

Monetary Policy & Anchored Expectations An Endogenous Gain Learning Model

Laura Gáti

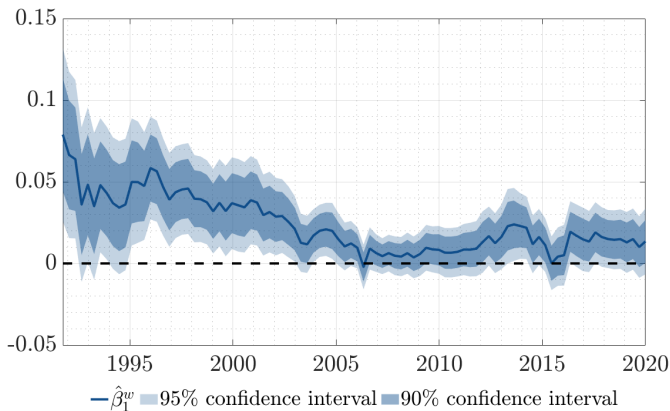
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Long-run expectations respond to short-run surprises

$$\Delta \underbrace{\mathbb{E}_t^i(\pi_{t+10\text{year}})}_{\substack{\text{10-year-ahead} \\ \text{inflation expectation} \\ \text{of SPF forecaster } i}} = \beta_0^w + \underbrace{\beta_1^w (\pi_t - \mathbb{E}_{t-1\text{year}}^i(\pi_t))}_{\substack{\text{1-year-ahead} \\ \text{inflation forecast error} \\ \text{of SPF forecaster } i}} + \epsilon_t^i$$



This paper

1. A model of **unanchored expectations**:

↪ sensitivity of long-run expectations to short-run fluctuations

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2. **Interaction** between **monetary policy** and **unanchored expectations**

- Embed anchoring theory in general equilibrium New Keynesian model
- Quantify unanchoring using data on inflation expectations
- Derive optimal monetary policy

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↪ sensitivity of long-run expectations to short-run fluctuations

2. **Interaction** between **monetary policy** and **unanchored expectations**

- Embed anchoring theory in general equilibrium New Keynesian model
- Quantify unanchoring using data on inflation expectations
- Derive optimal monetary policy

3. **Key takeaway**: optimal monetary policy

- anchors expectations to inflation target
- responds aggressively to movements in long-run expectations

NK model with long-run inflation expectations

- New Keynesian core: standard IS and Phillips curves

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n)$$
$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T)$$

Observables: (π, x, i) inflation, output gap, interest rate

Exogenous states: (r^n, u) natural rate and cost-push shock

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Observables: (π, x, i) inflation, output gap, interest rate

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- **Novelty of the paper:** inflation expectations process

$$\hat{\mathbb{E}}_t \pi_{t+1} = \bar{\pi}_{t-1} + \mathbb{E}_t \pi_{t+1}$$

\mathbb{E} : rational (model-consistent) expectations

$\hat{\mathbb{E}}$: nonrational expectations \rightarrow long-run inflation expectations $\bar{\pi}_{t-1}$

Evolution of long-run inflation expectations

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1}$$

$$f_{t|t-1} = \pi_t - \hat{\mathbb{E}}_{t-1} \pi_t$$

$k_t \in [0, 1]$ learning gain

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Endogenous gain as metric for unanchoring

$$k_t = \mathbf{g}(f_{t|t-1}), \quad \mathbf{g}'' > 0$$

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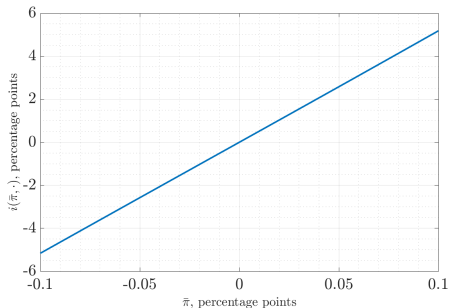
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Endogenous gain as metric for unanchoring

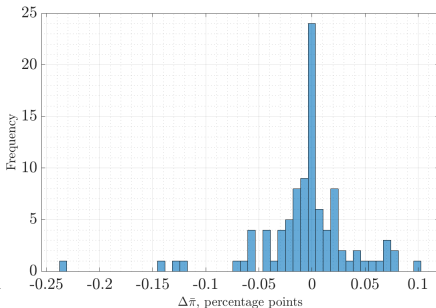
$$k_t = \mathbf{g}(f_{t|t-1}), \quad \mathbf{g}'' > 0$$

- If $k_t = 0 \rightarrow$ anchored expectations = rational expectations
- If $k_t = 1 \rightarrow$ short-run surprises translate 1:1 to long-run expectations

Optimal policy - responding to unanchoring



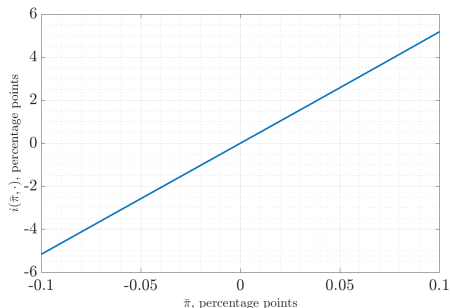
$i(\bar{\pi}, \text{all other states at their means})$



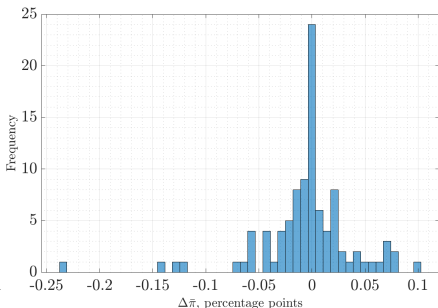
Stabilizing $\bar{\pi}$

5 bp movement in $\bar{\pi} \rightarrow 250$ bp movement in i

Optimal policy - responding to unanchoring



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Stabilizing $\bar{\pi}$

5 bp movement in $\bar{\pi} \rightarrow 250$ bp movement in i

Mode: 0.3 bp movement in $\bar{\pi}$

Unanchoring amplifies shocks

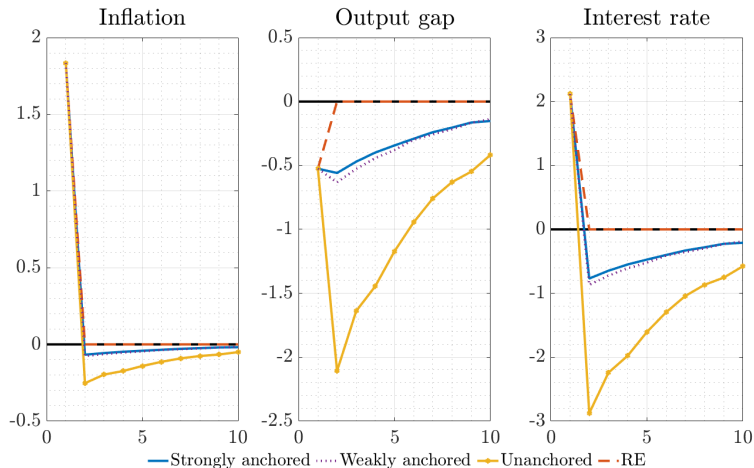


Figure: Impulse responses after a cost-push shock when policy follows a Taylor rule

Conclusion

First theory of **monetary policy** for potentially **unanchored expectations**

Model-based notion of **unanchoring**

- Sensitivity of long-run expectations to short-run fluctuations

Optimal **monetary policy**

- Anchors expectations by responding aggressively to long-run expectations

Thank you!

Questions? → gati@bc.edu

Appendix

Long-run expectations: responsive to short-run conditions?

Individual-level Survey of Professional Forecasters (SPF): for 1991-Q4 onward, estimate rolling regression

$$\Delta \bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \quad (1)$$

$\bar{\pi}_t$ 10-year ahead inflation expectation

$f_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1} \pi_t$ individual one-year-ahead forecast error

w indexes windows of 20 quarters

Time-varying responsiveness

$$\Delta \bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \quad (1)$$

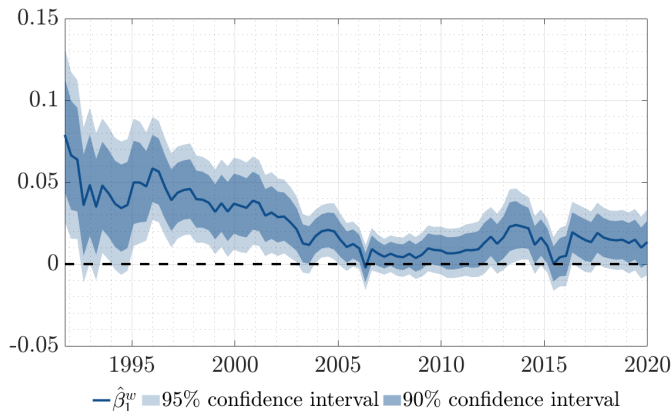


Figure: Time series of $\hat{\beta}_1^w$

Breakeven inflation



Figure: Market-based inflation expectations, various horizons, %

Correcting the TIPS from liquidity risk

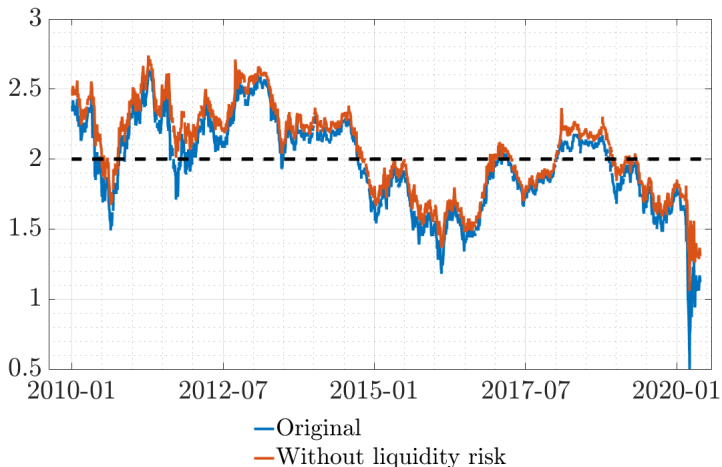


Figure: Market-based inflation expectations, 10 year, %

Robustness checks

$$\Delta \bar{\pi}_t = \beta_0 + \beta_1^w \pi_t + \epsilon_t \quad (1)$$

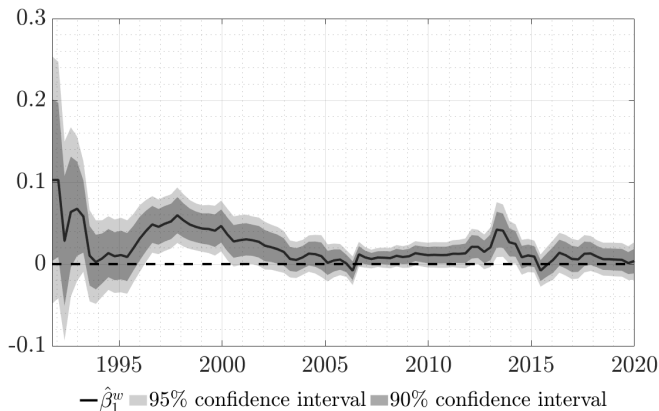


Figure: Time series of $\hat{\beta}_1^w$

Robustness checks - PCE core

$$\Delta \bar{\pi}_t = \beta_0^w + \beta_1^w f_{t|t-1} + \epsilon_t \quad (1)$$

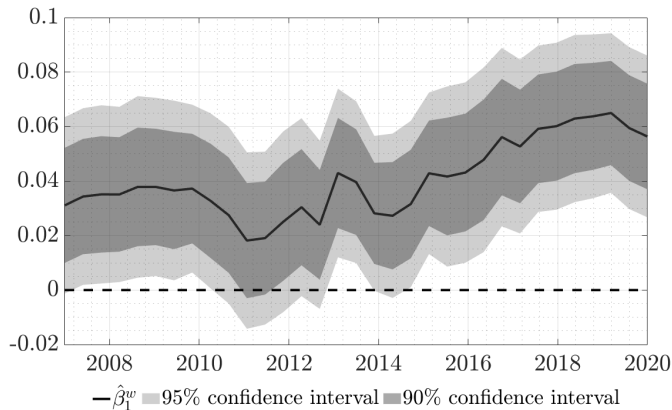


Figure: Time series of $\hat{\beta}_1^w$

Robustness checks - controlling for inflation levels

$$\Delta \bar{\pi}_t = \beta_0^w + \beta_1^w f_{t|t-1} + \beta_2^w \pi_t + \epsilon_t \quad (1)$$

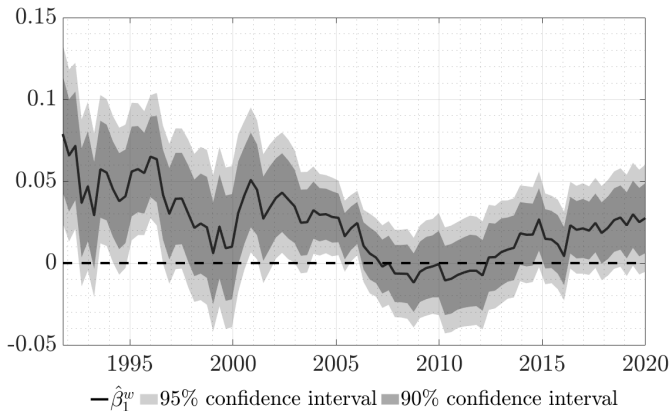


Figure: Time series of $\hat{\beta}_1^w$

Further evidence: disagreement

Figure: Livingston Survey of Firms:
Interquartile range of 10-year ahead inflation expectations

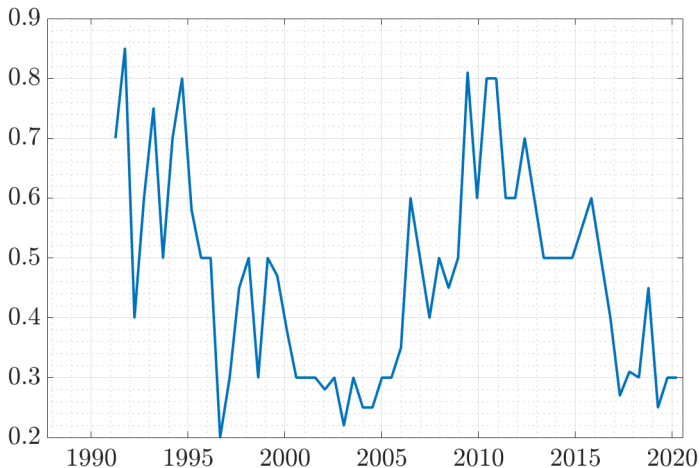
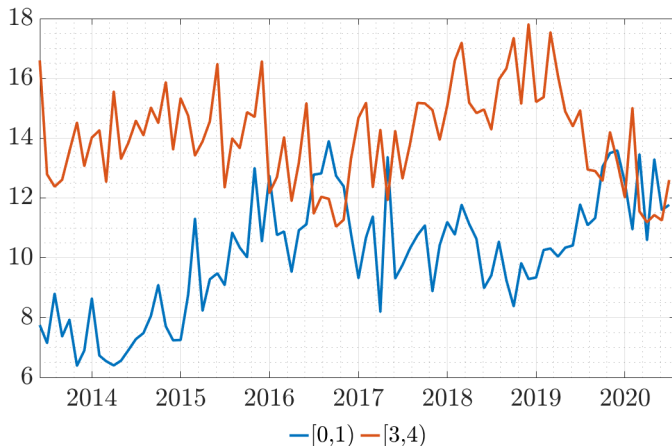
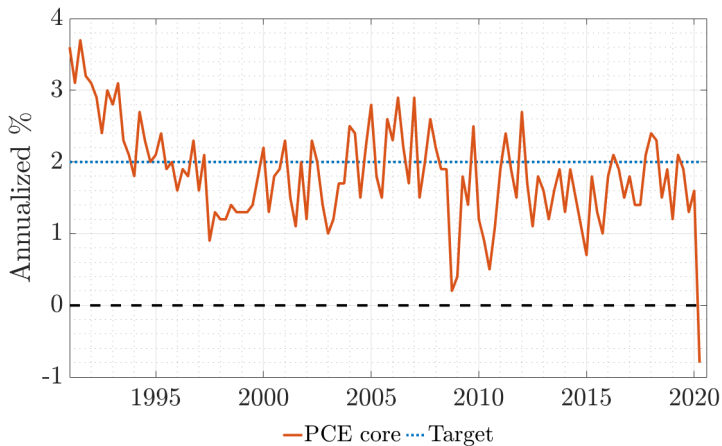


Figure: New York Fed Survey of Consumers:
Percent of respondents indicating 3-year ahead inflation will be in a particular range



Further evidence: introspection

Figure: PCE core inflation against the Fed's target



Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j)) dj \right] \quad (2)$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_t C_t^i \quad (3)$$

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t^j \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right] \quad (4)$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \quad (5)$$

Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \quad (6)$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \quad (7)$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \quad (8)$$

Solve for forecast error $f_t \equiv \pi_t - f_{t-1}$:

$$f_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k \rightarrow 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \quad (9)$$

Functional forms for \mathbf{g} in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \quad (10)$$

$p \equiv \text{Prob}(\text{liquidity trap regime})$
 y_{t-1} output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when } \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \quad (11)$$

θ_t criterion, $\bar{\theta}$ threshold value

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \quad (12)$$

Σ variance-covariance matrix of shocks

$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1} (f_{t|t-1}' f_{t|t-1}' - \omega_{t-1}) \quad (13)$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1} (f_{t|t-1}' \omega_t^{-1} f_{t|t-1} - \theta_{t-1}) \quad (14)$$

ω_t estimated forecast-error variance

General updating algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right) \right)' \quad (15)$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} [1 \quad s_{t-1}] - R_{t-1} \right) \quad (16)$$

Assumptions on $\mathbf{g}(\cdot)$

$$\mathbf{g}_{ff} \geq 0 \quad (17)$$

$\mathbf{g}(\cdot)$ convex in forecast errors.

Estimating form of gain function

- Calibrate parameters of New Keynesian core to literature
- Estimate flexible form of expectations process via simulated method of moments
(Duffie & Singleton 1990, Lee & Ingram 1991, Smith 1993)

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \mathbf{g}(f_{t|t-1}) f_{t|t-1} \quad (18)$$

- Moments: autocovariances of inflation, output gap, federal funds rate and 1-year ahead Survey of Professional Forecasters (SPF) inflation expectations at lags $0, \dots, 4$

Estimated expectations process

$$\bar{\pi}_t - \bar{\pi}_{t-1} = \mathbf{g}(f_{t|t-1}) f_{t|t-1} \quad (18)$$

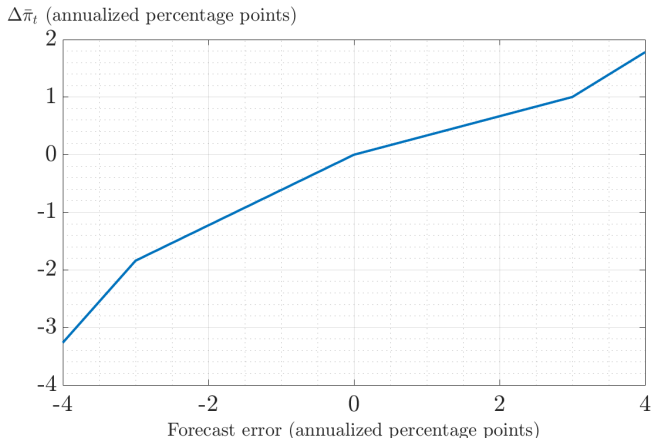


Figure: Changes in long-run inflation expectations as a function of forecast errors

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (18)$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \quad (19)$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \quad (20)$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \quad (21)$$

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \quad (22)$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \quad (23)$$

where ‘hats’ denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (22) backward to some date t , take expectations at t
 2. Sub in (23)
 3. Aggregate over households i
- Obtain (??)

Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \quad (24)$$

$$s_t = h s_{t-1} + \epsilon_t \quad (25)$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \quad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \quad (26)$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} y_{T+1} \quad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \quad (27)$$

Piecewise linear approximation to gain function

$$\mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}) \quad (28)$$

- $b_i(f_{t|t-1})$ = piecewise linear basis
- γ_i = approximating coefficient at node i

↪ Estimate $\hat{\gamma}$ via simulated method of moments

The expectation process over time

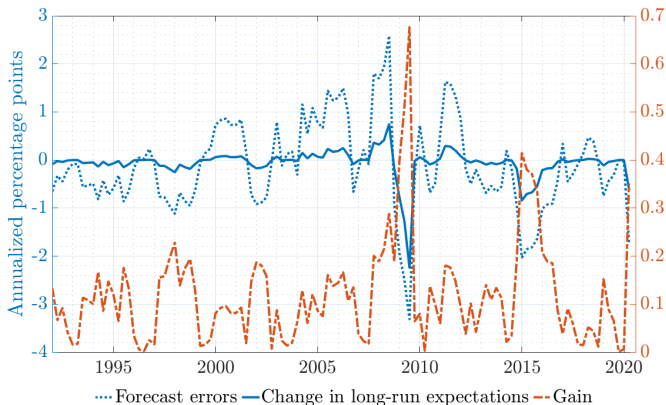


Figure: Time series of forecast errors, changes in long-run expectations and gain

Target criterion

Proposition

Let $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ at t . Then monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t$$

RE (discretion): move π_t and x_t to offset cost-push shocks

Target criterion

Proposition

Let $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial \mathbf{z}}$ at t . Then monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t - \Gamma(k) \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} = -\frac{\lambda_x}{\kappa} x_t$$

Adaptive learning: can move $\mathbb{E}_t x_{t+i}$ too if $k > 0$

► $\Gamma(k)$

Target criterion

Proposition

Let $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ at t . Then monetary policy optimally brings about the following target relationship between inflation and the output gap

$$\pi_t - \Omega \left(k_{t+f_t|t-1} \mathbf{g}_{\pi,t} \right) \left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j} - f_{t+1+j|t+j} \mathbf{g}_{\bar{\mathbf{b}},t+j}) \right) = -\frac{\lambda_x}{\kappa} x_t$$

Endogenous gain: ability to move $\mathbb{E}_t x_{t+i}$ depends on present and future degree of unanchoring

► Full expression, Ω

► No commitment

Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

► Why no commitment?

Corollary

Optimal policy under adaptive learning is time-consistent.

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t) \right. \\ \left. + \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \right\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa}x_t - \varphi_{2,t}(k_t + \mathbf{g}(\pi_t - f_{t-1})) = 0 \quad (29)$$

$$-2\beta\frac{\lambda}{\kappa}x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}(\pi_{t+1} - f_t)) = 0 \quad (30)$$

Target criterion system for anchoring function as changes of the gain

$$\begin{aligned} \varphi_{6,t} = & -cf_{t|t-1}x_{t+1} + \left(1 + \frac{f_{t|t-1}}{f_{t+1|t}}(1 - k_{t+1}) - f_{t|t-1}\mathbf{g}_{\pi,t}\right)\varphi_{6,t+1} \\ & - \frac{f_{t|t-1}}{f_{t+1|t}}(1 - k_{t+1})\varphi_{6,t+2} \end{aligned} \quad (31)$$

$$0 = 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{f_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{f_{t|t-1}}\varphi_{6,t+1} \quad (32)$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (32) is given by:

$$\varphi_{6,t} = -2\mathbb{E}_t \sum_{i=0}^{\infty} \left(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}\right) \prod_{j=0}^{i-1} \frac{\frac{k_{t+j}}{f_{t+j|t+j-1}}}{\frac{k_{t+j}}{f_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \quad (33)$$

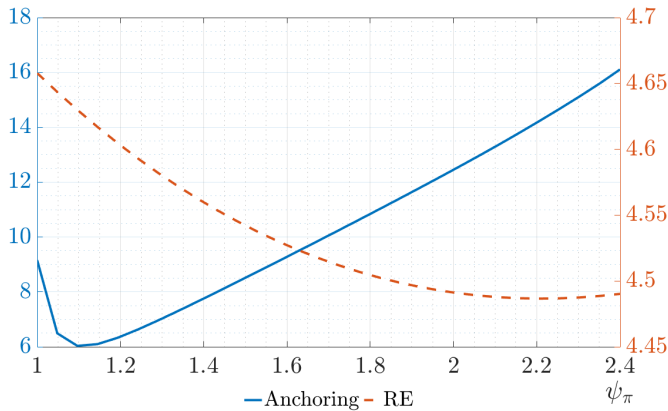
Optimal Taylor-coefficient on inflation

$$i_t = \psi_{\pi} \pi_t + \psi_x x_t \quad (34)$$

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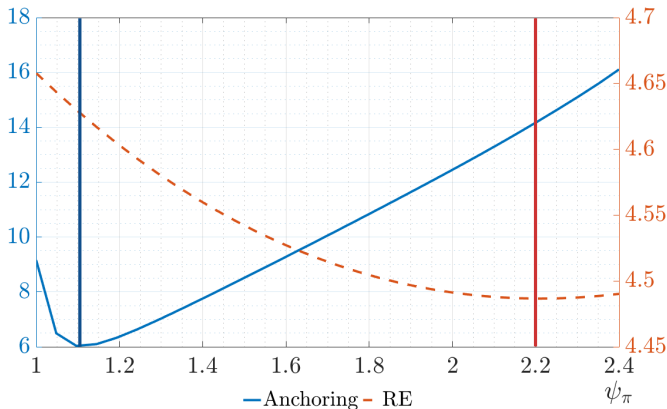
Figure: Central bank loss as a function of ψ_π



Optimal Taylor-coefficient on inflation

$$i_t = \psi_\pi \pi_t + \psi_x x_t \quad (34)$$

Figure: Central bank loss as a function of ψ_π



Anchoring-optimal coefficient: $\psi_\pi^A = 1.1$

RE-optimal coefficient: $\psi_\pi^{RE} = 2.2$

Why less aggressive? Future interest rate expectations

IS curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n)$$

- Current interest rate i_t : one channel of policy

Why less aggressive? Future interest rate expectations

IS curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} ((1-\beta)x_{T+1} - \sigma(\beta \textcolor{red}{i}_{T+1} - \pi_{T+1}) + \sigma r_T^n)$$

- Current interest rate i_t : one channel of policy
- Taylor rule implies interest rate expectation

$$\textcolor{red}{\hat{\mathbb{E}}}_t i_{t+k} = \psi_\pi \hat{\mathbb{E}}_t \pi_{t+k} + \psi_x \hat{\mathbb{E}}_t x_{t+k} \quad (35)$$

Why less aggressive? Future interest rate expectations

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- If private sector understands and believes Taylor rule, expected future interest rates additional channel of policy (Eusepi, Giannoni & Preston 2018)

Respond but not too much

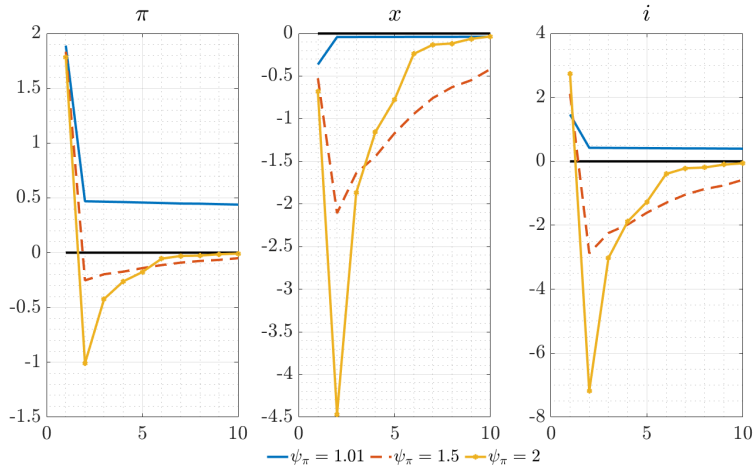


Figure: Impulse responses for unanchored expectations for various values of ψ_π