Materials 6 - More on IRFs

Laura Gáti

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Overview

1 Model summary, adding ρi_{t-1}

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \rho i_{t-1} + \bar{i}_t \tag{3}$$

$$\hat{\mathbb{E}}_t z_{t+h} = \begin{bmatrix} \bar{\pi}_{t-1} \\ 0 \\ 0 \end{bmatrix} + b\, hx^{h-1} s_t \quad \forall h \geq 1 \qquad b = gx\ hx \qquad \text{PLM} \tag{4}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \underbrace{\big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big)}_{\text{fcst error using (4)}} \qquad (b_1 \text{ is the first row of } b) \tag{5}$$

$$k_t = \mathbb{I} \times (k_{t-1} + 1) + (1 - \mathbb{I}) \times \bar{g}^{-1} \tag{6}$$

$$\mathbb{I} = \begin{cases} 1 & \text{if } \theta_t \leq \bar{\theta} \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

$$\theta_t = |\hat{\mathbb{E}}_{t-1}\pi_t - \mathbb{E}_{t-1}\,\pi_t|/\sigma_s \qquad \text{CEMP criterion for the gain} \tag{8}$$

The alternative criterion for the choice of gain is a recursive variant of the CUSUM-test (Brown, Durbin, Evans 1975):

1. Let FE_t denote the short-run forecast error, and ω_t firms' estimate of the FE variance.

2. Let $\kappa \in (0,1)$ and $\tilde{\theta}$ be the new threshold value for the criterion.

3. Then for initial (ω_0, θ_0), firms in every period estimate the criterion and the FEV as:

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1}(FE_t^2 - \omega_{t-1}) \tag{9}$$

$$\theta_t = \theta_{t-1} + \kappa k_{t-1}^{-1}(FE_t^2/\omega_t - \theta_{t-1}) \tag{10}$$

$$k_t = \mathbb{I} \times (k_{t-1} + 1) + (1 - \mathbb{I}) \times \bar{g}^{-1} \tag{11}$$

$$\mathbb{I} = 1 \quad \text{if} \quad \theta_t \leq \tilde{\theta} \tag{12}$$

2 Compact notation - with lagged interest rate term in TR

$$z_t = A_p^{RE}\,\mathbb{E}_t\, z_{t+1} + A_s^{RE} s_t \tag{13}$$

$$z_t = A_a^{LH} f_a(t) + A_b^{LH} f_b(t) + A_s^{LH} s_t \tag{14}$$

$$s_t = P s_{t-1} + \epsilon_t \qquad \rightarrow \quad s'_t = hx\; s'_{t-1} + \epsilon'_t \tag{15}$$

$$\text{where} \quad s'_t \equiv \begin{pmatrix} r_t^n \\ \bar{i}_t \\ u_t \\ i_{t-1} \end{pmatrix} \quad hx \equiv \begin{pmatrix} \rho_r & 0 & 0 & 0 \\ 0 & \rho_i & 0 & 0 \\ 0 & 0 & \rho_u & 0 \\ gx_{3,1} & gx_{3,2} & gx_{3,3} & gx_{3,4} \end{pmatrix} \quad \epsilon'_t \equiv \begin{pmatrix} \varepsilon_t^r \\ \varepsilon_t^i \\ \varepsilon_t^u \\ 0 \end{pmatrix} \quad \text{and} \quad \Sigma' = \begin{pmatrix} \sigma_r & 0 & 0 & 0 \\ 0 & \sigma_i & 0 & 0 \\ 0 & 0 & \sigma_u & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{16}$$

i_{t-1} is an endogenous state and breaks the link that previously had $P = hx$; now this is no longer true. In particular, using Matlabby notation, $P = hx(1:3, 1:3)$. What I don't get though is why $gx_{3,4} \neq \rho$? ?

Adding i_{t-1} to the state vector fortunately doesn't change A_p^{RE}, A_a^{LH} or A_b^{LH}, but it does change A_s^{RE} and A_s^{LH}. The latter two get an additional column to account for the new state variable. Also f_a and f_b need to be adjusted (replace P by hx). With $g_{i,j}$ $i = \pi, x,$ $j = a, b$ unchanged from Materials 4, the new coefficient matrices are given by (new elements highlighted in blue):

$$A_s^{RE} = \begin{pmatrix} \frac{\kappa\sigma}{w} & -\frac{\kappa\sigma}{w} & 1 - \frac{\kappa\sigma\psi_\pi}{w} & 0 \\ \frac{\sigma}{w} & -\frac{\sigma}{w} & -\frac{\sigma\psi_\pi}{w} & 0 \\ \psi_x(\frac{\sigma}{w}) + \psi_\pi(\frac{\kappa\sigma}{w}) & \psi_x(-\frac{\sigma}{w}) + \psi_\pi(-\frac{\kappa\sigma}{w}) + 1 & \psi_x(-\frac{\sigma\psi_\pi}{w}) + \psi_\pi(1 - \frac{\kappa\sigma\psi_\pi}{w}) & \rho \end{pmatrix} \tag{17}$$

$$A_s^{LH} = \begin{pmatrix} g_{\pi s} \\ g_{xs} \\ \psi_\pi g_{\pi s} + \psi_x g_{xs} + \begin{bmatrix} 0 & 1 & 0 & \rho \end{bmatrix} \end{pmatrix} \tag{18}$$

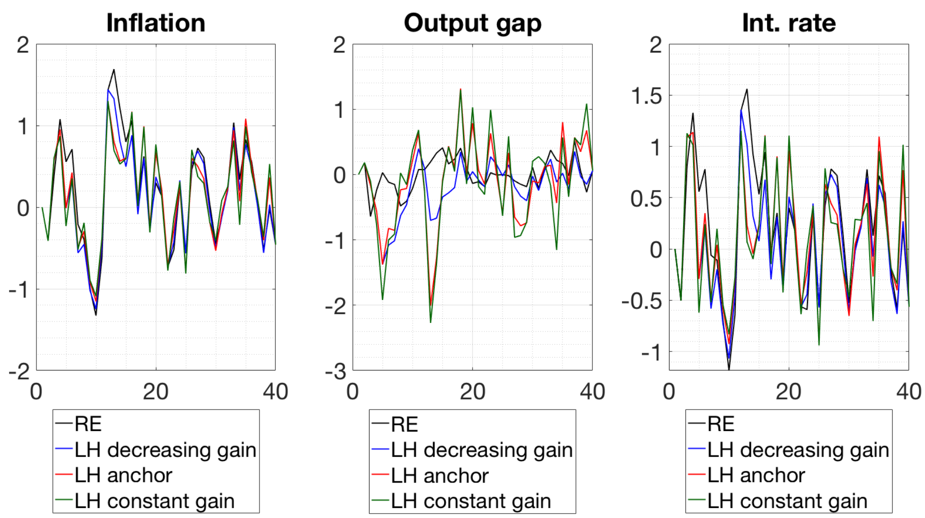
$$g_{\pi s} = (1 - \frac{\kappa\sigma\psi_\pi}{w}) \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} (I_4 - \alpha\beta hx)^{-1} - \frac{\kappa\sigma}{w} \begin{bmatrix} -1 & 1 & 0 & \rho \end{bmatrix} (I_4 - \beta hx)^{-1} \tag{19}$$

$$g_{xs} = \frac{-\sigma\psi_\pi}{w} \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} (I_4 - \alpha\beta hx)^{-1} - \frac{\sigma}{w} \begin{bmatrix} -1 & 1 & 0 & \rho \end{bmatrix} (I_4 - \beta hx)^{-1} \tag{20}$$

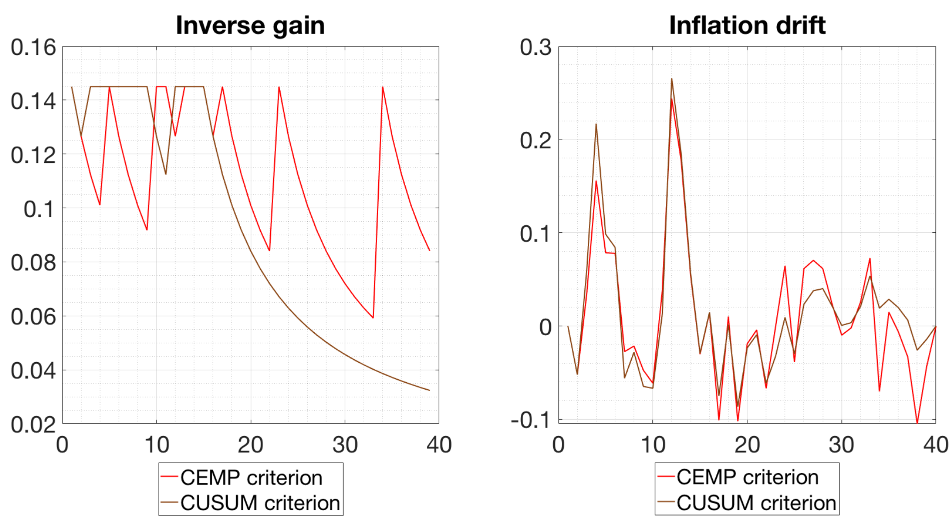
3 Small deviations in π, large ones in x, overshooting - IRFs

3.1 Baseline figures, $\psi_x = 0, \psi_\pi = 1.5, \rho = 0$

Figure 1: A baseline shock sequence,$\psi_x = 0, \psi_\pi = 1.5, \rho = 0$



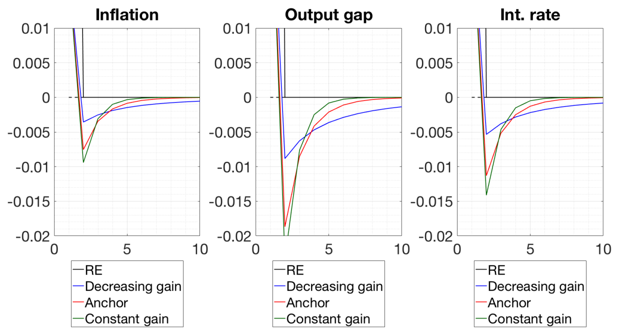
(a) The observables for the specific shock sequence



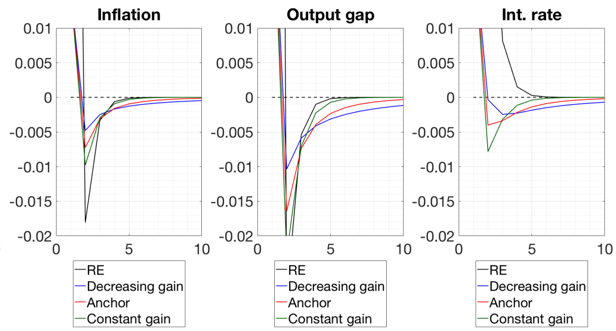
(b) Inverse gain and drift for the specific shock sequence, CEMP and CUSUM criterion

3.2 IRFs when changing ρ

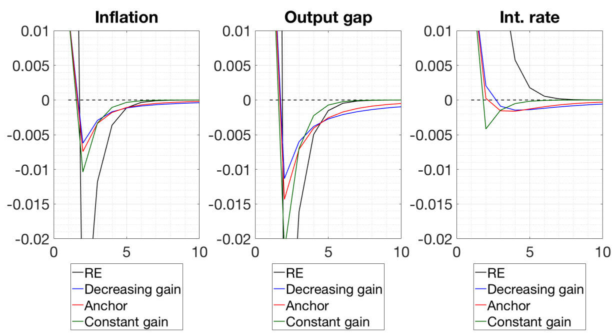
Figure 2: IRFs to a natural rate shock (r^n)



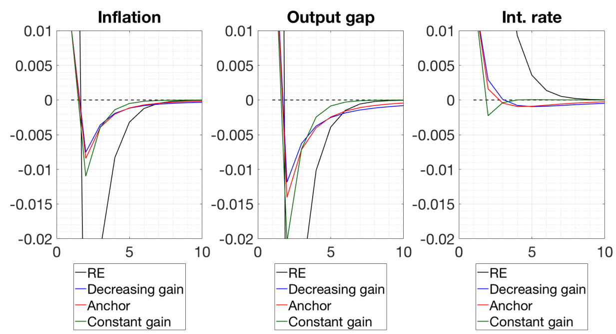
(a) $\rho = 0$



(b) $\rho = 0.3$

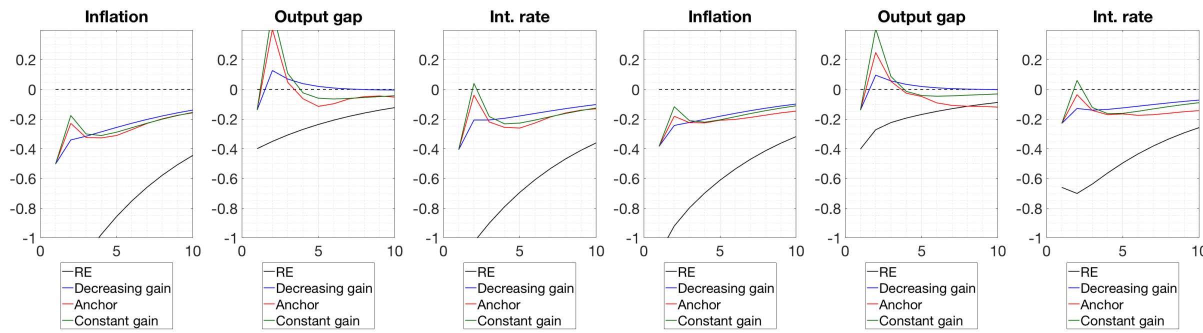


(c) $\rho = 0.6$

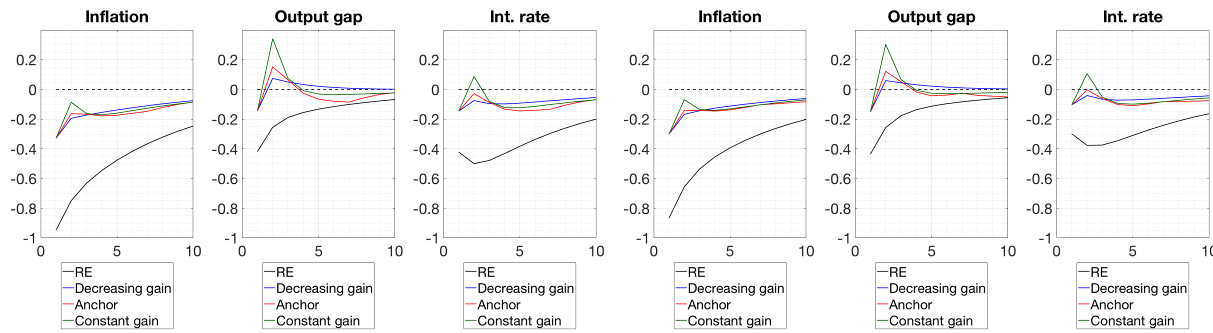


(d) $\rho = 0.9$

Figure 3: IRFs to a monetary policy shock ($\bar{i}$)

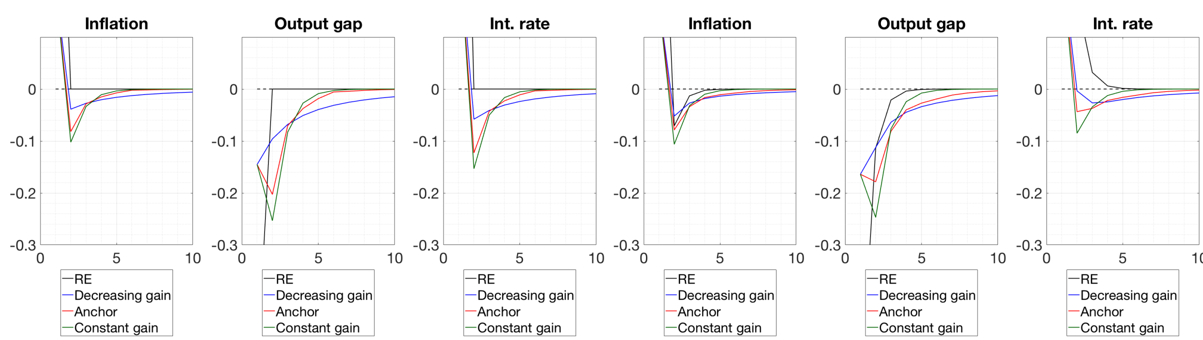


(a) $\rho = 0$ (b) $\rho = 0.3$

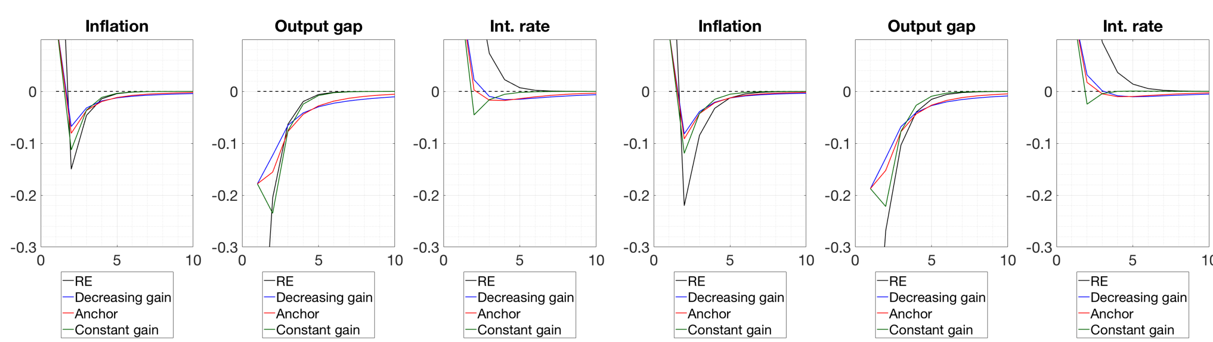


(c) $\rho = 0.6$ (d) $\rho = 0.9$

Figure 4: IRFs to a cost-push shock (u)

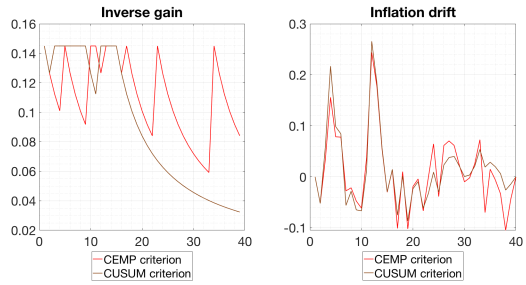


(a) $\rho = 0$ (b) $\rho = 0.3$

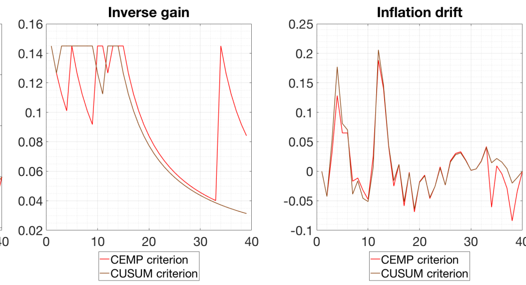


(c) $\rho = 0.6$ (d) $\rho = 0.9$

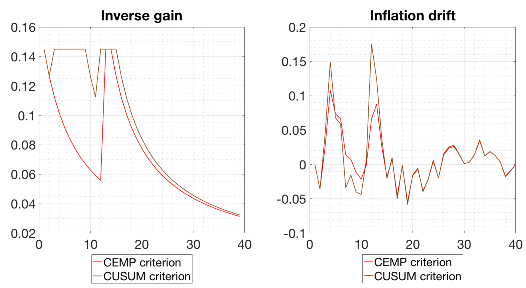
3.3 Inverse gain and drift when changing ρ, no shocks, CEMP vs CUSUM



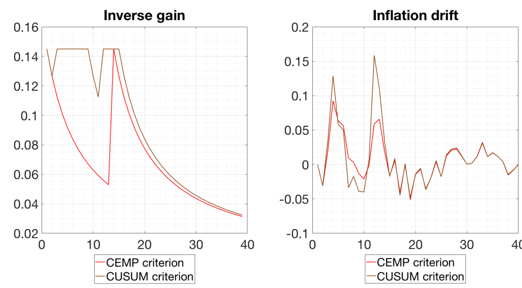
(a) $\rho = 0$



(b) $\rho = 0.3$



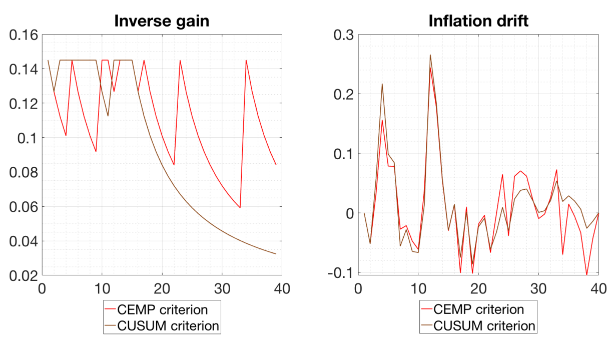
(c) $\rho = 0.6$



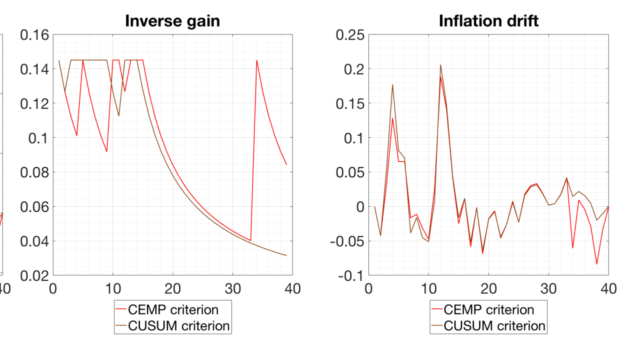
(d) $\rho = 0.9$

3.4 Gain and drift conditional on shocks when changing ρ

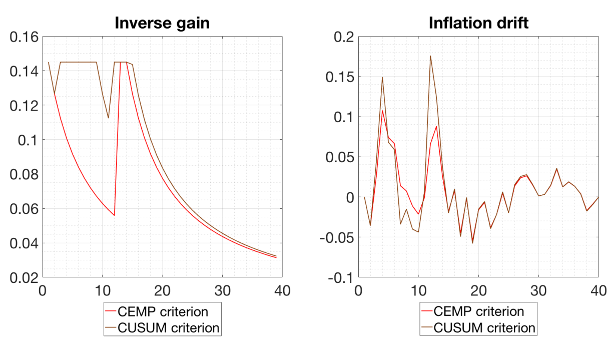
Figure 5: Mean gain and drift after a natural rate shock (r^n)



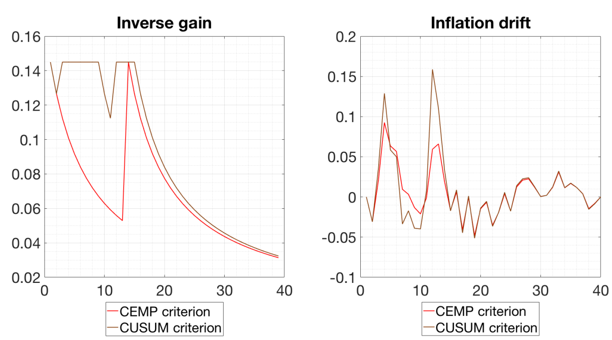
(e) $\rho = 0$



(f) $\rho = 0.3$

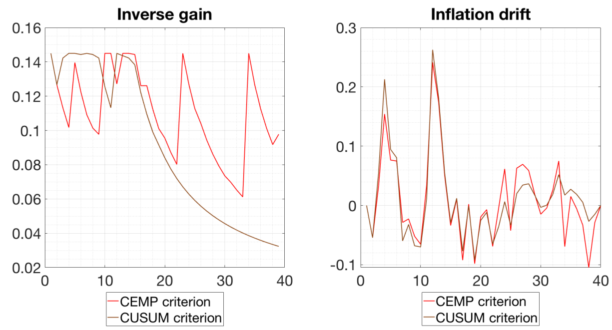


(g) $\rho = 0.6$

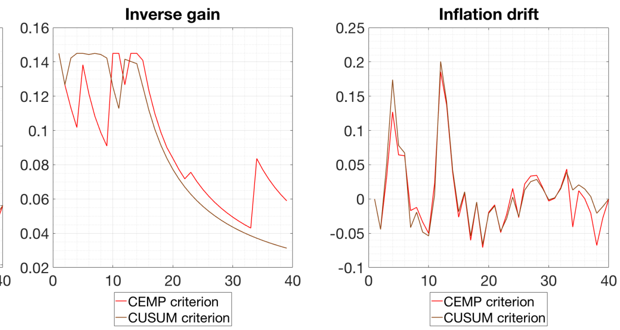


(h) $\rho = 0.9$

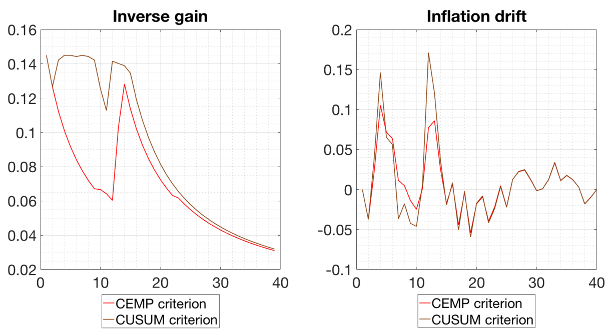
Figure 6: Mean gain and drift after a monetary policy shock ($\bar{i}$)



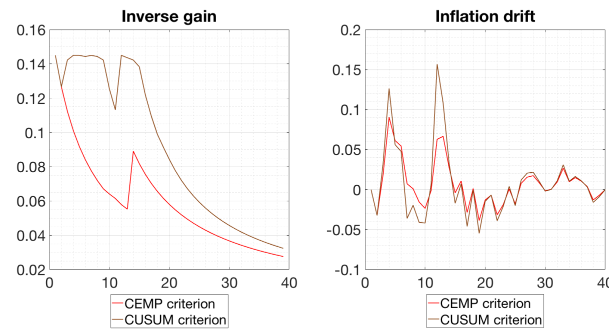
(a) $\rho = 0$



(b) $\rho = 0.3$

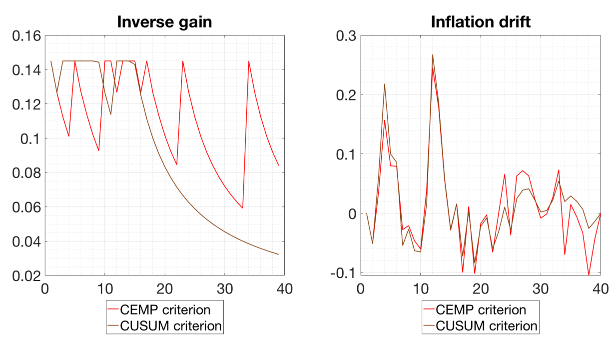


(c) $\rho = 0.6$

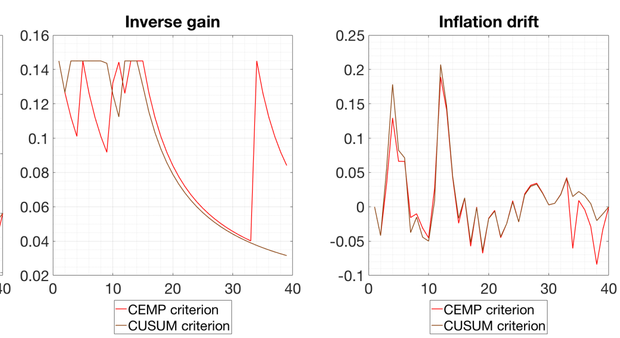


(d) $\rho = 0.9$

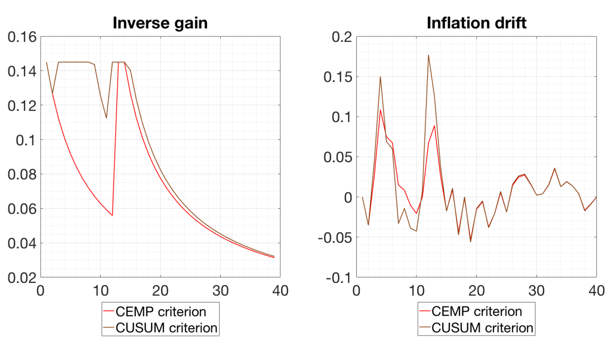
Figure 7: Mean gain and drift after a cost-push shock (u)



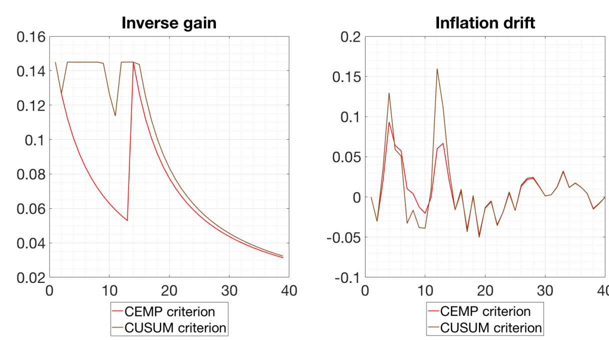
(a) $\rho = 0$



(b) $\rho = 0.3$



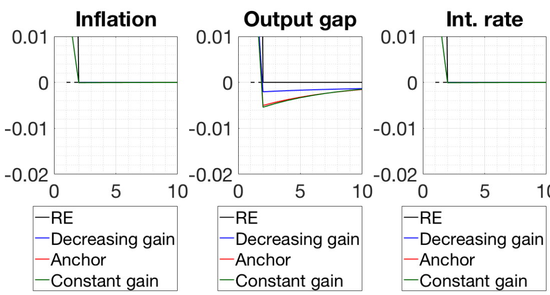
(c) $\rho = 0.6$



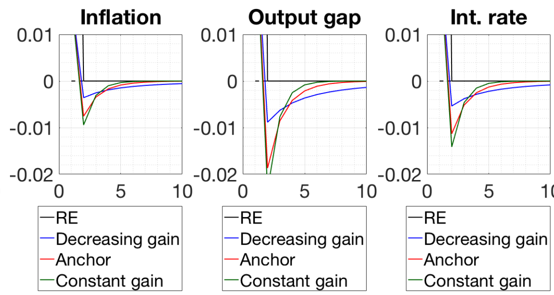
(d) $\rho = 0.9$

3.5 IRFs when changing $\psi_\pi(\rho = 0)$

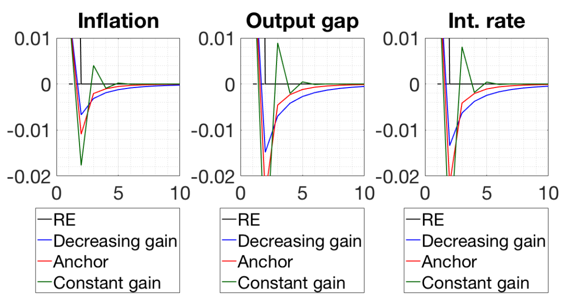
Figure 8: IRFs to a natural rate shock (r^n)



(a) $\psi_\pi = 1.1$

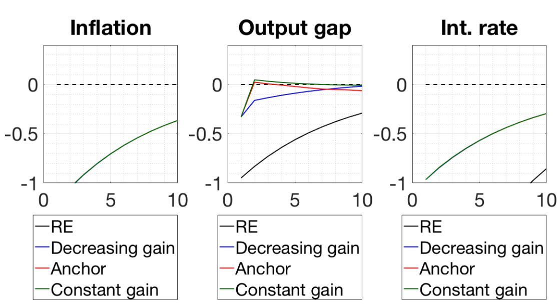


(b) $\psi_\pi = 1.5$

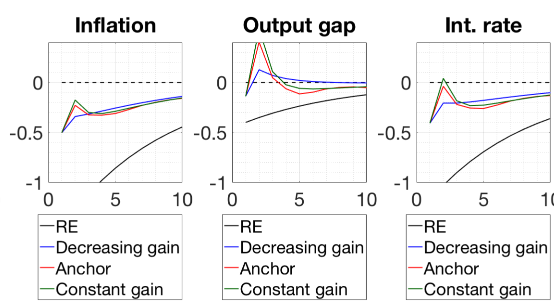


(c) $\psi_\pi = 2$

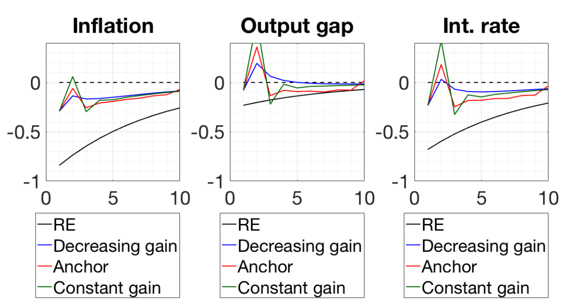
Figure 9: IRFs to a monetary policy shock ($\bar{i}$)



(a) $\psi_\pi = 1.1$

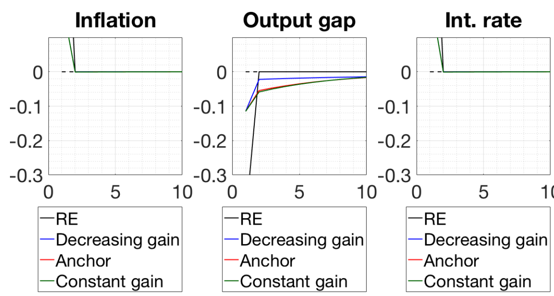


(b) $\psi_\pi = 1.5$

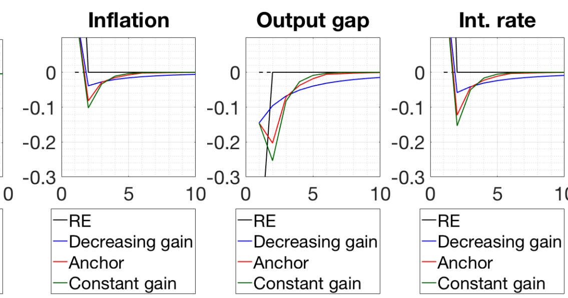


(c) $\psi_\pi = 2$

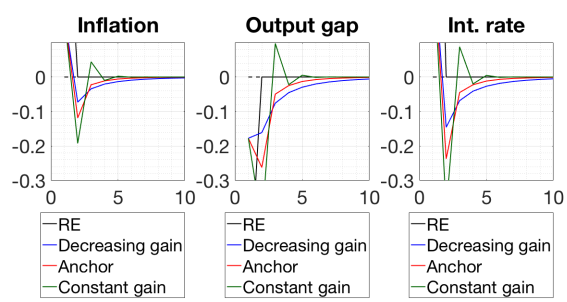
Figure 10: IRFs to a cost-push shock (u)



(a) $\psi_\pi = 1.1$



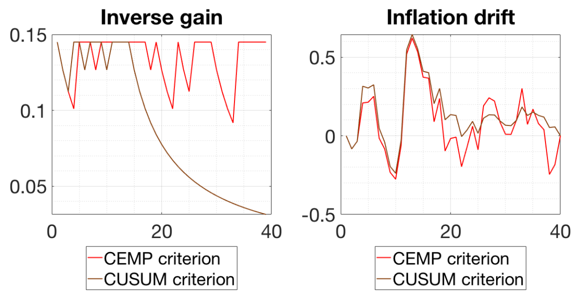
(b) $\psi_\pi = 1.5$



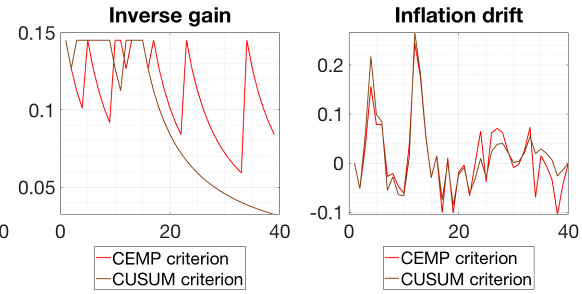
(c) $\psi_\pi = 2$

3.6 Gain and drift conditional on shocks when changing ψ_π

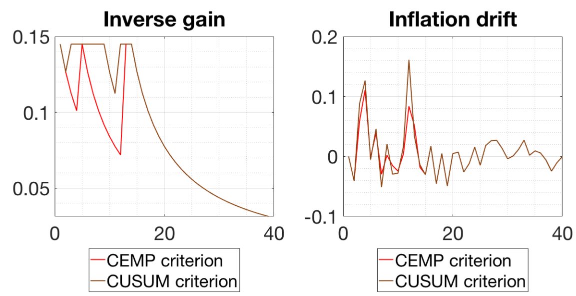
Figure 11: Mean gain and drift after a natural rate shock (r^n)



(a) $\psi_\pi = 1.1$

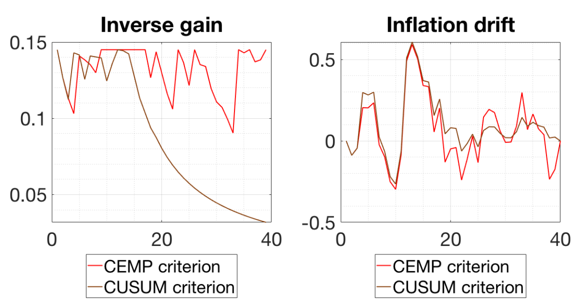


(b) $\psi_\pi = 1.5$

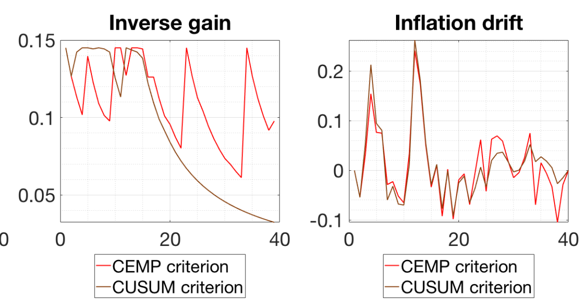


(c) $\psi_\pi = 2$

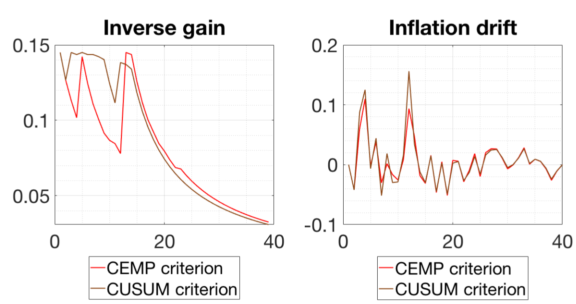
Figure 12: Mean gain and drift after a monetary policy shock ($\bar{i}$)



(a) $\psi_\pi = 1.1$

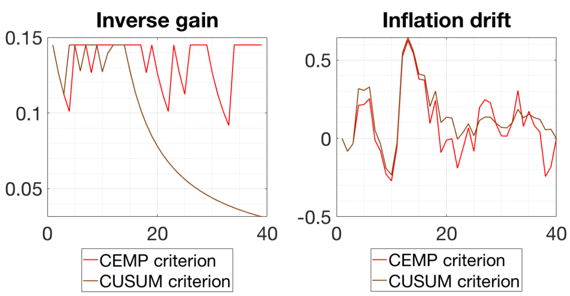


(b) $\psi_\pi = 1.5$

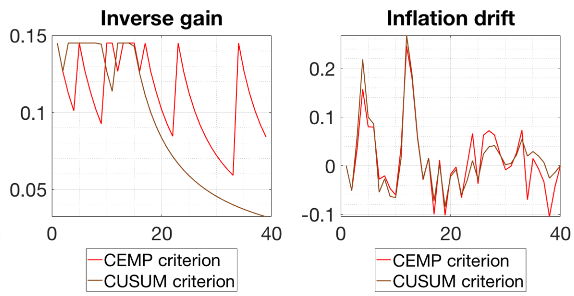


(c) $\psi_\pi = 2$

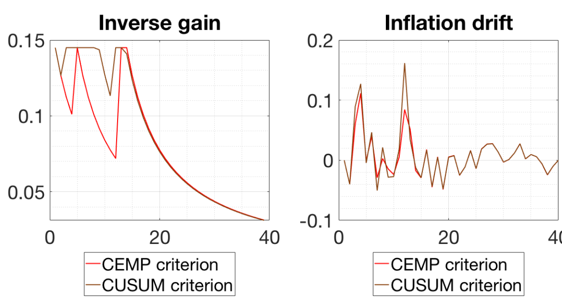
Figure 13: Mean gain and drift after a cost-push shock (u)



(a) $\psi_\pi = 1.1$



(b) $\psi_\pi = 1.5$



(c) $\psi_\pi = 2$

4 Discussion

Proposition 1. (Dampened impact of shocks)
Impact effects of structural shocks are smaller under learning than under rational expectations because expectations are a state variable and thus only respond with a lag. For this reason, there is no difference between decreasing gain and constant gain learning on impact.

Proposition 2. (Higher persistence after shocks)
Under learning, impulse responses exhibit more persistence to iid shocks than under rational expectations because shocks propagate via expectations, and under learning, expectations are slow-moving. Persistence is higher the less agents load on recent forecast errors. Thus decreasing gain learning leads to more persistence than constant gain learning.

Proposition 3. (Overshooting)
Under learning, impulse responses exhibit overshooting, i.e. change sign after impact. This is because the shock interrupts an ongoing learning process of the law of motion of the economy. Expectations incorporate not only the current shock, but also responses of jump variables to it, which feeds back into the law of motion itself. Overshooting is the more pronounced the more agents load on recent forecast errors. Thus constant gain learning produces stronger overshooting than decreasing gain learning.

Corollary 1. (Overshooting to monetary policy)
Under learning, a monetary policy coefficient on inflation of ψ_π leads to bigger responses than under rational expectations. In particular, a monetary policy that exactly closes inflation gaps under rational expectations overshoots under learning, opening a gap of the opposite sign. The size of overshooting depends on whether, given ψ_π, expectations are anchored or not.

Corollary 2. (Anchoring expectations diminishes overshooting)
Since anchoring expectations means decreasing gains, monetary policy will wish to anchor expectations in order to mitigate overshooting.

Proposition 4. (Comparative statics of anchoring)
In general, increasing any parameter that induces more stability in the system (makes inflation less volatile) leads to more anchoring. Thus increasing ρ or lowering κ c.p. leads to more anchoring.

Proposition 5. (Inflation aggressivity and anchoring)
The relationship between the probability of expectations becoming unanchored and ψ_π, the aggressiveness of the central bank on inflation, has a U-shape. Very low values of ψ_π lead to frequent unanchoring as agents doubt the central bank's commitment to the inflation target. Very high values also lead to frequent unanchoring because by moving the nominal interest rate often and strongly, the monetary authority contributes to a volatile environment.

Proposition 6. (Margin of adjustment)
Whether expectational differences between learning and rational expectations show up in inflation or the output gap depends on κ (or equivalently α), the extent of nominal rigidities. A lower κ (higher α) means higher price rigidity and translates to output gaps being the margin of adjustment.

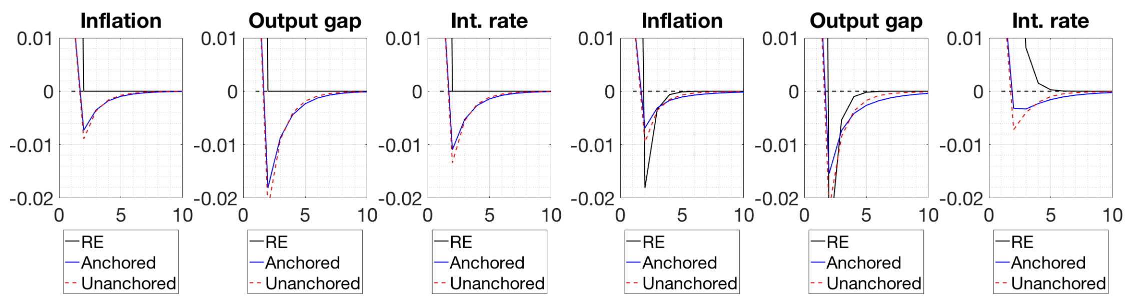
Conjecture 1. (Optimal monetary policy)
Monetary policy under learning with endogenous gain trades off overshooting with persistent impulse responses. On the one hand, monetary policy wishes to anchor expectations to mitigate overshooting. On the other hand, having unanchored expectations has the advantage that agents "learn away" the shock faster. However, unanchoring leads to more volatile observables since expectations are more volatile,

feeding back into endogenous variables. Thus optimal monetary policy needs to consider a) the type of shock and its severity, b) the extent of overshooting c) the desirability of getting out of the shock quickly d) and whether expectations are currently anchored, as well as how costly it would be to keep them or get them anchored.

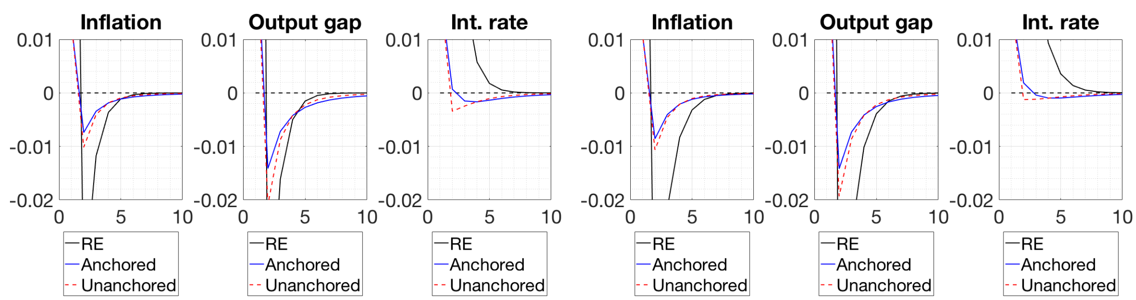
5 IRFs conditional on being anchored / unanchored

5.1 IRFs when changing ρ, anchored/unanchored

Figure 14: IRFs to a natural rate shock (r^n)

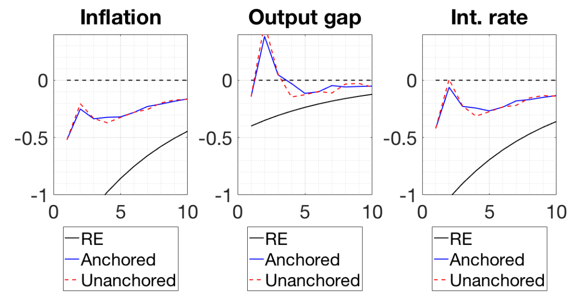


(a) $\rho = 0$ (b) $\rho = 0.3$

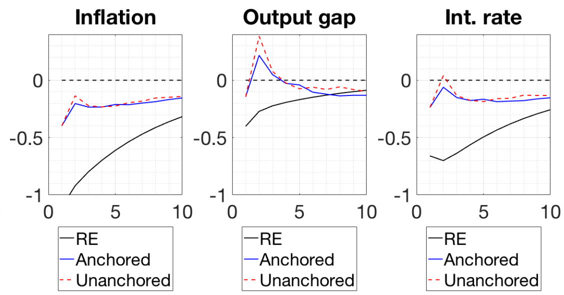


(c) $\rho = 0.6$ (d) $\rho = 0.9$

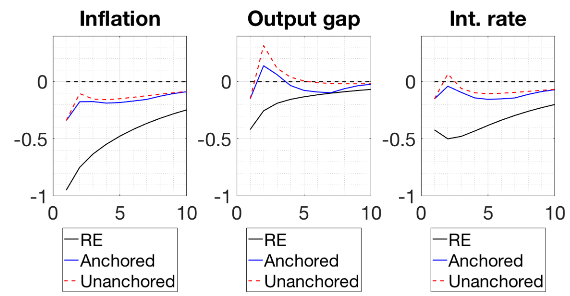
Figure 15: IRFs to a monetary policy shock ($\bar{i}$)



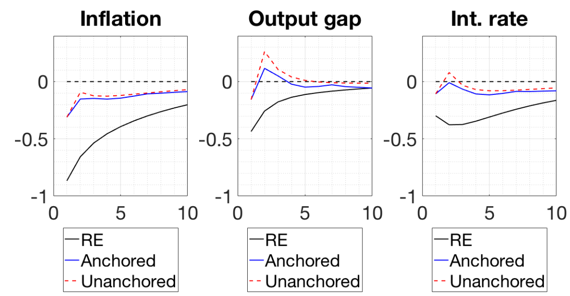
(a) $\rho = 0$



(b) $\rho = 0.3$

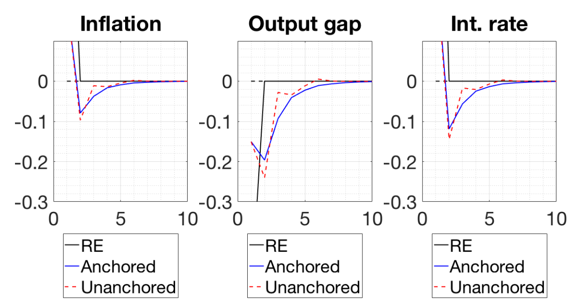


(c) $\rho = 0.6$

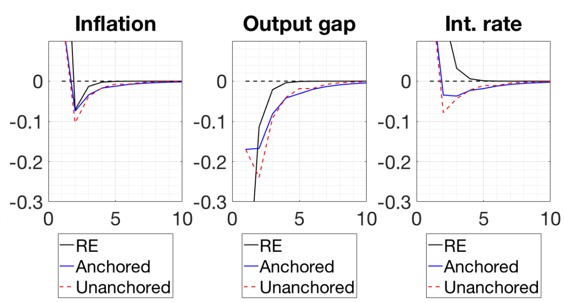


(d) $\rho = 0.9$

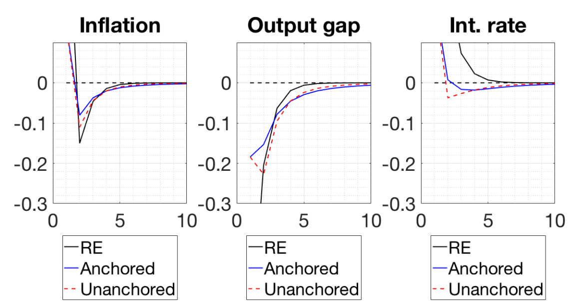
Figure 16: IRFs to a cost-push shock (u)



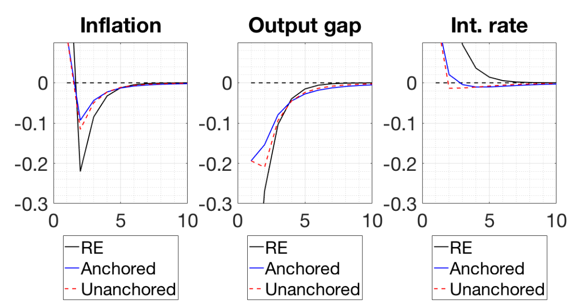
(a) $\rho = 0$



(b) $\rho = 0.3$



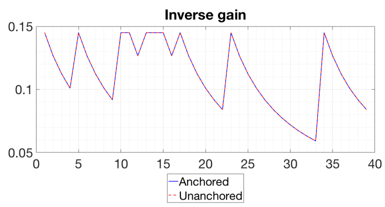
(c) $\rho = 0.6$



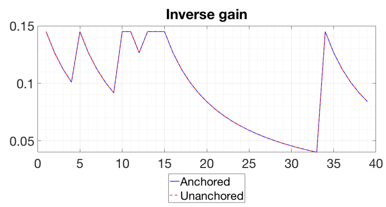
(d) $\rho = 0.9$

5.2 Gain conditional on shocks when changing ρ, anchored/unanchored

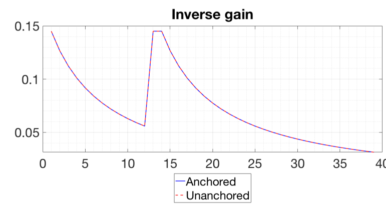
Figure 17: Mean gain and drift after a natural rate shock (r^n)



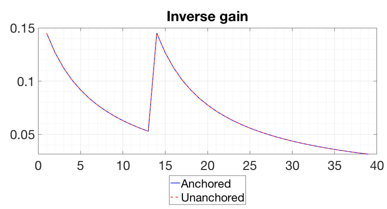
(a) $\rho = 0$



(b) $\rho = 0.3$

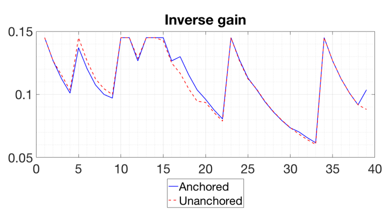


(c) $\rho = 0.6$

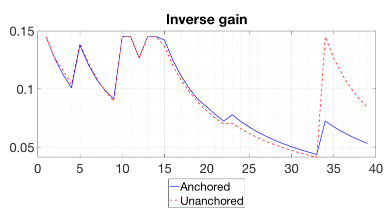


(d) $\rho = 0.9$

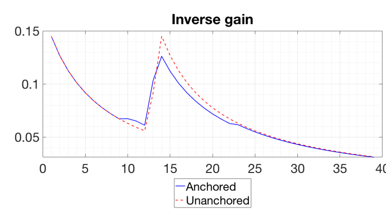
Figure 18: Mean gain and drift after a monetary policy shock ($\bar{i}$)



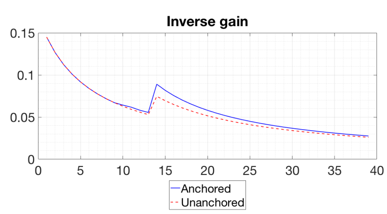
(a) $\rho = 0$



(b) $\rho = 0.3$

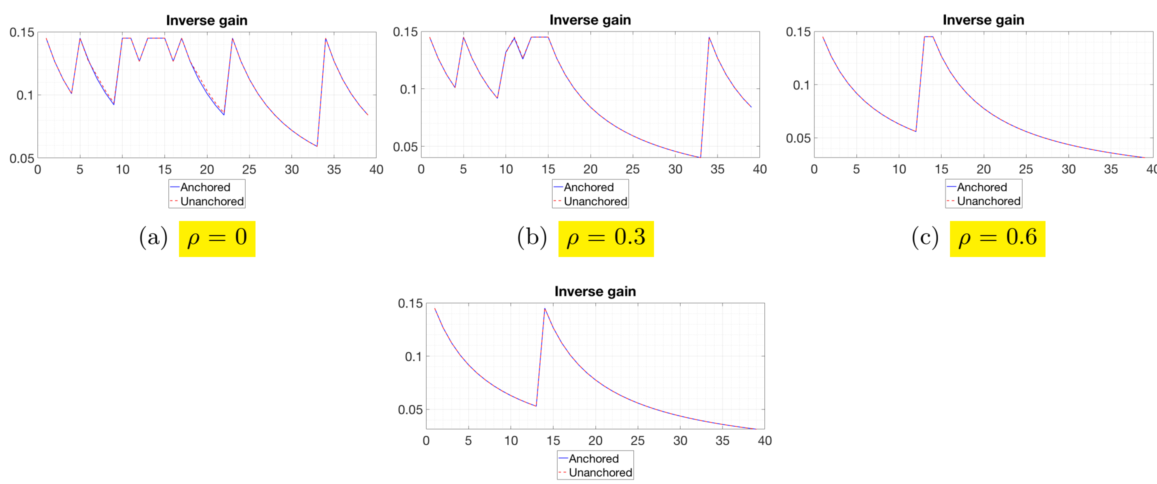


(c) $\rho = 0.6$



(d) $\rho = 0.9$

Figure 19: Mean gain and drift after a cost-push shock (u)



(a) $\rho = 0$

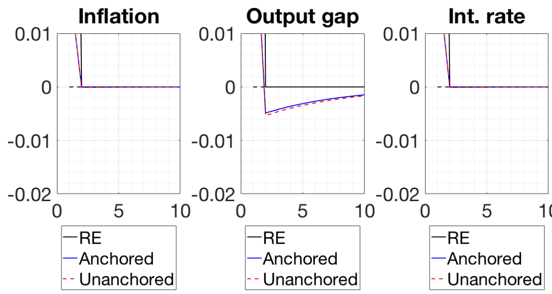
(b) $\rho = 0.3$

(c) $\rho = 0.6$

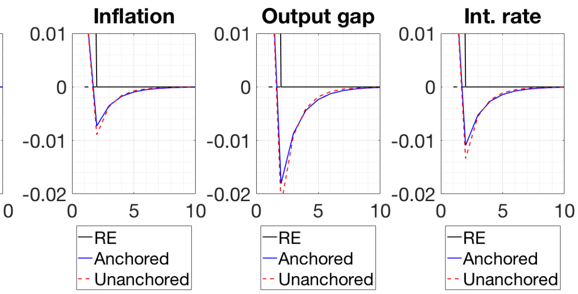
(d) $\rho = 0.9$

5.3 IRFs when changing $\psi_\pi(\rho = 0)$, anchored/unanchored

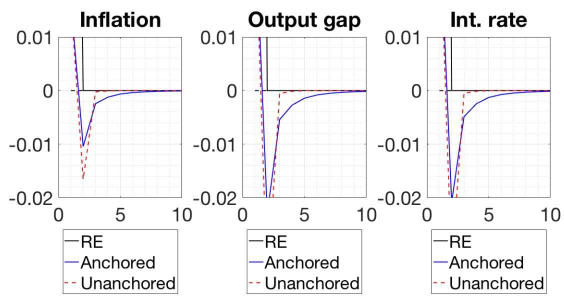
Figure 20: IRFs to a natural rate shock (r^n)



(a) $\psi_\pi = 1.1$

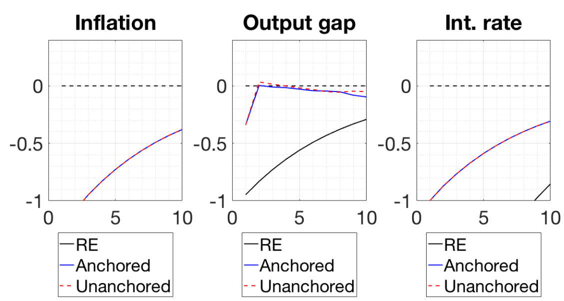


(b) $\psi_\pi = 1.5$

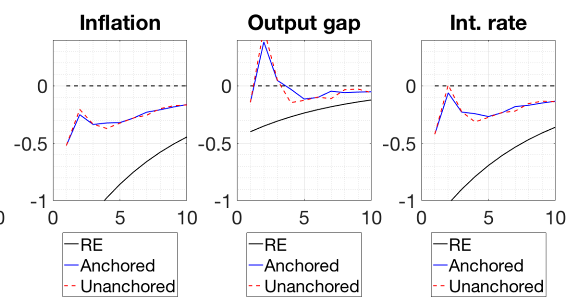


(c) $\psi_\pi = 2$

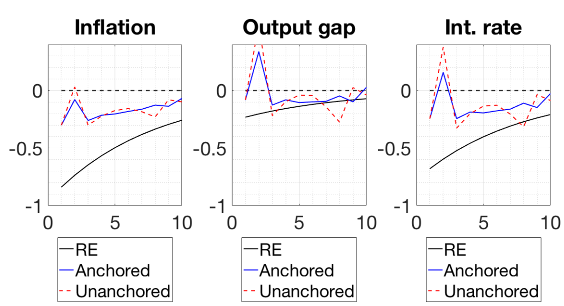
Figure 21: IRFs to a monetary policy shock ($\bar{i}$)



(a) $\psi_\pi = 1.1$

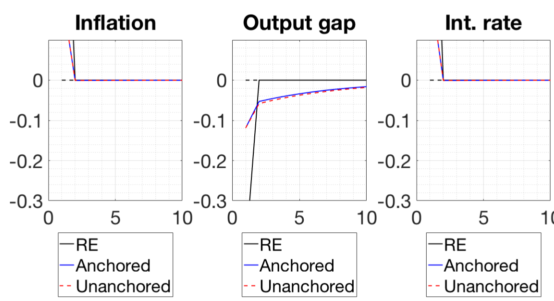


(b) $\psi_\pi = 1.5$

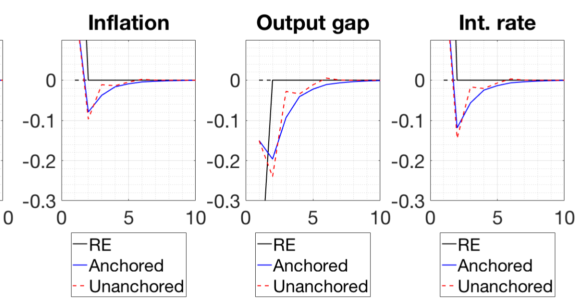


(c) $\psi_\pi = 2$

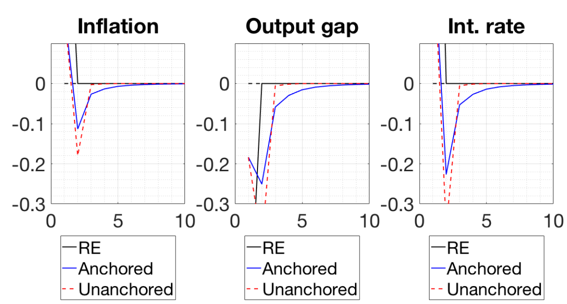
Figure 22: IRFs to a cost-push shock (u)



(a) $\psi_\pi = 1.1$



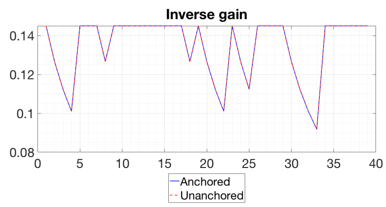
(b) $\psi_\pi = 1.5$



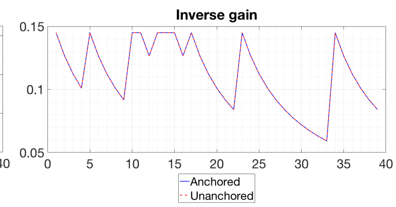
(c) $\psi_\pi = 2$

5.4 Gain conditional on shocks when changing ψ_π, anchored/unanchored

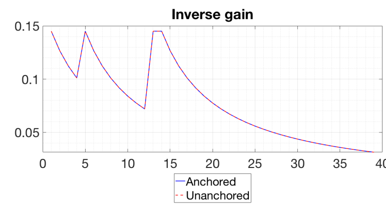
Figure 23: Mean gain and drift after a natural rate shock (r^n)



(a) $\psi_\pi = 1.1$

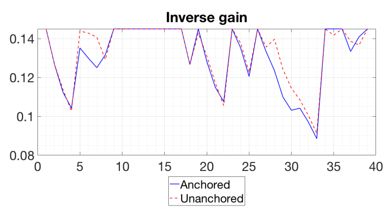


(b) $\psi_\pi = 1.5$

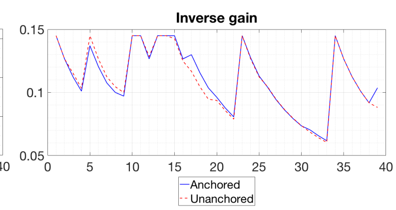


(c) $\psi_\pi = 2$

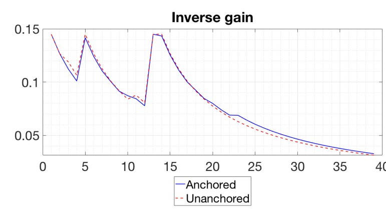
Figure 24: Mean gain and drift after a monetary policy shock ($\bar{i}$)



(a) $\psi_\pi = 1.1$



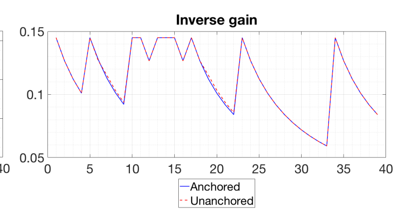
(b) $\psi_\pi = 1.5$



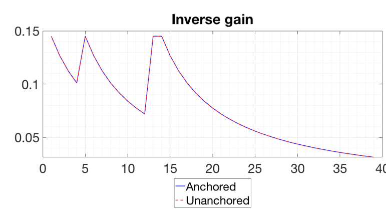
(c) $\psi_\pi = 2$

Figure 25: Mean gain and drift after a cost-push shock (u)

(a) $\psi_\pi = 1.1$



(b) $\psi_\pi = 1.5$



(c) $\psi_\pi = 2$