7
Self-Confirming Equilibria

Two literatures

This chapter takes up the quest for models that depart minimally from the basic Kydland-Prescott model, but that also can replicate a 1960's acceleration of inflation followed by a Volcker stabilization. I combine ideas from two literatures to build imperfect rational expectations equilibria of a kind constructed in the 1970's and 1980's in response to Lucas's Critique.[1] These models fail to generate the U.S. inflation pattern because they are too close to the basic Kydland-Prescott model. But when modified slightly in Chapter 8, they take us closer to success.

Directions of fit

I draw first on a literature that verifies the persistence of the Phillips curve in post-World War II U.S. time series data. King and Watson's (1994) perspective on this literature contributes a main ingredient. King and Watson carefully documented how impulse responses depend on which direction of fit is used to identify a Phillips curve. Simplifying their perspective slightly, they catalogued different inferences about dynamic responses that flow from regressing unemployment on inflation rather than inflation on unemployment.

[1] I use imperfect to distinguish these models from the perfect equilibria described in Chapter 4. The equilibria in this chapter will be characterized by subtle departures from the Chapter 4 assumptions about the government's understanding of the model.

Imperfect (1970's) rational expectations equilibria

I consider the issues raised by King and Watson's work in the context of another literature that formulates rational expectations models as fixed points of mappings from beliefs to population limits of statistical models.[2] I define and discuss a particular kind of imperfect rational expectations equilibrium, to be called a self-confirming equilibrium to admit the presence of what might be argued is irrationality on the part of the government.[3] The government has a wrong model in the sense that it misinterprets statistical regularities. But it fits its model to match the data according to least squares. I describe two examples of this type of rational expectations model which differ in the statistical model used by the government, but only in the direction of minimization used in estimation.

The equilibrium concept of this chapter identifies potential limit points for a system with adaptive agents who recursively update estimates of a model as new data become available. In the next chapter I describe adaptive models and how they approach a self-confirming equilibrium.

Self-confirming equilibria

Objects in Phelps problem

The key objects in this chapter appeared in our general statement of the Phelps problem listed in Table 2 from Chapter 5 and reproduced here. They include γ, a vector of regression coefficients in the government's Phillips curve, and h, the coefficients in the government's decision rule. The Phelps problem makes h a function of γ. The Phelps problem takes γ as given and delivers the mapping $h(\gamma)$.

[2] The tradition goes back to Muth (1961) and Lucas and Prescott (1971), and is exploited by Marcet and Sargent (1989) and Evans and Honkapohja (1995).

[3] Fudenberg and Levine (1993) use the term self-confirming equilibrium to refer to a related idea.

Table 2
Objects in Phelps problem

Object	Meaning
X_{Ut}	$[U_{t-1}, \dots, U_{t-m_u}]'$
X_{yt}	$[y_{t-1}, \dots, y_{t-m_y}]'$
X_t	$[X'_{Ut}, X'_{yt}, 1]'$
X_{Ct}	$[y_t \; X'_{t-1}]'$
X_{Kt}	$[U_t \; X'_{t-1}]'$
Class. Phillips curve	$U_t = [\gamma_1 \; \gamma'_{-1}]' X_{Ct} + \varepsilon_{Ct}$
Keynes Phillips curve	$y_t = [\beta_1 \; \beta{-1'}]' X_{Kt} + \varepsilon_{Kt}$
γ	coeffnts (classical)
β	coeffnts (Keynesian)
$h(\gamma)$	coeffnts of govt rule

Elements of self-confirming models

Self-confirming models have these parts: (i) the government's erroneous belief about the Phillips curve; (ii) the private sector's beliefs about the evolution of inflation; (iii) an optimum problem that determines the government's setting of the inflation rate; (iv) the actual expectational Phillips curve; and (v) some orthogonality conditions making beliefs consistent with the data. Unlike Chapters 3 and 4, this equilibrium concept gives the government a model that is not structural in the econometric sense of being invariant with respect to the type of intervention that it contemplates.

I describe two models that are identical in their first four elements. They differ only in element (v), the King-Watson statistical details used to interpret the estimated Phillips curve. I adopt King and Watson's classical and Keynesian identification schemes as terms to describe U on y and y on U regressions, respectively.

The actual Phillips curve

First, I describe the actual Phillips curve, which simply extends the one used in earlier chapters to allow for serially correlated shocks. I specify a statistical version of the natural-rate Phillips curve (3) with a geometric distributed lag in surprise inflation ($y_t - x_t = v_{2t}$) and a serially correlated disturbance.[4] Thus, assume that the underlying Phillips curve is

$$U_t = U^* - \frac{\theta}{1-\rho_2 L}(y_t - x_t) + \frac{v_{1t}}{1-\rho_1 L}$$

where $|\rho_1| < 1$ and $|\rho_2| < 1$. Using (33), we can rewrite this as

$$\begin{aligned} U_t =& U^*(1-\rho_1)(1-\rho_2) \\ &+(\rho_1+\rho_2)U_{t-1} - \rho_1\rho_2 U_{t-2} + (1-\rho_2 L)v_{1t} - \theta(1-\rho_1 L)\, v_{2t} \end{aligned} \tag{48}$$

where v_t is a 2×1 vector white noise with covariance matrix $Ev_tv_t' = V_v$ with diagonal components σ_i; L is the lag operator.[5] This specification allows separate dynamic responses to v_{1t}, the disturbance to the Phillips curve, and to $v_{2t} \equiv y_t - x_t$, the surprise in inflation. For much of this chapter and the next, we shall set $\rho_1 = \rho_2 = 0$ to make some theoretical points. In our empirical work in Chapter 9, we shall permit nonzero ρ_j's.

Self-confirmation

Self-confirmation reconciles beliefs with the environment. Recall how the Phelps problem is defined in terms of the government's perceived Phillips curve (32) with its arbitrary parameter vector γ. The following definition makes γ an outcome.

DEFINITION: A *self-confirming* equilibrium is a fixed vector γ describing government beliefs, a fixed government decision rule h for setting inflation, and an associated stationary stochastic process for (y_t, U_t, x_t) such that

(a) Inflation obeys $y_t = hX_{t-1} + v_{2t}$, where $h = h(\gamma)$. This means that up to the random disturbance v_{2t}, inflation solves the Phelps problem.

[4] Our specification of the Phillips curve is an adaptation of Chung's (1990).
[5] In this chapter, I assume that V_v is a diagonal matrix. The estimation work in Chapter 9 allows V_v not to be diagonal.

(b) The public sector optimally forecasts inflation: $x_t = hX_{t-1}$.[6]

(c) Unemployment is generated by the natural-rate hypothesis (48).

(d) Government's beliefs γ satisfy the orthogonality conditions

$$E\left[U_t - \gamma' X_{Ct}\right] X'_{Ct} = 0. \tag{50}$$

Here E is the mathematical expectation over the equilibrium probability distribution. Condition (d) makes the government's beliefs γ depend on the moment matrices $EU_tX'_{Ct}, EX_{Ct}X'_{Ct}$, as dictated by the least squares normal equations. Conditions (a), (b), (c) make these same moment matrices depend on the government's beliefs γ. The government's beliefs imply behavior that produces data whose moment matrices confirm the government's beliefs.

Direction of minimization

Condition (50) records the vector normal equations for least squares estimates of γ. These normal equations reconcile the government's beliefs with the environment. In the spirit of Chapter 6, the form of (50) emphasizes how a self-confirming equilibrium depends on the government's statistical model. A distinct self-confirming equilibrium results from replacing item (d) with the following alternative, which differs in the direction of minimization imposed on the government's beliefs:

[6] The public forms expectations of inflation by fitting a regression model $y_t = \alpha X_{t-1} + \epsilon_{yt}$, where ϵ_{yt} is a least squares residual that is orthogonal to X_{t-1}. Thus, the public's expectation x_t minimizes $E(y_t - x_t)^2$ by choice of the vector α in

$$x_t = \alpha X_{t-1}. \tag{49}$$

Notice that the form of the public's sector's rule coincides with the form of the government's decision rule for setting the predictable part of inflation. In a self-confirming equilibrium, $\alpha = h$.

(d′) Government's beliefs γ are obtained by first fitting the Keynesian Phillips curve (35), parameterized by β, then using the inversion formulas given in (36). The government's estimate of β satisfies the orthogonality conditions

$$E\left[y_t - \beta' X_{Kt}\right] X'_{Kt} = 0, \tag{51}$$

and X_{Kt} is defined beneath (35).

Because the government's beliefs affect its behavior and therefore the probability distribution for all observables, the direction of minimization affects outcomes.

Equilibria incorporating either (d) or (d′) close a self-referential loop. The government's beliefs – γ or β – determine its behavior rule $h(\gamma)$ and the public's forecasting rule. These outcomes determine the stochastic process with respect to which the expectation in (50) or (51) is evaluated.

Vanishing parameters

Equilibrium concepts differ in how they make parameters describing expectations appear or disappear. In the original version of the Phelps problem with adaptive expectations, a parameter λ describes the public's beliefs. Imposing rational expectations makes this parameter disappear. In the more general Phelps problem, parameters for the public's beliefs are absorbed into the reduced form Phillips curve, whose parameters now summarize the government's model. A self-confirming equilibrium makes parameters describing both the government's and the public's expectations disappear.[7]

The remainder of this chapter describes two types of self-confirming equilibria, links them to the econometric issues raised by King and Watson, and shows their outcomes.

[7] Remember Lucas's warning to 'beware of theorists bearing free parameters.'

Self-confirmation under classical direction

A self-confirming equilibrium closes the following circle. The γ from the perceived Phillips curve $U_t = \gamma' X_{Ct} + \epsilon_{Ct}$ induces, via the Phelps problem, the decision rule for inflation

$$y_t = h(\gamma) X_{t-1} + v_{2t}.$$

This, in conjunction with formulas for the moments and the normal equations (50), implies that the actual projection of U_t on X_{Ct} is

$$\hat{E}\, U_t | X_{Ct} = T(h)' X_{Ct},$$

where $\hat{E}(\cdot)$ is the linear least squares projection operator, and where $T(h)$ is induced by the normal equations and some moment formulas. A self-confirming equilibrium satisfies:

$$T(h(\gamma)) = \gamma. \tag{52}$$

Moment formulas

By way of characterizing $T(\cdot)$, I describe how to compute a self-confirming equilibrium. To calculate the second moments that appear in (50) under the system as it responds to the government's behavior, I apply elementary formulas for linear stochastic processes (e.g., see Anderson, Hansen, McGrattan, and Sargent (1996)). To describe the motion of the system, add v_t to the state vector. The state for the system is then

$$\tilde{X}_t = \begin{bmatrix} v_t \\ X_t \end{bmatrix} = \begin{bmatrix} v_t \\ X_{Ut} \\ X_{yt} \\ 1 \end{bmatrix}.$$

Let $\bar{X}_t$ be the state excluding the constant component 1. Then the motion of the system is

$$\bar{X}_{t+1} = c + \bar{A}\, \bar{X}_t + C v_{t+1}. \tag{53}$$

Let

$$\mu = E\bar{X}_t$$
$$V_{\bar{X}}(0) = E(\bar{X}_t - \mu)(\bar{X}_t - \mu)'$$
$$V_{\bar{X}}(1) = E(\bar{X}_t - \mu)(\bar{X}_{t-1} - \mu)'.$$

Then

$$\mu = (I - \bar{A})^{-1}c \tag{54a}$$
$$V_{\bar{X}}(0) = \bar{A}V_{\bar{X}}(0)\bar{A}' + CV_vC' \tag{54b}$$
$$V_{\bar{X}}(1) = \bar{A}V_{\bar{X}}(0), \tag{54c}$$

where recall that $V_v = Ev_t\, v_t'$.[8] These formulas give all the moments in the normal equations (50) that define the actual regressions used to form the perceived Phillips curve.

Keynesian direction of fit

What the government observes depends partly on what it believes. Studying how the idea of a self-confirming equilibrium interacts with the issues studied by King and Watson provides a practical illustration. This is relevant, because the renaissance of the U.S. Phillips curve stems from an econometric relation with y_t as the left side variable.[9] King and Watson (1994) call this a Keynesian identification scheme for the contemporaneous innovations in a vector autoregression for y_t, U_t because Robert Gordon and Robert Solow used it in widely cited studies.

I assume that the government overlooks the econometric details, sees the Phillips curve handed to it as an exploitable relationship, and solves the Phelps problem. Except for the direction of fit, all other aspects of the model agree with the previous version.

[8] Equation (54b) is a discrete Lyapunov or Sylvester equation and can be solved by algorithms described in Anderson, Hansen, McGrattan, and Sargent (1996).

[9] For example, see Jeffrey Fuhrer (1995). When I delivered a talk about this essay at Northwestern University and had finished describing self-confirmation with the classical direction of fit but had not yet presented the setup with the Keynesian direction of fit, Professor Robert Eisner helpfully remarked that the Phillips curve should be run with y_t on the left.

Government beliefs and behavior

The econometric department of the government continues to believe in an exploitable Phillips curve of the form (32), but now estimates it by applying least squares to the reverse relationship (35), namely, $y_t = \beta' X_{Kt} + \epsilon_{Kt}$. The normal equations are (51) or $\beta = E(X_{Kt}\, X'_{Kt})^{-1}\, E(X_{Kt} y_t)$.

For the Keynesian direction of minimization, a self-confirming equilibrium is defined like the classical one. The estimated Phillips curve (35) implies an inverted Phillips curve

$$U_t = \gamma' X_{Ct} + \tilde{\epsilon}_{2t}, \tag{55}$$

with associated $\gamma(\beta) = [\gamma_1\ \gamma'_{-1}] = [\beta_1^{-1} \quad -\beta'_{-1}/\beta_1]$; this is used in the Phelps problem to set the actual rate of inflation

$$y_t = hX_{t-1} + v_{2t}, \tag{56}$$

where $h = h(\gamma)$ solves the Phelps problem. Via formulas (54), this makes the actual projection of y_t on X_{Kt}, i.e., the actual Keynesian Phillips curve, become

$$\hat{E} y_t \mid X_{Kt} = S(h)'\, X_{Kt}.$$

A self-confirming equilibrium satisfies $S(h(\gamma(\beta))) = \beta$.

Calculation of S

Under the Keynesian identification scheme, it is easy to compute the operator $S(\beta)$ mapping the government's believed Phillips curve into the one that would be recovered by least squares in a large sample. Let $\hat{\beta} \equiv S(\beta)$. Express (56) as

$$y_t = \begin{bmatrix} 0 \\ h \end{bmatrix}' X_{Kt} + v_{2t},$$

substitute into the normal equation (51) and rearrange to get

$$EX_{Kt}\left(\begin{bmatrix} 0 \\ h \end{bmatrix}' X_{Kt} - \hat{\beta}' X_{Kt} + v_{2t}\right) = 0;$$

v_{2t} is orthogonal to all components of X_{kt} except for U_t, with which its covariance is from (48) equal to $-\theta\sigma_2^2$. Here h is implicitly a function of β, via the Phelps problem. Then the normal equations imply

$$S(\beta) = \hat{\beta} = \begin{bmatrix} 0 \\ h \end{bmatrix} + \phi \tag{57a}$$

where

$$\phi = \left(EX_{Kt}X'_{Kt}\right)^{-1} \begin{bmatrix} -\theta\sigma_2^2 \\ 0 \end{bmatrix}. \tag{57b}$$

Special case by hand

It is instructive to consider the special case where $\rho_1 = \rho_2 = 0$, which lets each of the self-confirming equilibria be computed by hand. The government's problem collapses to a sequence of static problems, independently of δ.[10] Accordingly, we take $X_{t-1} = 1$. The Phillips curve (48), the condition that v_{1t} is orthogonal to the right side variables, and the rest of the specification imply

$$\begin{aligned} \text{var}(U_t) &= \theta^2\sigma_2^2 + \sigma_1^2 \\ \text{var}(y_t) &= \sigma_2^2 \\ \text{cov}(U_t, y_t) &= -\theta\sigma_2^2. \end{aligned} \tag{58}$$

For the classical direction of fit (U_t on y_t),

$$\gamma_1 = \frac{\text{cov}(U_t, y_t)}{\text{var}(y_t)} = -\theta.$$

To determine the constant γ_{-1}, I use information about the government's decision rule. By solving the first-order conditions for the government's problem, we find

$$\hat{y}_t = x_t = \frac{-\gamma_{-1}\gamma_1}{\gamma_1^2 + 1},$$

[10] This observation rationalizes the control problem studied by Sims (1988).

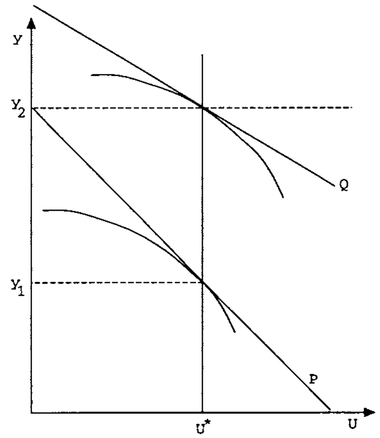

Figure 7.1. Two self-confirming equilibria.

which is a constant, call it $\bar{x}$. Because the means lie on the regression line, use the formula for $\bar{x}$ to compute $\gamma_{-1} = (\gamma_1^2 + 1)U^*$. For the Keynesian direction of fit (y_t on U_t), we have $\beta_1 = \frac{-\theta\sigma_2^2}{\sigma_1^2+\theta^2\sigma_2^2}$. The government now computes its rule using the Phillips curve $U_t = \frac{-\beta_{-1}}{\beta_1} + \beta_1^{-1} y_t$, which implies that $\hat{y}_t = \frac{\beta_{-1}}{\beta_1^2+1} = \bar{x}$, a constant. Imposing $\bar{x} = \beta_{-1} + \beta_1 U^*$ implies that $\beta_{-1} = \frac{-\beta_1^2+1}{\beta_1} U^*$. This then implies that in the perceived Phillips curve, $\gamma_{-1} = \frac{\beta_1^2+1}{\beta_1^2} U^*$.

I compute a numerical example depicted in Figure 7.1. I take an economy with $\theta = 1, U^* = 5, \sigma_1 = .3, \sigma_2 = .3$. The classical form of self-confirming equilibrium has $\gamma_{-1} = U^*(1+\theta^2) = 10, \gamma_1 = -\theta = -1$. The mean levels of unemployment and inflation are $(5,5)$ in this economy. For the Keynesian form, I compute $\beta_{-1} = 12.5, \beta_1 = -.5$, and $\gamma_{-1} = 25, \gamma_1 = -2$. The mean levels of unemployment and inflation in the self-confirming equilibrium are now $(5,10)$. The government's decision rule for inflation is simply $\hat{y}_t = 10$. In Figure 7.1, curve P is the self-confirming Phillips curve estimated with the classical direction of fit. Curve Q is the self-confirming Phillips curve estimated with the Keynesian direction of fit. Under the classical direction of fit, the inflation and unemployment on average are the Nash equilibrium pair (y_1, U^*). Under the Keynesian direction, the Phillips curve is estimated to be flatter, and the inflation rate is on average higher, at y_2.

The example shows how the direction of minimization affects the outcome. With the Keynesian direction, the government believes a larger constant and a larger effect of inflation on unemployment in the U_t-on-y_t relationship. Because this makes the government believe that the Phillips curve trade-off is more favorable than it is, it sets inflation twice as high. See Figure 7.1.

Why not Ramsey?

Each of these two computed self-confirming equilibria gives a mean outcome worse than the Ramsey outcome. This signifies a breakdown in one of the hypotheses of the Chapter 5 Proposition that the solution of the Phelps problem eventually sustains nearly a Ramsey outcome. What fails is the induction hypothesis restricting the sum of the weights on lagged inflation (either zero on current and lagged inflation in the classical version or one on lagged inflation in the Keynesian version).[11] In the examples, because $\rho_1 = \rho_2 = 0$, the self-confirming equilibria are serially uncorrelated processes for U_t, y_t, making lagged (U_t, y_t)'s disappear from empirical Phillips curves. Deactivating the induction hypothesis makes the government in effect solve a one-period problem, and shuts down the intertemporal avenue that promotes better outcomes in the Proposition. This is relevant for the simulations in the following chapter.

[11] These two example self-confirming equilibria exhibit the assertion of Lucas (1972) and Sargent (1971) that rational expectations and the natural-rate hypothesis do not imply the induction hypothesis.

Direction of minimization: caution

The outcomes of our two self-confirming equilibria depend on details of the stochastic specifications, and in particular, the orthogonality of the disturbance v_t to the other right side variables in the true expectational Phillips curve (48), and the orthogonality of v_{2t} to $\hat{y}_t$ in the inflation generator (33). These orthogonality conditions make our first self-confirming equilibrium on average recover the Nash outcome. The classical identifying conditions are more consistent with the environment, and this helps account for the better average outcome under the classical identification.

Evidently, it would be possible to affect the quality of outcomes in the two self-confirming equilibria by appropriately altering the assumed orthogonality conditions in (48). My purpose at this point is not to recommend one set of orthogonality conditions over another in (48), but to point out their influence on outcomes. In this way, though our behavioral model of the government differs from theirs, we are joining King and Watson's (1994) inquiry.

Equilibrium computation

Depending on the direction of fit, a self-confirming equilibrium is either the fixed point $\gamma = T(h(\gamma))$ for the classical direction or $\beta = S(h(\gamma(\beta)))$ for the Keynesian direction. To conserve notation at a cost of an inconsistency, I shall occasionally denote these two mappings as $T(\gamma)$ and $S(\beta)$, respectively. In practice, an equilibrium can be calculated by employing the relaxation algorithm

$$\beta_{j+1} = \kappa\beta_j + (1-\kappa)S(\beta_j), \tag{59}$$

where $\kappa \in [0,1)$ is the relaxation parameter. It is useful sometimes to represent (59) as the adaptive scheme

$$\beta_{j+1} = \beta_j + (1-\kappa)[S(\beta_j) - \beta_j]. \tag{60}$$

When $\kappa = 0$, (59) is equivalent to iterating on S. Setting $\kappa > 0$ can assist convergence. Notice the resemblance of (60) to the recursive representation of the least squares algorithm (8). In the next chapter, I transform the equilibrium computation algorithm (60) into real time dynamics for the government's beliefs by making the government estimate the Phillips curve recursively using the most recent data.

Messages

Self-confirming equilibria impose rational expectations by requiring the public to use an unimprovable forecasting scheme.[12] Different self-confirming equilibria attribute separate quantitative theories – really identification schemes – to the government. Those lead to different policies. The next chapter shows how least squares learning impels the system toward some self-confirming equilibrium but how it need not dispel an erroneous identification scheme nor lead to a Nash equilibrium outcome.

Equilibrium with misspecified beliefs

Readers of Kalai and Lehrer (1993) might have foreseen that the self-confirming equilibrium under classical identification puts the mean inflation rate at the Nash outcome. The result reflects convergence to the truth when decision makers' probability models encompass the Nash equilibrium. The classical identification scheme satisfies the required encompassing condition. But the Keynesian identification scheme gives the government a mistaken interpretation of the contemporaneous covariance between innovations to inflation and unemployment, and that worsens outcomes relative to the Nash outcome.

To explore how imputing a different wrong model might improve upon the Nash outcome, I now propose a model in which the public, not the government, makes a subtle specification error. This model links adaptive expectations, the induction hypothesis, and the Phelps problem. It will be useful in identifying

[12] The public forecasts with a conditional expectations over the equilibrium distribution.

features that will help interpret some simulations and empirical estimates.

The model has a misspecification like the one we put into Bray's model. Within the class of exponential smoothing forecasting rules, the public's beliefs must be optimal, just as in our modification of Bray's model.

An erroneous forecasting function

The government knows the correct model, but not the public. The government's model is

$$\begin{aligned} U_t &= U^* - \theta(y_t - x_t) + v_{1t} \\ x_t &= C y_{t-1} + (1 - C)x_{t-1}, \quad C \in (0,1). \end{aligned} \tag{61}$$

We continue to assume that v_t is a 2×1 vector white noise. The public has adaptive expectations but tunes the free parameter to fit the data. Thus, C will be determined as a fixed point of an operator mapping beliefs into optimal beliefs. By choice of a linear decision rule for y_t, the government solves the Phelps problem: maximize the criterion function

$$-E_0 \sum_{t=0}^{\infty} \delta^t [(U^* - \theta(y_t - x_t))^2 + y_t^2]$$

subject to the second equation of (61), with x_t being taken as a state variable, and y_t as the control. The optimum is a feedback rule

$$y_t = f_1 + f_2 x_t + v_{2t},$$

which has noise because the government cannot perfectly control inflation.

The government's behavior makes the actual rate of inflation

$$y_t = \frac{f_1}{1 - f_2} + \frac{1 - (1 - C)L}{1 - (1 - C(1 - f_2))L} v_{2t}. \tag{62}$$

This can be written

$$y_t = \nu + h(L)v_{2t}, \tag{63}$$

so that the true process has mean ν and spectrum

$$F(\omega; C) = h(\exp(i\omega))h(\exp(-i\omega))\sigma_2^2. \tag{64}$$

This reasoning induces a mapping from C to $F(\omega; C)$, via the Phelps problem.

Given C and a process of the form (62), we seek the best fitting model of the form

$$y_t = \frac{1-(1-c)L}{1-L}\epsilon_t \tag{65}$$

or

$$y_t = g(L)\epsilon_t. \tag{66}$$

The free parameters in (65) are c, σ_ϵ^2. The process has mean 0 and spectrum

$$G(\omega) = g(\exp(i\omega))g(\exp(-i\omega))\sigma_\epsilon^2.$$

Notice that $G(\omega)$ depends on c, through $g(\exp(i\omega))$. As in our analysis of Bray's model, given C and therefore (62), we can find the least squares value of c for the model (65).[13] The Phelps problem and the approximation problem thereby induce a best estimate mapping, $c = B(C)$. We define an equilibrium under forecast misspecification as a fixed point of B. This definition makes both the true and the approximating models equilibrium outcomes.

[13] Again, the best approximating c, σ_ϵ^2 can be computed by minimizing with respect to c, σ_ϵ^2 the expression (47) given above.

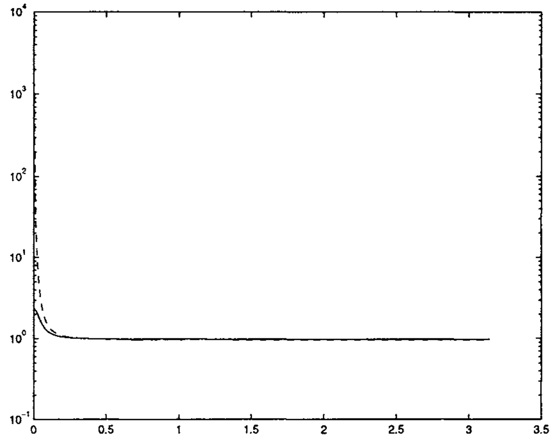

Figure 7.2. Log spectral density of the inflation for the true (solid) and misspecified (dotted line) models, $\delta = .97$.

Figure 7.2 shows the log spectra for inflation for the equilibrium true and approximating models, for the parameter values $U^* = 5, \theta = 1, \sigma_1 = \sigma_2 = .3, \delta = .97$. Associated equilibrium values are $C = .0673, \sigma_\epsilon = 1.0155, f_1 = 1.0243, f_2 = .3495$. The one-step ahead standard error of prediction for the true model is 1, compared with 1.0134 for the approximating model.

Approaching Ramsey

The most important outcome is that the implied mean inflation rate is 1.57, substantially down from the Nash value of 5. Calculations confirm that as δ approaches 1 from below, the mean inflation rate approaches the Ramsey value 0.[14] The true and approximating spectral densities match well at all but low frequencies. The true inflation rate is only moderately serially

[14] For values of $\delta = (.95, .96, .98, .99)$ equilibrium values are, respectively, $C = (.079, .074, .059, .019)$, $\sigma_\epsilon = (1.0182, 1.017, 1.0135, 1.0032)$, and mean inflation equal $(2.00, 1.80, 1.29, .25)$.

correlated, as indicated by the modest departure from the perfectly flat spectral density that characterizes a serially uncorrelated process. As in the Bray model, by simulating a mean with a unit root, the approximating model is using second moment properties to capture first moments.

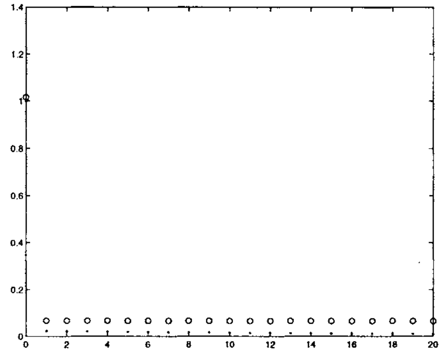

Figure 7.3. Impulse response functions for true (dots) and approximating (circles) models, $\delta = .97$. The circle at zero lag is just above a dot at zero lag at value 1.

Figure 7.3 reports impulse response functions for the true and approximating models. A unit root manifests itself in a nonzero asymptote in the impulse response for the approximating model.

This example separates the equilibrium true and approximating models only at very low frequencies. Figure 1 suggests that it would take many observations for the people living inside this model to detect that their model is wrong. Nevertheless, the average inflation outcome coming from this model is very different from the Nash outcome (i.e., 1.6 versus 5). The induction hypothesis incorporated in the adaptive expectations scheme and the high discount factor of $\delta = .97$ deliver the improved outcome. Because C is an outcome, the present model sharpens our earlier account of the workings of the induction hypothesis in the Phelps problem by removing C as a parameter that can be manipulated independently of δ.

Grounds for optimism

After disappointments from our self confirming equilibria, the equilibrium with forecast misspecification is heartening in supporting better than Nash outcomes. The equilibrium concept is not self-confirming, but has that spirit. It embodies a type of self-confirmation with a wrong model. That the approximation error in our example is small shows that there is a nearly self-confirming model with much better than Nash outcomes.

We take this optimism and the mechanism that generates it into the next chapter, where we construct adaptive versions of our self-confirming models.