

Finally back to work! To do

7 Oct 2019

- generate IRFs from all models } Susanto
- think about about slope, b

- ✓ - change α (and α) and see if π fluctuates as } Pedro
much as X then

$X - \hat{E}$ -operator

- θ_t as SR FE
- small T vs. larger $X \& i \rightarrow$ contrast to
rest of Lit to see what mechanism is
responsible

- ✓ - think about whether ppl know the TR?

- ✓ - read Thomas Lubik: Indeterminacy & Learning (JME)

- ✓ - derive Preston's IS curve → me.

Lubik: has a comment that b/c of anticipated utility

(Kreps 1998, Cosley & Sargent 2008), their model can be
solved using standard, RE algorithms

→ in Lubrile, the CB knows the econ and announces

$i = \gamma_n \pi_t + \gamma_x x_t$ w/ (γ_n, γ_x) each period, updated according to its estimates. Depending on $(\gamma_{x,t}, \gamma_{\pi,t})$ we can land in the determinacy or indeterminacy region. Ryan is right: this amounts to ppl not knowing the TR. And he's doubly right b/c in my setting, they know the Taylor-rule, the only thing they don't know are future things.

Let's first try to derive the IS curve in Preston 8 Oct 2015

$$\text{HH: } \max E_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[u(c_T^i, \xi_T) - \int_0^1 v(h_T^i(j), \xi_T) dj \right] \quad (1)$$

$$M_t^i + B_t^i \leq (1+i_{t+1}^m) M_{t-1}^i + (1+i_{t-1}) B_{t-1}^i + p_t \gamma_t^i - \tau_t - p_t c_t^i \quad (2)$$

$$\int [w_t(j) h_t^i(j) + \pi_t(j)] dj$$

so the HH chooses: $\{c_t^i(j), h_t^i(j), \pi_t^i, B_t^i\} \forall j \in [0, 1]$ so as to max (1) s.t. (2), taking as given $\{p_t(j), w_t(j), \pi_t, i_{t-1}^m, \tau_{t-1}, \xi_t\}$ $\forall T \geq t$.

In the App, Preston defines $W_{t+1}^i = (1+i_t^m) M_t^i + (1+i_t) B_t^i$ as

beginning-of-period wealth at time $t+1$

$$\Rightarrow (2) M_t^i + B_t^i \leq w_t^i + P_t Y_t^i - T_t - P_t C_t^i$$

Now let $\Delta_t = \frac{i_t - i_t^m}{1+i_t}$ recall: $w_{t+1}^i = (1+i_t^m)M_t^i + (1-i_t)B_t^i$

$$\Leftrightarrow P_t C_t^i + \underbrace{M_t^i + B_t^i}_{(1+i_t^m - i_t^m)M_t^i + \frac{1-i_t}{1+i_t} B_t^i} \leq w_t^i + [P_t Y_t^i - T_t]$$

$$(1+i_t^m - i_t^m)M_t^i + \frac{1-i_t}{1+i_t} B_t^i$$

$$= -i_t^m M_t^i + (1-i_t^m)M_t^i + \underbrace{(1+i_t)}_{1-i_t} B_t^i$$

$$= -i_t^m M_t^i + \underbrace{(1+i_t)(1-i_t^m)M_t^i}_{1-i_t} + \underbrace{\frac{(1-i_t)}{1+i_t} B_t^i}_{1-i_t}$$

$$= -i_t^m M_t^i + \underbrace{\frac{i_t(1+i_t^m)}{1+i_t} M_t^i}_{1-i_t} + \underbrace{\frac{(1+i_t^m)M_t^i + (1-i_t)B_t^i}{1+i_t}}_{1-i_t}$$

$$= \underbrace{(1+i_t)(-i_t^m)M_t^i}_{1-i_t} + \underbrace{i_t(1+i_t^m)M_t^i}_{1-i_t} = \frac{1}{1+i_t} M_{t+1}^i$$

$$= \underbrace{(-i_t^m - i_t i_t^m + i_t - i_t i_t^m)}_{1-i_t} M_t^i = \frac{i_t - i_t^m}{1+i_t} M_t^i = \Delta_t M_t^i$$

$$\text{So } \Rightarrow (2) P_t C_t^i + \Delta_t M_t^i + \frac{1}{1+i_t} M_{t+1}^i \leq w_t^i + [P_t Y_t^i - T_t] \quad (32)$$

Now solve this first.

$$w_t^i \geq p_t c_t^i + \Delta_t m_t^i - (p_t y_t^i - T_t) + \frac{1}{1+i_t} w_{t+1}^i$$

$$\Rightarrow w_t^i \geq p_t c_t^i + \Delta_t m_t^i - (p_t y_t^i - T_t)$$

$$\cdot \left(\frac{1}{1+i_t} \right) \left[p_{t+1} c_{t+1}^i + \Delta_{t+1} m_{t+1}^i - (p_{t+1} y_{t+1}^i - T_{t+1}) \right]$$

...
1 ...

$$\underbrace{\cdot \left(\frac{1}{1+i_t} \right) \cdot \dots \left(\frac{1}{1+i_t+j-1} \right)}_j \left[p_{t+j} c_{t+j}^i + \Delta_{t+j} m_{t+j}^i - (p_{t+j} y_{t+j}^i - T_{t+j}) \right]$$

$$=: R_{t,t+j} = \prod_{s=1}^j \left(\frac{1}{1+i_{t+s-1}} \right)$$

$$w_t^i \geq \sum_{j=0}^{\infty} R_{t,t+j} \left[p_{t+j} c_{t+j}^i + \Delta_{t+j} m_{t+j}^i - (R_{t+j} y_{t+j}^i - T_{t+j}) \right]$$

[The flow BC solved fwd to get the intertemporal BC, the IBC.

Let's now take the FOCs using the new flow BC, 32

$$\alpha = \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[h(c_t^i; \xi_T) - \int_0^1 v(h_T(j); \xi_T) dj \right] \\ + \lambda_t \left(-p_t c_t^i - \Delta_t m_t^i - \frac{1}{1+i_t} w_{t+1}^i + w_t^i + [p_t y_t^i - T_t] \right)$$

$$\text{c 1)} u_C = \lambda_t p_t \rightarrow \lambda_t = \frac{u_C}{p_t} \quad \left. \begin{array}{l} \frac{u_C}{p_t} = \frac{v_h}{p_t w_t} \\ \uparrow w_t^i \text{ hit} \end{array} \right\}$$

$$\text{c 2)} v_{hj} = \lambda_t p_t w_t^j \rightarrow \lambda_t = \frac{v_{hj}}{p_t w_t} \quad \left. \begin{array}{l} \Rightarrow \frac{v_h}{u_C} = w_t \text{ real wage} \\ \uparrow \end{array} \right\}$$

$$\text{B (3)} \quad \lambda_t (1+i_t) = \beta \hat{E}_t \lambda_{t+1} \rightarrow 1+i_t = \beta \hat{E}_t \frac{\lambda_{t+1}}{\lambda_t} = \beta \hat{E}_t \frac{p_t}{p_{t+1}} \frac{u_{C+t}}{u_{C_t}}$$

ok so I *almost* get what Preston gets: (difference)

$$\underline{i_{t+1}} = \beta E_t \frac{P_+}{P_{t+1}} \frac{U_C(C_{t+1})}{U_C(C_t)} \quad (33)$$

$$\frac{v'_n}{v'_c} = \underline{w_t} \quad (34)$$

- A cashless econ implies $i_t^* = i_t^m$ or $M_t^i = 0$

Since $M^S > 0$, we get $i_t^* = i_t^m \Rightarrow \Delta_t = 0 \quad \forall t$

- Market clearing implies $y_t(j) = c_t(j) \quad \forall j \Rightarrow C_t = Y_t$

- Zero debt fiscal policy: $B_t = 0 \Rightarrow T_t = (1 + i_{t-1})M_{t-1} - M_t$

↳ thus the IBC becomes *ain't so sure if this is right!*

$$w_t^i = \sum_{j=0}^{\infty} R_{t+j} [P_{t+j} c_{t+j} - (P_{t+j} Y_{t+j}^i - T_{t+j})]$$

$$w_t^i = \sum_{j=0}^{\infty} R_{t+j} [P_{t+j} c_{t+j} - P_{t+j} Y_{t+j}^i - (1 + i_{t+j-1})M_{t+j-1} + M_{t+j}]$$

Maybe the point is that there is no diff b/w bonds & money,

$$\text{so } w_{t+1}^i = (1 + i_t)M_t + (1 + i_t)B_t = (1 + i_t)[M_t + B_t]$$

so we can assume ppl only hold bonds, $M_t = 0$

$$\rightarrow w_{t+1}^i = (1 + i_t)B_t \rightarrow T_t = 0 \quad (\text{a little non-kosher but ok})$$

$$w_t^i \geq \sum_{j=0}^{\infty} R_{t+j} [P_{t+j} c_{t+j} - P_{t+j} Y_{t+j}^i]$$

Funnily, the App. stops here.

But it is clear that w/ a specific α -fit ans, loglin of the EE (33) gives rise to (3). But does loglin of the IBC $w_{t+1}^i \geq \sum_{j=0}^{\infty} R_{t+j+1} [P_{t+j} C_{t+j} - P_{t+j} Y_{t+j}]$ give rise

$$\text{to (4), } \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i = \bar{w}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T^i \quad ?$$

What's the loglin of $R_{t+j+1} = \prod_{s=1}^j \left(\frac{1}{1+i_{t+s-1}} \right)$?

$$= \sum_{s=1}^j \ln \left(\frac{1}{1+i_{t+s-1}} \right) = \sum_{s=1}^j -\ln(1+i_{t+s-1})$$

Take the gross interest rate $1+i_t$ as a const, call it R

$$\Rightarrow \sum_{s=1}^j -\ln(R_{t+s-1}) \text{ Total diff} = \sum_{s=1}^j -\frac{dR_{t+s-1}}{R} - \sum_{s=1}^j \hat{R}_{t+s-1}$$

$$\text{where } R = \beta^{-1} \text{ so } \bar{i} = i^* = \beta^{-1} - 1$$

Supp I write the IBC as

$$w_{t+1}^i \geq \sum_{j=0}^{\infty} R_{t+j+1} P_{t+j} [C_{t+j}^i - Y_{t+j}^i] \quad | : P_t Y$$

$$\bar{w}_t^i = \sum_{j=0}^{\infty} R_{t+j+1} \frac{P_{t+j}}{P_t} \left[\frac{C_{t+j}^i}{\bar{Y}} - \frac{Y_{t+j}^i}{\bar{Y}} \right]$$

$$\bar{w}_t^i = \sum_{j=0}^{\infty} R_{t+j+1} \bar{\Pi}_{t+j} [\hat{C}_{t+j}^i - \hat{Y}_{t+j}^i] \quad \text{To get eq (4),}$$

we need the logins of $R_{t,t+j} \pi_{t,t+j}$ to be β^j

Honestly, I don't think there's a way. Or?

$R_{t,t+j} \pi_{t,t+j}$ is the s.d. It's just like the firms' std in Calvo, $\phi_{t+2,t} = \beta \frac{p_t c_t}{p_{t+2} c_{t+2}}$, so then, analogously to here, it cancels and we're left w/ β every time.

→ yes, you can see it from eq (33)

⇒ ok so eq. (4) is good!

The next step is solving (3) backwards.

$$(3): \hat{c}_t^i = \hat{e}_t^i \hat{c}_{t+1}^i - \beta(\hat{i}_t^i - \hat{e}_t^i \hat{\pi}_{t+1}^i) + g_t - \hat{e}_t^i g_{t+1}$$

$$\hat{e}_t^i \hat{c}_{t+1}^i = \hat{c}_t^i - g_t + \hat{e}_t^i g_{t+1} + \beta(\hat{i}_t^i - \hat{e}_t^i \hat{\pi}_{t+1}^i)$$

$$\hat{e}_t^i \hat{c}_{t+1}^i - \hat{e}_t^i g_{t+1} = \hat{c}_t^i - g_t + \beta(\hat{i}_t^i - \hat{e}_t^i \hat{\pi}_{t+1}^i)$$

Let's call $t+1 = T$

$$\begin{aligned} \hat{e}_t^i \hat{c}_T^i - \hat{e}_t^i g_T &= \underbrace{\hat{c}_{T-1}^i - g_{T-1}}_{\hat{c}_T^i} + \underbrace{\beta(\hat{i}_{T-1}^i - \hat{e}_t^i \hat{\pi}_T^i)}_{\hat{c}_T^i - g_T + \beta(\hat{i}_T^i - \hat{e}_t^i \hat{\pi}_{T-1}^i)} \\ &= \hat{c}_{T-2}^i - g_{T-2} + \beta(\hat{i}_{T-2}^i - \hat{e}_t^i \hat{\pi}_{T-1}^i) - \dots \\ &= \dots \hat{c}_t^i - g_t + \beta \sum_{s=t}^{T-1} (\hat{i}_s^i - \hat{e}_t^i \hat{\pi}_{s+1}^i) \quad \checkmark \end{aligned}$$

$$\begin{aligned}\hat{E}_t^i \hat{C}_t^i &= \hat{E}_t^i g_t + \hat{c}_t^i - g_t + 3 \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \\ &= \hat{c}_t^i - g_t + \hat{E}_t^i \left[g_t + 3 \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \right]\end{aligned}$$

So now sub $\hat{E}_t^i \hat{C}_t^i$ (I guess) into the IBC, eq (b)

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i = \tilde{w}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{y}_T^i$$

$$\Rightarrow \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \hat{c}_t^i - g_t + \hat{E}_t^i \left[g_t + 3 \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \right] \right\}$$

$$= \tilde{w}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{y}_T^i$$

$$\Leftrightarrow \hat{c}_t^i - g_t = \tilde{w}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{y}_T^i - g_T - 3 \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) \right]$$

$$\text{Conjecture: } \sum_{T=t}^{\infty} \sum_{s=t}^{T-1} (\hat{i}_s - \hat{\pi}_{s+1}) = \sum_{T=t}^{\infty} (\hat{i}_T - \hat{\pi}_{T+1}).$$

$$\text{Why? BIC } \sum_{T=t}^{\infty} \sum_{s=t}^{T-1} \text{stuffs} = \sum_{s=t}^{T-1} \text{stuffs} + \sum_{s=t}^{\infty} \text{stuffs}$$

Yo what if I don't sub the (3) solved bwd into (b), but just
(3)?

$$(3) \quad \hat{C}_t^i = \hat{E}_t^i \hat{C}_{t+1}^i - \beta(\hat{i}_t^i - \hat{E}_t^i \hat{\pi}_{t+1}^i) + g_t - \hat{E}_t^i g_{t+1}$$

$$(4) \quad \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i = \tilde{w}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T^i$$

$$\rightarrow \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} [\hat{C}_{T+1}^i - \beta(\hat{i}_T^i - \hat{\pi}_{T+1}^i) + g_T - g_{T+1}] = \tilde{w}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T^i$$

that doesn't seem to do it at all!

Let's pause the issue of eq(5). Supp. we have it.

• Then integrate over $i \rightarrow \tilde{w}_t \Rightarrow 0$

$$C_t = \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)\hat{Y}_T^i - \beta\beta(\hat{i}_T^i - \hat{\pi}_{T+1}^i) + \beta(g_T - g_{T+1})].$$

$$\cdot \hat{Y}_t = \hat{C}_t$$

$$\hat{Y}_t = \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)\hat{Y}_T^i - \beta\beta(\hat{i}_T^i - \hat{\pi}_{T+1}^i) + \beta(g_T - g_{T+1})].$$

$$\cdot x_t := \hat{Y}_t - \hat{Y}_t^n$$

$$x_t = \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)x_T - \beta\beta(\hat{i}_T^i - \hat{\pi}_{T+1}^i) + \beta(g_T - g_{T+1}) - (1-\beta)\hat{Y}_T^n]$$

$$- \hat{Y}_t^n \quad \text{even that doesn't work out! damn!}$$

Ryan meeting

8 Oct 2019

• Invite Basuji / Prasad for macro seminar? Who organizes it?

↳ Ryan, Susanto & Pabbi for the spring, Rosen, Fabio & Jaron for fall

Ryan said that Susanto connects b (slope) w/ Missity Deflation b/c b transmits shocks to SR facts

Work after:

8 Oct 2019

So supp my previous conjecture is right. Then

$$G^i - g_T = \tilde{\omega}_T^i + \hat{E}_T^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}_T^i - g_T + \beta (\hat{i}_T^i - \hat{\pi}_{T+1}) \right]$$

and I still don't get (5).

But if I now do the last 3 steps:

$$\underbrace{\hat{Y}_T^i - \hat{Y}_T^n - g_T + \hat{Y}_T^n}_{\rightarrow x_T} = \hat{E}_T^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}_T^i - \hat{Y}_T^n - g_T + \hat{Y}_T^n + \beta (\hat{i}_T^i - \hat{\pi}_{T+1}) \right]$$

$$\Rightarrow x_T + \hat{Y}_T^n = \hat{E}_T^i \sum_{T=t}^{\infty} \beta^{T-t} [x_T]$$

↳ no, the problem is that that ain't right b/c

$$r_T^n = \hat{Y}_{T+1}^n - \hat{Y}_T^n \left[+ (g_T - g_{T+1}) \text{ if we have this preference shift} \right]$$

↳ and for me, $r_T^n = \frac{1}{\delta} (\hat{Y}_{T+1}^n - \hat{Y}_T^n)$, in line w/ Basu, SUM
so let's set all $g_T = 0$ $\forall t$ b/c I don't have it. (part 2, p.58)

→ then the EE is $\hat{G}_T^i = \hat{E}_T^i \hat{C}_{T+1}^i - \beta (\hat{i}_T^i - \hat{E}_T^i \hat{\pi}_{T+1})$

Solve back: $\hat{E}_T^i \hat{C}_{T+1}^i = \hat{G}_T^i + \beta (\hat{i}_T^i - \hat{E}_T^i \hat{\pi}_{T+1})$

$\hat{E}_T^i \hat{C}_T^i = \hat{C}_{T-1}^i + \beta (\hat{i}_{T-1}^i - \hat{E}_T^i \hat{\pi}_T)$

$$\hat{E}_t^i \hat{C}_t^i = b(\hat{i}_{T-1} - \hat{E}_t^i \bar{\pi}_T) + b(\hat{i}_{T-2} - \hat{E}_t^i \bar{\pi}_{T-1}) + \dots \\ + b(\hat{i}_+ - \hat{E}_t^i \bar{\pi}_{T+1}) + \hat{c}_t^i$$

$$\hat{E}_t^i \hat{C}_t^i = \hat{c}_t^i + b \sum_{s=t}^{T-1} (\hat{i}_s - \hat{E}_t^i \bar{\pi}_{s+1}) \quad (\text{EE simple solved bwd})$$

Sub this into (4)

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{c}_t^i + b \sum_{s=t}^{T-1} (\hat{i}_s - \hat{E}_t^i \bar{\pi}_{s+1}) \right] = \tilde{w}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{y}_T^i$$

$$\Leftrightarrow \hat{c}_t^i \sum_{T=t}^{\infty} \beta^{T-t} + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} b \sum_{s=0}^{T-1} (\hat{i}_s - \hat{E}_t^i \bar{\pi}_{s+1}) = \tilde{w}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{y}_T^i$$

$$\Leftrightarrow \underbrace{\hat{c}_t^i + (1-\beta)b \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \sum_{s=0}^{T-1} (\hat{i}_s - \hat{E}_t^i \bar{\pi}_{s+1})}_{\checkmark} = (1-\beta) \tilde{w}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} (1-\beta) \hat{y}_T^i \quad \checkmark \quad \checkmark$$

now I only need this to equal $\beta b(\hat{i}_T - \bar{\pi}_{T+1})$

But potentially, it should actually be $(1-\beta)(b?)$

Now I'm only missing the $\hat{i}_T - \bar{\pi}_{T+1}$ term 9 Oct 2019

in eq(5). So supp. again that I have eq(5), w/o g_s , and I do the last 3 steps:

aggregate, set $C_+ = Y_+$ and am about to get $x_+ = \hat{Y}_+ - \hat{V}_+^n$

$$\hat{Y}_+ = \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) \hat{Y}_T - \beta b(\hat{i}_T - \bar{\pi}_{T+1}) \right]$$

$$\hat{Y}_t - \hat{Y}_t^n = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) \hat{Y}_T - \beta b(\hat{i}_T - \hat{\pi}_{T+1}) \right] - \hat{Y}_t^n$$

(*) $x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)[\hat{Y}_T - \hat{Y}_T^n] - \beta b(\hat{i}_T - \hat{\pi}_{T+1}) + (1-\beta)\hat{Y}_T^n \right] - \hat{Y}_t^n$

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)(\hat{Y}_{T+1} - \hat{Y}_T^n) - \beta b(\hat{i}_T - \hat{\pi}_{T+1}) + (1-\beta)\hat{Y}_{T+1}^n \right]$$

$$- \hat{Y}_t^n + (1-\beta)\hat{Y}_t$$

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)x_{T+1} - \beta b(\hat{i}_T - \hat{\pi}_{T+1}) \right]$$

$$+ \underbrace{x_t - \beta \hat{Y}_t + (1-\beta)\hat{Y}_{t+1}^n + (1-\beta)\beta \hat{Y}_{t+2}^n + (1-\beta)\beta^2 \hat{Y}_{t+3}^n + \dots}$$

$$\underbrace{x_t - \beta \hat{Y}_t + \hat{Y}_{t+1}^n}_{\beta r_{t+1}^n} \underbrace{- \beta \hat{Y}_{t+2}^n + \beta \hat{Y}_{t+3}^n}_{\beta^2 r_{t+2}^n} \underbrace{- \beta^2 \hat{Y}_{t+4}^n + \beta^3 \hat{Y}_{t+5}^n}_{\beta^3 r_{t+3}^n} \underbrace{- \beta^3 \hat{Y}_{t+6}^n + \dots}_{\text{cool}}$$

$$\hat{Y}_t - \hat{Y}_t^n - \beta \hat{Y}_t + \hat{Y}_{t+1}^n$$

$$= \hat{Y}_t - \beta \hat{Y}_t + r_t^n$$

$$= \underbrace{(1-\beta)\hat{Y}_t}_{\text{good}} + r_t^n$$

good. let's retry, from step (*)

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)[\hat{Y}_T - \hat{Y}_T^n] - \beta b(\hat{i}_T - \hat{\pi}_{T+1}) + (1-\beta)\hat{Y}_T^n \right] - \hat{Y}_t^n$$

$$x_t = \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)x_T - \beta b(\hat{i}_T - \hat{\pi}_{T+1}) \right]$$

$$- \hat{Y}_t^n + (1-\beta)\hat{Y}_t^n - (1-\beta)\beta \hat{Y}_{t+1}^n + (1-\beta)\beta^2 \hat{Y}_{t+2}^n + \dots$$

$$\begin{aligned}
 \text{The last series is } & -\hat{Y}_t^n + (1-\beta)\hat{Y}_t^n - (1-\beta)\beta\hat{Y}_{t+1}^n + (1-\beta)\beta^2\hat{Y}_{t+2}^n + \\
 = & -\underbrace{\beta\hat{Y}_t^n + \beta\hat{Y}_{t+1}^n}_{\beta r_t^n} - \underbrace{\beta^2\hat{Y}_{t+1}^n + \beta^2\hat{Y}_{t+2}^n}_{\beta^2 r_{t+1}^n} - \underbrace{\beta^3\hat{Y}_{t+2}^n}_{\beta^3 r_{t+2}^n} + \dots \\
 = & \beta[r_t^n + \beta r_{t+1}^n + \beta^2 r_{t+2}^n + \dots] \\
 = & \beta \sum_{T=0}^{\infty} \beta^{T-t} r_T^n
 \end{aligned}$$

↑ now this guy is superfluous ...

and we have x_T , not x_{T+1} in the Σ , i.e.

$$x_t = \sum_{T=t}^{\infty} \beta^{T-t} [(-\beta)x_T - \beta b(\hat{i}_T - \hat{\pi}_{T+1}) + \beta r_T^n] \quad | -x_t$$

$$\beta x_t = \sum_{T=t}^{\infty} \beta^{T-t} [(-\beta)x_{T+1} - \beta b(\hat{i}_T - \hat{\pi}_{T+1}) + \beta r_T^n]$$

$$x_t = \sum_{T=t}^{\infty} \beta^{T-t} \left[\frac{1-\beta}{\beta} x_{T+1} - b(\hat{i}_T - \hat{\pi}_{T+1}) + r_T^n \right]$$

↑ hat jetzt neu horen el ... mostmar
wieder stimmt grade el neu ...

(Meg az $r_T^n = \beta r_T^n$ is jó, ha átdefinition

$$r_T^n = \frac{1}{\beta} (\hat{Y}_{T+1}^n - \hat{Y}_T^n) \text{ mit Basis Szen 2 p. 58)$$

So at the current point in time I'm inclined to say that if eq(5)
is right, then eq.(6) is too, except the coefficient of x_{T+1}
should be $\frac{1-\beta}{\beta}$, not $1-\beta$.

So we need to turn back to deriving eq (5) from (3).

$$\hat{C}_t^i = \hat{E}_t C_{t+1}^i - \beta(i_t^i - \hat{E}_t \hat{\pi}_{t+1}) \quad (3)$$

Drop i 's and hats for simplicity:

$$c_t - E_t c_{t+1} = -\beta(i_t - E_t \pi_{t+1})$$

$$(1 - L^{-1}) c_t = -\beta(i_t - E_t \pi_{t+1})$$

$$\text{or } -(1 - L) E_t c_{t+1} = -\beta(i_t - E_t \pi_{t+1})$$

$$\Leftrightarrow (1 - L) E_t c_T = \beta(i_T - E_t \pi_T) \quad \left| \begin{array}{l} \text{See rothemberg-pricing-peter} \\ \text{notes.pdf} \end{array} \right.$$

$$E_t c_T = (1 - L)^{-1} \beta(i_T - E_t \pi_{T+1}) \quad \left| \begin{array}{l} \text{- my notes.pdf} \\ \text{not trivial} \end{array} \right.$$

$$= \beta \sum_{i=0}^{\infty} i^i \text{ RHS} = \beta \sum_{T=t}^{T-1} (i_T - E_t \pi_{T+1})$$

I think this is pretty much what I got before, and it's also in accordance w/ Preston's (4.5)

Plugging this into the LHS of (4), we have

$$\begin{aligned} & \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\beta \sum_{s=t}^{T-1} (i_s - \hat{E}_s \hat{\pi}_{s+1}) \right] \\ &= \beta \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \sum_{s=t}^{T-1} (i_s - \hat{E}_s \pi_{s+1}) \end{aligned}$$

$$= \sum_{s=t}^{t-1} \text{stuff} + \beta \sum_{s=t}^t + \beta \sum_{s=t}^{t+1} + \beta^2 \sum_{s=t}^{t+2}$$

That don't look good.

$$= \beta \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \sum_{S=t}^{T-1} (i_s - \hat{E}_t \pi_{SM})$$

let's try to redefine the indices

$$\beta \hat{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{S=0}^{t-1} \text{stuffs}$$

$$= \beta \left[0 + \beta \sum_{S=0}^0 + \beta^2 \sum_{S=0}^1 \dots \text{mmm...} : S \right]$$

$$\quad \quad \quad t=0 \quad \quad \quad t=1 \quad \quad \quad t=2$$

Also I kinda feel that even if I don't take time t , but time 0 expectations, I can't get around the second sum ...

$$= \beta [\beta \cdot \text{stuffs}_0 + \beta^2 \text{stuffs}_0 + \beta^2 \text{stuffs}_1 + \beta^3 \text{stuffs}_0 + \beta^3 \text{stuffs}_1 + \beta^3 \text{stuffs}_2 + \dots]$$

$$= \beta [(\beta + \beta^2 + \dots) \text{stuffs}_0 + (\beta^2 + \beta^3 + \dots) \text{stuffs}_1 + \dots] \quad | \text{Ignore } \beta$$

$$= \beta (1 + \beta^2 + \beta^3 + \dots) \text{stuffs}_0 + \beta^2 (1 + \beta^2 + \beta^3 + \dots) \text{stuffs}_1 + \dots$$

$$= (\beta^1 \text{stuffs}_0 + \beta^2 \text{stuffs}_1 + \beta^3 \text{stuffs}_2 + \dots) \frac{1}{1-\beta} \quad (!)$$

$$= (\beta^0 \text{stuffs}_0 + \beta^1 \text{stuffs}_1 + \beta^2 \text{stuffs}_2 + \dots) \frac{\beta}{1-\beta} \quad (!!)$$

So the LHS is

$$\beta \hat{E}_t \sum_{T=t}^{\infty} \frac{\beta}{1-\beta} (i_T - \hat{E}_t \pi_{T+1}) \quad \text{Hmz tht!}$$

So (5) is

$$\frac{1}{1-\beta} C_t + \beta \hat{E}_t \sum_{T=t+1}^{\infty} \frac{\beta}{1-\beta} (i_T - \hat{E}_T \pi_{T+1}) = \hat{E}_t \sum_{T=t+1}^{\infty} \hat{Y}_T + \bar{w}_t$$

$$\Rightarrow \hat{C}_t + \beta \hat{E}_t \sum_{T=t+1}^{\infty} \beta^{T-t} \beta (i_T - \hat{E}_T \pi_{T+1}) = (1-\beta) \bar{w}_t + \hat{E}_t \sum_{T=t+1}^{\infty} (1-\beta) \hat{Y}_T$$

So ...

$$\hat{C}_t = (1-\beta) \bar{w}_t + \hat{E}_t \sum_{T=t+1}^{\infty} \beta^{T-t} \left[(1-\beta) \hat{Y}_T - \beta \beta (i_T - \hat{E}_T \pi_{T+1}) \right]$$

= Preston's (5). Yay!

The last 3 steps again: aggregate and set $\hat{C}_t = \hat{Y}_t$, drop hats.

$$Y_t = \hat{E}_t \sum_{T=t+1}^{\infty} \beta^{T-t} \left[(1-\beta) Y_T - \beta \beta (i_T - \pi_{T+1}) \right]$$

$$Y_t = \hat{E}_t \sum_{T=t+1}^{\infty} \beta^{T-t} \left[(1-\beta)(Y_t - Y_t^n) - \beta \beta (i_T - \pi_{T+1}) + (1-\beta) Y_t^n \right]$$

$$Y_t = \underbrace{\hat{E}_t \sum_{T=t+1}^{\infty} \beta^{T-t} \left[(1-\beta) X_{T+1} - \beta \beta (i_T - \pi_{T+1}) \right]}_{= CS} + (1-\beta) X_t + \hat{E}_t \sum_{T=t+1}^{\infty} \beta^{T-t} (1-\beta) Y_t^n$$

$$X_t = \text{correct-shift} - Y_t^n + (1-\beta) X_t + (1-\beta) [Y_t^n + \beta Y_{t+1}^n + \beta^2 Y_{t+2}^n + \dots]$$

$$\beta X_t = CS - \cancel{Y_t^n} - \cancel{X_t} - \beta Y_t^n + \beta Y_{t+1}^n - \beta^2 Y_{t+2}^n + \beta^3 Y_{t+3}^n - \beta^4 Y_{t+4}^n + \dots$$

$$\beta X_t = CS + \beta r_t^n + \beta^2 r_{t+1}^n + \dots$$

$$X_t = \frac{1}{\beta} CS + r_t^n + \beta^2 r_{t+1}^n + \dots$$

βr_t^n w/ my def.

$$X_t = \hat{E}_t \sum_{T=t+1}^{\infty} \beta^{T-t} \left[\frac{1-\beta}{\beta} X_{T+1} - \beta (i_T - \pi_{T+1}) + r_T^n \right].$$

Wait... can it be that I did one thing wrong:

$$\begin{aligned} \text{when I have } \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)x_T] &= (1-\beta) \hat{E}_t \sum_{T=t}^{\infty} x_T \\ &= (1-\beta)x_t + (1-\beta)\hat{E}_t [\beta x_{t+1} - \beta^2 x_{t+2} + \dots] \\ &= (1-\beta)x_t + (1-\beta)\hat{E}_t \left[\underbrace{\beta \sum_{T=t}^{\infty} \beta^{T-t} x_{T+1}}_{\text{Yeah!}} \right] \end{aligned}$$

So that means that my truly correct CS is:

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)\beta x_{T+1} - \beta^2 (i_T - \pi_{T+1})]$$

so that when I take $\frac{1}{\beta}$ CS I obtain

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)x_{T+1} - \beta(i_T - \pi_{T+1}) + \beta r_T^N]$$

which is Preston's (6) = (18), and the equation I had

in my DW prezi and in materials. Yay!

Great! We can go on to investigating the expectation-operator, \hat{E}_t . Obs. 1. Preston took derivatives from it no problem, w/o saying anything. So let's go to Evans & Honkapohja (2001).

Evans & Honk (2001) on the nonrational expectation-operator \hat{E}

- p. 68-69 Ramsey model: they derive them at no prob.
- Minimum-state-variable (MSV) solution (introduced in McCallum (1983))
- heterogeneous learning p. 223-225 (Evans, Honk & Marimon 2000)
- "regular" vs "irregular" models (Farmer 1999, Pesaran 1987)
 - ↳ unique REE (Blanchard-Kahn conditions)
- RE is an esp concept p. 11
- Expectation is a fit of observables, and so is the updating p. 17-18 → I think it's fair to say that we conjecture \hat{E} to be linear and of the form $\bar{\pi} + b s_t$, and $\bar{\pi}_t = Q(\bar{\pi}_{t-1}, \theta_t)$ nonlinear updating rule and to somehow verify this using undet. cons.

Reading the chapter on Nonlinear models
(Chapter 11)

10 Oct 2019

Let the nonlinear univariate model be of the form

$$y_t = F(y_{t+1})^e + v_t \quad (11.2)$$

$$\begin{array}{c} \text{nonlinear} \\ \uparrow \\ = E_t [F(y_{t+1})] \end{array} \quad \begin{array}{l} \text{shock } v_t \sim \text{iid}(0, \cdot) \text{ if } v_t = 0 \\ (11.2) = (11.1) \end{array}$$

A more general nonlinear model is

$$y_t = h(G(y_{t+1}, v_{t+1})^e, v_t) \quad (11.3)$$

Here's an argument (p. 273) for linearity:

1. Supp agents don't know $G(y_{t+1}, v_{t+1})^e$, but have data on its past values $G(y_j, v_j)$ $j=1, \dots, t$.
2. A natural estimator for $G(y_{t+1}, v_{t+1})^e$ is the sample mean:

$$\theta_t = \frac{1}{t} \sum_{j=1}^t G(y_j, v_j)$$

which is then updated using RLS as more data becomes available.

3. the sample mean is a linear operator.

Peter's argument for differentiating them:

- E is an integral over states

- Differentiation proceeds over different variables, not states.

By the way, Evans & Hauke derive from \hat{E} all over the place here w/o remarks

- That's all Elans & Honk had to say about \hat{E} .
- Liam Graham also simply differentiates them \tilde{E} (Mac p 5)

Moving on : small π , large x

- well, setting $\alpha=1$ and $\kappa=1$ still doesn't get π to move as much as x (and thus i). So what's going on?
- now what I did is on top of $(\alpha=1, \kappa=1)$, I shut off all shocks except u_t , so now only π is affected.

Now the magnitude of the responses is the same (almost)

↳ But even then, even then x & i fluctuate more!

- shocks shut off except u , α back to 0.5
 \rightarrow same as w/ $\alpha=1$ $\rightarrow \alpha$ doesn't matter
- -||-, κ back to 0.51.
 $\rightarrow \kappa$ matters a lot: this really brings in the gap between π and x .

Let's note a couple of things

1. You'd think that the higher the shock volatilities, the more wandering happens. But that is only partly so:
 - If I shut off \bar{t} -shocks, c.p., I'm always anchored
 - If I shut off the other shocks, c.p., I'm unanchored

→ What's going on?

3. What matters for the size of gaps in π & x is

- $\kappa \rightarrow$ but this doesn't explain it all

↳ Why?

- size of shocker DOESN'T really matter, α doesn't matter

→ Something else going on?

4. CEMP say about the criterion: Or just it's a reduced-form way to capture model misspecification tests

↳ They simulate a calibrated version where firms instead employ a t-test of shifting means

(Brown, Durbin, Evans 1975) (see CEMP p. 18)

↳ get nearly identical results but w/ more parameters to estimate.

Analysis of the issues:

$$1) k = \text{slope of NKPC} \quad \pi_t = \beta E_t \pi_{t+1} + k \hat{x}_t$$

- If $k=0$, no rel. blun $\pi \not\propto x \rightarrow$ money is strongly non-neutral, x persistently $\neq 0$.
- If $k \rightarrow \infty$, then flex prices: changes in x translate immediately and a lot into $\pi \rightarrow$ money is neutral

Note that $\frac{\partial k}{\partial \alpha} < 0 \rightarrow$ If firms are stuck w/ a price for longer ($\alpha \uparrow$), then $k \downarrow$ and money becomes less neutral!

- So if $\alpha \uparrow$ (from 0.5 to 1) \rightarrow we get anchoring where we previously didn't
 - if $\alpha \downarrow$ (from 0.5 to 0.1) \rightarrow we get deanchoring and the gaps in π are indeed larger
- \Rightarrow the lower α (higher k), the more deanchoring we get and the bigger the gaps in π are relatively to x
- \hookrightarrow I think what is happening is that when α high (k low), we

have a lot of price stickiness \Rightarrow so inflation cannot respond a lot to shocks, so the margin of adjustment is x

\hookrightarrow we need sufficiently flex. prices for inflation to move away from its RE path.

\Rightarrow cool, now we understand that fairly well ^{SOLVED}

 \Rightarrow a RESULT:

The more price stickiness in a learning world w/ anchoring, the more deanchored periods will show up in output gaps instead of inflation b/c x will be the margin of adjustment!

Let's do a survey of lit on learning w/ price stickiness to see if we have anything similar

- CEMP: can't address b/c no x in model
- Graham: no inflation in model
- Porton: doesn't do simulations

- Easypi et al., Lim'03 : doesn't get it
- Fenero (2007) : has $\pi \propto x$, but doesn't seem to discuss the relative role of those

• Orphanides & Williams (2004) is a little difficult to compare b/c the model isn't an NK model

→ there doesn't seem to be a clear mapping b/w

a parameter and price stickiness (maybe α ?)

they do have a related result: when inflation inertia is low (flex prices) then RLS w/ a constant gain produces very bad forecasts \Rightarrow i.e. TC moves far from ARE

→ it might be useful for me to

1) write out the ACM, in particular of $\bar{\pi}$ and π

2) look into the Lit on π -persistence (CEMP attitude

b/c if the conclusion is that there's

to it)

a drift in π , well, boom! that's our $\bar{\pi}$.

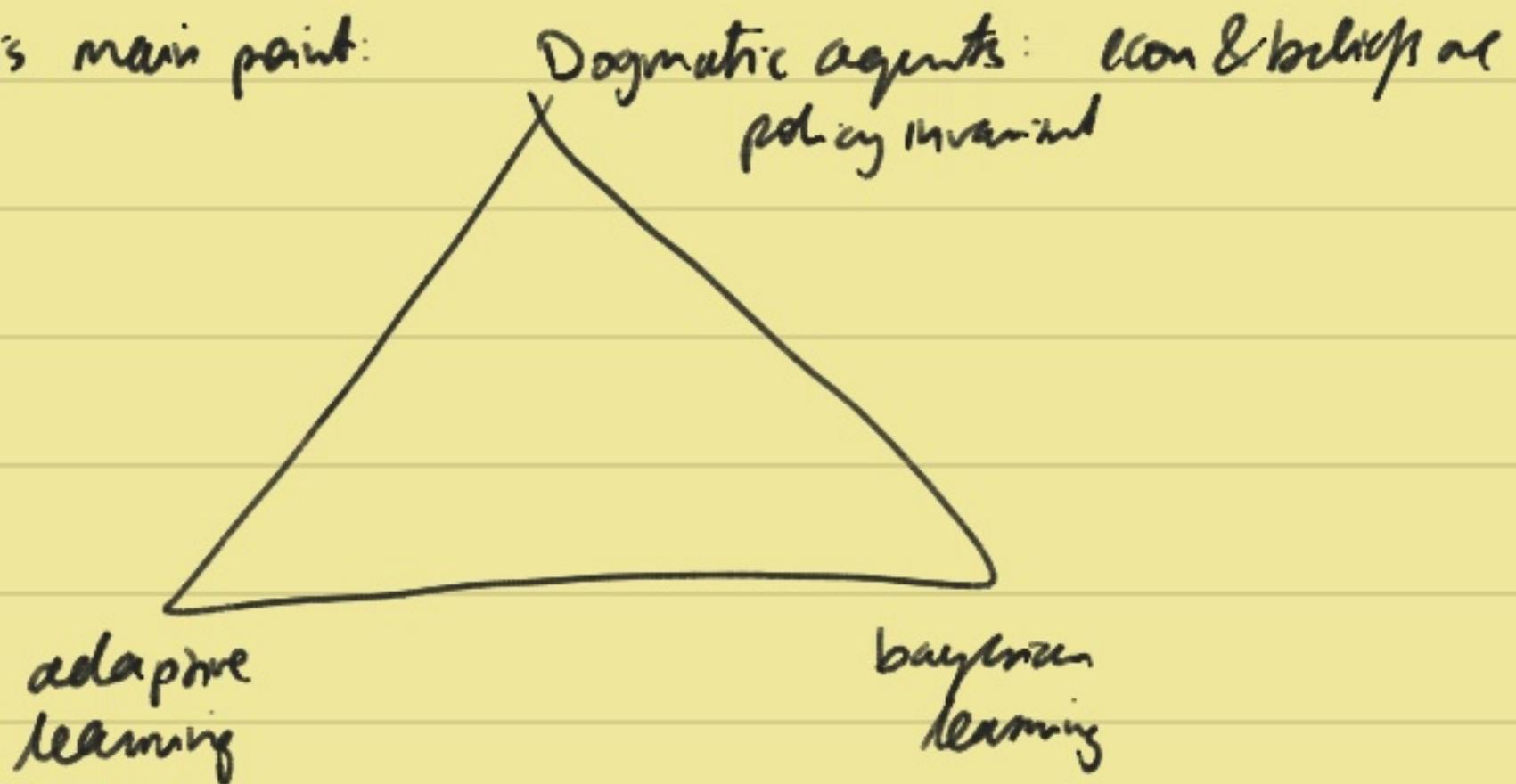
- Compare my IRFs w/ Orph & Will p. 15 (Mac)

Angelitos

10 Oct 2019

Eusepi Preston

This main point:



1) Why isn't dogmatic beliefs enough? (i.e. why learning?)

→ b/c w/ dogmatic E, E are policy invariant!

Mon-pol. might depart from TR to correct E, but can't influence E.

2) why adaptive and not Bayesian learning?

It might not matter?

And lastly: what evidence is there in the data for learning?

Send him paper when you have it!

Angelitos Talk

it is the same as NBER 51

- Angelitos & Lian 2018, AER

does "Preston": NK model w/o common knowledge
gives rise to LH plots (for IS & NKPL!) ^{key}

what mon pol does here is to anchor E .

"Android Beliefs" = lack of common knowledge

when policy says x ,

$$E_i[x] = x \quad \text{but} \quad E_i[\bar{E}[x]] = (1-\lambda)0 + \lambda E_i[x]$$

\uparrow_{prior}

i.e. I update my beliefs but I wrongly believe
that $(1-\lambda)$ fraction are "anchored to the prior", zero.

Estimate λ by taking the diff b/wn IRFs of beliefs
of C and IRFs of C (smart!)

→ Coibion & Goro do this, get $\lambda \approx 0.5$.
(but not for AB)

Back to my work

- Eusepi & Preston (2018)'s Fig 4 (p. 23, Mac)
shows volatilities of π $8 \times \frac{\sigma_\pi^2 + \lambda \cdot \sigma_x^2}{\sigma_x^2}$
as a function of λ in learning vs. RE.
 - 1.) Volatility is higher in learning
 - 2.) as $\lambda \uparrow$, volatility \uparrow i.e. $\sigma_x^2 > \sigma_\pi^2$ (like me!)
- Noah Williams (2003)'s Fig 2.
under a constant growth, π responds less than x
(to a shock to r^h , NK model)
 \rightarrow more support for my results!

Now we need to tackle the issue of IRFs 11 Oct 2019
in the learning world

Recall that in the linear RE, the impulse response is:

$$\text{IRF} = h x^\top \gamma \delta \quad [\text{where } x_t = h x_{t-1} + \gamma \epsilon_t]$$

But this is a special case of the generalized IR,
the GIR (Lect 3 (and 4))

$$GIR(j, \delta, \tilde{x}_{t-1}, \tilde{\epsilon}_t) = E_t[x_{t+j} | \tilde{x}_{t-1}, \tilde{\epsilon}_t + \underline{\delta}] - E_t[x_{t+j} | \tilde{x}_{t-1}, \tilde{\epsilon}_t]$$

↑ ↑ ↑
 periods econ up other
 short to now shocks

In the linear world, $E_t[x_{t+j} | \tilde{x}_{t-1}, \tilde{\epsilon}_t] = \sum_{\tau=j}^{\infty} h^{x^\tau} \gamma \tilde{\epsilon}_{t+j-\tau}$
(Wold-representation), and thus

$$E_t[x_{t+j} | \tilde{x}_{t-1}, \tilde{\epsilon}_t + \underline{\delta}] = \sum_{\tau=j}^{\infty} h^{x^\tau} \gamma \tilde{\epsilon}_{t+j-\tau} + h^{x^j} \gamma \delta, \text{ so}$$

$GIR^{\text{linear}} = h^{x^j} \gamma \delta.$

But in the nonlinear world, the Wold representation isn't valid b/c you can't neatly separate shocks.

→ a solution is to integrate out shocks: Ryan's IR:

$RIR = E[GIR]$ → don't need to know where in the state-space
the economy is.

- Simulate a bunch of histories \tilde{x}_{t-1} , calc GIR at all and take an average

Initially I thought that the learning IRFs will be a

similar issue, but I don't think so anymore b/c
the world is linear (works w/), but the problem is
that g_x isn't.

One part that will be constant is the effect on exogenous
states: $h_x^j \eta_S$ will be the same as for RE.

But every time I calculate the effect on the jumps,
 $g_x h_x^j \eta_S$, g_x will be different.

But what is a "GIR"-issue is that the econ's position
in the state-space matters b/c the stance of convergence
of learning to RE means that reaction to shocks will be
diff. So maybe I do have to do the RIR-thing!

One option: supp. $T=40$. For $t=1, \dots, T$, take a "GIR"
for every t and then calc the average (kind of like (I))
→ in fact, I think that might work ...

⇒ In particular:

for $t = 1 \dots T-h$ (so that IRFs are the same length, h)

$$[\text{sim_x}, \text{sim_y}] = \text{sim-learn}(\text{same}, t)$$

↑ add short
to inner in that
period

for the same shock sequence otherwise!

and then take

$$\text{sim_x} = x\text{-LR-anchor} \text{ and } \text{sim_y} = y\text{-LR-anchor}$$

⇒ let these diff's be $\text{ir-}h\text{-t}$ (h -period ahead IRFs started at time t), or GIR

[end

now we have $\overset{\text{GIR}}{\text{ir-}h\text{-t}}$ which is $h \times 3 \times T$

1 approach: take average over the 3rd dim, T .

2nd approach: sort and take median

- Have done IRFs

15 Oct 2019

- and casum-based alternative criterion for

Now I want to write out the ALM and the criterion at least in Mathematica to investigate the processes for π and $\bar{\pi}$.

→ materials5.nb. See ALM & criterion in materials3, eq (41) & (43).

Mathematica, materials5.nb shows that you can write the LOM of π as:

$$\pi_t = a_1 \cdot \bar{\pi}_{t-1} + a_2 r_t^n + a_3 \bar{i}_t + a_4 u_t$$

where a_i , $i=1, \dots, 4$ are scalars.

At the same time, the PLM is $\hat{E}_{t-1} \pi_t = \bar{\pi}_{t-2} + b s_{t-1}$

which for π reads $\hat{E}_{t-1} \pi_t = \bar{\pi}_{t-2} + b_{11} r_{t-1}^n + b_{12} \bar{i}_{t-1} + b_{13} u_{t-1}$

Then the criterion is $(\bar{\pi}_{t-2})$ check timing → for Ryan's PLM

$$|\hat{E}_{t-1} \pi_t - \hat{E}_{t-1} \bar{\pi}_t| = |\bar{\pi}_{t-2} + b_{11} r_{t-1}^n + b_{12} \bar{i}_{t-1} + b_{13} u_{t-1} - a_1 \bar{\pi}_{t-1} - a_2 p_r r_{t-1}^n - a_3 p_i \bar{i}_{t-1} - a_4 p_u u_{t-1}|$$

as usual I'm sidestepping the timing issues for now.

The point is that for $\text{LDM}(\pi)$ of

$$\pi_t = a_1 \cdot \bar{\pi}_{t-1} + a_2 r_t^n + a_3 i_t + a_4 u_t$$

check timing!

the "scalar extension" θ_t is

$$\theta_t(b_r + b_i + b_u) = \left| (1-a_1) \bar{\pi}_{t-1} + (b_{11}-a_2 p_r) r_{t-1}^n + (b_{12}-a_3 p_i) i_{t-1} + (b_{13}-a_4 p_u) u_{t-1} \right|$$

which we can summarize as

$$\pi_t = a_1 \bar{\pi}_{t-1} + [a_2 \ a_3 \ a_4] s_t$$

For Ryan's PLM it's $\bar{\pi}_{t-2}$,
for Honky's it's $\bar{\pi}_{t-1}$.

$$\theta_t(b_r + b_i + b_u) = \left| (1-a_1) \bar{\pi}_{t-1} + [b_{11}-a_2 p_r, b_{12}-a_3 p_i, b_{13}-a_4 p_u] s_t \right|$$

while we still have, as in (OMP), that

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} f_{t-1} \quad \text{but the FE } f_{t-1} \text{ is diff,}$$

in particular it is ALM-PLM, $\bar{\pi}_{t-1} - \hat{E}_{t-2} \bar{\pi}_{t-1}$

$$f_t = (a_1 - 1) \bar{\pi}_{t-1} + (a_2 - b_{11}) r_t^n + (a_3 - b_{12}) i_t + (a_4 - b_{13}) u_t$$

where I've again been sloppy w/ the timing.

$$f_t = (a_1 - 1) \bar{\pi}_{t-1} + [a_2 - b_{11}, a_3 - b_{12}, a_4 - b_{13}] s_t$$

\Rightarrow So supposing the timing is right, I'd get

$$\bar{\pi}_t = (1 + k_t^{-1}(a_1 - 1)) \bar{\pi}_{t-1} + k_t^{-1} [a_2 - b_{11}, a_3 - b_{12}, a_4 - b_{13}] s_{t-1}$$

check timing!

The only element that is def. true regardless of timing

$$\text{is } \bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} f_{t-1}$$

$$= \bar{\pi}_{t-2} + k_{t-1}^{-1} f_{t-2} + k_t^{-1} f_{t-1}$$

$$= \bar{\pi}_0 + \sum_{\tau=0}^{t-1} k_{\tau+1}^{-1} f_\tau$$

Obs. 1. The estimate of LR π -mean will be $\neq RE(0)$

when there have been 
a bunch of FE w/ the same sign
some very large FE.

The difference to COMP is that here, it is only $\bar{\pi}$ that causes a deviation between PLM-EALM (i.e.
 $\hat{E}_{t-1}\pi_t - E_{t-1}\pi_t$) whereas in my case there's an additional wedge that comes from the fact that the RE coefficient b isn't correct as long as we haven't converged to RE!

Look at timing one more time:

$$1) f_{t-1} = \pi_t - E_{t-1} \pi_t \quad (\text{index on FE refers to the time the fact was made})$$

This is NOT what CEMP have. For them,

$$\begin{aligned} f_{t-1} &= \pi_{t-1} - \hat{E}_{t-2} \pi_{t-1} \quad (\text{index refers to what you're forecasting}) \\ \text{and } \theta_t &= |\hat{E}_{t-1} \hat{\pi}_t - E_{t-1} \pi_t| \\ &= |E_{t-1} f_t| \end{aligned}$$

The criterion at t is the date $t-1$ expectation of time t FEs.

I'm really starting to think that Ryan's code isn't a 100% consistent. But so isn't CEMP either!

Let's go thru timing in words.

$$\text{In } t=2 : \text{ have } \bar{z}_1, s_2, k_1, f_1 \stackrel{\text{CEMP}}{=} z_1 - \underbrace{E_0 z_1}_{\bar{z}_1 + b s_0}$$

$$\text{Form } E_2 z_3 = \bar{z}_1 + b s_2 \quad \begin{cases} \text{if } z_2 \text{ realized} \\ \bar{z}_1 + b s_0 \end{cases}$$

$$\text{Form } \theta_2 = \hat{E}_1 z_2 - E_1 z_2 \quad \begin{cases} \downarrow \text{CEMP} \\ f_2 \text{ realized} \end{cases}$$

$$\text{Form } k_2 = f(\theta_2)$$

$$\text{Update } \bar{z}_2 = \bar{z}_1 + k_2^{-1}(f_1)$$

CEMP - that cannot be right!

Repeat

In $t=2$: have $\bar{z}_1, S_2, k_1, f_1^{\text{COMP}} = z_1 - \underbrace{E_0 z_1}_{\text{realized}}$

Form $E_2 z_3 = \underline{\bar{z}_1 + bS_2}$ if z_2 realized $\bar{z}_1 + bS_0$

Form $\theta_2 = \hat{E}_1 z_2 - E_1 z_2$ $\downarrow f_2^{\text{COMP}}$ $= f_1^{\text{my realized}}$

Form $k_2 = f(\theta_2)$

Update $\bar{z}_2 = \bar{z}_1 + k_2^{-1}(f_1^{\text{my}})$

Issues:

① timing of first \rightarrow COMP cannot be right, so let me use my notation $f_1^{\text{my}} = z_2 - E_1 z_2$ ($=$ Ryan's)

$$f_1^{\text{my}} = z_2 - (\bar{z}_0 + bS_1)$$

"The first made at period 1 of z_2 "

$f_2^{\text{assess}} = z_2 - (\bar{z}_1 + bS_1)$ "if I'd have to redo yesterday's expectation of z_2 given having updated \bar{z} , I'd do this"

② This brings us to our 2nd issue: timing of the PLM

I need to suffer w/ the assessment first b/c I'm using

Ryan's PLM: $E_t z_{t+1} = \bar{z}_{t-1} + bS_t$

But if I instead assume that ppl update \bar{z} at the beginning of the period,

Then: at $t=2$, have $\bar{z}_2, s_2, k_1, f_0^{\text{my}}$

Form $E_2 z_3 = \bar{z}_2 + b s_2$ } z_2 realized, $f_1^{\text{my}} = z_2 - E_1 z_2$

Form $\theta_2 = \hat{E}_1 \theta_2 - E_1 \theta_2$ realized

Form $k_2 = f(\theta_2)$

Update $\bar{z}_3 = \bar{z}_2 + k_2^{-1}(f_1^{\text{my}})$

$$\hookrightarrow z_2 - (\bar{z}_1 + b s_1)$$

Let's write the two approaches for time next to each other

Ryan:

At t , have: $\bar{z}_{t-1}, s_t, k_{t-1}, f_{t-2}^{\text{my}}$

Honey

Form: $E_t z_{t+1} = \bar{z}_{t-1} + b s_t \Rightarrow z_t$

$E_t z_{t+1} = \bar{z}_t + b s_t \Rightarrow z_t$

Form $\theta_t = \hat{E}_{t-1} z_t - E_{t-1} z_t$

Form $k_t = f(\theta_t)$

Update $\bar{z}_t = \bar{z}_{t-1} + k_t^{-1}(f_{t-1}^{\text{my}})$

$$\bar{z}_t - (\bar{z}_{t-1} + b s_{t-1})$$

$\bar{z}_{t+1} = \bar{z}_t + k_t^{-1}(f_{t-1}^{\text{my}})$

$$z_t - (\bar{z}_{t-1} + b s_{t-1})$$

→ clearly the two formulations are equivalent, except that for Ryan's code to work out, you need to use an assessment first $f_{t-1, \text{assess}}$ for updating \bar{z} instead of f_{t-1}^{my} .

= $f_{t, \text{moring}}$ (\Leftarrow before s_t realizes) \Rightarrow Does the assessment fast matter?

Let's see if the PLM matters for the ALM.

Ryan: $E_t \bar{z}_{t+1} = \bar{z}_{t-1} + bS_t$

\uparrow
available at t

$$f_{a,b} = \text{stuff } \bar{z}_{t-1} + \text{stuff } \cdot S_t$$

$$\bar{z}_t = \text{stuff } \bar{z}_{t-1} + \text{stuff } \cdot S_t$$

$$\underline{E_{t-1} \bar{z}_t = \text{stuff } \bar{z}_{t-2} + \text{stuff } \cdot P_{S_{t-1}}}$$

hummm...

$$\hat{E}_{t-1} \bar{z}_t = \underline{\bar{z}_{t-2}} + bS_{t-1}$$

$E_t \bar{z}_{t+1} = \bar{z}_t + bS_t$ Klarby

\uparrow
available at t

$$f_{a,b} = \text{stuff } \bar{z}_t + \text{stuff } \cdot S_t$$

$$\bar{z}_t = \text{stuff } \bar{z}_t + \text{stuff } \cdot S_t$$

$$\underline{E_t \bar{z}_t = \text{stuff } \bar{z}_{t-1} + \text{stuff } P_{S_{t-1}}}$$

$$\hat{E}_{t-1} \bar{z}_t = \underline{\bar{z}_{t-1}} + bS_{t-1}$$

Aaah... either way you get θ_t being a function of the \bar{z} at the same time: the point is it's the one that was available last period (whether you denote that \bar{z}_{t-1} or \bar{z}_{t-2} doesn't matter!)

$$\theta_t = \theta_1(\bar{z}_{t-2}, S_{t-1})$$

$$f_t = \text{ALM} - \text{PLM}$$

$$= \bar{z}_{t+1} - E_t \bar{z}_{t+1}$$

$$= \text{stuff } \bar{z}_{t-1} + \text{stuff } S_t$$

$$- (\bar{z}_{t-1} + bS_t)$$

$$\theta_t = \theta_1(\bar{z}_{t-1}, S_{t-1})$$

$$f_t = \bar{z}_{t+1} - E_t \bar{z}_{t+1}$$

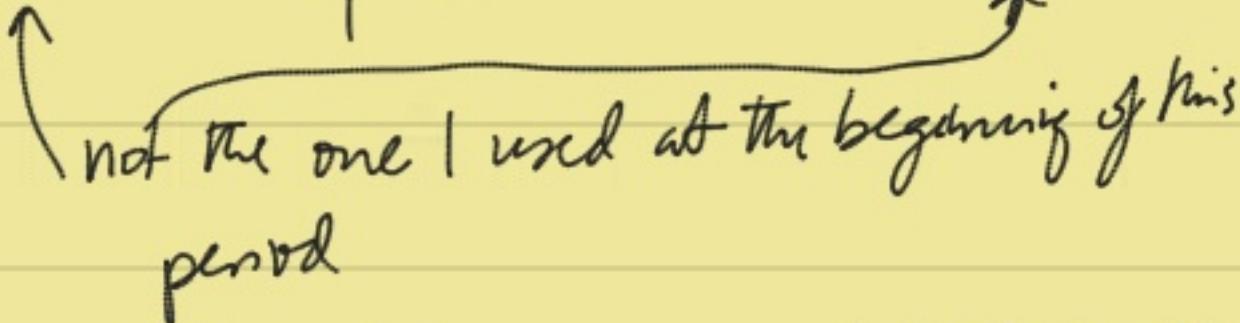
$$= \text{stuff } \bar{z}_t + \text{stuff } S_t$$

$$- (\bar{z}_t + \text{stuff } S_t)$$

→ These work out as well! \Rightarrow but:

in both cases if I have to update using \hat{f}_{t-1} , then

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} (z_t - (\bar{z}_{t-2} + bS_{t-1})) \quad | \quad \bar{z}_{t+1} = \bar{z}_t + k_t^{-1} (z_t - (\bar{z}_{t-1} + bS_{t-1}))$$



not the one I used at the beginning of this period

\Rightarrow so I have the assessment just trouble no matter what!

i.e. both for Ryan's & Honky's PGM

So is the assessment just the sol. to all my hiccups?

It's a new day and today I think

16 Oct 2019

The assessment just is fine: it's yesterday evening's forecast so it does use the most recent info available yesterday to just today's state. (See materials 5c.tex)

\rightarrow So I'm ok w/ using the assessment just to update \bar{u} (or \bar{z}) (Issue #1 is "solved").

But issue #2 is a bigger problem.

\rightarrow tomorrow: let's tackle that and see if Comp's being

can be written like mine!

Before I do that, just one note:

17 Oct 2015

$$\theta_t = \hat{E}_{t-1}(z_t) - E_{t-1}(z_t)$$

We can interpret both expectations as $t-1$ evening!

$$\text{then } \theta_t = \bar{z}_{t-1} + b s_{t-1} - (\text{stuff } \bar{z}_{t-1} + \text{stuff } s_{t-1}) \\ = \mathcal{X}(\bar{z}_{t-1}, s_{t-1})$$

My feeling is that the CEMP formulation (when agents at time $t-1$ just time t stuff) works out only if

$$\begin{aligned} \theta_t &= |\hat{E}_{t-2}\pi_{t-1} - E_{t-2}\bar{\pi}_{t-1}| \\ &= |r\pi_{t-2} + (1-r)\bar{\pi}_{t-1} + \text{stuff} - (r\pi_{t-2} + (1-r)\bar{\pi}_{t-1} + \text{stuff})| \\ &= |\text{stuff} \cdot \bar{\pi}_{t-1}| \end{aligned}$$

that seems to work. But let's try to change the notation so that $\bar{\pi}$ makes more sense in light of standard learning things.

Suppose: PLM: $\pi_t = \bar{\pi}_{t-2} + \rho \gamma_{t-1}$ (ignoring indexing)

This maps perfectly to my PLM.

Then: at time $t-1$

$$\bar{\pi}_t = \bar{\pi}_{t-2} + p\varphi_{t-1}$$

Moving $\bar{\pi}_{t-1}$ realized.

$\rightarrow \bar{\pi}_{t-2}$ was formed in $t-2$ evening.

Now we wanna update to $\bar{\pi}_{t-1}$

$$\bar{\pi}_{t-1} = \bar{\pi}_{t-2} + k_{t-1}^{-1} (\underbrace{\bar{\pi}_{t-2} - \hat{E}_{t-3}\bar{\pi}_{t-2}}_{\text{this is my } FE_{t-2}, \text{ should be } \hat{E}_{t-1}})$$

this is my FE_{t-2} , should be \hat{E}_{t-1}

$\bar{\pi}_{t-1} - \hat{E}_{t-2}\bar{\pi}_{t-1}$ is what I'd have.

\rightarrow even my FE_{t-1} only works out if it's the evening:

$$FE_{t-2}^e = \bar{\pi}_{t-1} - (\bar{\pi}_{t-2} + p\varphi_{t-2})$$

$$k_{t-1} = f(\theta_{t-1}, k_{t-2})$$

$$\theta_{t-1} = |\hat{E}_{t-2}\bar{\pi}_{t-1} - E_{t-2}\bar{\pi}_{t-1}|$$

$$\text{Ok, if I am } |\hat{E}_{t-2}\bar{\pi}_{t-1} - E_{t-2}\bar{\pi}_{t-1}| \text{ Igw}$$

$$\bar{\pi}_{t-2} + p\varphi_{t-2} - (\text{stuff } \bar{\pi}_{t-2} + \text{stuff } \varphi_{t-2}) \checkmark$$

- Ok, so changing the notation for $\bar{\pi}_t$ to $\bar{\pi}_{t-2}$
 - imposing evening parts in $\bar{\pi}$ -update and criterion θ
 - changing the timing of COMP's first $\bar{\pi}$ -update
- \rightarrow Igw (Thg that 1) maps to mine 2) makes sense 3) works out.

Honestly, I think I won't even ask Ryan about this b/c the important thing is that I know what I'm assuming. I'll still send him materials, but I don't think I'll talk about it.

Ryan meeting

17 Oct 2019

- IRFs: present Susanto ones conditional on being anchored or not before the shock hits
- kill γ_x to clarify story telling
- after: add persistence term to TR \rightarrow kills overshooting?

Pit-1

• For Susanto:

- only show mean lines (not all - at most, confidence bands)
- have IRFs conditional on being $\begin{cases} \text{anchored} \\ \text{unanchored} \end{cases}$ before shock
- understand why & how the overshooting happen
- as for θ_t : keep both specifications, CEMP's θ_t and the CUSUM one - right now they're tied b/c the CUSUM is closer to

what firms do and isn't technically harder to implement (as long as we're not estimating - which is not clear to him. But that's where we're headed ... mmm...)

⇒ decide whether they lead to different results

- as far as \hat{E} :

keep this question at the back of your mind and when you get to talk to some of these people, ask them!

→ then you can say, in a job talk or when you've asked this question, gingerly, that "Experts in the field told me that we don't worry about it."

Then you've shown that you've at least thought about it.

Work after materials6

18 Oct 2015

Adding ρi_{t-1} to TR isn't that simple b/c it changes a bunch of things in the LR models.

i_{t-1} is a new state variable, so should it show up

in s_t ? $s_t = \begin{bmatrix} r_t^n \\ i_t \\ u_t \\ i_{t-1} \end{bmatrix}$? \rightarrow that's what I did
for the RE world.

In materials6, eqs (1)-(3) are Duttar's (18), (19) & (22), NKIS, NKPC & TR. Now that I modified the TR, to get the compact notation I need to redo the steps w/ the new Taylor rule.

$$x_t = -\beta(\gamma_\pi \pi_t + \gamma_x x_t + \rho i_{t-1} + \bar{i}_t) + \Sigma(\cdot)$$

$$(1+\beta\gamma_x)x_t = -\beta\gamma_\pi \pi_t - \beta\rho i_{t-1} - \beta\bar{i}_t$$

$$+\sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} + \beta\pi_{T+1} + \beta r_T^n - \beta\beta(\gamma_\pi \pi_{T+1} + \gamma_x x_{T+1} + \rho i_T + \bar{i}_{T+1}))$$

$$(1+\beta\gamma_x)x_t = -\beta\gamma_\pi \pi_t - \beta\rho i_{t-1} - \beta\bar{i}_t$$

$$+\sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta-\beta\beta\gamma_x)x_{T+1} + (b-\beta\beta\gamma_\pi)\pi_{T+1} + \beta r_T^n - \beta\beta\bar{i}_{T+1} - \beta\beta\rho i_T)$$

$$(1+bi_X)x_t = -b\gamma_{\pi} \pi_t - b\rho i_{t-1} - bi_t$$

$$+ \sum_{T=t}^{\infty} \beta^{T-t} \left((1-\rho - b\beta\gamma_X) x_{T+1} + (b - b\beta\gamma_{\pi}) \pi_{T+1} - b\gamma_T^N - b\bar{i}_{T+1} - b\rho i_T \right)$$

$$\Leftrightarrow (1+bi_X)x_t = -b\gamma_{\pi} \pi_t$$

$$+ \sum_{T=t}^{\infty} \beta^{T-t} \left((1-\rho - b\beta\gamma_X) x_{T+1} + (b - b\beta\gamma_{\pi}) \pi_{T+1} - b\gamma_T^N - b\bar{i}_T - b\rho i_{T-1} \right)$$

The i_{t-1} terms

$$-b\rho i_{t-1} - b\rho\beta i_t - b\rho\beta^2 i_{t+1} + \dots$$

$$-b\rho [i_{t-1} + \beta i_t + \beta^2 i_{t+1} + \dots]$$

$$= -b\rho \sum_{T=t}^{\infty} \beta^{T-t} i_{T-1} \quad \text{de!} \quad \text{Now plug } \pi_t$$

$$\Rightarrow (1+bi_X)x_t = -b\gamma_{\pi} \left[Kx_t + \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T) \right] \\ + \sum_{T=t}^{\infty} \beta^{T-t} \left((1-\rho - b\beta\gamma_X) x_{T+1} + (b - b\beta\gamma_{\pi}) \pi_{T+1} - b\gamma_T^N - b\bar{i}_T - b\rho i_{T-1} \right)$$

$$(1+bi_X - b\beta\gamma_{\pi})x_t = \text{stuff}$$

$$x_t = \frac{-b\gamma_{\pi}}{w} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\alpha\beta x_{T+1} + (1-\alpha)\beta \pi_{T+1} + u_T)$$

$$+ \frac{1}{w} \sum_{T=t}^{\infty} \beta^{T-t} \left((1-\rho - b\beta\gamma_X) x_{T+1} + (b - b\beta\gamma_{\pi}) \pi_{T+1} - b\gamma_T^N - b\bar{i}_T - b\rho i_{T-1} \right)$$

$$\begin{aligned}\pi_t &= \left(1 - \frac{K\beta Y_t}{W}\right) \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (K\alpha\beta X_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \\ &\quad + \frac{K}{W} \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta - 2\beta Y_t)X_{T+1} + (b - 2\beta Y_t)\pi_{T+1} + b\bar{r}_T - 2\bar{r}_{t-1} - 2\beta i_{T-1})\end{aligned}$$

Ah yeah, so all I need to do is to expand the S -vector
 $\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = f(f_a, f_b) + f_f \underbrace{\text{which is different}}_{\text{but only to the extent that it has}} \\ \text{a } 4^m \text{ row for } i_{T-1}, \text{ and } A_S \text{ is different}}$

$$f_f = E_t \sum_{T=t}^{\infty} \beta^{T-t} S_T = (I_t - \beta P)^{-1} S_t \quad W/ \quad S_t = \begin{bmatrix} \bar{r}_t \\ i_t \\ u_t \\ i_{t-1} \end{bmatrix}$$

↑
Does P include ρ ? I think so!

$$P = \begin{bmatrix} p_{rr} & 0 \\ 0 & p_{ii} \\ 0 & p_{uu} \\ 0 & p_{ii} \end{bmatrix}$$

And A_S^{LR} looks as follows:

$$\text{for } x_t \Rightarrow \frac{-bY_t}{W} [1 - \alpha\beta P]^{-1} [0 \ 0 \ 1 \ 0] S_t + \frac{b}{W} [1 - \beta P]^{-1} [1 \ -1 \ 0 \ -\bar{r}] S_t$$

$$\text{for } \pi_t \Rightarrow \left(1 - \frac{K\beta Y_t}{W}\right) [1 - \alpha\beta P]^{-1} [0 \ 0 \ 1 \ 0] S_t + \frac{K\beta}{W} [1 - \beta P]^{-1} [1 \ -1 \ 0 \ -\bar{r}] S_t$$

$$\text{and for } i_t \Rightarrow \underbrace{\gamma_X \text{ stuff } x + \gamma_\pi \text{ stuff } \pi + \rho i_{t-1} + \bar{i}_t}_{-1(1-)} + [0 \ 1 \ 0 \ \bar{r}] S_t$$

Yeah so I only need to adjust A_S^{LR} & A_S^{RE} . 19 Oct 2019

It's now 3×4 instead of 3×3 .

I can summarize the previous as: (new in blue)

$$g_{X_S} = \left(\frac{-b\pi_1}{w} [1-\alpha\beta P]^{-1} [0 \ 0 \ 1 \ 0] - \frac{b}{w} [1-\beta P]^{-1} [-1 \ 1 \ 0 \ P] \right)$$

$$g_{\pi_S} = \left(\left(1 - \frac{kb\pi_1}{w} \right) [1-\alpha\beta P]^{-1} [0 \ 0 \ 1 \ 0] - \frac{k^2}{w} [1-\beta P]^{-1} [-1 \ 1 \ 0 \ P] \right)$$

$$\text{and for } i_t \Rightarrow \gamma_X g_{X_S} + \gamma_{\pi} g_{\pi_S} + [0 \ 1 \ 0 \ P]$$

Of course it doesn't work, even if I set $P=0$ I don't get what I had before, not even for the RE model.

So:

$$x_t = h_x \cdot x_{t-1} + \eta \cdot \epsilon_t \quad \text{Is this right? I don't think so...}$$

$$y_t = g_x \cdot x_t$$

$$\begin{bmatrix} r_{t+1} \\ i_t \\ u_t \\ i_{t-1} \end{bmatrix} = \begin{bmatrix} p_r & & 0 \\ p_i & & \\ 0 & p_u & \\ \end{bmatrix} \begin{bmatrix} r_{t+1} \\ i_{t+1} \\ u_{t+1} \\ i_{t-2} \end{bmatrix} + \begin{bmatrix} b_r & & 0 \\ b_i & & \\ 0 & b_u & \\ \end{bmatrix} \begin{bmatrix} \epsilon_r \\ \epsilon_i \\ \epsilon_u \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \left[\begin{array}{c} \\ \\ 3 \times 4 \end{array} \right] \begin{bmatrix} r_{t+1} \\ i_t \\ u_t \\ i_{t-1} \end{bmatrix} \quad \text{This stuff ain't right!}$$

Everything looks correct. So I'll take a big step back:

go back to materials5.m and

✓ 1) implement the learning & IRFs w/ a single code,
making sure you keep obtaining the same thing

2) change RE to account for the smoothing in it
making sure you obtain the same thing as before

Ok, did that, retested.

→ Let's think this thru one more time:

$$\begin{bmatrix} f_t \\ \vdots \\ u_t \\ i_{t-1} \end{bmatrix} = \begin{bmatrix} h_x \\ \vdots \\ ir_i i_n p \end{bmatrix} \begin{bmatrix} r_{t-1} \\ \vdots \\ u_{t-1} \\ i_{t-2} \end{bmatrix} + \begin{bmatrix} \overset{n}{\underset{0}{\dots}} \\ 0000 \end{bmatrix} \begin{bmatrix} \epsilon_r \\ \epsilon_i \\ \epsilon_u \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} g_x \\ \vdots \\ ir_i i_n p \end{bmatrix} \begin{bmatrix} r_t \\ \vdots \\ u_t \\ i_{t-1} \end{bmatrix}$$

→ it makes sense that the last row of $h_x = \text{last of } g_x$!

→ and that does look like this is where roads part between P and h_x b/c i_{t-1} is an endogenous state!

→ so I don't think I need to redefine P or Σ ,
but in f_a & f_b , agents will use h_x instead of P
to test statistics! Well, Σ maybe...

↳ will need to correct `materialsb.tex`!

↳ still need to think it over - this isn't done yet!

But maybe the 1st step is to replace P w/ h_x in
 f_{ab} , f_k (both CEMP & CUSUM) and let n adjust
and make sure we still get the same things.

To be sure I'd do new functions for everything!

Analyzing IRFs (all of the above done & works!) 20 Oct 2015

1.) CUSUM criterion doesn't respond a whole lot to p , to
the point of me suspecting that something isn't quite right
there.

2) In general, when $\uparrow p$, CEMP criterion gives more anchoring
→ makes sense b/c then it isn't moving around as
much, as so they aren't making as large errors.

3) Same for $\Psi_{it}\uparrow$, except after a while it reverses

Focus for a sec on constant gain learning:

- 1) overshooting ↑ when $\gamma_{\pi} \uparrow$
- 2) $\rho \uparrow$ doesn't really do much to overshooting
- 3) $\rho \uparrow$ or $\gamma_{\pi} \uparrow \rightarrow$ anchoring \rightarrow AM behaves more like decreasing gain learning

Both cgain & dgain have overshooting if γ_{π} high enough
but cgain has a more "sign-switching convergence"-
behavior in the sense that it's initially further from
RE and also moving further away, but then it
overtakes dgain learning.

→ It's as if the overshooting was happening b/c the
shock interrupt an ongoing learning process
⇒ That's why there are these reversals in
 π b/c cgain learners change their mind about
where π is headed!

$$b = g_x \cdot h_x \quad y \quad ny \cdot nx$$

$ny \cdot nx \quad nx \cdot nx$

3×4

20 Oct 2019

$$M_1 \text{ is } 3 \times 3 \quad M_2 \text{ is } 3 \times 4 \Rightarrow M_2 \cdot s = 3 \times 1$$

$3 \times 4 \quad 4 \times 1$

Still trying to understand IRFs:

- $p \uparrow$ just means that RE starts overshooting too
the learning models also overshoot more, but the change
is way less pronounced than for RE.

It seems like $p > 0$ in RE is a substitute for learning!

RE responses closest to learning when $p \approx 0.3$ (save for monpol shock but I'm ignoring that a bit b/c it's persistent)

But what is quite diff is the int rate, even under $p=0.3$

→ it's more quick to react

it switches sign

→ why is i doing this? It's not the case that the impact effect of shocks would be greater under learning, in fact,

the impact is always greater in RE on everything
→ in RE, a bigger X is cancelled by a bigger i
whereas in learning a smaller X is reward by a
smaller i

So it seems like the same γ_π has a bigger effect in
a learning model, especially if you have constant gain
learning → you get overshooting b/c of that
and so the CB reverses course to correct it

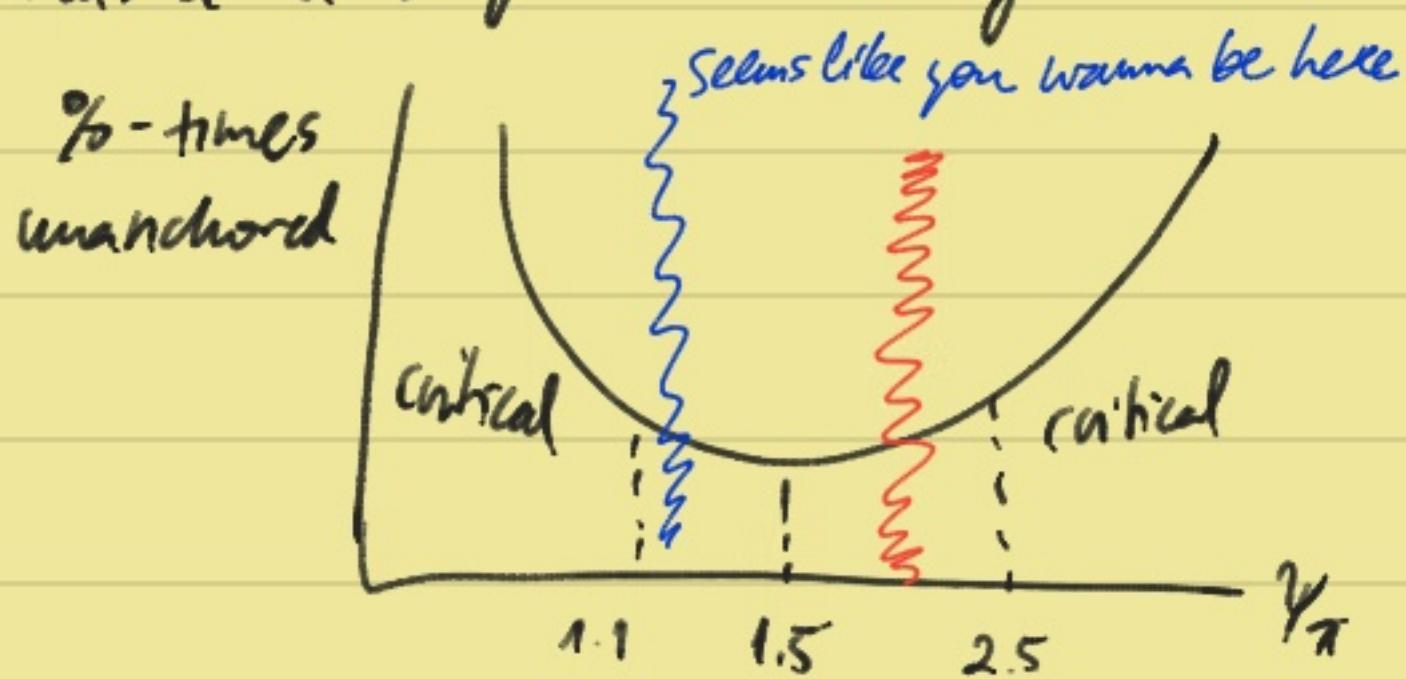
Why does the same γ_π have a bigger effect under
learning, especially under gain?

→ b/c agents use it strongly to infer the π^*
⇒ this is the same reason why a too high γ_π leads
to constant gains: it leads to "excess volatility"
which suggests to agents that they need to monitor
the site.

What doesn't make sense is that $\gamma_\pi = 1.1$ which seems
to do the optimal cancelling, gives you gains!

Maybe here's what's going on:

- impact effects are smaller in learning b/c E are slow-moving (states feed from the system slower)
- γ_{π} has a bigger effect though somehow (...? :5)
(maybe jumps feed faster than the system)
- γ_{π} has a U-shaped anchoring curve



→ it seems like you're trading off excess volatility and anchoring

Ver: you could $\uparrow \gamma_{\pi}$ to get more anchoring, but then you get more overshooting

Here that badoff is gone b/c $\gamma_{\pi} \uparrow$ means less anchoring and more overshooting.

⇒ It seems like there's some "interior And" of anchoring that's

optimal in that you both learn fast enough (gain)
but it doesn't lead to excess volatility.

But why does learning respond

- slower to states $\rightarrow b/c E = f(\text{states})$

- faster to jumps? $\rightarrow b/c E \text{ doesn't absorb } m's$

Result 1. Under learning, shocks to states propagate slower through the system b/c expectations are functions of states and thus slow-moving.

Cor. 1. Impact effects of structural shocks are smaller than under RE.

Result 2. Under learning, changes to jump variables (such as $\uparrow i_t$ when γ_{π} is high) propagate faster through the system b/c expectations take time to respond & absorb them (this is like prices when money isn't neutral)

Cor. 2. The same γ_{π} leads to bigger responses (overshooting).

- Result 1 implies that mon. pol. should anchor b/c you can dampen responses.
- Result 2 implies that mon. pol. shouldn't anchor too much b/c the more anchored, the more overshooting you get. ~~X~~ That's not true.
Monetary policy trades off damping works on impact versus overshooting dynamics.
This tradeoff might disappear if you have $\alpha \neq 0$ that can raise anchoring w/o γ_{π} ($\rho \uparrow$ or $\alpha \uparrow$).

Result 3. The $\downarrow \alpha$ ($\alpha \uparrow$), the more is π the margin of adjustment. This is not just b/c in this case, prices are more flexible, but also b/c flex prices lead to more overshooting as α moves more.

I'm a little sceptical about the overshooting 22 Oct 2019
result's interpretation b/c if it's the case that
we're overshooting b/c $E(\cdot)$ is slow to adapt,
then shouldn't D_{gain} have the biggest overshoot?
But currently, clearly c_{gain} does!

(c_{gain} learning updates $E(\cdot)$ by much more each period
→ this is why especially long after the shock, d_{gain}
converges to RE much slower than c_{gain} does.
⇒ so the overshooting has to have sthg to do w/
learning happening.

↳ from the perspective of overshooting, you want to
be anchored b/c that gets you closer to d_{gain} .
But initially (early after impulse), c_{gain} & d_{gain}
are close: so why does c_{gain} lead to more
overshooting then? B/c there are avg IRFs, so c_{gain}
& d_{gain} aren't actually close!

Ryan

24 Oct 2015

- Error in RE $g\chi_{3,4} \neq p$

E.g. if we only have i_{t-1} as a state:

$$i_t = \phi \pi_t + p i_{t-1}$$

$$\pi_t = g i_{t-1}$$

$$\Rightarrow i_t = (\phi g + p) i_{t-1} \quad \text{The jump picks up effects of the state } \pi_t \text{ via a) direct b) indirect.}$$

- Revisit RE effects on impact of net rate shock.

- IRF of expectations & FE \rightarrow plot 'em!

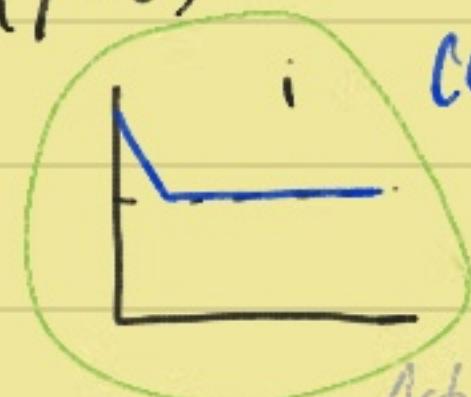
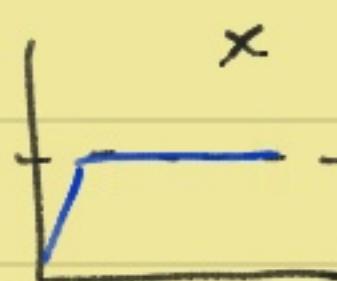
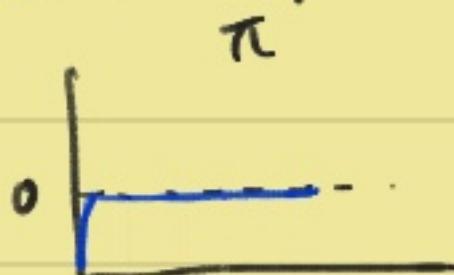
- Check MP shock. fall in i can't happen w/o capital.

work after

27 Oct 2019

Let's work out what these shocks should be doing in the RE NK model:

- ① mon. pol. shock ($\bar{i} \uparrow$) ($\rho = 0$)

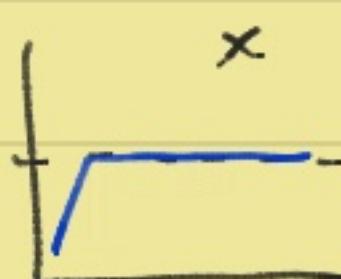
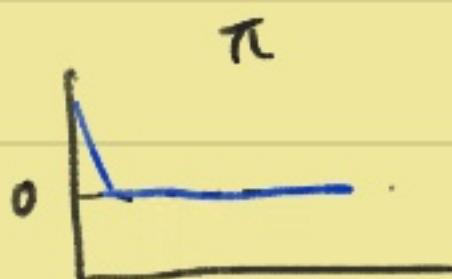


(by def b/c $\bar{i} \uparrow$, so $i \uparrow$)

I get the opposite.

Actually, I got this if i is iid.

- ② cost-push shock ($u \uparrow$) ✓ I get this.



I guess b/c $u \neq 0$

This is why Ryan called it a stagflation shock.

- ③ nat. rate shock (a news shock?) ($r^* \uparrow$)



✓
What is what I get

It's a stand-in for all kinds of demand shocks

However, all shocks are fine when AR(1) except monpol.

The net rate of interest shock

There are two competing interpretations of this shock

- 1) $r^n \uparrow$ as a tech shock $\hat{z} \uparrow$

In Peter Ireland's notation, $r^n = (1-\omega)(1-p_a)\hat{a}_t$

where \hat{a}_t is the tech process

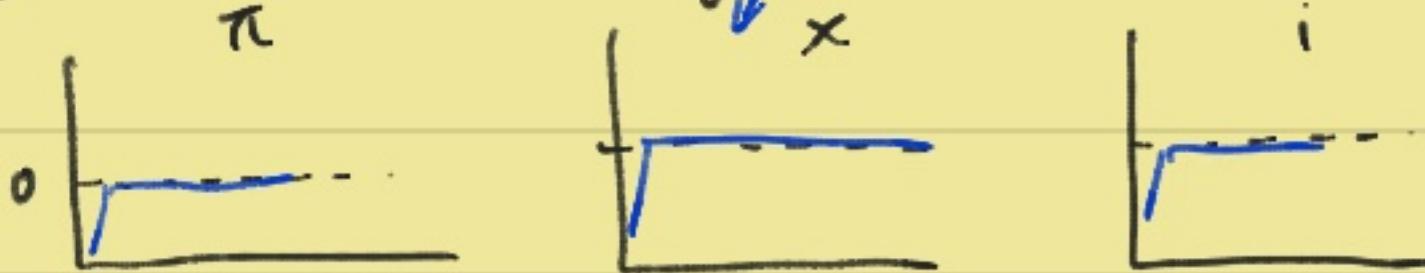
- 2) $r^n \uparrow$ as a news shock

since $r^n = \beta(E(Y_{t+1}^f) - Y_t^f)$

(clearly, w/o perfect foresight $E(Y_{t+1}^n) = Y_t^n$ and

r^n is a tech shock, but not today but tomorrow!)

- ④ Tech shocks $\hat{z} \uparrow$ today b/c $Y_t^f \uparrow$



- ⑤ Tech shock tomorrow (news shock)



This is what I get

I think a net rate shock is best thought of as a news shock, yielding these responses.

Walsh p. 251 (Mac 272) writes

$$x_t = E_t x_{t+1} - \tilde{\beta}^{-1} (i_t^* - E_t \pi_{t+1}) + \xi_t \quad (\text{NKIS})$$

where $\tilde{\beta}^{-1} = \beta^{-1} (c/\gamma)^{ss}$ (used to extend $y=c$ w/ γ)

then we can interpret the shock in NKIS, ξ_t , as

- 1.) a gov-spnd shock
- 2.) a preference/kick shock (\rightarrow hits $u'(c)$)
- 3.) expected change in y_t

\Rightarrow are all these just stand-ins for a demand shock?

I think so!

Shocks in NKIS \rightarrow demand shocks (G, taste, news, nat. etc.)

Shock in NKPC \rightarrow supply shock (add-prsn)

So why do we get this bubble w/ the monpol shock (only) when it is persistent?

In fact, the impact of the monpol shock is correct up

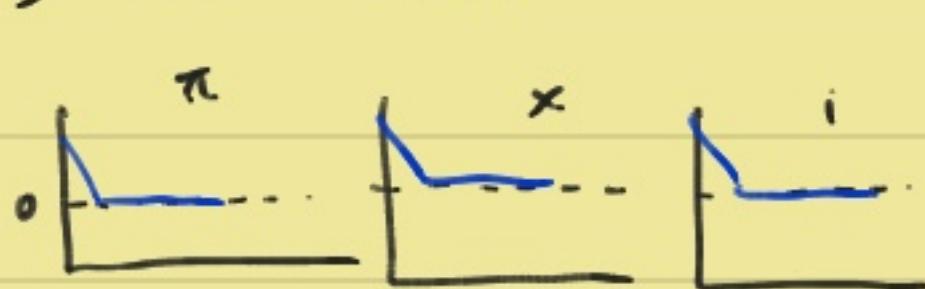
to and including $p_i = 0.6$. If $p_i > 0.6$, then it's an impact.

$$i_t = \gamma_\pi \pi_t + \beta_1 \bar{i}_t$$

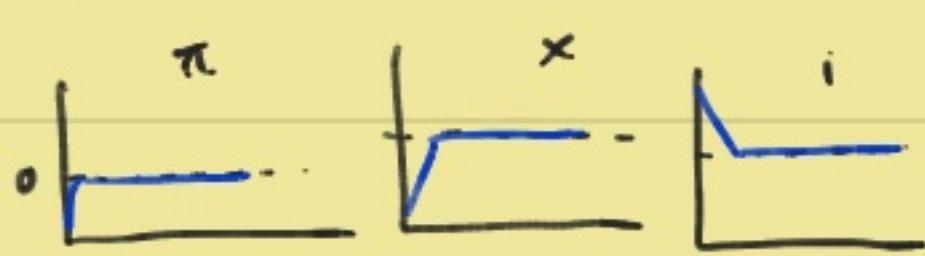
28 Oct 2019

Why do I get the wrong sign on impact if the shock is persistent but only for the mon pol shock?

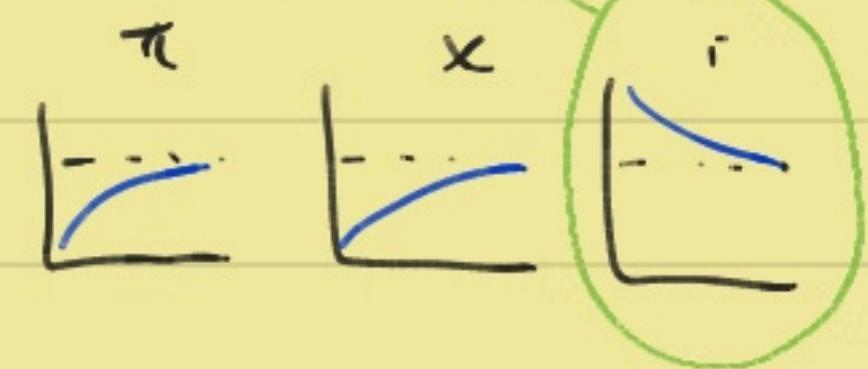
1) nat rate shock



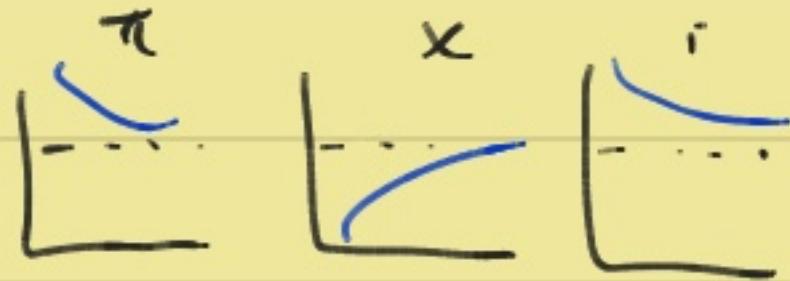
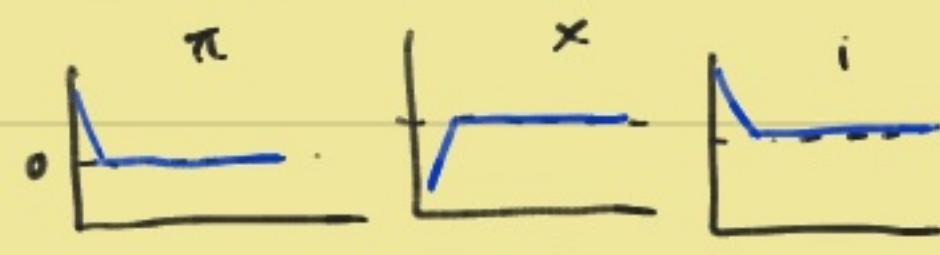
2) mon pol shock



I got all except this:



3) cost push



I obtain the same IRFs if I consider w/ or w/o int rate smoothing, so it's not that

rate smoothing, so it's not that

I can reverse the result if $\alpha \uparrow \downarrow k$

Initially, I had $\alpha = 0.5$ ($\rightarrow \kappa = 0.51$)
 but you need $\alpha \geq 0.87$ ($\rightarrow \kappa \leq 0.022$) to
 get $i \uparrow$ on impact!

Let me reconsider IRFs in the linear (RE) world.

Ryan writes (Lect 3, p. 9)

$$x_t = h_x \cdot x_{t-1} + \eta \varepsilon_t$$

$$= \dots$$

$$x_t = \sum_{\tau=0}^{\infty} h_x^\tau \eta \varepsilon_{t-\tau}$$

So on impact that is on jumps

$$x_t = \eta \varepsilon_t \quad (\text{impact})$$

$$y_t = g_x \cdot \eta \varepsilon_t$$

$$x_{t+1} = h_x \cdot \eta \varepsilon_t \quad (\text{1-period out})$$

$$y_{t+1} = g_x \cdot h_x \cdot \eta \varepsilon_t$$

\vdots

So since my h_x only has >20 elements, the problem seems to be g_x

$g_{x,2} < 0$, so the response of i to $\bar{i} \uparrow$ is \downarrow .

(if I set $\alpha \geq 0.87$, $g_{x,2}$ turns >0)

$$Y_t = \underbrace{- [f_y \ f_{xp}]^{-1} [f_{yp} \ f_x]}_{\text{top left block}} E_t Y_{t+1}$$

$$- [f_y \ f_{xp}]^{-1} [f_{yp} \ f_x] X_t$$

top right block

A_p^{RE} = top left block, A_s^{RE} = top right block.

$$\therefore z_t = A_p^{RE} E_t z_{t+1} + A_s^{RE} s_t$$

Let bigguy := $- [f_y \ f_{xp}]^{-1} [f_{yp} \ f_x]$
 $(nx+ny, nx+ny)$

then $A_p^{RE} = \text{bigguy} (1:ny, 1:ny) \quad ny \times ny$

$$A_s^{RE} = \text{bigguy} (1:ny, ny+1:end) \quad ny \times nx$$

Yeah, these guys are right!

this bigguy is what Ryan in Leet 2 called B

so then he says

$$\begin{bmatrix} Y_t \\ X_{t+1} \end{bmatrix} = B \begin{bmatrix} Y_{t+1} \\ X_t \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} Y_{t+1} \\ X_t \end{bmatrix} \quad \begin{array}{l} (\text{where } A_p = B_{11} \\ A_s = B_{12}) \end{array}$$

Looking at Basin Sum 2, NK w/ capital 29 Oct 2019

- For some reason we have a fix cost which means we have IRS I think.
- We also have 2 assets: bonds (B) & capital (K)
Bonds pay $1+r_t$. Capital pays $1+R_t - \delta$ (net of depreciation)

Arbitrage between the two assets requires:

$$\text{in st st: } R = r + \delta$$

$$\text{outside st st: } \hat{r}_t = \frac{r+\delta}{\hat{R}_t} \hat{R}_t$$

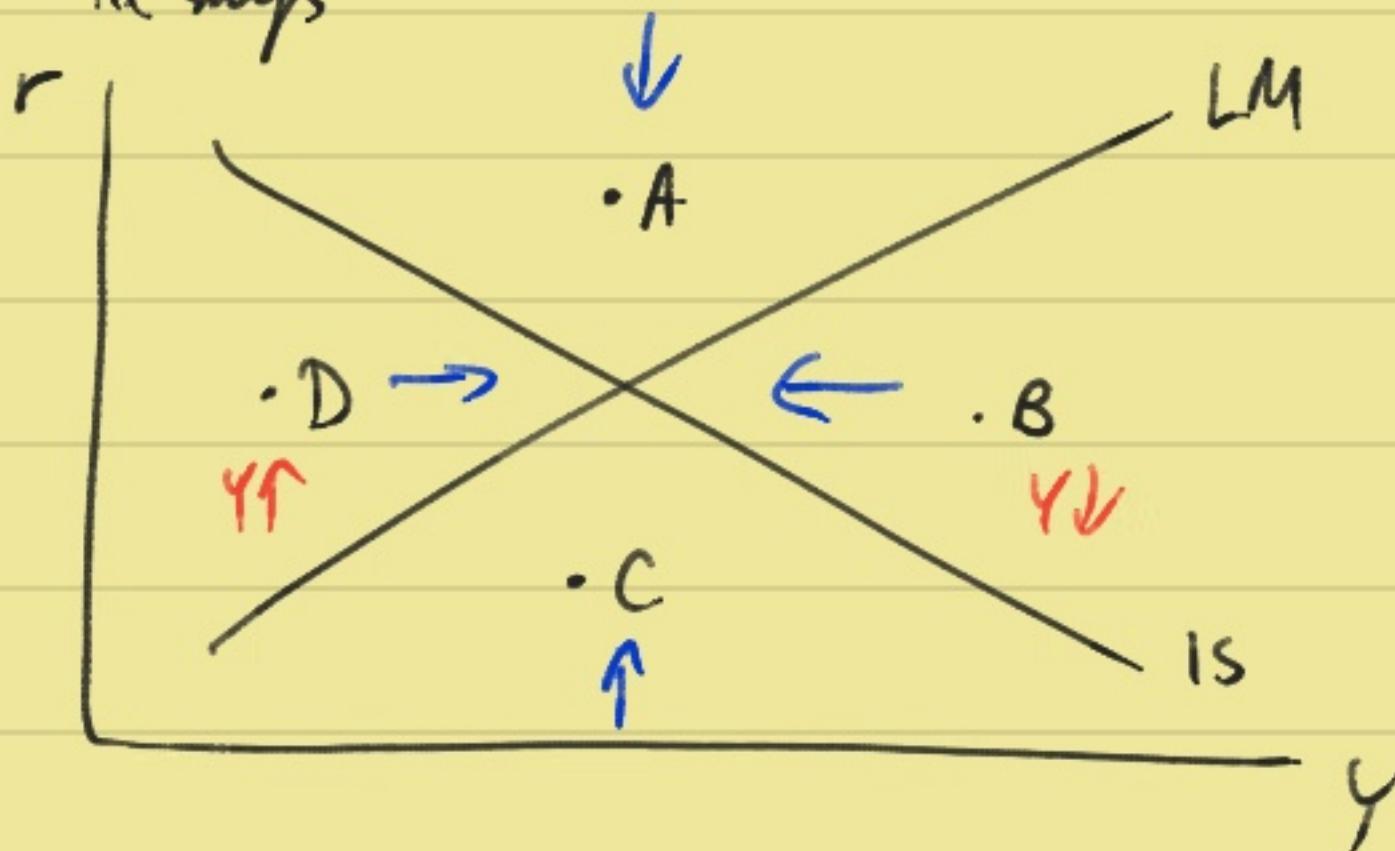
We switch out \hat{R}_t from factor demands to obtain

$$\hat{r}_C = \frac{r+\delta}{\hat{r}} (w_t^* + \pi_t^* - k_t^*) \quad (\text{NRR curve})$$

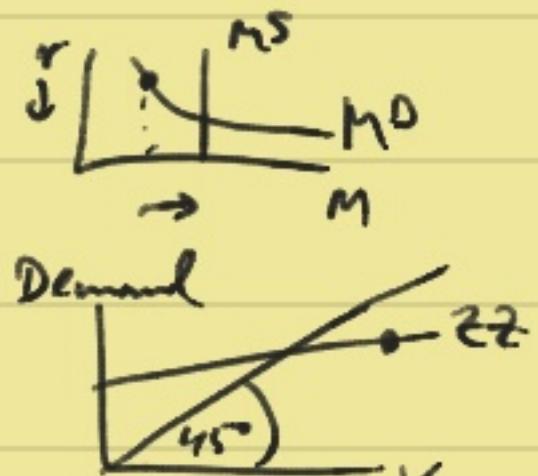
instead of the int. rate being pinned down by the NKIS (one EE, here it's two)

I think the point is that this NRR is the "new IS-curve"
which is why Basin focuses on the intersection of
IS-LM:

He says



A: excess supply of money & goods



$r \downarrow$ from money market

$r \downarrow$ from loanable funds market

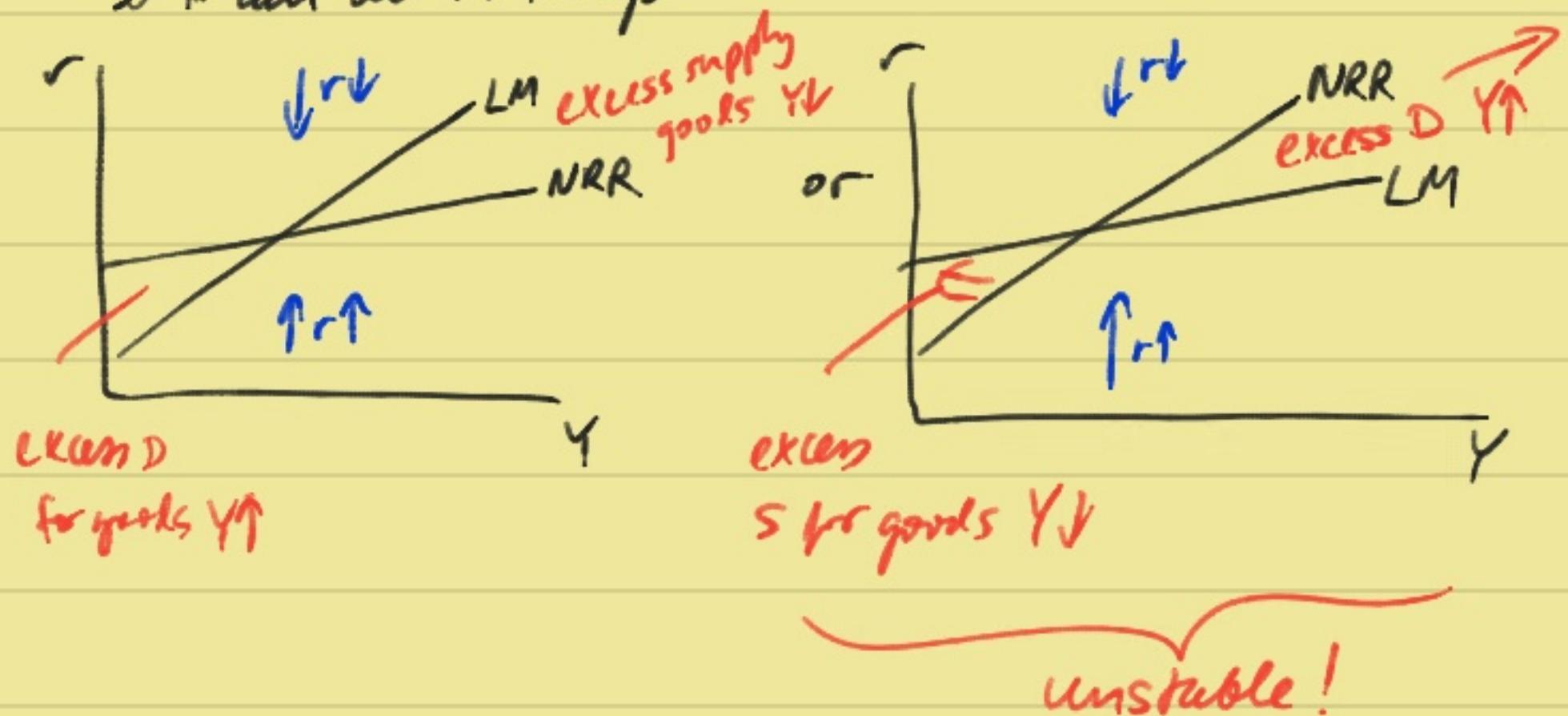
B: excess supply of goods $r \downarrow$ } Y (supply?) \downarrow
excess demand of money $r \uparrow$ } to equilibrate out

C: excess demand of both $r \uparrow$

D: excess supply of money $r \downarrow$ } Y (Supply?) \uparrow
excess demand of goods $r \uparrow$ } to equilibrate out

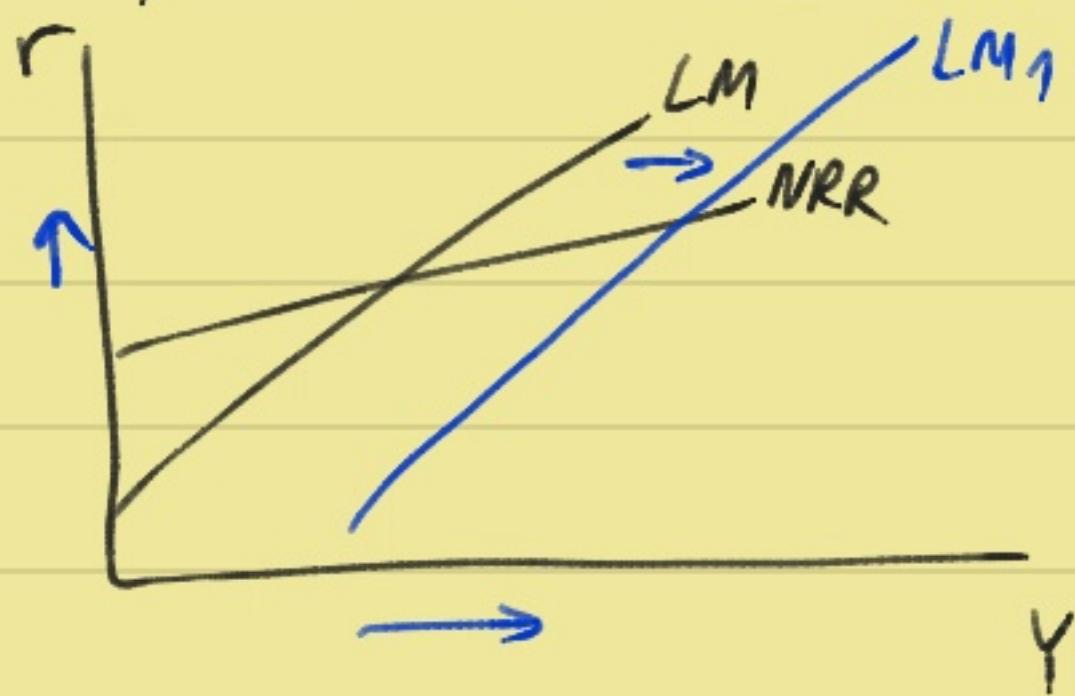
But unlike IS, NRR is upward-sloping.

So it can do 2 things:



the channel seems to be SR-supply adjusting!

Ok, and this means: $M \uparrow \rightarrow LM \rightarrow R$



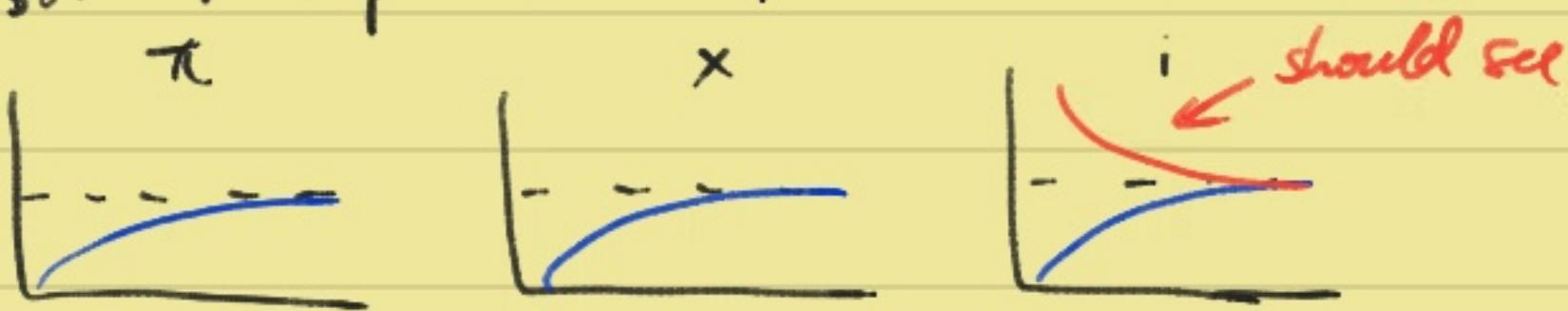
$\Rightarrow Y \uparrow$ and $r \uparrow$ (!) Expansionary monpol causes $r \uparrow$

Why is this happening?

b/c $M \uparrow$ raises $E_t \pi_{t+1}$, so money will be taxed by future inflation \Rightarrow the asset K becomes more valuable relative to money, but no arbitrage, so must bonds! $(R_B \uparrow)$

Ok, so let's turn back to our NK model w/o cap. inv.

so: Mon. pol. shock $\bar{i} \uparrow$



Can it be that only the nom int rate \downarrow , the real \uparrow ?

Nah...

So back to thinking: why is $g_x \chi_{3,2} < 0$?

In particular, why does $g_x \chi_{3,2}$ depend on whether the shock is iid or persistent?

$g_x \chi_{3,2} > 0$ if $\rho_i = 0$

$g_x \chi_{3,2} < 0$ if $\rho_i > 0$

I mean, it's clear that h_x changes when ρ_i changes
 In the plain (no-int-rate-smoothing) NK, there
 are only exog states and so

$$h_x = P = \begin{pmatrix} p_r & 0 & 0 \\ 0 & p_i & 0 \\ 0 & 0 & p_u \end{pmatrix}$$

If all shocks iid, then $h_x = \mathbf{0}$, which is why IRFs
 are all zeros after the impact ($= g_x \cdot \eta \varepsilon_+$)

So what is g_x ?

$$Y_t = g_x X_t, \text{ in this case}$$

$$\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} & & & <0 \\ & g_{12} & & \\ & g_{22} & & \\ & & & \\ & g_{31} & g_{32} & g_{33} \end{bmatrix} \begin{bmatrix} r^n \\ -\tau \\ \tau \\ u_t \end{bmatrix}$$

All other $g_{x,ij} > 0$. And the point is that $g_{x,ij}$ don't
 ever change sign, except for $g_{x,32}$, which

$g_{x,32}$	$\rho_i = 0$	$\rho_i = 0.5$	$\rho_i = 0.6$	$\rho_i = 0.7$	$\rho_i = 0.9$
	0.72	0.25	0.03	-0.28	-1.32

Setting $ES = \beta = 1$ (instead of my 0.5) lowers these numbers?

Ok so what about the Frisch? For Bern, the Frisch

is denoted either ϵ_{HW} (e.g. Lect 11 slide 9 Mac)

or η (sum 2, p. 48)

and it's set to 1, 2 (micro) or 4 (macro).

→ is it missing from my α ?

$$\alpha = \frac{(\eta + \beta)(1 - \omega)(1 - \beta\omega)}{\beta} \quad \beta = \text{IES}, \eta = \text{Frisch}$$

$$\text{In COMP, } \alpha = \frac{\omega}{(1 - \omega)(1 - \beta\omega)} \quad (\alpha = \omega)$$

b/c there ain't no demand side!

$$\text{In Preston, } \alpha = \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \left(\frac{\omega + \beta^{-1}}{1 + \omega\theta} \right)$$

where $\omega > 0$ is the "elasticity of a firm's real marginal function wrt its own output, $y_t(i)$." (?)

And θ seems to be the elast. of demand.

4 questions I think so!

- 1) Is Preston really using β as Basu $\frac{1}{\beta}$? (if $\beta=1$ don't matter)
- 2) Why does η (Frisch) show up in Basu, not in Preston?
- 3) Why does θ (price elast. of demand) show up in Preston?
- 4) What's ω in Preston and why is it there?

King & Rebelo 1999, Table 2 Calibration

β	η	θ	α	γ	δ	σ	ρ	β_E
1	0.984	3.48	1	1.004	0.667	0.025	0.975	0.0072
\uparrow IES = CREA (sdf)	\uparrow β	\uparrow		\uparrow grt rate	\uparrow	\uparrow	\uparrow	\uparrow tech shock

↑ Fresh of tech

↑ k-share depreciation

disability of labor?

$$U(c_t, l_t) = \log(c_t) + \theta \log(l_t) \Rightarrow (\beta=1, \eta=1)$$

Baum says: $\beta+\eta = GE$ elasticity of $\frac{w}{P}$ wrt ΔH

In this model, real wage = real MC

$\Rightarrow \beta+\eta = \text{elasticity of real MC wrt output}$

Susanto also says somewhere that we need $\kappa = 0.1$
or even $\kappa \approx 0.01$ to get reasonable persistence.

(Lect 22, slide 3 Mac)

It also says that w/ $\beta \in [1, 2]$, $\eta \in [1, 4]$, $\beta = 0.95$, $w = \text{avg duration} = 6$, you

don't get a low enough k

6 months =

2 quarters

Wiki says: if $h = \text{Prob}(\text{can reset}) = 1 - \alpha$ for Preston.

Then the prob to last i periods is

$$\Pr[i] = (1-h)^{i-1} h \quad (\text{geometric distb})$$

Expected duration is h^{-1}

then if expected duration is 2 quarters, then

$$\text{expected duration} = (1-\alpha)^{-1} = \frac{1}{1-\alpha}$$

$$2 = \frac{1}{1-\alpha} \Rightarrow 1-\alpha = \frac{1}{2} \rightarrow \alpha = 1 - \frac{1}{2} = 0.5$$

If it was 6 months (if the model was monthly)

$$6 = \frac{1}{1-\alpha} \Rightarrow 1-\alpha = \frac{1}{6} \rightarrow \alpha = 1 - \frac{1}{6} = \frac{5}{6}$$

New set of default params: $\alpha = 0.5$ (= as before)

$\beta = 1$ (instead of 0.5, although that would converge to Baum's 2)

Frisch = 1.

But even w/o worrying about questions (2)-(1),
we have that the correctly specified α makes
the problem worse!

Let's see Galí: (Monetary Policy & Theory) $y_t(i) = A_t N_t(i)^{1-\alpha}$

$$K = \frac{(1-\theta)(1-\beta\varphi)}{\theta} \left(\underbrace{\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}}_{=: \Theta} \right) \left(\beta + \frac{\varphi + \alpha}{1-\alpha} \right)$$

Collard:

$$\left(\beta + \frac{\varphi + 1 - \alpha}{\alpha} \right)$$

$$\frac{2\alpha + \nu + 1 - \alpha}{\alpha}$$

$= \lambda$

θ = labor part of shock

$\beta = 1ES$

Collard's λ is
only right if $\beta = 1$.

ε = price elast. of demand

$\varphi = \text{Friction}$

$1-\alpha$ = labor share (in Baum, $\alpha=0$, w) that you get PF: $y_t = z_t b_t$

$$K = \frac{(1-\theta)(1-\beta\varphi)}{\theta} \cdot (1) \cdot (\beta + \varphi) = \text{Baum!}$$

Check Collard (Montiel 4, p. 16 Mac)

PF: $y_t = A_t h_t^\alpha$

$$\text{slope} = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p} \left(\underbrace{\frac{\alpha}{\alpha + (1-\alpha)\theta}}_{=: \Theta} \right) \left(\underbrace{\frac{1+\nu}{\alpha}}_{=: \chi} \right) \overset{\beta \text{ (log } u)}{=} \chi$$

In this notation, Baum has $\alpha=1 \Rightarrow \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p} (1+\nu)$

So let's use Collard's notes to interpret the parts of the slope of the NKPC:

$$\text{slope(NKPC)} = \frac{(1-\omega)(1-\beta\omega)}{\omega} \cdot \frac{\alpha}{\alpha + (1-\alpha)\theta} \cdot \frac{3 - \alpha_2 + \gamma + \alpha}{\alpha}$$

$$= \kappa \cdot \Theta \cdot \chi$$

ω = Calvo prob of stuck , β = sly

α = labor share (or: $Y = A_t \cdot H_t^\alpha$)

θ = price elasticity of demand for varieties

γ = IES/CRRA of utility function

α_2 = inverse of labor supply elasticity

Collard's interpretation:

κ = dynamics of price setting (frequency of adjustment & sly)

Θ = relates firm's MC to the average MC

χ = relates avg MC to the output gap

More interpretations..

Θ • $\alpha \uparrow, \theta \uparrow$ α measures returns to scale. Since

there's no capital in this model $Y = A \cdot N^\alpha$

is a DRS PF. The higher α , the higher returns to scale. In Baum's case, $\alpha=1$, the PF is CRS.

$$\frac{\partial \theta}{\partial \alpha} = \frac{\partial}{\partial \alpha} \frac{\alpha}{\alpha + (1-\alpha)\theta} = \frac{1}{\alpha + (1-\alpha)\theta} \cdot (-1) \frac{1}{(\alpha + (1-\alpha)\theta)^2} (1-\theta)$$

$$\alpha \geq 0, \theta > 1 \text{ since } \mu^* = \frac{\theta}{\theta-1} > 0$$

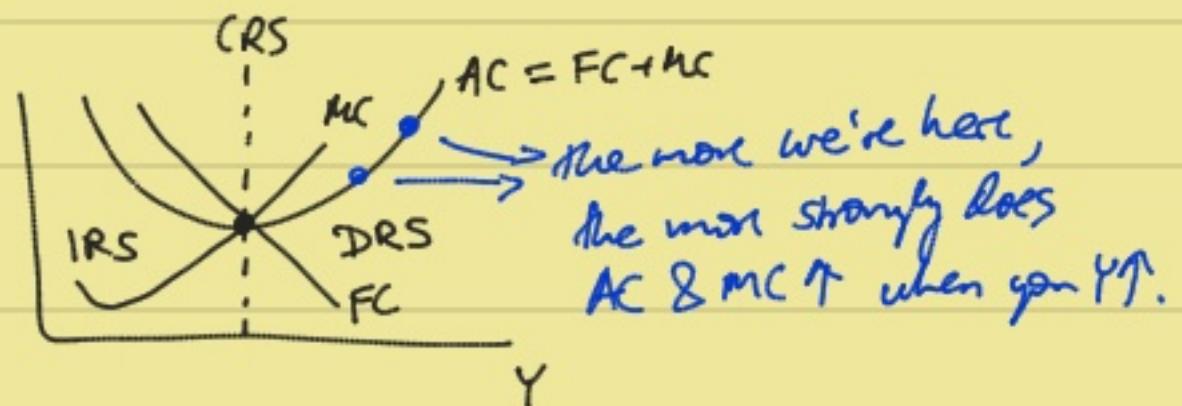
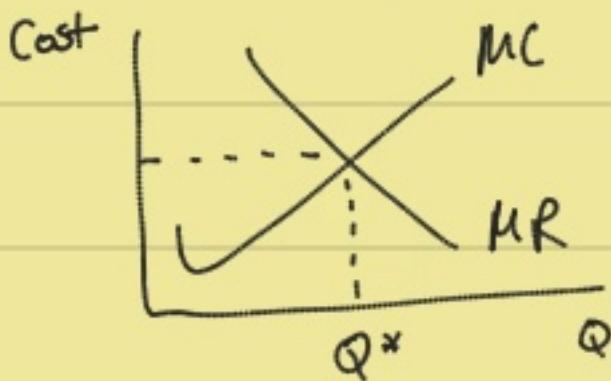
↳ Yes! Baum Sum 1 p. 81 Mac & p. 84.

$$\frac{-\overbrace{(1-\theta)}^{>0}}{\underbrace{(\alpha + (1-\alpha)\theta)^3}_{>0}} > 0 \Rightarrow \theta \uparrow \text{ w/ } \alpha \text{ indeed!}$$

So the higher returns to scale, the less flat the NKPC.

The more DRS, the more flat the NKPC.

I think the point is that



So when α low, θ low, then $\epsilon_{MC/Y}$ high, in other words, MC fluctuates more (out-of-stat.) when α & θ low.

→ This point isn't quite clear right now! Get back to it later!

- $\Theta \downarrow$ when $\Theta \uparrow$

when closer to perfect comp (Θ high), then $\uparrow P$ means demand falls more (flatter D-curve)

\Rightarrow so firms won't adjust prices as much, more stickiness, flatter NKPC

$$\underline{\chi} = \frac{2\alpha + \nu + 1 - \alpha}{\alpha} = \beta - 1 + \frac{\nu + 1}{\alpha}$$

- $\frac{\partial \chi}{\partial \beta} = \frac{1-\alpha}{\alpha} > 0$ when IES higher, slope NKPC higher

$$\frac{u'(c)}{u'(c')} = \left(\frac{c_{t+1}}{c_t} \right)^\beta \rightarrow \text{sub more across time}$$

\rightarrow firms change prices less

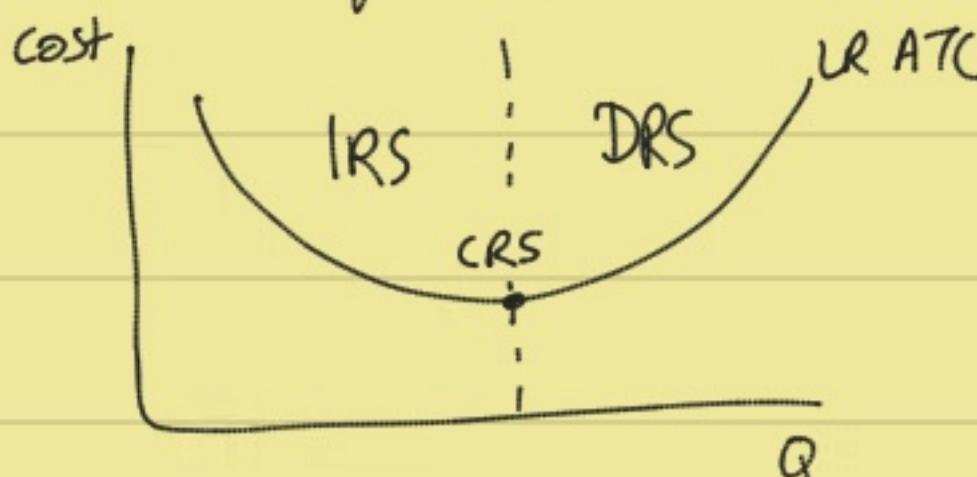
- $\frac{\partial \chi}{\partial \eta} = \frac{1}{\alpha} > 0 \rightarrow$ when η higher, χ higher \rightarrow NKPC flatter
 L^S -elasticity low, i.e. when wages need to move a lot to affect L^S . (cost & turnover)

- $\frac{\partial \chi}{\partial \alpha} = -\frac{\nu+1}{\alpha^2} < 0$ Now we have the opposite:

low α (more DRS) leads to more flexible prices.

Bazmug!

Review from micro:



In Basu: $Y = Z \cdot h \rightarrow \text{CRS}$

$$\text{OR} = Y = Z \cdot h + \Phi \rightarrow \text{IRS}$$

\uparrow
fix cost

Collard & Gali: $Y = A \cdot h^\alpha \rightarrow \text{DRS}$

\Rightarrow seems like a monopoly can be any of these.

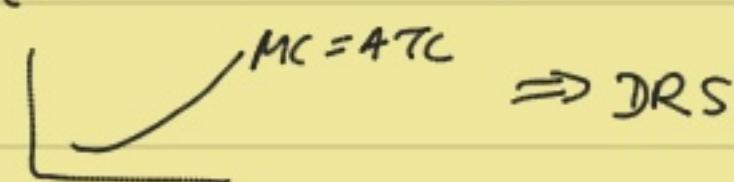
Basu Sudh p. 1 p. 86 Thm: $\gamma = AC/MC$ for a cost-minimizing firm
 returns to scale

and $\mu = \gamma \Rightarrow$ st. st. markup = returns to scale.

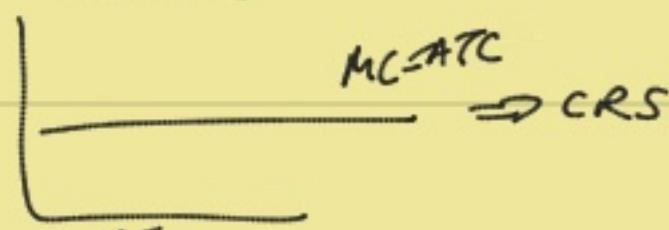
Jeff just explained this as the following:

1.) If there's no fix cost (as is the case in NK model)

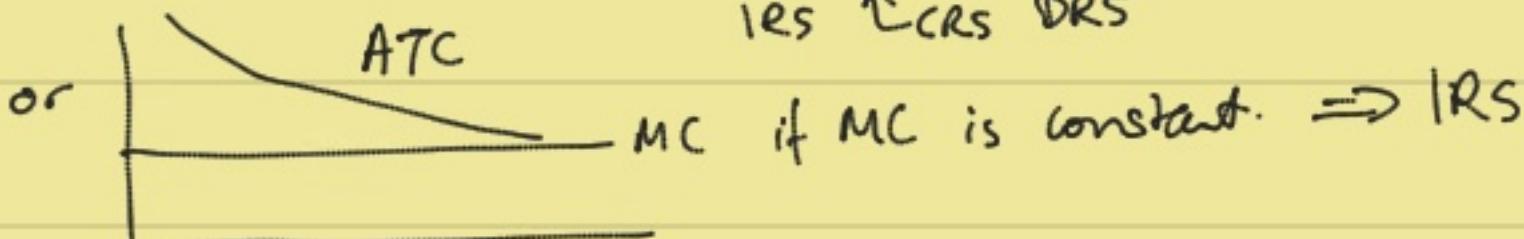
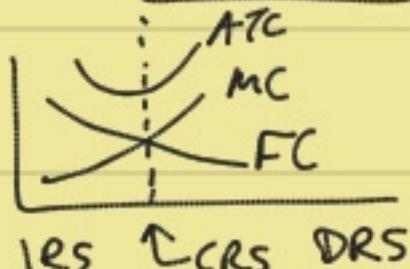
$$MC = AVC = ATC$$



or if MC constant



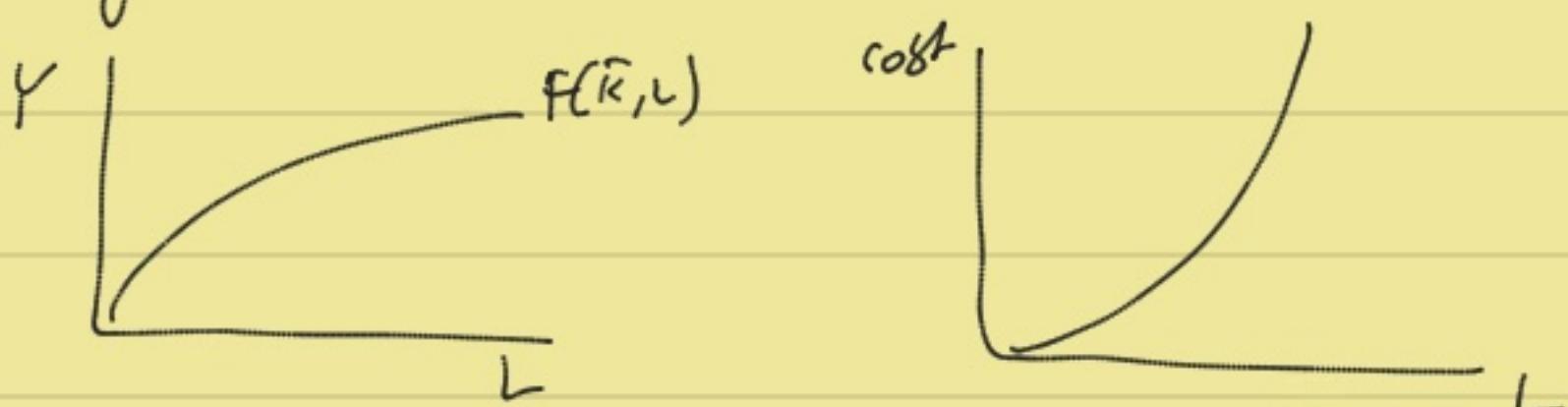
2) If there is fix cost



3) Normally we think that MC is upward-sloping, in fact it's convex. The reason is diminishing maz. prod.

If MPL is

then MC of labor is



b/c to get 1 unit of Y extra, you need to use more & more labor.

Using this, let's check connection b/w returns to scale & MC.

$$\text{Let } Y_t = Z_t h_t^\alpha \quad \frac{W_t}{P_t} = \alpha \cdot Z_t h_t^{\alpha-1}$$

If $\alpha = 1$ (CRS), then

$$\frac{W_t}{P_t} = Z_t \rightarrow \text{constant (given technology)}$$

wrt output or scale of prod.

$$\frac{W_t}{P_t} = \alpha \cdot \frac{Y_t}{h_t} \quad \text{if } \alpha \neq 1 \quad \text{MC depends } \oplus \text{ on scale of prod.}$$

I'm not sure I get that b/c if I pay my workers more

MP, and that's diminishing, then MC diminishes too?
 What if I call MC the things I need to pay to get another unit of output?

$$1 = F'(\cdot) ?$$

$$1 = \underbrace{\alpha \cdot z_t h_{it}^{\alpha-1}}_{\text{constant}} \cdot \underbrace{dh_{it}}_{\text{MP of 1 labor}} \downarrow$$

→ overall I may pay each hour less, but I need a lot of additional hours

In the CRS case dh_{it} is always the same b/c MPL is constant, making MC constant.

This means that having CRS ($\alpha=1$) let's $\Theta=1$ meaning that a firm i's MC equal avg MC.

- $\frac{\partial \Theta}{\partial \alpha} > 0 \rightarrow \text{DRS}$ means that α falls below 1, so $\Theta < 1$ too, i.e. NRPC flatter \Rightarrow a difference in firm i's MC to the avg MC means a bigger diff of MCs and prices
 \Rightarrow you get more procyclical MC!

- $\frac{\partial \theta}{\partial \theta} < 0$ More competition means demand reacts more when you change prices, so firms will abstain from doing so if they can.

If CRS, then I don't care about this.

$$\underline{\underline{\chi = \frac{3\alpha + \gamma + 1 - \alpha}{\alpha}}}$$

2) Let's set $\alpha = 1 \rightarrow 3 + \gamma$ Barro's

1) Let's set $\beta = 1 \Rightarrow \chi = \frac{1 + \gamma}{\alpha} = \text{Collard's}$

- RTS has opposite effects: $\chi \downarrow$ as $\alpha \uparrow$

The higher RTS, the more MC is scale-dependent

Susanto calls $\beta + \gamma$ = Elasticity of MC to the
change in the output gap

\hookrightarrow comes from changes in H

\hookrightarrow comes from: $\Delta L^D \rightarrow \gamma$
 $\Delta L^S \rightarrow \beta$

I think it's the other way around

γ is the LS-elasticity

β is IES \rightarrow will show up in $\Delta C \rightarrow \Delta Y \rightarrow$ firms' demand of L

Θ relates firm's MC to the average MC

χ relates avg MC to the output gap

Bali says that the slope of NKPC is \downarrow in α and θ
 \rightarrow I should check the Woodford book!

I'm still not quite clear on the intuition here

but at least I've understood a lot and need to
be more careful w/ writing the slope.

Woodford - Interest & Prices

31 Oct 2019

$$\pi_t = \kappa (\hat{p}_t - \hat{p}_t^n) + \beta E_t \pi_{t+1} \quad (2.12)$$

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \gamma > 0 \quad (2.13)$$

↑
the smaller γ , the larger the strategic complementarity in price setting, the flatter the NKPC (ahn!)

The discusses sources of strategic complementarity in Sec 1.4

p. 164

$$g = \frac{\omega + \beta^{-1}}{1 + \omega\theta} \quad \text{so } K \text{ becomes}$$

$$K = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{\omega + \beta^{-1}}{1 + \omega\theta} \quad \text{which is exactly what}$$

Preston has! But how to map this to Basu, Gali & Gollard's framework?

Determinants of strategic complementarity (δ)

Conclusions summed up in Table 3.1. (p. 172)

$\beta^{-1} = 1$ and $\omega = 1.25$ seems reasonable.

- homogeneous factor markets $\rightarrow s_y = 0, s_Y = \omega + \beta^{-1}$
- specific factor markets $\rightarrow s_y = \omega, s_Y = \beta^{-1}$
- constant derived markup $\rightarrow \theta \cdot \epsilon_\mu = 0$
- variable derived markup $\rightarrow \theta \epsilon_\mu = 1$
- no intermediate inputs ($s_m = 0$) vs. $\mu s_m = 0.6$

$$\text{Prop 3.3. } g = \frac{(1-\mu s_m)(s_y + s_Y)}{1 + \theta [\epsilon_\mu + (1-\mu s_m) s_y]} \quad (1.43)$$

The underlined $\delta \downarrow$, leading to more strat. comp.

Supp no intermediate inputs & specific factor markets.

then: $y = \frac{\omega + \beta^{-1}}{1 + \theta [\epsilon_p + \omega]}$

To get Preston's thing, then you need to assume constant desired markups: $\epsilon_p = 0$.

Why does specific vs homogeneous/common factor markets matter so much, and how?

ϵ_p = Elasticity of markup function $\mu(x)$ at x

(how much do target markups change at different levels of output)

s_y = elasticity of real marginal cost function wrt. y

$s_y = -1/\text{wrt } k_t$ ← exogenous disturbances

Real MC function $s(y, Y, \tilde{\xi})$ is given by (1.10) (p. 149)

Ok, let's walk thru the real MC fit, $s(y, Y, \tilde{\xi})$

Peter meeting

31 Oct 2019

He's saying it's b/c $E[\bar{a}]$ -effect is stronger when shock persistent
→ leads $\pi \downarrow$ more so that i drops (on net). But real $i \uparrow$
Easy fix:

- lagged inflation in TR → might cause trouble in learning models though, but Peter
- int. rate smoothing term usually uses this when estimating an NK model
- iid shock or low persistence

Justify: "in NK model \exists lq effect & $E[\bar{a}]$ effect. In the SR, the lq effect is stronger than the $E[\bar{a}]$ -effect"

1980s VAR Lit: couldn't find lq effect
b/c they used data from 1970s
when non pd shock were persistent
and so $E[\bar{\pi}]$ -effect dominated

↪ (Gib-Story: MP raised $i \uparrow$ but not sufficiently to $r \uparrow$, so opened door to $E[\bar{\pi}] \uparrow$)

Back to Woodford

The real marginal cost function (p. 148)

Variable cost of supplying quantity $y_+(i)$ of good i :

is $w_t(i) \cdot f^{-1}(y_+(i)/A_+)$ where $f^{-1} \approx h$

where PF is $y_+(i) = A_+ f(h_+(i))$

$\frac{\partial VC}{\partial y_+(i)}$ = nominal MC of supplying good i

$$\Rightarrow S_t(i) = w_t(i) \cdot \frac{1}{f^{-1}(\cdot)} \cdot f' \cdot \frac{1}{A_+} = \underbrace{w_t(i)}_{A_+} \cdot \underbrace{\frac{f'}{f'^{-1}(\cdot)}}_{\psi(j)}$$

$$S_t(i) = \underbrace{w_t(i)}_{A_+} \cdot \underbrace{\psi(y_+(i)/A_+)}_{\psi(j)} \quad (1.8)$$

↑ "how much do we need to ↑ labor"
wage of that labor (nominal)

→ Real MC:

$$s_t(i) = \frac{S_t(i)}{P_t} = s(y_+(i), Y_+, \tilde{\xi}_t)$$

$$s(y, Y, \tilde{\xi}) = \frac{V_h(f^{-1}(y/A); \tilde{\xi})}{u_c(Y; \tilde{\xi})A} \psi(y/A) \quad (1.10)$$

Let's interpret the real marginal cost function:

$$s(y, \gamma, \bar{s}) = \frac{v_h(f^{-1}(y/A); \bar{s})}{u_c(Y, \bar{s})A} \psi(y/A)$$

The elasticity of $L \uparrow$ for different levels of L and for stocks \bar{s}

\uparrow "utility of L "

Coming from extra consumption

by how much do I need to increase labor to get 1 output
→ RTS param

↳ Fisch - inverse of

\rightarrow CES param β^{-1}

\uparrow α
MPL of labor

Log-linear approx to this gives

$$\hat{s}_y = \omega \hat{y}_t(i) + \beta^{-1} \hat{\psi}_t - (\omega + \beta^{-1}) \hat{Y}_t^\alpha \quad (1.15)$$

elasticity of $s(y, \gamma, \bar{s})$ wrt y ($\rightarrow s_y$)

The above makes clear that has elasticity,

$$s_y = \omega = \omega_w + \omega_p \quad (1.16)$$

elasticity of work
(holding u_c fixed)

elasticity of ψ (how much I need to adjust L to increase prod)

Let's understand specific factor markets by looking at MC:

$$\hat{s}_t = \omega \hat{y}_t(i) + \beta^T \hat{\gamma}_t - (\omega + \beta^T) \hat{\gamma}_t''$$

As Woodford details starting p. 164, common economy-wide factor markets means that the MC of supply is equal for all goods i.v.t.

In English: "1 unit of labor costs the same, no matter in which industry it's used to produce" (see p 166)

And look: Basu, Lec 21, p. 5 Mac:

"Common MC result rests on four important ass's:

1. CRS

2. Common technology

3. Economy-wide factor markets ← there it is!

4. No adj-cost for inputs (i.e. labor)

Woodford says that in this case,

$$S_t = \frac{w_t}{A_t + f'(h_t)}$$

b/c this way $w_t(i) = w_t + h_i$
and $h_t(i) = h_t + v_i$.

Woodford goes on to say that in this case,

$$\xi = \omega + \beta^{-1} > 0$$

and since in RBC, $\beta^{-1} = 1$, $\xi > 1$, i.e.

by design, pricing decisions are strategic substit.

(sic!)

This implies that $\omega = \eta$: somehow there still is a disconnect between ξ and ω in the sense that "which one is actually the MC elasticity"?

Where are the specific factor markets ass introduced?

It's in the PF: $y_i(i) = A_i f(h_i(i))$

"All producers of good i hire type i -labor, and get that labor from a labor market that's distinct from type j -labor markets." This doesn't mean that firms have mon. power on these specific labor markets b/c they're still a continuum in each industry.

w/ specific factor markets (but no intermediate goods
and no variation in desired markups)

$$S = \frac{\omega + \beta^{-1}}{1 + \theta\omega} \quad \text{exactly as in Preston.}$$

(2000)

Now take Chari et al assumptions for the values of these
params: $\omega = 1.25$, $\beta = 1$, $\theta = 10$

$$S = \frac{2.25}{1 + 12.5} = 0.166$$

Now all I need to do is to interpret 1) why
specific factor gives this S (lower) 2) what is ω

1) specific factor markets gives more strat comp in pricing
Maybe it's b/c one way of interpreting specific factor markets
is that it's as if factors were "quasi-fixed" \rightarrow if I can't
adjust my quantity, I will find it more important to
price as others are.

\rightarrow this is why higher strat comp "amplifies price stickiness"

b/c if a fraction α of the economy can't adjust,
 Then Stab comp makes the other $1-\alpha$ fraction not to
 adjust as well!

2) What is ω ?

The two conflicting things are these:

Basm

$$\begin{aligned} b + \eta &= \text{El of MC wrt output} \\ &= s_y \quad (\text{and } s_y = 0) \end{aligned}$$

in Woodford's terms

$$= \zeta$$

$$\Rightarrow \omega = \eta$$

Woodford

$$\begin{aligned} \omega &= \omega_w + \omega_p \\ &\uparrow \\ &(\text{El. of MC wrt. } y(i)) \end{aligned}$$

$\omega_w = \text{El of real wage demands}$
 $(\rightarrow \eta)$ wrt output $y(i)$

$\omega_p = (-1) \cdot \text{Elasticity of MPL}$
 wrt output $y(i)$

$$\text{w/ CRS, } MPL = 1, \omega_p = 0 \rightarrow \omega = \omega_w = \eta$$

w/ DRS, $MPL = \text{decreasing fct of labor}$, so $\omega_p = \text{some number}$

$$\Rightarrow \omega = \eta + \omega_p$$

I think that Woodford's ω takes out (doesn't include) u_c ,
 which is why b^{-1} only shows up in s_y .

⇒ Ok so big conclusion for me:

1.) Need specific labor markets

Use Chari, Kehoe & McGrattan (2000) & Woodford 2003/2011

values of $\theta = 10$, $\beta = 1$, $\omega = 1.25$ (ω/γ inverse
Frisch worked into ω I think)

2) I think Woodford is assuming DRS (I'm not sure and
I'm not sure it's necessary to make that assumption.)

I don't think Preston assumes anything here and
again, I don't think it's necessary b/c ω
(or s_y & s_r) will subsume it.

So then

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \quad \omega/\gamma = \frac{\omega + \beta^{-1}}{1 + \omega\theta}$$

$$\omega/\theta = 10, \beta = 1, \omega = 1.25, \quad \beta = 0.95, \kappa = \text{whatever}$$

But even that isn't sufficient to get $g_{X3,2} > 0$ when $p_i \geq 0.8$

But it is if $p_i < 0.8$! So I set it to $p_i = 0.7$.

Follow Peter: check what happens if we add int rate smoothing!

→ it doesn't do anything b/c the value of $g_x(i, \bar{i})$ remains the same!

To do:

- check the interest rate smoothing w/ $\rho > 0$ (idiot!)
- understand how the learning models behave

especially when moving between $(\alpha, \rho_i) = (0.75, 0.7)$

$$\& (\alpha, \rho_i) = (0.5, 0.6)$$

→ restudy dynamics of learning, together w/ expectations.

Interpretation of IRFs

1 Nov 2019

- 1) RE:
 - ρ only makes IRFs hump-shaped but does nothing on output
 - You can move ρ_i up to 0.78 if you $\uparrow \alpha$ to 0.75.
- 2) Differences between learning IRFs is mat8 vs materials?
 - not much, only anchoring is diff b/c $\rho_i \downarrow$ to 0.6 from 0.877

→ less shocks, more anchoring, so anchoring responses resemble decreasing gain ones.

- nonpol shock looks diff b/c I decreased P_i and so i^{\uparrow} on impact

⇒ but it seems to me that we still get the same conclusions as in materials:

- overshooting, especially for constant gain learning

Actually, what I find is that taking α for the gain, all gain sequences are the same. This means that the shock didn't knock the econ out of whatever its gain sequence was. ⇒ return to this!

- Moving P doesn't do a lot in the learning models b/c 1) dynamics was already sluggish
2) since we endog. choose d gain, P doesn't help us there

⇒ Need to understand 3 things:

1) What moves the endogenous gain / why don't infl. shocks move it? → shock IS moving mean gain just not much
→ gain IRFs & equations

2) It seems to be equivalent to compare

a) Δgain vs. $c\text{gain}$

b) endog. gain which was anchored vs. unanchored
when the shock hit / If the shock doesn't influence
the further evolution of the gain. But my feeling
is that it should!

3) Overshooting: why is it happening in learning?

Is stg like this happening:

time 1: shock hits, $\pi \uparrow$, $E_t^m(\pi_{t+1})$ hasn't yet
responded

time 2: $E_2^m(\pi_3) \uparrow$, but $\pi \downarrow$

This is the key question

CAMP's criterion: $\theta_t^{\text{CAMP}} = |(1-\delta)(P-1)\bar{\pi}_{t+1}|$

mine: $\theta_t = |\text{stuff } \bar{\pi}_{t+2} + \text{stuff } s_{t+1}|$

In CAMP: once you've converged, ain't nothing gon' move you out

But mine should be moved out when large enough
shorts hit.

→ and I think they are: the point is: not much.

1) tiny changes 2) they happen early in the sample

(makes sense: once $\bar{\pi} \rightarrow 0$, the shorts don't have
a large chance to hit it out of anchordness)

3) they seem even smaller b/c we're talking short

gains: $|1-2| > |1-\frac{1}{2}|$

⇒ so actually, to do conf. bands is quite tough
for the gain

My feeling is that overshooting may indeed be coming from a messed-up timing, and here's my hand:

IRFs: for each t , I shock the econ at $t+1$
so I shouldn't subtract t from t ?

✓ To do: plot $\bar{\pi}$ -IRF

4 Nov 2019

"what happens to LH-expectations?"

✓ figured out why gain IRF changed \rightarrow it was b/c the shock δ was bigger

• timing: morning vs. evening tests, timing of IRF

→ 1) I'm doing it correctly: for $t=2 \dots T-h$

- I impose the shock $\delta=1$ at time t

- I subtract $\text{sim}(\text{w/shock}) - \text{sim}(\text{w/o shock})$

for time: $t \dots t+h-1$ (to get a h -horizon IRF)

Now also evening tests adjust on impact too, which is correct, and by more than morning ones b/c $\bar{\pi}$ gets updated in the meanwhile

\Rightarrow could it be that some deviation between
the morning and evening post causes the
overshooting?

A shortle hits: $i \uparrow \rightarrow s_t \uparrow$

$$\text{morning post} = \bar{\pi}_{t-1} + b \cdot s_t \downarrow$$

$\Rightarrow (\pi, x, i)$ realized

$\Rightarrow \bar{\pi}$ updated to $\bar{\pi}_t$

$$\text{evening post} = \bar{\pi}_t + b \cdot s_t \downarrow \downarrow$$

But starting from $t=2$, there should be no surprises
coming from s_t b/c b incorporates the P-matrix.

The thing is: observables are realized based on
morning post, but updating is done based
on yesterday's evening post

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k^{-1} \left(\bar{\pi}_t - \underbrace{(\bar{\pi}_{t-1} + b s_{t-1})}_{f_{t-1}^e} \right)$$

On impact ($t=1$)

$$f_{t+1|t}^m > f_{t+1|t}^e \text{ b/c } \bar{\pi} \text{ was raised down}$$

At time $t=2$

$$f_{t+1|t}^m < f_{t+1|t}^e \rightarrow \bar{\pi} \text{ is being rented up}$$

At time $t=3$ and onward the two are basically the same

But here's another observation:

- evening and morning rents 1-period ahead are always negative
 - f_a & f_b for $\bar{\pi}$ are always negative
- ... so why on all earth do $(\bar{\pi}, x)$ revert to positive?

\Rightarrow it makes no sense unless A_a^{LN} , A_b^{LN} (or A_s^{LN}) are wrong!

But the fact that this only happens for gain indicates that the problem is w/ the expectations,

f_a , f_b and the rents e & m

It does come from expectations:

5 Nov 2019

when $\gamma_n = 2$, and again yields oscillations, both 1-period ahead jets and $f_a \& f_b$ exhibit oscillations.

It seems like what is happening is that again gets into oscillations after a shock - how long those last depend on γ_n ($n \uparrow$ the longer).

Oscillations arise when morning & evening jets

cross-cross. What induces differences between

morning & evening jets? \rightarrow the updating of $\bar{\pi}$

? Observables at $t = \text{fcst}_t^m\left(\bar{\pi}_{t-1}, \underline{s}_t\right)$ when shock hits
the period after

$$\bar{\pi}_t = \text{function}\left(\text{fcst}_{t-1}^e\left(\bar{\pi}_{t-1}, \underline{s}_{t-1}\right)\right) \downarrow \leftarrow$$

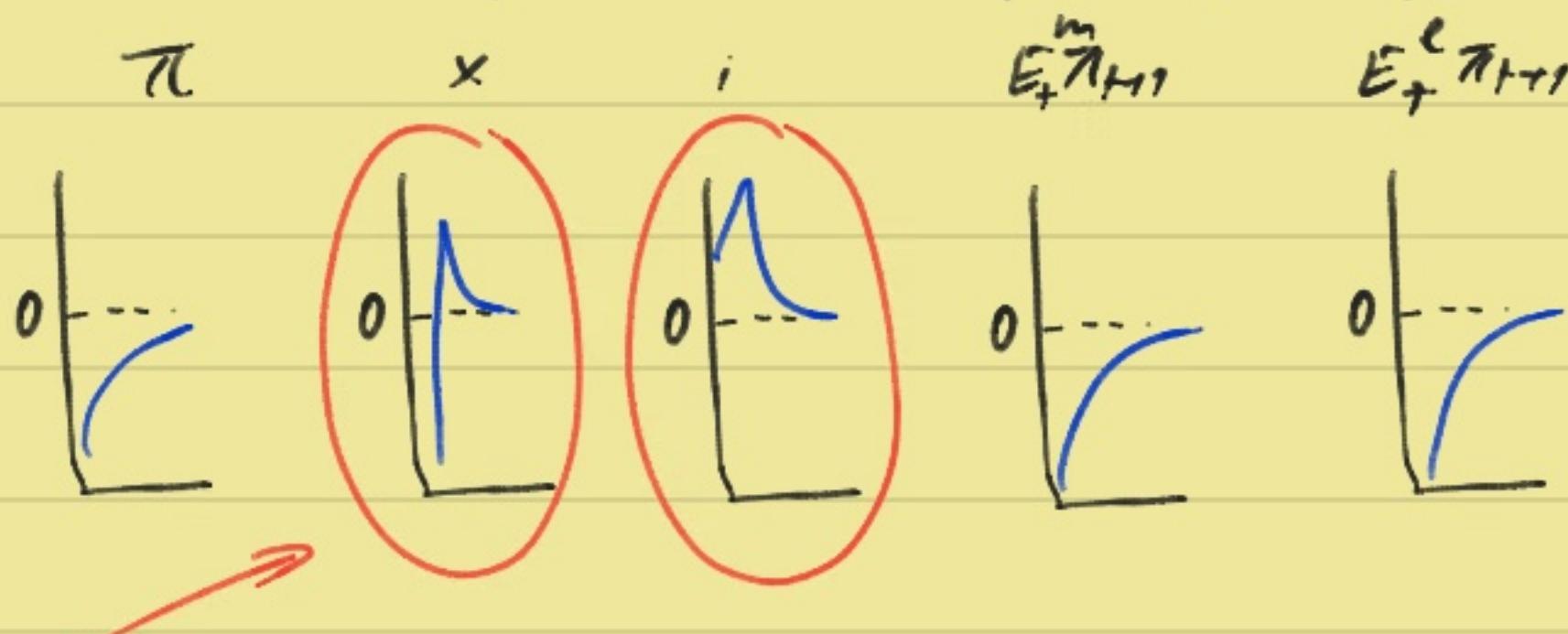
$$\text{fcst}_t^e\left(\bar{\pi}_t, \underline{s}_t\right) \quad \Downarrow$$

it's gotta be
that at this stage,

w/ gain, your expectation
"overadjusted," so you keep readjusting

w/ opposite sign \Rightarrow so the oscillations make sense,
but not (π_t, x) shooting into \oplus territory!

Wait a sec ... for $\gamma_{\pi} = 1.2$ I get a puzzling result



why is this happening? Why $x \uparrow$ when $i \uparrow$? And why does $i \uparrow$ in period 2 even though $\pi \downarrow$ and $\gamma_{\pi} = 0$?

You'd think g_x & h_x ain't right, but this is only happening in the gain world, so it has to do w/ expectations.

No cost-push shock.

$$x_t = -\beta i_t + \hat{E}_t \sum_{T=1}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \beta(\beta i_{T+1} - \pi_{T+1}) + \beta r_T)$$

$$x_t = -\beta i_t + \sum_{T=1}^{\infty} \beta^{T-t} \left[\underbrace{(1-\beta)\hat{E}_t x_{T+1}}_{\text{This needs to } \uparrow \text{ by more than } \downarrow \text{ and } \downarrow} + \underbrace{\beta \hat{E}_t (\pi_{T+1})}_{\text{and } \downarrow} - \underbrace{\beta \beta \hat{E}_t (i_{T+1})}_{\text{and } \downarrow} \right]$$

This needs to \uparrow by more than \downarrow and \downarrow

But that ain't happening, and in particular, $f_b \downarrow$ by a lot!

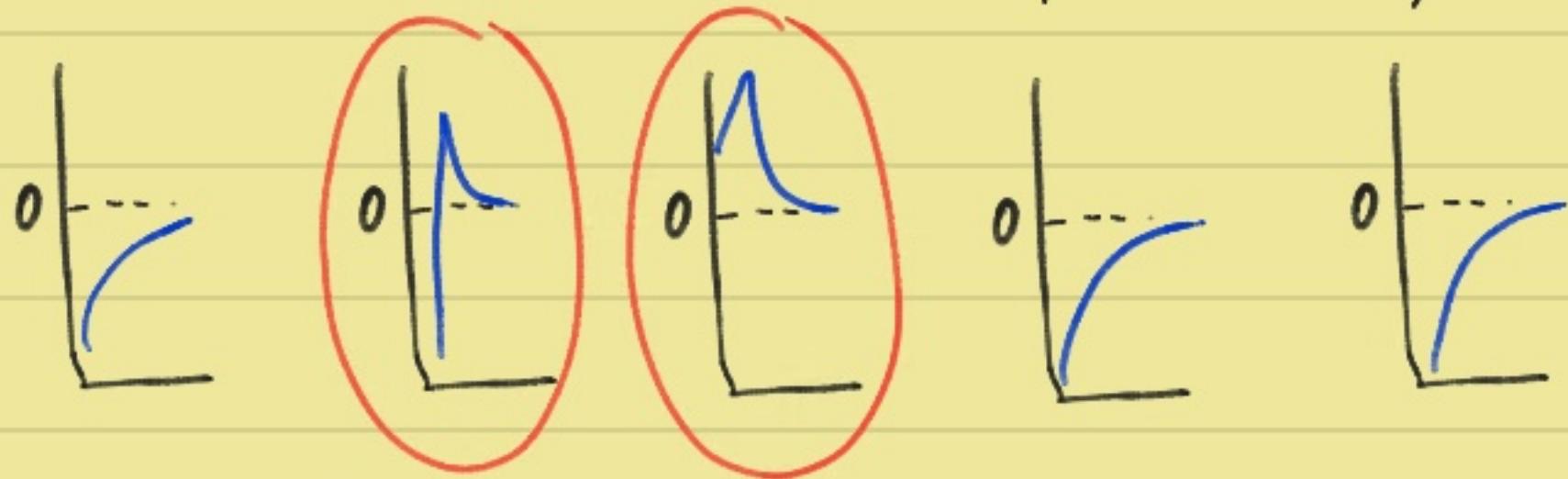
→ I'm starting to suspect that maybe a constraint

IRFs wrong (maybe only for ϵ_{gov}). Maybe a negative sign slips in there somewhere...
 → but that doesn't seem to be the case either!

Now I decreased $\bar{g} \rightarrow$ even 0.1 decreases the sign-switch, and 0.01 tames things completely.

$\bar{g} = 0.05$ gives the following responses:

$$\pi \quad x \quad ; \quad E_f^m \pi_{t+1} \quad E_f^e \pi_{t+1}$$



⇒ just like $\gamma_\pi = 1.2$

Let's analyse A_a, A_b, A_s :

$$A_s = \begin{pmatrix} + & - & + & - \\ + & - & - & - \\ + & + & + & + \end{pmatrix} \begin{matrix} r^n \\ i \\ u \\ i_{t-1} \end{matrix}$$

$$\begin{pmatrix} \pi \\ x \\ i \end{pmatrix} = \begin{bmatrix} + & + & - \\ + & + & - \\ + & + & -? \end{bmatrix} \underbrace{\alpha \beta}_{\propto} \cdot \hat{E}_t \begin{bmatrix} \pi \\ x \\ i \end{bmatrix}$$

*should be from a theoretical perspective

$$A_b - A_a$$

$$A_a = \begin{array}{ccc|cc} + & + & 0 & - & - & 0 \\ \textcircled{-} & \textcircled{-} & 0 & + & - & 0 \\ + & + & 0 & - & - & 0 \end{array}$$

$$A_b = \begin{array}{ccc|c} \textcircled{-} & + & 0 & \text{hmm...} \\ \textcircled{-} & \textcircled{+} & 0 \\ \textcircled{-} & + & 0 \end{array}$$

\uparrow this may be why $i \uparrow$ when $\hat{E}(\pi) \downarrow$

this may be why $x \uparrow$ when $\hat{E}(\pi) \downarrow$

\Rightarrow check g_{xa} (the $-2 \frac{\psi_n}{w}$ part!)

\Rightarrow check g_{xb} (the $\beta - \beta \beta \psi_n$ part!) } responsible for
and g_{xb} (-11-) } $A_b(2:3, 1) < 0$

Check A_a & A_b (g_{Xb} & $g_{\pi b} \rightarrow 3(1-\beta \psi_\pi)$) are particularly important, and $g_{xa} \rightarrow -\frac{3\psi_\pi}{W}$ a little bit
One troubling thing though is that Peter has exactly these values in proton 2005-potter.

Setting all θ 's to 0 for simplicity

$$x_t = -\gamma i_t + E_t^* \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)x_{T+1} - 3(\beta i_{T+1} - \pi_{T+1})] \quad (1)$$

$$\pi_t = \kappa x_t + E_t^* \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} [\kappa \alpha \beta x_{T+1} + (1-\alpha) \beta \pi_{T+1}] \quad (2)$$

$$i_t = \psi_\pi \pi_t + \gamma_x x_t \quad (3)$$

\hookrightarrow set to 0 for simplicity

$$x_t = -\gamma(\psi_\pi \pi_t) + E_t^* \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)x_{T+1} - 3(\beta \psi_\pi \pi_{T+1} - \pi_{T+1})]$$

$$x_t = -\gamma \psi_\pi \pi_t + E_t^* \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)x_{T+1} - 3(\beta \psi_\pi - 1)\pi_{T+1}]$$

$$x_t = -\gamma \psi_\pi \pi_t + [3(1-\beta \psi_\pi), (1-\beta), 0] f_\beta \quad \begin{matrix} \uparrow \\ \text{this also} \\ \text{means that part} \end{matrix}$$

$$\pi_t = \kappa x_t + [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_\alpha \quad \begin{matrix} \uparrow \\ \text{of } f_\alpha \text{ has a } \ominus \text{ offset} \\ \text{via expected } i \uparrow \end{matrix}$$

$$\rightarrow x_t = -\gamma \psi_\pi [\kappa x_t + [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_\alpha] + [3(1-\beta \psi_\pi), (1-\beta), 0] f_\beta$$

$$(1+\gamma \psi_\pi \kappa) x_t = \underbrace{-\gamma \psi_\pi}_{=W} [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_\alpha + [3(1-\beta \psi_\pi), (1-\beta), 0] f_\beta$$

this means that x_t depends \ominus on f_α b/c higher f_α implies higher i !

$$x_t = -\frac{2\gamma_{\pi}}{\omega} [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + \frac{1}{\omega} [b(1-\beta\gamma_{\pi}), (1-\beta), 0] f_{\beta}$$

$\ominus b/c E[i] \uparrow$

$E[i] \uparrow$

$$\begin{aligned}\pi_t &= \kappa \left[-\frac{2\gamma_{\pi}}{\omega} [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + \frac{1}{\omega} [b(1-\beta\gamma_{\pi}), (1-\beta), 0] f_{\beta} \right] \\ &\quad + [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a \\ &= -\frac{k\gamma_{\pi}}{\omega} [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + \frac{k}{\omega} [b(1-\beta\gamma_{\pi}), (1-\beta), 0] f_{\beta} \\ &\quad + [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a\end{aligned}$$

$$\pi_t = \left(1 - \frac{k\gamma_{\pi}}{\omega}\right) [(1-\alpha)\beta, \kappa\alpha\beta, 0] f_a + \frac{k}{\omega} [b(1-\beta\gamma_{\pi}), (1-\beta), 0] f_{\beta}$$

\uparrow
 \ominus effect of f_b on π
mediated here by $\frac{k}{\omega}$

$$\text{b=1, } \omega > 2 \rightarrow \max(k\gamma_{\pi}) = \bar{\gamma}_{\pi}$$

which is usually < 2

In that case this expression > 0

$\rightarrow \pi$ depends \oplus on f_a

However, if γ_{π} crosses a threshold value, $\bar{\gamma}_{\pi}$, then

$1 - \frac{k\gamma_{\pi}}{\omega} < 0$ and π depends negatively on f_a !

$$\begin{aligned}1 < \frac{k\gamma_{\pi}}{\omega} &\Rightarrow \frac{\omega}{k\beta} < \bar{\gamma}_{\pi} \\ \Rightarrow \frac{1 + \beta\gamma_x + k\beta\bar{\gamma}_{\pi}}{k\beta} &< \bar{\gamma}_{\pi} \quad | \beta = 1, \gamma_x = 0\end{aligned}$$

$$\frac{1 + k\bar{\gamma}_{\pi}}{k} < \bar{\gamma}_{\pi} \Rightarrow 1 + k\bar{\gamma}_{\pi} < k\bar{\gamma}_{\pi} \quad \text{oh, it can never happen!}$$

Alright so let's recap:

$$x_t = \underbrace{-\frac{\kappa b \gamma_\pi}{\omega} [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_\alpha + \frac{1}{\omega} [b(1-\beta \gamma_\pi), (1-\beta), 0] f_\beta}_{x \text{ depends on } \hat{E}(\pi, x) \text{ thru } E[i]}$$

x depends on $\hat{E}(\pi, x)$ thru $E[i]$
via $\hat{E}[\pi](.)$

x depends on $\hat{E}[\pi]$ b/c $E[i]$
but \oplus on $\hat{E}[x]$

$$\pi_t = \underbrace{\left(1 - \frac{\kappa b \gamma_\pi}{\omega}\right) [(1-\alpha)\beta, \kappa \alpha \beta, 0] f_\alpha + \frac{1}{\omega} [b(1-\beta \gamma_\pi), (1-\beta), 0] f_\beta}_{\oplus \text{ effect of } \hat{E}(\pi, x) \text{ on } \pi}$$

\uparrow
 \ominus effect of $\hat{E}[\pi]$ on
 $\hat{E}[x]$ mediated by
NKPC slope

2 notes:

$$\begin{aligned} 1) \frac{\partial \frac{\kappa b \gamma_\pi}{\omega}}{\partial \gamma_\pi} &= \frac{\partial}{\partial \gamma_\pi} (\kappa b \gamma_\pi) (1 + b \gamma_x + \kappa b \gamma_\pi)^{-1} \\ &= \underbrace{\frac{\kappa b}{1 + b \gamma_x + \kappa b \gamma_\pi}}_{>0} \cdot (-1) \frac{\kappa b}{(1 + b \gamma_x + \kappa b \gamma_\pi)^2} < 0 \end{aligned}$$

So when $\gamma_\pi \uparrow$, $\frac{\kappa b \gamma_\pi}{\omega} \downarrow$, so \oplus effect of f_α on π stronger!

2) Is $\beta \gamma_\pi < 1$ ever? For $\beta = 0.99 \rightarrow \gamma_\pi < 1.0101$

So the point is that non pol would have to be very weak on π for the effect that higher $\pi \rightarrow E[i] \uparrow \rightarrow x \downarrow$ to disappear!

~~An interface 220~~

6 Nov 2019

~~Do materials 8b.m to use sam-team.m to simply generate simulations of EE vs LHI learning w/ constant l slope learning~~

→ Actually, soap that - it's well implemented correctly in materials 3!

$$w = 1 + b\psi_x + k b\psi_\pi$$

7 Nov 2019

Then $\frac{-b\psi_\pi}{w} = -\frac{b\psi_\pi}{1 + b\psi_x + k b\psi_\pi}$

$$\frac{\partial}{\partial \psi_x} \left[\frac{b\psi_\pi}{1 + b\psi_x + k b\psi_\pi} \right] = -\frac{b\psi_\pi}{(1 + b\psi_x + k b\psi_\pi)^2} > 0$$

→ so $-\frac{b\psi_\pi}{w} \uparrow$ (approaches 0 from below) when $\psi_x \uparrow$

$$\frac{\partial}{\partial \psi_\pi} \left[\frac{b\psi_\pi}{1 + b\psi_x + k b\psi_\pi} \right] = \frac{b}{w} \cdot (-1) \frac{b\psi_\pi}{w^2} \cdot k b < 0$$

→ same for $\psi_\pi \uparrow$

x_t depends Θ on $E_t(\pi_{t+h}, x_{t+h})$ no matter what, and

$\psi_x \uparrow$ or $\psi_\pi \uparrow$ mitigate this a bit (especially ψ_x) but don't solve it.

$$1 - \frac{k_3 \gamma_{\pi}}{w} = \frac{w - k_3 \gamma_{\pi}}{w} = \frac{1 + b \gamma_X + k_3 \gamma_{\pi} - k_3 \gamma_{\pi}}{1 + b \gamma_X + k_3 \gamma_{\pi}}$$

$$= \frac{1 + b \gamma_X}{1 + b \gamma_X + k_3 \gamma_{\pi}} = C_{\pi, f_a}$$

$$\frac{\partial C_{\pi, f_a}}{\partial \gamma_X} = \frac{b}{w} (-1) \cdot \frac{b \gamma_X}{w^2} \geq 0$$

$$\frac{\partial C_{\pi, f_a}}{\partial \gamma_{\pi}} = (-1) \frac{1 + b \gamma_X}{w^2} \cdot k_3 < 0$$

$\Rightarrow \pi_t$ depends \oplus on $\hat{E}_t[\pi_{t+h}, x_{t+h}]$ (direct effect)
but this is \downarrow the higher k_3 & γ_X (especially γ_{π}).

$f_b \rightarrow$ direct effect of CH-E on X , indirect on π

$f_a \rightarrow$ direct effect — II — on π , indirect on X

$$x_t = -\beta i_t + [b, 1-\beta, -\beta\beta] f_a \quad \rightarrow \text{shares}$$

$$\pi_t = k x_t + [(1-\alpha)\beta, k\alpha\beta, 0] f_b \quad \rightarrow \text{borrows}$$

$$i_t = \gamma_{\pi} \pi_t + \gamma_X x_t \quad \rightarrow \text{shares}$$

if $E[X] \uparrow \Rightarrow \pi_t \uparrow \rightarrow i \uparrow$

Why isn't this happening for again?

$$-\frac{3\sqrt{\pi}}{w} = -1.2656$$

$$(1-\alpha)\beta = 0.495$$

$$\kappa\alpha\beta = 0.0833$$

Aha! But the point is that the diff between again and cagain is really fb, not fa

Peter meeting \rightarrow ^{misses estimates} of $1/4 \times 115$ \rightarrow Nov 2019
You usually estimate $b \ll 1$ \rightarrow ^{and pricing gets} $b = 1/100$
 \rightarrow that can decrease the punishing response!
AD doesn't depend on hours if U is separable

Other ways of coping w/ inconsistency w/
balanced growth

e.g. leisure is th production

"in RE you can get away w/ an. log utility
but not in learning"

all of HANK research agenda
is motivated by the idea that NK relies
on intertemp switch, whereas NK's no
evidence for it in data

Theory Need of BC measurement
or Kydland & Prescott

→ "you can't just change $E(\cdot)$ and leave
params, you need to rethink
calibration"

Work after

7 Nov 2019

- Basu SUM p. 1 p. 20 \Rightarrow Lect. 3 p. 8.

Equity premium puzzle implies $\beta \approx 30$
(This β^{-1} for me, that is $\beta^{\text{mine}} \approx 1/30$.

The higher β^{Basu} (lower β^{mine}), the more risk-averse.

$$\text{CES utility: } u(C_t) = \frac{C_t^{1-\beta}}{1-\beta} \rightarrow u'(C_t) = C_t^{-\beta}$$

Or in my formulation (Preston-Woodfords)

$$u(C_t) = \frac{C_t^{1-\beta^{-1}}}{1-\beta^{-1}} \rightarrow u'(C_t) = C_t^{-\beta^{-1}}$$

$$\text{Back to Baum-notation: } u''(C_t) = -\beta C_t^{1-\beta} < 0$$

$$\frac{\partial u''(C_t)}{\partial \beta} = -C_t^{1-\beta} \cdot \dots \quad ?$$

~~we~~ call this "stuff"

use "logarithmic derivative" (Simon & Blume p. 79 Mac)

to compute $\frac{\partial}{\partial \beta} C_t^{1-\beta}$

$$(19) \text{ S&B: } u'(x) = (\ln u(x))' u(x)$$

Where does this come from?

Take $u(x)$ and $u'(x)$ | $\ln(\cdot)$

$\ln(u(x))$ | Take $\frac{\partial}{\partial x}$

$\frac{1}{u(x)} \cdot u'(x)$ by chain rule. So:

$$\frac{\partial}{\partial x} \ln(u(x)) = \frac{u'(x)}{u(x)}$$

$$\Rightarrow u'(x) = \underline{\underline{\frac{\partial}{\partial x} [\ln(u(x))] \cdot u(x)}}$$

Derivative of a^x

Logarithmic derivative would say

$$\frac{\partial}{\partial x} a^x = \frac{\partial}{\partial x} \ln(a^x) \cdot a^x = \ln(a) \cdot a^x$$

Here we have $g(b) = -b c_+^{1-b}$ $f(b) = c_+^{1-b}$

$$\begin{aligned} g'(b) &= \ln(c_+^{1-b})' \cdot f(b) \\ &= [(1-b) \ln(c_+)]' \cdot c_+^{1-b} \\ &= -\ln(c_+) \cdot c_+^{1-b} \quad \text{from PR} \end{aligned}$$

So then stuff = $[-\ln(c_+) c_+^{1-b}] [-c_+^{1-b}] > 0$

$$\begin{aligned} g'(b) &= \text{stuff} = \ln(-b c_+^{1-b})' \cdot (-b c_+^{1-b}) \\ &= [\ln(-b) + (1-b) \ln(c_+)]' \cdot (-b c_+^{1-b}) \\ &= \left[\frac{1}{-b} (-1) - \ln(c_+) \right] [-b c_+^{1-b}] \quad \begin{matrix} M \text{ tells me} \\ \ln 5 \text{ is right} \end{matrix} \\ &= [1 - b \ln(c_+)] [-c_+^{1-b}] > 0 \end{aligned}$$

I don't know, the two don't look the same, but both are > 0 . $b \uparrow$, $u''(c) \rightarrow 0$ from below.

I wanted to derive the IES and interpret its magnitude, but I'm not getting at it from Barro's SHM.

But there is something in Macro HW 1 Corr (1st Barro PS, p. 3)

NB. 2. $\text{sign}\left(\frac{\partial C_t}{\partial r}\right) = \text{sign}\left(\frac{1-\beta}{\beta}\right)$ Why?

$\Rightarrow b/c$ $r \uparrow$ has 2 effects on C_t , and 2 decisions which dominates:

wealth effect/income effect (IE):

since $r \uparrow$, your savings are worth more (\rightarrow richer) $\rightarrow C_t \uparrow$
(and $C_{t+1} \uparrow$)

substitution effect (SE)

since $r \uparrow$, C_{t+1} is worth more $\rightarrow C_t \downarrow$ (and $C_{t+1} \uparrow$)

Wiki also says that IES is a measure for how the grt of

cons, $\frac{C_{t+1}}{C_t}$, responds to r $b/c \Delta C_t \uparrow = \Delta C_{t+1} \uparrow$
 $\text{IES} \rightarrow 0$ $\beta > 1$ when $\beta \rightarrow \infty$, only IE, $\Delta \frac{C_{t+1}}{C_t} = 0$ (same)

$\beta = 1$ $\text{IE} = \text{SE} \Leftrightarrow \frac{\partial C_t}{\partial r} = 0 \quad \Delta \frac{C_{t+1}}{C_t} > 0$

$\beta \rightarrow \infty$ $\beta < 1 \quad \Delta C_t \downarrow \quad \Delta C_{t+1} \uparrow$ $\Delta \frac{C_{t+1}}{C_t} > 0$ b/c the SE
"slows" C_t down.

Let's be precise:

$$\text{For } u(c_t) = \frac{c_t^{1-\beta}}{1-\beta}$$

$$IES = \frac{1}{\beta} \quad (\text{so for me, } \beta = IES)$$

$$CRRA = \beta \quad (\text{so for me, } \frac{1}{\beta} = CRRA) \quad \text{when CES pref.}$$

Acknowledg's slides say

$$E_u(c(t)) = -\frac{u''(c(t)) \cdot c(t)}{u'(c(t))} \quad \text{is the inverse of IES.}$$

He also writes $\frac{\dot{c}(t)}{c(t)} = \left(\frac{1}{\beta}\right) \cdot (r(t) - \rho)$

\uparrow real income \uparrow discount rate

so finally! If this $= IES$,

- Then the higher this value, the more the govt of cons reacts to $r \uparrow$ ($IES \rightarrow \infty$)
- ... the stronger the SE (relative to IE)

• ... the more c_t moves in opposite dir to c_{t+1}

In my (Preston's) world, this is $\beta \rightarrow \infty \Rightarrow$ I need to kill the SE!

NKIS:

$$X_t = \text{stuff} - \frac{1}{\beta} (i_t - E_t \pi_{t+1}) + \text{stuff}$$

$$X_t = -\text{IES}(r_t) + \text{stuff}$$

⇒ in the NK world, output responds negatively to r thru its effects on borrowing

Actually, reading Hall, 1988, JPE, the first few sentences, is very illuminating

"The IES = response of the change rate in C to changes in expected real int. rate"

"If cons. can be induced to postpone cons. by modest increases in the int. rates, then (1) movements in int. rates will make cons decline whenever other components of AD rise"

For this channel to be at work in NK, where $I=0$,

$SE > IE$ so that $C \downarrow$ when $r \uparrow$

→ Yes, so I need a lower IES which lowers the SE.

Check this & Angelos

↓ at Yale

his work (maybe JNP) is the one that solves
the ∞ state-space problem of higher-order
beliefs using frequency domain.

Ryan meeting

7 Nov 2019

- 1) Redo shades & IRF for shock at $t=5$ & $t=25$
for 400 different underlying sequences of
shocks \rightarrow average across cross-section
 \rightarrow need to see diff b/wn cgain & dgain, or, they
should be the same early on in sample!
- 2) wanna see FE's (IRFs)
they should flip sign for Cgain for
the criss-crossing to happen
- 3) gain \bar{g} \rightarrow play w/ it \rightarrow Ryan has less
prior on that
one way to choose is to see where
the dgain is after 50 periods.