Materials 13 - Still looking for a version of the model w/o overshooting

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Overview

1 Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

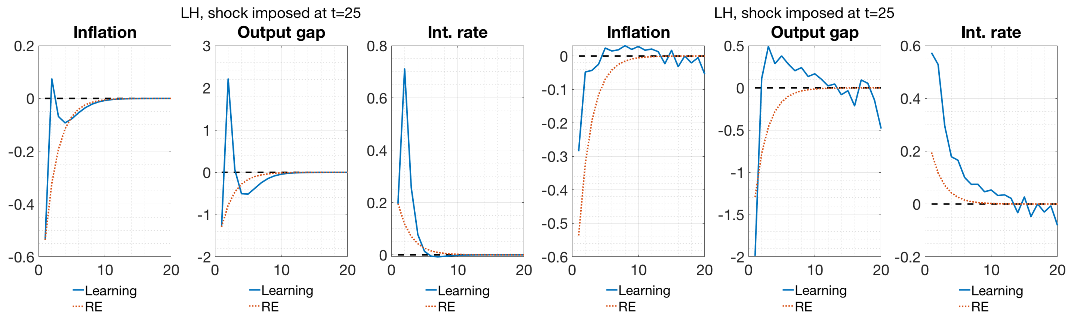
$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \tag{3}$$

$$\hat{\mathbb{E}}_t z_{t+h} = \begin{bmatrix} \bar{\pi}_{t-1} \\ 0\ (\bar{x}_{t-1}) \\ 0\ (\bar{i}_{t-1}) \end{bmatrix} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\, h_x \qquad \text{PLM} \tag{4}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \underbrace{\big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big)}_{\text{fcst error using (4)}} \qquad (b_1 \text{ is the first row of } b) \tag{5}$$

$$k_t = \begin{cases} k_{t-1} + 1 & \text{for decreasing gain learning} \\ \bar{g}^{-1} & \text{for constant gain learning.} \end{cases} \tag{6}$$

Figure 1: Reference: baseline model



(a) Learning constant only (b) Learning slope and constant

2 Ideas

1. Check ψ_π above but close to 1

   $\rightarrow$ works but only quantitatively; qualitatively, the overshooting is still there, likely because this only cancels out one of the two channels through which $\mathbb{E}\,\pi$ affects x_t negatively.

2. Fix shock for simulation

   Indeed the issue was that for learning, I accidentally scaled down the shock by $\sigma_i < 1$, while for RE I had maintained $\sigma_i = 1$.

3. Interest rate smoothing as $i_t = \rho i_{t-1} + (1-\rho)(\psi_\pi \pi_t + \psi_x x_t) + \bar{i}_t$

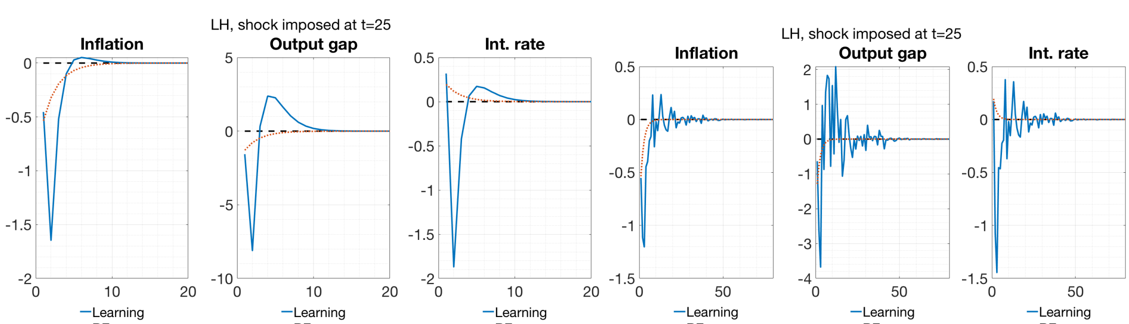
   Doesn't work either - it doesn't change the model except reduces ψ_π.

4. Indexation in NKPC

   Doesn't work either - same model dynamics.

5. Learn h_x

Figure 2: Learning h_x, baseline



(a) Learning constant only (b) Learning slope and constant

Like learning the Taylor rule b/c agents initially don't know if the shock will continue.

6. Central bank's $\mathbb{E}\,\pi_{t+1}$ in TR?

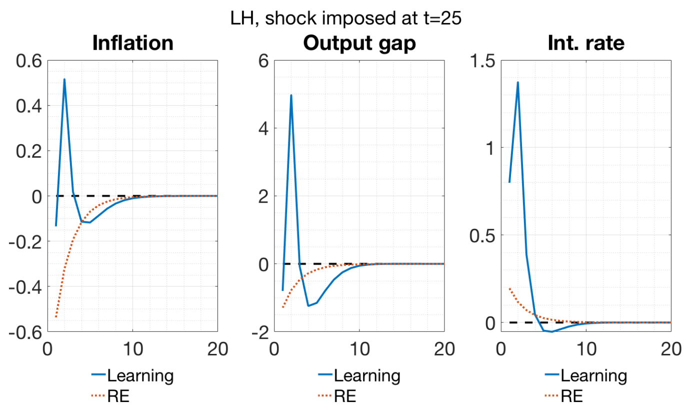
   Done a correction for $\hat{\mathbb{E}}\pi_{t+1}$ in TR, now both are stable, but overshooting is still there in both. Not so dissimilar to baseline except that the periods are shifted.

7. Initialize beliefs away from RE somehow

   Slobodyan & Wouters do this, but in an estimation context, which I think is necessary because you need pre-sample data to condition priors on.

8. Slobodyan & Wouters' "VAR-learning": use lagged observables to learn from, not from states.

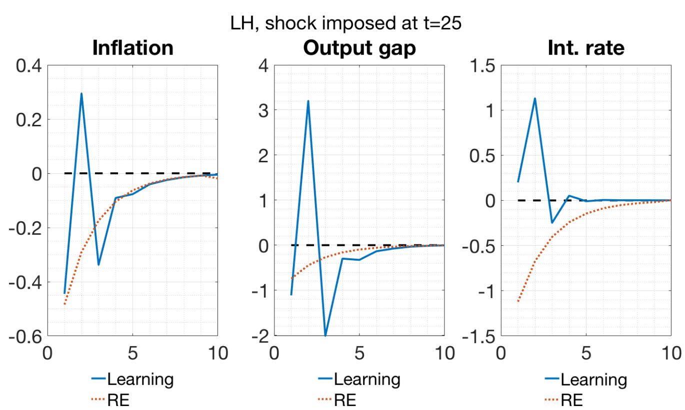
Figure 3: VAR learning, baseline, learning only constant



For learning both slope and constant, not E-stable. Kind of makes sense since I'd think that this amplifies positive feedback.

9. Davig & Leeper-style switching Taylor rule where only long-run Taylor principle holds?

Figure 4: Markov-switching Taylor rule, baseline, learning only constant (slope learning unstable, why?)



10. Some kind of moving average of inflation (or average) in the TR?

A quick question on projection facility: checking `eig(phi)` when ϕ isn't square? Right now I'm splitting up ϕ as `a = phi(:,1), b = phi(:, 2:end)` and then checking `eig(b)` and `eig(diag(a))`.

3 Details on the Markov-switching setup

Model equations remain the same, except the Taylor rule now is:

$$i_t = \psi_\pi(r_t)\pi_t + \psi_x x_t + \bar{i}_t \tag{7}$$

$$r_t = \begin{cases} 1 \quad \text{active regime} \quad \psi_\pi = \psi_1 = 2.19 \\ 2 \quad \text{passive regime} \quad \psi_\pi = \psi_2 = 0.89 \end{cases} \tag{8}$$

$$r_{t+1} = \begin{cases} p_{11}1 + (1-p_{11})2 \quad \text{if} \quad r_t = 1 \\ (1-p_{22})1 + p_{22}2 \quad \text{if} \quad r_t = 2 \end{cases} \qquad \text{where} \quad p_{ji} \equiv Prob(s_{t+1} = j | s_t = i) \tag{9}$$

So I solve the RE model by introducing the new jump variables $\pi_{it}, x_{it}, i_{it},\ i = 1, 2$ and writing the model equations as

$$x_{it} = (p_{1i}\,\mathbb{E}_t\,x_{1t+1} + p_{2i}\,\mathbb{E}_t\,x_{2t+1}) - \sigma(i_{it} - (p_{1i}\,\mathbb{E}_t\,\pi_{1t+1} + p_{2i}\,\mathbb{E}_t\,\pi_{2t+1})) + \sigma r_t^n \tag{10}$$

$$\pi_{it} = \kappa x_{it} + \beta(p_{1i}\,\mathbb{E}_t\,\pi_{1t+1} + p_{2i}\,\mathbb{E}_t\,\pi_{2t+1})) + u_t \tag{11}$$

$$i_{it} = \psi_i \pi_{it} + \psi_x x_{it} + \bar{i}_t \tag{12}$$

Now I unleash the usual method of solving for the observable and state transition matrix g_x, h_x. The only difference will be that since the number of jumps now is double the old number, g_x will be $2n_y \times n_x$. Is it correct to interpret $g_x(1) \equiv$ `gx(1:ny,:)` as pertaining to regime 1, and $g_x(2) \equiv$ `gx(ny+1:end,:)` to regime 2?

Then, generating an exogenous regime sequence r, I compute RE IRFs as usual for the state block, but depending on the state, I use the corresponding block of g_x. With x_0 being the impulse, so that $IR_1^x = x_0$, I do the following:

$$IR_t^y = \begin{cases} g_x(1)x_t \quad \text{if} \quad r_t = 1 \\ g_x(2)x_t \quad \text{if} \quad r_t = 2 \end{cases} \tag{13}$$

$$IR_{t+1}^x = h_x x_t \tag{14}$$

As for the learning model, the compact notation for the model was:

$$z_t = A_a f_a(t) + A_b f_b(t) + A_s s_t \tag{15}$$

where the A-matrices are functions of the parameters, including ψ_π. The LH expectations f_a and f_b are only functions of the learning coefficients a, b and of the state transition matrix h_x. So the only salient difference is that agents react to expectations differently depending on the regime: the A-matrices become state-dependent.

Is it correct then to simulate the model as

$$z_t = A_a(i) f_a(t) + A_b(i) f_b(t) + A_s(i) s_t \quad \text{for} \quad i = 1, 2 \tag{16}$$

?