Materials 12f3 - "il"-extension of baseline model - Interest rate smoothing using the "suboptimal forecaster" info assumption

See Notes 8 Jan 2020

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Note: the Matlab codes `matrices_A_intrate_smoothing.m` and `matrices_A_intrate_smoothing3.m` do the "myopic info" informational assumption, for which the MN method works. So both do the MN method, in particular 3 does it explicitly. For the "suboptimal forecaster" info assumption, the MN solution doesn't exist.

Compare Mathematica (`materials12f3.nb`).

Blue stuff are changes compared to the baseline model.

1 Model equations

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1} \tag{3}$$

Compact notation

$$z_t = \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = A_a f_a + A_b f_b + A_s s_t \quad \text{with} \quad s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{bmatrix} \tag{4}$$

2 MN matrices

In principle do not exist.

3 PQ matrices

$$\underbrace{\begin{bmatrix} 0 & 1 & \sigma+\frac{\sigma\beta\rho}{1-\rho\beta} \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix}}_{\equiv P} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \underbrace{\begin{bmatrix} \begin{bmatrix} \sigma, & 1-\beta, & \beta(-\sigma) \end{bmatrix} f_b + c_{x,s} s_t \\ \begin{bmatrix} (1-\alpha)\beta, & \alpha\beta\kappa, & 0 \end{bmatrix} f_a + c_{\pi,s} s_t \\ c_{i,s} s_t \end{bmatrix}}_{\equiv Q} \tag{5}$$

where

$$c_{x,s} = \sigma \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} .\text{InxBhx}; \tag{6}$$

$$c_{\pi,s} = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} .\text{InxABhx} \tag{7}$$

$$c_{i,s} = \begin{pmatrix} 0 & 1 & 0 & \rho \end{pmatrix} = d_{i,s} \tag{8}$$

where InxABhx and InxBhx are the same as before. The (*)-relation is

$$f_b(3) = \frac{1}{1-\rho\beta}\left(\psi_\pi f_b(1) + \psi_x f_b(2) + \frac{1}{\beta}\{\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} (I_{nx} - \beta h_x)^{-1} s_t - \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} s_t\}\right) \tag{*}$$

There was a ρi_t term in this equation which was moved to the LHS of the PQ equation and shows up in $P(1,3)$.

The Matlab code that uses this is `matrices_A_12f3.m`.