

# Monetary Policy & Anchored Expectations An Endogenous Gain Learning Model

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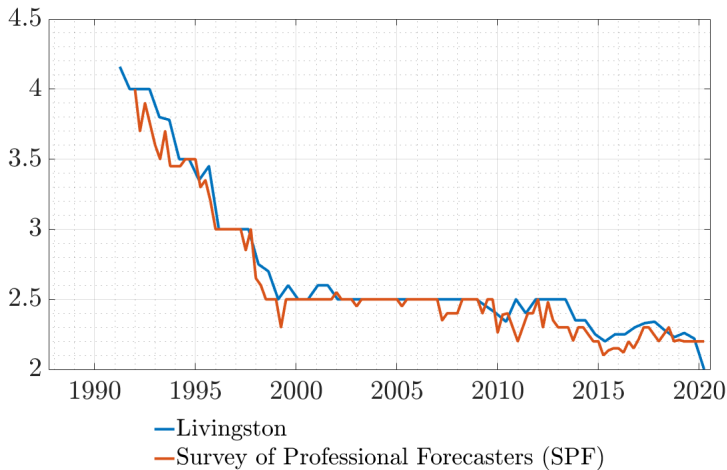
*Inflation that runs below its desired level can lead to an unwelcome fall in longer-term inflation expectations, which, in turn, can pull actual inflation even lower, resulting in an adverse cycle of ever-lower inflation and inflation expectations. [...] Well-anchored inflation expectations are critical[.]*

*Jerome Powell, Chairman of the Federal Reserve*<sup>1</sup>

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<sup>1</sup>“New Economic Challenges and the Fed’s Monetary Policy Review,” August 27, 2020.

Figure: Expectations of average inflation over 10 years



# This project

- How to conduct monetary policy in interaction with the anchoring expectation formation?
- Model of anchoring expectation formation as an endogenous gain adaptive learning scheme
- Estimation of the anchoring function: when do expectations become unanchored?

# Preview of results

- Optimal monetary policy responsiveness time-varying
  - ↪ Optimal policy aggressive when expectations unanchor, dovish when anchored
- Taylor rule policy less aggressive on inflation than under rational expectations
  - ↪ Anchoring-optimal Taylor rule eliminates 90% of loss from volatility

## Related literature

- **Optimal monetary policy in New Keynesian models**

Clarida, Gali & Gertler (1999), Woodford (2003)

- **Adaptive learning**

Evans & Honkapohja (2001, 2006), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Eusepi & Preston (2011), Milani (2007, 2014), Lubik & Matthes (2018), Mele et al (2019)

- **Anchoring and the Phillips curve**

Sargent (1999), Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Reis (2020), Gobbi et al (2019), Carvalho et al (2019)

# Structure of talk

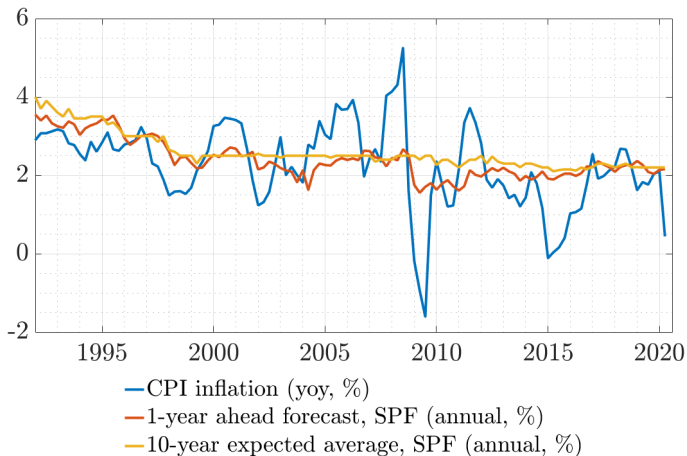
1. Unanchoring in the data
2. Model of anchoring expectations
3. Solving the Ramsey problem
4. Implementing optimal policy

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Figure: Inflation and expectations



For 1999-Q1 onward, estimate

$$\bar{\pi}_t = \beta_0 + \beta_1 fe_{t|t-1} + \epsilon_t \quad (1)$$

where

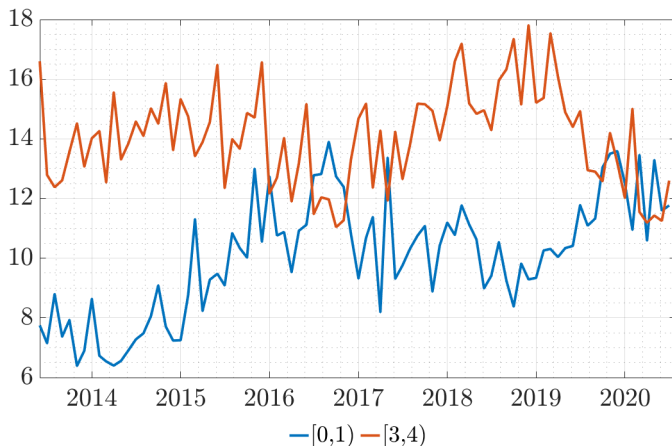
$$\bar{\pi}_t \equiv \mathbb{E}_t(\pi_{t+10}) \quad (2)$$

$$fe_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1}(\pi_t) \quad (3)$$

$$\hat{\beta}_1 = 0.06 \quad (\text{p-value: } 0.000017)$$

1 pp forecast error  $\rightarrow$  6 bp revision in long-run expectations

Figure: New York Fed Survey of Consumers:  
Percent of respondents indicating 3-year ahead inflation will be in a particular range



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# Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ U(C_T^i) - \int_0^1 v(h_T^i(j)) dj \right] \quad (4)$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_t C_t^i \quad (5)$$

► Consumption, price level

# Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[ \Pi_t^j(p_t(j)) \right] \quad (6)$$

subject to demand

$$y_t(j) = Y_t \left( \frac{p_t(j)}{P_t} \right)^{-\theta} \quad (7)$$

► Profits, stochastic discount factor

## Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- If use  $\mathbb{E}$  (rational expectations, RE)

Model solution

$$s_t = hs_{t-1} + \epsilon_t \quad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \quad (8)$$

$$y_t = gs_t \quad (9)$$

$s_t \equiv$  states

$y_t \equiv$  jumps

$\epsilon_t \equiv$  disturbances

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$y_t \equiv$  jumps

$\epsilon_t \equiv$  disturbances

- If use  $\hat{\mathbb{E}} \rightarrow$  private sector does not know (9)

$\hookrightarrow$  estimate using observed states & knowledge of (8)



# Adaptive learning

- Postulate linear functional relationship instead of (9):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \quad (10)$$

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- Estimate  $a, b$  using recursive least squares (RLS)

# Recursive least squares

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Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g h \quad \forall t \quad (11)$$

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$\bar{\pi}_{t-1}$ : long-run inflation expectations  $\rightarrow$  anchoring

$\rightarrow$  RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))}_{\equiv fe_{t|t-1}, \text{ forecast error}} \quad (12)$$

$k_t \in (0, 1)$  gain

$b_1$  first row of  $b$



# Decreasing versus constant gain

Decreasing gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} fe_{t|t-1} \quad (13)$$

→ consider sample mean of full sample of forecast errors

Constant gain learning:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k fe_{t|t-1} \quad (14)$$

→ consider sample mean of most recent observations only

## Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})) \quad (15)$$

$k_t = \mathbf{g}(fe_{t|t-1})$ : anchoring function

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$k_t = \mathbf{g}(fe_{t|t-1})$ : anchoring function

$$\mathbf{g}(fe_{t|t-1}) = \sum_i \alpha_i b_i(fe_{t|t-1}) \quad (16)$$

$b_i(fe_{t|t-1})$  = basis, here: second order spline (piecewise linear)

$\alpha_i$  = approximating coefficients, here: use  $\hat{\alpha}$  from estimation

# Anchoring function in the data

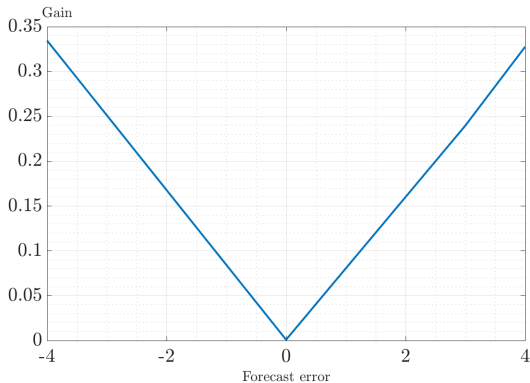


Figure: Learning gain as a function of forecast errors in inflation (pp)

# Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (17)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (18)$$

► Derivations

► Actual laws of motion

- Expectations evolve according to RLS with the endogenous gain given by (16)

→ How should  $\{i_t\}$  be set?

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# Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

- $\mathbb{E}$  is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

# Target criterion

## Proposition

*In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap*

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_{\pi,t} \right)$$
$$\left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\pi,t+j}) \right)$$

where  $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$  at  $t$ ,  $\prod_{j=0}^0 \equiv 1$  and  $b_1$  is the first row of  $b$ .



# Two layers of intertemporal stabilization tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ - \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t + fe_{t|t-1} \mathbf{g}_{\pi,t} \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j} + fe_{t+1+j|t+j} \mathbf{g}_{\pi,t+j})$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

## Lemma

*The discretion and commitment solutions of the Ramsey problem coincide.*

► Why no commitment?

## Corollary

*Optimal policy under adaptive learning is time-consistent.*

↪ Foreshadow: optimal policy aggressiveness time-varying

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# Solution procedure

Solve system of model equations + target criterion

↪ solve using parameterized expectations (PEA)

↪ obtain a cubic spline approximation to optimal policy function

# Calibration - parameters from the literature

$\beta$	0.98	stochastic discount factor
$\sigma$	1	intertemporal elasticity of substitution
$\alpha$	0.5	Calvo probability of not adjusting prices
$\kappa$	0.0842	slope of the Phillips curve
$\psi_\pi$	1.5	coefficient of inflation in Taylor rule*
$\psi_x$	0.3	coefficient of the output gap in Taylor rule*
$\bar{g}$	0.145	initial value of the gain
$\lambda_x$	0.05	weight on the output gap in central bank loss
$\rho_r$	0	persistence of natural rate shock
$\rho_i$	0	persistence of monetary policy shock*
$\rho_u$	0	persistence of cost-push shock

\* pertains to sections where Taylor rule is in effect

# Calibration - matching moments

$\sigma_r$	0.01	standard deviation, natural rate shock
$\sigma_i$	0.01	standard deviation, monetary policy shock*
$\sigma_u$	0.5	standard deviation, cost-push shock
$\hat{\alpha}_i$	(0.33; 0.25; 0.001; 0.24; 0.33)	coefficients in anchoring function

Calibrated ( $\sigma_j$ ,  $j = r, i, u$ ) or estimated ( $\hat{\alpha}_i$ ) to match the autocovariances of inflation, output gap, interest rate and one-period ahead inflation expectations for lags  $0, \dots, 4$ .

\* pertains to sections where Taylor rule is in effect

# Optimal policy - responding to unanchoring

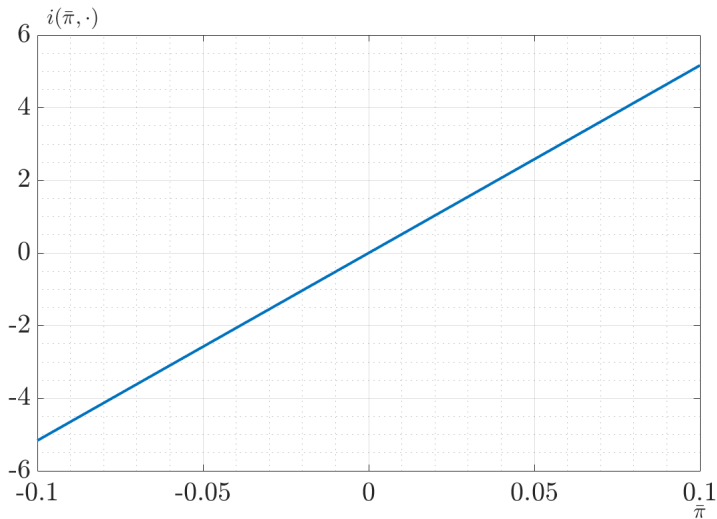


Figure: Policy function:  $i(\bar{\pi}, \text{all other states at their means})$

# The intertemporal volatility tradeoff

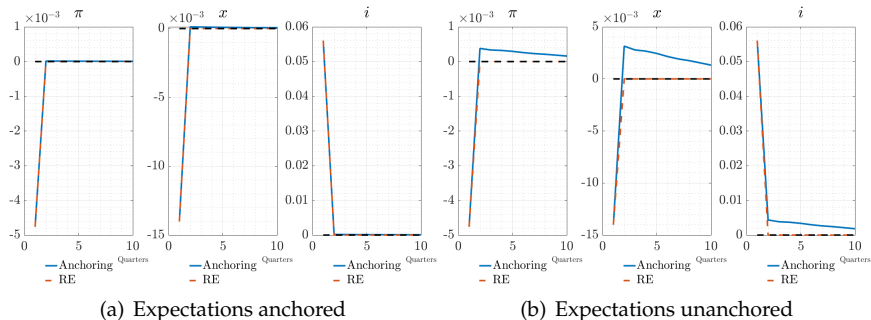


Figure: Impulse responses after a contractionary monetary policy shock



# Intertemporal volatility tradeoff: term structure of expectations

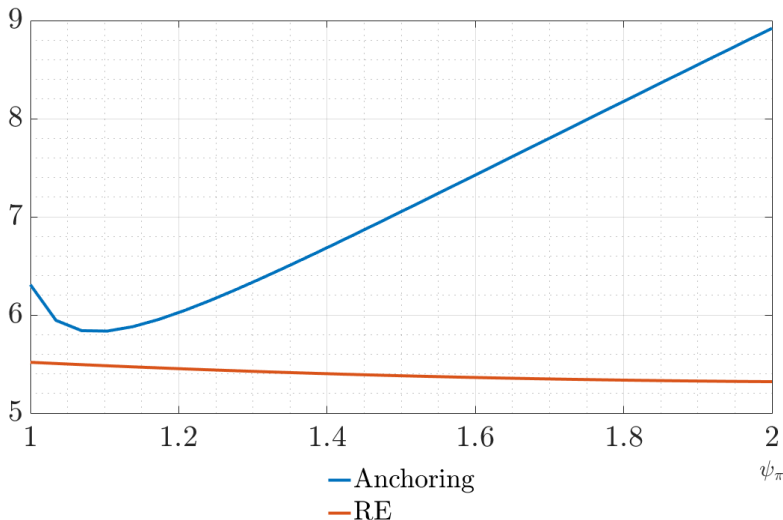
IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n)$$

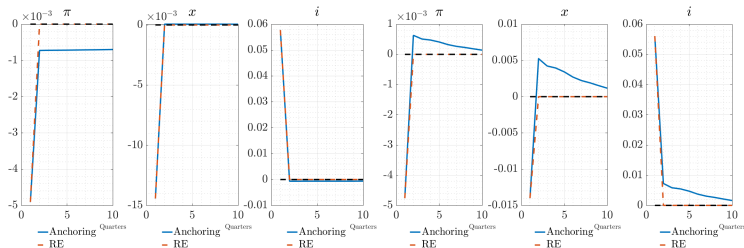
$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T)$$

# Optimal Taylor-coefficient on inflation

Figure: Central bank loss as a function of  $\psi_\pi$

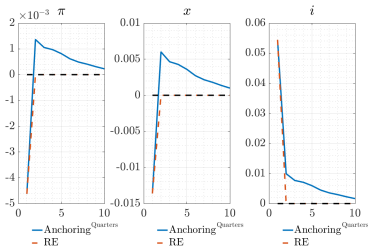


# The intertemporal volatility tradeoff - again



(a)  $\psi_\pi = 1.01$

(b)  $\psi_\pi = 1.5$



(c)  $\psi_\pi = 2$

# Losses for optimal Taylor-rule coefficient on inflation

RE-optimal coefficient:  $\psi_{\pi}^{RE} = 2.21$

Anchoring-optimal coefficient:  $\psi_{\pi}^A = 1.09$

Table: Loss for RE and anchoring models for choice of RE- or anchoring-optimal  $\psi_{\pi}$

Anchoring, $\psi_{\pi}^{RE}$	Anchoring, $\psi_{\pi}^A$	RE, $\psi_{\pi}^{RE}$
9.6901	5.8296	5.3148

→ If model is anchoring, anchoring-optimal  $\psi_{\pi}^A$  gets 90% of the distance to RE-optimal  $\psi_{\pi}^{RE}$  under RE

# Conclusion

- First theory of monetary policy for potentially unanchored expectations
- Optimal policy conditions on stance of current and expected future anchoring
  - ↔ determine intertemporal tradeoffs
- Frontloads aggressive interest rate response to suppress potential unanchoring
- Matters: already anchoring-optimal Taylor rule reduces losses by 50%
- Future work: how to anchor at zero-lower bound?

## Appendix

# Correcting the TIPS from liquidity risk

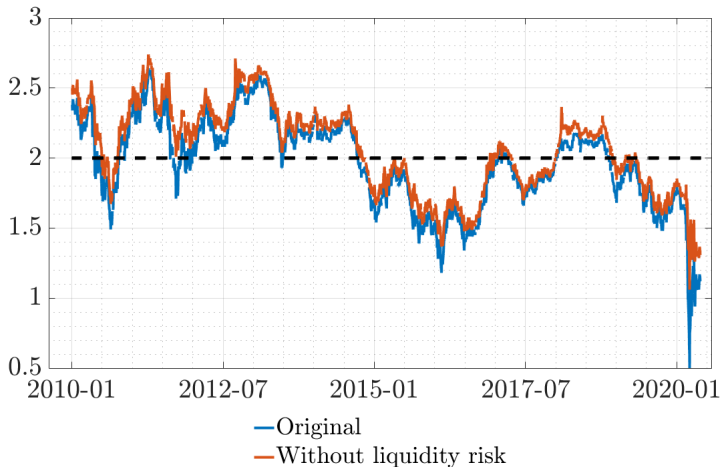


Figure: Market-based inflation expectations, 10 year, average, %

# Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \quad (19)$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \quad (20)$$

Solve for the time series of expectations  $f_t$

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \quad (21)$$

Solve for forecast error  $fe_t \equiv \pi_t - f_{t-1}$ :

$$fe_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k \rightarrow 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \quad (22)$$



# Functional forms for $g$ in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \quad (23)$$

$p \equiv \text{Prob}(\text{liquidity trap regime})$   
 $y_{t-1}$  output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when } \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \quad (24)$$

$\theta_t$  criterion,  $\bar{\theta}$  threshold value

# Choices for criterion $\theta_t$

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \quad (25)$$

$\Sigma$  variance-covariance matrix of shocks

$T(\phi)$  mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1} (fe_{t|t-1} fe'_{t|t-1} - \omega_{t-1}) \quad (26)$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1} (fe'_{t|t-1} \omega_t^{-1} fe_{t|t-1} - \theta_{t-1}) \quad (27)$$

$\omega_t$  estimated forecast-error variance

# Recursive least squares algorithm

$$\phi_t = \left( \phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left( y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right) \right)' \quad (28)$$

$$R_t = R_{t-1} + k_t \left( \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} [1 \quad s_{t-1}] - R_{t-1} \right) \quad (29)$$

# Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \quad (30)$$

$$s_t = h s_{t-1} + \epsilon_t \quad (31)$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \quad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \quad (32)$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} y_{T+1} \quad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \quad (33)$$

# No commitment - no lagged multipliers

Simplified version of the model: planner chooses  $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$  to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t) \right. \\ \left. + \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \right\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa}x_t - \varphi_{2,t}(k_t + \mathbf{g}_{\pi}(\pi_t - f_{t-1})) = 0 \quad (34)$$

$$-2\beta\frac{\lambda}{\kappa}x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_f(\pi_{t+1} - f_t)) = 0 \quad (35)$$

# Target criterion system for anchoring function as changes of the gain

$$\begin{aligned} \varphi_{6,t} = & -cfe_{t|t-1}x_{t+1} + \left(1 + \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1}) - fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\varphi_{6,t+1} \\ & - \frac{fe_{t|t-1}}{fe_{t+1|t}}(1 - k_{t+1})\varphi_{6,t+2} \end{aligned} \quad (36)$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{fe_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{fe_{t|t-1}}\varphi_{6,t+1} \quad (37)$$

$\varphi_{6,t}$  Lagrange multiplier on anchoring function

The solution to (37) is given by:

$$\varphi_{6,t} = -2\mathbb{E}_t \sum_{i=0}^{\infty} (\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{fe_{t+j|t+j-1}}}{\frac{k_{t+j}}{fe_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \quad (38)$$

# Details on households and firms

Consumption:

$$C_t^i = \left[ \int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (39)$$

$\theta > 1$ : elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[ \int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \quad (40)$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \quad (41)$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \quad (42)$$

# Derivations

## Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \quad (43)$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \quad (44)$$

where ‘hats’ denote log-linear approximation and  $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$ .

1. Solve (43) backward to some date  $t$ , take expectations at  $t$
  2. Sub in (44)
  3. Aggregate over households  $i$
- Obtain (17)