Materials 44 - I think I've got the estimation

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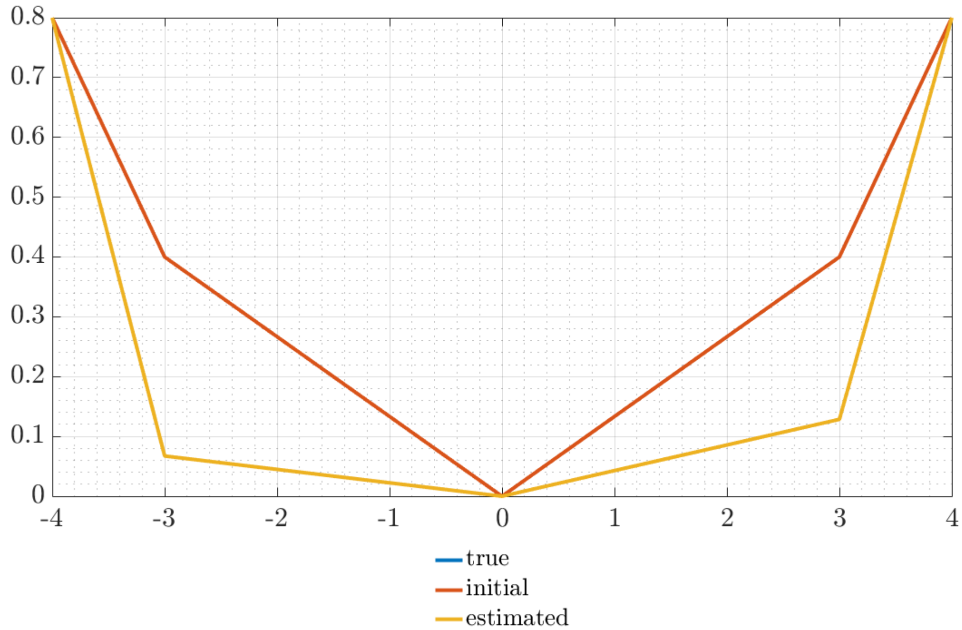
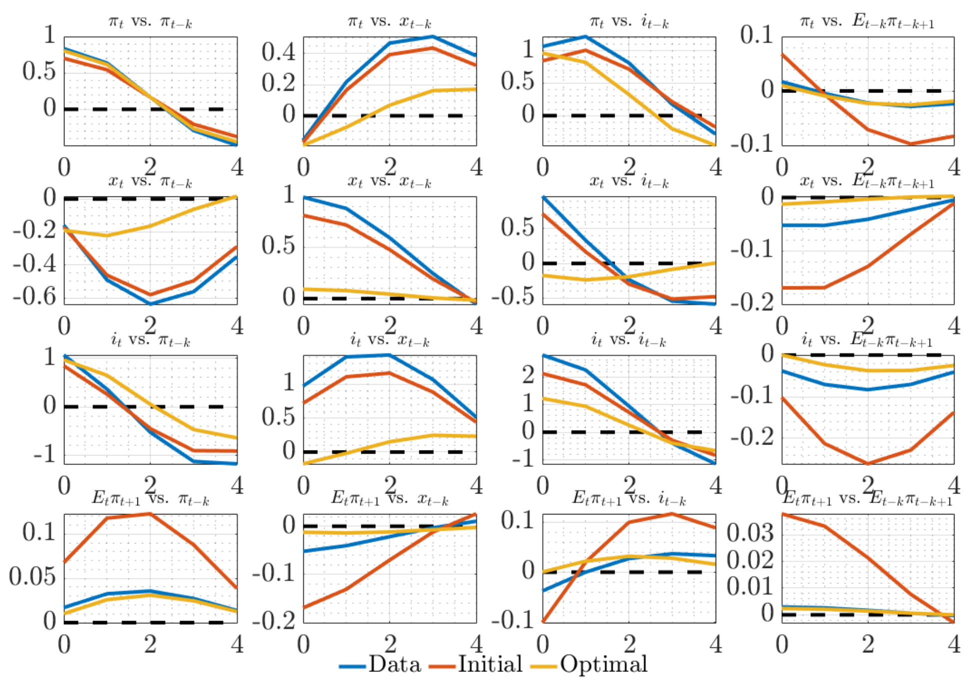
Overview

1 Inconsistency was: didn't annualize expectations in true data

Figure 1: Calibration C, use expectations, ridge tuning = 0.01, initialize at truth

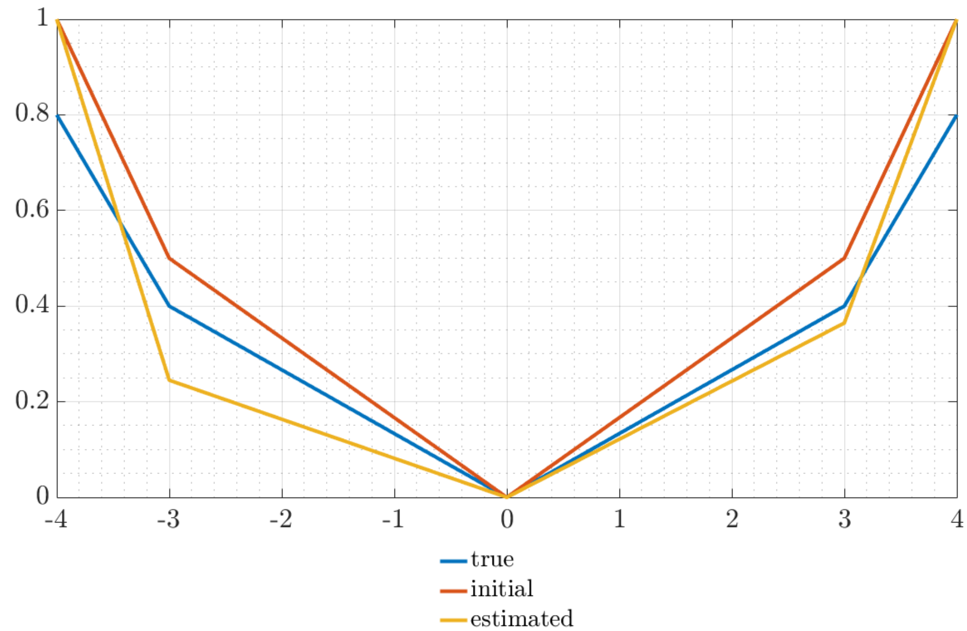


(a)

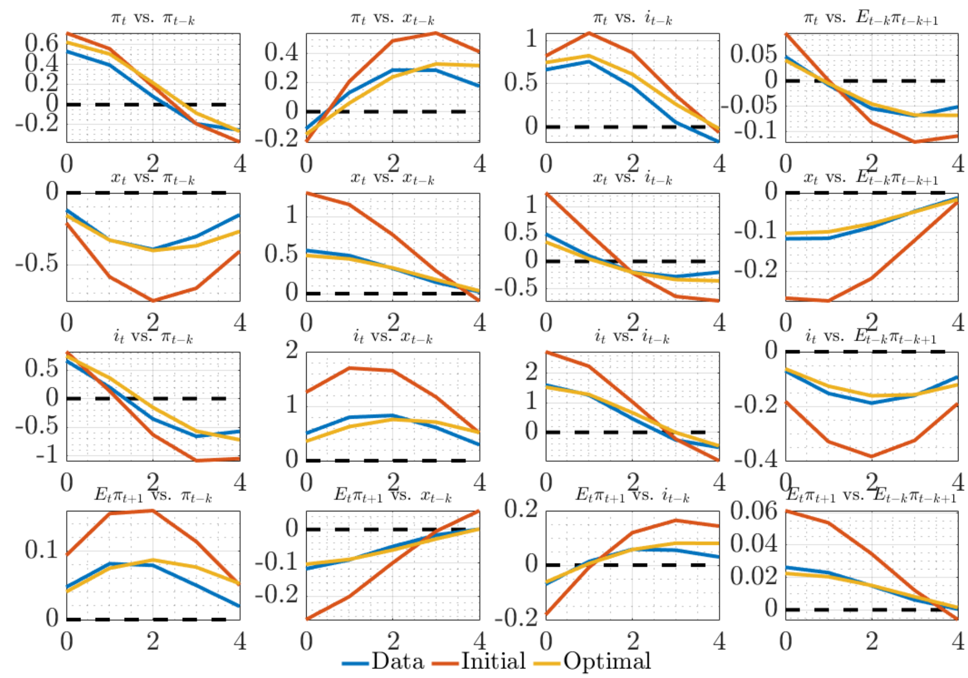


(b)

Figure 2: Calibration C, use expectations, ridge tuning = 0.01, initialize above truth, annualize expectations in true data



(a)

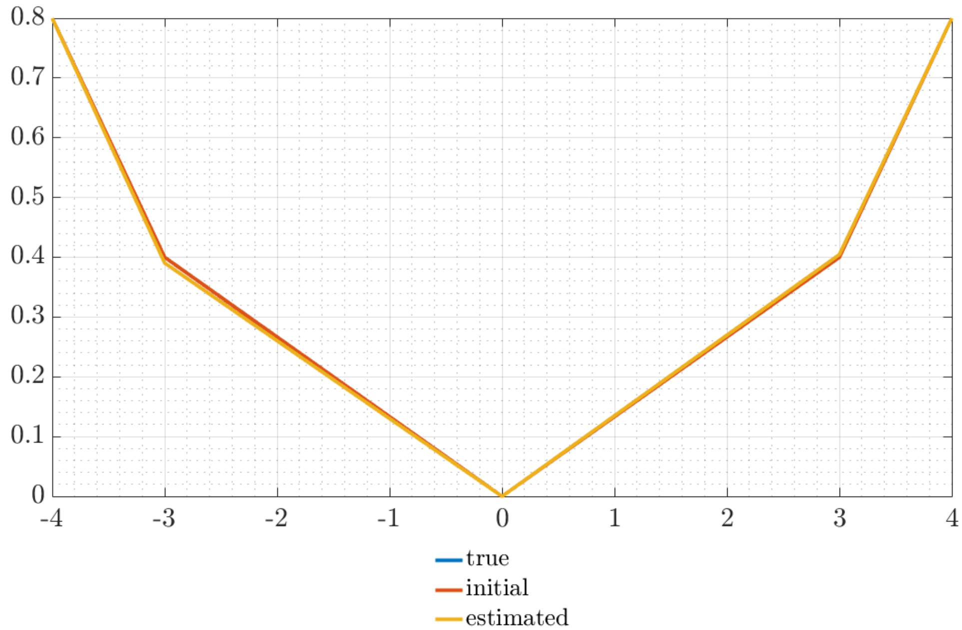


(b)

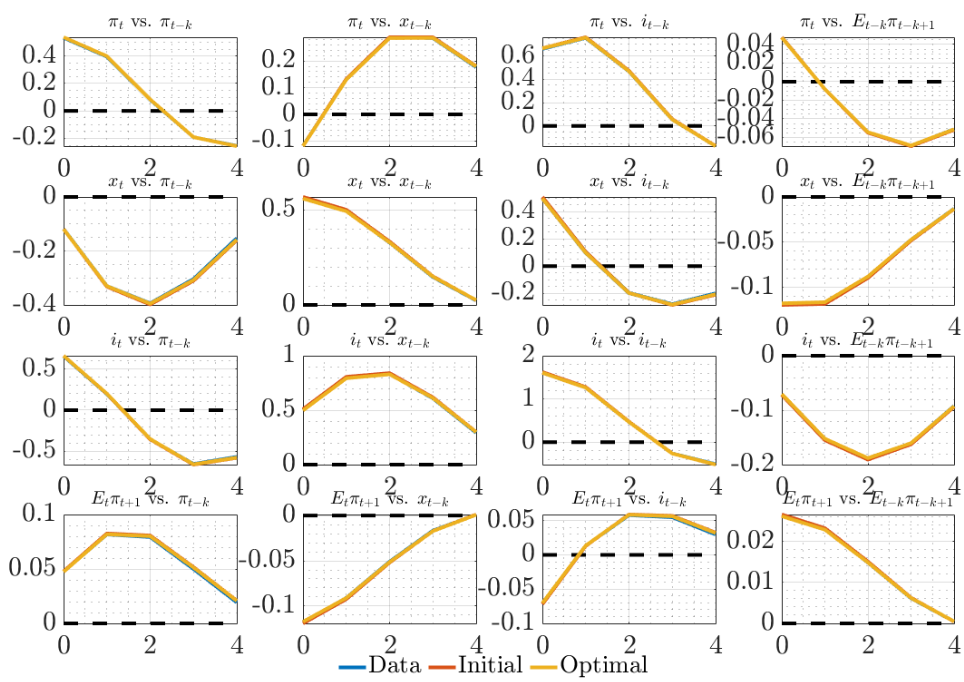
I think that's success!

1.1 Check with same seed as true data and $N = 1$

Figure 3: Calibration C, use expectations, ridge tuning = 0.01, initialize at truth, annualize expectations in true data



(a)

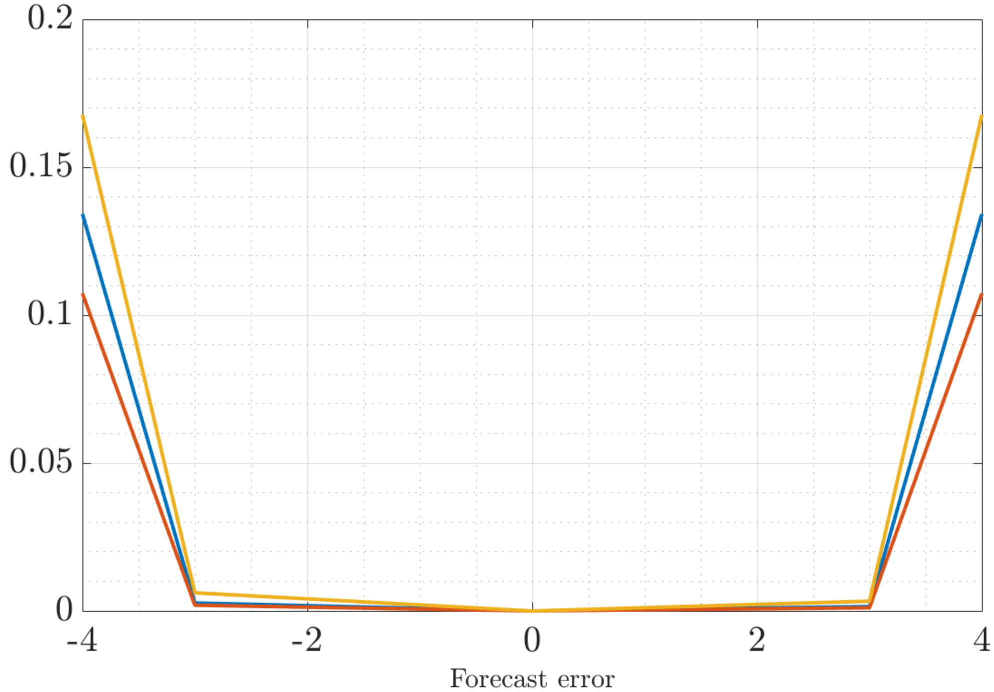


(b)

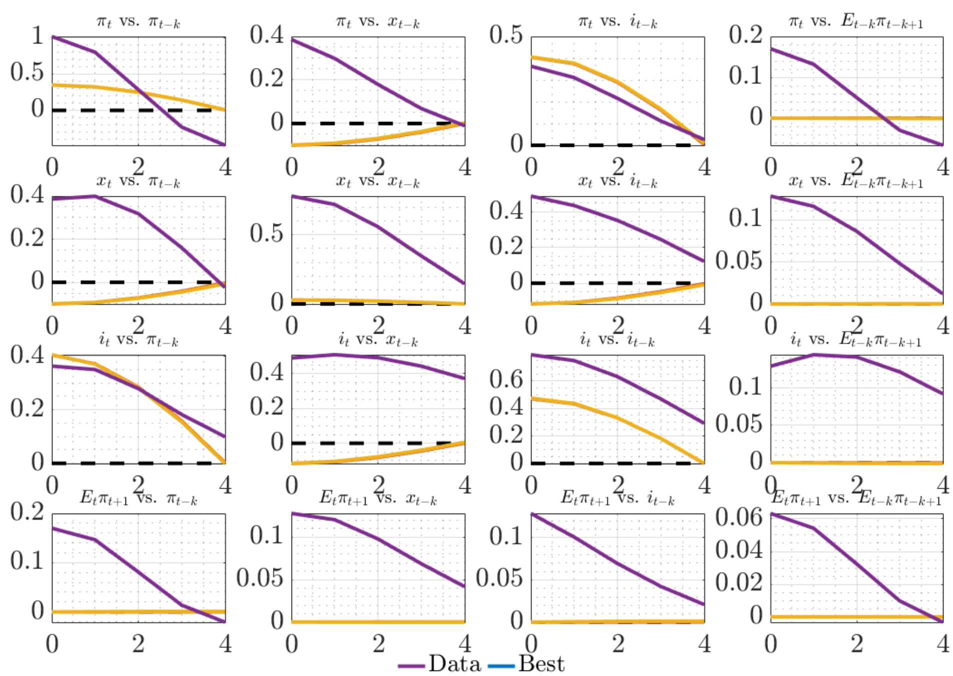
Yes!

2 Real data

Figure 4: Settings from Fig 2, 10 different starting points, showing top 3 (blue is best), $N = 100$



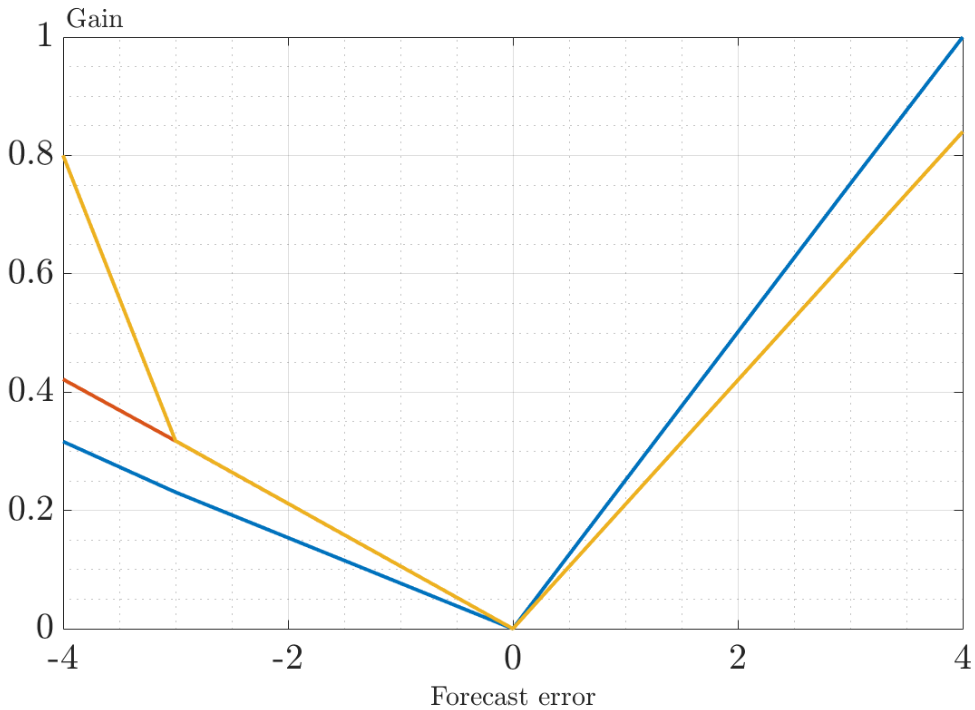
(a)



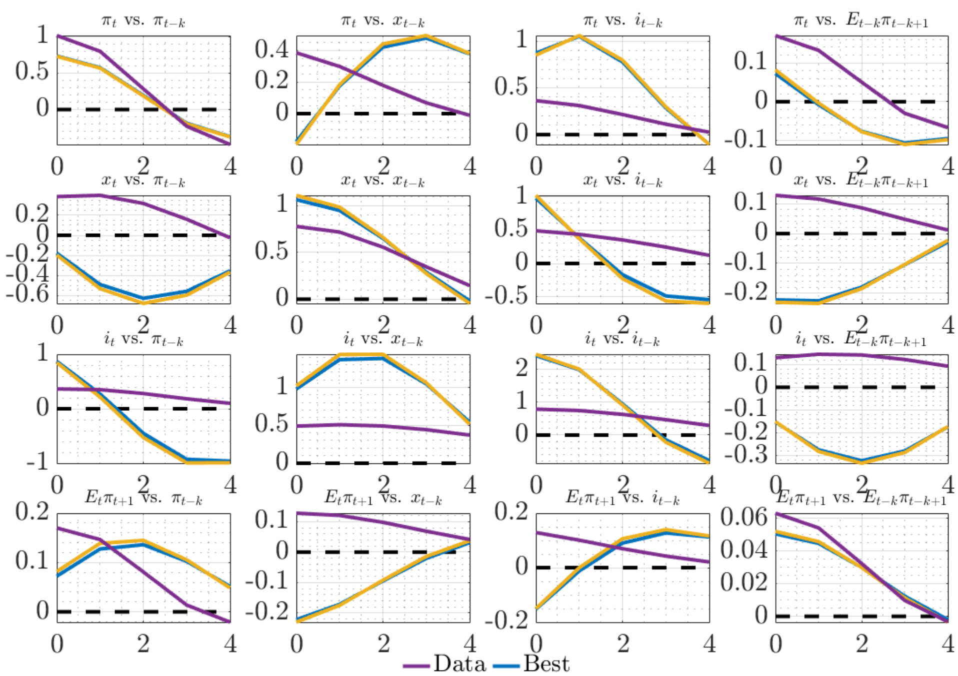
(b)

Darn. It doesn't hit the moments I want it to. So let's manually re-weight the own-autocovariances.

Figure 5: Settings from Fig 2, 5 different starting points, showing top 3 (blue is best), $N = 100$, manually putting more weight on own autocovariances

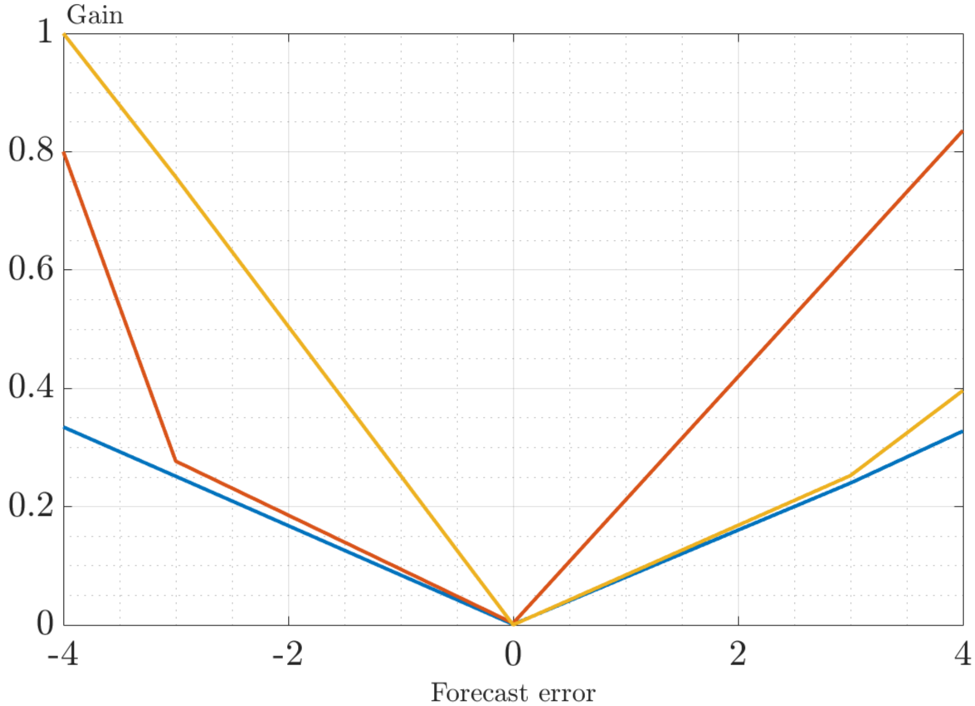


(a)

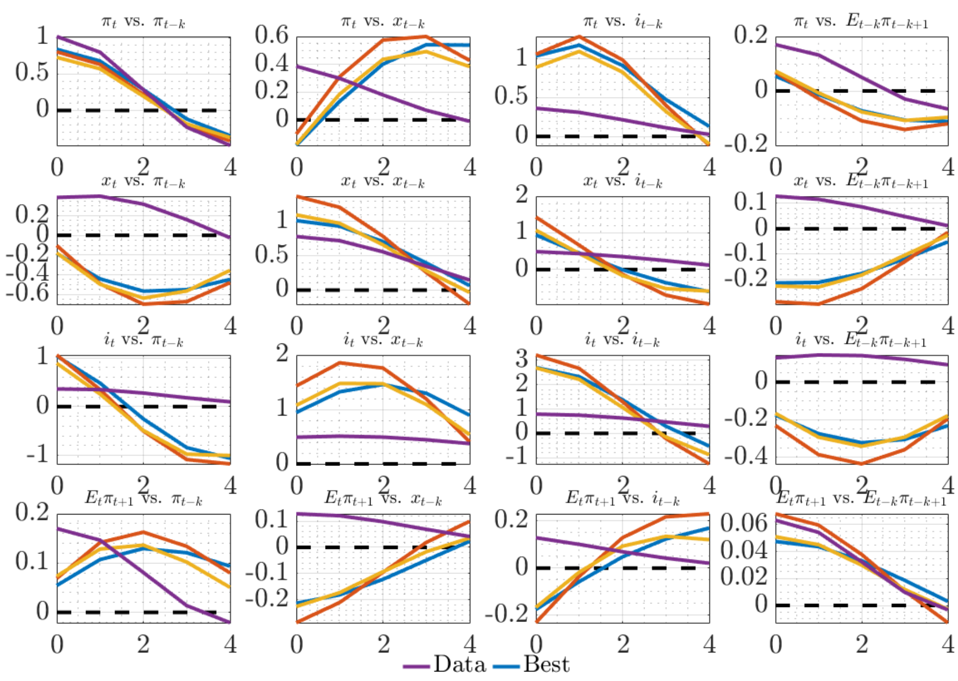


(b)

Figure 6: Settings from Fig 2, 5 different starting points, showing top 3 (blue is best), $N = 500$,manually putting more weight on own autocovariances



(a)

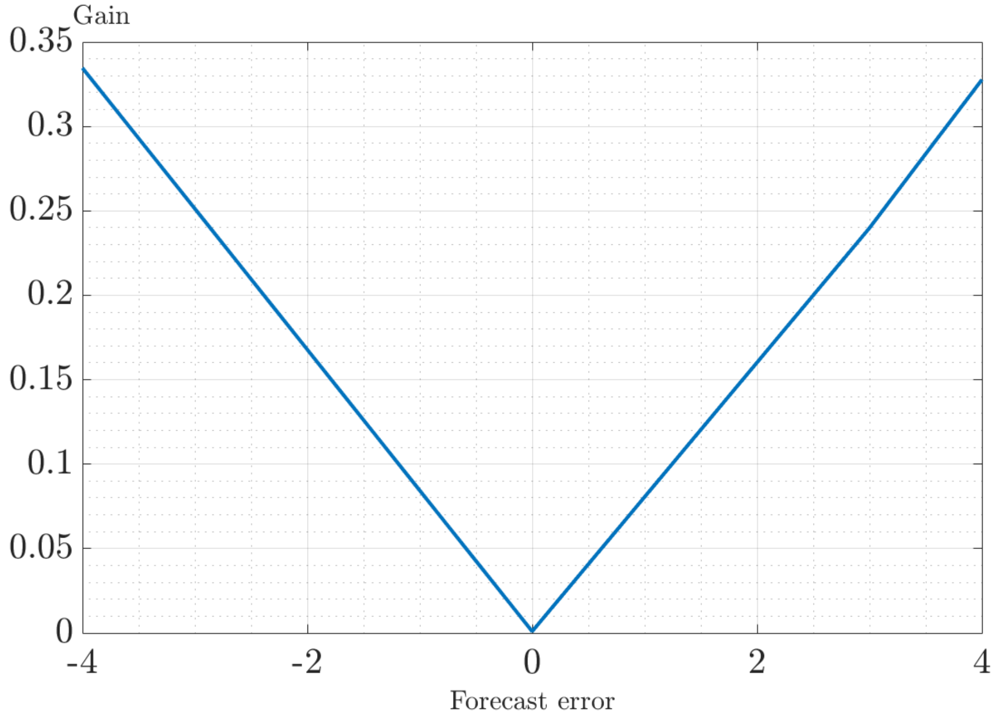


(b)

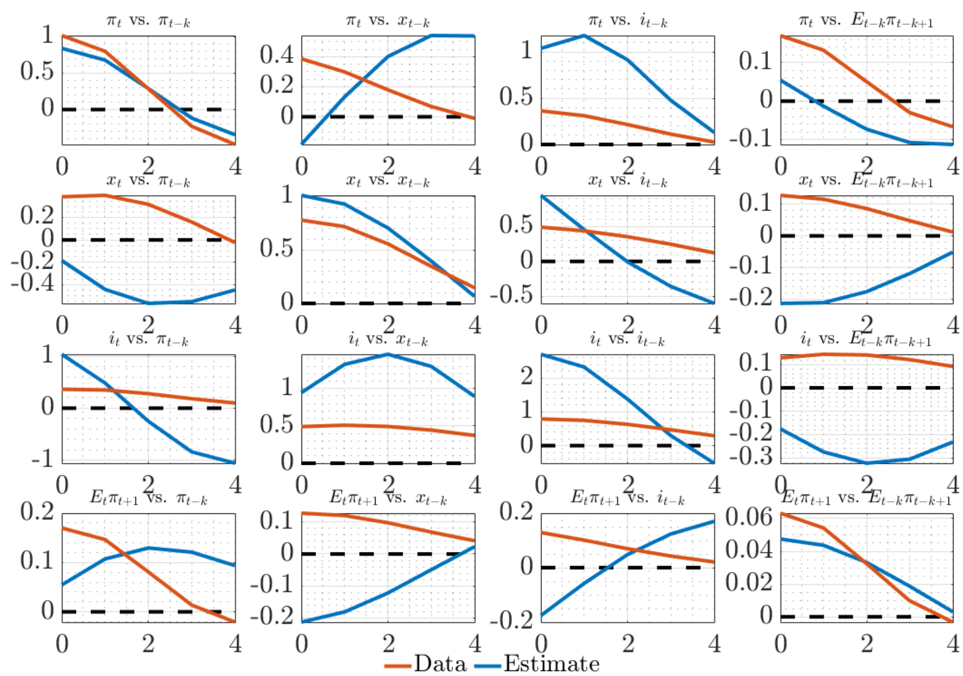
This took 5h:12min.

$\hat{\alpha}_i = (0.3346; 0.2513; 0.001; 0.2399; 0.3277)$

Figure 7: Same as the previous, just the best candidate alone

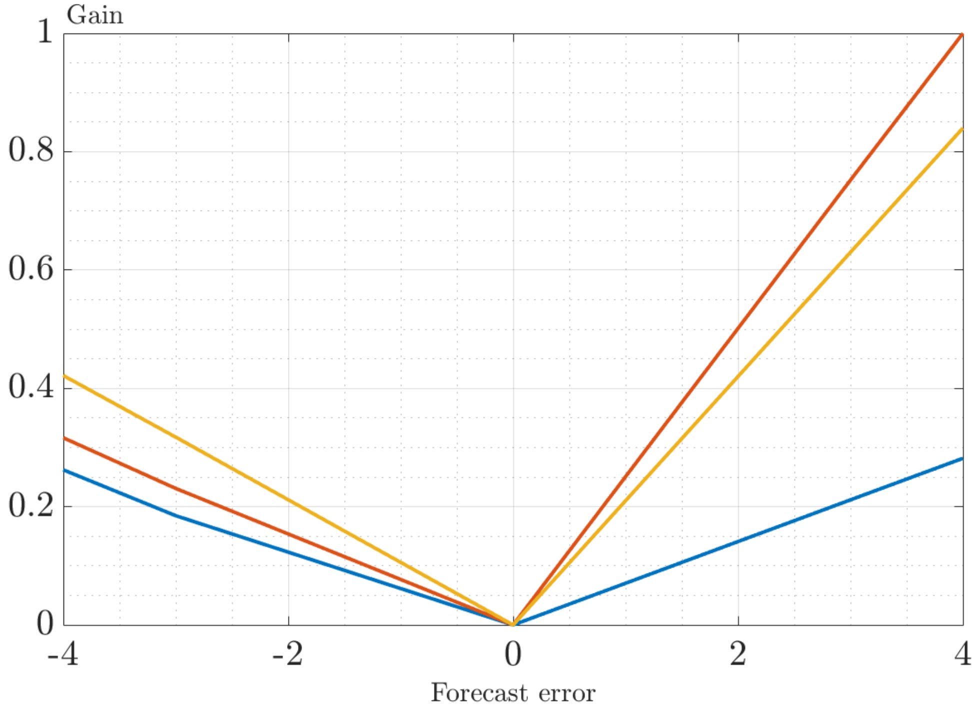


(a)

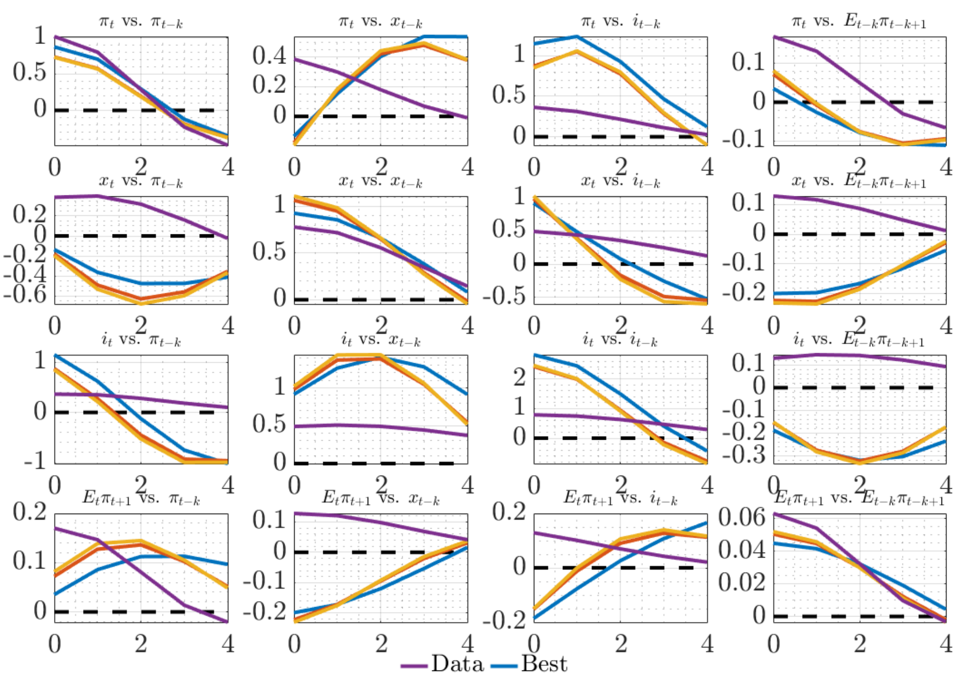


(b)

Figure 8: Settings from Fig 2, 20 different starting points, showing top 3 (blue is best), $N = 100$, manually putting more weight on own autocovariances



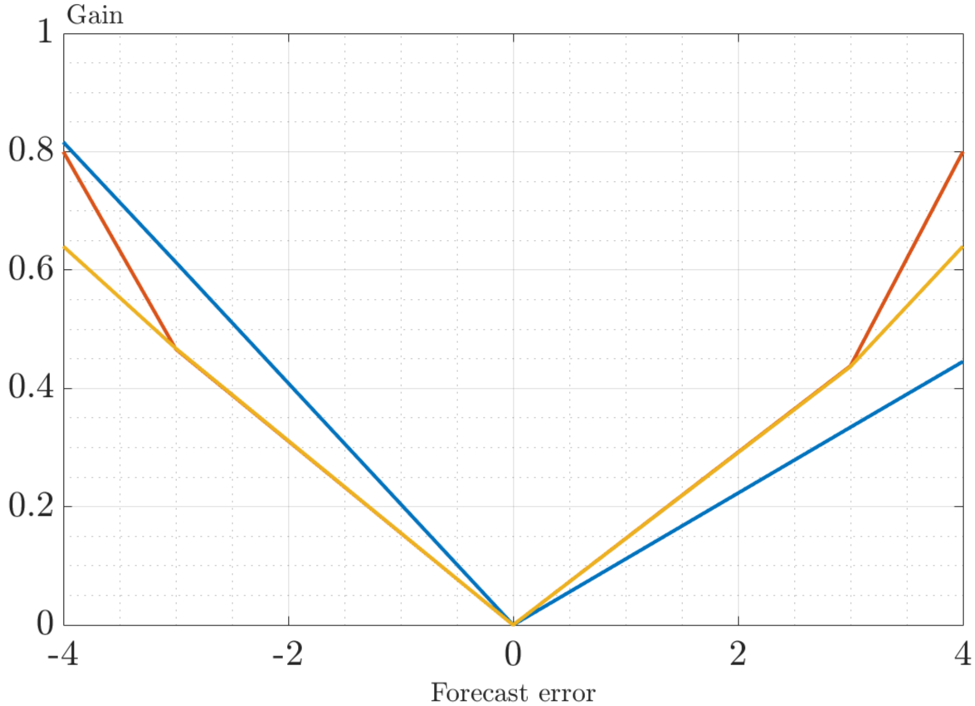
(a)



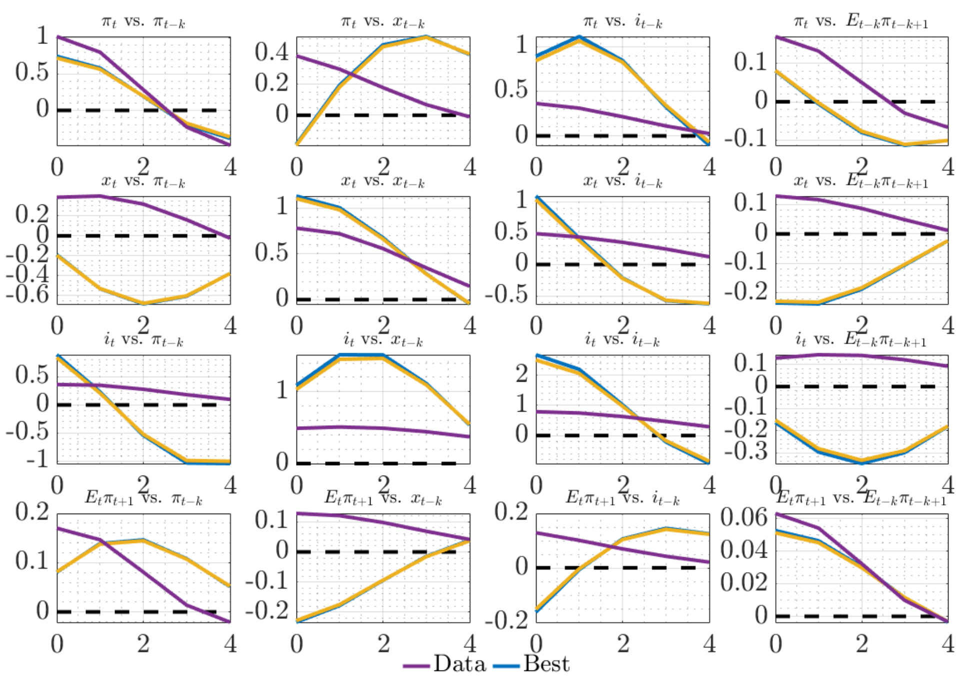
(b)

$\hat{\alpha}_i = (0.2621; 0.1847; 0; 0.2115; 0.2817)$

Figure 9: Settings from Fig 2, 10 different starting points, showing top 3 (blue is best), $N = 1000$, manually putting more weight on own autocovariances



(a)



(b)

$\hat{\alpha}_i = (0.8161; 0.61330; 0.3342; 0.4452)$

(Saved as `estim_LOMgain_outputs_univariate_coax15_Sep_2020_16_14_00.mat`)

3 Policy isn't a function of k_t^{-1}

The anchoring function is (A.6): $k_t^{-1} = \sum_i \alpha_i b_i(fe_{t|t-1})$. This essentially eliminates k as a state variable.

A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{A.1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{A.2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \qquad \text{(if imposed)} \tag{A.3}$$

$$\text{PLM:} \qquad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\, h_x \tag{A.4}$$

$$\text{Updating:} \qquad a_t = a_{t-1} + k_t^{-1}\big(z_t - (a_{t-1} + b s_{t-1})\big) \tag{A.5}$$

$$\text{Anchoring function:} \qquad k_t^{-1} = \sum_i \alpha_i b_i (fe_{t|t-1}) \tag{A.6}$$

$$\text{Forecast error:} \qquad fe_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \tag{A.7}$$

$$\text{LH expectations:} \qquad f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \qquad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \tag{A.8}$$

This notation captures vector learning (z learned) for intercept only. For scalar learning, $a_t = \begin{pmatrix}\bar{\pi}_t & 0 & 0\end{pmatrix}'$ and b_1 designates the first row of b. The observables (π, x) are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix}\sigma & 1-\beta & -\sigma\beta\end{bmatrix} f_b + \sigma \begin{bmatrix}1 & 0 & 0\end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \tag{A.9}$$

$$\pi_t = \kappa x_t + \begin{bmatrix}(1-\alpha)\beta & \kappa\alpha\beta & 0\end{bmatrix} f_a + \begin{bmatrix}0 & 0 & 1\end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \tag{A.10}$$

B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only, $k_t^{-1} = \mathbf{g}(fe_{t-1})$):

$$\pi_t = -\frac{\lambda_x}{\kappa}\Bigg\{x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_\pi(t)\Big)$$
$$\Big(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi}}(t+j))\Big)\Bigg\} \tag{B.1}$$

where I'm using the notation that $\prod_{j=0}^{0} \equiv 1$. For interpretation purposes, let me rewrite this as follows:

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + fe_{t|t-1}^{eve}\mathbf{g}_\pi(t)\Big)\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i}$$
$$-\frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + fe_{t|t-1}^{eve}\mathbf{g}_\pi(t)\Big)\Big(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j}^{-1} + fe_{t+1+j|t+j}^{eve})\mathbf{g}_{\bar{\pi}}(t+j)\Big) \tag{B.2}$$

Interpretation: tradeoffs from discretion in RE + effect of current level and change of the gain on future tradeoffs + effect of future expected levels and changes of the gain on future tradeoffs