**Internet Appendix to “Diagnostic Expectations and Credit Cycles”**

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Section I of this Appendix contains proofs of the results presented in the paper. Section II offers a discussion of alternative specifications of the reference group –*G* used to define representativeness.

**I. Proofs**

*Proof of Proposition 1:* Let be an AR(1) process, , with i.i.d. normal shocks . We now compute diagnostic expectations at a generic horizon . Writing as a function of plus subsequent shocks, we find

so that the true distribution of given , , is a normal distribution with mean and variance given by

The reference distribution is , which is characterized by:

The diagnostic distribution then reads (up to normalization constants)

The quadratic and linear terms in are as follows (the constant terms being absorbed by a normalization constant)

It follows that the diagnostic distribution is also a normal distribution with mean

In particular, for we get

It is clear from the above that the proof carries through to a generic autoregressive process, provided that the distributions and are normal and have the same variance. Here we present a generalization to GARCH processes. Suppose that the state of the world follows the GARCH(1,1) process

where is white noise. Then the true conditional distribution is given by , while the expected distribution is given by . This is because, in the absence of news at , variance takes its baseline value .

For convenience, denote by the increase in variance of given news relative to the comparison context where . Then, provided the variance does not increase excessively,[[2]](#footnote-2) that is, , diagnostic expectations at are also described by a normal distribution, given by , where

with distortions being modulated by the effective (time-dependent) diagnostic parameter

Diagnostic variance is given by

For normal distributions, the kernel of truth property of diagnostic expectations applies to both the first and second moments. Suppose there is a shock at , that is,, so that . Then the diagnostic mean exaggerates the shock, as in Proposition 1, except that the effective diagnostic parameter is amplified, . The intuition for this effect is clear: when there is a positive shock to the variance, the tails get thicker and the diagnostic tail gets more overweighted, thus compounding any average movement of the distribution.

Moreover, diagnostic expectations also exaggerate the variance of , . In a world with risk aversion, this leads to a systematic overreaction to bad news, but a more dampened reaction to good news.

*Proof of Corollary 1:* Equation (3) follows from the general proof given for Proposition 1. We now compute iterated diagnostic expectations. From the perspective of period , the expectation , for any , is a normal variable with mean and variance . Moreover, again from the perspective of period , this variable is independent of the expectation in the previous period. As a consequence, we have that

is itself a normally distributed normal variable. Thus, the representation of diagnostic expectations from Proposition 1 can be applied. We find that

where we applied Proposition 1 in the second step. We now use linearity and the law of iterated expectations for the operator to find

Intuitively, future distortions are in the kernel of the diagnostic expectations operator, because on average there is no news. As a result, the term structure of diagnostic expectations is fully consistent.

It is important to stress that the linear representation (2) of diagnostic expectations can be applied to the linear combination of variables only because the latter is itself a normal variable. Being defined in terms of representativeness, diagnostic expectations do not satisfy linearity in the following sense: . In fact, representativeness must be defined with respect to the distribution of , which yields:

In the case above, linearity breaks down because is determined at time . As a result, when computing diagnostic expectations of , we find that its infinitely representative state is itself (formally, we represent with a delta distribution). Thus, the distribution of does not enter the diagnostic expectation . In general, however, linearity holds for combinations of nondegenerate normal random variables. In particular, whenever and are non-degenerate.

*Proof of Proposition 2:*  For point (a), write

where the last term is negative. Using the shorthand , the first term reads

where is the Gaussian density function. Expanding the derivative and rearranging, we have

The second term in parentheses is equal to . To compute the first term, we use the identity to write

where . We thus find

which is negative and hence .

For point (b), use equations (4) and (5) to write

The first term is negative. The second term is proportional to

which is positive for any . In the first line, we use and in the second line we use the identity derived above.

*Proof of Proposition 3:* From equation (9) we have:

where we used the AR(1) condition . Rearranging the first line (valid for all ), we find

Inserting this above, we then get

Spreads thus follow an ARMA(1,1) process.

*Proof of Lemma 1:* For notational convenience, rewrite the stochastic process driving credit spreads as , with , , and . The -periods-ahead diagnostic forecast of the spread is given by

Note that for any , because rational expectations of future shocks are always zero. Thus, for , becomes

Inserting the coefficients, we get

Consider now the case . Using (9) and the law of iterated expectations for diagnostic expectations, write . Inserting and above, we obtain the result.

*Proof of Proposition 4:* The forecast error at is . The first result follows immediately from Lemma 1. Alternatively, write

The first term is , while the second term is . Taking expectations on the difference, we find that . Thus, positive news today narrows the spread today and the predicted spread tomorrow, but the realized spread tomorrow is systematically larger than predicted.

Similarly, we can write

Using the representation of Corollary 1, this becomes

Again, positive news today compresses expected spreads in the future, and these expectations systematically widen going forward.

*Proof of Lemma 2:* Given normal and i.i.d. errors, the OLS regression coefficient of credit spread forecast error on current spread levels is given by the maximum likelihood estimator

From (9) and (10), the forecast error in the spread over the next 12 months is given by

so that .

To derive the sample variance , we write spreads iteratively in terms of the time series of shocks. Let , with , , and . For large we have

where we use . It then follows that . Replacing in the expression for gives the result.

*Proof of Corollary 2:*Defining as the credit spread that obtains under rational expectations, where , it follows immediately from equation (9) that

Moreover,

*Proof of Proposition 5:* Assume that at spreads are low due to recent good news, . It follows from the ARMA(1,1) structure for spreads derived in Proposition 3 that the expected future path of spreads is

from which the result follows.

Aggregate investment at and aggregate production at are strictly decreasing functions of the average credit spread . It follows from point (a) that, under the assumptions of the proposition and controlling for fundamentals at , there is a predictable drop in these quantities from the perspective of .

*Proof of Lemma 3:* We start from

Regressing on yields coefficients

Explained variance is therefore equal to

As was shown in Lemma 2, the actual variance is . The result follows from taking the ratio between explained and actual variance, or . Note that for or , but for and .

*Proof of Proposition 6:* Let be the riskiness threshold, at time , below which firms can issue safe bonds whose default probability at is less than , and above which only risky debt can be issued. As in Section III, we adopt Assumption 1, which holds that wealth is sufficiently large that the debt of firm is priced in such a way as to make investors indifferent between investing in the firm and consuming in the current period (i.e., firm optimization is binding for the total investment).

The result now follows immediately from equations (14) and (15). The rate of return that makes the household indifferent between consuming and investing in safe debt is given by equation (5), that is, . For risky debt, the corresponding rate of return is given by , that is, .

*Proof of Proposition 7:* From Proposition 6, the value of debt for firm at time is given by the inverse of the equilibrium interest rate . In the interim period, the default probability is updated from to . For firms that are perceived as safe both prior and during the interim period, , firm value changes by the factor . For firms that are perceived as risky both prior and during the interim period, , firm value changes by the (same) factor . Finally, for marginally safe firms, , which are perceived as safe following good news but as risky in the interim period, firm value changes by .

**II. Alternative Specifications**

We briefly consider two alternative specifications of the reference group –*G* used to define representativeness.

*A Lagged Diagnostic Expectations as Reference*

We start by specifying in terms of diagnostic expectations . Assume that the agent compares the current distribution to the one implied by his past diagnostic expectation of , namely, . Diagnostic expectations at time are then given by

The agent is overly optimistic when news points to an outcome that is sufficiently good as compared with his past expectations, , and overly pessimistic otherwise. By iterating equation (IA1) backwards, for we obtain

Diagnostic expectations are a weighted average of current and past one-period-ahead rational expectations, with weights that depend on . Again, when , expectations are rational. In equation (IA2) the signs on rational expectations obtained in odd and even past periods alternate. This is an intuitive consequence of (IA1) and implies that news exerts a nonmonotonic effect on future expectations. Agents overreact on impact, but this overreaction implies that reference expectations are higher the next period, causing a reversal to pessimism (which in turn generates future optimism and so on). Specifying in terms of diagnostic expectations thus preserves the two key properties of our basic model: expectations display overreaction to news on impact but also a reversal in the future.

*B. Slow Moving*

In our main specification, context is the immediate past. This assumption starkly illustrates our results and buys significant tractability. It is possible, however, that remote but remarkable memories influence the agent’s background context. Our model can be easily enriched to capture this feature by defining representativeness in terms of a mixture of current and past likelihood ratios.

Let representativeness be defined as

where capture the weights attached to present and past representativeness. In this case we have that

The benchmark model in the paper has and for . A constant rate of decay would have for all . Intermediate specifications, capturing recency effects but with some memory of past representativeness, would feature .

In models with a “slow moving” , that is, where for some , the agent can remain too optimistic even after minor bad news, provided he experienced major good news in the past, for instance, . This feature can yield underreaction to early warnings of crises (see Gennaioli, Shleifer, and Vishny (2015) for a related formulation). At the same time, the main properties of overreaction and reversal continue to hold in this specification with respect to repeated news in the same direction, which are plausible in the case of credit cycles.

In general, the robust predictions of the model remain overreaction and reversals. Different specifications of yield different ancillary predictions that may make it possible to uncover the structure of in the data. This is an important avenue for future work.

**REFERENCES**

# Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny. 2015. “Neglected Risks: The Psychology of Financial Crises.” *American Economic Review, Papers and Proceedings* 105 (5): 310-314.

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2. If the variance increases, the tails become overweighted, and increasingly so as increases. If is sufficiently large, the resulting density function becomes U-shaped and is not normalizable. The condition ensures that diagnostic expectations are normalizable. This condition always holds in the limit of rational expectations, . [↑](#footnote-ref-2)