Monetary Policy & Anchored Expectations An Endogenous Gain Learning Model

Laura Gáti

Boston College

Green Line Macro Meeting

September 18, 2020

Inflation that runs below its desired level can lead to an unwelcome fall in longer-term inflation expectations, which, in turn, can pull actual inflation even lower, resulting in an adverse cycle of ever-lower inflation and inflation expectations. [...] Well-anchored inflation expectations are critical[.]

Jerome Powell, Chairman of the Federal Reserve ¹ (Emphases added.)

¹"New Economic Challenges and the Fed's Monetary Policy Review," August 27, 2020.

Anchored expectations

Anchored expectations: long-run expectations at Fed's target (π *)

Anchored expectations

Anchored expectations: long-run expectations at Fed's target (π *)

→ short-run expectations "anchored" to stable mean:

$$\mathbb{E}_t \, \pi_{t+1} = \pi^* + f(shocks) \tag{1}$$

Anchored expectations

Anchored expectations: long-run expectations at Fed's target (π *)

→ short-run expectations "anchored" to stable mean:

$$\mathbb{E}_t \, \pi_{t+1} = \pi^* + f(shocks) \tag{1}$$

Unanchored expectations: long-run expectations deviate systematically from the target

$$\mathbb{E}_t \, \pi_{t+1} = \bar{\pi}_{t-1} + f(shocks) \tag{2}$$

$$\bar{\pi}_{t-1} \neq \pi^*$$
 and $\bar{\pi}_{t-1} = h(shocks)$

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \, \mathbb{E}_t \, \pi_{t+1} + u_t \tag{3}$$

$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t \tag{4}$$

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \, \mathbb{E}_t \, \pi_{t+1} + u_t \tag{3}$$

$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t \tag{4}$$

The Fed's uneasiness: public thinks inflation averages only 1%

$$\pi_t = \kappa x_t + \beta(1\% + f(shocks)) + u_t \tag{5}$$

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \, \mathbb{E}_t \, \pi_{t+1} + u_t \tag{3}$$

$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t \tag{4}$$

The Fed's uneasiness: public thinks inflation averages only 1%

$$\pi_t = \kappa x_t + \beta(1\% + f(shocks)) + u_t \tag{5}$$

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \, \mathbb{E}_t \, \pi_{t+1} + u_t \tag{3}$$

$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t \tag{4}$$

The Fed's uneasiness: public thinks inflation averages only 1%

$$\pi_t = \kappa x_t + \beta(1\% + f(shocks)) + u_t \tag{5}$$

A standard Phillips curve with expectations anchored at the 2% target

$$\pi_t = \kappa x_t + \beta \, \mathbb{E}_t \, \pi_{t+1} + u_t \tag{3}$$

$$\pi_t = \kappa x_t + \beta(2\% + f(shocks)) + u_t \tag{4}$$

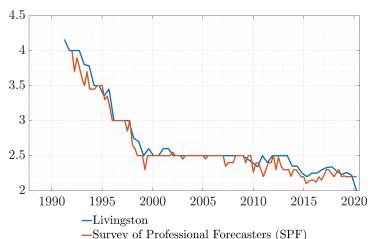
The Fed's uneasiness: public thinks inflation averages only 1%

$$\pi_t = \kappa x_t + \beta(1\% + f(shocks)) + u_t \tag{5}$$

→ unanchored expectations can cause a deflationary spiral

Long-run expectations drifting down?

Figure: Expectations of average inflation over 10 years ($\bar{\pi}$ in data)





Long-run expectations moving systematically?

SPF: for 1999-Q1 onward, estimate

$$\bar{\pi}_t = \beta_0 + \beta_1 f e_{t|t-1} + \epsilon_t \tag{6}$$

where
$$\bar{\pi}_t \equiv \mathbb{E}_t(\pi_{t+10})$$
 and $fe_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1}(\pi_t)$ (forecast error)

Long-run expectations moving systematically?

SPF: for 1999-Q1 onward, estimate

$$\bar{\pi}_t = \beta_0 + \beta_1 f e_{t|t-1} + \epsilon_t \tag{6}$$

where
$$\bar{\pi}_t \equiv \mathbb{E}_t(\pi_{t+10})$$
 and $fe_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1}(\pi_t)$ (forecast error)

$$\Rightarrow \hat{\beta}_1 = 0.06$$
 (p-value: 0.000017)

 \Rightarrow 1 pp forecast error \rightarrow 6 bp revision in long-run expectations

Long-run expectations moving systematically?

SPF: for 1999-Q1 onward, estimate

$$\bar{\pi}_t = \beta_0 + \beta_1 f e_{t|t-1} + \epsilon_t \tag{6}$$

where $\bar{\pi}_t \equiv \mathbb{E}_t(\pi_{t+10})$ and $fe_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1}(\pi_t)$ (forecast error)

$$\Rightarrow \hat{\beta}_1 = 0.06$$
 (p-value: 0.000017)

 \Rightarrow 1 pp forecast error \rightarrow 6 bp revision in long-run expectations

Likely understatement: professional forecasters may run stationary models



This project

 How to conduct monetary policy in interaction with the anchoring expectation formation?

- Model of anchoring expectation formation as an extension to adaptive learning
 - \hookrightarrow twist: systematic fluctuations in long-run expectations

• Estimation of the anchoring function: when do expectations become unanchored?

Preview of results

 Optimal policy aggressive when expectations unanchor, accommodates when anchored

• Taylor rule policy less aggressive on inflation than under rational expectations

 \hookrightarrow Anchoring-optimal Taylor rule eliminates 75% of loss from volatility

Related literature

 Optimal monetary policy in New Keynesian models Clarida, Gali & Gertler (1999), Woodford (2003)

• Adaptive learning

Evans & Honkapohja (2001, 2006), Eusepi & Preston (2011), Milani (2007, 2014), Lubik & Matthes (2018), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Mele et al. (2019)

• Anchoring and the Phillips curve

Sargent (1999), Williams (2006), Svensson (2015), Afrouzi et al. (2015), Hooper et al. (2019), Hebden et al. (2020), Afrouzi & Yang (2020), Reis (2020), Gobbi et al (2019), Carvalho et al (2019)

Today's talk

1. Model of anchoring expectations: key elements

2. Features of optimal policy under anchoring expectation formation

Today's talk

1. Model of anchoring expectations: key elements

2. Features of optimal policy under anchoring expectation formation

The New Keynesian backbone

• IS- and Phillips curve under rational expectations:

$$x_t = \mathbb{E}_t x_{t+1} - \sigma i_t + \mathbb{E}_t \pi_{t+1} + \sigma r_t^n$$
 (7)

$$\pi_t = \kappa x_t + \beta \, \mathbb{E}_t \, \pi_{t+1} + u_t \tag{8}$$

The New Keynesian backbone

• IS- and Phillips curve under rational expectations:

$$x_t = \mathbb{E}_t x_{t+1} - \sigma i_t + \mathbb{E}_t \pi_{t+1} + \sigma r_t^n$$
 (7)

$$\pi_t = \kappa x_t + \beta \, \mathbb{E}_t \, \pi_{t+1} + u_t \tag{8}$$

• E: Agents know they are identical and can compute (7)-(8)

The New Keynesian backbone

• IS- and Phillips curve under rational expectations:

$$x_t = \mathbb{E}_t x_{t+1} - \sigma i_t + \mathbb{E}_t \pi_{t+1} + \sigma r_t^n$$
 (7)

$$\pi_t = \kappa x_t + \beta \, \mathbb{E}_t \, \pi_{t+1} + u_t \tag{8}$$

- \mathbb{E} : Agents know they are identical and can compute (7)-(8)
- Here instead: adaptive learning

$$\hat{\mathbb{E}}_t \pi_{t+1} = \overline{\pi}_{t-1} + b^{RE} s_t \tag{9}$$

 $(s_t = exogenous states)$

Recursive least squares

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \underbrace{k_t}_{\in (0,1), \text{ gain}} \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b^{RE} s_{t-1})\right)}_{fe_{t|t-1}, \text{ forecast error}}$$
 (10)

Recursive least squares

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \underbrace{k_t}_{\in (0,1), \text{ gain}} \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b^{RE} s_{t-1})\right)}_{fe_{t|t-1}, \text{ forecast error}}$$
(10)

Decreasing gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f e_{t|t-1} \tag{11}$$

 \rightarrow sample mean of full sample of forecast errors

Recursive least squares

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \underbrace{k_t}_{\in (0,1), \text{ gain}} \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b^{RE} s_{t-1})\right)}_{fe_{t|t-1}, \text{ forecast error}}$$
(10)

Decreasing gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f e_{t|t-1} \tag{11}$$

 \rightarrow sample mean of full sample of forecast errors

Constant gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k \, f e_{t|t-1}$$
 (12)

→ sample mean of most recent observations only

Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{k_t}{k_t} \left(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}) \right)$$
 (13)

$$k_t = \mathbf{g}(fe_{t|t-1})$$
: anchoring function

Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \left(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}) \right) \tag{13}$$

 $k_t = \mathbf{g}(fe_{t|t-1})$: anchoring function

$$\mathbf{g}(fe_{t|t-1}) = \sum_{i} \alpha_i b_i (fe_{t|t-1}) \tag{14}$$

 $b_i(fe_{t|t-1}) = \text{basis}$, here: second order spline (piecewise linear)

 α_i = approximating coefficients, here: use $\hat{\alpha}$ from estimation



Today's talk

1. Model of anchoring expectations: key elements

2. Features of optimal policy under anchoring expectation formation

Optimal policy - responding to unanchoring

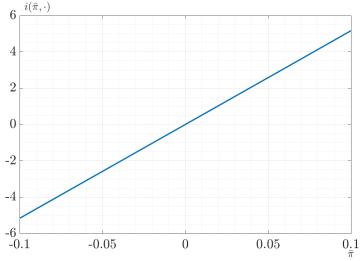


Figure: Policy function: $i(\bar{\pi}, \text{all other states at their means})$

The intertemporal volatility tradeoff

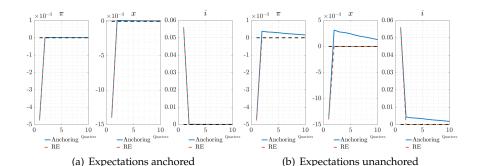
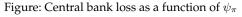
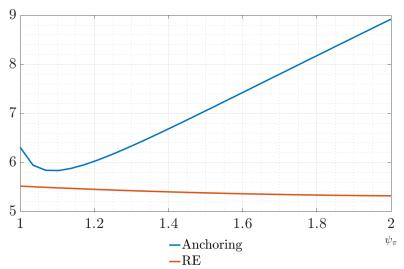


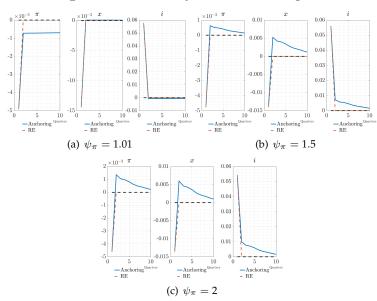
Figure: Impulse responses after a contractionary monetary policy shock

Optimal Taylor-coefficient on inflation





The intertemporal volatility tradeoff - again



Losses for optimal Taylor-rule coefficient on inflation

RE-optimal coefficient: $\psi_{\pi}^{RE} = 2.21$

Anchoring-optimal coefficient: $\psi_{\pi}^{A} = 1.09$

Table: Loss for RE and anchoring models for choice of RE- or anchoring-optimal ψ_π

Anchoring, ψ_{π}^{RE}	Anchoring, ψ_{π}^{A}	RE, ψ_{π}^{RE}
9.6901	5.8296	5.3148

 \to If model is anchoring, anchoring-optimal ψ_π^A gets 75% of the distance to RE-optimal ψ_π^{RE} under RE

Conclusion

- First theory of monetary policy for potentially unanchored expectations
- Optimal policy frontloads aggressive interest rate response to suppress potential unanchoring
- $\bullet\,$ Matters: already anchoring-optimal Taylor rule reduces losses by 75%
- Future work: how to anchor at zero-lower bound?



Calibration - parameters from the literature

β	0.98	stochastic discount factor	
$\overline{\sigma}$	1	intertemporal elasticity of substitution	
α	0.5	Calvo probability of not adjusting prices	
κ	0.0842	slope of the Phillips curve	
ψ_{π}	1.5	coefficient of inflation in Taylor rule*	
$\overline{\psi_{x}}$	0.3	coefficient of the output gap in Taylor rule*	
\bar{g}	0.145	initial value of the gain	
λ_x	0.05	weight on the output gap in central bank loss	
ρ_r	0	persistence of natural rate shock	
$-\rho_i$	0	persistence of monetary policy shock*	
ρ_u	0	persistence of cost-push shock	

 $[\]ensuremath{^*}$ pertains to sections where Taylor rule is in effect

Calibration - matching moments

σ_r	0.01	standard deviation, natural rate shock
σ_i	0.01	standard deviation, monetary policy shock*
σ_u	0.5	standard deviation, cost-push shock
$\hat{\alpha}_i$	(0.33; 0.25; 0.001; 0.24; 0.33)	coefficients in anchoring function

Calibrated $(\sigma_j, j = r, i, u)$ or estimated $(\hat{\alpha}_i)$ to match the autocovariances of inflation, output gap, interest rate and one-period ahead inflation expectations for lags $0, \dots, 4$.

^{*} pertains to sections where Taylor rule is in effect

Breakeven inflation



Figure: Market-based inflation expectations, various horizons, %



Correcting the TIPS from liquidity risk



Figure: Market-based inflation expectations, 10 year, %



Further evidence

Figure: Livingston Survey of Firms: Interquartile range of 10-year ahead inflation expectations

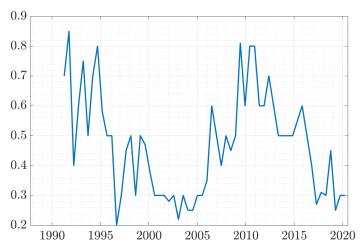
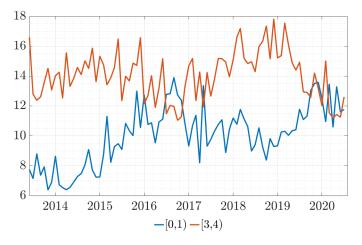




Figure: New York Fed Survey of Consumers: Percent of respondents indicating 3-year ahead inflation will be in a particular range





Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{15}$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{16}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \tag{17}$$

Solve for forecast error $fe_t \equiv \pi_t - f_{t-1}$:

$$fe_{t} = \underbrace{-\frac{1-\beta}{1-k\beta}}_{\lim_{t \to 1} = -1} f_{t-1} + \frac{1}{1-k\beta} u_{t}$$
(18)

Functional forms for g in the literature

• Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2}$$
 (19)

 $p \equiv Prob(\text{liquidity trap regime})$ y_{t-1} output gap

• Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when } \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases}$$
 (20)

 θ_t criterion, $\bar{\theta}$ threshold value



Choices for criterion θ_t

• Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))|$$
 (21)

 Σ variance-covariance matrix of shocks $T(\phi)$ mapping from PLM to ALM

CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1} (f e_{t|t-1} f e'_{t|t-1} - \omega_{t-1})$$
(22)

$$\theta_t^{\text{CUSUM}} = \theta_{t-1} + \kappa k_{t-1} (f e'_{t|t-1} \omega_t^{-1} f e_{t|t-1} - \theta_{t-1})$$
 (23)

 ω_t estimated forecast-error variance



Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right)' \right)' \tag{24}$$

$$R_{t} = R_{t-1} + k_{t} \begin{pmatrix} 1 \\ s_{t-1} \end{pmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1}$$
 (25)



Actual laws of motion

$$s_t = h s_{t-1} + \epsilon_t$$

 $y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \tag{28}$$

(26)

(27)

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T-t}^{\infty} (\alpha \beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T-t}^{\infty} (\beta)^{T-t} y_{T+1}$$
 (29)

Anchoring function in the data

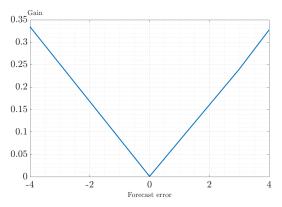


Figure: Learning gain as a function of forecast errors in inflation (pp)



No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t} (\pi_t - \kappa x_t - \beta f_t + u_t) + \varphi_{2,t} (f_t - f_{t-1} - k_t (\pi_t - f_{t-1})) + \varphi_{3,t} (k_t - \mathbf{g}(\pi_t - f_{t-1})) \right\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa}x_t - \varphi_{2,t}(k_t + \mathbf{g}_{\pi}(\pi_t - f_{t-1})) = 0$$
 (30)

$$-2\beta \frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1} (1 - k_{t+1} - \mathbf{g_f}(\pi_{t+1} - f_t)) = 0$$
 (31)



Target criterion system for anchoring function as changes of the gain

$$\varphi_{6,t} = -cfe_{t|t-1}x_{t+1} + \left(1 + \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1}) - fe_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1}$$
$$-\frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1})\varphi_{6,t+2} \tag{32}$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{fe_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{fe_{t|t-1}}\varphi_{6,t+1}$$
(33)

 $\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (33) is given by:

$$\varphi_{6,t} = -2 \, \mathbb{E}_t \sum_{i=0}^{\infty} (\pi_{t+i} + \frac{\lambda_x}{\kappa} x_{t+i}) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{f e_{t+j|t+j-1}}}{\frac{k_{t+j}}{f e_{t+j|t+j-1}}} + \mathbf{g}_{\pi,t+j}$$
(34)



Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta - 1}{\theta}} dj \right]^{\frac{\theta}{\theta - 1}}$$
(35)

 $\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{36}$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t)$$
(37)

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)}$$
(38)



Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1})$$
(39)

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i$$
(40)

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_tY^*}$.

- 1. Solve (39) backward to some date *t*, take expectations at *t*
- 2. Sub in (40)
- 3. Aggregate over households *i*
- \rightarrow Obtain (7)



Target criterion

Proposition

In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_{\pi,t} \right)$$

$$\left(\mathbb{E}_{t} \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_{1}s_{t+j}) \mathbf{g}_{\bar{\pi}, \mathbf{t}+\mathbf{j}})\right)$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ at t, $\prod_{i=0}^{0} \equiv 1$ and b_1 is the first row of b.



Two layers of intertemporal stabilization tradeoffs

$$\pi_{t} = -\frac{\lambda_{x}}{\kappa} x_{t} + \frac{\lambda_{x}}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_{t} + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_{t} \sum_{i=1}^{\infty} x_{t+i}$$
$$-\frac{\lambda_{x}}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_{t} + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_{t} \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j} + fe_{t+1+j|t+j} \mathbf{g}_{\pi,t+j})$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

▶ Why no commitment?

Corollary

Optimal policy under adaptive learning is time-consistent.

 \hookrightarrow Foreshadow: optimal policy aggressiveness time-varying