Materials 16 - Preparing Clough rough draft, simulation-based results and abstract-ish something

Laura Gáti

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Overview

1 Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \tag{3}$$

$$\hat{\mathbb{E}}_t z_{t+h} = \bar{z}_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\ h_x \qquad \text{PLM} \tag{4}$$

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} \underbrace{\big(z_t - (\bar{z}_{t-1} + b s_{t-1})\big)}_{\text{fcst error using (4)}} \tag{5}$$

(Vector learning. For scalar learning, $\bar{z} = \begin{pmatrix} \bar{\pi} & 0 & 0 \end{pmatrix}'$. I'm also not writing the case where the slope b is also learned.)

$$k_t = \begin{cases} k_{t-1} + 1 & \text{when} \quad \theta^{CEMP} < \bar{\theta} \quad \text{or} \quad \theta_t < \tilde{\theta} \\ \bar{g}^{-1} & \text{otherwise.} \end{cases} \tag{6}$$

1.1 The CEMP vs. the CUSUM criterion

CEMP's criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi - \begin{bmatrix} F & G \end{bmatrix})| \tag{7}$$

where Σ is the VC matrix of shocks, ϕ is the estimated matrix, $[F, G]$ is the ALM.

CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1}(f_t f_t' - \omega_{t-1}) \tag{8}$$

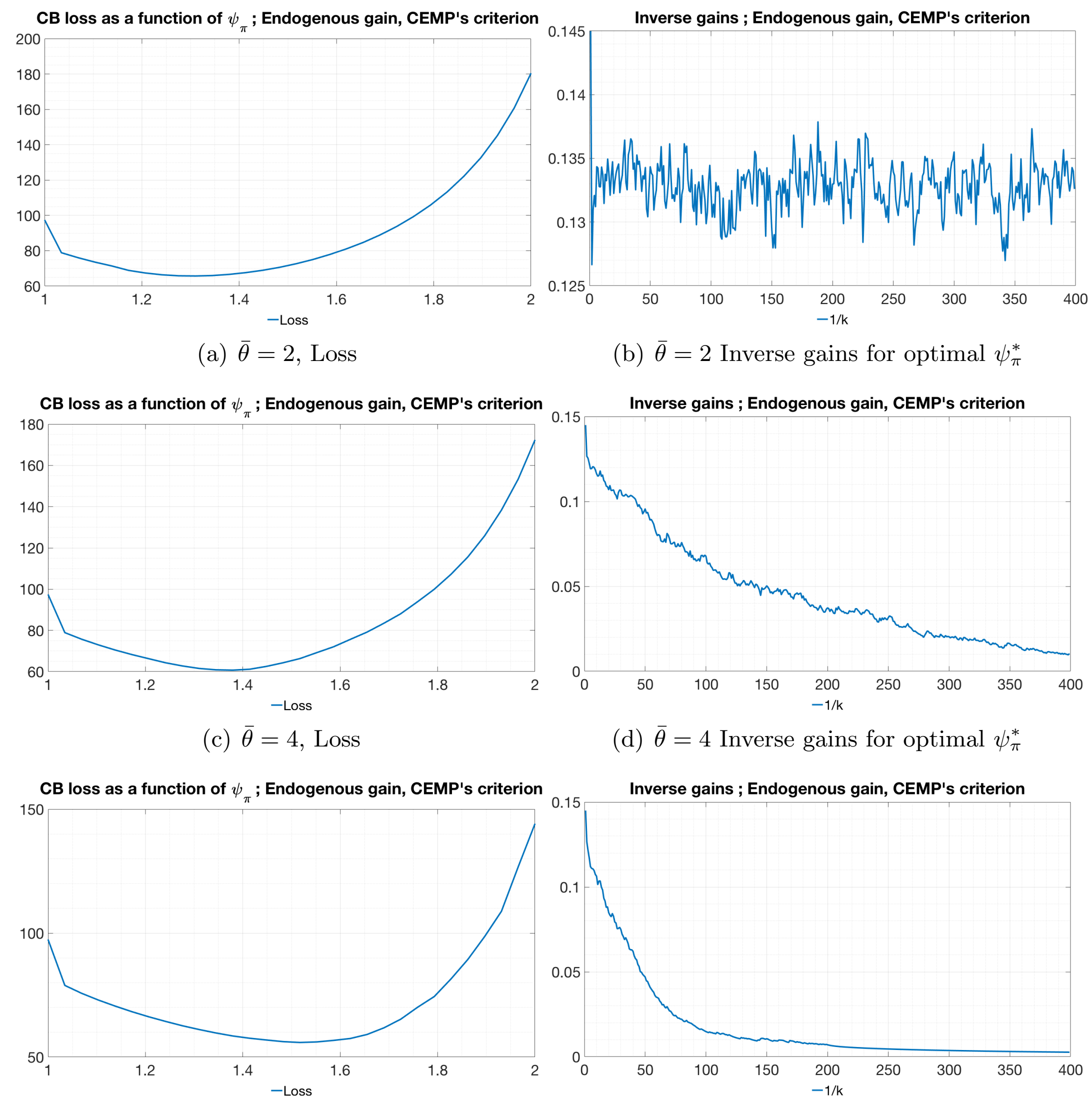
$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}^{-1}(f_t' \omega_t^{-1} f_t - \theta_{t-1}) \tag{9}$$

where f is the most recent forecast error and ω is the estimated FEV.

2 Simulated ψ_π^* and CB losses, RE against learning, fixing $\psi_x = 0$

2.1 RE against CEMP-criterion

Figure 1: CB losses, RE against learning with CEMP's criterion



(a) $\bar{\theta} = 2$, Loss

(b) $\bar{\theta} = 2$ Inverse gains for optimal ψ_π^*

(c) $\bar{\theta} = 4$, Loss

(d) $\bar{\theta} = 4$ Inverse gains for optimal ψ_π^*

(e) $\bar{\theta} = 6$, Loss

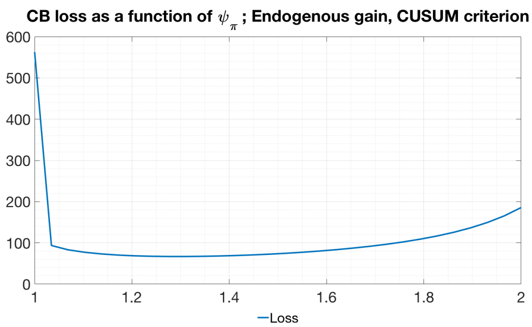
(f) $\bar{\theta} = 6$ Inverse gains for optimal ψ_π^*

- When $\bar{\theta} = 2$, you get unanchored for or $\psi_\pi \geq 1.25$
- When $\bar{\theta} = 4$, you get unanchored for or $\psi_\pi \geq 1.8$
- When $\bar{\theta} = 6$, you get unanchored for $\psi_\pi \geq 2.5$

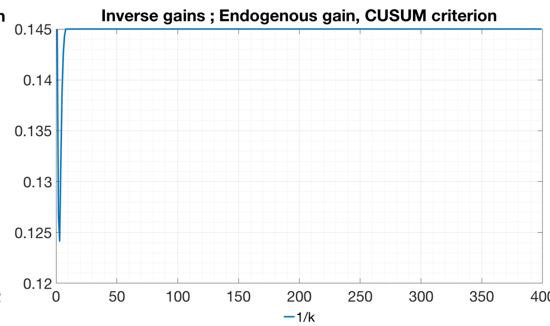
$\rightarrow$ so usually when the choice of aggressiveness on inflation matters for anchoring, mon pol chooses to anchor. But not when this would involve a "too low" ψ_π.

2.2 RE against CUSUM-criterion

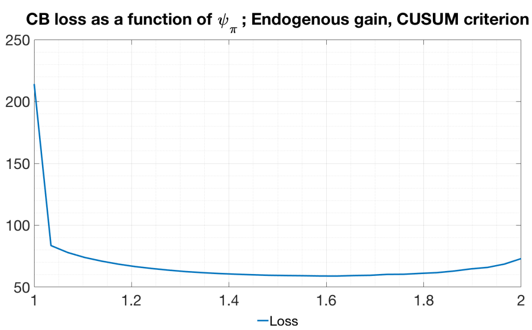
Figure 2: CB losses, RE against learning with CUSUM criterion



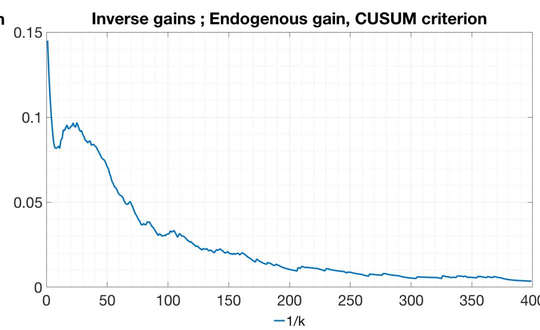
(a) $\tilde{\theta} = 1$, Loss



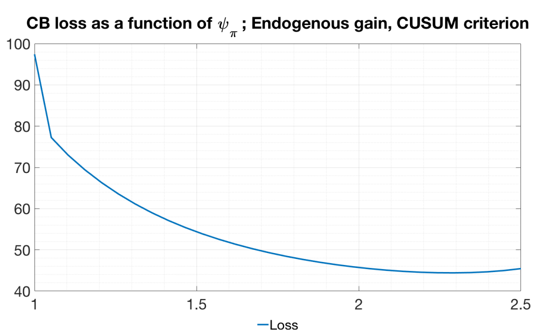
(b) $\tilde{\theta} = 1$ Inverse gains for optimal ψ_π^*



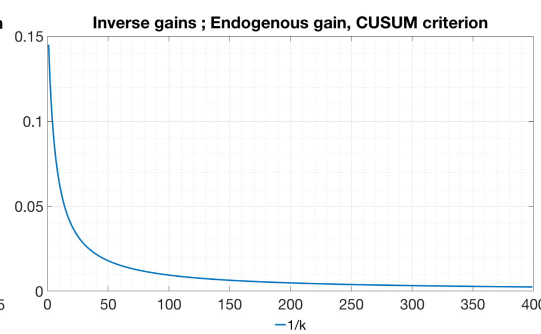
(c) $\tilde{\theta} = 2.5$, Loss



(d) $\tilde{\theta} = 2.5$ Inverse gains for optimal ψ_π^*



(e) $\tilde{\theta} = 4$, Loss



(f) $\tilde{\theta} = 4$ Inverse gains for optimal ψ_π^*

Note:

- When $\tilde{\theta} = 1$, you never get anchoring for any value of $\psi_\pi \in (1, 2]$
- When $\tilde{\theta} = 2.5$, you're unanchored for low ψ_π, anchored for high
- When $\tilde{\theta} = 4$, you always get anchoring for any value of $\psi_\pi \in (1, 2.5]$

$\rightarrow$ so $\tilde{\theta} = 2.5$ is the interesting case because this is where the choice of aggressiveness on inflation matters for anchoring. As we can see, when mon pol can, it chooses to anchor.

3 "Best-case" abstract

Abstract

This paper analyzes optimal monetary policy in a model where expectation formation is characterized by potential anchoring of expectations. In the spirit of Carvalho et al. 2019, I model anchored expectations using an endogenous gain adaptive learning framework. Expectations are anchored if the private sector's forecast errors are sufficiently small such that agents choose a decreasing gain. Anchored expectations are thus a metric for the public's trust in the central bank's commitment to the long-run target. I embed the anchoring expectation formation in an otherwise standard macro model with nominal rigidities and solve for optimal monetary policy. I find that the central bank trades off the costs and benefits of anchoring expectations. Having anchored expectations reduces the volatility of observables, but anchoring expectations is costly in terms of inducing volatility. Optimal policy is therefore conditioned on the stance of expectations.

4 Optimal policy

1. You seem to have doubts.

2. Endogenous gain framework is not differentiable.