Materials 23

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Overview

A   Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \qquad \text{(if imposed)} \tag{3}$$

$$\text{PLM:} \qquad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + bh_x^{h-1}s_t \quad \forall h \geq 1 \qquad b = g_x\, h_x \tag{4}$$

$$\text{Updating:} \qquad a_t = a_{t-1} + k_t^{-1}\big(z_t - (a_{t-1} + bs_{t-1})\big) \tag{5}$$

$$\text{Anchoring function:} \qquad k_t = k_{t-1} + \mathbf{g}(fe_{t-1}^2) \tag{6}$$

$$\text{Forecast error:} \qquad fe_{t-1} = z_t - (a_{t-1} + bs_{t-1}) \tag{7}$$

This notation captures vector learning (z learned) for intercept only. For scalar learning, $a_t = \big(\bar{\pi}_t \;\; 0 \;\; 0\big)'$ and b_1 designates the first row of b. The learning setup allows me to evaluate the long-horizon expectations f_a, f_b, and then the observables (π, x) are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix} \sigma & 1-\beta & -\sigma\beta \end{bmatrix} f_a + \sigma \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \tag{8}$$

$$\pi_t = \kappa x_t + \begin{bmatrix} (1-\alpha)\beta & \kappa\alpha\beta & 0 \end{bmatrix} f_b + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \tag{9}$$

B   Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only):

$$\pi_t = -\frac{\lambda_x}{\kappa}\bigg\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\bigg(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_\pi(t) \bigg)$$
$$\bigg(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=1}^{i-1}(1 - k_{t+j}^{-1}(\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})) \bigg) \bigg\} \tag{10}$$