Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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Policymakers came out of the Great Inflation era with a clear understanding that it was essential to anchor inflation expectations at some low level.
Jerome Powell, Chairman of the Federal Reserve [1]

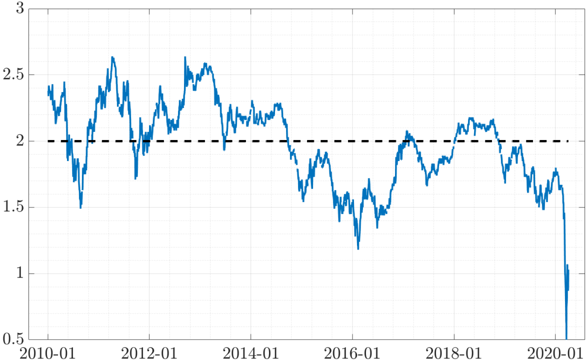

Figure: Market-based inflation expectations, 10 year, average, %

TIPS

[1] Federal Reserve "Challenges for Monetary Policy," August 23, 2019.

This project

- How to conduct optimal monetary policy in interaction with the anchoring expectation formation?
- Model of anchoring expectation formation as an endogenous gain adaptive learning scheme
- Estimation of the anchoring function: when do expectations become unanchored?

Preview of results

- 1 pp forecast error unanchors expectations

- Optimal monetary policy responsiveness time-varying

↪ Unanchored expectations introduce intertemporal stabilization and volatility tradeoffs, latter matters more

↪ Optimal policy aggressive when expectations unanchor, dovish when anchored

Related literature

- **Optimal monetary policy in New Keynesian models**

  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Econometric learning**

  Evans & Honkapohja (2001, 2006), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Eusepi & Preston (2011), Milani (2007, 2014), Lubik & Matthes (2018), Mele et al (2019)

- **Anchoring and the Phillips curve**

  Sargent (1999), Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Reis (2020), Gobbi et al (2019), Carvalho et al (2019)

Structure of talk

1. Unanchoring in the data

2. Model of anchoring expectations

3. Solving the Ramsey problem

4. Implementing optimal policy

Unanchoring in the data

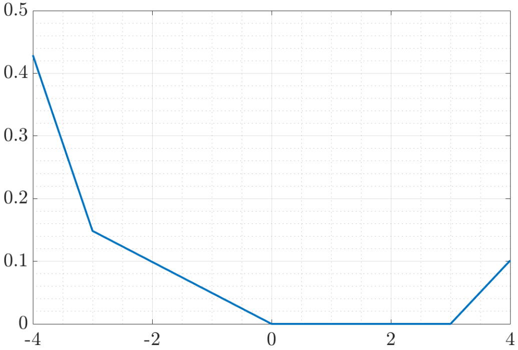

Figure: Learning gain as a function of forecast errors in inflation (pp)

Structure of talk

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Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j))dj \right] \tag{1}$$

Budget constraint

$$B_t^i \leq (1+i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_t C_t^i \tag{2}$$

▸ Consumption, price level

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \tag{3}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{4}$$

▸ Profits, stochastic discount factor

Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- If use $\mathbb{E}$ (rational expectations, RE)

  Model solution

$$s_t = hs_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \tag{5}$$
$$y_t = gs_t \tag{6}$$

  $s_t \equiv$ states
  $y_t \equiv$ jumps
  $\epsilon_t \equiv$ disturbances

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  $s_t \equiv$ states
  $y_t \equiv$ jumps
  $\epsilon_t \equiv$ disturbances

- If use $\hat{\mathbb{E}} \rightarrow$ private sector does not know (6)

  $\hookrightarrow$ estimate using observed states & knowledge of (5)

Adaptive learning

- Postulate linear functional relationship instead of (6):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{7}$$

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- Note: misspecified $\rightarrow$ not model-consistent (not RE)
- Estimate a, b using recursive least squares (RLS)

Recursive least squares

Jumps are: $(\pi, x, i)'$

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Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\, h \quad \forall t \tag{8}$$

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$\rightarrow$ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\right)}_{\equiv fe_{t|t-1}, \text{ forecast error}} \tag{9}$$

$k_t \in (0,1)$ gain
b_1 first row of b

▸ General RLS algorithm

Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}) \big) \tag{10}$$

$k_t = \mathbf{g}(fe_{t|t-1})$: anchoring function

Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big) \tag{10}$$

$k_t = \mathbf{g}(fe_{t|t-1})$: anchoring function

$$\mathbf{g}(fe_{t|t-1}) = \alpha b(fe_{t|t-1}) \tag{11}$$

$b(fe_{t|t-1})$ = basis, here: second order spline (piecewise linear)

α = approximating coefficients, here: use $\hat{\alpha}$ from estimation

▸ Functional forms in literature

Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \Big) \tag{12}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \Big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \Big) \tag{13}$$

▸ Derivations ▸ Actual laws of motion

- Expectations evolve according to RLS with the endogenous gain given by (11)

→ How should $\{i_t\}$ be set?

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Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

- $\mathbb{E}$ is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

Target criterion

Result

In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_{\pi,t} \right.$$

$$\left. \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi},\mathbf{t+j}}) \right) \right)$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t, $\prod_{j=0}^{0} \equiv 1$ *and* b_1 *is the first row of* b.

General case

Two layers of intertemporal stabilization tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa}x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t + fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\mathbb{E}_t\sum_{i=1}^{\infty}x_{t+i}$$

$$-\frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t + fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\mathbb{E}_t\sum_{i=1}^{\infty}x_{t+i}\prod_{j=0}^{i-1}(k_{t+1+j} + fe_{t+1+j|t+j}\mathbf{g}_{\bar{\pi},\mathbf{t+j}})$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

▸ Why no commitment?

Corollary

Optimal policy under adaptive learning is time-consistent.

$\hookrightarrow$ Foreshadow: optimal policy aggressiveness time-varying

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Solution procedure

Solve system of model equations + target criterion

$\hookrightarrow$ solve using parameterized expectations (PEA) and value function iteration (VFI)

$\hookrightarrow$ obtain a cubic spline approximation to optimal policy function

Optimal policy I - responding to unanchoring

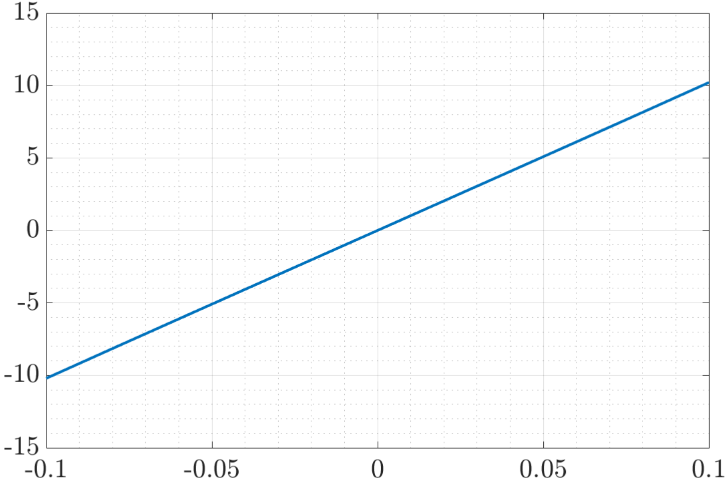
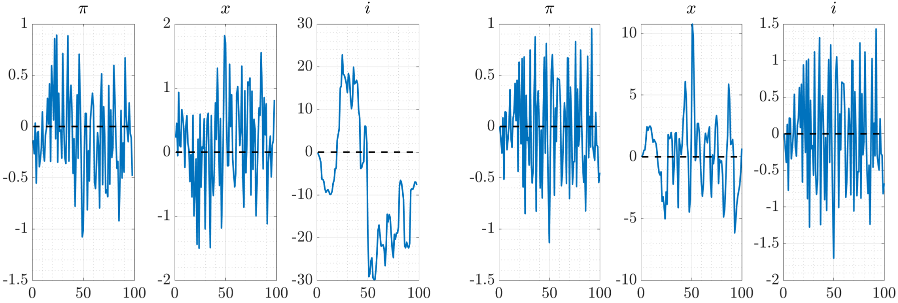


Figure: Comparative statics of the policy function: $\partial i / \partial \bar{\pi}$

Optimal policy II - a particular history

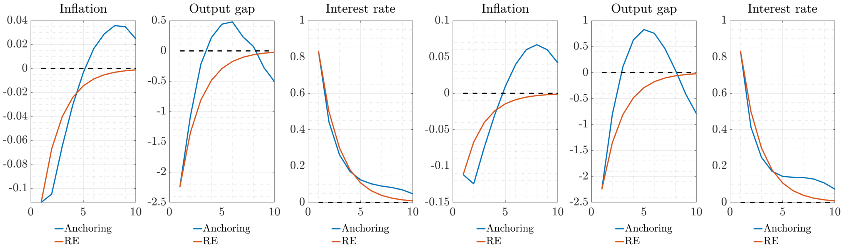


(a) Optimal policy

(b) Taylor rule with 1.5 and 0 coefficients on inflation and the output gap

Figure: Observables conditional on a particular evolution of shocks

The intertemporal volatility tradeoff



(a) Expectations anchored

(b) Expectations unanchored

Figure: Impulse responses after a contractionary monetary policy shock

Optimal policy III - optimal Taylor-rule coefficients

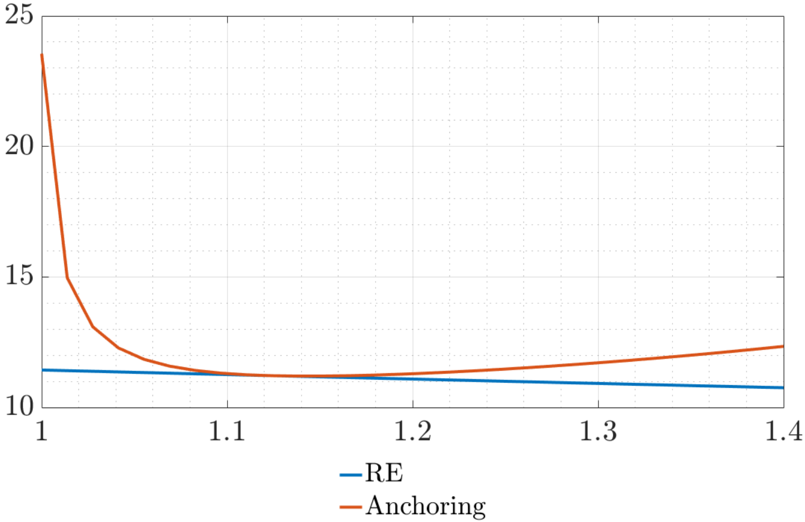
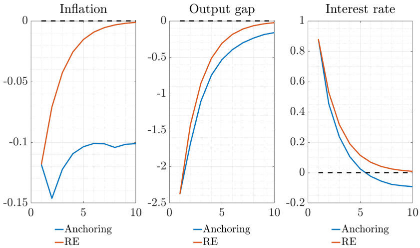
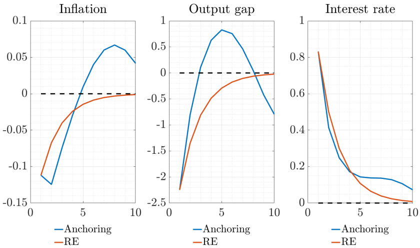


Figure: CB loss as a function of $\psi_{\pi} (\lambda_x = 0)$

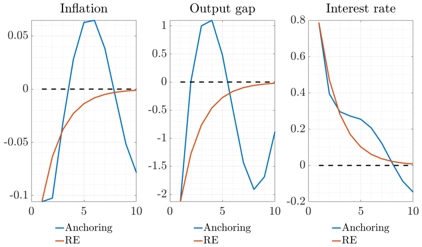
The intertemporal volatility tradeoff - again



(a) $\psi_\pi = 1.01$



(b) $\psi_\pi = 1.5$



(c) $\psi_\pi = 2$

Conclusion

- Interaction between monetary policy and anchoring

- Optimal policy conditions on stance of current and expected future anchoring

  $\hookrightarrow$ determine intertemporal tradeoffs

- Frontloads aggressive interest rate response to suppress potential unanchoring

- For a 1 pp positive (negative) forecast error, raises (lowers) interest rate by 11.61 pp

Appendix

Correcting the TIPS from liquidity risk

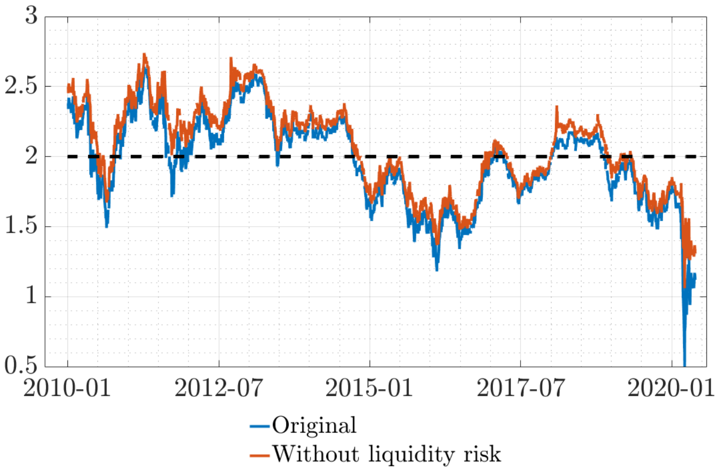


Figure: Market-based inflation expectations, 10 year, average, %

Return

Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{14}$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{15}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1-k^{-1}}{1-k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1-k^{-1}\beta} u_t \tag{16}$$

Solve for forecast error $fe_t \equiv \pi_t - f_{t-1}$:

$$fe_t = \underbrace{-\frac{1-\beta}{1-k\beta}}_{\lim_{k\to 1}=-1} f_{t-1} + \frac{1}{1-k\beta} u_t \tag{17}$$

Functional forms for **g** in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \tag{18}$$

$p \equiv Prob(\text{liquidity trap regime})$
y_{t-1} output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{19}$$

θ_t criterion, $\bar{\theta}$ threshold value

Return

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \tag{20}$$

Σ variance-covariance matrix of shocks
$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(fe_{t|t-1}fe'_{t|t-1} - \omega_{t-1}) \tag{21}$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(fe'_{t|t-1}\omega_t^{-1}fe_{t|t-1} - \theta_{t-1}) \tag{22}$$

ω_t estimated forecast-error variance

Return

Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right)' \right)' \tag{23}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{24}$$

◂ Return

Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{25}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{26}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \tag{27}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{28}$$

Return

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t)$$

$$+ \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \Big\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t + \mathbf{g}_\pi(\pi_t - f_{t-1})) = 0 \tag{29}$$

$$-2\beta\frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g_f}(\pi_{t+1} - f_t)) = 0 \tag{30}$$

Return

Target criterion system for anchoring function as changes of the gain

$$
\begin{aligned}
\varphi_{6,t} &= -cfe_{t|t-1}x_{t+1} + \left(1 + \frac{fe_{t|t-1}}{fe_{t+1|t}}(1-k_{t+1}) - fe_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1} \\
&\quad - \frac{fe_{t|t-1}}{fe_{t+1|t}}(1-k_{t+1})\varphi_{6,t+2} \qquad (31) \\
0 &= 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{fe_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{fe_{t|t-1}}\varphi_{6,t+1} \qquad (32)
\end{aligned}
$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (32) is given by:

$$
\varphi_{6,t} = -2\,\mathbb{E}_t \sum_{i=0}^{\infty}\left(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}\right)\prod_{j=0}^{i-1}\frac{\frac{k_{t+j}}{fe_{t+j|t+j-1}}}{\frac{k_{t+j}}{fe_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \qquad (33)
$$

Return

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{34}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{35}$$

Profits:

$$\Pi_t^j = p_t(j) y_t(j) - w_t(j) f^{-1}(y_t(j)/A_t) \tag{36}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{37}$$

Return

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{38}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{39}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (38) backward to some date t, take expectations at t
2. Sub in (39)
3. Aggregate over households i

$\rightarrow$ Obtain (12)

Return