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INSTRUMENT INSTABILITY AND SHORT-TERM MONETARY CONTROL

Timothy D. LANE*

University of Iowa, Iowa City, IA 52242, USA

The literature on instrument instability lends support to a policy of smoothing interest rates: it contends that rigid adherence to a monetary rule would bring about explosive interest-rate movements. This contention is examined using a simple model which incorporates rational expectations; the results suggest that instrument instability is associated with interest-rate smoothing rather than with short-term control of the money supply. Furthermore, policy that attempts to stabilize interest rates may itself account for empirical findings which have hitherto been viewed as evidence that instrument instability would occur if the money supply were closely controlled.

1. Introduction

How would interest rates behave if the money supply were controlled rigidly in accordance with a constant-growth-rate rule? In the familiar static analysis [e.g. Poole (1970)], interest rates obviously fluctuate more widely if the money supply is controlled than if the interest rate is controlled. Some simple dynamic analysis suggests a concern of a different order of importance: it has been argued that the lag structure of interest rates in the demand-for-money function is such that, if the money supply were controlled continuously according to a constant-growth-rate rule, interest rates would follow an explosive path. If this were true, rigid adherence to such a rule would clearly be impracticable: it would be necessary, even in principle, for the authorities to smooth interest rates, at least to some extent, in the short run.[1]

The analysis leading to this conclusion is based on the idea of 'instrument instability'. In general, instrument instability is a problem that arises when a

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[1]This proposition is worked out formally in Sims (1974) for the general case in which using the instrument to minimize the variance of the target would lead the instrument to follow an unstable path. Sims' solution is for the authorities to minimize a loss function in which variances of both the target and the instrument are given some weight.

policy instrument is manipulated to control a target variable whose value depends on both current and lagged values of the instrument. In this case, the appropriate setting of the instrument at any moment in time depends on that instrument's own previous values: the instrument thus follows a difference (or differential) equation which may or may not be stable [Holbrook (1972)].

A commonly-used method of controlling the money stock is viewed as consisting of 'picking a point on a money-demand curve' – that is, of using an interest rate as a policy instrument, and setting it in such a way that predicted money demand equals the target level of the money stock [Parkin (1978)]. The instrument instability problem may then arise if both current and lagged interest rates affect the demand for money. Suppose, for instance, that the money-demand equation is

$$m_t^{\mathrm{d}} = \alpha + \gamma y_t^{\mathrm{P}} - \delta r_t + \sum_{k=1}^{\infty} \omega'_k r_{t-k} + p_t + \varepsilon_t, \tag{1}$$

where m_t^{d}, p_t and y_t^{P} are the logarithms of money demand, the price level and permanent income,[2] respectively, where r_t is the interest rate, where α, γ, δ' and ω'_k are coefficients, and where ε_t is a random disturbance term. In order to achieve a target money stock m^*, the authorities then set

$$r_t - \frac{1}{\delta'} \sum_{k=1}^{\infty} \omega'_k r_{t-k} = \frac{1}{\xi'} \left[\alpha + \gamma y_t^{\mathrm{P}} + p_t + \mathrm{E}\varepsilon_t - m^* \right]. \tag{2}$$

Next, it is assumed that the monetary control problem is set in the Keynesian 'very short run', in which the money market is in equilibrium but the goods market is not yet affected by the interest rate: y^{P} and p are thus treated as predetermined, and eq. (2) is examined as a difference equation in r_t. Empirical evidence on the demand for money (generally based on monthly data) has been adduced to show that the lag structure $\omega_1, \omega_2, \ldots$ is such that the roots of eq. (2) do not all lie in the unit circle: on this basis, it has been argued that controlling the money supply too rigidly would set interest rates on an explosive path [see Pierce and Thompson (1972), Ciccolo (1974), White (1976)].

In this paper, this contention will be examined in the light of an explicit explanation of the lag structure[3] of interest rates in the demand-for-money function. In many empirical estimates of the demand for money, lagged

[2]The use of permanent income here would not meet with universal agreement, but does correspond to widespread practice [see Laidler (1977, pp. 139–148)]. The results that follow are not substantially altered by using current instead of permanent income, or by using a double-log instead of a semi-log functional form.

[3]Another consideration is that econometric estimates of the lag structure may give misleading impressions of the possibility of instrument instability; this may occur especially if the empirical evidence consists of a discrete-time approximation of continuous lag distribution and thus (if the lag structure is one-sided) underestimates the weight on the current period [Sims (1974a)].

interest rates play some role.[4] Explanations of this role generally run along either of two lines: it is sometimes held that past interest rates influence expected rates, and that these affect the demand for money [e.g. White (1976)]; sometimes, alternatively, it is claimed that what is being observed is some kind of partial adjustment behavior, which leads cash balances to depend on their own past levels [e.g. Feige (1967)]. In this paper, the explanation based on interest-rate expectations will be considered, and its implications for the instrument instability argument will be traced.

2. Interest-rate expectations and money demand

Why should the expected rate of interest influence the demand for money? The simplest explanation is that the interest rate which is used as a measure of the opportunity cost of holding money is the yield on an asset whose term to maturity is longer than the holding period over which individuals' decisions to hold cash are made. If this is the case, the expected holding period yield on the alternative asset, in which consists the opportunity cost of holding money, differs from the yield to maturity by some element of capital gain or loss. For example, if cash balances are adjusted continuously, the opportunity cost of holding money is the instantaneous yield on the alternative asset; if this asset is a bill whose term to maturity is T months, this opportunity cost is

$$R_t^{\mathrm{h}} = r_{T,t} - T\frac{\mathrm{d}r_{T,t}}{\mathrm{d}t}, \tag{3}$$

where $r_{T,t}$ is the T-month rate of interest. Using the mean-value theorem, this can be written as

$$\begin{aligned} R_t^{\mathrm{h}} &= r_t - T(r_{t+1} - r_t) + T\left(\frac{\mathrm{d}r(t+\psi)}{\mathrm{d}t} - \frac{\mathrm{d}r(t)}{\mathrm{d}t}\right) \\ &\simeq r_{T,t} - T(r_{t+1} - r_t), \end{aligned} \tag{4}$$

for some ψ, where $0 < \psi < 1$. In short, if the yield on the asset alternative to money is expected to change, the opportunity cost of holding money depends not only on the current yield, but also on the yield expected in the future.[5]

Accordingly, one formulation of the demand-for-money function might be

$$m_t^{\mathrm{d}} = \alpha + \gamma y_t^{\mathrm{P}} - \delta r_t + \kappa \hat{r}_{t+1} + p_t + \varepsilon_t, \tag{5}$$

where $\hat{r}_{t+1}$ is the interest rate now expected to prevail next period. When the

[4] This role may take the form of a lagged dependent variable, which in a simple money-demand equation is equivalent to including lagged income and interest rates in the function (subject to the restriction that the coefficients be proportional to those on current income and interest rates). Alternatively, distributed lags on income and interest rates may be estimated separately [Laidler (1977)].

[5] The expression derived is an approximation rather than an exact relationship in discrete time because (3) compounds a movement along a yield curve with a movement through time, while the expected change in the interest rate consists only of the latter.

expected interest rate is given as a distributed lag on current and past interest rates,

$$\hat{r}_{t+1} = \sum_{k=0}^{\infty} \omega_k r_{t-k}, \tag{6}$$

this would give a formulation of the demand for money equivalent to eq. (1), where $\delta' = \delta - \kappa\omega_0$ and $\omega'_k = \kappa\omega_k$, $k = 1, \ldots$. The approximation given in eq. (4) suggests that

$$\kappa = (T + 1/T)\delta, \tag{4'}$$

so that in general $\kappa < \delta$.

One might then ask what determines the lag structure given by eq. (6). Such a lag structure is often explained in terms of some ad hoc regressive-*cum*-extrapolative expectations mechanism [e.g. White (1976)]. Such a mechanism can, of course, be criticized from a rational-expectations perspective: under rational expectations, expectations would only be formed as in (6) if (6) also described the actual behavior of the interest rate (up to a white-noise disturbance term). This, of course, puts restrictions on the lag structure, and also implies that the lag structure would depend on the policy regime that is in effect – and in particular on whether the authorities are attempting to smooth interest rates.

The implications of rationally-formed expectations for the possibility of instrument instability will now be considered in a descriptive rational-expectations framework.[6] In a model in which expected interest rates affect money demand, it is a straightforward step to make the formation of expectations endogenous and rational.

3. A 'very-short-run' analysis

The implications of rational expectations can first be considered in the context in which instrument instability is usually discussed: the very short run in which both prices and output are treated as predetermined. Of course, this case is broadly inconsistent with rational expectations; however, it is useful to examine it initially as a 'strong case', in order to show that any rejection of the instrument instability argument does not hinge on the assumption of perfect price flexibility and instantaneous market clearing.[7]

[6] Using descriptive models, rather than attempting to set up a complete model of optimizing behavior under rational expectations, clearly limits the purposes to which the results can be used. Since the purpose of this analysis is essentially critical – to assess the validity of the instrument-instability argument when the lag structure is endogenous rather than, for example, to compare alternative policies – the use of descriptive models seems to be justified.

[7] Furthermore, it might be argued that this case is not a bad caricature of much neo-Keynesian analysis, according to which financial markets use information efficiently but behavior in other markets is myopic and/or constrained by contracts and other institutions.

If the authorities controlled the money supply rigidly at m^*, and if the money-demand function were given by (5), the path of the interest rate would be given by

$$r_t - \frac{\kappa}{\delta}\hat{r}_{t+1} = \frac{1}{\delta}\left(\alpha + \gamma y_t^{\mathrm{P}} + p_t + \varepsilon_t - m_t^*\right). \tag{7}$$

Here, as in many rational-expectations models, the potential of multiple solutions arises. In this paper, this issue is resolved following Blanchard and Kahn (1980): the stable solution is chosen if one exists that satisfies any constraints imposed by initial conditions. In the present context, this criterion corresponds to the one proposed by McCallum (1983), namely that the solution path chosen is the one that is conditional on a minimal set of state variables.

Using this criterion, then, and taking account of the fact that $\kappa/\delta < 1$, one can write the solution for the interest rate as

$$r_t = \sum_{i=0}^{\infty}\left(\frac{\kappa}{\delta}\right)^i\left(\alpha + \gamma\hat{y}_{t+i}^{\mathrm{P}} + \hat{p}_{t+i} - \hat{m}_{t+i}^*\right) + \frac{1}{\gamma}\varepsilon_t. \tag{8}$$

Thus, in this case there seems to be no reason to suppose that the interest rate would follow an unstable path if the money stock were controlled: eq. (8) implies that if income, prices and the target money stock were constant, any fluctuations of the interest rate would be white noise.

What would happen, in this case, if the authorities attempted to smooth interest rates? One case that could be considered – and which corresponds to what the authorities frequently claim they are doing [e.g. White (1978)] – is that in which, instead of adhering rigidly to a monetary rule, the authorities manipulate the interest rate according to an interest-rate rule with feedback from deviations of the money stock from its target level. A simple representation of such a rule would be[8]

$$r_t = r_{t-1} + \theta\left(m_{t-1} - m_t^*\right). \tag{9}$$

[8] In the present context, this rule is used only as an example of an intuitively plausible feedback rule [cf. McCallum (1981)]. However, it could also be justified as a rule that appears to be optimal (given an appropriate choice of θ) in the light of the authorities' misperception of the lag structure of interest rates in the money-demand function as part of an invariant structure of the economy. Substituting (29) below into (5) yields an equation equivalent to (1), in which money demand depends on current and one-period-lagged interest rates. If the authorities used this reduced-form money-demand equation to minimize a weighted sum of the variances of the money supply and the interest rate [as prescribed in Sims (1974) and solved numerically in White (1976)], they would be solving a linear-quadratic optimization problem which turns out in most cases to have a saddlepoint solution. That solution implies that the interest rate is manipulated according to a first-order process as in (9). Unfortunately, the expression for θ so derived is too cumbersome to be analyzed.

If this rule were followed, and if the money-demand function were as described by (5), then the interest rate would follow,

$$r_t - \frac{(1-\theta\delta)}{(1-\theta\kappa)} r_{t-1} = -\frac{\theta}{(1-\theta\kappa)}\left(m_t^* - \alpha - \gamma y_{t-1}^{\mathrm{P}} - p_{t-1}\right). \tag{10}$$

In solving for the interest rate in this case, there is one important difference: the interest rate is being set as an instrument of policy, using a rule that takes account of the previous period's interest rate. This policy rule constrains the initial interest rate, implying that it is no longer free to take an initial value required for stability [see Blanchard and Kahn (1980)]. Such being the case, the non-uniqueness found in the solution to (7) does not arise.

Four possible results emerge. If the authorities adjust the interest rate very quickly (if, that is, $\theta > 1/\kappa$), the interest rate follows an explosive path. If the interest rate is adjusted slowly enough ($\theta < 1/\delta$), it converges to the level given by (8). In the intermediate case ($1/\kappa > \lambda > 1/\delta$) there are oscillations which may be either convergent or explosive (see appendix).[9]

In this simplified case in which permanent income, the price level and the target money stock are constant, the money stock follows a path given by

$$m_t = m^* + \frac{1}{\theta}\left(\alpha + \gamma y^{\mathrm{P}} + p - r_0\lambda(1-\lambda) - m^*\right)\lambda^t + \sum_{i=0}^{t-1} \lambda^i \varepsilon_{t-i}. \tag{11}$$

Thus, although a feedback rule such as (9) is an intuitively plausible interpretation of what the policy-makers often claim that they are doing – that is, resisting interest-rate fluctuations in the short run while gradually adjusting the interest rate to offset departures of the money stock from its target path, with a view to adhering to monetary targets in the long run – such a policy may, in this model, give rise to explosive movements of both the interest rate and the money stock. This occurs even in a model chosen as the most favorable to the case for interest-rate smoothing, a model in which departures of the money stock from its target do not cause income and the price level to fluctuate.

What is the reason for this result? If the authorities adjust the interest rate gradually in response to money-stock deviations, starting from a historically-given initial rate, this means that future movements of interest rates can be anticipated; expectations of interest-rate movements in turn influence the demand for money, and the resulting process is unstable. Also, introducing a feedback rule designed to reduce the period-to-period fluctuations of the interest rate constrains the behavior of the interest rate, preventing it from attaining the path associated with stability.

[9]Of course, none of these dynamic paths would arise if the authorities began by setting the interest rate at the level given by (8); the case considered above is that in which the authorities are attempting to resist sudden interest-rate movements in all periods, including the first one.

4. Variable prices and output

The next step is to drop the assumption that the price level and output are given. This can be done in a simple ad hoc rational expectations model like the following:

$$m_t^{\mathrm{d}} = \alpha + \gamma y_t^{\mathrm{P}} - \delta r_t + \kappa \hat{r}_{t+1} + p_t + \varepsilon_t,$$

$$y_t^{\mathrm{d}} = \bar{y}_t + \xi\left(p_{t+1}^t - p_t + \rho - r_t\right) + \nu_t, \tag{12}$$

$$y_t^{\mathrm{s}} = \bar{y}_t + \mu\left(p_t - p_t^{t-1}\right) + \eta_t, \tag{13}$$

$$y_t^{\mathrm{d}} = y_t^{\mathrm{s}}. \tag{14}$$

Here the first equation is the money-demand function (5) of section 2, (12) is an aggregate demand function expressing aggregate demand in terms of the deviation of the real interest rate from its natural rate, (13) is a Lucas 'surprise' aggregate supply function [Lucas (1973)], and (14) is the condition that aggregate supply and demand be equal. The trend level of output is $\bar{y} = y^{\mathrm{P}}$. For simplicity, it is assumed that trend output is constant, that monetary targets specify a constant level of the money stock m^*, and that the disturbance terms ε_t, ν_t and η_t are mutually and serially uncorrelated.

If the authorities do not attempt to smooth interest rates, but simply attempt to set $m_t = m^*$ for all t, the result is a first-order system, which can be written

$$\kappa \hat{p}_{t+2}^t - (\kappa + \delta)\hat{p}_{t+1}^t + (1+\delta)p_t + \left(\frac{\delta\mu}{\xi} + \delta\mu\right)\left(p_t - \hat{p}_t^{t-1}\right)$$

$$= \left(m^* - \alpha - \gamma y_t + (\delta - \kappa)\rho\right) - \frac{\delta}{\xi}(\eta_t + \nu_t) - \varepsilon_t. \tag{15}$$

The solution to this system [again following Blanchard and Kahn (1980) and McCallum (1983)] is

$$p_t = m^* - \alpha - \gamma y + (\delta - \kappa)\rho$$

$$-\left(1 + \delta + \frac{\delta\mu}{\xi} + \gamma\mu\right)^{-1}\left[\frac{\delta}{\xi}(\eta_t + \nu_t) - \varepsilon_t\right]. \tag{16}$$

This implies that in the absence of interest-rate smoothing, the price level fluctuates randomly around its steady-state level; the interest rate will accordingly fluctuate randomly around the natural rate ρ,

$$r_t = \rho + \left(1 + \frac{1}{\xi}\right)\left(1 + \delta + \frac{\delta\mu}{\xi} + \gamma\right)^{-1}\left[\frac{\delta}{\xi}(\eta_t + \nu_t) - \varepsilon_t\right] + \frac{1}{\xi}(\eta_t + \nu_t). \tag{17}$$

Thus, in this simple rational expectations model, there is no need for the interest rate to follow an unstable path if the money stock is controlled according to a constant-growth-rate rule. Under this regime, output is given by

$$y_t = \bar{y} - \mu\left(1 + \delta + \frac{\delta\mu}{\xi} + \gamma\mu\right)^{-1}\left[\frac{\delta}{\xi}(\eta_t + \nu_t) - \varepsilon_t\right] + \eta_t, \tag{18}$$

that is, it will deviate from the natural level as a result of shocks to aggregate supply and demand and to money demand.

What would happen in this model if the authorities attempted to smooth interest rates? In this case, the system would consist of eqs. (5), (12), (13), (14) and (9). Solving these simultaneously gives a difference equation for the price level which can be written

$$\begin{aligned}
&(1-\theta\kappa)\hat{p}_{t+1}^{t-1} - [2-\theta(\kappa+\delta)]\,\hat{p}_t^{t-1} + (1-\theta\delta-\theta)\,p_{t-1} \\
&\quad + (\hat{p}_{t+1}^{t} - \hat{p}_{t+1}^{t-1}) - \left(1+\frac{\mu}{\xi}\right)(p_t - \hat{p}_t^{t-1}) + \frac{(1-\theta\delta)\mu}{\xi}(p_{t-1} - \hat{p}_{t-1}^{t-2}) \\
&= \theta(\alpha + \gamma\bar{y} - (\delta-\kappa)\rho - m^*) + \theta\varepsilon_{t-1} \\
&\quad + \frac{1}{\xi}(\eta_t - \nu_t) - \frac{1-\theta\delta}{\xi}(\eta_{t-1} - \nu_{t-1}).
\end{aligned} \tag{19}$$

Taking expectations as of time $t-2$, we have

$$\begin{aligned}
&(1-\theta\kappa)\hat{p}_{t+1}^{t-2} - [2-\theta(\kappa+\delta)]\,\hat{p}_t^{t-2} + (1-\theta\delta-\theta)\,\hat{p}_{t-1}^{t-2} \\
&= \theta(\alpha + \gamma\bar{y} - (\delta-\kappa)\rho - m^*),
\end{aligned} \tag{20}$$

a second-order equation for the time path of the expected price level. Its solution can be written as

$$\hat{p}_{t+k}^{t} = m^* - \alpha - \gamma\bar{y}_t + (\delta-\kappa)\rho + A_1\lambda_1^{t+k} + A_2\lambda_2^{t+k}, \tag{21}$$

where λ_1 and λ_2 are characteristic roots, and A_1 and A_2 are constants. Subtracting eq. (20) from eq. (19) gives

$$\begin{aligned}
&(1-\theta\kappa)(\hat{p}_{t+1}^{t-1} - \hat{p}_{t+1}^{t-2}) - [2-\theta(\kappa+\delta)](\hat{p}_t^{t-1} - \hat{p}_t^{t-2}) \\
&\quad + \left[1 - \theta\delta - \theta + \frac{(1-\theta\delta)\mu}{\xi}\right](p_{t-1} - \hat{p}_{t-1}^{t-2}) \\
&\quad + (\hat{p}_{t+1}^{t} - \hat{p}_{t+1}^{t-1}) - \left(1+\frac{\mu}{\xi}\right)(p_t - \hat{p}_t^{t-1}) \\
&= \theta\varepsilon_{t-1} + \frac{1}{\xi}(\eta_t - \nu_t) - \frac{(1-\theta\delta)}{\xi}(\eta_{t-1} - \nu_{t-1}).
\end{aligned} \tag{22}$$

Taking expectations as of time $t-1$ in eq. (22) yields

$$(1-\theta\kappa)\left(\hat{p}_{t+1}^{t-1}-\hat{p}_{t+1}^{t-2}\right)-[2-\theta(\kappa+\delta)]\left(\hat{p}_t^{t-1}-\hat{p}_t^{t-2}\right)$$
$$+\left[1-\theta\delta-\theta+\frac{(1-\theta\delta)\mu}{\xi}\right]\left(p_{t-1}-\hat{p}_{t-1}^{t-2}\right)$$
$$=\theta\varepsilon_{t-1}-\frac{1-\theta\delta}{\xi}(\eta_{t-1}-\nu_{t-1}), \tag{23}$$

which can be subtracted from (22) to yield

$$\left(\hat{p}_{t+1}^{t}-\hat{p}_{t+1}^{t-1}\right)-\left(1+\frac{\mu}{\xi}\right)\left(p_t-\hat{p}_t^{t-1}\right)=\frac{1}{\xi}(\eta_t-\nu_t). \tag{24}$$

Finally, using eqs. (21) to (24), the following equation[10] for the time path of the price level can be found:

$$p_t=m^*-\alpha-\gamma\bar{y}+(\delta-\kappa)\rho+A_1\lambda_1^t+A_2\lambda_2^t+\sum_{j=1}^{t-1}\left(\phi_1\lambda_1^{t-j}+\phi_2\lambda_2^{t-j}\right)u_j$$
$$+\frac{\xi}{\xi+\mu}(\lambda_1\phi_1+\lambda_2\phi_2)u_t-\frac{1}{\xi+\mu}(\eta_t-\nu_t), \tag{25}$$

$$u_t\equiv\frac{1}{1-\theta\kappa}\left[\theta\varepsilon_t+\left(\frac{1}{\xi+\mu}-\frac{1-\theta\delta}{\xi}\right)\right](\eta_t-\nu_t),$$

$$\phi_m\equiv\left\{\lambda_m\left[\lambda_m-\left(1+\frac{\theta\xi}{(1-\theta\kappa)(\xi+\mu)}\right)\right]\right.$$
$$-\lambda_n\left[\lambda_n-\left(1+\frac{\theta\xi}{(1-\theta\kappa)(\xi+\mu)}\right)\right]$$
$$\left.\times\frac{[(\xi+\mu)(1-\theta\delta)-\theta\xi]\lambda_m-(1-\theta\xi-\theta)}{[(\xi+\mu)(1-\theta\delta)-\theta\xi]\lambda_n-(1-\theta\delta-\theta)]}\right\}^{-1}$$
$$\times\left\{1-\frac{(1-\theta\kappa)(\xi+\mu)}{[(\xi+\mu)(1-\theta\delta)-\theta\xi]\lambda_n-(1-\theta\delta-\theta)(1-\theta\kappa)(\xi+\mu)}\right\}.$$

As can be seen, the smoothing of interest rates implies that the price level

[10] Eq. (25) describes the time path of the price level if the larger root $\lambda_1<1$; if $\lambda_1>1$, then $A_1=0$ and $\phi_1=0$; in addition, the expression for ϕ_2 will be different from the one given here.

follows an autoregressive process.[11] This time path may or may not be stable, depending on the values of the characteristic roots. If interest-rate smoothing is introduced starting with a historically-given interest rate, the constants A_1 and A_2 cannot both take values which ensure stability [see Blanchard and Kahn (1980)]: only one of them can be chosen independently, since even though the price level is free to take any initial level, the relationship between two initial expected price levels is pinned down by an initial interest rate through eq. (12). Thus if both the characteristic roots λ_1 and λ_2 lie outside the unit circle, the price level will be subject to explosive oscillations. Whether this will occur depends on the values of all the parameters of the model: the possible cases are given in the appendix. An examination of the characteristic roots indicates that although the system will not necessarily be unstable because of the smoothing policy (9), it is quite possible that such a policy will be destabilizing in such a rational-expectations framework.[12]

If the time path of the price level is given by (25), then the time path of the interest rate can be found by substituting from (25) into (12) and using (13),

$$r_t = \rho + (\lambda_1 - 1)\sum_{j=1}^{t-1} \phi_1 \lambda_1^{t-j} u_j + (\lambda_2 - 1)\sum_{j=1}^{t-1} \phi_2 \lambda_2^{t-j} u_j$$

$$+ A_1(\lambda_1 - 1)\lambda_1^t + A_2(\lambda_2 - 1)\lambda_2^t. \tag{26}$$

It is clear that if price level's path is unstable, the interest rate's path will also be unstable.

In this case, the time path of the money stock is given by

$$m_t = m^* + \frac{1}{\theta}\left[(\lambda_1 - 1)\lambda_1\phi_1 + (\lambda_2 - 1)\lambda_2\phi_2\right]u_t$$

$$+ \frac{1}{\theta}\left[(\lambda_1 - 1)^2\phi_1 \sum_{j=1}^{t-1} \lambda_1^{t-j} u_j + (\lambda_2 - 1)^2 \phi_2 \sum_{j=1}^{t-1} \lambda_2^{t-j} u_j\right]$$

$$+ \frac{1}{\theta}\left[A_1(1 - \lambda_1)^2 \lambda_1^{t-1} + A_2(1 - \lambda_2)^2 \lambda_2^{t-1}\right]. \tag{27}$$

Thus, if interest rates are smoothed, the money stock also follows an autoregressive process; this process will be unstable if the process generating the

[11] More precisely, the disturbance term is the sum of two autoregressive terms and a white-noise term.

[12] The most plausible cases are those in which $\theta > 1/\kappa$; if this is so, at least one and possibly both of the roots lie outside the unit circle, so that it is quite possible that the system will be unstable.

price level is unstable. Thus, attempting to resist movements in the interest rate may, by causing predictable patterns of interest-rate movement, have effects opposite to those intended: it may lead to instability of the interest rate, the money supply and the price level.

Nevertheless, it should be noted that in this case, national income is not unstable; it is given by

$$y_t = \bar{y} + \mu\left\{\frac{\xi}{\xi+\mu}(\lambda_1\phi_1 + \lambda_2\phi_2)u_t - \frac{1}{\xi+\mu}\left(\frac{\xi}{\mu}\eta_t - \nu_t\right)\right\}. \tag{28}$$

Thus, fluctuations in national income are still white noise as in the case (18) where the money supply is controlled.[13] This is simply a feature of the 'surprise' aggregate supply function.

5. Spurious instrument instability?

A further question can also be considered: what conclusion would be drawn from data generated by this model if these data were analyzed from the perspective of the existing literature on instrument instability? This question can be elaborated as follows: suppose that the system consisted of eqs. (5), (12), (13) and (14), and that the policy regime consisted of manipulating interest rates according to (9). If expectations were formed rationally, expected future interest rates would be related to past and current rates in a way implied by taking the expected value in eq. (26). Suppose that this relationship were interpreted as 'structural', as reflected in the weights of current and lagged interest rates in the demand-for-money function. Suppose that these weights were then used to infer the behavior of interest rates which would occur under an alternative policy regime involving rigid control of the money supply. What misleading conclusions would then be drawn about the danger of interest-rate instability under a monetary rule?

This question can be addressed by using (26) to find the next period's interest rate in terms of current and past rates.

$$\begin{aligned} r_t = {} & \frac{2-\theta(\kappa+\delta)}{1-\theta\kappa} r_{t-1} + \frac{1-\theta\delta-\theta}{1-\theta\kappa} r_{t-2} - \frac{\theta}{1-\theta\kappa}\rho \\ & + (\lambda_1 - 1)\phi_1\left(\lambda_1 - \frac{2-\lambda(\kappa+\delta)}{1-\theta\kappa}\right)u_{t-2} + (\lambda_2 - 1)\phi_2 u_{t-1}. \end{aligned} \tag{29}$$

Taking expectations, substituting into the money-demand function (5) and

[13] Unfortunately, the expressions for ϕ_1, ϕ_2, λ_1 and λ_2 are too cumbersome to permit any interesting comparisons between the variance of output with and without smoothing.

solving for r_t as in (2) gives the following difference equation describing the time path which the interest rate would supposedly follow if the money supply were controlled:

$$r_t - \frac{\kappa(1-\theta\delta-\theta)}{\delta-2\kappa+\kappa^2\theta} r_{t-1} = \alpha + \gamma\bar{y} + p\frac{1}{1-\theta\kappa}\rho - m^*. \tag{30}$$

This difference equation may or may not be stable. It is interesting to note that in the case in which, according to the analysis presented earlier, interest-rate smoothing is most likely to cause explosive movements of prices and interest rates (specifically when $\theta > 1/\kappa$), eq. (30) will imply instrument instability. Ironically, in the very case in which interest-rate smoothing is most likely to be destabilizing, empirical evidence generated under that regime would – if interpreted as it has been in the literature – lead one to infer that abandoning such a regime would cause the interest rate to explode. This result, which is related to the 'Lucas critique' [Lucas (1976)], suggests that intervention to limit interest-rate movements may be, in some sense, a self-justifying policy: under such a regime, empirical evidence is generated which suggests that abandoning the regime would be catastrophic; this occurs in a model in which the system can be stable without such intervention, and under conditions under which the intervention is itself likely to be destabilizing.

6. Conclusion

Because of the ad hoc nature of the model used in this paper, the results obtained therefrom are obviously not conclusive. They are, however, suggestive. They suggest that intervention to stabilize interest rates within a framework of long-run control of the money supply, far from being necessary to prevent interest-rate instability, may well be itself a source of instability. It may lead to movements of interest rates; moreover, it may cause widening divergences of the money supply from its target. Moreover, such intervention may be self-justifying: interest-rate smoothing lends some predictability to movements in interest rates; such policy-induced interest-rate trends will be incorporated in the lag structure of interest rates in the demand-for-money function; if this lag structure is interpreted as 'structural', it would lead one to infer that interest rates would follow an unstable path if the money supply were controlled rigidly in the short run.

The foregoing analysis does not, of course, absolutely disprove the instrument-instability argument against tight short-term control of the money supply, but it does seem to weaken it considerably: it indicates that barring a stronger case to the contrary, the instrument-instability problem should not be taken seriously as a danger of controlling the money supply 'too well'.

Appendix

In this appendix, some of the mathematical results in the paper will, for the sake of completeness, be established.

(1) Eq. (10) is a first-order difference equation in r_t. Its characteristic root is

$$\lambda = \frac{1-\theta\delta}{1-\theta\kappa}. \tag{A.1}$$

Thus, as asserted in the text, if $\theta < 1/\delta$, then $\lambda > 0$, but that clearly also implies that $0 < \lambda < 1$. In the intermediate case in which $1/\kappa > \theta > 1/\delta$, we have $\lambda < 0$, with $\lambda < -1$ if $\theta > 2/(\delta+\kappa)$. If $\theta > 1/\kappa$, then $\lambda > 0$, but that obviously means that $\lambda > 1$ since $\delta > \kappa$.

(2) Eq. (15), a first-order system in p_t, has characteristic roots

$$\lambda = \frac{\kappa+\delta}{2\kappa} \pm \left\{\left(\frac{\kappa+\delta}{\kappa}\right)^2 - \frac{4}{\kappa}\left(1+\delta+\frac{\delta\mu}{\xi}+\gamma\mu\right)\right\}^{\frac{1}{2}}. \tag{A.2}$$

It is not clear whether or not these roots are real: they will be real if and only if

$$(\kappa-\delta)^2 \geq 4\kappa\left[1+\delta+\mu\left(\frac{\delta}{\xi}+\gamma\right)\right]. \tag{A.3}$$

However, both roots can be shown to lie outside the unit circle regardless of whether they are real or complex. Thus, eq. (15) has the 'forward-looking' solution given in eq. (16).

(3) The characteristic roots of the difference equation (20) are

$$\lambda = \frac{2-\theta(\kappa+\delta)}{2(1-\theta\kappa)} \pm \left\{\left[\frac{2-\theta(\kappa+\delta)}{1-\theta\kappa}\right]^2 - 4\frac{1-\theta\delta-\theta}{1-\theta\kappa}\right\}^{\frac{1}{2}}. \tag{A.4}$$

Unfortunately, little can be said in general terms about these roots. The roots are both real if $\theta\kappa < 1$; they are also real if

$$\theta(\delta-\kappa)^2 > 4(\theta\kappa - 1). \tag{A.5}$$

Using the approximation (4) this implies that, where T is the term to maturity of the alternative asset, the roots are real unless

$$1 < \theta\kappa < \frac{T}{T-\frac{1}{4}}, \tag{A.6}$$

i.e., almost always. Should λ_1, λ_2 be non-real, their modulus would be

$$R = \left\{ \frac{1 - \theta\delta - \theta}{1 - \theta\kappa} \right\}^{\frac{1}{2}}, \tag{A.7}$$

which exceeds unity provided that $\delta + 1 > \kappa$, i.e., in general. If the roots are real, there are no fewer than ten non-vacuous subcases:

(a) $\lambda_1 > \lambda_2 > 1$,
(b) $\lambda_1 > 1 > \lambda_2 > 0$,
(c) $\lambda_1 > 1 > 0 > \lambda_2 > -1$,
(d) $\lambda_1 > 1 > -1 > \lambda_2$,
(e) $1 > \lambda_1 > \lambda_2 > 0$,
(f) $1 > \lambda_1 > 0 > \lambda_2 > -1$,
(g) $1 > \lambda_1 > 0 > -1 > \lambda_2$,
(h) $0 > \lambda_1 > \lambda_2 > -1$,
(i) $0 > \lambda_1 > -1 > \lambda_2$,
(j) $-1 > \lambda_1 > \lambda_2$.

Each of these cases has different implications for the behavior of the price level, the interest rate and the money supply. Three of these cases – (a), (d) and (j) – give rise to instability. These cases occur as follows:

(a) $\lambda_1 > \lambda_2 > 1$ if $\theta > 1/\kappa$ and $(1 - \theta)(\delta - \kappa) < \theta$,

(d) $\lambda_1 > 1 > -1 > \lambda_2$ if $\theta < 1 - \theta\kappa < (1/4)(\delta - \kappa)$

and $(\theta^2 - 1)(\delta - \kappa)^2 > -4\theta(1 - \theta\kappa)$,

or $0 < 1 - \theta\kappa < (1/3)[\theta + \delta - \kappa - (1 - \theta\delta)]$

and $(\theta^2 - 1)(\delta - \kappa)^2 > -4\theta(1 - \theta\kappa)$,

(j) $-1 > \lambda_1 > \lambda_2$ if $(1 - \theta\kappa)/(1 - \theta) > (1/4)(\delta - \kappa) > 4(1 - \theta\kappa) > 0$.

The cases in which $\theta > 1/\kappa$ appear to be the more plausible ones, since, under plausible assumptions about the parameters, $1/\kappa$ is likely to be extremely small.

(4) The characteristic root of eq. (30) is

$$\omega = \frac{\kappa(1 - \theta\delta - \theta)}{\theta - 2\kappa + \kappa^2\theta}. \tag{A.8}$$

It will be found that $\omega < -1$ if $\theta > 1/\kappa$; otherwise, the result is ambiguous, with four different cases:

(a) $(1-\theta\kappa) > 0,\quad (\delta-\kappa) > (1-\theta\kappa),$

(i) if $(1-\theta\delta) < \theta$, then

$$\omega < -1 \quad \text{iff} \quad (1-\theta\kappa) < 1/\delta,$$

$$0 > \omega > -1 \quad \text{iff} \quad (1-\theta\kappa) > 1/\delta;$$

(ii) if $(1-\theta\delta) > \theta$, then

$$\omega > 1 \quad \text{iff} \quad 2\kappa(1-\theta)+1 > \delta(1+\theta\kappa),$$

$$0 < \omega < 1 \quad \text{iff} \quad 2\kappa(1-\theta)+1 < \delta(1+\theta\kappa).$$

(b) $(1-\theta\kappa) > 0,\quad (\delta-\kappa) < (1-\theta\kappa),$

(i) if $(1-\theta\delta) < \theta$, then

$$\omega > 1 \quad \text{iff} \quad 2\kappa(1-\theta)+\delta(1+\theta\kappa) > 1,$$

$$0 < \omega < 1 \quad \text{iff} \quad 2\kappa(1-\theta)+\delta(1+\theta\kappa) < 1;$$

(ii) if $(1-\theta\delta) > \theta$, then

$$\omega < -1 \quad \text{iff} \quad (1-\theta\kappa) > 1/\delta,$$

$$0 > \omega > -1 \quad \text{iff} \quad (1-\theta\kappa) < 1/\delta.$$

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