Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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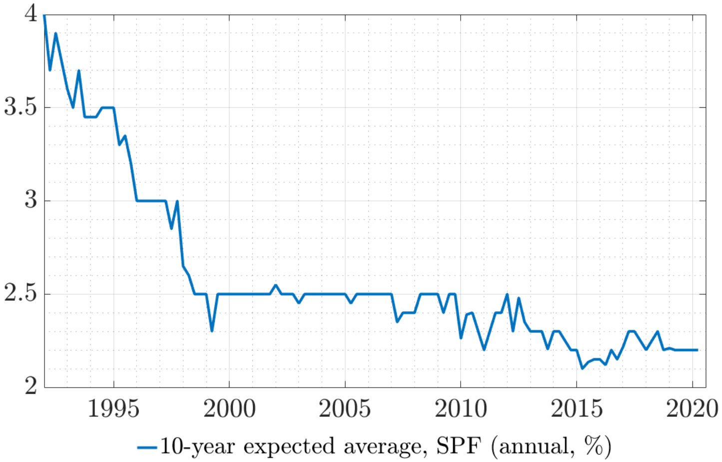
Inflation that runs below its desired level can lead to an unwelcome fall in longer-term inflation expectations, which, in turn, can pull actual inflation even lower, resulting in an adverse cycle of ever-lower inflation and inflation expectations. [...] Well-anchored inflation expectations are critical[.]
Jerome Powell, Chairman of the Federal Reserve [1]

[1] "New Economic Challenges and the Fed's Monetary Policy Review," August 27, 2020.

Figure: Expectations of average inflation over 10 years



TIPS

This project

- How to conduct monetary policy in interaction with the anchoring expectation formation?

- Model of anchoring expectation formation as an endogenous gain adaptive learning scheme

- Estimation of the anchoring function: when do expectations become unanchored?

Preview of results

- Optimal monetary policy responsiveness time-varying

↪ Optimal policy aggressive when expectations unanchor, dovish when anchored

- Taylor rule policy less aggressive on inflation than under rational expectations

↪ Anchoring-optimal Taylor rule eliminates 90% of loss from volatility

Related literature

- **Optimal monetary policy in New Keynesian models**
  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Adaptive learning**
  Evans & Honkapohja (2001, 2006), Bullard & Mitra (2002), Preston (2005, 2008), Ferrero (2007), Molnár & Santoro (2014), Eusepi & Preston (2011), Milani (2007, 2014), Lubik & Matthes (2018), Mele et al (2019)

- **Anchoring and the Phillips curve**
  Sargent (1999), Svensson (2015), Hooper et al (2019), Afrouzi & Yang (2020), Reis (2020), Gobbi et al (2019), Carvalho et al (2019)

Structure of talk

1. Unanchoring in the data
2. Model of anchoring expectations
3. Solving the Ramsey problem
4. Implementing optimal policy

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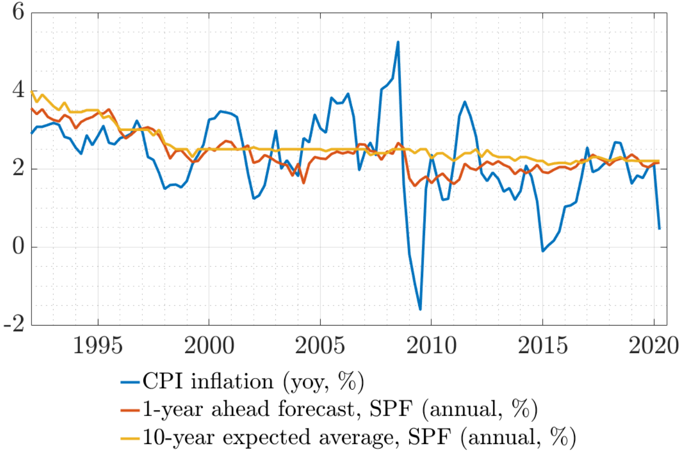
Figure: Inflation and expectations



Second half of sample: estimate

$$\bar{\pi}_t = \beta_0 + \beta_1 fe_{t|t-1} + \epsilon_t \tag{1}$$

$$\bar{\pi}_t \equiv \mathbb{E}_t(\pi_{t+10}) \tag{2}$$

$$fe_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1}(\pi_t) \tag{3}$$

$\hat{\beta}_1 = 0.04$ (p-value: 0.0016)

1 pp forecast error $\rightarrow$ 4 bp revision in long-run expectations

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Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j)) dj \right] \tag{4}$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1}) B_{t-1}^i + \int_0^1 w_t(j) h_t^i(j) dj + \Pi_t^i(j) dj - T_t - P_t C_t^i \tag{5}$$

Consumption, price level

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \tag{6}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{7}$$

▸ Profits, stochastic discount factor

Expectations: $\hat{\mathbb{E}}$ instead of $\mathbb{E}$

- If use $\mathbb{E}$ (rational expectations, RE)

  Model solution

$$s_t = hs_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \tag{8}$$

$$y_t = gs_t \tag{9}$$

$s_t \equiv$ states
$y_t \equiv$ jumps
$\epsilon_t \equiv$ disturbances

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  $s_t \equiv$ states
  $y_t \equiv$ jumps
  $\epsilon_t \equiv$ disturbances

- If use $\hat{\mathbb{E}} \rightarrow$ private sector does not know (9)

  $\hookrightarrow$ estimate using observed states & knowledge of (8)

Adaptive learning

- Postulate linear functional relationship instead of (9):

$$\hat{\mathbb{E}}_t y_{t+1} = a_{t-1} + b_{t-1} s_t \tag{10}$$

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- Estimate a, b using recursive least squares (RLS)

Recursive least squares

Jumps are: $(\pi, x, i)'$

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Special case: learn only intercept of inflation:

$$a_{t-1} = (\bar{\pi}_{t-1}, 0, 0)', \quad b_{t-1} = g\, h \quad \forall t \tag{11}$$

Recursive least squares

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$\bar{\pi}_{t-1}$: long-run inflation expectations $\rightarrow$ anchoring

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$\bar{\pi}_{t-1}$: long-run inflation expectations $\rightarrow$ anchoring

$\rightarrow$ RLS

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \underbrace{\left(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\right)}_{\equiv fe_{t|t-1}\text{, forecast error}} \tag{12}$$

$k_t \in (0, 1)$ gain
b_1 first row of b

General RLS algorithm

Anchoring mechanism: endogenous gain

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t \big(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1})\big) \tag{13}$$

$k_t = \mathbf{g}(fe_{t|t-1})$: anchoring function

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$k_t = \mathbf{g}(fe_{t|t-1})$: anchoring function

$$\mathbf{g}(fe_{t|t-1}) = \sum_i \alpha_i b_i(fe_{t|t-1}) \tag{14}$$

$b_i(fe_{t|t-1})$ = basis, here: second order spline (piecewise linear)

α_i = approximating coefficients, here: use $\hat{\alpha}$ from estimation

▸ Functional forms in literature

Model summary

- IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{15}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{16}$$

Derivations
Actual laws of motion

- Expectations evolve according to RLS with the endogenous gain given by (14)

→ How should $\{i_t\}$ be set?

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Ramsey problem

$$\min_{\{y_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

s.t. evolution of expectations

- $\mathbb{E}$ is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

Target criterion

Proposition

In the model with anchoring, monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_{\pi,t} \right)$$

$$\left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\bar{\pi},\mathbf{t+j}}) \right)$$

where $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t*,* $\prod_{j=0}^{0} \equiv 1$ *and* b_1 *is the first row of* b.

General case

Two layers of intertemporal stabilization tradeoffs

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t + fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\mathbb{E}_t\sum_{i=1}^{\infty} x_{t+i}$$
$$-\frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\left(k_t + fe_{t|t-1}\mathbf{g}_{\pi,t}\right)\mathbb{E}_t\sum_{i=1}^{\infty} x_{t+i}\prod_{j=0}^{i-1}(k_{t+1+j} + fe_{t+1+j|t+j}\mathbf{g}_{\bar{\pi},\mathbf{t+j}})$$

Intratemporal tradeoffs in RE (discretion)

Intertemporal tradeoff: current level and change of the gain

Intertemporal tradeoff: future expected levels and changes of the gain

Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

▸ Why no commitment?

Corollary

Optimal policy under adaptive learning is time-consistent.

$\hookrightarrow$ Foreshadow: optimal policy aggressiveness time-varying

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Solution procedure

Solve system of model equations + target criterion

$\hookrightarrow$ solve using parameterized expectations (PEA)

$\hookrightarrow$ obtain a cubic spline approximation to optimal policy function

Calibration - parameters from the literature

β	0.98	stochastic discount factor
σ	1	intertemporal elasticity of substitution
α	0.5	Calvo probability of not adjusting prices
κ	0.0842	slope of the Phillips curve
ψ_π	1.5	coefficient of inflation in Taylor rule*
ψ_x	0.3	coefficient of the output gap in Taylor rule*
$\bar{g}$	0.145	initial value of the gain
λ_x	0.05	weight on the output gap in central bank loss
ρ_r	0	persistence of natural rate shock
ρ_i	0	persistence of monetary policy shock*
ρ_u	0	persistence of cost-push shock

Calibration - matching moments

σ_r	0.01	standard deviation of natural rate shock
σ_i	0.01	standard deviation of monetary policy shock*
σ_u	0.5	standard deviation of cost-push shock
$\hat{\alpha}$	(0.8; 0.4; 0; 0.4; 0.8)	approximating coefficients in anchoring function

Picked to match the autocovariances of inflation, output gap, interest rate and one-period ahead inflation expectations for lags $0, \ldots, 4$.

Optimal policy - responding to unanchoring

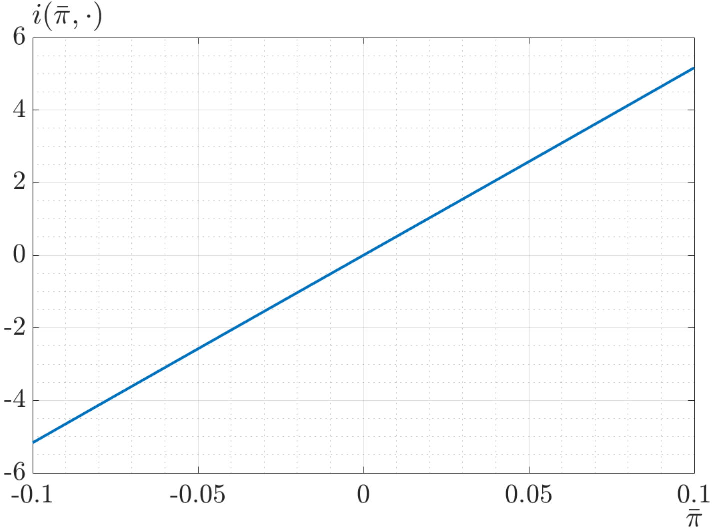
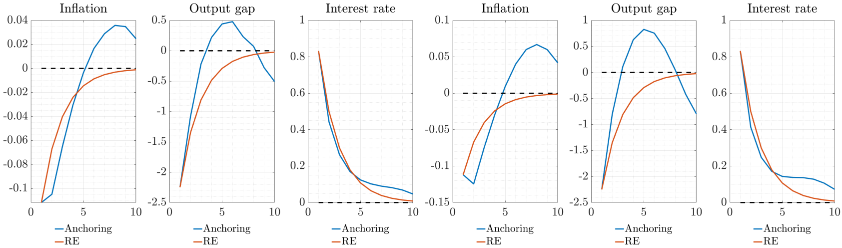


Figure: Policy function: $i(\bar{\pi}$, all other states at their means)

The intertemporal volatility tradeoff



(a) Expectations anchored (b) Expectations unanchored

Figure: Impulse responses after a contractionary monetary policy shock

Intertemporal volatility tradeoff: term structure of expectations

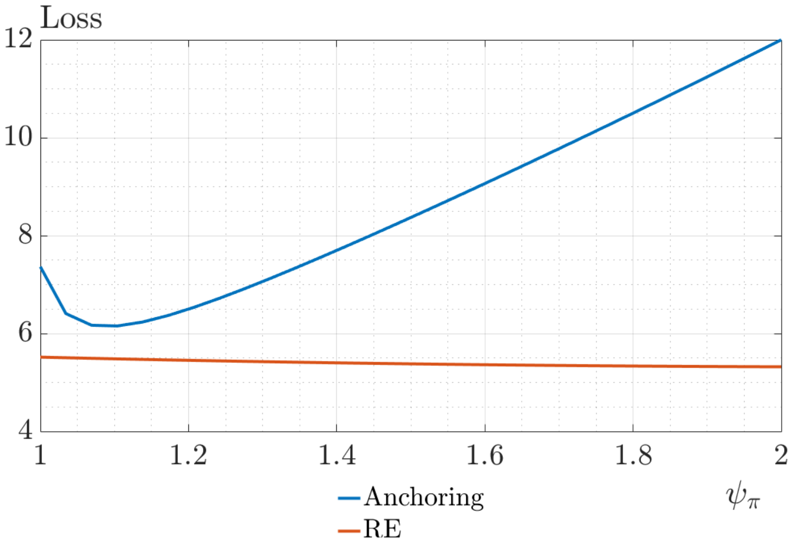
IS- and Phillips curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \big((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \big)$$

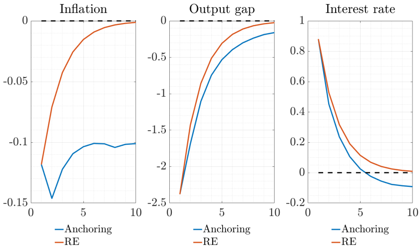
$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T \big)$$

Optimal Taylor-coefficient on inflation

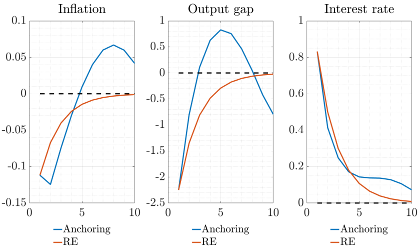
Figure: Central bank loss as a function of ψ_π



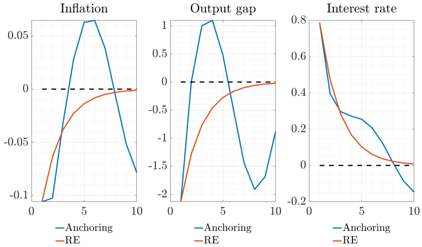
The intertemporal volatility tradeoff - again



(a) $\psi_\pi = 1.01$



(b) $\psi_\pi = 1.5$



(c) $\psi_\pi = 2$

Losses for optimal Taylor-rule coefficient on inflation

RE-optimal coefficient: $\psi_\pi^{RE} = 2.21$

Anchoring-optimal coefficient: $\psi_\pi^{A} = 1.1$

Table: Loss for RE and anchoring models for choice of RE- or anchoring-optimal ψ_π

Anchoring, ψ_π^{RE}	Anchoring, ψ_π^{A}	RE, ψ_π^{RE}
13.6380	6.1446	5.3148

$\rightarrow$ If model is anchoring, anchoring-optimal ψ_π^{A} gets 90% of the distance to RE-optimal ψ_π^{RE} under RE

Conclusion

- First theory of monetary policy for potentially unanchored expectations
- Optimal policy conditions on stance of current and expected future anchoring

  $\hookrightarrow$ determine intertemporal tradeoffs
- Frontloads aggressive interest rate response to suppress potential unanchoring
- Matters: already anchoring-optimal Taylor rule reduces losses by 50%
- Future work: how to anchor at zero-lower bound?

Appendix

Correcting the TIPS from liquidity risk

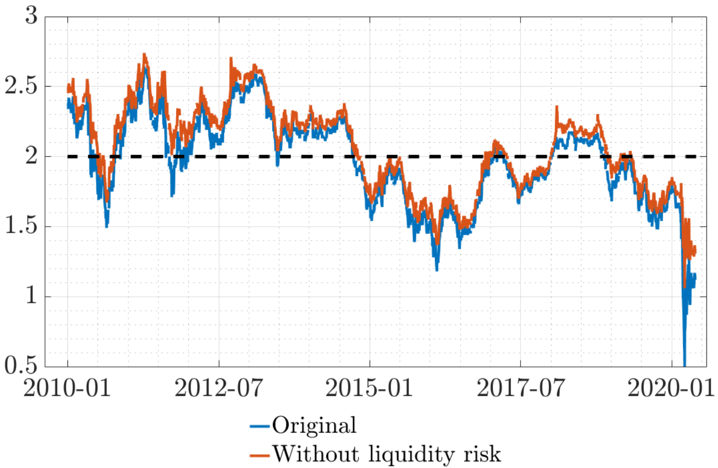


Figure: Market-based inflation expectations, 10 year, average, %

Return

Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{17}$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{18}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1-k^{-1}}{1-k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1-k^{-1}\beta} u_t \tag{19}$$

Solve for forecast error $fe_t \equiv \pi_t - f_{t-1}$:

$$fe_t = \underbrace{-\frac{1-\beta}{1-k\beta}}_{\lim_{k\to 1}=-1} f_{t-1} + \frac{1}{1-k\beta} u_t \tag{20}$$

Functional forms for **g** in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \tag{21}$$

$p \equiv Prob$(liquidity trap regime)
y_{t-1} output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{22}$$

θ_t criterion, $\bar{\theta}$ threshold value

Return

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \tag{23}$$

Σ variance-covariance matrix of shocks
$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(fe_{t|t-1} fe'_{t|t-1} - \omega_{t-1}) \tag{24}$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(fe'_{t|t-1} \omega_t^{-1} fe_{t|t-1} - \theta_{t-1}) \tag{25}$$

ω_t estimated forecast-error variance

Return

Recursive least squares algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right)' \right)' \tag{26}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{27}$$

Return

Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{28}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{29}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \tag{30}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{31}$$

Return

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t) + \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \Big\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t + \mathbf{g}_\pi(\pi_t - f_{t-1})) = 0 \tag{32}$$

$$-2\beta\frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g_f}(\pi_{t+1} - f_t)) = 0 \tag{33}$$

Return

Target criterion system for anchoring function as changes of the gain

$$\varphi_{6,t} = -cfe_{t|t-1}x_{t+1} + \left(1 + \frac{fe_{t|t-1}}{fe_{t+1|t}}(1-k_{t+1}) - fe_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1}$$

$$- \frac{fe_{t|t-1}}{fe_{t+1|t}}(1-k_{t+1})\varphi_{6,t+2} \tag{34}$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{fe_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{fe_{t|t-1}}\varphi_{6,t+1} \tag{35}$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (35) is given by:

$$\varphi_{6,t} = -2\,\mathbb{E}_t \sum_{i=0}^{\infty}(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{fe_{t+j|t+j-1}}}{\frac{k_{t+j}}{fe_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \tag{36}$$

Return

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{37}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{38}$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \tag{39}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{40}$$

Return

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{41}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{42}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (41) backward to some date t, take expectations at t
2. Sub in (42)
3. Aggregate over households i

$\rightarrow$ Obtain (15)

◂ Return