Materials 40 - Still trying to understand why not identified

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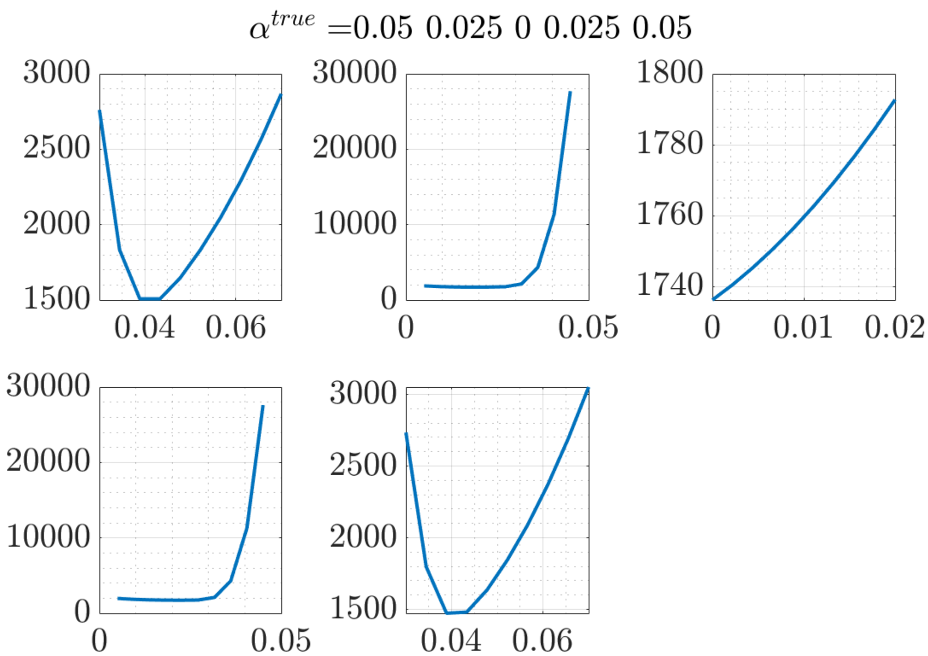
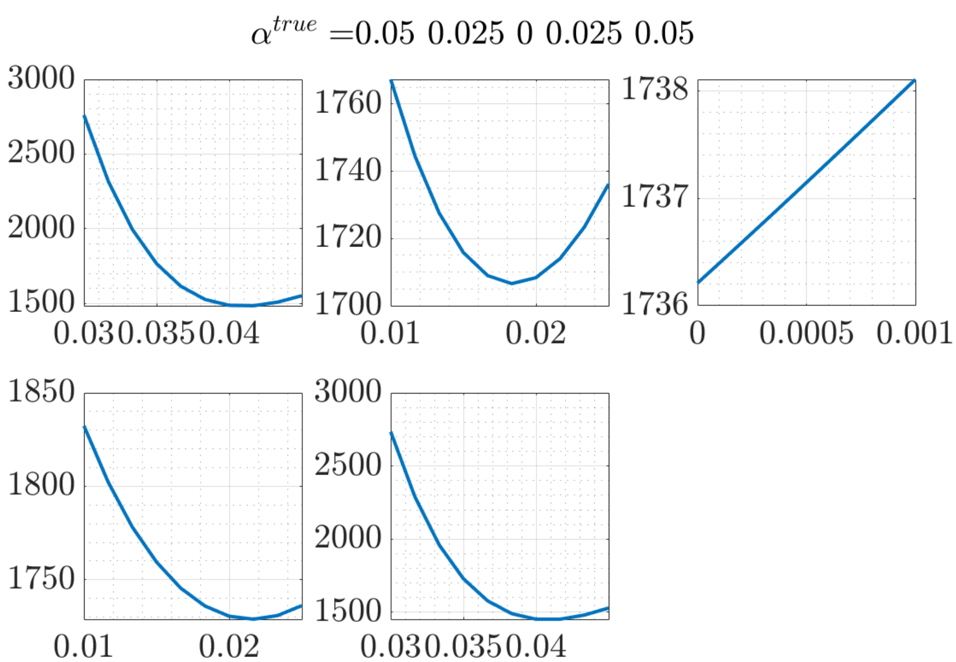
Overview

1 Loss when varying one parameter, more details

Figure 1: Loss for $N = 100$, NOT using 1-step ahead forecasts of inflation, estimate mean moments once, imposing convexity with weight 100K, w/o measurement error, truth with $nfe = 5, fe \in (-2, 2)$

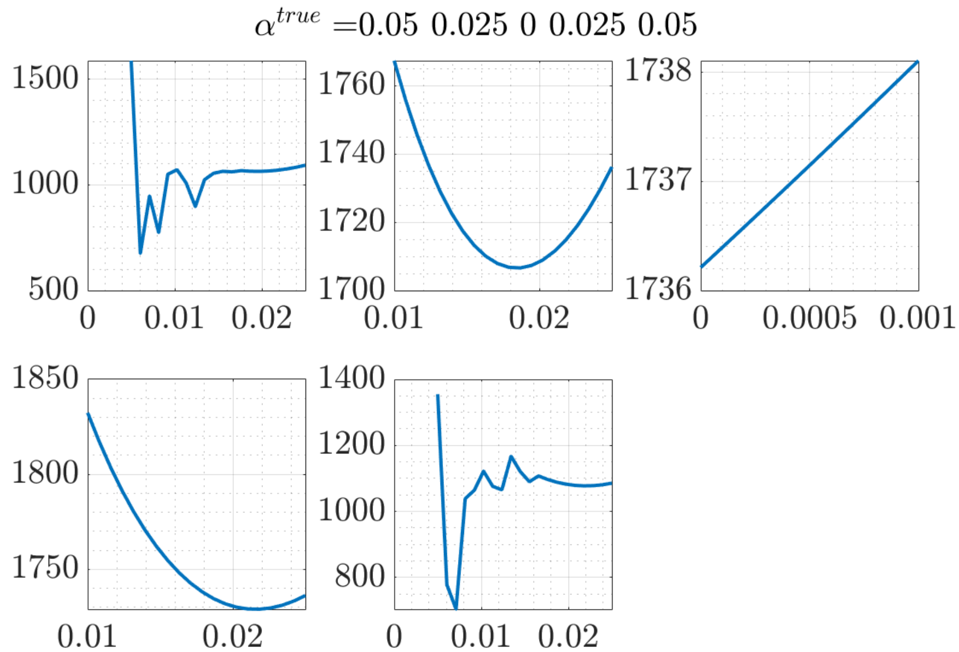


(a) Reference figure from Materials 39: increment around true value 0.02

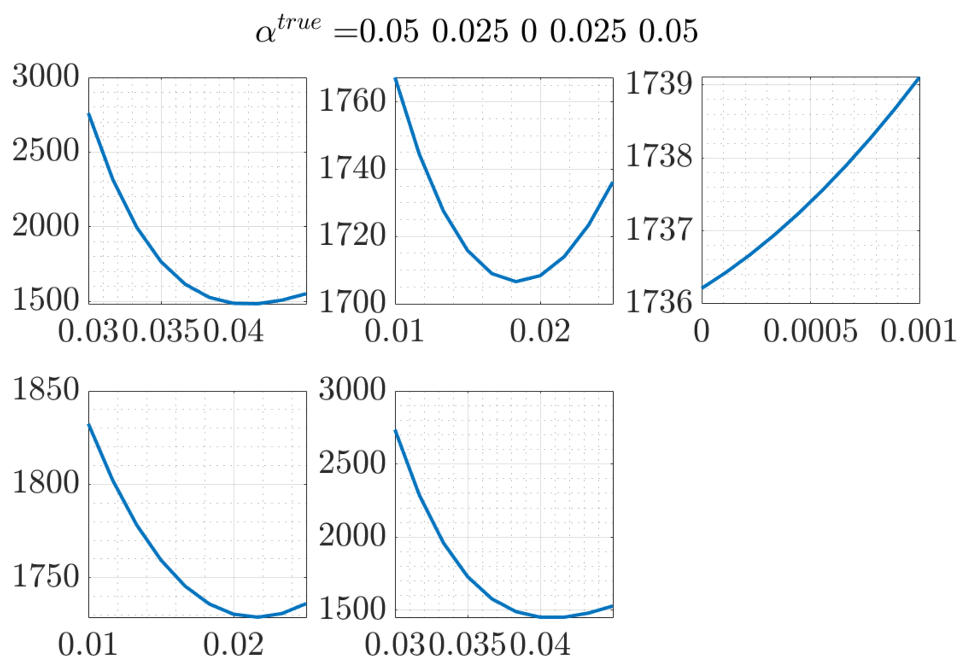


(b) Minima: $(0.041; 0.0187; 0; 0.0218; 0.041)$

Figure 2: Variations I

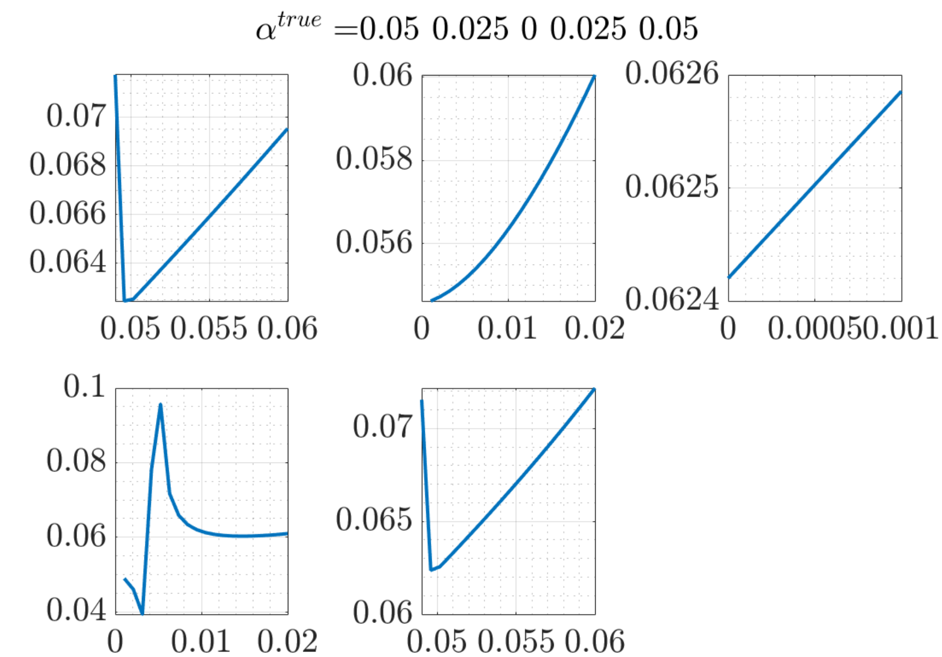


(a) Shutting of the convexity moment (weight 0)

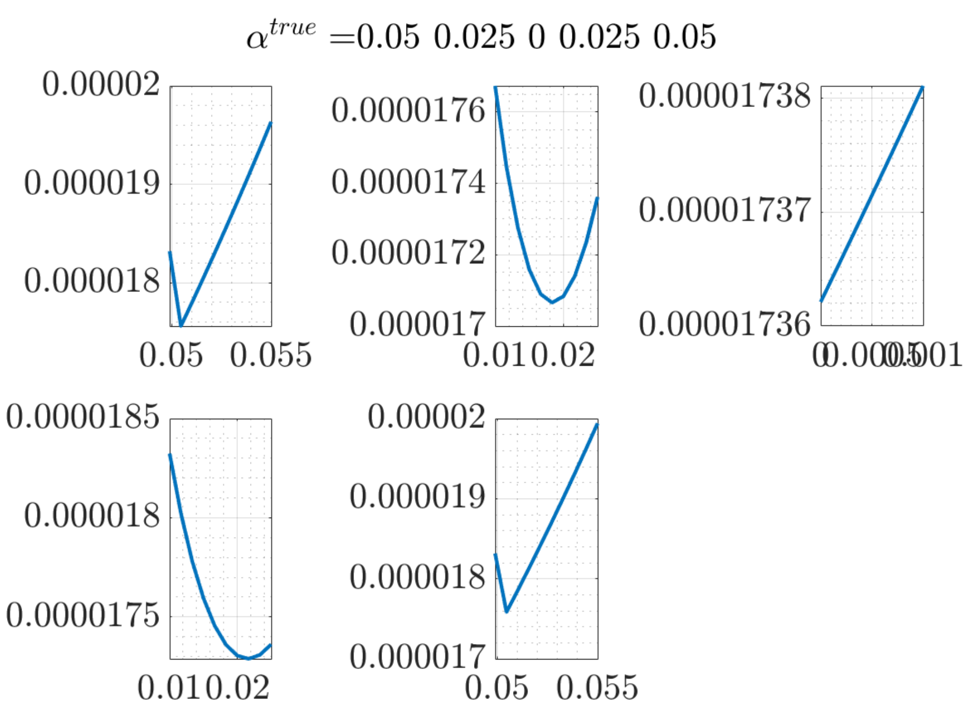


(b) Adding the 0 at 0 restriction (weight 1000)

Figure 3: Variations II

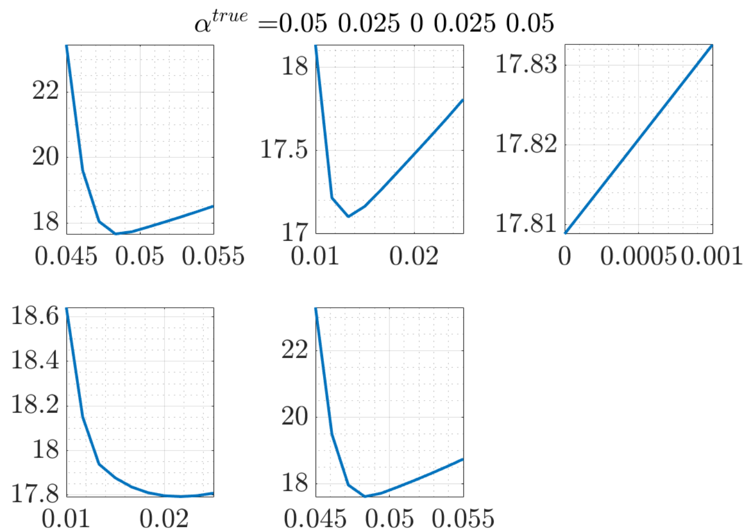


(a) Add measurement error

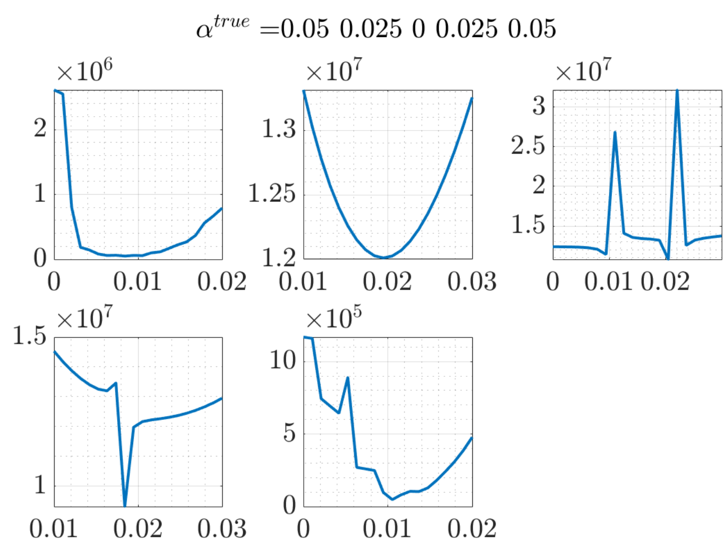


(b) Rescale W

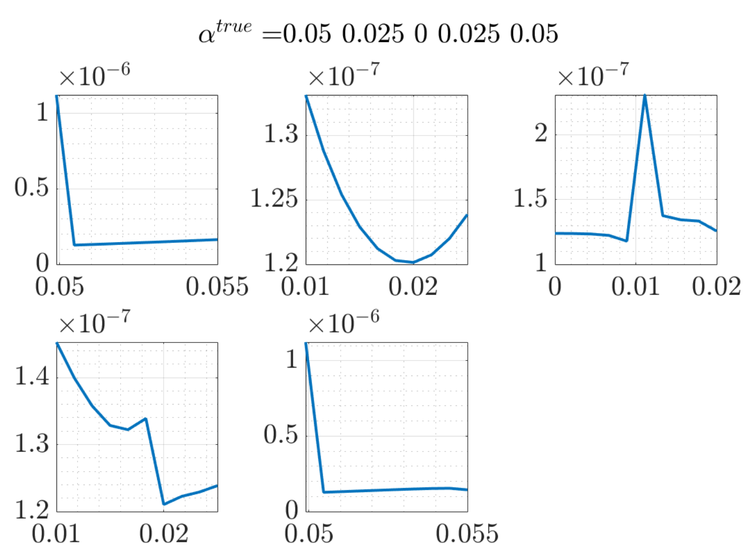
Figure 4: Variations III



(a) A shorter dataset w/o expectations

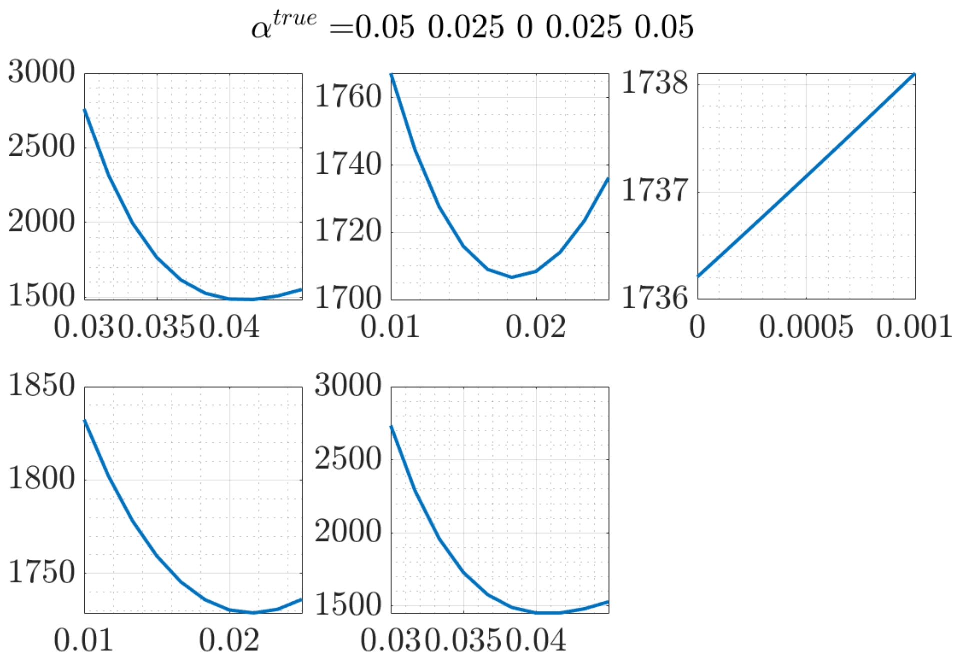


(b) A shorter dataset adding expectations

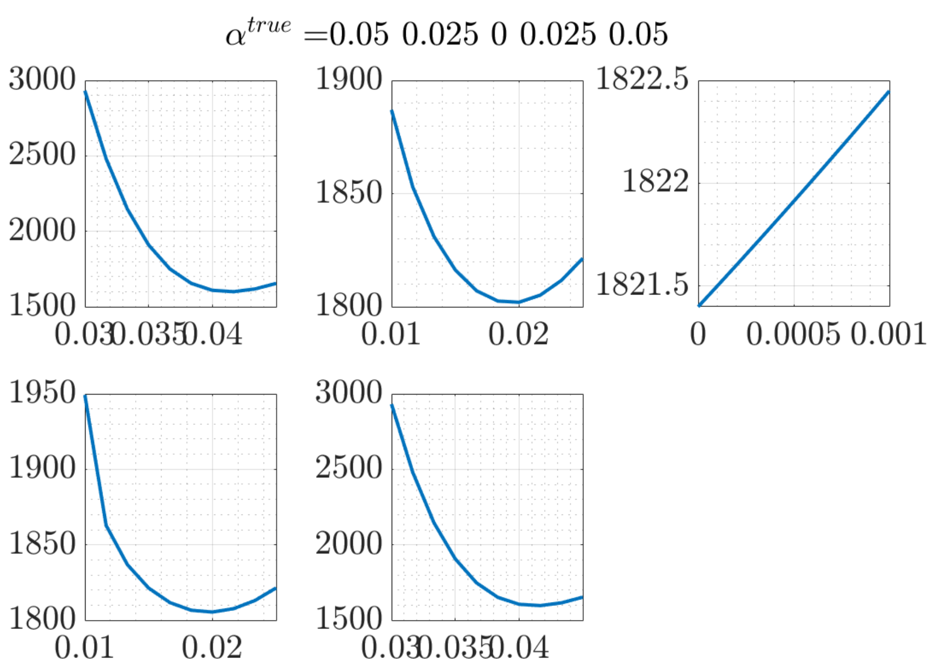


(c) A shorter dataset adding expectations, rescaling W

Figure 5: Variations IV



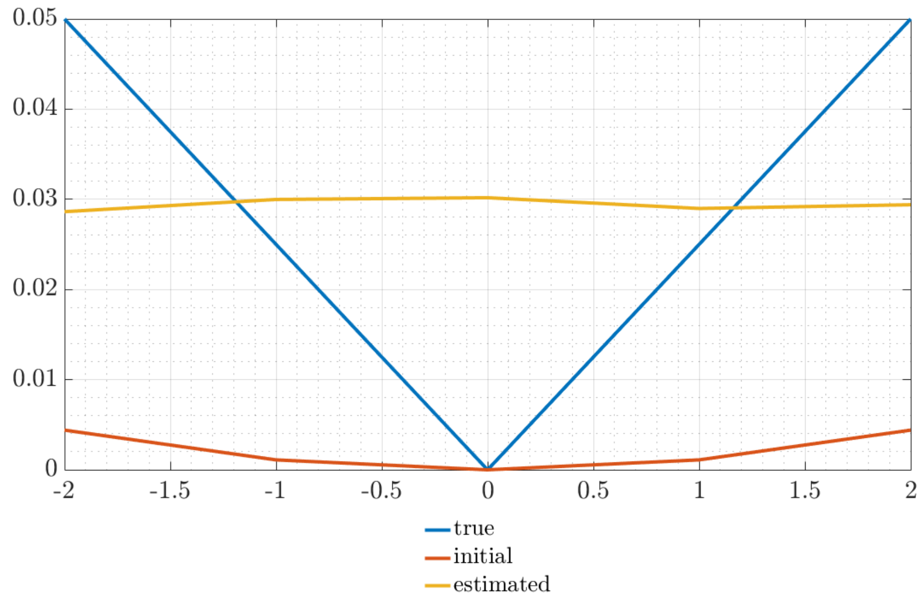
(a) Reproduces Fig 1, Panel b



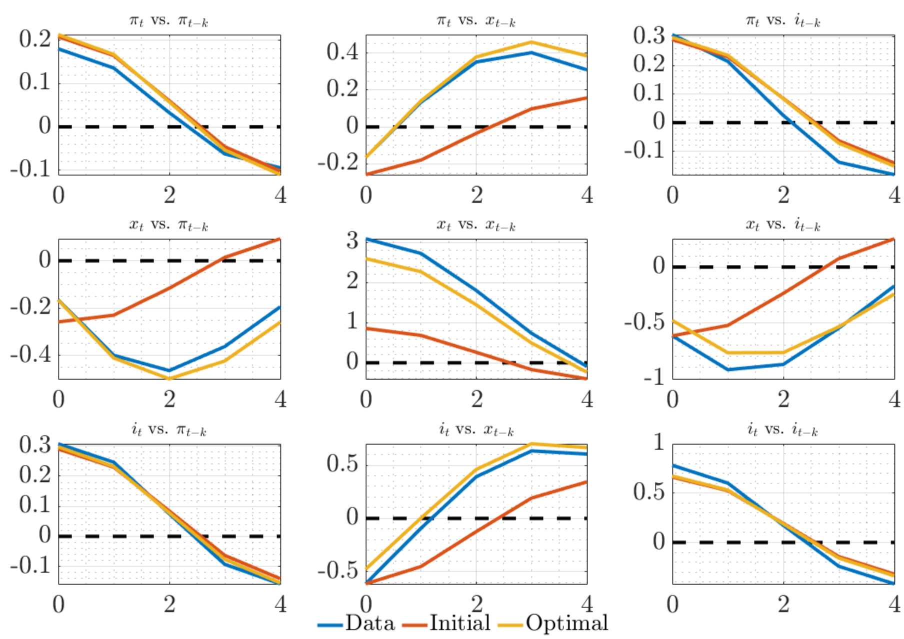
(b) N=1000

1.1 $N = 1000$ for N simulations, one estimation

Figure 6: NOT using 1-step ahead forecasts of inflation, estimate mean moments once, imposing convexity with weight 100K, w/o measurement error, truth with $nfe = 5, fe \in (-2, 2)$



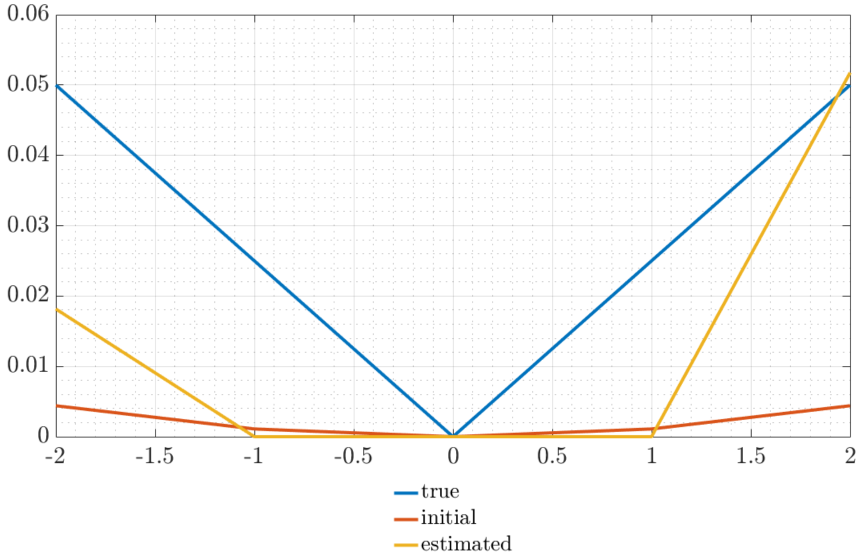
(a) Estimated parameters



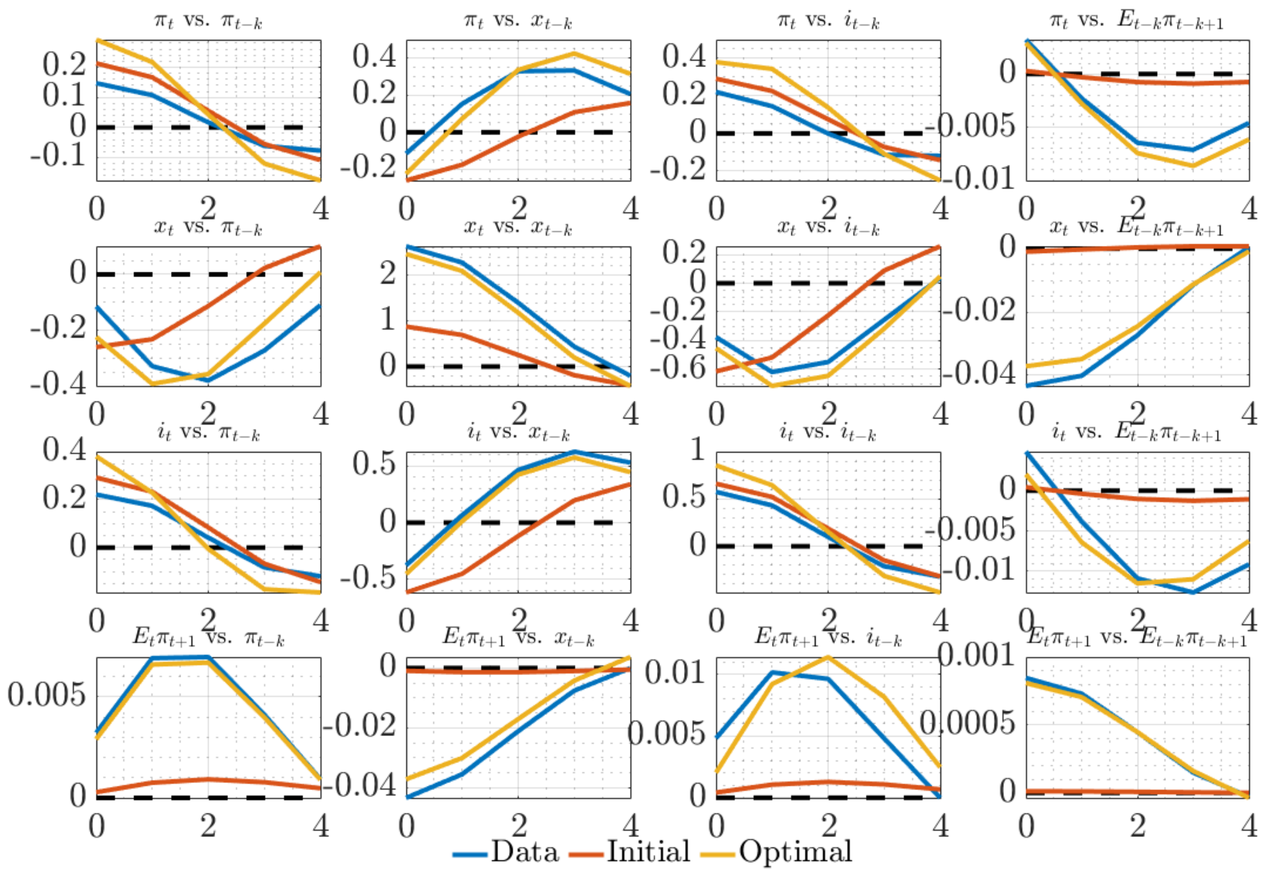
(b) Autocovariogram

1.2 Expectations

Figure 7: using 1-step ahead forecasts of inflation, estimate mean moments once, imposing convexity with weight 100K, w/o measurement error, truth with $nfe = 5, fe \in (-2, 2)$



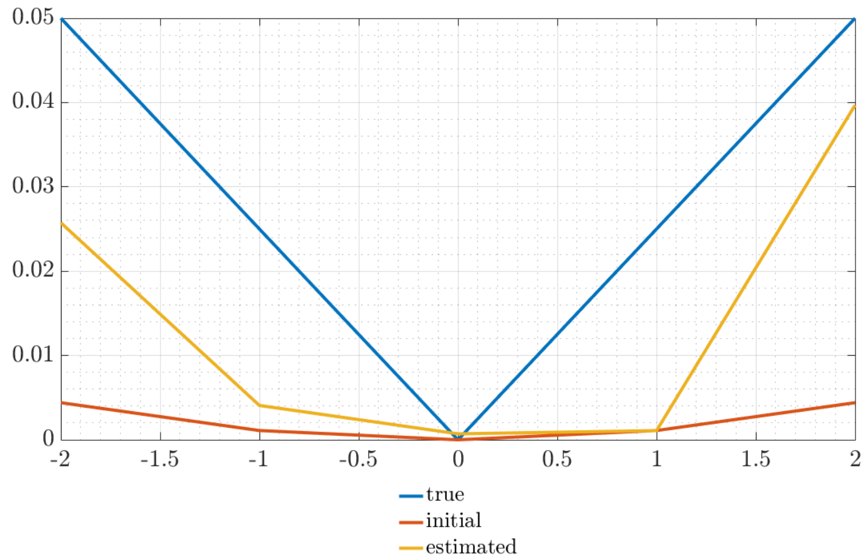
(a) Estimated parameters



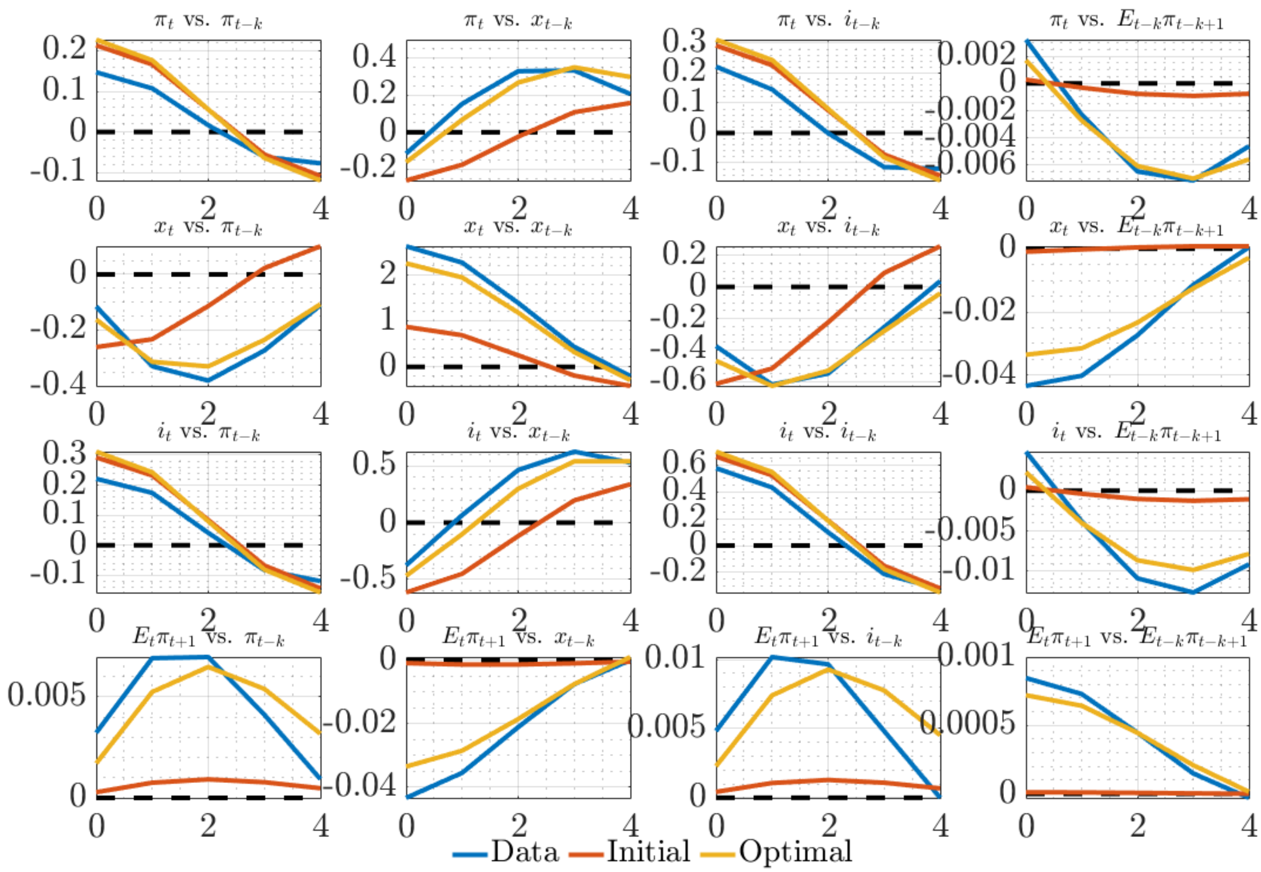
(b) Autocovariogram

1.3 Expectations and rescaling

Figure 8: using 1-step ahead forecasts of inflation, rescaling W , estimate mean moments once, imposing convexity with weight 100K, w/o measurement error, truth with $nfe = 5, fe \in (-2, 2)$



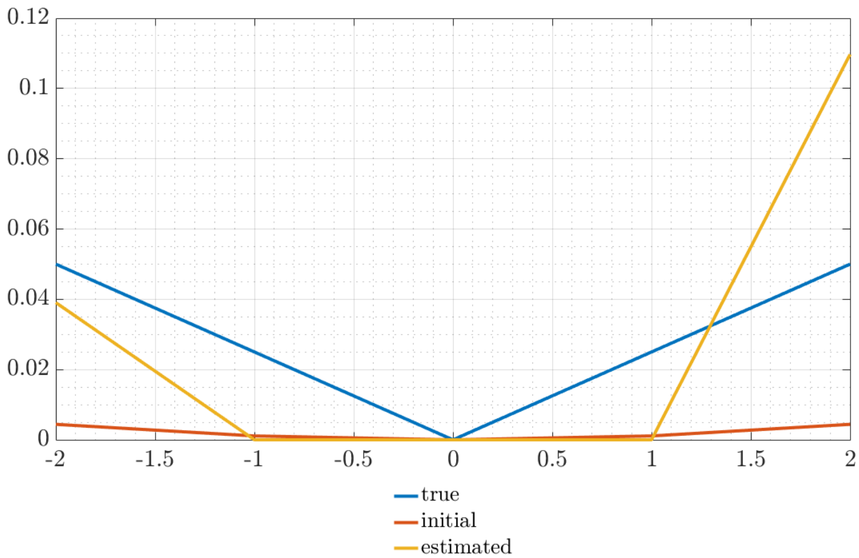
(a) Estimated parameters



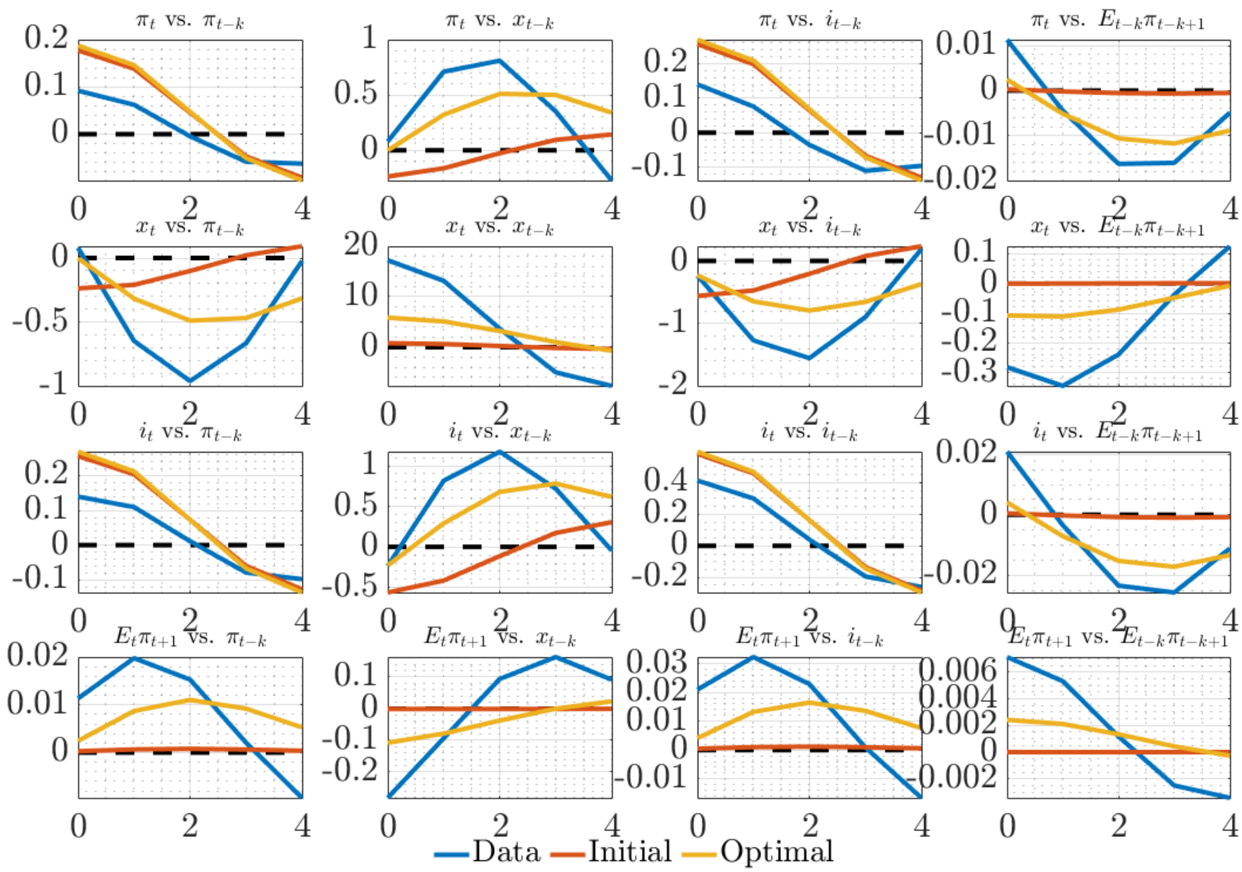
(b) Autocovariogram

1.4 Expectations and ridge

Figure 9: using 1-step ahead forecasts of inflation, ridge regression for data generation and estimation , estimate mean moments once, imposing convexity with weight 100K, w/o measurement error, truth with $nfe = 5, fe \in (-2, 2)$



(a) Estimated parameters



(b) Autocovariogram

1.5 Take a deep breath: what have I learned?

1. Some indication that the measurement error is screwed up, but I can bypass it, so ignore for now.

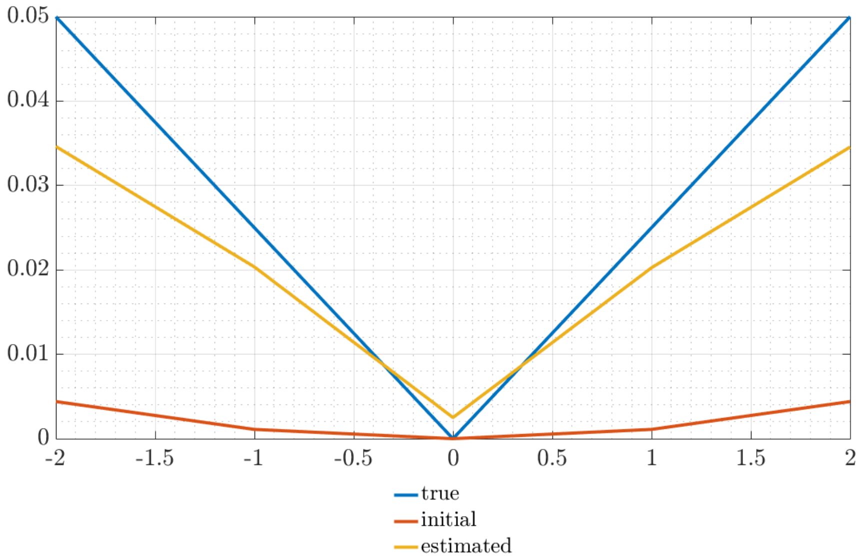
2. Rescaling might exit too soon. Main problem is it shouldn't change the *shape* of the loss function, but does. Yet no indication of numerical matrix inversion problems. I don't understand.

3. Indication that something is screwed up with the expectations, potentially connected with the rescaling. Ridge didn't really help either.

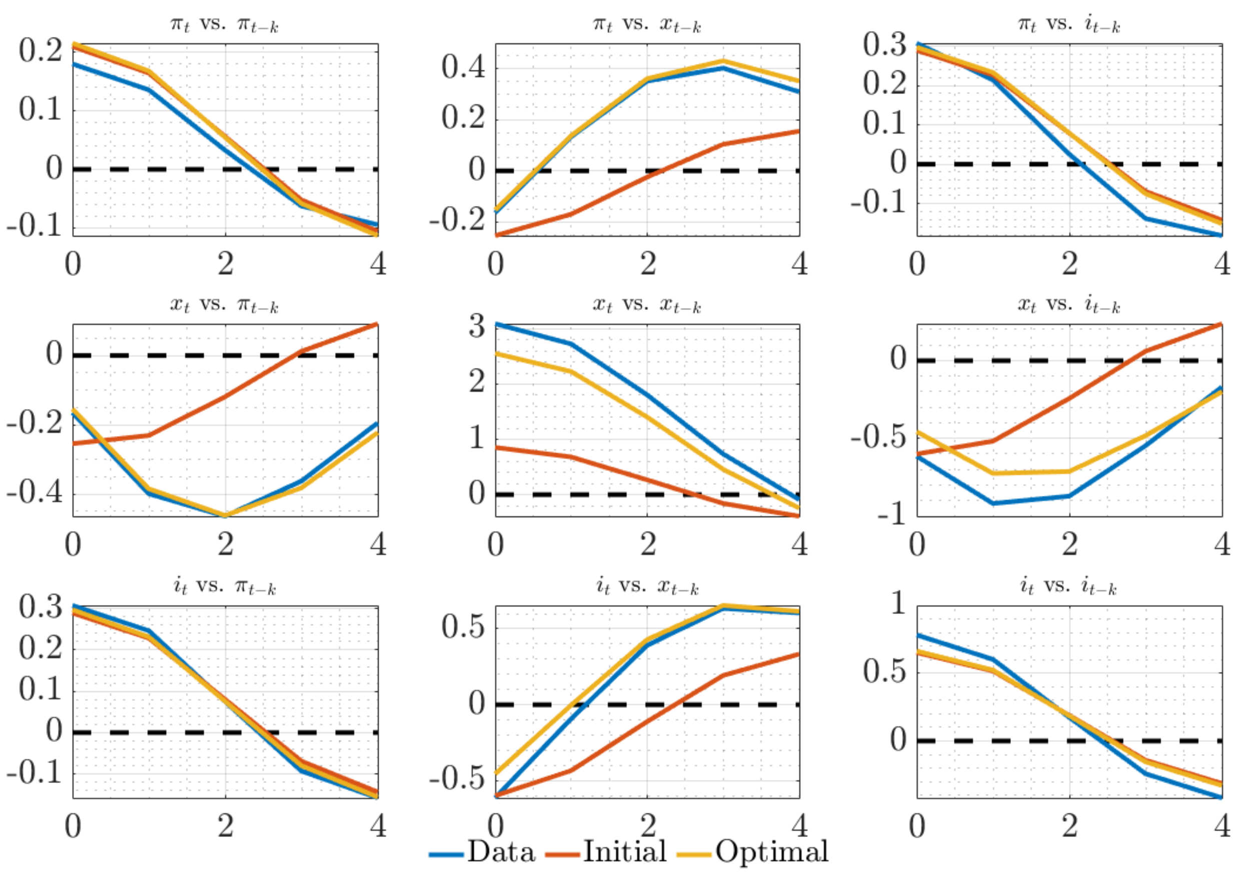
4. Loss function indicates that the parameters *are* identified. However, since loss is greater at true values than at estimated ones, it seems that the truth is a local, not a global min. I need to i) use some tricks to find this min ii) understand why this min isn't the global. I have a hypothesis: I think expectations in the true data aren't very large, and thus also aren't fluctuating enough. This screws up the moments somehow, but it also means that the estimation wants to set the αs corresponding to large forecast errors to a low value, b/c otherwise it would cause fluctuations that aren't there in the data. Combined with the zero-neighborhood problem, the flat estimate is the result.

1.6 0 at 0

Figure 10: 0 at 0 imposed with weight 1000 not using 1-step ahead forecasts of inflation, not rescaling W, estimate mean moments once, imposing convexity with weight 100K, w/o measurement error, truth with $nfe = 5, fe \in (-2, 2)$



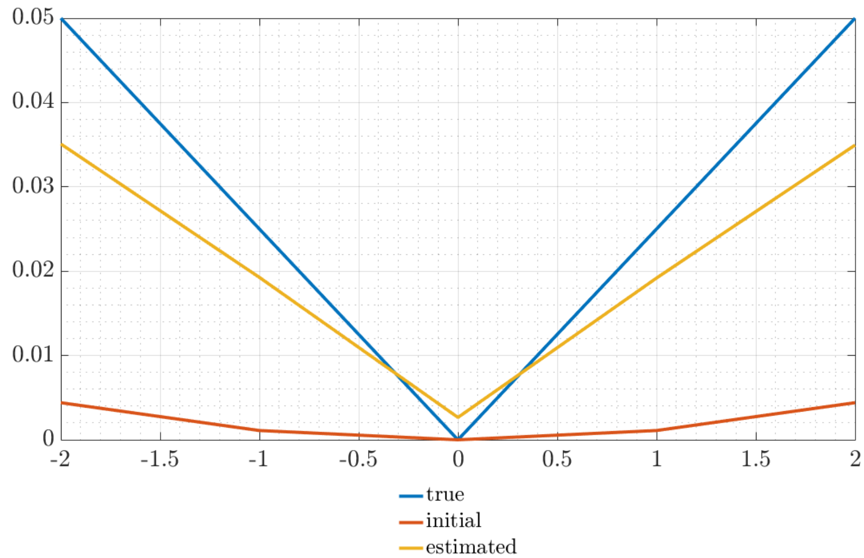
(a) Estimated parameters



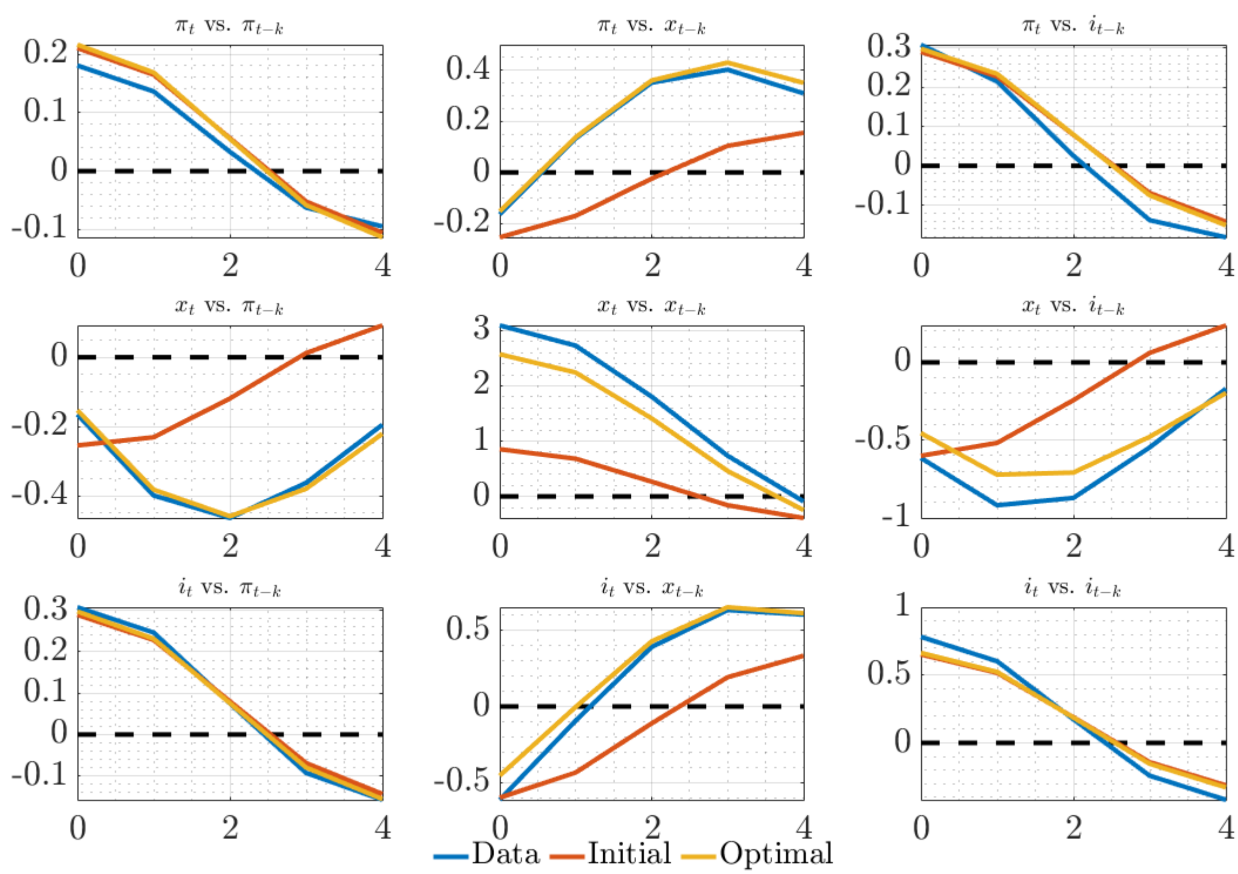
(b) Autocovariogram

1.7 0 at 0, more convexity

Figure 11: 0 at 0 imposed with weight 1000 not using 1-step ahead forecasts of inflation, not rescaling W, estimate mean moments once, imposing convexity with weight 1000K, w/o measurement error, truth with $nfe = 5, fe \in (-2, 2)$



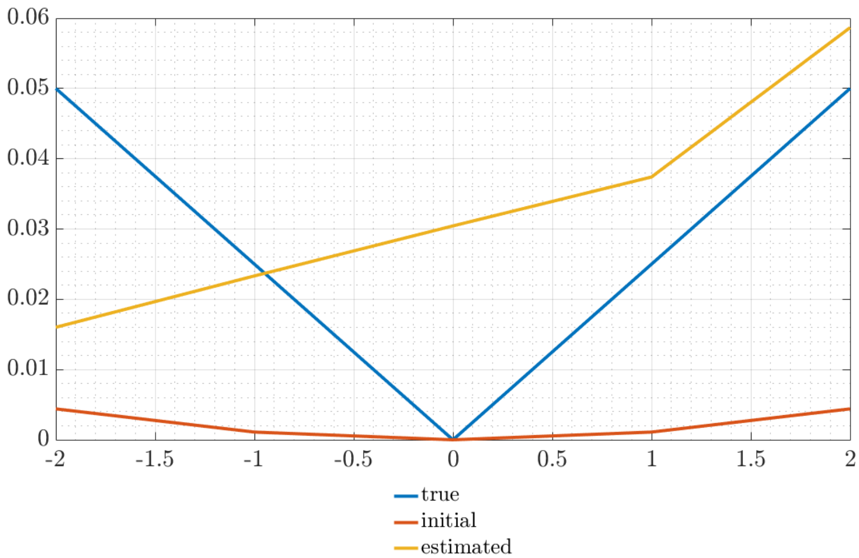
(a) Estimated parameters



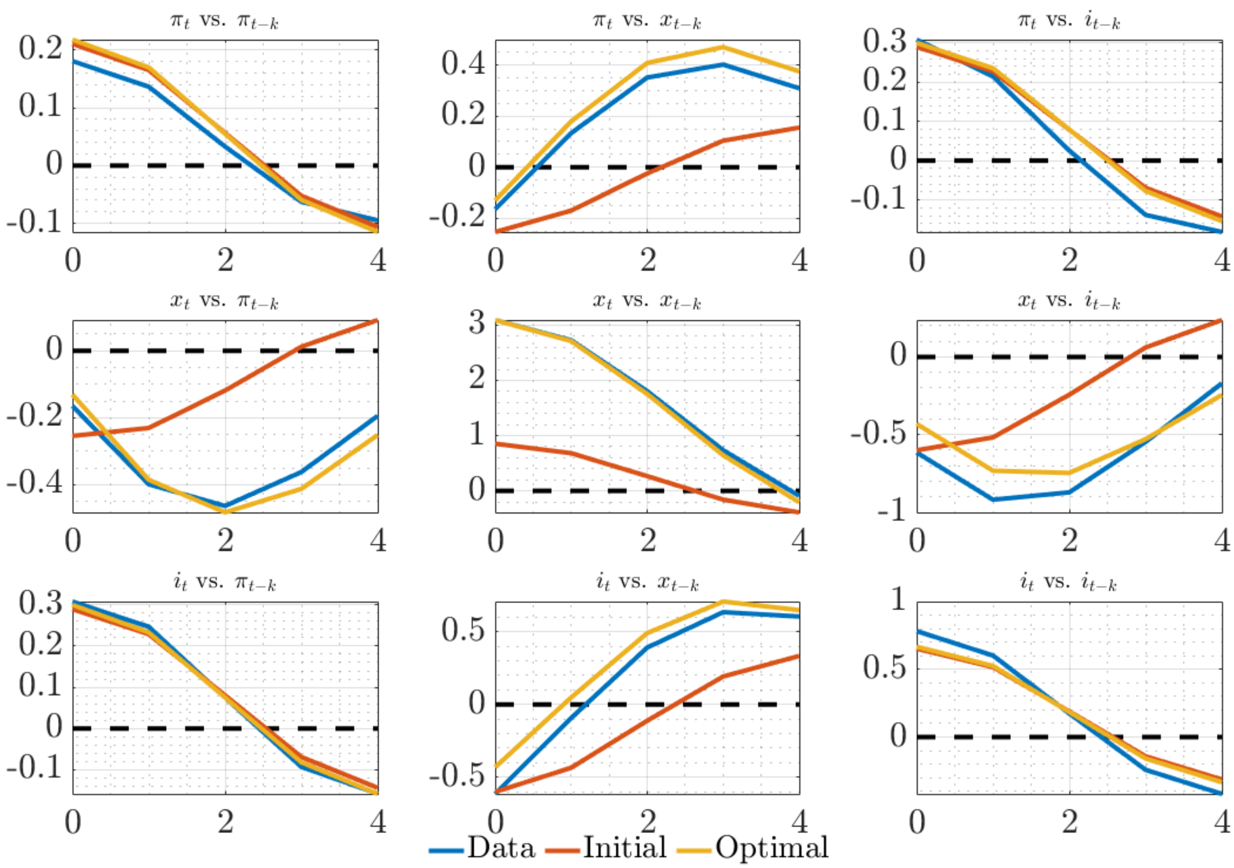
(b) Autocovariogram

1.8 Identity weighting matrix

Figure 12: identity weighting matrix, not using 1-step ahead forecasts of inflation, not rescaling W, estimate mean moments once, imposing convexity with weight 1000K, w/o measurement error, truth with $nfe = 5, fe \in (-2, 2)$



(a) Estimated parameters



(b) Autocovariogram

A Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{A.1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{A.2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \qquad \text{(if imposed)} \tag{A.3}$$

$$\text{PLM:} \qquad \hat{\mathbb{E}}_t z_{t+h} = a_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\, h_x \tag{A.4}$$

$$\text{Updating:} \qquad a_t = a_{t-1} + k_t^{-1}\big(z_t - (a_{t-1} + b s_{t-1})\big) \tag{A.5}$$

$$\text{Anchoring function:} \qquad k_t^{-1} = \rho_k k_{t-1}^{-1} + \gamma_k f e_{t-1}^2 \tag{A.6}$$

$$\text{Forecast error:} \qquad f e_{t-1} = z_t - (a_{t-1} + b s_{t-1}) \tag{A.7}$$

$$\text{LH expectations:} \qquad f_a(t) = \frac{1}{1-\alpha\beta} a_{t-1} + b(\mathbb{I}_{nx} - \alpha\beta h)^{-1} s_t \qquad f_b(t) = \frac{1}{1-\beta} a_{t-1} + b(\mathbb{I}_{nx} - \beta h)^{-1} s_t \tag{A.8}$$

This notation captures vector learning (z learned) for intercept only. For scalar learning, $a_t = \begin{pmatrix}\bar{\pi}_t & 0 & 0\end{pmatrix}'$ and b_1 designates the first row of b. The observables (π, x) are determined as:

$$x_t = -\sigma i_t + \begin{bmatrix}\sigma & 1-\beta & -\sigma\beta\end{bmatrix} f_b + \sigma \begin{bmatrix}1 & 0 & 0\end{bmatrix} (\mathbb{I}_{nx} - \beta h_x)^{-1} s_t \tag{A.9}$$

$$\pi_t = \kappa x_t + \begin{bmatrix}(1-\alpha)\beta & \kappa\alpha\beta & 0\end{bmatrix} f_a + \begin{bmatrix}0 & 0 & 1\end{bmatrix} (\mathbb{I}_{nx} - \alpha\beta h_x)^{-1} s_t \tag{A.10}$$

B Target criterion

The target criterion in the simplified model (scalar learning of inflation intercept only, $k_t^{-1} = \mathbf{g}(f e_{t-1})$):

$$\begin{aligned}\pi_t = -\frac{\lambda_x}{\kappa}\Bigg\{ x_t - \frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + ((\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1}))\mathbf{g}_\pi(t)\Big) \\ \Bigg(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (1 - k_{t+1+j}^{-1} - (\pi_{t+1+j} - \bar{\pi}_{t+j} - b_1 s_{t+j})\mathbf{g}_{\bar{\pi}}(t+j))\Bigg)\Bigg\}\end{aligned} \tag{B.1}$$

where I'm using the notation that $\prod_{j=0}^{0} \equiv 1$. For interpretation purposes, let me rewrite this as follows:

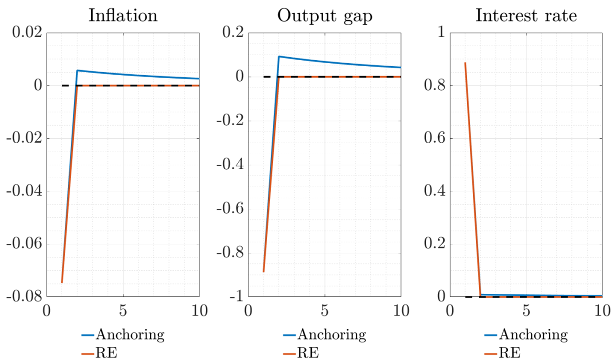
$$\begin{aligned}\pi_t = {} & -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + f e_{t|t-1}^{eve}\mathbf{g}_\pi(t)\Big)\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \\ & -\frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta}{1-\alpha\beta}\Big(k_t^{-1} + f e_{t|t-1}^{eve}\mathbf{g}_\pi(t)\Big)\Bigg(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} (k_{t+1+j}^{-1} + f e_{t+1+j|t+j}^{eve})\mathbf{g}_{\bar{\pi}}(t+j)\Bigg)\end{aligned} \tag{B.2}$$

Interpretation: tradeoffs from discretion in RE + effect of current level and change of the gain on future tradeoffs + effect of future expected levels and changes of the gain on future tradeoffs

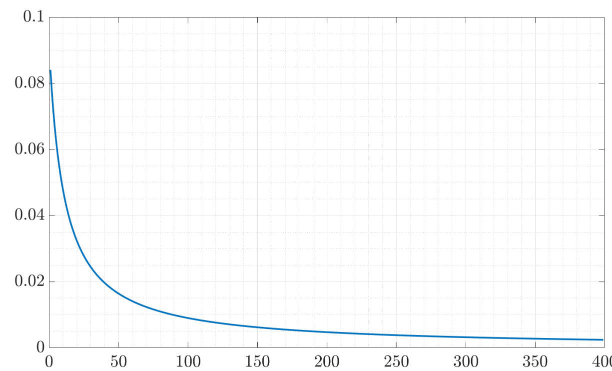
C Impulse responses to iid monpol shocks across a wide range of learning models

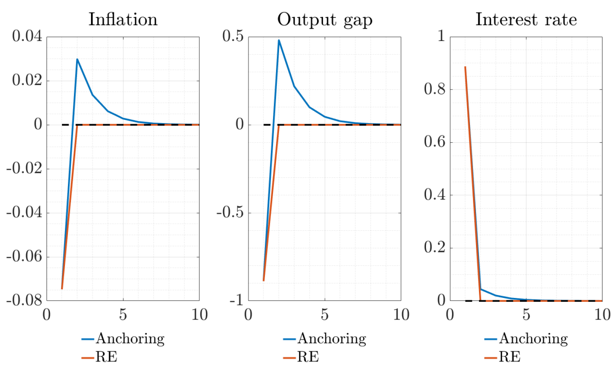
$T = 400, N = 100, n_{drop} = 5$, shock imposed at $t = 25$, calibration as above, Taylor rule assumed to be known, PLM = learn constant only, of inflation only.

Figure 13: IRFs and gain history (sample means)

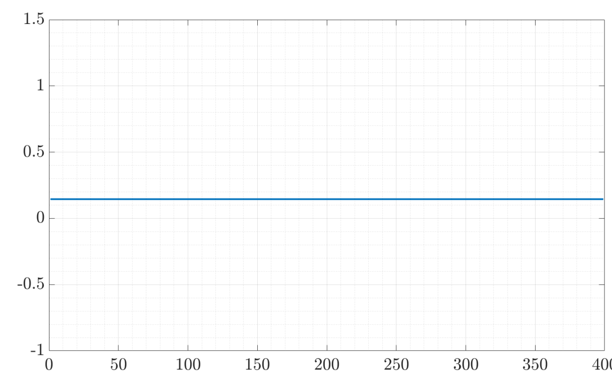


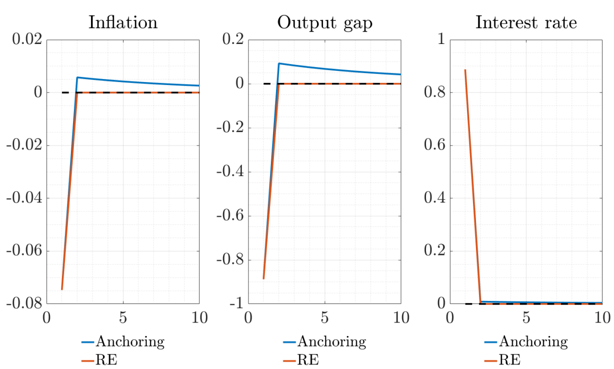
(a) Decreasing gain learning


(b) Mean gain



(c) Constant gain learning


(d) Mean gain



(e) CEMP criterion (vector)

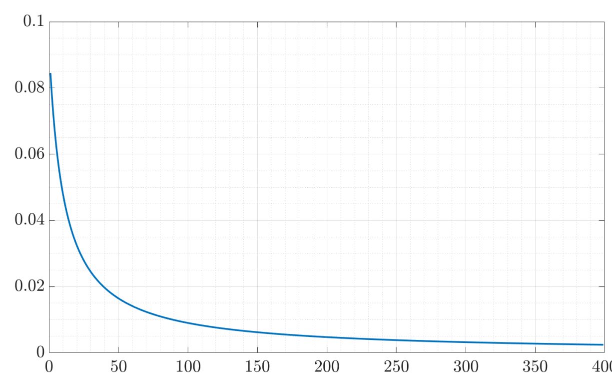
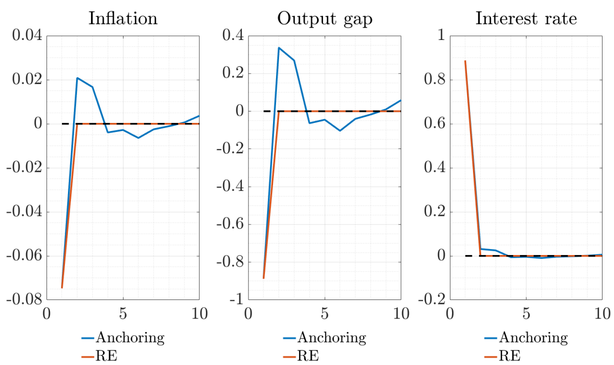
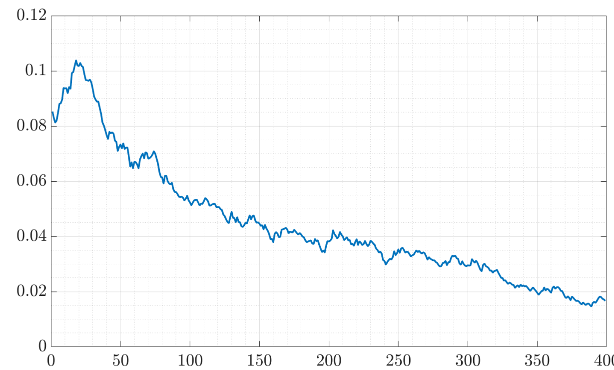

(f) Mean gain

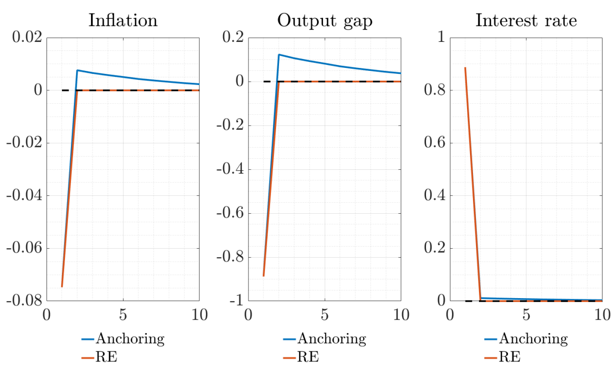
Figure 14: IRFs and gain history (sample means), continued



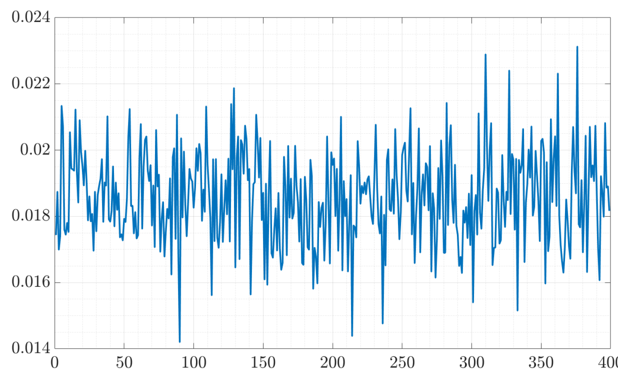
(a) CUSUM criterion (vector)



(b) Mean gain



(c) Smooth criterion, approximated, using $\alpha^{true} = (0.05; 0.025; 0; 0.025; 0.05)$, on $fe \in (-2, 2)$.



(d) Mean gain