

Materials 12c - A general solution method for the learning model that works

- WORKING MN method as long as  $\rho = 0$

The goal is to have a flexible, automated method so one can tweak the model and avoid errors.

See Notes 11 Dec 2019

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January 4, 2020

## 1 Model equations and goal

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (1)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (2)$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \rho i_{t-1} + \bar{i}_t \quad (3)$$

Goal: obtain endogenous stuff as a function of expectations and states:

$$z_t = \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = A_a f_a + A_b f_b + A_s s_t \quad (4)$$

where I already have expectations  $f_a, f_b$  and the state vector can vary by model, but in this default case it is

$$s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{bmatrix} \quad (5)$$

That is, we want the matrices  $A_a, A_b$  and  $A_s$ .

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## 2 Step 1 - eliminate the interest rate and introduce LH expectations for observables and states

$$\underbrace{\begin{bmatrix} \sigma\psi_\pi & 1 + \sigma\psi_x \\ 1 & -\kappa \end{bmatrix}}_{\equiv M} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \underbrace{\begin{bmatrix} c_{x,b}f_b + c_{x,s}s_t \\ c_{\pi,a}f_a + c_{\pi,s}s_t \end{bmatrix}}_{\equiv N} \quad (6)$$

where  $M$  and the  $c$  are model-specific. In the baseline model they are given by

$$c_{x,b} = \begin{bmatrix} \sigma(1 - \beta\psi_\pi), & 1 - \beta - \sigma\beta\psi_x, & 0 \end{bmatrix} \quad (7)$$

$$c_{x,s} = -\sigma \begin{bmatrix} -1 & 1 & 0 & \rho \end{bmatrix} (I_{nx} - \beta h_x)^{-1} \quad (8)$$

$$c_{\pi,a} = \begin{bmatrix} (1 - \alpha)\beta, & \kappa\alpha\beta, & 0 \end{bmatrix} \quad (9)$$

$$c_{\pi,s} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} (I_{nx} - \alpha\beta h_x)^{-1} \quad (10)$$

$$c_{i,s} = \begin{bmatrix} 0 & 1 & 0 & \rho \end{bmatrix}$$

## 3 Step 2 - solve for observables as a function of $cs$ , expectations and states

Now unleash Mathematica (`materials12.nb`, after “Restart 12 Dec 2019”) to solve for  $x$  and  $\pi$  as

$$\begin{bmatrix} \pi_t^* \\ x_t^* \end{bmatrix} = M^{-1}N \quad (11)$$

Then the solution for the interest rate will just be

$$i_t = \psi_\pi \pi_t^* + \psi_x x_t^* + c_{is}s_t \quad (12)$$

The last step is to gather the matrices  $g_{i,j}$ , the coefficients of  $i$  on  $j$ ,  $i = x, \pi, i, j = f_a, f_b, s$ . Mathematica will output these  $g$ -matrices and stack them appropriately to give the  $A$ -matrices:

$$\underbrace{A_a}_{ny \times ny} = \begin{pmatrix} g_{\pi,a} \\ g_{x,a} \\ g_{i,a} \end{pmatrix} \quad \underbrace{A_b}_{ny \times ny} = \begin{pmatrix} g_{\pi,b} \\ g_{x,b} \\ g_{i,b} \end{pmatrix} \quad \underbrace{A_s}_{ny \times nx} = \begin{pmatrix} g_{\pi,s} \\ g_{x,s} \\ g_{i,s} \end{pmatrix} \quad (13)$$

which can be copied directly into Matlab (for the default model `matrices_A.intrate_smoothing3.m`).