Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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October 21, 2020

Inflation that runs below its desired level can lead to an unwelcome fall in ***longer-term inflation expectations****, which, in turn, can pull actual inflation even lower, resulting in an adverse cycle of ever-lower inflation and inflation expectations. [...]* ***Well-anchored inflation expectations*** *are critical[.]*
Jerome Powell, Chairman of the Federal Reserve [1]
(Emphases added.)

[1] "New Economic Challenges and the Fed's Monetary Policy Review," August 27, 2020.

Long-run expectations: responsive to short-run conditions?

Individual-level Survey of Professional Forecasters (SPF): for 1991-Q4 onward, estimate rolling regression

$$\Delta \bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{1}$$

$\bar{\pi}_t$ 10-year ahead inflation expectation

$f_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1}\, \pi_t$ individual one-year-ahead forecast error

w indexes windows of 20 quarters

Time-varying responsiveness

$$\Delta\bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \quad (1)$$

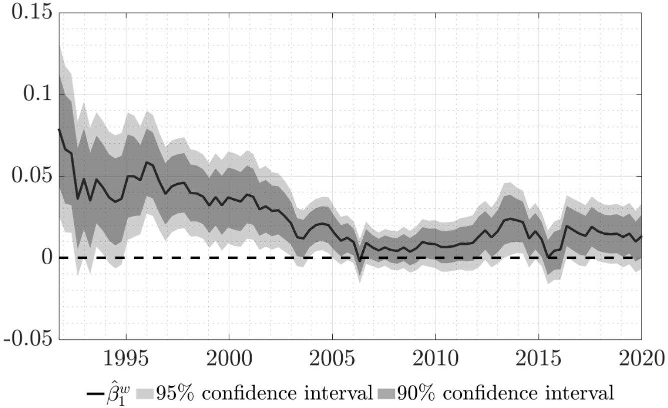

Figure: Time series of $\hat{\beta}_1^w$

Robustness

This project

- How to conduct monetary policy when expectations can become unanchored?

- Model of expectations unanchoring
  $\hookrightarrow$ extension to adaptive learning that captures time-varying responsiveness of long-run expectations

- Estimate how unanchoring takes place in data
  $\hookrightarrow$ quantify novel anchoring channel

- Analyze monetary policy
  $\hookrightarrow$ analytically and numerically using novel model disciplined by data

Preview of results

1. Estimation
   - Larger mistakes unanchor more

2. Optimal policy
   - Responds aggressively to inflation when unanchored, accommodates inflation when anchored

3. Taylor rule
   - Less aggressive on inflation than under rational expectations

Related literature

- **Optimal monetary policy in the New Keynesian model**
  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Adaptive learning**
  Evans & Honkapohja (2001, 2006), Sargent (1999), Primiceri (2006), Lubik & Matthes (2018), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Mele et al (2019), Eusepi & Preston (2011), Milani (2007, 2014), Marcet & Nicolini (2003)

- **Anchoring and the Phillips curve**
  Goodfriend (1993), Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Reis (2020), Hebden et al 2020, Gobbi et al (2019), Carvalho et al (2019)

Structure of talk

1. Model of anchoring expectations
2. Quantification of learning channel
3. Solving the Ramsey problem
4. Implementing optimal policy
5. Approximating optimal policy with a Taylor rule

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Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j))dj \right] \tag{2}$$

Budget constraint

$$B_t^i \leq (1+i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_tC_t^i \tag{3}$$

▸ Consumption, price level

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}^j_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi^j_t(p_t(j)) \right] \tag{4}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{5}$$

▸ Profits, stochastic discount factor

Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- Model implies mapping between exogenous states s_t and observables $y_t \equiv (\pi_t, x_t, i_t)'$

$$y_t = g s_t \tag{6}$$

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- Under rational expectations (RE), private sector knows model
$\rightarrow$ knows (6)

$$\mathbb{E}_t\, y_{t+1} = g\, \mathbb{E}_t\, s_{t+1} \tag{7}$$

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- Under rational expectations (RE), private sector knows model $\rightarrow$ knows (6)

$$\mathbb{E}_t \, y_{t+1} = g \, \mathbb{E}_t \, s_{t+1} \tag{7}$$

- $\hat{\mathbb{E}}$: agents do not internalize that identical $\rightarrow$ do not know aggregate model $\rightarrow$ do not know (6)

Adaptive learning

- Agents know exogenous evolution of states

$$s_{t+1} = hs_t + \epsilon_{t+1} \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \tag{8}$$

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- Estimate $\bar{\pi}_{t-1}$ using recursive least squares (RLS) using observed states and knowledge of (8)

Recursive least squares (RLS)

Let b_1 denote first row of gh.

One-period ahead inflation forecast:

$$\hat{\mathbb{E}}_{t-1}\pi_t = \bar{\pi}_{t-1} + b_1 s_{t-1} \tag{10}$$

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One-period ahead inflation forecast error:

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$\rightarrow$ Recursive least squares update for long-run inflation expectations:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1} \tag{12}$$

$k_t \in (0, 1)$ gain

General RLS algorithm

Alternatives for the gain

1. Decreasing gain:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f_{t|t-1} \tag{13}$$

2. Constant gain:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k\, f_{t|t-1} \tag{14}$$

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   Optimal monetary policy: -

Model summary

- New Keynesian core: IS and Phillips curves

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \Big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \Big) \tag{16}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \Big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \Big) \tag{17}$$

▸ Derivations ▸ Actual laws of motion

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→ How should $\{i_t\}$ be set?

Structure of talk

1. Model of anchoring expectations

2. Quantification of learning channel

3. Solving the Ramsey problem

4. Implementing optimal policy

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Estimating form of gain function

- Calibrate parameters of New Keynesian core to literature
- Estimate flexible form of expectations process via simulated method of moments
  (Duffie & Singleton 1990, Lee & Ingram 1991, Smith 1993)

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \mathbf{g}(f_{t|t-1})\, f_{t|t-1} \tag{18}$$

- Moments: autocovariances of inflation, output gap, federal funds rate and 1-year ahead SPF inflation expectations at lags $0, \ldots, 4$

Details

Calibration - parameters from the literature

β	0.98	stochastic discount factor
σ	1	intertemporal elasticity of substitution
α	0.5	Calvo probability of not adjusting prices
κ	0.0842	slope of the Phillips curve
ψ_π	1.5	coefficient of inflation in Taylor rule
ψ_x	0.3	coefficient of the output gap in Taylor rule
σ_r	0.01	standard deviation, natural rate shock
σ_i	0.01	standard deviation, monetary policy shock
σ_u	0.5	standard deviation, cost-push shock
$\bar{g}$	0.145	initial value of the gain

Chari et al 2000, Woodford 2003, Nakamura & Steinsson 2008
Carvalho et al 2019

Estimated expectations process

$$\bar{\pi}_t - \bar{\pi}_{t-1} = \mathbf{g}(f_{t|t-1})\, f_{t|t-1} \tag{18}$$

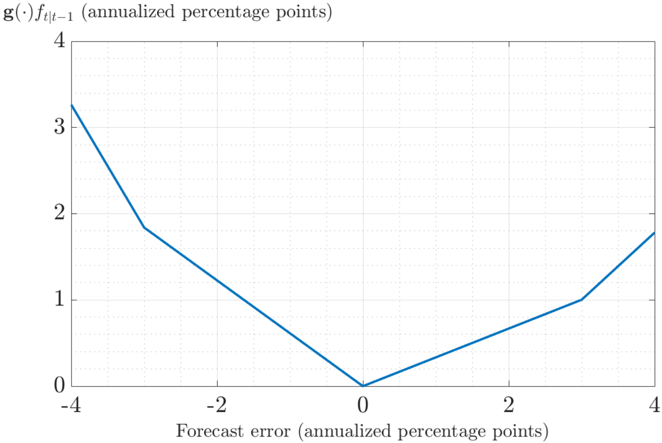

Figure: Changes in long-run inflation expectations as a function of forecast errors

The expectation process over time

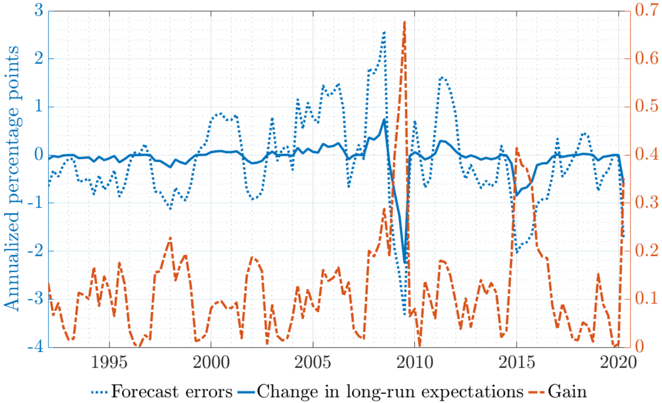

Figure: Time series of forecast errors, changes in long-run expectations and gain

Structure of talk

Ramsey problem

$$\min_{\{y_t,\bar{\pi}_{t-1},k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0}(\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

- $\mathbb{E}$ is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

Target criterion

Proposition

Let $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t*. Then monetary policy optimally brings about the following target relationship between inflation and the output gap*

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t$$

RE (discretion): move π_t and x_t to offset cost-push shocks

Target criterion

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Let $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t*. Then monetary policy optimally brings about the following target relationship between inflation and the output gap*

$$\pi_t - \Gamma(k)\, \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} = -\frac{\lambda_x}{\kappa} x_t$$

Adaptive learning: can move $\mathbb{E}_t\, x_{t+i}$ too if $k > 0$

▸ $\Gamma(k)$

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Let $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t. *Then monetary policy optimally brings about the following target relationship between inflation and the output gap*

$$\pi_t \; - \Omega\left(k_t + f_{t|t-1}\mathbf{g}_{\pi,t}\right)\left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(1 - k_{t+1+j} - f_{t+1+j|t+j}\mathbf{g}_{\bar{\pi},\mathbf{t+j}})\right) = -\frac{\lambda_x}{\kappa} x_t$$

Endogenous gain: ability to move $\mathbb{E}_t \, x_{t+i}$ depends on present and future degree of unanchoring

▸ Full expression, Ω ▸ No commitment

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Numerical solution procedure

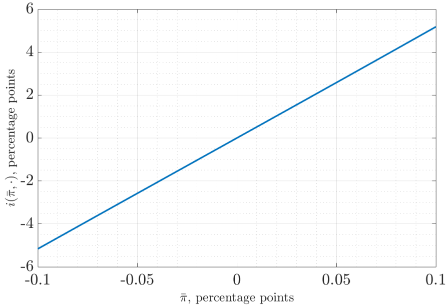
Solve system of model equations + target criterion

For calibrated model with $\lambda_x = 0.05$ (Rotemberg & Woodford 1997),

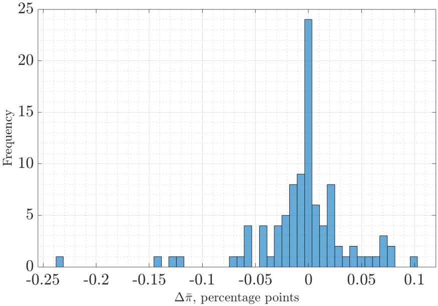
$\hookrightarrow$ solve using parameterized expectations algorithm

$\hookrightarrow$ obtain a cubic spline approximation to optimal policy function

Optimal policy - responding to unanchoring



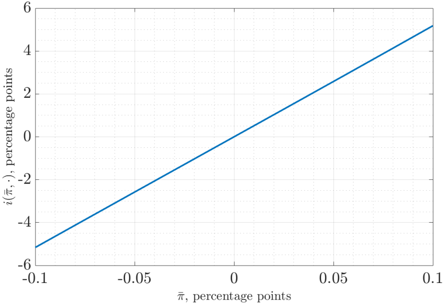
$i(\bar{\pi}$, all other states at their means)



Stabilizing $\bar{\pi}$

5 bp movement in $\bar{\pi} \rightarrow 250$ bp movement in i

Optimal policy - responding to unanchoring



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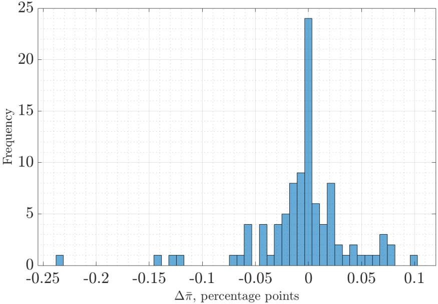

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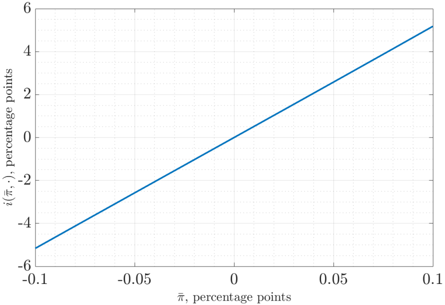

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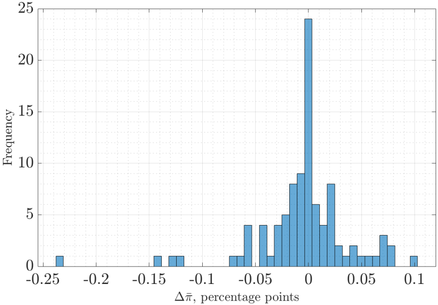

Stabilizing $\bar{\pi}$

5 bp movement in $\bar{\pi} \to$ 250 bp movement in i (1 standard dev. change in $\bar{\pi}$)

Mode: 0.3 bp movement in $\bar{\pi} \to$ 15 bp movement in i

Unanchoring causes volatility

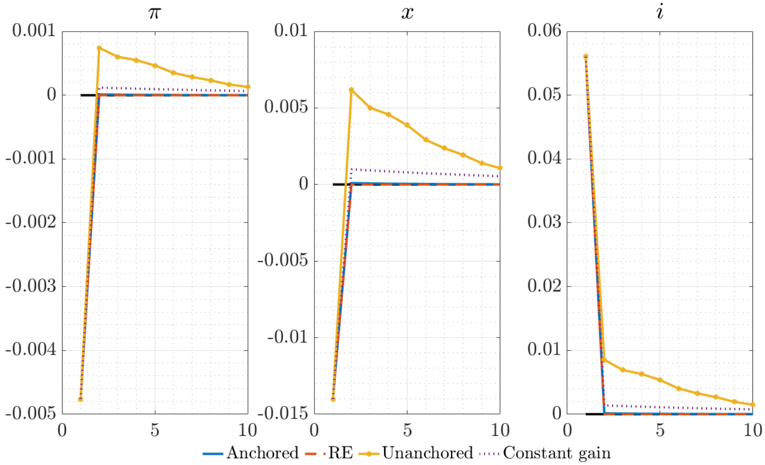

Figure: Impulse responses after a contractionary monetary policy shock when policy follows a Taylor rule

▸ Why oscillatory?

Volatility comes from endogenous gain

- Constant gain:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k\, f_{t|t-1} \tag{13}$$

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Shocks raise the gain → central bank needs to anchor

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Optimal Taylor-coefficient on inflation

$$i_t = \psi_\pi \pi_t + \psi_x x_t \tag{20}$$

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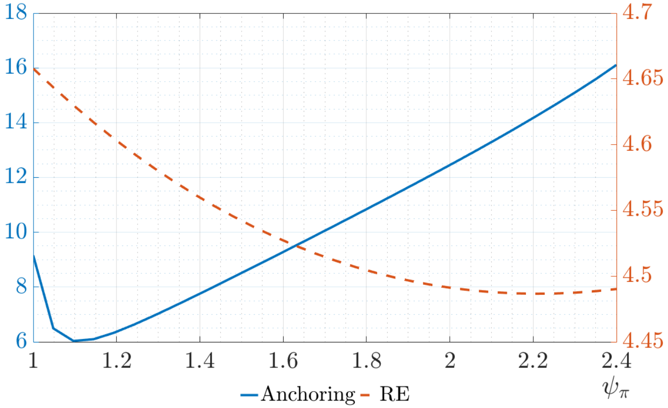
Figure: Central bank loss as a function of ψ_π



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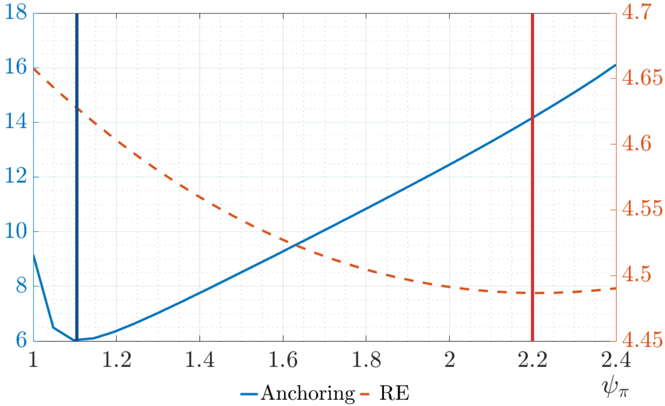

Anchoring-optimal coefficient: $\psi_\pi^A = 1.1$ RE-optimal coefficient: $\psi_\pi^{RE} = 2.2$

Why less aggressive? Future interest rate expectations

IS curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \right)$$

- Current interest rate i_t: one channel of policy

IRFs for various ψ_π

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- Current interest rate i_t: one channel of policy
- Taylor rule implies interest rate expectation

$$\hat{\mathbb{E}}_t i_{t+k} = \psi_\pi \hat{\mathbb{E}}_t \pi_{t+k} + \psi_x \hat{\mathbb{E}}_t x_{t+k} \tag{21}$$

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- If private sector understands and believes Taylor rule, expected future interest rates additional channel of policy (Eusepi, Giannoni & Preston 2018)

IRFs for various ψ_π

Conclusion

First theory of monetary policy for potentially unanchored expectations

Estimation of novel unanchoring channel

- Large and negative surprises unanchor more

Monetary policy

- Degree of expectations unanchoring determines extent of smoothing shocks
- Optimal policy aggressive when unanchored, accommodates otherwise
- Taylor rule less aggressive than under rational expectations

Future work

$\hookrightarrow$ How to anchor at zero-lower bound?

$\hookrightarrow$ Other applications: currency crises

Appendix

Breakeven inflation

Figure: Market-based inflation expectations, various horizons, %

Return

Correcting the TIPS from liquidity risk

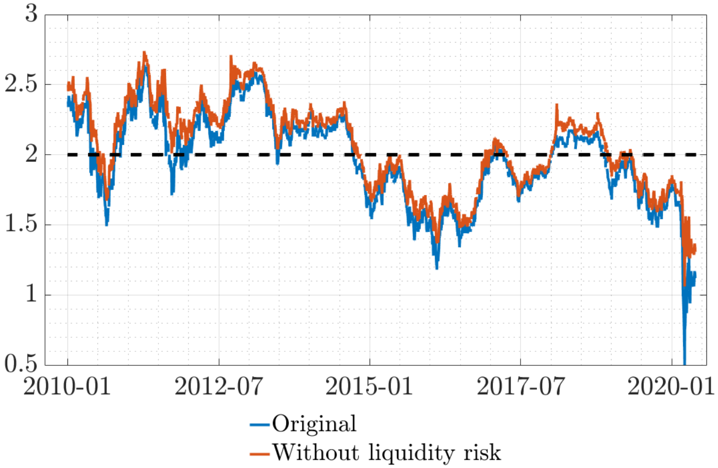

Figure: Market-based inflation expectations, 10 year, %

Return

Robustness checks

$$\Delta\bar{\pi}_t = \beta_0 + \beta_1^w \pi_t + \epsilon_t \quad (1)$$

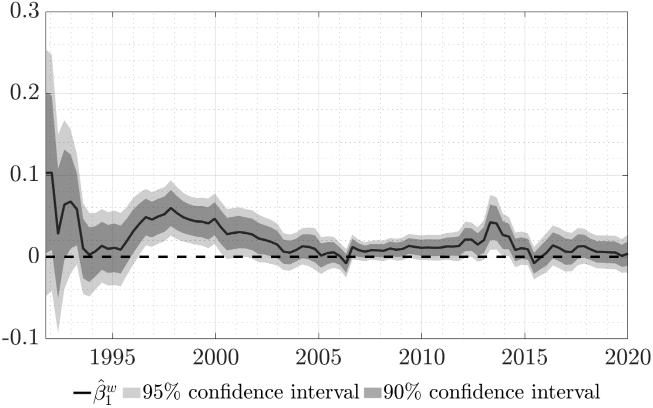

Figure: Time series of $\hat{\beta}_1^w$

Return

Robustness checks - PCE core

$$\Delta\bar{\pi}_t = \beta_0^w + \beta_1^w f_{t|t-1} + \epsilon_t \tag{1}$$

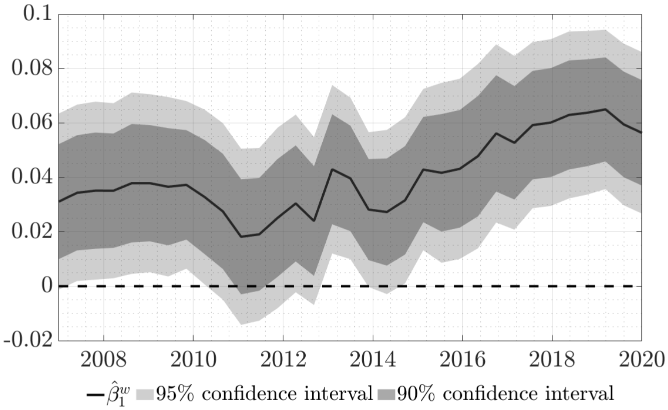

Figure: Time series of $\hat{\beta}_1^w$

Return

Robustness checks - controlling for inflation levels

$$\Delta \bar{\pi}_t = \beta_0^w + \beta_1^w f_{t|t-1} + \beta_2^w \pi_t + \epsilon_t \tag{1}$$

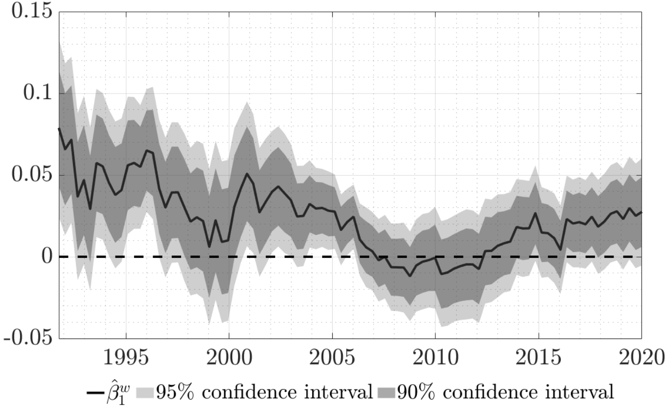

Figure: Time series of $\hat{\beta}_1^w$

Return

Further evidence: disagreement

Figure: Livingston Survey of Firms:
Interquartile range of 10-year ahead inflation expectations

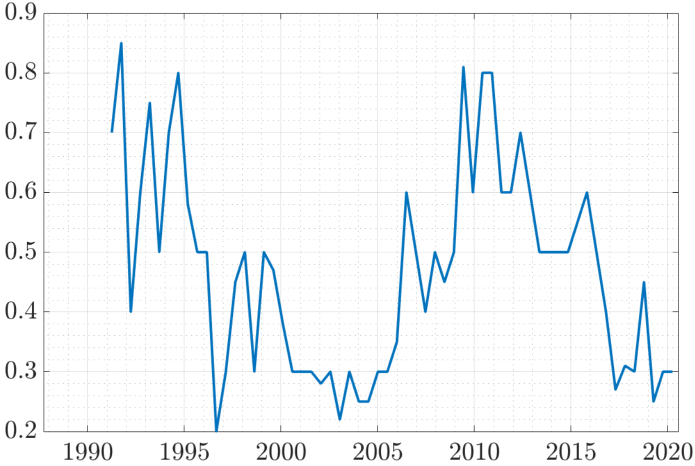

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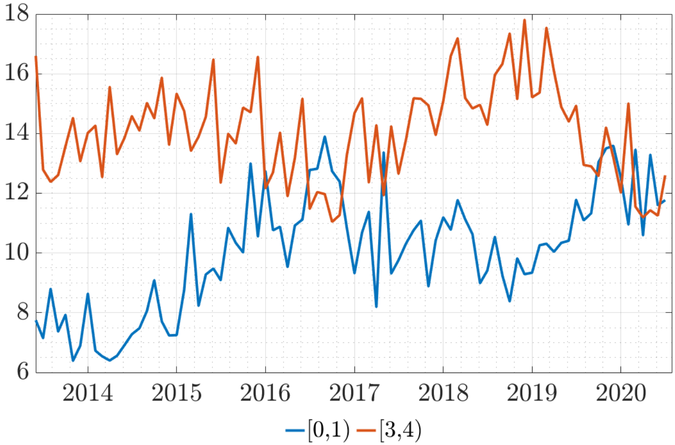
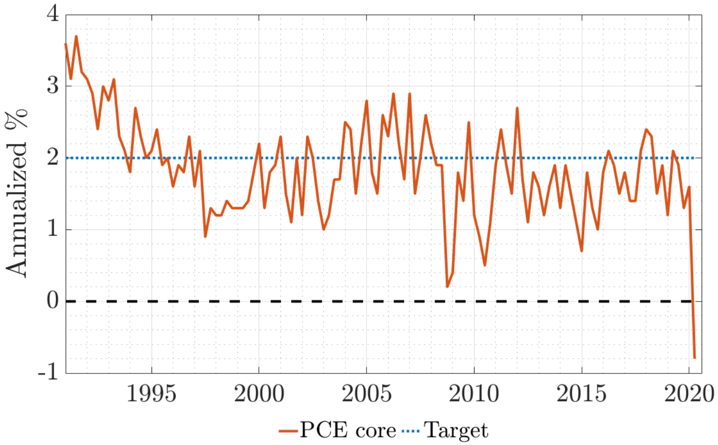
Figure: New York Fed Survey of Consumers:
Percent of respondents indicating 3-year ahead inflation will be in a particular range



◂ Return

Further evidence: introspection

Figure: PCE core inflation against the Fed's target



Return

Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{22}$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{23}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1-k^{-1}}{1-k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1-k^{-1}\beta} u_t \tag{24}$$

Solve for forecast error $f_t \equiv \pi_t - f_{t-1}$:

$$f_t = \underbrace{-\frac{1-\beta}{1-k\beta}}_{\lim_{k\to 1}=-1} f_{t-1} + \frac{1}{1-k\beta} u_t \tag{25}$$

Return

Functional forms for **g** in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \tag{26}$$

$p \equiv Prob(\text{liquidity trap regime})$
y_{t-1} output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{27}$$

θ_t criterion, $\bar{\theta}$ threshold value

Return

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \tag{28}$$

Σ variance-covariance matrix of shocks
$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(f_{t|t-1} f'_{t|t-1} - \omega_{t-1}) \tag{29}$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(f'_{t|t-1} \omega_t^{-1} f_{t|t-1} - \theta_{t-1}) \tag{30}$$

ω_t estimated forecast-error variance

Return

Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}\right)'\right)' \tag{31}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{32}$$

Return

Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{33}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{34}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \tag{35}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{36}$$

Return

Piecewise linear approximation to gain function

$$\mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}) \tag{37}$$

- $b_i(f_{t|t-1})$ = piecewise linear basis
- γ_i = approximating coefficient at node i

$\hookrightarrow$ Estimate $\hat{\gamma}$ via simulated method of moments

Return

Target criterion

Proposition

In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + f_{t|t-1} \mathbf{g}_{\pi,t} \right) \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - f_{t+1+j|t+j} \mathbf{g}_{\bar{\pi},\mathbf{t+j}}) \right)$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t*, and* b_1 *is the first row of* b*.*

◂ Return

▸ General case

Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

▸ Why no commitment?

Corollary

Optimal policy under adaptive learning is time-consistent.

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t)$$

$$+ \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \Big\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t + \mathbf{g}_\pi(\pi_t - f_{t-1})) = 0 \tag{38}$$

$$-2\beta\frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_\mathbf{f}(\pi_{t+1} - f_t)) = 0 \tag{39}$$

Return

Target criterion system for anchoring function as changes of the gain

$$\begin{aligned}\varphi_{6,t} &= -cf_{t|t-1}x_{t+1} + \left(1 + \frac{f_{t|t-1}}{f_{t+1|t}}(1-k_{t+1}) - f_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1} \\ &\quad - \frac{f_{t|t-1}}{f_{t+1|t}}(1-k_{t+1})\varphi_{6,t+2} \qquad (40)\\ 0 &= 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{f_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{f_{t|t-1}}\varphi_{6,t+1} \qquad (41)\end{aligned}$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (41) is given by:

$$\varphi_{6,t} = -2\,\mathbb{E}_t \sum_{i=0}^{\infty}(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{f_{t+j|t+j-1}}}{\frac{k_{t+j}}{f_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \qquad (42)$$

Return

Respond but not too much

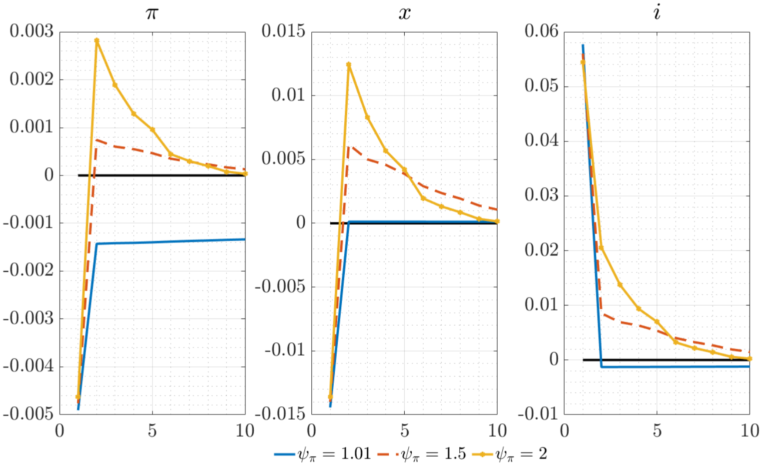


Figure: Impulse responses for unanchored expectations for various values of ψ_π

Return

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{43}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{44}$$

Profits:

$$\Pi_t^j = p_t(j) y_t(j) - w_t(j) f^{-1}(y_t(j)/A_t) \tag{45}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{46}$$

Return

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{47}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{48}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (47) backward to some date t, take expectations at t
2. Sub in (48)
3. Aggregate over households i

→ Obtain (16)

◂ Return