On the Limits of Monetary Policy*

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Abstract

This paper provides theory and evidence that distorted long-term interest-rate expectations limit the effectiveness of monetary policy. Beliefs that depart from rational expectations break the tight link between policy rates and long-term interest rates, even when determined by the expectations hypothesis of the yield curve. Because bond prices are excessively sensitive to short-term interest rates, optimal policy is less aggressive relative to rational expectations. More aggressive policy leads to sub-optimal volatility in long-term interest rates and aggregate demand through standard intertemporal substitution effects. These effects are quantitatively important in the United States over the Great Inflation and Great Moderation periods.

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1 Introduction

This paper argues that distorted long-term interest rates limit the effectiveness of monetary policy. We build a New Keynesian model where departures from rational expectations introduce an endogenous, time-varying wedge between subjective and objective expectations. This wedge makes optimal policy more activist when long-run expectations are stable (anchored) and less activist when long-run expectations are less stable. We estimate this model on US data and show via historical decompositions that this mechanism is quantitatively important. There are fundamental limits to what monetary policy can do.

Acknowledging this has consequences for how we interpret the historical performance of the Federal Reserve and for what we can reasonably expect monetary policy to achieve. Indeed, Clarida, Gali, and Gertler (1999) and Woodford (2003) articulate a research program which envisages a highly effective central bank, unhindered by informational problems. The framework variously exemplifies the great power of monetary policy as a stabilization tool. Optimal policy can completely stabilize inflation and the output gap when faced with movements in the natural real rate of interest. And should the zero lower bound on nominal interest rates be a relevant constraint on policy actions, commitment to conduct future policy in a certain way can largely mitigate recession (Eggertsson and Woodford 2003).[1]

To model expectations anchoring we introduce a theory of distorted beliefs in which agents have imperfect information about long-run economic fundamentals.[2] These fundamentals are the long-run equilibrium outcomes implied by the economic model, including the inflation rate, the real interest rate and the nominal interest rate. Agents use a Kalman filter to form inferences about these unknown objects, making long-run beliefs respond to short-run forecast errors, consistent with survey data on expectations (Crump, Eusepi, and Moench 2016 and Carvalho, Eusepi, Moench, and Preston 2019). The strength of this connection measures the degree of anchoring of expectations. The weaker the sensitivity, the better anchored expectations. When long-run expectations are independent of short-run forecast errors we have rational expectations. Carvalho, Eusepi, Moench, and Preston (2019) document substantial time-variation in the degree to which expectations are anchored.

We embed this expectations formation mechanism into a standard New Keynesian model. Equilibrium subjective beliefs are more persistent than objective beliefs and display greater sensitivity to short-term forecast errors relative to what would be optimal given the true data-

[1] These predictions are not only a property of simple models: monetary policy can eliminate almost all inefficient fluctuations in models with multiple nominal and real frictions (Justiniano, Primiceri, and Tambalotti 2013), financial imperfections (Furlanetto, Gelain, and Sanjani 2017) and heterogeneous agents (Challe 2017); and in such models, commitments about future policy are, if anything, too effective, giving rise to the forward guidance puzzle (Del Negro, Giannoni, and Patterson 2012).

[2] In the language of Kozicki and Tinsley (2001) a 'shifting end-points' model.

generating process. Extrapolation bias is a general equilibrium outcome of the model — a model in which the macroeconomic variables being forecast are themselves endogenous and determined in part by expectations. This wedge between subjective and objective beliefs is endogenous, time-varying and depends on monetary policy and economic disturbances. The size of the wedge indexes the economic and quantitative importance of the information friction. When the wedge is equal to zero the dynamics are given by a rational expectations analysis. Our approach is consistent with evidence that observed measures of expectations exhibit extrapolation bias (Fuster, Laibson, and Mendel 2010, Bordalo, Gennaioli, Ma, and Shleifer 2018 and Angeletos, Huo, and Sastry 2020).

In response to a surprise increase in nominal interest rates, subjective beliefs of long-term interest rates rise by more than under objective beliefs, persistently so. Similarly a surprise fall in short-run interest rates, generates a larger fall in long-term rates. The term structure of interest rates, determined by the expectations hypothesis, is therefore flatter. This information friction makes the central bank's control of long-term interest rates and aggregate demand imprecise.[3] This imposes a constraint on stabilization policy the severity of which depends on how anchored are expectations.[4]

In a simple version of the model, we study how distorted beliefs affect optimal monetary policy. In general the rational expectations optimal policy in which the nominal interest rates track the natural rate of interest one-for-one is not feasible. Optimal policy is progressively less activist the more sensitive long-run beliefs are to short-run forecast errors — that is, as long-run expectations become less well anchored. We show that the stabilization trade-off arises from uncertainty about the location of the aggregate demand curve. If the central bank can control aggregate demand as the instrument of policy then the optimal policy under imperfect information and rational expectations is identical. Hence the information friction affects the transmission mechanism of monetary policy which operates through long-term interest rates. For this reason our results contrast with Orphanides and Williams (2005b), Ferrero (2007) and Molnar and Santoro (2013) which find optimal policy under learning is more aggressive than rational expectations.

To establish the quantitative relevance of the information friction, we estimate a medium-scale New Keynesian model on US data over the sample 1964Q1-2007Q3. Over this sample, the data exhibit substantial low-frequency movement. While valuable to identify our mechanism, it also raises challenges. Our theory emphasizes the sensitivity of long-run beliefs to

[3]Here we use imprecise to mean that long-term interest rates and interest rate expectations are more variable than rational expectations.

[4]Gurkaynak, Sack, and Swanson (2005), Crump, Eusepi, and Moench (2016), Campbell, Fisher, Justiniano, and Melosi (2017) and Nakamura and Steinsson (2018) document evidence that monetary disturbances have sizable and significant effects on long-term nominal and real rates of interest.

short-run forecast errors. Because this relationship depends on the stance of monetary policy and the size of economic shocks — both central to the debate about the causes of the Great Moderation — we allow for structural breaks in these model parameters prior to the Great Moderation sample 1984Q1-2007Q3. Within the Great moderation period we also allow for a third regime in which expectations become better anchored after the late 1990s, consistent with evidence from Carvalho et al. (2019). In addition to standard macroeconomic variables we use measures of short- and long-term expectations from professional forecasters to discipline beliefs. Following Orphanides (2001), we also use a real time measure of the output gap to account for the information available to policy makers at the time that decisions were made and to estimate output gap measurement error.

Using Bayesian inference, we find strong evidence in support of a model in which monetary policy coefficients are stable over the full sample, but has a substantial fall in both the volatility of disturbances in the Great Moderation — of around 40 percent on average — and also the sensitivity of beliefs to short-term surprises after the late 1990s.[5] Policy shocks — the sum of policy-rule shocks and output gap mis-measurement — largely account for the wedge between subjective and objective expectations, with the sign and size of the wedge informative about economic developments. Long-term interest rate expectations and interest rates were too low through much of the 1970s, delivering a negative 'interest rate wedge' and large positive output gaps. These patterns reverse during the 1980s and early 1990s. After this time, both smaller disturbances and more stable long-term expectations cause the the wedge to narrow considerably, with greater nominal and real stability. Variance decompositions reveal that policy shocks explain little short-run variation, but the bulk of long-run variation in inflation and output.

Without either monetary policy shocks or extrapolation bias, the model cannot explain the low-frequency developments in inflation and nominal interest rates. Monetary policy shocks are critical to the rise in long-term inflation expectations over the Great Inflation. As such our findings provide evidence that long-term expectations are themselves 'trend inflation' and that the non-systematic component of monetary policy is a vital determinant of this trend. This distinguishes our research from other theories in which the inflation trend is exogenously determined (Smets and Wouters 2003, Cogley, Primiceri, and Sargent 2010 and Cogley and Sbordone 2008).

Having established an important role for the non-systematic component of policy for long-term expectations and macroeconomic dynamics, we then evaluate the systematic conduct

[5] Our findings are consistent with Primiceri (2005a), Justiniano and Primiceri (2008a) and Sims and Zha (2006) which argue that declines in the volatility of economic shocks better account for the fall in aggregate volatility than does monetary policy.

of policy. We compute optimal policy counterfactuals under both learning and rational expectations. There are four key results. First, optimal policy under extrapolation bias is always less activist than rational expectations. Second, these differences are quantitatively important. In contrast to rational expectations, policy can moderate to some degree, but not eliminate, demand short falls in major recessions. There are fundamental limits to stabilization policy. Third, the degree to which policy is less activist depends on the shocks and the degree to which expectations are anchored. Smaller shocks and better anchored long-term expectations make the predictions under learning closer to rational expectations. Fourth, the empirical model confirms the insights of the simple example: if the central bank can control aggregate demand as the instrument of policy then the predictions under learning and rational expectations are almost identical. The trade-off confronting policy is located in the aggregate demand curve.

Related literature. A growing literature addresses optimal monetary policy design under a wide range of information frictions, such as rational inattention (Paciello and Wiederholt 2013, Adam 2007), noisy information (Lorenzoni 2010, Angeletos and La'O 2020), bounded rationality (Gabaix 2020) and robust control (Woodford 2010). Our paper departs from existing work in two ways. First, we explore the effects of informational frictions on the monetary transmission mechanism, in particular the link between short-term and long-term interest rates and how monetary policy shapes long-run expectations. Second, existing results find that full stabilization in response to efficient fluctuations is always achievable by appropriate choice of policy even if not always desirable from the point of view of the policymaker. Conversely in our framework full stabilization is desirable but not a *feasible* goal for the central bank.

To give focus to the consequences of long-run uncertainty for short-run stabilization policy, we assume the monetary authority formulates optimal policy using the correct model of the economy. However, we assume that the output gap is imperfectly measured. This latter informational friction is an example of the kind that pre-occupied both the early monetarist controversies (Friedman 1968, Modigliani 1977 and Meltzer 1987) and much subsequent literature on policy under imperfect information, centered on fundamental questions about a central bank's ability to assess accurately the current state of the economy (Orphanides 2001, Svensson and Woodford 2003 and Orphanides and Williams 2012).

Empirically, our results contribute to the literature on the identification of monetary policy shocks. The identified policy shocks are consistent with narrative evidence about historical policy and the policy-rule shocks highly correlated with the Kuttner (2001) and Romer and Romer (2004) monetary policy shocks, which employ different identification strategies. That temporary policy shocks have long-lasting effects on the economy once entrenched in long-run inflation and nominal interest rate expectations is consistent with Òscar Jordà, Singh, and Taylor (2020). Like Orphanides and Williams (2012), Justiniano, Primiceri, and Tambalotti (2013) and Melosi (2016) we find an important role for these shocks in the Great Inflation.[6] However, broader policy implications differ. Justiniano, Primiceri, and Tambalotti (2013) rational expectations analysis finds optimal monetary policy can achieve near full stabilization of the economy. In our analysis, optimal policy, while improving upon historical policy, cannot achieve full stabilization. In Orphanides and Williams (2012), and other research in the learning literature, optimal monetary policy prescribes a more aggressive response to inflation and muted activism toward output gap stabilization compared to rational expectations. We show instead that optimal policy is more gradual and overall less aggressive than rational expectations.

Outline. The paper proceeds as follows. Section 2 elucidates the constraints that extrapolation bias impose on policy, in a simple example. Section 3 lays out a medium-scale New Keynesian model with features required for a plausible account of aggregate data. Section 4 develops the theory of beliefs. Section 5 estimates the model and discusses basic properties. Section 6 reveals core mechanisms and the central role of monetary policy shocks. Section 7 explores optimal policy in the empirical model. Finally, section 8 concludes and speculates about the implications of extrapolation bias in settings other than the New Keynesian model.

2 Extrapolation Bias as a Policy Constraint

This section develops a simple New Keynesian model with extrapolation bias and discusses the implications for monetary policy. We show: i) extrapolation bias drives a wedge between subjective and objective probability models; ii) subjective beliefs display less mean reversion than objective beliefs; iii) this belief distortion constrains monetary policy relative to a full information rational expectations analysis. In contrast to rational expectations analysis, the aggregate demand curve is a binding constraint on policy choice.

Consider the canonical New Keynesian model with arbitrary beliefs. The aggregation of optimal household consumption and firm pricing decisions gives the demand and supply

[6] Melosi (2016) studies a New Keynesian model with disperse (imperfect) information: the policy response to negative demand shocks in the 1970s is interpreted as both an expansionary monetary policy shock and an overestimation of potential output by the central bank, triggering a persistent increase in inflation and inflation expectations.

equations

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\, x_{T+1} - (R_T - \pi_{t+1} - r_T^n)\right] \tag{1}$$

$$\pi_t = \hat{E}_t \sum_{T=t}^{\infty} (\xi\beta)^{T-t} \left[\kappa x_T + (1-\xi)\,\beta\pi_{T+1}\right] \tag{2}$$

where x_t is the output gap; π_t inflation; R_t the nominal interest rate.[7] The household's discount factor satisfies $0 < \beta < 1$; the probability that firms cannot re-optimize their price $0 < \xi < 1$; the slope of the aggregate supply curve $\kappa \equiv (1-\xi\beta)(1-\xi)/\xi > 0$; and the natural rate of interest r_T^n is for simplicity an i.i.d. process. The operator $\hat{E}_t$ denotes household expectations which we discuss below. All variables are expressed in log-deviations from their non-stochastic steady-state values.

Monetary policy influences aggregate demand through intertemporal substitution. Not only does the contemporaneous interest rate, R_t, matter, but the entire future anticipated sequence of rates, $\hat{E}_t\{R_T\}_{T=t+1}^{\infty}$. The link between these two objects depends on the monetary policy regime and the expectations formation process. To frame ideas, suppose the central bank minimizes the loss function

$$\mathcal{L}_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \pi_T^2.$$

Given this objective the optimal policy is the targeting criterion $\pi_t = 0$ for all t. Assume the monetary authority has full information about the economy. Using (1), (2) and the target criterion, reveals the implicit instrument rule

$$\begin{aligned} R_t \;=\; & r_t^n + \hat{E} \sum_{T=t}^{\infty} (\xi\beta)^{T-t} \left[\xi\beta x_{T+1} + (1-\xi)\beta\kappa^{-1}\pi_{T+1}\right] + \\ & + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)x_{T+1} - \left(\beta R_{T+1} - \pi_{T+1} - \beta r_{T+1}^n\right)\right] \end{aligned} \tag{3}$$

implements optimal policy. We now show that the feasibility of this policy depends on the expectations formation mechanism.

Rational Expectations. Rational expectations equilibrium implies $\pi_t = x_t = 0$ in every period with the implicit instrument rule, (3), reducing to $R_t = r_t^n$: the expected path of the real interest rate tracks the natural rate of interest. With long-run expectations 'anchored',

[7] See Appendix A for the microfoundations.

so that

$$\lim_{T\to\infty} E_t\pi_T = 0 \text{ and } \lim_{T\to\infty} E_t R_T = 0,$$

monetary policy can aggressively offset any change in the current and expected change in the natural rate. The central bank then achieves both inflation and output stabilization — see Clarida, Gali and Gertler (1999).

Subjective beliefs. Now introduce the information friction. Households and firms are uncertain about the long-run mean of inflation and of the nominal interest rate. This might reflect imperfect credibility of the monetary authority or the belief that the economy is undergoing structural change. We assume households project a zero output gap in all future periods.[8] Each agent forecasts using an identical 'shifting end-points' model. Average beliefs are then described by

$$\begin{aligned} \begin{bmatrix} \pi_t \\ R_t \end{bmatrix} &= \bar{\omega}_t + e_t \\ \bar{\omega}_{t+1} &= \bar{\omega}_t + u_{t+1} \end{aligned} \tag{4}$$

where prior beliefs about transitory shocks and innovations to the low-frequency components satisfy $J = E\left[e_t e_t'\right]$ and $Q = E\left[u_t u_t'\right] = \bar{g}^2 J$ for some constant $0 < \bar{g} < 1$. Agents face a classic signal extraction problem. They observe the current interest rate and inflation but they cannot disentangle their persistent and temporary components. An estimate $\omega_{t+1} \equiv \hat{E}_t\left(\bar{\omega}_{t+1}|\pi_t, R_t, \omega_t\right)$ is obtained using the steady-state Kalman filter[9]

$$\begin{aligned} \omega_{t+1}^{\pi} &= \omega_t^{\pi} + \bar{g}\left(\pi_t - \omega_t^{\pi}\right) \\ \omega_{t+1}^{R} &= \omega_t^{R} + \bar{g}\left(R_t - \omega_t^{R}\right). \end{aligned} \tag{5}$$

The learning gain $\bar{g}$, measuring the perceived volatility of the unobserved drift $\bar{\omega}_t$ relative to short-term noise, regulates the information friction. For example, a sequence of positive surprises in the short-term inflation forecast leads to an upward revision in the estimated long-run mean of inflation, ω_t^{π}. The size of the gain determines how sensitive these revisions are to recent forecast errors. Note that for $\bar{g} = 0$ the model nests rational expectations where $\omega_t^R = \omega_t^{\pi} = 0$ in every period.

[8]The appendix demonstrates this is without loss of generality.

[9]See Appendix C for the derivation of the learning gain. Note that the natural rate, r_t^n, is not included in *individual* agents' information set. This is a simple way to introduce the signal extraction problem without adding more shocks relative to the rational expectations benchmark. This is a standard assumption in the literature. For more discussion see Eusepi and Preston (2011)

Equilibrium Dynamics. Evaluating expectations in the aggregate demand and supply relations gives

$$\begin{aligned} x_t &= -\left(R_t - r_t^n\right) - \frac{1}{1-\beta}\left(\beta\omega_t^R - \omega_t^\pi\right) \\ \pi_t &= \kappa x_t + \frac{(1-\xi)\,\beta}{1-\xi\beta}\omega_t^\pi. \end{aligned}$$

Under this belief structure the aggregate decision rules represent the optimal Bayesian solution to household and firm decision problems in the New Keynesian model. The solutions are an example of internal rationality — see Preston (2005) and Adam and Marcet (2011). Shifts in long-run beliefs about the real interest rate feed back into aggregate demand; shifts in long-term inflation expectations shift inflation. In turn, observed nominal interest rates and inflation are used to update long-run beliefs, an example of what Marcet and Sargent (1989a) call self-referential dynamics.

To implement the inflation target, $\pi_t = 0$, solve for equilibrium interest rates using (3) to get

$$R_t = r_t^n + \left(\frac{1}{1-\beta} + \frac{(1-\xi)\,\beta\kappa^{-1}}{1-\xi\beta}\right)\omega_t^\pi - \frac{\beta}{1-\beta}\omega_t^R. \tag{6}$$

This implicit instrument rule introduces a trade-off that was absent under rational expectations. Shifting long-run interest-rate beliefs require policy responses above and beyond off-setting movements in the natural rate of interest. Policy cannot fully off-set fluctuations in the natural rate for all possible interest-rate beliefs. The problem is movements in the policy rate lead to shifts in long-term beliefs and so too the term structure of interest rates. These movements can be destabilizing.

To see this, substitute (6) into the belief updating equations (5) to give

$$\begin{aligned} \omega_{t+1}^\pi &= (1-\bar{g})\,\omega_t^\pi \qquad (7) \\ \omega_{t+1}^R &= \left(1 - \frac{\bar{g}}{1-\beta}\right)\omega_t^R + \bar{g}\left(\frac{1}{1-\beta} + \frac{(1-\xi)\,\beta\kappa^{-1}}{1-\xi\beta}\right)\omega_t^\pi + \bar{g}r_t^n. \end{aligned}$$

Innovations in the natural rate r_t^n trigger an interest-rate response. The surprise $R_t - \omega_t^R$ shifts the whole term structure of expectations. The size of the learning gain $\bar{g}$ regulates both their persistence and volatility. Long-run expectations therefore display 'excess-sensitivity' to movements in the natural rate of interest relative to rational expectations. Inspecting (7), our first result is immediate:

Result 1. *The central bank cannot implement the targeting criterion* $\pi_t = 0$ *if*

$$\bar{g} > 2\,(1-\beta)\,. \tag{8}$$

As the sensitivity of beliefs to short-term surprises increases so does the volatility of long-run beliefs; when it is too high long-term expectations become disconnected from the natural rate, leading to swings in aggregate demand and an explosive interest-rate path. In contrast, under rational expectations beliefs are anchored by movements in the natural rate of interest. Movements in the policy rate have precise effects on aggregate demand by shifting the entire expected path for the short rate in lock-step with the expected path of the natural rate of interest. Aggressive short-run stabilization policy is feasible.

This result begs the question: if the central bank can't implement the inflation target, what can it do? Consider implementing policy using the policy rule

$$R_t = \phi \pi_t$$

which nests the inflation target criterion as the special case $\phi \to \infty$. For simplicity, assume prices are close to full flexibility, $\xi \to 0$. Calculations similar to the inflation target criterion give our second result:

Result 2. *Policy rules of the form* $R_t = \phi\pi_t$ *promote a stable equilibrium provided*

$$\bar{g} < \frac{2\,(1-\beta)}{1-\phi^{-1}}.$$

For a given learning gain, the policy coefficient cannot be too large; and the higher $\bar{g}$ the tighter the constraint on policy choice. Since monetary policy cannot fully off-set movements in the natural rate of interest in this economy, complete stabilization of inflation and the output gap is not achievable. The result illustrates the limits of monetary policy for the particular case where the central bank adopts a set of rules. This is also the approach we take in our quantitative analysis below. However, this result is general: Appendix J shows that even in the case of fully optimal policy under learning — which takes account of the effects of policy on the evolution of beliefs — monetary policy faces the same limits.

Extrapolation bias. The information friction fundamentally alters the behavior of the economy. Expectations display *extrapolation bias* as an equilibrium outcome. Longer-term forecasts are excessively influenced by short-term fluctuations and overstate the persistence of the true data-generating process. This feature stems from the belief that inflation and the nominal interest rate have a low-frequency non-stationary component (equation 5), while feasible policies induce equilibrium dynamics displaying mean reversion (equation 7). In equilibrium, extrapolation bias generates a wedge between subjective and objective probability models.[10] This wedge is endogenous and time-varying.

To illustrate this point, consider again the model in the neighborhood of the flexible price equilibrium and assume the Taylor rule is known so that interest rate and inflation beliefs satisfy $\omega_t^R = \phi\omega_t^\pi$. Taking the difference between agents' interest-rate forecasts and model-consistent projections (using the true data-generating process) gives the 'wedge'

$$\begin{aligned} \hat{E}_t R_T - E_t R_T &= \left(1 - \frac{\phi^{-1} - \beta}{1 - \beta}\right)\omega_t^R \\ &= \bar{g} \times \left(1 - \frac{\phi^{-1} - \beta}{1 - \beta}\right)\sum_{j=0}^{\infty}\left(1 - \bar{g}\frac{1 - \phi^{-1}}{1 - \beta}\right)^j r_{t-1-j}^n \end{aligned}$$

in any future period $T > t$. Both the size of the learning gain and monetary policy regulate the degree of extrapolation bias. The stance of policy matters for both the size of the wedge for given beliefs (the first line), but also the size of the deviation of beliefs from their true long-term mean of zero (the second line). As the gain $\bar{g}$ decreases the wedge shrinks towards zero, the rational expectations equilibrium. Depending on beliefs and monetary policy, transitory natural-rate shocks may have long-lived effects. For example, a sequence of positive shocks to the natural rate leads to an increase in the wedge, leading to a steeper path of the interest rate compared to that path which would be expected under the true data-generating process.[11]

Role of aggregate demand. The conclusion that imperfect knowledge requires a less aggressive inflation policy depends on the monetary transmission mechanism. If the gain $\bar{g}$ is large then extrapolation bias leads to large movements in interest-rate beliefs and therefore long-term interest rates, through the term structure of interest-rate expectations. Hence it is the location of the aggregate demand relation that is a constraint on policy.

To isolate this mechanism, consider a central bank that can control the output gap directly as the instrument of policy. This assumption describes an economy where the central bank is in full control the term structure of interest rates. Since aggregate demand (and interest-rate expectations) no longer constrains policy, the model is summarized by the

[10] This is a general property of models with learning dynamics and doesn't depend on the assumption of a perceived unit root, as the empirical model will make clear.

[11] We consider the, empirically relevant, case where the coefficient ϕ is sufficiently small that $0 < \left(1 - \bar{g}\frac{1-\phi^{-1}}{1-\beta}\right) < 1$

targeting criterion and the aggregate supply curve. Substituting for expectations we get

$$\begin{aligned}\pi_t &= 0 \\ \pi_t &= \kappa x_t + \frac{(1-\xi)\beta}{1-\xi\beta}\omega_t^{\pi}.\end{aligned}$$

Because the inflation target is enforced in every period, the evolution of inflation beliefs is

$$\omega_{t+1}^{\pi} = (1-\bar{g})\,\omega_t^{\pi},$$

resulting in a stable equilibrium:

Result 3. *If the central bank fully controls the output gap, the inflation target $\pi_t = 0$ can always be implemented.*

When the central bank can control aggregate demand directly, the information friction only produces uncertainty about the location of the aggregate supply curve: this uncertainty stems from long-term inflation expectations. The central contribution of this paper is to demonstrate that uncertainty about the location of the aggregate demand curve, sourced to long-term interest rate expectations, represents a more significant constraint on monetary policy.

Most of the imperfect knowledge and learning literature focuses on cases in which information frictions only affect the location to the aggregate supply curve. Analyses of this kind, such as Bomfim, Tetlow, von zur Muehlen, and Williams (1997), Ferrero (2007), Gaspar, Smets, and Vestin (2006), Orphanides and Williams (2005c), Orphanides and Williams (2007), Bianchi (2013) and Molnar and Santoro (2013), demonstrate that policies which are *more* aggressive towards inflation and less *activist* towards output gap stabilization relative to rational expectations predictions help restrain inflation expectations, and improve short-run stabilization outcomes. These papers reach this conclusion because they assume the central bank has tight control of aggregate demand.[12] Our simple example suggests failure to model the transmission mechanism of monetary policy is not without consequence. We now quantify the relevance of this constraint for monetary history.

[12]In most of these papers aggregate demand depends only on the current interest rate: the term structure of interest rates play no role. See Appendix A for more results and Eusepi and Preston (2018b) for additional discussion.

3 A Medium-Scale New Keynesian Model

This section states a version of the New Keynesian model proposed by Giannoni and Woodford (2004) and widely used for monetary policy analysis.[13] The principle modeling innovation concerns the treatment of expectations formation.

Firms. A continuum of monopolistically competitive firms $f \in [0,1]$ each produce differentiated goods, $Y_t(f)$, using the linear production technology in composite labor services, $N(f)$,

$$Y_t(f) = A_t [Z_t N_t(f)]$$

where Z_t, labor-augmenting technological progress, evolves deterministically as $Z_t = \gamma Z_{t-1}$, with $\gamma > 1$, and A_t denotes a stationary technology shock

$$\log A_t = \rho_a \log A_{t-1} + \sigma_a \varepsilon_t^a$$

where ε_t^a is IID $N(0,1)$, $\sigma_a > 0$, and $0 < \rho_a < 1$. Each firm faces a demand curve

$$Y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\theta_{p,t}} Y_t$$

where $\theta_{p,t} > 1$, the elasticity of substitution across differentiated goods, follows an exogenous process

$$\log \left(\frac{\theta_{p,t}}{\theta_p} \right) = \rho_{\theta_p} \log \left(\frac{\theta_{p,t-1}}{\theta_p} \right) + \sigma_{\theta_p} \varepsilon_t^{\theta_p}$$

where $\varepsilon_t^{\theta_p}$ is IID $\mathrm{N}(0,1)$, $\sigma_{\theta_p} > 0$, $0 < \rho_{\theta_p} < 1$ and $E[\theta_{p,t}] = \theta_p$.

Following Calvo (1983) and Yun (1996) a fraction of firms $0 < \xi_p < 1$ cannot optimally choose their price, but reset it according to the indexation rule

$$P_t(f) = P_{t-1}(f)\, \pi_{t-1}^{\iota_p}$$

where $\pi_t = P_t/P_{t-1}$ is the inflation rate, and $0 < \iota_p < 1$. The remaining fraction of firms choose a price $P_t(f)$ to maximize the expected discounted value of profits

$$\hat{E}_t^f \sum_{T=t}^{\infty} \xi_p^{T-t} Q_{t,T} \Gamma_T^f(f)$$

[13] The model abstracts from capital accumulation. While this is an important feature of these model, it introduces a degree complexity that would obscure the central mechanism of the model. Estimation and evaluation of this larger scale model is left for further research.

where the stochastic discount factor, $Q_{t,T} = \beta^{T-t}\lambda_T/\lambda_t$, values future profits

$$\Gamma_T^f(f) = Y_T(f)\left((1-\tau_f)\frac{P_t(f)}{P_T}\left(\frac{P_{T-1}}{P_{t-1}}\right)^{\iota_p} - \frac{W_T}{A_T P_T Z_T}\right)$$

for constant sales revenue tax τ_f, and λ_t the marginal value of household wealth. The conditional expectations of firms, $\hat{E}_t^f$, is discussed below.

Households. A continuum of households $i \in [0,1]$ maximize intertemporal utility

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t}\left[\frac{C_{H,T}(i)^{1-\sigma}}{1-\sigma} - \varphi_T \int \frac{N_T(i,j)^{1+\phi^{-1}}}{1+\phi^{-1}} dj\right]$$

where

$$C_{H,t}(i) = \frac{C_t(i)}{Z_t} - b\frac{C_{t-1}(i)}{Z_{t-1}}$$

with $\sigma, \phi > 0$, $0 < b < 1$. Each household comprises a large family, whose members $j \in [0,1]$ supply specialized labor, $N(i,j)$, to the production of each differentiated good j. The large family assumption insures each household member against labor market risk from nominal wage contracting. The dis-utility of labor supply shock is a stationary exogenous process

$$\log\left(\frac{\varphi_t}{\varphi}\right) = \rho_\varphi \log\left(\frac{\varphi_{t-1}}{\varphi}\right) + \sigma_\varphi \varepsilon_t^\varphi$$

ε_t^φ is IID N$(0,1)$, $\sigma_\varphi > 0$, $0 < \rho_\varphi < 1$ and $E[\varphi_t] = \varphi$. The conditional expectations of households, $\hat{E}_t^i$, is discussed below.

The household's flow budget constraint is

$$C_t(i) + \frac{B_t(i)}{P_t} \leq R_{t-1}\pi_t^{-1}\frac{B_{t-1}(i)}{P_{t-1}} + (1-\tau_w)\int \frac{W_t(j)}{P_t} N_t(i,j)\, dj + \Gamma_t^f - T_t + T_t^w + T_t^f$$

where: R_t is the gross one-period nominal interest rate; $B_t(i)$ holdings of one-period nominal government debt; Γ_t^f dividend payments net of sales taxes; τ_w the labor income tax rate whose proceeds are rebated lump-sum to households as T_t^w; T^f the lump-sum rebate of sales revenue taxes; and T_t lump-sum taxes.[14] Household's optimal consumption and portfolio choice must also satisfy the No-Ponzi condition

$$\lim_{T\to\infty} \hat{E}_t^i \left(\prod_{s=0}^{T-t} R_{t+s}\pi_{t+s}^{-1}\right)^{-1} \frac{B_T(i)}{P_T} \geq 0.$$

[14]The assumptions on tax policy ensure an efficient steady state level of output.

Households have market power in the supply of differentiated labor inputs.[15] The demand for labor type j by firm f is

$$N_t(j,f) = \left(\frac{W_t(j)}{Wt}\right)^{-\theta_{w,t}} N_t(f) \tag{9}$$

where

$$N_t(f) = \left[\int_0^1 N_t(j,f)^{\frac{\theta_{w,t}-1}{\theta_{w,t}}} dj\right]^{\frac{\theta_{w,t}}{\theta_{w,t}-1}} \text{ and } W_t = \left[\int_0^1 W_t(j)^{1-\theta_{w,t}} dj\right]^{\frac{1}{1-\theta_{w,t}}}$$

define the composite labor input used in production and the associated wage rate. The elasticity of demand across differentiated labor inputs satisfies the exogenous process

$$\log\left(\frac{\theta_{w,t}}{\theta_w}\right) = \sigma_{\theta_w}\varepsilon_t^{\theta_w}$$

and $\theta_{w,t} > 1$ and $\varepsilon_t^{\theta_w}$ is IID $\mathrm{N}(0,1)$, $\sigma_{\theta_w} > 0$ and $E[\theta_{w,t}] = \theta_w$. Following Erceg, Henderson, and Levin (2000) a fraction of household members $0 < \xi_w < 1$ cannot optimally reset their wage, but follow the indexation rule

$$W_t(j) = W_{t-1}(j)\,\pi_{t-1}^{\iota_w}\gamma \tag{10}$$

for $0 < \iota_w < 1$. For the remaining fraction, ξ_w, each member j of household i, choose optimally their nominal wage, $W_t(j)$, to maximize

$$\hat{E}_t^i \sum_{T=t}^{\infty} (\xi_w\beta)^{T-t}\left[Q_{t,T}(i)\frac{W_t(j)}{P_T}\left(\frac{P_{T-1}}{P_t-1}\right)^{\iota_w}\frac{Z_{T-1}}{Z_{t-1}}N_T(i) - \varphi_T\frac{N_T(j)^{1+\phi^{-1}}}{1+\phi^{-1}}\right]$$

subject to (9).[16]

Government Policy. The central bank implements monetary policy using the interest rate rule

$$R_t = (R_{t-1})^{\rho_R}\left[R\,(P_t/P_{t-1})^{1+\phi_\pi}\left(X_t^{CB}\right)^{\phi_x}\right]^{1-\rho_R}\left(\Delta X_t^{CB}\right)^{\phi_{\Delta x}} m_t$$

where $\phi_\pi,\ \phi_x \geq 0$, R the steady-state gross interest rate, and $X_t = Y_t/Y_t^n$ denotes the model-theoretic output gap, where Y_t is the level of output and Y_t^n the natural rate of output in

[15] The assumption is interpreted as follows. For each type of labor, which is sourced from all households, there is an employment agency that has market power. See Giannoni and Woodford (2004) and Justiniano, Primiceri, and Tambalotti (2013).

[16] Members supplying labor of type j, being represented by an employment agency, re-optimize at the same time in all households i.

a flexible-price equilibrium of the model. The central bank sets policy in response to its measure of the output gap $X_t^{CB} = X_t e^{\nu_t}$, where

$$\nu_t \;\; = \;\; \rho_\nu \nu_{t-1} + \sigma_\nu \varepsilon_t^\nu$$

denotes a persistent measurement error. Interest-rate policy exhibits inertia and responds to deviations of inflation and output gap from steady-state levels. The steady-state inflation rate is zero; $\log m_t = \sigma_m \varepsilon_t^m$ denotes a mean-zero IID monetary shock.

To give focus to how learning dynamics constrain monetary policy, we assume fiscal policy is Ricardian, and that this is understood by agents. Eusepi and Preston (2018) show that in general learning will imply departures from Ricardian equivalence, with holdings of the public debt perceived as net wealth. The associated wealth effects on aggregate demand can be sizable, which impairs the standard intertemporal substitution channel of monetary policy. We also assume that agents know the tax rules in place, including the rebate of sales and income taxes. Together these assumptions imply agents do not need to forecast various taxes and that debt will not have monetary consequences. This permits focus on how belief distortions affect long-term interest rates and monetary policy design, understanding that imperfect knowledge about fiscal and monetary policy both serve to complicate inflation policy. With this in mind, we consider an economy with zero government debt and balanced budget policy

$$T_t = G_t$$

where exogenous government purchases, $g_t = G_t / Z_t$ satisfy

$$\frac{g_t - g}{y} = \rho_g \left(\frac{g_{t-1} - g}{y} \right) + \sigma_g \varepsilon_t^g$$

where ε_t^g is IID $\mathrm{N}(0,1)$, $\sigma_g > 0$, $0 < \rho_g < 1$ and $E\left[g_t\right] = g$. Motivated by empirical fit we follow Smets and Wouters (2007) and permit correlation between government purchases and technology shocks.

Market Clearing and Equilibrium. We consider a symmetric equilibrium in which all households are identical, even though they do not know this to be true. Given that households have the same initial asset holdings, preferences, and beliefs, and face common constraints, they make identical state-contingent decisions. Similarly, all firms having the opportunity to re-optimize choose an identical re-set price. Equilibrium requires all goods, labor and asset markets to clear providing the restrictions

$$\begin{aligned}\int_0^1 C_t(i)\,di + G_t &= \int_0^1 Y_t(f)\,df \\ \int_0^1\int_0^1 N_t(i,j)\,didj &= \int_0^1 N_t(f)\,df\end{aligned}$$

and

$$\int_0^1 B_t(i)\,di = 0$$

with initial condition $B_{-1}(i) = 0$. Given exogenous processes $\{G_t, \theta_{p,t}, \theta_{w,t}, m_t, Z_t, A_t, \varphi_t\}$, equilibrium then is a sequence of prices $\{P_t, W_t, R_t\}$ and allocations $\left\{C_t, N_t, Y_t, T_t, T_t^w, T_t^f, \Gamma_t^f\right\}$ satisfying individual optimality and market clearing conditions. Appendix B summarizes a log-linear approximation to the complete model.

4 Beliefs

Appendix C shows a first-order approximation to optimal decisions and market clearing conditions give aggregate dynamics

$$A_0 z_t = \sum_{s=1}^{4} A_s \hat{E}_t \sum_{T=t}^{\infty} \lambda_s^{-(T-t)} z_{T+1} + A_5 z_{t-1} + A_6 \varepsilon_t \tag{11}$$

where the vector z_t collects all model variables, the vector ε_t collects exogenous innovations and the matrices A_i, for $i \in 1, ..., 6$, collect relevant model coefficients. This representation holds for arbitrary beliefs, including rational expectations.

Dynamics depend on a set of projections into the indefinite future, reflecting the intertemporal decision problems solved by households and firms. The projected variables are those macroeconomic objects taken as given and beyond the control of each decision maker. Firms must forecast real wages and goods price inflation; households must forecast goods price inflation, wage inflation, the real wage, nominal interest rates, and aggregate demand. In all, agents forecast five endogenous variables and all exogenous variables. Households and firms know the long-run mean of exogenous variables. This has no relevance to our conclusions. The discount factors λ_s are the unstable eigenvalues associated with individual decisions, so that the infinite sums encode the usual forward recursion to suppress the effects of explosive roots.

Subjective beliefs. Consistent with the assumption of a symmetric equilibrium, each agent has a common forecasting model

$$z_t = S\bar{\omega}_t + \Phi z_{t-1} + e_t \tag{12}$$
$$\bar{\omega}_{t+1} = \rho\bar{\omega}_t + u_{t+1} \tag{13}$$

where Φ is a matrix to be discussed; $0 \leq \rho \leq 1$ a parameter; e_t and u_t IID with $J = E\left[e_t e_t'\right]$ and $Q = E\left[u_t u_t'\right]$. The matrix S is a selection matrix mapping the subset of drifts $\bar{\omega}_t$ to a larger vector of drifts of the same size of z_t. These extra elements of the vector are zeros, corresponding to variables that are either not forecast or exogenous variables whose mean is known. The beliefs nest rational expectations as a special case: $\bar{\omega}_t = \bar{\omega}_{t-1} = 0$ when $Q = 0$ — that is the prior belief about the variance-covariance matrix of the drift terms is zero.

The forecasting model implies conditional expectations satisfy

$$E_t z_{t+n} = \Phi^n z_t + \sum_{j=0}^{n} \Phi^j \rho^{n-j} S\bar{\omega}_t. \tag{14}$$

Medium- to long-term forecasts are determined by two components: the first term is the conventional auto-regressive impact of the current state. The second term captures the effects of drifting beliefs on conditional expectations. In the special case $\rho = 1$ we have an example of a shifting end-point model in the language of Kozicki and Tinsley (2001). Beliefs then satisfy

$$\lim_{n\to\infty} E_t z_{t+n} = (I - \Phi)^{-1} S\bar{\omega}_t. \tag{15}$$

Objective Beliefs. Define $\omega_{t+1} \equiv \hat{E}_t\left(\bar{\omega}_{t+1} | z_t, z_{t-1}, \omega_t\right)$ to be agents' estimate of the unobserved drift. Using (14) to evaluate expectations in (11) returns the true data-generating process

$$z_t = T(\Phi) S\omega_t + \Phi z_{t-1} + \Phi_\varepsilon \varepsilon_t. \tag{16}$$

We assume agents understand the true dynamics of aggregate variables up to the unobserved mean so that the matrices Φ and Φ_ε correspond to the rational expectations solution.

This assumption ensures tractability in estimation and optimal policy exercises. A linear state-space representation permits use of standard likelihood methods and a linear-quadratic optimal policy problem. Moreover, endowing agents with knowledge about transitional dynamics gives focus to the information friction — imperfect knowledge about the long run —

and its effects on policy design.[17] Drifts in beliefs are encoded into the intercept of the true data-generating process, and represent the only difference between the subjective (equation 12) and objective (equation 16) data-generating processes of the model.[18]

Subjective belief updating. Beliefs are updated using the the Kalman filter. Appendix C shows the estimate of the unobserved state is updated following

$$\omega_{t+1} = \rho\omega_t + \bar{g}\mathcal{F}_t$$

where

$$\mathcal{F}_t \quad = \quad S' \left(z_t - S\omega_t - \Phi z_{t-1}\right)$$

denotes the period-t prediction errors and again S' selects the forecast errors of the endogenous variables agents forecast. Prior beliefs satisfy the restriction $Q = \hat{c}^2 J$ for scalar $\hat{c}$, so that the learning gain, $\bar{g} = \bar{g}(\hat{c}, \rho)$, depends on the volatility of the perceived drift relative to the structural innovations and their perceived persistence.

Evaluating the forecast error implies beliefs evolve according to

$$\omega_{t+1} \quad = \quad \left[\rho + \bar{g}S' \left(T\left(\Phi\right) - I\right) S\right] \omega_t + \bar{g}S'\Phi_\varepsilon\varepsilon_t. \tag{17}$$

The evolution of beliefs depends on two key components. Feed-back effects from learning — captured by $T(\Phi)$ — regulate their persistence, while the learning gain directly affect both persistence and the size of innovations. Subsequent estimation and policy evaluation exercises require beliefs to be stationary, so we focus on parameter values that respect this requirement.

5 Estimation and Model Implications

5.1 Estimation

The Data. To estimate model parameters we use fourteen US time series. Five are standard macroeconomic variables: the log-difference of the GDP deflator, the output gap (as measured by the Congressional Budget Office), the three-month Treasury-Bill interest rate,

[17] Eusepi and Preston (2011, 2018a, 2018b) adduce theoretical and empirical evidence that together demonstrate learning about intercepts generates empirically relevant variation and creates policy challenges. Learning about the coefficients Φ would make the filtering problem and the state-space representation of the model non-linear.

[18] When $T(\Phi) = I$ beliefs are perfectly validated by the data, generating a self-confirming equilibrium — see Sargent (1999). If $T(\Phi) = 0$ we have rational expectations. For intermediate cases, beliefs are partially self-confirming.

and, following Justiniano, Primiceri, and Tambalotti (2013), two measures of nominal wage growth from NIPA and the BLS Establishment survey.[19] Eight time series are short- and long-term professional forecasts of the three-month Treasury-bill rate and inflation. We use these series to discipline beliefs and, in particular, to draw inference about the learning gain. For each of these two variables, the one-quarter- and four-quarter-ahead forecasts from the Survey of Professional forecasts measure short-term forecasts. The mean one-to-ten-years-ahead and the five-to-ten-years-ahead forecasts from Blue Chip Economics and Financial measure long-term beliefs.

We use available quarterly data for the period 1964Q1 to 2007Q3. Short-term forecasts of inflation are available from 1968Q3; short-term forecasts of nominal interest rates from 1981Q3; long-term forecasts of inflation from 1979Q3 and long-term interest-rate forecasts from 1985Q1. As a direct measure of the output gap measurement error (the process ν_t), we use the difference between the real-time estimates of the output gap from Orphanides (2003) and the CBO measure. These real time estimates were available to the Federal Reserve at the time of monetary policy decisions. The series is available at quarterly frequency since 1965Q4.[20]

Three Regimes. Over this sample, the US economy experienced structural change that can impact our inference about the proposed expectations formation mechanism. The Great Moderation literature documents a substantial decline in macroeconomic volatility from around 1984.[21] One line of research sources the decline in volatility to more favorable exogenous disturbances — falling macroeconomic volatility simply reflects good luck.[22] Another line of research emphasizes changes in the conduct of policy, with the Volcker Federal Reserve adopting a much more aggressive policy towards inflation and less emphasis on output gap stabilization, relative to the 1970s.[23] Toward the end of the sample we also observe greater stability in long-term inflation expectations: Carvalho, Eusepi, Moench, and Preston (2019) document diminished sensitivity of long-run beliefs to short-term developments.

These types of structural change are relevant to evaluating our model of long-term expectations for three reasons. First, because agents revise beliefs in response to short-term forecast errors, the size of shocks matters. Second, in equilibrium, monetary policy shapes the persistence and volatility of beliefs by regulating the degree of feedback from beliefs to outcomes. Third, the learning gain is determined by agents' priors about the relative

[19] We use the CBO measure of the output gap to detrend output, not to fit the model-theoretic output gap.

[20] We thank Athanasios Orphanides for sharing an updated series of the one used in his paper. See Orphanides (2003) for details on the construction of the series.

[21] See Perez-Quiros and McConnell (2000) and Kim and Nelson (1999) for example.

[22] See, among others, Stock and Watson (2002) and Sims and Zha (2006).

[23] See Clarida, Gali, and Gertler (2000), Boivin and Giannoni (2006) and Orphanides and Williams (2012).

variance of low-frequency drift to high-frequency economic disturbances. Again structural changes to the variance of shocks and monetary policy can affect the size of the gain, as shown in Carvalho, Eusepi, Moench, and Preston (2019). Collectively, these properties regulate the relationship between short-run forecast errors and long-run expectations. A proper test of our behavioral theory of expectations formation must account for them.

For this reason our empirical model permits three types of regime change. The dates of regime change are taken as exogenous and known. We estimate the model using data on two separate subsamples, 1964Q1-1979Q3 and 1984Q1-2007Q3. The sample choice reflects the Great Moderation. The near-five year gap between subsamples allows possible changes in monetary policy regime without having to model explicitly the transition between different regimes.[24] The assumption of a discrete one-time change is made for simplicity and is likely to deliver an inferior fit compared to stochastic regime-switching, as shown by Sims and Zha (2006). Bianchi (2013) estimates a New Keynesian model with shifting monetary policy rules and shock volatilities in a Markov-switching setup, where agents have full information about the policy regimes. While this latter approach is preferable, its application to our model would require introducing non-linearities to model transition dynamics as agents learn about a new policy regime. We leave this to future research. The end of the sample is chosen to exclude the period when the policy rate is at the zero lower bound on nominal interest rates.

We exploit these subsamples to study sources of structural change. Most model parameters are held fixed across subsamples. Parameter estimates for two models are reported in the main text: i) a model which holds monetary policy fixed, but permits the volatility of structural shocks to change; and ii) a model which allows both monetary policy and structural shocks to change. In each case, we allow for a third regime entailing a one-time change in the learning gain in the second subsample from 1999Q1. This break-point approximates when Carvalho, Eusepi, Moench, and Preston (2019) estimate the gain to fall substantially.[25]

State Space and Observation Equations. Conditional on each regime $i = 1, 2, 3$, the economy has a linear state-space representation

$$Z_t = F^i(\Theta) Z_{t-1} + Q^i(\Theta) \varepsilon_t \tag{18}$$

where $Z_t = (z_t, \omega_t, \mathcal{F}_t)'$ includes all the state variables and the vector Θ defines the set of model parameters. This permits use of standard likelihood-based estimation. The measure-

[24] The implicit assumption is that any meaningful transition would be completed during this gap.

[25] Carvalho, Eusepi, Moench, and Preston (2019) estimate a non-linear model of inflation dynamics and long-term inflation expectations in which the gain is endogenous and time varying. They adduce evidence the long-term inflation expectations are anchored from the late 1990s, displaying little sensitivity to inflation surprises. We treat this regime change as exogenous to preserve the linearity of the model.

ment equation

$$Y_t = \mu_t(\Theta) + H_t^i(\Theta) Z_t + o_t$$

attaches ten measurement errors, o_t, to the eight survey forecasts and the two measures of the nominal wage growth. The vector μ_t contains the long-run mean of the observables. The matrix H_t^i and μ_t are time varying because of missing observations. We estimate the model using Bayesian inference.[26]

Calibrated Parameters. The long-run mean of inflation is constant over the full sample, reflecting central bank time-invariant preference for a stable inflation process: the mean inflation rate is set to be about 2.2% per annum, in line with long-term expectations at the end of the sample. Rather than assuming an exogenous time-varying inflation target, this paper explains low-frequency properties of inflation endogenously. The quarterly growth rate in technical progress $\gamma = 1.04$ matches the average GDP per-capita growth over the sample. Elasticity of demand across differentiated goods and labor services, θ_p and θ_w, are both set equal to 5. The parameter ρ which determines the persistence in beliefs is 0.995.[27] And the government spending-to-output ratio is G/Y=0.16.

Prior Distributions. Tables 1 and 2 provide information about the priors, posterior and the marginal likelihood for our two main model specifications. Table 1 reports information on parameters held constant across subsamples. Table 2 reports: i) our baseline specification in which the monetary policy is fixed, but permits changes in the volatility of structural shocks (top panel); and ii) an alternative specification which allows structural change in both monetary policy and structural shocks (bottom panel).

The priors for the exogenous shock processes are the same across variables. The persistence of the auto-correlated processes have a beta distribution with mean 0.5 and standard deviation 0.1; the standard deviations of the innovations and all measurement errors have an inverse-gamma distribution with mean 0.1 and standard deviation of 2. The priors on the parameters of the monetary policy reaction function are based on the Taylor rule — we define the coefficient on inflation as $1 + \phi_\pi$. Given evidence in Hall (1988) and Ravina (2011), the inverse intertemporal elasticity of substitution, σ, has a gamma distribution with mean 1.5 and fairly large standard deviation of 0.6, while the degree of habit persistent has a beta prior with mean 0.35 and standard deviation of 0.1. The Calvo adjustment parameter on goods prices, ξ_p has a prior mean which implies contracts have an average duration of two quarters, with a fairly diffuse prior. In contrast, the wage rigidity parameter, ξ_w, is

[26] Details are in the appendix.

[27] Because households effectively discount future outcomes by $\beta\rho$, a value of ρ slightly below unity ensures the expected present discounted value of income is well defined, while permitting jointly fitting persistent expectations data and a low steady-state real rate.

set to be fairly high (over a year duration) and with a fairly tight prior. The parameters capturing price and wage indexation, ι_p and ι_w, have means 0.5. Following Slobodyan and Wouters (2012), the constant-gain coefficients g and g_{1999} have a gamma distribution with mean 0.035 and standard deviation 0.03.

	Prior			**Posterior: 1964-1979 & 1984-2007**		
	Dist.	Mean	Std	Mode	Mean	[5% - 95%]
$100 \times (\beta^{-1} - 1)$	G	0.50	0.10	0.16	0.16	[0.13 , 0.18]
σ	G	1.50	0.60	7.63	8.72	[6.73 , 10.9]
ϕ_n	G	0.50	0.10	0.12	0.15	[0.10 , 0.22]
b	B	0.35	0.10	0.82	0.74	[0.61 , 0.84]
ξ_w	B	0.85	0.01	0.88	0.87	[0.86 , 0.89]
ι_w	B	0.50	0.15	0.32	0.36	[0.23 , 0.51]
ξ_p	B	0.50	0.10	0.86	0.83	[0.72 , 0.91]
ι_p	B	0.50	0.15	0.06	0.08	[0.03 , 0.13]
ϕ_π	G	0.50	0.25	0.18	0.22	[0.13 , 0.34]
ρ_R	B	0.50	0.10	0.88	0.87	[0.85 , 0.89]
ϕ_x	G	0.10	0.09	0.01	0.01	[0.00 , 0.02]
$\phi_{\Delta x}$	G	0.15	0.09	0.07	0.07	[0.06 , 0.09]
$10 \times \bar{g}$	G	0.35	0.30	0.67	0.63	[0.53 , 0.73]
$10 \times \bar{g}_{1999}$	G	0.35	0.30	0.09	0.10	[0.08 , 0.12]
ρ_{θ_p}	B	0.50	0.10	0.17	0.16	[0.10 , 0.22]
ρ_g	B	0.50	0.10	0.88	0.87	[0.84 , 0.90]
ρ_a	B	0.50	0.10	0.94	0.94	[0.91 , 0.95]
ρ_φ	B	0.50	0.10	0.55	0.41	[0.24 , 0.60]
ρ_ψ	B	0.50	0.10	0.93	0.92	[0.90 , 0.95]
$\sigma_{g,a}$	B	0.50	0.20	0.29	0.27	[0.09 , 0.52]
$\sigma_{o,\pi^{1Q}}$	IG	0.10	2.00	0.08	0.08	[0.07 , 0.10]
$\sigma_{o,R^{1Q}}$	IG	0.10	2.00	0.05	0.04	[0.04 , 0.05]
$\sigma_{o,\pi^{4Q}}$	IG	0.10	2.00	0.10	0.10	[0.09 , 0.11]
$\sigma_{o,R^{4Q}}$	IG	0.10	2.00	0.07	0.06	[0.05 , 0.08]
$\sigma_{o,R^{510Y}}$	IG	0.10	2.00	0.04	0.05	[0.03 , 0.07]
$\sigma_{o,\pi^{510Y}}$	IG	0.10	2.00	0.03	0.03	[0.02 , 0.03]
$\sigma_{o,R^{110Y}}$	IG	0.10	2.00	0.05	0.05	[0.04 , 0.06]
$\sigma_{o,\pi^{110Y}}$	IG	0.10	2.00	0.03	0.03	[0.02 , 0.04]
σ_{o,w_1}	IG	0.10	2.00	0.51	0.52	[0.47 , 0.57]
σ_{o,w_2}	IG	0.10	2.00	0.29	0.30	[0.26 , 0.34]
Γ	N	1.00	0.50	0.80	0.80	[0.77 , 0.83]

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table 1: **Prior and Posterior Distribution: Baseline model**

Posterior Distributions: time-invariant parameters. Table 1 shows the mean, the mode and the 5 and 95 percentiles of the posterior distribution of parameters for the

	Prior			Posterior: 1964-1979			Posterior: 1984-2007		
	Dist.	Mean	Std	Mode	Mean	[5% - 95%]	Mode	Mean	[5% - 95%]
Baseline									
σ_{θ_p}	IG	0.10	2.00	0.27	0.28	[0.24 , 0.34]	0.17	0.18	[0.16 , 0.22]
σ_{θ_w}	IG	0.10	2.00	0.05	0.08	[0.03 , 0.15]	0.10	0.10	[0.07 , 0.13]
σ_g	IG	0.10	2.00	0.93	0.92	[0.76 , 1.10]	0.52	0.55	[0.47 , 0.64]
σ_m	IG	0.10	2.00	0.16	0.16	[0.13 , 0.20]	0.09	0.11	[0.09 , 0.14]
σ_a	IG	0.10	2.00	0.94	0.90	[0.44 , 1.67]	0.44	0.44	[0.23 , 0.77]
σ_φ	IG	0.10	2.00	0.11	0.13	[0.08 , 0.19]	0.05	0.07	[0.04 , 0.10]
σ_ψ	IG	0.10	2.00	1.21	1.24	[1.06 , 1.45]	0.65	0.66	[0.58 , 0.74]
MLik	30.8								
MP rule									
ϕ_π	G	0.50	0.25	0.13	0.23	[0.13 , 0.37]	0.36	0.45	[0.23 , 0.71]
ρ_R	B	0.50	0.10	0.91	0.91	[0.88 , 0.93]	0.87	0.85	[0.83 , 0.87]
ϕ_x	G	0.10	0.09	0.00	0.01	[0.00 , 0.03]	0.02	0.01	[0.00 , 0.03]
$\phi_{\Delta x}$	G	0.15	0.09	0.06	0.07	[0.05 , 0.10]	0.08	0.08	[0.06 , 0.10]
σ_{θ_p}	IG	0.10	2.00	0.27	0.29	[0.24 , 0.35]	0.17	0.19	[0.16 , 0.22]
σ_{θ_w}	IG	0.10	2.00	0.10	0.10	[0.03 , 0.16]	0.10	0.09	[0.07 , 0.12]
σ_g	IG	0.10	2.00	0.89	0.89	[0.74 , 1.07]	0.51	0.54	[0.47 , 0.62]
σ_m	IG	0.10	2.00	0.13	0.13	[0.09 , 0.17]	0.10	0.12	[0.10 , 0.15]
σ_a	IG	0.10	2.00	0.99	0.79	[0.41 , 1.54]	0.44	0.39	[0.22 , 0.71]
σ_φ	IG	0.10	2.00	0.09	0.12	[0.08 , 0.19]	0.04	0.07	[0.05 , 0.10]
σ_ψ	IG	0.10	2.00	1.22	1.24	[1.06 , 1.46]	0.65	0.66	[0.58 , 0.74]
MLik	21.2								
Other (Mlik)									
Rational exp.	5.81								
Constant vol.	13.9								
Single gain	-1.1								

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table 2: **Prior and Posterior Distribution: sub-samples**

Baseline model, which holds the policy regime constant, but allows the shock volatility and learning gain to vary. The data are informative. The intertemporal elasticity of substitution is remarkably low, within the range 0.1 to 0.15. The Frisch elasticity of labor supply is around 0.1. The price and wage stickiness parameters are about 0.9, implying a long duration of price contracts, common to most estimated New Keynesian DSGE models. However, because of real rigidities, the implied slope of the wage Phillips curve is an order of magnitude smaller than the price Phillips curve, with important implications for monetary policy. The shocks have lower persistence than usually found in DSGE models. This reflects the role of learning in soaking up low-frequency variation in data. The small observation errors on survey data indicate the expectation formation mechanism is consistent with observed measures of expectations, with a tight mapping between short-run forecast errors and long-term beliefs. This stands as important validation of the expectations formation mechanism central to our model.

Posterior Distributions: regime-dependent parameters. For our benchmark model Table 1 shows the posterior distributions for the policy parameters. The estimated policy parameters are quite different from prior values. The inflation response coefficient has 90 percent confidence set of $1.15 - 1.30$, lower than usually estimated. The annualized response to the output gap level (growth) is about $0.0 - 0.08$ $(0.24 - 0.36)$, slightly below standard Taylor rule specifications. Finally, the rule displays a significant degree of inertia.

Table 2 shows, for the same model, the shock volatilities change significantly across samples. The volatility, as measured by the standard deviations, falls more than 40% in 1984-2007 compared to 1964-1979. The only exception is the wage markup shock whose volatility increases across samples.

The bottom panel of Table 2 gives estimates for the alternative specification that allows structural change in both policy and shocks. The policy parameter estimates display little change across samples. The output gap coefficients are nearly identical, with the inflation response coefficient slightly higher in the post Volcker sample rising from 1.2 to 1.4. This suggests that shifts in the policy regime are not the main force behind the medium-term evolution of the economy over the sample. The marginal likelihood supports this conclusion, with the benchmark model with a fixed policy rule providing a superior fit. These findings are consistent with a large literature using VAR and DSGE models such as Primiceri (2005a), Justiniano and Primiceri (2008a) and Sims and Zha (2006). Orphanides (2003) and Orphanides and Williams (2005a) estimate a monetary policy rule using real time data and show that monetary aggressiveness to inflation had not changed significantly between samples but emphasize the key role of decreased activism towards output stabilization in the Great Moderation period. Compared to the latter, we do not find strong evidence of changes in responsiveness to the output gap over the two samples.

While the data do not support structural change in policy, evidence for other regime change is considerably stronger. Our baseline model has a higher marginal likelihood when compared with a specification that holds shock volatilities constant across samples; and a specification where the learning gain is constant. The learning gain before 1999, $\bar{g}$, is estimated to be 0.07. This value implies a short-term forecast error of 1 percent leads to a 7 basis point revision in long-term beliefs and that a data that is five-years old receives a weight of about 15% percent when estimating drifts. The post-1999 gain, $\bar{g}_{1999}$, is about 0.01 which implies very little sensitivity to new information (equivalently, a much longer memory of old data). The baseline model also has higher likelihood than an equivalent model with rational expectations.

5.2 Model Predictions

Using the parameter estimates of the baseline specification we obtain the model predictions for the *whole sample*. We filter the distribution of the model's unobserved states for the period 1964-2007 which includes the three regimes. The low-volatility regime starts in 1984Q3, while the low-gain regime starts in 1999Q1. The assumption of invariant monetary policy justifies agents including an invariant set of parameters, Φ, to forecast in the short term. While a shift in the policy rule would lead to a change in Φ, time-varying shock volatilities or learning gains do not. Since agents' forecasting model in (12) does not change, no transition dynamics occur between regimes. Figure 1 provides model predictions for long-term inflation and real interest-rate forecasts together with the output gap, defined as the difference between output and its flexible price and wage counterpart. In the top and middle panel, we plot the realized variable, the predictions for expectations at both the one-to-ten-year and five-to-ten-year horizon, along with the corresponding survey data. For the real rate we use an ex post measure, the difference of the nominal interest rate and inflation, and compute associated real-rate expectations as the difference between the forecasts for nominal interest rates and inflation for each horizon.

The model captures low-frequency developments characteristic of the Great Inflation and Great Moderation periods, as well as the subsequent stabilization of long-term expectations from the late 1990s on-wards. The one-to-ten year (blue) and five-to-ten year (black) model-implied expectations tend to move very closely throughout the sample. Recalling expectations satisfy

$$E_t z_{t+n} = \Phi^n z_t + \sum_{j=0}^{n} \Phi^j \rho^{n-j} S\omega_t,$$

this reveals that the evolution of long-term expectations is mainly driven by the second component: drifts, ω_t, affect beliefs at very long-horizons, as opposed to first component, which reflects short-run dynamics driven by persistent exogenous shocks. One exception is the downward shift in the one-to-ten-year real rate expectations in the mid-2000, which is not accompanied by a similar drop in the five-to-ten-year forecast. This behavior holds true both in the model and in the data and reflects the relative stability of long-run beliefs in the post-1999 period. The bottom panel shows the output gap (black line, with the associated 95% bands in grey) and de-trended output as measured by the CBO (blue line). The output gap has the expected business cycle properties, generally declining during conventional recession dates. The output gap is positive over the first decade of the sample, but becomes persistently negative after 1980, before stabilizing for much of the Great Moderation period. The conduct of monetary policy will account for these features.

6 Monetary Policy With Extrapolation Bias

The previous section provides evidence that the systematic component of policy does not exhibit structural change over the sample. We now establish a central role for the non-systematic components of monetary policy in explaining the real and nominal economy, especially in the medium term. Accommodative monetary shocks and output gap mismeasurement flatten the term structure of real interest rate expectations in the 1970s generating consistently positive output gaps which fuel the Great inflation. The Volker disinflation in the early 1980s produced a persistent increase in long-term interest rates reducing inflation expectations through a period of negative output gaps. The excess sensitivity of long-term rates to temporary monetary policy shocks stems from the extrapolation bias that is at the heart of this model. Absent this information friction the model predicts stable long-term inflation expectations throughout the sample.

The monetary transmission mechanism. Figure 2 offers key insights on the expectations formation mechanism, the sources of variation in observed long-term interest rates and their effects on economic activity. The top panel displays the evolution of both the short-term nominal interest rate (light grey line) and the long-term rate (dashed red line), measured by the yield on the ten-year Treasury bond. In addition, the panel shows the model's predicted yield for a nominal bond of ten-year duration when priced using the expectations hypothesis of the term structure under both *subjective* (black line) and *objective* (blue line) beliefs. Objective beliefs are model-consistent expectations: they define expectations from the point of view of the econometrician knowing the true data-generating process.

Because of extrapolation bias subjective yields display weaker mean reversion than objective yields. An outside observer who knows the true data-generating process would correctly predict short-term rates to fall more quickly from the peak of the Great Inflation over the subsequent Great Moderation period: a lower expected path of the short-term rate in turn would deliver lower equilibrium long-term yields. In contrast to yields priced with model-consistent expectations, subjective yields move closely with both observed short- and long-interest rates. Moreover, they exhibit persistent deviations from objective yields.[28]

The middle panel of Figure 2 provides a shock decomposition of the wedge between subjective and objective yields (evaluated at modal estimates). This wedge provides a measure of the importance of the information friction. When the wedge is small beliefs are close to rational expectations. To ease interpretation we combine the shocks into four categories. The first includes all exogenous disturbances that move the natural level of output (equivalently the natural real interest rate), such as productivity and labor supply; the second includes wage and price markup-shocks; the third, conventional monetary policy shocks; and the fourth central bank errors in measuring the output gap. We call the final two policy shocks.

All shocks can potentially explain the discrepancy between subjective and objective yields (as they trigger surprise changes in the interest rate path). However, the decomposition reveals that policy disturbances play a central role. Monetary policy shocks (red bars) and output gap measurement errors (yellow bars) combine to give a negative wedge, on average, over the early part of the sample. After the late 1970s, these two policy shocks operate at different times. Monetary policy shocks cause a sustained increase in the wedge in the early 1980s and then a smaller rise in the late 1990s and early 2000s. Measurement error sustains the elevated subjective yields in the intervening years of the late 1980s. Shocks to the natural rate also play an important role, though tend to work in opposition to the policy shocks for much of the 1970s and 1980s. Markup shocks play a minor role in driving the wedge.

The bottom panel plots the shock decomposition of the output gap. The discrepancy between subjective and objective beliefs explains movements in real economic activity through monetary policy shocks. Policy disturbances have an important role in lifting the output gap during the mid-1970s. The degree of monetary accommodation reverses in the early 1980 with the Volcker disinflation and during the second half of the 1990s, with sizable negative effects on the output gap. That the average output gap is negative for the remainder of the sample reflects the fact the wedge is on average positive: from the peak of the Great Inflation to the end of the Great Moderation subjective yields are on average higher than objective

[28]That said, consistent with Crump, Eusepi, and Moench (2016), we find that while long-term expectations of the short-rate move substantially over the sample, there remains still a gap between yields implied by the expectations hypothesis and observed yields, particularly in the period from the early 1980s to the late 1990s, which the model does not explain. After this time, the gaps between subjective, objective and observed yields narrows and gradually close — consistent with the fall in learning gain and more stable long-term inflation expectations.

yields. Only at the end of the sample, the wedge vanishes as the the low-gain regime is established.

Overall, the three panels illustrate the monetary transmission mechanism. Extrapolation bias flattens the yield curve so that policy tends to be tighter or looser relative to rational expectations. Expansionary monetary policy shocks in the early 1970s were transmitted trough the entire term structure of interest rates as agents' extrapolated short-term surprises to a belief in persistently lower interest rates. This flattening of the curve provided stimulus to aggregate demand through lower expected real rates leading to persistently higher output gaps and, as we discuss below, persistently higher inflation. Tight monetary policy under Volcker led agents to expect persistently high interest rates, providing a sharp and long-lasting economic contraction.

Dynamics. How do monetary policy shocks affect economic dynamics? The transmission of monetary policy shock depends on the learning gain, which determines the degree of extrapolation bias. Figure 4 shows the impulse response distribution to a one standard deviation expansionary monetary policy shock in both the high- and low-gain regimes. The high-gain regime is denoted by black (median) and shaded grey lines (95% interval), while the low-gain regime is in blue. Long-term inflation expectations, as measured by the five-to-ten ahead yearly average are shown as dashed lines.

In the high-gain regime the monetary policy shock produces a large and long-lasting expansion in output (black line). This expansion is accompanied with a large and persistent increase in inflation and long-term inflation expectations. Surprised by the expansionary shock, agents update their beliefs about long-term inflation, generating an upward trend in actual inflation. The behavior of the ten-year interest yield in the high-gain regime shows a monetary surprise produces a substantial downward shift in the long-term rate, coming from extrapolation bias. Interestingly, the expansion reverses in the medium term as the central bank is forced to reduce inflation. The same monetary policy surprise under the low-gain regime is more familiar. It produces a mild expansion in output and has virtually no effect on long-term inflation expectations.[29]

Table 3 shows the fraction of variance in the short- and long-run that is attributable to the monetary policy shock in each sample. The variance decompositions corroborate the key role of monetary policy shocks in generating low-frequency movements in the economy. Regardless of regime, in the short-term monetary policy shocks do not have a prominent role in explaining the variation in output and inflation. However, in the high-gain regime, from 1964 to 1998, monetary policy shocks explain over 40% of inflation and the output

[29] These highly persistent effects of monetary policy are consistent with Òscar Jordà, Singh, and Taylor (2020).

gap variance over the long term. Consistent with the impulse responses, in the final low-gain regime the role of monetary policy shocks declines sharply, explaining about 5% of the inflation variance and 20% of the variation in the output gap. Throughout the entire sample, monetary policy remains an important factor in explaining the variance of long-term yields, both in the short and in the long run.

	Short-Term			Long-Term		
	1964-1983	1984-1998	1999-2007	1964-1983	1984-1998	1999-2007
Inflation	0.7	0.8	0.1	37.8	45.1	5.5
	(0.3,1.9)	(0.3,2.7)	(0.0,0.3)	(26.4,50.2)	(32.0,58.0)	(2.6,11.8)
Output	9.5	11.8	0.9	38.6	48.2	19.6
	(5.6,15.2)	(7.3,19.7)	(0.4,2.0)	(26.9,50.9)	(37.8,58.1)	(12.3,29.5)
10-year R	39.3	48.9	47.3	50.8	59.6	37.9
	(27.0,52.3)	(35.8,62.1)	(32.6,63.6)	(35.8,64.2)	(47.4,69.9)	(24.4,54.7)

The Table report median percentage fraction of the variance explained by the monetary shock, together with the 5 and 95 percentile. Short-term is the $1-4$ quarters average; long-term is the $36-100$ quarters average.

Table 3: **Variance Decomposition: Monetary Policy Shocks.**

Monetary Shocks. Are our identified monetary policy shocks plausible? Figure 3 plots the smoothed estimates of output gap mis-measurement shocks (top panel), defined as

$$\text{Mis-measurement shock}_t = (\phi_x + \phi_{\Delta x}) \times \nu_t - \phi_{\Delta x} \times \nu_{t-1},$$

and monetary shocks (bottom panel). The definition of the measurement shock comes directly from the assumed monetary policy rule. The black line measures the median prediction, and the grey area includes the 95% coverage interval. Notice that while the process ν_t is perfectly observed, at least starting in 1965Q4, the measurement error shock still depends on the estimated distribution of the policy parameters. Starting from the top panel, measurement errors are large and negative over the 1970s, resulting in overly accommodative policy. While the shocks are sizable between the end of 1970s and beginning of the 1980s, they do not appear to show much persistence in either direction. The only episode where the Fed appeared to be making persistently negative mistakes is the end of the 1980s.

In the bottom panel, the estimated monetary policy shocks display a mild auto-correlation over the sample (in the range $0.08-0.34$) suggesting at times the Federal Reserve surprised market participants over several quarters. The estimated shock sequence, however, does not seem to be at odds with other estimates of monetary surprises. We also plot the extended quarterly measure of the Romer and Romer (2004) shock series (red line), identified using historical Federal Reserve records and the Greenbook forecasts to eliminate sources of endogeneity or anticipatory effects, and the Kuttner (2001) shock series (blue line), based on future contracts. Both measures have fairly strong correlations with our estimated shocks series.[30] Perhaps more importantly, our estimated monetary policy shocks conform to narratives offered by the literature. Following Romer and Romer (2004), Appendix E discusses some relevant episodes.

The Great Inflation. The interplay between monetary policy shocks and learning is crucial to generate the model's persistent endogenous inflation trend. Figure 5 shows counterfactual simulations of long-term inflation expectations under different assumptions about shocks and expectations, together with the survey data. The grey line shows the five-to-ten year expectations under the benchmark model. Absent monetary policy shocks (red line), long-term expectations rise modestly in the 1970s and remain below 3.5% through the whole sample. Strikingly, under rational expectations (blue line) long-term inflation expectations do not move at all, even if all shocks are included in this counterfactual simulation.[31] The experiment reveals the self-referential mechanism that is at the core of the model: extrapolation bias amplifies and propagates disturbances to endogenously deliver the observed inflation trend as an equilibrium outcome.

7 Optimal Policy

We have adduced evidence that extrapolation bias and the non-systematic component of monetary policy together explain a sizable portion of economic fluctuations over the sample. Here we revisit the systematic component of policy. Given imperfect control of the term structure of interest-rate expectations, could the Federal Reserve have achieved superior macroeconomic stabilization using a different policy rule? This section provides a quantitative evaluation of the trade-off that extrapolation bias presents for policy design. We show that even under ideal conditions, when the central bank knows the true data-generating process, optimal policy is unable to jointly stabilize, output, wage and price inflation. Consistent with the simple example, optimal policy accommodates the constraints imposed by the information friction by implementing a less aggressive interest-rate policy relative to rational expectations.

[30] In line with other DSGE estimates such as Smets and Wouters (2007) and Justiniano and Primiceri (2008b), the volatility of our estimated shocks is higher than these alternative measures.

[31] A similar outcome would obtain if the counterfactual assumed the low Kalman gain that we estimate for the post-1999 period.

7.1 The Loss Function

Under arbitrary beliefs, the period welfare-theoretic loss is

$$L_t = \lambda_p \left(\pi_t - \iota_p \pi_{t-1}\right)^2 + \lambda_w \left(\pi_t^w - \iota_w \pi_{t-1}\right)^2 + \lambda_x \left(x_t - \bar{b} x_{t-1}\right)^2,$$

where the weights

$$(\lambda_p,\ \lambda_w,\ \lambda_x) \equiv \left(\theta_p \kappa_p^{-1} + \theta_w \kappa_w^{-1}\right)^{-1} \times \left(\theta_p \kappa_p^{-1},\ \theta_w \kappa_w^{-1},\ \tilde{\vartheta}\right)$$

determine the relative priority given to stabilizing prices, wages and output, and are functions of the slopes of the wage and price Phillips curves, κ_p and κ_w. The parameters $\bar{b} \leq b$ and $\tilde{\vartheta}$ are a function structural parameters.[32]

The derivation of the second-order approximation to household utility is valid under both rational expectations and learning. The architecture of the loss function reflects well-understood sources of inefficiency which arise from monopoly power in goods and labor markets. In our model, equilibrium price and wage markups can vary for two reasons. First, exogenous time variation in the elasticity of demand across differentiated goods and labor services shifts firms' and workers' desired markups. Second, staggered price setting in goods and labor markets means prevailing prices depart from the optimal flexible-price levels, which lead to endogenous variation in markups in response to all aggregate disturbances. Optimal policy mitigates this second source of variation due to nominal rigidities. By stabilizing endogenous variation in markups, policy reduces cross sectional dispersion in price and wage setting, and the associated inefficiencies in supply of goods and labor.

7.2 The Policy Problem

Using the empirical model, we now replicate the analysis of the simple model. Regardless of how agents form beliefs, the central bank minimizes the expected discounted loss

$$\mathcal{L} = E_t \sum_{T=t}^{\infty} \beta^{T-t} L_T. \tag{19}$$

taking as given the optimal decisions of households and firms. The central bank has rational expectations, knows the objective probability distribution, and takes account of the fact that the output gap is measured with error. This gives the central bank the greatest possible ability to improve upon the historical performance of monetary policy. The only instrument

[32] Additional details can be found in Giannoni and Woodford (2004) and the Appendix

of policy is the short-term interest rate. Whether additional policy instruments, such as balance-sheet policy, can improve stabilization is left to future work.

Rational Expectations. We assume the central bank seeks to implement a targeting rule of the form

$$0 = \phi_\pi \left(\pi_t - \iota_p \pi_{t-1}\right) + \left(\pi_t^w - \iota_w \pi_{t-1}\right) + \phi_{\Delta x} \left(x_t^{CB} - x_{t-1}^{CB}\right). \tag{20}$$

The central bank policy problem is to choose the parameters $\{\phi_\pi, \phi_{\Delta x}\}$ to minimize the loss (19). This target criterion, which includes macroeconomic variables that directly relate to the loss function, is not the fully optimal policy under rational expectations, but delivers stabilization outcomes indistinguishable from optimal policy under the timeless perspective — see Woodford (2003).

Subjective beliefs. Informed by the analysis of the simple example, when beliefs exhibit extrapolation bias the central bank implements the policy rule

$$R_t = R_{t-1} + \phi \times \left[\phi_\pi \left(\pi_t - \iota_p \pi_{t-1}\right) + \left(\pi_t^w - \iota_w \pi_{t-1}\right) + \phi_{\Delta x} \left(x_t^{CB} - x_{t-1}^{CB}\right)\right]. \tag{21}$$

This permits gradual implementation of the target criterion (20). The parameter ϕ regulates how aggressively the policy rule is implemented. The central bank's problem is to choose the policy parameter ϕ to minimize the expected loss, taking the optimized targeting criterion under rational expectations as given. As anticipated by the simple example, the optimal parameter $\phi^*(\bar{g})$ is a function of the learning gain $\bar{g}$. As the gain shrinks to zero beliefs converge to rational expectations. The optimal policy $\phi^*(\bar{g})$ becomes arbitrarily large and the central bank gets closer to achieving the optimized target criterion in every period. Only in this limiting case does extrapolation bias not represent a constraint on policy.

While this policy is not optimal under learning, it has the following desirable features. First, the structure of the rule permits direct comparison to the optimal target criterion under rational expectations. Differences in policy are summarized by a single parameter, ϕ. Second, Orphanides and Williams (2007) show this type of "difference rule" performs well under imperfect information and learning. It is robust to mis-measurement in the output gap and to different expectations formation mechanisms. These advantages of an highly inertial rule extend to our model. Third, we experimented with alternative formulations of this targeting criterion. None improved performance significantly.[33] The main implication of the information friction is that the target criterion should be achieved *less aggressively* and

[33] For example, we allowed the policy maker to choose the parameters $\{\phi, \phi_\pi, \phi_{\Delta x}\}$ to minimize the loss (19) under learning. This specification gave coefficients that are remarkably similar to our baseline specification. Other examples are given in Appendix I.

gradually over time. Fourth, the specific choice of the targeting rule is not driving the result. We show that even in a simpler model environment without the additional trade-off induced by wage stickiness the optimal policy fails to fully stabilize the economy in our setup. (These an additional results are discussed in Appendix J.)

To give emphasis to this final point, we also consider a specification in which the central bank has direct control over aggregate demand. In this case, the central bank is able to implement the targeting criterion (20) in every period.[34] This captures a model environment where the transmission mechanism of monetary policy works flawlessly. Aggregate demand is no longer a constraint to policy.

A maintained assumption in the optimal policy exercises is that the gain coefficient is policy invariant. While this means the perceived signal-to-noise ratio is invariant across policy regimes (that is, agents perceive the same volatility of long-term drift *relative* to short-term disturbances), it doesn't imply beliefs are invariant to policy. Because short-run forecast errors are endogenous to policy, long-term expectations will adjust. Policy can't exploit beliefs to deliver any equilibrium of its choosing. While some papers such as Marcet and Nicolini (2003) and Carvalho, Eusepi, Moench, and Preston (2019) explicitly model the learning gain to be endogenous to the policy regime in simple model environments, addressing this issue in a medium-scale model with multiple regimes is outside the scope of this paper.

Results. Table 4 records the loss in each of the three regimes. The losses are sample-specific, because both the volatility of shocks and the learning gain vary. Optimal policy and the associated losses are computed using modal estimates for each regime. To get a sense of the historical policy performance we add the associated loss for the estimated policy rule without policy shocks.

When the location of the aggregate demand curve is a constraint on central bank actions, the efficacy of monetary policy declines substantially. Regardless of the regime, the loss under imperfect knowledge is significantly higher than under full information. The optimal policy coefficients are $\phi^* = 1.74$ in the period 1965-1983, $\phi^* = 0.44$ in 1984-1998, and $\phi^* = 19.1$ in 1999-2007. The first two high-gain periods require gradual implementation of the rational expectations optimal target criterion. In the third, low-gain regime, beliefs are much closer to rational expectations — more aggressive policy is both feasible and desirable.

When the central bank has direct control of aggregate demand losses are almost identical to rational expectations. The central bank can implement the optimal allocation despite fluctuating long-run beliefs about inflation and economic activity. This quantitative result mirrors the conclusions from the simple example. Intuitively the Federal Reserve prevents

[34] Here $\{\phi_\pi, \phi_{\Delta x}\}$ is chosen to minimize the loss (19) under learning. However the optimal parameters are very close to the parameters chosen under rational expectations

	$\mathcal{L}$		
	1964-1983	**1984-1998**	**1999-2007**
Historical policy [$\hat{m}_t = 0$]	159.8	65.2	57.2
Optimal Policy	72.5	48.0	37.9
Optimal Policy AD control	9.5	20.7	20.2
Optimal Policy full info	8.6	20.1	20.1

The table summarizes the performance of different policy rules. The first column measures the unconditional loss as in equation (19). The second column show standard deviations for each argument in the loss function in counterfactuals conducted using filtered shocks. These counterfactuals include output gap mis-measurement shocks by the central bank.

Table 4: **Optimal Policy**.

beliefs from drifting by aggressively responding to the state of the economy. Not surprisingly, the historical rule delivers a higher loss compared to the optimal rule under learning. But what is interesting is that, even though the central bank has full information, optimal policy can only close about half of the gap between the historical rule and rational expectations.

Lastly, because markup shocks are more prominent over 1984-2007, the stabilization trade-off worsens: this explains why optimal policy under learning is now closer to the historical policy than the optimal policy under rational expectations during this sub-sample.

7.3 The Counterfactuals

To illustrate further the properties of optimal policy we use counterfactual analysis, making the following assumptions. With the exception of monetary policy shocks, the economy experiences the same sequence of shocks and has the same initial state estimated in the benchmark model. Monetary policy shocks are set equal to zero since purely exogenous variation in interest rates reduces welfare. However, output gap mis-measurement shocks are included so that we can properly evaluate the robustness of optimal policy.

We consider optimal policy in each of the three subsamples. Sample-specific policies imply different transitional dynamics. Here we assume agents taking decisions within each regime know the new transition dynamics associated with the policy regime. We consider this scenario to reflect the best conditions for optimal policy. The question of how to design the optimal transition from one regime to another is left for future research. To construct the counterfactuals we must specify the initial beliefs in each regime. In the first regime we take the estimated state as the initial conditions. To give continuity to the figures, in the second and third regimes we assume the initial conditions are given by the terminal values of the simulated data from the prior regime. This is not important for the results.

Figure 6 shows the optimal policy counterfactuals. Each column reports results for one of the three regimes. Each row reports the dynamics for wages, inflation and the output gap. Consistent with the welfare-theoretic loss function, we report wage inflation net of inflation indexation and goods-price inflation net of indexation. Both variables are reported as the average over four-quarters. Each panel includes the variable under the historical policy rule (grey line), the optimal policy under rational expectations (blue line) and the optimal policy under learning (red line). In the case of inflation we also report the 5-10 year average inflation expectation under the optimal learning policy. This is informative about the extent to which good policy provides a nominal anchor.

The first regime, 1964-1983, offers several observations. Reflecting the relatively higher costs of nominal distortions in the labor market, optimal policy stabilizes wages to a greater extent than goods prices.[35] While the optimal policy under rational expectations almost completely stabilizes wages and the output gap, the optimal policy under learning accepts some variation in wages and considerably more variation in the output gap. We will show that this reflects the constraint that aggregate demand places on optimal policy when expectations are subject to an information friction.

The second regime, 1984-1998, confirms these insights with differences that reflect the fact that the average volatility shocks falls substantially, with the exception of wage markup shocks which are more volatile. Indeed, the volatility of inflation falls relative to the first regime, but more volatile markup shocks increase variation in wages and the output gap regardless of expectation assumption.[36] Sizable movements in natural-rate shocks over the period 1985-1995 also make stabilization policy more challenging under learning. The results for the final regime underscore the role the combined effects of the decline in shock volatility and better anchored expectations. The lower gain makes beliefs much closer to rational expectations: long-term expectations display little sensitivity to forecast errors permitting more aggressive policy actions. Consistent with the simple example, monetary policy has more precise control of the macroeconomy under learning and can be more aggressive without creating output gap variation.

Finally, note that the optimal learning policy predicts paths for each macroeconomic

[35] Recall the slope of the wage Phillips curve is an order of magnitude smaller than the goods-price Phillips curve.

[36] To get an idea about the shocks, see shock decomposition in figure (2).

variable that lie in between those under the historical rule and rational expectations optimal policy. The optimal learning policy ameliorates but by no means eliminates business cycle variation observed under the historical policy. This casts a more positive light on the historical performance of the Federal Reserve than does a rational expectation analysis. Having said this, in each regime, optimal policy provides a nominal anchor by stabilizing long-term inflation expectations.

There are limits to what monetary policy can do. To drive this point home, Figure 7 plots the same counterfactuals but assuming the central bank can control the output gap directly. The predictions for wages and inflation are almost identical, while there are some minor differences for the output gap. The aggregate demand curve fundamentally constrains policy.

These results relate to a number of papers in the literature. Our rational expectations and learning results (when the central bank has direct control of aggregate demand) have much in common with Justiniano, Primiceri, and Tambalotti (2013), particularly in the first regime. Optimal policy almost completely stabilizes output gap and wage variation, while permitting some variation in inflation. Most variation in the data stem from efficient movements in the natural rate of output, meaning optimal policy approximates so-called Divine Coincidence. The results for the second regime differ more substantially as we find a more prominent role for inefficient movements in wage markups.

When the central bank controls aggregate demand, our results beg the question of why learning (even if only about wages, inflation and output) doesn't generate large differences compared to rational expectations. For example, Orphanides and Williams (2008, 2012) find significant differences in optimal policy in models of learning and rational expectations. Why don't we find similar implications? The explanation lies in the weight given to output gap stabilization in the welfare-theoretic loss function. Consistent with earlier work in the rational expectations literature, our parameter estimates imply a small weight on output gap stabilization. As in the simple example of section 2, optimal policy under rational expectations and learning then deliver similar results. But if the weight on output gap stabilization rises, then larger differences emerge. Indeed, it becomes optimal to aggressively stabilize inflation expectations. Instability in inflation expectations requires substantial variation in aggregate demand to stabilize inflation — and these movements are costly for welfare. If our model had a greater weight on output gap stabilization, we would have both this trade-off and that from aggregate demand.[37] The contribution of this paper it to provide evidence that the transmission mechanism of monetary policy is itself a constraint, and, at least with our estimate, the more relevant constraint.

[37]See Eusepi and Preston (2018b) for related discussion.

8 Conclusions

This paper proposes and quantitatively evaluates a new information friction confronting monetary policy: the stability of long-term interest rate expectations. As such this friction is fundamentally different to papers that consider other important practical constraints on policy, including uncertainty about the current state of the economy and the ability to construct reliable forecasts. That the stability of expectations are endogenous to interest-rate policy means the central bank optimally adjusts policy rates less aggressively than a full information rational expectations analysis in response to shocks, including movements in the natural rate of interest. This differentiates our work from other imperfection information models in which price stability is feasible but not desirable. Here it is desirable but not feasible. This property also distinguishes our work from a long literature on policy design in New Keynesian models which incorporate a range of non-information based frictions, but nonetheless conclude monetary policy can largely eliminate inefficient fluctuations in economic activity.

The central message then is that we should not over promise on what monetary policy can achieve. There are fundamental limits to stabilization policy. While we have explored the consequences of extrapolation bias for monetary policy in a New Keynesian environment, this type of constraint is likely to have implications for policy more generally. The basic mechanisms of this paper imply that asset prices will display excess volatility to short run developments. How this matters for macro-prudential and monetary policy is surely of interest. Similarly, Eusepi and Preston (2018a) demonstrate in a New Keynesian model with fiscal policy, that distorted beliefs about future tax policy re-weights income and substitution effects to make monetary policy less effective. Understanding how instability in long-term tax and debt expectations matter for budget policy will likely be vital given the state government finances we have witnessed this year.

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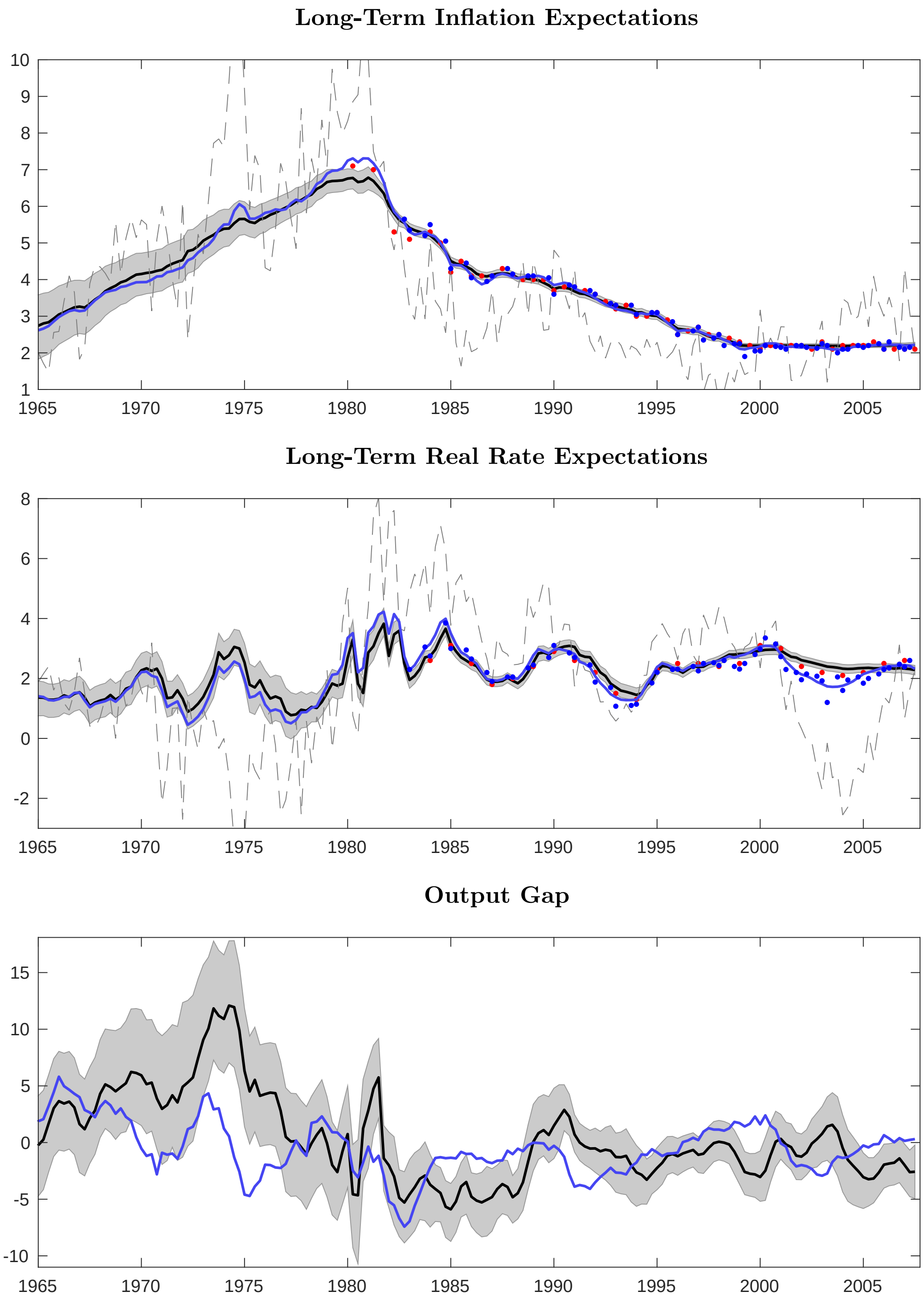

Figure 1: **Baseline Predictions.**

The top and middle panels show the evolution of long-term survey expectations data for inflation and the short-term real rate of interest. Actual variable (dashed black), the two survey expectations measures (red and blue dots), the model implied 1-10 year average expectations (the blue line); and the model implied 5-10 year average expectations with 95% posterior probability band (black line). The bottom panel shows the model implied output gap (black line), Y_t/Y_t^n, and the de-trended output, Y_t/Z_t, from the CBO (blue line).

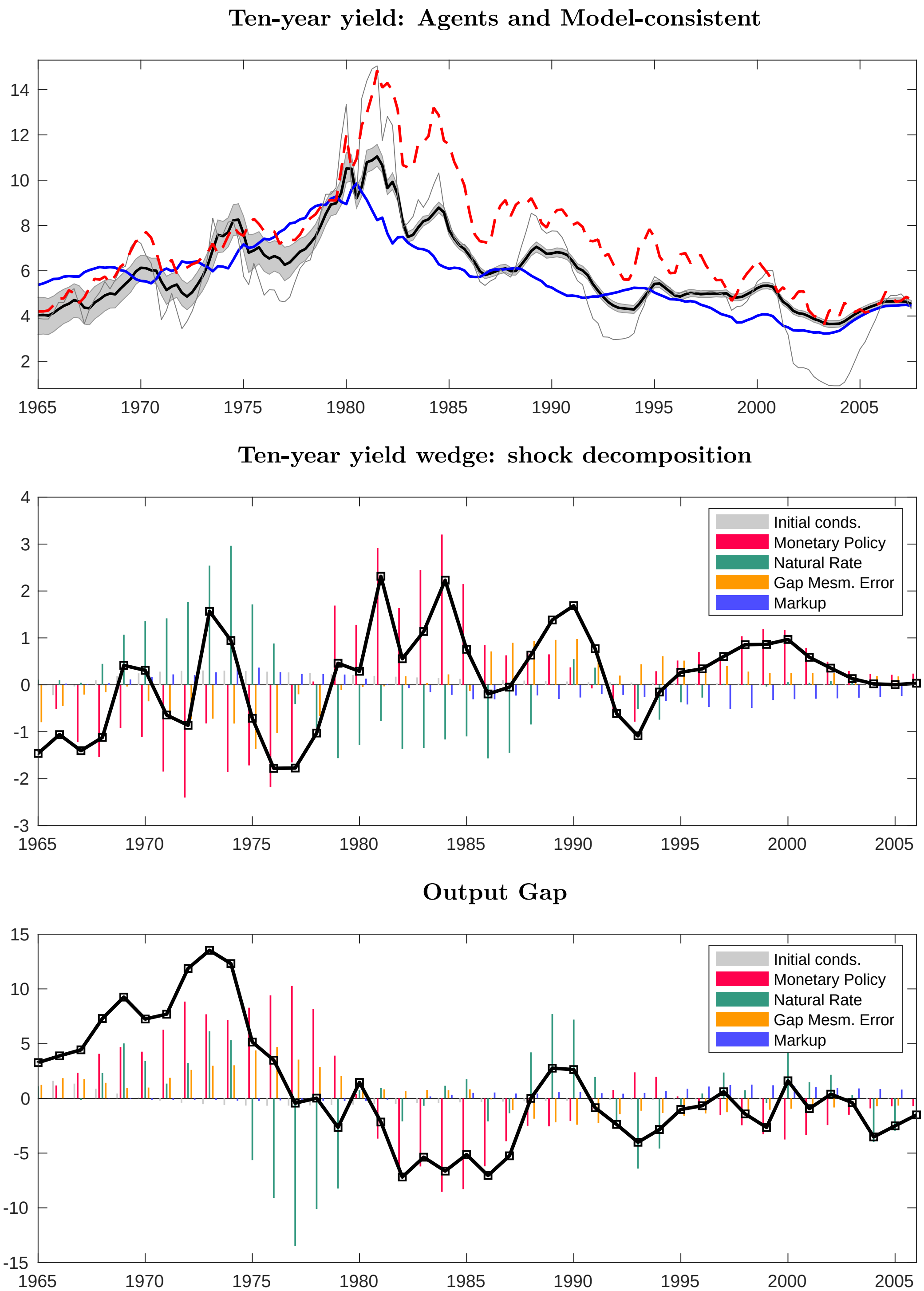

Figure 2: **Over-extrapolation and Aggregate Demand.**

Top panel: the black line defines the 10-year interest rate in the model, while the blue line is the rate that would prevail if the bond was prices under model-consistent expectations. The dashed grey line denotes the short-term interest rate. *Middle panel*: shows the shock decomposition of the wedge between the subjective and objective equilibrium yields– last quarter realization for each year. *Bottom panel*: shows the shocks decomposition for model-implied output gap – last quarter realization for each year.

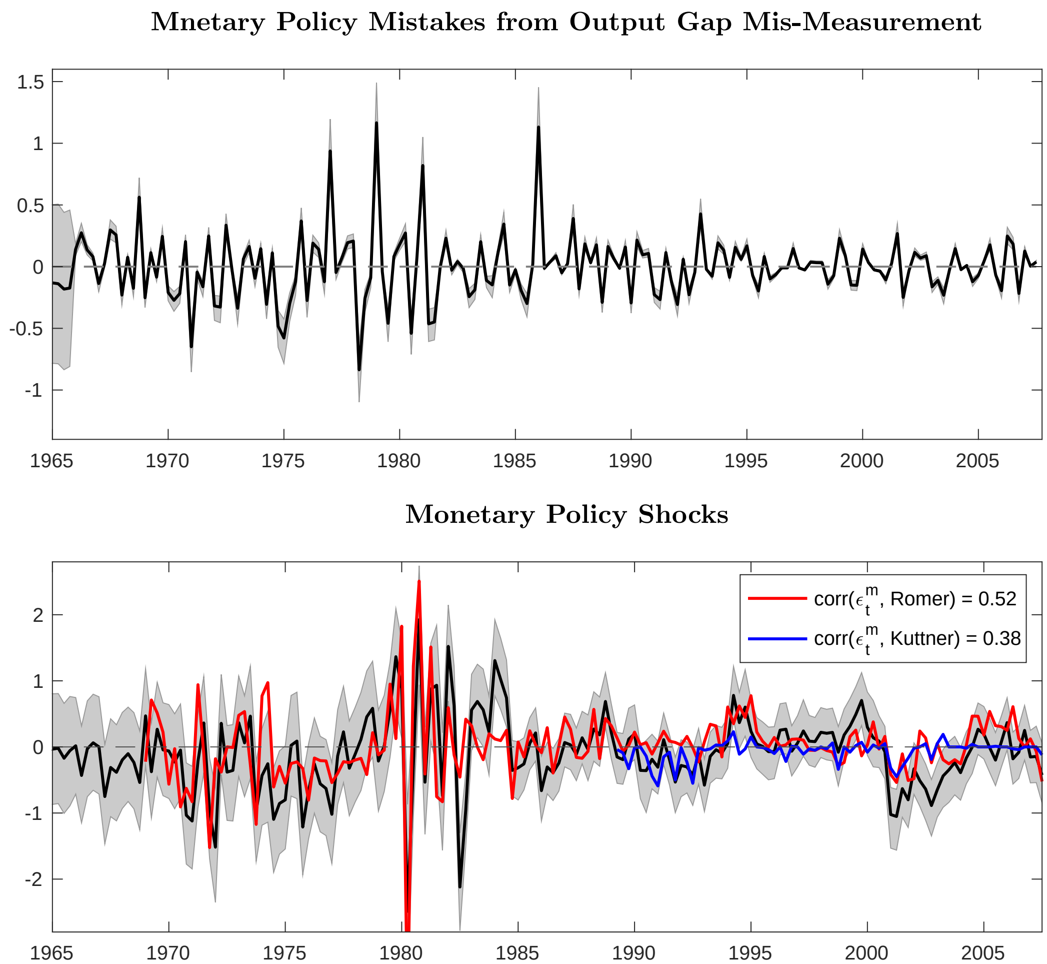

Figure 3: **Monetary Policy: non-systematic components**

Top panel: the black line defines monetary policy surprises originating from output gap mis-measurement. The grey area measures the 95% coverage interval. *Bottom panel*: The black line (and shaded area) shows the evolution of (annualized) monetary shocks as implied by the model (smoothed estimates, black line); the red line measures the quarterly measure of the Romer and Romer (2004); the blue displays (quarterly) monetary shocks from Kuttner (2002). The legend shows the (median) correlation between the model's monetary policy shocks and those two alternative measures.

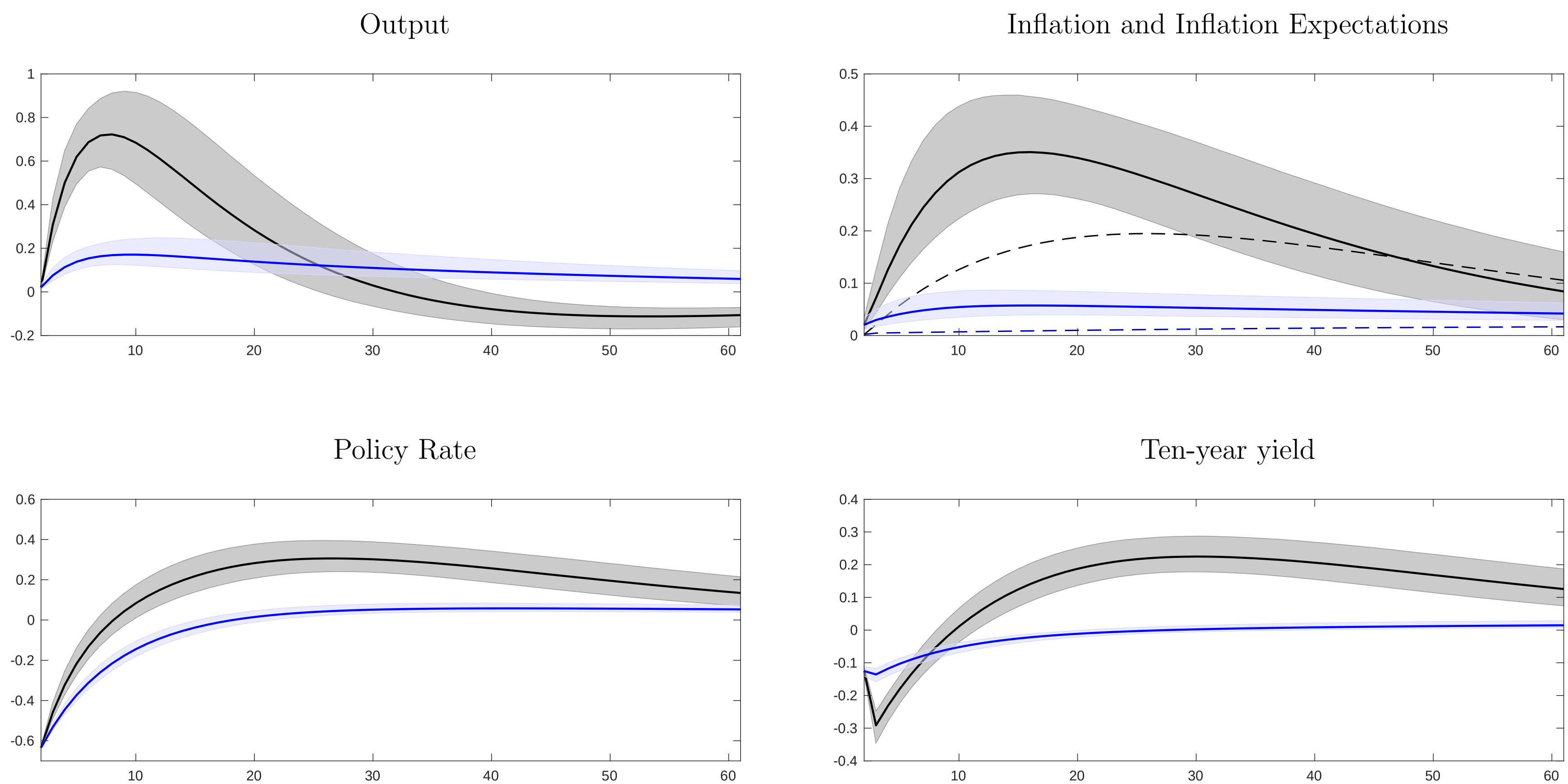

Figure 4: **Monetary Transmission Mechanism**.

The figure shows the impulse response to a one standard deviation expansionary monetary policy shock.

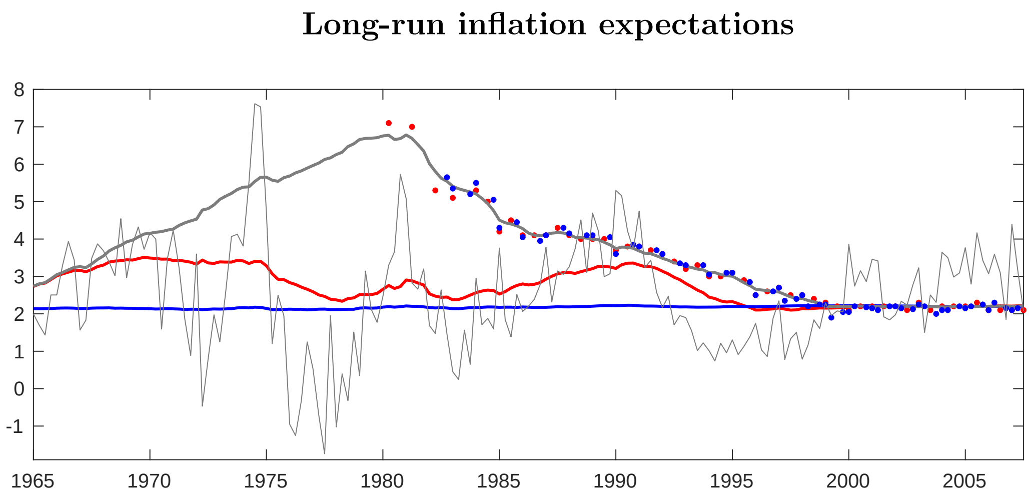

Figure 5: **Long-term inflation expectations.**

The panel shows counterfactual simulations for five-to-ten year ahead inflation expectations. Baseline model predictions (grey line); no monetary policy shocks (red line); rational expectations (blue line).

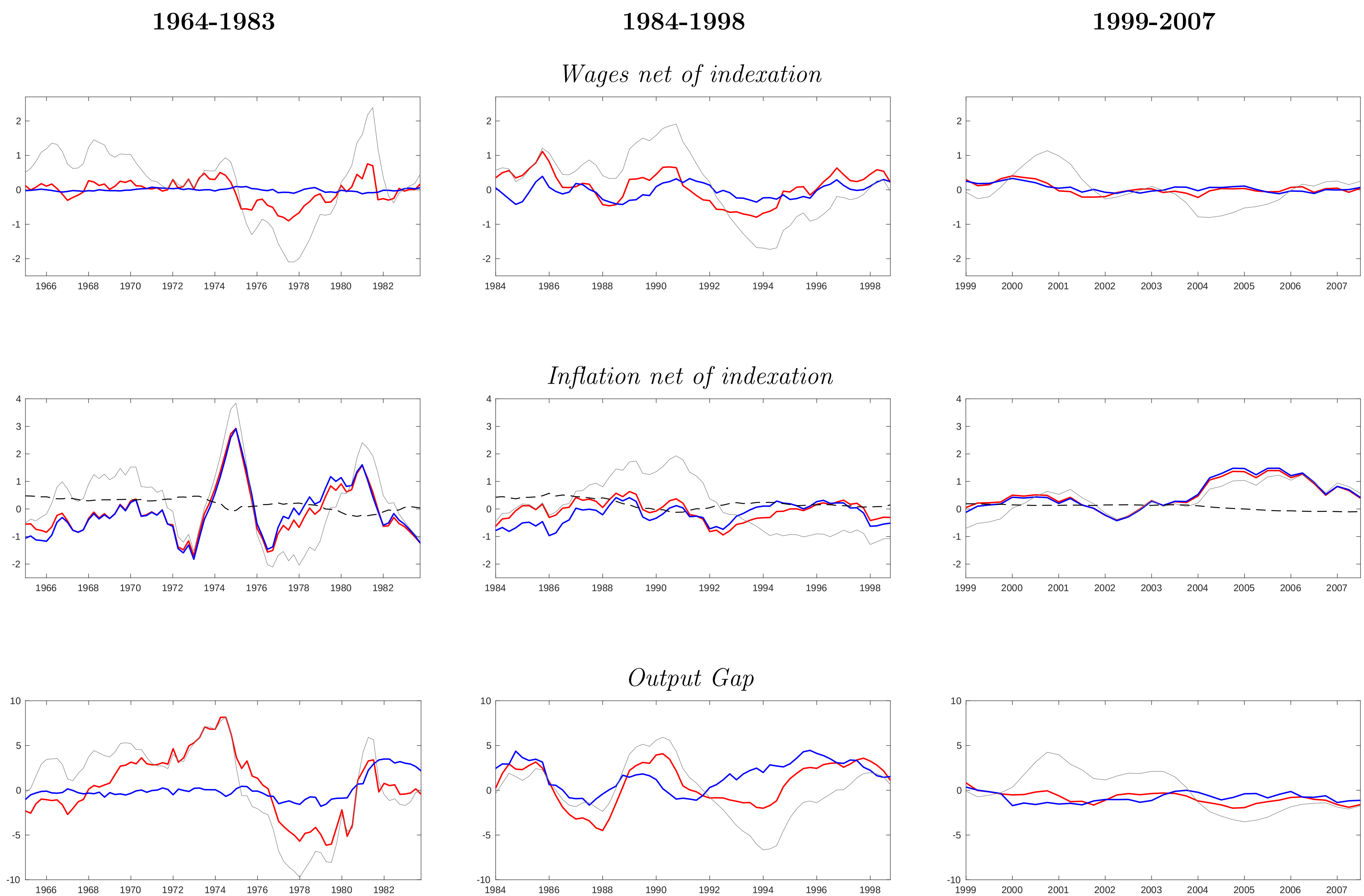

Figure 6: **Optimal Policy**

The panels describe the evolution of wages net of indexation (top), inflation net of indexation (middle) and output gap (bottom). Baseline model with the historical policy rule (grey line); optimal policy under learning (red line); optimal policy under rational expectations (blue line); finally the black dashed line in the middle panels denote the five-to-ten years ahead inflation expectations under optimal policy.

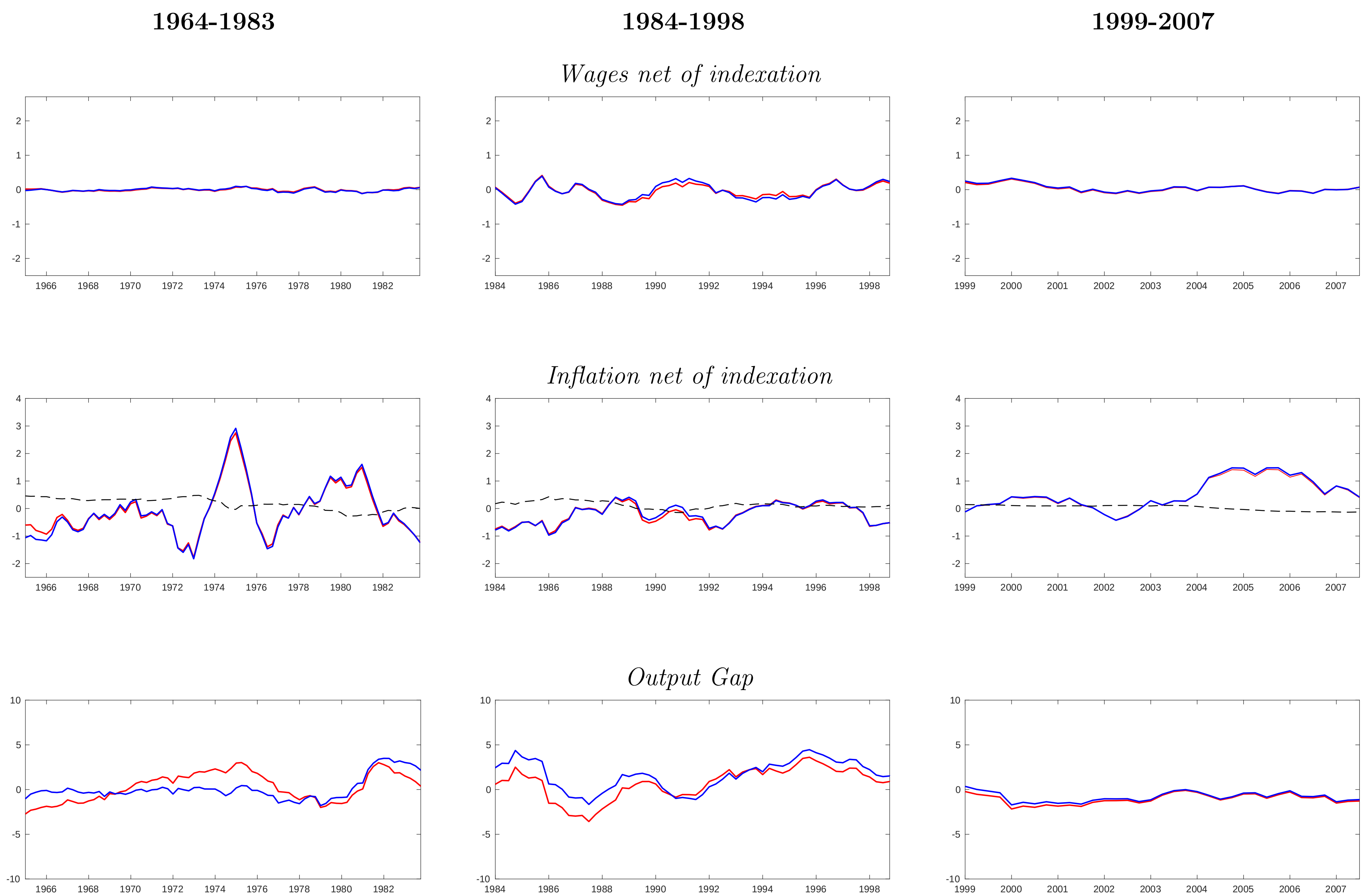

Figure 7: **OptimalPolicy: Central bank has full control over output gap.**
The panels describe the evolution of wages net of indexation (top), inflation net of indexation (middle) and output gap (bottom). Baseline model with the historical policy rule (grey lineP); optimal policy under learning (red line); optimal policy under rational expectations (blue line); finally the black dashed line in the middle panels denote the five-to-ten years ahead inflation expectations under optimal policy.

A Simple Example: Further Details

This section develops a parsimonious version of the canonical New Keynesian model widely used for monetary policy analysis and some analytical details for the results in section 2. A range of assumptions are made for expositional simplicity — for example log utility and linear disutility of labor supply. Further details on the microfoundations can be found in Woodford (2003) and Gali (2008).

Microfoundations. A continuum of households i on the unit interval maximize utility

$$\hat{E}_t^i \sum_{T=t}^{\infty} \bar{C}_T \beta^{T-t} \left[\ln c_T(i) - \chi n_T(i)\right],$$

where $0 < \beta < 1$ and $\chi > 0$, by choice of sequences for consumption, $c_t(i)$, and labor supply, $n_t(i)$, subject to the flow budget constraint

$$c_t(i) + b_t(i) \leq (1 + R_{t-1}) \pi_t^{-1} b_{t-1}(i) + W_t n_t(i)/P_t + \Gamma_t(i)/P_t$$

and the No-Ponzi condition

$$\lim_{T\to\infty} \hat{E}_t^i \left(\prod_{s=0}^{T-t} (1 + R_{t+s}) \pi_{t+s+1}^{-1} \right)^{-1} b_{T+1}(i) \geq 0.$$

The variable $b_t(i) \equiv B_t(i)/P_t$ denotes real bond holdings (which in equilibrium are in zero net supply), R_t the nominal interest rate, $\pi_t \equiv P_t/P_{t-1}$ the inflation rate, W_t is the hourly wage, $\Gamma_t(i)$ dividends from equity holdings of firms and $\bar{C}_T$ exogenous preference shifter. The operator $\hat{E}_t^i$ denotes subjective expectations, which might differ from rational expectations.

A continuum of monopolistically competitive firms maximize profits

$$\hat{E}_t^j \sum_{T=t}^{\infty} \xi^{T-t} Q_{t,T} \left[p_t(j)\, y_T(j) - W_T n_T(j)\right]$$

by choice of $p_t(j)$ subject to the production technology and demand function $y_T(j) = n_T(j) = (p_t(j)/P_T)^{-\theta} Y_T$ for all $T \geq t$, with the elasticity of demand across differentiated goods an exogenous process satisfying $\theta > 1$; and exogenous probability $0 < \xi < 1$ of not being able to reset their price in any subsequent period. When setting prices in period t, firms are assumed to value future streams of income at the marginal value of aggregate income in terms of the marginal value of an additional unit of aggregate income today giving the stochastic discount factor $Q_{t,T} = \beta^{T-t}(P_t Y_t)/(P_T Y_T)$.

For any beliefs satisfying standard probability laws, to a first-order log-linear approximation in the neighborhood of a zero-inflation steady state, optimal individual consumption and pricing decisions can be expressed as

$$\hat{c}_t(i) = \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\hat{w}_T - \beta \left(\hat{R}_T - \hat{\pi}_{T+1} - (\bar{c}_T - \bar{c}_{T+1}) \right) \right] \tag{22}$$

$$\hat{p}_t(j) = \hat{E}_t^j \sum_{T=t}^{\infty} (\xi\beta)^{T-t} \left[(1-\xi\beta)(\hat{w}_T) + \xi\beta\hat{\pi}_{T+1} \right] \tag{23}$$

where for any variable z_t, $\hat{z}_t = \ln(z_t/\bar{z})$ the log-deviation from steady state $\bar{z}$, with the exceptions $\hat{p}_t(j) = \ln(p_t(j)/P_t)$, $\hat{R}_t = \ln\left[(1+R_t)/\left(1+\bar{R}\right)\right]$, and $\bar{c}_t = \ln\left(\bar{C}_t/\bar{C}\right)$. With a slight abuse of notation, the caret denoting log deviation from steady state is dropped for the remainder, so long as no confusion results.

In a symmetric equilibrium $c_t(i) = c_t = w_t \equiv W_t/P_t = n_t = Y_t$ for all i, $p_t(j) = p_t(j)$ and $b_t(i) = b_t(j) = 0$ for all i, j. Aggregating across the continuum of households and firms, and imposing market-clearing conditions, the economy is described by the aggregate demand and supply equations

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - (R_T - \pi_{T+1} - r_T^n) \right] \tag{24}$$

$$\pi_t = \hat{E}_t \sum_{T=t}^{\infty} (\xi\beta)^{T-t} \left[\kappa x_T + (1-\xi)\beta\pi_{T+1} \right] \tag{25}$$

where the output gap is defined as

$$x_t = y_t - y_t^n = w_t$$

the difference between output and the natural rate of output, the level of output determined by a flexible price economy: here $y_t^n = 0$. In this simple example we assume agents understand this equilibrium relationship between wages and the output gap. This is without loss of generality. The associated natural rate of interest $r_t^n = (\bar{c}_t - \hat{E}_t\bar{c}_{t+1})$ is determined by fluctuations in the propensity to consume, assumed to be an i.i.d. processes. Average beliefs are defined as

$$\int_0^1 \hat{E}_t^i di = \int_0^1 \hat{E}_t^j dj = \hat{E}_t.$$

The aggregate demand equation determines the output gap as the discounted expected value of future wages, with the second term capturing variations in the real interest rate, applied in future periods, due to changes in the nominal interest rate and goods price inflation. That expected future dividends are irrelevant to consumption plans, to the first-order, reflects the assumption of an infinite Frisch elasticity of labor supply. The aggregate supply curve determines inflation as the discounted future sequence of marginal costs and the inflation rate. The slope of the Phillips curve is measured by $\kappa = (1-\xi\beta)(1-\xi)/\xi$.

Consider a central bank pursuing optimal policy under rational expectations of $\pi_t = 0$ for all t. Assume beliefs are identical for all agents and given as

$$\begin{aligned} \begin{bmatrix} \pi_t \\ R_t \\ x_t \end{bmatrix} &= \bar{\omega}_t + e_t \\ \bar{\omega}_{t+1} &= \bar{\omega}_t + u_{t+1} \end{aligned}$$

with beliefs updated using the steady state Kalman filter

$$\begin{aligned} \omega_{t+1}^{\pi} &= \omega_t^{\pi} + \bar{g}\left(\pi_t - \omega_t^{\pi}\right) \\ \omega_{t+1}^{R} &= \omega_t^{R} + \bar{g}\left(R_t - \omega_t^{R}\right) \\ \omega_{t+1}^{x} &= \omega_t^{x} + \bar{g}\left(x_t - \omega_t^{x}\right). \end{aligned}$$

Evaluating expectations in the aggregate demand and supply equations gives

$$x_t = -\left(R_t - r_t^n\right) + \omega_{t-1}^{x} - \frac{1}{1-\beta}\left(\beta\omega_{t-1}^{R} - \omega_{t-1}^{\pi}\right) \tag{26}$$

$$\pi_t = \kappa x_t + \frac{\xi\beta\kappa}{1-\xi\beta}\omega_{t-1}^{x} + \frac{(1-\xi)\beta}{1-\xi\beta}\omega_{t-1}^{\pi}. \tag{27}$$

Before proceeding, note that this set of calculations is equivalent to working directly with (22) and (23). Evaluating beliefs in these expressions gives

$$\hat{c}_t(i) = (1-\beta)x_t + \beta\omega_{t-1}^{x}(i) - \beta(\hat{R}_t - r_t^n) - \beta\left(\frac{\beta\omega_{t-1}^{R}(i) + \omega_{t-1}^{\pi}(i)}{1-\beta}\right) \tag{28}$$

$$\hat{p}_t(j) = (1-\xi\beta)x_t + \xi\beta\omega_{t-1}^{x}(j) + \frac{\xi\beta}{1-\xi\beta}\omega_{t-1}^{\pi}(j) \tag{29}$$

where we have used the assumption that agents know $x_t = w_t$ for all t and explicitly indexed the beliefs of each household i and firm j. These are the optimal decision rules for consumption demand and price-setting for each agent given their beliefs. Goods market clearing and a log-linear approximation to the evolution of the price index gives

$$\int_0^1 \hat{c}_t(i)di = x_t \text{ and } \int_0^1 \hat{p}_t(j)dj = \frac{\xi}{1-\xi}\pi_t$$

and the assumption that beliefs are identical across all agents implies

$$\omega_{t-1}(i) = \omega_{t-1}(j) = \omega_{t-1}$$

for all i and j. Using these conditions in (28) and (29) then delivers the same expressions as (26) and (27).

Policy and stability analysis. Implementing the inflation target criterion of price stability we have

$$\begin{aligned}
\pi_t &= 0 \\
x_t &= -\frac{\xi\beta}{1-\xi\beta}\omega^x_{t-1} - \frac{(1-\xi)\,\beta\kappa^{-1}}{1-\xi\beta}\omega^\pi_{t-1} \\
(R_t - r^n_t) &= \frac{\xi\beta}{1-\xi\beta}\omega^x_{t-1} - \frac{(1-\xi)\,\beta\kappa^{-1}}{1-\xi\beta}\omega^\pi_{t-1} + \omega^x_{t-1} - \frac{1}{1-\beta}\left(\beta\omega^R_{t-1} - \omega^\pi_{t-1}\right).
\end{aligned}$$

Substituting into beliefs and dropping the natural rate term which is irrelevant to stability conditions gives

$$\begin{aligned}
\omega^\pi_t &= (1-\bar{g})\,\omega^\pi_{t-1} \\
\omega^x_t &= \omega^x_{t-1} + \bar{g}\left(-\frac{\xi\beta}{1-\xi\beta}\omega^x_{t-1} - \frac{(1-\xi)\,\beta\kappa^{-1}}{1-\xi\beta}\omega^\pi_{t-1} - \omega^x_{t-1}\right) \\
\omega^R_t &= \omega^R_{t-1} + \bar{g}\left(\frac{\xi\beta}{1-\xi\beta}\omega^x_{t-1} - \frac{(1-\xi)\,\beta\kappa^{-1}}{1-\xi\beta}\omega^\pi_{t-1} + \omega^x_{t-1} - \frac{1}{1-\beta}\left(\beta\omega^R_{t-1} - \omega^\pi_{t-1}\right) - \omega^R_{t-1}\right)
\end{aligned}$$

or in matrix notation

$$\begin{bmatrix} \omega^\pi_t \\ \omega^x_t \\ \omega^R_t \end{bmatrix} = \begin{bmatrix} (1-\bar{g}) & 0 & 0 \\ -\frac{\bar{g}(1-\xi)\beta\kappa^{-1}}{1-\xi\beta} & 1-\bar{g}-\frac{\bar{g}\xi\beta}{1-\xi\beta} & 0 \\ -\frac{\bar{g}(1-\xi)\beta\kappa^{-1}}{1-\xi\beta} + \frac{\bar{g}}{1-\beta} & \frac{\bar{g}\xi\beta}{1-\xi\beta} + \bar{g} & 1-\frac{\bar{g}}{1-\beta} \end{bmatrix} \begin{bmatrix} \omega^\pi_{t-1} \\ \omega^x_{t-1} \\ \omega^R_{t-1} \end{bmatrix}$$

with eigenvalues:

$$\frac{1}{\beta-1}\left(\bar{g}+\beta-1\right), 1-\bar{g}$$

and

$$\frac{1}{\xi\beta-1}\left(\bar{g}+\xi\beta-1\right).$$

This means we require

$$\bar{g} < 2\left(1-\beta\right)$$

and

$$\bar{g} < 2\left(1-\xi\beta\right)$$

for stability. The first condition is more stringent since $0 < \xi < 1$.

Now consider the case when the central bank controls the output gap directly as the instrument of policy. The model comprises the aggregate supply curve and target criterion — the aggregate demand curve is not long relevant. Evaluating beliefs gives

$$\begin{aligned} \pi_t &= 0 \\ x_t &= -\frac{\xi\beta}{1-\xi\beta}\omega_{t-1}^x - \frac{\left(1-\xi\right)\beta\kappa^{-1}}{1-\xi\beta}\omega_{t-1}^\pi \end{aligned}$$

Substituting into beliefs gives

$$\begin{aligned} \omega_t^\pi &= \left(1-\bar{g}\right)\omega_{t-1}^\pi \\ \omega_t^x &= \omega_{t-1}^x + \bar{g}\left(-\frac{\xi\beta}{1-\xi\beta}\omega_{t-1}^x - \frac{\left(1-\xi\right)\beta\kappa^{-1}}{1-\xi\beta}\omega_{t-1}^\pi - \omega_{t-1}^x\right) \end{aligned}$$

and

$$\begin{bmatrix} \omega_t^\pi \\ \omega_t^x \end{bmatrix} = \begin{bmatrix} \left(1-\bar{g}\right) & 0 \\ -\frac{\bar{g}(1-\xi)\beta\kappa^{-1}}{1-\xi\beta} & 1-\bar{g}-\frac{\bar{g}\xi\beta}{1-\xi\beta} \end{bmatrix} \begin{bmatrix} \omega_{t-1}^\pi \\ \omega_{t-1}^x \end{bmatrix}$$

which requires only that

$$\bar{g} < 2\left(1-\xi\beta\right)$$

In the neighborhood of flexible price equilibrium we have $\bar{g} < 2$ which is always satisfied. When prices are nearly fixed we have $\bar{g} < 2\left(1-\beta\right)$ as in the more general case. For empirically plausible parameter values this restriction will always be satisfied

Now consider the case in which the central bank uses a taylor rule. We have

$$\begin{aligned} x_t &= -\left(R_t - r_t^n\right) + \omega_{t-1}^x - \frac{1}{1-\beta}\left(\beta\omega_{t-1}^R - \omega_{t-1}^\pi\right) \\ \pi_t &= \kappa x_t + \frac{\xi\beta\kappa}{1-\xi\beta}\omega_{t-1}^x + \frac{\left(1-\xi\right)\beta}{1-\xi\beta}\omega_{t-1}^\pi \\ R_t &= \phi\pi_t + r_t^n \end{aligned}$$

Solving

$$x_t = -\phi\pi_t + \omega_{t-1}^x - \frac{1}{1-\beta}\left(\beta\omega_{t-1}^R - \omega_{t-1}^\pi\right)$$

and substituion into the aggregate supply curve gives

$$\pi_t = (1+\kappa\phi)^{-1}\left[\left(\kappa + \frac{\xi\beta\kappa}{1-\xi\beta}\right)\omega^x_{t-1} + \left(\frac{\kappa}{1-\beta} + \frac{(1-\xi)\,\beta}{1-\xi\beta}\right)\omega^\pi_{t-1} - \frac{\kappa\beta}{1-\beta}\omega^R_{t-1}\right]$$

$$\phi^{-1}\left(1+\frac{\phi^{-1}}{\kappa}\right)^{-1} = \frac{1}{\phi}\left(\frac{\phi\kappa+1}{\phi\kappa}\right)^{-1} = \frac{1}{\phi}\frac{\phi\kappa}{1+\phi\kappa} = \frac{\kappa}{1+\phi\kappa}$$

and

$$\begin{aligned}
x_t &= \kappa^{-1}\pi_t - \frac{\xi\beta}{1-\xi\beta}\omega^x_{t-1} - \frac{(1-\xi)\,\beta\kappa^{-1}}{1-\xi\beta}\omega^\pi_{t-1} \\
&= \frac{1}{\kappa\,(1+\kappa\phi)}\left[\left(\kappa + \frac{\xi\beta\kappa}{1-\xi\beta}\right)\omega^x_{t-1} + \left(\frac{\kappa}{1-\beta} + \frac{(1-\xi)\,\beta}{1-\xi\beta}\right)\omega^\pi_{t-1} - \frac{\kappa\beta}{1-\beta}\omega^R_{t-1}\right] \\
&\quad - \frac{\xi\beta}{1-\xi\beta}\omega^x_{t-1} - \frac{(1-\xi)\,\beta\kappa^{-1}}{1-\xi\beta}\omega^\pi_{t-1}
\end{aligned}$$

and

$$R_t = \phi\,(1+\kappa\phi)^{-1}\left[\left(\kappa + \frac{\xi\beta\kappa}{1-\xi\beta}\right)\omega^x_{t-1} + \left(\frac{\kappa}{1-\beta} + \frac{(1-\xi)\,\beta}{1-\xi\beta}\right)\omega^\pi_{t-1} - \frac{\kappa\beta}{1-\beta}\omega^R_{t-1}\right]$$

where we drop the exogenous natural rate term for simplicity.

Use the change of variables

$$z_t = R_t - \phi\pi_t$$

so that

$$\omega^z_t = \omega^R_t - \phi\omega^\pi_t$$

to give

$$\begin{aligned}
z_t &= 0 \\
\pi_t &= (1+\kappa\phi)^{-1}\left[\left(\kappa + \frac{\xi\beta\kappa}{1-\xi\beta}\right)\omega^x_{t-1} + \left(\frac{\kappa}{1-\beta} + \frac{(1-\xi)\,\beta}{1-\xi\beta}\right)\omega^\pi_{t-1} - \frac{\kappa\beta}{1-\beta}\left(\omega^z_{t-1} + \phi\omega^\pi_{t-1}\right)\right] \\
x_t &= \frac{1}{\kappa\,(1+\kappa\phi)}\left[\left(\kappa + \frac{\xi\beta\kappa}{1-\xi\beta}\right)\omega^x_{t-1} + \left(\frac{\kappa}{1-\beta} + \frac{(1-\xi)\,\beta}{1-\xi\beta}\right)\omega^\pi_{t-1} - \frac{\kappa\beta}{1-\beta}\left(\omega^z_{t-1} + \phi\omega^\pi_{t-1}\right)\right] \\
&\quad - \frac{\xi\beta}{1-\xi\beta}\omega^x_{t-1} - \frac{(1-\xi)\,\beta\kappa^{-1}}{1-\xi\beta}\omega^\pi_{t-1}
\end{aligned}$$

In matrix form we have

$$\begin{bmatrix} \omega_t^\pi \\ \omega_t^x \\ \omega_t^z \end{bmatrix} = \begin{bmatrix} B_1 & B_2 & -\bar{g}\left(1+\kappa\phi\right)^{-1}\frac{\kappa\beta}{1-\beta} \\ B_3 & B_4 & -\frac{\bar{g}}{\kappa(1+\kappa\phi)}\frac{\kappa\beta}{1-\beta} \\ 0 & 0 & 1-\bar{g} \end{bmatrix} \begin{bmatrix} \omega_{t-1}^\pi \\ \omega_{t-1}^x \\ \omega_{t-1}^z \end{bmatrix}$$

where

$$\begin{aligned} B_1 &= = 1+\bar{g}\left(\left(1+\kappa\phi\right)^{-1}\left(\frac{\kappa}{1-\beta}+\frac{\left(1-\xi\right)\beta}{1-\xi\beta}-\frac{\phi\kappa\beta}{1-\beta}\right)-1\right) \\ B_2 &= \bar{g}\left(1+\kappa\phi\right)^{-1}\left(\kappa+\frac{\xi\beta\kappa}{1-\xi\beta}\right) \\ B_3 &= \frac{\bar{g}}{\kappa\left(1+\kappa\phi\right)}\left(\frac{\kappa}{1-\beta}+\frac{\left(1-\xi\right)\beta}{1-\xi\beta}-\frac{\phi\kappa\beta}{1-\beta}\right)-\frac{\bar{g}\left(1-\xi\right)\beta\kappa^{-1}}{1-\xi\beta} \\ B_4 &= 1+\bar{g}\left(\frac{1}{\kappa\left(1+\kappa\phi\right)}\left(\kappa+\frac{\xi\beta\kappa}{1-\xi\beta}\right)-\frac{\xi\beta}{1-\xi\beta}-1\right) \end{aligned}$$

In the flex-price limit we have

$$\begin{bmatrix} \omega_t^\pi \\ \omega_t^x \end{bmatrix} = \begin{bmatrix} 1+\bar{g}\left(\phi^{-1}\left(\frac{1}{1-\beta}-\frac{\phi\beta}{1-\beta}\right)-1\right) & \bar{g}\phi^{-1} \\ 0 & 1+\bar{g}\left(\phi^{-1}-1\right) \end{bmatrix} \begin{bmatrix} \omega_{t-1}^\pi \\ \omega_{t-1}^x \end{bmatrix}$$

Hence we have the two eigenvalues

$$\begin{gathered} 1+\bar{g}\left(\phi^{-1}\left(\frac{1}{1-\beta}-\frac{\phi\beta}{1-\beta}\right)-1\right) \\ 1+\bar{g}\left(\phi^{-1}-1\right) \end{gathered}$$

So if $\phi > 1$ then the second is less than unity. The first is the familiar one. Stability requires

$$\bar{g} < \frac{2\left(1-\beta\right)}{\phi^{-1}-1}$$

as before.

No transmission mechanism. Earlier papers such as Orphanides and Williams (2005b), Ferrero (2007) and Molnar and Santoro (2013), implicitly assumed models in which monetary policy directly controls demand. For example, consider a model with aggregate demand and supply conditions

$$\begin{aligned} x_t &= \hat{E}_t x_{t+1} - \left(R_t - \hat{E}_t \pi_{t+1} - r_t^n\right) \\ \pi_t &= \kappa x_t + \beta \hat{E}_t \pi_{t+1}. \end{aligned}$$

The term structure of interest rates are absent and given inflation and output beliefs, interest rates map one for one into variations in output. Assuming the same belief structure we have

$$\begin{aligned} x_t &= \omega^x_{t-1} - \left(R_t - \omega^\pi_{t-1} - r^n_t\right) \\ \pi_t &= \kappa x_t + \beta \omega^\pi_{t-1}. \end{aligned}$$

Using the target criterion and solving for period t gives

$$\begin{aligned} \pi_t &= 0 \\ x_t &= -\frac{\beta}{\kappa}\omega^\pi_{t-1} \\ R_t &= \omega^x_{t-1} + \left(\frac{\beta}{\kappa}+1\right)\omega^\pi_{t-1} + r^n_t. \end{aligned}$$

An immediate different is the implicit instrument rule does not feature any dependence on interest rate beliefs.

Substituting into beliefs gives

$$\begin{aligned} \omega^\pi_t &= (1-\bar{g})\,\omega^\pi_{t-1} \\ \omega^x_t &= \omega^x_{t-1} + \bar{g}\left(-\frac{\beta}{\kappa}\omega^\pi_{t-1} - \omega^x_{t-1}\right) \\ \omega^R_t &= \omega^R_{t-1} + \bar{g}\left(\omega^x_{t-1} + \left(\frac{\beta}{\kappa}+1\right)\omega^\pi_{t-1} + r^n_t - \omega^R_{t-1}\right) \end{aligned}$$

or in matrix notation

$$\begin{bmatrix} \omega^\pi_t \\ \omega^x_t \\ \omega^R_t \end{bmatrix} = \begin{bmatrix} 1-\bar{g} & 0 & 0 \\ -\bar{g}\frac{\beta}{\kappa} & (1-\bar{g}) & 0 \\ \bar{g}\left(\frac{\beta}{\kappa}+1\right) & \bar{g} & 1-\bar{g} \end{bmatrix} \begin{bmatrix} \omega^\pi_{t-1} \\ \omega^x_{t-1} \\ \omega^R_{t-1} \end{bmatrix} + \bar{g}\begin{bmatrix} 0 \\ 0 \\ r^n_t \end{bmatrix}.$$

This system is stable provided

$$|1-\bar{g}| < 1.$$

This condition is always satisfied under maintained assumptions. The absence of the term structure of interest rates is critical to stability.

B Summary Equations

The model is comprised by the following list of endogenous variables: $w^{opt}_t(i)$, w^{opt}_t, w_t, $\pi_t, \pi^w, N_t,\ \lambda_t,\ c_t(i),\ c_t,\ R_t,\ y_t$, $b_t(i), b_t,\ c^{nr}_t,\ \lambda^{nr}_t,\ r^{nr}_t$— 16 variables; and by the following exogenous disturbances: $a_t, g_t, \varphi_t, \theta_{p,t}, \nu_t, \theta_{w,t}, m_t$— 7 variables. The first five are AR(1) processes, the last two i.i.d. processes. The list of equations below involves variables expressed in log-deviation from their deterministic steady-state. Exceptions are the real value of nominal bonds, $\hat{b}_t = (b_t - b)/y$, and government spending where $\hat{g}_t = (g_t - g)/y$. Last, recall consumption, output, government spending, debt and the real wage are expressed in efficiency units.

1. Optimal reset wage: $w_t^{opt}(i)$

$$\begin{aligned}\hat{w}^{opt}(j) &= \xi_w \beta \hat{E}_t^j \left(\pi_{t+1}^w - \iota_w \pi_t\right) \\ &+ \frac{1-\xi_w\beta}{1+\theta_w\phi_n^{-1}} \left[\hat{\varphi}_t + \phi_n^{-1}\left(\frac{c}{y}\hat{c}_t + \hat{g}_t - \hat{a}_t\right) - \hat{w}_t^r - \hat{\lambda}_t(j) - \frac{\hat{\theta}_{w,t}}{(\theta_w - 1)}\right] + \xi_w\beta\hat{E}_t^j \hat{w}_{t+1}^{opt}(j)\end{aligned}$$

2. Wage inflation: π_t^w

$$\hat{w}_t^{opt} = \frac{\xi_w}{1-\xi_w}\left(\pi_t^w - \iota_w \pi_{t-1}\right)$$

3. Real Wage: w_t

$$\hat{w}_t^r = \pi_t^w - \pi_t + \hat{w}_{t-1}^r$$

4. Consumption: $c_t(i)$

$$(1-\beta b)\hat{\lambda}_t(i) = \frac{b\sigma}{1-b}\hat{c}_{t-1}(i) - \frac{\sigma}{1-b}\left(1+\beta b^2\right)\hat{c}_t(i) + \beta b\frac{\sigma}{1-b}\hat{E}_t\hat{c}_{t+1}(i).$$

5. Marginal utility: $\lambda_t(i)$

$$\hat{\lambda}_t(i) - R_t = E_t^i\left(\hat{\lambda}_{t+1}(i) - \pi_{t+1}\right)$$

6. Bond holdings: $b_t(i)$ (will assume in zero supply)

$$\hat{b}_t(i) = \beta^{-1}\left(\hat{b}_{t-1}(i)\right) - \frac{c}{y}\hat{c}_t(i) + \frac{c}{y}\hat{c}_t$$

7. Aggregate Production: $\hat{N}_t$

$$\hat{y}_t = \hat{a}_t + \hat{N}_t$$

8. Inflation: π_t

$$\pi_t - \iota_p \pi_{t-1} = \kappa_p E_t \sum_{T=t}^{\infty} (\xi_p \beta)^{T-t} \left[\frac{\xi_p \beta}{1-\xi_p \beta} (\pi_{T+1} - \iota_p \pi_T) + \hat{w}_T - \hat{a}_T - \frac{\hat{\theta}_{p,T}}{(\theta_p - 1)} \right]$$

where $\kappa_p = \frac{(1-\xi_p)(1-\xi_p\beta)}{\xi_p}$.

9. Market Clearing Goods: $\hat{y}_t$

$$\hat{y}_t = \frac{c}{y} \hat{c}_t + \hat{g}_t$$

10. Monetary Policy: R_t

$$R_t = \rho_R R_{t-1} + (1-\rho_R) \left[\phi_\pi \pi_t + \phi_x \frac{c}{y} (\hat{c}_t - \hat{c}_t^{nr} + \nu_t) \right] + \phi_{\Delta x} \left(\hat{c}_t - \hat{c}_t^{nr} + \nu_t - \hat{c}_{t-1} + \hat{c}_{t-1}^{nr} - \nu_{t-1} \right) + \hat{m}_t$$

11. Market clearing bonds: b_t

$$\hat{b}_t = \int \hat{b}_t (i) \, di = 0$$

12. Definition of aggregate optimal wage: w_t^{opt}

$$\hat{w}_t^{opt} = \int \hat{w}_t^{opt} (i) \, di$$

13. Definition of aggregate consumption:

$$\hat{c}_t = \int \hat{c}_t (i) \, di$$

14. Natural rate of consumption: c_t^{nr}

$$(1-\beta b) \hat{\lambda}_t = \frac{b\sigma}{1-b} \hat{c}_{t-1}^{nr} - \frac{\sigma}{1-b} \left(1 + \beta b^2 \right) \hat{c}_t^{nr} + \beta b \frac{\sigma}{1-b} \hat{E}_t \hat{c}_{t+1}^{nr}$$

15. Natural rate of marginal utility: λ_t^{nr}

$$\phi_n^{-1} \left(\frac{c}{y} \hat{c}_t^{nr} + \hat{g}_t \right) = \hat{\lambda}_t^{nr} - \hat{\varphi}_t + \left(1 + \phi_n^{-1} \right) \hat{a}_t$$

16. Natural rate of real interest: r_t^{nr}

$$\hat{\lambda}_t^{nr} = \hat{r}_t^{nr} + \hat{E}_t \hat{\lambda}_{t+1}^{nr}.$$

The assumptions that agents know the dividend and tax policies permits simplification of the flow budget constraint as follows. The terms

$$\frac{\Gamma_{p,t} + T_{p,T}}{Z_t} + (1 - \tau_w)\, w_t^r N_t + T_{w,t} = \int_0^1 \frac{Y_t(f)}{Z_t} df$$

satisfies

$$\hat{\gamma}_{p,t} + \hat{\tau}_{p,t} + (1 - \tau_w) \frac{w^r N}{y} \left(\hat{w}_t^r + \hat{N}_t\right) + \hat{\tau}_{w,t} = \hat{y}_t$$

$\hat{\gamma}_{p,t}$ and $\hat{\tau}_{p,t}$ are the dividends and taxes from firms and $\hat{\tau}_{w,t}$ the taxes from labor income in deviations from steady state (as a fraction of steady-state income). Using this in the flow budget constraint gives

$$\begin{aligned}
\hat{b}_t(i) &= \beta^{-1} \hat{b}_{t-1}(i) - \frac{c}{y} \hat{c}_t(i) - \hat{\tau}_t + \hat{y}_t \\
&= \beta^{-1} \hat{b}_{t-1}(i) - \frac{c}{y} \hat{c}_t(i) + \frac{c}{y} \hat{c}_t
\end{aligned}$$

under the assumption of a balanced budget policy $\hat{\tau}_t = \hat{g}_t$

When solving under rational expectations we replace equation 8 with the familiar and equivalent difference equation:

$$\begin{aligned}
\pi_t - \iota_p \pi_{t-1} &= \kappa_p E_t \sum_{T=t}^{\infty} (\xi_p \beta)^{T-t} \left[\frac{\xi_p \beta}{1 - \xi_p \beta} (\pi_{T+1} - \iota_p \pi_T) + \hat{w}_T - \hat{a}_T - \frac{\hat{\theta}_{p,T}}{(\theta_p - 1)} \right] \\
&= \kappa_p E_t \left[\frac{\xi_p \beta}{1 - \xi_p \beta} (\pi_{T+1} - \iota_p \pi_T) + \hat{w}_T - \hat{a}_T - \frac{\hat{\theta}_{p,T}}{(\theta_p - 1)} \right] \\
&\quad + \xi_p \beta E_t (\pi_{t+1} - \iota_p \pi_t) \\
&= \kappa_p \left[\hat{w}_t - \hat{a}_t - \frac{\hat{\theta}_{p,t}}{(\theta_p - 1)} \right] + \beta E_t (\pi_{t+1} - \iota_p \pi_t).
\end{aligned}$$

The system then can be solved using standard solution methods.

C Solving the Model under Learning

This section provides an overview of the general approach to solving DSGE models under non-rational expectations. We show how to write the system in canonical form

$$A_0 z_t = \sum_{s=1}^{4} A_s \left(\hat{E}_t \sum_{T=t}^{\infty} \lambda_s^{T-t} z_{T+1} \right) + A_5 z_{t-1} + A_6 \varepsilon_t \tag{30}$$

where the vector z_t collects all model variables defined above; the vector ε_t collects exogenous innovations; the discounts λ_s for $s \in 1, ..., 4$ index the models unstable eigenvalues; A_i for $i \in 1, ..., 6$ coefficient matrices; and

$$\hat{E}_t = \int_0^1 \hat{E}_t^i di$$

average beliefs. This representation holds for arbitrary beliefs, including rational expectations. Dynamics depend on a set of projections into the indefinite future, reflecting the intertemporal decision problems solved by households and firms. The projected variables are those macroeconomic objects taken as given and beyond the control of each decision maker. We then close the model with a theory of belief formation to evaluate expectations and derive the state-space representation to exploit in estimation.

The solution procedure starts by solving for the optimal intertemporal decisions of each type of agent in the model. There are two blocks of forward-looking agents: households and firms. Taking expectations as given, we solve for optimal decisions as a function of expectations about variables that are exogenous to each agent's decision problem. Aggregation of these decisions together with the remaining equilibrium conditions which don't involve expectations — such as market clearing conditions, definitions and policy assumptions — yields the system (30).

C.1 Decision Rule 1: Households

We want to write the optimal decision of household i in the form

$$\Psi_0^{HH} \hat{E}_t^i \begin{bmatrix} \hat{w}_{t+1}^{opt}(i) \\ \hat{c}_{t+1}(i) \\ \hat{\lambda}_{t+1}(i) \\ \hat{c}_t(i) \\ \hat{b}_t(i) \end{bmatrix} = \Psi_1^{HH} \begin{bmatrix} \hat{w}_t^{opt}(i) \\ \hat{c}_t(i) \\ \hat{\lambda}_t(i) \\ \hat{c}_{t-1}(i) \\ \hat{b}_{t-1}(i) \end{bmatrix} + \delta_t$$

where the vector

$$\delta_t = \begin{bmatrix} \delta_t^{w^{opt}} \\ \delta_t^c \\ \delta_t^\lambda \\ 0 \\ \delta_t^b \end{bmatrix}$$

collects aggregate variables which are exogenous to the individual household's decision problem and taken as given. This system can be solved using standard difference equation methods. Let's rearrange the first-order conditions one by one.

The first-order conditions can be re-stated as

1. Optimal reset wage: $\hat{w}_t^{opt}(i)$

$$\hat{w}^{opt}(i) = \xi_w \beta \hat{E}_t^i \left(\pi_{t+1}^w - \iota_w \pi_t\right) + \frac{1-\xi_w\beta}{1+\theta_w\phi_n^{-1}} \left[\hat{\varphi}_t + \phi_n^{-1}\hat{N}_t - \hat{w}_t^r - \hat{\lambda}_t(i) - \frac{\hat{\theta}_{w,t}}{(\theta_w - 1)}\right] + \xi_w\beta\hat{E}_t^i \hat{w}_{t+1}^{opt}(i)$$

or

$$\begin{aligned}
\xi_w\beta\hat{E}_t^i\hat{w}_{t+1}^{opt}(i) &= \hat{w}^{opt}(i) - \xi_w\beta\hat{E}_t^i\left(\pi_{t+1}^w - \iota_w\pi_t\right) \\
&\quad - \frac{1-\xi_w\beta}{1+\theta_w\phi_n^{-1}}\left[\hat{\varphi}_t + \phi_n^{-1}\hat{N}_t - \hat{w}_t^r - \hat{\lambda}_t(i) - \frac{\hat{\theta}_{w,t}}{(\theta_w-1)}\right] \\
&= \hat{w}^{opt}(j) + \frac{1-\xi_w\beta}{1+\theta_w\phi_n^{-1}}\hat{\lambda}_t(i) + \delta_t^{w^{opt}} \\
\delta_t^{w^{opt}} &= -\xi_w\beta\hat{E}_t^i\left(\pi_{t+1}^w - \iota_w\pi_t\right) - \frac{1-\xi_w\beta}{1+\theta_w\phi_n^{-1}}\left[\hat{\varphi}_t + \phi_n^{-1}\hat{N}_t - \hat{w}_t^r - \frac{\hat{\theta}_{w,t}}{(\theta_w-1)}\right]
\end{aligned}$$

2. Marginal Utility: $\hat{c}_t(i)$

$$(1-\beta b)\hat{\lambda}_t(i) = \frac{b\sigma}{1-b}\hat{c}_{t-1}(i) - \frac{\sigma}{1-b}\left(1+\beta b^2\right)\hat{c}_t(i) + \beta b\frac{\sigma}{1-b}\hat{E}_t^i\hat{c}_{t+1}(i)$$

or

$$\begin{aligned}
\beta b\frac{\sigma}{1-b}\hat{E}_t^i\hat{c}_{t+1}(i) &= (1-\beta b)\hat{\lambda}_t(i) + \frac{\sigma}{1-b}\left(1+\beta b^2\right)\hat{c}_t(i) - \frac{b\sigma}{1-b}\hat{c}_{t-1}(i) \\
&= (1-\beta b)\hat{\lambda}_t(i) + \frac{\sigma}{1-b}\left(1+\beta b^2\right)\hat{c}_t(i) - \frac{b\sigma}{1-b}\hat{c}_{t-1}(i) + \delta_t^c \\
\delta_t^c &= 0
\end{aligned}$$

3. Bond holdings: $\hat{\lambda}_t(i)$

$$\hat{\lambda}_t(i) - R_t = \hat{E}_t^i\left(\hat{\lambda}_{t+1}(i) - \pi_{t+1}\right)$$

or

$$\begin{aligned}\hat{E}_t^i\hat{\lambda}_{t+1}(i) &= \hat{\lambda}_t(i) - R_t + \hat{E}_t^i\pi_{t+1} = \hat{\lambda}_t(i) + \delta_t^{\lambda} \\ \delta_t^{\lambda} &= R_t + \hat{E}_t^i\pi_{t+1}\end{aligned}$$

4. Consumption identity: $\hat{c}_t(i) = \hat{c}_t(i)$

5. Flow budget constraint: $\hat{b}_t(i)$

$$\begin{aligned}\beta\hat{b}_t(i) &= \hat{b}_{t-1}(i) - \frac{c}{y}\hat{c}_t(i) + \frac{c}{y}\hat{c}_t \\ &= \hat{b}_{t-1}(i) - \frac{c}{y}\hat{c}_t(i) + \delta_t^b \\ \delta_t^c &= \frac{c}{y}\hat{c}_t.\end{aligned}$$

Hence we have

$$\Psi_0^{HH} = \begin{bmatrix} \xi_w\beta & 0 & 0 & 0 & 0 \\ 0 & \beta b\frac{\sigma}{1-b} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \beta \end{bmatrix} \quad \text{and} \quad \Psi_1^{HH} = \begin{bmatrix} 1 & 0 & \frac{1-\xi_w\beta}{1+\theta_w\phi_n^{-1}} & 0 & 0 \\ 0 & \frac{\sigma}{1-b}(1+\beta b^2) & (1-\beta b) & -\frac{b\sigma}{1-b} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{c}{y} & 0 & 0 & 1 \end{bmatrix}$$

where terms, exogenous to the household, are collected in the vector

$$\delta_t = \begin{bmatrix} \delta_t^{w^{opt}} \\ \delta_t^c \\ \delta_t^{\lambda} \\ 0 \\ \delta_t^b \end{bmatrix} = \begin{bmatrix} -\xi_w\beta\hat{E}_t^i\left(\pi_{t+1}^w - \iota_w\pi_t\right) - \frac{1-\xi_w\beta}{1+\theta_w\phi_n^{-1}}\left[\hat{\varphi}_t + \phi_n^{-1}\left(\frac{c}{y}\hat{c}_t + \hat{g}_t - \hat{a}_t\right) - \hat{w}_t^r - \frac{\hat{\theta}_{w,t}}{(\theta_w-1)}\right] \\ 0 \\ -R_t + \hat{E}_t^i\pi_{t+1} \\ 0 \\ \frac{c}{y}\hat{c}_t \end{bmatrix}$$

The solution to this system is

$$\begin{bmatrix} \hat{w}_t^{opt}(i) \\ \hat{c}_t(i) \\ \hat{\lambda}_t(i) \end{bmatrix} = \bar{D}_k \begin{bmatrix} \hat{c}_{t-1}(i) \\ \hat{b}_{t-1}(i) \end{bmatrix} + \bar{D}_{\delta 1} \sum_{T=t}^{\infty} \Lambda^{-(T-t)} \bar{D}_{\delta 2} \hat{E}_t^i \delta_T$$

where the fourth column of $\bar{D}_{\delta 2}$ can be eliminated as it is multiplied by zero (the fourth element of the vector δ_t). We can rewrite in the form

$$\begin{aligned} \begin{bmatrix} \hat{w}_t^{opt}(i) \\ \hat{c}_t(i) \\ \hat{\lambda}_t(i) \end{bmatrix} &= \bar{D}_k \begin{bmatrix} \hat{c}_{t-1}(i) \\ \hat{b}_{t-1}(i) \end{bmatrix} + D_{\lambda_1} \sum_{T=t}^{\infty} \lambda_1^{-(T-t)} E_t \bar{\delta}_T \\ &\quad + D_{\lambda 2} \sum_{T=t}^{\infty} \lambda_2^{-(T-t)} E_t \bar{\delta}_T + D_{\lambda_3} \sum_{T=t}^{\infty} \lambda_3^{-(T-t)} E_t \bar{\delta}_T \end{aligned} \tag{31}$$

where

$$D_{\lambda_1} = \bar{D}_{\delta 1} \begin{bmatrix} \bar{D}_{\delta 2}^1 \\ 0 \\ 0 \end{bmatrix}; D_{\lambda_2} = \bar{D}_{\delta 1} \begin{bmatrix} 0 \\ \bar{D}_{\delta 2}^2 \\ 0 \end{bmatrix}; D_{\lambda_3} = \bar{D}_{\delta 1} \begin{bmatrix} 0 \\ 0 \\ \bar{D}_{\delta 2}^3 \end{bmatrix}$$

where $\bar{D}_{\delta 2}^i$ denotes the ith row of the matrix $\bar{D}_{\delta 2}$.

Write the solution in the following form (imposing the equilibrium condition that debt is in zero supply, with all households having an equal share)

$$\begin{aligned} \hat{w}_t^{opt}(i) - \sum_{s=1}^{3} \sum_{j}^{\{c,\lambda,w^{opt},b\}} P_t^{0,1}(\lambda_s, \delta_j) &= \bar{D}_k(1,1)\, \hat{c}_{t-1}(i) + \sum_{s=1}^{3} \sum_{j}^{\{c,\lambda,w^{opt},b\}} P_t^{e,1}(\lambda_s, \delta_j) \\ \hat{c}_t(i) - \sum_{s=1}^{3} \sum_{j}^{\{c,\lambda,w^{opt},b\}} P_t^{0,2}(\lambda_s, \delta_j) &= \bar{D}_k(2,1)\, \hat{c}_{t-1}(i) + \sum_{s=1}^{3} \sum_{j}^{\{c,\lambda,w^{opt},b\}} P_t^{e,2}(\lambda_s, \delta_j) \\ \hat{\lambda}_t(i) - \sum_{s=1}^{3} \sum_{j}^{\{c,\lambda,w^{opt},b\}} P_t^{0,3}(\lambda_s, \delta_j) &= \bar{D}_k(3,1)\, \hat{c}_{t-1}(i) + \sum_{s=1}^{3} \sum_{j}^{\{c,\lambda,w^{opt},b\}} P_t^{e,3}(\lambda_s, \delta_j) \end{aligned}$$

where $j = \{c, \lambda, w^{opt}, b\}$ index the collections of exogneous terms in δ_t appearing in the original system. Un-packing the terms we get the following:

1. *Contemporaneous terms*

$$
\begin{aligned}
P_t^{0,l}(\lambda_s,\delta_w) &= D_{\lambda_s}(l,1)\left(\xi_w\beta\hat{E}_t^i(\iota_w\pi_t)-\frac{1-\xi_w\beta}{1+\theta_w\phi_n^{-1}}\left[\hat{\varphi}_t+\phi_n^{-1}\left(\frac{c}{y}\hat{c}_t+\hat{g}_t-\hat{a}_t\right)-\hat{w}_t^r-\frac{\hat{\theta}_{w,t}}{(\theta_w-1)}\right]\right)\\
P_t^{0,l}(\lambda_s,\delta_c) &= 0\\
P_t^{0,l}(\lambda_s,\delta_\lambda) &= -D_{\lambda_s}(l,3)\,R_t\\
P_t^{0,l}(\lambda_s,\delta_b) &= D_{\lambda_s}(l,4)\,\frac{c}{y}\hat{c}_t.
\end{aligned}
$$

2. *Expected (discounted) terms*

$$
\begin{aligned}
P_t^{e,l}(\lambda_s,\delta_w) &= \lambda_s^{-1}D_{\lambda_s}(l,1)\sum_{T=t}^{\infty}\lambda_s^{-(T-t)}\left(\begin{array}{c}-\xi_w\beta\left(\lambda_s\pi_{T+1}^w-\iota_w\pi_{T+1}\right)-\\ \frac{1-\xi_w\beta}{1+\theta_w\phi_n^{-1}}\left[\begin{array}{c}\hat{\varphi}_{T+1}+\phi_n^{-1}\left(\frac{c}{y}\hat{c}_{T+1}+\hat{g}_{T+1}-\hat{a}_{T+1}\right)-\\ \hat{w}_{T+1}^r-\frac{\hat{\theta}_{w,T+1}}{(\theta_w-1)}\end{array}\right]\end{array}\right)\\
P_t^{0,l}(\lambda_s,\delta_c) &= 0\\
P_t^{0,l}(\lambda_s,\delta_\lambda) &= \lambda_s^{-1}D_{\lambda_s}(l,3)\sum_{T=t}^{\infty}\lambda_s^{-(T-t)}\left(-R_{T+1}+\lambda_s\pi_{t+1}\right)\\
P_t^{e,l}(\lambda_s,\delta_b) &= \lambda_s^{-1}D_{\lambda_s}(l,4)\sum_{T=t}^{\infty}\lambda_s^{-(T-t)}\frac{c}{y}\hat{c}_{T+1}
\end{aligned}
$$

C.2 Decision Rule 2: Firms

Recall firms' pricing gives

$$
\pi_t-\iota_p\pi_{t-1}=\kappa_pE_t\sum_{T=t}^{\infty}(\xi_p\beta)^{T-t}\left[\frac{\xi_p\beta}{1-\xi_p\beta}(\pi_{T+1}-\iota_p\pi_T)+\hat{w}_T-\hat{a}_T-\frac{\hat{\theta}_{p,T}}{(\theta_p-1)}\right]
$$

which can be rearranged as

$$
\begin{aligned}
(1+(1-\xi_p)\,\beta\iota_p)\pi_t-\iota_p\pi_{t-1} &= \kappa_p\left(\hat{w}_t-a_t-\frac{\hat{\theta}_{p,t}}{(\theta_p-1)}\right)+\\
&\quad \kappa_pE_t\sum_{T=t}^{\infty}(\xi_p\beta)^{T-t}\left[\begin{array}{c}\frac{\xi_p\beta(1-\iota_p\xi_p\beta)}{1-\xi_p\beta}\pi_{T+1}\\ +\xi_p\beta\left(\hat{w}_{T+1}-\hat{a}_{T+1}-\frac{\hat{\theta}_{p,T+1}}{(\theta_p-1)}\right)\end{array}\right].
\end{aligned}
$$

C.3 Beliefs

Agents have the forecasting model

$$z_t = S\bar{\omega}_t + \Phi z_{t-1} + e_t \tag{32}$$

$$\bar{\omega}_t = \rho\bar{\omega}_{t-1} + u_t \tag{33}$$

where $0 \leq \rho \leq 1$ a parameter; e_t and u_t IID with $J = E\left[e_t e_t'\right]$ and $Q = E\left[u_t u_t'\right]$. Beliefs are updated using the the Kalman filter. Consider the recursion

$$\begin{aligned} \omega_{t+1} &= \rho\omega_t + \rho P_t \left(P_t + J\right)^{-1} \mathcal{F}_t \\ P_{t+1} &= \rho^2 P_t - \rho^2 P_t \left(P_t + J\right)^{-1} P_t + Q \end{aligned}$$

where the matrix P_t is the mean square error associated with the estimate ω_{t+1}. The vector F_t denotes the current prediction error

$$\mathcal{F}_t = S'\left(z_t - S\omega_{t-1} - \Phi^* z_{t-1}\right)$$

where S is the selection matrix discussed in the main text.

Following Sargent and Williams (2005), we make the following simplifying assumptions. Re-scale the posterior estimate as $P_t = \Xi_t J$ and use the approximation $\left(I + \Xi_t\right)^{-1} \simeq I$ for small Ξ_t to give

$$\begin{aligned} \omega_{t+1} &= \rho\omega_t + \rho\Xi_t\mathcal{F}_t \\ \Xi_{t+1} &= \rho^2\Xi - \rho^2\Xi_t\Xi_t + QJ^{-1}. \end{aligned}$$

We restrict the analysis to the steady state of this filter assuming prior beliefs satisfy the restriction $Q = \hat{c}^2 J$ for scalar $\hat{c}$. We want to solve for Ξ in

$$\Xi\Xi + (1-\rho^2)\Xi - \hat{c}^2 I = 0.$$

The solution has the form $\Xi = bI$ where α solves

$$\alpha^2 I + (1-\rho^2)\alpha - \hat{c}^2 I = 0.$$

to give

$$\alpha = \frac{-(1-\rho^2) + \sqrt{(1-\rho^2)^2 + 4\hat{c}^2}}{2}.$$

Under these assumptions the updating equation becomes

$$\omega_{t+1} = \rho\omega_t + \rho\alpha\mathcal{F}_t$$

where $0 < \alpha < 1$ is a function of the parameters ρ and $\hat{c}$. The leaning gain is then $\bar{g} \equiv \rho\alpha$. In the special (and more common) case $\rho = 1$ and $\bar{g} \equiv \hat{c}$.

C.4 Solution and state space representation

Given an estimate of the unobserved state, ω_t, we can evaluate expectations required for optimal decisions as

$$E_t \sum_{T=t}^{\infty} \lambda_s^{-(T-t)} z_{T+1} = F_0\left(\lambda_s, \Phi\right) S\omega_t + F_1\left(\lambda_s, \Phi\right) z_t$$

where $F_0\left(\lambda_s, \Phi\right)$ and $F_1\left(\lambda_s, \Phi\right)$ are composites of structural parameters and eigenvalues λ_s. The structural equations (11) then provide

$$\begin{aligned} z_t &= \left(A_0 - \sum_{j=1}^{4} A_s F_1\left(\lambda_s, \Phi\right)\right)^{-1} \left[\sum_{j=1}^{4} A_s F_0\left(\lambda_s, \Phi\right) S\omega_t + A_5 z_{t-1} + A_6\varepsilon_t\right] \\ &= T\left(\Phi^*\right) S\omega_t + \Phi^* z_{t-1} + \Phi_\varepsilon^*\varepsilon_t \end{aligned}$$

where

$$\begin{aligned} \Phi^* &\equiv \left(A_0 - \sum_{j=1}^{4} A_s F_1\left(\lambda_s, \Phi^*\right)\right)^{-1} A_5 \\ \Phi_\varepsilon^* &\equiv \left(A_0 - \sum_{j=1}^{4} A_s F_1\left(\lambda_s, \Phi^*\right)\right)^{-1} A_6 \end{aligned}$$

represent a fixed point of the beliefs which corresponds to the rational expectations solution. So agents' forecasting models uses $\Phi = \Phi^*$.[38]

Finally, combining aggregate dynamics with beliefs, provides the linear state-space representation of the model

$$Z_t = F\left(\Theta\right) Z_{t-1} + Q\left(\Theta\right)\varepsilon_t$$

[38]Formally an example of the method of undetermined coefficients.

where Θ defines the set of model parameters with

$$F(\Theta) = \begin{bmatrix} \Phi^* & T(\Phi^*) S\rho & T(\Phi^*) S\bar{g} \\ 0 & \rho I & \bar{g} I \\ 0 & S'[T(\Phi^*) - I] S\rho & S'[T(\Phi^*) - I] S\bar{g} \end{bmatrix}$$

and

$$Z_t = \begin{bmatrix} z_t \\ \omega_t \\ \mathcal{F}_t \end{bmatrix} \text{ and } Q(\Theta) = \begin{bmatrix} \Phi^*_\varepsilon \\ 0 \\ S'\Phi^*_\varepsilon \end{bmatrix}.$$

D Estimation

We estimate the model with Bayesian methods using two subsamples. The estimation setup is as follows. We observe two samples $Y_1 \equiv Y_{1...T}$ and $Y_2 \equiv Y_{T+K...T+N}$ where $K > 0$. The first sample covers the years 1964Q1:1979Q2 (sample 1) and the second sample spans 1984Q1-2007Q3 (sample 2). The middle part of the sample is left out because it accounts for transitional dynamics when there are shifts in the policy rule. We are interested in estimating the parameters and comparing the performance of alternative models over both samples. Each model is characterized by three sets of parameters: $\Theta_{1,}, \Theta_2, \Theta_{3.}$ The set Θ_3 includes parameters that are invariant across the two samples, while the other two sets include parameters that are allowed to be different in sample 1 and sample 2. Different models are defined by the sets of parameters that remain constant across samples, as defined in the main text.

Consider first the joint probability of the two samples conditional on the parameters:

$$\begin{aligned} P(Y_1, Y_2 | \Theta_{1,}, \Theta_{2,} \Theta_3) &= P(Y_2 | Y_1, \Theta_{1,}, \Theta_{2,} \Theta_3) P(Y_1 | \Theta_{1,}, \Theta_{2,} \Theta_3) \\ &= L_2(Y_2 | \Theta_{2,} \Theta_3) L_1(Y_1 | \Theta_{1,}, \Theta_3) \end{aligned}$$

where the last line shows the product of the Likelihood functions for each sample. The independence assumption is based the fact that these are two possibly different regimes separated by about 5 years of data. The DSGE's unobserved states are integrated out using the Kalman filter, where initial states have distribution corresponding to the invariant stationary distribution corresponding the the sample-specific parameters. The posterior distribution is

$$\begin{aligned} P\left(\Theta_{1,},\Theta_{2,}\Theta_3|Y_1,Y_2\right) &= P\left(Y_1,Y_2|\Theta_{1,},\Theta_{2,}\Theta_3\right)P\left(\Theta_1\right)P\left(\Theta_2\right)P\left(\Theta_3\right) \\ &= L_2\left(Y_2|\Theta_{2,}\Theta_3\right)L_1\left(Y_1|\Theta_{1,},\Theta_3\right)P\left(\Theta_1\right)P\left(\Theta_2\right)P\left(\Theta_3\right) \end{aligned}$$

where $P\left(\Theta_i\right), i = 1,2,3$ denotes prior densities. The Posterior is then computed using standard MCMC methods. Finally, we can evaluate the marginal Likelihood (computed using the posterior sample draws) as

$$P\left(Y_1,Y_2|M_i\right) = \int L_2\left(Y_2|\Theta_{2,}\Theta_3;M_i\right)L_1\left(Y_1|\Theta_{1,},\Theta_3;M_i\right)P\left(\Theta_1\right)P\left(\Theta_2\right)P\left(\Theta_3\right)d\Theta_1 d\Theta_2 d\Theta_3$$

where, as mentioned above, different models correspond to different sets of parameters that are allowed to vary. As mentioned in the main text, one model allows both the monetary policy rules's parameters to vary and the shocks' volatility. A different model allows only the shocks to vary. We can evaluate model performance (or the cost of allowing a parameter shift) by comparing marginal likelihoods.

E Monetary policy shocks

The estimated monetary policy shocks are consistent with extant narratives about monetary policy. Here we single out five episodes that, through the lens of our model, have shaped the behavior of long-term rates and the economy. In the period spanning the mid-1960s to the early 1970s, monetary shocks are generally negative. Much work has sourced these shocks to policymakers' incorrect assumptions about the Phillips curve: in particular the belief in an exploitable long-run policy trade-off between economic activity and inflation, coupled with the belief that inflation was not responsive to economic slack.[39] An alternative view in Rotemberg (2013) and Modigliani (1977) explains the expansionary surprises during this period with political considerations. Scarred by the economic contractions from aggressive inflation policy in the 1950s until the early 1960s, the Federal Reserve responded weakly to rising inflation in the mid-1960s.[40]

The second set of episodes comprise the monetary tightening shocks in 1969, 1973-74 and 1979-82. Both measures of monetary shocks are mostly positive during these years. The

[39] Romer and Romer (2002) use historical document to advance this point, while Sargent (1999) and Primiceri (2005b) show with estimated structural models that optimal policies designed using the 'wrong model' of the economy generated persistent policy mistakes.

[40] "[...] This brought withering criticism for the Federal Reserve on the grounds that the recessions of 1957 and 1960 had been unnecessary." Rotemberg (2013), p. 65.

sources of these shocks point at periods when the Federal Reserve opted for a more aggressive response to inflationary pressures, whether that was due to a shift in its objectives (Romer and Romer, 1989) or a reformed view about the workings of the economy (Sargent (1999) and Primiceri (2005b). The third of these episodes captures the Volcker disinflation.[41] Romer and Romer (2004) source the string of easing shocks around 1977 to Arthur Burns's political motives, and the large negative surprises around 1984-85 to the value of the dollar. The final episode with large movements in the shock series covers the series of positive shocks in the late 1980s and the stream of negative surprises in the early 1990s, when Kuttner (2001) shows that accommodative policy took the market largely by surprise.[42]

F Optimal policy under the timeless perspective

To derive the rational expectations optimal policy we write the model in terms of the output gap. The marginal utility of consumption

$$\frac{b\sigma}{1-b}\hat{c}_{t-1} - \frac{\sigma}{1-b}\left(1+\beta b^2\right)\hat{c}_t + \beta b\frac{\sigma}{1-b}E_t\hat{c}_{t+1} = (1-\beta b)\,\hat{\lambda}_t.$$

can be written as

$$\hat{\lambda}_t \quad = \quad -\frac{\sigma}{(1-b)(1-\beta b)}\left[(\hat{c}_t - b\hat{c}_{t-1}) - \beta b E_t\left(\hat{c}_{t+1} - b\hat{c}_t\right)\right].$$

Define

$$\varphi = \frac{\sigma\left(\frac{c}{y}\right)^{-1}}{(1-b)(1-\beta b)}$$

to get

$$\hat{\lambda}_t \quad = \quad -\varphi\left[\left(\frac{c}{y}\hat{c}_t - b\frac{c}{y}\hat{c}_{t-1}\right) - \beta b E_t\left(\frac{c}{y}\hat{c}_{t+1} - b\frac{c}{y}\hat{c}_t\right)\right].$$

Now use: $\hat{y}_t = \frac{c}{y}\hat{c}_t + \hat{g}_t$. Substituting gives

$$\hat{\lambda}_t \quad = \quad -\varphi\left[(\hat{y}_t - \hat{g}_t - b\left(\hat{y}_{t-1} - \hat{g}_{t-1}\right)) - \beta b E_t\left(\hat{y}_{t+1} - \hat{g}_{t+1} - b\left(\hat{y}_t - \hat{g}_t\right)\right)\right].$$

[41] Notice that our shock series, in line with Romer and Romer (2004) displays a substantial increase in volatility during the period 1979-82 due to changing operating procedures.

[42] Poole, Rasche and Thorton (2002) also document tightening surprises over the years 1988-1989 and large negative surprises over the period 1990-92.

We can then re-define

$$\hat{\lambda}_t = -\varphi \left(\tilde{y}_t - \tilde{g}_t\right)$$

where we define

$$\tilde{y}_t = \left(\hat{y}_t - b\hat{y}_{t-1}\right) - \beta b E_t \left(\hat{y}_{t+1} - b\hat{y}_t\right)$$

$$\tilde{g}_t = \left(\hat{g}_t - b\hat{g}_{t-1}\right) - \beta b E_t \left(\hat{g}_{t+1} - b\hat{g}_t\right).$$

The natural rate is defined as (from the intertemporal equation for λ_t)

$$r_t^{nr} = \varphi \left[E_t \left(\tilde{y}_{t+1}^{nr} - \tilde{g}_{t+1}\right) - \left(\tilde{y}_t^{nr} - \tilde{g}_t\right)\right].$$

We can then write the output gap as

$$\tilde{x}_t = \left(x_t - bx_{t-1}\right) - \beta b E_t \left(x_{t+1} - bx_t\right),$$

where the output gap can also be expressed as $x_t = \frac{c}{y}\left(\hat{c}_t - \hat{c}_t^{nr}\right)$.

Recalling the wage Phillips curve is

$$\hat{\pi}_t^w - \iota_w \hat{\pi}_{t-1} = \kappa_w \left(\phi_n^{-1}\left(\hat{y}_t - \hat{a}_t\right) - \hat{\lambda}_t + \hat{\varphi}_t - \frac{\hat{\theta}_{w,t}}{\theta_w - 1} - \hat{w}_t^r\right) + \beta E_t \left(\hat{\pi}_{t+1}^w - \iota_w \hat{\pi}_t\right)$$

where

$$\kappa_w = \frac{\left(1 - \beta\xi_w\right)\left(1 - \xi_w\right)}{\xi_w \left(1 + \theta_w \phi_n^{-1}\right)}.$$

substitute for the marginal utility of consumption

$$\hat{\pi}_t^w - \iota_w \hat{\pi}_{t-1} = \kappa_w \left(\phi_n^{-1}\left(\hat{y}_t - \hat{a}_t\right) + \varphi\left(\tilde{y}_t - \tilde{g}_t\right) + \hat{\varphi}_t - \frac{\hat{\theta}_{w,t}}{\theta_w - 1} - \hat{w}_t^r\right) + \beta E_t \left(\hat{\pi}_{t+1}^w - \iota_w \hat{\pi}_t\right).$$

In absence of price/wage rigidities where $\hat{w}_t^{nr} = \hat{a}_t$ and natural output is

$$\phi_n^{-1}\left(\hat{y}_t^{nr} - \hat{a}_t\right) + \varphi\left(\tilde{y}_t^{nr} - \tilde{g}_t\right) + \hat{\varphi}_t - \hat{w}_t^{nr} = 0$$

$$\phi_n^{-1}\hat{y}_t^{nr} - \left(\phi_n^{-1} + 1\right)\hat{a}_t + \varphi\left(\tilde{y}_t^{nr} - \tilde{g}_t\right) + \hat{\varphi}_t = 0$$

Using the first equation, we can write the wage Phillips curve

$$\hat{\pi}_t^w - \iota_w \hat{\pi}_{t-1} = \kappa_w \left(\phi_n^{-1} \left(\hat{y}_t - \hat{y}_t^{nr} \right) + \varphi \left(\tilde{y}_t - \tilde{y}_t^{nr} \right) - \frac{\hat{\theta}_{w,t}}{\theta_w - 1} - \left(\hat{w}_t^r - \hat{w}_t^{nr} \right) \right) + \beta E_t \left(\hat{\pi}_{t+1}^w - \iota_w \hat{\pi}_t \right)$$

or

$$\hat{\pi}_t^w - \iota_w \hat{\pi}_{t-1} = \kappa_w \left(\phi_n^{-1} x_t + \varphi \tilde{x}_t - \mu_t^w - \left(\hat{w}_t^r - \hat{w}_t^{nr} \right) \right) + \beta E_t \left(\hat{\pi}_{t+1}^w - \iota_w \hat{\pi}_t \right)$$

where

$$\mu_t^w = \frac{\hat{\theta}_{w,t}}{\theta_w - 1}.$$

Moving to the price Phillips curve,

$$\left(1 + \beta \iota_p\right) \hat{\pi}_t = \kappa_p \left(\hat{w}_t^r - \hat{a}_t \right) + \iota_p \hat{\pi}_{t-1} + \beta E_t \hat{\pi}_{t+1} + \left(1 + \beta \iota_p\right) \tilde{\theta}_{p,t}$$

or

$$\hat{\pi}_t - \iota_p \hat{\pi}_{t-1} = \kappa_p \left(\hat{w}_t^r - \hat{a}_t \right) + \beta E_t \left(\hat{\pi}_{t+1} - \iota_p \hat{\pi}_t \right) + \left(1 + \beta \iota_p\right) \tilde{\theta}_{p,t}.$$

In terms of natural rates

$$\hat{\pi}_t - \iota_p \hat{\pi}_{t-1} = \kappa_p \left(\hat{w}_t^r - \hat{w}_t^{nr} \right) + \beta E_t \left(\hat{\pi}_{t+1} - \iota_p \hat{\pi}_t \right) + \mu_t^p.$$

The central bank optimal problem is then

$$\mathcal{L} = E \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{2} \left[\lambda_p \left(\pi_t - \iota_p \pi_{t-1} \right)^2 + \lambda_w \left(\pi_t^w - \gamma_w \pi_{t-1} \right)^2 + \lambda_x \left(x_t - \delta x_{t-1} \right)^2 \right] \\ + \varphi_{1,t} \left[\pi_t - \iota_p \pi_{t-1} - \kappa_p \left(\hat{w}_t - \hat{w}_t^{nr} \right) - \beta \left(\pi_{t+1} - \iota_p \pi_t \right) - \mu_t^p \right] \\ + \varphi_{2,t} \left[\begin{array}{c} \pi_t^w - \iota_w \pi_{t-1} - \kappa_w \left(\phi_n^{-1} x_t + \varphi \left(\left(x_t - b x_{t-1} \right) - \beta b E_t \left(x_{t+1} - b x_t \right) \right) - \mu_t^w - \left(\hat{w}_t^r - \hat{w}_t^{nr} \right) \right. \\ - \beta E_t \left(\hat{\pi}_{t+1}^w - \iota_w \hat{\pi}_t \right) \end{array} \right. \\ + \varphi_{3,t} \left[\hat{w}_t^r - \hat{w}_{t-1}^r - \pi_t^w + \pi_t . \right] \end{array} \right.$$

where $\delta, \lambda_x, \lambda_p, \lambda_w$ are defined as

$$\lambda_p \equiv \frac{\theta_p \kappa_p^{-1}}{\theta_p \kappa_p^{-1} + \theta_w \kappa_w^{-1}}$$

$$\lambda_w \equiv \frac{\theta_w \kappa_w^{-1}}{\theta_p \kappa_p^{-1} + \theta_w \kappa_w^{-1}}$$

$$\lambda_x \equiv \frac{\tilde{\vartheta}}{\theta_p \kappa_p^{-1} + \theta_w \kappa_w^{-1}}$$

where $\tilde{\vartheta} = \vartheta\varphi$ and ϑ solves

$$P(\vartheta) \equiv \beta^{-1}\vartheta^2 - \chi\vartheta + b^2 = 0$$

where

$$\vartheta \equiv b\delta^{-1}$$

$$\chi \equiv \frac{\phi_n^{-1} + \varphi\left(1 + \beta b^2\right)}{\beta\varphi}.$$

Pick the largest root

$$\vartheta = \vartheta_2 = \frac{\beta}{2}(\chi + \sqrt{\chi^2 - 4b^2\beta^{-1}}) > 1.$$

First-order conditions for the policy problem with respect to inflation, nominal wage inflation, real wages and the output gap are:

$$\begin{aligned}
0 &= \lambda_p\left(\pi_t - \iota_p\pi_{t-1}\right) - \beta\lambda_p\iota_p E_t\left(\pi_{t+1} - \iota_p\pi_t\right) - \beta\lambda_w\iota_w E_t\left(\pi^w_{t+1} - \iota_w\pi_t\right) \\
&\quad + \varphi_{1,t} - \beta\iota_p E_t\varphi_{1,t+1} - \left(\varphi_{1,t-1} - \beta\iota_p\varphi_{1,t}\right) - \beta\iota_w E_t\varphi_{2,t+1} + \beta\iota_w\varphi_{2,t} + \varphi_{3,t} \\
0 &= \lambda_w\left(\pi^w_t - \iota_w\pi_{t-1}\right) + \varphi_{2,t} - \varphi_{2,t-1} - \varphi_{3,t} \\
0 &= -\kappa_p\varphi_{1,t} + \kappa_w\varphi_{2,t} + \varphi_{3,t} - \beta E_t\varphi_{3,t+1} \\
0 &= \lambda_x\left(x_t - \delta x_{t-1}\right) - \beta\lambda_x\delta E_t\left(x_{t+1} - \delta x_t\right) \\
&\quad - \kappa_w\left(\left(\phi_n^{-1} + \varphi + \varphi\beta b^2\right)\varphi_{2,t} - \varphi b\beta E_t\varphi_{2,t+1} - \varphi b\varphi_{2,t-1}\right).
\end{aligned}$$

We can stack all the model's equations in the vector Z_t. Additionally, the monetary policy rule is substituted by the four equations above. We then obtain the system

$$Z_t = FZ_{t-1} + S_C\epsilon_t$$

where the vector Z_t includes lagged variables and where innovations ϵ_t are standardized.

G Evaluating policy

When evaluating alternative targeting rules collect the policy rule together with the other model equations state space form. The loss function in terms of this vector is given by

$$L = E\sum_{t=0}^{\infty}\beta^t y_t' W y_t$$

where

$$y_t = \begin{pmatrix} \pi_t - \iota_p\pi_{t-1} \\ \pi^w_t - \iota_w\pi_{t-1} \\ x_t - \delta x_{t-1} \end{pmatrix} = \begin{pmatrix} \pi_t - \iota_p\pi_{t-1} \\ \hat{w}^r_t - \hat{w}^r_{t-1} + \pi_t - \iota_w\pi_{t-1} \\ \frac{c}{y}\left(\hat{c}_t - \hat{c}^{nr}_t\right) - \delta\frac{c}{y}\left(\hat{c}_{t-1} - \hat{c}^{nr}_{t-1}\right) \end{pmatrix}$$

and

$$W = diag\begin{pmatrix} \lambda_p \\ \lambda_w \\ \lambda_x \end{pmatrix}$$

and where finally $y_t = DZ_t$ for some appropriately chosen matrix. We can then evaluate the discounted loss as

$$\begin{aligned}
\mathcal{L} &= E\sum_{t=0}^{\infty}\beta^t y_t' W y_t \\
&= E\sum_{t=0}^{\infty}\beta^t\left[D\left(\sum_{\tau=0}^{t-1}F^{\tau}S_C\epsilon_{t-\tau}+F^{\tau}Z_0\right)\right]' W\left[D\left(\sum_{\tau=0}^{t-1}F^{\tau}S_C\epsilon_{t-\tau}+F^{\tau}Z_0\right)\right] \\
&= \sum_{t=0}^{\infty}\beta^t\left\{\sum_{\tau=0}^{t-1}tr\left[F^{\tau}D'WDF^{\tau}S_C E\left[\epsilon_{t-\tau}\epsilon_{t-\tau}'\right]S_C'\right]+tr\left[F^tD'WDF^tE\left[Z_0Z_0\right]\right]\right\} \\
&= \frac{1}{1-\beta}\sum_{t=0}^{\infty}tr\left[\beta^tF^tD'WDF^tS_CS_C^{'}\right]+\sum_{t=0}^{\infty}tr\left[\beta^tF^tD'WDF^t\Sigma_Z\right] \\
&= \frac{1}{1-\beta}tr\left[\beta\bar{L}S_CS_C^{'}\right]+tr\left[\bar{L}\Sigma_Z\right]
\end{aligned}$$

where

$$\Sigma_Z = F\Sigma_Z F^{'} + S_C S_C^{'}$$

$$\bar{L} = \beta F^{'}\bar{L}F + D'WD.$$

H Direct control of aggregate demand

When the central bank is in full control of aggregate demand, the marginal utility of income becomes *exogenous* from the point of view of individual households. So

$$\begin{aligned}
\hat{\lambda}_t(j) &= \hat{\lambda}_t \\
&= \frac{b\sigma}{(1-\beta b)(1-b)}\hat{c}_{t-1}-\frac{\sigma}{(1-\beta b)(1-b)}\left(1+\beta b^2\right)\hat{c}_t+\beta b\frac{\sigma}{(1-\beta b)(1-b)}\hat{E}_t^j\hat{c}_{t+1}.
\end{aligned}$$

Substituting into the wage Phllips curve

$$\hat{w}_t^{opt}(j) = \xi_w \beta \hat{E}_t^j \left(\pi_{t+1}^w - \iota_w \pi_t\right) + \frac{1-\xi_w\beta}{1+\theta_w\phi_n^{-1}}\left[\hat{\varphi}_t + \phi_n^{-1}\hat{N}_t - \hat{w}_t^r - \hat{\lambda}_t(j) - \frac{\hat{\theta}_{w,t}}{(\theta_w - 1)}\right] + \xi_w\beta\hat{E}_t^j \hat{w}_{t+1}^{opt}(i)$$

we get

$$\hat{w}_t^{opt}(j) = \xi_w\beta\hat{E}_t^j\hat{w}_{t+1}^{opt}(j) + \delta_t^w$$

where

$$\begin{aligned}
\delta_t^w = {} & \xi_w\beta\hat{E}_t^j\left(\pi_{t+1}^w - \iota_w\pi_t\right) + \\
& + \frac{1-\xi_w\beta}{1+\theta_w\phi_n^{-1}}\left[\hat{\varphi}_t + \phi_n^{-1}\left(\hat{g}_t - \hat{a}_t\right) - \hat{w}_t^r - \frac{\hat{\theta}_{w,t}}{(\theta_w-1)}\right] + \\
& - \frac{1-\xi_w\beta}{1+\theta_w\phi_n^{-1}}\frac{b\sigma}{(1-\beta b)(1-b)}\hat{c}_{t-1} + \\
& + \frac{1-\xi_w\beta}{1+\theta_w\phi_n^{-1}}\left[\left(\phi_n^{-1}\frac{c}{y} + \frac{\sigma}{(1-\beta b)(1-b)}\left(1+\beta b^2\right)\right)\hat{c}_t - \beta b\frac{\sigma}{(1-\beta b)(1-b)}\hat{E}_t^j\hat{c}_{t+1}\right]
\end{aligned}$$

The solution for optimal wages becomes

$$\hat{w}_t^{opt}(j) = \hat{E}_t^j\sum_{T=t}^{\infty}(\xi_w\beta)^{T-t}\delta_T^w.$$

In equilibrium every agent that gets to reset chooses the same wage so use

$$\hat{w}_t^{opt} = \frac{\xi_w}{1-\xi_w}\left(\pi_t^w - \iota_w\pi_{t-1}\right)$$

to get

$$\pi_t^w - \iota_w\pi_{t-1} = \frac{1-\xi_w}{\xi_w}\hat{E}_t^j\sum_{T=t}^{\infty}(\xi_w\beta)^{T-t}\delta_T^w.$$

The other equations in this problem are the price Philips curve

$$\pi_t - \iota_p\pi_{t-1} = \kappa_p E_t^i\sum_{T=t}^{\infty}(\xi_p\beta)^{T-t}\left[\frac{\xi_p\beta}{1-\xi_p\beta}\left(\pi_{T+1} - \iota_p\pi_T\right) + \hat{w}_T - \hat{a}_T - \frac{\hat{\theta}_{p,T}}{(\theta_p-1)}\right];$$

The targeting rule

$$0 = \phi_\pi (\pi_t - \zeta_p \pi_{t-1}) + (\pi_t^w - \zeta_w \pi_{t-1}) + \phi_{\Delta x} \left(x_t^{CB} - x_{t-1}^{CB} \right);$$

and the remaining intra-temporal equations

$$\pi_t^w = \hat{w}_t^r - \hat{w}_{t-1}^r + \pi_t$$

and

$$\hat{y}_t = \hat{a}_t + \hat{N}_t$$

and

$$\hat{y}_t = \frac{c}{y} \hat{c}_t + \hat{g}_t$$

together with the same exogenous processes defined above.

I Optimal policy: additional results

In this section we evaluate the loss for alternative targeting rules. We also compare the targeting rule under rational expectations with the optimal policy under the timeless perspective. The label "Optimal Policy" refers to implemented targeting rules. The results are summarized in Table 5. The first line reports the baseline targeting criterion where only the coefficient ϕ is chosen to minimize the central loss function. The second line corresponds to a targeting rule of the same form

$$R_t = \tilde{\rho}_R R_{t-1} + \tilde{\phi}_\pi (\pi_t - \iota_p \pi_{t-1}) + \tilde{\phi}_w (\pi_t^w - \iota_w \pi_{t-1}) + \tilde{\phi}_{\Delta x} \left(x_t^{CB} - x_{t-1}^{CB} \right).$$

where the coefficients are jointly chose to minimize the loss function. The third line captures a targeting rule as above but with an additional response to the level of the output gap

$$R_t = \tilde{\rho}_R R_{t-1} + \tilde{\phi}_\pi (\pi_t - \iota_p \pi_{t-1}) + \tilde{\phi}_w (\pi_t^w - \iota_w \pi_{t-1}) + \tilde{\phi}_x x_t^{CB} + \tilde{\phi}_{\Delta x} \left(x_t^{CB} - x_{t-1}^{CB} \right).$$

This improves the performance marginally. The next line shows a targeting rule with no smoothing but where all coefficients are optimized. This rule performs considerably worse. Finally the last two lines compare the optimized targeting rule under rational expectations with the optimal policy under the timeless perspectives, in the case of no output gap measurement error. These policies deliver identical losses.

	$\mathcal{L}$		
	1964-1983	**1984-1998**	**1999-2007**
Optimal Policy: baseline	72.5	48.0	37.9
Optimal Policy: optimized coeffs.	72.0	47.7	37.8
Optimal Policy: with gap level	70.3	45.0	37.0
Optimal Policy: $\rho_R = 0$	87.7	52.2	46.8
Optimal Policy: full info, no m.e.	4.6	16.7	16.7
Timeless Perspective: full info, no m.e.	4.6	16.7	16.7

The table summarizes the performance of different policy rules. The first column measures the unconditional loss. The second column show standard deviations for each argument in the loss function in counterfactuals conducted using filtered shocks. These counterfactuals include output gap mis-measurement shocks by the central bank.

Table 5: **Optimal Policy: additional results**.

J Intertemporal Trade-offs under Optimal Policy

This section provides some final formal results, to complement the simple example and empirical findings. We do this in a special case of our empirical model, in which there is a frictionless labor market, and purely forward-looking optimal pricing and consumption decisions. The analysis of optimal monetary policy shows that in general the aggregate demand curve is a binding constraint on feasible choices of interest-rate paths, even though this is never true of the equivalent model with rational expectations.

J.1 The Policy Problem

The policymaker minimizes the period loss function

$$L_t = \pi_t^2 + \lambda_x x_t^2$$

where $\lambda_x > 0$ determines the relative weight given to output gap versus inflation stabilization. Feasible sequences of inflation and the output gap must satisfy the aggregate demand and supply equations

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - (R_T - \pi_{T+1} - r_T^n)\right] \tag{34}$$

$$\pi_t = \hat{E}_t \sum_{T=t}^{\infty} (\xi_p \beta)^{T-t} \left[\kappa x_T + (1-\alpha) \beta \pi_{T+1}\right] \tag{35}$$

where all variables are interpreted as log-deviations from steady state; x_t is the output gap; r_t^n the natural rate of interest an exogenous process; and $\kappa = (1-\xi_p\beta)(1-\xi_p)/\xi_p$ the slope of the short-run trade-off between inflation and the output gap.[43] Optimal consumption and price-setting requires households and firms to forecast future output, interest rates and inflation. Assume agents have a forecasting model of the form () and (), with

$$z_t = \begin{bmatrix} \pi_t \\ x_t \\ R_t \end{bmatrix} \text{ and } \omega_t = \begin{bmatrix} \omega_t^\pi \\ \omega_t^x \\ \omega_t^R \end{bmatrix}$$

and where $\Phi = 0$ and $\rho = 1$ to give a shifting end-point model.[44]

Subject to aggregate demand and supply, and the evolution of beliefs, the central bank solves the problem

$$\min_{\{x_t, \pi_t, R_t,\ \omega_t\}} E_t^{RE} \sum_{T=t}^{\infty} \beta^{T-t} L_T \tag{36}$$

taking as given initial beliefs, ω_{-1}. Assume that the central bank has rational expectations and has complete information about the true structural relations describing household and firm behavior. The Lagrangian is:

$$\max_{\left\{\pi_t, x_t, R_t, \omega_t^\pi, \omega_t^x, \omega_t^R\right\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{2}\left[\pi_t^2 + \lambda_x (x_t - x^*)^2\right] \\ +\lambda_{1,t}\left(-\pi_t + \kappa x_t + \frac{\kappa\xi\beta}{1-\xi\beta}\omega_{t-1}^x + \frac{(1-\xi)\beta}{1-\xi\beta}\omega_{t-1}^\pi\right) \\ +\lambda_{2,t}\left(-x_t - (R_t - \hat{r}_t^n) - \frac{1}{1-\beta}\left(\beta\omega_{t-1}^R - \omega_{t-1}^\pi\right) + \omega_{t-1}^x\right) \\ +\lambda_{3,t}\left(-\omega_t^\pi + \omega_{t-1}^\pi + g\left(\pi_t - \omega_{t-1}^\pi\right)\right) \\ +\lambda_{4,t}\left(-\omega_t^x + \omega_{t-1}^x + g\left(x_t - \omega_{t-1}^x\right)\right) \\ +\lambda_{5,t}\left(-\omega_t^R + \omega_{t-1}^R + g\left(R_t - \omega_{t-1}^R\right)\right) \end{array} \right\}.$$

[43] Derivation of these expressions assume a unity elasticity of intertemporal substitution, and infinite Frisch elasticity of labor supply.

[44] Under rational expectations, because the model is purely forward looking, the minimum state variables solution is a linear function of aggregate disturbances. We therefore assuming a belief structure consistent with this solution.

The first-order conditions are:

$$
\begin{aligned}
\pi_t - \lambda_{1,t} + g\lambda_{3,t} &= 0 && (37)\\
\lambda_x (x_t - x^*) + \lambda_{1,t}\kappa - \lambda_{2,t} + \lambda_{4,t} g &= 0 && (38)\\
\frac{\kappa\xi\beta}{1-\xi\beta}\beta E_t\lambda_{1,t+1} - \lambda_{4,t} + \beta(1-g)E_t\lambda_{4,t+1} + \beta E_t\lambda_{2,t+1} &= 0 && (39)\\
\frac{(1-\xi)\beta}{1-\xi\beta}\beta E_t\lambda_{1,t+1} + \frac{\beta}{1-\beta}E_t\lambda_{2,t+1} - \lambda_{3,t} + \beta(1-g)E_t\lambda_{3,t+1} &= 0 && (40)\\
g\lambda_{5,t} - \lambda_{2,t} &= 0 && (41)\\
-\lambda_{5,t} + \beta(1-g)E_t\lambda_{5,t+1} - \frac{\beta^2}{1-\beta}E_t\lambda_{2,t+1} &= 0. && (42)
\end{aligned}
$$

Because beliefs are state variables there is no distinction between optimal commitment and discretion. The policy maker can only influence expectations through current and past actions — not through announced commitments to some future course of action.

The first-order conditions constitute a linear rational expectations model.[45] The system can be solved using standard methods. Using results from Giannoni and Woodford (2017), Eusepi, Giannoni, and Preston (2018) we establish conditions on beliefs for a unique bounded rational expectations equilibrium.

Proposition 1. *Let* $\bar{g} = \frac{(1-\xi\beta)(\lambda_x+\kappa^2)}{\lambda_x(1-\beta)+\kappa^2}$. *For beliefs* $g \in (0,\bar{g})$ *that satisfy either* $g < 2(1-\beta)$ *or* $g > \beta^{-1}-\beta$ *the optimal policy problem has a unique bounded solution. When* $g < 2(1-\beta)$ *the aggregate demand constraint is not binding, and the associated Lagrange multiplier is equal to zero. When* $g > \beta^{-1}-\beta$ *the aggregate demand constraint is binding, and the associated Lagrange multiplier is strictly positive.*

Proof. This is an application of results in Eusepi, Giannoni, and Preston (2018). □

This result formalizes the central insight of the paper. When long-term interest rate beliefs are sufficiently sensitive to short-run forecast errors, aggregate demand limits the movements in interest rates. The central bank has imprecise control of long-term interest rates, even though the model satisfies the expectations hypothesis of the term structure. Belief distortions prevent changes in short-term rates being efficiently transmitted to long-term rates relevant for aggregate demand.

A further implication concerns a special case of beliefs. When the gain coefficient converges to zero the optimal policy coincides with optimal discretion under rational expecta-

[45] In an innovative study, Molnar and Santoro (2013) explore optimal policy under learning in a model where only one-period-ahead expectations matter to the pricing decisions of firms. Gaspar, Smets, and Vestin (2006) provide a global solution to the same optimal policy problem but under a more general class of beliefs.

tions. This result is intuitive: for small gains beliefs are almost never revised. Because policy cannot influence beliefs, which is precisely the assumption of optimal discretion, dynamics will correspond to those predicted by optimal discretion. For sufficiently small gains, policy is well approximated by rational expectations equilibrium analysis, and the central bank will have precise control of long-term inflation expectations.

Corollary 1. *In the special case $g \to 0$ optimal policy will give the same dynamic responses to disturbances as optimal discretion under rational expectations.*

This type of result has been discussed by Sargent (1999), Molnar and Santoro (2013) and Eusepi, Giannoni, and Preston (2018). The empirical model reflects this property. After 1998, when agent's beliefs display relatively little sensitivity to forecast errors, monetary policy ensures greater stability of long-term inflation expectations. By providing a nominal anchor, policy permits better stabilization outcomes.

J.2 A Simple Example

To appreciate further aggregate demand as a constraint confronting policy, consider a central bank faced only with i.i.d. shocks to the natural rate r_t^n, and private agent beliefs initially consistent with rational expectations equilibrium so that $\omega_{t-1} = 0$. Because initial forecasts satisfy $E_t z_T = 0$ for all $T > t$, period t equilibrium is determined by the aggregate demand and supply curves (34) and (35) which simplify to

$$\pi_t = \kappa x_t \qquad \text{and} \qquad x_t = -\left(R_t - r_t^n\right).$$

Given a disturbance to the natural rate of interest, complete stabilization is possible in period t. Nominal interest-rate policy must track the natural rate, $R_t = r_t^n$, giving $\pi_t = x_t = 0$. But this implies subsequent movements in long-run interest-rate beliefs according to

$$\omega_t^R = \omega_{t-1}^R + g\left(r_t^n - \omega_{t-1}^R\right).$$

The next-period's stabilization problem — and every subsequent period — is given by the pair of equations

$$\begin{aligned} \pi_{t+1} &= \kappa x_{t+1} \\ x_{t+1} &= -\left(R_{t+1} - r_{t+1}^n\right) - \frac{1}{1-\beta}\beta\omega_t^R \end{aligned}$$

where the final term in the demand equation reflects the restraining effects of long-term interest rates on aggregate demand. Complete stabilization of inflation and the output gap is again possible by having nominal interest rates track long-run expectations and the natural rate of interest.

But is this interplay sustainable? Imposing full stabilization, $x_{t+1} = \pi_{t+1} = 0$, the aggregate demand constraint defines the implicit policy rule

$$R_{t+1} = r_{t+1}^{n} - \frac{\beta}{1-\beta}\omega_t^R \tag{43}$$

in every period t. Optimal policy not only responds to natural-rate disturbances, but also movements in long-term interest rates, driven by expectations.[46] Substituting into the updating rule for beliefs, ω_t^R, gives

$$\omega_{t+1}^R = \left(1 - \frac{g}{1-\beta}\right)\omega_t^R + g r_{t+1}^n$$

which is a first-order difference equation. Sustainable policy requires the dynamics of beliefs to be stationary. The following restriction must hold

$$g < 2\left(1-\beta\right).$$

For larger gains, stability is not feasible, implying beliefs and interest rates are explosive. This is not a permissible, or at least desirable, feature of optimal policy if only because the zero lower bound on interest rates obviates such solutions.[47]

This restriction is the limit of the condition derived for the simple endowment economy when $\phi \to \infty$, and defines one region of the parameter space for which the optimal Ramsey problem has a unique bounded rational expectations equilibrium in Proposition 1. An important lesson emerges: complete stabilization of inflation is infeasible. A central bank charged with implementing the target criterion $\pi_t = 0$ will fail, because it requires large movements in nominal interest rates. This result holds more generally, placing important bounds to arguments made by Evans and Honkapohja (2006), Woodford (2007) and Preston (2008), that the target criterion approach to implementing policy is robust to alternative assumptions about belief formation.[48] Of course, should the condition on the gain be violated, Proposition 1 shows such beliefs are still consistent with equilibrium, but one in which the central bank optimally accepts some variation in inflation.

[46]The implied interest rates of a bond of any maturity can be shown to be a function of the long-term interest rate belief. This is an example of the expectations hypothesis of the term structure of interest rates.

[47]While some might not object to nominal explosions, if beliefs about real activity depend on nominal interest-rate forecast errors, there would also be unbounded paths for real variables.

[48]For example, it is equally true when using a Taylor-type rule to implement the target criterion $\pi_t = -\theta^{-1}(x_t - x_{t-1})$ the optimal commitment policy in the canonical New Keynesian model.

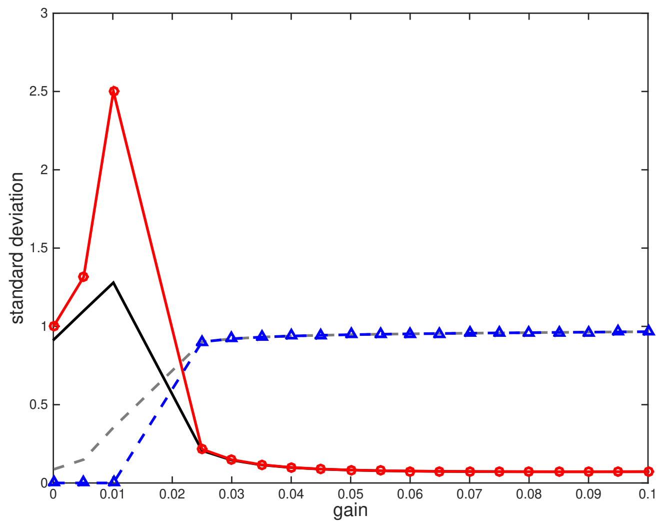

Figure 8: **Appendix: Volatility as a function of the constant gain.**

This figure show the volatility of output and interest rates as a function of the constant gain $\bar{g}$. The welfare theoretic loss gives the volatility of the interest rate (red circles) and the output gap (blue triangles); while a policy maker with a concern of interest rate volatility delivers the interest rate shown by the black line, and the output gap given by the grey dashed line.

Figure 8 provides numerical illustration, plotting the standard deviation of the output gap and interest rate as a function of the constant gain g under optimal policy. Assume the discount factor is $\beta = 0.995$; the frequency of price changes determined by $\xi_p = 0.8$; and the weight on output gap stabilization $\lambda_x = 0.05$. Under these assumptions there is relatively small variation in inflation, so it matters little whether we plot the sum of the output gap and inflation variation or the output gap alone. Only variations in the natural rate, r_t^n, drive economic fluctuations. The figure describes outcomes under the welfare-theoretic loss (36), and under a loss function

$$L_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_R R_t^2$$

that also penalizes volatility in the interest rate. Recall optimal discretion corresponds to the case $g = 0$. Under the standard loss function a knife-edge result obtains: for $g < 0.01$ the output gap is fully stabilized even if this induces substantial volatility in the interest rate. For large values of g, the policy maker loses the ability to stabilize the output gap. Feasible policy restricts variation in the policy rate, translating into increasing volatility in the output gap. If the policy maker has some preference for interest-rate stabilization, perhaps reflecting zero-lower bound considerations, then the increase in output volatility occurs continuously with the size of the gain. Even relatively small values of the gain lead to output gap volatility.

Proposition 2. *In the model given by (34) and (35), Divine Coincidence will in general not* *hold even in a model with only disturbances to the natural real rate of interest.*

The inability of the central bank to stabilize both output gap and inflation in the face of aggregate demand shocks stems from agents' expectations about the policy rate. For example, suppose as in Molnar and Santoro (2013) the policymaker can directly control the output gap as the instrument of policy, and solves the problem

$$\min_{\{x_t,\pi_t,\omega_t^x,\omega_t^\pi\}} E_t^{RE} \sum_{T=t}^{\infty} \beta^{T-t} L_T$$

subject only to the Phillips curve (35), taking as given initial beliefs ω_{-1}^x and ω_{-1}^π. Equivalently, suppose interest-rate beliefs are anchored so that $\omega_t^R = 0$ for all t, giving households rational expectation forecasts of interest rates. Then the Divine Coincidence holds, despite long-term drift in expectations about inflation and real activity.

Corollary 2. *Absent low-frequency drift in interest-rate beliefs, the central bank can directly control aggregate demand, and the Divine Coincidence holds.*

In the case that $\omega_t^R = 0$ for all t the Lagrangian is:

$$\max_{\{\pi_t,x_t,R_t,\omega_t^\pi,\omega_t^x\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{2}\left[\pi_t^2 + \lambda_x (x_t - x^*)^2\right] \\ +\lambda_{1,t}\left(-\pi_t + \kappa x_t + \frac{\kappa\xi\beta}{1-\xi\beta}\omega_{t-1}^x + \frac{(1-\xi)\beta}{1-\xi\beta}\omega_{t-1}^\pi\right) \\ +\lambda_{2,t}\left(-x_t - (R_t - \hat{r}_t^n) - \frac{1}{1-\beta}\left(-\omega_{t-1}^\pi\right) + \omega_{t-1}^x\right) \\ +\lambda_{3,t}\left(-\omega_t^\pi + \omega_{t-1}^\pi + g\left(\pi_t - \omega_{t-1}^\pi\right)\right) \\ +\lambda_{4,t}\left(-\omega_t^x + \omega_{t-1}^x + g\left(x_t - \omega_{t-1}^x\right)\right) \end{array} \right\}.$$

The first-order conditions are:

$$\begin{aligned} \pi_t - \lambda_{1,t} + g\lambda_{3,t} &= 0 \\ \lambda_x (x_t - x^*) + \lambda_{1,t}\kappa - \lambda_{2,t} + \lambda_{4,t} g &= 0 \\ \frac{\kappa\xi\beta}{1-\xi\beta}\beta E_t\lambda_{1,t+1} - \lambda_{4,t} + \beta(1-g)E_t\lambda_{4,t+1} + \beta E_t\lambda_{2,t+1} &= 0 \\ \frac{(1-\xi)\beta}{1-\xi\beta}\beta E_t\lambda_{1,t+1} + \frac{\beta}{1-\beta}E_t\lambda_{2,t+1} - \lambda_{3,t} + \beta(1-g)E_t\lambda_{3,t+1} &= 0 \\ -\lambda_{2,t} &= 0. \end{aligned}$$

Because $\lambda_{2,t} = 0$ the aggregate demand curve does not restrict the choice of R_t. Hence for any path of beliefs the central bank can determine an interest rate to achieve the desired output gap. Hence this is equivalent to controlling the output gap directly. For the same reason policy can always insulate the economy from movements in the natural rate. The optimal policy satisfies

$$R_t = x_t + \hat{r}_t^n + \frac{1}{1-\beta}\omega_{t-1}^{\pi} + \omega_{t-1}^{x}.$$

The result underscores the importance of jointly modeling monetary policy, long-term expectations and policy credibility. Central banks which provide a credible nominal anchor — that is, they stabilize long-run inflation expectations — will have tighter control of the macroeconomy. If policy lacks credibility, so that long-term expectations display high sensitivity to short-run surprises, stabilization policy becomes more difficult, as the term structure of interest rates constrains policy actions.