Materials 15 - More on the CEMP vs. CUSUM criteria

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January 29, 2020

Overview

1 Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \tag{3}$$

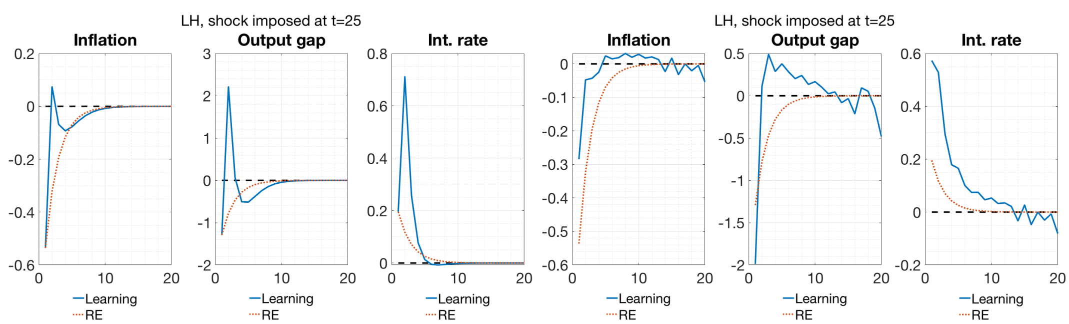
$$\hat{\mathbb{E}}_t z_{t+h} = \bar{z}_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\, h_x \qquad \text{PLM} \tag{4}$$

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} \underbrace{\big(z_t - (\bar{z}_{t-1} + b s_{t-1})\big)}_{\text{fcst error using (4)}} \tag{5}$$

(Vector learning. For scalar learning, $\bar{z} = \begin{pmatrix} \bar{\pi} & 0 & 0 \end{pmatrix}'$. I'm also not writing the case where the slope b is also learned.)

$$k_t = \begin{cases} k_{t-1} + 1 & \text{for decreasing gain learning} \\ \bar{g}^{-1} & \text{for constant gain learning.} \end{cases} \tag{6}$$

Figure 1: Reference: baseline model



(a) Learning constant only (b) Learning slope and constant

2 The CEMP vs. the CUSUM criterion

CEMP's criterion

$$\theta_t = |\hat{\mathbb{E}}_{t-1}\pi_t - \mathbb{E}_{t-1}\,\pi_t|/(\mathrm{Var}(\text{shocks})) \tag{7}$$

$$\text{i.e.} \quad \text{PLM- E[ALM], scaled by shocks} \tag{8}$$

For my version of CEMP's criterion, I rewrite the ALM

$$z_t = A_a f_a + A_b f_b + A_s s_t \tag{9}$$

$$\text{as} \quad z_t = F + G s_t \tag{10}$$

$$\Leftrightarrow \quad z_t = \begin{bmatrix} F & G \end{bmatrix} \begin{bmatrix} 1 \\ s_t \end{bmatrix} \tag{11}$$

Then, since the PLM is $z_t = \phi \begin{bmatrix} 1 \\ s_t \end{bmatrix}$, the generalized CEMP criterion becomes

$$\theta_t = \max |\Sigma^{-1}(\phi - \begin{bmatrix} F & G \end{bmatrix})| \tag{12}$$

where Σ is the VC matrix of shocks. As for the CUSUM criterion, what I did in Materials 5 was

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1}(FE_t^2 - \omega_{t-1}) \tag{13}$$

$$\theta_t = \theta_{t-1} + \kappa k_{t-1}^{-1}(FE_t^2/\omega_t - \theta_{t-1}) \tag{14}$$

where FE_t is the most recent short-run forecast error ($ny \times 1$), and ω_t is the agents' estimate of the forecast error variance ($ny \times ny$). To take into account that these are now matrices, I now write

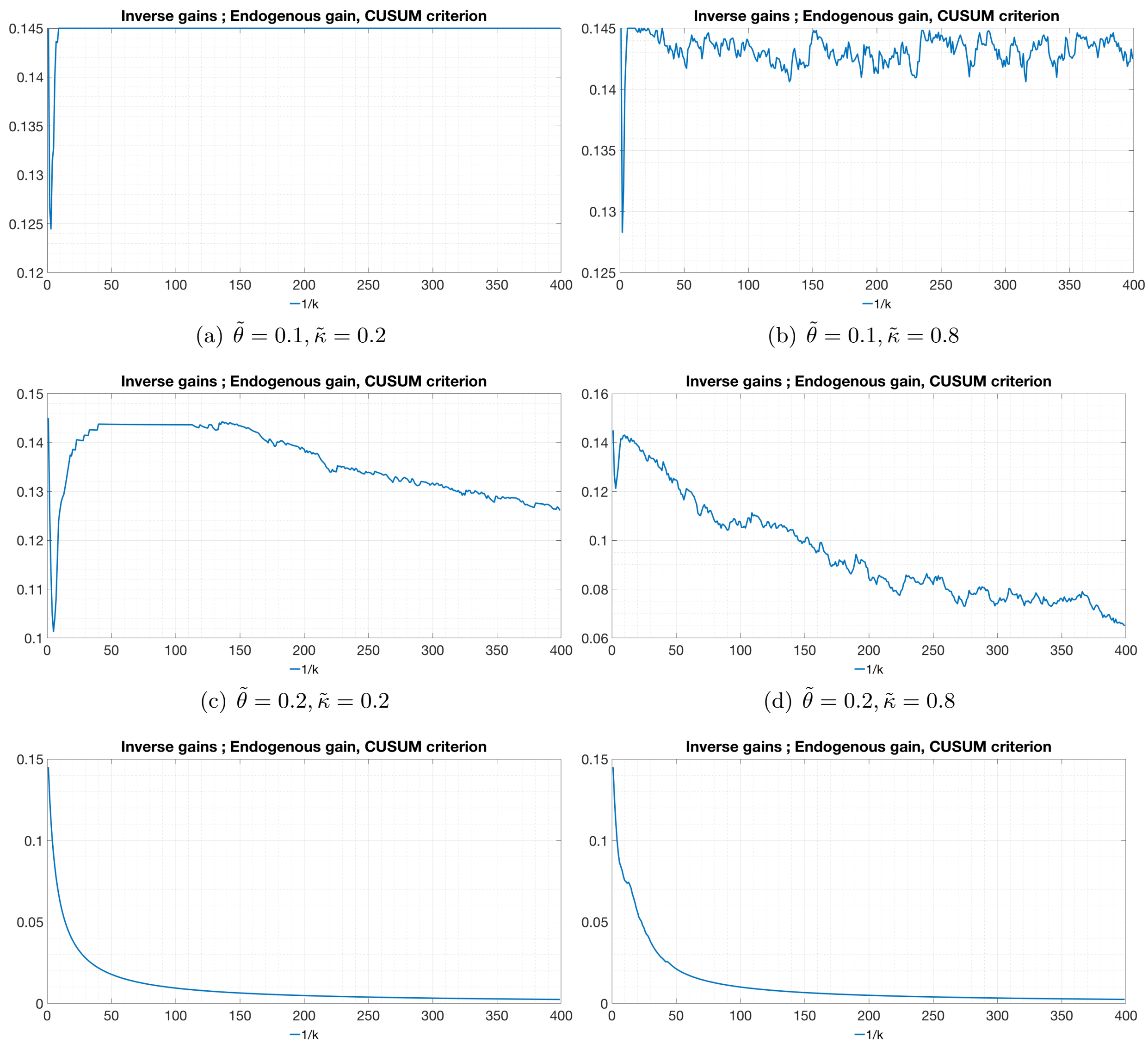
$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1}(FE_t FE_t' - \omega_{t-1}) \tag{15}$$

$$\theta_t = \theta_{t-1} + \kappa k_{t-1}^{-1}\mathrm{mean}((\omega_t^{-1} FE_t FE_t' - \theta_{t-1})) \tag{16}$$

3 Behavior of CEMP and CUSUM criteria as functions of their parameters

$\psi_\pi = 1.5, \psi_x = 0.$

Figure 2: Inverse gains CUSUM, learning the constant only



(a) $\tilde{\theta} = 0.1, \tilde{\kappa} = 0.2$

(b) $\tilde{\theta} = 0.1, \tilde{\kappa} = 0.8$

(c) $\tilde{\theta} = 0.2, \tilde{\kappa} = 0.2$

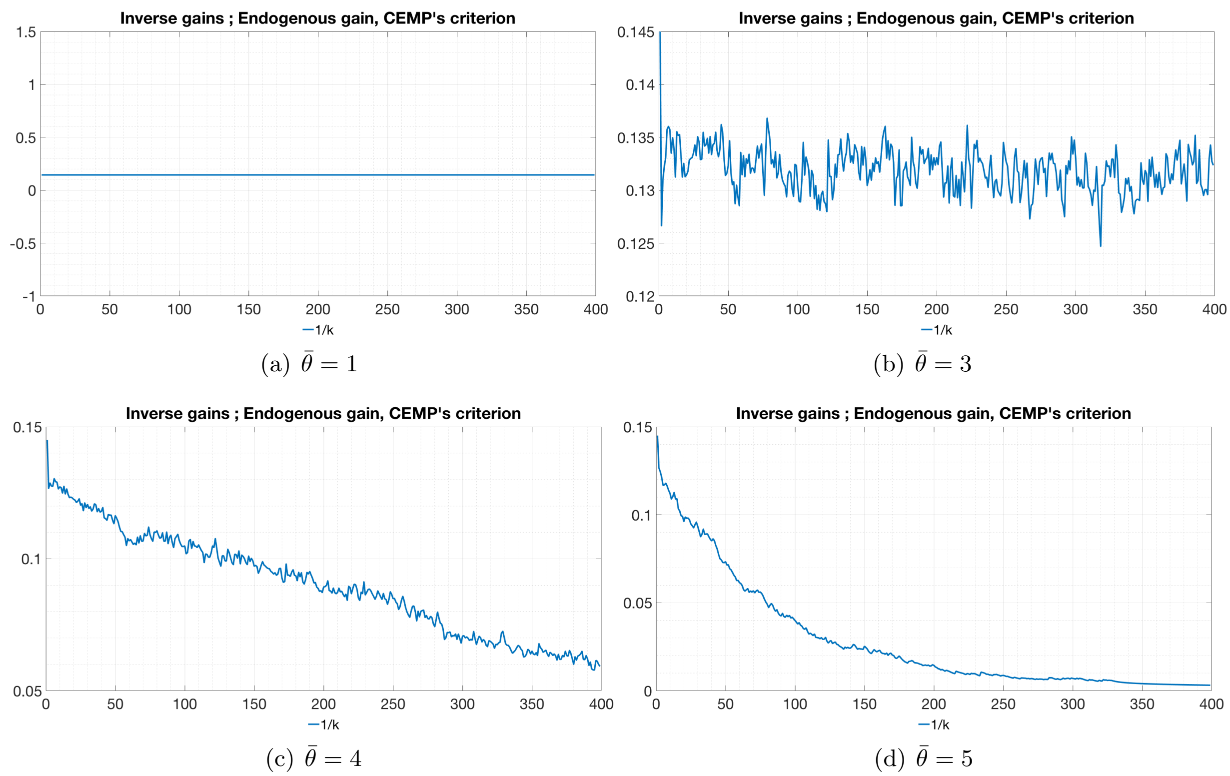
(d) $\tilde{\theta} = 0.2, \tilde{\kappa} = 0.8$

(e) $\tilde{\theta} = 0.4, \tilde{\kappa} = 0.2$

(f) $\tilde{\theta} = 0.4, \tilde{\kappa} = 0.8$

A higher $\tilde{\kappa}$ just increases the action.

Figure 3: Inverse gains CEMP, learning the constant only



(a) $\bar{\theta} = 1$

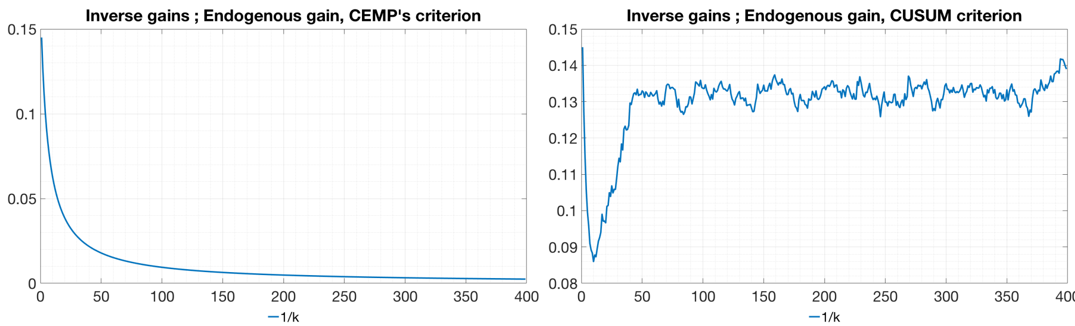
(b) $\bar{\theta} = 3$

(c) $\bar{\theta} = 4$

(d) $\bar{\theta} = 5$

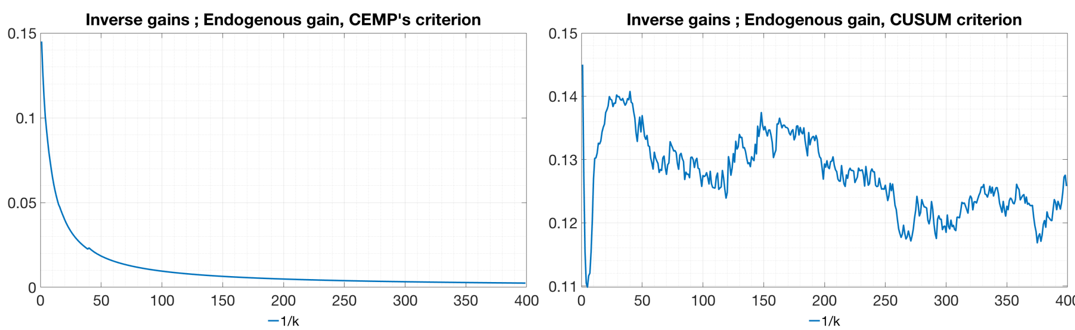
4 Anchoring as a function of ψ_π

Figure 4: Inverse gains, $\psi_\pi = 1.01, \psi_x = 0$



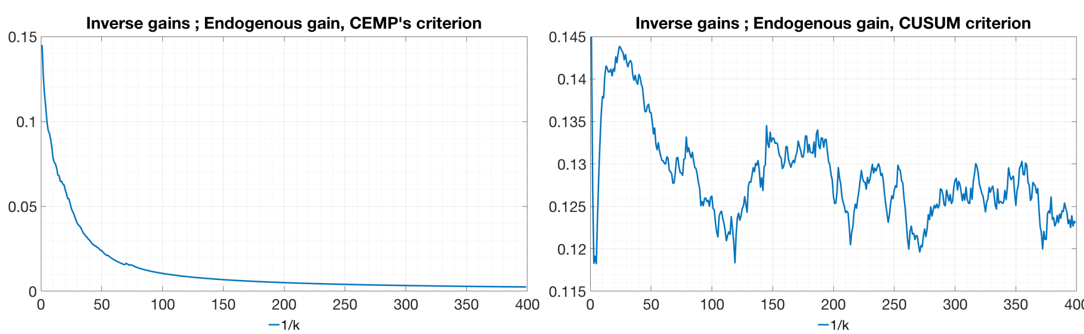
(a) CEMP's criterion, $\bar{\theta} = 4$ (b) CUSUM criterion, $\tilde{\theta} = 0.2$

Figure 5: Inverse gains, $\psi_\pi = 1.1, \psi_x = 0$



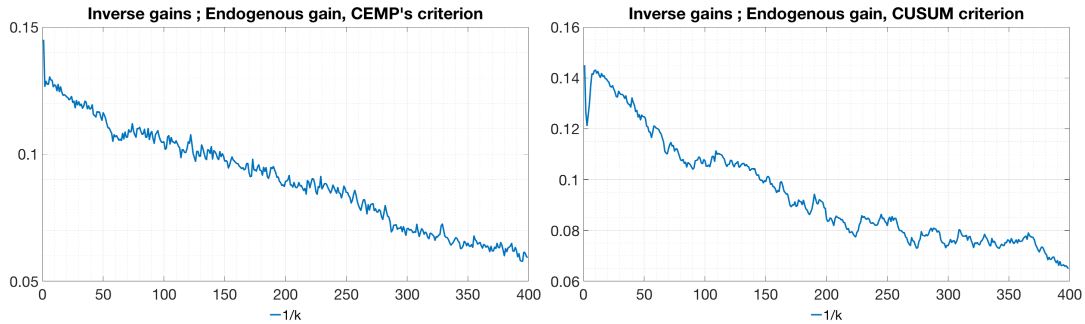
(a) CEMP's criterion, $\bar{\theta} = 4$ (b) CUSUM criterion, $\tilde{\theta} = 0.2$

Figure 6: Inverse gains, $\psi_\pi = 1.2, \psi_x = 0$



(a) CEMP's criterion, $\bar{\theta} = 4$ (b) CUSUM criterion, $\tilde{\theta} = 0.2$

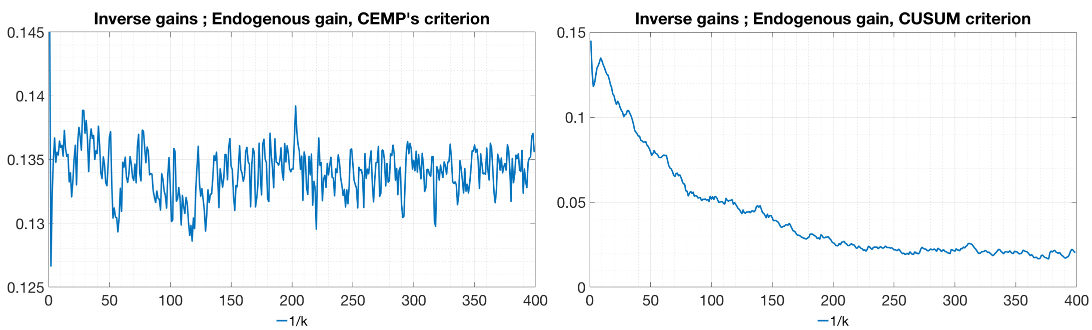
Figure 7: Inverse gains, $\psi_\pi = 1.5, \psi_x = 0$



(a) CEMP's criterion, $\bar{\theta} = 4$

(b) CUSUM criterion, $\tilde{\theta} = 0.2$

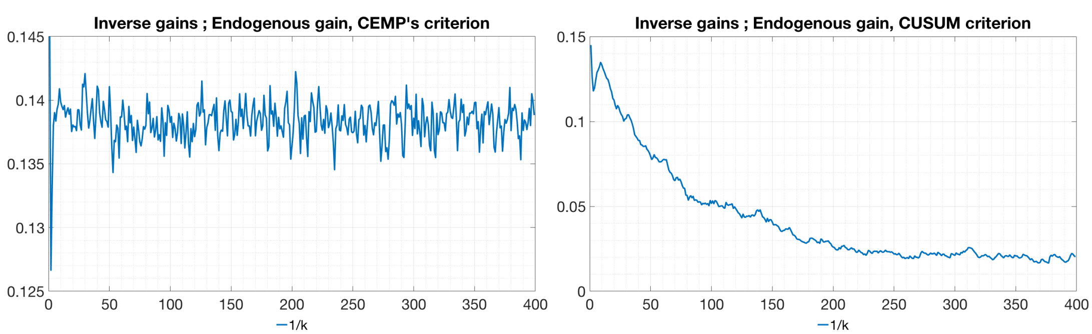
Figure 8: Inverse gains, $\psi_\pi = 1.8, \psi_x = 0$



(a) CEMP's criterion, $\bar{\theta} = 4$

(b) CUSUM criterion, $\tilde{\theta} = 0.2$

Figure 9: Inverse gains, $\psi_\pi = 2, \psi_x = 0$



(a) CEMP's criterion, $\bar{\theta} = 4$

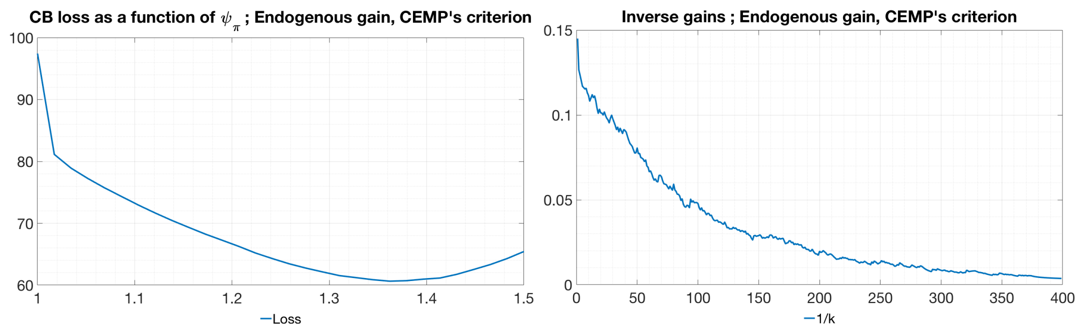
(b) CUSUM criterion, $\tilde{\theta} = 0.2$

5 The central bank's loss function

Relying on Clarida, Gali and Gertler (1999), I start with the simple quadratic loss:

$$\mathcal{L} = \frac{1}{2}\,\mathbb{E}_t \sum_{T=t}^{\infty} \left(\alpha^{CB} x_T^2 + \pi_T^2 \right) \tag{17}$$

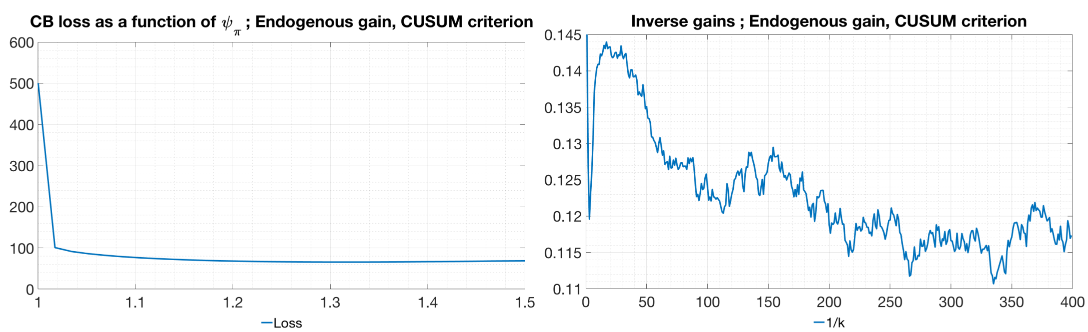
Figure 10: Central bank loss for $\psi_x = 0, \alpha^{CB} = 0, \bar{\theta} = 4$, computed as a cross-sectional average, $N = 100, T = 400$.



(a) Loss (b) Inverse gains associated with ψ_π^*

Fmincon says $\psi_\pi^* = 1.3650$.

Figure 11: Central bank loss for $\psi_x = 0, \alpha^{CB} = 0, \tilde{\theta} = 0.2$, computed as a cross-sectional average, $N = 100, T = 400$.



(a) Loss (b) Inverse gains associated with ψ_π^*

Fmincon says $\psi_\pi^* = 1.3$.

An issue with CUSUM: ω often becomes singular, so for fmincon to be able to run, I took the average element value of ω.

A potential problem is that these optimal values are super-sensitive to threshold criterion values which are completely arbitrary!