

# Materials 34 - Still estimating the anchoring function

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## 1 Estimation procedure

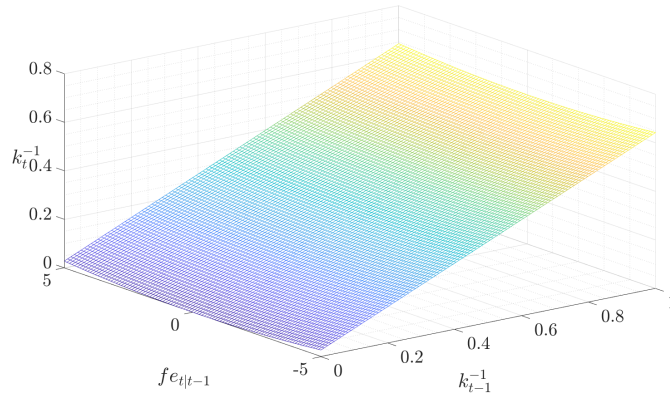
Instead of the AR(1) anchoring function used so far (Equation A.6), I use the following equation

$$k_t^{-1} = \alpha s(X) \quad (1)$$

where  $X = (k_{t-1}^{-1}, fe_{t|t-1})$  and I use piecewise linear interpolation. I initialize  $\alpha_0$  by specifying a grid for  $X$ , passing the grid through Equation (A.6) to generate  $k_t^{-1}$ -values, and approximating by fitting the grid to the  $k_t^{-1}$ -values. See Fig. 1.

Then I estimate  $\alpha$  using GMM, targeting the autocovariance structure of inflation, the output gap and the nominal interest rate (federal funds rate) in the data.

**Figure 1:** Initialization via Equation (A.6) implies this functional relationship

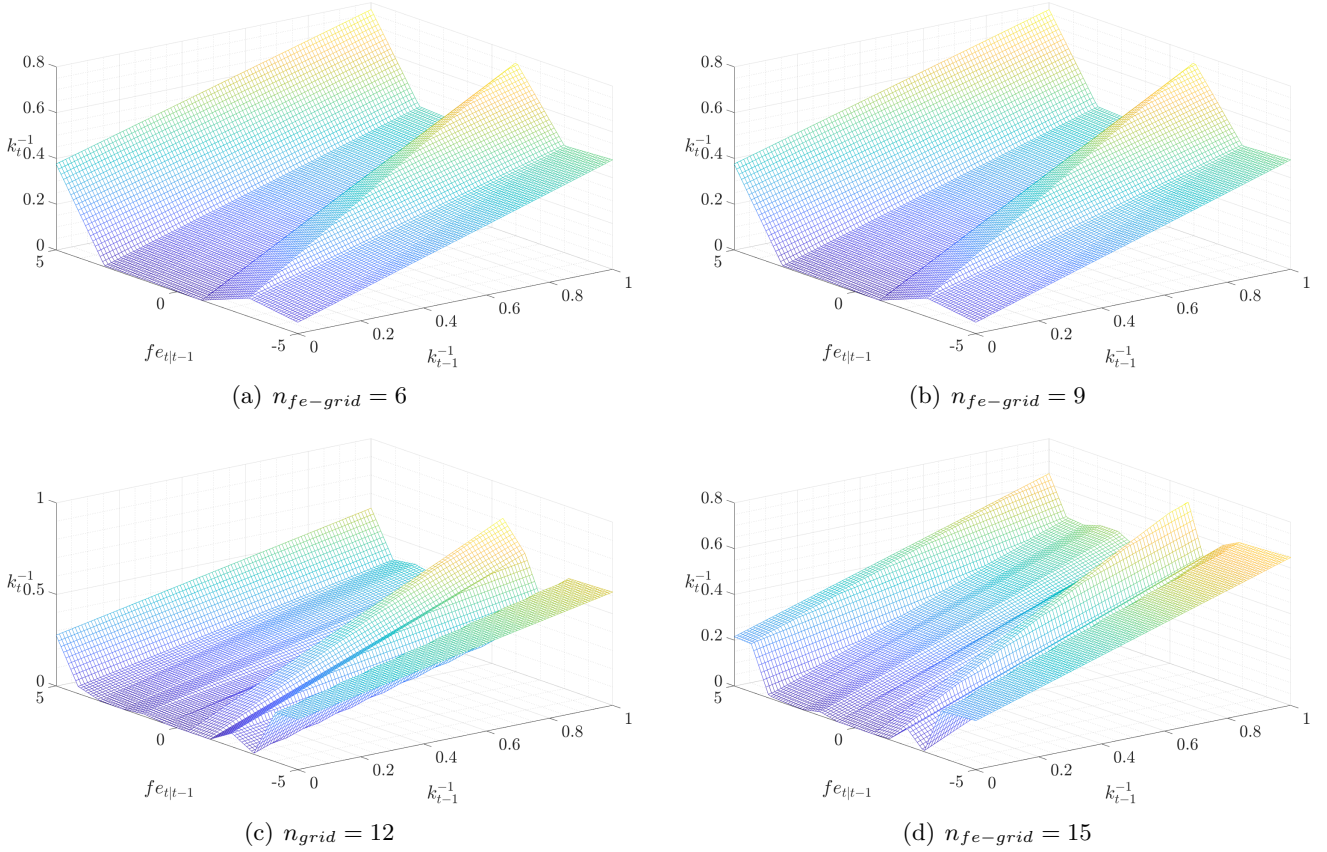


$T = 233$  before BK-filtering,  $T = 209$  after BK-filtering. Using the “constant-only, inflation-only” learning PLM. I drop the  $ndrop = 5$  initial values. I restrict  $\alpha \in (0, 1)$ , the support of  $k^{-1}$  in the grid. I target the lag  $0, \dots, 4$  autocovariance matrices, dropping repeated entries at lag 0, leaving me with 42 moments.

## 2 Estimation issues

1. Target criterion relies on the assumption  $k_t^{-1} = \mathbf{g}(fe)$ . Need a 1D estimate to implement the target criterion.

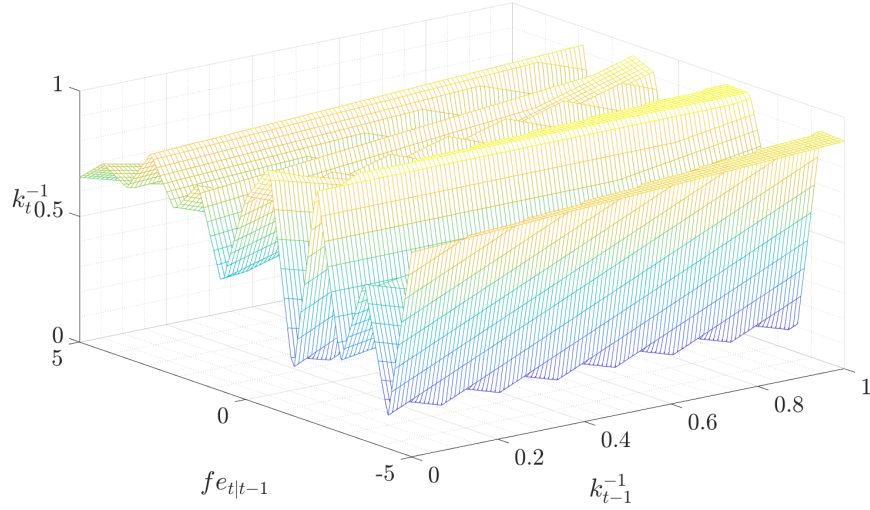
**Figure 2:**  $k_t^{-1}$  as a function of  $k_{t-1}^{-1}$  and  $fe_{t|t-1}$  given  $\hat{\alpha}^{GMM} \in (0, 1)$ . The dimension of the gain-grid is 2.



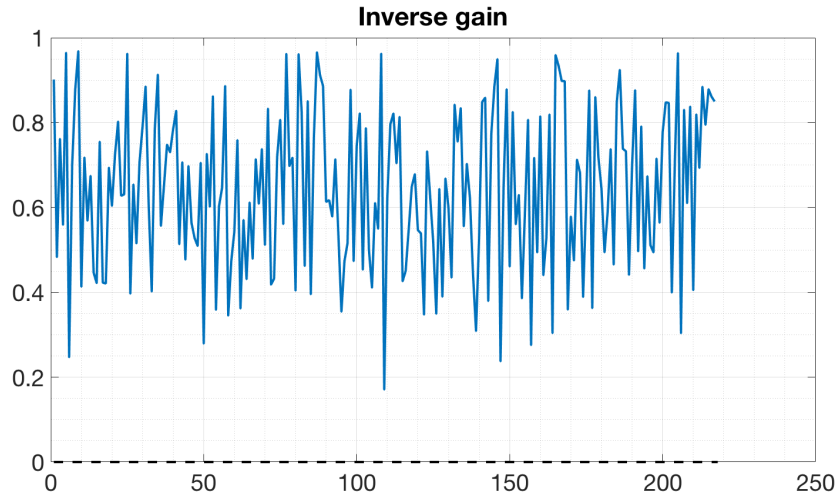
This actually looks quite robust i) to the gridsize, ii) to starting from random points. All I have done at this point is to increase the support of the gain, decrease its gridpoints to 2, increase the forecast error gridpoints, and implement `lsqnonlin`. Increasing the support of the gain also had the feature that now there are no explosions. So all these measures seem to have been beneficial.

### 3 Estimating $\alpha$ on simulated data

**Figure 3:** The truth using 2 gridpoints on  $[0, 1]$  for the gain, 15 on  $[-5, 5]$  for the forecast error

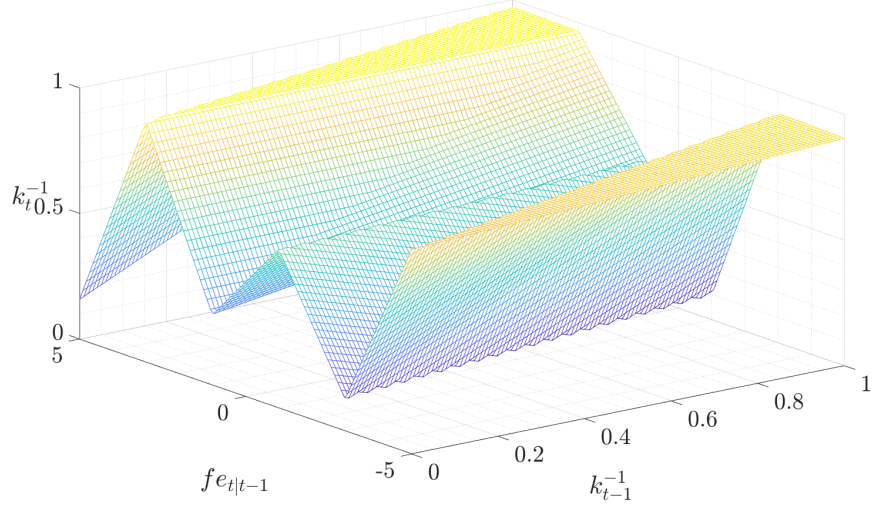


(a)  $k_t^{-1}$  as a function of  $k_{t-1}^{-1}$  and  $fe_{t|t-1}$  given  $\hat{\alpha}^{true}$



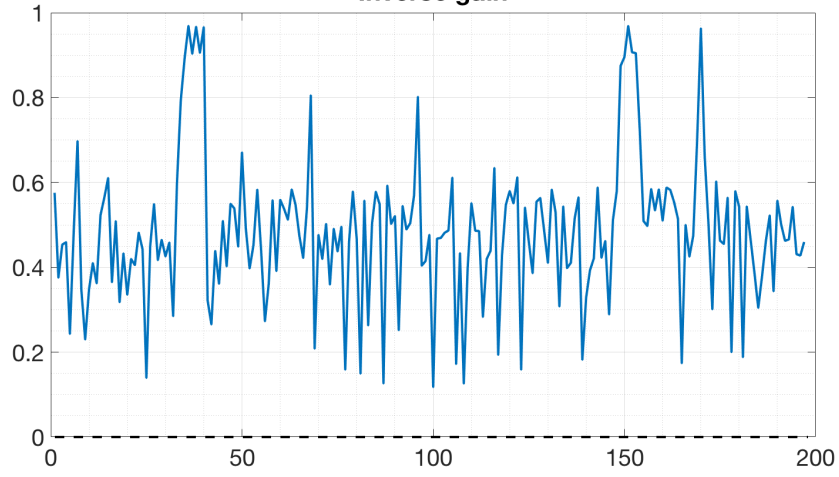
(b) A simulated history of  $k_t^{-1}$  given  $\hat{\alpha}^{true}$

**Figure 4:** The truth using 2 gridpoints on  $[0, 1]$  for the gain, 6 on  $[-5, 5]$  for the forecast error



(a)  $k_t^{-1}$  as a function of  $k_{t-1}^{-1}$  and  $f e_{t|t-1}$  given  $\hat{\alpha}^{true}$

Simulation using estimated LOM-gain approx  
**Inverse gain**



(b) A simulated history of  $k_t^{-1}$  given  $\hat{\alpha}^{true}$

## A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (\text{A.1})$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (\text{A.2})$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \quad (\text{if imposed}) \quad (\text{A.3})$$

$$\text{PLM:} \quad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \quad b = g_x h_x \quad (\text{A.4})$$

$$\text{Updating:} \quad a_t = a_{t-1} + k_t^{-1} (z_t - (a_{t-1} + b s_{t-1})) \quad (\text{A.5})$$

$$\text{Anchoring function:} \quad k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k f e_{t-1}^2 \quad (\text{A.6})$$

$$\text{Forecast error:} \quad f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \quad (\text{A.7})$$

$$\text{LH expectations:} \quad f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \quad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \quad (\text{A.8})$$

This notation captures vector learning ( $z$  learned) for intercept only. For scalar learning,  $a_t = (\bar{\pi}_t \ 0 \ 0)'$  and  $b_1$  designates the first row of  $b$ . The observables  $(\pi, x)$  are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix} \sigma & 1-\beta & -\sigma\beta \end{bmatrix} f_b + \sigma \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \quad (\text{A.9})$$

$$\pi_t = \kappa x_t + \begin{bmatrix} (1-\alpha)\beta & \kappa\alpha\beta & 0 \end{bmatrix} f_a + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \quad (\text{A.10})$$

## B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only,  $k_t^{-1} = \mathbf{g}(f e_{t-1})$ ):

$$\pi_t = -\frac{\lambda_x}{\kappa} \left\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_\pi(t) \right) \right. \\ \left. \left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\bar{\pi}}(t+j)) \right) \right\} \quad (\text{B.1})$$

where I'm using the notation that  $\prod_{j=0}^0 \equiv 1$ . For interpretation purposes, let me rewrite this as follows:

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ - \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j}^{-1} + f e_{t+1+j|t+j}^{eve} \mathbf{g}_{\bar{\pi}}(t+j)) \right) \quad (\text{B.2})$$

Interpretation: **tradeoffs from discretion in RE** + **effect of current level and change of the gain on future tradeoffs** + **effect of future expected levels and changes of the gain on future tradeoffs**