Materials 39 - Combing through the code for bugs

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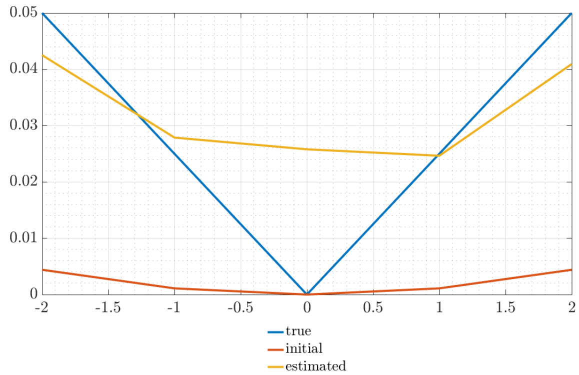
Overview

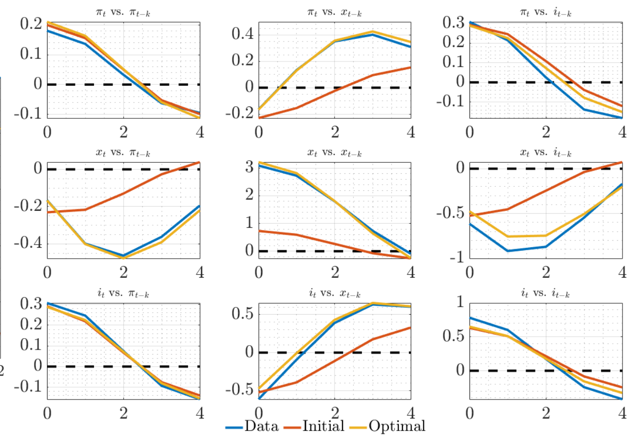
1 Look into measurement error

1.1 Autocovariograms w/ and w/o measurement error, not using expectations data

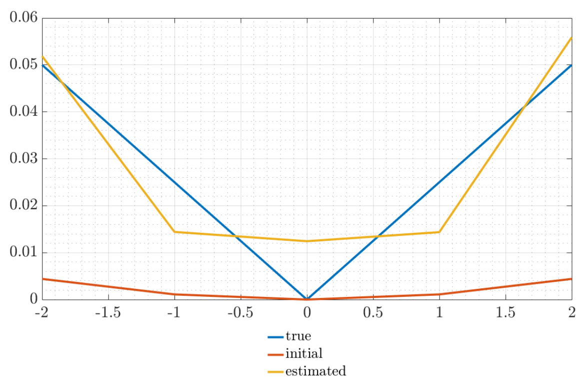
Figure 1: $N = 100$, w/o expectations data, estimate moments N times (N estimations), imposing convexity with weight 100K, truth with $nfe = 5, fe \in (-2, 2)$



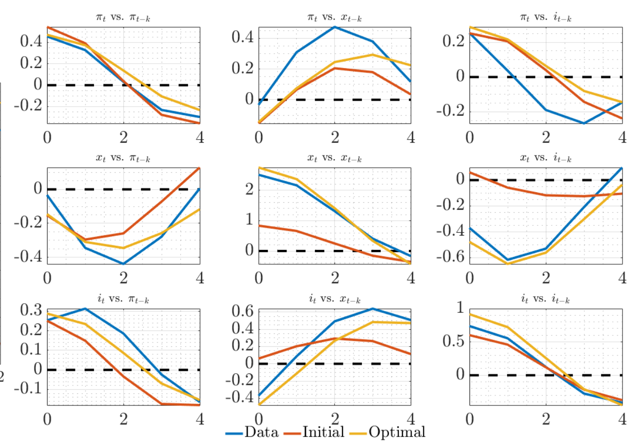
(a) w/o measurement error
nearly = Fig. 1 of Materials 37, loss $\approx$ 783



(b) w/o measurement error



(c) with measurement error, loss ≈ 0

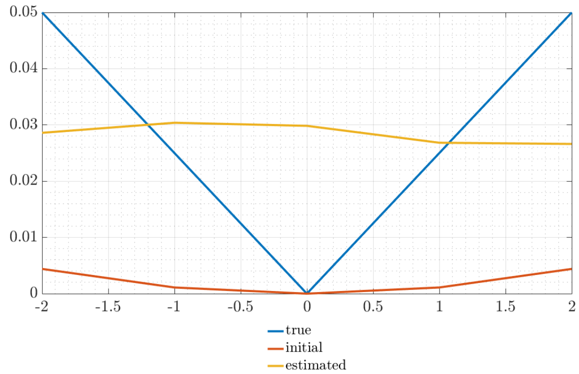


(d) with measurement error (in true data and in estimation)

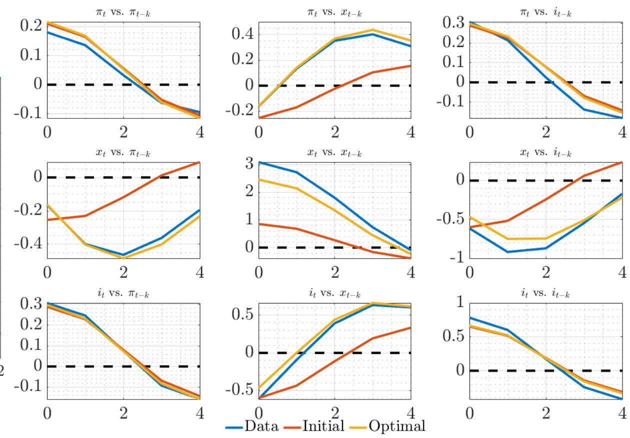
The measurement error definitely shows up.

- It should change the true moments, b/c those are just one simulation.
- For N estimations, it should change the initial moments, and it does.

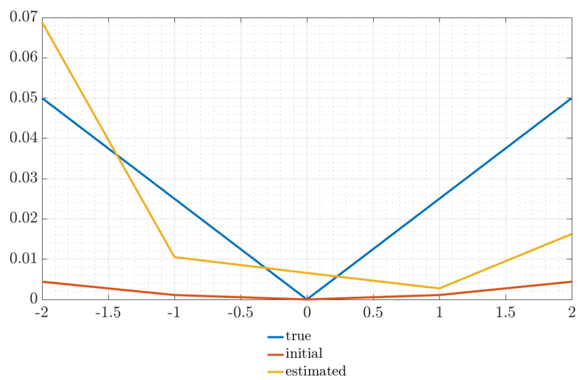
Figure 2: $N = 100$, w/o expectations data, estimate mean moments once (N simulations), imposing convexity with weight 100K, truth with $nfe = 5, fe \in (-2, 2)$



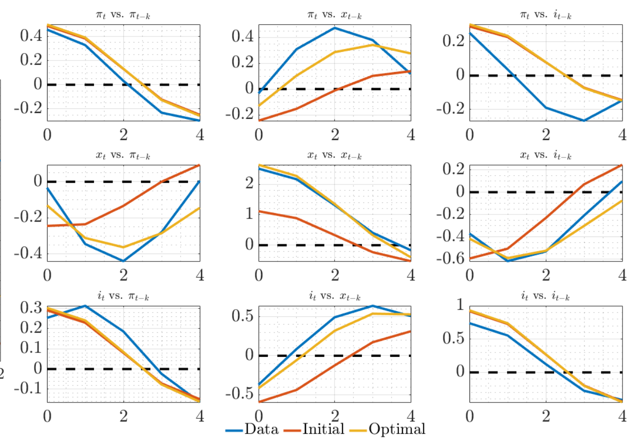
(a) w/o measurement error, loss ≈ 944



(b) w/o measurement error



(c) with measurement error, loss ≈ 0

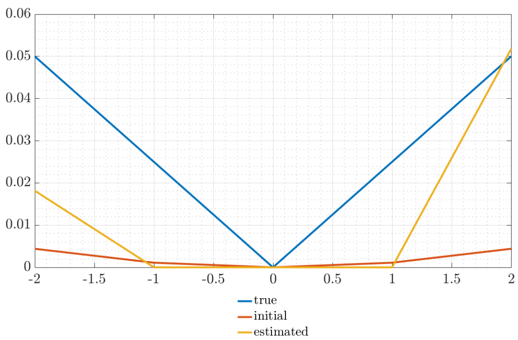


(d) with measurement error (in true data and in estimation)

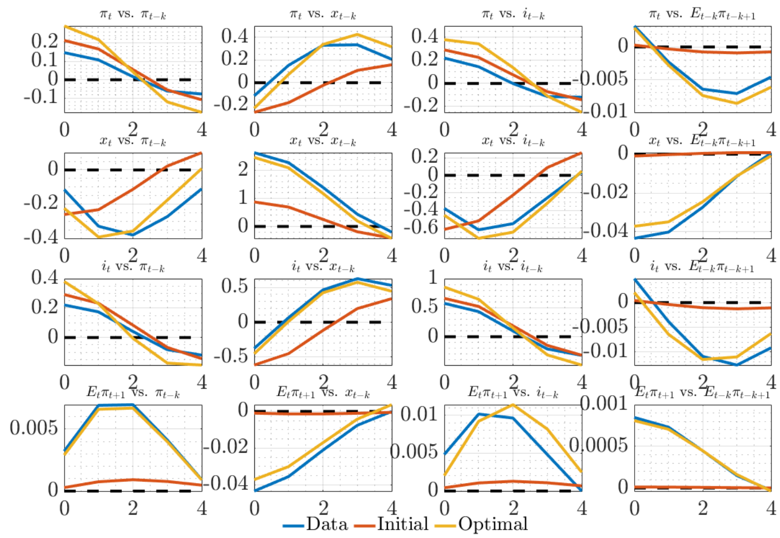
- For N simulations, it shouldn't change the initial moments, and it doesn't either.
- But it shouldn't affect the procedure's ability to match the moments. It does though.
- A phenomenon I also can't explain is why losses become orders of magnitude smaller just by using the data that was generated with measurement error in it.

1.1.1 Put expectations back in: w/ measurement error, or w/o

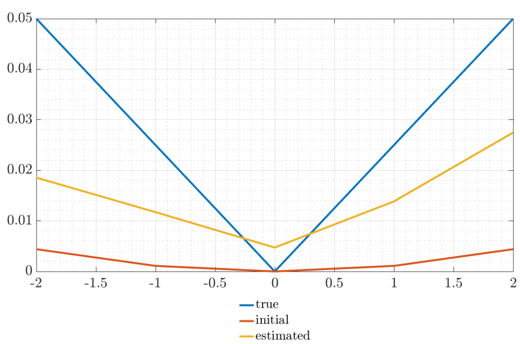
Figure 3: $N = 100$, w/ expectations data, estimate mean moments once (N simulations), imposing convexity with weight 100K, truth with $nfe = 5, fe \in (-2, 2)$



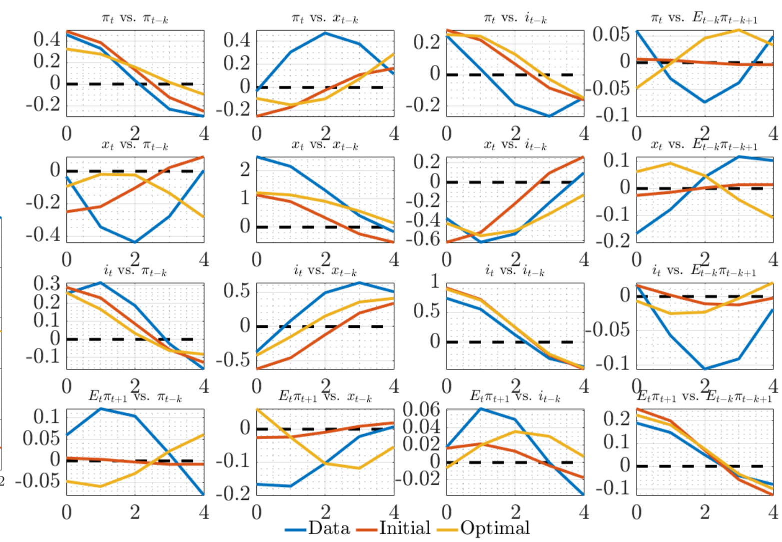
(a) w/o measurement error, loss ≈ 5243



(b) w/o measurement error



(c) with measurement error, loss ≈ 0 This figure is Fig 3 from Materials 38



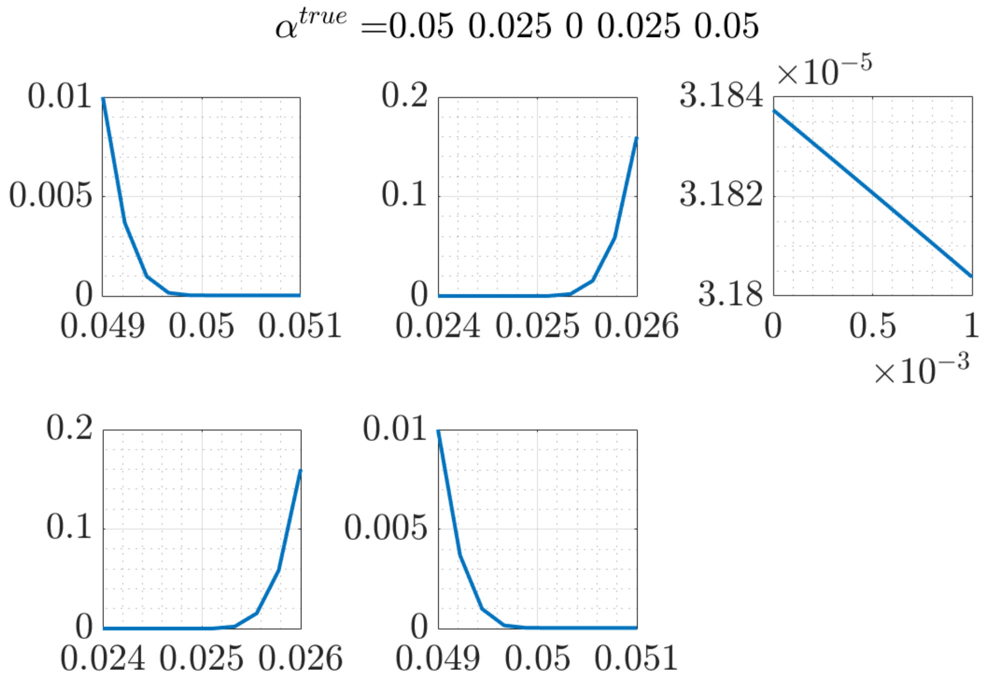
(d) with measurement error (in true data and in estimation)

Clearly the measurement error is responsible for the misses!

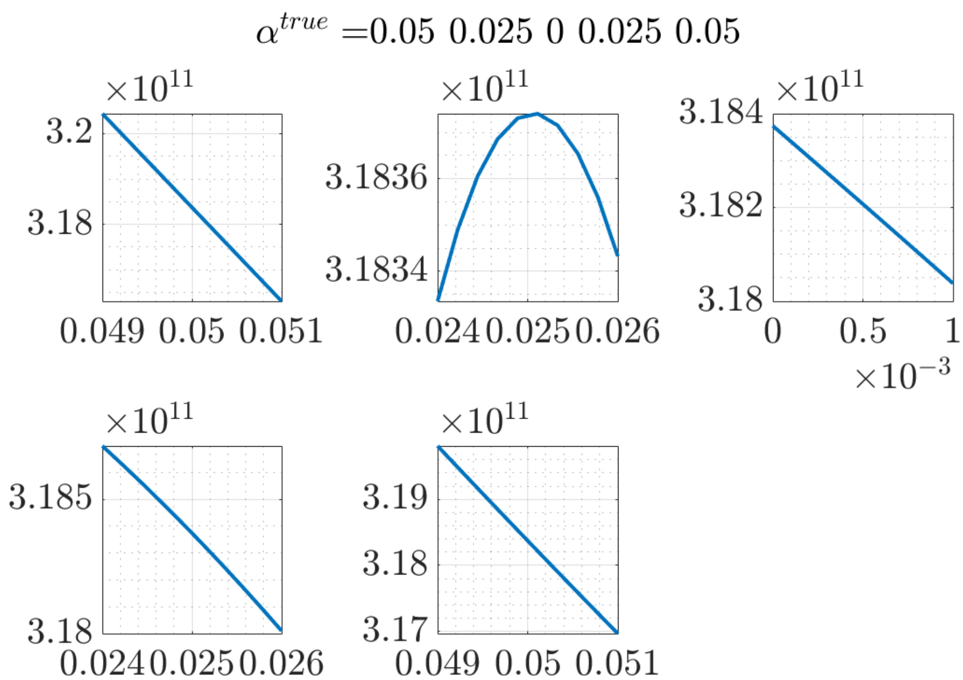
1.2 Loss for holding αs at true values, and varying one at a time

1.2.1 Reference: materials 38

Figure 4: Loss for $N = 100$, incl. 1-step ahead forecasts of inflation, estimate mean moments once (N simulations), imposing convexity with weight 100K, with measurement error imposed, truth with $nfe = 5, fe \in (-2, 2)$



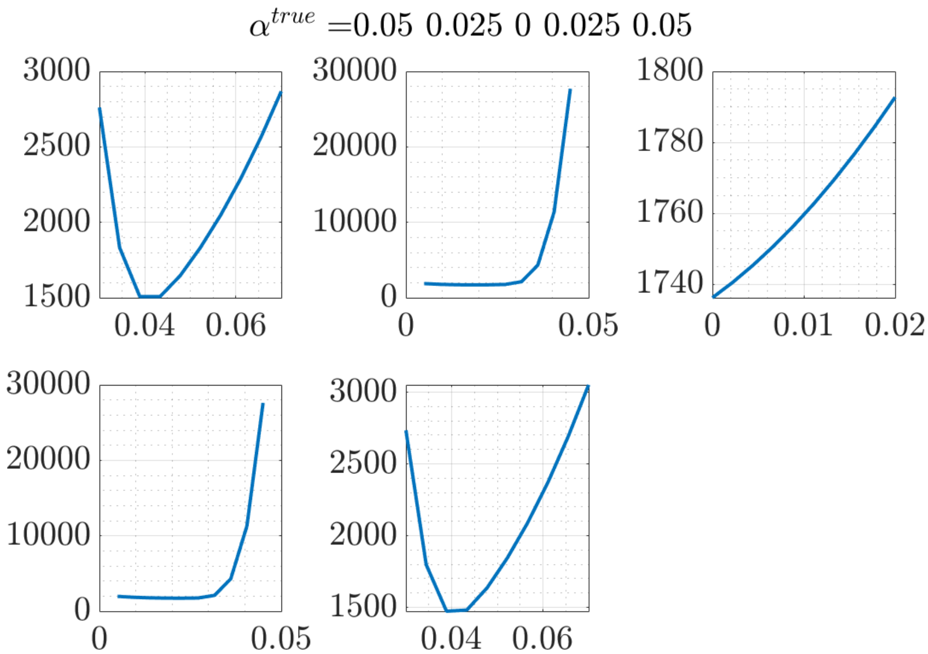
(a) Rescale W



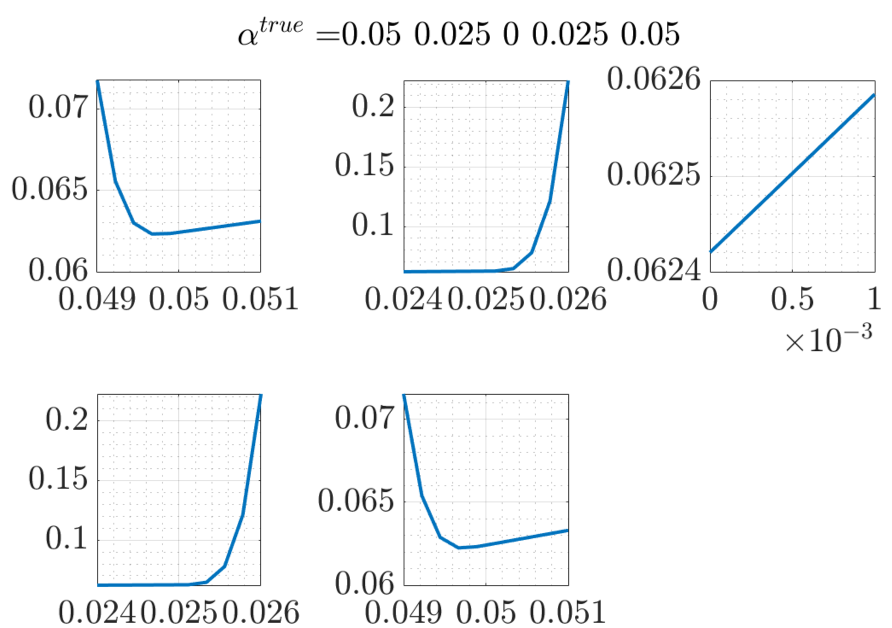
(b) Don't rescale W (Rescaling but dropping the convexity moment looks like this too, except the order of magnitude is 10^{-5})

1.2.2 Loss when not using expectations data, all else same as Materials 38

Figure 5: Loss for $N = 100$, NOT using 1-step ahead forecasts of inflation, estimate mean moments once, imposing convexity with weight 100K, , truth with $nfe = 5, fe \in (-2, 2)$



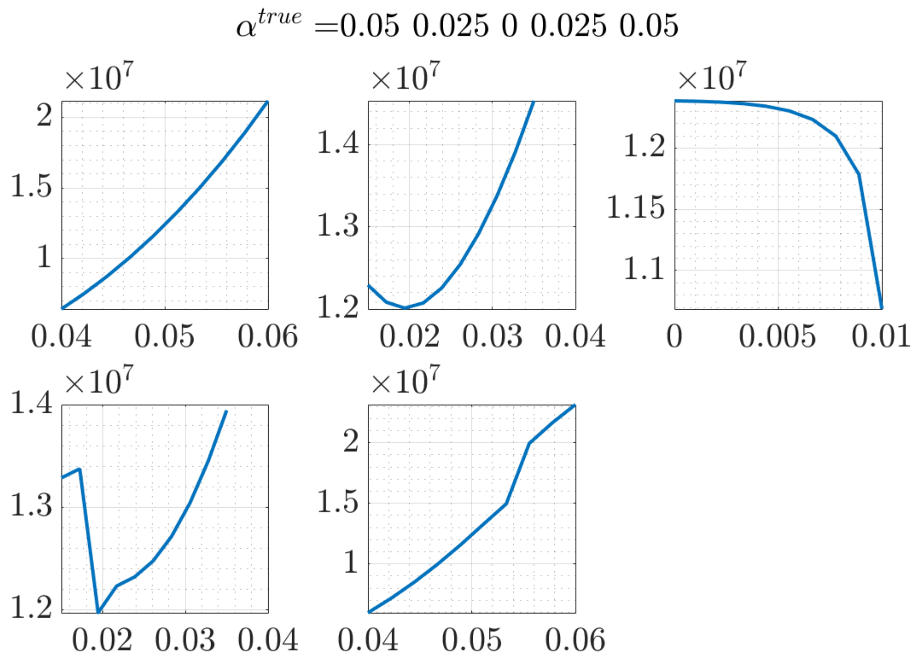
(a) w/o measurement error



(b) with measurement error imposed in true data and in estimation

Note: If I don't use expectations variables, the tiny W issue disappears. So those are the moments that cause the rescaling issue in the first place. Therefore I'm not rescaling here.

Figure 6: Loss for $N = 100$, using 1-step ahead forecasts of inflation, estimate mean moments once, imposing convexity with weight 100K, , truth with $nfe = 5, fe \in (-2, 2)$



(a) w/o measurement error

- the 0-forecast-error α was reversed
- also reversed: now middle αs identified, although a little too low, but the sides don't seem so

A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{A.1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{A.2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \qquad \text{(if imposed)} \tag{A.3}$$

$$\text{PLM:} \quad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\, h_x \tag{A.4}$$

$$\text{Updating:} \quad a_t = a_{t-1} + k_t^{-1}\big(z_t - (a_{t-1} + b s_{t-1})\big) \tag{A.5}$$

$$\text{Anchoring function:} \quad k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k f e_{t-1}^2 \tag{A.6}$$

$$\text{Forecast error:} \quad f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \tag{A.7}$$

$$\text{LH expectations:} \quad f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \qquad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \tag{A.8}$$

This notation captures vector learning (z learned) for intercept only. For scalar learning, $a_t = \begin{pmatrix} \bar{\pi}_t & 0 & 0 \end{pmatrix}'$ and b_1 designates the first row of b. The observables (π, x) are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix} \sigma & 1-\beta & -\sigma\beta \end{bmatrix} f_b + \sigma \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \tag{A.9}$$

$$\pi_t = \kappa x_t + \begin{bmatrix} (1-\alpha)\beta & \kappa\alpha\beta & 0 \end{bmatrix} f_a + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \tag{A.10}$$

B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only, $k_t^{-1} = \mathbf{g}(f e_{t-1})$):

$$\pi_t = -\frac{\lambda_x}{\kappa}\Bigg\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_\pi(t)\Big) \\ \Big(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi}}(t+j))\Big)\Bigg\} \tag{B.1}$$

where I'm using the notation that $\prod_{j=0}^{0} \equiv 1$. For interpretation purposes, let me rewrite this as follows:

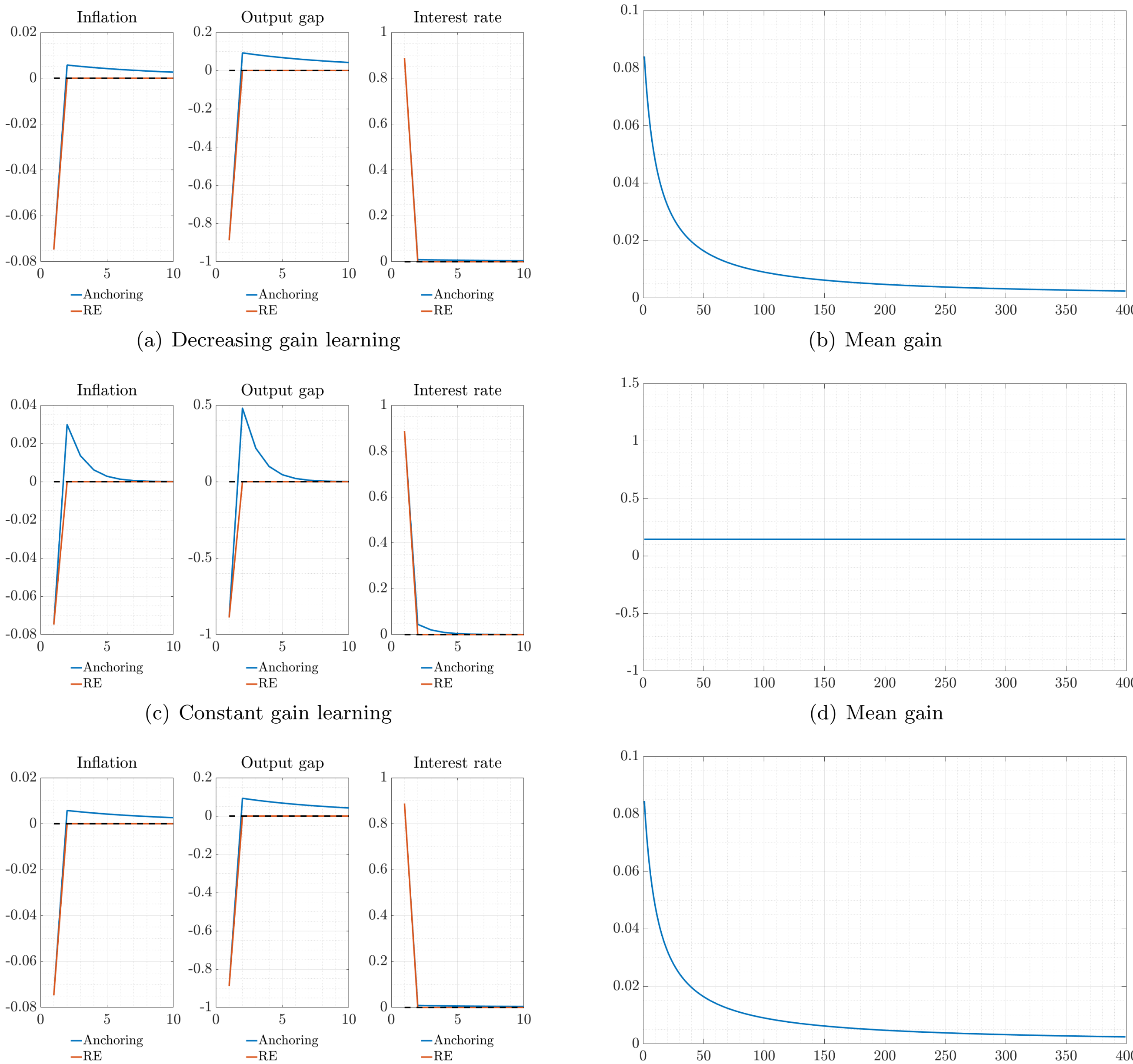
$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + f e_{t|t-1}^{eve}\mathbf{g}_\pi(t)\Big)\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ -\frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + f e_{t|t-1}^{eve}\mathbf{g}_\pi(t)\Big)\Big(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(k_{t+1+j}^{-1} + f e_{t+1+j|t+j}^{eve})\mathbf{g}_{\bar{\pi}}(t+j)\Big) \tag{B.2}$$

Interpretation: tradeoffs from discretion in RE + effect of current level and change of the gain on future tradeoffs + effect of future expected levels and changes of the gain on future tradeoffs

C Impulse responses to iid monpol shocks across a wide range of learning models

$T = 400, N = 100, n_{drop} = 5$, shock imposed at $t = 25$, calibration as above, Taylor rule assumed to be known, PLM = learn constant only, of inflation only.

Figure 7: IRFs and gain history (sample means)



(a) Decreasing gain learning

(b) Mean gain

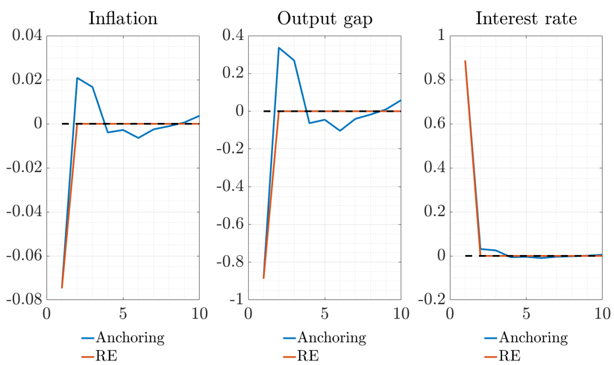
(c) Constant gain learning

(d) Mean gain

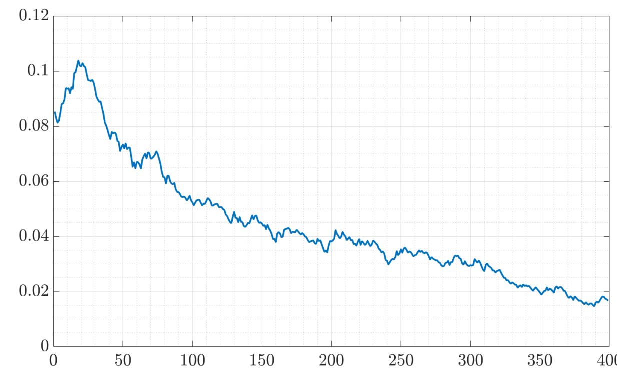
(e) CEMP criterion (vector)

(f) Mean gain

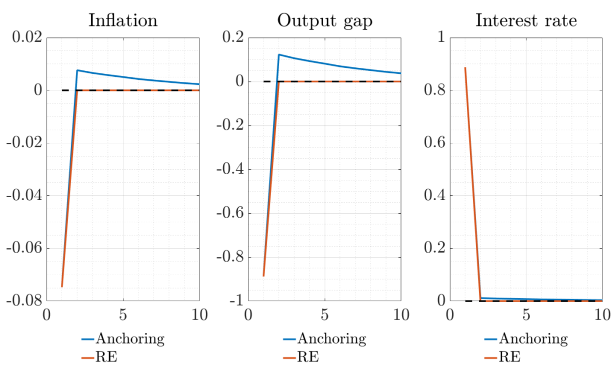
Figure 8: IRFs and gain history (sample means), continued



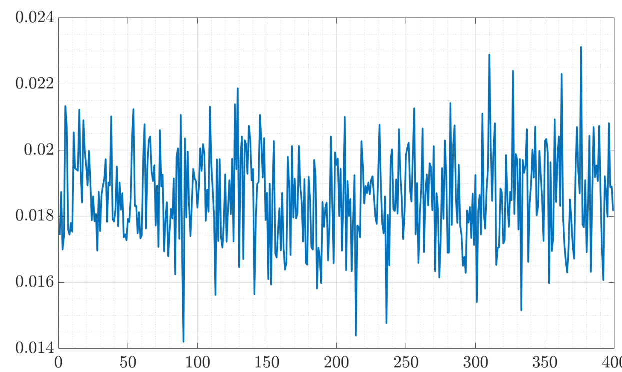
(a) CUSUM criterion (vector)



(b) Mean gain



(c) Smooth criterion, approximated, using $\alpha^{true} = (0.05; 0.025; 0; 0.025; 0.05)$, on $fe \in (-2, 2)$.



(d) Mean gain