# Materials 36 - Convince that estimation is robust

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## Overview

# 1 Calibration issues

## 1.1 $\alpha = Prob($**keep same price**$) = $ ?

So far I've used 0.5.

- Nikolay Hristov notes: expected duration of contract $= \frac{1}{1-\alpha}$ periods.
- Evidence on average duration of prices:
  - Bils & Klenow (2004): 4.3 months
  - Klenow & Kryvstov (2008): mean 7-9 months, median 4-7 months
  - Nakamura & Steinsson (2008): 7-9 months
  - Klenow & Malin (2010): 6.9 months
  - Eichenbaum, Jaimovitch & Rebelo (2008, published as 2011): 10.6 months

$\rightarrow$ On average this gives us 7.56 months, a little more than two quarters. The implied $\alpha \approx 0.6$. Rotemberg & Woodford (1997) calibrate $\alpha = 0.66$. To err on the flexible price side, I set $\alpha = 0.5$.

## 1.2 The composite parameter $\kappa$ once and for all

I've used $\kappa = \frac{(1-\alpha\beta)}{\alpha}\zeta$ where I should have used $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\zeta$. In Preston (2005), and I think this is also Woodford's favorite specification, $\zeta = \frac{\omega+\sigma^{-1}}{1+\omega\theta}$. Let's define terms:

- $\alpha$: Prob(keep price unchanged)
- $\beta$: discount factor
- $\zeta$: measure of strategic complementarity in price setting. The smaller $\zeta$, the more complementarity. This depends on a bunch of things:
  - homogenous vs. specific factor markets ($s_y = 0$ or not)
  - constant vs. variable desired markup ($\epsilon_\mu = 0$ or not)
  - no vs. intermediate inputs ($s_m = 0$ or not)

  In particular (Prop 3.3 in Woodford 2011, equation 1.43, Chapter 3, p. 171):

$$\zeta = \frac{(1-\mu s_m)(s_y + s_Y)}{1+\theta[\epsilon_\mu + (1-\mu s_m)s_y]} \tag{1}$$

  with

- $\theta$: price elasticity of demand
- $\mu(x)$: markup function
- $\epsilon_\mu$: elasticity of markup function (how much do target markups change at different levels of output)
- $s(y, Y, \xi)$: real marginal cost function
- $s_y$: elasticity of real marginal cost function wrt firm i's output, $y_t(i)$
- $s_Y$: elasticity of real marginal cost function wrt aggregate output, $y_t$
- $s_m$: elasticity of real marginal cost function wrt intermediate inputs, $m_t(i)$

Then expression $\zeta = \frac{\omega+\sigma^{-1}}{1+\omega\theta}$ is obtained by assuming no intermediate inputs, constant desired markups wrt. output levels and specific factor markets, so that

$$\zeta = \frac{s_y + s_Y}{1 + s_y\theta} \tag{2}$$

What is $\omega$? It's the derivative of the MC function wrt own output, but this only coincides with $s_y$ for specific factor markets. Woodford shows that for specific factor markets, $s_y = \omega, s_Y = \sigma^{-1}$, while for common factor markets $s_Y = \omega + \sigma^{-1}$ because for the latter, there is no distinction between own and aggregate output for the purpose of wage setting and thus marginal cost. So, more broadly, $\omega$ is a measure of how marginal cost reacts to some wage-relevant measure of output. Woodford, Chapter 3, (1.16), p. 152:

$$\omega = \underbrace{\omega_w}_{=\eta,\text{ Frisch elasticity of disutility of labor wrt output}} + \underbrace{\omega_p}_{\text{elasticity of MPL wrt output}} \tag{3}$$

Denoting the Frisch elasticity as $\eta$, and noting that for Cobb-Douglas, $\partial MPL/\partial y_t = 0$,

$$\omega = \eta \tag{4}$$

- Chari, Kehoe & McGrattan (2000) and Woodford (2011) values: $\theta = 10, \sigma = 1, \omega = 1.25, \beta = 0.99$ These values are not controversial. I just want to check that the Frisch elasticity is Kosher, because by setting $\omega = 1.25$, we are implicitly setting the Frisch. According to Susanto (compare my summaries Part 1, p. 56 Mac and Part 2 p. 46-47 Mac), the inverse Frisch elasticity, $\varepsilon_{H,W} = \eta^{-1}$ needs to be 4 to flatten the labor supply curve (Barro-King comovement puzzle), but estimates suggests it's $< 1$. Here, $\omega = 1.25 = \eta$ implies $\eta^{-1} = 4/5 < 1$. I guess that's at least in line with estimates.

- The value of $\kappa$

  Susanto suggests (somewhere in my notes) that for a NK model to display reasonable dynamics, $\kappa$ needs to be below 0.1, but preferably even less than 0.01.

  Chari, Kehoe & McGrattan (2000) and Woodford (2011) values with $\alpha = 0.5 \rightarrow \kappa = 0.0842$.

  Chari, Kehoe & McGrattan (2000) and Woodford (2011) values with $\alpha = 0.6 \rightarrow \kappa = 0.0451$.

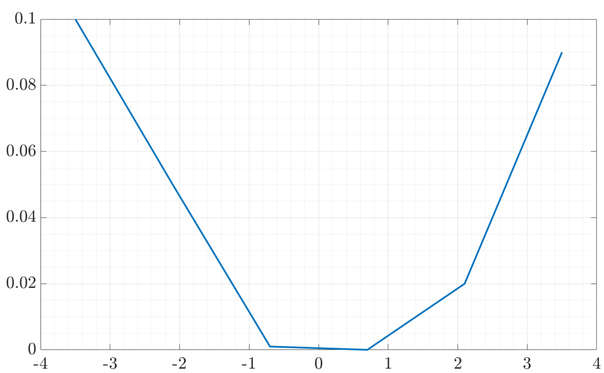
  Chari, Kehoe & McGrattan (2000) and Woodford (2011) values with $\alpha = 0.66 \rightarrow \kappa = 0.0298$.

  Chari, Kehoe & McGrattan (2000) and Woodford (2011) values with $\alpha = 0.7 \rightarrow \kappa = 0.0219$.

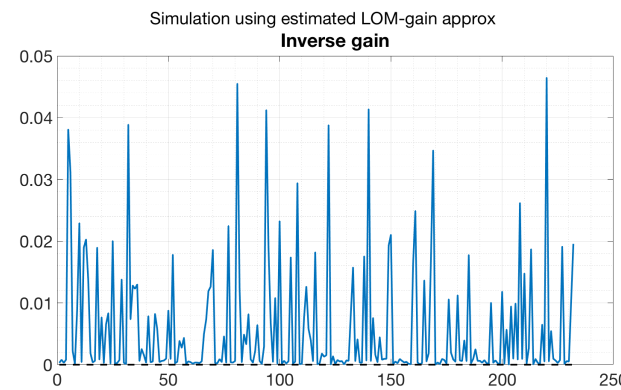
# 2 Back to estimation

## 2.1 Simulated data with different seed

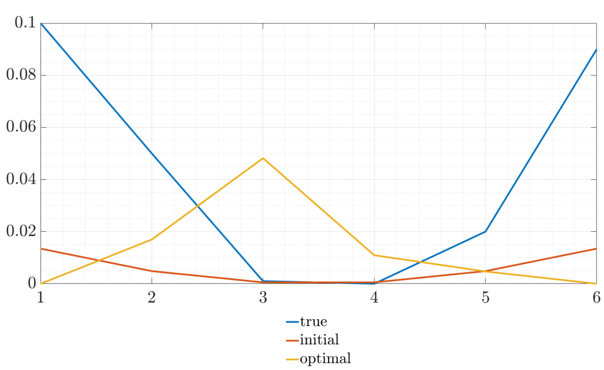
**Figure 1:** A seed for shocks of `rng(1)` when true data was generated using `rng(0)`



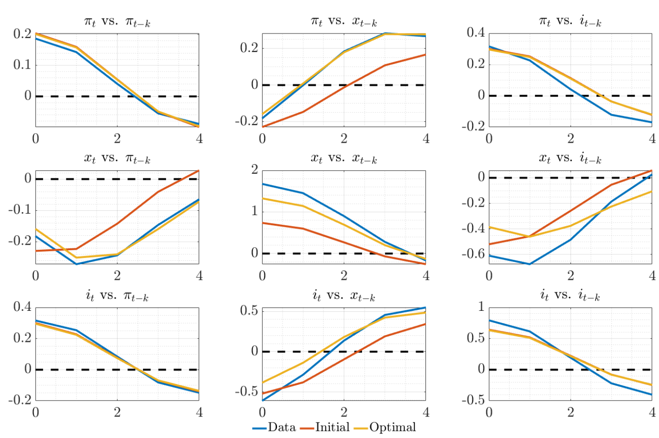
(a) $\alpha^{true} = (0.1; 0.05; 0.001; 0.00001; 0.02; 0.09)$




(b) Gain for a simulation using $\alpha^{true}$

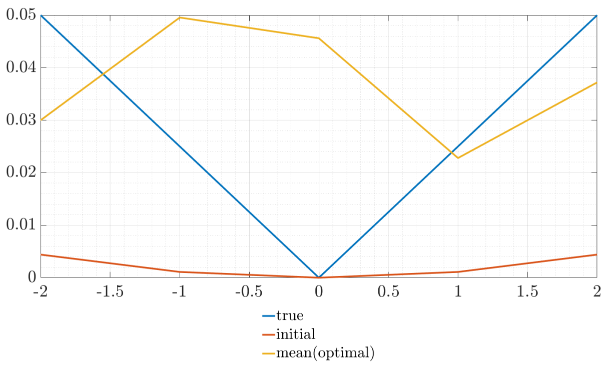



(c) $\alpha^{true}, \alpha_0, \hat{\alpha}$, right now for nonconvergent solution using default configs

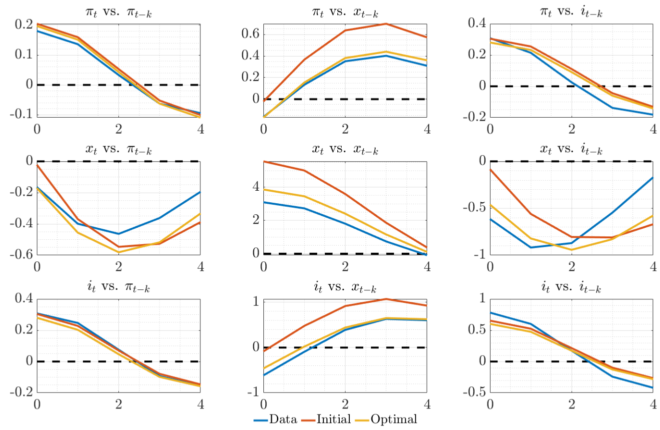



(d) Autocovariogram, right now for nonconvergent solution using default configs

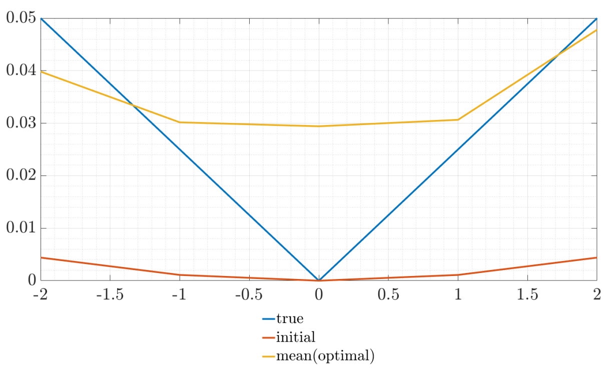
**Figure 2:** Mean of top 10 estimates for 20 draws of shock histories when true data was generated using `rng(0)` and $\alpha^{true} = (0.05; 0.025; 0; 0.025; 0.05)$, $fe \in (-2, 2)$



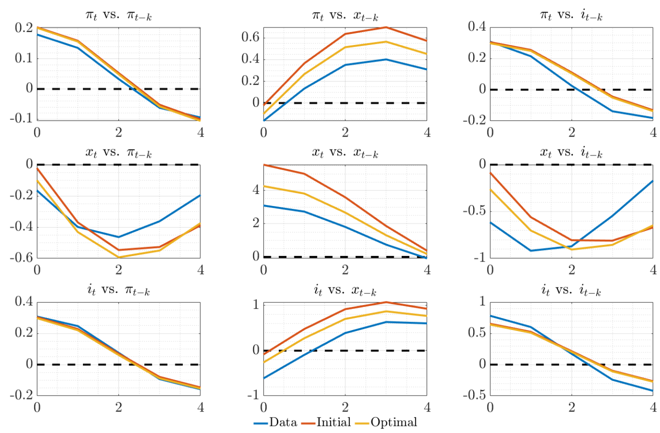

(a) $\alpha^{true}, \alpha_0, mean(\hat{\alpha}^{top10})$, no additional moments




(b) Autocovariogram, no additional moments

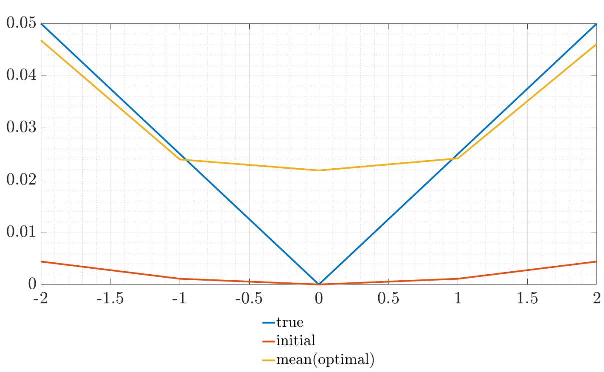



(c) $\alpha^{true}, \alpha_0, mean(\hat{\alpha}^{top10})$, convexity moments imposed with weight 10M

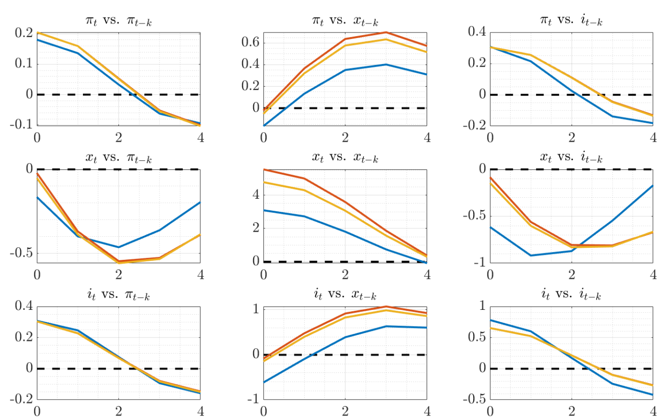



(d) Autocovariogram, convexity moments imposed with weight 10M

**Figure 3:** Mean estimates for increasing $N$, imposing convexity

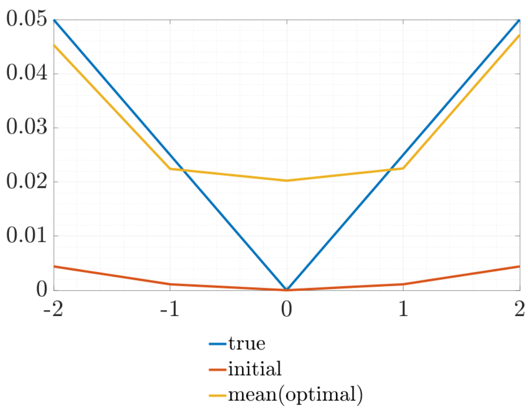



(a) $\alpha^{true}, \alpha_0, mean(\hat{\alpha})$, N=1000, convexity moments imposed with weight 10M

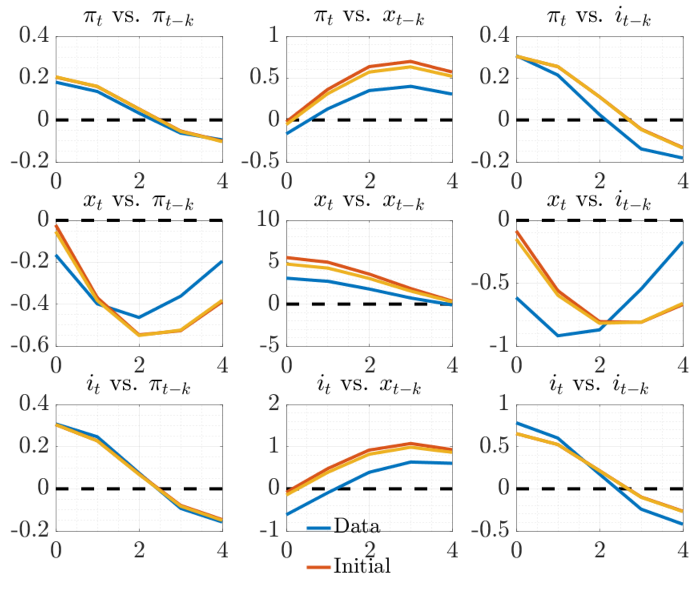



(b) Autocovariogram, convexity moments imposed with weight 10M

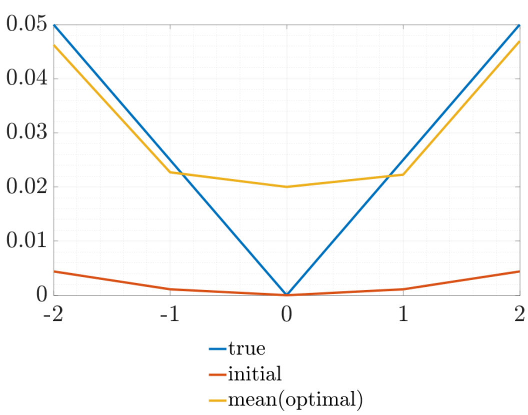
**Figure 4:** Mean estimates for increasing $N$, imposing convexity, continued



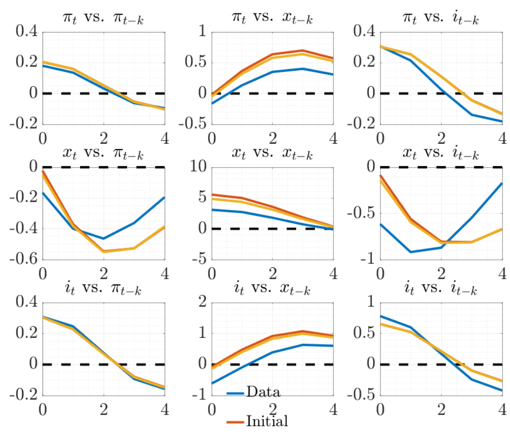

(a) $\alpha^{true}, \alpha_0, mean(\hat{\alpha})$, N=2000, convexity moments imposed with weight 10M




(b) Autocovariogram, convexity moments imposed with weight 10M



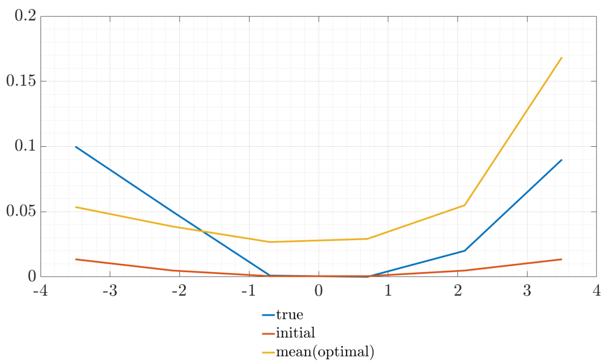

(c) $\alpha^{true}, \alpha_0, mean(\hat{\alpha})$, N=10000, convexity moments imposed with weight 10M, $mean(\hat{\alpha}) = 0.0463; 0.0227; 0.02; 0.0223; 0.0469)$



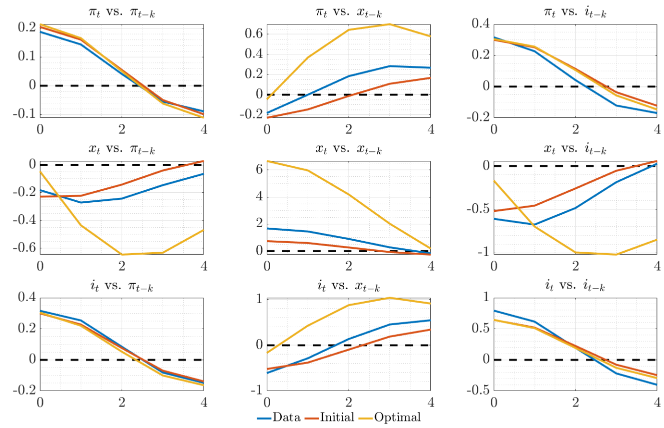

(d) Autocovariogram, convexity moments imposed with weight 10M

### 2.1.1 The range $fe \in (-1, 1)$ seems ill identified

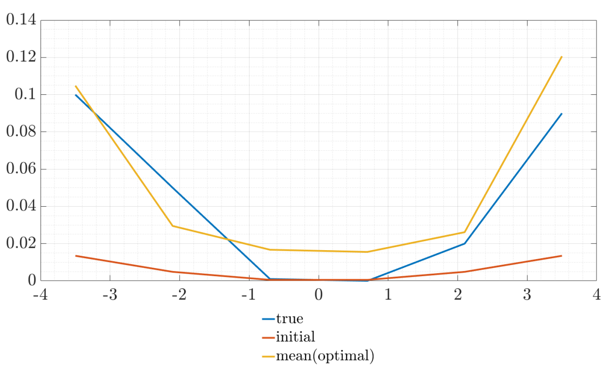
**Figure 5:** Mean estimates for increasing $N$, imposing convexity with weight 100K, truth with $nfe = 6, fe \in (-3.5, 3.5)$



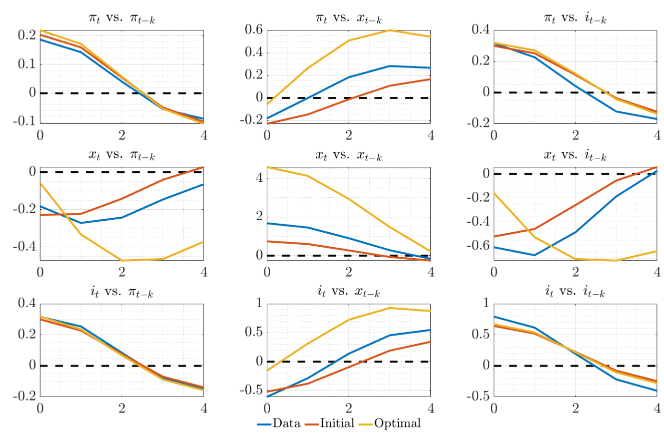

(a) $\alpha^{true}, \alpha_0, mean(\hat{\alpha})$, N=10



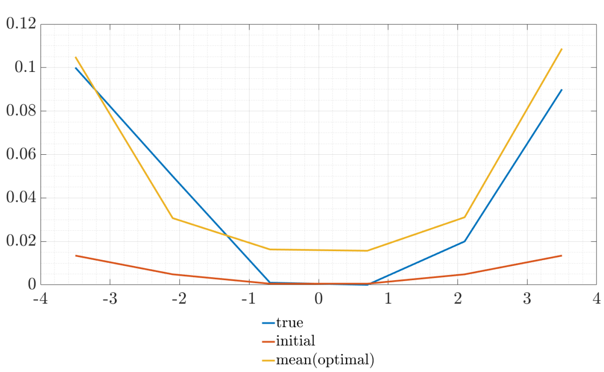

(b) Autocovariogram



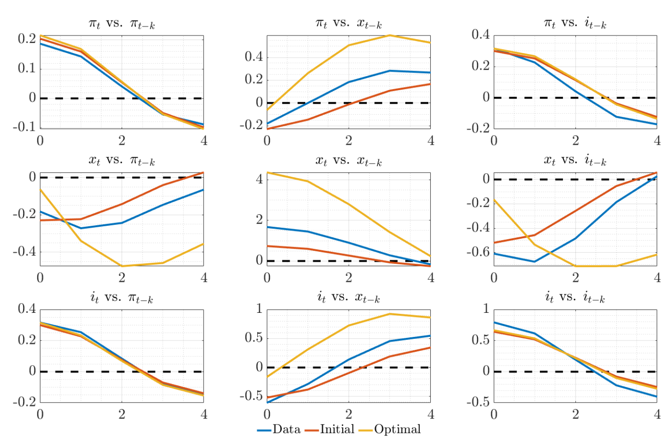

(c) $\alpha^{true}, \alpha_0, mean(\hat{\alpha})$, N=100, $mean(\hat{\alpha}) = (0.1048; 0.0295; 0.0167; 0.0155; 0.0261; 0.1206)$




(d) Autocovariogram

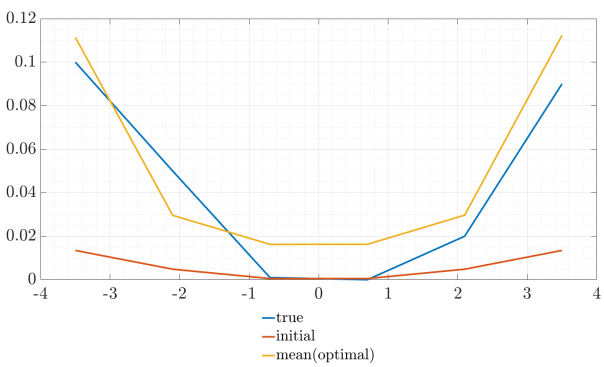



(e) $\alpha^{true}, \alpha_0, mean(\hat{\alpha})$, N=1000, $mean(\hat{\alpha}) = (0.1049; 0.0307; 0.0163; 0.0157; 0.0311; 0.1087)$

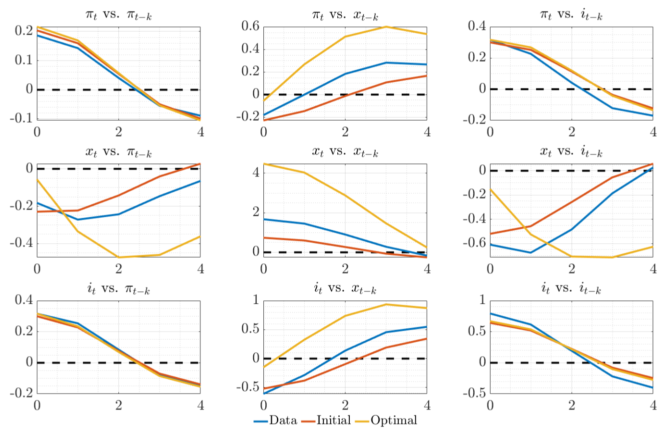



(f) Autocovariogram

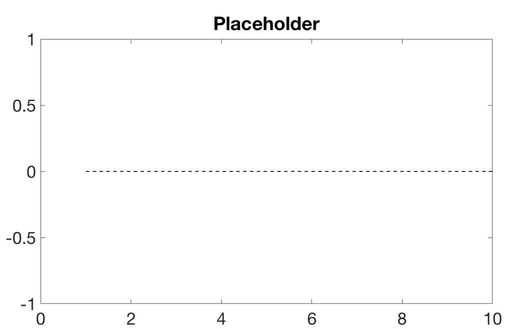
**Figure 6:** Mean estimates for increasing $N$, imposing convexity with weight 100K, truth with $nfe = 6, fe \in (-3.5, 3.5)$, continued



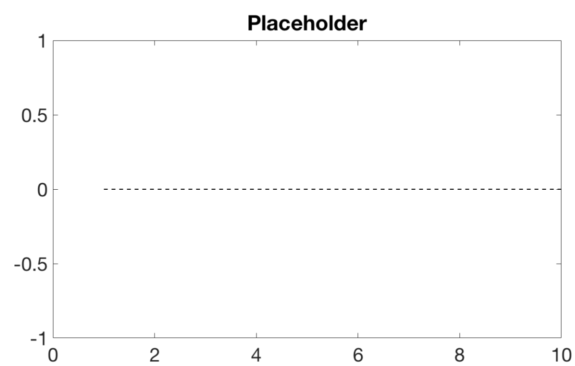

(a) $\alpha^{true}, \alpha_0, mean(\hat{\alpha})$, N=10000 , $mean(\hat{\alpha}) = (0.1114; 0.0296; 0.0163; 0.0163; 0.0297; 0.1123)$




(b) Autocovariogram



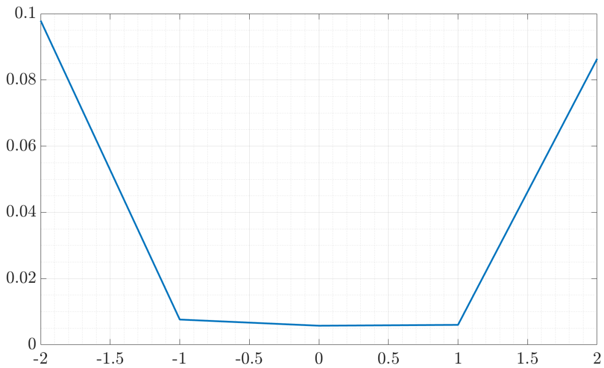

(c) $\alpha^{true}, \alpha_0, mean(\hat{\alpha})$, N=100000, $mean(\hat{\alpha}) = ()$



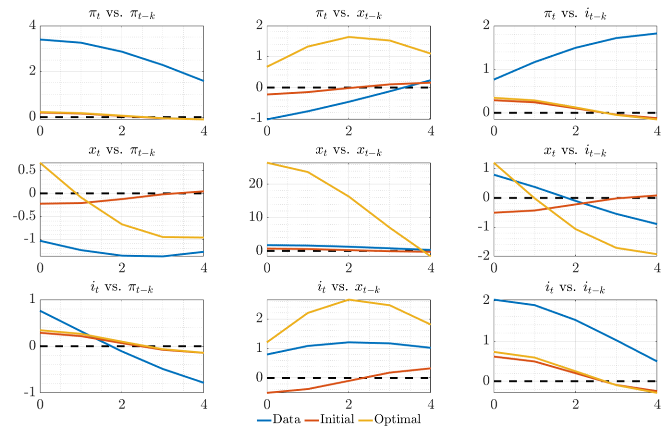

(d) Autocovariogram

## 2.2 Autocovariogram for real data

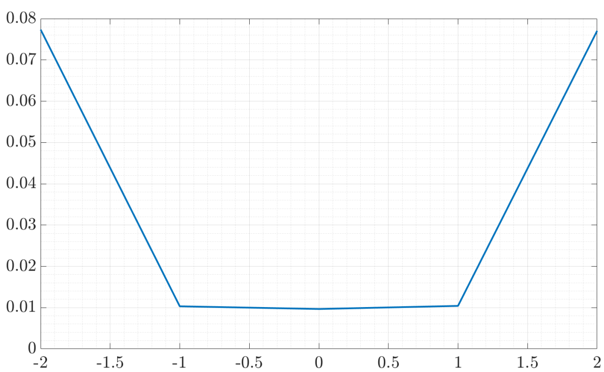
**Figure 7:** Mean estimated parameters over the cross-section of size $N$, convexity imposed, mean moment not imposed



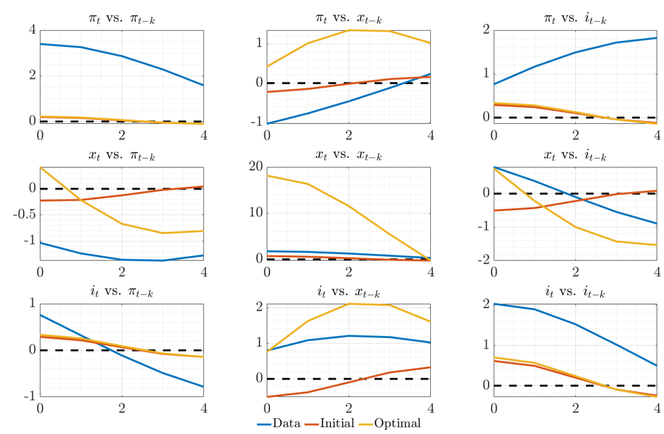
(a) $mean(\hat{\alpha}) = (0.0979; 0.0076; 0.0058; 0.006; 0.0864)$, N=1000, convexity weight 10M




(b) Autocovariogram



(c) $mean(\hat{\alpha}) = (0.0773; 0.0103; 0.0097; 0.0104; 0.0771)$, N=1000, convexity weight 100K

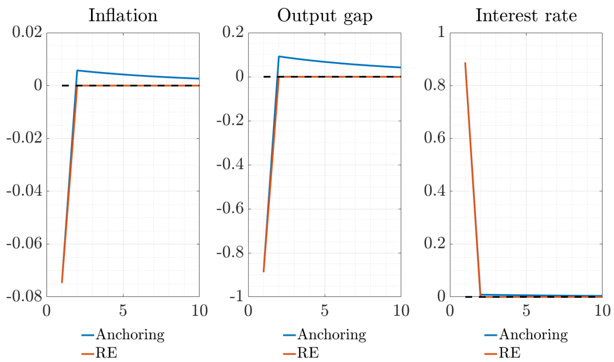



(d) Autocovariogram

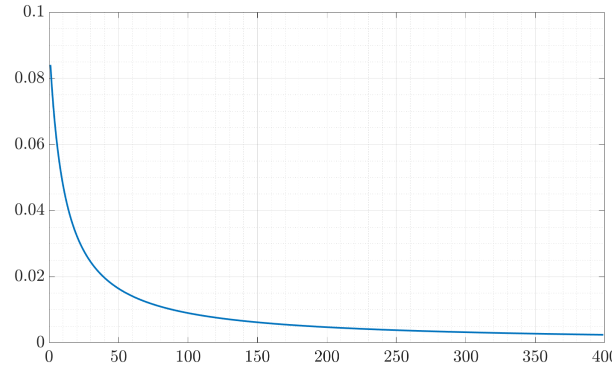
# 3 Impulse responses to iid monpol shocks across a wide range of learning models

$T = 400, N = 100, n_{drop} = 5$, shock imposed at $t = 25$, calibration as above, Taylor rule assumed to be known, PLM = learn constant only, of inflation only.

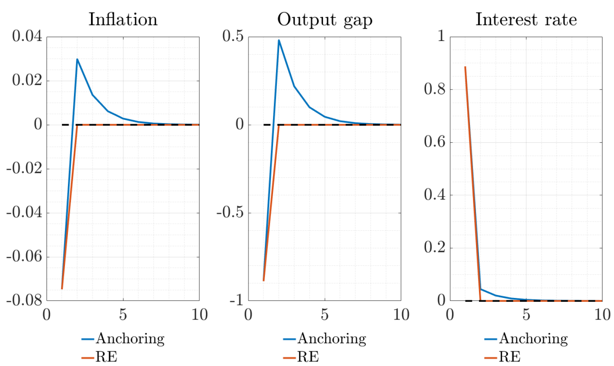
**Figure 8:** IRFs and gain history (sample means)



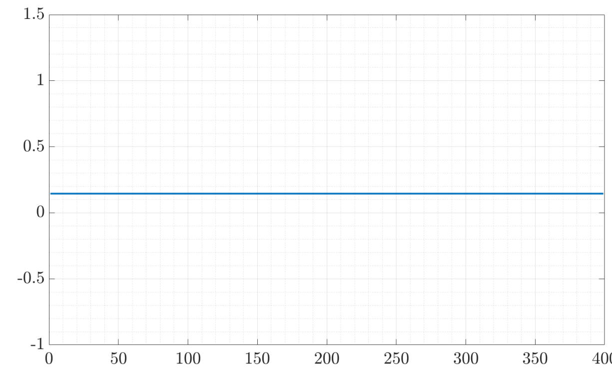

(a) Decreasing gain learning



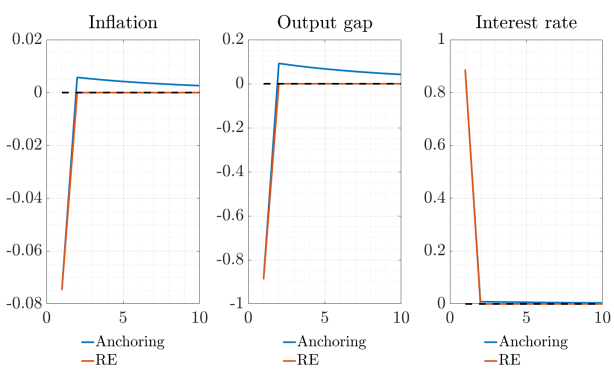
(b) Mean gain



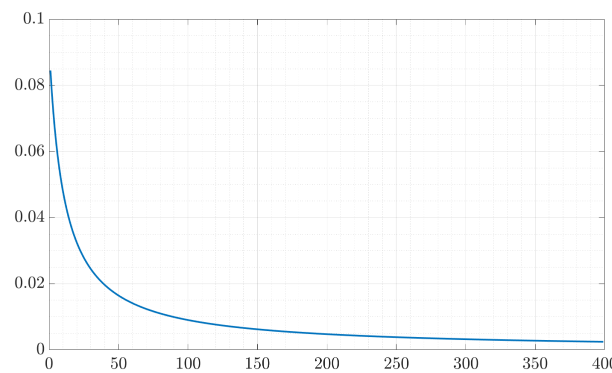

(c) Constant gain learning



(d) Mean gain

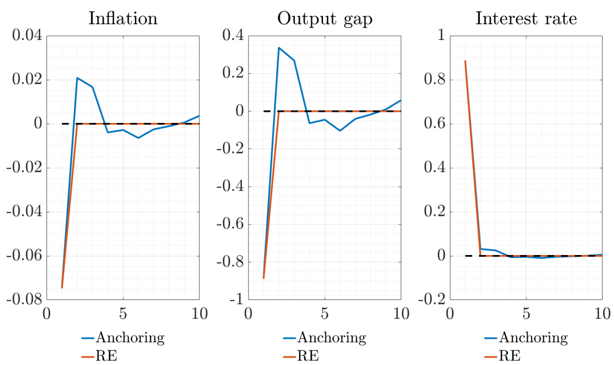



(e) CEMP criterion (vector)

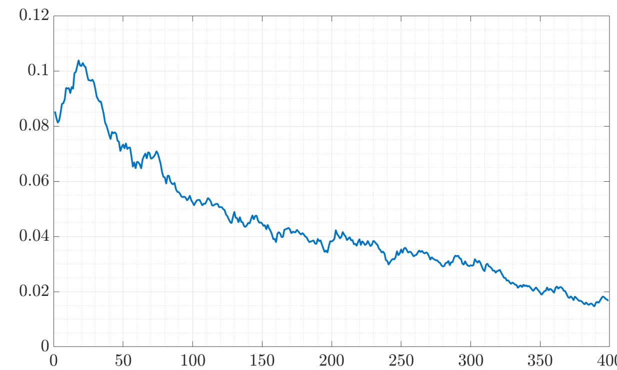


(f) Mean gain

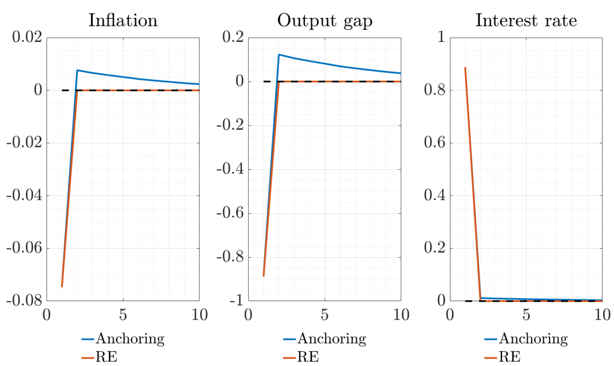
**Figure 9:** IRFs and gain history (sample means), continued



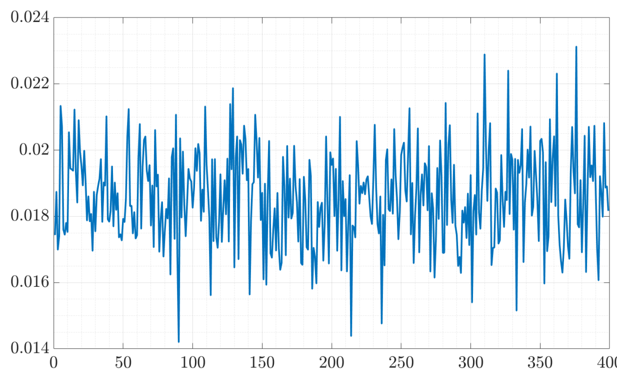

(a) CUSUM criterion (vector)



(b) Mean gain




(c) Smooth criterion, approximated, using $\alpha^{true} = (0.05; 0.025; 0; 0.025; 0.05)$, on $fe \in (-2, 2)$.



(d) Mean gain

## A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r^n_T\big) \tag{A.1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{A.2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \qquad \text{(if imposed)} \tag{A.3}$$

$$\text{PLM:} \qquad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\, h_x \tag{A.4}$$

$$\text{Updating:} \qquad a_t = a_{t-1} + k_t^{-1}\big(z_t - (a_{t-1} + b s_{t-1})\big) \tag{A.5}$$

$$\text{Anchoring function:} \qquad k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k f e_{t-1}^2 \tag{A.6}$$

$$\text{Forecast error:} \qquad fe_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \tag{A.7}$$

$$\text{LH expectations:} \qquad f_a(t) = \frac{1}{1-\alpha\beta}a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1}s_t \qquad f_b(t) = \frac{1}{1-\beta}a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1}s_t \tag{A.8}$$

This notation captures vector learning ($z$ learned) for intercept only. For scalar learning, $a_t = \begin{pmatrix}\bar{\pi}_t & 0 & 0\end{pmatrix}'$ and $b_1$ designates the first row of $b$. The observables $(\pi, x)$ are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix}\sigma & 1-\beta & -\sigma\beta\end{bmatrix} f_b + \sigma \begin{bmatrix}1 & 0 & 0\end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \tag{A.9}$$

$$\pi_t = \kappa x_t + \begin{bmatrix}(1-\alpha)\beta & \kappa\alpha\beta & 0\end{bmatrix} f_a + \begin{bmatrix}0 & 0 & 1\end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \tag{A.10}$$

## B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only, $k_t^{-1} = \mathbf{g}(fe_{t-1})$):

$$\pi_t = -\frac{\lambda_x}{\kappa}\Bigg\{x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_\pi(t)\Big)$$
$$\Bigg(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi}}(t+j))\Bigg)\Bigg\} \tag{B.1}$$

where I'm using the notation that $\prod_{j=0}^{0} \equiv 1$. For interpretation purposes, let me rewrite this as follows:

$$\pi_t = -\frac{\lambda_x}{\kappa}x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + fe^{eve}_{t|t-1}\mathbf{g}_\pi(t)\Big)\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i}$$
$$-\frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + fe^{eve}_{t|t-1}\mathbf{g}_\pi(t)\Big)\Bigg(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(k_{t+1+j}^{-1} + fe^{eve}_{t+1+j|t+j})\mathbf{g}_{\bar{\pi}}(t+j)\Bigg) \tag{B.2}$$

Interpretation: tradeoffs from discretion in RE + effect of current level and change of the gain on future tradeoffs + effect of future expected levels and changes of the gain on future tradeoffs