

# Materials 44 - I think I've got the estimation

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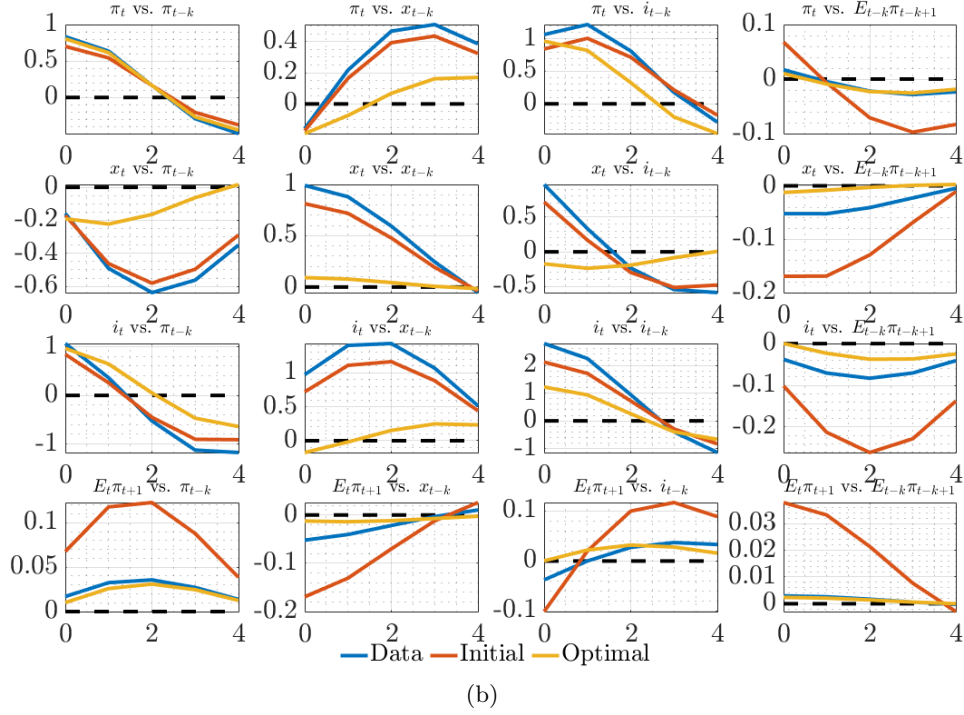
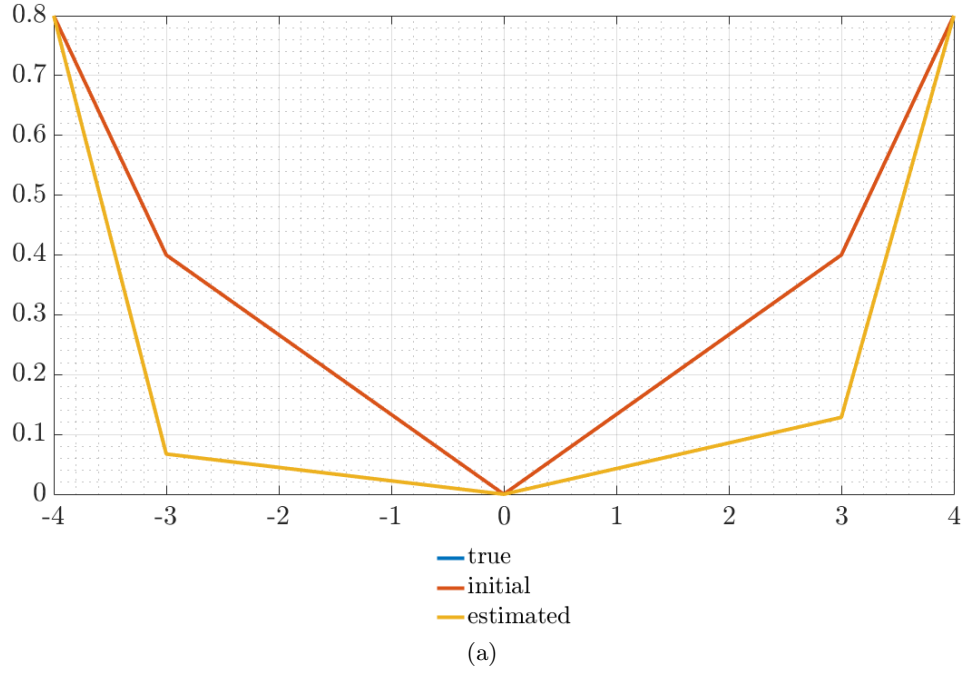
September 12, 2020

## Overview

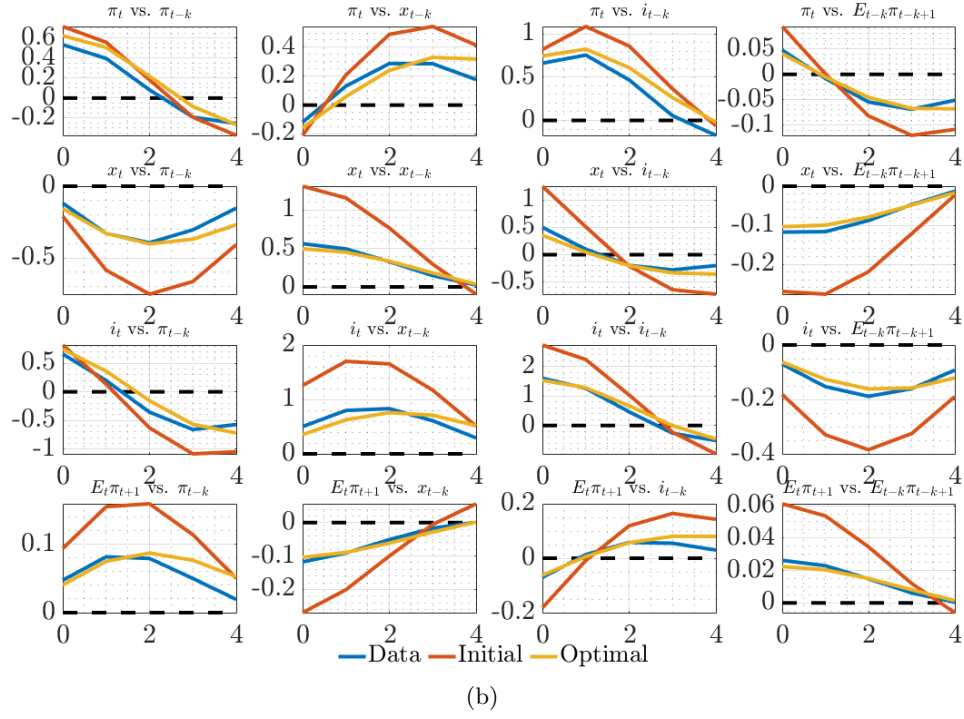
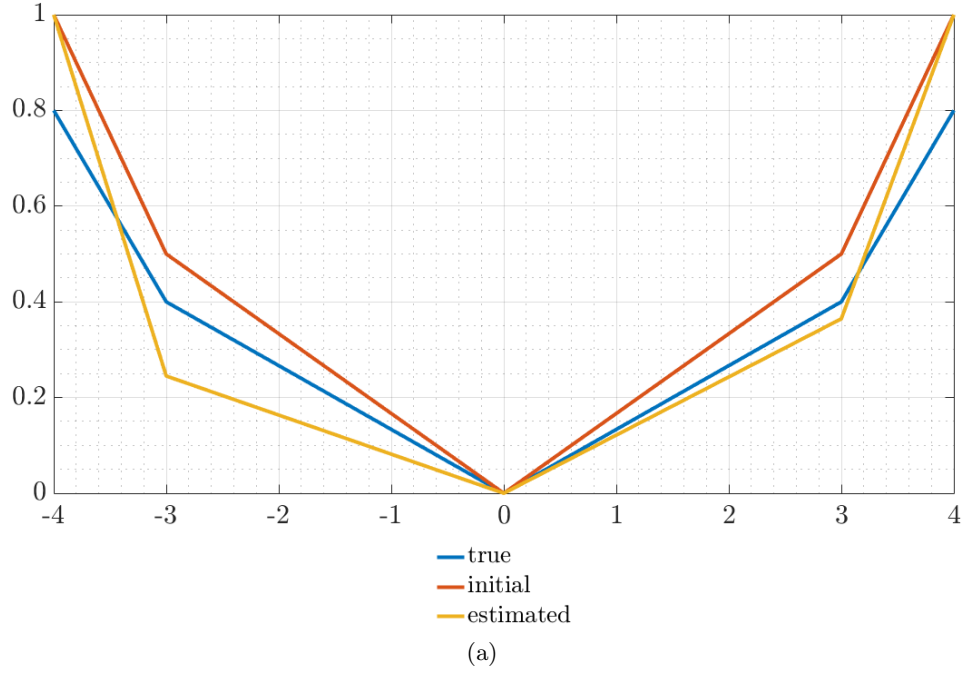
<b>1</b>	<b>Inconsistency was: didn't annualize expectations in true data</b>	<b>2</b>
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# 1 Inconsistency was: didn't annualize expectations in true data

**Figure 1:** Calibration C, use expectations, ridge tuning = 0.01, initialize at truth



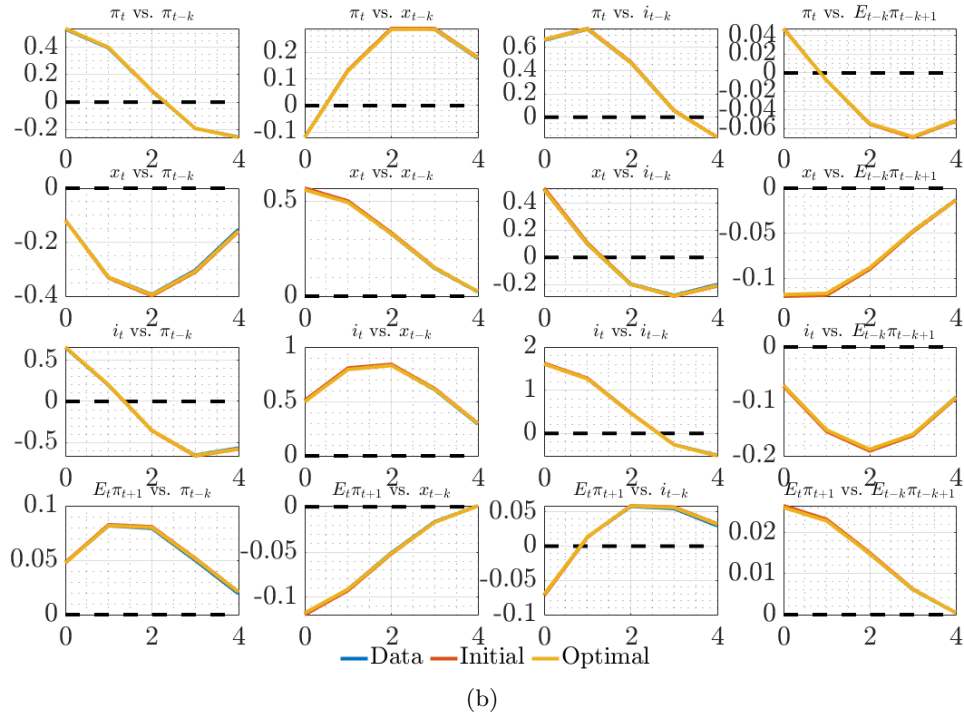
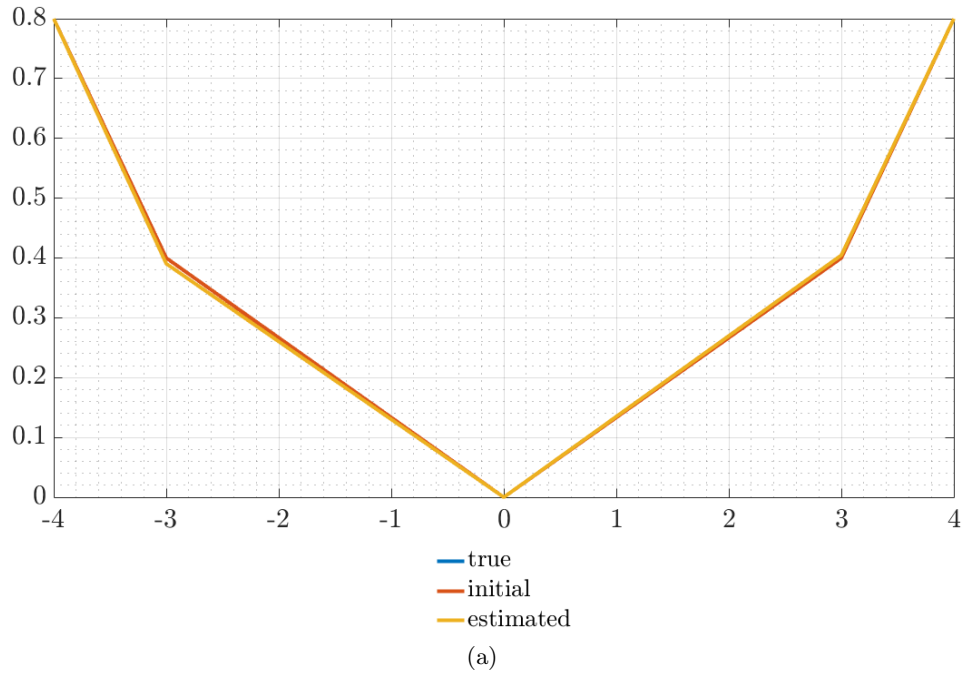
**Figure 2:** Calibration C, use expectations, ridge tuning = 0.01, initialize above truth, annualize expectations in true data



I think that's success!

## 1.1 Check with same seed as true data and $N = 1$

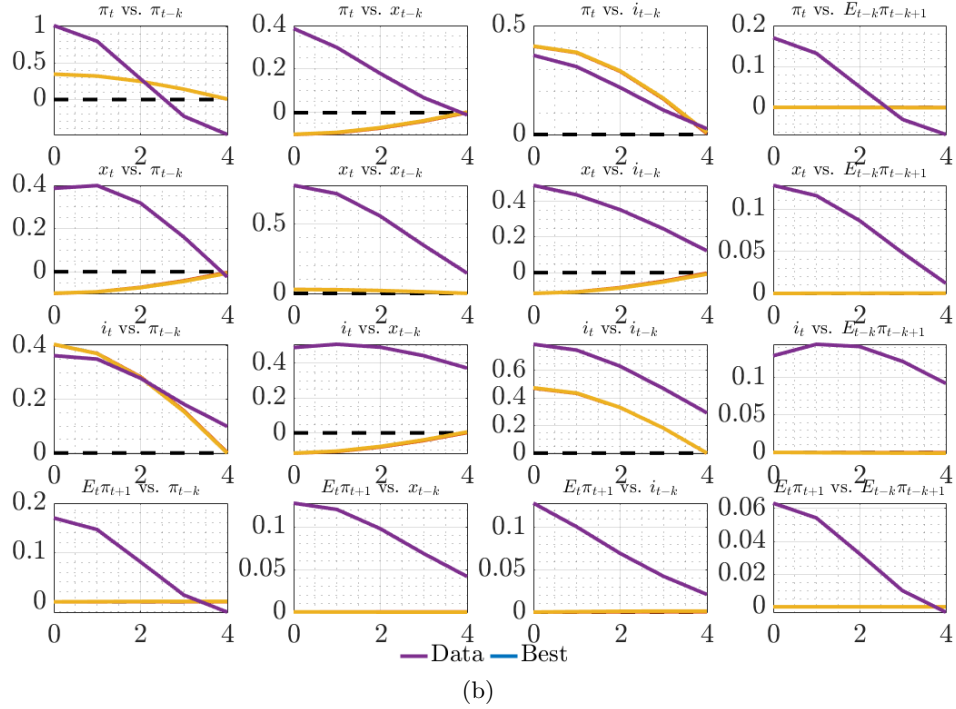
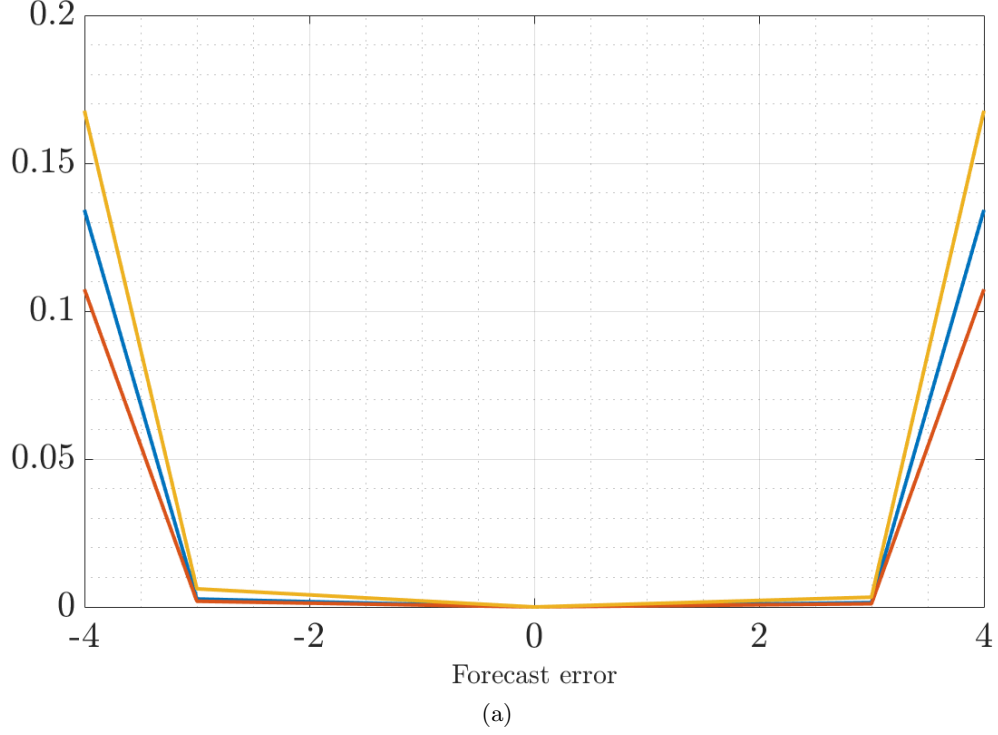
**Figure 3:** Calibration C, use expectations, ridge tuning = 0.01, initialize at truth, annualize expectations in true data



Yes!

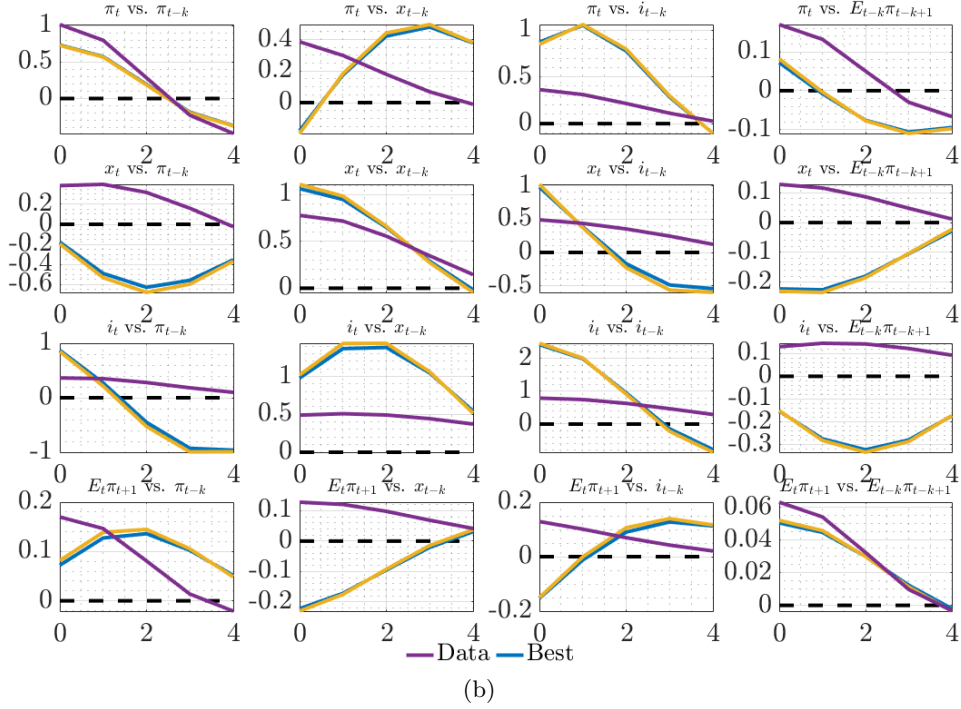
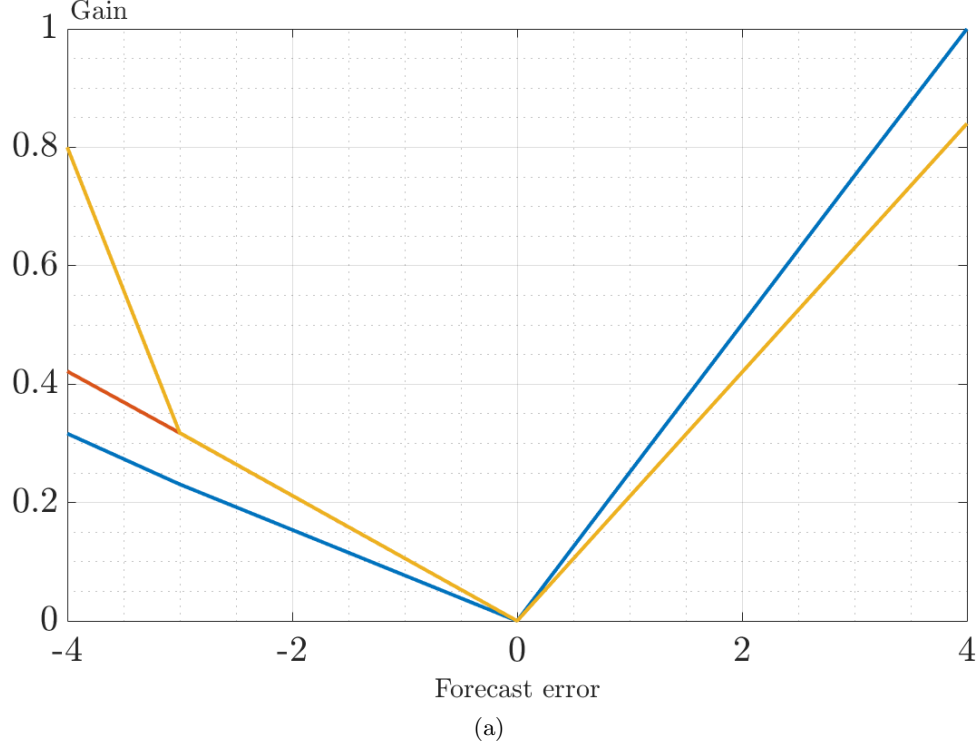
## 2 Real data

**Figure 4:** Settings from Fig 2, 10 different starting points, showing top 3 (blue is best),  $N = 100$

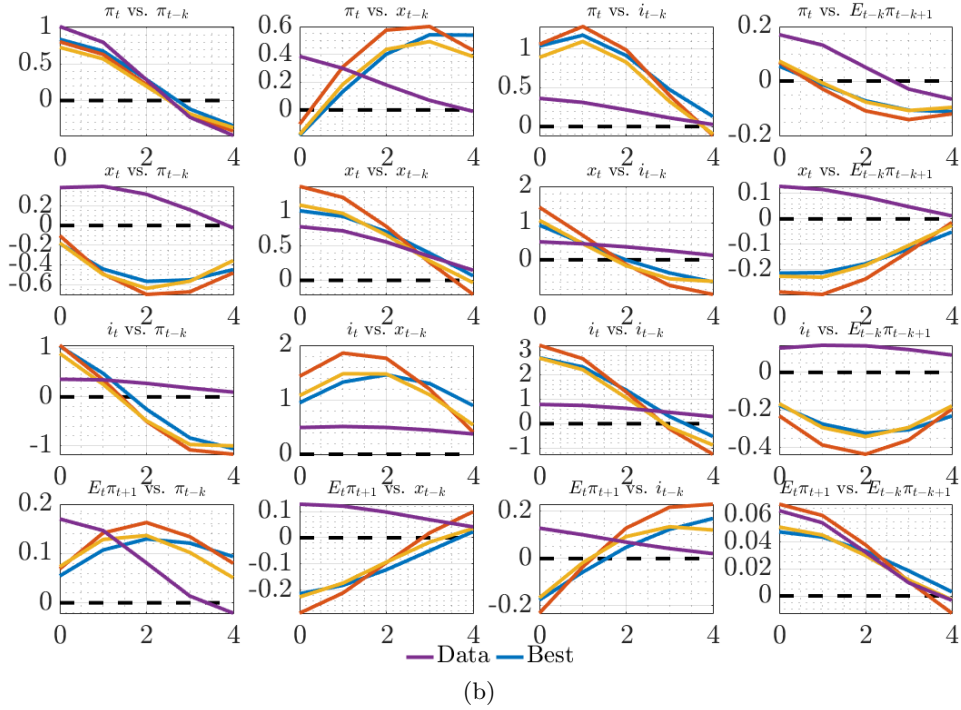
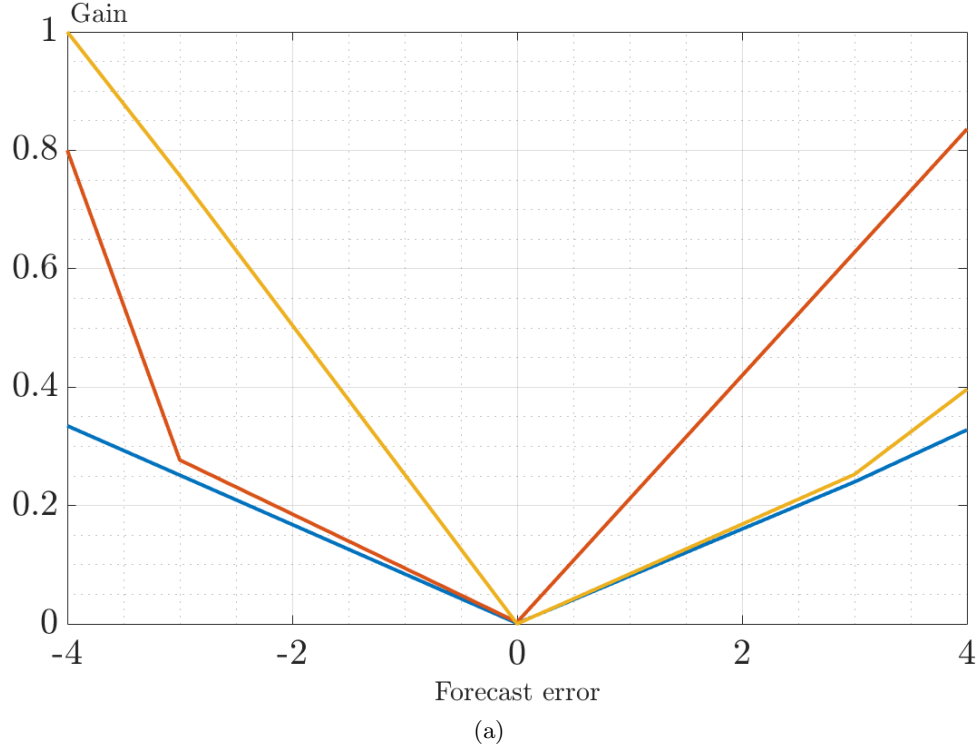


Darn. It doesn't hit the moments I want it to. So let's manually re-weight the own-autocovariances.

**Figure 5:** Settings from Fig 2, 5 different starting points, showing top 3 (blue is best),  $N = 100$ , manually putting more weight on own autocovariances



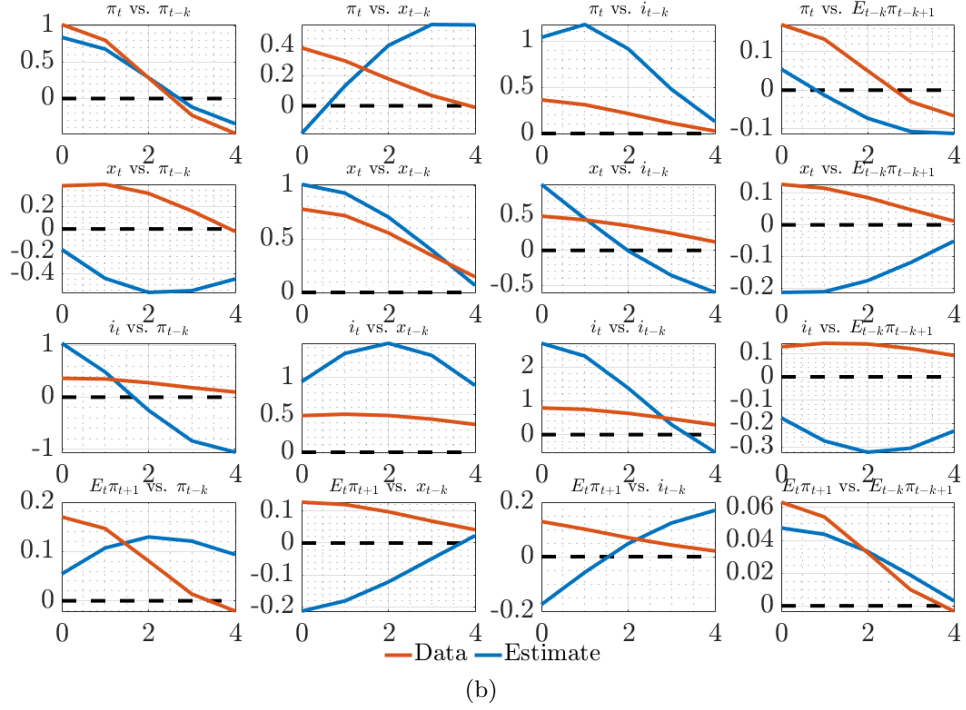
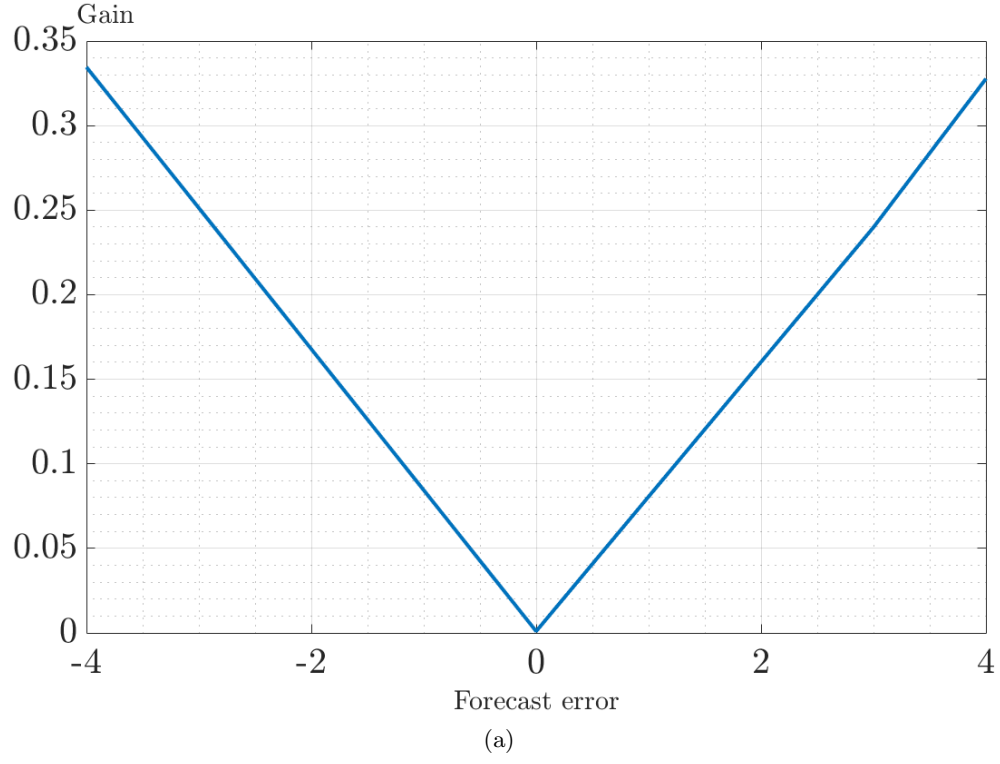
**Figure 6:** Settings from Fig 2, 5 different starting points, showing top 3 (blue is best),  $N = 500$ , manually putting more weight on own autocovariances



This took 5h:12min.

$$\hat{\alpha}_i = (0.3346; 0.2513; 0.001; 0.2399; 0.3277)$$

**Figure 7:** Same as the previous, just the best candidate alone





### 3 Policy isn't a function of $k_t^{-1}$

The anchoring function is (A.6):  $k_t^{-1} = \sum_i \alpha_i b_i(fe_{t|t-1})$ . This essentially eliminates  $k$  as a state variable.



## A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (\text{A.1})$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (\text{A.2})$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \quad (\text{if imposed}) \quad (\text{A.3})$$

$$\text{PLM:} \quad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \quad b = g_x h_x \quad (\text{A.4})$$

$$\text{Updating:} \quad a_t = a_{t-1} + k_t^{-1} (z_t - (a_{t-1} + b s_{t-1})) \quad (\text{A.5})$$

$$\text{Anchoring function:} \quad k_t^{-1} = \sum_i \alpha_i b_i (f e_{t|t-1}) \quad (\text{A.6})$$

$$\text{Forecast error:} \quad f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \quad (\text{A.7})$$

$$\text{LH expectations:} \quad f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \quad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \quad (\text{A.8})$$

This notation captures vector learning ( $z$  learned) for intercept only. For scalar learning,  $a_t = (\bar{\pi}_t \quad 0 \quad 0)'$  and  $b_1$  designates the first row of  $b$ . The observables  $(\pi, x)$  are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix} \sigma & 1-\beta & -\sigma\beta \end{bmatrix} f_b + \sigma \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \quad (\text{A.9})$$

$$\pi_t = \kappa x_t + \begin{bmatrix} (1-\alpha)\beta & \kappa\alpha\beta & 0 \end{bmatrix} f_a + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \quad (\text{A.10})$$

## B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only,  $k_t^{-1} = \mathbf{g}(f e_{t-1})$ ):

$$\begin{aligned} \pi_t = & -\frac{\lambda_x}{\kappa} \left\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_\pi(t) \right) \right. \\ & \left. \left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_\pi(t+j)) \right) \right\} \end{aligned} \quad (\text{B.1})$$

where I'm using the notation that  $\prod_{j=0}^0 \equiv 1$ . For interpretation purposes, let me rewrite this as follows:

$$\begin{aligned} \pi_t = & -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ & - \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left( k_t^{-1} + f e_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \left( \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j}^{-1} + f e_{t+1+j|t+j}^{eve} \mathbf{g}_\pi(t+j)) \right) \end{aligned} \quad (\text{B.2})$$

Interpretation: **tradeoffs from discretion in RE** + **effect of current level and change of the gain on future tradeoffs**  
+ **effect of future expected levels and changes of the gain on future tradeoffs**