Info assumptions 8 Jun 2020 cont'd

Let me give names to the info-assumptions:

The particular question is: supposing agents don't know the NKPC, NKIS, do know the TR, do they know the linking equation between the jump (i_t or π_t) and the endogenous lagged value (il_t or pil_t), and if yes, what do they use to fcst them?

- No: fcst i_t using gx and il_t using hx → "myopic info"
- Yes: fcst i_t using gx and il_t using hx → "schizophrenic"
- Yes: both using gx → "suboptimal forecaster info ass"
- Yes: both using hx → "optimal forecaster info ass"

→ these are the two that at least are internally consistent

The irony is that the old approach to il did the "myopic info", and the corrected MN & PQ methods outlined in materials12f1-3 all do the suboptimal forecaster.

So -lol- 12f is not internally consistent!

A "middle road" was ...intrate_smoothing3.m , b/c it does the MN approach w/ the myopic info ass, which is why it always coincides w/ older approaches.

At least what I'm more & more converging on is the idea that, concerning the big-picture question, agents

- do NOT know NKPC & NKIS → b/c they don't know they are identical
- do know the TR → b/c that doesn't require knowledge that they are identical, so CB can just announce it.

But as for whether they know the linking equation:

- both myopic info & optimal forecaster ass are consistent
- but my hunch is that ass. myopic info at the same time as I ass. that they know the TR is weird b/c it amounts to ass-ing that the CB announces the TR but people don't understand that the i_t on the RHS is i_{t-1}.

So from a consistency-realisticness standpoint the "optimal forecaster info" ass seems to be the best.

But in terms of desirability of model dynamics, which should we prefer? In particular, can some overcome the overshooting?

- materials6 introduced int-rate smoothing, now I know using the "myopic info" ass and it didn't do much to dampen the overshooting
- materials12 introduced learning the slope and found it was desirable
  → "optimal forecaster" ass would undo some of this
  ↳ this suggests that having them know less slows down things

This is confusing b/c the "myopic info" ass endowes them w/ less info than the "optimal forecaster". So that suggests that I need to withdraw more info: make 'em learn h_x too.

The reason that makes me anxious is that over-shooting is already happening b/c $E(\cdot)$ moves so much. So if I increase the role of $E(\cdot)$ b/c hx isn't known either, then will that not make more overshooting?

Step back 1 sec. $E(\cdot)$ moving wouldn't necess be a prob if it wasn't moving so fast or if the TR wasn't known. → No! Whether the TR is known might not matter actually b/c whether agents know how i_t is set, i_t just affects stuff.

→ No that's not true either b/c LH fcsts matter and thus fcsts of i_{t+k} matter, and how you do those depends on whether you know the TR.

→ Does that mean that I've been ass-ing the TR is known all along?

I think so b/c otherwise the (*)-conditions wouldn't be valid.

Do you agree w/ the statement $\hat{E}_t^i(i_{t+1}) = \hat{E}_t(i_{t+1})$?

9 Jan 2020

For this to be true, it has to be that agents first aggregates the same way: $\hat{E}_t^i(y_{t+k}) \overset{!}{=} \hat{E}_t^j(y_{t+k})$ for y being an aggregate variable. Given that agents are identical and observe the same aggregates, this should be true. In fact, it may be true for disagg. variables too, except agents don't realize that either.

The fact that agents know the TR allows me to use the (*)-relations to write
$$\hat{E}_t(i_{t+k}) = \psi_\pi \hat{E}_t(\pi_{t+k}) + \psi_x \hat{E}_t(x_{t+k}) + \hat{E}_t(\text{shocks})$$

But in a certain sense this doesn't help HHs b/c they don't know $\hat{E}_t(\pi_{t+k})$ nor $\hat{E}_t(x_{t+k})$, and so they still estimate $\hat{E}_t(i_{t+k})$ as they estimate gx. Recall that you could solve the model using the PQ-method w/o using (*); then you get a different soln of course when HHs don't know the

TR. But the MN-method is not valid if HHs don't know the TR b/c then you can't sub in $\hat{E}_t\left[\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \text{shocks}_{T+1}\right]$ into $\hat{E}_t i_{T+1}$.
So, since Preston uses the MN method, it means that he ass-s that agents know the Taylor-rule!

Ok: so big-picture info ass are cleared up:
- agents do <u>not</u> know NKIS, NKPC
- agents <u>do</u> know the TR

Now we just need to solve the in-depth issue: do agents internalize the linking equation?
- I think it's tough to tell what has the most appealing dynamics.
- But intuitively I'd say yes.

⇒ Resolve pil & il using the "optimal forecasters" info ass.

⇒ materials 12g2-3.

pil

$$x_t = -b i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + b\pi_{T+1} - b\beta i_{T+1} + b r_T^{n} \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t$$

If agents use hx to fcst π, then $f_\beta(1)$ or $f_\alpha(1)$ will not be used.

PQ:

$$\frac{b}{\beta} + b\beta\left(\beta\psi_\pi \frac{b}{\beta}\right) = \frac{b}{\beta} + b^2\beta\psi_\pi$$

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -\kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{cases} [0, 1-\beta, -b\beta] f_\beta + b[1, 0, 0, 1](I_{nx} - \beta hx)^{-1} s_t - [0\ 0\ 0\ b] s_t \\ [0, \kappa\alpha\beta, 0] f_\alpha + [0, 0, 1, (1-\alpha)\beta](I_{nx} - \alpha\beta hx)^{-1} s_t - [0\ 0\ 0\ (1-\alpha)\beta] s_t \\ [0, 1, 0, \psi_\pi] s_t \end{cases}$$

$$b \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \pi_{T+1} + r_T^n \right\} = b \sum_{T=t}^{\infty} \beta^{T-t} \hat{E}_t \left\{ pil_{T+2} + r_T^n \right\}$$

$$= b\left[pil_{t+2} + r_t^n + \beta\, pil_{t+3} + \beta r_{t+1}^n + \beta^2 pil_{t+4} + \beta^2 r_{t+2}^n + \dots \right]$$

$$= b\left[\frac{1}{\beta^2}\left(\beta^2 pil_{t+2} + \beta^3 pil_{t+3} + \dots \right) + \left(r_t^n + \beta r_{t+1}^n + \beta^2 r_{t+2}^n + \dots \right) \right]$$

$$= b\left[\frac{1}{\beta^2}\left(pil_t + \beta\, pil_{t+1} + \beta^2 pil_{t+2} + \dots \right) + \left(\text{— ॥ —} \right) - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1} \right]$$

$$= b\left[\sum_{T=t}^{\infty} \beta^{T-t} \left(\frac{1}{\beta^2} pil_T + r_T^n \right) - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1} \right]$$

$$= -\frac{b}{\beta}\pi_t + b \sum_{T=t}^{\infty} \beta^{T-t} \left(\frac{1}{\beta^2} pil_T + r_T^n \right) - \frac{b}{\beta^2}[0, 0, 0, 1] s_t$$

The problem is that $\hat{E}_t\, pil_t = \hat{E}_t \pi_{t-1}$ & $\hat{E}_t\, pil_{t+1} = \hat{E}_t \pi_t$

I actually now think it's fine to write

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} pil_T = \frac{1}{1-\beta(hx_{\pi})} pil_t$$

b/c this is $pil_t (= \pi_{t-1}) + \beta pil_{t+1} (= \beta hx_{\pi} pil_t = \pi_t)$

$+ \beta^2 pil_{t+2} \; (= \beta \pi_{t+1})$.

With this logic, $\hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \{ (1-\alpha)\beta \pi_{T+1} + u_T \}$

$= \hat{E}_t \left[u_t + \alpha\beta u_{t+1} + (\alpha\beta)^2 u_{t+2} + \dots + pil_{t+2} + (\alpha\beta)^2 pil_{t+3} + \dots \right]$

$= \hat{E}_t \left[-{\mid\mid}- + \frac{1}{(\alpha\beta)^2} \left[pil_t + \alpha\beta pil_{t+1} + (\alpha\beta)^2 pil_{t+2} + \dots \right] - \frac{1}{(\alpha\beta)^2} pil_t - \frac{1}{\alpha\beta} pil_{t+1} \right]$

$= \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[0, 0, 1, \frac{1}{(\alpha\beta)^2} \right] s_t - \frac{1}{(\alpha\beta)^2} [0, 0, 0, 1] s_t - \frac{1}{\alpha\beta} \pi_t$

$= \left[0, 0, 1, \frac{1}{(\alpha\beta)^2} \right] (I_{nx} - \alpha\beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0, 0, 0, 1] s_t - \frac{1}{\alpha\beta} \pi_t$

So using

$$-\frac{b}{\beta}\pi_t + b\sum_{T=t}^{\infty}\beta^{T-t}\left(\frac{1}{\beta^2}pil_T + r_T^n\right) - \frac{b}{\beta^2}[0,0,0,1]s_t$$

and using

$$[0,0,1,\frac{1}{\kappa\beta\delta}](I_{nx}-\alpha\beta hx)^{-1}s_t - \frac{1}{\kappa\beta\delta}[0,0,0,1]s_t - \frac{1}{\alpha\beta}\pi_t$$

we get

$$\begin{bmatrix} \frac{b}{\beta} & 1 & b \\ (1+\frac{1}{\alpha\beta}) & \kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix}\begin{bmatrix}\pi_t \\ x_t \\ i_t\end{bmatrix} = \begin{bmatrix} [0,1-\beta,-b\beta]f_\beta + b[1,0,0,\frac{1}{\beta^2}](I_{nx}-\beta hx)^{-1}s_t - \frac{b}{\beta^2}[0,0,0,1]s_t \\ [0,\kappa\alpha\beta,0]f\alpha + [0,0,1,\frac{1}{\kappa\beta\delta}](I_{nx}-\alpha\beta hx)^{-1}s_t - \frac{1}{\kappa\beta\delta}[0,0,0,1]s_t \\ [0,1,0,\psi_\pi]s_t \end{bmatrix}$$

$(1+\frac{1}{\alpha\beta})$

And we need to modify (*) as well.

But wait a sec: using the above, can I not, instead of rewriting everything, simply add two linking equations that relate $f_\beta(1)$ and $f\alpha(1)$ to $s_t(4)$?

Something like: $f_\beta(1) = \hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\pi_{T+1}$

$$= \hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}pil_{T+2} = \hat{E}_t\left[pil_{t+2} + \beta pil_{t+3} + \ldots\right]$$

$$= \hat{E}_t \left[\frac{1}{\beta^2} \left(pil_t + \beta\, pil_{t+1} + \beta^2 pil_{t+2} + \dots \right) - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1} \right]$$

$$= \frac{1}{\beta^2} \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} pil_T \quad - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} \pi_t$$

$$= \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t - \frac{1}{\beta} \pi_t$$

L1: $f_\beta(1) = \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t - \frac{1}{\beta} \pi_t$

L2: $f_\alpha(1) = \frac{1}{(\alpha\beta)^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0,0,0,1] s_t - \frac{1}{\alpha\beta} \pi_t$

OR: simply add $\underbrace{\text{sky} \times \left(\frac{1}{\beta}\right) \pi_t}$ and $\underbrace{\text{sky} \times \left(\frac{1}{\alpha\beta}\right) \pi_t}$ to the LHS, $\quad P(1,1) = \frac{2}{\beta} \qquad P(2,1) = \left(1 + \frac{(1-\alpha)\beta}{\alpha\beta}\right) = \frac{1}{\alpha}$

L1': $f_\beta(1) = \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t$

L2': $f_\alpha(1) = \frac{1}{(\alpha\beta)^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0,0,0,1] s_t$

and then to deal w/ (*), add sky $\times\, \psi_\pi \pi_t$ to the LHS

and lastly there's a $\beta \psi_\pi f_\beta(1)$ in (*), which from the NKIS relation has a -2β coefficient, so put things together:

$$-2\beta \cdot \beta \psi_\pi f_\beta(1) \Rightarrow 2\beta \psi_\pi \cdot \pi_t \Rightarrow \text{add } -2\beta \psi_\pi \pi_t$$

to the LHS, i.e. to $P(1,1)$

$\underset{\text{final}}{P(1,1)} = \left(\frac{2}{\beta} - 2\beta\psi_\pi\right)$, don't change anything else but add L1' & L2'.

MN (the red highlighted stuff is correct, see Mathematica)

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \underline{\pi_{t-1}} + \psi_x x_t + \bar{i}_t$$

$$\rightarrow x_t = -\sigma(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta(\psi_\pi \pi_T + \bar{i}_{T+1}) + \sigma r_T^N \right\}$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \pi_{t-1}$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta\, \psi_\pi \pi_T + \sigma(r_T^N + \bar{i}_T) \right\}$$

$$(1+\sigma\psi_x) x_t =$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma \pi_{T+1} - \sigma\psi_\pi\, pil_T + \sigma(r_T^N - \bar{i}_T) \right\}$$

$$\begin{bmatrix} 0 & 1+\sigma\psi_x \\ 1 & -\kappa \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta-\sigma\beta\psi_x, 0] f_\beta + \sigma[1,-1,0,-\psi_\pi](I_{nx}-\beta h x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx}-\alpha\beta h x)^{-1} s_t \end{bmatrix}$$

and

$$L1: \; f_\beta(1) = \frac{1}{\beta^2}[0,0,0,1](I_{nx}-\beta h x)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1] s_t - \frac{1}{\beta}\pi_t$$

$$L2: \; f_\alpha(1) = \frac{1}{(\alpha\beta)^2}[0,0,0,1](I_{nx}-\beta h x)^{-1} s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1] s_t - \frac{1}{\alpha\beta}\pi_t$$

$$\begin{bmatrix} \frac{\sigma}{\beta} & 1+\sigma\psi_x \\ \frac{1}{\alpha} & -\kappa \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta-\sigma\beta\psi_x, 0] f_\beta + \sigma[1,-1,0,-\psi_\pi](I_{nx}-\beta h x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx}-\alpha\beta h x)^{-1} s_t \end{bmatrix}$$

$$1 + \frac{(1-\alpha)\cancel{\beta}}{\alpha\cancel{\beta}} = \frac{\alpha+1-\alpha}{\alpha} = \frac{1}{\alpha}$$

w/ L1' & L2'.

Another attempt at PQ:

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \underline{\pi_{t-1}} + \psi_x x_t + \bar{i}_t$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma [1,0,0,0] (I_{nx} - \beta h x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0] (I_{nx} - \alpha\beta h x)^{-1} s_t \\ [0, 1, 0, \psi_\pi] s_t \end{bmatrix}$$

$P(1,1)$ $\quad \frac{1}{\alpha}$

$$L1: \quad f_\beta(1) = \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta h x)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t - \frac{1}{\beta} \pi_t$$

$$L2: \quad f_\alpha(1) = \frac{1}{(\alpha\beta)^2} [0,0,0,1] (I_{nx} - \beta h x)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0,0,0,1] s_t - \frac{1}{\alpha\beta} \pi_t$$

$$P(2,1): \quad 1 - \frac{(1-\alpha)\beta}{\alpha\beta}(-1) = 1 + \frac{1-\alpha}{\alpha} = \frac{\alpha + 1 - \alpha}{\alpha} = \underline{\underline{\frac{1}{\alpha}}} \qquad P(2,1)$$

$$P(1,1): \quad \sigma f_\beta(1) \qquad | \; (*) \text{ But need to rewrite } (*)$$

$$= \sigma\left(-\frac{1}{\beta}\right) \quad | \text{ over to LHS} \rightarrow \frac{\sigma}{\beta} \qquad P(1,1)$$

$(*)$ needs to take the "optimal forecaster" ass into account

$$f_\beta(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [i_{T+1}] = \hat{E}_t \sum_{T=t} \beta^{T-t} \left\{ \psi_\pi \pi_T + \psi_x x_{T+1} + \bar{i}_{T+1} \right\}$$

$$= \psi_x f_\beta(2) + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [0,1,0,\psi_\pi] s_{T+1}$$

$$f_\beta(3) = \psi_x f_\beta(2) + \frac{1}{\beta} [0,1,0,\psi_\pi] (I_{nx} - \beta h x)^{-1} s_t - \frac{1}{\beta} [0,1,0,\psi_\pi] s_t$$

Ok, having pil, let's now do it w/ "optimal forecasters."

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}$$

Now the linking equations have to relate $f_a(3)$ & $f_b(3)$ to the errors.

$$f_b(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1}$$

| Don't plug the Taylor-rule b/c that is less info than using hx_1.

$$= \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{t+2} = \hat{E}_t \left[i_{t+2} + \beta i_{t+3} + \dots \right]$$

$$= \frac{1}{\beta^2} \hat{E}_t \left[i_t + \beta i_{t+1} + \dots \right] - \frac{1}{\beta^2} i_t - \frac{1}{\beta} i_{t+1}$$

$\underline{\underline{L1}}$

$$f_b(3) = \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t - \frac{1}{\beta} i_t$$

$\underline{\underline{L2}}$

$$f_a(3) = \frac{1}{(\alpha\beta)^2} [0,0,0,1] (I_{nx} - \alpha\beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0,0,0,1] s_t - \frac{1}{\alpha\beta} i_t$$

I wonder what happens to $(*)$ ...

MN

$$x_t = -\sigma\left(\psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}\right)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta\left(\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \bar{i}_T + \rho i_T\right) \sigma r_T^n \right\}$$

But again, this doesn't make sense b/c you've got sthg better than the TR to fcst i:

PQ:

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}$$

$$\begin{bmatrix} 0 & 1 & -\sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0,0](I_{nx} - \beta h x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + [0,0,1,0](I_{nx} - \alpha\beta h x)^{-1} s_t \\ [0,1,0,\rho] s_t \end{bmatrix}$$

$\underline{\underline{L1}}$ s.t. L1 instead of (*) [and L2 is obsolete!]

$$f_\beta(3) = \frac{1}{\beta^2}[0,0,0,1](I_{nx} - \beta h x)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1] s_t - \frac{1}{\beta} i_t$$

So in the first equation, coeffs of i are:

$$-\sigma - (-\sigma\beta) f_\beta = -\sigma + \sigma\beta\left(-\frac{1}{\beta}\right) = \underline{-2\sigma} \leftarrow P(1,3)$$

I just have to hope that this is right!

I guess it's time to compare results and check work. 10 Jan 2020

But first: implement pil w/ "myopic" info ass!
(which actually in terms of forecasting equals the "schizophrenic" info ass b/c either you're myopic and so you don't realize $pil_{t+1} = \pi_t$, so you fcst pil using hx and π_t using gx; or you do realize this but you're schizophrenic and still forecast them separately).

pil - myopic → materials12h2

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \underline{\pi_{t-1}} + \psi_x x_t + \bar{i}_t$$

MN

$$x_t = -\sigma(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$

agents don't realize this is the same var

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta \left[\psi_\pi \pi_T + \bar{i}_{T+1} \right] + \sigma r_T^n \right\}$$

$$(1+\sigma\psi_x) x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x) x_{T+1} + \sigma\pi_{T+1} - \sigma\left(\psi_\pi \pi_{T-1} + \bar{i}_T\right] + \sigma r_T^n \right\}$$

$$(1+\sigma\psi_x)x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\psi_x)x_{T+1} + \sigma\pi_{T+1} - \sigma(\psi_\pi \pi_{T-1} + \bar{i}_T] + \sigma r_T^n \right\}$$

$$\begin{bmatrix} 0 & 1+\sigma\psi_x \\ 1 & -k \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta-\sigma\beta\psi_x, 0] f_\beta + \sigma[1,-1,0,-\psi_\pi](I_{nx}-\beta hx)^{-1} s_t \\ [(1-\alpha)\beta, \overset{k\alpha\beta}{\cancel{\ }}, 0] f_\alpha + [0,0,1,0](I_{nx}-\alpha\beta hx)^{-1} s_t \end{bmatrix}$$

PQ

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -k & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma[1,0,0,0](I_{nx}-\beta hx)^{-1} s_t \\ [(1-\alpha)\beta, \overset{k\alpha\beta}{\ }, 0] f_\alpha + [0,0,1,0](I_{nx}-\alpha\beta hx)^{-1} s_t \\ [0, 1, 0, \psi_\pi] s_t \end{bmatrix}$$

(x)

$$f_\beta(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \psi_\pi pi l_{T+1} + \psi_x x_{T+1} + \bar{i}_{T+1}$$

$$= \psi_x f_\beta(2) + \frac{1}{\beta} \left\{ [0,1,0,\psi_\pi](I_{nx}-\beta hx)^{-1} s_t - [0,1,0,\psi_\pi] s_t \right\}$$

Ok, so look at IRFs in materials12.tex, quickly:
(Note: should check all work to make sure they are really correct → much of this will be my task on Mon, Tues & Wed.)

1) Baseline: learning slope & constant seems better b/c since agents don't know gx(2:end,:), the Ball-effect can't pan out.

2) Epi:

constant only → instrument instability

slope & constant → not E-stable (visually)

To me it makes intuitive sense that Epi should exhibit more instability than baseline b/c instability in the baseline also comes from the Ball-effect, E(.) moving a lot and mattering a lot due to fwd-lookingness.
In Epi, they get to matter more b/c now if E(.) are unstable, i becomes unstable too ⇒ that's why you get instr-instability. When agents are learning all of gx, then

an exploding i is not sufficient to keep $E(\cdot)$ at bay, leading agents to "learn the wrong thing" and so they don't converge to RE.

3) pil

"myopic": for both learning PLMs, seems not E-stable

In a sense it makes sense b/c their forecasting is not consistent w/ RE. Gets worse the less they know of gx. Why don't they learn that $gx = hx$ for π? Maybe b/c this assumes that pil $\neq \pi$ in RE.

"suboptimal fcst": both behave nicely, in particular the

& "optimal fcst": latter

In the former, learning only constant or slope & constant matters of course b/c you're using gx to fcst.

In the latter, this distinction doesn't matter b/c you're using hx to fcst anyway.

I think the reason this model works so nicely in general is b/c it takes out π_t as a jump (and that's what "myopic" fails to do), and since π_t is gone as a jump, $E(\pi)$ don't misbehave, and so the Ball-type disinflationary boom effect doesn't occur.

4) il

"myopic": behaves a lot like the baseline, where learning the slope too dampens the Ball-effect.

"suboptimal" vs "optimal fcst": again, optimal fcst is more stable b/c your fcst is closer to RE since you're using hx (which comes from RE). Learning slope too makes things worse → it seems that the gx you're learning here is unstable and b/c you use it to fcst x & π in both cases, you diverge.

⇒ The overarching theme seems to be that gx is not E-stable and so the only way to restore stability is to effectively make things known: preferably π (pil) b/c it's more $E(\pi)$ that lead to instability → need to check E-stability!