

# Monetary Policy & Anchored Expectations

## An Endogenous Gain Learning Model

Preliminary and Incomplete

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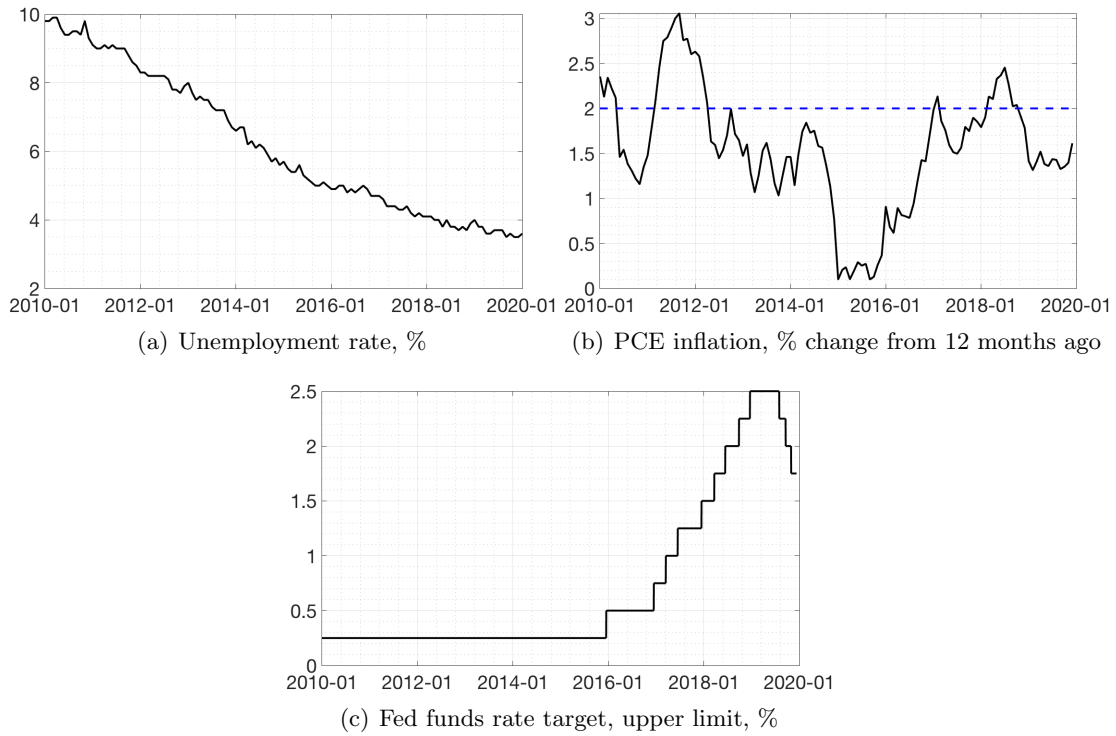
### **Abstract**

This paper investigates optimal monetary policy when expectation formation is characterized by potential anchoring of expectations. Using the adaptive learning literature approach, firms and households do not know the correct model of the economy and thus form expectations using a forecasting rule that they update in light of incoming data. Within this framework, the anchoring mechanism corresponds to an endogenous learning gain in the spirit of [Carvalho et al. \(2019\)](#). Expectations are said to be anchored when forecasting performance is sufficiently good such that a decreasing gain is chosen. Expectations thus become a state variable from the viewpoint of monetary policy that requires proper monitoring and management. In particular, optimal policy trades off the cost and benefit of anchoring expectations.

# 1 Introduction

The current stance of the United States business cycle is boldly defiant of mainstream macroeconomic theory. The historically low unemployment level, portrayed on panel (a) of Fig. 1, has not resulted in rising inflation. On the contrary, personal consumption expenditures (PCE) inflation has persistently undershot the Federal Reserve’s 2% target, prompting the Fed to be expansionary despite the economy experiencing a boom (panels (b) and (c) of Fig. 1).

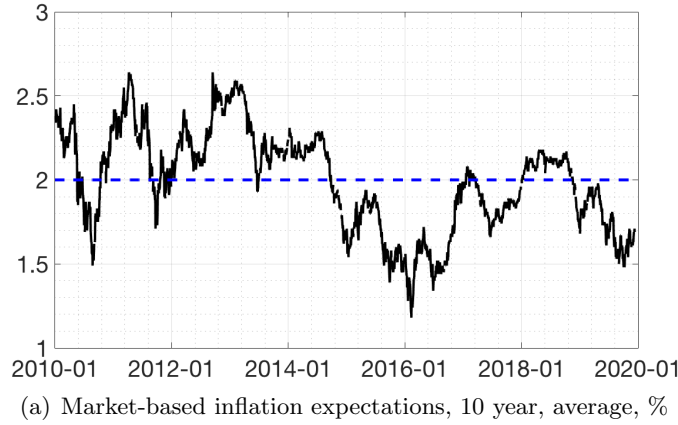
**Figure 1**



In this paper I argue that the key to understanding both the puzzling behavior of inflation as well as the Fed’s response to it is the time series of long-run inflation expectations. As Fig. 2 shows, long-run inflation expectations of the public, averaging a little above the 2% target prior to 2015, display a marked downward drift since 2015. This indicates that the public has doubts whether the Fed is able to restore inflation to the target. Confronted with what it sees as a changing environment, the public revises its predictions about the future course of the economy.

Macroeconomic theory that seeks to understand this phenomenon thus needs to account for

Figure 2



expectation formation that features a notion of stability of forecasting behavior. I propose a model in the adaptive learning tradition where the public sector's choice of learning gain is endogenous, as in [Carvalho et al. \(2019\)](#). This captures the idea that in normal times, when firms and households observe economic data that confirms their previous predictions, agents choose a decreasing gain and thus do not change their forecasting rules by much. By contrast, when incoming data suggests that the current forecasting rule is incorrect, agents switch to a constant gain, updating their forecasting rule strongly. I refer to the former case as *anchored* and to the latter as *unanchored expectations*.

I embed the anchoring mechanism in an otherwise standard New Keynesian model of the type widely used for monetary policy analysis in academia and at central banks. This allows a crisp comparison between optimal monetary policy in the standard rational expectations model and the model with the expectation-anchoring mechanism. As is well known in the adaptive learning literature, learning models turn the evolution of beliefs into an endogenous state variable. Since expectations about the future determine current outcomes, learning models exhibit positive feedback effects.

The contribution of this paper is to investigate how this affects the optimal conduct of monetary policy. It turns out that optimal monetary policy takes the stance of expectations explicitly into account. In particular, the central bank finds it optimal to anchor expectations whenever it is not too costly to do so. However, anchoring expectations comes at a cost which introduces tradeoffs in the conduct of monetary policy, presenting a novel case for violations of the divine coincidence. This insight allows us to interpret the Fed's fall 2019 decision to lower interest rates despite a strong

economy as an attempt to anchor expectations or to keep them from becoming unanchored.

## 1.1 Related literature

My work draws on two strands of macroeconomic research. The first is the extensive literature on optimal monetary policy. Most of this literature, such as [Clarida et al. \(1999\)](#) or [Woodford \(2011\)](#), analyzes optimal monetary policy in the rational expectations New Keynesian (NK) model. As a result, the nature of optimal monetary policy in the NK model is well understood, which is the reason I adopt this framework as my benchmark of comparison. Optimal monetary policy in the NK model stabilizes the price level and, in the absence of shocks to desired markups (cost-push shocks), the divine coincidence holds. That is, stabilizing the price level coincides with stabilizing output around potential output. Moreover, if monetary policy is characterized as using a rule of the form advocated by [Taylor \(1993\)](#), the condition for determinacy in the NK model is that the Taylor principle is satisfied. My work revisits these principles from the lens of an NK model with anchoring replacing rational expectations.

A branch of the optimal monetary policy literature has stressed the importance of commitment to the policy rule. [Kydland and Prescott \(1977\)](#) shows that a discretionary policy leads to inflationary bias: the central bank engineers suboptimally high average levels of inflation because it ignores the effect of its policy on expectations altogether. Since the seminal work by [Barro and Gordon \(1983\)](#), the discussion around discretion versus commitment has also been connected to the idea of central bank credibility, or, in the case of discretion, the lack thereof. Also [Ball \(1994\)](#) concludes that the only way the NK model can be used to justify that we do not observe disinflationary booms in data is to assume that central banks face credibility issues. Additionally, in endogenous gain adaptive learning models such as [Carvalho et al. \(2019\)](#), the gain serves as a metric for central bank credibility.<sup>1</sup>

The second branch of related work is the adaptive learning literature. Following the book by [Evans and Honkapohja \(2001\)](#), this literature replaces the rational expectations assumption by postulating

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<sup>1</sup>I follow [Carvalho et al. \(2019\)](#) in using the term “credibility” loosely here. A more precise formulation would be to say that the gain captures the extent to which the public trusts the central bank to deliver the known long-run targets.

an ad-hoc forecasting rule, the perceived law of motion (PLM), as the expectation-formation process. Agents use the PLM to form expectations and update it in every period using recursive estimation techniques. The intuition behind adaptive learning models is the idea that firms and households do not know the laws that govern the evolution of economic variables. Therefore they use the PLM to forecast instead, but as their sample of observed data grows, they refine the PLM and thus learn the true underlying laws of motion.

The early learning literature concentrated on the question of under what conditions learning converges to rational expectations. Indeed, much of [Evans and Honkapohja \(2001\)](#) is devoted to this question.<sup>2</sup> The vast majority of the learning literature still focuses on expectational stability (E-stability) and various related aspects such as the speed of convergence. Examples are [Evans and Honkapohja \(2003\)](#), [Marcet and Sargent \(1989\)](#), [Ferrero \(2007\)](#) or [Eusepi and Preston \(2018\)](#). In the context of the New Keynesian model, [Bullard and Mitra \(2002\)](#) and [Preston \(2005\)](#) both investigate the conditions for an E-stable REE. Perhaps surprisingly, despite the methodological divide between the so-called Euler-equation learning approach advocated by the former and the long-horizon learning approach advanced by the latter, the two papers obtain the same result: the REE in the NK model is E-stable if the Taylor principle holds.

The quantitative dynamics of models with learning has received less attention in the literature. One reason may be that quantitative studies such as [Williams \(2003\)](#), [Eusepi and Preston \(2011\)](#), [Lubik and Matthes \(2016\)](#) or [Winkler \(2019\)](#) need to assign an appropriate value to the learning gain. And since only a handful of studies have estimated the gain ([Branch and Evans \(2006\)](#), [Milani \(2007\)](#), [Eusepi et al. \(2018a\)](#) and [Carvalho et al. \(2019\)](#) appears to be an exhaustive list), calibration of this parameter is difficult.

Another potential reason is that learning models with a sufficiently high constant gain exhibit oscillatory impulse responses. Since this is at odds with empirical studies of impulse responses obtained from structural vector autoregressions (SVAR), the tendency of learning models to produce

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<sup>2</sup>As [Evans and Honkapohja \(2001\)](#) show, the question can be answered by studying the map between the PLM and the data-generating process under learning, the actual law of motion (ALM). In their terminology, a REE is expectationally stable (E-stable) if the differential equation obtained by differentiating the map between PLM and ALM is stable. The latter can be examined using standard methods to determine the stability of dynamic systems.

oscillations in response to shocks has not been widely discussed. Yet oscillations show up in nearly all of the quantitative studies cited above (except for [Williams \(2003\)](#)). In fact, an early precursor to the learning literature, [Townsend \(1983\)](#), is the first to observe that higher-order expectations can result in oscillatory dynamics. [Evans and Honkapohja \(2001\)](#) and [Evans et al. \(2013\)](#) therefore acknowledge oscillations as a potential problem of learning but do not attempt to reconcile it with data or to provide an economic interpretation of the phenomenon.

A small subset of the learning literature has reevaluated optimal monetary policy from the lens of a learning model ([Orphanides and Williams \(2004\)](#), [Ferrero \(2007\)](#), [Molnár and Santoro \(2014\)](#), [Eusepi and Preston \(2018\)](#) and [Eusepi et al. \(2018a\)](#) are some examples). There is no consensus on how learning affects optimal monetary policy. In the case of the New Keynesian model, for example, [Eusepi and Preston \(2018\)](#) and [Molnár and Santoro \(2014\)](#) conclude that optimal monetary policy is more aggressive on inflation than under rational expectations, yet [Eusepi et al. \(2018a\)](#) find the exact opposite.

An even smaller subset of the learning literature has considered learning models with an endogenous gain. This set consists of [Marcet and Nicolini \(2003\)](#), [Milani \(2014\)](#), and [Carvalho et al. \(2019\)](#). Having an endogenous gain allows the modeler to capture an essential element of learning: the public’s confidence that it has found the correct model of the economy. Choosing a decreasing gain corresponds neatly to the notion that firms and households no longer see the need to update their forecasting models because current models are performing sufficiently well. Choosing a constant gain, by contrast, captures the opposite case where agents, dissatisfied with the forecasting performance of their current model, rely on recent data very strongly to update their PLM.

This is the sense in which [Lubik and Matthes \(2016\)](#) and [Carvalho et al. \(2019\)](#) interpret the choice of gain as a metric for the credibility of policy authorities in the eye of the public. In fact, [Carvalho et al. \(2019\)](#) interpret the choice of a decreasing gain as the public’s trust in the central bank’s inflation target and they are the first to label this trust *anchored expectations*. My work takes the [Carvalho et al. \(2019\)](#) model of anchored expectations as a starting point and asks what the implications for optimal monetary policy are in a full-fledged New Keynesian model with an

anchoring mechanism.

The paper is structured as follows. Section 2 contains a concise refresher of the New Keynesian model, both under rational expectations and in a learning version with microfoundations. Section 3 introduces the learning framework and spells out the anchoring mechanism. Section 4 lays out the problem of the central bank. Section 5 presents results and provides intuition. Section 6 concludes.

## 2 The underlying New Keynesian model

Apart from expectation formation, the model is a standard New Keynesian model where the rational expectations (RE) assumption is replaced by the expectation-anchoring mechanism. The advantage of having a standard NK backbone to the model is that one can neatly isolate the way the anchoring mechanism alters the behavior of the model. Since the mechanics of the rational expectations version of this model are well understood, I only lay out the model and pinpoint the few places where the assumption of nonrational expectations matters.<sup>3</sup> To ease comparison, I follow the notation in Woodford (2011).

### 2.1 Households

The representative household is infinitely-lived and maximizes expected discounted lifetime utility from consumption net of the disutility of supplying labor hours:

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ U(C_T^i) - \int_0^1 v(h_T^i(j)) dj \right] \quad (1)$$

Here  $U(\cdot)$  and  $v(\cdot)$  denote the utility of consumption and disutility of labor, respectively, and  $\beta$  is the discount factor of the household.  $h_T^i(j)$  denotes the supply of labor hours of household  $i$  at time  $T$  to the production of good  $j$  and the household participates in the production of all goods  $j$ . Similarly, household  $i$ 's consumption bundle at time  $T$ ,  $C_T^i$ , is a Dixit-Stiglitz composite of all goods in the

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<sup>3</sup>For the specifics of the NK model the reader is referred to Woodford (2011).

economy:

$$C_t^i = \left[ \int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (2)$$

$\theta > 1$  denotes the elasticity of substitution between the varieties of consumption goods. Denoting by  $p_T(j)$  the time- $T$  price of good  $j$ , the aggregate price level in the economy can then be written as

$$P_t = \left[ \int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \quad (3)$$

The budget constraint of household  $i$  is given by

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j) + \Pi_t^i(j)dj - T_t - P_t C_t^i \quad (4)$$

where  $\Pi_t^i(j)$  denotes profits from firm  $j$  remitted to household  $i$ ,  $T_t$  taxes, and  $B_t^i$  the riskless bond purchases at time  $t$ .<sup>4</sup>

The only difference to the standard New Keynesian model thus far is the expectation operator,  $\hat{\mathbb{E}}$ . This is the subjective expectation operator that differs from its rational expectations counterpart,  $\mathbb{E}$ , in that it does not encompass knowledge of the model. In particular, households have no knowledge of the fact that they are identical and by extension they also do not internalize that they hold identical beliefs about the evolution of the economy. As we will see in Section 2.3, this has implications for their forecasting behavior and will result in decision rules that differ from those of the rational expectations version of the model.

## 2.2 Firms

Firms are monopolistically competitive producers of the differentiated varieties  $y_t(j)$ . The production technology of firm  $j$  is  $y_t(j) = A_t f(h_t(j))$ , whose inverse,  $f^{-1}(\cdot)$ , signifies the amount of labor input.

Noting that  $A_t$  is the level of technology and that  $w_t(j)$  is the wage per labor hour, firm  $j$  profits at

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<sup>4</sup>For ease of exposition I have suppressed potential money assets here. This has no bearing on the model implications since it represents the cashless limit of an economy with explicit money balances.



time  $t$  can be written as

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \quad (5)$$

Firm  $j$ 's problem then is to set the price of the variety it produces,  $p_t(j)$ , to maximize the present discounted value of profit streams

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[ \Pi_t^j(p_t(j)) \right] \quad (6)$$

subject to the downward-sloping demand curve

$$y_t(j) = Y_t \left( \frac{p_t(j)}{P_t} \right)^{-\theta} \quad (7)$$

where

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \quad (8)$$

is the stochastic discount factor from households. Nominal frictions enter the model through the parameter  $\alpha$  in Equation (6). This is the Calvo probability that firm  $j$  is not able to adjust its price in a given period.

Analogously to households, the setup of the production side of the economy is standard up to the expectation operator. Also here the rational expectations operator  $\mathbb{E}$  has been replaced by the subjective expectations operator  $\hat{\mathbb{E}}$ . This implies that firms, like households, do not know the model equations and fail to internalize that they are identical. Thus their decision rules, just like those of the households, will be distinct from their rational expectations counterparts.

## 2.3 Aggregate laws of motion

The model solution procedure entails deriving first order conditions, taking a loglinear approximation around the nonstochastic steady state and imposing market clearing conditions to reduce the system to two equations, the New Keynesian Phillips curve (NKPC) and IS curve (NKIS). The presence of subjective expectations, however, implies that firms and households are not aware of the fact

that they are identical. Thus, as [Preston \(2005\)](#) takes pains to point out, imposing market clearing conditions in the expectations of agents is inconsistent with the assumed information structure.<sup>5</sup>

Instead, I follow [Preston \(2005\)](#) in preventing firms and households from internalizing market clearing conditions. As [Preston \(2005\)](#) shows, this leads to long-horizon forecasts showing up in firms' and households' first order conditions. As a consequence, instead of the familiar expressions, the NKIS and NKPC take the following form:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (9)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (10)$$

Here  $x_t$ ,  $\pi_t$  and  $i_t$  are the log-deviations of the output gap, inflation and the nominal interest rate from their steady state values, and  $\sigma$  is the intertemporal elasticity of substitution. The variables  $r_T^n$  and  $u_T$  are exogenous disturbances representing a natural rate shock and a cost-push shock respectively.

The laws of motion (9) and (10) are obtained by deriving individual firms' and households' decision rules, which involve long-horizon expectations, and aggregating across the cross-section. Importantly, agents in the economy have no knowledge of these relations since they don't know that they are identical and thus are not able to impose market clearing conditions required to arrive at (9) and (10). Thus, although the evolution of the observables  $(\pi, x)$  is governed by the exogenous state variables  $(r^n, u)$  and long-horizon expectations via these two equations, agents in the economy are unaware of this. As I will spell out more formally in [Section 3](#), it is indeed the equilibrium mapping between states and jump variables the agents are attempting to learn.<sup>6</sup>

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<sup>5</sup>The target of [Preston \(2005\)](#)'s critique is the Euler-equation approach as exemplified for example by [Bullard and Mitra \(2002\)](#). This approach involves writing down the loglinearized first order conditions of the model, and simply replacing the rational expectations operators with subjective ones. In a separate paper, I demonstrate that the Euler-equation approach is not only inconsistent on conceptual grounds as [Preston \(2005\)](#) shows, but also delivers substantially different quantitative dynamics in a simulated New Keynesian model. Thus relying on the Euler-equation approach when investigating the role of learning can be misleading.

<sup>6</sup>The learning of (9) and (10) is complicated by the fact that the current stance of expectations figures into the equations, resulting in the well-known positive feedback effects of learning.

The model is closed by the standard specification of monetary policy as a Taylor rule:

$$i_t = \psi_\pi(\pi_t - \bar{\pi}) + \psi_x(x_t - \bar{x}) + \bar{i}_t \quad (11)$$

where  $\psi_\pi$  and  $\psi_x$  represent the responsiveness of monetary policy to inflation and the output gap respectively,  $\bar{\pi}$  and  $\bar{x}$  are the central bank's targets. Lastly,  $\bar{i}_t$  is a monetary policy shock. I assume that the central bank publicly announces the Taylor rule. Thus Equation (11) is common knowledge and is therefore not the object of learning.<sup>7</sup>

Next, to simplify notation, I gather the exogenous state variables in the vector  $s_t$  and jump variables in the vector  $z_t$  as

$$s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{bmatrix} \quad z_t = \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} \quad (12)$$

Then, denoting long-horizon expectations as

$$f_a(t) \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} z_{T+1} \quad f_b(t) \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} z_{T+1} \quad (13)$$

I write the laws of motion of jump variables (Equations (9), (10) and (11)) compactly as

$$z_t = A_a f_a(t) + A_b f_b(t) + A_s s_t \quad (14)$$

where the matrices  $A_i$ ,  $i = \{a, b, s\}$  gather coefficients and are given in App. A. Assuming that exogenous variables evolve according to independent AR(1) processes, I write the state transition matrix equation as

$$s_t = h s_{t-1} + \epsilon_t \quad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \quad (15)$$

where  $h$  gathers the autoregressive coefficients  $\rho_j$ ,  $\epsilon_t$  the Gaussian innovations  $\varepsilon_t^j$  and  $\eta$  the standard

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<sup>7</sup>In an extension I consider the case where the Taylor rule is not known (or not believed) by the public and therefore is learned together with the relations (9) and (10). This dampens intertemporal expectation effects as long as the Taylor rule is not learned; afterwards, the model dynamics are identical to those of the baseline.

deviations  $\sigma_t^j$ , for  $j = \{r, i, u\}$ .  $\Sigma = \eta\eta'$  is the variance-covariance matrix of disturbances.

$$h \equiv \begin{pmatrix} \rho_r & 0 & 0 \\ 0 & \rho_i & 0 \\ 0 & 0 & \rho_u \end{pmatrix} \quad \epsilon_t \equiv \begin{pmatrix} \varepsilon_t^r \\ \varepsilon_t^i \\ \varepsilon_t^u \end{pmatrix} \quad \text{and} \quad \eta \equiv \begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_i & 0 \\ 0 & 0 & \sigma_u \end{pmatrix} \quad (16)$$

Thus, while the state-space form of the solution for the rational expectations version of the model takes the form

$$s_t = hs_{t-1} + \epsilon_t \quad (17)$$

$$z_t = g^{RE} s_t \quad (18)$$

the model with learning leaves the state transition equation (17) unchanged, but replaces (18) with the law of motion for observables (14). Once I have specified expectation formation, I will revisit this formulation to highlight more formally the difference between the rational expectations and learning versions of the model.

### 3 Learning with anchored expectations

The main informational assumption of the model is that agents have no knowledge of the equilibrium mapping between states and jumps in the model. Therefore they are not able to form rational expectations forecasts. To see this, observe that an agent with rational expectations would internalize the rational expectations state-space system (17) - (18) and would therefore forecast future jumps as  $\mathbb{E}_t z_{t+h} = g^{RE} \mathbb{E}_t s_{t+h} = g^{RE} h^h s_t$ . Agents in the learning model however don't know  $g^{RE}$  and are thus indeed unable to form the rational expectations forecast.

How do they then form expectations about future jumps? Assuming that agents know the evolution of states, that is they have knowledge of Equation (17), they postulate an ad-hoc relationship between states and jumps and seek to refine it in light of incoming data.<sup>8</sup>

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<sup>8</sup>Allowing agents to know the state transition equation is a common assumption in the learning literature. In an

### 3.1 Perceived law of motion

I assume agents consider a forecasting model for jumps of the form

$$\hat{\mathbb{E}}_t z_{t+1} = a_{t-1} + b_{t-1} s_t \quad (19)$$

where  $a$  and  $b$  are estimated coefficients of dimensions  $3 \times 1$  and  $3 \times 3$  respectively. This perceived law of motion (PLM) reflects the assumption that agents forecast jumps using a linear function of current states and a constant, with last period's estimated coefficients. I also assume that

$$\hat{\mathbb{E}}_t \phi_{t+h} = \phi_t \quad \forall h \geq 0 \quad (20)$$

This assumption, known in the learning literature as anticipated utility, means that agents fail to internalize that they will update the forecasting rule in the future.<sup>9</sup> The PLM together with anticipated utility implies that  $h$ -period ahead forecasts are constructed as

$$\hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b_{t-1} h^{h-1} s_t \quad \forall h \geq 1 \quad (21)$$

Summarizing the estimated coefficients as  $\phi_{t-1} \equiv \begin{bmatrix} a_{t-1} & b_{t-1} \end{bmatrix}$ , here  $3 \times 4$ , I can rewrite Equation (19) as

$$\hat{\mathbb{E}}_t z_{t+1} = \phi_{t-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix} \quad (22)$$

The timing assumptions of the model are as follows. In the beginning of period  $t$ , the current state  $s_t$  is realized. Agents then form expectations according to (19) using last period's estimate  $\phi_{t-1}$  and the current state  $s_t$ . Given exogenous states and expectations, today's jump vector  $z_t$  is realized.

This allows agents to evaluate the most recent forecast error  $f_{t-1} \equiv z_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}$  to update

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extension, I relax this assumption and find that it has similar implications as having agents learn the Taylor rule: initial responses to shocks lack intertemporal expectation effects, but these reemerge as the evolution of state variables is learned.

<sup>9</sup>This is a conventional assumption in the learning literature and serves to simplify the algebra. As [Sargent \(1999\)](#) shows, similar results obtain upon relaxing anticipated utility.

their forecasting rule. The estimate is updated according to the following recursive least-squares algorithm:

$$\phi_t = \left( \phi'_{t-1} + k_t^{-1} R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left( z_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right) \right)' \quad (23)$$

$$R_t = R_{t-1} + k_t^{-1} \left( \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \quad (24)$$

where  $R_t$  is the  $4 \times 4$  variance-covariance matrix of the regressors and  $k_t$  is the learning gain, specifying to what extent the updated estimate loads on the forecast error. Clearly, a high gain implies high loadings and thus strong changes in the estimated coefficients  $\phi$ . A low gain, by contrast, means that the current forecast error only has a small effect on  $\phi_t$ .

The vast majority of the learning literature specifies the gain either as a constant,  $\bar{g}$ , or decreasing with time so that  $k_t^{-1} = (k_{t-1} + 1)^{-1}$ . Instead, in the spirit of [Carvalho et al. \(2019\)](#), I allow firms and households in the model to choose whether to use a constant or a decreasing gain using the following endogenous gain specification: let  $\omega_t$  denote agents' time  $t$  estimate of the forecast error variance and  $\theta_t$  be a statistic evaluated by agents in every period as

$$\omega_t = \omega_{t-1} + \tilde{\kappa} k_{t-1}^{-1} (f_{t-1} f'_{t-1} - \omega_{t-1}) \quad (25)$$

$$\theta_t = \theta_{t-1} + \tilde{\kappa} k_{t-1}^{-1} (f'_{t-1} \omega_t^{-1} f_{t-1} - \theta_{t-1}) \quad (26)$$

where  $\tilde{\kappa}$  is a parameter that allows agents to scale the gain compared to the previous estimation and  $f_{t-1}$  is the most recent forecast error, realized at time  $t$ . Then for a specified threshold  $\tilde{\theta}$ , the gain is determined endogenously as

$$k_t = \begin{cases} k_{t-1} + 1 & \text{if } \theta_t < \tilde{\theta} \\ \bar{g}^{-1} & \text{otherwise.} \end{cases} \quad (27)$$

In other words, agents choose a decreasing gain when the criterion  $\theta_t$  is lower than the threshold  $\tilde{\theta}$ ; otherwise they choose a constant gain. This framework, which I refer to as an anchoring mechanism, captures the intuition that when current squared forecast errors are large compared to agents' estimated forecast error variance, agents conclude that the forecasting performance of their current PLM  $\phi_{t-1}$  is poor. Since  $\phi_{t-1}$  appears to provide an inaccurate description of the evolution of observables, agents choose a constant gain, reflecting their desire to update  $\phi$  strongly using the most recent data. By contrast, if  $\theta_t < \tilde{\theta}$ , current squared forecast errors are not sizable compared the estimated forecast error variance; it seems that  $\phi_{t-1}$  is close to the data-generating process and so agents see no need to change it, thus opting for a decreasing gain.

Moreover, I refer to the situation when  $\theta_t < \tilde{\theta}$  as anchored expectations. As outlined above, choosing a decreasing gain reflects that firms and households believe that they have found the correct model of the economy. In other words, they have confidence that the PLM  $\phi_{t-1}$  is the true underlying mapping between states and jumps in the economy.

Now recall that  $\phi = [a, b]$ , where  $a$  is the estimate of the constant and  $b$  the estimate of the slope in the law of motion of jumps. Believing the estimate  $b$  to be the correct one means that agents think they know how the observables respond to shocks. Analogously, thinking the estimate  $a$  to be correct has the interpretation of agents being confident about the long-run average values of the observables. But that is equivalent to agents trusting that the central bank is able to implement the targets in the long run. In this way, anchored expectations has a natural interpretation as trust in the central bank's ability to achieve the long-run target.<sup>10</sup>

Having thus established anchored expectations as a metric of trust in the central bank implementing the announced target, it becomes intuitive why a monetary authority would want to make sure that expectations do not become unanchored. Clearly, unanchored expectations have the opposite interpretation to anchored ones: they reflect that the public has doubts whether the central bank is committed or able to achieve the long-run target. And due to the feedback from expectations to observables (recall Equation (14)), unanchored expectations can have a self-confirming effect in that

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<sup>10</sup>For this reason, a learning specification in which only the constant is learned ( $b = b^{RE}$ ) is sufficient to analyze anchored expectations. The figures of Section 5.1 correspond to this case.

they cause observables to drift away from the announced target. Thus a central bank that doesn't come across as committed to the target may fail to anchor expectations and will thus face additional difficulty in implementing the target.

### 3.2 Actual law of motion

Having laid out the expectation formation, I can now characterize the evolution of the jump variables under learning. Using the PLM from Equation (19), I write the long-horizon expectations in (28) as

$$f_a(t) = \frac{1}{1 - \alpha\beta} a_{t-1} + b_{t-1}(I_3 - \alpha\beta h)^{-1} s_t \quad f_b(t) = \frac{1}{1 - \beta} a_{t-1} + b_{t-1}(I_3 - \beta h)^{-1} s_t \quad (28)$$

Substituting these into the law of motion of observables (Equation (14)) yields the actual law of motion (ALM):

$$z_t = g^l \begin{bmatrix} 1 \\ s_t \end{bmatrix} \quad (29)$$

where  $g^l$  is a  $3 \times 4$  matrix given in App. B. Thus, instead of the state-space solution of the RE version of the model (Equations (17) and (18)), the state-space solution for the learning model is characterized by the pair of equations (17) and (29).

## 4 The monetary policy problem

I assume the monetary authority seeks to maximize welfare of the representative household under commitment. As shown in Woodford (2011), a second-order Taylor approximation of household utility delivers a central bank loss function of the form

$$L^{CB} = \mathbb{E}_t \sum_{T=t}^{\infty} \{ \pi_T^2 + \lambda_x (x_T - x^*)^2 + \lambda_i (i_T - i^*) \} \quad (30)$$

where  $\lambda_j$   $j = \{x, i\}$  is the weight the central bank assigns to stabilizing variable  $j$  and  $j^*$  is its target value. The central bank's problem, then, is to determine paths for inflation, the output gap and the



interest rate that minimize the loss in Equation (30).

I tackle this problem in two ways. In a first approach, I restrict my attention to a given policy rule of the form of Equation (11) as the policy function of the central bank. In this case, the central bank's problem becomes the choice of  $(\psi_\pi, \psi_x)$  that minimize the loss in Equation (30). Following Woodford (2011)'s terminology, this is the “optimal noninertial plan.” The noninertial solution involves splitting the problem into a deterministic part and a part that specifies optimal responses to unexpected shocks. One thus conjectures a solution for the observables of the form  $z_t = \bar{z} + F s_t$  where  $\bar{z}$  is the vector of optimal long-run average values and the elements of the matrix  $F$  specify the optimal responses of the observables to shocks. Analogously, one can decompose (30) as  $L^{CB} = L^d + L^s$ , where

$$L^d = \sum_{T=t}^{\infty} \beta^{T-t} \{ \mathbb{E}_t \pi_T^2 + \lambda_x (\mathbb{E}_t x_T - x^*)^2 + \lambda_i (\mathbb{E}_t i_T - i^*)^2 \} \quad (31)$$

$$L^s = \sum_{T=t}^{\infty} \beta^{T-t} \{ \text{var}_t(\pi_T) + \lambda_x \text{var}_t(x_T) + \lambda_i \text{var}_t(i_T) \} \quad (32)$$

One then solves for  $\bar{z}$  as the value that minimizes the deterministic part of the loss,  $L^d$ , subject to the constraints that the conjecture is compatible with the model equations. Similarly,  $F$  is the solution to the minimization of  $L^s$  subject to the compatibility constraints. Having in this way obtained an optimal path for the observables  $(\pi, x, i)$ , one can then perform coefficient comparison on the Taylor rule to determine the optimal values of  $(\psi_\pi, \psi_x)$ .

The second approach is to consider the unconstrained Ramsey solution to this problem, subject to the laws of motion of expectations.

## 5 Results

### 5.1 Simulations

#### 5.1.1 Calibration

In this section I simulate the rational expectations and learning versions of the model and compute the optimal Taylor rule coefficients numerically.<sup>11</sup> Table 1 summarizes the calibrated parameter values. For most of the parameters, I assign values commonly used in the macroeconomic literature. In particular, I follow [Woodford \(2011\)](#)'s calibration. For this section, I shut off the monetary policy parameters  $\lambda_i$ ,  $\lambda_x$  and  $\psi_x$  in order to focus on the role of inflation in the central bank's problem and thus on the optimal choice of inflation aggressiveness,  $\psi_\pi$ .

The learning parameters  $\bar{g}$ ,  $\tilde{\theta}$  and  $\tilde{\kappa}$  require some discussion. While the choice of  $\tilde{\kappa}$  only matters for the smoothness of the endogenous gain choice and thus can be set relatively freely, the threshold  $\tilde{\theta}$  has more bearing on the behavior of the model. Intuitively, the higher  $\tilde{\theta}$ , the more forecast error volatility agents in the economy are willing to tolerate before switching to a constant gain. Experimenting with different values reveals that once  $\tilde{\theta}$  is higher than a particular threshold, expectations are anchored for any value of  $\psi_\pi$ . Analogously if  $\tilde{\theta}$  is below a lower threshold, expectations are always unanchored regardless of the value of  $\psi_\pi$ . My choice of  $\tilde{\theta} = 2.5$  is thus motivated by assigning a value for which the choice of  $\psi_\pi$  has an effect on the anchoring behavior of the model.

The choice of  $\bar{g}$  is far from innocent as it has considerable implications for model dynamics. In particular, as I address in the Introduction, constant gain learning models have a tendency to produce impulse responses that exhibit damped oscillations. The reason is that under an adaptive learning framework, forecast errors following an impulse are oscillatory. In fact, the higher the learning gain, the higher the amplitude of forecast error oscillations. The oscillations can even become explosive if the gain is high enough.

Unfortunately, the model gives no guidance on the appropriate value for  $\bar{g}$ .<sup>12</sup> I thus turn to the

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<sup>11</sup>For simplicity, in this section I implement the learning algorithm such that only the constant is learned. The general formulation has qualitatively similar features but is more impacted by small-sample concerns prevalent in simulations.

<sup>12</sup>The analogy of the Kalman gain from the Kalman filter does not prove helpful either because it requires a steady state forecast error variance matrix which is not available in a learning context.

**Table 1:** Calibrated parameters

$\beta$	0.99	stochastic discount factor
$\sigma$	1	intertemporal elasticity of substitution
$\alpha$	0.5	Calvo probability of not adjusting prices
$\psi_\pi$	1.5	coefficient of inflation in Taylor rule
$\psi_x$	0	coefficient of the output gap in Taylor rule
$\bar{g}$	0.145	value of the constant gain
$\tilde{\theta}$	2.5	threshold value for criterion of endogenous gain choice
$\tilde{\kappa}$	0.2	scaling parameter of gain for forecast error variance estimation
$\rho_r$	0	persistence of natural rate shock
$\rho_i$	0.6	persistence of monetary policy shock
$\rho_u$	0	persistence of cost-push shock
$\sigma_i$	1	standard deviation of natural rate shock
$\sigma_r$	1	standard deviation of monetary policy shock
$\sigma_u$	1	standard deviation of cost-push shock
$\lambda_x$	0	weight on the output gap in central bank loss
$\lambda_i$	0	weight on the interest rate in central bank loss

admittedly thin literature on estimating learning gains. I therefore assign the value 0.145, obtained by [Carvalho et al. \(2019\)](#), to my knowledge the only study to estimate the value of the constant gain for an endogenous gain model. It has to be observed, however, that this is a significantly higher value than what was otherwise found in the literature. [Branch and Evans \(2006\)](#)’s number of 0.062 is quite close to [Eusepi et al. \(2018a\)](#)’s estimate of 0.05, while [Milani \(2007\)](#) finds an even lower number of 0.0183. Studies that use calibrated gains such as [Williams \(2003\)](#) or [Orphanides and Williams \(2005\)](#) tend to experiment with a range of values in the  $[0.01, 0.1]$  interval. The value of 0.05 seems to have attained particular prominence, but also much lower numbers have been used, such as 0.002 in [Eusepi and Preston \(2011\)](#).

### 5.1.2 Simulated dynamics

Having thus assigned values to the parameters, I turn to the model’s behavior. Table 2 presents an overview of the optimal Taylor rule coefficient  $\psi_\pi$ , obtained via grid-search, for the rational expectations and learning models. The table also compares the baseline parameterization with several alternatives. One notices that if the central bank has no concern to stabilize the output gap ( $\lambda_x = 0$ ) or the nominal interest rate ( $\lambda_i = 0$ ),  $\psi_\pi^{*,RE}$  is infinity. This is because if the central bank

suffers no loss upon output variation, then the fact that the divine coincidence doesn't hold does not pose a problem. Similarly, if the monetary authority is willing to allow the nominal interest rate to fluctuate vastly in order to stabilize inflation, this also allows the central bank to be infinitely aggressive on inflation.

**Table 2:** Optimal coefficient on inflation, RE against learning for alternative parameters

	$\psi_{\pi}^{*,RE}$	$\psi_{\pi}^{*,learn}$
Baseline	$\infty$	1.6243
$\lambda_x = 1$	2.1042	1.0571
$\lambda_i = 1$	1.1	1.0978

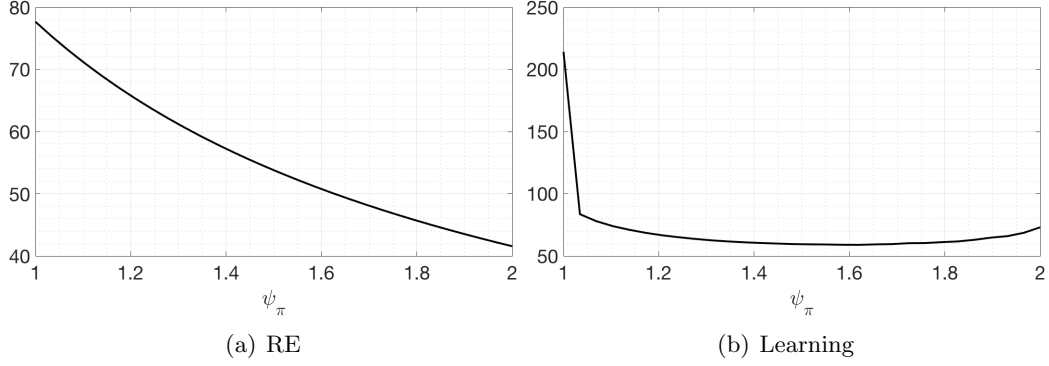
The main observation however is that  $\psi_{\pi}$  is always lower for the learning model than for the RE model. This is reinforced in Fig. 3 which plots the central bank's loss in the RE and learning models for various values of  $\psi_{\pi}$  but otherwise the baseline specification. The message is clear: while for rational expectations, the loss is strictly decreasing in  $\psi_{\pi}$ , this is not the case for the anchoring model. Why does the anchoring expectation formation induce the central bank to optimally be less aggressive on inflation?

The answer is that the anchoring mechanism introduces a novel tradeoff for the central bank. On the one hand, as we will see shortly in Fig. 4, having unanchored expectations increases the volatility of the observables. This results in the central bank wishing to anchor expectations. As Fig. 5 makes clear, this requires raising  $\psi_{\pi}$ . But in an environment where agents know the Taylor rule, a higher coefficient on inflation will lead to initially higher volatility due to the agents anticipating the endogenous responses of the nominal interest rate far in the future.

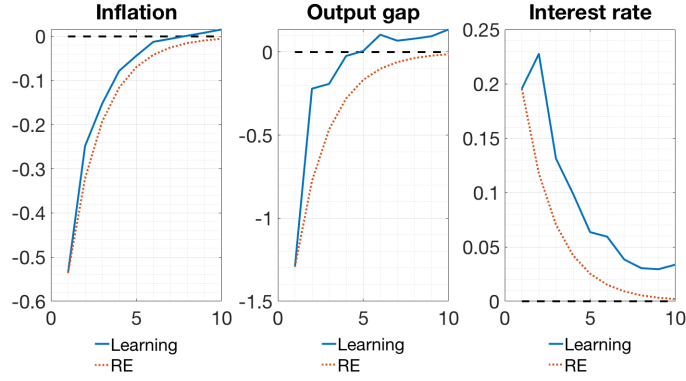
To understand what's going on in the model in detail, consider Fig. 4, portraying the impulse responses of the model after a contractionary monetary policy shock. The red dashed lines show the responses of the observables in the rational expectations version of the model. The blue lines show the responses in the learning model, on panel (a) conditional on expectations being anchored when the shock hits, on panel (b) being unanchored upon the arrival of the shock.

Not only do the impulse responses show the usual behavior of learning models - dampened responses and increased persistence. More importantly, responses differ strongly depending on whether

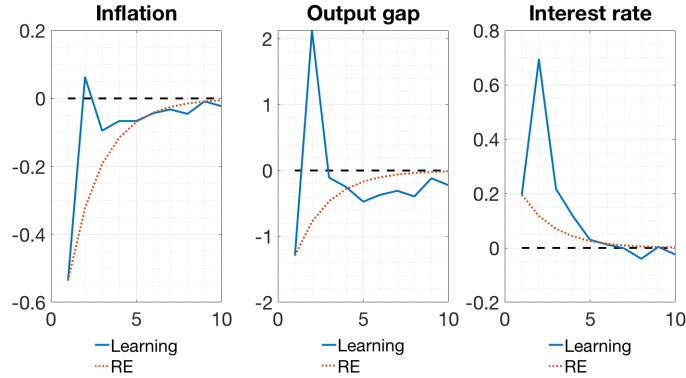
**Figure 3:** Central bank loss function as a function of  $\psi_\pi$



**Figure 4:** Impulse responses after a contractionary monetary policy shock



(a) RE against learning, expectations anchored



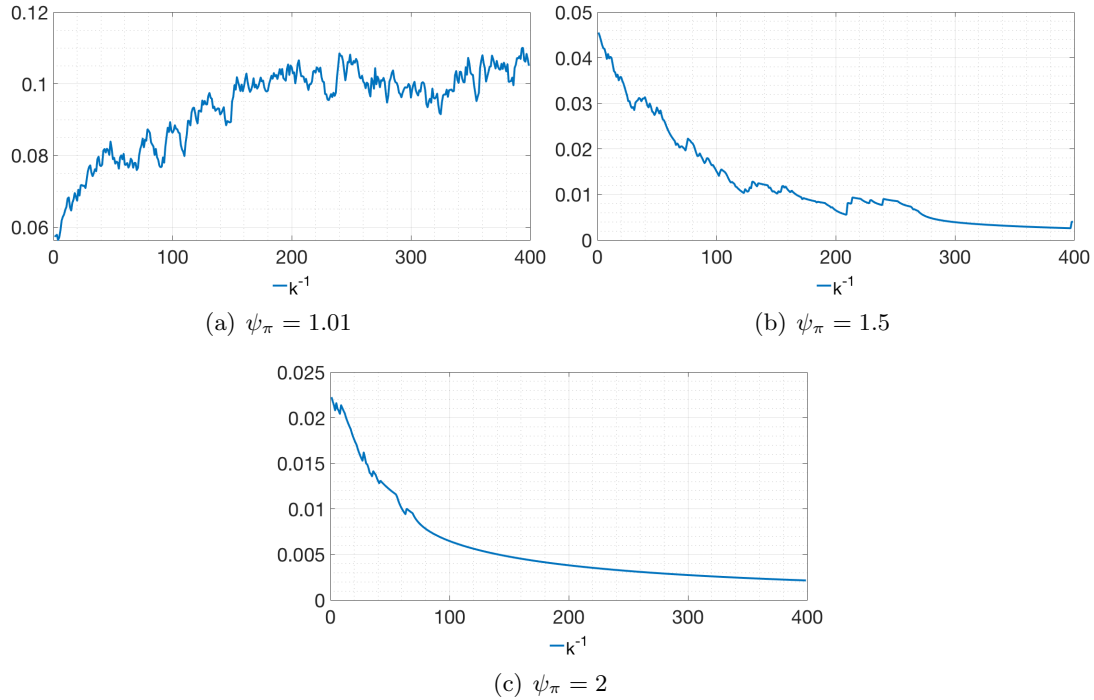
(b) RE against learning, expectations unanchored

Shock imposed at  $t = 25$  of a sample length of  $T = 400$  (with 100 initial burn-in periods), cross-sectional average with a cross-section size of  $N = 100$ . For the rest of the paper, I keep these simulation values unless otherwise stated. For the learning model, the remark refers to whether expectations are anchored at the time the shock hits.

expectations are anchored or not when the shock hits. In particular, if expectations are anchored, responses are closer to rational expectations than when expectations are unanchored. Moreover,

when expectations are unanchored, the endogenous responses of the observables become much more volatile, indeed, oscillatory. This makes intuitive sense: expectations being unanchored reflects the fact that firms and households are confronted with an environment that does not line up with their currently held perceived law of motion. They thus believe that a structural change has occurred and are therefore revising their expectations. Expectations are therefore fluctuating strongly, and as they feed back to the observables, the latter inherit their volatility.

**Figure 5:** Cross-sectional average inverse gains for various values of  $\psi_\pi$



Therefore, to avoid the volatility that results from unanchored expectations, the central bank wishes to anchor expectations. What choice of  $\psi_\pi$  will do the job? Fig. 5 provides the answer. The figure shows the cross-sectional average of inverse gains that result when  $\psi_\pi$  takes on different values. Clearly, a higher  $\psi_\pi$  results in lower and decreasing gains. Thus a central bank aiming to anchor expectations needs to employ a high  $\psi_\pi$ . This is also intuitive. A more aggressive central bank can signal to the public that it is determined to achieve the announced inflation target. Thus agents can rest assured that their believed inflation target is indeed the one the central bank can

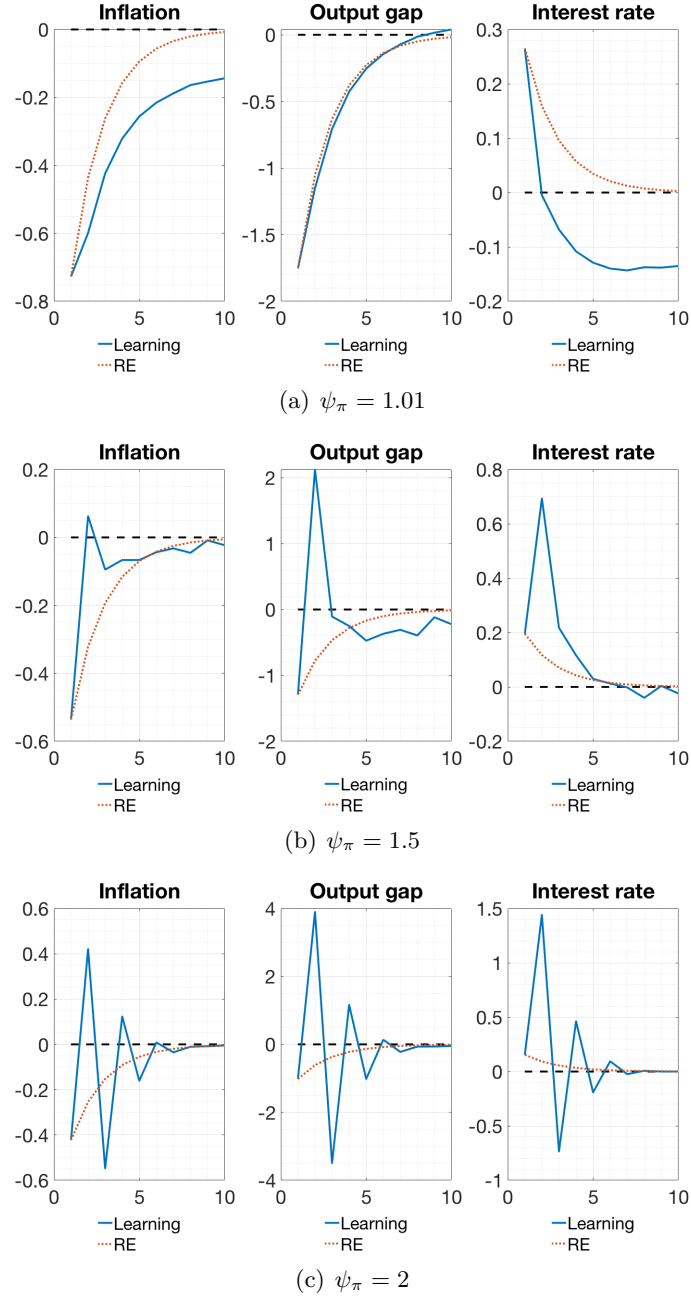
and will implement. Whenever  $\psi_\pi$  is low, however, the central bank is willing to tolerate bigger deviations from the target. This opens the door to speculation about whether the central bank is really committed to the target. In this case, deviations from the believed target of the same magnitude can cast doubt on the central bank's commitment, so that agents decide to monitor recent data closely to learn the seemingly shifting average value of inflation.

But if unanchored expectations cause heightened volatility, and being aggressive on inflation is able to anchor expectations, why does the monetary authority optimally choose a lower value for  $\psi_\pi$  than for rational expectations? The reason is that conditional on being unanchored, a higher  $\psi_\pi$  actually causes higher volatility than a lower one. This can be seen on Fig. 6 which depicts the same impulse responses to a contractionary monetary policy shock as Fig. 4, focusing however only on responses conditional on expectations being unanchored upon the shock. It shows these responses for three different values of  $\psi_\pi$ .

As the figure shows, a high  $\psi_\pi$  leads to more volatility than a low one does. The intuition is a little subtle. Since expectations are unanchored, they are also volatile. This implies that inflation far ahead in the future is expected to fluctuate strongly. Since agents know the Taylor rule, this also means that they expect the nominal interest rate far in the future to respond. The more aggressive the central bank, the stronger an interest rate response will the agents expect. This however feeds back into current output gaps and thus inflation. Higher overall volatility is the result.

Thus the presence of an expectation formation that allows for the anchoring and unanchoring of expectations introduces unexpected new features to the New Keynesian model. Unsurprisingly, it is desirable for the central bank to anchor expectations. It is also intuitive that being aggressive on inflation helps to anchor expectations. However, less intuitive is the fact that the optimal degree of aggressiveness on inflation is lower than under rational expectations. This has to do with the heightened volatility of the expectations process when  $\psi_\pi$  is high. A higher  $\psi_\pi$  increases the response of future nominal interest rate expectations, thus raising the feedback from expectations to current observables. Thus the central bank has to trade off the cost with the benefit of anchoring expectations.

**Figure 6:** Impulse responses for unanchored expectations for various values of  $\psi_\pi$



## 6 Conclusion

Central bankers frequently voice a concern to anchor expectations. The fact that rational expectations New Keynesian models have nothing to say about this aspect of monetary is a gap in the macroeconomic literature. Absent a theory of anchored expectations, it is difficult for macroe-



conomists to understand periods where central banks are clearly off the Taylor rule. The current stance of US monetary policy is an example of such an episode: the business cycle calls for monetary tightening, yet inflation lags below the target and if anything, the Fed is expansionary. My work suggests that the Fed reads the downward drift of long-run inflation expectations as a threat that expectations may become unanchored. In order to prevent that from happening, the Fed therefore is anxious to signal that it is determined to achieve its 2% inflation target. Its action are thus not intended to stimulate an already tight labor market; instead the Fed's present objective is to keep expectations anchored.

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## A Coefficient matrices in NK model

$$A_a = \begin{pmatrix} g_{\pi a} \\ g_{xa} \\ \psi_\pi g_{\pi a} + \psi_x g_{xa} \end{pmatrix} \quad A_b = \begin{pmatrix} g_{\pi b} \\ g_{xb} \\ \psi_\pi g_{\pi b} + \psi_x g_{xb} \end{pmatrix} \quad A_s = \begin{pmatrix} g_{\pi s} \\ g_{xs} \\ \psi_\pi g_{\pi s} + \psi_x g_{xs} + \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \end{pmatrix} \quad (33)$$

$$g_{\pi a} = \left(1 - \frac{\kappa\sigma\psi_\pi}{w}\right) \begin{bmatrix} (1-\alpha)\beta, \kappa\alpha\beta, 0 \end{bmatrix} \quad (34)$$

$$g_{xa} = \frac{-\sigma\psi_\pi}{w} \begin{bmatrix} (1-\alpha)\beta, \kappa\alpha\beta, 0 \end{bmatrix} \quad (35)$$

$$g_{\pi b} = \frac{\kappa}{w} \begin{bmatrix} \sigma(1-\beta\psi_\pi), (1-\beta-\beta\sigma\psi_x), 0 \end{bmatrix} \quad (36)$$

$$g_{xb} = \frac{1}{w} \begin{bmatrix} \sigma(1-\beta\psi_\pi), (1-\beta-\beta\sigma\psi_x), 0 \end{bmatrix} \quad (37)$$

$$g_{\pi s} = \left(1 - \frac{\kappa\sigma\psi_\pi}{w}\right) \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (I_3 - \alpha\beta P)^{-1} - \frac{\kappa\sigma}{w} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} (I_3 - \beta P)^{-1} \quad (38)$$

$$g_{xs} = \frac{-\sigma\psi_\pi}{w} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (I_3 - \alpha\beta P)^{-1} - \frac{\sigma}{w} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} (I_3 - \beta P)^{-1} \quad (39)$$

$$w = 1 + \sigma\psi_x + \kappa\sigma\psi_\pi \quad (40)$$

## B The observation matrix for learning

$$g^l = \begin{bmatrix} F & G \end{bmatrix} \quad (41)$$

with

$$F = \left( A_a \frac{1}{1-\alpha\beta} + A_b \frac{1}{1-\beta} \right) a_{t-1} \quad (42)$$

$$G = A_a b_{t-1} \left( I_3 - \alpha\beta h \right)^{-1} + A_b b_{t-1} \left( I_3 - \beta h \right)^{-1} + A_s \quad (43)$$