Materials 12f3 - “il”-extension of baseline model - Interest rate smoothing using the “optimal forecaster” info assumption

See Notes 9 Jan 2020

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Note: the Matlab codes `matrices_A_intrate_smoothing.m` and `matrices_A_intrate_smoothing3.m` do the “myopic info” informational assumption, for which the MN method works. So both do the MN method, in particular 3 does it explicitly. For the “suboptimal forecaster” info assumption, the MN solution doesn’t exist. And it seems that for the optimal forecaster assumption, the MN solution doesn’t make sense.

Compare Mathematica (`materials12g3.nb`).

1 Model equations

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1} \tag{3}$$

Compact notation

$$z_t = \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = A_a f_a + A_b f_b + A_s s_t \quad \text{with} \quad s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{bmatrix} \tag{4}$$

2 MN matrices

In principle do not exist.

3 PQ matrices

$$\underbrace{\begin{bmatrix} 0 & 1 & -2\sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix}}_{\equiv P} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \underbrace{\begin{bmatrix} \begin{bmatrix} \sigma, & 1-\beta, & \beta(-\sigma) \end{bmatrix} f_b + c_{x,s} s_t \\ \begin{bmatrix} (1-\alpha)\beta, & \alpha\beta\kappa, & 0 \end{bmatrix} f_a + c_{\pi,s} s_t \\ c_{i,s} s_t \end{bmatrix}}_{\equiv Q} \tag{5}$$

where

$$c_{x,s} = \sigma \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} .\text{InxBhx}; \tag{6}$$

$$c_{\pi,s} = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} .\text{InxABhx} \tag{7}$$

$$c_{i,s} = \begin{pmatrix} 0 & 1 & 0 & \rho \end{pmatrix} = d_{i,s} \tag{8}$$

where InxABhx and InxBhx are the same as before.

The (*)-relation gets replaced by $L1'$ which gives agents more info to forecast!

$$f_b(3) = \frac{1}{\beta^2} \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} (I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{\beta^2} \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} s_t \tag{L1'}$$

The Matlab code that uses this is `matrices_A_12g3.m`.