Info assumptions 8 Jan 2020 cont'd

Let me give names to the info-assumptions:

The particular question is: supposing agents don't know the NKPC, NKIS, do know the TR, do they know the lending equation between the jump (i_t or π_t) and the endogenous lagged value (il_t or pil_t), and if yes, what do they use to fest them?

- No: fest i_t using gx and il_t using hx → "myopic info"
- Yes: fest i_t using gx and il_t using hx → "schizophrenic"
- Yes: both using gx → "suboptimal forecaster info ass"
- Yes: both using hx → "optimal forecaster info ass"

→ these are the two that at least are internally consistent

The irony is that the old approach to il did the "myopic info", and the corrected MN & PQ methods outlined in materials12f1-3 all do the suboptimal forecaster.

So-lol- 12f is not internally consistent!

A "middle road" was ...intrate_smoothing3.m, b/c it does the MN approach w/ the myopic info ass, which is why it always coincides w/ older approaches.

At least what I'm more & more converging on is the idea that, concerning the big-picture question, agents

- do NOT know NKPC & NKIS → b/c they don't know they are identical
- <u>do</u> know the TR → b/c that doesn't require knowledge that they are identical, so CB can just announce it.

But as for whether they know the linking equation:

- both myopic info & optimal forecaster ass are consistent
- but my hunch is that ass. myopic info at the same time as I ass. that they know the TR is weird b/c it amounts to ass-ing that the CB announces the TR but people don't understand that the ile on the RHS is i_{t-1}.

So from a consistency-realisticness standpoint the "optimal forecaster info" ass seems to be the best.

But in terms of desirability of model dynamics, which should we prefer? In particular, can some overcome the overshooting?

- materials6 introduced int-rate smoothing, now I know using the "myopic info" ass and it didn't do much to dampen the overshooting
- materials12 introduced learning the slope and found it was desirable
  - → "optimal forecaster" ass would undo some of this
  - ↳ this suggests that having them know less slows down things

This is confusing b/c the "myopic info" ass endowes them w/ less info than the "optimal forecaster". So that suggests that I need to withdraw more info: make 'em learn hx too.

The reason that makes me anxious is that overshooting is already happening b/c $E(\cdot)$ moves so much. So if I increase the role of $E(\cdot)$ b/c h_x isn't known either, then will that not make more overshooting?

Step back 1 sec: $E(\cdot)$ moving wouldn't necessarily be a prob if i_t wasn't moving so fast or if the TR wasn't known. → No! Whether the TR is known might not matter actually b/c whether agents know how i_t is set, i_t just affects stuff.

→ No that's not true either b/c LH fcsts matter and thus fcsts of i_{t+k} matter, and how you do those depends on whether you know the TR.

→ Does that mean that I've been ass-ing the TR is known all along?

I think so b/c otherwise the (*)-conditions wouldn't be valid.

Do you agree w/ the statement 9 Jan 2020

$\hat{E}_t^i(i_{t+1}) = \hat{E}_t(i_{t+1})$?

For this to be true, it has to be that agents first aggregates the same way: $\hat{E}_t^i(y_{t+k}) \overset{!}{=} \hat{E}_t^j(y_{t+k})$ for y being an aggregate variable. Given that agents are identical and observe the same aggregates, this should be true. In fact, it may be true for disagg. variables too, except agents don't realize that either.

The fact that agents know the TR allows me to use the (*)-relations to write $\hat{E}_t(i_{t+k}) = \psi_\pi \hat{E}_t(\pi_{t+k}) + \psi_x \hat{E}_t(x_{t+k}) + \hat{E}_t(\text{shocks})$

But in a certain sense this doesn't help HHs b/c they don't know $\hat{E}_t(\pi_{t+k})$ nor $\hat{E}_t(x_{t+k})$, and so they still estimate $\hat{E}_t(i_{t+k})$ as they estimate gx. Recall that you could solve the model using the PQ-method w/o using (*); then you get a different sol of course where HHs don't know the

TR. But the MN-method is not valid if HHs don't know the TR b/c then you can't sub in $\hat{E}_t[\gamma_\pi \pi_{T+1} + \gamma_x x_{T+1} + \text{shocks}_{T+1}]$ into $\hat{E}_t i_{T+1}$.

So, since Preston uses the MN method, it means that he ass-s that agents know the Taylor-rule!

Ok: so big-picture info ass are cleared up:

- agents do <u>not</u> know NKIS, NKPC
- agents <u>do</u> know the TR

Now we just need to solve the in-depth issue: do agents internalize the linking equation?

- I think it's tough to tell what has the most appealing dynamics.
- But intuitively I'd say yes.

$\Rightarrow$ Resolve pit & it using the "optimal forecasters" info ass.

$\Rightarrow$ materials 12g2-3.

pil

$$x_t = -b i_t + \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta) x_{\tau+1} + b\pi_{\tau+1} - b\beta i_{\tau+1} + b r_\tau^n \right\}$$

$$\pi_t = k x_t + \hat{E}_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \left\{ k\alpha\beta x_{\tau+1} + (1-\alpha)\beta \pi_{\tau+1} + u_\tau \right\}$$

$$i_t = \psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t$$

If agents use bx to fest π, then $f_\beta(1)$ or $f_\alpha(1)$ will not be used.

PQ:

$$\frac{b}{\beta} + b\beta\left(\beta\psi_\pi \frac{b}{\beta}\right) = \frac{b}{\beta} + b^2\beta\psi_\pi$$

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -k & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{cases} [0, 1-\beta, -b\beta] f_\beta + b[1, 0, 0, 1](I_{nx} - \beta L_{nx})^{-1} s_t - [0\ 0\ 0\ b] s_t \\ [0, k\alpha\beta, 0] f_\alpha + [0, 0, 1, (1-\alpha)\beta](I_{nx} - \alpha\beta L_{nx})^{-1} s_t - [0\ 0\ 0\ (1-\alpha)\beta] s_t \\ [0, 1, 0, \psi_\pi] s_t \end{cases}$$

$$b\hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ \pi_{\tau+1} + r_\tau^n \right\} = b \sum_{\tau=t}^{\infty} \beta^{\tau-t} \hat{E}_t \left\{ pil_{\tau+2} + r_\tau^n \right\}$$

$$= b\left[pil_{t+2} + r_t^n + \beta pil_{t+3} + \beta r_{t+1}^n + \beta^2 pil_{t+4} + \beta^2 r_{t+2}^n + \dots \right]$$

$$= b\left[\frac{1}{\beta^2}\left(\beta^2 pil_{t+2} + \beta^3 pil_{t+3} + \dots \right) + \left(r_t^n + \beta r_{t+1}^n + \beta^2 r_{t+2}^n + \dots \right) \right]$$

$$= b\left[\frac{1}{\beta^2}\left(pil_t + \beta pil_{t+1} + \beta^2 pil_{t+2} + \dots \right) + (- \| -) - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1} \right]$$

$$= b\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\frac{1}{\beta^2} pil_\tau + r_\tau^n \right) - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1} \right]$$

$$= -\frac{b}{\beta}\pi_t + b \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\frac{1}{\beta^2} pil_\tau + r_\tau^n \right) - \frac{b}{\beta^2}[0, 0, 0, 1] s_t$$

The problem is that $\hat{E}_t pil_t = \hat{E}_t \pi_{t-1}$ & $\hat{E}_t pil_{t+1} = \hat{E}_t \pi_t$

I actually now think it's fine to write

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \, pil_T = \frac{1}{1-\beta(hx_\pi)} \, pil_t$$

b/c this is $pil_t \left(= \pi_{t-1}\right) + \beta \, pil_{t+1} \left(= \beta \, hx_\pi \, pil_t = \pi_t\right)$

$$+ \beta^2 pil_{t+2} \left(= \beta \pi_{t+1}\right).$$

With this logic, $\hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ (1-\alpha)\beta \, \pi_{T+1} + u_T \right\}$

$$= \hat{E}_t \left[u_t + \alpha\beta u_{t+1} + (\alpha\beta)^2 u_{t+2} + \dots + pil_{t+2} + (\alpha\beta)^2 pil_{t+3} + \dots \right]$$

$$= \hat{E}_t \left[-||- + \frac{1}{(\alpha\beta)^2} \left[pil_t + \alpha\beta \, pil_{t+1} + (\alpha\beta)^2 pil_{t+2} + \dots \right] - \frac{1}{(\alpha\beta)^2} pil_t - \frac{1}{\alpha\beta} pil_{t+1} \right]$$

$$= \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[0, 0, 1, \frac{1}{(\alpha\beta)^2} \right] s_t - \frac{1}{(\alpha\beta)^2} [0, 0, 0, 1] s_t - \frac{1}{\alpha\beta} \pi_t$$

$$= \left[0, 0, 1, \frac{1}{(\alpha\beta)^2} \right] (I_{nx} - \alpha\beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0, 0, 0, 1] s_t - \frac{1}{\alpha\beta} \pi_t$$

So using

$$-\frac{b}{\beta}\pi_t + b\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left(\frac{1}{\beta^2}pil_\tau + r_\tau^n\right) - \frac{b}{\beta^2}[0,0,0,1]\,s_t$$

and using

$$[0,0,1,\tfrac{1}{(\alpha\beta)^2}](I_{nx}-\alpha\beta h_x)^{-1}s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1]s_t - \frac{1}{\alpha\beta}\pi_t$$

we get

$$\begin{bmatrix} \frac{b}{\beta} & 1 & b \\ (1+\frac{1}{\alpha\beta})k & 0 & \\ 0 & -\psi_x & 1 \end{bmatrix}\begin{bmatrix}\pi_t \\ x_t \\ i_t\end{bmatrix} = \begin{bmatrix} [0,1-\beta,-b\beta]f_\beta + b[1,0,0,\frac{1}{\beta^2}](I_{nx}-\beta h_x)^{-1}s_t - \frac{b}{\beta^2}[0,0,0,1]s_t \\ [0,k\alpha\beta,0]f_\alpha + [0,0,1,\frac{1}{(\alpha\beta)^2}](I_{nx}-\alpha\beta h_x)^{-1}s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1]s_t \\ [0,1,0,\psi_\pi]\,s_t \end{bmatrix}$$

$(1+\frac{1}{\alpha\beta})$

And we need to modify (*) as well.

But wait a sec: using the above, can I not, instead of rewriting everything, simply add two linking equations that relate $f_\beta(1)$ and $f_\alpha(1)$ to $s_t(4)$?

Something like: $f_\beta(1) = \hat{E}_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}\pi_{\tau+1}$

$= \hat{E}_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}pil_{\tau+2} = \hat{E}_t\left[pil_{t+2} + \beta pil_{t+3} + \dots\right]$

$$= \hat{E}_t\left[\frac{1}{\beta^2}\left(pil_t + \beta\, pil_{t+1} + \beta^2 pil_{t+2} + \dots\right) - \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1}\right]$$

$$= \frac{1}{\beta^2}\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\, pil_T \quad - \frac{1}{\beta^2} pil_t - \frac{1}{\beta}\pi_t$$

$$= \frac{1}{\beta^2}[0,0,0,1](I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1]s_t - \frac{1}{\beta}\pi_t$$

L1: $f_\beta(1) = \frac{1}{\beta^2}[0,0,0,1](I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1]s_t - \frac{1}{\beta}\pi_t$

L2: $f_\alpha(1) = \frac{1}{(\alpha\beta)^2}[0,0,0,1](I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1]s_t - \frac{1}{\alpha\beta}\pi_t$

OR: simply add $\underbrace{\text{stuff} \times \left(\frac{1}{\beta}\right)\pi_t}$ and $\underbrace{\text{stuff} \times \left(\frac{1}{\alpha\beta}\right)\pi_t}$ to the LHS,

$P(1,1) = \frac{2}{\beta}$ $\qquad P(2,1) = \left(1 + \frac{(1-\alpha)\beta}{\alpha\beta}\right) = \frac{1}{\alpha}$

L1': $f_\beta(1) = \frac{1}{\beta^2}[0,0,0,1](I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1]s_t$

L2': $f_\alpha(1) = \frac{1}{(\alpha\beta)^2}[0,0,0,1](I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1]s_t$

and then to deal w/ (*), add stuff $\times\, \psi_\pi \pi_t$ to the LHS and lastly there's a $\beta\psi_\pi f_\beta(1)$ in (*), which from the NKIS relations has a -2β coefficient, so put things together:

$-2\beta \cdot \beta\psi_\pi f_\beta(1) \Rightarrow 2\beta\psi_\pi \cdot \pi_t \Rightarrow$ add $-2\beta\psi_\pi\pi_t$ to the LHS, i.e. to $P(1,1)$

$P(1,1)_{\text{final}} = \left(\frac{2}{\beta} - 2\beta\psi_\pi\right)$, don't change anything else but add L1' & L2'.

MN (the red highlighted stuff is correct, see Mathematica)

$$x_t = -b i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + b \pi_{T+1} - b\beta i_{T+1} + b r_T^n \right\}$$

$$\pi_t = k x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ k\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \underline{\pi_{t-1}} + \psi_x x_t + \bar{i}_t$$

$$\Rightarrow x_t = -b(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta - b\beta\psi_x) x_{T+1} + b \pi_{T+1} - b\beta(\psi_\pi \pi_T + \bar{i}_{T+1}) + b r_T^N \right\}$$

$$(1+b\psi_x) x_t = -b\psi_\pi \pi_{t-1}$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta - b\beta\psi_x) x_{T+1} + b \pi_{T+1} - b\beta \psi_\pi \pi_T + b(r_T^N + \bar{i}_T) \right\}$$

$$(1+b\psi_x) x_t =$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta - b\beta\psi_x) x_{T+1} + b \pi_{T+1} - b\psi_\pi \beta i_T + b(r_T^N - \bar{i}_T) \right\}$$

$$\begin{bmatrix} 0 & 1+b\psi_x \\ 1 & -k \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [b, 1-\beta-b\beta\psi_x, 0] f_\beta + b[1,-1,0,-\psi_\pi](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t \end{bmatrix}$$

and

$$L1: \quad f_\beta(1) = \frac{1}{\beta^2}[0,0,0,1](I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1] s_t - \frac{1}{\beta}\pi_t$$

$$L2: \quad f_\alpha(1) = \frac{1}{(\alpha\beta)^2}[0,0,0,1](I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1] s_t - \frac{1}{\alpha\beta}\pi_t$$

$$\begin{bmatrix} \frac{b}{\beta} & 1+b\psi_x \\ \frac{1}{\alpha} & -k \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [b, 1-\beta-b\beta\psi_x, 0] f_\beta + b[1,-1,0,-\psi_\pi](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t \end{bmatrix}$$

$$1 + \frac{(1-\alpha)\cancel{\beta}}{\alpha\cancel{\beta}} = \frac{\alpha + 1 - \alpha}{\alpha} = \frac{1}{\alpha}$$

w/ L1' & L2'.

Another attempt at PQ:

$$x_t = -b i_t + \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta) x_{\tau+1} + b\pi_{\tau+1} - b\beta i_{\tau+1} + b r_\tau^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \left\{ \kappa\alpha\beta x_{\tau+1} + (1-\alpha)\beta \pi_{\tau+1} + u_\tau \right\}$$

$$i_t = \psi_\pi \underline{\pi_{t-1}} + \psi_x x_t + \bar{i}_t$$

$$\Rightarrow \begin{array}{c} \\ \frac{1}{\alpha} \end{array} \begin{array}{c} \leftarrow P(1,1) \\ \\ \\ \end{array} \begin{bmatrix} 0 & 1 & b \\ 1 & -\kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [b, 1-\beta, -b\beta] f_\beta + b[1,0,0,0](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t \\ [0, 1, 0, \psi_\pi] s_t \end{bmatrix}$$

$$L1: \; f_\beta(1) = \frac{1}{\beta^2}[0,0,0,1](I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1] s_t - \frac{1}{\beta}\pi_t$$

$$L2: \; f_\alpha(1) = \frac{1}{(\alpha\beta)^2}[0,0,0,1](I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1] s_t - \frac{1}{\alpha\beta}\pi_t$$

$$P(2,1): \; 1 - \frac{(1-\alpha)\beta}{\alpha\beta}(-1) = 1 + \frac{1-\alpha}{\alpha} = \frac{\alpha + 1 - \alpha}{\alpha} = \underline{\underline{\frac{1}{\alpha}}} \quad P(2,1)$$

$$P(1,1): \; b f_\beta(1) \quad \Big| \; (*) \text{ But need to rewrite } (*)$$

$$= b\left(-\frac{1}{\beta}\right) \quad \Big| \text{ over to LHS} \rightarrow \frac{b}{\beta} \quad P(1,1)$$

(*) needs to take the "optimal forecaster" ass into account

$$f_\beta(3) = \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} [i_{\tau+1}] = \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ \psi_\pi \pi_\tau + \psi_x x_{\tau+1} + \bar{i}_{\tau+1} \right\}$$

$$= \psi_x f_\beta(2) + \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} [0, 1, 0, \psi_\pi] s_{\tau+1}$$

$$f_\beta(3) = \psi_x f_\beta(2) + \frac{1}{\beta}[0,1,0,\psi_\pi](I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{\beta}[0,1,0,\psi_\pi] s_t$$

Ok, having pil, let's now do it w/ "optimal forecasters."

$$x_t = -b i_t + \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta) x_{\tau+1} + b\pi_{\tau+1} - b\beta i_{\tau+1} + b r_\tau^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \left\{ \kappa\alpha\beta x_{\tau+1} + (1-\alpha)\beta \pi_{\tau+1} + u_\tau \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}$$

Now the linking equations have to relate $f_\alpha(3)$ & $f_\beta(3)$ to the errors.

$$f_\beta(3) = \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} i_{t+1}$$

Don't plug the Taylor-rule b/c that is less info than using hx_i.

$$= \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} i_{t+2} = \hat{E}_t \left[i_{t+2} + \beta i_{t+3} + \dots \right]$$

$$= \frac{1}{\beta^2} \hat{E}_t \left[i_t + \beta i_{t+1} + \dots \right] - \frac{1}{\beta^2} i_t - \frac{1}{\beta} i_{t+1}$$

$\underline{\underline{L1}}$

$$f_\beta(3) = \frac{1}{\beta^2} [0,0,0,1] (I_{nx} - \beta hx)^{-1} s_t - \frac{1}{\beta^2} [0,0,0,1] s_t - \frac{1}{\beta} i_t$$

$\underline{\underline{L2}}$

$$f_\alpha(3) = \frac{1}{(\alpha\beta)^2} [0,0,0,1] (I_{nx} - \alpha\beta hx)^{-1} s_t - \frac{1}{(\alpha\beta)^2} [0,0,0,1] s_t - \frac{1}{\alpha\beta} i_t$$

I wonder what happens to (*) ...

MN

$$x_t = -b\left(\psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}\right)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta)x_{T+1} + b\pi_{T+1} - b\beta\left(\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \bar{i}_T + \rho i_T\right) + r_T^n\right\}$$

But again, this doesn't make sense b/c you've got stg better than the TR to first i:

PQ:

$$x_t = -bi_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{(1-\beta)x_{T+1} + b\pi_{T+1} - b\beta i_{T+1} + r_T^n\right\}$$
$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\left\{\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T\right\}$$
$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}$$

$$\begin{bmatrix} 0 & 1 & -b \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [b, 1-\beta, -b\beta] f_\beta + b[1,0,0,0](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + [0,0,1,0](I_{nx} - a\beta h_x)^{-1} s_t \\ [0,1,0,\rho] s_t \end{bmatrix}$$

L1 s.t. L1 instead of (*) [and L2 is obsolete!]

$$f_\beta(3) = \frac{1}{\beta^2}[0,0,0,1](I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1] s_t - \frac{1}{\beta} i_t$$

So in the first equation, coeff of i are:

$$-b - (-b\beta) f_\beta = -b + b\beta\left(-\frac{1}{\beta}\right) = -2b \leftarrow P(1,3)$$

I just have to hope that this is right!

I guess it's time to compare results and check work. 10 Jan 2020

But first: implement pil w/ "myopic" infp ass!
(which actually in terms of forecasting equals the "schizophrenic" infp ass b/c either you're myopic and so you don't realize $pil_{t+1} = \pi_t$, so you fcst pil using hx and π_t using gx; or you do realize this but you're schizophrenic and still forecast them separately).

<u>pil - myopic</u> → materials12h2

$$x_t = -bi_t + \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta) x_{\tau+1} + b\pi_{\tau+1} - b\beta i_{\tau+1} + br_\tau^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \left\{ \kappa\alpha\beta x_{\tau+1} + (1-\alpha)\beta \pi_{\tau+1} + u_\tau \right\}$$

$$i_t = \psi_\pi \underline{\pi_{t-1}} + \psi_x x_t + \bar{i}_t$$

MN

agents don't realize this is the same var

$$x_t = -b(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$
$$+ \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta-b\beta\psi_x) x_{\tau+1} + b\pi_{\tau+1} - b\beta\left[\psi_\pi \pi_\tau + \bar{i}_{\tau+1}\right] + br_\tau^n \right\}$$

$$(1+b\psi_x) x_t = \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta-b\beta\psi_x) x_{\tau+1} + b\pi_{\tau+1} - b\left(\psi_\pi \pi_{\tau-1} + \bar{i}_\tau\right] + br_\tau^n \right\}$$

$$(1+b\psi_x)x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-b\beta\psi_x)\lambda_{T+1} + b\pi_{T+1} - b(\psi_\pi \pi_{T-1} + \tilde{i}_T) + 2r_T^n \right\}$$

$$\begin{bmatrix} 0 & 1+b\psi_x \\ 1 & -k \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [b, 1-\beta-b\beta\psi_x, 0] f_\beta + b[1, -1, 0, -\psi_\pi](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + [0, 0, 1, 0](I_{nx} - \alpha\beta h_x)^{-1} s_t \end{bmatrix}$$

PQ

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -k & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [b, 1-\beta, -b\beta] f_\beta + b[1, 0, 0, 0](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + [0, 0, 1, 0](I_{nx} - \alpha\beta h_x)^{-1} s_t \\ [0, 1, 0, \psi_\pi] s_t \end{bmatrix}$$

(*)

$$f_\beta(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \tilde{i}_{T+1}$$

$$= \psi_x f_\beta(2) + \frac{1}{\beta} \left\{ [0, 1, 0, \psi_\pi](I_{nx} - \beta h_x)^{-1} s_t - [0, 1, 0, \psi_\pi] s_t \right\}$$

Ok, so look at IRFs in materials12.tex, quickly:
(Note: should check all work to make sure they are really correct → much of this will be my task on Mon, Tues & Wed.)

1) Baseline: learning slope & constant seems better b/c since agents don't know gx(2:end,:), the Ball-effect can't pass out.

2) Epi:

constant only → instrument instability

slope & constant → not E-stable (visually)

To me it makes intuitive sense that Epi should exhibit more instability than baseline b/c instability in the baseline also comes from the Ball-effect, E(·) moving a lot and mattering a lot due to fwd-looingness.

In Epi, they get to matter more b/c now if E(·) are unstable, i becomes unstable too ⇒ that's why you get inter-instability. When agents are learning all of gx, then

an exploding i is not sufficient to keep $E(\cdot)$ at bay, leading agents to "learn the wrong thing" and so they don't converge to RE.

3) pil

"myopic": for both learning PLMs, seems not E-stable

In a sense it makes sense b/c their forecasting is not consistent w/ RE. Gets worse the less they know of g_x. Why don't they learn that $g_x = h_x$ for π? Maybe b/c this assumes that pil $\neq \pi$ in RE.

"suboptimal test": both behave nicely, in particular the

& "optimal test": latter

In the former, learning only constant or slope & constant matters of course b/c you're using g_x to test.

In the latter, this distinction doesn't matter b/c you're using h_x to test anyway.

I think the reason this model works so nicely in general is b/c it takes out π_x as a jump (and that's what "myopic" fails

to do), and since π_t is gone as a jump, $E(\pi)$ don't misbehave, and so the Ball-type disinflationary boom effect doesn't occur.

4) i.l

"myopic": behaves a lot like the baseline, where learning the slope too dampens the Ball-effect.

"suboptimal" vs "optimal fcst": again, optimal fcst is more stable b/c your fcst is closer to RE since you're using hx (which comes from RE). learning slope too makes things worse → it seems that the gx you're learning here is unstable and b/c you use it to fcst x & π in both cases, you diverge.

⇒ The overarching theme seems to be that gx is not E-stable and so the only way to restore stability is to effectively make things known: preferably π (pi'l) b/c it's more $E(\pi)$ that lead to instability → need to check E-stability!

Check Firm work

14 Jan 2020

(1) Eqi

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma \beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa \alpha \beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \varphi_\pi \hat{E}_t \pi_{t+1} + \varphi_x x_t + \bar{i}_t$$

MN

$$x_t = -\sigma(\varphi_\pi \hat{E}_t \pi_{t+1} + \varphi_x x_t + \bar{i}_t)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta(\varphi_\pi \hat{E}_T \pi_{T+2} + \varphi_x x_{T+1} + \bar{i}_{T+1}) + \sigma r_T^n \right\}$$

$$(1+\sigma\varphi_x) x_t = -\sigma \varphi_\pi \hat{E}_t \pi_{t+1}$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\varphi_x) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta\varphi_\pi \hat{E}_T \pi_{T+2} + \sigma[1,-1,0] s_T \right\}$$

$$\hat{E}_t \hat{E}_T \pi_{T+1} = \hat{E}_t \pi_{T+1}$$

I've been ass-ing all along that $\hat{E}$, not just $\hat{E}^i$, satisfies LIE

$$(1+\sigma\varphi_x) x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\sigma\beta\varphi_x) x_{T+1} + \sigma(1-\varphi_\pi)\pi_{T+1} + \sigma[1,-1,0] s_T \right\}$$

$$\begin{bmatrix} 0 & 1+\sigma\varphi_x \\ 1 & -\kappa \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [\sigma(1-\varphi_\pi), 1-\beta-\sigma\beta\varphi_x, 0] f_\beta + \sigma[1,-1,0](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] + [0,0,1](I_{nx} - \alpha\beta h_x)^{-1} s_t \end{bmatrix}$$

materials 12 f1. tex ✓ nb ✓

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \hat{E}_t \pi_{t+1} + \psi_x x_t + \bar{i}_t$$

$+0 \cdot \hat{E}_t \pi_{t+1}$

<u>PB</u>

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ 0 & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma [1,0,0](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1](f_{nx} - \alpha\beta h_x)^{-1} s_t \\ [0,1,0] s_t + \psi_\pi \cdot \hat{E}_t \pi_{t+1} \end{bmatrix}$$

materials 12 f1. tex ✓ nb ✓

From now on: only MN and only the x-equation!

② <u>Pil</u> <u>first "myopic"</u> (1242 - which I apparently never typed up on LaTeX)

$$i_t = \psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t$$

<u>MN</u>

$$x_t = -\sigma(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta(\psi_\pi \pi_T + \psi_x x_{T+1} + \bar{i}_{T+1}) + \sigma r_T^n \right\}$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \pi_{t-1}$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta\sigma\psi_x) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta(\psi_\pi \pi_T) + \sigma [1,-1,0,0] s_T \right\}$$

$-\psi_\pi$

✓

Pil - second "subopt fasters" (fast both using gx)

MN Step 1 is the same:

$$x_t = -b(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta) x_{\tau+1} + b\pi_{\tau+1} - b\beta(\psi_\pi \pi_\tau + \psi_x x_{\tau+1} + \bar{i}_{\tau+1}) + b r_\tau^n \right\}$$

$$(1+b\psi_x) x_t = -b\psi_\pi \pi_{t-1} - b\beta\psi_\pi \pi_t$$

$$+ \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta-\beta b\psi_\pi) x_{\tau+1} + \underbrace{b\pi_{\tau+1} - b\beta^2\psi_\pi \pi_{\tau+1}}_{b(1-\beta^2\psi_\pi)} + b[1,-1,0,0]\cdot z_\tau \right\}$$

.tex ✓ .nb ✓

Pil - third "opt fasters" (fast both using gx)

MN Step 2 is even the same!

$$(1+b\psi_x) x_t = -b\psi_\pi \pi_{t-1} - b\beta\psi_\pi \pi_t$$

$$+ \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta-\beta b\psi_\pi) x_{\tau+1} + b(1-\beta^2\psi_\pi)\pi_{\tau+1} + b[1,-1,0,0]\cdot z_\tau \right\}$$

The diff is that the $b(1-\beta^2\psi_\pi)\pi_{\tau+1}$ term now goes into the error.

$$\hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} b(1-\beta^2\psi_\pi)\pi_{\tau+1} = b(1-\beta^2\psi_\pi)\hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} pil_{\tau+2}$$

$$= b(1-\beta^2\psi_\pi)\hat{E}_t\left(pil_{t+2} + \beta pil_{t+3} + \dots \right)$$

$$= b(1-\beta^2\psi_\pi)\hat{E}_t \frac{1}{\beta^2}\left(pil_t + \beta pil_{t+1} + \beta^2 pil_{t+2} + \beta^3 pil_{t+3} + \dots \right)$$

$$- \frac{1}{\beta^2} pil_t - \frac{1}{\beta} pil_{t+1}$$

$$= b(1-\beta^2\psi_\pi)\,\hat{E}_t\,\frac{1}{\beta^2}\left(\pi_t + \beta\pi_{t+1} + \beta^2\pi_{t+2} + \beta^3\pi_{t+3} + \dots\right)$$

$$- \frac{1}{\beta^2}\pi_t - \frac{1}{\beta}\pi_{t+1}$$

$$= b(1-\beta^2\psi_\pi)\,[0,0,0,1]\,(I_{nx} - \beta h_x)^{-1} s_t - \frac{1}{\beta^2}\pi_{t-1} - \frac{1}{\beta}\pi_t$$

So then

$$(1+b\psi_x)x_t = -b\psi_\pi\pi_{t-1} - b\beta\psi_\pi\pi_t$$

$$+\hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{(1-\beta-\beta b\psi_\pi)x_{T+1} + b(1-\beta^2\psi_\pi)\pi_{T+1} + b[1,-1,0,0]\cdot s_T\right\}$$

becomes

$$(1+b\psi_x)x_t = -\left(b\psi_\pi + \frac{1}{\beta^2}\right)\pi_{t-1} - \left(b\beta\psi_\pi + \frac{1}{\beta}\right)\pi_t$$

$$+\hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{(1-\beta-\beta b\psi_\pi)x_{T+1} + b[1,1,0,1-\beta^2\psi_\pi]\,s_T\right\}$$

which is NOT what I have at all! LOL!

And using this, you can also rewrite the NKPC

$$\pi_t - \kappa x_t = \hat{E}_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\,\hat{E}_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\pi_{T+2}$$

$$+ [0,0,1,0]\,(I_{nx} - \alpha\beta h_x)^{-1} s_t$$

Note that the linking equations do exactly this.

So if I just write the expectations separately, but impose the linking eqs L1 & L2, do I get the same?

MN, Step 1:

$$x_t = -\sigma(\psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t)$$

$$+ \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta) x_{\tau+1} + \sigma \pi_{\tau+1} - \sigma\beta(\psi_\pi \pi_\tau + \psi_x x_{\tau+1} + \bar{i}_{\tau+1}) + \sigma r_\tau^N \right\}$$

$$(1+\sigma\psi_x) x_t = -\sigma\psi_\pi \pi_{t-1}$$

$$+ \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta-\beta\sigma\psi_\pi) x_{\tau+1} \underbrace{+\sigma\pi_{\tau+1} - \sigma\beta\psi_\pi \pi_\tau}_{\sigma\pi_{\tau+1} - \sigma\psi_\pi \rho i d_t \text{ (pulling in } \pi_{t-1})} + \sigma[1,-1,0,0] s_\tau \right\}$$

$$(1+\sigma\psi_x) x_t = \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta-\beta\sigma\psi_\pi) x_{\tau+1} + \sigma\pi_{\tau+1} + \sigma[1,-1,0,-\psi_\pi] s_\tau \right\}$$

$$+ L1$$

Yes!

$$1 + \frac{(1-\alpha)\beta}{\alpha\beta} = \frac{\alpha + (1-\alpha)}{\alpha} = \frac{1}{\alpha}$$

okidoki, this is pure tov!

(3) <u>il</u>: here I'll only check subopt & opti fixtures.

First: subopt fixtures

$$x_t = -bi_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + b\pi_{T+1} - b\beta i_{T+1} + b r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t + \rho i_{t-1}$$

<u>PQ only</u>

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [b, 1-\beta, -b\beta] f_\beta + b[1,0,0,0](I_{4\times} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{4\times} - \alpha\beta h_x)^{-1} s_t \\ [0, 1, 0, \rho] s_t \end{bmatrix}$$

This initial setup is true for both info assumptions. The two will only change the (*)-condition which relates $f_\beta(3)$ to $f_\beta(1)$ & $f_\beta(2)$, and in the case of "opti fixtures", to $s(4)$

$$f_\beta(3) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \bar{i}_{T+1} + \rho i_T$$

$$= \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \hat{E}_t(\bar{i}_{t+1} + \beta \bar{i}_{t+2} + \dots) + \rho \hat{E}_t \sum_{T=t}^{\infty} i_T$$

$$= \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \frac{1}{\beta} \hat{E}_t(\bar{i}_t + \beta \bar{i}_{t+1} + \dots) - \frac{1}{\beta} \bar{i}_t + \rho [i_t + \beta i_{t+1} + \dots]$$

$$= \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \frac{1}{\beta} \hat{E}_t(\bar{i}_t + \beta \bar{i}_{t+1} + \dots) - \frac{1}{\beta} \bar{i}_t + \rho\beta [i_{t+1} + \beta i_{t+2} + \dots] + \rho i_t$$

$f_\beta(3)$

$$= \gamma_\pi f_\beta(1) + \gamma_x f_\beta(2) + \frac{1}{\beta}\hat{E}_t\left(\bar{i}_t + \beta \bar{i}_{t+1} + \dots\right) - \frac{1}{\beta}\bar{i}_t + \rho\beta\overbrace{\left[i_{t+1} + \beta i_{t+2} + \dots\right]}^{f_\beta(3)} + \rho i_t$$

$$(1-\rho\beta) f_\beta(3) = -\text{''}-$$

$$f_\beta(3) = \frac{1}{1-\rho\beta}\left(\gamma_\pi f_\beta(1) + \gamma_x f_\beta(2) + \frac{1}{\beta}\left\{[0,1,0,0](I_{nx} - \beta h_{xx})^{-1} s_t - [0,1,0,0] s_t\right\} + \rho i_t\right)$$

The (*)-condition is the same.

Use this on the $-b\beta f_\beta(3) = -b\beta \cdot \left(\frac{\rho}{1-\rho\beta} i_t\right)$ is on the RHS $= \underline{\frac{b\beta\rho}{1-\rho\beta}}$ → so on the LHS we'll have

$$\Rightarrow b + \frac{b\beta\rho}{1-\rho\beta}$$

→ subopt is correct!

Now: "opt factors"

$$f_\beta(3) = \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} i_{\tau+1} = \hat{E}_t\left[i_{t+1} + \beta i_{t+2} + \beta^2 i_{t+3} + \dots\right]$$

$$= \frac{1}{\beta^2}\left[i_{t-1} + \beta i_t + \beta^2 i_{t+1} + \dots\right] - \frac{1}{\beta^2} i_{t-1} - \frac{1}{\beta} i_t$$

$$= \frac{1}{\beta^2}\left\{[0,0,0,1](I_{nx} - \beta h_{xx})^{-1} s_t - [0,0,0,1] s_t\right\} = L1' \checkmark$$

And on the RHS we have $-b\beta f_\beta(3) = b\beta \frac{1}{\beta} i_t = b i_t$

→ so on the LHS we'll have $(b - b) i_t = 0$.

LHS: $b - \left[-b\beta\left(-\frac{1}{\beta}\right)\right] = b - b = 0$. $\Rightarrow -2b$ was wrong!

$P(1,3) = 0$.

Ok so we've checked thru everything - it should be all correct! So now: E-stability & interpretation of phs.

conclusions / questions

1) Epi is never E-stable

I think this is b/c although i could dampen things by reacting to $E(\pi)$, $E(\pi)$ internalizes this

→ feedback effects

2) Why is myopic the only unstable for pil, why is it the only stable for il?

- I think pil explodes for myopic b/c $E(\pi)$ are governed by gx, so explosive, and $E(pil)$ are governed by hx, so stable, so they don't counterbalance.
- For pil the opposite happens b/c when $E(i)$ & $E(il)$ are governed by the same thing, then ppl internalize that i is unstable, either in an oscillatory way (gx) or in a smooth way (hx). By contrast, when they don't

realize that $i = i^d$, then they think that i is driven by something stable, so it's balanced out.

$\Rightarrow$ it seems like behind all of this are (at least initially) unstable $E(\cdot)$ that need to be balanced out somehow. When the TR fails to do that, you explode, either smoothly or via oscillations.

Somehow I'm not so hot on deriving E-stab. analytically, maybe I'm just tired. I'm more fixated on seeing where to go from here, and what we learn.

Preston (2005, p. 112) suggests that E-stability requires that the TR itself be not an additional source of instability.

→ but that is exactly what it seems to be when for i^d the agents recognize $i = i^d$, b/c this opens a new feedback

→ and also for p^d when agents don't recognize $\pi = p^d$

↳ then it's as if they missed a dampening, "negative" feedback.

il (as opposed to pil) makes the TR recursive which generates instability unless you shut off recursiveness by making agents not realize it! (i.e. when they don't internalize that $il = i$, then you effectively shut off the recursiveness.)

The situ is different for pil b/c it doesn't make the TR recursive. Instead, when agents don't realize that $\pi = pil$, then the TR is effectively independent of endog stuff ($\gamma_x = 0$) and thus not E-stable, as Preston found.

$\Rightarrow$ does pil, fosting using h_x for both ("opt. fosters") work when agents are learning h_x too?

Overshooting comes from $E(\pi)$ moving around a lot and pushing around $E(i)$ due to knowledge of the TR.

$\Rightarrow$ So actually a cheap way to make 'em not know the

TR is simply not to impose (*)!

So take the baseline for example

$$x_t = -b i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + b \pi_{T+1} - b\beta i_{T+1} + b r_T^n \right\}$$

$$\pi_t = k x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ k\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

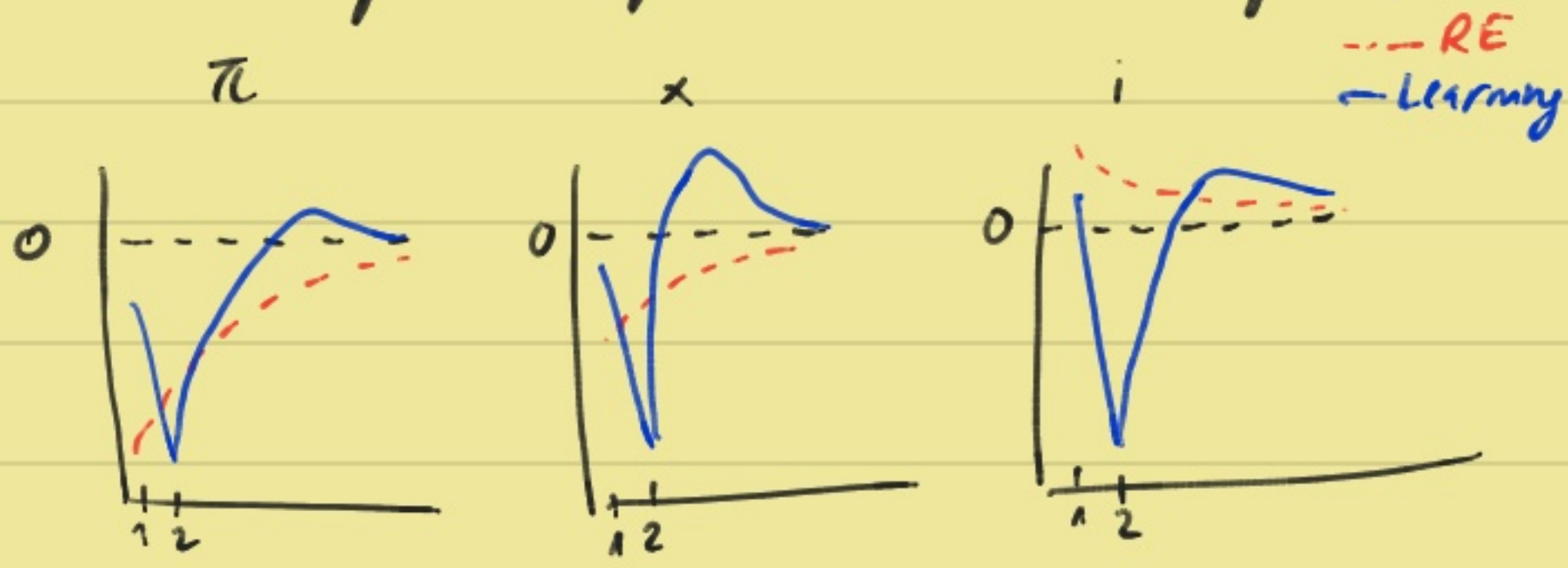
PQ

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -k & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} = \begin{bmatrix} [b, 1-\beta, -b\beta] f_\beta + b[1,0,0](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + [0,0,1] s_t \\ [0,1,0] s_t \end{bmatrix}$$

and no (*) imposed!

→ <u>12i</u>

↳ very interesting IRFs! (constant-only PLM)



→ You can literally see the overshooting be born as they learn the TR..!

Ryan meeting 15 Jan 2020

Macro seminars this semester: Last date: Apr 23 or 30?

→ CEMP might be next semester so you're good.

Josephine de Karman: Needs to be received by Jan 31

in one package by snail mail. You'll need to sign

my application document too!

CSWEP Summer due Feb 1 → gonna try Boston Fed

↳ I'll send the request soon.

Is it an issue that for pil, mon pol shock isn't 1

on impact?

→ looks like a scaled-down version of RE.

Not knowing the TR is like Angeletos: "agents don't

understand GE".

check in data whether EE learning fits data better

check Fabian Winkler's work → he'll have dealt w/ these issues!

<u>Work after</u>

Looking at lit w/ estimation of learning and Winkler:

- <u>Winkler 2019</u>: Beautifully <u>oscillating IRFs</u>! p. 12 (Fig 2)

Should try int. rate smoothing as <u>16 Jan 2020</u>

- $i_t = \rho i_{t-1} + (1-\rho)[\psi_\pi \pi_t + \psi_x x_t] + \varepsilon_t$ (Milani 2007, p. 4 Mac)

- Indexation in NKPC (<u>Milani 2004b</u> p 27 Mac)
- set κ^* = optimal κ from Kalman filter = relative variances of transitory & permanent shocks (Orphanides & W, 2005 footnote 18, p. 13 Mac)

- <u>Durig & Llepez</u>'s generalized Taylor principle suggests that maybe I could have an alternating, state-contingent TR.
- or have initial beliefs different (not RE) like Lubik
  - Add burn-in!

- Milani (2004b) finds for LH learning a gain of 0.028
  - also contrasts EE & LH learning, and fits the fit of the former better.

- Slobodyan & Wouters' "VAR learning" may be similar to my "let them learn hx too".
  It seems like VAR learning uses a smaller set of observables in the RLS algorithm than MSV.
  ↳ could also be an alternative to consider

- Evans & Honkapohja 2003 do "3-horizon learning" and take differentiable from $\tilde{E}$ no problem :)
  But they have a Kalman-filter type of "distinguish temporary from persistent component" learning.
  → not entirely sure about distinction btwn constant gain learning and the Kalman filter.

- Preston et al "Limits": int. rate expectations induce instability in the model ⇒ aggressive policy leads to learning about i which counteracts instability ⇒ related to what I find!

- Evagorou & Preston (2011, AER) actually do compare LH & EE, finding that EE yields much worse properties (no amplification or propagation, moments are too close to RE) and they also cite another paper to reach similar conclusions (p. 25 Mac)

  But Lubik also gets story like this: EE learning only matters a lot when initial beliefs are not RE.

- Evagorou & Preston (2018) also has stuff that resembles "limits" and my instability results. But finds that you should be more aggressive on π.

  But Result 6 (p. 30 Mac) makes clear that it's learning i and not the internalization of $E(\pi) \to E(i)$ that drives this, so when $i_t \uparrow \to$ agents expect LR-i to be higher ($\to$ in my case, lower b/c $E(\pi) \downarrow$)

- Graham: as I remembered, he prefers LH to EE b/c the former converges faster. EE is more volatile than LH.

Peter meeting 16 Jan 2020

- Modify the NKPC
- Dring & Leeper : $\psi_\pi = 1.01$ would that work?
- Learn hx

  Somehow add states as "copies" to jump vector

  → some accounting trick to relabel stocks as flows

  Like you can rewrite an AR(1) WLOG to AR(2), there might be some trick to rewrite the eq system $\begin{cases} \text{states} \\ \text{flows} \end{cases}$ as one equation $\begin{bmatrix} \text{states} \\ \text{flows} \end{bmatrix}$ and then use the existing learning machinery on that matrix.
- low gain is ok: if high gain produces these counterfactual IRFs, then an MLE would give you a low gain estimate to make sure that IRFs align w/ those in data.
- not spend too much time (more than 1 month)

  ⇒ You want the best model possible, yes, but don't want to spend too much time finding it!

Work after

Interest rate smoothing w/

$$i_t = \rho i_{t-1} + (1-\rho)(\psi_\pi \pi_t + \psi_x x_t) + \bar{i}_t$$

"weighted il"

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma\beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

PQ

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ \underline{-(1-\rho)\psi_\pi} & \underline{-(1-\rho)\psi_x} & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + \sigma [1,0,0,0] (I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0] s_t \\ [0,1,0,\rho] s_t \end{bmatrix}$$

And (*) will be different

$$f_\beta(3) = \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} i_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \overset{\beta^{T-t}}{\rho} i_t + (1-\rho)\psi_\pi \pi_{T+1} + (1-\rho)\psi_x x_{T+1} + \bar{i}_{T+1}$$

→ should be the same as previously except that instead of ψ_π and ψ_x, we'll have $(1-\rho)\psi_\pi$ and $(1-\rho)\psi_x$.

→ Implement via changing the params to $(1-\rho)\psi$.

Identical: doesn't work! ☹

What's interesting tho is that this effectively makes $\psi_\pi < 1$, but

RE still doesn't complain! And thus you can for subopt. fosters w/ il get damped oscillations.

Analog to Kalman gain:

$k = P\, gx'\, \Omega^{-1}$ (evaluated in st st, so I'll just take gx^{RE})

FEV(states) FEV(observables)

↑ gonna be easier since hx is known

↳ $\Omega = gx\, P\, gx' + R$

↑ Cov(meas. err) = 0 here.

↳ $= \Sigma_x$, unconditional variance of states

$\Rightarrow k^* = \Sigma_x \cdot gx' \left(gx\, \Sigma_x\, gx' \right)^{-1}$

So of course I'm getting a 3x3 and I don't know how to interpret it. I think this isn't conceptually right b/c "it's the other way around": states are known and jumps aren't.

Let's tackle it from another angle:

$$k = \beta^{proj} = \frac{Cov(X,Y)}{Var(X)}$$

In the learning updating step

$\phi' = \phi + k(FE)$ so $Y = \phi$ and $X = FE$

So $$k = \frac{Cov(\phi, FE)}{Var(FE)} = \frac{Cov(\phi, FE)}{FEV}$$

And I see the issue: neither ϕ, nor FE, nor FEV is constant $\rightarrow$ k varies in every iteration, in every history, depending on shocks.

And by the way

$$k = \frac{Cov(FE, \phi)}{Var(FE)} = \overbrace{P g x'}^{Cov(FE,\phi)} \cdot \Omega^{-1} \quad \rightsquigarrow \frac{1}{Var(FE)}$$

↳ But what one could try is to use, in every t,

$$k = P_{t|t-1} g x' \Omega_{t|t-1}^{-1}$$

or some sensible analogy thereof. I wonder if that would amount to decreasing gains? I guess so if the learning is E-stable.

I've decided not to pursue the Kalman gain thing for now. Instead, the π-indexation in NKPC. We know from CEMP (and b/c we've derived it) that the NKPC w/ exog markups & indexation is:

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$$\pi_t - \gamma\pi_{t-1} = \mu_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\xi_p s_T + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) \right]$$

Ok. Rewrite a bit:

$$\pi_t - \gamma\pi_{t-1} = \mu_t + \xi_p s_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\xi_p \alpha\beta s_{T+1} + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) \right]$$

My NKPC looks like

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \right]$$

So w/ indexation, my NKPC will be:

$$\pi_t - \gamma\pi_{t-1} = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) + u_T \right]$$

Now the question is: in the baseline model, how to write the compact notation? In particular, we run into the usual info assumption question: do agents realize that $\hat{\pi} = \pi_{t-1}$, and if yes, what do they use to forecast it?

Baseline w/ indexation:

$$x_t = -b i_t + \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta) x_{\tau+1} + b\pi_{\tau+1} - b\beta i_{\tau+1} + b r_\tau^n \right\}$$

$$\pi_t - \gamma \pi_{t-1} = \kappa x_t + \hat{E}_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \left[\kappa\alpha\beta x_{\tau+1} + (1-\alpha)\beta(\pi_{\tau+1} - \gamma \pi_\tau) + u_\tau \right]$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

So yes, π_{t-1} is another state, so

<u>DQ</u>

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [b, 1-\beta, -b\beta] f_\beta + b[1,0,0,0](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t \underline{- \Sigma (1-\alpha)\beta\gamma \pi_t + \gamma \pi_{t-1}} \\ [0,1,0,0] s_t \end{bmatrix}$$

this is where the info assumptions come to bear.

<u>MN</u>

$$x_t = -b(\psi_\pi \pi_t + \psi_x x_t + \bar{i}_t)$$
$$+ \hat{E}_t \sum_{\tau=1}^{\infty} \beta^{\tau-t} \left\{ (1-\beta) x_{\tau+1} + b\pi_{\tau+1} - b\beta(\psi_\pi \pi_{\tau+1} + \psi_x x_{\tau+1} + \bar{i}_{\tau+1}) + b r_\tau^n \right\}$$

$$b\psi_\pi \pi_t + (1 + b\psi_x) x_t =$$
$$\hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta-\beta b\psi_x) x_{\tau+1} + b(1-\beta\psi_\pi)\pi_{\tau+1} + b[1,-1,0,0] s_\tau \right\}$$

(same as baseline)

But the NKPC will be diff.

$$\pi_t - \gamma \pi_{t-1} = \kappa x_t + \hat{E}_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \left[\kappa\alpha\beta x_{\tau+1} + (1-\alpha)\beta(\pi_{\tau+1} - \gamma \pi_\tau) + u_\tau \right]$$

$$\pi_t - \kappa x_t = \gamma \pi_{t-1} + \hat{E}_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \left[\kappa\alpha\beta x_{\tau+1} + (1-\alpha)\beta \pi_{\tau+1} - (1-\alpha)\beta\gamma \pi_\tau \right]$$

$$+ [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$

Now the diff info assumptions on RHS:

"Myopic":

$$\gamma \pi_{t-1} + [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$

$$- (1-\alpha)\beta\gamma \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} pil_{\tau+1}$$

$$= \gamma \pi_{t-1} + [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$

$$- (1-\alpha)\beta\gamma \left[pil_{t+1} + \beta pil_{t+2} + \dots \right]$$

$$= \gamma pil_t + [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$

$$- (1-\alpha)\beta\gamma \frac{1}{\alpha\beta} \left[pil_t + \alpha\beta pil_{t+1} + (\alpha\beta)^2 pil_{t+2} + \dots \right] + (1-\alpha)\frac{\gamma}{\alpha} pil_t$$

$$= [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$

$$- (1-\alpha)\frac{\gamma}{\alpha} [0,0,0,1](I_{nx} - \alpha\beta h_x)^{-1} s_t + \underbrace{\left(\gamma + (1-\alpha)\frac{\gamma}{\alpha}\right)}_{\frac{\cancel{\gamma\alpha} + \gamma - \cancel{\alpha\gamma}}{\alpha} = \frac{\gamma}{\alpha}} pil_t$$

So

$$\pi_t - \kappa x_t = [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$

$$- (1-\alpha)\frac{\gamma}{\alpha} [0,0,0,1](I_{nx} - \alpha\beta h_x)^{-1} s_t + [0,0,0,\tfrac{\gamma}{\alpha}] s_t$$

$$\pi_t - kx_t = [(1-\alpha)\beta,\ k\alpha\beta,\ 0]\, f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$

$$-(1-\alpha)\frac{\gamma}{\alpha}[0,0,0,1](I_{nx} - \alpha\beta h_x)^{-1} s_t + [0,0,0,\tfrac{\gamma}{\alpha}]\, s_t$$

$$\Rightarrow \pi_t - kx_t = [(1-\alpha)\beta,\ k\alpha\beta,\ 0]\, f_\alpha + [0,0,1,-\tfrac{(1-\alpha)\gamma}{\alpha}](I_{nx} - \alpha\beta h_x)^{-1} s_t + [0,0,0,\tfrac{\gamma}{\alpha}]\, s_t \quad \text{(myopic)}$$

"Suboptimal fisters"

$$\pi_t - kx_t = \gamma \pi_{t-1} + \hat{E}_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \left[k\alpha\beta x_{\tau+1} + (1-\alpha)\beta\, \pi_{\tau+1} - (1-\alpha)\beta\gamma\, \pi_\tau \right] + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$

$$\pi_t - kx_t = \gamma \pi_{t-1} - (1-\alpha)\beta\gamma\, \pi_t + \hat{E}_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \left[k\alpha\beta x_{\tau+1} + (1-\alpha)\beta\, \pi_{\tau+1} - (1-\alpha)\beta\gamma(\alpha\beta)\, \pi_{\tau+1} \right] + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$

$$[1 + (1-\alpha)\beta\gamma]\, \pi_t - kx_t = \gamma \pi_{t-1} + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t + \hat{E}_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \left[k\alpha\beta x_{\tau+1} + (1-\alpha)\beta(1-\alpha\beta\gamma)\, \pi_{\tau+1} \right] \quad (\star)$$

$$[1 + (1-\alpha)\beta\gamma]\, \pi_t - kx_t = \gamma \pi_{t-1} + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t + [(1-\alpha)\beta(1-\alpha\beta\gamma),\ k\alpha\beta,\ 0]\, f_\alpha \quad \text{(subopt fisters)}$$

For "opt fixtes", you just go back to step (⊠):

$$[1+(1-\alpha)\beta\gamma]\pi_t - kx_t = \pi_{t-1} + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[k\alpha\beta x_{T+1} + (1-\alpha)\beta(1-\alpha\beta\gamma)\pi_{T+1} \right]$$

and rewrite $\hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \pi_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} pil_{T+2}$

$$= pil_{t+2} + \alpha\beta\, pil_{t+3} + \dots$$
$$= \frac{1}{(\alpha\beta)^2}\left[pil_t + \alpha\beta\, pil_{t+1} + (\alpha\beta)^2 pil_{t+2} + \dots \right] - \frac{1}{(\alpha\beta)^2} pil_t - \frac{1}{\alpha\beta} pil_{t+1}$$

So the eq becomes:

$$[1+(1-\alpha)\beta\gamma]\pi_t - kx_t = \pi_{t-1} + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$
$$+ [0, k\alpha\beta, 0] f_\alpha$$
$$+ (1-\alpha)\beta(1-\alpha\beta\gamma)\left\{ \frac{1}{(\alpha\beta)^2} [0,0,0,1](I_{nx} - \alpha\beta h_x)^{-1} s_t - \frac{1}{(\alpha\beta)^2} pil_t - \frac{1}{\alpha\beta} pil_{t+1} \right\}$$

which is:

$$[1+(1-\alpha)\beta\gamma]\pi_t - kx_t = \pi_{t-1} + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t$$
$$+ [0, k\alpha\beta, 0] f_\alpha + (1-\alpha)\beta(1-\alpha\beta\gamma) \frac{1}{(\alpha\beta)^2} [0,0,0,1](I_{nx} - \alpha\beta h_x)^{-1} s_t$$
$$- (1-\alpha)\beta(1-\alpha\beta\gamma)\frac{1}{(\alpha\beta)^2}\pi_{t-1} - (1-\alpha)\beta(1-\alpha\beta\gamma)\frac{1}{\alpha\beta}\pi_t$$

$$\boxed{\begin{aligned} &\left[1+(1-\alpha)\beta\gamma + (1-\alpha)\beta(1-\alpha\beta\gamma)\frac{1}{\alpha\beta}\right]\pi_t - kx_t = \left[\gamma - (1-\alpha)\beta(1-\alpha\beta\gamma)\frac{1}{(\alpha\beta)^2}\right]\pi_{t-1} \\ &+ [0, k\alpha\beta, 0] f_\alpha + \left[0,0,1,(1-\alpha)\beta(1-\alpha\beta\gamma)\frac{1}{(\alpha\beta)^2}\right](I_{nx} - \alpha\beta h_x)^{-1} s_t \end{aligned}}$$

(opti fixtes)

So I'm not gonna write these up on Latex (yet),
but in Mathematica I'll call 'em

materials13indexation_myopic.nb

materials13indexation_subopt.nb

materials13indexation_opt.nb

=> and analogously in Matlab.

What I need to do still is a RE-version w/ hybrid MKK.

Do the "pull out 1 term at a time" thing (Preston, p. 95)

First for Preston's NKIS: (for practice) Use RE-E(·)

$$x_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta) x_{\tau+1} - \sigma(i_\tau + \pi_{\tau+1}) + r_\tau^n \right\}$$

lead & take E(·)

$$E_t x_{t+1} = E_t E_{t+1} \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \left\{ (1-\beta) x_{\tau+1} - \sigma(i_\tau + \pi_{\tau+1}) + r_\tau^n \right\}$$

So $E_t x_{t+1} = E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \left\{ (1-\beta) x_{\tau+1} - \sigma(i_\tau + \pi_{\tau+1}) + r_\tau^n \right\}$ (1)

Now take x_t and pull all $t+1$ terms out (and t)

$$x_t = E_t \left[(1-\beta) x_{t+1} - \sigma(i_t + \pi_{t+1}) + r_t^n \right] + \underbrace{E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \beta \left\{ (1-\beta) \cdot x_{\tau+1} - \sigma(i_\tau + \pi_{\tau+1}) + r_\tau^n \right\}}_{= \beta E_t x_{t+1} \text{ by (1)}} \checkmark$$

Now do the same thing for the NKPC w/ indexation:

$$\pi_t - \gamma\pi_{t-1} = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) + u_T \right]$$

Rearrange a bit and replace $\hat{E}$ w/ E:

$$\pi_t - \gamma\pi_{t-1} = E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa x_T + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) + u_T \right]$$

Lead and take E_t:

$$E_t\left[\pi_{t+1} - \gamma\pi_t\right] = E_t \sum_{T=t+1}^{\infty} (\alpha\beta)^{T-t} \left[\kappa x_T + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) + u_T \right] \quad (1)$$

Now take out first terms in NKPC

$$\pi_t - \gamma\pi_{t-1} = E_t\left[\kappa x_t + (1-\alpha)\beta(\pi_{t+1} - \gamma\pi_t) + u_t\right]$$

$$+ (\alpha\beta) \cdot E_t \sum_{T=t+1}^{\infty} (\alpha\beta)^{T-t} \left[\kappa x_T + (1-\alpha)\beta(\pi_{T+1} - \gamma\pi_T) + u_T \right]$$

$$= \kappa x_t + (1-\alpha)\beta\left(E_t\pi_{t+1} - \gamma\pi_t\right) + u_t$$

$$+ \alpha\beta \cdot \underbrace{E_t\left[\pi_{t+1} - \gamma\pi_t\right]}_{\text{by (1)}}$$

$$\Rightarrow \pi_t - \gamma\pi_{t-1} = \kappa x_t + \beta\left[E_t(\pi_{t+1}) - \gamma\pi_t\right] + u_t$$

$$\pi_t = \gamma\pi_{t-1} + \kappa x_t + \beta E_t(\pi_{t+1}) - \beta\gamma\pi_t + u_t$$

$$(1+\beta\gamma)\pi_t = \gamma\pi_{t-1} + \kappa x_t + \beta E_t(\pi_{t+1}) + u_t$$

$$\pi_t = \frac{1}{1+\beta\gamma}\left[\gamma\pi_{t-1} + \kappa x_t + \beta E_t\pi_{t+1} + u_t\right] \quad \text{RE-NKPC}$$

for indexation.

Oh wonder! Subopti fcst w/ indexation seems to work! The reason I'm focusing on that is b/c

1) it's not dumb like the myopic info ass
2) it doesn't do away w/ learning
(opti also works)

⇒ So then next step is to check using PQ whether the model solutions for subopt & opt fcsters are indeed correct.

<u>PQ</u>

Ok so the baseline model w/ indexation is:

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma \beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t - \gamma \pi_{t-1} = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa \alpha \beta x_{T+1} + (1-\alpha)\beta \left(\pi_{T+1} - \gamma \pi_T \right) + u_T \right]$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

$$\Rightarrow \pi_t = \gamma \pi_{t-1} + \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa \alpha \beta x_{T+1} + (1-\alpha)\beta \left(\pi_{T+1} - \gamma \pi_T \right) + u_T \right]$$

$$\pi_t = \gamma \pi_{t-1} - (1-\alpha)\beta\gamma \pi_t + \kappa x_t + \hat{E}_t \left[\kappa \alpha \beta x_{T+1} + (1-\alpha)\beta(1-\alpha\beta\gamma) \pi_{T+1} + u_T \right]$$

$$\left[1 + (1-\alpha)\beta\gamma \right] \pi_t + \kappa x_t = \gamma \pi_{t-1} + \left[(1-\alpha)\beta(1-\alpha\beta\gamma), \kappa\alpha\beta, 0 \right] f_\alpha + \left[0, 0, 1, 0 \right] [A]_{\alpha x}$$

This means

$$\begin{pmatrix} 0 & 1 & b \\ 1+(1-\alpha)\beta\gamma & -k & 0 \\ -\psi_\pi & -\psi_x & 1 \end{pmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [b, 1-\beta, -b\beta] f_\beta + b[1,0,0,0](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta(1-\alpha\beta\gamma), k\alpha\beta, 0] f_\alpha + [0,0,1,0](I_{nx} - \alpha\beta h_x)^{-1} s_t + [0,0,0,\gamma] s_t \\ [0,1,0,0] s_t \end{bmatrix}$$

s.t. (*) and linking equations

In subopt, I only need (*), linking $f_\beta(3)$ to $f_\beta(1)$ & $f_\beta(2)$.

In opt fcters, I need (*) and L1 & L2, linking $f_\alpha(1)$ & $f_\beta(1)$ to the shocks.

$$(*) \quad \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} i_{T+1} = \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \bar{i}_{T+1}$$

the last term is $\bar{i}_{t+1} + \beta \bar{i}_{t+2} + \dots = \frac{1}{\beta}\left[\bar{i}_t + \beta \bar{i}_{t+1} + \dots\right] - \frac{1}{\beta}\bar{i}_t$

$$(*) \quad f_\beta(3) = \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \frac{1}{\beta}\left\{[0,1,0,0](I_{nx} - \beta h_x)^{-1} s_t - [0,1,0,0] s_t\right\}$$

(as in baseline)

Looking:

$$fa(1) = \hat{E}_1 \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \pi_{T+1} = \hat{E}_1 \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} pi_{T+2}$$

$$= pi_{t+2} + \alpha\beta\, pi_{t+2} + \dots$$

$$= \frac{1}{(\alpha\beta)^2}\left[pi_t + \alpha\beta\, pi_{t+1} + (\alpha\beta)^2 pi_{t+2} + \dots\right] - \frac{1}{(\alpha\beta)^2} pi_t - \frac{1}{\alpha\beta} pi_{t+1}$$

L1 & L2

$$fa(1) = \frac{1}{(\alpha\beta)^2}[0,0,0,1](I_{nx} - \alpha\beta h_x)^{-1} s_t - \frac{1}{(\alpha\beta)^2}[0,0,0,1] s_t - \frac{1}{\alpha\beta}\pi_t$$

$$f_\beta(1) = \frac{1}{\beta^2}[0,0,0,1](I_{nx} - \alpha\beta h_x)^{-1} s_t - \frac{1}{\beta^2}[0,0,0,1] s_t - \frac{1}{\beta}\pi_t$$

$\Rightarrow$ For "opt," we'll need to modify ρ

$$P(1,1) = 0 - [z f_\beta(1) + -z\beta \cdot f_\beta(\beta)] = -z f_\beta(1) + z\beta\psi_\pi f_\beta(1)$$

$$= z(\beta\psi_\pi - 1)\left(-\frac{1}{\beta}\right) = \underline{\underline{\frac{z}{\beta}(1-\beta\psi_\pi)}}$$

$$P(2,1) = 1 + (1-\alpha)\beta\gamma + (1-\alpha)\beta(1-\gamma) f_\alpha(1) = 1 + (1-\alpha)\beta\gamma + (1-\alpha)\beta(1-\gamma)\left(-\frac{1}{\alpha\beta}\right)$$

$$= \underline{\underline{1 + (1-\alpha)\beta\gamma - (1-\alpha)\beta(1-\gamma)\frac{1}{\alpha\beta}}}$$

"Subopt fisers" works and IRF is ok!

Crazy! X Unfortunately, that's not right: I had kept $\psi_\pi = 1.01$, that's why IRFs were nice!

My notes here for "opt fest" may not be quite correct, but what I have found is that you can simply take the MN & PQ sols of "subopt fest" and simply add the linking equations L1 & L2 to them.

The only thing you need to do is to modify $P(1,1)$ & $P(2,1)$ $M(1,1)$ & $M(2,1)$

$$M11RHS = z\cdot(1-\beta\cdot\psi)f_\beta(1) = -z(1-\beta\psi)\frac{1}{\beta} \qquad (L1)$$

$$M21RHS = (1-\alpha)\beta(1-\alpha\beta\gamma)f_\alpha(1) = -(1-\alpha)\beta(1-\alpha\beta\gamma)\frac{1}{\alpha\beta} \qquad (L2)$$

$$P11RHS = z f_\beta(1) - z\beta f_\beta(3) = z(1-\beta\psi)f_\beta(1) = -z(1-\beta\psi)\frac{1}{\beta}$$

$$= M11RHS$$

$$P21RHS = (1-\alpha)\beta(1-\alpha\beta\cdot\gamma)f_\alpha(1) = M21RHS$$

$\Rightarrow$ IRFs are good too! X

Indexation is what it takes to get rid of the overshooting ...

No it doesn't → it doesn't work either ☹

Alright, so then I won't do more extensions, at least not for now. Instead I'll turn to the issue of learning h_x.

Learning hx

18 Jan 2020

Let's go back to the baseline version.

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma \beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa \alpha \beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

Instead of the old compact form, $z_t = A_a f_a + A_b \cdot f_b + A_c \cdot s_t$,

let's write it like this: $z_t = A_a f_a + A_b \cdot f_b + B_a \cdot g_a + B_b \cdot g_b + A_c \cdot s_t$

where g_a & g_b are LH-expectations of the states.

Let's do PQ:

$$\begin{bmatrix} 0 & 1 & \sigma \\ 1 & -\kappa & 0 \\ -\psi_\pi & -\psi_x & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [\sigma, 1-\beta, -\sigma\beta] f_\beta + [\sigma, 0, 0] g_\beta \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0, 0, 1] g_\alpha \\ [0, 1, 0] s_t \end{bmatrix}$$

Several questions:

1) If hx is learned, what are agents regressing on? Still s_t?

An alternative way to ask this question is: why weren't agents regressing z_t on past values, z_{t-1}? $\Rightarrow$ b/c that's

wrt the minimum-state-variable (MSV) solution of the model. ⇒ Slobodyan & Wouters' "VAR-learning" goes in this direction where agents use past jumps too. My hunch is that if learning converges, then in the end the coeffs on z_{t-1} should $\to 0$ (or to $gx\, hx\, gx^{-1}$) b/c in the state space

$$\left.\begin{aligned} Y_t &= gx\, X_t \\ X_t &= hx\, X_{t-1} + \eta\, \varepsilon_t \end{aligned}\right\} \quad Y_t = gx\, hx \underbrace{X_{t-1}}_{gx^{-1} Y_{t-1}} + \eta\, \varepsilon_t$$

Y_t doesn't depend on Y_{t-1} (or only indirectly).

Slobodyan & Wouters' VAR-learning lets agents learn gx by regressing not on X_t but on Y_{t-1}. The argument is that states are <u>un</u>observed and observables are, well, observed (in reality). The MSV-sol learning implicitly assumes that at least current states are observed. So in MSV learning, agents use a subset of observed vars to learn from b/c somehow they know the functional form of the state-space and so they know that adding observables to their regressor wouldn't add info. →

So maybe it's fine if also to test X agents use past values of X. So I keep the same regressor vector.

But the states s_t are still exogenous... So they will still develop according to $X_t = h_x X_{t-1} + \eta \varepsilon_t$; only the endog variables will be affected by the estimated $\hat{h_x}$ and thus by $\hat{E}_t X_{t+k}$ $k \geq 1$.

So maybe all I need to change is

Mathematica materials13 _learnhx_baseline.nb

1. compact notation to $z_t = A_a f_a + A_b f_b + B_a g_a + B_b g_b + A_c s_t$
2. ysim in learning code will be $\begin{bmatrix} z_t \\ s_t \end{bmatrix}$ → sim_learnUH_learnhx.m
3. fafb_gagb.m which will give the UH expectations of states too analytically.

1. Sol of model PQ

$$(*) \quad \hat{E}_t \sum_{\tau=1}^{\infty} i_{\tau+1} = \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \bar{i}_{\tau+1}$$

$$\underline{\underline{f_\beta(3) = \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + g_\beta(2) \qquad (*)}}$$

→ which highlights that I need to modify the system to have $\hat{E}_t s_{t+1}$

$$x_t = -b i_t + \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta) x_{\tau+1} + b\pi_{\tau+1} - b\beta i_{\tau+1} + b r_\tau^n \right\}$$

$$\pi_t = k x_t + \hat{E}_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \left\{ k\alpha\beta x_{\tau+1} + (1-\alpha)\beta \pi_{\tau+1} + u_\tau \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

$$\Rightarrow x_t = -b i_t + b r_t^n + \hat{E}_t \sum_{\tau=t}^{\infty} \left\{ (1-\beta) x_{\tau+1} + b\pi_{\tau+1} - b\beta i_{\tau+1} + b\beta r_{\tau+1}^n \right\}$$

$$\pi_t = k x_t + u_t + \hat{E}_t \sum_{\tau=t}^{\infty} \left\{ k\alpha\beta x_{\tau+1} + (1-\alpha)\beta \pi_{\tau+1} + \alpha\beta u_{\tau+1} \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -k & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [b, 1-\beta, -b\beta] f_\beta + b\beta [1,0,0] g_\beta + b[1,0,0] s_t \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + \alpha\beta [0,0,1] g_\alpha + [0,0,1] s_t \\ [0,1,0] s_t \end{bmatrix} \quad (PQ)$$

Learnbx Baseline

$$\underline{\underline{f_\beta(3) = \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + g_\beta(2)}} \quad (*)$$

$\beta^{\tau-t}$

MN $$x_t = -b(\psi_\pi \pi_t + \psi_x x_t + \bar{i}_t) + b r_t^n + \hat{E}_t \sum_{\tau=t}^{\infty} \left\{ (1-\beta) x_{\tau+1} + b\pi_{\tau+1} - b\beta(\psi_\pi \pi_{\tau+1} + \psi_x x_{\tau+1} + \bar{i}_{\tau+1}) + b\beta r_{\tau+1}^n \right\}$$

$$b\psi_\pi \pi_t + (1 + b\psi_x) x_t = b[1,-1,0] s_t + \hat{E}_t \sum_{\tau=1}^{\infty} \beta^{\tau-t} \left\{ (1-\beta-b\beta\psi_x) x_{\tau+1} + b(1-\beta\psi_\pi)\pi_{\tau+1} + b\beta[1,-1,0] s_{\tau+1} \right\}$$

So for MN, the system is:

$$\left.\begin{aligned}
&b\psi_\pi \pi_t + (1+b\psi_x)x_t = b[1,-1,0]s_t + \hat{E}_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}\Big\{(1-\beta-b\beta\psi_x)x_{\tau+1} \\
&\quad + b(1-\beta\psi_\pi)\pi_{\tau+1} + b\beta[1,-1,0]s_{\tau+1}\Big\} \\
&\pi_t = kx_t + u_t + \hat{E}_t\sum_{\tau=t}^{\infty}\Big\{k\alpha\beta x_{\tau+1} + (1-\alpha)\beta\pi_{\tau+1} + \alpha\beta u_{\tau+1}\Big\}
\end{aligned}\right\}$$

$\Rightarrow$

$$\begin{bmatrix} b\psi_\pi & 1+b\psi_x \\ 1 & -k \end{bmatrix}\begin{bmatrix}\pi_t \\ x_t\end{bmatrix} = \begin{bmatrix} [b(1-\beta\psi_\pi),(1-\beta-b\beta\psi_x),0]f_\beta + b\beta[1,-1,0]g_\beta + b[1,-1,0]s_t \\ [(1-\alpha)\beta, k\alpha\beta, 0]f_\alpha + \alpha\beta[0,0,1]g_\alpha + [0,0,1]s_t \end{bmatrix}$$

MN learn hx baseline.

coeffs of f_α — coeffs of f_β — coeffs of s

$$A_a = \begin{pmatrix} aa\pi \\ aax \\ aai \end{pmatrix} \qquad A_b = \begin{pmatrix} abp \\ abx \\ abi \end{pmatrix} \qquad A_s = \begin{pmatrix} asx \\ asp \\ asi \end{pmatrix}$$

coeffs of g_a — coeffs of g_b

$$B_a = \begin{pmatrix} bap \\ bax \\ bai \end{pmatrix} \qquad B_b = \begin{pmatrix} bbp \\ bbx \\ bbi \end{pmatrix}$$

3. fafb_gagb.m

need to initialize $b = gx$ in sim_learnLH_learnhx.m

PLM: $\hat{E}_t z_{t+h} = a + b \cdot h_x^{h-1} s_t$

I think what I'll have to do is to split this up as

(PLM_1) $\hat{E}_t z_{t+h} = a + b \hat{E}_t s_{t+h}$ $\qquad \phi = [a, b]$

where $\hat{E}_t s_{t+h} = c + d^h \cdot s_t$ $\qquad (PLM_2)$ $\Gamma = [c, d]$

$\hat{E}_t s_{\tau+1} = c + d^{\tau-t+1} \cdot s_t$ $\qquad$ or stng...

$$g_\beta = \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} s_{\tau+1} = \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ c + d^{\tau-t+1} s_t \right\}$$

$$= \frac{1}{1-\beta} c + \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} d^{\tau-t} \cdot d s_t$$

$$g_\beta = \frac{1}{1-\beta} c + (I_{nx} - \beta d)^{-1} \cdot d s_t$$

$$g_\alpha = \frac{1}{1-\alpha\beta} c + (I_{nx} - \alpha\beta d)^{-1} \cdot d s_t$$

Update 23 Jan 2020

The way I'm treating PLM_1 & $\hat{E}_t s_{t+h}$ makes clear that fa & fb w/o the additional hx-term are really wrong!

Update 29 Jan 2020

$$f_\beta = \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} z_{\tau+1} = \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ a + b \hat{E}_t s_{\tau+1} \right\}$$

Don't panic. Keep fafb_gagb.m

$$\Rightarrow \frac{1}{1-\beta} a + b \cdot \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} s_{\tau+1}$$

$$f_\beta = \frac{1}{1-\beta} a + b \cdot g_\beta \qquad f_\alpha = \frac{1}{1-\alpha\beta} a + b \cdot g_\alpha$$

So IRFs look a <u>lot</u> like IRFs from when they are learning the Taylor-rule!

Initially, endog. vars jump back expectations 1:1, for which reason they drop a lot at period 2. Therefore, the interest rate has to drop a lot too. It only recovers as agents learn the evolution of states and w/ it, the TR.

But why is the state evolution so closely related to the TR?

→ Maybe b/c this is a shock to the TR: Recall that (*) was modified to be $f_\beta(3) = \gamma_\pi f_\beta(1) + \gamma_x f_\beta(2) + g_\beta(2)$

they're learning this

So the story isn't quite the same: it's not that they don't internalize the GE effect, it's only that they are not sure which way the shock will go → they think initially that the CB will keep being discretionarily contractionary! Then they learn that the shock is dying out! Wow!

One thing that's been nagging me: code for baseline

$$x_t = -b i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + b \pi_{T+1} - b\beta i_{T+1} + b r_T^n \right\}$$

$$\pi_t = k x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ k\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t$$

MN

$$x_t = -b(\psi_\pi \pi_t + \psi_x x_t + \bar{i}_t) + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big\{ (1-\beta) x_{T+1} + b \pi_{T+1} + b r_T^n$$
$$- b\beta(\psi_\pi \pi_{T+1} + \psi_x x_{T+1} + \bar{i}_{T+1}) \Big\}$$

$$b\psi_\pi \pi_t + (1+b\psi_x) x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big\{ (1-\beta-b\beta\psi_x) x_{T+1} + b(1-\beta\psi_\pi)\pi_{T+1} + b r_T^n - b\bar{i}_T \Big\}$$

$$\begin{bmatrix} b\psi_\pi & 1+b\psi_x \\ 1 & -k \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [b(1-\beta\psi_\pi), 1-\beta-b\beta\psi_x, 0] f_\beta + b[1,-1,0](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + [0,0,1](I_{nx} - \alpha\beta h_x)^{-1} s_t \end{bmatrix}$$

MN, true-baseline

PQ

PQ, true-baseline

$$\begin{bmatrix} 0 & 1 & b \\ 1 & -k & 0 \\ -\psi_\pi & -\psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [b, 1+\beta, -b\beta] f_\beta + b[1,0,0](I_{nx} - \beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, k\alpha\beta, 0] f_\alpha + [0,0,1](I_{nx} - \alpha\beta h_x)^{-1} s_t \\ [0,1,0] s_t \end{bmatrix}$$

s.t. (*)

$$f_\beta(3) = \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \frac{1}{\beta} \Big\{ [0,1,0](I_{nx} - \beta h_x)^{-1} s_t - [0,1,0] s_t \Big\}$$

Ok, so now try CB's $E(\pi_{t+1})$ in TR

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma \beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa \alpha \beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \gamma_\pi E_t \pi_{t+1} + \gamma_x x_t + \bar{i}_t$$

Two tricky questions arise:

1) How to evaluate $E_t \pi_{t+1}$?

2) How do agents evaluate $E_t \pi_{t+1}$? $\Rightarrow \hat{E}_t E_t \pi_{t+1} = ?$

↳ I think for agents it must hold that $\hat{E}_t E_t \pi_{t+1} = \hat{E}_t \pi_{t+1}$ b/c otherwise we'd be saying that agents think π will go one way but are aware that model-consistent E are going another way. Another way of saying this is to say that agents can't evaluate $E_t \pi_{t+1}$

↳ Although in the criterion, θ, they do evaluate it :D

1) The ALM is $z_t = A_a f_\alpha + A_b f_\beta + A_s s_t$

$\Rightarrow$ Model-consistent expectations are $E(z_t)$, or

$$E_t(z_{t+1}) = E_t \left[A_a f_\alpha(t+1) + A_b f_\beta(t+1) + A_s \cdot s_{t+1} \right]$$

$$= A_a E_t f_\alpha(t+1) + A_b E_t f_\beta(t+1) + A_s E_t s_{t+1}$$

$= A_a E_t f_\alpha(t+1) + A_b E_t f_\beta(t+1) + A_s hx \cdot s_t$

What is $E_t f_\beta(t+1)$?

$E_t f_\beta(t+1) = E_t \hat{E}_{t+1} \sum_{T=t+1}^{\infty} \beta^{T-t-1} z_{T+1}$:S

Or you could say that $f_\beta(t) = \frac{1}{1-\beta} a + b(I_{nx} - \beta hx)^{-1} s_t$

so $E_t f_\beta(t+1) = \frac{1}{1-\beta} a + b(I_{nx} - \beta hx)^{-1} E_t s_{t+1}$

The problem is $E_t(a_t)$ & $E_t(b_t)$ which the CB can't evaluate until z_t has realized and so HHs update ϕ_{t-1} to ϕ_t. So as a shortcut I can assume a version of "anticipated-utility" for the CB, so the CB assumes that ϕ won't be updated.

$\rightarrow$ In that case $E_t f_\beta(t+1) = \frac{1}{1-\beta} a + b(I_{nx} - \beta hx)^{-1} hx\, s_t$

$$\left[\begin{aligned} E_t(z_{t+1}) = {} & A_a \left[\frac{1}{1-\alpha\beta} a + b(I_{nx} - \alpha\beta hx)^{-1} hx\, s_t \right] \\ & + A_b \left[\frac{1}{1-\beta} a + b(I_{nx} - \beta hx)^{-1} hx\, s_t \right] \\ & + A_s hx \cdot s_t \end{aligned} \right]$$

[and $E_t(\pi_{t+1}) = e_{11} \cdot E_t(z_{t+1})$ (first el.)]

Ok: so w/ the 2 assumptions that $k \geq 0$

1. CB has a version of anticipated utility: $E_t(\phi_{t+k}) = E_t(\phi_t)$
2. agents do not internalize that CB has model-consistent expectations: $\hat{E}_t E_t(\pi_{t+1}) = \hat{E}_t(\pi_{t+1})$,

we have that the MN & PQ sols are identical to "Epi", only the compact notation (ALM) of the econ is different. It is:

$$z_t = A_a f_x + A_b \cdot f_x + A_s \cdot s_t + A_c \cdot E_t(\pi_{t+1}), \quad A_c = \begin{bmatrix} 0 \\ 0 \\ * \\ [\pi] \end{bmatrix}$$

b/c the Taylor-rule is different.

and $E_t(\pi_{t+1}) = e_{11} \cdot E_t(z_{t+1})$

$$E_t(z_{t+1}) = A_a \left[\frac{1}{1-\alpha\beta} a + b(I_{nx} - \alpha\beta h_x)^{-1} h_x s_t \right]$$

$$+ A_b \left[\frac{1}{1-\beta} a + b(I_{nx} - \beta h_x)^{-1} h_x s_t \right]$$

$$+ A_s h_x \cdot s_t$$

- Can use matrices from Epi
- need to use a different learning code:

  Sim_learnLH_Epi_CB.m

I corrected sthg in Epi

$$z_t = A_\alpha f_\alpha + A_\beta \cdot f_\beta + A_s \cdot s_t + \begin{bmatrix} 0 \\ 0 \\ \varphi \end{bmatrix} \hat{E}_t \pi_{t+1}$$

$\Rightarrow$ Now that's stable!

Epi_CB is also stable, but both keep the overshooting!

Alright, so "VAR-learning". 20 Jan 2020

Supp agents only observe observables z_t and use those to forecast their future selves proper:

$$\hat{E}_t z_{t+1} = \underset{n_y \times (n_y+1)}{\varphi_{t-1}} \cdot \underset{(n_y+1)\times 1}{\begin{bmatrix} 1 \\ z_{t-1} \end{bmatrix}}$$

with φ updating exactly as ϕ used to:

$$\varphi_t = \left(\varphi_{t-1}' + \kappa^{-1} R_t^{-1} \begin{bmatrix} 1 \\ z_{t-1} \end{bmatrix} \left(z_t - \varphi_{t-1} \begin{bmatrix} 1 \\ z_{t-1} \end{bmatrix} \right)' \right)'$$

and

$$R_t = R_{t-1} + \kappa^{-1} \left(\begin{bmatrix} 1 \\ z_{t-1} \end{bmatrix} \begin{bmatrix} 1 & z_{t-1} \end{bmatrix} - R_{t-1} \right)$$

R should be initialized as Σ_Y from sol of RE model.

and φ according to $Y_t = gx\, X_t = gx(hx\, X_{t-1} + \eta \varepsilon_t)$

$\Rightarrow \varphi_0 = gx\, hx\, gx^{-1}$. $\quad = gx\, hx\, gx^{-1} Y_{t-1} + gx\, \eta \underbrace{\varepsilon_t}_{=0 \text{ in st. st.}}$

(b)

Now what we need to figure out here is "fafb", the LH expectations.

$$\text{If } f_\beta = \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} z_{\tau+1} = \hat{E}_t \sum_{k=0}^{\infty} \beta^k z_{t+1+k}$$

$$= \hat{E}_t z_{t+1} + \hat{E}_t \beta z_{t+2} + \dots$$

The PLM is $\hat{E}_t z_{t+1} = \varphi_{t-1}[\hat{z}_t]$ but z_t hasn't realized yet, unlike for MSV learning, where that's the first thing that happens.

Let's write this as $\hat{E}_t z_{t+1} = \overset{a_{t-1}}{a} + \overset{b_{t-1}}{b} z_t$

$$= a + b\hat{E}_t z_t = a + b[a + b z_{t-1}]$$

$$= a + \underset{3\times3}{b} \cdot \underset{3\times1}{a} + b z_{t-1}$$

$$\hat{E}_t z_{t+2} = a + b(\hat{E}_t z_{t+1})$$

$$= a + b(a + ba + b z_{t-1})$$

$$= a + ba + b^2 a + b^2 z_{t-1}$$

$$= [I + b + b^2]a + b^2 z_{t-1}$$

$$\hat{E}_t z_{t+k} = [I + b + b^2 + \dots + b^k]a + b^k z_{t-1}$$

$$\hat{E}_t z_{t+k+1} = [I + b + b^2 + \dots + b^{k+1}]a + b^{k+1} z_{t-1}$$

So then $\hat{E}_t \sum_{k=0}^{\infty} \beta^k z_{t+1+k} = \{[I + b]a + b z_{t-1}\} + \dots$

$$\hat{E}_t \sum_{k=0}^{\infty} \beta^k z_{t+1+k} = \{[I+b]a + b z_{t-1}\}$$
$$+ \beta\{[I+b+b^2]a + b^2 z_{t-1}\}$$
$$+ \beta^2\{[I+b+b^2+b^3]a + b^3 z_{t-1}\}$$
$$+ \dots$$

$$= [I+b]a + \beta[I+b+b^2]a + \beta^2[I+b+b^2+b^3]a + \dots$$
$$+ b z_{t-1} + \beta b^2 z_{t-1} + \beta^2 b^3 z_{t-1} + \dots$$
$$= [I+b]a + \beta[I+b+b^2]a + \beta^2[I+b+b^2+b^3]a + \dots$$
$$+ b(1-\beta b)^{-1} z_{t-1}$$

Erm... this is unwieldy. But remember that the PLM is VAR: $Y_{t+1} = a + bY_t$ $(+\varepsilon_t(?))$

→ so Hamilton can tell us how to find that.

Hamilton, p. 93 (Mac):

$$\hat{E}_t[Y_{t+s} \mid Y_t, Y_{t-1}, \dots] = \mu + \left[\frac{\psi(L)}{L^s}\right]_+ \eta(L)(Y_t - \mu)$$

or

$$\hat{E}_t[Y_{t+s} \mid Y_t, Y_{t-1}, \dots] = \mu + \left[\frac{\psi(L)}{L^s}\right]_+ \frac{1}{\psi(L)}(Y_t - \mu)$$

where $[\cdot]_+$ is the annihilator, and this formula is the Wiener-Kolmogorov formula.

p. 94 Mac: Forecasting an AR(1):

For $Y_t = c + \phi Y_{t-1} + \varepsilon_t$ (p. 66 Mac)

$$\hat{E}_t[Y_{t+s} \mid Y_t, Y_{t-1}, \ldots] = \mu + \phi^s (Y_t - \mu)$$

where $\mu = E[Y]$ (unconditional mean) $= \frac{c}{1-\phi}$ (p. 67 Mac)

Ok so then $z_{t+1} = a + b z_t$ ($+ \varepsilon_{t+1}$?)

$$\hat{E}_t z_t = \mu + b(z_{t-1} - \mu) \quad \text{w/ } \mu = (I_{n_y} - b)^{-1} \cdot a$$

$$\hat{E}_t z_{t+1} = \mu + b^2 (z_{t-1} - \mu)$$

$$\hat{E}_t z_{t+k} = \mu + b^{k+1} (z_{t-1} - \mu)$$

So now $\hat{E}_t \sum_{k=0}^{\infty} \beta^k z_{t+1+k} = \sum_{k=0}^{\infty} \beta^k \left\{ \mu + b^{k+2} (z_{t-1} - \mu) \right.$

$$= \frac{1}{1-\beta} \mu + (I_{n_y} - \beta b)^{-1} b b' (z_{t-1} - \mu) = f_\beta$$

f_β for "VAR-learning"

But now a question arises, which is, do they know h_x? If I don't change anything in the model further, then I'm assuming they do. → the compact notation is the same. ("know_hx")

An alternative would be to say that they don't know h_x. How would they forecast states? E.g. in the NKPC, they need to evaluate $\hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} u_T$. An interesting extension would be where they don't learn h_x and simply assume no shocks in the future: $\hat{E}_t s_{t+k} = s_t$ $\forall k$. ("dont_know_hx")

↳ I didn't pursue this.

A Markov-switching Taylor-rule (à la Davig & Leeper)

$i_t = \alpha(s_t)\,\pi_t + \gamma(s_t)\,x_t$ (e.g. (29) D&L p. 10 Mac)

where s_t is the underlying state and it evolves according to a Markov chain.

Transition matrix P w/ elements

$p_{ij} = \text{Prob}\left[s_t = j \mid s_{t-1} = i\right]$

$\alpha = \begin{cases}\alpha_1 \\ \alpha_2\end{cases}$ and $\gamma = \begin{cases}\gamma_1 \\ \gamma_2\end{cases}$ Ass: st. st. is invariant.

Let me simplify:

$i_t = \psi_\pi(s_t)\,\pi_t + \psi_x x_t + \tilde{i}_t$ w/ $s_t = s_1$ or s_2

What this does though is that it changes $E(\cdot)$:

$\hat{E}_t(z_{t+1})$ needs to be taken conditional on the state.

$\hat{E}_t(z_{t+1}) = p_{1|1}\,\hat{E}_t(z_{t+1} \mid s_1) + p_{1|2}\,\hat{E}_t(z_{t+1} \mid s_1)$

where $p_{i|j} = \text{prob}(s_t = i \mid s_{t-1} = j)$

Lol - it's not quite right yet, but it's getting there.

Let me put the intuition in words:

W/ state-dependent TR, $E(\cdot)$ are also state-dep

Supp. the TR is in regime 1. Then $\hat{E}_t(\pi_{t+1})$ depends on what regime I expect for tomorrow.

Since this is true for regime 2 as well, $\hat{E}_t(\pi_{t+1})$ can have 4 different values:

$\hat{E}_t(\pi_{t+1} | s_1)$ and tomorrow $s = s_1 \rightarrow \hat{E}_t(\pi_{t+1} | s_1, s_1)$

$\hat{E}_t(\pi_{t+1} | s_1)$ and tomorrow $s = s_2 \rightarrow \hat{E}_t(\pi_{t+1} | s_2, s_1)$

$\hat{E}_t(\pi_{t+1} | s_2)$ and tomorrow $s = s_1 \rightarrow \hat{E}_t(\pi_{t+1} | s_1, s_2)$

$\hat{E}_t(\pi_{t+1} | s_2)$ and tomorrow $s = s_2 \rightarrow \hat{E}_t(\pi_{t+1} | s_2, s_2)$

That means that in a simulation we'll have to keep track of s_t and form for regime s_i

$$\hat{E}_t(\pi_{t+1} | s_i) = p_{1|s_t} \hat{E}_t(\pi_{t+1} | s_1, s_i) + p_{2|s_t} \hat{E}_t(\pi_{t+1} | s_2, s_i)$$

→ For the baseline model, this shouldn't change the A-matrices except that this s is a new state...

darn. But let's call this little s, "r_t" as in "regime".

If this evolves exogenously then agents might even know it.

What I don't know though is what to do w/ LH-E?

$f_\beta = \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} z_{\tau+1}$ ... but each 1-period-ahead fcst is state-dependent ...

So what one can do is a kind-of "Markov-switching anticipated utility": assume that agents think that the TR may switch regime tomorrow but then stay the same ever after. This would give you

$f_{\beta,i} = P_{1|i} f_{\beta 1,i} + P_{2|i} \cdot f_{\beta 2,i}$ for regime i.

LH expectations given that we're in regime i today and 1 tomorrow and ever after.

I can maybe simplify a bit in that I can make $f_{\beta j,i}$ independent of today's state

$f_{\beta,i} = P_{1|i} f_{\beta,1} + P_{2|i} \cdot f_{\beta 2,i}$

But wait a sec ... ψ_n doesn't actually show up in f_β

So scrap that!

Instead it's the A-matrices that change!

$$z_t = A_a \cdot f_a + A_b \cdot f_b + A_s \cdot s_t$$

$$\Rightarrow z_t = A_a(\psi_{11})\, f_a + A_b(\psi_{11})\, f_b + A_s(\psi_{11})\, s_t$$

Is that it to say

$$z_t = P_{11|i} \left\{ A_a(r_1) f_a + A_b(r_1) f_b + A_s(r_1) s_t \right\}$$
$$+ P_{21|i} \left\{ A_a(r_2) f_a + A_b(r_2) f_b + A_s(r_2) s_t \right\} \quad ?$$

And then include $\mathrm{LOM}(r)$?

Or actually $z_t = A_a(i) f_a + A_b(i) f_b + A_s(i) s_t \quad i = 1,2$.

Shit: RE also needs to be extended to accomodate for this!

<u>Markov-switching RE</u> <u>21 Jan 2020</u>

$$z_t = A_0^{RE} E_t z_{t+1} + A_s^{RE} s_t$$

The difficulty here is that $E_t z_{t+1}$ will actually be conditional on s.

At least this way of writing the model is close to Davig & Leeper 2007's system (35) (p. 11 Mac)

$$\underset{\substack{\uparrow \\ \text{jumps}}}{AY_t} = BY_{t-1} + \underset{\substack{\uparrow \\ \text{PEs}}}{A\eta_t} + \underset{\substack{\uparrow \\ \text{states}}}{Cu_t}$$

The RE model is (w/ Markov-switching ψ_π)

$$x_t = E_t x_{t+1} - \sigma i_t + \sigma E_t \pi_{t+1} + \sigma r_t^n$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t$$

$$i_t = \psi_\pi(r) \pi_t + \psi_x x_t + \bar{i}_t$$

To follow Davig & Leeper (2007), do the MN-style substit and sub i_t out:

$$\left.\begin{aligned} x_t &= -\sigma\left(\psi_\pi(r)\pi_t + \psi_x x_t + \bar{i}_t\right) + E_t x_{t+1} + \sigma E_t \pi_{t+1} + \sigma r_t^n \\ \pi_t &= \kappa x_t + \beta E_t \pi_{t+1} + u_t \end{aligned}\right\}$$

This is the system to be solved.

Denote w/ $x_{it} := x_t \mid s_t = i \quad i = 1, 2$. shock (real int rate)

$\Omega_t = \{ s_t, s_{t-1}, \dots, r_t, r_{t-1}, \dots \}$ = info set

$\Omega_t^{-s} = \{ s_{t-1}, s_{t-2}, \dots, r_t, r_{t-1}, \dots \}$, a smaller info set that doesn't include today's s (which is weird — I think it should)

As on p. 5 (Mac),

$$E_t \pi_{t+1} = E[\pi_{t+1} \mid s_t = i, \Omega_t^{-s}]$$

$$= p_{1i} E[\pi_{1t+1} \mid \Omega_t^{-s}] + p_{2i} E[\pi_{2t+1} \mid \Omega_t^{-s}]$$

$$= p_{1i} E_t \pi_{1t+1} + p_{2i} E_t \pi_{2t+1}$$

The issue is computing the RE-expectations.

Ah... but w/ method of undet coeffs you're conjectured the state-space as a set, and given that, E should be easy to compute.

So if I extend the state-space kind of like Davig & Leeper, I can maybe conjecture an "extended nx & gx"

Let $Y_t = \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \\ x_{1t} \\ x_{2t} \end{bmatrix}$ $E_t Y_{t+1} = \begin{bmatrix} E_t \pi_{1t+1} \\ E_t \pi_{2t+1} \\ E_t x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix}$ $S_t = \begin{bmatrix} r_t^n \\ u_t \end{bmatrix}$

As if there were 4 jumps instead of two.

Using these 4 jumps, rewrite the model as for $i=1,2$

(82) $\pi_{it} = \beta(\rho_{1i} E_t \pi_{1t+1} + \rho_{2i} E_t \pi_{2t+1}) + k x_{it} + u_t$

(83) $x_{it} = \rho_{1i} E_t x_{1t+1} + \rho_{2i} E_t x_{2t+1} - \sigma(\psi_i \pi_{it} + \psi_x x_{it})$
$+ \sigma(\rho_{1i} E_t \pi_{1t+1} + \rho_{2i} E_t \pi_{2t+1}) + \sigma r_t^n$

Now rewrite (82)-(83) in MN form:

$$\pi_{it} - k x_{it} = \beta \rho_{1i} E_t \pi_{1t+1} + \beta \rho_{2i} E_t \pi_{2t+1} + u_t$$

$$b\psi_i \pi_{it} + (1 + b\psi_x) x_{it} = \rho_{1i} E_t x_{1t+1} + \rho_{2i} E_t x_{2t+1}$$

$$+ b \rho_{1i} E_t \pi_{1t+1} + b \rho_{2i} E_t \pi_{2t+1} + b r_t^n$$

The order is $\pi_{1t}, \pi_{2t}, x_{1t}, x_{2t}$

$$\underbrace{\begin{bmatrix} 1 & 0 & -k & 0 \\ b\psi_1 & 0 & 1+b\psi_x & 0 \\ 0 & 1 & 0 & -k \\ 0 & b\psi_2 & 0 & 1+b\psi_x \end{bmatrix}}_{=:A} \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \\ x_{1t} \\ x_{2t} \end{bmatrix} = \overbrace{\begin{bmatrix} \beta\rho_{11} & 0 & \beta\rho_{21} & 0 \\ b\rho_{11} & \rho_{11} & b\rho_{21} & \rho_{21} \\ \beta\rho_{12} & 0 & \beta\rho_{22} & 0 \\ b\rho_{12} & \rho_{12} & b\rho_{22} & \rho_{22} \end{bmatrix}}^{=:B} \begin{bmatrix} E_t \pi_{1t+1} \\ E_t \pi_{2t+1} \\ E_t x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix}$$

$$+ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & b \\ 1 & 0 \\ 0 & b \end{bmatrix}}_{=:C} \begin{bmatrix} u_t \\ r_t^n \end{bmatrix}$$

$$\Rightarrow A Y_t = B E_t Y_{t+1} + C \cdot X_t$$

Erm... why isn't there a X_{t+1} term?

Erm, another thing: can we simply input these equations into gx_hx? Like:

Model equations:

$$\pi_{it} = \beta(\rho_{1i} E_t \pi_{1t+1} + \rho_{2i} E_t \pi_{2t+1}) + \kappa x_{it} + u_t$$

$$x_{it} = \rho_{1i} E_t x_{1t+1} + \rho_{2i} E_t x_{2t+1} - \sigma i_{it} + \sigma(\rho_{1i} E_t \pi_{1t+1} + \rho_{2i} E_t \pi_{2t+1}) + \sigma r_t^n$$

$$i_{it} = \psi_i \pi_{it} + \psi_x x_{it} + \bar{i}_t$$

$\Rightarrow$ 6 eqs, 6 unknowns

↳ ok, works for now, need to discuss w/ Ryan though!

Ok, so last extension I'll attempt is the "(moving) average π in TR" "MA-TR"

$\rightarrow$ in a way this is related to pil if the MA-terms are bw-looking, or to Epi if they are fw-looking.

A (in terms of lag structure) not so demanding version is

$$i_t = \theta \pi_{t-1} + \psi_\pi \pi_t + \theta^{-1} \hat{E}_t \pi_{t+1} + \psi_x x_t + \bar{i}_t$$

$$x_t = -\sigma i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma \beta i_{T+1} + \sigma r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa \alpha \beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \theta \pi_{t-1} + \psi_\pi \pi_t + \theta^{-1} \hat{E}_t \pi_{t+1} + \psi_x x_t + \bar{i}_t$$

<u>MN</u>

$$x_t = -\sigma \left[\theta \pi_{t-1} + \psi_\pi \pi_t + \theta^{-1} \hat{E}_t \pi_{t+1} + \psi_x x_t + \bar{i}_t \right]$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma \pi_{T+1} - \sigma \beta i_{T+1} + \sigma r_T^n \right\}$$

$$\sigma \psi_\pi \pi_t + (1+\sigma \psi_x) x_t = -\sigma \theta \pi_{t-1} - \sigma \theta^{-1} \hat{E}_t \pi_{t+1} - \sigma \bar{i}_t$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + \sigma r_T^N - \sigma \beta i_{T+1} + \sigma \pi_{T+1} \right.$$
$$\left. - \sigma \beta \left[\theta \pi_T + \psi_\pi \pi_{T+1} + \theta^{-1} \hat{E}_T \pi_{T+2} + \psi_x x_{T+1} + \bar{i}_T \right] \right\}$$

$$\sigma \psi_\pi \pi_t + (1+\sigma \psi_x) x_t = -\sigma \theta \pi_{t-1} - \sigma \theta^{-1} \hat{E}_t \pi_{t+1}$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta\sigma\psi_x) x_{T+1} + \sigma r_T^N - \sigma \bar{i}_T + \sigma \pi_{T+1} \right.$$
$$\left. - \sigma \beta \theta \pi_T - \sigma \beta \psi_\pi \pi_{T+1} - \sigma \beta \theta^{-1} \pi_{T+2} \right\}$$

$$1 - \beta^2 \psi_\pi > 0$$
$$1 > \beta^2 \psi_\pi$$
$$1/\beta^2 > \psi_\pi$$

$$\sigma \psi_\pi \pi_t + (1+\sigma \psi_x) x_t = -\sigma \theta \pi_{t-1} - \sigma \beta \theta \pi_t$$
$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-\beta\sigma\psi_x) x_{T+1} + \sigma r_T^N - \sigma \bar{i}_T + \sigma \pi_{T+1} \right.$$
$$\left. \underbrace{- \sigma \beta^2 \theta \pi_{T+1} - \sigma \beta \psi_\pi \pi_{T+1} - \sigma \theta^{-1} \pi_{T+1}}_{\sigma \left[1 - \beta^2 \theta - \beta \psi_\pi - \theta^{-1} \right] \pi_{T+1}} \right\}$$

This comes from pil
-> so only that extension should in theory have a chance.

Timeout: more thoughts on why pill can work:

Suppose we had π_{t-2} in TR.

Then $\hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} i_{\tau+1} = \text{stuff} + \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} -b\beta(\pi_{\tau-1})$

$= \text{stuff} + \pi_{t-1} + \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \beta \pi_\tau (-b\beta)$

$= \text{stuff} + \pi_{t-1} + \beta \pi_t + \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \beta^2 \pi_t (-b\beta)$

$\downarrow$

$\underline{\underline{-b\beta^3}}$

So for one lag, if $\psi_\pi = 1.5$, then β has to be

$\psi_\pi < \frac{1}{\beta^2} \Rightarrow \quad \beta < \left(\frac{1}{\psi_\pi}\right)^{\frac{1}{2}} = 0.8165$ hmm...

For two lags, $\beta < \left(\frac{1}{\psi_\pi}\right)^{\frac{1}{3}} = 0.8736$ hmm...

↳ So sorry I won't pursue the MA-TR extension!

Instead, try 2-lag π in TR! "pill"

$i_t = \psi_\pi \pi_{t-2} + \psi_x x_t + \bar{i}_t$

w/ suboptimal testers, and then maybe opt using looking.

"pill" - extension using "subopt feders"

$$x_t = -b i_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + b \pi_{T+1} - b\beta i_{T+1} + b r_T^n \right\}$$

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa \alpha \beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

$$i_t = \psi_\pi \pi_{t-2} + \psi_x x_t + \bar{i}_t$$

MN

$$x_t = -b(\psi_\pi \pi_{t-2} - \psi_x x_t + \bar{i}_t) + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta) x_{T+1} + b \pi_{T+1} - b\beta i_{T+1} + b r_T^n \right\}$$

$$(1+b\psi_x) x_t = -b\psi_\pi \pi_{t-2} + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-b\beta\psi_x) x_{T+1} + b[1,-1,0,0,0] s_T + b \pi_{T+1} - b\beta \psi_\pi \pi_{T-1} \right\}$$

$$(1+b\psi_x) x_t = -b\psi_\pi \pi_{t-2} - b\beta\psi_\pi \pi_{t-1} + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-b\beta\psi_x) x_{T+1} + b[1,-1,0,0,0] s_T + b \pi_{T+1} - b\beta^2 \psi_\pi \pi_T \right\}$$

$$(1+b\psi_x) x_t = -b\psi_\pi \pi_{t-2} - b\beta\psi_\pi \pi_{t-1} - b\beta^2\psi_\pi \pi_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-b\beta\psi_x) x_{T+1} + b[1,-1,0,0,0] s_T + b \pi_{T+1} - b\beta^3 \psi_\pi \pi_{T+1} \right\}$$

$$2\beta^2\psi_\pi \pi_t + (1+2\psi_x)x_t = -2\psi_\pi \pi_{t-2} - 2\beta\psi_\pi \pi_{t-1}$$
$$+ \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ (1-\beta-2\beta\psi_x)x_{\tau+1} + 2(1-\beta^2\psi_\pi)\pi_{\tau+1} + 2[1,-1,0,0,0]\, s_\tau \right\}$$

So, together w/

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \left\{ \kappa\alpha\beta\, x_{\tau+1} + (1-\alpha)\beta\, \pi_{\tau+1} + u_\tau \right\}$$

This gives:

$$\begin{bmatrix} 2\beta^2\psi_\pi & 1+2\psi_x \\ 1 & -\kappa \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} [2(1-\beta^3\psi_\pi), 1-\beta-2\beta\psi_x, 0] f_\beta + 2[1,-1,0,0,0](I_{nx}-\beta h_x)^{-1} s_t + [0,0,0,-2\beta\psi_\pi,-2\psi_\pi] s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0,0](I_{nx}-\alpha\beta h_x)^{-1} s_t \end{bmatrix}$$

— MN, "prle", "opt festers" —

<u>PQ</u>

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & -\kappa & 0 \\ 0 & \psi_x & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} [0, 1-\beta, -2\beta] f_\beta + 2[1,0,0,0,0](I_{nx}-\beta h_x)^{-1} s_t \\ [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_\alpha + [0,0,1,0,0](I_{nx}-\alpha\beta h_x)^{-1} s_t \\ [0,1,0,0,\psi_\pi] s_t \end{bmatrix}$$

s.t. (*)

$$(*)\quad \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} i_{\tau+1} = \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ \psi_\pi \pi_{\tau-1} + \psi_x x_{\tau+1} + \bar{i}_{\tau+1} \right\}$$

$f_\beta(3)$

$$= \psi_x f_\beta(2) + \frac{1}{\beta}\left\{ [0,1,0,0,0](I_{nx} - \beta h_x)^{-1} s_t - [0,1,0,0,0] s_t \right\}$$

$$+ \underbrace{\hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \psi_\pi \pi_{\tau-1}}$$

$$\underbrace{\hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \psi_\pi \pi_\tau \beta + \psi_\pi \pi_{t-1}}$$

$$= \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \psi_\pi \beta^2 \pi_{\tau+1} + \psi_\pi \pi_{t-1} + \beta \psi_\pi \pi_t$$

So $(*)$:

$$\left[f_\beta(3) = \beta^2 \psi_\pi f_\beta(1) + \psi_x f_\beta(2) + \frac{1}{\beta}\left\{ [0,1,0,0,0](I_{nx} - \beta h_x)^{-1} s_t - [0,1,0,0,0] s_t \right\} + \psi_\pi \pi_{t-1} + \beta \psi_\pi \pi_t \right.$$

↳ So LHS $P(1,1)$ becomes:

$$0 - (-3\beta \cdot \beta \psi_\pi) = \underline{\underline{3\beta^2 \psi_\pi}} \quad \leftarrow P(1,1)$$

2 discoveries: <u>22 Jan 2020</u>

1) It's not 2x lagged π in TR which works ... it's β b/c

2) lagged jumps as endog states is still not quite correct b/c in the learning sim code, the code just

advances endog states as if they were evolving according to the RE hx, whereas they are evolving according to $\hat{g}$.

<u>Ryan meeting</u> <u>22 Jan 2020</u>

→ check Ryan's code to see what he did for projection facility

↳ none of the approaches I had really correspond to getting rid of explosive stuff

- Plot RE for
  - only passive ← that's the usual thing ppl do!
  - only active regime

- Can have them learn the regime (a separate constant (& slope) for each regime)

→ try the passive initialization

- RE w/ π_{t-1} has a unique eqb? Check!

check if

$$Y^e_{t+h} = \hat{g}\left(h^{RE}\right)^h \begin{bmatrix} x_t \\ \pi_{t-1} \end{bmatrix}$$

i.e. does the learning test equal the estimated $\hat{g} \cdot h_x \cdot s_t$? $\Rightarrow$ For lagged jumps, you wanna input $\hat{g}$ here

For proj facility:

$$Y_t = \hat{g}\, x_t$$

$$\begin{bmatrix} x_{t+1} \\ \pi_t \end{bmatrix} = \underbrace{\begin{bmatrix} h^{RE} & 0 \\ g_\pi^{RE} & \end{bmatrix}}_{\text{check eig of this whole matrix}} \begin{bmatrix} x_t \\ \pi_{t-1} \end{bmatrix}$$

What Evans & Honk might have meant w/ "overparametrize" = too many degrees of freedom. So that oscillations are more likely if agents are allowed to estimate too many things!

→ like overfitting a model when you're estimating
→ it's bad b/c you fit well but bad fcst.

→ like overdifferencing in a sense:
everyone knows that it's a sin and you shouldn't do it but no one really knows why.

Work after 23 Jan 2020

1. checked Ryan's code: he doesn't do the proj. facility
2. Did all the learning the Markov-switching TR plots
   1. conditional on the mixed regime sequence r, w/ initialization of learning both ways
   2. conditional on passive-only regime sequence w/ initialization of learning as passive
   3. conditional on active-only regime sequence w/ initialization of learning as passive
3. wanted to remark on Ryan's comment that "you can make agents be aware of S_t and estimate a separate constant (and slope) for each regime": In the current setup, the learning agents aren't aware

that there are 2 regimes: they are trying to learn a single one. So when the regime <u>is</u> indeed switching, no matter when you initialize, you confuse them. W/ constant-only-learning, I guess they can figure out that gx doesn't have a constant, but they fail to get the slope b/c it's changing.

→ if they recognized that there are two regimes, then their estimates of $\hat{gx}_1$ & $\hat{gx}_2$ would converge (hopefully) to gx_1 & gx_2.

→ It makes sense that the sign doesn't change too much when only active regime is realized b/c the RE model, gx^{RE}, is different from what is realized. The only commonality between gx^{RE} & the realized regime-sequence is that gx^{RE} has no constant. So I'd expect constant-only learning to be stable also in the only-passive-r-realized case.

↳ it could be that it's a projection facility issue!

4. Check whether RE w/ π_{t-1} has unique eqb:

- 1 too many stable roots → indeterminacy
- 1 too many explosive roots → explosive

⇒ Need as many explosive roots as fw-looking vars!

Baseline RE NKPC w/ TR subbed in

NKPC → fw-looking in π
NKIS → fw-looking in x
} 2 explosive eigs are needed.

Baseline RE NKPC w/ TR subbed in,

but TR is $i_t = \psi_\pi \pi_{t-1} + \psi_x x_t + \bar{i}_t$

NKPC → fw-looking in π

NKIS → fw-looking in x

Looking: $pi1_t = \pi_{t-1}$ → bw-looking in pi1.

⇒ Need 2 explosive (>1) and 1 stable (<1) eig.

↳ see, I don't why 1 shouldn't have those?

All I'm finding in the lit. is Bernanke & Woodford 1997 who investigate a "fwd-looking TR" of the type

$i_t = \delta E_t \pi_{t+1}$ + stuff ⇒ what they find is

interesting b/c unexpected:

1.) "Epi" is not a good Taylor rule b/c it renders the inflation fcst $E_t \pi_{t+1}$ uninformative ("it kills itself")

2) it's also not good b/c it leads to indeterminacy

Use Bernanke & Woodford's analysis to see what pi1 means for uniqueness of eqlo.

BW gather pimps into the vector x_t, states into the vector z_t, and write the compact notation of the RE model as (once they've subbed the TR in)

$$E_t x_{t+1} = M x_t + z_t \qquad (35)$$

This system has a unique sol if both eig(M) > 1 (b/c Blanchard & Kahn 1980 tells us).

So
$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t$$
$$x_t = E_t x_{t+1} - \sigma i_t + \sigma E_t \pi_{t+1} + \sigma r_t^n$$
$$i_t = \psi_\pi pi1_t + \psi_x x_t + \bar{i}_t$$
$$pi1_t = \pi_{t-1}$$

$$\Rightarrow \quad \pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t$$

$$x_t = E_t x_{t+1} + \sigma E_t \pi_{t+1} - \sigma \psi_\pi pil_t - \sigma \psi_x x_t - \sigma \hat{i}_t + \sigma r_t^n$$

$$pil_t = \pi_{t-1}$$

$$\Leftrightarrow \quad \beta E_t \pi_{t+1} = \pi_t - \kappa x_t - u_t$$

$$E_t x_{t+1} + \sigma E_t \pi_{t+1} = (1+\sigma\psi_x) x_t + \sigma \psi_\pi pil_t + \sigma \hat{i}_t - \sigma r_t^n$$

$$pil_t = \pi_{t-1} \quad \text{or} \quad pil_{t+1} = \pi_t$$

$$\overbrace{\begin{bmatrix} \beta & 0 & 0 \\ \sigma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{=:B} E_t \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \\ pil_{t+1} \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & -\kappa & 0 \\ 0 & 1+\sigma\psi_x & \sigma\psi_\pi \\ 1 & 0 & 0 \end{bmatrix}}^{=:C} \begin{bmatrix} \pi_t \\ x_t \\ pil_t \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ \sigma & -\sigma & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_t \\ \hat{i}_t \\ r_t^n \end{bmatrix}$$

$$E_t \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \\ pil_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \beta & 0 & 0 \\ \sigma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -\kappa & 0 \\ 0 & 1+\sigma\psi_x & \sigma\psi_\pi \\ 1 & 0 & 0 \end{bmatrix}}_{=A} \begin{bmatrix} \pi_t \\ x_t \\ pil_t \end{bmatrix} + \begin{bmatrix} \beta & 0 & 0 \\ \sigma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 & 0 & 0 \\ \sigma & -\sigma & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_t \\ \hat{i}_t \\ r_t^n \end{bmatrix}$$

In Basu's notation (Lec 10, s. 12), this is the "interacting system":

$$z_t = A z_{t-1} + m$$

which we make non-interacting by replacing A with $E \Omega E^{-1}$

where $\Omega = \text{eig}(A)$ and E = matrix of eigvectors(A).

The trick two is that in the Ramsey example, Basa "multiplies out the eigenvector E" so

$z_t = A\, z_{t-1}$ (w/o shocks) becomes

$z_t = E \Omega E^{-1} z_{t-1}$

$\Leftrightarrow \underbrace{E^{-1} z_t}_{z_t' = \Omega z_{t-1}'} = \Omega E^{-1} z_{t-1}$

So it's sufficient to check Ω, the matrix of eigvals(A).

How to compute eig(A)? (call 'em ω_i).

ω_i $i=1,2,3$ solve

$$\det \underbrace{\begin{bmatrix} a_{11}-\omega_1 & a_{12} & a_{13} \\ a_{21} & a_{22}-\omega_2 & a_{23} \\ a_{31} & a_{32} & a_{33}-\omega_3 \end{bmatrix}}_{=:A_p} = 0$$

$\Rightarrow$ Solve using Mathematica!

matematisk.nb $\rightarrow$ you can see that 2 eigs >1, 1 eig <1, so we're good.

<u>The pil extension is unique.</u>

Do the same spiel for the std RENK model

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1}$$

$$x_t = E_t x_{t+1} - \sigma i_t + \sigma E_t \pi_{t+1}$$

$$i_t = \psi \pi_t$$

to simplify my life and no shocks

$$\Leftrightarrow \quad \pi_t = \kappa x_t + \beta E_t \pi_{t+1}$$

$$x_t = E_t x_{t+1} + \sigma E_t \pi_{t+1} - \sigma \psi \pi_t$$

$$\Leftrightarrow \quad \beta E_t \pi_{t+1} = \pi_t - \kappa x_t$$

$$\sigma E_t \pi_{t+1} + E_t x_{t+1} = \sigma \psi \pi_t + x_t$$

$$\underbrace{\begin{bmatrix} \beta & 0 \\ \sigma & 1 \end{bmatrix}}_{=:F} E_t \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -\kappa \\ \sigma\psi & 1 \end{bmatrix}}_{=:G} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}$$

$$H = F^{-1} G.$$

$$\operatorname{eig}(H)_{1,2} = \frac{1+\beta+\kappa\sigma \pm \sqrt{(-1-\beta-\kappa\sigma)^2 - 4\beta(1+\kappa\sigma\psi)}}{2\beta}$$

(Two sols, ω_1 & ω_2.)

It'd be kinda cool if we could derive the Taylor-principle:

$$\mathrm{eig}(H)_{1,2} = \frac{1+\beta+kb \pm \sqrt{(-1-\beta-kb)^2 - 4\beta(1+kb\psi)}}{2\beta} \overset{!}{>} 1$$

when $\psi = 1$,

$$\mathrm{eig}(H) = \frac{1+\beta+kb \pm \sqrt{(\beta-1-kb)^2}}{2\beta}$$

$$= \frac{\beta+(1+kb) \pm \left(\beta-(1+kb)\right)}{2\beta}$$

$$\frac{\beta+(1+kb)-\beta+(1+kb)}{2\beta} \qquad \frac{\beta+(1+kb)+\beta-(1+kb)}{2\beta}$$

$$= \frac{2(1+kb)}{2\beta} \qquad = \frac{2\beta}{2\beta} = 1$$

Ok, a half-baked proof. :S ;)

5. The joint issue of the pil-learning code and the projection facility.

A change: $b = gx$ so PLM is

$$\hat{E}_t z_{t+h} = \bar{z}_{t-1} + b\, hx^h s_t \qquad b = gx$$

instead of $\hat{E}_t z_{t+h} = \bar{z}_{t-1} + b\, hx^{h-1} s_t \qquad b = gx\, hx$

Need to check if this doesn't wreak havoc everywhere.

$\hat{E}_t s_{t+1}$?

$$\hat{E}_t z_{t+1} = a_{t-1} + b_{t-1} s_t \qquad \text{(Morning facts)}$$

$$\hat{E}_t \hat{E}_{t+1} z_{t+2} = a_{t-1} + b_{t-1} \hat{E}_t s_{t+1}$$

$$hx\, s_t$$

$$\hat{E}_t z_{t+2} = a_{t-1} + b_{t-1}\, hx\, s_t$$

$$\rightarrow \hat{E}_t z_{t+h} = a_{t-1} + b_{t-1}\, hx^{h-1} s_t$$

Ok so it <u>is</u> to the $h-1$ (that's cool b/c then my facts are still correct.)

But in RE, $Y_t = gx\, X_t$

so $E_t Y_{t+1} = gx \cdot \hat{E}_t X_{t+1}$

$= gx \cdot hx\, X_t$ Ah weird

But if that's the case, so that $\hat{E}_t z_{t+h} = a_{t-1} + b_{t-1} hx^h s_t$

then $\hat{E}_t z_T = a_{t-1} + b_{t-1} hx^{T-t} s_t$

So $\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} z_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} a_{t-1}$ (same)

$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} b_{t-1} hx^{T-t+1} s_t$

$\Rightarrow f_\beta = \frac{1}{1-\beta} a_{t-1} + b_{t-1}(I_{nx} - \beta hx)\, hx\, s_t$

Ok, here's the deal:

If the PLM is $z_t = \phi_{t-1} s_t = a_{t-1} + b_{t-1} s_t$

then $\hat{E}_t z_T = a_{t-1} + b_{t-1} \hat{E}_t s_T = a_{t-1} + b_{t-1} hx^{T-t} s_t$

and $\hat{E}_t z_{T+1} = a_{t-1} + b_{t-1} hx^{T-t+1} s_t$

and $b = gx$ b/c $E_t z_{T+1} = gx \cdot hx^{T-t+1} s_t$

and $f_a f_b$ must be changed

If the PLM is $z_t = \phi_{t-1} s_{t-1} = a_{t-1} + b_{t-1} s_{t-1}$

then $\hat{E}_t z_T = a_{t-1} + b_{t-1} \hat{E}_t s_T = a_{t-1} + b_{t-1} hx^{T-t+1} s_{t-1}$

and $\hat{E}_t z_{T+1} = a_{t-1} + b_{t-1} hx^{T-t+2} s_{t-1}$ so that doesn't

make sense at all! And I fear this is the one I've been

using ...

So that means I've uncovered a new error in Preston: (40) & (29) don't correspond

wherefore f_α & f_β are incorrect (an hx or C is missing)

$\Rightarrow$ materials14

call this fafb_materials14.

So what needs to change:

① fafb_materials14

② initialization and treatment of b as $= \hat{gx}$ in sim_learnLH.m

③ morning & evening fcsts at time t need to condition on s_t not s_{t-1} in sim_learnLH.m

What is fine:

fafb_gagb.m (I checked)

maybe pill... but that needs more changes.

VARlearn

learn_hx

Markov-switching

indexation, Epi_CB,

So to do:

- Reproduce IRFs for all extensions: baseline, pil, il, Epi
- Reproduce IRFs of FEs to make sure they are fine
  - Check convergence to RE by simple simulation

Rethink the fa & fb-correction. 29 Jan 2020

The PLM is: agents think that the jumps evolve instead of $y_t = gx \cdot x_t$, as $y_t = \hat{g}\, x_t$.

$\Rightarrow$ $z_t = \phi s_t$ and they use last period's ϕ to do their morning fest: ϕ_{t-1}, and ϕ_t to do their evening fest.

$\Rightarrow$ then $z_{t+1} = \phi s_{t+1}$, so they need to fest s_{t+1} too! $\Rightarrow \hat{E}_t z_{t+1} = \phi \hat{E}_t s_{t+1} = \phi\, hx\, s_t$

$$\Rightarrow \hat{E}_t z_{t+h} = \phi\, hx^h s_t$$

$$\Rightarrow \hat{E}_t z_T = \phi\, hx^{T-t} s_t \quad \text{if } T = t+h,\ T-t = h \checkmark$$

$$\Rightarrow \hat{E}_t z_{T+1} = \phi\, hx^{T-t} \underline{hx}\, s_t$$

Morning fest of $z_{T+1} = \hat{E}_t^m z_{T+1} = \phi_{t-1}\, hx^{T-t} \underline{hx}\, s_t$

Evening fest of $z_{T+1} = \hat{E}_t^e z_{T+1} = \phi_t\, hx^{T-t} \underline{hx}\, s_t$

$\rightarrow$ use this to form fa/fb.

Use yesterday evening's fest to update ϕ:

$$\hat{E}_{t-1}^e z_t = \phi_{t-1}\, hx\, s_{t-1}$$

which suggests that even the updating eq is wrong.

The funny thing though is that even Ryan has in his sim_learn.m as updating eq

$$q_t - \hat{q}_{t-1} \cdot x_{t-1}$$

→ ah... but he <u>is</u> initializing q as $gx \cdot hx$.

⇒ Ok, so here's the deal:

I think that the two approaches are equivalent; you just need to be consequent.

→ just fafb_gagb.m may need to be corrected,

<u>and</u> in the old approach $\phi \to gx\, hx$

so to get gx back, you need to say $\phi\, hx^{-1}$

Check fafb_gagb.m

→ I kept fafb_gagb.m the same, just modified the updating of ϕ and its initialization.

→ it doesn't make a big diff! Relieved! :)

So to the endogenous states issue

Ryan asked: check if $Y^e_{t+h} = \hat{g} \cdot h^{RE^h} \begin{bmatrix} x_t \\ \pi_{t-1} \end{bmatrix}$ s_t

Now in my case

$\hat{g} = gx \cdot hx$ so the question is whether

$Y^e_{t+h} = \hat{g}\, h^{RE^{h-1}}\, s_t$

→ I'm not sure how to check this, but this is what fafb_.... m is doing.

So what we wanna do for the endog states is

$$\underbrace{\begin{bmatrix} x_{t+1} \\ \pi_t \end{bmatrix}}_{s_{t+1}} = \underbrace{\begin{bmatrix} h^{RE} & 0 \\ \hat{g}_\pi & \end{bmatrix}}_{=:HX} \underbrace{\begin{bmatrix} x_t \\ \pi_{t-1} \end{bmatrix}}_{s_t}$$

Problem #1: $b \neq gx$. $b \to gx \cdot hx$

$\Rightarrow \hat{g} \approx gx \cdot hx \cdot hx^{-1}$

Problem #2: hx^{-1} doesn't exist. So it take the pseudoinverse, pinv(hx). :S

Problem #3: the constant

A new understanding: I need R even when the constant only is learned. Why? B/c R scales the regressors – and those are still a vector → CEMP doesn't need R b/c for them the regressor is a scalar. They would scale by var(regressor two).

$$\underset{n_y \times (n_x+1)}{\phi_t} = \Big(\underset{(n_x+1,\, n_y)}{\phi_{t-1}'} + \underbrace{k\, R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}}_{(n_x+1)\times 1} \underbrace{\Big(\overbrace{z_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}}^{n_y \times 1} \Big)'}_{\substack{n_y \times 1 \\ 1 \times n_y}} \Big)'$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{n_x+1 \times n_y}$$

If

$$\underset{n_y,1}{a_t} = \underset{n_y,1}{a_{t-1}} + \underset{n_x+1 \times n_y}{\cdots}$$

↳ Nope that wouldn't work! So I was right in materials 3 to claim that if you only learn the constant, you don't need R!

Ok so projection facility is ok. – except for il... ugh...
Now to those endog. states. Is RE ok?

↳ Yeah, RE is ok: i↓ endog but is ↑ exog in period 3.

↳ what I'm doing is to replace xesim(e, t+1) with ysim(e, t)

→ This is the only way out of the several I've tried that actually generates xsim(e, :) corresponding to ysim(e, :) lagged.

Ok so I think I've done what I can – no extension really works. I don't want them to learn slopes & constants for two states. I think I wanna go back to the baseline model and reintroduce anchoring [maybe also for learn TR]. Do IRFs, tell stories and finally start thinking about optimal mon. pol.

I want to write an alternative 25 Jan 2020

learning simulation code that implements the

$b \to \hat{g}x$ version

- Uses fafb_materials14.m
- uses sim_learnLH_materials14.m (to be written)

which - initializes b as gx

- an breaks expectations differently

In the updating step, $a + b \cdot \underline{\underline{hx}} \cdot s_{t-1}$.

→ I should get the same baseline IRF

⇒ and I do!

If e is the endog state, $h_e = g_e$, that is,

x_e is evolving according to the same LOM as g_e.

The question is the timing.

$y_{e,t} = g(a_{t-1}, b_{t-1})$.

$x_{e,t+1} = y_{e,t} = g(a_{t-1}, b_{t-1})$.

→ $\underline{\underline{\phi_{t-1}}}$!

What is reassuring is that I get the same as for $\hat{g} = b \cdot \text{Pseudoinverse}(hx)$

and $x_{e,t+1} = a_e + \hat{g}_e \cdot x_t$

I can check which is the correct one by simply calculating what i_t should be at $t=3$ where $t=1$ is the impact of the shock.

$$i_t = \psi_\pi \pi_{t-1} + \bar{i}_t$$

$$i_3 = 1.5\, \pi_2 + \rho^2 \bar{i}_1$$

$$i_3 \approx 1.5 \cdot 0.1 + (0.6)^2 \cdot 1$$

$$= 0.15 + 0.36$$

$$= \underbrace{0.51}$$

this is where the "brute force"-method is (i.e. when I force $x_{e,t} = y_{e,t-1}$)

but not the $x_{e,t+1} = a_e + b_e x_t$.

Maybe the problem is that y doesn't evolve as simply as $\hat{g}_x \cdot x_t$ ... or in the transition,

$$\hat{gx} \approx \underbrace{\left[A_a(f_a - s_t) + A_b(f_b - s_t) + A_s\right]}_{\text{sloppy notation but what I want to say}} s_t$$

is $\{A_a \cdot \text{Factor}[f_a, s_t] + A_b \text{Factor}[f_b, s_t) + A_s\}$

is maybe the in-transition approx of $\hat{g}$ b/c

$$z_t = \underbrace{\{A_a \cdot \text{Factor}[f_a, s_t] + A_b \text{Factor}[f_b, s_t) + A_s\}}_{=: \hat{G}} \cdot s_t$$

$$z_t = A_a\left[\frac{1}{1-\alpha\beta} a + b(I_{nx} - \alpha\beta hx)^{-1} s_t\right]$$
$$+ A_b\left[\frac{1}{1-\beta} a + b(I_{nx} - \beta hx)^{-1} s_t\right]$$
$$+ A_s \cdot s_t$$

$$z_t = \underbrace{\underbrace{\left[A_a \frac{1}{1-\alpha\beta} + A_b \frac{1}{1-\beta}\right]}_{3\times 3} \underbrace{a}_{3\times 1}}_{=: \hat{F}_{3\times 1}} + \underbrace{\left[A_a b(I_{nx} - \alpha\beta hx)^{-1} + A_b \cdot b(I_{nx} - \beta hx)^{-1} + A_s\right]}_{=: \hat{G}_{3\times 3}} s_t$$

So you can say

$$z_t = \hat{F} + \hat{G} s_t \quad \text{or} \quad z_t = \underbrace{[\hat{F}, \hat{G}]}_{3\times 4} \cdot \begin{bmatrix} 1 \\ s_t \end{bmatrix}$$

Exactly! Holly smolly, when you let

$\underline{\underline{Xesim_e = \hat{F}_e + \hat{G}_e \cdot s_t}}$, $\underline{\underline{\text{you get the right thing!}}}$

Ok so analysis of results:

- Marbor-switching: why is there so much volatility in the passive initialization / sequence?
  → if you condition on an all-active regime, it doesn't matter where you initialize, it's stable anyhow.

Maybe what happens in the passive sequence is that they learn the wrong thing: they learn the passive regime, but that's not RE.

Why doesn't that happen in the all-active sequence?

Maybe the difference is that the passive gx couldn't be an RE, while the active can, and the mixed can too.

Maybe that explains the additional volatility coming from initializing at the passive state: it seems to be further from RE somehow.

↳ Yep: if I set $\psi_2 > 1$, E-stability is restored.

⇒ baseline, pil, indexation all work for me
learn TR or learn bx too

So maybe to be preferred for simplicity are
baseline & learn TR

Optimal mon. pol. — The loss function

Peter said on 5 Sep 2019 (p. 62 Notes 1) that initially one can do a quadratic loss fct.

"Quadratic on π & x"

↳ I guess $\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \alpha (\pi_t - \pi^*)^2 + (1-\alpha)(x_t - x^*)^2 \right\}$

Peter also said that a result in FIRE NK is that mon. pol. can offset 1:1 real rate shocks. This pertains to Clarida, Galí, Gertler's result that opt. mon pol in NK offsets demand shocks $(r_t^n, \bar{i}_t)$ and accomodates supply shocks (u_t). Why? B/c the divine coincidence holds for demand shocks, not for supply shocks.

→ so Peter is saying: do divine coincidence-related results still hold in the NK model w/ learning?

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \alpha (\pi_t - \pi^*)^2 + (1-\alpha)(x_t - x^*)^2 \right\}$$

where $\pi^* = x^* = 0$, so

$$\mathcal{L} = E \sum_{t=0}^{\infty} \beta^t \left\{ \alpha \pi_t^2 + (1-\alpha) x_t^2 \right\}$$

CGG minimizes $\mathcal{L}^{CGG} = \frac{1}{2} E_t \sum_{s=0}^{\infty} \beta^s \left[\alpha x_{t+s}^2 + \pi_{t+s}^2 \right]$

So to be consistent w/ CGG & Preston, I write

$$\mathcal{L} = \frac{1}{2} E_t \sum_{T=1}^{\infty} \beta^{T-1} \left[\alpha x_T^2 + \pi_T^2 \right]$$

↳ what we can already do is what Peter suggested: a grid search over $\{\psi_\pi, \psi_x\}$ w/ this $\mathcal{L}$ to see what's ψ_π^*, ψ_x^*.

Can we solve this analytically?

↳ Check Woodford 2003 (2011)

Revisit f_k → the CEMP & CUSUM criterion, augmented for vectors

26 Jan 2020

As I noted in Notes 3, 2 Dec 2019, p. 34,

$T(\phi) = E(ALM)$ ⟹ So $T(\phi)$ = model-consistent expectations.

The E-stability condition is

$$\frac{d}{d\tau}(\phi) := T(\phi) - \phi = E(ALM) - PLM$$

$$= -1\,[\underbrace{PLM - E(ALM)}]$$

$$= -1\,[\theta_t]$$

↑ CEMP's criterion

Ok: so yesterday I found that the ALM

$z_t = A_a f_a + A_b f_b + A_s s_t$

can be written as

$z_t = \hat{F} + \hat{G}\, s_t$ or $z_t = [\hat{F}, \hat{G}]\begin{bmatrix}1\\ s_t\end{bmatrix}$

The PLM instead is

$z_t = a + b\, s_t$ or $z_t = \phi\begin{bmatrix}1\\ s_t\end{bmatrix}$

I'm not sure how to take $E(ALM)$ in this case,

or, to be more precise, I feel $E(ALM) = ALM$

b/c $E(A_a) = A_a$, $E(A_b) = A_b$, $E(A_s) = A_s$, $E(s_t) = s_t$

and $E_t(fa) = E_t(\text{stuff}(a_{t-1}, b_{t-1}, \text{param}))$

But wait: CEMP's $\theta_t = \dfrac{|\hat{E}_{t-1}\pi_t - E_{t-1}\pi|}{\sigma_\eta}$

Yesterday evening's feat − yesterday evening's expected ALM

$$\phi_{t-1}\begin{bmatrix}1\\ s_{t-1}\end{bmatrix} - [\hat{F}, \hat{G}]\begin{bmatrix}1\\ s_{t-1}\end{bmatrix}$$

In any event, what you get is

$$\frac{dT}{d\tau} = \phi_{t-1} - [\hat{F}, \hat{G}]_{t-1}$$

$\underline{\underline{Var(\text{shocks}) = \Sigma}}$ or $\text{sum}(\text{diag}(\Sigma)) = \sigma_r + \sigma_i + \sigma_y$

(I'm doing this b/c I think that corresponds better.)

$\Rightarrow$ CEMP's criterion becomes

$$\theta_t = |\Sigma^{-1}(\phi_{t-1} - [\hat{F}, \hat{G}]_{t-1})|$$

or potentially $\max(\text{abs}(\,?\,))$

It works, $\bar{\theta} = 5$ seems to be a value where for $\Sigma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ a sim is initially unanchored and then anchors.

I've implemented CUSUM again but now I have the problem that CUSUM either becomes d gain forever, or there's one shock that knocks it out and then it's c gain forever.

Yesterday I solved the CUSUM problems, but there's a new one: ω explodes a lot. Also I implemented a grid search for (ψ_π, ψ_x). It works, but since CUSUM is unstable, those results aren't reliable yet.

27 Jan 2020

Now I've stepped back to optimizing only over ψ_π, with $\psi_x = 0$. Goal: 1) wanna solve the CUSUM issue.

2) Wanna see how optimal ψ_π is. (for CEMP & CUSUM)

3) Wanna see loss as a function of ψ_π (for CEMP & CUSUM)

4) Wanna see how anchoring behaves as a function of ψ_π (for CEMP & CUSUM)

<u>Reading Woodford 2011</u> <u>28 Jan 2020</u>

Woodford's loss fct is a 2nd order Taylor approx to the level of expected utility of the representative HH (p. 383).

For the baseline model, Woodford's U-based criterion is

$$U_t = -\frac{\bar{Y}\alpha_c}{2}\left\{(\sigma^{-1}+\omega)(x_t - x^*)^2 + (\theta^{-1}+\omega)\,\mathrm{var}_i\,\hat{y}_t(i)\right\}$$

$+$ t.i.p. $+\ o(\|\text{shock}\|^3)$

"terms independent of policy"

variance of sectoral output i

p. 395 (2.11)

Subbing in for ego output for different goods i, we get:

$$u_t = -\frac{\bar{Y} u_c}{2}\left\{(\theta^{-1}+\omega)(x_t+x^*)^2 + \theta(1+\omega\theta)\,\mathrm{var}_i \log(p_t(i))\right\}$$

+ rest.

reducing price dispersion is also optimal for policy!

$\Rightarrow$ stabilizing the general price level is desirable in NK model

But what reducing $\mathrm{var}_i(\log(p_t(i)))$ actually means depends on the details of nominal frictions in the model. (of price setting)

In the case of a NK model w/ a NKPC of the form

$\pi_t = \kappa x_t + \beta E_t \pi_{t+1}$,

Woodford rewrites the dispersion measure

$\Delta_t \equiv \mathrm{var}_i \log p_t(i)$ as

$$\Delta_t = \alpha \Delta_{t-1} + \frac{\alpha}{1-\alpha} \pi_t^2 + O(\|\text{stuff}\|^3) \qquad (2.20)$$

Integrating fwd,

the normalized quadratic loss function for the NK model is given by

$$\mathcal{L}_t = \pi_t^2 + \lambda (x_t - x^*)^2 \qquad (2.22, p.400)$$

w/ $\lambda = \frac{\kappa}{\theta}$ where $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{(\sigma^{-1}+\omega)}{(1+\omega\theta)} > 0$ (2.19)

(which is what I have)

and so $\sum_{t=0}^{\infty} \beta^t u_t = \underbrace{-\Omega}_{>0} \sum_{t=0}^{\infty} \beta^t L_t + \text{tip} + O(\|\cdot\|^3)$ (2.21)

$\Rightarrow$ So you get <u>exactly</u> the form of Clarida, Galí & Gertler, except that $\alpha^{CB} = \lambda$, and here that is κ/θ, which for my param values is $= 0.01683$ (quite small).

It's interesting to interpret this.

$\lambda = \frac{\kappa}{\theta}$ $\leftarrow$ nom. frictions

θ $\leftarrow$ price elast of demand.

$\lambda \downarrow$ if $\kappa \downarrow$ or $\theta \uparrow$

$\rightarrow$ more nominal frictions

$\rightarrow$ more competition

The relative importance of stabilizing π (vs. x) grows if prices are more sticky or if there's more competition. $\longrightarrow$ makes sense b/c more distortions come from prices in these 2 cases!

Note: Rotemberg & Woodford's (1997) estimate of $\lambda_x = 0.05$

For NKPC w/ indexation, the loss modifies to

$$L_t = (\pi_t - \gamma \pi_{t-1})^2 + \lambda (x_t - x^*)^2 \qquad (2.25) \text{ p. } 402$$

and $\lambda = \frac{\kappa}{\theta}$

<u>Optimal path of policy</u>

Just looking at

$$\mathcal{L}_t = \pi_t^2 + \lambda (x_t - x^*)^2$$

it is immediate that if $x^* = 0$, $(\pi_t, x_t) = (0,0)$ $\forall t$ is opt.

Interestingly, if $x^* > 0$ (as is the case in the NK model due to monopolistic competition), this conclusion remains largely unchanged:

$$L_t = \pi_t^2 - \lambda x^* x_t + \lambda x_t^2$$

So $$E_0 \sum_{t=0}^{\infty} L_t = -2\theta^{-1} x^* \pi_0 + \underbrace{E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda x_t^2 \right] \right\}}_{\text{the part for } x^* = 0}$$

So apart from $-2\theta^{-1} x^* \pi_0$, opt. mon. pol. is the same! This initial term says that there is a welfare benefit from an initial spurt of inflation, but the optimal plan <u>is still</u> $\pi_t \to 0$ as $t \to \infty$ and in particular, your opt. response to shocks is the same.

Furthermore, as Woodford's "timeless perspective" will suggest, if you commit to conduct policy "as one would have optimally committed to do in the far past" then $\pi_t = 0$ $\forall t$ is opt & you resist the temptation of $\pi_0 > 0$.

p. 415.

However, this optimal plan does not hold for shocks that introduce a time-varying difference between natural output & efficient output. In other words, for <u>cost-push shocks</u>

↳ nice def (p. 419): "a time-varying gap between the efficient level of output and the flexible-price equilibrium level"

⇒ of course: divine coincidence breaks down

p. 451 treats this in more detail:

$$L_t = \pi_t^2 + \lambda(\log Y_t - \log Y_t^e)^2$$

efficient output vs. Y_t^n = flex-price output

The output gap is now

$$x_t = \log Y_t - (\log Y_t^n + (\omega + \sigma^{-1})\hat{\mu}_t)$$

exog. time-var. markup factor

$$= \log Y_t - \log Y_t^e + x^*$$

(instead of $Y_t - Y_t^n$)

and the NKPC is

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t$$

$$\hookrightarrow \equiv \frac{\kappa}{\omega + \sigma^{-1}} \hat{\mu}_t$$ = CGG's cost-push shock.

NB. Table 6.1 Woodford 2011 p. 431 has calibration

Another aside: Woodford 2011, p. 261 ff., Chap. 4, Section 2.3 analyzes "Stability under Learning Dynamics"

Argument: Milton Friedman (1968): "interest-rate targeting can leave the econ vulnerable to instability due to self-fulfilling expectations" (p. 262)

→ Woodford interprets this as saying that a determinate REE may not be converged to by expectations (as emphasized by Howitt (1992) "Interest Rate Control and Nonconvergence to Rational Expectations")

And then Woodford intuitively explains E-stability (incl. "notional time")! Bestaude!

p. 272 ff. looks at Preston (2002a)'s critique!

↑ this is the manuscript to 2005

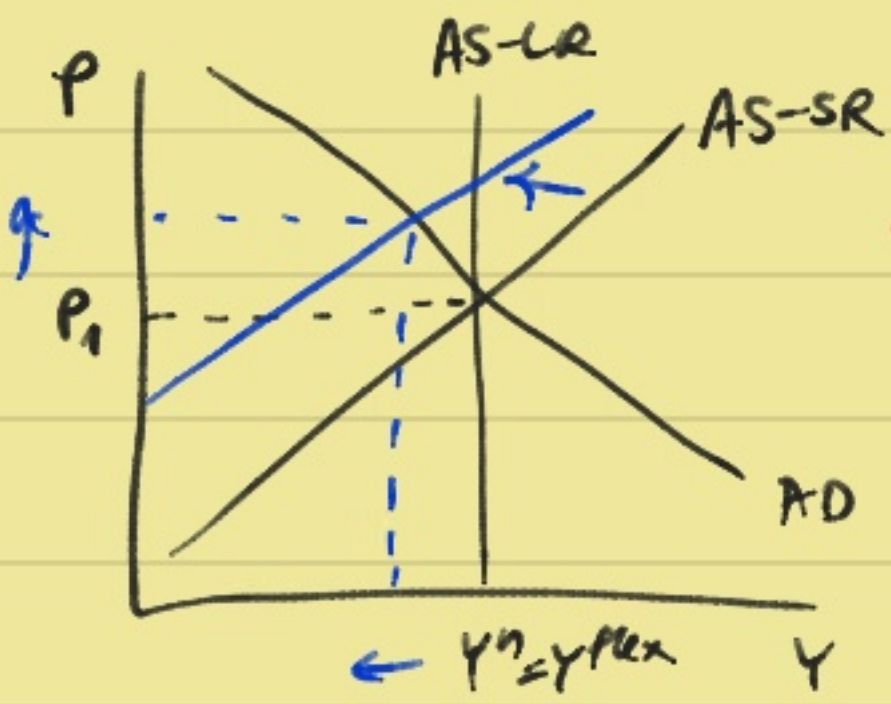
and he makes 'em learn hx... bestaude, komolyan!

See also Chap. 7, Sec. 5 for opt. mon. pol. under learning (p. 528 ff.)

The Chapter 7 Sect. 5 discussion also reminds us that "a policy of strongly responding to ... expected inflation ... can be counterproductive, as it may actually enable instability of inflation due to self-fulfilling expectations." (p. 531)

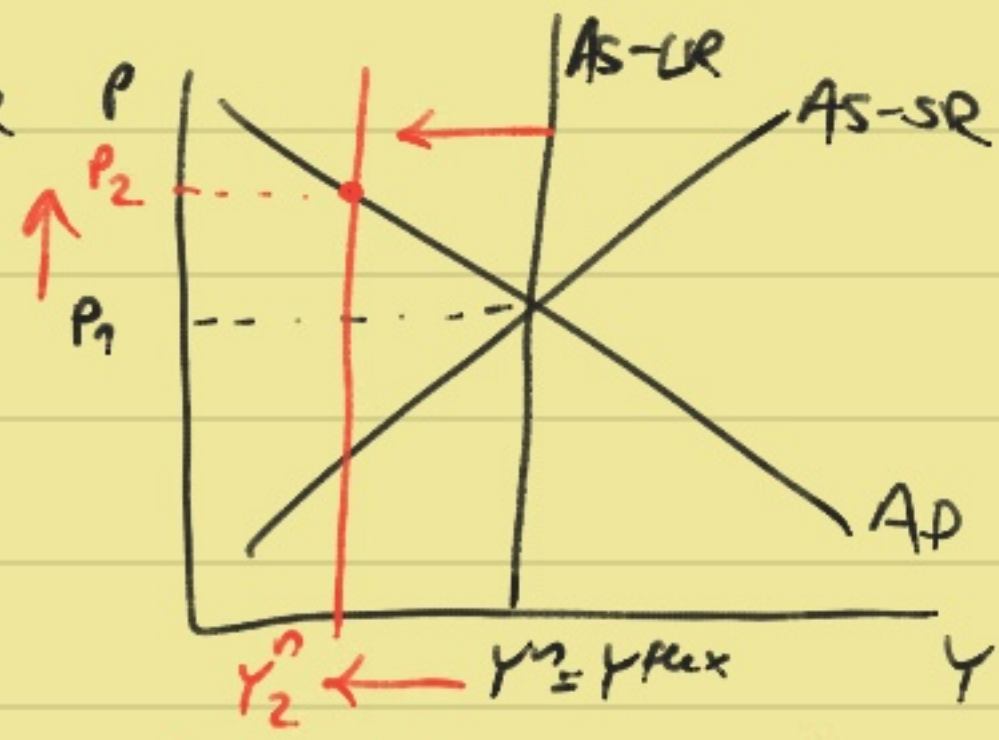
↳ Preston (2002b) evaluates opt. mon. pol. under LH expectations.

Back to cost-push shocks. p. 454 says I think:



is a supply shock, but not a cost-push shock b/c it doesn't change the distance btwn Y^{flex} and $Y^{efficient}$.

Efficient supply shock



I think this is a cost-push shock b/c it ↑ prices, ↓ Y^n and increases the distance $Y^{efficient} - Y^n$

Inefficient supply shock.

Chap 7. Sect 3 "Optimal Simple Policy Rules"

is where Woodford turns to optimal feedback Rules (instead of the optimal state-contingent evolution of (x, π))

1. $\min_{(\psi_\pi, \psi_x)} \mathcal{L}$ often leads to a time-inconsistent choice b/c $\mathcal{L}$ depends on the current stance of (π_t, x_t).

→ 2. Woodford proposes, for a loss fct. of the form

$$\mathcal{L}_t = \pi_t^2 + \lambda (x_t - x^*)^2$$ the following criterion:

$$\boxed{\begin{aligned} \mathcal{L}^{det} &= \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[(E_{t_0} \pi_t)^2 + \lambda (E_{t_0} x_t - x^*)^2 \right] \\ \mathcal{L}^{stab} &= \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[var_{t_0}(\pi_t) + \lambda \, var_{t_0}(x_t) \right] \end{aligned}}$$

$\mathcal{L}^{det}$: depends only on the deterministic component of the exp. paths of (π, x).

$\mathcal{L}^{stab}$: depends only on the exp. responses to unexpected shocks in periods after t_0.

⇒ Woodford's claim is that $(\psi_\pi, \psi_x)^* = \arg\min \mathcal{L}^{stab}$ is opti from a timeless perspective b/c it's independent from the state of the econ at time t_0.

<u>Prop. 7.12</u> on p. 514 establishes optimal $(\gamma_\pi^*, \gamma_x^*)$

Woodford says that you obtain $(\gamma_\pi^*, \gamma_x^*)$ by:

1. Take the sol. for the "optimal noninertial plan", given in App. F, Sect. F.3.; (p. 712)
2. Sub it into the TR (I think he means F.4 on p. 713)

> Note: opt. TRs are history-dependent (i.e. have lags in them

<u>App. F, F.4</u>

NK model w/ 2 disturbances of the form: Chap. 7

$$r_t^n = (1-\rho_r)\bar{r} + \rho_r r_{t-1}^n + \varepsilon_t^r \qquad (2.27)$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \qquad (2.18)$$

Let $z_t \equiv \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}$ $\quad e_t \equiv \begin{bmatrix} \hat{r}_t^n \\ u_t \end{bmatrix}$

Then the <u>optimal noinertial plan</u> is

$$z_t = \bar{z} + F e_t \quad \text{and} \quad i_t = \bar{i} + f_i e_t$$

LR-avg response to shocks LR-avg response to shocks

But yo - how do you obtain this "opt. nonmonetary plan"? → p. 510

<u>optimal nonmonetary plan:=</u>

a state-contingent evolution of endog vars $z_t = (\pi, x)$ that depends only on (s_t, ξ_t), where s_t is the vector of exog. states, ξ_t = exog. shocks, with LR average values $\bar{z} = (\bar{\pi}, \bar{x})$ that are optimal from a timeless perspective.

Formally:

(i) $\bar{z}$ optimal from a timeless perspective

(ii) fluctuations in response to shocks minimize $\mathcal{L}^{stab}$ s.t. z_t depends only on (s_t, ξ_t).

Example: Suppose the model is

$$\pi_t = k x_t + \beta E_t \pi_{t+1} + u_t \qquad (2.1)$$

$$\text{Welfare} = \min \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda (x_t - x^*)^2 \right] \qquad (1.2)$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u$$

Step 1 $\Rightarrow$ in this ex., $(s_t, z_t) = \{u_t\}$, and the constraint that z_t depend only on (s_t, z_t) [in fact, linearly] means that we consider plans of the form

$$\pi_t = \bar{\pi} + f_\pi u_t \quad \& \quad x_t = \bar{x} + f_x u_t \qquad (2.19)$$

Step 2. Make (2.19) consistent w/ the model eq (2.1) which I guess means plug (2.19) into (2.1)

$$\pi_t = k x_t + \beta E_t \pi_{t+1} + u_t \qquad (2.1)$$

$$(\bar{\pi} + f_\pi u_t) = k(\bar{x} + f_x u_t) + \beta E_t[\bar{\pi} + f_\pi u_{t+1}] + u_t$$

$$\bar{\pi} + f_\pi u_t = k\bar{x} + k f_x u_t + \beta\bar{\pi} + \beta f_\pi \rho_u u_t + u_t$$

$$(1-\beta)\bar{\pi} = k\bar{x} - f_\pi u_t + k f_x u_t + \beta f_\pi \rho_u u_t + u_t$$

$$(1-\beta)\bar{\pi} = k\bar{x} + [-f_\pi + k f_x + \beta f_\pi \rho_u + 1]$$

$$\Leftrightarrow (1-\beta)\bar{\pi} = k\bar{x} + \underbrace{[k f_x + 1 - (1-\beta\rho_u) f_\pi]}_{\stackrel{!}{=} 0} u_t \qquad (3.3)$$

For some reason, Woodford requires that so as to get $(1-\beta)\bar{\pi} = k\bar{x}$.

Maybe the requirement comes from the fact that (2.1) in the LR implies: $\bar{\pi} = k\bar{x} + \beta\bar{\pi}$.

(So this is an undet. coefficients argument.)

Step 3. Derive $\mathcal{L}^{stab}$. Recall that it's given by:

$$\mathcal{L}^{stab} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[var_{t_0}(\pi_t) + \lambda\, var_{t_0}(x_t) \right]$$

Plug (2.19)

$$\mathcal{L}^{stab} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[var_{t_0}(\bar{\pi} + f_\pi u_t) + \lambda\, var_{t_0}(\bar{x} + f_x u_t) \right]$$

$$\boxed{Var(A+B) = Var(A) + Var(B) + 2Cov(A,B) \quad \text{I think } \bar{z} \perp u_t}$$

$$= \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\overbrace{Var_{t_0}(\bar{\pi})}^{=0} + f_\pi^2 Var_{t_0}(u_t) + \lambda \underbrace{Var_{t_0}(\bar{x})}_{=0} + \lambda f_x^2 Var_{t_0}(u_t) \right]$$

$$= \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[(f_\pi^2 + \lambda f_x^2)\, Var_{t_0}(u_t) \right]$$

$$= (f_\pi^2 + \lambda f_x^2) \sum_{t=t_0}^{\infty} \beta^{t-t_0} \underbrace{Var_{t_0}(u_t)}_{?}$$

the unconditional $Var(u_t)$ would be $\gamma_0 = E(u_t)^2 = \frac{\sigma_u^2}{1-\rho_u^2}$

If $Var_{t_0}(u_t)$ would correspond to this, we'd get

$$= (f_\pi^2 + \lambda f_x^2) \frac{1}{1-\beta} \frac{1}{1-\rho_u^2} \sigma_u^2$$

which isn't exactly what Woodford gets.

Rewrite $\sum_{t=t_0}^{\infty} \beta^{t-t_0} var_{t_0}(u_t)$ as $\sum_{T=t}^{\infty} \beta^{T-t} var_t(u_T)$

Maybe $var_t(\cdot)$ signifies that we only care about u_t, u_{t+1}, u_{t+2} ... etc.

$$var_t(u_t) + \beta var_t(u_{t+1}) + \beta^2 var_t(u_{t+2}) + \dots$$

$$= var_t(u_t) + \beta var_t(\rho u_t + \varepsilon_{t+1}) + \beta^2 var_t(\rho u_{t+1} + \varepsilon_{t+2}) + \dots$$

$$= var_t(u_t) + \beta\rho^2 var_t(u_t) + \beta b^2 + \beta^2 var_t(\rho^2 u_t + \rho\varepsilon_{t+1} + \varepsilon_{t+2}) + \dots$$

should be 0 b/c u_t is known at t.

$$= [1 + \beta\rho^2 + (\beta\rho)^2 + \dots] \overbrace{var_t(u_t)} + \beta b^2 + \beta^2\rho^2 b^2 + \beta^2 b^2 + \dots$$

$$\frac{1}{1-\beta\rho^2} \quad \frac{b^2}{1-\rho^2} \qquad + b^2[\beta + \beta^2\rho^2 + \beta^2 + \dots]$$

Errors: $\beta var_t(\varepsilon_{t+1}) + \beta^2 var_t(\rho\varepsilon_{t+1} + \varepsilon_{t+2})$

$$+ \beta^3 var_t(\rho^2\varepsilon_{t+1} + \rho\varepsilon_{t+2} + \varepsilon_{t+3})$$

$$+ \beta^4 var_t(\rho^3\varepsilon_{t+1} + \rho^2\varepsilon_{t+2} + \rho\varepsilon_{t+3} + \varepsilon_{t+4}) + \dots$$

$$= \beta b^2 + \beta^2(\rho^2+1)b^2 + \beta^3(\rho^4+\rho^2+1)b^2$$

$$+ \beta^4(\rho^6+\rho^4+\rho^2+1)b^2 + \dots$$

$$= \beta z^2 + \beta^2(\rho^2+1)z^2 + \beta^3(\rho^4+\rho^2+1)z^2$$
$$+ \beta^4(\rho^6+\rho^4+\rho^2+1)z^2 + \dots$$

Again, call $r := \rho^2$

$$= z^2\left\{\beta + \beta^2(r^1+r^0) + \beta^3(r^2+r^1+r^0) + \beta^4(r^3+r^2+r^1+r^0)\right\}$$

$$\sum_{k=0}^{n-1} r^k = \frac{1-r^n}{1-r}$$

$$= z^2\left\{\beta + \beta^2\frac{1-r^2}{1-r} + \beta^3\frac{1-r^3}{1-r} + \beta^4\frac{1-r^4}{1-r} + \dots\right\}$$

$$\frac{z^2}{1-r}\left\{\beta(1-r) + \beta^2(1-r^2) + \beta^3(1-r^3) + \beta^4(1-r^4) + \dots\right\}$$

$$\frac{z^2}{1-r}\left\{\beta+\beta^2+\beta^3+\beta^4+\dots - \left[\beta r + \beta^2 r^2 + \beta^3 r^3 + \dots\right]\right\}$$

$$= \frac{z^2}{1-r}\left\{1+\beta+\beta^2+\dots - \left[1+\beta r+\beta^2 r^2+\dots\right]\right\}$$

$$= \frac{z^2}{1-r}\left\{\frac{1}{1-\beta} - \frac{1}{1-\beta r}\right\}$$

$$= \frac{z^2}{1-r}\,\frac{1-\beta r-(1-\beta)}{(1-\beta)(1-\beta r)} = \frac{z^2}{\cancel{(1-r)}}\,\frac{\beta\cancel{(1-r)}}{(1-\beta)(1-\beta r)}$$

$$= z^2 \frac{\beta}{(1-\beta)(1-\beta r)} \qquad \Big| \; r = \rho^2$$

$$= \frac{\beta}{1-\beta} \frac{1}{1-\beta\rho^2} z^2$$ and I'm missing a $(1-\rho^2)$-term.

Ok, whatever, the point is I see how he obtains $\mathcal{L}^{stat}$.

<u>Step 4.</u> $(f_\pi^*, f_x^*) = \arg\min \mathcal{L}^{stat}$ s.t. (3.3)

or equivalently

$\min f_\pi^2 + \lambda f_x^2$ s.t. $(1-\beta f_\pi) f_\pi = \kappa f_x + 1$

$\rightarrow$ obtain (f_π^*, f_x^*).

Exciting!

Now let's try to do that for my model...

Finding the "optimal nonintertial plan" for my model. 29 Jan 2020

<u>Step 1</u>. Postulate solutions of the form $z_t = F s_t$

$\pi_t = \bar{\pi} + f_\pi s_t$ $\qquad$ $x_t = \bar{x} + f_x s_t$

$s_t = (r_t^n, i_t, u_t)'$ and I need to write the baseline model in MN-form (copied from before)

(1) $$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T \right\}$$

(2) $$b\psi_\pi \pi_t + (1+b\psi_x) x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-b\beta\psi_x) x_{T+1} + b(1-\beta\psi_\pi)\pi_{T+1} + b r_T^n - b\bar{i}_T \right\}$$

<u>Step 2</u>. Plug postulated sol. into model equations

(1) $$\pi + f_\pi s_t = \kappa(\bar{x} + f_x s_t) + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \kappa\alpha\beta(\bar{x} + f_x s_{T+1}) + (1-\alpha)\beta(\bar{\pi} + f_\pi s_{T+1}) + [0,0,1] s_T \right\}$$

(2) $$b\psi_\pi(\bar{\pi} + f_\pi s_t) + (1+b\psi_x)(\bar{x} + f_x s_t) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\beta-b\beta\psi_x)(\bar{x} + f_x s_{T+1}) + b(1-\beta\psi_\pi)(\bar{\pi} + f_\pi s_{T+1}) + b[1,-1,0] s_T \right\}$$

Collect s's on RHS. We'll have to use LOM-s to make 'em all time t.

(1) $$\bar{\pi} - \kappa\bar{x} - \frac{\kappa\alpha\beta}{1-\alpha\beta}\bar{x} - \frac{(1-\alpha)\beta}{1-\alpha\beta}\bar{\pi} = -f_\pi s_t + \kappa f_x s_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ (\kappa\alpha\beta f_x + (1-\alpha)\beta f_\pi) s_{T+1} + [0,0,1] s_T \right\}$$

(2) $$b\psi_\pi \bar{\pi} + (1+b\psi_x)\bar{x} - \frac{1-\beta-b\beta\psi_x}{1-\beta}\bar{x} - \frac{b(1-\beta\psi_\pi)}{1-\beta}\bar{\pi} = RHS_2$$

$$RHS_2 = \left[-b\psi_\pi f_\pi - (1+b\psi_x) f_x\right] s_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big\{ \left[(1-\beta-b\beta\psi_x) f_x + b(1-\beta\psi_\pi) f_\pi\right] s_{T+1} + b[1,-1,0]\, s_T \Big\}$$

So the 2 eqs become:

$$\bar{\pi} - \kappa\bar{x} - \frac{\kappa\alpha\beta}{1-\alpha\beta}\bar{x} - \frac{(1-\alpha)\beta}{1-\alpha\beta}\bar{\pi} = -f_\pi s_t + \kappa f_x s_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \Big\{ \left(\kappa\alpha\beta f_x + (1-\alpha)\beta f_\pi\right) s_{T+1} + [0,0,1]\, s_T \Big\}$$

$$b\psi_\pi \bar{\pi} + (1+b\psi_x)\bar{x} - \frac{1-\beta-b\beta\psi_x}{1-\beta}\bar{x} - \frac{b(1-\beta\psi_\pi)}{1-\beta}\bar{\pi}$$

$$= \left[-b\psi_\pi f_\pi - (1+b\psi_x) f_x\right] s_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big\{ \left[(1-\beta-b\beta\psi_x) f_x + b(1-\beta\psi_\pi) f_\pi\right] s_{T+1} + b[1,-1,0]\, s_T \Big\}$$

LOM-s: $s_{t+1} = h_x s_t + \eta \varepsilon_t$ so $s_{T+1} = h_x s_T$

$$s_{t+1} = h_x^1 s_t \rightarrow s_{t+k} = h_x^k s_t \qquad s_T = h_x^{T-t} s_t$$

So RHSs become:

(1) $\left(-f_\pi + \kappa f_x\right) s_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-1} \Big\{ \left[\left(\kappa\alpha\beta f_x + (1-\alpha)\beta f_\pi\right) h_x + [0,0,1]\right] s_T \Big\}$

(2) $\left[-b\psi_\pi f_\pi - (1+b\psi_x) f_x\right] s_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big\{ \left[\left[(1-\beta-b\beta\psi_x) f_x + b(1-\beta\psi_\pi) f_\pi\right] h_x + b[1,-1,0]\right] s_T \Big\}$

which is

$$\left(-f_\pi + \kappa f_x\right) s_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-1} \Big\{ \left[\left(\kappa\alpha\beta f_x + (1-\alpha)\beta f_\pi\right) h_x + [0,0,1]\right] h_x^{T-t} s_t$$

$$\left[-b\psi_\pi f_\pi - (1+b\psi_x) f_x\right] s_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big\{ \left[\left[(1-\beta-b\beta\psi_x) f_x + b(1-\beta\psi_\pi) f_\pi\right] h_x + b[1,-1,0]\right] h_x^{T-t} s_t$$

$$(-f_\pi + \kappa f_x) s_t + \hat{E}_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-1} \left\{ \left[(\kappa\alpha\beta f_x + (1-\alpha)\beta f_\pi) h_x + [0,0,1] \right] h_x^{\tau-t} s_t \right.$$

$$[-b\psi_\pi f_\pi - (1+b\psi_x) f_x] s_t + \hat{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ \left[[(1-\beta-b\beta\psi_x) f_x + b(1-\beta\psi_\pi) f_\pi] h_x + b[1,-1,0] \right] h_x^{\tau-t} s_t \right.$$

$\Leftrightarrow$

$$(-f_\pi + \kappa f_x) s_t + \left[(\kappa\alpha\beta f_x + (1-\alpha)\beta f_\pi) h_x + [0,0,1] \right] \cdot (I_{nx} - \alpha\beta h_x)^{-1} s_t$$

$$[-b\psi_\pi f_\pi - (1+b\psi_x) f_x] s_t + \left[[(1-\beta-b\beta\psi_x) f_x + b(1-\beta\psi_\pi) f_\pi] h_x + b[1,-1,0] \right] (I_{nx} - \beta h_x)^{-1} s_t$$

$\Leftrightarrow$

$$\left\{ -f_\pi + \kappa f_x + \left[(\kappa\alpha\beta f_x + (1-\alpha)\beta f_\pi) h_x + [0,0,1] \right] \cdot (I_{nx} - \alpha\beta h_x)^{-1} \right\} s_t \qquad (RHS_1)$$

$$\left\{ -b\psi_\pi f_\pi - (1+b\psi_x) f_x + \left[[(1-\beta-b\beta\psi_x) f_x + b(1-\beta\psi_\pi) f_\pi] h_x + b[1,-1,0] \right] (I_{nx} - \beta h_x)^{-1} \right\} s_t \qquad (RHS_2)$$

LHSs were

$$\bar{\pi} - \kappa\bar{x} - \frac{\kappa\alpha\beta}{1-\alpha\beta}\bar{x} - \frac{(1-\alpha)\beta}{1-\alpha\beta}\bar{\pi} = \left(1 - \frac{(1-\alpha)\beta}{1-\alpha\beta}\right)\bar{\pi} + \left(-\kappa - \frac{\kappa\alpha\beta}{1-\alpha\beta}\right)\bar{x} \qquad (LHS_1)$$

$$b\psi_\pi\bar{\pi} + (1+b\psi_x)\bar{x} - \frac{1-\beta-b\beta\psi_x}{1-\beta}\bar{x} - \frac{b(1-\beta\psi_\pi)}{1-\beta}\bar{\pi} = \left(b\psi_\pi - \frac{b(1-\beta\psi_\pi)}{1-\beta}\right)\bar{\pi} + \left((1+b\psi_x) - \frac{1-\beta-b\beta\psi_x}{1-\beta}\right)\bar{x} \qquad (LHS_2)$$

Now I follow Woodford to require that RHSs $\overset{!}{=} 0$, and thus LHSs $\overset{!}{=} 0$

For the LHSs, this means

$$\frac{1-\alpha\beta-(1-\alpha)\beta}{1-\alpha\beta}\bar{\pi} = \frac{\kappa(1-\alpha\beta)+\kappa\alpha\beta}{1-\alpha\beta}\bar{x}$$

$$\Rightarrow (1-\cancel{\alpha\beta}-\beta+\cancel{\alpha\beta})\bar{\pi} = (\kappa-\cancel{\kappa\alpha\beta}+\cancel{\kappa\alpha\beta})\bar{x} \quad \Rightarrow \quad (1-\beta)\bar{\pi} = \kappa\bar{x} \qquad (I)$$

(I get the RE thing. B/c in LR they're the same. :))

$$\left(b\psi_\pi - \frac{b(1-\beta\psi_\pi)}{1-\beta}\right)\bar{\pi} + \left((1+b\psi_x) - \frac{1-\beta-b\beta\psi_x}{1-\beta}\right)\bar{x}$$

$$\frac{b\psi_\pi(1-\beta) - b + b\beta\psi_\pi}{1-\beta}\,\bar{\pi} = \frac{1-\beta-b\beta\psi_x - (1+b\psi_x)(1-\beta)}{1-\beta}\,\bar{x}$$

$$\Leftrightarrow \left(b\psi_\pi - \cancel{b\psi_\pi\beta} - b + \cancel{b\beta\psi_\pi}\right)\bar{\pi} = \left(1-\beta-b\beta\psi_x - (1-\beta+b\psi_x - b\psi_x\beta)\right)\bar{x}$$

$$b(\psi_\pi - 1)\bar{\pi} = (\cancel{1}-\cancel{\beta}-\cancel{b\beta\psi_x} - \cancel{1} + \cancel{\beta} - b\psi_x + \cancel{b\beta\psi_x})\bar{x}$$

$$b(\psi_\pi - 1)\bar{\pi} = -b\psi_x\bar{x} \qquad (\mathrm{II})$$

So Step 2 gives me 4 equations (instead of Woodford's 2):

$$(1-\beta)\bar{\pi} = \kappa\bar{x} \qquad (\mathrm{I})$$

$$b(\psi_\pi - 1)\bar{\pi} = -b\psi_x\bar{x} \qquad (\mathrm{II})$$

$$\left\{-f_\pi + \kappa f_x + \left[\left(\kappa\alpha\beta f_x + (1-\alpha)\beta f_\pi\right)h_x + [0,0,1]\right]\cdot\left(I_{nx} - \alpha\beta h_x\right)^{-1}\right\} = 0 \qquad (\mathrm{III})$$

$$\left\{-b\psi_\pi f_\pi - (1+b\psi_x)f_x + \left[(1-\beta-b\beta\psi_x)f_x + b(1-\beta\psi_\pi)f_\pi\right]h_x + b[1,-1,0]\left[I_{nx} - \beta h_x\right]^{-1}\right\} = 0 \qquad (\mathrm{IV})$$

By the way, (I) & (II) allow me to solve for $(\bar{\pi}, \bar{x})$

$b(\psi_\pi - 1)\frac{\kappa}{1-\beta}\cancel{\bar{x}} = -b\psi_x\cancel{\bar{x}}$ darn. They are redundant.

So that only gives me parameter restrictions, unless $\bar{\pi} = \bar{x} = 0$, b/c then it doesn't do that either. But I think that's fine b/c $(\bar{\pi}, \bar{x})$ need to be argmin $\mathcal{L}^{det}$, which $= 0$ (obviously :))

Step 3. Derive $\mathcal{L}^{stab}$.

$$\mathcal{L}^{stab} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[var_{t_0}(\pi_t) + \lambda\, var_{t_0}(x_t) \right]$$

Fortunately the sols have the same form, only now s_t is a vector while u_t was a scaler.

Let's rewrite L^{stab} as

$$\mathcal{L}^{stab} = \sum_{T=t}^{\infty} \beta^{T-t} \left[var_t(\pi_T) + \lambda\, var_t(x_T) \right]$$

$$var_t(\pi_T) = var_t(\bar{\pi} + \underset{1\times3}{f_\pi} \underset{3\times1}{s_T}) = \underset{3\times1}{f_\pi'} \underset{1\times3}{f_\pi} var_t(s_T)$$

$$\underset{3\times3}{f_\pi' f_\pi}$$

$$var_t(s_t) = E_t[\underset{3\times1}{s_t} \underset{1\times3}{s_t'}]$$

$$\underset{3\times3}{}$$

But then $\mathcal{L}^{stab}$ will be 3x3 too!

$$var(\underset{1\times3}{f_\pi} \underset{3\times1}{s_T}) = E_t[\underset{1\times1}{(f_\pi s_T)} \underset{1\times1}{(f_\pi s_T)'}]$$

Maybe I can try to "keep $f_\pi s_T$ together" ☺

$$\rightarrow \mathcal{L}^{stab} = \sum_{T=t}^{\infty} \beta^{T-t} \left[var_t(f_\pi s_T) + \lambda\, var_t(f_x s_T) \right]$$

I don't know about that!

Let's go back to the scalar version:

$$\sum_{T=t}^{\infty} \beta^{T-t} \, var_t(u_T)$$

$$= var_t(u_t) + \beta var_t(u_{t+1}) + \beta^2 var_t(u_{t+2}) + \beta^3 var_t(u_{t+3}) + \ldots$$

$$var_t(u_t) = E_t[u_t^2] = E_t[[\rho u_{t-1} + \varepsilon_t][\rho u_{t-1} + \varepsilon_t]]$$

$$= E_t[\rho^2 u_{t-1}^2 + 2\rho\varepsilon_t + \varepsilon_t^2]$$

$$= \rho^2 u_{t-1}^2 + 2\rho\varepsilon_t + \varepsilon_t^2$$

But that's not really the point. The point was to get to $\mathcal{L}^{stat} = [f_\pi^2 + \lambda f_x^2] \sum_{T=t}^{\infty} \beta^{T-t} var_t(u_T)$

$$= [f_\pi^2 + \lambda f_x^2] \cdot (\text{stuff independent of } f_\pi, f_x)$$

$$\propto [f_\pi^2 + \lambda f_x^2]$$

→ so I don't want to "keep $f_i S_T$ $i = \pi, x$ together"!

Ok so let's write

$$\mathcal{L}^{stat} = \sum_{T=t}^{\infty} \beta^{T-t} [var_t(f_\pi S_T) + \lambda \, var_t(f_x S_T)]$$

as $\mathcal{L}^{stat} = \sum_{T=t}^{\infty} \beta^{T-t} [(\underset{3\times3}{f_\pi' f_\pi} + \lambda \underset{3\times3}{f_x' f_x}) \underbrace{var_t(S_T)}_{\text{and not derive this.}}]$

Instead, observe that $\alpha^{stat} \propto (f_\pi' f_\pi + \lambda f_x' f_x)$

and thus now find f_π^{oni} & f_x^{oni} s.t.

$$(f_\pi^{oni}, f_x^{oni}) = \arg\min (f_\pi' f_\pi + \lambda f_x' f_x)$$

s.t. (III) and (IV).

$$\left\{ -f_\pi + \kappa f_x + \left[(\kappa\alpha\beta f_x + (1-\alpha)\beta f_\pi) h_x + [0,0,1] \right] \cdot (I_{nx} - \alpha\beta h_x)^{-1} \right\} = 0 \quad \text{(III)}$$

$$\left\{ -b\psi_\pi f_\pi - (1+b\psi_x) f_x + \left[(1-\beta-b\beta\psi_x) f_x + b(1-\beta\psi_\pi) f_\pi \right] h_x + b[1,-1,0] \right] (I_{nx} - \beta h_x)^{-1} \Big\} = 0 \quad \text{(IV)}$$

Matrix derivatives.

Econometrics Sum Part 1, p. 1 (lol!)

$$\mathrm{Cov}(\underset{j\times j}{A}\,\underset{j\times 1}{X} + \underset{j\times 1}{c},\ \underset{j\times j}{B}\,\underset{j\times 1}{Z} + \underset{j\times 1}{d}) = A\,\mathrm{Cov}(X, Z)\,B'$$

A, B, c, d constants.

My case was similar but not identical to

$$\mathrm{Cov}(AX, AX) = A\,\mathrm{Cov}(X,X)\,A' = A\,\mathrm{Var}(X)\,A'$$

which is bad news b/c it will mean that I might have to

evel $\mathrm{var}_t(s_T)$.

$$Cov(f_\pi s_T, f_\pi s_T) = \underset{1\times 3}{f_\pi} \underset{3\times 3}{Var(s_T)} \underset{3\times 1}{f_\pi'} = 1\times 1$$

ok works!

So that means

$$\mathcal{L}^{star} = \sum_{T=t}^{\infty} \beta^{T-t} \left[var_t(f_\pi s_T) + \lambda\, var_t(f_x s_T) \right]$$

$$\mathcal{L}^{star} = \sum_{T=t}^{\infty} \beta^{T-t} \left[f_\pi\, var_t(s_T) f_\pi' + \lambda f_x\, var_t(s_T) f_x' \right]$$

Ok so reconsider scalar case. $var_t(u_t) = 0$ b/c u_t known at t.

$$var_t(u_{t+1}) = var_t[\rho u_t + \varepsilon_{t+1}] = \sigma^2$$

$$= E_t[u_{t+1}^2] = E_t[\rho^2 u_t^2 + \varepsilon_{t+1}^2 + 2\rho\varepsilon_{t+1}]$$

$$= E_t[\varepsilon_{t+1}^2] = \sigma^2 = \sigma^2(1)$$

$$var_t(u_{t+2}) = var_t[\rho u_{t+1} + \varepsilon_{t+2}]$$

$$= var_t(\rho^2 u_t + \rho\varepsilon_{t+1} + \varepsilon_{t+2})$$

$$= \rho^2\sigma^2 + \sigma^2 = (\rho^2 + 1)\sigma^2$$

$$var_t(u_{t+k}) = \left((\rho^2)^k + (\rho^2)^{k-1} + \dots + (\rho^2) + (\rho^2)^0\right)\sigma^2$$

$$var_t(u_T) = \left((\rho^2)^{T-t} + (\rho^2)^{T-t-1} + \dots + \rho^2 + 1\right)\sigma^2 = \frac{1-(\rho^2)^{T-t+1}}{1-\rho^2}\sigma^2$$

$$\sum_{T=t}^{\infty} \beta^{T-t}\, var_t(u_T) = \sum_{T=1}^{\infty} \beta^{T-t}\left(1-(\rho^2)^{T-t+1}\right)\frac{\sigma^2}{1-\rho^2}$$

$$\sum_{\tau=1}^{\infty} \beta^{\tau-t}\left(1-(\rho^2)^{\tau-t+1}\right)\frac{\sigma^2}{1-\rho^2}$$

$$= \frac{\sigma^2}{1-\rho^2}\left[\sum_{\tau=t}^{\infty}\beta^{\tau-t} - \sum_{\tau=t}^{\infty}\beta^{\tau-t}(\rho^2)^{\tau-t+1}\right]$$

$$= \frac{\sigma^2}{1-\rho^2}\left[\frac{1}{1-\beta} - \rho^2\sum_{\tau=t}^{\infty}(\beta\rho^2)^{\tau-t}\right]$$

$$= \frac{\sigma^2}{1-\rho^2}\left[\frac{1}{1-\beta} - \frac{\rho^2}{1-\beta\rho^2}\right]$$

$$= \frac{\sigma^2}{1-\rho^2}\left[\frac{1-\beta\rho^2-(1-\beta)\rho^2}{(1-\beta)(1-\beta\rho^2)}\right]$$

$$= \frac{\sigma^2}{1-\rho^2}\left[\frac{1-\beta\rho^2-\rho^2+\beta\rho^2}{(1-\beta)(1-\beta\rho^2)}\right]$$

$$= \frac{\sigma^2}{1-\rho^2}\,\frac{1-\rho^2}{(1-\beta)(1-\beta\rho^2)}$$

$$= \frac{\sigma^2}{(1-\beta)(1-\beta\rho^2)}$$

which is, apart from one β-term, the same as I obtained last time.

So the scalar case was

$$var_t(u_T) = \left((\rho^2)^{T-t} + (\rho^2)^{T-t-1} + \dots + \rho^2 + 1\right)\sigma^2$$

Ok, so given this, the vector case will be something like

$$var_t(s_T) = \left[(hxhx')^{T-t} + (hxhx')^{T-t-1} + \dots + hxhx' + I_{nx}\right]\underbrace{\eta\eta'}_{\text{or } \Sigma}$$

So the matrix generalization of the geom. sum

$$\sum_{k=0}^{n-1} r^k = \frac{1-r^n}{1-r} \Rightarrow (I_{nx} - R)^{-1}(I_{nx} - R^n)$$

(this seems to be right)

So then

$$\mathcal{L}^{star} = \sum_{T=t}^{\infty} \beta^{T-t}\left[f_\pi var_t(s_T) f_\pi' + \lambda f_x var_t(s_T) f_x'\right]$$

Consider just first part

$$\sum_{T=t}^{\infty} \beta^{T-t}\left[f_\pi var_t(s_T) f_\pi'\right]$$

$$= \sum_{T=t}^{\infty} \beta^{T-t} f_\pi\left[(I_{nx} - hxhx')^{-1}(I_{nx} - (hxhx')^{T-t+1})\right] f_\pi'$$

$$= \sum \beta^{T-t} f_\pi\left[(I_{nx} - hxhx')^{-1} I_{nx} - (I_{nx} - hxhx')^{-1}(hxhx')^{T-t+1}\right] f_\pi'$$

$$= \sum \beta^{T-t} f_\pi (I_{nx} - hxhx')^{-1} I_{nx} f_\pi' - \sum \beta^{T-t} f_\pi (I_{nx} - hxhx')^{-1}(hxhx')^{T-t+1} f_\pi'$$

$$= \sum \beta^{T-t} f_\pi (I_{n_x} - h_x h_x')^{-1} I_{n_x} f_\pi' - \sum \beta^{T-t} f_\pi (I_{n_x} - h_x h_x')^{-1} (h_x h_x')^{T-t+1} f_\pi'$$

$$= \frac{1}{1-\beta} f_\pi (I_{n_x} - h_x h_x')^{-1} I_{n_x} f_\pi' - f_\pi (I_{n_x} - h_x h_x')^{-1} \sum \beta^{T-t} (h_x h_x')^{T-t} (h_x h_x') f_\pi'$$

$$= \frac{1}{1-\beta} f_\pi (I_{n_x} - h_x h_x')^{-1} I_{n_x} f_\pi' - f_\pi (I_{n_x} - h_x h_x')^{-1} (I_{n_x} - \beta h_x h_x')^{-1} (h_x h_x') f_\pi'$$

$$=: \mathcal{L}_\pi$$

and analogously, $\mathcal{L}_x$ will be

$$\frac{1}{1-\beta} f_x (I_{n_x} - h_x h_x')^{-1} I_{n_x} f_x' - f_x (I_{n_x} - h_x h_x')^{-1} (I_{n_x} - \beta h_x h_x')^{-1} (h_x h_x') f_x'$$

$$=: \mathcal{L}_x$$

So then $\mathcal{L}^{stab} = \mathcal{L}_\pi + \lambda \mathcal{L}_x$

Fortunately, since f_π and f_x enter $\mathcal{L}_\pi$ & $\mathcal{L}_x$ symmetrically,

$\frac{\partial \mathcal{L}_\pi}{\partial f_\pi}$ will be symmetric to $\frac{\partial \mathcal{L}_x}{\partial f_x}$.

Ryan meeting 29 Jan 2020

- Macro lunch?
- financing next year?
- you on sabbatical next year?
- Clough travel grant?

For CUSUM, instead of mean, try taking det(·) b/c it appears in info theory some times, including for entropy.

Is $\omega^{-1} FEFE' - \theta_{t-1}$ pos. semidef?
(check entropy of normal RC matrix or stuff like that?)
Sims 2003: eq (8) p. 673

Sensitivity to $\bar{\theta}/\tilde{\theta}$:
"As a CB, I don't know the threshold parameter, so I wanna have a policy that's robust to that parameter"

- check if for CEMP the proj-facility is looked every time so you look like you're converging but actually you're not.

For CUSUM: big mistakes will count a lot b/c of squaring, vs. for CEMP, it's not squared, it's linear

$\Rightarrow$ check differences in length of a spell

For the scalar version, checking $\sqrt{\text{CUSUM}}$ would also allow to check.