

Materials 33 - Estimating the anchoring function

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Instead of the AR(1) anchoring function used so far (Equation A.6), I use the following equation

$$k_t^{-1} = \beta s(X) \quad (1)$$

where $X = (k_{t-1}^{-1}, fe_{t|t-1})$. I'm trying to implement piecewise linear interpolation (to preserve shape).
Initialization of β ? If I had initial X !

A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n) \quad (A.1)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T) \quad (A.2)$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \quad (\text{if imposed}) \quad (A.3)$$

$$\text{PLM:} \quad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \quad b = g_x h_x \quad (A.4)$$

$$\text{Updating:} \quad a_t = a_{t-1} + k_t^{-1} (z_t - (a_{t-1} + b s_{t-1})) \quad (A.5)$$

$$\text{Anchoring function:} \quad k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k f e_{t-1}^2 \quad (A.6)$$

$$\text{Forecast error:} \quad f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \quad (A.7)$$

$$\text{LH expectations:} \quad f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \quad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \quad (A.8)$$

This notation captures vector learning (z learned) for intercept only. For scalar learning, $a_t = (\bar{\pi}_t \ 0 \ 0)'$ and b_1 designates the first row of b . The observables (π, x) are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix} \sigma & 1-\beta & -\sigma\beta \end{bmatrix} f_b + \sigma \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \quad (A.9)$$

$$\pi_t = \kappa x_t + \begin{bmatrix} (1-\alpha)\beta & \kappa\alpha\beta & 0 \end{bmatrix} f_a + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \quad (A.10)$$

B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only, $k_t^{-1} = \mathbf{g}(fe_{t-1})$):

$$\pi_t = -\frac{\lambda_x}{\kappa} \left\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})) \mathbf{g}_\pi(t) \right) \right. \\ \left. \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j}) \mathbf{g}_{\bar{\pi}}(t+j)) \right) \right\} \quad (\text{B.1})$$

where I'm using the notation that $\prod_{j=0}^0 \equiv 1$. For interpretation purposes, let me rewrite this as follows:

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t^{-1} + fe_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ - \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta}{1-\alpha\beta} \left(k_t^{-1} + fe_{t|t-1}^{eve} \mathbf{g}_\pi(t) \right) \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j}^{-1} + fe_{t+1+j|t+j}^{eve}) \mathbf{g}_{\bar{\pi}}(t+j) \right) \quad (\text{B.2})$$

Interpretation: **tradeoffs from discretion in RE** + **effect of current level and change of the gain on future tradeoffs** + **effect of future expected levels and changes of the gain on future tradeoffs**