Materials 18 - Optimal monpol under learning is an ∞ prisoner's dilemma w/o the grim trigger

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Take a very simple optimal policy problem where the planner chooses $\{\pi_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \varphi_t(\pi_t - \beta\pi_{t+1}) \Big\} \tag{1}$$

Consider this same problem under three cases:

1. **RE and commitment**

   FOC:

$$2\pi_t + \varphi_t - \varphi_{t-1} = 0 \tag{2}$$

2. **RE and discretion**

   Write expected inflation as a variable f_t that the authority takes as given:

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \varphi_t(\pi_t - \beta f_t) \Big\} \tag{3}$$

   FOC:

$$2\pi_t + \varphi_t = 0 \tag{4}$$

3. **Learning and commitment**

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \varphi_{1,t}(\pi_t - \beta f_t) + \varphi_{2,t}(f_t - f_{t-1} - k^{-1}(\pi_t - f_{t-1})) \Big\} \tag{5}$$

   FOCs:

$$2\pi_t + \varphi_{1,t} - \varphi_{2,t}k^{-1} = 0 \tag{6}$$

$$-\beta\varphi_{1,t} + \varphi_{2,t} + \mathbb{E}_t\, \varphi_{2,t+1}(-1 + k^{-1}) = 0 \tag{7}$$

   $\rightarrow$ no lagged multiplier despite taking formation of expectations into account!

Mele, Molnár & Santoro (2019): optimal monetary policy w/ learning does not involve commitment! Indeed it cannot because the learning mechanism takes away the CB's commitment technology. (In their lingo, learning agents don't have "off-equilibrium strategies.") This is a Stackelberg infinitely repeated game where b/c of learning, the private sector is not strategic: it looses access to the grim trigger threat strategy. Therefore the CB will always play its best response and we land in the suboptimal Nash in the long-run!

In a sense, this was anticipated in the "Ramsey policy is indeterminate under RE" literature:

- Ramsey policy is time-inconsistent (Kydland & Prescott, 1977, Barro & Gordon, 1983). So invent RE to endow agents with threats for deviating → a commitment device.
- Now the problem is that the commitment solution is a Nash, but it's indeterminate (b/c purely forward-looking).
- Invent learning to introduce backward-lookingness: allows you to select the RE with commitment solution (focus of Evans & Honkapohja 2001 on E-stability).
- Problem: that solution is no longer optimal b/c the PS cannot enforce it! ("machine")
- Circumvent that by introducing backward-looking expectation formation that satisfies some sense of optimality (Cho and Matsui, 1995, "inductive expectations") so that the PS regains some of its strategic nature.

Here's a step of bold interpretation:

- Maybe Ramsey under RE is indeterminate because of the folk theorem: any feasible and individually rational payoff profile can be a Nash or a subgame perfect equilibrium of the infinitely repeated prisoner's dilemma.
- Learning breaks the folk theorem b/c the assumption of credible threats is gone when one of the players is an automaton and in particular, it's not an optimal one.