Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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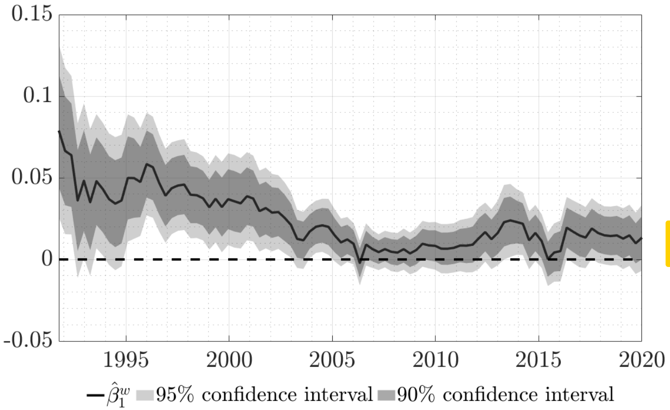
Varying responsiveness of long-run expectations

$$\Delta \bar{\pi}_t^i = \beta_0^w + \beta_1^w f_{t|t-1}^i + \epsilon_t^i \qquad (1)$$



$\bar{\pi}_t^i$: 10-year-ahead inflation expectation of SPF forecaster i

$f_{t|t-1}^i$: 1-year-ahead inflation forecast error of SPF forecaster i

This paper

1. A model of unanchored expectations:
   $\hookrightarrow$ sensitivity of long-run expectations to short-run fluctuations

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   $\hookrightarrow$ analytically and numerically using novel model disciplined by data

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1. A model of unanchored expectations:
   ↪ sensitivity of long-run expectations to short-run fluctuations

2. Analyze monetary policy
   ↪ analytically and numerically using novel model disciplined by data

3. **Key takeaway**: optimal monetary policy
   - anchors expectations to inflation target
   - responds aggressively to movements in long-run expectations

NK model with long-run inflation expectations

- New Keynesian core: standard IS and Phillips curves

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r^n_T) \quad (2)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (3)$$

Observables: (π, x, i) inflation, output gap, interest rate
Exogenous states: (r^n, u) natural rate and cost-push shock

NK model with long-run inflation expectations

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$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\left((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\right) \tag{2}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\left(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\right) \tag{3}$$

  Observables: (π, x, i) inflation, output gap, interest rate
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- Novelty of the paper: inflation expectations process

$$\hat{\mathbb{E}}_t \pi_{t+1} = \bar{\pi}_t + \mathbb{E}_t\, \pi_{t+1} \tag{4}$$

  $\mathbb{E}$: rational (model-consistent) expectations
  $\hat{\mathbb{E}}$: nonrational expectations $\to$ long-run inflation expectations $\bar{\pi}_{t-1}$

Evolution of long-run inflation expectations

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1} \tag{5}$$

$f_{t|t-1} = \pi_t - \hat{\mathbb{E}}_{t-1}\pi_t$
$k_t \in (0,1)$ learning gain

Evolution of long-run inflation expectations

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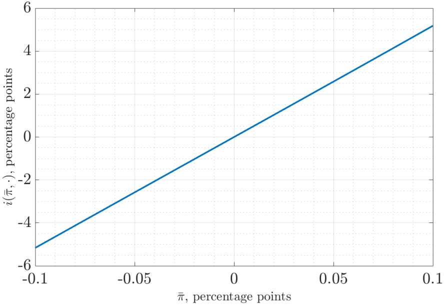
$f_{t|t-1} = \pi_t - \hat{\mathbb{E}}_{t-1}\pi_t$
$k_t \in (0, 1)$ learning gain

Endogenous gain as metric for unanchoring

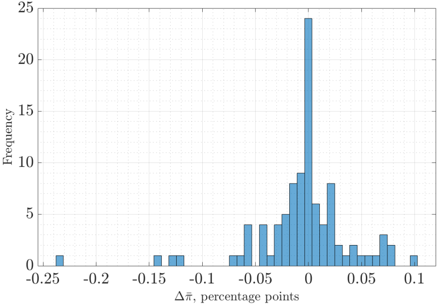
$$k_t = \mathbf{g}(f_{t|t-1}) \tag{6}$$

$\mathbf{g}(f_{t|t-1})$ continuous, smooth, convex

Optimal policy - responding to unanchoring



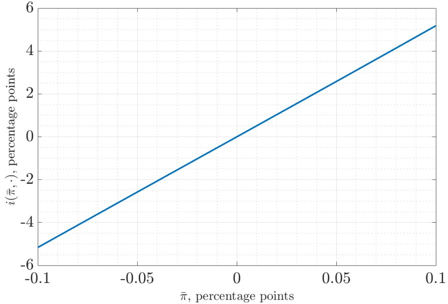
$i(\bar{\pi}$, all other states at their means)



Stabilizing $\bar{\pi}$

5 bp movement in $\bar{\pi} \rightarrow$ 250 bp movement in i

Optimal policy - responding to unanchoring



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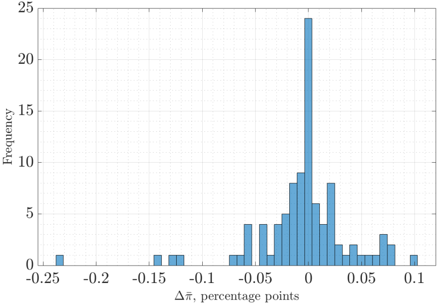

Stabilizing $\bar{\pi}$

5 bp movement in $\bar{\pi} \rightarrow$ 250 bp movement in i

Mode: 0.3 bp movement in $\bar{\pi}$

Unanchoring amplifies shocks

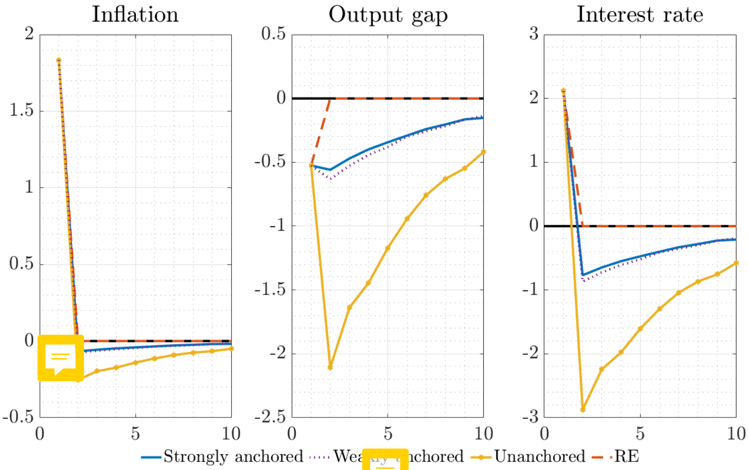

Figure: Impulse responses after a cost-push shock when policy follows a Taylor rule

Conclusion

First theory of monetary policy for potentially unanchored expectations

Model-based notion of unanchoring

- Sensitivity of long-run expectations to short-run fluctuations

Optimal monetary policy

- Anchors expectations by responding aggressively to long-run expectations

Thank you!

Questions? → gati@bc.edu

Appendix

Long-run expectations: responsive to short-run conditions?

Individual-level Survey of Professional Forecasters (SPF): for 1991-Q4 onward, estimate rolling regression

$$\Delta\bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{7}$$

$\bar{\pi}_t$ 10-year ahead inflation expectation

$f_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1}\, \pi_t$ individual one-year-ahead forecast error

w indexes windows of 20 quarters

Time-varying responsiveness

$$\Delta\bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{1}$$

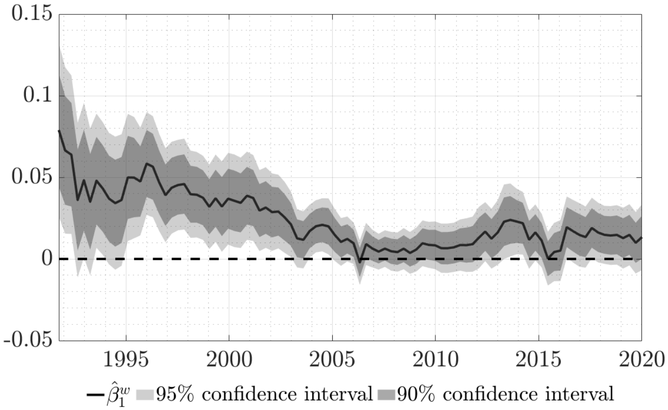

Figure: Time series of $\hat{\beta}_1^w$

Robustness

Breakeven inflation

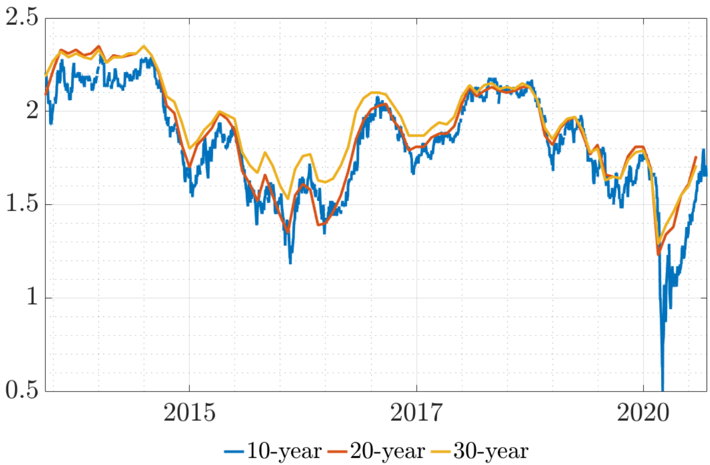

Figure: Market-based inflation expectations, various horizons, %

Return

Correcting the TIPS from liquidity risk

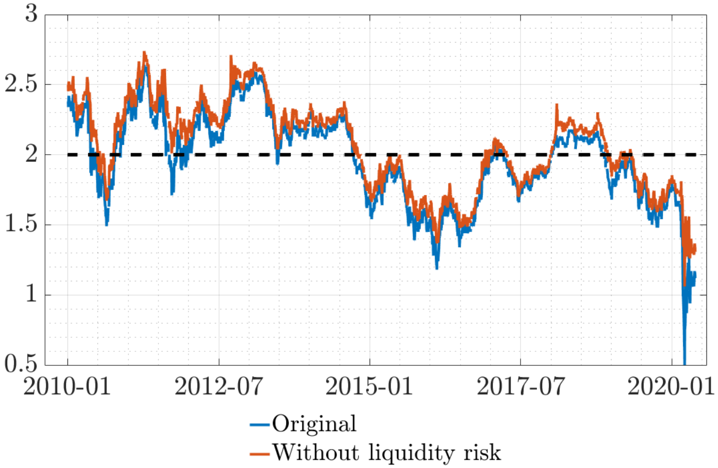

Figure: Market-based inflation expectations, 10 year, %

Return

Robustness checks

$$\Delta \bar{\pi}_t = \beta_0 + \beta_1^w \pi_t + \epsilon_t \quad (1)$$

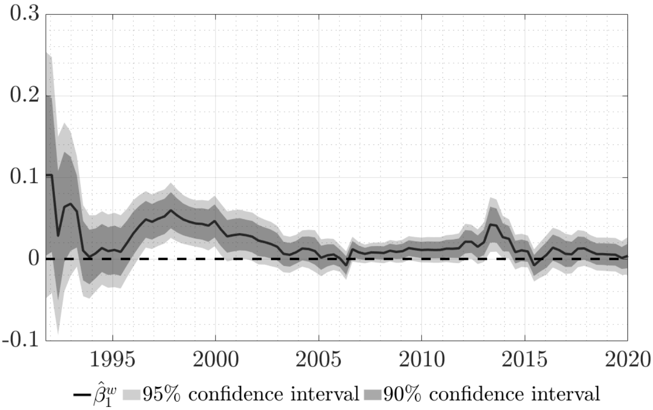

Figure: Time series of $\hat{\beta}_1^w$

Return

Robustness checks - PCE core

$$\Delta\bar{\pi}_t = \beta_0^w + \beta_1^w f_{t|t-1} + \epsilon_t \quad (1)$$

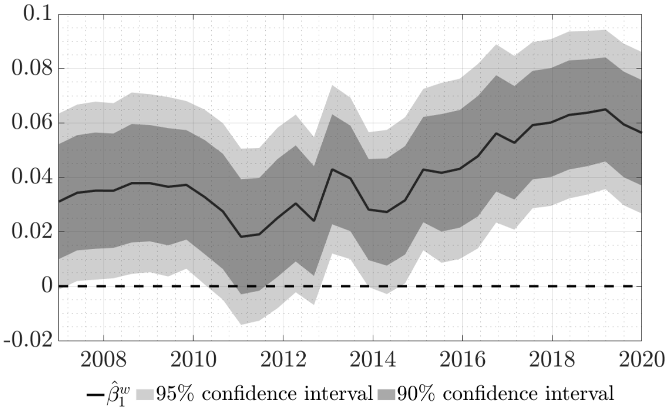

Figure: Time series of $\hat{\beta}_1^w$

Return

Robustness checks - controlling for inflation levels

$$\Delta \bar{\pi}_t = \beta_0^w + \beta_1^w f_{t|t-1} + \beta_2^w \pi_t + \epsilon_t \tag{1}$$

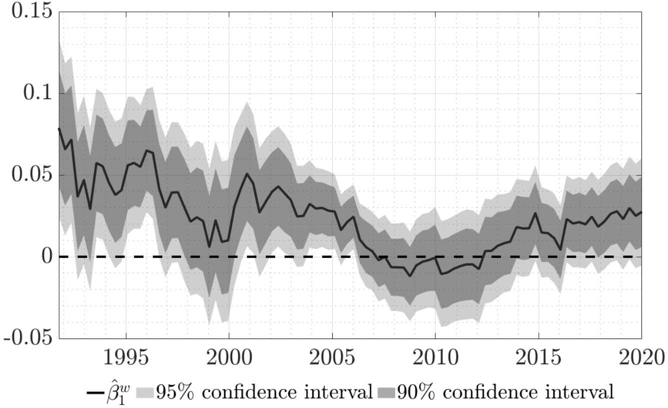

Figure: Time series of $\hat{\beta}_1^w$

Return

Further evidence: disagreement

Figure: Livingston Survey of Firms:
Interquartile range of 10-year ahead inflation expectations

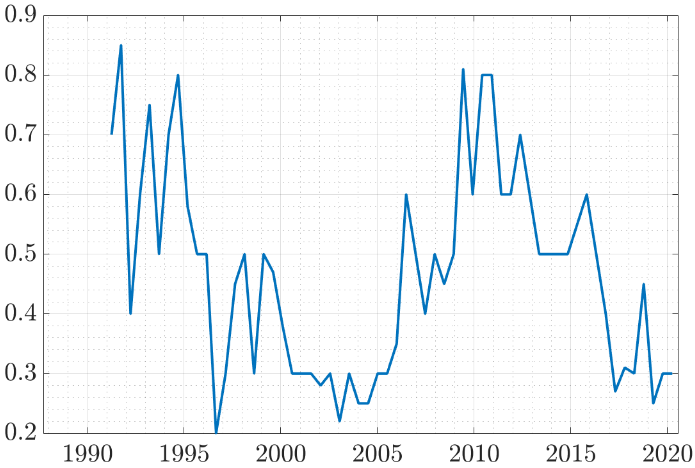

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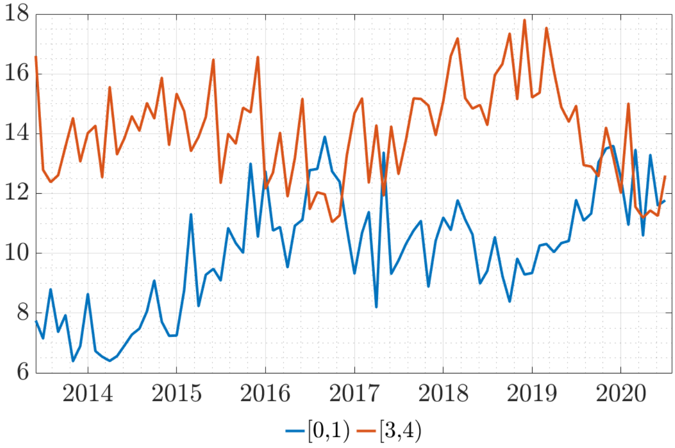
Figure: New York Fed Survey of Consumers:
Percent of respondents indicating 3-year ahead inflation will be in a particular range



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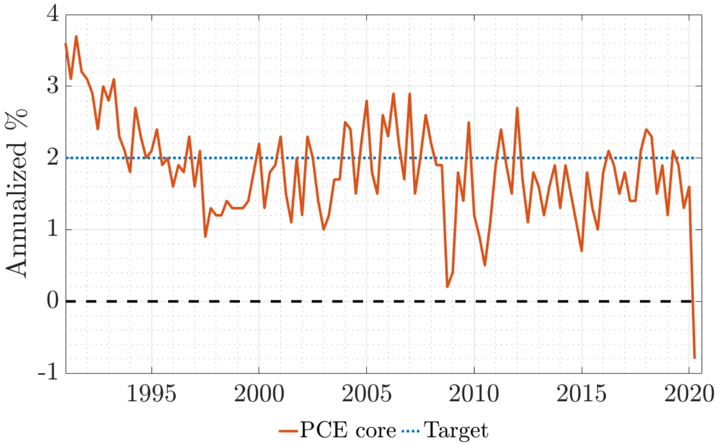
Further evidence: introspection

Figure: PCE core inflation against the Fed's target



Return

Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j)) dj \right] \tag{8}$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1}) B_{t-1}^i + \int_0^1 w_t(j) h_t^i(j) dj + \Pi_t^i(j) dj - T_t - P_t C_t^i \tag{9}$$

Return

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t^j \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \tag{10}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{11}$$

Return

Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{12}$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{13}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \tag{14}$$

Solve for forecast error $f_t \equiv \pi_t - f_{t-1}$:

$$f_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k \to 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \tag{15}$$

Return

Functional forms for **g** in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \tag{16}$$

$p \equiv Prob(\text{liquidity trap regime})$
y_{t-1} output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{17}$$

θ_t criterion, $\bar{\theta}$ threshold value

Return

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max|\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \tag{18}$$

Σ variance-covariance matrix of shocks
$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(f_{t|t-1} f'_{t|t-1} - \omega_{t-1}) \tag{19}$$
$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(f'_{t|t-1} \omega_t^{-1} f_{t|t-1} - \theta_{t-1}) \tag{20}$$

ω_t estimated forecast-error variance

Return

General updating algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right)' \right)' \tag{21}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{22}$$

Return

Assumptions on $\mathbf{g}(\cdot)$

$$\mathbf{g}_{ff} \geq 0 \tag{23}$$

$\mathbf{g}(\cdot)$ convex in forecast errors.

Return

Estimating form of gain function

- Calibrate parameters of New Keynesian core to literature
- Estimate flexible form of expectations process via simulated method of moments
  (Duffie & Singleton 1990, Lee & Ingram 1991, Smith 1993)

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \mathbf{g}(f_{t|t-1})\, f_{t|t-1} \tag{18}$$

- Moments: autocovariances of inflation, output gap, federal funds rate and 1-year ahead Survey of Professional Forecasters (SPF) inflation expectations at lags $0, \ldots, 4$

Return
Details

Estimated expectations process

$$\bar{\pi}_t - \bar{\pi}_{t-1} = \mathbf{g}(f_{t|t-1})\, f_{t|t-1} \tag{18}$$

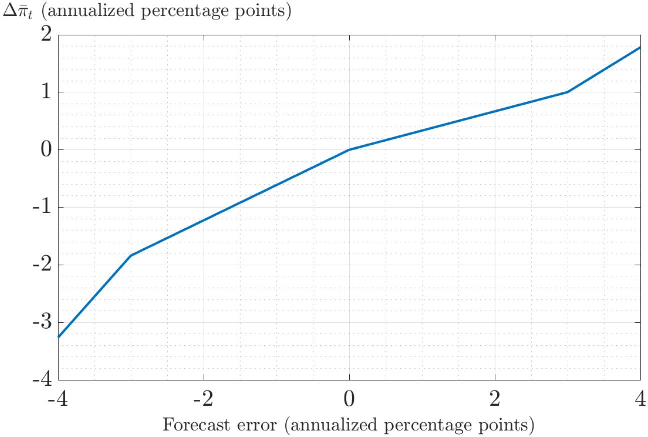


Figure: Changes in long-run inflation expectations as a function of forecast errors

Return

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{24}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{25}$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \tag{26}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{27}$$

Return

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{28}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{29}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (28) backward to some date t, take expectations at t
2. Sub in (29)
3. Aggregate over households i

$\rightarrow$ Obtain (2)

◂ Return

Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{30}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{31}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \tag{32}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{33}$$

Return

Piecewise linear approximation to gain function

$$\mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}) \tag{34}$$

- $b_i(f_{t|t-1})$ = piecewise linear basis
- γ_i = approximating coefficient at node i

$\hookrightarrow$ Estimate $\hat{\gamma}$ via simulated method of moments

Return

The expectation process over time

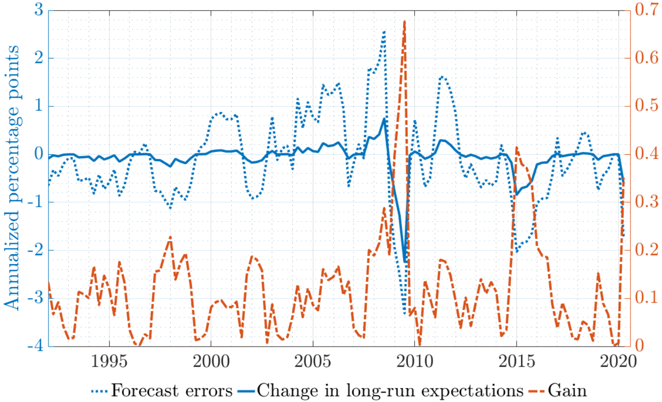


Figure: Time series of forecast errors, changes in long-run expectations and gain

Target criterion

Proposition

Let $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t*. Then monetary policy optimally brings about the following target relationship between inflation and the output gap*

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t$$

RE (discretion): move π_t and x_t to offset cost-push shocks

Return

Target criterion

Proposition

Let $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t*. Then monetary policy optimally brings about the following target relationship between inflation and the output gap*

$$\pi_t - \Gamma(k)\,\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} = -\frac{\lambda_x}{\kappa} x_t$$

Adaptive learning: can move $\mathbb{E}_t\, x_{t+i}$ too if $k > 0$

▸ $\Gamma(k)$

◂ Return

Target criterion

Proposition

Let $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ at t. Then monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t \ - \Omega\Big(k_t + f_{t|t-1}\mathbf{g}_{\pi,t}\Big)\left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(1 - k_{t+1+j} - f_{t+1+j|t+j}\mathbf{g}_{\bar{\mathbf{ß}},\mathbf{t+j}})\right) = -\frac{\lambda_x}{\kappa} x_t$$

Endogenous gain: ability to move $\mathbb{E}_t\, x_{t+i}$ depends on present and future degree of unanchoring

Full expression, Ω No commitment

Return

Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

▸ Why no commitment?

Corollary

Optimal policy under adaptive learning is time-consistent.

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$
\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t)
$$

$$
+ \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \Big\}
$$

$$
2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t + \mathbf{g}_{\textbf{ß}}(\pi_t - f_{t-1})) = 0 \tag{35}
$$

$$
-2\beta \frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_{\mathbf{f}}(\pi_{t+1} - f_t)) = 0 \tag{36}
$$

Return

Target criterion system for anchoring function as changes of the gain

$$\varphi_{6,t} = -cf_{t|t-1}x_{t+1} + \left(1 + \frac{f_{t|t-1}}{f_{t+1|t}}(1-k_{t+1}) - f_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1}$$
$$- \frac{f_{t|t-1}}{f_{t+1|t}}(1-k_{t+1})\varphi_{6,t+2} \tag{37}$$
$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{f_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{f_{t|t-1}}\varphi_{6,t+1} \tag{38}$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (38) is given by:

$$\varphi_{6,t} = -2\,\mathbb{E}_t\sum_{i=0}^{\infty}(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i})\prod_{j=0}^{i-1}\frac{\frac{k_{t+j}}{f_{t+j|t+j-1}}}{\frac{k_{t+j}}{f_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \tag{39}$$

Return

Optimal Taylor-coefficient on inflation

$$i_t = \psi_\pi \pi_t + \psi_x x_t \tag{40}$$

Optimal Taylor-coefficient on inflation

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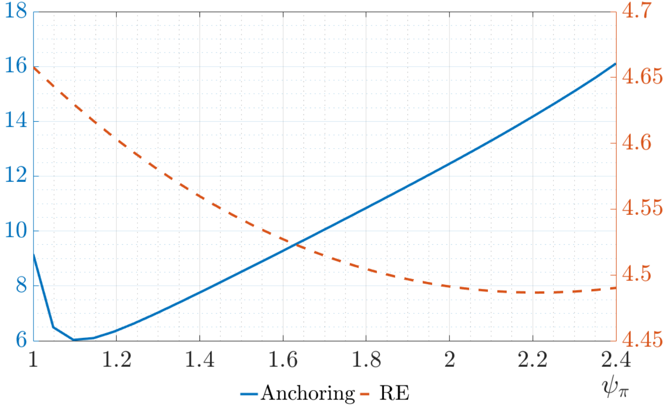
Figure: Central bank loss as a function of ψ_π



Optimal Taylor-coefficient on inflation

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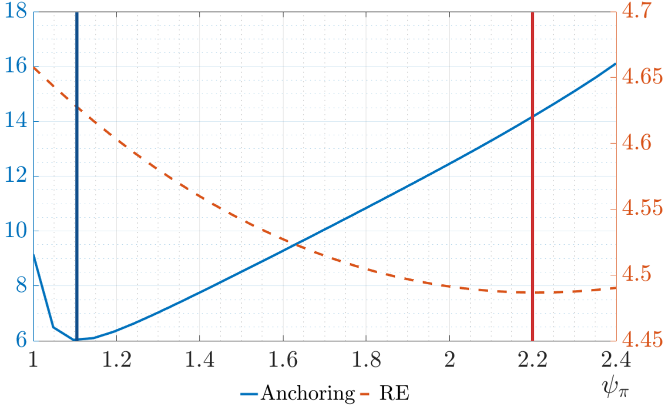

Anchoring-optimal coefficient: $\psi_\pi^A = 1.1$ RE-optimal coefficient: $\psi_\pi^{RE} = 2.2$

Why less aggressive? Future interest rate expectations

IS curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n)$$

- Current interest rate i_t: one channel of policy

Return

IRFs for various ψ_π

Why less aggressive? Future interest rate expectations

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$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n)$$

- Current interest rate i_t: one channel of policy
- Taylor rule implies interest rate expectation

$$\hat{\mathbb{E}}_t i_{t+k} = \psi_\pi \hat{\mathbb{E}}_t \pi_{t+k} + \psi_x \hat{\mathbb{E}}_t x_{t+k} \tag{41}$$

Return

IRFs for various ψ_π

Why less aggressive? Future interest rate expectations

IS curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left((1-\beta) x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n \right)$$

- Current interest rate i_t: one channel of policy
- Taylor rule implies interest rate expectation

$$\hat{\mathbb{E}}_t i_{t+k} = \psi_\pi \hat{\mathbb{E}}_t \pi_{t+k} + \psi_x \hat{\mathbb{E}}_t x_{t+k} \tag{41}$$

- If private sector understands and believes Taylor rule, expected future interest rates additional channel of policy (Eusepi, Giannoni & Preston 2018)

Return

IRFs for various ψ_π

Respond but not too much

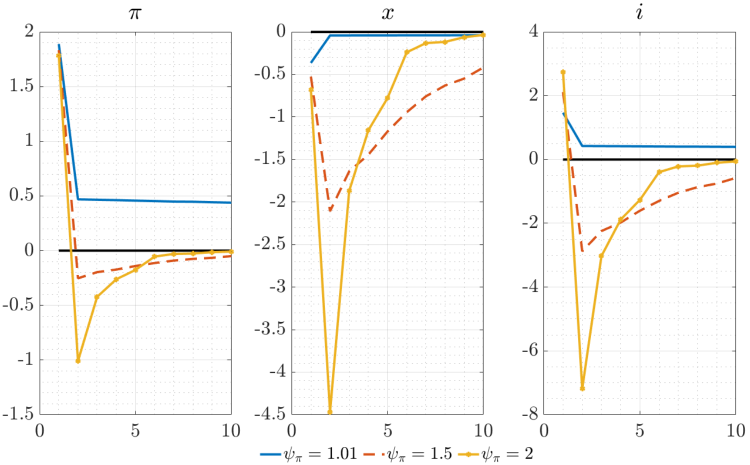

Figure: Impulse responses for unanchored expectations for various values of ψ_π