Materials 17 - Analytical optimal monetary policy (a.k.a. making sense of Woodford and all those that cite him)

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Overview

1 Model summary

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n\big) \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big) \tag{2}$$

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \bar{i}_t \tag{3}$$

$$\hat{\mathbb{E}}_t z_{t+h} = \bar{z}_{t-1} + b h_x^{h-1} s_t \quad \forall h \geq 1 \qquad b = g_x\ h_x \qquad \text{PLM} \tag{4}$$

$$\bar{z}_t = \bar{z}_{t-1} + k_t^{-1} \underbrace{\big(z_t - (\bar{z}_{t-1} + b s_{t-1})\big)}_{\text{fcst error using (4)}} \tag{5}$$

(Vector learning. For scalar learning, $\bar{z} = \begin{pmatrix}\bar{\pi} & 0 & 0\end{pmatrix}'$. I'm also not writing the case where the slope b is also learned.)

$$k_t = \begin{cases} k_{t-1} + 1 & \text{when} \quad \theta^{CEMP} < \bar{\theta} \quad \text{or} \quad \theta_t < \tilde{\theta} \\ \bar{g}^{-1} & \text{otherwise.} \end{cases} \tag{6}$$

1.1 The CEMP vs. the CUSUM criterion

CEMP's criterion

$$\theta_t^{CEMP} = \max |\Sigma^{-1}(\phi - \begin{bmatrix} F & G \end{bmatrix})| \tag{7}$$

where Σ is the VC matrix of shocks, ϕ is the estimated matrix, $[F, G]$ is the ALM.

CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}^{-1}(f_t f_t' - \omega_{t-1}) \tag{8}$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}^{-1}(f_t' \omega_t^{-1} f_t - \theta_{t-1}) \tag{9}$$

where f is the most recent forecast error and ω is the estimated FEV.

2 Optimal policy - dimensions of conceptual confusion

1. Commitment vs. time-consistency

   My understanding of Woodford is that commitment and time-consistency are two separate binaries. In words:

$$\textit{commitment} = \begin{cases} \text{discretion: } \max \text{period utility, s.t. stuff taken as given} \\ \text{commitment: } \max \text{expected lifetime utility, s.t. model equations \& initial condition} \end{cases}$$

$$\textit{time-consistency} = \begin{cases} \text{time-inconsistent: initial condition is that multiplier} = 0 \\ \text{time-consistent: } \max \text{initial condition is that endogenous stuff is at target initially} \end{cases}$$

   This gives rise to 3 different optimality concepts:

$$\begin{cases} \text{optimal discretionary policy } \rightarrow \text{time-inconsistent} \\ \text{optimal commitment } (t_0\text{-optimal}) \rightarrow \text{time-inconsistent} \\ \text{timelessly optimal commitment } \rightarrow \text{time-consistent} \end{cases}$$

2. Constraints of the problem / endogenous variables of interest

   Mainly model equations. But why doesn't Woodford take the IS-relation of the NK model as a constraint? He says: "if there is no welfare loss resulting from nominal interest-rate variation, one may omit the constraint terms corresponding to the IS relation, as these constraints never bind."
   → but what about fluctuations in the output gap? Gaspar et al 2011 at least claim to ignore the IS curve because they assume that the CB controls the output gap directly.
   And what about expectations? It seems like Gaspar et al 2011 treat those the same as jumps.

3. Optimal plan vs. implementation

   My understanding is that solving the above policy problem just gives you the optimal plan: a time path for the endogenous variables that the policy maker wants to bring about. It's a separate question to ask how to implement that policy.

4. Implementation of policy: "simple rules," reaction functions and "targeting rules"