in an oscillatory way (gx) or in a smooth way (hx). By contrast, when they don't

realize that $i = il$, then they think that i is driven by something stable, so it's balanced out.

$\Rightarrow$ it seems like behind all of this are (at least initially) unstable $E(\cdot)$ that need to be balanced out somehow. When the TR fails to do that, you explode, either smoothly or via oscillations.

Somehow I'm not so hot on deriving E-stab. analytically, maybe I'm just tired. I'm more fixated on seeing where to go from here, and what we learn.

Preston (2005, p. 112) suggests that E-stability requires that the TR itself be not an additional source of instability.

$\rightarrow$ but that is exactly what it seems to be when for il the agents recognize $i = il$, b/c this opens a new feedback

$\rightarrow$ and also for pil when agents don't recognize $\pi = pil$

↳ then it's as if they missed a dampening, "negative" feedback.

il (as opposed to pil) makes the TR recursive which generates instability unless you shut off recursiveness by making agents not realize it! (i.e. when they don't internalize that il = i, then you effectively shut off the recursiveness)
The situ is different for pil b/c it doesn't make the TR recursive. Instead, when agents don't realize that π = pil, then the TR is effectively independent of endog stuff ($\gamma_x = 0$) and thus not E-stable, as Preston found.

⇒ does pil, fcsting using hx for both ("opt. fcsters") work when agents are learning hx too?

Overshooting comes from E(π) moving around a lot and pushing around E(i) due to knowledge of the TR.
⇒ So actually a cheap way to make 'em not know the

TR is simply not to impose (*)!

So take the baseline for example

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

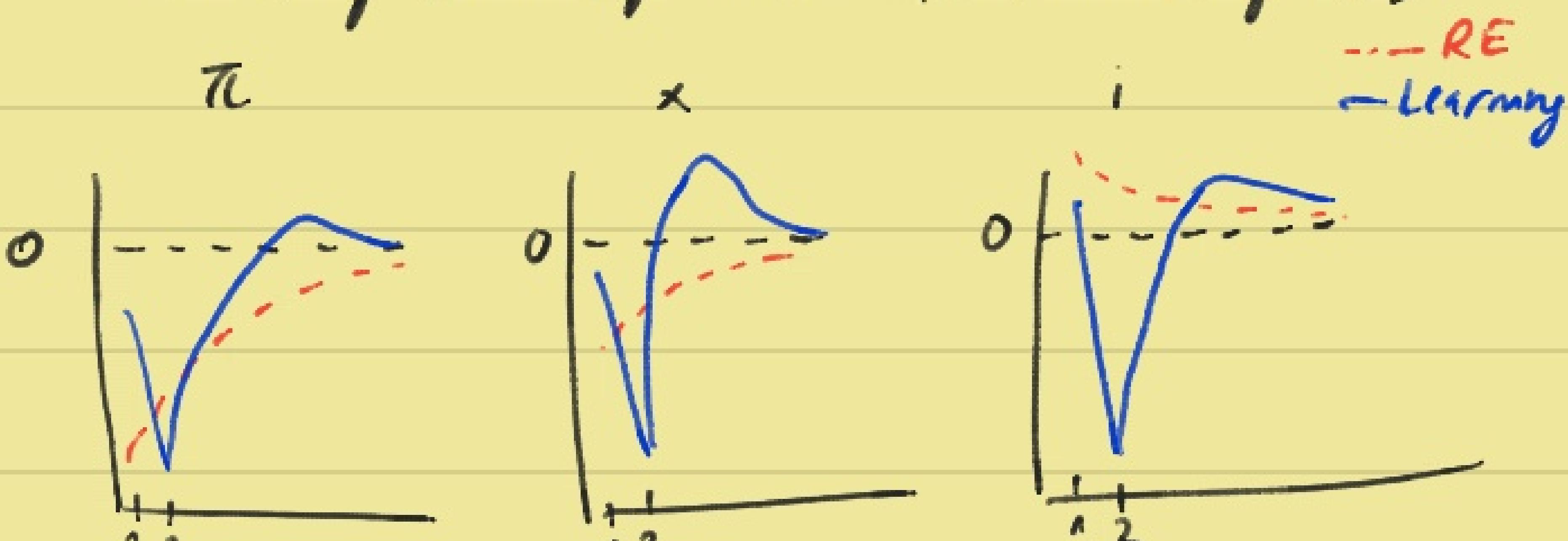
PQ

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma [1,0,0] (I_{nx} - \beta h x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1] s_t \\ [0,1,0] s_t \end{bmatrix}$$

and no (*) imposed!

→ 12i

↳ very interesting IRFs! (constant-only PLM)



→ You can literally see the overshooting be born as they learn the TR!

Ryan meeting 15 Jan 2020

Macro seminars this semester: Last date: Apr 23 or 30?

→ CEMP might be next semester so you're good.

Josephine de Karman: Needs to be received by Jan 31

in one package by snail mail. You'll need to sign

my application document too!

CSWEP due Feb 1 → gonna try Boston Fed

Summer

↳ I'll send the request soon.

Is it an issue that for pil, mon pol shock isn't 1

on impact?

→ looks like a scaled-down version of EE.

Not knowing the TR is like Angeletos: "agents don't

understand GE".

check in data whether EE learning fits data better check Fabian Winkler's work → he'll have dealt w/ these issues!

Work after

Looking at lit w/ estimation of learning and Winkler:

- Winkler 2019: Beautifully oscillating IRFs! p. 12 (Fig 2)

Should try int. rate smoothing as   16 Jan 2020

- $i_t = \rho i_{t-1} + (1-\rho)[\psi_\pi \pi_t + \psi_x x_t] + \varepsilon_t$ (Milani 2007, p. 4 Mac)

- Indexation in NKPC (Milani 2004b p 27 Mac)
- set $k^* =$ optimal k from Kalman filter = relative variances of transitory & permanent shocks (Orphanides & W, 2005 footnote 18, p. 13 Mac)

- Durdu & Leeper's generalized Taylor principle suggests that maybe I could have an alternating, state-contingent TR
- or have initial beliefs different (not RE) like Leeble
  - Add burn-in!

- Milani (2006b) finds for LH learning a gain of 0.028
  - also contrasts EE & LH learning, and finds the fit of the former better.
- Slobodyan & Wouters' "VAR learning" may be similar to my "let them learn hx too".
  It seems like VAR learning uses a smaller set of observables in the RLS algorithm than MSV.
  ↳ could also be an alternative to consider
- Evans & Leeuw 2003 do "3-horizon learning" and then differentiate them Ẽ no problem :)
  But they have a Kalman-filter type of "distinguish temporary from persistent component" learning.
  → not entirely sure about distinction btw constant gain learning and the Kalman filter.
- Preston et al "Limits": int. rate expectations induce instability in the model ⇒ aggressive policy leads to learning about i which creates instability ⇒ related to what I find!

- Eusepi & Preston (2011, AER) actually do compare LH & EE, finding that EE yields much worse properties (no amplification or propagation, moments are too close to RE) and they also cite another paper to reach similar conclusions (p. 25 Mac)

  But Lubik also got sthg like this: EE learning only matters a lot when initial beliefs are not RE.

- Eusepi & Preston (2018) also has stuff that resembles "Limits" and my instability results. But finds that you should be more aggressive on π.

  But Result 6 (p. 30 Mac) makes clear that it's learning i and not the internalization of $E(\pi) \to E(i)$ that drives this, so when $i_t \uparrow \to$ agents expect LR-i to be higher ($\to$ in my case, lower b/c $E(\pi)\downarrow$)

- Graham: as I remembered, he prefers LH to EE b/c the former converges faster. EE is more volatile than LH.

Peter meeting

16 Jan 2020

- Modify the NKPC
- Dávig & Leeper : $\psi_\pi = 1.01$ would that work?
- Learn hx

Somehow add states as "copies" to jump vector

→ some accounting trick to relabel stocks as flows

Like you can rewrite an AR(1) WLOG to AR(2), there might be some trick to rewrite the eq system $\begin{cases} \text{states} \\ \text{flows} \end{cases}$ as one equation $\begin{bmatrix} \text{states} \\ \text{flows} \end{bmatrix}$ and then use the existing learning machinery on that matrix.

- low gain is ok: if high gain produces these counterfactual IRFs, then an MLE would give you a low gain estimate to make sure that IRFs align w/ those in data.
- not spend too much time (more than 1 month)

⇒ You want the best model possible, yes, but don't want to spend too much time finding it!

Work after

Interest rate smoothing w/

$$i_t = \rho i_{t-1} + (1-\rho)(\psi_\pi \pi_t + \psi_x x_t) + \bar{i}_t$$

"weighted il"

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \}$$

PQ

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ -(1-\rho)\psi_\pi & -(1-\rho)\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0,0](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0] s_t \\ [0,1,0,\rho] s_t \end{bmatrix}$$

And (*) will be different β^{T-t}

$$f_\beta(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \rho i_t + (1-\rho)\psi_\pi \pi_{T+1} + (1-\rho)\psi_x x_{T+1} + \bar{i}_{T+1}$$

→ should be the same as previously except that instead of ψ_π and ψ_x, we'll have $(1-\rho)\psi_\pi$ and $(1-\rho)\psi_x$.

→ Implement via changing the params to $(1-\rho)\psi$.

Jdenken: doesn't work! ☹

What's interesting tho is that this effectively makes $\psi_\pi < 1$, but

RE still doesn't complain! And thus you can for suboptimal. festers w/ it get damped oscillations.

Analog to Kalman gain:

$K = P\, gx'\, \Omega^{-1}$ (evaluated in st st, so I'll just take gx^{RE})

FEV(states) FEV(observables)

↑ gonna be easier since hx is known

↳ $= \Sigma_x$, unconditional variance of states

↳ $\Omega = gx\, P\, gx' + R$

↑ Cov(meas. err) = 0 here.

$$\Rightarrow K^* = \Sigma_x \cdot gx' \left(gx\, \Sigma_x\, gx' \right)^{-1}$$

So of course I'm getting a 3x3 and I don't know how to interpret it. I think this isn't conceptually right b/c "it's the other way around": states are known and jumps aren't.

Let's tackle it from another angle:

$$k = \beta^{proj} = \frac{Cov(X,Y)}{Var(X)}$$

In the learning updating step

$\phi' = \phi + k(FE)$ so $Y = \phi$ and $X = FE$

So $k = \frac{Cov(\phi, FE)}{Var(FE)} = \frac{Cov(\phi, FE)}{FEV}$

And I see the issue: neither ϕ, nor FE, nor FEV is constant $\rightarrow$ k varies in every iteration, in every history, depending on shocks.

And by the way

$$k = \frac{Cov(FE, \phi)}{Var(FE)} = \underbrace{P g x'}_{Cov(FE,\phi)} \cdot \underbrace{\Omega^{-1}}_{\frac{1}{Var(FE)}}$$

$\rightarrow$ But what one could try is to use, in every t,

$k = P_{t|t-1} \, g x' \, \Omega_{t|t-1}^{-1}$

or some sensible analogy thereof. I wonder if that would amount to decreasing gains? I guess so if the learning is E-stable.

I've decided not to pursue the Kalman gain thing for now. Instead, the π-indexation in NKPC. We know from CEMP (and b/c we've derived it) that the NKPC w/ exog markups & indexation is:

$$\pi_t - \gamma\pi_{t-1} = \mu_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\xi_p s_T + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) \right]$$

Ok. Rewrite a bit:

$$\pi_t - \gamma\pi_{t-1} = \mu_t + \xi_p s_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\xi_p \alpha\beta s_{T+1} + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) \right]$$

My NKPC looks like

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \right]$$

So w/ indexation, my NKPC will be:

$$\pi_t - \gamma\pi_{t-1} = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) + u_T \right]$$

Now the question is: in the baseline model, how to write the compact notation? In particular, we run into the usual info assumption question: do agents realize that pil $= \pi_{t-1}$, and if yes, what do they use to forecast it?

Baseline w/ indexation:

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t - \gamma \pi_{t-1} = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta(\pi_{T+1} - \gamma \pi_T) + u_T \right]$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

So yes, π_{t-1} is another state, so

<u>PQ</u>

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0,0](I_{nx} - \beta h x)^{-1} s_T \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h x)^{-1} s_T \underline{- \Sigma (1-\alpha)\beta\gamma\pi_T + \gamma\pi_{t-1}} \\ [0,1,0,0] s_t \end{bmatrix}$$

this is where the

exp assumptions come to bear.

<u>MN</u>

$$x_t = -\sigma(\psi_\pi \pi_t + \psi_x x_t + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta(\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \bar{i}_{T+1}) + \sigma r_T^n \right\}$$

$$\sigma\psi_\pi \pi_t + (1+\sigma\psi_x) x_t =$$

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta\sigma\psi_x) x_{T+1} + \sigma(1-\beta\psi_\pi)\pi_{T+1} + \sigma[1,-1,0,0] s_T \right\}$$

(same as baseline)

But the NKPC will be diff:

$$\pi_t - \gamma\pi_{t-1} = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) + u_T \right]$$

$$\pi_t - \kappa x_t = \gamma\pi_{t-1} + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\,\pi_{T+1} - (1-\alpha)\beta\gamma\,\pi_T \right]$$

$$+ [0,0,1,0](I_{nx} - \alpha\beta hx)^{-1} s_t$$

Now the diff info assumptions on RHS:

"Myopic":

$$\gamma\pi_{t-1} + [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta hx)^{-1} s_t$$

$$- (1-\alpha)\beta\gamma \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\, pil_{T+1}$$

$$= \gamma\pi_{t-1} + [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta hx)^{-1} s_t$$

$$- (1-\alpha)\beta\gamma \left[pil_{t+1} + \beta\, pil_{t+2} + \dots \right]$$

$$= \gamma\, pil_t + [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta hx)^{-1} s_t$$

$$- (1-\alpha)\beta\gamma \frac{1}{\alpha\beta} \left[pil_t + \alpha\beta\, pil_{t+1} + (\alpha\beta)^2 pil_{t+2} + \dots \right] + (1-\alpha)\frac{\gamma}{\alpha}\, pil_t$$

$$= [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta hx)^{-1} s_t$$

$$- (1-\alpha)\frac{\gamma}{\alpha} [0,0,0,1](I_{nx} - \alpha\beta hx)^{-1} s_t + \underbrace{\left(\gamma + (1-\alpha)\frac{\gamma}{\alpha} \right)}_{\frac{\cancel{\gamma\alpha} + \gamma - \cancel{\alpha\gamma}}{\alpha} = \frac{\gamma}{\alpha}} pil_t$$

So

$$\pi_t - \kappa x_t = [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta hx)^{-1} s_t$$

$$- (1-\alpha)\frac{\gamma}{\alpha} [0,0,0,1](I_{nx} - \alpha\beta hx)^{-1} s_t + \left[0,0,0,\frac{\gamma}{\alpha}\right] s_t$$

$$\pi_t - kx_t = [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$

$$-(1-\alpha)\frac{\gamma}{\alpha}[0,0,0,1](I_{nx} - \alpha\beta h_x)^{-1} s_t + [0,0,0,\tfrac{\gamma}{\alpha}] s_t$$

$$\boxed{\Rightarrow \pi_t - kx_t = [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + [0,0,1,-\tfrac{(1-\alpha)\gamma}{\alpha}](I_{nx} - \alpha\beta h_x)^{-1} s_t + [0,0,0,\tfrac{\gamma}{\alpha}] s_t}$$

(myopic)

"Suboptimal fisters"

$$\pi_t - kx_t = \gamma\pi_{t-1} + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[k\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} - (1-\alpha)\beta\gamma\pi_T \right]$$
$$+ [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$

$$\pi_t - kx_t = \gamma\pi_{t-1} - (1-\alpha)\beta\gamma\pi_t$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[k\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} - (1-\alpha)\beta\gamma(\alpha\beta)\pi_{T+1} \right]$$
$$+ [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$

$$[1+(1-\alpha)\beta\gamma]\pi_t - kx_t = \gamma\pi_{t-1} + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[k\alpha\beta x_{T+1} + (1-\alpha)\beta(1-\alpha\beta\gamma)\pi_{T+1} \right] \quad (\star)$$

$$\boxed{[1+(1-\alpha)\beta\gamma]\pi_t - kx_t = \gamma\pi_{t-1} + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t + [(1-\alpha)\beta(1-\alpha\beta\gamma), k\alpha\beta, 0] f_\alpha}$$

(suboptimal fisters)

For "opt fcstcs", you just go back to step (✡):

$$\left[1+(1-\alpha)\beta\gamma\right]\pi_t - kx_t = \gamma\pi_{t-1} + [0,0,1,0](I_{nx}-\alpha\beta hx)^{-1}s_t$$
$$+\hat{E}_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[k\alpha\beta x_{T+1} + (1-\alpha)\beta(1-\alpha\beta\gamma)\pi_{T+1}\right]$$

and rewrite $\hat{E}_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\pi_{T+1} = \hat{E}_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}pil_{T+2}$

$$= pil_{t+2} + \alpha\beta\, pil_{t+3} + \dots$$

$$= \frac{1}{(\alpha\beta)^2}\left[pil_t + \alpha\beta\, pil_{t+1} + (\alpha\beta)^2 pil_{t+2} + \dots\right] - \frac{1}{(\alpha\beta)^2}pil_t - \frac{1}{\alpha\beta}pil_{t+1}$$

So the eq becomes:

$$\left[1+(1-\alpha)\beta\gamma\right]\pi_t - kx_t = \gamma\pi_{t-1} + [0,0,1,0](I_{nx}-\alpha\beta hx)^{-1}s_t$$
$$+[0,k\alpha\beta,0]f\alpha$$
$$+(1-\alpha)\beta(1-\alpha\beta\gamma)\left\{\frac{1}{(\alpha\beta)^2}[0,0,0,1](I_{nx}-\alpha\beta hx)^{-1}s_t - \frac{1}{(\alpha\beta)^2}pil_t - \frac{1}{\alpha\beta}pil_{t+1}\right\}$$

which is:

$$\left[1+(1-\alpha)\beta\gamma\right]\pi_t - kx_t = \gamma\pi_{t-1} + [0,0,1,0](I_{nx}-\alpha\beta hx)^{-1}s_t$$
$$+[0,k\alpha\beta,0]f\alpha + (1-\alpha)\beta(1-\alpha\beta\gamma)\frac{1}{(\alpha\beta)^2}[0,0,0,1](I_{nx}-\alpha\beta hx)^{-1}s_t$$
$$-(1-\alpha)\beta(1-\alpha\beta\gamma)\frac{1}{(\alpha\beta)^2}\pi_{t-1} - (1-\alpha)\beta(1-\alpha\beta\gamma)\frac{1}{\alpha\beta}\pi_t$$

$$\left[1+(1-\alpha)\beta\gamma + (1-\alpha)\beta(1-\alpha\beta\gamma)\frac{1}{\alpha\beta}\right]\pi_t - kx_t = \left[\gamma - (1-\alpha)\beta(1-\alpha\beta\gamma)\frac{1}{(\alpha\beta)^2}\right]\pi_{t-1}$$
$$+[0,k\alpha\beta,0]f\alpha + \left[0,0,1,(1-\alpha)\beta(1-\alpha\beta\gamma)\frac{1}{(\alpha\beta)^2}\right](I_{nx}-\alpha\beta hx)^{-1}s_t$$

(opti fcstcs)

So I'm not gonna write these up on LaTeX (yet),
but in Mathematica I'll call 'em

materials13indexation_myopic.nb

materials13indexation_subopt.nb

materials13indexation_opt.nb

=> and analogously in Matlab.

What I need to do still is a RE-version w/ hybrid MKPC.
Do the "pull out 1 term at a time" thing (Preston, p. 95)
First for Preston's NK IS: (for practice) Use RE-E(·)

$$x_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} - \sigma(i_T + \pi_{T+1}) + r_T^n \right\}$$

Lead & take E(·)

$$E_t x_{t+1} = E_t E_{t+1} \sum_{T=t+1}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} - \sigma(i_T + \pi_{T+1}) + r_T^n \right\}$$

So
$$E_t x_{t+1} = E_t \sum_{T=t+1}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} - \sigma(i_T + \pi_{T+1}) + r_T^n \right\} \quad (1)$$

Now take x_t and pull all t+1 terms out (and t)

$$x_t = E_t \left[(1-\beta) x_{t+1} - \sigma(i_t + \pi_{t+1}) + r_t^n \right] + \underbrace{E_t \sum_{T=t+1}^{\infty} \beta^{T-t} \beta \left\{ (1-\beta) \cdot x_{T+1} - \sigma(i_T + \pi_{T+1}) + r_T^n \right\}}_{= \beta E_t x_{t+1} \text{ by } (1)} \checkmark$$

Now do the same thing for the NKPC w/ indexation:

$$\pi_t - \gamma\pi_{t-1} = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) + u_T\right]$$

Rearrange a bit and replace $\hat{E}$ w/ E:

$$\pi_t - \gamma\pi_{t-1} = E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa x_T + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) + u_T\right]$$

Lead and take E_t:

$$E_t\left[\pi_{t+1} - \gamma\pi_t\right] = E_t \sum_{T=t+1}^{\infty} (\alpha\beta)^{T-t} \left[\kappa x_T + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) + u_T\right] \quad (1)$$

Now take out first terms in NKPC

$$\begin{aligned}
\pi_t - \gamma\pi_{t-1} &= E_t\left[\kappa x_t + (1-\alpha)\beta(\pi_{t+1} - \gamma\pi_t) + u_t\right] \\
&\quad + (\alpha\beta) \cdot E_t \sum_{T=t+1}^{\infty} (\alpha\beta)^{T-t} \left[\kappa x_T + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) + u_T\right] \\
&= \kappa x_t + (1-\alpha)\beta(E_t\pi_{t+1} - \gamma\pi_t) + u_t \\
&\quad + \alpha\beta \cdot \underbrace{E_t\left[\pi_{t+1} - \gamma\pi_t\right]}_{\text{by (1)}}
\end{aligned}$$

$$\Rightarrow \pi_t - \gamma\pi_{t-1} = \kappa x_t + \beta\left[E_t(\pi_{t+1}) - \gamma\pi_t\right] + u_t$$

$$\pi_t = \gamma\pi_{t-1} + \kappa x_t + \beta E_t(\pi_{t+1}) - \beta\gamma\pi_t + u_t$$

$$(1+\beta\gamma)\pi_t = \gamma\pi_{t-1} + \kappa x_t + \beta E_t(\pi_{t+1}) + u_t$$

$$\pi_t = \frac{1}{1+\beta\gamma}\left[\gamma\pi_{t-1} + \kappa x_t + \beta E_t\pi_{t+1} + u_t\right]$$ RE-NKPC for indexation.

Oh wonder! Subopti fcst w/ indexation seems to work! The reason I'm focusing on that is b/c

1) it's not dumb like the myopic info ass

2) it doesn't do away w/ learning

(opti also works)

⇒ So then next step is to check using PQ whether the model solutions for subopt & opt fcsters are indeed correct.

<u>PQ</u>

Ok so the baseline model w/ indexation is:

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t - \gamma \pi_{t-1} = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta(\pi_{T+1} - \gamma \pi_T) + u_T \right]$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

$$\Rightarrow \pi_t = \gamma \pi_{t-1} + \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta(\pi_{T+1} - \gamma \pi_T) + u_T \right]$$

$$\pi_t = \gamma \pi_{t-1} - (1-\alpha)\beta\gamma \pi_t + \kappa x_t + \hat{E}_t \left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta(1-\alpha\beta\gamma) \pi_{T+1} + u_T \right]$$

$$\left[1 + (1-\alpha)\beta\gamma \right] \pi_t - \kappa x_t = \gamma \pi_{t-1} + \left[(1-\alpha)\beta(1-\alpha\beta\gamma), \kappa\alpha\beta, 0 \right] f_a + \left[\underline{0}, 0, 1, 0 \right] (I - A h)x$$

this means

$$\begin{pmatrix} 0 & 1 & \sigma \\ 1+(1-\alpha)\beta\gamma & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{pmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0,0](I_{nx} - \beta hx)^{-1} s_t \\ [(1-\alpha)\beta(1-\alpha\beta\gamma), \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta hx)^{-1} s_t + [0,0,0,\gamma] s_t \\ [0,1,0,0] s_t \end{bmatrix}$$

s.t. (*) and linking equations

In subopt, I only need (*), linking $f_\beta(3)$ to $f_\beta(1)$ & $f_\beta(2)$.

In opt filters, I need (*) and L1 & L2, linking $f_\alpha(1)$ & $f_\beta(1)$ to the shocks.

$$(*) \quad \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \bar{i}_{T+1}$$

the last term is $\quad \bar{i}_{t+1} + \beta \bar{i}_{t+2} + \dots = \frac{1}{\beta}\left[\bar{i}_t + \beta \bar{i}_{t+1} + \dots\right] - \frac{1}{\beta}\bar{i}_t$

$$(*) \quad f_\beta(3) = \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \frac{1}{\beta}\left\{[0,1,0,0](I_{nx} - \beta hx)^{-1} s_t - [0,1,0,0] s_t\right\}$$

(as in baseline)

Lökning:

$$fa(1) = \hat{E}_1 \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \pi_{T+1} = \hat{E}_1 \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} pil_{T+2}$$

$$= pil_{t+2} + \alpha\beta\, pil_{t+2} + \dots$$

$$= \frac{1}{(\alpha\beta)^2}\left[pil_t + \alpha\beta\, pil_{t+1} + (\alpha\beta)^2 pil_{t+2} + \dots\right] - \frac{1}{(\alpha\beta)^2} pil_t - \frac{1}{\alpha\beta} pil_{t+1}$$

L1 & L2

$$fa(1) = \frac{1}{(\alpha\beta)^2}[0,0,0,1](I_{nx} - \alpha\beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1] s_t - \frac{1}{\alpha\beta}\pi_t$$

$$f_\beta(1) = \frac{1}{\beta^2}[0,0,0,1](I_{nx} - \alpha\beta hx)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1] s_t - \frac{1}{\beta}\pi_t$$

$\Rightarrow$ For "opt," we'll need to modify P

$$P(1,1) = 0 - [\,3 f_\beta(1) + -3\beta \cdot f_\beta(3)\,] = -3 f_\beta(1) + 3\beta\psi_\pi f_\beta(1)$$

$$= 3(\beta\psi_\pi - 1)\left(-\frac{1}{\beta}\right) = \underline{\underline{\frac{3}{\beta}(1-\beta\psi_\pi)}}$$

$$P(2,1) = 1 + (1-\alpha)\beta\gamma + (1-\alpha)\beta(1-\gamma) f_\alpha(1) = 1 + (1-\alpha)\beta\gamma + (1-\alpha)\beta(1-\gamma)\left(-\frac{1}{\alpha\beta}\right)$$

$$= \underline{\underline{1 + (1-\alpha)\beta\gamma - (1-\alpha)\beta(1-\gamma)\frac{1}{\alpha\beta}}}$$

"Subopt fests" works and IRF is ok!

Crazy! X Unfortunately, that's not right: I had kept $\psi_\pi = 1.01$, that's why IRFs were nice!

My notes here for "opt fest" may not be quite correct, but what I have found is that you can simply take the MN & PQ sols of "subopt fest" and simply add the linking equations L1 & L2 to them.

The only thing you need to do is to modify P(1,1) & P(2,1) M(1,1) & M(2,1)

$$M11RHS = \tau \cdot (1-\beta \cdot \psi) f_\beta(1) = \underline{-\tau(1-\beta\psi)\frac{1}{\beta}} \qquad (L1)$$

$$M21RHS = (1-\alpha)\beta(1-\alpha\beta\gamma) f_\alpha(1) = \underline{-(1-\alpha)\beta(1-\alpha\beta\gamma)\frac{1}{\alpha\beta}} \qquad (L2)$$

$$P11RHS = \tau f_\beta(1) - \tau\beta f_\beta(3) = \tau(1-\beta\psi) f_\beta(1) = -\tau(1-\beta\psi)\frac{1}{\beta}$$

$$= M11RHS$$

$$P21RHS = (1-\alpha)\beta(1-\alpha\beta\cdot\gamma) f_\alpha(1) = M21RHS$$

$\Rightarrow$ IRFs are good too! X

Indexation is what it takes to get rid of the overshooting ...

No it doesn't → it doesn't work either :(

Alright, so then I won't do more extensions, at least not for now. Instead I'll turn to the issue of learning hx.

Learning hx 18 Jan 2020

Let's go back to the baseline version.

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

Instead of the old compact form, $z_t = A_a f_a + A_b \cdot f_b + A_c \cdot s_t$,

let's write it like this: $z_t = A_a f_a + A_b \cdot f_b + B_a \cdot g_a + B_b \cdot g_b + A_c \cdot s_t$

where g_a & g_b are LH-expectations of the states.

Let's do PQ:

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_b + [\sigma, 0, 0] g_b \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + [0, 0, 1] g_a \\ [0, 1, 0] s_t \end{bmatrix}$$

Several questions:

1) If hx is learned, what are agents regressing on? Still s_t?

An alternative way to ask this question is: why aren't agents regressing z_t on past values, z_{t-1}? ⇒ b/c that's not the minimum-state-variable (MSV) solution of the model. ⇒ Slobodyan & Wouters' "VAR-learning" goes in this direction when agents use past jumps too. My hunch is that if learning converges, then in the end the coeffs on z_{t-1} should → 0 (or to $gx\, hx\, gx^{-1}$) b/c in the state space

$$\left.\begin{aligned} Y_t &= gx\, X_t \\ X_t &= hx\, X_{t-1} + \eta\, \varepsilon_t \end{aligned}\right\} \; Y_t = gx\, hx \underbrace{X_{t-1}}_{gx^{-1} Y_{t-1}} + \eta\, \varepsilon_t$$

Y_t doesn't depend on Y_{t-1} (or only indirectly).

Slobodyan & Wouters' VAR-learning lets agents learn gx by regressing not on X_t but on Y_{t-1}. The argument is that states are <u>un</u>observed and observables are, well, observed (in reality). The MSV-sol learning implicitly assumes that at least current states are observed. So in MSV learning, agents use a subset of observed vars to learn from b/c somehow they know the functional form of the state-space and so they know that adding observables to their regressor wouldn't add info. →

So maybe it's fine if also to first X agents use past values of X. So I keep the same regressor vector.

But the states s_t are still exogenous... So they will still develop according to $X_t = h_x X_{t-1} + \eta \varepsilon_t$; only the endog variables will be affected by the estimated $\hat{h_x}$ and thus by $\hat{E}_t X_{t+k}$ $k \geq 1$.

So maybe all I need to change is ↗ Mathematica materials13 _learnhx_baseline.nb

1. compact notation to $z_t = A_a f_a + A_b \cdot f_b + B_a \cdot g_a + B_b \cdot g_b + A_c \cdot s_t$
2. ysim in learning code will be $\begin{bmatrix} z_t \\ s_t \end{bmatrix}$ → sim_learnLH_learnhx.m
3. fafb_gagb.m which will give the LH expectations of states too analytically.

1. Sol of model <u>PQ</u>

$$(*)\ \hat{E}_t \sum_{T=t}^{\infty} i_{T+1} = \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\, \bar{i}_{T+1}$$

$$\underline{\underline{f_\beta(3) = \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + g_\beta(2)}} \quad (*)$$

→ which highlights that I need to modify the system to have $\underline{\underline{\hat{E}_t s_{t+1}}}$

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

$$\Rightarrow x_t = -\sigma i_t + \sigma r_t^n + \hat{E}_t \sum_{T=t}^{\infty} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma\beta r_{T+1}^n \right\}$$

$$\pi_t = \kappa x_t + u_t + \hat{E}_t \sum_{T=t}^{\infty} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + \alpha\beta u_{T+1} \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma\beta [1,0,0] g_\beta + \sigma [1,0,0] s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + \alpha\beta [0,0,1] g_\alpha + [0,0,1] s_t \\ [0,1,0] s_t \end{bmatrix} \quad (PQ)$$

Learning Baseline

$$\underline{\underline{f_\beta(3) = \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + g_\beta(2)}} \quad (*)$$

<u>MN</u>

$$x_t = -\sigma(\psi_\pi \pi_t + \psi_x x_t + \bar{i}_t) + \sigma r_t^n + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta(\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \bar{i}_{T+1}) + \sigma\beta r_{T+1}^n \right\}$$

$$\sigma\psi_\pi \pi_t + (1+\sigma\psi_x) x_t = \sigma[1,-1,0] s_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma(1-\beta\psi_\pi)\pi_{T+1} + \sigma\beta[1,-1,0] s_{T+1} \right\}$$

So for MN, the system is:

$$b\psi_\pi \pi_t + (1+b\psi_x) x_t = b[1,-1,0] s_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \{ (1-\beta-b\beta\psi_x) x_{T+1} + b(1-\beta\psi_\pi)\pi_{T+1} + b\beta[1,-1,0] s_{T+1} \}$$

$$\pi_t = \kappa x_t + u_t + \hat{E}_t \sum_{T=t}^{\infty} \{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + \alpha\beta u_{T+1} \}$$

$\Rightarrow$

$$\begin{bmatrix} b\psi_\pi & 1+b\psi_x \\ 1 & -\kappa \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [b(1-\beta\psi_\pi), (1-\beta-b\beta\psi_x), 0] f_\beta + b\beta[1,-1,0] g_\beta + b[1,-1,0] s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + \alpha\beta[0,0,1] g_\alpha + [0,0,1] s_t \end{bmatrix}$$

MN Learning baseline.

coeffs of f_a coeffs of f_b coeffs of s

$$A_a = \begin{pmatrix} aa_\pi \\ aa_x \\ aa_i \end{pmatrix} \quad A_b = \begin{pmatrix} abp \\ abx \\ abi \end{pmatrix} \quad A_s = \begin{pmatrix} asx \\ asp \\ asi \end{pmatrix}$$

coeffs of g_a coeffs of g_b

$$B_a = \begin{pmatrix} bap \\ bax \\ bai \end{pmatrix} \quad B_b = \begin{pmatrix} bbp \\ bbx \\ bbi \end{pmatrix}$$

3. fafb_gagb.m

PLM: $\hat{E}_t z_{t+h} = a + b \cdot h_x^{h-1} s_t$

I think what I'll have to do is to split this up as

(PLM$_1$) $\hat{E}_t z_{t+h} = a + b\,\hat{E}_t s_{t+h}$ $\qquad \phi = [a, b]$

where $\hat{E}_t s_{t+h} = c + d^h \cdot s_t$ $\qquad$ (PLM$_2$) $\quad \Gamma = [c, d]$

$\hat{E}_t s_{T+1} = c + d^{T-t+1} \cdot s_t$ $\qquad$ or sthg ...

$$g_\beta = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} s_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ c + d^{T-t+1} s_t \right\}$$

$$= \frac{1}{1-\beta} c + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} d^{T-t} \cdot d s_t$$

$$\underline{\underline{g_\beta = \frac{1}{1-\beta} c + \left(I_{nx} - \beta d\right)^{-1} d s_t}}$$

$$\underline{\underline{g_\alpha = \frac{1}{1-\alpha\beta} c + \left(I_{nx} - \alpha\beta d\right)^{-1} \cdot d s_t}}$$

$$f_\beta = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} z_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ a + b\,\hat{E}_t s_{T+1} \right\}$$

$$\Rightarrow \frac{1}{1-\beta} a + b \cdot \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} s_{T+1}$$

$$\underline{\underline{f_\beta = \frac{1}{1-\beta} a + b \cdot g_\beta}} \qquad \underline{\underline{f_\alpha = \frac{1}{1-\alpha\beta} a + b \cdot g_\alpha}}$$

So IRFs look a <u>lot</u> like IRFs from when they are learning the Taylor-rule!

Initially, endog. vars just track expectations 1:1, for which reason they drop a lot at period 2. Therefore, the interest rate has to drop a lot too. It only recovers as agents learn the evolution of states and w/ it, the TR.

But why is the state evolution so closely related to the TR?

→ Maybe b/c this is a shock to the TR. Recall that (*) was modified to be $f_\beta(3) = \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + g_\beta(2)$

they're learning this

So the story isn't quite the same: it's not that they don't internalize the GE effect, it's only that they are not sure which way the shock will go → they think initially that the CB will keep being discretionarily contractionary! Then they learn that the shock is dying out! Wow!

One thing that's been nagging me: code for baseline

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

MN

$$x_t = -\sigma(\psi_\pi \pi_t + \psi_x x_t + \bar{i}_t) + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} + \sigma r_T^n - \sigma\beta(\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \bar{i}_{T+1}) \right\}$$

$$\sigma\psi_\pi \pi_t + (1+\sigma\psi_x) x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma(1-\beta\psi_\pi)\pi_{T+1} + \sigma r_T^n - \sigma \bar{i}_T \right\}$$

$$\begin{bmatrix} \sigma\psi_\pi & 1+\sigma\psi_x \\ 1 & -\kappa \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [\sigma(1-\beta\psi_\pi), 1-\beta-\sigma\beta\psi_x, 0] f_\beta + \sigma[1,-1,0](I_{nx} - \beta hx)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1](I_{nx} - \alpha\beta hx)^{-1} s_t \end{bmatrix}$$

MN, true-baseline

PQ

PQ, true-baseline

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0](I_{nx} - \beta hx)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1](I_{nx} - \alpha\beta hx)^{-1} s_t \\ [0,1,0] s_t \end{bmatrix}$$

s.t. (*)

$$f_\beta(3) = \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \frac{1}{\beta}\left\{ [0,1,0](I_{nx} - \beta hx)^{-1} s_t - [0,1,0] s_t \right\}$$

Ok, so now try CB's $E(\pi_{t+1})$ in TR

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi E_t \pi_{t+1} + \psi_x x_t + \bar{i}_t$$

Two tricky questions arise:

1) How to evaluate $E_t \pi_{t+1}$?

2) How do agents evaluate $E_t \pi_{t+1}$? $\Rightarrow \hat{E}_t E_t \pi_{t+1} = ?$

↳ I think for agents it must hold that $\hat{E}_t E_t \pi_{t+1} = \hat{E}_t \pi_{t+1}$ b/c otherwise we'd be saying that agents think π will go one way but are aware that model-consistent E are going another way. Another way of saying this is to say that agents can't evaluate $E_t \pi_{t+1}$

↳ Although in the criterion, θ, they do evaluate it :D

1) The ALM is $z_t = A_a f_a + A_b f_\beta + A_s s_t$

$\Rightarrow$ Model-consistent expectations are $E(z_t)$, or

$$E_t(z_{t+1}) = E_t\left[A_a f_\alpha(t+1) + A_b \cdot f_\beta(t+1) + A_s \cdot s_{t+1}\right]$$

$$= A_a E_t f_\alpha(t+1) + A_b E_t f_\beta(t+1) + A_s E_t s_{t+1}$$

$- A_a E_t f_\alpha(t+1) + A_b E_t f_\beta(t+1) + A_s\, hx \cdot s_t$

What is $E_t f_\beta(t+1)$?

$$E_t f_\beta(t+1) = E_t \hat{E}_{t+1} \sum_{T=t+1}^{\infty} \beta^{T-t-1} z_{T+1} \qquad :S$$

Or you could say that $f_\beta(t) = \frac{1}{1-\beta} a + b(I_{nx} - \beta hx)^{-1} s_t$

so $E_t f_\beta(t+1) = \frac{1}{1-\beta} a + b(I_{nx} - \beta hx)^{-1} E_t s_{t+1}$

The problem is $E_t(a_t)$ & $E_t(b_t)$ which the CB can't evaluate until z_t has realized and so HHs update ϕ_{t-1} to ϕ_t. So as a shortcut I can assume a version of "anticipated-utility" for the CB, so the CB assumes that ϕ won't be updated

$\rightarrow$ In that case $E_t f_\beta(t+1) = \frac{1}{1-\beta} a + b(I_{nx} - \beta hx)^{-1} hx\, s_t$

$$\left[\begin{aligned} E_t(z_{t+1}) = {} & A_a \left[\frac{1}{1-\alpha\beta} a + b(I_{nx} - \alpha\beta hx)^{-1} hx\, s_t\right] \\ & + A_b \left[\frac{1}{1-\beta} a + b(I_{nx} - \beta hx)^{-1} hx\, s_t\right] \\ & + A_s\, hx\, s_t \end{aligned}\right]$$

[and $E_t(\pi_{t+1}) = e_{11} \cdot E_t(z_{t+1})$ (first el.)]

Ok: so w/ the 2 assumptions that $\qquad k \geq 0$

1. CB has a version of anticipated utility: $E_t(\phi_{t+k}) = E_t(\phi_t)$
2. agents do not internalize that CB has model-consistent expectations $\hat{E}_T E_t(\pi_{t+1}) = \hat{E}_t(\pi_{t+1})$,

we have that the MN & PQ sols are identical to "Epi", only the compact notation (ALM) of the econ is different. It is

$$z_t = A_a f_a + A_b \cdot f_a + A_s s_t + A_e \cdot E_t(\pi_{t+1}), \quad A_e = \begin{bmatrix} 0 \\ 0 \\ \psi_\pi \end{bmatrix}$$

b/c the Taylor-rule is different

and $E_t(\pi_{t+1}) = e_1 \; E_t(z_{t+1})$

$$E_t(z_{t+1}) = A_a \left[\frac{1}{1-\alpha\beta} a + b(I_{nx} - \alpha\beta h x)^{-1} h x \; s_t \right]$$

$$+ A_b \left[\frac{1}{1-\beta} a + b(I_{nx} - \beta h x)^{-1} h x \; s_t \right]$$

$$+ A_s \, h x \cdot s_t$$

- Can use matrices from Epi
- Need to use a different learning code

sim_learnLH_Epi_CB.m

I corrected sthg in Epi

$z_t = A_\alpha f_\alpha + A_\beta \cdot f_\beta + A_s s_t + \begin{bmatrix} 0 \\ 0 \\ \psi \end{bmatrix} \hat{E}_t \pi_{t+1}$

$\Rightarrow$ Now that's stable!

Epi_CB is also stable, but both keep the overshooting!

Alright, so "VAR-learning". 20 Jan 2020

Supp agents only observe observables z_t and use those to forecast their future selves proper:

$$\hat{E}_t z_{t+1} = \underset{n_y \times (n_y+1)}{\psi_{t-1}} \cdot \underset{(n_y+1)\times 1}{\begin{bmatrix} 1 \\ z_{t-1} \end{bmatrix}}$$

with ψ updating exactly as ϕ used to:

$$\psi_t = \left(\psi_{t-1}' + k^{-1} R_t^{-1} \begin{bmatrix} 1 \\ z_{t-1} \end{bmatrix} \left(z_t - \psi_{t-1} \begin{bmatrix} 1 \\ z_{t-1} \end{bmatrix}\right)'\right)'$$

and

$$R_t = R_{t-1} + k^{-1} \left(\begin{bmatrix} 1 \\ z_{t-1} \end{bmatrix} \begin{bmatrix} 1 & z_{t-1} \end{bmatrix} - R_{t-1} \right)$$

R should be initialized as Σ_Y from sol of RE model.

and ψ according to $Y_t = gx\, X_t = gx(hx X_{t-1} + \eta \varepsilon_t)$

$\Rightarrow \underset{(b)}{\psi_0} = gx\, hx\, gx^{-1}$. $\qquad = gx\, hx\, gx^{-1} Y_{t-1} + gx\, \eta\, \varepsilon_t^{=0 \text{ in st.st.}}$

Now what we need to figure out here is "fafb", The LH expectations.

$$fb = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} z_{T+1} = \hat{E}_t \sum_{k=0}^{\infty} \beta^k z_{t+1+k}$$

$$= \hat{E}_t z_{t+1} + \hat{E}_t \beta z_{t+2} + \dots$$

The PLM is $\hat{E}_t z_{t+1} = \varphi_{t-1} \begin{bmatrix} 1 \\ z_t \end{bmatrix}$ but z_t hasn't realized yet, unlike for MSV learning, where that's the first thing that happens.

Let's write this as $\hat{E}_t z_{t+1} = \overset{a_{t-1}}{a} + \overset{b_{t-1}}{b} z_t$

$$= a + b \hat{E}_t z_t = a + b \left[a + b z_{t-1} \right]$$

$$= a + \underset{3\times3}{b} \cdot \underset{3\times1}{a} + b z_{t-1}$$

$$\hat{E}_t z_{t+2} = a + b \left(\hat{E}_t z_{t+1} \right)$$

$$= a + b \left(a + ba + b z_{t-1} \right)$$

$$= a + ba + b^2 a + b^2 z_{t-1}$$

$$= \left[I + b + b^2 \right] a + b^2 z_{t-1}$$

$$\hat{E}_t z_{t+k} = \left[I + b + b^2 + \dots + b^k \right] a + b^k z_{t-1}$$

$$\hat{E}_t z_{t+k+1} = \left[I + b + b^2 + \dots + b^{k+1} \right] a + b^{k+1} z_{t-1}$$

So then $\hat{E}_t \sum_{k=0}^{\infty} \beta^k z_{t+1+k} = \left\{ \left[I + b \right] a + b z_{t-1} \right\} + \dots$

$$\hat{E}_t \sum_{k=0}^{\infty} \beta^k z_{t+1+k} = \left\{[I+b]a + b z_{t-1}\right\}$$
$$+\beta\left\{[I+b+b^2]a + b^2 z_{t-1}\right\}$$
$$+\beta^2\left\{[I+b+b^2+b^3]a + b^3 z_{t-1}\right\}$$
$$+ \ldots$$

$$= [I+b]a + \beta[I+b+b^2]a + \beta^2[I+b+b^2+b^3]a + \ldots$$
$$+ b z_{t-1} + \beta b^2 z_{t-1} + \beta^2 b^3 z_{t-1} + \ldots$$
$$= [I+b]a + \beta[I+b+b^2]a + \beta^2[I+b+b^2+b^3]a + \ldots$$
$$+ b \quad (1-\beta b)^{-1} z_{t-1}$$

Erm... this is unwieldy. But remember that the PLM is VAR: $Y_{t+1} = a + bY_t \quad (+\varepsilon_t(?))$

$\rightarrow$ so Hamilton can tell us how to do that.

Hamilton, p. 93 (Mac):

$$\hat{E}_t\left[Y_{t+s} \mid Y_t, Y_{t-1}, \ldots\right] = \mu + \left[\frac{\psi(L)}{L^s}\right]_+ \eta(L)(Y_t - \mu)$$

or

$$\hat{E}_t\left[Y_{t+s} \mid Y_t, Y_{t-1}, \ldots\right] = \mu + \left[\frac{\psi(L)}{L^s}\right]_+ \frac{1}{\psi(L)} (Y_t - \mu)$$

where $[\cdot]_+$ is the annihilator, and this formula is the Wiener-Kolmogorov formula.

p. 94 Mac: Forecasting an AR(1):

For $Y_t = c + \phi Y_{t-1} + \varepsilon_t$ (p. 66 Mac)

$$\hat{E}_t[Y_{t+s} \mid Y_t, Y_{t-1}, \dots] = \mu + \phi^s (Y_t - \mu)$$

where $\mu = E[Y]$ (unconditional mean) $= \frac{c}{1-\phi}$ (p. 67 Mac)

Ok so then $z_{t+1} = a + b z_t \; (+ \varepsilon_{t+1}?)$

$$\hat{E}_t z_t = \mu + b(z_{t-1} - \mu) \quad w/\ \mu = (I_{ny} - b)^{-1} \cdot a$$

$$\hat{E}_t z_{t+1} = \mu + b^2 (z_{t-1} - \mu)$$

$$\hat{E}_t z_{t+k} = \mu + b^{k+1} (z_{t-1} - \mu)$$

So now $\hat{E}_t \sum_{k=0}^{\infty} \beta^k z_{t+1+k} = \sum_{k=0}^{\infty} \beta^k \{ \mu + b^{k+2} (z_{t-1} - \mu)$

$$= \frac{1}{1-\beta} \mu + (I_{ny} - \beta b)^{-1} b b' (z_{t-1} - \mu) = f_\beta$$

f_β for "VAR-learning"

But now a question arises, which is, do they know hx? If I don't change anything in the model further, then I'm assuming they do. → the compact notation is the same. ("know_hx")

An alternative would be to say that they don't know hx. How would they forecast states? E.g. in the NKPC, they need to evaluate $\hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} u_T$. An interesting extension would be where they don't learn hx and simply assume no shocks in the future: $\hat{E}_t s_{t+k} = s_t$ $\forall k$. ("dont_know_hx")

↳ I didn't pursue this.

A Markov-switching Taylor-rule (à la Davig & Leeper)

$$i_t = \alpha(s_t)\,\pi_t + \gamma(s_t)\,x_t$$ (e.g. (29) D&L p.10 Mac)

where s_t is the underlying state and it evolves according to a Markov chain.

Transition matrix P w/ elements

$$p_{ij} = \text{Prob}\left[s_t = j \mid s_{t-1} = i\right]$$

$$\alpha = \begin{cases}\alpha_1\\ \alpha_2\end{cases} \quad \text{and} \quad \gamma = \begin{cases}\gamma_1\\ \gamma_2\end{cases}$$ Ass: st.st. is invariant.

Let me simplify:

$$i_t = \psi_\pi(s_t)\,\pi_t + \psi_x x_t + \bar{i}_t \qquad \text{w/ } s_t = s_1 \text{ or } s_2$$

What this does though is that it changes $E(\cdot)$:

$\hat{E}_1(z_{t+1})$ needs to be taken conditional on the state.

$$\hat{E}_1(z_{t+1}) = p_{1|1}\,\hat{E}_1(z_{t+1} \mid s_1) + p_{1|2}\,\hat{E}_1(z_{t+1} \mid s_1)$$

where $P_{i|j} = \text{prob}(s_t = i \mid s_{t-1} = j)$

Lol – it's not quite right yet, but it's getting there.

Let me put the intuition in words:

W/ state-dependent TR, E(.) are also state-dep

Supp. the TR is in regime 1. Then $\hat{E}_t(\pi_{t+1})$ depends on what regime I expect for tomorrow.

Since this is true for regime 2 as well, $\hat{E}_t(\pi_{t+1})$ can have 4 different values:

$\hat{E}_t(\pi_{t+1} | s_1)$ and tomorrow $s = s_1 \rightarrow \hat{E}_t(\pi_{t+1} | s_1, s_1)$

$\hat{E}_t(\pi_{t+1} | s_1)$ and tomorrow $s = s_2 \rightarrow \hat{E}_t(\pi_{t+1} | s_2, s_1)$

$\hat{E}_t(\pi_{t+1} | s_2)$ and tomorrow $s = s_1 \rightarrow \hat{E}_t(\pi_{t+1} | s_1, s_2)$

$\hat{E}_t(\pi_{t+1} | s_2)$ and tomorrow $s = s_2 \rightarrow \hat{E}_t(\pi_{t+1} | s_2, s_2)$

That means that in a simulation we'll have to keep track of s_t and form for regime s_i

$$\hat{E}_t(\pi_{t+1} | s_i) = p_{1|s} \hat{E}_t(\pi_{t+1} | s_1, s_i) + p_{2|s_t} \hat{E}_t(\pi_{t+1} | s_2, s_i)$$

→ For the baseline model, this shouldn't change the A-matrices except that this s is a new state... darn. But let's call this little s, "r_t" as in "regime". If this evolves exogenously then agents might even know it.

What I don't know though is what to do w/ LH-E?

$f_\beta = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} z_{T+1}$ ... but each 1-period-ahead fcst is state-dependent ...

So what one can do is a kind-of "Markov-switching anticipated utility": assume that agents think that the TR may switch regime tomorrow but then stay the same ever after. This would give you

$f_{\beta,i} = p_{1|i} \, f_{\beta\, 1,i} + p_{2|i} \cdot f_{\beta\, 2,i}$ for regime i.

LH expectations given that we're in regime i today and 1 tomorrow and ever after.

I can maybe simplify a bit in that I can make $f_{\beta\, j,i}$ independent of today's state

$f_{\beta,i} = p_{1|i} \, f_{\beta,1} + p_{2|i} \cdot f_{\beta\, 2,i}$

But wait a sec ... ψ_π doesn't actually show up in f_β

So scrap that!

Instead it's the A-matrices that change!

$$z_t = A_a \cdot f_a + A_b \cdot f_b + A_s \cdot s_t$$

$$\Rightarrow z_t = A_a(\psi_\pi)\, f_a + A_b(\psi_\pi)\, f_b + A_s(\psi_\pi)\, s_t$$

Is that it to say

$$z_t = p_{1|i} \left\{ A_a(r_1) f_a + A_b(r_1) f_b + A_s(r_1) s_t \right\}$$
$$+ p_{2|i} \left\{ A_a(r_2) f_a + A_b(r_2) f_b + A_s(r_2) s_t \right\} \quad ?$$

And then include com(r) ?

Or actually $z_t = A_a(i) f_a + A_b(i) f_b + A_s(i) s_t \quad i = 1,2$

Shit: RE also needs to be extended to accommodate for this!

Markov-switching RE — 21 Jan 2020

$$z_t = A_0^{RE} E_t z_{t+1} + A_s^{RE} s_t$$

The difficulty here is that $E_t z_{t+1}$ will actually be conditional on s.

At least this way of writing the model is close to Davig & Leeper 2007's system (35) (p. 17 Mac)

$$\underset{\substack{\uparrow \\ \text{jumps}}}{AY_t} = BY_{t-1} + \underset{\substack{\uparrow \\ \text{FEs}}}{A\eta_t} + \underset{\substack{\uparrow \\ \text{states}}}{Cu_t}$$

The RE model is (w/ Markov-switching ψ_π)

$$x_t = E_t x_{t+1} - \sigma i_t + \sigma E_t \pi_{t+1} + \sigma r_t^n$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t$$

$$i_t = \psi_\pi(r)\pi_t + \psi_x x_t + \bar{i}_t$$

To follow Davig & Leeper (2007), do the MN-style subst.

and sub i_t out:

$$\left.\begin{aligned} x_t &= -\sigma\left(\psi_\pi(r)\pi_t + \psi_x x_t + \bar{i}_t\right) + E_t x_{t+1} + \sigma E_t \pi_{t+1} + \sigma r_t^n \\ \pi_t &= \kappa x_t + \beta E_t \pi_{t+1} + u_t \end{aligned}\right\}$$

This is the system to be solved.

Denote w/ $x_{it} := x_t | s_t = i \quad i = 1, 2$ shock (real int. rate)

$\Omega_t = \{ s_t, s_{t-1}, \dots, r_t, r_{t-1}, \dots \}$ = info set

$\Omega_t^{-s} = \{ s_{t-1}, s_{t-2}, \dots, r_t, r_{t-1}, \dots \}$, a smaller

info set that doesn't include today's s

(which is weird — I think it should)

As on p. 5 (Mac),

$$E_t \pi_{t+1} = E[\pi_{t+1} | s_t = i, \Omega_t^{-s}]$$

$$= p_{1i} E[\pi_{1t+1} | \Omega_t^{-s}] + p_{2i} E[\pi_{2t+1} | \Omega_t^{-s}]$$

$$= p_{1i} E_t \pi_{1t+1} + p_{2i} E_t \pi_{2t+1}$$

The issue is computing the RE-expectations.

Ah... but w/ method of undet coeffs you're conjectured the state-space as a sol, and given that, E should be easy to compute.

So if I extend the state-space kind of like Davig & Leeper, I can maybe conjecture an "extended hx & gx"

Let $Y_t = \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \\ x_{1t} \\ x_{2t} \end{bmatrix}$ $E_t Y_{t+1} = \begin{bmatrix} E_t \pi_{1t+1} \\ E_t \pi_{2t+1} \\ E_t x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix}$ $S_t = \begin{bmatrix} r_t^n \\ u_t \end{bmatrix}$

As if there were 4 jumps instead of two.

Using these 4 jumps, rewrite the model as for $i=1,2$

(82) $\pi_{it} = \beta(p_{1i} E_t \pi_{1t+1} + p_{2i} E_t \pi_{2t+1}) + \kappa x_{it} + u_t$

(83) $x_{it} = p_{1i} E_t x_{1t+1} + p_{2i} E_t x_{2t+1} - \sigma(\psi_i \pi_{it} + \psi_x x_{it})$
$+ \sigma(p_{1i} E_t \pi_{1t+1} + p_{2i} E_t \pi_{2t+1}) + \sigma r_t^n$

Now rewrite (82)-(83) in MN form:

$$\pi_{it} - \kappa x_{it} = \beta\rho_{1i} E_t \pi_{1t+1} + \beta\rho_{2i} E_t \pi_{2t+1} + u_t$$

$$\sigma\psi_i \pi_{it} + (1+\sigma\psi_x) x_{it} = \rho_{1i} E_t x_{1t+1} + \rho_{2i} E_t x_{2t+1} + \sigma\rho_{1i} E_t \pi_{1t+1} + \sigma\rho_{2i} E_t \pi_{2t+1} + \sigma r_t^n$$

The order is $\pi_{1t}, \pi_{2t}, x_{1t}, x_{2t}$

$$\underbrace{\begin{bmatrix} 1 & 0 & -\kappa & 0 \\ \sigma\psi_1 & 0 & 1+\sigma\psi_x & 0 \\ 0 & 1 & 0 & -\kappa \\ 0 & \sigma\psi_2 & 0 & 1+\sigma\psi_x \end{bmatrix}}_{=:A} \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \\ x_{1t} \\ x_{2t} \end{bmatrix} = \overbrace{\begin{bmatrix} \beta\rho_{11} & 0 & \beta\rho_{21} & 0 \\ \sigma\rho_{11} & \rho_{11} & \sigma\rho_{21} & \rho_{21} \\ \beta\rho_{12} & 0 & \beta\rho_{22} & 0 \\ \sigma\rho_{12} & \rho_{12} & \sigma\rho_{22} & \rho_{22} \end{bmatrix}}^{=:B} \begin{bmatrix} E_t\pi_{1t+1} \\ E_t\pi_{2t+1} \\ E_t x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix}$$

$$+ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \sigma \\ 1 & 0 \\ 0 & \sigma \end{bmatrix}}_{=:C} \begin{bmatrix} u_t \\ r_t^n \end{bmatrix}$$

$$\Rightarrow A Y_t = B E_t Y_{t+1} + C \cdot X_t$$

Erm... why isn't there a X_{t+1} term?

Erm, another thing: can we simply input these equations into gx_hx? Like:

Model equations:

$$\pi_{it} = \beta(p_{1i} E_t \pi_{1t+1} + p_{2i} E_t \pi_{2t+1}) + \kappa x_{it} + u_t$$

$$x_{it} = p_{1i} E_t x_{1t+1} + p_{2i} E_t x_{2t+1} - \sigma i_{it} + \sigma(p_{1i} E_t \pi_{1t+1} + p_{2i} E_t \pi_{2t+1}) + \sigma r_t^n$$

$$i_{it} = \psi_i \pi_{it} + \psi_x x_{it} + \bar{i}_t$$

$\Rightarrow$ 6 eqs, 6 unknowns

$\hookrightarrow$ ok, works for now, need to discuss w/ Ryan though!

Ok, so last extension I'll attempt is the "(moving) average π in TR" "MA-TR"

$\rightarrow$ in a way this is related to pil if the MA-terms are bw-looking, or to Epi if they are fw-looking.

A (in terms of lag structure) not so demanding version is

$$i_t = \theta \pi_{t-1} + \psi_\pi \pi_t + \theta^{-1} \hat{E}_t \pi_{t+1} + \psi_x x_t + \bar{i}_t$$

$$
\left.
\begin{aligned}
x_t &= -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\} \\
\pi_t &= \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\} \\
i_t &= \theta \pi_{t-1} + \psi_\pi \pi_t + \theta^{-1} \hat{E}_t \pi_{t+1} + \psi_x x_t + \bar{i}_t
\end{aligned}
\right\}
$$

<u>MN</u>

$$
x_t = -\sigma \left[\theta \pi_{t-1} + \psi_\pi \pi_t + \theta^{-1} \hat{E}_t \pi_{t+1} + \psi_x x_t + \bar{i}_t \right]
$$
$$
+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}
$$

$$
\sigma\psi_\pi \pi_t + (1+\sigma\psi_x) x_t = -\sigma\theta \pi_{t-1} - \sigma\theta^{-1} \hat{E}_t \pi_{t+1} - \sigma \bar{i}_t
$$
$$
+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} + \sigma r_T^N
$$
$$
- \sigma\beta \left[\theta \pi_T + \psi_\pi \pi_{T+1} + \theta^{-1} \hat{E}_T \pi_{T+2} + \psi_x x_{T+1} + \bar{i}_T \right] \Big\}
$$

$$
\sigma\psi_\pi \pi_t + (1+\sigma\psi_x) x_t = -\sigma\theta \pi_{t-1} - \sigma\theta^{-1} \hat{E}_t \pi_{t+1}
$$
$$
+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big\{ (1-\beta-\beta\sigma\psi_x) x_{T+1} + \sigma r_T^N - \sigma \bar{i}_T + \sigma \pi_{T+1}
$$
$$
- \sigma\beta\theta \pi_T - \sigma\beta\psi_\pi \pi_{T+1} - \sigma\beta\theta^{-1} \pi_{T+2} \Big\}
$$

$1-\beta^2\psi_\pi > 0$

$1 > \beta^2\psi_\pi$

$1/\beta^2 > \psi_\pi$

$$
\sigma\psi_\pi \pi_t + (1+\sigma\psi_x) x_t = -\sigma\theta \pi_{t-1} - \sigma\beta\theta \pi_t
$$
$$
+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big\{ (1-\beta-\beta\sigma\psi_x) x_{T+1} + \sigma r_T^N - \sigma \bar{i}_T + \sigma \pi_{T+1}
$$
$$
\underbrace{- \sigma\beta^2\theta \pi_{T+1} - \sigma\beta\psi_\pi \pi_{T+1} - \sigma\,\theta^{-1} \pi_{T+1}}_{\sigma\left[1-\beta^2\theta-\beta\psi_\pi-\theta^{-1}\right]\pi_{T+1}} \Big\}
$$

This comes from pil
→ so only that extension
should in theory have a
chance.

Timeout: more thoughts on why pill can work:

Suppose we had π_{t-2} in TR.

then $\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \text{stuff} + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} - 3\beta(\pi_{T-1})$

$= \text{stuff} + \pi_{t-1} + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \beta \pi_T (-3\beta)$

$= \text{stuff} + \pi_{t-1} + \beta\pi_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \beta^2 \pi_t (-3\beta)$

$\underline{\underline{-3\beta^3}}$

So for one lag, if $\psi_\pi = 1.5$, then β has to b

$\psi_\pi < \frac{1}{\beta^2} \Rightarrow \quad \beta < \left(\frac{1}{\psi_\pi}\right)^{\frac{1}{2}} = 0.8165$ hmm...

For two lags, $\quad \beta < \left(\frac{1}{\psi_\pi}\right)^{\frac{1}{3}} = 0.8736$ hmm...

$\hookrightarrow$ So sorry I won't pursue the MA-TR extension!

Instead, try 2-lag π in TR! "pill"

$i_t = \psi_\pi \pi_{t-2} + \psi_x x_t + \bar{i}_t$

w/ suboptimal festers, and then maybe opt using linking.

"pill" - extension using "subopt forecasts"

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma \beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_{t-2} + \psi_x x_t + \bar{i}_t$$

MN

$$x_t = -\sigma(\psi_\pi \pi_{t-2} - \psi_x x_t + \bar{i}_t)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma \beta i_{T+1} + \sigma r_T^n \right\}$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \pi_{t-2}$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma[1,-1,0,0,0] S_T$$
$$+ \sigma \pi_{T+1} - \sigma\beta \psi_\pi \pi_{T-1} \Big\}$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \pi_{t-2} - \sigma\beta\psi_\pi \pi_{t-1}$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma[1,-1,0,0,0] S_T$$
$$+ \sigma \pi_{T+1} - \sigma\beta^2 \psi_\pi \pi_T \Big\}$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \pi_{t-2} - \sigma\beta\psi_\pi \pi_{t-1} - \sigma\beta^2\psi_\pi \pi_t$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma[1,-1,0,0,0] S_T$$
$$+ \sigma \pi_{T+1} - \sigma\beta^3 \psi_\pi \pi_{T+1} \Big\}$$

$$b\beta^2\psi_\pi\pi_t + (1+b\psi_x)x_t = -b\psi_\pi\pi_{t-2} - b\beta\psi_\pi\pi_{t-1}$$
$$+\hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{(1-\beta-b\beta\psi_x)x_{T+1} + b(1-\beta^3\psi_\pi)\pi_{T+1} + b[1,-1,0,0,0]s_T\right\}$$

So, together w/

$$\pi_t = \kappa x_t + \hat{E}_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left\{\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\right\}$$

This gives:

$$\begin{bmatrix} b\beta^2\psi_\pi & 1+b\psi_x \\ 1 & -\kappa \end{bmatrix}\begin{bmatrix}\pi_t \\ x_t\end{bmatrix} = \begin{bmatrix} [b(1-\beta^3\psi_\pi), 1-\beta-b\beta\psi_x, 0]f_\beta + b[1,-1,0,0,0](I_{nx}-\beta hx)^{-1}s_t + [0,0,0,-b\beta\psi_\pi,-b\psi_\pi]s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0]f_\alpha + [0,0,1,0,0](I_{nx}-\alpha\beta hx)^{-1}s_t \end{bmatrix}$$

MN, "pile", "opt factors"

<u>PQ</u>

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -\kappa & 0 \\ 0 & \psi_x & 1 \end{bmatrix}\begin{bmatrix}\pi_t \\ x_t \\ i_t\end{bmatrix} = \begin{bmatrix} [b, 1-\beta, -b\beta]f_\beta + b[1,0,0,0,0](I_{nx}-\beta hx)^{-1}s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0]f_\alpha + [0,0,1,0,0](I_{nx}-\alpha\beta hx)^{-1}s_t \\ [0,1,0,0,\psi_\pi]s_t \end{bmatrix}$$

s.t. (*)

$$(*)\quad \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \psi_\pi \pi_{T-1} + \psi_x x_{T+1} + \bar{i}_{T+1} \right\}$$

$f_\beta(3)$

$$= \psi_x f_\beta(2) + \frac{1}{\beta}\left\{ [0,1,0,0,0](I_{nx} - \beta hx)^{-1} s_t - [0,1,0,0,0] s_t \right\}$$

$$+ \underbrace{\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \psi_\pi \pi_{T-1}}$$

$$\underbrace{\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \psi_\pi \pi_T \beta + \psi_\pi \pi_{t-1}}$$

$$= \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \psi_\pi \beta^2 \pi_{T+1} + \psi_\pi \pi_{t-1} + \beta \psi_\pi \pi_t$$

So $(*)$:

$$\left[\; f_\beta(3) = \beta^2 \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \frac{1}{\beta}\left\{ [0,1,0,0,0](I_{nx} - \beta hx)^{-1} s_t - [0,1,0,0,0] s_t \right\} + \psi_\pi \pi_{t-1} + \beta \psi_\pi \pi_t \right.$$

↳ So LHS $P(1,1)$ becomes:

$$0 - (-\gamma\beta \cdot \beta\psi_\pi) = \underline{\underline{\gamma\beta^2\psi_\pi}} \leftarrow P(1,1)$$

2 discoveries: <u>22 Jan 2020</u>

1) It's not 2x lagged π in TR which works ... it's $\beta\downarrow$

2) lagged jumps as endog states is still not quite correct b/c in the learning sim code, the code just

advances endog states as if they were evolving according to the RE lrx, whereas they are evolving according to $\hat{g}$.

Ryan meeting — 22 Jan 2020

→ check Ryan's code to see what he did for projection facility

↳ none of the approaches I had really correspond to getting rid of explosive stuff

- Plot RE for
  - only passive ← that's the usual thing ppl do!
  - only active regime

- Can have them learn the regime (a separate constant (& slope) for each regime)

→ try the passive initialization

- RE w/ π_{t-1} has a unique eqb? Check!

check if

$$Y^e_{t+h} = \hat{g}\left(h^{RE}\right)^h \begin{bmatrix} x_t \\ \pi_{t-1} \end{bmatrix}$$

i.e. does the learning fcst equal the estimated

$\hat{g} \cdot hx \cdot s_t$? $\Rightarrow$ For lagged jumps, you wanna input $\hat{g}$ here

For proj facility:

$$Y_t = \hat{g}\, x_t$$

$$\begin{bmatrix} x_{t+1} \\ \pi_t \end{bmatrix} = \underbrace{\begin{bmatrix} h^{RE} & 0 \\ g^{RE}_{\pi} & \end{bmatrix}}_{\text{check eig of this whole matrix}} \begin{bmatrix} x_t \\ \pi_{t-1} \end{bmatrix}$$

What Evans & Honk might have meant w/ "overparametrize" = too many degrees of freedom. So that oscillations are more likely if agents are allowed to estimate too many things!

$\rightarrow$ like overfitting a model when you're estimating

$\rightarrow$ it's bad b/c you fit well but bad fcst.

→ Like overdifferencing in a sense:
everyone knows that it's a sin and you
shouldn't do it but no one really knows why.