Materials 12f - “pil”-extension of baseline model - Lagged inflation in TR

See Notes 7 Jan 2020

Laura Gáti

January 7, 2020

Compare Mathematica (`materials12f2.nb`).

Red stuff are changes compared to the baseline model.

1 Model equations and goal

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t \tag{3}$$

Compact notation

$$z_t = \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = A_a f_a + A_b f_b + A_s s_t \quad \text{with} \quad s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{bmatrix} \tag{4}$$

2 MN matrices

$$\underbrace{\begin{bmatrix} \sigma\psi_\pi\beta & 1+\sigma\psi_x \\ 1 & -\kappa \end{bmatrix}}_{\equiv M} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \underbrace{\begin{bmatrix} \begin{bmatrix} \sigma(1-\beta^2\psi_\pi), & 1-\beta-\sigma\beta\psi_x, & 0 \end{bmatrix} f_b + d_{x,s}s_t + \begin{bmatrix} 0, & 0, & 0, & -\sigma\psi_\pi \end{bmatrix} s_t \\ \begin{bmatrix} (1-\alpha)\beta, & \kappa\alpha\beta, & 0 \end{bmatrix} f_a + d_{\pi,s}s_t \end{bmatrix}}_{\equiv N} \tag{5}$$

where

$$d_{x,s} = -\sigma \begin{bmatrix} -1 & 1 & 0 & 0 \end{bmatrix} InxBhx \qquad InxBhx \equiv (I_{nx} - \beta h_x)^{-1} \tag{6}$$

$$d_{\pi,s} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} InxABhx \qquad InxABhx \equiv (I_{nx} - \alpha\beta h_x)^{-1} \tag{7}$$

$$d_{i,s} = \begin{bmatrix} 0 & 1 & 0 & \psi_\pi \end{bmatrix} \tag{8}$$

3 PQ matrices

$$\underbrace{\begin{bmatrix} \sigma\beta\psi_\pi & 1 & \sigma \\ 1 & -\kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix}}_{\equiv P} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \underbrace{\begin{bmatrix} \begin{bmatrix} \sigma, & 1-\beta, & \beta(-\sigma) \end{bmatrix} f_b + c_{x,s} s_t \\ \begin{bmatrix} (1-\alpha)\beta, & \alpha\beta\kappa, & 0 \end{bmatrix} f_a + c_{\pi,s} s_t \\ c_{i,s} s_t \end{bmatrix}}_{\equiv Q} \tag{9}$$

where

$$c_{x,s} = \sigma \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \text{.InxBhx;} \tag{10}$$

$$c_{\pi,s} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \text{.InxABhx} \tag{11}$$

$$c_{i,s} = \begin{pmatrix} 0 & 1 & 0 & \psi_\pi \end{pmatrix} = d_{i,s} \tag{12}$$

where InxABhx and InxBhx are the same as before. The (*)-relation is

$$f_b(3) = \beta\psi_\pi f_b(1) + \psi_x f_b(2) + \frac{1}{\beta}\{\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} (I_{nx} - \beta h_x)^{-1} s_t - \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} s_t\} \tag{*}$$

where in (*) there was a $\psi_\pi \pi_t$ term that I moved to the LHS of the PQ equation ($P(1,1)$).

The Matlab code that uses this is `matrices_A_12f2.m`.