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Indeterminacy and learning: An analysis of monetary policy in the Great Inflation



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ABSTRACT

The Great Inflation of the 1970s can be understood as the result of equilibrium indeterminacy in which loose monetary policy engendered excess volatility in macroeconomic aggregates and prices. The Federal Reserve inadvertently pursued policies that were not anti-inflationary enough because it did not fully understand the economic environment it was operating in. Specifically, it had imperfect knowledge about the structure of the economy and was subject to data misperceptions. The combination of learning about the economy and the use of mis-measured data resulted in policies, which the Federal Reserve believed to be optimal, but when implemented led to equilibrium indeterminacy.

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1. Introduction

There are three narratives about the Great Inflation and the Great Moderation in the academic literature. At opposite ends of the spectrum are the good/bad luck and good/bad policy stories. The 1970s were a time of economic upheaval with strong and persistent exogenous shocks that occurred with high frequency. It was simply bad luck to being a central banker at that time since despite best intentions the incidence of shocks proved too difficult to handle. In the 1980s, however, the reduced incidence and persistence of shocks rang in the Great Moderation. This view is exemplified by Sims and Zha (2006). An almost orthogonal narrative argues that the Federal Reserve conducted bad policy in the 1970s in that it was not aggressive enough in fighting inflation. It is only through a high-interest rate policy, commonly labelled the Volcker disinflation, that the Great Inflation was reigned in. This view of policy having been bad is associated with Clarida et al. (2000) and Lubik and Schorfheide (2004) who argue that policy that is not sufficiently anti-inflationary leads to equilibrium indeterminacy in the economy and excess fluctuations in output and inflation. A third narrative, typically associated with Orphanides (2001), rests on the idea that the Federal Reserve did not perceive the economic scenario of the 1970s correctly. Data misperceptions led it to implement policies that delivered bad outcomes and that only abated in the 1980s with a better understanding of the state of the economy.

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Our paper attempts to integrate the bad policy narrative with the data misperception narrative. More specifically, the paper provides an explanation why the Federal Reserve engaged at first in monetary policy that led to bad outcomes (the Great Inflation), but subsequently pursued a policy that resulted in good outcomes (the Great Moderation). What appears in the data as good and bad outcomes is the result of an optimal policy problem under imperfect information. This framework sits at the intersection between the view espoused by Clarida et al. (2000) and Lubik and Schorfheide (2004) on the one hand and the optimal policy analysis under central bank learning of Primiceri (2006) and Sargent et al. (2006) on the other hand. Relative to the latter, we use a forward-looking private-sector model, which allows us to think seriously about the issue of equilibrium indeterminacy as a cause of the Great Inflation. Relative to the former, we provide an explanation of why the switch from indeterminacy to determinacy, from the Great Inflation to the Great Moderation, occurs at a certain point in time.

The key element that is added to these frameworks is that the Federal Reserve operates in a real-time data environment, where initial data releases are subject to measurement error. The main finding is that shifts in the type of equilibrium are driven by changing perceptions of inflation and output dynamics in the economy, which translates into shifts in optimal monetary policy coefficients. The fact that the central bank misperceives the true state of the economy can lead to policy and equilibrium outcomes that would not be implemented if the true final data are known. This framework not only rationalizes the presence of indeterminate equilibria during the 1970s, but also the switch to a determinate equilibrium in the Volcker disinflation. In this model it is precisely the specific pattern of mis-measured data that is important in explaining post-war U.S. economic history. When there are no data misperceptions, so that the central bank observes the true data contemporaneously, the indeterminacy period extends well into the 1990s.

The model assumes a central bank that does not know the true data-generating process and that observes all data with error. It gathers information by estimating a backward-looking model, and then updates its beliefs about the state of the world and the underlying economic model using least-squares learning. The central bank then chooses monetary policy in a linear-quadratic optimal policy problem. Every period, the optimal rule is communicated to the private sector, which is represented by a standard New Keynesian framework. Private agents assume that the policy rule is time-invariant and form rational expectations conditional on that rule. The source of indeterminacy that arises from this rational expectations system is the same as in Bullard and Mitra (2002), Woodford (2003), and Lubik and Schorfheide (2004), namely a violation of the Taylor principle, which is tied to the value of the policy coefficients in an interest-rate rule. The model is estimated on real-time and final data using Bayesian methods.

The paper thus provides a rationale for why the central bank may choose policy coefficients that inadvertently induce indeterminate outcomes. Given the learning mechanism, the estimated coefficients of the central bank's model, and therefore the optimal policy coefficients, change period by period. The values that these coefficients attain depend on the degree of misperception of the data due to measurement issues. The equilibrium that arises each period is either unique or indeterminate given the policy rule in place. It is the endogenous shifts of the policy coefficients for fixed private sector parameters that move the economy across the threshold between the determinate and indeterminate regions of the parameter space. 'Bad policy', that is, indeterminacy, arises not because of intent but because of incomplete knowledge of the economy on part of the central bank.

We identify two especially prominent turning points. The largest change in policy, based on our estimated policy coefficients, occurred at the end of 1974, at the height of stagflation in the wake of the abandonment of price controls earlier that year. We find that the Federal Reserve under Burns pursued an aggressively anti-inflationary policy that resulted in a determinate equilibrium in the middle of the Great Inflation decade. The Federal Reserve reversed course, when it was confronted with a situation where a decline in growth in 1975 implied a lessening of inflationary pressures. It consequently shifted to a more accommodative stance that led to an indeterminate equilibrium. This set in motion a shift towards an increasingly less accommodative policy stance that culminated in what has come to be known as the Volcker disinflation. A central result of our framework is that the policy change under Volcker is not an abrupt move to an aggressive policy regime, but rather the culmination of a gradual process that started under Burns.

Traditionally, DSGE models for the analysis of monetary policy have been estimated using final data. It is only very recently that the importance of real-time data for understanding monetary policy decisions is being considered in this literature. Collard and Dellas (2010) demonstrate in an, albeit calibrated, New Keynesian DSGE model that monetary misperceptions, interpreted as the difference between real-time and revised data, are an important driver of observed economic fluctuations through a monetary policy transmission channel. Neri and Ropele (2011) substantiate these insights by estimating a similar model for Euro area real-time data using Bayesian Methods. They find that data misperceptions lead to estimated interest-rate smoothing coefficients that are higher than in the standard model. This finding parallels our results since an increasingly more inertial policy rule was one of the drivers of the switch from indeterminacy to determinacy in the early 1980s.

¹ This is notwithstanding earlier contributions, such as Orphanides and Williams (2005), which use reduced-form models and non-system based empirical methods to understand the implications of data misperceptions.

² Collard et al. (2009) estimate this model using Bayesian methods and find strong support for the data mismeasurement specification in terms of overall fit. However, they do not use real-time data in their estimation. Consequently, measurement error takes on the role of a residual that is not disciplined by the relevant data concept in the empirical model.

These papers model monetary policy in terms of an ad-hoc interest-rate feedback rule. This specification is by definition not designed to address the question that is central to the Lubik and Schorfheide (2004) interpretation of the Great Inflation, namely, why a central bank would choose a suboptimal policy that leads to indeterminacy. For this to happen, the central bank needs to face imperfect knowledge about the structure of the economy and the data. Pruitt (2012) develops a model along these lines by modifying Sargent et al. (2006) to take account of the real-time data issue that the Federal Reserve faced in the 1970s and 1980s. Pruitt's model is in reduced form, in which the central bank chooses inflation and unemployment directly by minimizing quadratic loss in these two variables subject to a backward-looking and not micro-founded Phillipscurve relationship. The issue of indeterminacy is left unaddressed since the private sector is not modelled explicitly.³

This paper also speaks to the literature that attempts to explain the Federal Reserve's seemingly lackluster response to inflation in the 1970s as a result of imperfect knowledge and misperceptions about the output gap. Following the contribution by Orphanides (2001), Cukierman and Lippi (2005) introduce imperfect information about the output gap in a semi-reduced form rational expectations model and solve for the optimal monetary policy. While they do not estimate their model, they derive results that are qualitatively similar to ours in that the information problem caused by data (or output gap) uncertainty appears more relevant in the 1970s and leads to larger policy changes. In the same vein, and in a similar modelling setup, Orphanides and Williams (2007) study optimal policy over the set of linear feedback rules from a robust control perspective under rational expectations and learning. Bullard and Eusepi (2005) discuss these issues in a richer New Keynesian model with uncertainty about trend productivity growth under learning and find that this can rationalize inflation dynamics during the 1970s. The key difference between our paper and this literature is that we rationalize bad policy outcomes in terms of indeterminacy through imperfect knowledge and thereby follow the literature established by Clarida et al. (2000) and Lubik and Schorfheide (2004).

The paper is structured as follows. In the next section, the theoretical model is presented and the timing and information assumptions are discussed in detail. This includes an explanation how equilibrium dynamics are computed in our framework, how indeterminate equilibria are chosen, and how the estimation of the model is implemented. Section 3 introduces an analytical example of the mechanism in our framework that leads from data misperceptions to equilibrium determinacy. Section 4 presents the baseline estimation results, while Section 5 studies the monetary policy outcomes during the period 1974–75, which is a key episode for understanding the results in this paper. Section 6 discusses in more detail how the insights from our framework rest on the notion of measurement error in central bank decision making. Section 7 contains a bevy of robustness checks. Section 8 concludes and lays out a path for future research.

2. The model

The model consists of two agents: a central bank and a private sector. The central bank does not know the true model of the economy, but gains knowledge about the state of the economy by employing a learning mechanism. In doing so, it only has access to economic data that are measured with error. However, it is not aware of the mismeasurement. The central bank treats the observed data as if they are measured without error. Furthermore, the central bank does not know the structure of the data-generating process. Instead, it uses a reduced-form specification to conduct inference. The central bank's policy choices are guided by a quadratic loss function, which is minimized every period to derive a linear optimal policy rule. This period-by-period updating of the optimal policy as new data is coming in results in time variation in the policy coefficients.

The private sector provides the data-generating process for the true aggregate data that are not available to the central bank. It consists of a set of structural equations and the monetary policy rule that is communicated by the central bank. The private sector therefore knows the central bank's current period policy rule and determines inflation and output accordingly. It does not face the same data mismeasurement problem as the central bank since it observes the data perfectly. At the same time, the private sector is aware of and understands the central bank's data issues. However, it is myopic in that it treats the policy coefficients, which are varying period by period, as fixed indefinitely.

The timing of the model is such that the central bank estimates its perceived model of the economy at the beginning of period t using data up to and including period t-1. The central bank then minimizes its loss function subject to its estimated law of motion for the private sector, treating the parameter estimates as fixed. This results in optimal policy coefficients for a linear rule, which is then communicated to the public. The private sector observes the true state of the world and the policy coefficients, which it believes to be time invariant. Shocks are realized and equilibrium outcomes are formed,

³ In a more recent contribution, Givens and Salemi (2015) estimate a simple forward-looking New Keynesian framework with real-time data and data misperception. The central bank solves optimal policy under discretion, but does not have to learn the structure of the economy. They only estimate the model from the early 1980s on and do not consider indeterminate equilibria.

⁴ We consider alternative specifications in which the central bank has access to final data at different time lags as a robustness exercise.

⁵ The assumption that the private sector is better informed than the central bank may seem unconvincing. In fact, a majority of learning papers is based on the opposite assumption. However, we choose to focus on the obvious mismeasurement and data revision problem that the Federal Reserve faces. In order to highlight this channel for endogeneous changes in policy parameters, we preserve the structure of the private sector system to be close to the literature. An extension to a two-side learning framework is an obvious, but non-trivial next step.

⁶ This "anticipated utility" assumption that the private sector shares with the central bank is discussed in more detail below.

which result in final data. The central bank's policy rule, taken as given by the private sector, and the structural equations of the private sector form a linear rational expectations model that can have either a determinate or an indeterminate solution, depending in which region of the parameter space the estimates fall. The central bank observes these new outcomes with error and updates its estimates at the beginning of the next period.

2.1. The central bank

The central bank operates in an environment that deviates from rational expectations in two critical aspects. First, it does not know the structure of the economy. It hence conducts inference based on a reduced-form model, which is similar to the specification in Primiceri (2006). The central bank uses the nominal interest rate as its policy instrument. It employs a learning mechanism, namely least-squares learning with constant gain, to update its model of the economy. The second key aspect of our approach is that the central bank observes the actual data with error. This is designed to capture the problems central banks face when data arrive in real time that are potentially riddled with error.

The central bank observes X_t , a noisy measurement of the true state X_t^{true} :

$$X_t^{true} = X_t + \nu_t, \tag{1}$$

where ν_t is a measurement error. The error is serially correlated of order one:

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu,\tag{2}$$

where the Gaussian innovation ε_t^{ν} has zero mean and is independent of X_t^{true} . While it may be problematic to justify autocorrelated measurement errors on a priori grounds, this is a key finding in Orphanides' (2001) analysis of monetary policy during the Great Inflation. It is further assumed that the central bank does not learn about the measurement error, which therefore persists during the estimation period.⁷

The central bank sets the interest rate target:

$$i_t^{CB} = i_t + \varepsilon_t^i, \tag{3}$$

based on a policy rule of the form:

$$i_t^{CB} = \sum_{k=1}^K \alpha_t^k X_{t-k} + \gamma_t i_{t-1}, \tag{4}$$

where ε_t^i is a zero-mean i.i.d. monetary policy implementation error. The policy coefficients $\{\alpha_t\}_{k=1}^K$ and γ_t are chosen from an optimal policy problem. Time variation in the coefficients arises from the learning problem described below. We follow Primiceri (2006) and Sargent et al. (2006) in assuming that the central bank chooses the policy coefficients to minimize a quadratic loss function:

$$W_t = E_t \sum_{j=t}^{\infty} \beta^{(j-t)} \left[\left(\pi_j - \pi^{target} \right)^2 + \lambda_y \left(\Delta y_j - \Delta y^{target} \right)^2 + \lambda_i (i_t - i_{t-1})^2 \right], \tag{5}$$

subject to estimated laws of motion for the relationship between the state variables, inflation π_t and output growth Δy_t , the policy variable i_t^{CB} , and the definition of the policy instrument (3). $0 < \beta < 1$ is the constant discount factor, $\lambda_y, \lambda_i \ge 0$ are weights in the loss function that we treat as structural parameters. 8 π^{target} and Δy^{target} are fixed target values for inflation and output growth, respectively.

In order to learn about the structure of the economy, the central bank estimates the following two-equation model:

$$\pi_i = c_{\pi,t} + a_t(L)\pi_{i-1} + b_t(L)\Delta y_{i-1} + u_t^{\pi},\tag{6}$$

$$\Delta y_i = c_{v,t} + d_t(L)\Delta y_{i-1} + \delta_t i_{t-1} + u_t^{\gamma}. \tag{7}$$

We thus have $X_t^{true} = [\pi_t, \Delta y_t]'$ as the nominal interest rate is not observed with error. All coefficients in the lag-polynomials $a_t(L)$, $b_t(L)$, and $d_t(L)$, and the interest-rate coefficient δ_t are potentially changing over time, as are the intercepts $c_{\pi,t}$ and $c_{y,t}$.

The central bank estimates its empirical model equation by equation, which is a standard assumption in the literature. Given the estimates, the central bank updates its beliefs about the state of the economy. In line with much of the learning literature (see Evans and Honkapohja, 2001), we assume that it uses recursive least-squares learning. The algorithm works as follows. Suppose the central bank wants to estimate an equation of the following form:

$$q_t = p'_{t-1}\phi_t + \xi_t \tag{8}$$

where q_t is the dependent variable or a vector of dependent variables, p_{t-1} a vector or matrix of regressors, ξ_t the residual

⁷ Alternative assumptions are considered in a robustness exercise below.

⁸ A loss function of this kind can be derived from a representative household's utility function within a New Keynesian framework. In this case, λ_y and λ_i would be functions of underlying structural parameters. While it is conceptually possible to derive a loss function within our learning framework, it is beyond the scope of our paper. Nevertheless, using a welfare-based loss function with a reduced-form model of the economy might be problematic since it raises the question how the central bank can calculate the welfare-based loss function without knowledge of the structure of the economy.

(s) and ϕ_t the vector of parameters of interest. The least-squares learning algorithm can be written as:

$$R_t = R_{t-1} + \chi_t (p_{t-1} p'_{t-1} - R_{t-1}), \tag{9}$$

$$\phi_t = \phi_{t-1} + \chi_t R_t^{-1} p_{t-1} (q_t - p'_{t-1} \phi_{t-1}), \tag{10}$$

which are the updating formulas for recursive least-squares estimation, and where R_t is an estimate of the second-moment matrix of the data.

A key parameter in least-squares learning is the gain χ_t . The standard assumption in the literature (Primiceri, 2006) is to use a constant gain $\chi_t = \chi$. This amounts to assuming that the agents whose estimate using constant gain believe that parameters drift over time. The size of this gain determines how much estimates are updated in the light of new data. It also captures how much signal about the coefficients and how much noise is contained in a data point. R_t and ϕ_t are initialized using a training sample, which spans 10 quarters of real-time data.

2.2. The private sector

The behavior of the private sector is described by a New Keynesian Phillips curve that captures inflation dynamics using both forward- and backward-looking elements:

$$\pi_t - \overline{\pi}_t = \beta \left[\alpha_\pi E_t \pi_{t+1} + (1 - \alpha_\pi) \pi_{t-1} - \overline{\pi}_t \right] + \kappa V_t - Z_t. \tag{11}$$

 $0 \le \alpha_\pi \le 1$ is the coefficient determining the degree of inflation indexation, while $\kappa > 0$ is a coefficient determining the slope of the Phillips curve. As before, $0 < \beta < 1$ is the constant discount factor. z_t is a serially correlated shock with law of motion $z_t = \rho_z z_{t-1} + \varepsilon_t^z$, where $0 < \rho_z < 1$, and ε_t^z a zero-mean i.i.d. innovation. Output dynamics are governed by an Euler-equation:

$$y_{t} = E_{t}y_{t+1} - \sigma^{-1} \left[i_{t} - \bar{i}_{t} - E_{t}(\pi_{t+1} - \overline{\pi}_{t}) \right] + g_{t}, \tag{12}$$

where $\sigma > 0$ is the coefficient of relative risk aversion. g_t is a serially correlated shock with law of motion $g_t = \rho_g g_{t-1} + \varepsilon_t^g$, where $0 < \rho_g < 1$, and ε_t^g a zero-mean i.i.d. innovation. The innovations to both AR(1) processes are assumed to be Gaussian. y_t can be interpreted as output relative to a stochastic trend. Shocks to the latter are captured by the generic process g_t . y_t in the structural private sector equations is connected to output growth Δy_t in the central bank's VAR via the measurement equation in the model's state-space representation as in An and Schorfheide (2007).

The private sector equations have a similar structure as those in Lubik and Schorfheide (2004) to facilitate comparison. The equations can be derived from an underlying utility and profit maximization problem of, respectively, a household and a firm. Since these steps are well known the derivations are not reported explicitly. This specification includes the timevarying inflation target $\bar{\pi}_t$ separately because the views the private sector holds about the steady-state level of inflation change as the central bank changes its policy rule. The private sector knows the steady-state real interest rate and can thus infer the implied steady-state level of inflation from the current period monetary policy rule.

The private sector equation system is closed by the monetary policy reaction function (3). This results in the three-equation model that forms the backbone of the standard DSGE model used in the analysis of monetary policy (Smets and Wouters, 2003). The central bank communicates the policy rule to the private sector after the central bank has solved its optimal policy problem. The private sector thus knows the time t policy rule when making its decision at time t. The private sector believes that the policy rule will not change in the future. This is akin to the anticipated utility assumption that the central bank is making and that is more generally often made in the learning literature. Private agents are rational in the model in respect to everything but the future evolution of policy. To wit, the private sector realizes that the central bank makes a mistake in terms of basing the policy rule decision on mismeasured data. Specifically, it understands the nature of the measurement problem in that it includes the law of motion for the measurement error in its structural equation system. Yet, it is myopic in the sense that it does not assign any positive probability to changes in that policy rule when making decisions.

2.3. Model solution, data and estimation

The model is given by the private sector equilibrium conditions (which are the same for all time periods) and the central bank's policy rule (which varies each period). Using the anticipated utility assumption for both the central bank and the private agents, each period the private sector equilibrium conditions and that period's monetary policy rule are stacked in a system of expectational difference equations, which (because of the anticipated utility assumption) can be solved using standard algorithms for linear rational expectations models. The solution thus takes the form of a VAR with time-varying coefficients and stochastic volatility. This VAR forms the state equation in a conditionally linear Gaussian state space system to calculate the likelihood function. The observation equation of the state space system picks the observable variables

⁹ An alternative is to use a decreasing gain. For instance, a recursive version of OLS would set the gain equal to a decreasing function of t.

¹⁰ The standard reference for the anticipated utility assumption is Kreps (1998). Cogley and Sargent (2008) present an extensive discussion in a macroeconomic context and elaborate on how it relates to Bayesian decision making.

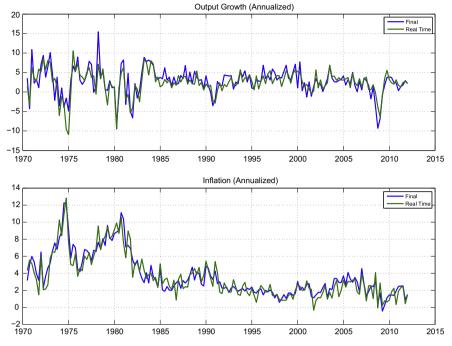


Fig. 1. Real-time and final data: real GDP growth and GDP deflator.

Table 1

Parameter		Value	Source
Inflation target	π^{target}	2.00%	Implied FOMC target
Output target	Δy^{target}	0.75%	Q/Q sample average
Discount factor	β	0.99	Standard value
Indexation NKPC	α_{π}	0.50	Pre-sample estimate
Habit parameter	$\delta_{ m \scriptscriptstyle V}$	0.00	Pre-sample estimate
Lag length in CB regression	n	3	Primiceri (2006)
Gain parameter	χ	0.01	Primiceri (2006)

(described below) from the entire vector of state variables. Details on the derivation of the equilibrium dynamics and the calculation of the likelihood function can be found in the online appendix.

In model, there are two data concepts. The key assumption is that the central bank only has access to real-time data. That is, its decisions are based on data releases as they first become available. The first releases are then subject to revisions later on, but the central bank never sees the final data or any vintage other than the first release. That is, over the entire sample period, the central bank only uses the initial data release in its policy problem. We use real-time data from the Federal Reserve Bank of Philadelphia for the estimation problem of the central bank. The sample period starts in 1968:Q3, based on data availability. The last data point is 2012:Q2. The first 10 quarters of data are used for a pre-sample analysis to initialize the prior. The effective sample period over which the model is estimated therefore starts in 1970:Q2. The data are collected at quarterly frequency.

The private sector, on the other hand, serves as data-generating process for the final true data. Our estimation combines real-time and final observations on output growth and the inflation rate in addition to the nominal interest rate which is observed without error (since it is the policy instrument of the central bank). The Federal Funds rate serves as the policy rate, whereas output growth is measured as the growth rate of real GDP, and inflation is the percentage change in the GDP deflator. Fig. 1 depicts the real-time and the final data for the growth rate in real GDP and in the GDP deflator. The online appendix contains further details on the construction of the data series.

In the estimation exercise, it is convenient to calibrate some parameters. Table 1 lists the calibrated parameter values and their source. The inflation target π^{target} in the central bank's loss function is set to an annual rate of 2%. While the Federal Reserve did not have an official inflation target for much of the sample period, we take it to be commonly understood, and

¹¹ Aruoba (2008) documents the statistical properties of data revisions, and thus the measurement errors, for major macroeconomic variables in the U. S. He points out that the revisions are quite large, which is in line with our interpretation of the central bank's policy changes as driven by large revisions.

even mandated by the (revision to the) Federal Reserve Act of 1977, that it pursued stable prices, a proxy for which we consider an inflation rate of 2%. The output growth target Δy^{target} is set to a quarter-over-quarter rate of 0.75%, which is roughly the sample average. The discount factor is fixed at $\beta = 0.99$.

In a preliminary model assessment, we found that the estimation was sensitive to the assumptions on the backward-looking New Keynesian Phillips curve and Euler equations in the private sector system. We therefore experimented with various specifications of the lag terms in these equations. The specification that fit best in a likelihood sense was one with a backward-looking coefficient of 0.5 in the New Keynesian Phillips curve and no backward-looking dynamics for the output gap in the Euler-equation. The former is consistent with much prior evidence in the Phillips curve, while the latter may seem more unusual in light of some evidence on the presence of habit formation. However, estimation results generally hinge on the model specification, whether the data are in levels or in growth rates and the pattern of exogenous shocks. In order to speed up computation, these two parameters are fixed at the given values by essentially imposing a very tight prior. Robustness checks for variations in these values resulted in lower likelihoods.

We assume that the lag length in all central bank regressions is 3. In preliminary investigation, we found that for shorter lag lengths most of the draws from the posterior distribution would have implied indeterminacy throughout the sample, which we did not find plausible. We fix the gain for the regressions at 0.01, which is at the lower end of the values used in the learning literature. When we estimated this parameter (while restricting it to be no smaller than 0.01) all estimates clustered around this value.

3. A simple illustration of our mechanism

We now illustrate the mechanism in our framework by means of a simple example for which analytical results can be derived. There are three distinct elements in the model that interact in a non-trivial manner. The first element is that the central bank uses recursive least squares to learn about the state of the economy. This is a standard assumption in models of learning (see Primiceri, 2006). In the framework of this paper, however, the central bank observes macroeconomic aggregates only with error. This feature introduces bias in the parameter estimates, which affects policy behavior. The second element is that the central bank solves an optimal policy problem, given the perceived model of the economy and the parameter estimates. The outcome of this optimization problem is a linear policy rule that maps the potentially biased coefficient estimates into reaction coefficients. The final element is that the private sector is described by a forward-looking rational expectations model, where the policy coefficients determine whether the equilibrium is determinate or indeterminate. Using a simple example, we thus trace out how the three modeling elements are connected, starting with the incidence of a measurement error to the potential switch between equilibria.

This simple example is drawn from Primiceri (2005) and adapted for the purposes of this paper. What is different from his specification is the idea of measurement error affecting optimal outcomes. Moreover, Primiceri (2005) uses a backward-looking model for the private sector and can therefore not speak to the question of equilibrium determinacy in a rational expectations environment. Consider a simple version of the 3-equation New Keynesian monetary policy framework. The central bank's perceived law of motion for inflation π_t is:

$$\pi_t = c_\pi + \widehat{\alpha} \pi_{t-1} + \theta(y_{t-1} - \overline{y}) + \varepsilon_\tau^\pi, \tag{13}$$

where y_t is output and \overline{y} is its natural level. Inflation is thus explained by the lagged output gap $(y_{t-1} - \overline{y})$ and lagged inflation. This specification can be understood as a restricted version of the inflation equation, where c_{π} , θ , and \overline{y} are known parameters, so that the central bank only estimates the unknown lagged coefficient $\widehat{\alpha}$.

The output gap is determined as a function of the real rate of interest r_t :

$$y_t - \overline{y} = c_y - r_t + \varepsilon_t^y, \tag{14}$$

where c_y is known. ε_t^r , ε_t^y are perceived shocks that are beyond control of the central bank. Eq. (13) resembles a Phillips curve with an activity variable as driving force, while (14) is akin to an Euler-equation. These two perceived relationships have their counterparts in the private sector's structural equations.

The central bank's loss function is quadratic in inflation and the output gap:

$$L = \widehat{E} \sum_{t=s}^{\infty} \beta^{t-s} \left[\pi_t^2 + (y_t - \overline{y})^2 \right], \tag{15}$$

where β is the discount factor and \widehat{E} the central bank's expectation operator. Optimal policy is found by minimizing the loss function subject to Eqs. (13) and (14) for all r_t , π_t , and y_t . As in Primiceri (2005), the linear feedback rule, that is, the perceived optimal policy rule, is as follows:

$$r_t = c_V + a(\widehat{\alpha}) + b(\widehat{\alpha})\pi_t,\tag{16}$$

¹² For instance, Sargent and Surico (2011) find almost purely backward-looking dynamics in their rational-expectations model.

¹³ Since the central bank estimates its perceived model with recursive least squares, the intuition for the implications of classical measurement error does not apply easily. Instead, we will rely on an instructive example.

where the coefficients $a(\widehat{\alpha})$ and $b(\widehat{\alpha})$ are given by:

$$a(\widehat{\alpha}) = \frac{c_{\pi} \left[1 + b(\widehat{\alpha}) \frac{\widehat{\alpha}}{\theta} \right]}{\theta \left[1 + b(\widehat{\alpha}) \frac{\widehat{\alpha}}{\theta} \right] - \left(\frac{\widehat{\alpha}}{\theta} - \frac{1}{\beta \theta} \right)},$$
(17)

$$b(\widehat{\alpha}) = \frac{1}{2\widehat{\alpha}} \left[-\left(\frac{1}{\beta\theta} + \theta - \frac{\widehat{\alpha}^2}{\theta}\right) + \sqrt{\left(\frac{1}{\beta\theta} + \theta - \frac{\widehat{\alpha}^2}{\theta}\right)^2 + 4\widehat{\alpha}^2} \right]. \tag{18}$$

The next step in disentangling the mechanism in our framework is to study the effects of implementing the policy rule in the structural forward-looking rational expectations environment of the private sector. Its behavior is governed by a New Keynesian Phillips curve:

$$\pi_t - \overline{\pi} = \beta E_t(\pi_{t+1} - \overline{\pi}) + \kappa(y_t - \overline{y}) + z_t, \tag{19}$$

and an Euler-equation:

$$y_t - \overline{y} = E_t(y_{t+1} - \overline{y}) - \tau r_t + g_t. \tag{20}$$

 κ and τ are structural parameters, and z_t and g_t are fundamental supply and demand shocks, respectively. E_t is the private sector rational expectations operator.

The equation system is closed by adding the policy rule (16). The three equations then form a rational expectations model in inflation π_t , output y_t , and the real interest rate r_t that is entirely forward looking and has to be solved out. Moreover, depending on the model's structural parameters the equilibrium can be either determinate or indeterminate. It is straightforward to verify that there is a unique rational expectations equilibrium if and only if $b(\widehat{\alpha}) > 0$, that is, when the central bank increases the real rate in response to higher inflation. The equilibrium is indeterminate otherwise. This is, in fact, the Taylor-principle with the difference that in this simple example the central bank directly controls the real rate instead of the nominal interest rate.

The sign and size of the optimal policy coefficient $b(\widehat{\alpha})$ depends on the perceived inflation persistence parameter $\widehat{\alpha}$. One can show that $\frac{\partial b(\widehat{\alpha})}{\partial \widehat{\alpha}} > 0$ and $\lim_{\widehat{\alpha} \to 0} b(\widehat{\alpha}) = 0$. The more the central bank perceives inflation to be persistent (consistent with a high and positive estimate of $\widehat{\alpha}$), the more strongly it leans against the wind by choosing a positive $b(\widehat{\alpha})$. The wrinkle in this

high and positive estimate of $\widehat{\alpha}$), the more strongly it leans against the wind by choosing a positive $b(\widehat{\alpha})$. The wrinkle in this story is that if $\widehat{\alpha} > 0$, then $b(\widehat{\alpha}) > 0$ and the resulting rational expectations equilibrium in the private sector is unique, whereby the solution for inflation π_t is i.i.d. (Lubik and Schorfheide, 2003). In the model, the true data-generating process is the private sector system where the type of equilibrium is determined period by period by the optimal policy rule (16). Switches in the equilibrium between determinacy and indeterminacy thus occur when $\widehat{\alpha}$ switches sign. This is where the third element of our framework comes in, namely the assumption that the central bank observes the true data only with error.

Consider the following thought experiment. Suppose that the central bank currently has an estimate $\widehat{\alpha}_{t-1}$, and all the other parameters of the model are fixed and known. Also consider two hypothetical scenarios, one where the policymaker observes the true value of inflation π_t , whereas in the other scenario the policymaker observes a noisy measurement $\widetilde{\pi}_t$. Up to time t the central bank has observed the same data in both scenarios and thus the estimates up to time t have been the same. The central bank obtains an estimate of $\widehat{\alpha}_t$ from a recursive least-squares regression of the perceived law of motion (13). The two equations for the recursive least-squares update are given by:

$$R_t = R_{t-1} + \chi(\pi_{t-1}^2 - R_{t-1}), \tag{21}$$

$$\widehat{\alpha}_{t} = \widehat{\alpha}_{t-1} + \chi R_{t-1}^{-1} \pi_{t-1} \left[\pi_{t} + \theta(\gamma_{t-1} - \overline{\gamma}) - c_{\pi} - \widehat{\alpha}_{t-1} \pi_{t-1} \right]$$
(22)

for starting values R_{t-1} and $\widehat{\alpha}_{t-1}$, and gain χ . R_t , the estimate of the data's second moments, is predetermined at time t, as it does not depend on π_t or any other variable dated t. We can thus focus on the recursive computation of $\widehat{\alpha}_t$. To be consistent with both the timing in the model we estimate later and the literature on learning in macroeconomics in general, it is assumed that the central bank updates its estimate $\widehat{\alpha}$ at the beginning of each period and then uses that estimate to form its policy rule coefficients at the beginning of next period; that is, a and b in period t depend on $\widehat{\alpha}_{t-1}$. The central bank's policy rule at time t is thus¹⁴:

$$r_t = c_v + a(\widehat{\alpha}_{t-1}) + b(\widehat{\alpha}_{t-1})\pi_t, \tag{23}$$

 $^{^{14}}$ In the case of mismeasured inflation, the central bank reacts to $\tilde{\pi}_t$ instead of true inflation π_t . Since the measurement error is a mean zero stochastic process, this introduces an error term in the policy rule when it is written in terms of actual inflation. The error term does not influence the determinacy properties we study here.

Denote the value associated with the mismeasured data $\tilde{\alpha}_t$. Using the second recursive least squares equation then yields:

$$\tilde{\alpha}_t - \hat{\alpha}_t = \chi R_t^{-1} \pi_{t-1} (\tilde{\pi}_t - \pi_t). \tag{24}$$

This relationship shows that the measurement error embedded in $\tilde{\pi}_t$ moves the biased parameter estimate $\tilde{\alpha}_t$ in the same direction relative to the true estimate $\hat{\alpha}_t$. More specifically, if the initial real-time reading of inflation $\tilde{\pi}_t$ is higher than its true value π_t (a positive measurement error), then the central bank responds by increasing its policy coefficient by more than it otherwise would have. Similarly, negative measurement error results in a smaller policy coefficient under mismeasured data. The key insight is if the measurement error is large enough and if the policy parameter is close enough to the boundary between determinacy and indeterminacy, then a switch between the two types of equilibria can occur. This insight extends to the larger model in the empirical application, for which analytical expressions for the policy coefficients are not obtainable. The results can, however, be interpreted in light of this simple example. In particular, the introduction of the measurement error in a real-time data environment is key for understanding the sequence of events that led to the Great Inflation and the Great Recession.

4. Estimation results

Fig. 2 shows the marginal posterior distributions for each estimated parameter, while Table 2 reports the median and the 5th and 95th percentile. The dotted line in each graph represents the prior distribution. The data appear quite informative as the posteriors are generally more concentrated than the priors and often exhibit a shift in location. The "supply" and "demand" shocks, z_t and g_t , respectively, show a high degree of persistence at $\hat{\rho}_z = 0.93$ and $\hat{\rho}_g = 0.73$. These numbers are very close to those found by Lubik and Schorfheide (2004) and other papers in the literature for this sample period. While the measurement error in the inflation rate is small, not very volatile, and especially not very persistent ($\hat{\rho}_\pi = 0.08$), the picture is different for output growth. Its median AR(1)-coefficient is estimated to be $\hat{\rho}_{ygrowth} = 0.48$. Finally, the estimates of the weights in the central bank's loss function reveal a low weight on output growth and a considerably stronger emphasis on interest rate smoothing.

Fig. 3 contains the key result in the paper. It shows the model-based evaluation of which type of equilibrium the U.S. economy was in over the sample period. For this purpose, a determinacy indicator is defined as follows. A value of '1' indicates a unique equilibrium, while a value of '0' means indeterminacy. The indicator is computed by drawing from the posterior distribution of the estimated model at each data point, whereby each draw results in either a determinate or an indeterminate equilibrium. Averaging over all draws, the indicator can thus be interpreted as a probability similar to the concept of a transition probability in the regime-switching literature. As it turns out, the estimation results are very unequivocal as far as equilibrium determinacy is concerned since the indicator attains either zero or one. ¹⁸

Two observations stand out from Fig. 3. First, the U.S. economy has been in a unique equilibrium since the Volcker disinflation of 1982:Q3, which, according to conventional wisdom, implemented a tough anti-inflationary stance through sharp interest-rate hikes. In the literature, these are interpreted as a shift to a policy rule with a much higher feedback coefficient on the inflation term (see Clarida et al., 2000). The second observation is that before the Volcker disinflation the economy alternated between a determinate and an indeterminate equilibrium. The longest indeterminate stretch was from 1977:Q1 until 1980:Q4 which covers the end of Burns' chairmanship of the Federal Reserve, Miller's short tenure, and the early Volcker period of a policy of non-borrowed reserve targeting. This was preceded by a short determinacy period starting at the end of 1974. At the beginning of our effective sample period, the U.S. economy was operating under an indeterminate equilibrium.

Impulse response functions to a monetary policy shock at the posterior mode (an innovation to the central bank's interest rate target in Eq. (3)) are reported in Fig. 4. Since the optimal policy rule changes period by period, there is a set of

¹⁵ This insight rests on the assumption that prior observations (π_{t-1} and the covariance matrix R_{t-1}) are not subject to measurement error and may thus run counter to the standard intuition that OLS-estimators are biased downward with error in the data. This bias is still present in our setting through its influence on the covariance matrix R_t , but it is lessened and potentially overturned by other factors namely the serial correlation of the errors in addition to the specific incidence of shocks every period. Moreover, the logic of this simple analytical example is consistent with the numerical example in the working paper version of this manuscript (see Lubik and Matthes, 2014).

¹⁶ The ultimate source of equilibrium switches is the time variation introduced into the private sector's reduced-form coefficients via central bank learning. The recursive least-squares update can be such that the economy drifts across the determinacy boundary even when the central bank has access to final data. However, it is the specific pattern of measurement errors that underlies a consistent pattern of equilibria in the data.

 $^{^{17}}$ This estimate implies that the measurement error process for GDP growth is far from classical. Moreover, the contemporaneous correlation between the two measurement errors is -0.4. Therefore, the standard OLS logic for biases under classical measurement error does not apply for our model. Incorporating contemporaneously correlated measurement errors via correlated innovations to the AR processes for measurement error would change neither the central bank's decision problem nor the private agents' decision rules.

¹⁸ The indicators are not exactly zero or one since there are some draws that put the equilibrium on the other side of the determinacy boundary.

¹⁹ Impulses responses to the other shocks are available in the online appendix. They are consistent with the types of pattern displayed in Fig. 4. Supply shocks raise output growth and lower inflation, while demand shocks lead to increases in both. The interest rate has a stabilizing effect by going up in response to expansionary shocks in accordance with the feedback mechanism embodied in Eq. (4). The exception is the pattern for 1979, the reason for which we discuss in this section.

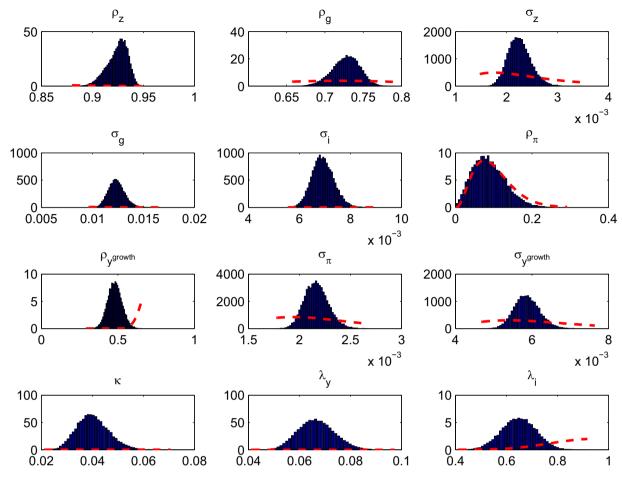


Fig. 2. Prior and posterior parameter density (benchmark specification).

impulse responses for each data point. We focus on four dates, the first quarter each of 1975, 1979, 1990 and the last data point in 2012. As established before, the U.S. economy was operating in a determinate equilibrium in 1975, 1990 and 2012. In these periods, a monetary policy shock raises the Federal Funds rate, lowers inflation, and lowers output growth, just as the intuition for the basic New Keynesian framework would suggest.²⁰ The extent of the individual responses depends solely on the policy coefficients since the other structural parameters of the model are treated as fixed for the entire sample.

The pattern for 1979 is strikingly different, however. In response to a positive interest rate shock inflation and output growth both increase with a prolonged adjustment for the former variable. Moreover, the Federal Funds rate remains persistently high for several years, as opposed to its response in 1975. The key difference is that the equilibrium in 1979 is indeterminate. This finding is consistent with the observation in Lubik and Schorfheide (2003) that indeterminacy changes the way a model's variables respond to *fundamental* shocks.²¹ Furthermore, a quick calculation shows that the Taylor principle, in terms of the response of the real interest rate (that is, the nominal rate less one-step ahead inflation), is violated in 1979 despite the strong and persistent Federal Funds rate response.

Our benchmark results show that the analysis provided by Clarida et al. (2000) and Lubik and Schorfheide (2004) is essentially correct. The U.S. economy was in an indeterminate equilibrium for much of the 1970s, which it escaped from only in the early 1980s. This switch in equilibrium coincided with changes in the Federal Reserve's operating procedures under Volcker's chairmanship; hence, the moniker "Volcker disinflation". We now dig deeper into our model's mechanism to understand the origins of the Great Inflation and Volcker's disinflation.

²⁰ This statement obviously has to be qualified for 2012 since the U.S. economy was operating under a zero lower bound for the Federal Funds rate, which we do not impose in order to stay within a linear framework. In this specific case, the interest rate response can be seen as that of a shadow interest rate that would obtain in the absence of the lower bound.

²¹ The working paper version of this paper, <u>Lubik and Matthes</u> (2014), contains a simple univariate example that shows analytically how the impulse responses functions are affected by the choice of a specific equilibrium under indeterminacy.

Table 2 Posterior mean estimates.

	Parameters	5th Percentile	Median	95th Percentile
Shocks	$ ho_{z}$	0.91	0.93	0.94
	$ ho_{ m g}$	0.70	0.73	0.76
	σ_z	0.002	0.002	0.003
	σ_g	0.011	0.012	0.014
	σ_i	0.006	0.007	0.008
Measurement	$ ho_\pi$	0.03	0.08	0.17
	$ ho_{_{V\!\! m growth}}$	0.41	0.48	0.56
	σ_{π}	0.0020	0.0022	0.0024
	$\sigma_{y ext{ iny growth}}$	0.0054	0.0059	0.0064
Structural	κ	0.03	0.04	0.05
	λ_y	0.06	0.07	0.08
	λ_i	0.54	0.65	0.76

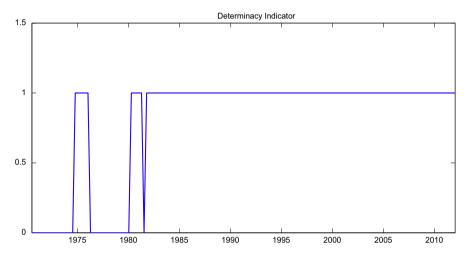


Fig. 3. Determinacy indicator: benchmark specification.

5. The Volcker disinflation of 1974

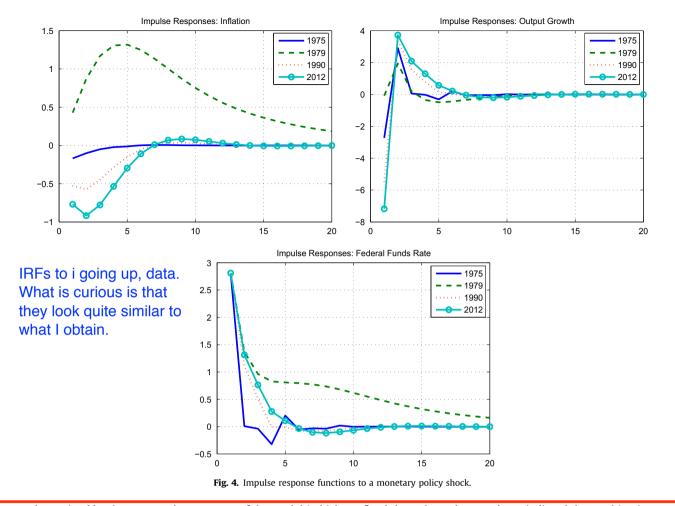
The determinacy indicator in Fig. 3 shows that two key events characterize U.S. monetary policy in the 1970s and 1980s. First, a shift from an indeterminate to a determinate equilibrium and back in 1974; second, the return to a determinate equilibrium in the early 1980s, which coincides with the Volcker disinflation and the onset of the Great Moderation. We now show that these two events are connected and that the Volcker disinflation has its origins in policy actions taken in 1974. Moreover, we show how the Federal Reserve's perceptions of the economy drive the switches between the two types of equilibria.

Whether an equilibrium is determinate or indeterminate is determined by the private sector equations once the central bank has communicated the policy rule for this period.²² At the same time, the optimal policy coefficients can change every period.²³ What drives this time variation is the fact that the central bank re-estimates its perceived model of the economy and then re-optimizes the policy rule. As shown analytically in the simple example above, there is a direct link between the estimated coefficients of the central bank's model and the optimal policy coefficients which determine the equilibrium.

In order to illustrate this link one can back out time series for the policy coefficients from the estimated model. Since the specified form of the policy rule contains more lags, namely three, than is usual for the simple New Keynesian framework upon which most of our intuition is built, we report the normalized sum of these coefficients, that is, the long-run coefficients, to gauge the effective stance of policy in Fig. 5. The boundary between determinacy and indeterminacy is

²² This is where the assumption of anticipated utility bears most weight since we can solve the linear rational expectations model in the usual manner (Sims, 2002) and do not have to account for the potential future switches in policy in every period.

²³ This feature marks a key difference from much of the literature, which describes policy via a time-invariant rule that is at best subject to exogenous breaks or regime-switches. At the same time, we treat the structural parameters of the private sector as invariant over the entire sample period. This allows us to focus on the changing nature of the policy coefficients as the source of changes in equilibrium.



determined by the structural parameters of the model (which are fixed throughout the sample period) and the combination of policy coefficients. Since analytical results cannot be obtained in the benchmark model, the boundary is explored numerically. Given the output coefficients, the long-run response to inflation needs to be higher than (slightly below) one.

At the beginning of the sample, the inflation coefficients are essentially zero. Consequently, the resulting equilibrium in the economy is indeterminate. The switch to a determinate equilibrium in 1974:Q4 is evident from the sharp rise in the long-run inflation coefficient. This is accompanied by a smaller, yet still considerable increase in the output coefficient. The switch back to an indeterminate equilibrium a year later appears as a knife-edge case as both inflation and output coefficients decline, but to levels that might not be considered a priori inconsistent with a determinate equilibrium. The behavior of the coefficient on the lagged interest rate in the third row of the figure offers a clue to this outcome. As Woodford (2003) shows, a highly inertial policy contributes to equilibrium determinacy even if the inflation coefficients are not large. The bottom graph of Fig. 5 shows that the rule becomes less inertial as the early 1970s progress, reaching almost zero in 1976. It then climbs only gradually, which is consistent with the indeterminate equilibrium occurring in the late 1970s. That is, a proximate cause of the indeterminate equilibrium during this period is the unwillingness of the Burns Federal Reserve to implement a more inertial policy.

After 1975 all policy coefficients gradually move upwards. Almost all of this movement is driven by the normalizing factor in the long-run coefficients, that is, the coefficient on the lagged nominal interest rate. Individual policy coefficients show virtually no variation after the 1980s. What is striking from the graphs is that the presence of the Volcker disinflation cannot be gleaned from the behavior of the output and inflation coefficients. It appears only as the endpoint of the gradual rise in the lagged interest-rate coefficient in 1982. The Volcker disinflation can therefore be interpreted not as an abrupt change in the Federal Reserve's responsiveness to inflation, but rather as the culmination of a policy that moves towards a super-inertial rule.²⁴ A more pointed explanation is that the Volcker disinflation happened in 1974 under Burns' chairmanship. The Federal Reserve sharply increased its feedback coefficients and then gradually implemented a more inertial

²⁴ Coibion and Gorodnichenko (2011) offer a similar interpretation.

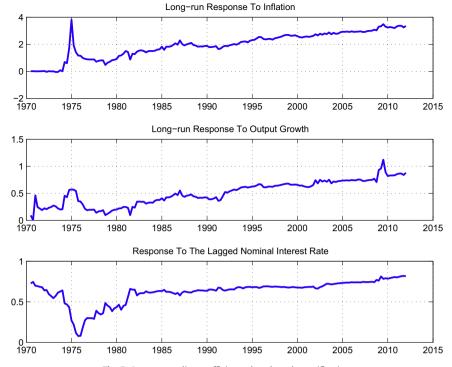


Fig. 5. Long-run policy coefficients: benchmark specification.

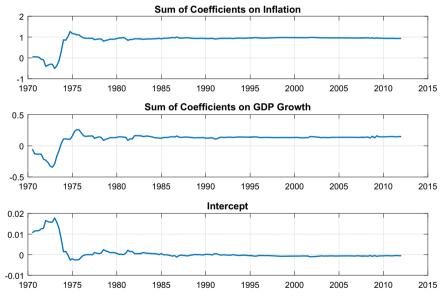


Fig. 6. Perceived inflation equation: benchmark coefficient estimates.

regime. It reached its long-run value just in time for what the literature has identified as the onset of the Volcker disinflation. The groundwork was prepared, however, by Burns in 1974.

The source of these shifts in policy coefficients lies in the changing estimates of the central bank's model. As shown in the simple example above, there is a direct mapping between these coefficients and the optimal policy coefficients. Figs. 6 and 7 depict the time-series of the estimated coefficients in Eqs. (6) and (7), the Phillips curve and the output growth relationship, respectively. The estimates of the inflation equation show that after 1975 there is almost no movement in the regression coefficients, with the sum on lagged inflation being close to one and on GDP growth roughly at 0.15. Perhaps surprisingly, the central bank's changing perception of the inflation process does not seem to be associated the gradual rise of the policy coefficients toward the determinate equilibrium in the early 1980s.

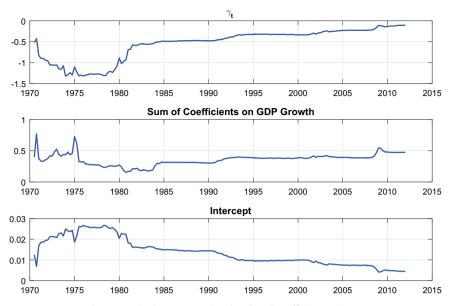


Fig. 7. Perceived output equation: benchmark coefficient estimates.

The switch to a much more aggressive policy in 1974:Q4, on the other hand, reflects changes in the estimated coefficients of the Federal Reserve's model. What stands out is that over the course of 1974 the estimated sum of coefficients on inflation and on GDP growth in the Phillips-curve relationship (6) gradually increase. This translates into an increase in the optimal policy coefficients (see the first two panels in Fig. 5). In terms of equilibrium determination, this pattern means that the policy coefficients move closer to the boundary between indeterminacy and determinacy during 1974. What pushes policy across the threshold is a final rise in the estimate of $a_t(L)$ in the inflation Eq. (6).

The determinacy period lasts for a year. Although the inflation coefficients in the policy rule decline after the initial sharp rise, they remain large enough to preserve a determinate equilibrium. A similar pattern can be detected for the output coefficients. When the switch back to indeterminacy occurs in 1976:Q1, the interest-smoothing coefficient reaches its lowest value before beginning a gradual rise. These patterns are mirrored by the estimates of the central bank's perceived model. The Phillips curve coefficients decline relative to their peak in the determinacy period, as do the coefficients in the output equation. A looser policy is thus associated with a relative decline in persistence.

What role does the measurement error play in the real-time environment of the model? As the simple example showed, the presence of a measurement error can affect the central bank estimates of its perceived model of the economy in such a way that it shifts the policy coefficients across the boundary between determinacy and indeterminacy. The next section establishes that measurement error did not play a big role in explaining the U.S. experience in 1974–1975, but that it was central for understanding the monetary policy history throughout the later part of the sample period.

In conclusion, the middle of the 1970s, that is, the time period for which we identified switches between indeterminacy and determinacy, coincides with one of the most tumultuous episodes in U.S. economic history. The sharp run-up in inflation throughout 1974 that is visible in the data (see Fig. 1) is commensurate with the end of price controls on April 30, 1974. The devaluation of the U.S. Dollar and the first oil price shocks were also contributing factors to the inflationary picture. Moreover, the winter of 1974–1975 marked the most acute period of stagflation in U.S. history. The Burns Federal Reserve rose to the occasion by hiking interest rates to combat inflation (see also Hetzel, 2008, pp. 108). Facing political pressures in favor of further stimulus to combat the ensuing recession the Federal Reserve relented and relaxed its tightening stance in mid-1975 which we pick up in our framework as a switch back to indeterminacy. Yet, throughout the remainder of the 1970s monetary policy, as Hetzel (2008, p. 113) argues, remained disinflationary, which we pick up in terms of a gradual shift to a more inertial policy rule. While Burns' record as Chairman of the Federal Reserve may not quite deserve the exalted status attributed to Volcker, we would argue that his performance during the 1970s warrants a more congratulatory second look.

6. The role of measurement error in the federal reserve's decision making

Central bank learning introduces time variation in the optimal policy coefficients. By itself, this mechanism raises the possibility that the economy crosses the threshold between determinacy and indeterminacy. In the benchmark specification, this feature explains the pattern of equilibria in the 1970s and then the final switch to determinacy in the early 1980s that has come to be known as the Volcker disinflation. This section demonstrates that measurement error plays a key role in generating the determinacy patterns in the data. It does so by exploring the effects of endowing the central bank with

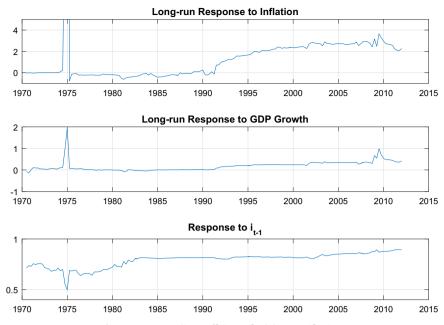


Fig. 8. Long-run policy coefficients: final data specification.

knowledge of the final data with various lags. The main alternative specification is one where we give the Federal Reserve access to the true data, but it still has to learn about the structure of the economy.

The role of the measurement error in our framework is subtle. It is well known that the presence of classical measurement error biases coefficient estimates downward. This might mean that the coefficients on lagged inflation and output in the central bank's empirical model imply lower perceived inflation persistence and a flatter slope of the Phillips curve. As highlighted by Primiceri (2005), albeit in a somewhat different framework, this bias translates into a weaker inflation response when compared to the model without measurement error. In contrast, when there is no measurement error and the central bank has access to final data, it implements policies that lead to indeterminate outcomes for much longer. It is precisely the presence of the measurement error that generates the determinacy pattern consistent with the Volcker disinflation in the early 1980s.

6.1. The information set of the central bank

In the benchmark case the central bank does not use final data. Instead, its decision making process relies on the initial data release only. As an alternative information set of the central bank, this section considers a specification where it uses final data throughout.²⁵ This assumption on the immediate availability of final data is a priori unreasonable since it implies knowledge that the central bank could not possibly have. However, it serves as an illustration of the role that data mismeasurement plays in learning environments like ours. In this model specification, the policy parameters still exhibit time variation because of the learning problem by the central bank. Therefore, switches between determinacy and indeterminacy can occur, as demonstrated in the introductory example. This is, in fact, the case as Fig. 11 shows. The figure reports the determinacy indicator obtained from the re-estimated model under the assumption that the central bank uses final data only. The first switch from indeterminacy to determinacy and back in 1974–1975 is still present, but the second, and final switch to indeterminacy occurs only in 1993. In the specification without measurement error, the Volcker disinflation does not happen at all.

Fig. 8shows the optimal policy coefficients. The increase in the long-run response to inflation and output is much larger than in the benchmark, and it reverts back immediately. The policy coefficients cross the threshold for a determinate equilibrium only in 1993. The bottom panel shows that the coefficient on the lagged interest rate rises slowly, but gradually, as in the benchmark specification. The delayed onset of the final determinacy period is thus largely driven by the inflation and output coefficients. In order to understand the underlying driving forces, the estimated coefficients of the perceived model are computed as before. These are depicted in Figs. 9 and 10.

Comparing the regression coefficients between specifications, the estimates for the output Eq. (7) are very similar. The coefficients on the interest rate in the top panel are almost identical, especially after 1975. There are differences for the lag

²⁵ This specification is estimated on final data only. In Section 7.1, the model is estimated on real-time and final data to make a marginal likelihood comparison possible, since the benchmark model includes both data sets. For the posterior of structural parameters, that is, all parameters except those governing the measurement error process, it does not matter whether this model is estimated on final data only or on both real-time and final data.

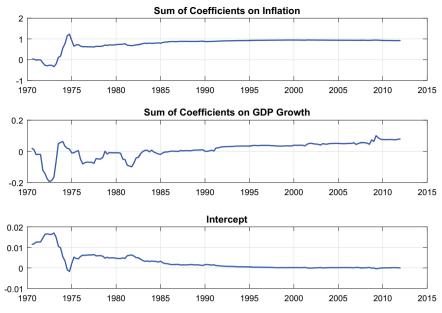


Fig. 9. Perceived inflation equation: final data coefficient estimates.

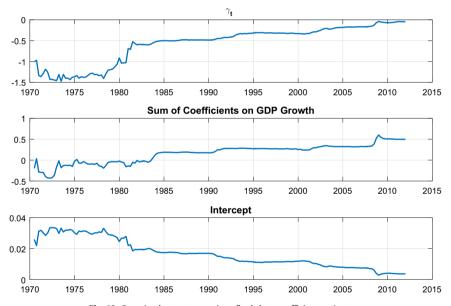


Fig. 10. Perceived output equation: final data coefficient estimates.

coefficients of output in the middle panel, but these manifest themselves only in a level shift before the first determinacy switch in 1974. Much larger differences emerge, however, for the estimates of the inflation Eq. (6). The sum of inflation coefficients in the alternative specification is considerably below that for the benchmark. In the latter, the Federal Reserve perceives inflation to be highly persistent, with a sum of coefficient estimate of near one, while the model with final data yields an estimate of 0.6 throughout the 1970 and early 1980s. It does not reach 0.9 until the switch to determinacy occurs in 1992. A similar picture emerges for the sum of coefficients on output growth in the Phillips curve in Figs. 6 and 9. When there is no measurement error in the data used by the central bank, it estimates output to have a negative impact on inflation between 1975 and 1985. The estimate turns positive only from then on and reaches a level in 1992 that maps into a large enough policy coefficient to ensure a determinate equilibrium at that time.

The role of the measurement error in the initial data release is such that it gives the central bank the perception that inflation is more persistent than it actually is and that output growth is a stronger determinant of inflation dynamics than it actually is. While it is the gradually increasing estimate of inflation persistence that eventually leads to a determinate

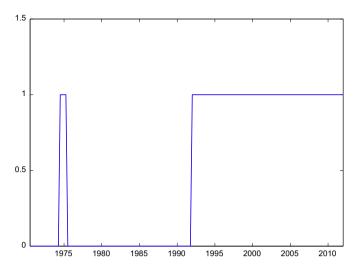


Fig. 11. Determinacy indicator without measurement error.

equilibrium after the period of indeterminacy this occurs earlier in the benchmark specification since the presence of measurement error biases the Federal Reserve toward a more aggressively anti-inflationary policy.²⁶

This result shows the centrality of data misperceptions in understanding the transition from the Great Inflation to the Great Moderation. The first switch in 1974 occurs since inflation is running high and the Federal Reserve moves to combat it aggressively irrespective of final data indicating that the initial belief was excessive. It reverses course in early 1976 because of the sharp decline in output growth irrespective of the initial belief. The central role of the measurement error emerges when considering the later behavior of the U.S. economy. Without mismeasurement, the switch to determinacy occurs later because of different central bank beliefs on the persistence of inflation. The main caveat to this analysis is that the benchmark model with a real-time data environment does not necessarily deliver the best fit against plausible alternatives. The answer to this is affirmative, as will be shown in the robustness section.

6.2. The role of measurement error

The importance of the measurement error can also be assessed from a different angle. Instead of giving the central bank access to the final data at different time horizons, which requires reestimating the model, one can vary the size of the estimated benchmark error in every period and trace out its effects on the determinacy indicator and the policy coefficients. Define $\{\nu_t\}_{t=1}^T$ as the estimated measurement error series from our benchmark specification, conditional on the set of estimated parameters. By replacing the series $\{\nu_t\}_{t=1}^T$ by a scaled series, $x*\{\nu_t\}_{t=1}^T$, where x=1 is the benchmark and x=0 is the case of the central bank observing the final data without error, the effects of the measurement error can be traced out.²⁷

Fig. 12 plots the determinacy indicator against the scale of the measurement error for each data point. At a scale of x=1 at the bottom of the graph, the pattern replicates that found in our benchmark case (see Fig. 3). The dark coloring indicates determinate equilibria. There is the 1-year determinacy period in the middle of the 1970s and then the eventual switch to a determinate equilibrium for the rest of the sample during the Volcker disinflation in the early 1980s. At the opposite end of the scale parameter, x=0 at the top of the graph, the determinacy period in 1974 is still present, albeit shorter. What is different is that the eventual switch to a determinate equilibrium occurs much later in 1992. As discussed in the next section, the fit of the case without measurement error is worse than in the benchmark and also less plausible since it imposes information on the central bank that it could not have. The figure also shows that for intermediate values of x, the onset of the determinacy period moves closer to the benchmark as the scale parameter increases.

Fig. 13 shows how the optimal policy rate changes as the scale is varied. Because of the large spike in the 1974, the figure only shows the time period that zooms in for the Volcker disinflation. We report our benchmark case, the specification with only final data and an intermediate case for x = 0.4. The pattern is consistent with that in the previous figure. Once all specifications imply a determinacy equilibrium from 1992 on, the optimal policy paths overlap. Before that date, the benchmark policy rate is above the other two rates, which indicates the earlier move towards a determinate equilibrium.

²⁶ In terms of the simple example of Section 3, the central bank estimates a positive persistence parameter \hat{a} , which necessitates a strong interest rate response in the policy rule to preserve a unique equilibrium.

²⁷ In this experiment, the actual Federal Funds rate and all other final data are taken as given. What changes as the scale is varied are the central bank's estimates of its model, its implied reaction function and the implied monetary policy error. Alternatively, one could leave the estimated monetary policy error the same, but one would then have to simulate new endogenous outcomes, since the observed interest rate would change. This would in turn change output growth and inflation.

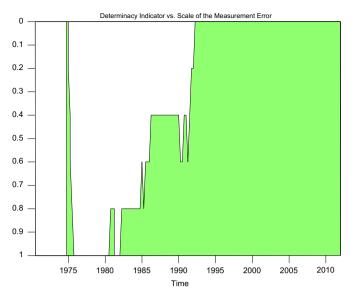


Fig. 12. Determinacy indicator against scale of the measurement error.

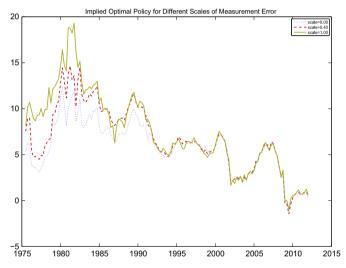


Fig. 13. Optimal policy paths at different scale of measurement error.

7. Robustness

It is well known that models with learning are sensitive to specification assumptions. We therefore conduct a broad range of robustness checks to further study the validity of our interpretation of the Great Inflation. Our results are broadly robust. This section first assesses how well the model fits the data against an alternative specification that allows for data mismeasurement. The second exercise studies the sensitivity of the baseline results to changes in individual parameters based on the posterior median estimates. This gives an idea how significant, in a statistical sense, the determinacy results are. The two previous exercises confirm the robustness of the benchmark findings. These are sensitive, however, to a modification of how the central bank's initial beliefs at the beginning of the sample are captured. Finally, it is shown how alternative assumptions change the determinacy pattern considerably over the full sample period.

7.1. Model fit

One question that arises in assessing the robustness of our framework is to what extent the benchmark model fits the data better than alternative explanations. In a quantitative sense, this is a well defined question: in a Bayesian setting competing models can be ranked based on their ability to fit given data using the marginal data density. A natural model comparison in this context involves the benchmark model with mismeasured data and a model in which the central bank has access to final data. A key difficulty for this particular comparison is that the second model does not require nor have

any implications for real-time data. Consequently, the data sets on which to estimate these two models are very different, with the second model being estimated using only final data. To estimate these two models on the same dataset, one would either have to re-estimate the main model using only final data or add a measurement equation for mismeasured data to the second model.

We opt for the second approach for three reasons: First, adding a set of observation equations for mismeasured data in the second model is straightforward. The mismeasured data in this model do not influence the dynamics of the final data, so that the estimates of the structural parameters not related to mismeasured data are the same as they would be if only final data are used for estimation. Second, estimating the benchmark model on final data only could substantially alter the estimates for all parameters and thus the implications discussed so far. This specification is, in fact, a much different model because private agents in the benchmark are aware of the measurement error process and take it into account when forming expectations of future monetary policy actions. Since they know that the central bank reacts to mismeasured data, this aspect generally influences the dynamics of the model. Estimating the model without real-time data would lead to estimates of the measurement error process that are not directly disciplined by real-time data. Finally, central banks do make real-time policy decisions based on initial data releases. In order to describe policy, the use of ex-post final data can therefore be misleading. Hence, a dataset that includes both real-time and final data would be the natural benchmark for model comparison.

The alternative model is estimated by appending the measurement equation for mismeasured data using both real-time and final data.²⁸ For purposes of model comparison the marginal likelihoods are computed via the method described in Geweke (1999). The resulting difference in log marginal likelihoods in favor of the benchmark model is approximately 400. The evidence in favor of the main model is thus very strong. We also consider alternative information sets of the central bank, specifically the length of time after which it gains access to the final data. In the benchmark, the central bank does not use final data at all. This is obviously an extreme assumption since data revisions occur frequently and the revised data generally get closer to the final data.²⁹ We first ask what would happen if the central bank had access to real-time data with a one-period lag. The indeterminacy indicator behaves quite erratically in this case (not reported). Moreover, the model's posterior mode takes on a value that is 600 log points lower than for the benchmark case. This implies that the alternative timing assumption is rejected by the data in favor of the benchmark specification.

7.2. Sensitivity to parameters

The determinacy indicators are fairly unequivocal in terms of which equilibrium obtains at each data point over the sample period. Probabilities of a determinate equilibrium are either zero or one. As pointed out above, the determinacy indicator is an average over the draws from the posterior distribution at each data point, which appears highly concentrated in either the determinacy or the indeterminacy region of the parameter space. A traditional coverage region to describe the degree of uncertainty surrounding the determinacy indicator would therefore be not very informative.

To give a sense of the robustness of the indicator with respect to variations in the parameters, we therefore perform an exercise where all parameters are fixed at their posterior means. Each parameter is then varied one by one for each data point and each imputed realization of the underlying shocks and measurement errors. The algorithm records whether the resulting equilibrium is determinate or indeterminate. As the results of Bullard and Mitra (2002) for the New Keynesian framework indicate, the boundary between determinacy and indeterminacy typically depends on all parameters of the model, but specifically on the Phillips curve parameter κ and the indexation parameter α_{π} . While this certainly is the case in our model as well,³⁰ we find, however, that the determinacy indicator is sensitive to almost none of the parameters in the model, the exception being the two weights in the central bank's loss function, λ_{γ} and λ_{i} .³¹

The simulation results for the two parameters are reported in Figs. 14 and 15, respectively. Each parameter is varied over the range [0, 1]. Each point in the underlying grid in these figures is a combination of a quarterly calendar date and a value of the parameter within this range. Indeterminate equilibria are depicted in white and determinate equilibria in grey. The posterior median of λ_y is 0.065. The horizontal cross-section at this value replicates Fig. 3. Indeterminacy in the early 1970s is followed by a determinate period around 1975, after which another bout of indeterminacy until the late 1970s is eradicated by the Volcker disinflation.

Fig. 14 shows that a higher weight on output growth in the Federal Reserve's loss function would generally tilt the economy towards indeterminacy, other things being equal. This is because a higher weight on output reduces the relative weight on inflation so that the central bank responds less strongly to inflation, either true or mismeasured, than in the benchmark policy rule. A second observation is that the indeterminacy and determinacy regimes in the early to mid 1970s

²⁸ Detailed results from this exercise are not reported, but are available upon request.

²⁹ The data vintage of 2012:Q3 is treated here as final, which it may not necessarily be since the Bureau of Economic Analysis periodically revises its procedures. In any case, the actual Federal Reserve during the third quarter of 2012 was certainly aware of the latest vintage of data as of this date as opposed to the central bank in our stylized environment.

³⁰ New analytical results by Bhattarai et al. (2014) in a New Keynesian model with a rich lag structure support this conjecture.

³¹ This finding is reminiscent of the results in Dennis (2006), who estimates these weights using likelihood-based methods in a similar model to ours, albeit without learning and measurement error. He finds that the main determinant of fit and of the location of the likelihood function in the parameter space is the central bank's preference parameters.

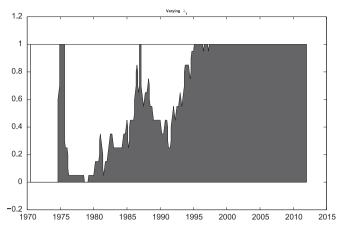


Fig. 14. Sensitivity of determinacy indicator to parameters: output.

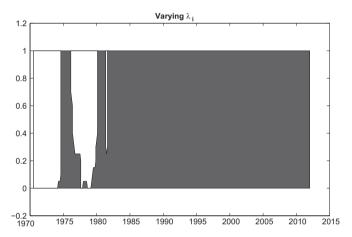


Fig. 15. Sensitivity of determinacy indicator to parameters: interest-rate smoothing.

are largely independent of the central bank's preferences. Similarly, the pattern of determinate equilibria from the mid-1990s on appears robust in the sense that even a relatively stronger preference for output growth would not have resulted in indeterminacy. The pattern for variations in the weight on interest-rate smoothing λ_i is similar. At the posterior median of 0.65 the determinacy indicator is not sensitive to large variations in this parameter.

7.3. The role of initial beliefs

A key determinant of the model's learning dynamics is the choice of initial beliefs held by the central bank. Since updating the parameter estimates in the face of new data can be quite slow, initial beliefs can induce persistence and therefore make switching less likely, everything else equal. There is no generally accepted way to choose initial beliefs. In our baseline specification, we pursued the to us most plausible approach in that we use a training sample to estimate initial beliefs as part of the overall procedure. Initial mean beliefs are set to zero before the start of the training sample and *R* (the recursively estimated second moment matrix of the data) is initialized to be of the same order of magnitude as the second moment matrix in the training sample. As an alternative, a model variant estimates the scale of the initial second-moment matrix by estimating a scale factor that multiplies both initial *R* matrices. Results (not reported) are unchanged from our benchmark.

When the magnitude of *R* is substantially by making the initial values an order of magnitude larger, the indeterminacy indicator changes, but the value at the posterior mode of that specification is 30 log points lower than in the benchmark. The determinacy indicator for this specification is depicted in Fig. 16. Indeterminacy lasts throughout the 1970s and well into the middle of the 1980s. Initial beliefs are such that policy is too accommodative and the data pattern in the 1970s is not strong enough to lead to different policies. Moreover, the learning mechanism is moving slowly so that initial beliefs need not be dispersed quickly. In this specification, it takes a while for the Federal Reserve to catch up after the period that is commonly associated with the Volcker disinflation. For the rest of the Volcker–Greenspan period a determinate equilibrium obtains.

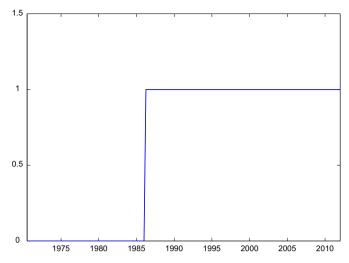


Fig. 16. Determinacy indicator under alternative initial beliefs.

8. Conclusion

We argue in this paper that the Great Inflation of the 1970s can be understood as the result of equilibrium indeterminacy in which loose monetary policy engendered excess volatility in macroeconomic aggregates and prices. The Federal Reserve inadvertently pursued policies that were not anti-inflationary enough because it did not fully understand the economic environment it was operating in. Specifically, it had imperfect knowledge about the structure of the U.S. economy and it was subject to data misperceptions. It is the combination of learning about the economy and the measurement error that resulted in policies that the Federal Reserve believed to be optimal, but when implemented led to an indeterminate equilibrium in the economy.

Our paper combines the insights of Clarida et al. (2000) and Lubik and Schorfheide (2004) about the susceptibility of New Keynesian modelling frameworks to sub-optimal interest rate rules with the observation of Orphanides (2001) that monetary policy operates in a real-time environment with an imperfect understanding of the same. This paper refines the interpretation of the Great Inflation and the Great Moderation in two directions. First, the recession of 1974–1975 coincided with a switch to an aggressively anti-inflationary monetary policy stance which led to a determinate equilibrium. The Federal Reserve responded to a perceived strong increase in inflation persistence, which it leaned against. It reversed course, however, in late 1975 when the data indicated a relative decline in inflation persistence. The reduction in optimal policy coefficients was then enough to induce indeterminacy. The second finding is that the Volcker disinflation appears not as a shift in policy during a short time period, but rather as the endpoint of a gradual adjustment process that made policy more inertial after 1975. Interestingly, these results should also offer comfort to the good luck/bad luck viewpoint, as espoused by, for instance, Sims and Zha (2006) since in a model without data misperceptions, that is, where the central bank has access to the true, final data, a stable determinate equilibrium does not occur until 1993. It was the pattern of measurement errors in the 1980s that led the Federal Reserve to a perception of persistent inflation and output dynamics, which required an aggressive policy stance.

The main criticism to be levelled against this approach is that the private sector behaves in a myopic fashion despite forming expectations rationally. The estimation algorithm relies on the anticipated utility assumption of Sargent et al. (2006). This means that the private sector in the model maintains the belief, despite all evidence to the contrary, that policy, which is changing period by period, will be fixed forever. A key extension of this paper would therefore include private sector learning of the central bank's learning problem. A second issue is that the central bank is not aware that it potentially generates indeterminacy since it essentially tries to learn the reduced-form of a structural model through its empirical backward-looking model, whereas indeterminacy is a property of a structural rational expectation system. Another extension would therefore endow the central bank with an awareness of indeterminacy. This could be done through using a forward-looking model in place of the backward-looking model in the learning problem.³² Moreover, if the central bank entertains the possibility that expectations are important and that the equilibrium can be indeterminate, then it might conduct policy more conservatively and have a preference for robustness that could be captured in the loss function.

³² A similar notion is present in the learning literature, where Tetlow and von zur Muehlen (2009) show the incentives for policymakers to shift policy such as to robustly guarantee e-stability.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jmoneco. 2016.07.006.

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