

Monetary Policy & Anchored Expectations An Endogenous Gain Learning Model

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Policymakers came out of the Great Inflation era with a clear understanding that it was essential to anchor inflation expectations at some low level.

Jerome Powell, Chairman of the Federal Reserve ¹

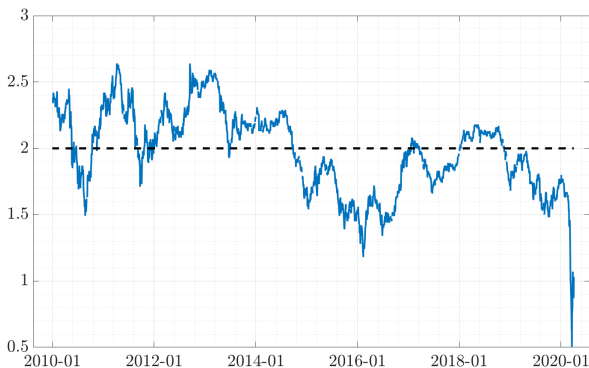


Figure: Market-based inflation expectations, 10 year, average, %

¹Federal Reserve “Challenges for Monetary Policy,” August 23, 2019.

This project

- Estimation of the anchoring function: when do expectations become unanchored?
 - Model of anchoring expectation formation as an endogenous gain adaptive learning scheme
- How to conduct optimal monetary policy in interaction with the anchoring expectation formation?

Preview of results

- A 1%-point forecast error unanchors expectations
 - Optimal monetary policy responsiveness time-varying
- ↪ Unanchored expectations introduce an intertemporal volatility tradeoff
- ↪ Illustrate analytically in special case: target criterion

Related literature

- **Optimal monetary policy in New Keynesian models**

Clarida, Gali & Gertler (1999), Woodford (2003)

- **Econometric learning**

Evans & Honkapohja (2001, 2006), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Eusepi & Preston (2011), Milani (2007, 2014), Lubik & Matthes (2018), Mele et al (2019)

- **Anchoring and the Phillips curve**

Sargent (1999), Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Gobbi et al (2019), Carvalho et al (2019)

Structure of talk

1. Unanchoring in the data
2. Model of anchoring expectations
3. Solving the Ramsey problem
4. Implementing optimal policy

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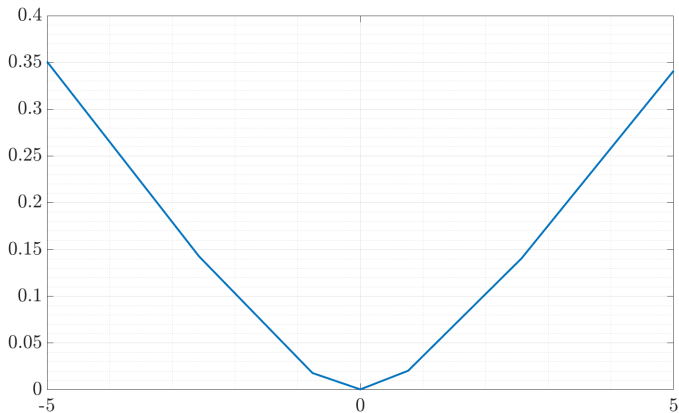


Figure: Unanchoring as a function of forecast errors in inflation (%-point)

Structure of talk

1. Unanchoring in the data
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Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j)) dj \right] \quad (1)$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j) + \Pi_t^i(j)dj - T_t - P_t C_t^i \quad (2)$$

► Consumption, price level

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \quad (3)$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \quad (4)$$

► Profits, stochastic discount factor

Expectations: $\hat{\mathbb{E}}$ instead of \mathbb{E}

- If use \mathbb{E} (rational expectations, RE)

Model solution

$$s_t = hs_{t-1} + \epsilon_t \quad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \quad (5)$$

$$y_t = gs_t \quad (6)$$

$s_t \equiv$ states

$y_t \equiv$ jumps

$\epsilon_t \equiv$ disturbances

Expectations: $\hat{\mathbb{E}}$ instead of \mathbb{E}

- If use \mathbb{E} (rational expectations, RE)

Model solution

$$s_t = h s_{t-1} + \epsilon_t \quad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \quad (5)$$

$$y_t = g s_t \quad (6)$$

$s_t \equiv$ states

$y_t \equiv$ jumps

$\epsilon_t \equiv$ disturbances

- If use $\hat{\mathbb{E}} \rightarrow$ private sector does not know (6)

\hookrightarrow estimate using observed states & knowledge of (5)

Adaptive learning

- Postulate linear functional relationship instead of (6):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \quad (7)$$

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- Note: **misspecified** \rightarrow not model-consistent (not RE)
- Estimate a, b using recursive least squares (RLS)

Recursive least squares

Jumps are: $(\pi, x, i)'$

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Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g h \quad \forall t \quad (8)$$

Recursive least squares

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Special case: learn only intercept of inflation:

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→ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))}_{\equiv fe_{t|t-1}, \text{ forecast error}} \quad (9)$$

$k_t \in (0, 1)$ gain
 b_1 first row of b

Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})) \quad (10)$$

$k_t = \mathbf{g}(fe_{t|t-1})$: anchoring function

Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})) \quad (10)$$

$k_t = \mathbf{g}(fe_{t|t-1})$: anchoring function

$$\mathbf{g}(fe_{t|t-1}) = \alpha b(fe_{t|t-1}) \quad (11)$$

$b(fe_{t|t-1})$ = basis, here: second order spline (piecewise linear)

α = approximating coefficients, here: use $\hat{\alpha}$ from estimation

Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (12)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (13)$$

► Derivations

► Actual laws of motion

- Expectations evolve according to RLS with the endogenous gain given by (11)

→ How should $\{i_t\}$ be set?

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Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

- \mathbb{E} is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

Target criterion

Result

In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa}x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_{\pi,t} \right) \\ \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\bar{\pi},t+j}) \right)$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ at t , $\prod_{j=0}^0 \equiv 1$ and b_1 is the first row of b .

Two layers of intertemporal tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ - \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j} + fe_{t+1+j|t+j} \mathbf{g}_{\pi,t+j})$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

► Why no commitment?

Corollary

Optimal policy under adaptive learning is time-consistent.

↪ Foreshadow: optimal policy aggressiveness time-varying

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Solution procedure

Solve system of model equations + target criterion

↪ solve using parameterized expectations (PEA) and value function iteration (VFI)

↪ obtain a cubic spline approximation to optimal policy function

Optimal policy I - responding to unanchoring

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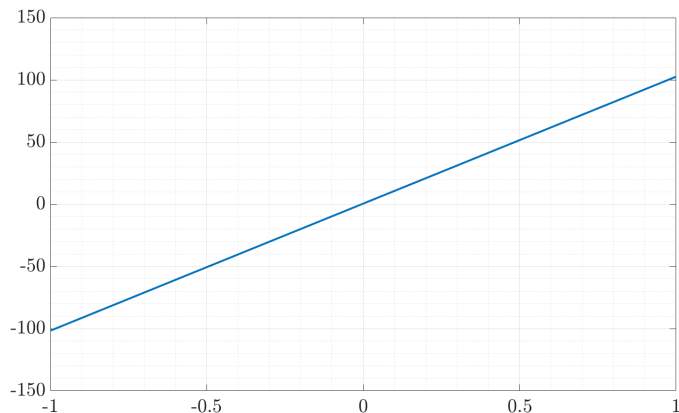
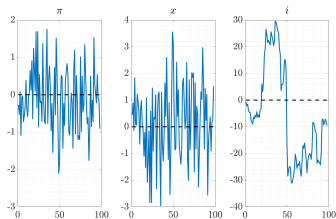


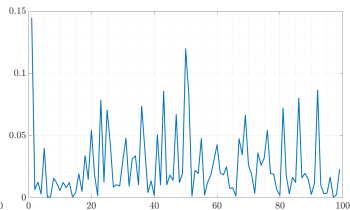
Figure: Comparative statics of the policy function: $\partial i / \partial \bar{\pi}$ if all other states are kept at their mean

Optimal policy II - a particular history

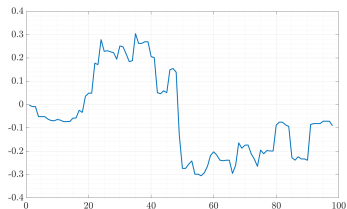
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(a) Observables



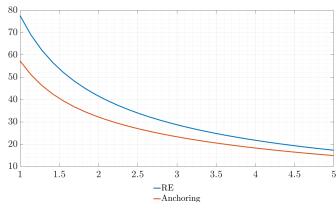
(b) Gain



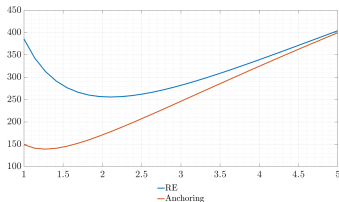
(c) $\bar{\pi}$

Optimal policy III - optimal Taylor-rule coefficients

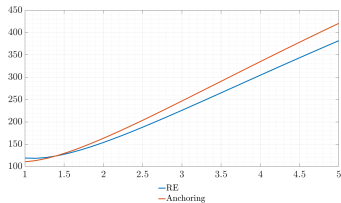
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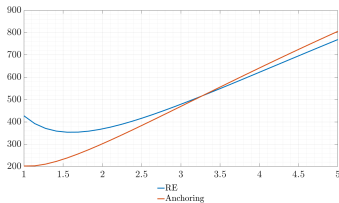
(a) $\lambda_x = 0, \lambda_i = 0$



(b) $\lambda_x = 1, \lambda_i = 0$



(c) $\lambda_x = 0, \lambda_i = 1$



(d) $\lambda_x = 1, \lambda_i = 1$

Conclusion

- Interaction between monetary policy and anchoring
- Optimal policy conditions on stance of current and expected future anchoring
 - ↪ determine intertemporal tradeoffs
- Frontloads aggressive interest rate response to suppress potential unanchoring
- For a 1%-point positive (negative) forecast error, raises (lowers) interest rate by %-point

Appendix

Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \quad (14)$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \quad (15)$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \quad (16)$$

Solve for forecast error $fe_t \equiv \pi_t - f_{t-1}$:

$$fe_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k \rightarrow 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \quad (17)$$

Functional forms for \mathbf{g} in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \quad (18)$$

$p \equiv Prob(\text{liquidity trap regime})$

y_{t-1} output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when } \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \quad (19)$$

θ_t criterion, $\bar{\theta}$ threshold value

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \quad (20)$$

Σ variance-covariance matrix of shocks

$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(fe_{t|t-1}fe'_{t|t-1} - \omega_{t-1}) \quad (21)$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(fe'_{t|t-1}\omega_t^{-1}fe_{t|t-1} - \theta_{t-1}) \quad (22)$$

ω_t estimated forecast-error variance

Recursive least squares algorithm

$$\phi_t = \left(\phi_{t-1}' + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right) \right)' \quad (23)$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} [1 \quad s_{t-1}] - R_{t-1} \right) \quad (24)$$

Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \quad (25)$$

$$s_t = h s_{t-1} + \epsilon_t \quad (26)$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \quad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \quad (27)$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \quad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \quad (28)$$

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t) \right. \\ \left. + \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \right\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa}x_t - \varphi_{2,t}(k_t + \mathbf{g}_{\pi}(\pi_t - f_{t-1})) = 0 \quad (29)$$

$$-2\beta\frac{\lambda}{\kappa}x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_f(\pi_{t+1} - f_t)) = 0 \quad (30)$$

Target criterion system for anchoring function as changes of the gain

$$\begin{aligned} \varphi_{6,t} = & -cfe_{t|t-1}x_{t+1} + \left(1 + \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1}) - fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\varphi_{6,t+1} \\ & - \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1})\varphi_{6,t+2} \end{aligned} \quad (31)$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{fe_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{fe_{t|t-1}}\varphi_{6,t+1} \quad (32)$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (36) is given by:

$$\varphi_{6,t} = -2\mathbb{E}_t \sum_{i=0}^{\infty} \left(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}\right) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{fe_{t+j|t+j-1}}}{\frac{k_{t+j}}{fe_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \quad (33)$$

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (34)$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \quad (35)$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \quad (36)$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \quad (37)$$

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \quad (38)$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \quad (39)$$

where ‘hats’ denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (42) backward to some date t , take expectations at t
 2. Sub in (43)
 3. Aggregate over households i
- Obtain (12)