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## 4

# *Credible Government Policies*

## *Perfection*

This chapter uses Kydland and Prescott's example to convey the theory of sustainable or credible government policies. We examine whether this theory can moderate or extend the pessimism from Kydland and Prescott's analysis.<sup>1</sup> My conclusion is that the multiplicity of credible plans replaces pessimism with agnosticism. The theory described in this chapter is difficult if a reader has not seen it before. A reader who is not interested in the theory of credible policy will lose little of the flow of the main argument in this book if he accepts my judgment on the meaning of this literature and skips to chapter 5 at this point.

The economy repeats itself for each  $t \geq 1$ . In this section, we let  $Y = [0, y^\#]$ . The lowest possible setting of the inflation rate is the Ramsey outcome 0. The upper bound on inflation  $y^\#$  makes the government's problem non-trivial. The government evaluates outcome paths  $(x, y) \equiv \{x_t, y_t\}_{t=1}^\infty$  according to

$$V^g(x, y) = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} r(x_t, y_t), \quad \delta \in (0, 1). \quad (18)$$

The government uses a strategy with a recursive representation. This substantially restricts the space of strategies because most history-dependent strategies cannot be represented recursively.

<sup>1</sup> For descriptions of theories of credible government policy see Chari and Kehoe (1990), Stokey (1989, 1991), Rogoff (1989), and Chari, Kehoe, and Prescott (1989). For applications of the framework of Abreu, Pearce, and Stacchetti, see Chang (1998), Phelan and Stacchetti (1998), and Ljungqvist and Sargent (1998, chapter 7).

Nevertheless, this class of strategies excludes no equilibrium payoffs  $V^g$ . We use the following:

DEFINITION: A *recursive government strategy* is a pair of functions  $\sigma = (\sigma_1, \sigma_2)$  and an initial condition  $v_1$  with the following structure:

$$\begin{aligned} v_1 &\in \mathbb{R} \text{ is given;} \\ y_t &= \sigma_1(v_t); \text{ and} \\ v_{t+1} &= \sigma_2(v_t, x_t, y_t), \end{aligned} \tag{19}$$

where  $v_t$  is a state variable designed to summarize the history of outcomes before  $t$ .

This form of strategy operates much like an autoregression to let the government's choice  $y_t$  depend on the history  $\{y_s, x_s\}_{s=1}^{t-1}$ , as mediated through the state variable  $v_t$ . Representation (19) induces history-dependent government policies, and thereby allows for reputation. We shall soon see that beyond its role in tracking histories,  $v_t$  also summarizes the future.<sup>2</sup>

Each member of the private sector forms an expectation about the government's action according to

$$x_t = \sigma_1(v_t), \tag{20}$$

where  $v_t$  is known to each private agent. Equation (20) builds in rational expectations, because the private sector knows both the state variable  $v_t$  and the government's decision rule  $\sigma$ .

A strategy  $(\sigma, v_1)$  recursively generates an entire outcome path expressed as  $(x, y) = (x, y)(\sigma, v_1)$ . By substituting the outcome path into (18), we find that strategy  $(\sigma, v_1)$  induces a value for the government, which we write as

$$V^g((x, y)(\sigma, v)) = V^g(\sigma, v). \tag{21}$$

<sup>2</sup> By iterating (19), we can construct a sequence of functions indexed by  $t \geq 1$   $\{\sigma_{3t}(h_t)\}$ , mapping histories that are augmented by initial conditions  $h_t = (\{x_s, y_s\}_{s=1}^{t-1}, v_1)$  into government time- $t$  actions  $y_t \in Y$ . A strategy for the repeated economy is a sequence of such functions without the restriction that it have a recursive representation.

So far, we have not interpreted the state variable  $v$ , except as a particular measure of the history of outcomes. The theory of credible policy ties past and future together by making the state variable  $v$  a promised value, an outcome to be expressed:

$$v = V^g(\sigma, v). \quad (22)$$

Equations (19), (20), (21), and (22) assert a dual role for  $v$ . In (19),  $v$  accounts for past outcomes. In equations (21) and (22),  $v$  looks forward. The state  $v_t$  is a discounted future value with which the government enters time  $t$  based on past outcomes. Depending on the outcome  $y$  and the entering promised value  $v$ ,  $\sigma_2$  updates the promised value with which the government leaves the period. The strategy gives current and future outcomes that make the government choose as expected. We postpone struggling with which of two valid interpretations of the government's strategy should be emphasized: something chosen by the government, or a description of a system of public expectations to which the government conforms.

### *Historical antecedents*

Credibility means public awareness that each period the government wants to execute its plans. Formalizing the idea of credibility was an important achievement from the 1980's, although a sophisticated understanding of credibility existed as early as the 1780's. The great finance minister Jacques Necker used this idea to explain to King Louis XVI why it was difficult for him to borrow more even though he was nominally an absolute monarch. After reminding him how 18th century French institutions limited his power to raise taxes but left him free to renege on debt payments, in 1784 Necker told the King:<sup>3</sup>

‘Therefore one can rekindle or sustain public trust only by giving reassurances on the sovereign's intentions, and

<sup>3</sup> This quotation is part of a longer passage cited by Sargent and Velde (1995).

by proving that no motive can incite him to fail his obligations.'

The formal definition of a subgame perfect equilibrium builds in every aspect of Necker's sentence. A king could prove that he would never fail his obligations by never wanting to fail them; that is, by conforming to a forecasting scheme that incorporates his incentive constraints. These incentive constraints assure that at each time and with each contingency, the King's current payoff plus continuation value would be higher if he confirmed those expectations than if he did not. Necker expresses frustration over the role of French institutions in making his sovereign's debt problem worse than it could be under improved institutions. The King could not choose his own behavior rule.<sup>4</sup>

These ideas are reflected in the following definition:

DEFINITION: A recursive strategy (19) with promised value  $v$  is a subgame perfect equilibrium (SPE) if and only if (a)  $\sigma_2(v, \sigma_1(v), \eta)$  is a value for a subgame perfect equilibrium  $\forall \eta \in Y$ ; and (b)

$$\begin{aligned} v &= (1 - \delta)r(\sigma_1(v), \sigma_1(v)) + \delta\sigma_2(v, \sigma_1(v), \sigma_1(v)) \\ &\geq (1 - \delta)r(\sigma_1(v), \eta) + \delta\sigma_2(v, \sigma_1(v), \eta), \quad \forall \eta \in Y. \end{aligned} \quad (23)$$

Conditions (a) and (b) associate with a subgame perfect equilibrium four objects: a promised value  $v$ ; a first-period outcome pair  $(y, y)$ , where  $y = \sigma_1(v)$ ; a continuation value  $v' = \sigma_2(v, \sigma_1(v), \sigma_1(v))$  if the required first-period outcome is observed; and another continuation value  $\tilde{v} = \sigma_2(v, \sigma_1(v), \eta)$  if the required first-period outcome is not observed. All of the continuation values must themselves be attained with subgame perfect equilibria. In terms of these objects, (b) is an incentive constraint

<sup>4</sup> The King was complaining about an expectations trap; see Chari, Christiano, and Eichenbaum (1998).

inspiring the government to adhere to the equilibrium:

$$\begin{aligned} v &= (1 - \delta)r(y, y) + \delta v' \\ &\geq (1 - \delta)r(y, \eta) + \delta \tilde{v}, \quad \forall \eta \in Y. \end{aligned}$$

This states that the government receives more if it adheres to an action called for by its strategy than if it departs. Part (a) of the definition requires that the continuation values be values for subgame perfect equilibria. The definition is circular, because the same class of objects, namely equilibrium values  $v$ , occur on each side of (23). Circularity comes with recursivity.

Abreu, Pearce, and Stacchetti (APS) (1986, 1990) showed how recursive equilibria of form (19) can attain all subgame perfect equilibrium values. APS's innovation was to shift the focus away from the set of equilibrium strategies and toward the set of values  $V$  attainable with subgame equilibrium strategies. They described a set  $V$  such that for all  $v \in V$ ,  $v$  is the value associated with a subgame perfect equilibrium. APS developed a recursive algorithm for computing the set  $V$ .

### *The method of Abreu-Pearce-Stacchetti*

Dynamic programming solves single-agent recursive optimization problems by finding an optimal value function. The optimal value function is computed by iterating on the Bellman equation, which maps continuation values giving tomorrow's discounted value as a function of tomorrow's state variables into today's discounted value as a function of today's state variables. The mapping from continuation values to current values embodies the backward induction of dynamic programming. Iterations on the Bellman equation converge to make continuation value (functions) equal value (functions).<sup>5</sup>

<sup>5</sup> It is helpful to indicate this structure within John McCall's celebrated model of sequential labor market search. In the McCall model, each period an unemployed worker receives an offer to work forever at a wage  $w$  drawn from the same distribution with cumulative density function  $F(w)$ . While unemployed,

APS adapted dynamic programming to characterize the set of equilibrium values in terms of an operator mapping sets of continuation values into sets of values. They computed the set of equilibrium values by iterating to convergence on that operator. Their method starts with a candidate set  $W \subset \mathbb{R}$ , where any  $w \in W$  can be assigned as a continuation value for adhering to or for deviating from the recommendation of the strategy. The idea is to start with a large but bounded set  $W$ ,<sup>6</sup> then to select a first period rational expectations outcome  $(y, y)$  and two associated continuation values drawn from  $W$ , a  $w_1$  for adhering to  $(y, y)$ , and a  $w_2$  associated with deviating from  $(y, y)$ .<sup>7</sup> The candidate continuation value  $w_1$  must be high enough, and  $w_2$  low enough, to make the government want to adhere to  $(y, y)$ , given those continuation values. Out of the two continuation values  $w_1, w_2 \in W \times W$ , this construction produces

the worker receives  $c$  in unemployment compensation. The worker wants to maximize his expected discounted income, with discount factor  $\delta \in (0, 1)$ , where income equals  $c$  if unemployed, and the wage  $w$  if employed. Once he accepts a job, the worker stays in it forever. Let  $v$  be the optimal value of this problem for an unemployed worker who is about to draw an offer for this period. The Bellman equation for this problem is

$$v = \int_w \max \left\{ \frac{w}{1 - \delta}, \delta v + c \right\} dF(w).$$

Letting  $T(v)$  be the function defined on the right side, this equation can be expressed as the fixed point

$$v = T(v).$$

The fixed point can be computed by iterating to convergence on

$$v_{j+1} = T(v_j),$$

starting from  $v_0 = 0$ . Here  $T$  maps a continuation value  $v_j$  into a current value  $v_{j+1}$ . This iteration does backward induction.

<sup>6</sup> The set has to be bounded because with an unbounded set of prospective continuation values, any value could seemingly be supported; but there would exist no associated outcome path that could attain it via equation (18).

<sup>7</sup> Remember that a rational expectations outcome is  $(y, y)$  because the private sector knows the government's strategy.

a value  $w$  satisfying  $w = (1 - \delta)r(y, y) + \delta w_1 \geq (1 - \delta)r(y, \eta) + \delta w_2$ ,  $\forall \eta \in Y$ . Thus, the operator produces candidate values from candidate continuation values.

The construction of values out of continuation values includes the incentive constraints imposed in condition (b) of the definition of a subgame perfect equilibrium, but it ignores condition (a) because the continuation values are arbitrary. APS thus conceive an operator  $B$  that maps an arbitrary compact set of potential continuation values into a set of values. They let  $B(W)$  denote this set.

To find subgame perfect equilibrium values, condition (a) of the definition must be incorporated. The challenge is to guarantee that each continuation value comes from a SPE and thus to assure that the set of subgame equilibrium values  $V$  satisfies  $V = B(V)$ . This expresses that subgame perfect equilibrium values are supported by subgame perfect equilibrium continuation values. The set  $V$  is a fixed point of the operator  $B$  mapping continuation values to values. APS showed that  $V$  is the largest fixed point of  $B$  and how it can be computed by iterating on  $B$ , starting from a large enough initial set  $W_0$  of candidate continuation values.

APS use two concepts. Admissibility denotes a value and an associated first period outcome  $y$  attainable with an arbitrary set of continuation values. Self-generation is about closure under the  $B$  operator.

**ADMISSIBILITY:** A pair  $(y, w)$  is *admissible* with respect to the set of continuation values  $W$  if there exist continuation values  $w_1, w_2 \in W \times W$  such that  $w = (1 - \delta)r(y, y) + \delta w_1 \geq (1 - \delta)r(y, \eta) + \delta w_2$ ,  $\forall \eta \in Y$ .<sup>8</sup>

For an arbitrary set of candidate continuation values  $W$ , the  $B(W)$  operator of APS selects the associated  $w$  pieces for all

<sup>8</sup> An equivalent definition formally closer to APS's would be to state the definition in terms of the triple  $(x, y, w)$  and to add the requirement that  $(x, y)$  forms a rational expectations equilibrium.



admissible pairs  $(y, w)$ . A candidate first-period outcome  $y$  is associated with each element  $\tilde{w} \in W$ .

**SELF-GENERATION:** The set  $W$  of prospective continuation values is *self-generating* if  $W \subseteq B(W)$ .

Each value in a self-generating set is supported by continuation values drawn from the set. Admissibility builds in feature (b) and self generation builds in feature (a) of the definition of a SPE. It follows that the set of SPE values is self-generating.

Imitating dynamic programming, APS iterate on  $B$  to find the largest self-generating set. APS show that: (1) the set of SPE values is the largest self-generating set; (2)  $B$  maps compact sets into compact sets; (3)  $B$  is monotone, meaning that if  $W_2 \subseteq W_1$ , then  $B(W_2) \subseteq B(W_1)$ ; and (4) starting from any  $W_0$  such that  $W_1 = B(W_0) \subseteq W_0$ , the algorithm  $W_j = B(W_{j-1})$  converges monotonically to  $V = B(V)$ .

Below we describe another way to find the set of SPE values.

### *Examples of recursive SPE*

To illustrate possible outcomes, I construct several SPE equilibria using a guess and verify technique. First guess  $(v_1, \sigma_1, \sigma_2)$  in (19), then verify parts (a) and (b) of the definition of a SPE.

The examples parallel the historical development of the theory. (1) The first example is infinite repetition of a one-period Nash outcome. (2) Barro and Gordon (1982) and Stokey (1989) used the value from infinite repetition of the Nash outcome as a continuation value to deter deviation from the Ramsey outcome. For sufficiently high discount factors, the continuation value associated with repetition of the Nash outcome can deter the government from deviating from infinite repetition of the Ramsey outcome, but not for low discount factors. (3) Abreu (1986) and Stokey (1991) showed that Abreu's 'stick and carrot'

strategy can have a value lower than repetition of the Nash outcome. (4) We display a pair of simple programming problems to find the best and worst SPE values.

*Infinite repetition of Nash outcome*

It is easy to construct an equilibrium whose outcome path forever repeats the one-period Nash outcome. Let  $v^N = r(y^N, y^N)$ . The proposed equilibrium is

$$\begin{aligned} v_1 &= v^N, \\ \sigma_1(v_t) &= y^N \quad \forall v_t, \text{ and} \\ \sigma_2(v_t, x_t, y_t) &= v^N, \quad \forall (v_t, x_t, y_t). \end{aligned}$$

Here for each  $t$ ,  $v^N$  simultaneously plays the roles of  $v, v'$ , and  $\bar{v}$  in condition (b). Condition (a) is satisfied by construction, and condition (b) collapses to

$$r(y^N, y^N) \geq r(y^N, B(y^N)),$$

which is satisfied at equality by the definition of a best response function. The equilibrium outcome forever repeats Kydland and Prescott's time consistent equilibrium.

*Infinite repetition of a better-than-Nash outcome*

Let  $v^b$  be a value associated with outcome  $y^b$  such that  $v^b = r(y^b, y^b) > v^N$ . Suppose further that

$$r(y^b, B(y^b)) - r(y^b, y^b) \leq \frac{\delta}{1 - \delta}(v^b - v^N). \quad (24)$$

The left side is the one-period return to the government from deviating from  $y^b$ ; the right side is the difference in present values associated with conforming to the plan versus reverting forever to the Nash equilibrium. When the inequality is satisfied, the

equilibrium presents the government with an incentive not to deviate from  $y^b$ . Then a SPE is

$$\begin{aligned} v_1 &= v^b; \\ \sigma_1(v) &= \begin{cases} y^b, & \text{if } v = v^b; \\ y^N, & \text{otherwise;} \end{cases} \\ \sigma_2(v, x, y) &= \begin{cases} v^b, & \text{if } (v, x, y) = (v^b, y^b, y^b); \\ v^N, & \text{otherwise.} \end{cases} \end{aligned}$$

This strategy specifies outcome  $(y^b, y^b)$  and continuation value  $v^b$  so long as  $v^b$  is the value promised at the beginning of the period. Any deviation from  $y^b$  generates continuation value  $v^N$ . Inequality (24) validates condition (b) of the definition of SPE.

Barro and Gordon (1982) considered a version of this equilibrium in which (24) is satisfied with  $v^b = v^R, y^b = y^R$ . In this case, anticipated reversion to Nash forever supports Ramsey forever. When (24) is *not* satisfied for  $v^b = v^R, y = y^R$ , we can solve for the best SPE value  $\tilde{v}$  supportable by infinite reversion to Nash (with associated action  $\tilde{y}$ ) from

$$\tilde{v} = r(\tilde{y}, \tilde{y}) = (1 - \delta)r(\tilde{y}, B(\tilde{y})) + \delta v^N > v^N. \quad (25)$$

The payoff from following the strategy equals that from deviating and reverting to Nash. Any value lower than this can be supported, but none higher.

In a related context, Abreu (1986) searched for a way to support something better than  $v^b$  when  $v^b < v^R$ . First, one must construct an equilibrium that yields a value worse than permanent repetition of the Nash outcome. The expectation of reverting to this equilibrium supports something better than  $v^b$  in (25).

Somehow the government must be induced temporarily to generate inflation higher than the Nash outcome, meaning that

the government is tempted to lower the inflation rate. An equilibrium system of expectations has to be constructed that makes the government expect to do better in the future only by conforming to expectations that it temporarily generate higher inflation than the Nash level.

*Something worse: a stick and carrot strategy*

We want a continuation value  $v^*$  for deviating to support the first-period outcome  $(y^\#, y^\#)$  and attain the value

$$\tilde{v} = (1 - \delta)r(y^\#, y^\#) + \delta v^R \geq (1 - \delta)r(y^\#, B(y^\#)) + \delta v^*. \quad (26)$$

Abreu (1986) proposed to set  $v^* = \tilde{v}$ . That is, the continuation value caused by deviating from the first-period action equals the original value. This 'stick and carrot' strategy attains a value worse than repetition of Nash by promising a continuation value that is better.

A strategy that attains  $\tilde{v}$  is

$$\begin{aligned} v_1 &= \tilde{v} \\ \sigma_1(v) &= \begin{cases} y^R & \text{if } v = v^R; \\ y^\# & \text{otherwise;} \end{cases} \\ \sigma_2(v, x, y) &= \begin{cases} v^R & \text{if } (x, y) = (\sigma_1(v), \sigma_1(v)); \\ \tilde{v} & \text{otherwise.} \end{cases} \end{aligned} \quad (27)$$

The consequence of deviating from the bad prescribed first-period government action  $y^\#$  is to restart the equilibrium.

DEFINITION: A recursive SPE  $(\sigma, v_1)$  is *self-enforcing* if in (27)

$$\sigma_2(v, \sigma_1(v), \eta) = v_1 \quad \forall \quad \eta \neq \sigma_1(v).$$

DEFINITION: A recursive SPE  $(\sigma, v_1)$  is *self-rewarding* if in (27),

$$\sigma_2(v, \sigma_1(v), \sigma_1(v)) = v.$$

*The worst SPE*

APS (1990) showed how to find the entire set of equilibrium values  $V$ . In the current setting, their ideas imply:

1. The set of equilibrium values  $V$  attainable by the government is a compact subset  $[\underline{v}, \bar{v}]$  of  $[r(y^\#, y^\#), 0]$ .
2. The worst equilibrium value can be computed from a simple programming problem.
3. Given the worst equilibrium value, the best equilibrium value can be computed from a programming problem.
4. Given a  $v \in [\underline{v}, \bar{v}]$ , it is easy to construct an equilibrium that attains it.

Here is how these ideas apply.

PROPOSITION: The worst SPE is self-enforcing.

Let  $\underline{v}$  be the minimum value associated with a SPE with value  $\bar{v}$  the maximum value and  $V = [\underline{v}, \bar{v}]$ . Evidently,  $\underline{v}$  satisfies

$$\underline{v} = \min_{y \in Y, v_1 \in V} [(1 - \delta)r(y, y) + \delta v_1]$$

subject to

$$(1 - \delta)r(y, y) + \delta v_1 \geq (1 - \delta)r(y, B(y)) + \delta \underline{v}. \quad (28)$$

In (28), we use the worst SPE as the continuation value in the event of a deviation. The minimum will be attained when the constraint is binding, which implies that  $\underline{v} = r(y, B(y))$ , for some government action  $y$ . Thus, the problem of finding the worst SPE reduces to solving

$$\underline{v} = \min_{y \in Y} r(y, B(y));$$

then computing  $v_1$  from  $(1 - \delta)r(y^\#, y^\#) + \delta v_1 = \underline{v}$  where  $y^\# = \arg \min r(y, B(y))$ ; and finally checking that  $v_1$  is itself a value

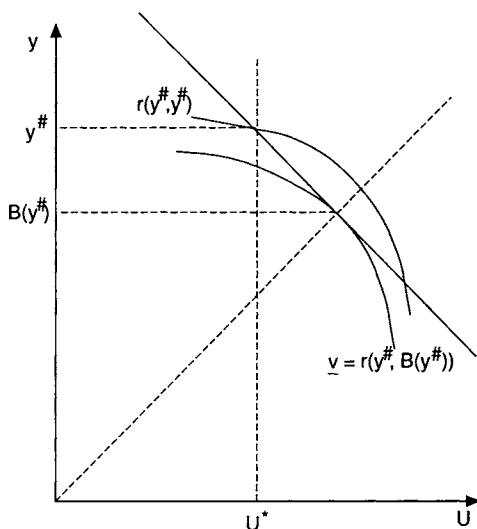
associated with a SPE. To check this condition, we need to know  $\bar{v}$ . Before we show how to check this, we note that this construction shows the following:

PROPOSITION: The best SPE is self-rewarding.

After we have computed a candidate for the worst SPE value  $\underline{v}$ , we can compute a candidate for the *best* value  $\bar{v}$  by solving the programming problem

$$\begin{aligned} \bar{v} = \max_{y \in Y} & r(y, y) \\ \text{s.t. } & r(y, y) \geq (1 - \delta)r(y, B(y)) + \delta \underline{v}. \end{aligned}$$

Here we are assuming that  $\bar{v}$  is the maximizing continuation value available to reward adherence to the policy, so that  $\bar{v} = (1 - \delta)r(y, y) + \delta \bar{v}$ . Let  $y^b$  be the maximizing value of  $y$ . Once we have computed  $\bar{v}$ , we can check that the continuation value  $v_1$  for supporting the worst value is within our candidate set  $[\underline{v}, \bar{v}]$ . If it is, we have succeeded in constructing  $V$ .



**Figure 4.1.** Calculation of the worst subgame equilibrium value  $\underline{v}$ .

### *Multiplicity*

Two layers of multiplicity inhabit this theory. First, there is a continuum of equilibrium values. Second, many outcome paths have the same value. To illustrate how multiple outcome paths  $(y, y)$  can attain the same equilibrium value, we construct several equilibria that attain the worst equilibrium value.

*Attaining the worst, method 1.*

Many SPE's attain the worst value  $\underline{v}$ . To compute one such SPE strategy, we can use the following recursive procedure:

- (i) Set the first-period promised value  $v_1 = \underline{v} = r(y^\#, B(y^\#))$ . See Figure 4.1. The highest feasible inflation rate is  $y^\#$ . The worst one-period value that is consistent with rational expectations is  $r(y^\#, y^\#)$ . Given expectations  $x = y^\#$ , the government is tempted toward  $B(y^\#) < y^\#$ , which yields one-period utility to the government of  $r(y^\#, B(y^\#))$ .

Then use  $\underline{v}$  as continuation value in the event of a deviation, and construct an increasing sequence of continuation values to reward adherence, as follows.

- (ii) Solve  $\underline{v} = (1 - \delta)r(y^\#, y^\#) + \delta v_2$  for continuation value  $v_2$ .
- (iii) For  $j = 2, 3, \dots$ , continue solving  $v_j = (1 - \delta)r(y^\#, y^\#) + \delta v_{j+1}$  for the continuation values  $v_{j+1}$  so long as  $v_{j+1} \leq \bar{v}$ . If  $v_{j+1}$  threatens to violate this constraint at step  $j = \bar{j}$ , then go to step (iv).
- (iv) Use  $\bar{v}$  as the continuation value, and solve  $v_j = (1 - \delta)r(\tilde{y}, \tilde{y}) + \delta \bar{v}$  for the prescription  $\tilde{y}$  to be followed if promised value  $v_j$  is encountered.
- (v) Set  $v_{j+s} = \bar{v}$  for  $s \geq 1$ .

*Attaining the worst, method 2.*

To construct another equilibrium supporting the worst SPE value, follow steps (i) and (ii) above, and follow step (iii) also, except that we continue solving  $v_j = (1 - \delta)r(y^\#, y^\#) + \delta v_{j+1}$  for the continuation values  $v_{j+1}$  only so long as  $v_{j+1} < v^N$ . As soon as  $v_{j+1} = v^{**} > v^N$ , we use  $v^{**}$  as both the promised value and the continuation value there after. Whenever  $v^{**} = r(y^{**}, y^{**})$  is the promised value,  $\sigma_1(v^{**}) = y^{**}$ .



Attaining the worst, method 3.

Here is another subgame perfect equilibrium that supports  $\underline{v}$ . Proceed as in step (i) to find continuation value  $v_2$ . Now set all the subsequent values and continuation values to  $v_2$ , with associated first-period outcome  $\tilde{y}$  which solves  $v_2 = r(\tilde{y}, \tilde{y})$ . It can be checked that the incentive constraint is satisfied with  $\underline{v}$  the continuation value in the event of a deviation.

Numerical examples

Set  $[\delta \ \theta \ U^* \ y^\#] = [.95 \ 1.2500 \ 5.5000 \ 10.0000]$ . Compute  $[x^N \ x^R] = [6.8750 \ 0]$ ,  $[v^R \ v^N \ \underline{v} \ v_{abreu}] = [-15.1250 \ -38.7578 \ -63.2195 \ -17.6250]$ . We attain the worst subgame perfect equilibrium value  $\underline{v}$  with any of the sequences of time- $t$  (promised value, action) pairs depicted in Figures 4.2, 4.3, or 4.4. These figures illustrate the preceding three types of equilibria supporting the worst equilibrium value  $\underline{v}$ .

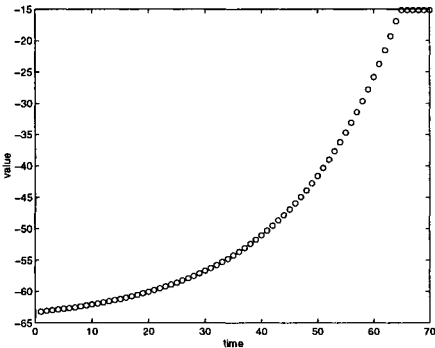


Figure 4.2a. Continuation values (on ordinate axis) of a SPE that attains  $\underline{v}$ .

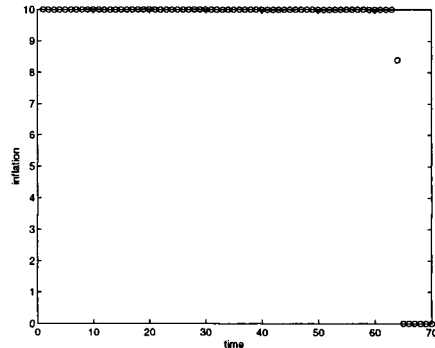


Figure 4.2b. Inflation values associated with those continuation values.

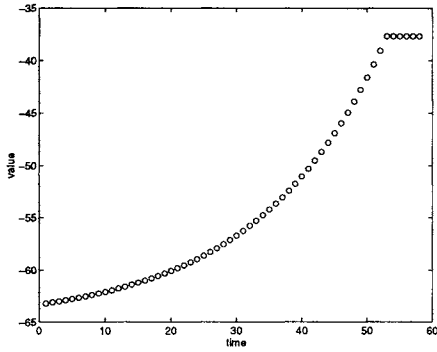


Figure 4.3a. Continuation values (on ordinate axis) of a SPE that attains  $\underline{v}$ .

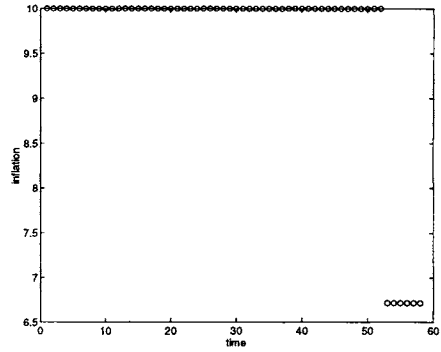


Figure 4.3b. Inflation values associated with those continuation values.

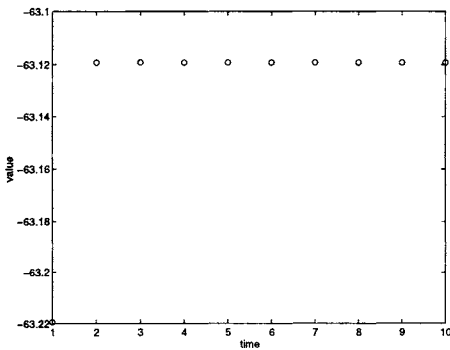


Figure 4.4a. Continuation values (on ordinate axis) of a SPE that attains  $\underline{v}$ .

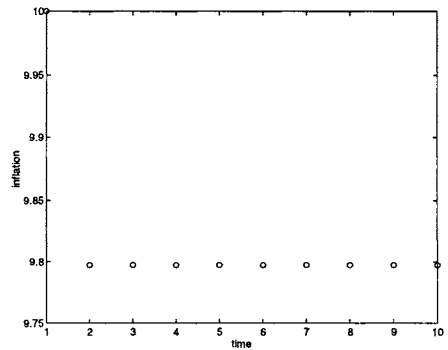


Figure 4.4b. Inflation values associated with those continuation values.

The cutoff value of  $\delta$  below which reversion to Nash fails to support Ramsey forever is .2807. For  $\delta = .2$ , Abreu's stick and carrot equilibrium that calls for the outcome  $(y^\#, y^\#)$  for one period to be followed by  $(0, 0)$  forever yields value  $v_{\text{Abreu}} = -55.125$ . For  $\delta = .2$ , the simple Abreu stick-and-carrot strategy supports repetition of Ramsey even though infinite reversion to Nash fails to do so.

*Interpretations*

The literature on credible plans bears mixed prospects for better than Nash outcomes and for our triumph of natural-rate theory story. The theory has many equilibria, some better and others worse than Kydland and Prescott's. The multitude of outcomes mutes the model empirically and undermines the intention of early researchers to use the rational expectations hypothesis to eliminate parameters describing expectations.

In his 1979 review of an OECD report on macroeconomic policies edited by Paul McCracken (1977), Lucas protested the report's recommendation that 'Governments should try to promote good expectations' as though expectations were an extra set of policy instruments. That recommendation misinterpreted what rational expectations meant for macroeconomics in 1979. Most rational expectations macroeconomic models then assumed exogenous government policies and made expectations functions of government policies. Cross-equation restrictions linked expectations to laws of motion for government instruments and other exogenous variables.<sup>9</sup>

The literature on credible government policy turns systems of expectations into free parameters that influence outcomes but that cannot necessarily be chosen by the government or anybody else inside the model. The government complies with equilibrium expectations about its behavior. These models isolate systems of expectations in which governments have reputations that they want to sustain. They do not model how a reputation is acquired or altered. The government's strategy plays two roles: as a decision rule, and as a description of the private sector's expectations, which restrain the government. It is impossible to disentangle them within an equilibrium.

<sup>9</sup> For some environments, this statement has to be qualified in light of the possibility that equilibria are indexed by sunspots. See Woodford (1990) for references to that literature, and for a theory of how adaptive agents might settle upon such an equilibrium.

The authors of the McCracken report believed in multiplicity and manipulation while Lucas doubted both. The literature on credible plans supports multiplicity but not manipulation.

### *Remedies*

Reputation alone is a weak foundation for anti-inflation policy. This fact has inspired proposals to change the game to assure a good outcome. Kenneth Rogoff (1985) proposed altering the government's preferences by delegating conduct of inflation policy to someone who doesn't care about unemployment.<sup>10</sup> Assigning inflation policy to someone who is unaware even of a temporary trade-off between inflation and unemployment would be equally effective. Having sets of central bankers indexed by different inflation–unemployment preferences<sup>11</sup> can also improve the outcome.<sup>12</sup>

I now explore a different route to better outcomes beginning with the notion that the multiplicity of equilibria within models of credible policy stems from the rationality assigned to all participants in the system. To eradicate multiplicity, I retreat from perfection and move to models in which some people have a more limited understanding of the economy. These models are closer to what Lucas imagined when he criticized the McCracken report.

<sup>10</sup> Alan Blinder (1998) suggested a related idea: that the monetary authority be someone who knows the natural rate of unemployment each period, and who never wants unemployment to differ from it.

<sup>11</sup> Some of the potential policy makers are like Rogoff's conservative and others are like the authority in Kydland and Prescott's original model.

<sup>12</sup> See Barro and Gordon (1983) and Ball (1995).