Materials for Susanto - IRFs for learning

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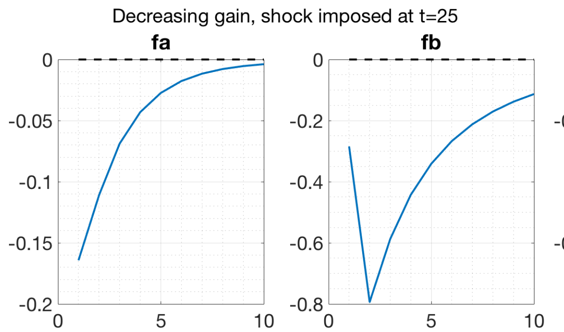
November 19, 2019

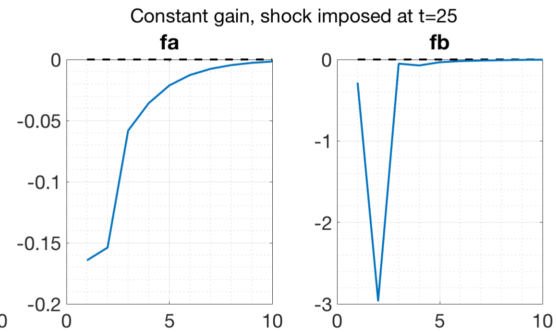
Overview

1 Observables for 3 shocks

Figure 1: Natural rate shock

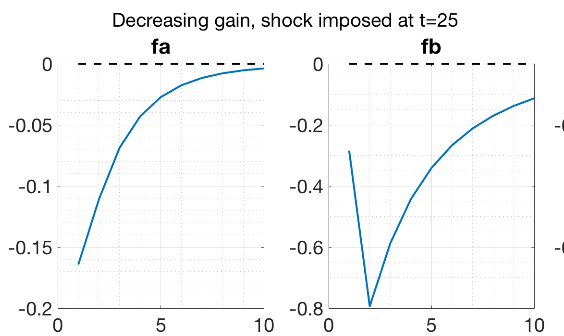


(a) Decreasing gain, $t = 25$

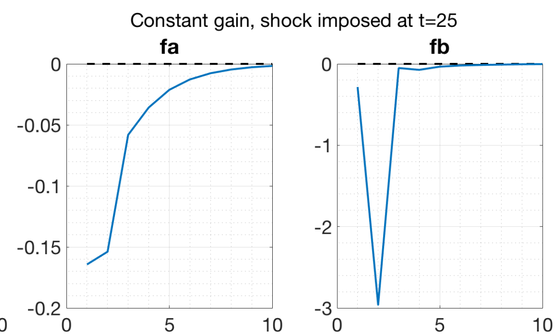


(b) Constant gain, $t = 25$

Figure 2: Monetary policy shock

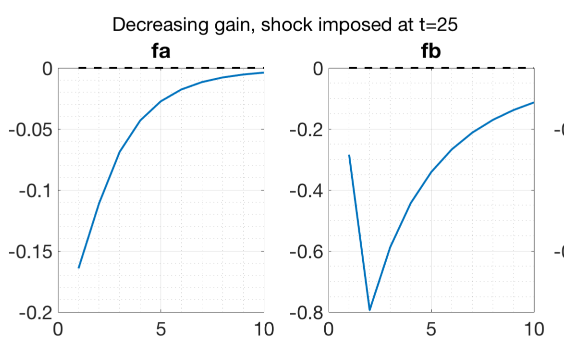


(a) Decreasing gain, $t = 25$

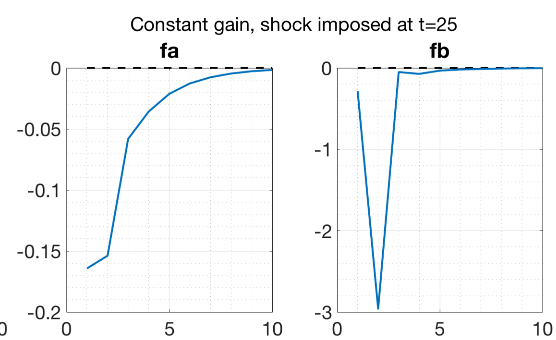


(b) Constant gain, $t = 25$

Figure 3: Cost-push shock



(a) Decreasing gain, $t = 25$



(b) Constant gain, $t = 25$

2 How observables respond to expectations - RE vs. learning

Ignoring shocks and setting $\psi_x = 0$, so the Taylor rule is just $i_t = \psi_\pi \pi_t$, the two systems are (throughout I'm using blue to denote negative values).

$$\begin{aligned}
&RE \\
&x_t = -\sigma\psi_\pi \pi_t + \mathbb{E}_t\, x_{t+1} + \sigma\, \mathbb{E}_t\, \pi_{t+1} \\
&\pi_t = \kappa x_t + \beta\, \mathbb{E}_t\, \pi_{t+1} \\
&Learning \\
&x_t = -\sigma\psi_\pi \pi_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} + \sigma(1-\beta\psi_\pi)\pi_{T+1}\big) \\
&\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1}\big)
\end{aligned}$$

Expressing x, π as functions of expectations alone, this gives:

$$\begin{aligned}
&RE \\
&x_t = \frac{\sigma(1-\beta\psi_\pi)}{1+\sigma\psi_\pi\kappa}\, \mathbb{E}_t\, \pi_{t+1} + \frac{1}{1+\sigma\psi_\pi\kappa}\, \mathbb{E}_t\, x_{t+1} \\
&\pi_t = \overbrace{\left(\frac{\kappa\sigma(1-\beta\psi_\pi)}{1+\sigma\psi_\pi\kappa} + \beta\right)}^{+} \mathbb{E}_t\, \pi_{t+1} + \frac{\kappa}{1+\sigma\psi_\pi\kappa}\, \mathbb{E}_t\, x_{t+1} \\
&Learning \\
&x_t = \frac{-\sigma\psi_\pi}{w} \begin{bmatrix}(1-\alpha)\beta & \kappa\alpha\beta & 0\end{bmatrix} f_a + \frac{1}{w} \begin{bmatrix}\sigma(1-\beta\psi_\pi) & 1-\beta & 0\end{bmatrix} f_b \\
&\pi_t = (1 - \frac{\kappa\sigma\psi_\pi}{w}) \begin{bmatrix}(1-\alpha)\beta & \kappa\alpha\beta & 0\end{bmatrix} f_a + \frac{\kappa}{w} \begin{bmatrix}\sigma(1-\beta\psi_\pi) & 1-\beta & 0\end{bmatrix} f_b
\end{aligned}$$

This yields the stylized representation of how endogenous variables respond to expectations in the two formulations:

RE

$$\begin{aligned}
x_t &= \overset{-}{\mathbb{E}}(\pi) + \overset{+}{\mathbb{E}}(x) \\
\pi_t &= \overset{+}{\mathbb{E}}(\pi) + \overset{+}{\mathbb{E}}(x)
\end{aligned}$$

Learning

$$\begin{aligned}
x_t &= \overset{-}{\mathbb{E}_a}(\pi) + \overset{-}{\mathbb{E}_b}(\pi) + \overbrace{\overset{-}{\mathbb{E}_a}(x) + \overset{+}{\mathbb{E}_b}(x)}^{+ \text{ since } f_a < f_b} \\
\pi_t &= \underbrace{\overset{+}{\mathbb{E}_a}(\pi) + \overset{-}{\mathbb{E}_b}(\pi)}_{+ \text{ since } \kappa \text{ tiny}} + \overset{+}{\mathbb{E}_a}(x) + \overset{+}{\mathbb{E}_b}(x)
\end{aligned}$$

Where f_a and f_b denote long-horizon expectations and are given by

$$f_a(t) \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \begin{bmatrix} \pi_{T+1} \\ x_{T+1} \\ i_{T+1} \end{bmatrix} \qquad f_b(t) \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} \begin{bmatrix} \pi_{T+1} \\ x_{T+1} \\ i_{T+1} \end{bmatrix} \tag{1}$$

$$f_a(t) = \frac{1}{1-\alpha\beta} \begin{bmatrix} \bar{\pi}_t \\ 0 \\ 0 \end{bmatrix} + b(I_4 - \alpha\beta h_x)^{-1} s_t \qquad f_b(t) = \frac{1}{1-\beta} \begin{bmatrix} \bar{\pi}_t \\ 0 \\ 0 \end{bmatrix} + b(I_4 - \beta h_x)^{-1} s_t \tag{2}$$

(And $b = g_x h_x$, where h_x is the state transition matrix and g_x is the observation matrix from the RE model solution.)