

Econometrics 3

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Assignment 3

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Question 1

You are interested in the response of output growth $y_{1,t}$ to fiscal policy shocks $e_{2,t}$. The structural model is as follows:

$$\begin{aligned}y_{1,t} &= \gamma_1 y_{2,t} + \alpha_{11} y_{1,t-1} + \alpha_{12} y_{2,t-1} + e_{1,t} \\y_{2,t} &= \gamma_2 y_{1,t} + \alpha_{21} y_{1,t-1} + \alpha_{22} y_{2,t-1} + e_{2,t}\end{aligned}\tag{1}$$

Question 1.a

a) Under what conditions can this structural model be transformed in structural VAR form? Write down the conditions in terms of the parameters above and explain.

The conditions that we need to transform this structural model to structural VAR form are that the parameters do not change with the time. In other words γ_1 , α_{11} , α_{12} , γ_2 , α_{21} and α_{22} should be the same for all periods of time in the model to estimate them. Another assumptions are $E(e_t) = 0$, $E(e_t e_t') = \Sigma_e$ and Γ is invertible where:

$$\begin{aligned}\Gamma &= \begin{bmatrix} 1 & -\gamma_1 \\ -\gamma_2 & 1 \end{bmatrix} \\e_t &= \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix} \\\Sigma_e &= \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}\end{aligned}\tag{2}$$

Question 1.b

b) Assuming the model can be written in a structural VAR form, write down the structural VAR formulation.

First we rewrite in lag terms:

$$\begin{aligned}y_{1,t} &= \gamma_1 y_{2,t} + \alpha_{11} y_{1,t-1} + \alpha_{12} y_{2,t-1} + e_{1,t} = \gamma_1 y_{2,t} + \alpha_{11}(L)y_{1,t} + \alpha_{12}(L)y_{2,t} + e_{1,t} \\y_{2,t} &= \gamma_2 y_{1,t} + \alpha_{21} y_{1,t-1} + \alpha_{22} y_{2,t-1} + e_{2,t} = \gamma_2 y_{1,t} + \alpha_{21}(L)y_{1,t} + \alpha_{22}(L)y_{2,t} + e_{2,t}\end{aligned}\tag{3}$$

Rewrite as:

$$\underbrace{\begin{bmatrix} 1 & -\gamma_1 \\ -\gamma_2 & 1 \end{bmatrix}}_{\Gamma} \underbrace{\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} \alpha_{11}(L) & \alpha_{12}(L) \\ \alpha_{21}(L) & \alpha_{22}(L) \end{bmatrix}}_{A(L)} \underbrace{\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix}}_{Y_t} + e_t \quad (4)$$

$$\Gamma Y_t = A Y_{t-1} + e_t$$

And the reduce form is:

$$\begin{aligned} \Gamma Y_t &= A(L) Y_t + e_t \\ Y_t &= \underbrace{\Gamma^{-1} A(L)}_{A^*(L)} Y_t + \underbrace{\Gamma^{-1} e_t}_{u_t} \\ Y_t &= A^*(L) Y_t + u_t \end{aligned} \quad (5)$$

Question 1.c

c) Based on b), use backward substitution to derive the expression for the IRF of output growth to a fiscal policy shock at horizons 0, 1 (i.e. $\frac{\partial y_{1,t+h}}{\partial e_{2,t}}$, $h = 0, 1$).

For derive IRF we should rewrite the expression of b):

$$\begin{aligned} Y_t &= A^*(L) Y_t + u_t \\ Y_t - A^*(L) Y_t &= u_t \\ (I_2 - A^*(L)) Y_t &= u_t \\ Y_t &= (I_2 - A^*(L))^{-1} u_t \\ Y_t &= (I_2 - A^*(L))^{-1} \Gamma^{-1} e_t \end{aligned} \quad (6)$$

using backward substitution, first we found the matrix Gamma inverse

$$\Gamma^{-1} = \begin{bmatrix} 1 & -\gamma_1 \\ -\gamma_2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{1-\gamma_1\gamma_2} & \frac{\gamma_1}{1-\gamma_1\gamma_2} \\ \frac{\gamma_2}{1-\gamma_1\gamma_2} & \frac{1}{1-\gamma_1\gamma_2} \end{bmatrix} \quad (7)$$

With this we found $A^*(L)$

$$\begin{aligned} A^*(L) &= \Gamma^{-1} A(L) = \begin{bmatrix} \frac{1}{1-\gamma_1\gamma_2} & \frac{\gamma_1}{1-\gamma_1\gamma_2} \\ \frac{\gamma_2}{1-\gamma_1\gamma_2} & \frac{1}{1-\gamma_1\gamma_2} \end{bmatrix} \begin{bmatrix} \alpha_{11}(L) & \alpha_{12}(L) \\ \alpha_{21}(L) & \alpha_{22}(L) \end{bmatrix} \\ A^*(L) &= \begin{bmatrix} \frac{\alpha_{11}(L) + \alpha_{21}(L)\gamma_1}{1-\gamma_1\gamma_2} & \frac{\alpha_{12}(L) + \gamma_1\alpha_{22}(L)}{1-\gamma_1\gamma_2} \\ \frac{\gamma_2\alpha_{11}(L) + \alpha_{21}(L)}{1-\gamma_1\gamma_2} & \frac{\gamma_2\alpha_{12}(L) + \alpha_{22}(L)}{1-\gamma_1\gamma_2} \end{bmatrix} \end{aligned} \quad (8)$$

Using $A^*(L)$ we can calculate $I_2 - A^*(L)$:

$$\begin{aligned} I_2 - A^*(L) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{\alpha_{11}(L) + \alpha_{21}(L)\gamma_1}{1-\gamma_1\gamma_2} & \frac{\alpha_{12}(L) + \gamma_1\alpha_{22}(L)}{1-\gamma_1\gamma_2} \\ \frac{\gamma_2\alpha_{11}(L) + \alpha_{21}(L)}{1-\gamma_1\gamma_2} & \frac{\gamma_2\alpha_{12}(L) + \alpha_{22}(L)}{1-\gamma_1\gamma_2} \end{bmatrix} \\ I_2 - A^*(L) &= \begin{bmatrix} \frac{1-\gamma_1\gamma_2 - \alpha_{11}(L) - \alpha_{21}(L)\gamma_1}{1-\gamma_1\gamma_2} & -\frac{\alpha_{12}(L) + \gamma_1\alpha_{22}(L)}{1-\gamma_1\gamma_2} \\ -\frac{\gamma_2\alpha_{11}(L) + \alpha_{21}(L)}{1-\gamma_1\gamma_2} & \frac{1-\gamma_1\gamma_2 - \gamma_2\alpha_{12}(L) - \alpha_{22}(L)}{1-\gamma_1\gamma_2} \end{bmatrix} \end{aligned} \quad (9)$$

The inverse of $I_2 - A^*(L)$ is:

$$[I_2 - A^*(L)]^{-1} = \begin{bmatrix} \frac{1-\gamma_1\gamma_2-\gamma_2\alpha_{12}(L)-\alpha_{22}(L)}{\gamma_2\alpha_{11}(L)+\alpha_{21}(L)} & \frac{\alpha_{12}(L)+\gamma_1\alpha_{22}(L)}{\theta} \\ \frac{\theta}{\gamma_2\alpha_{11}(L)+\alpha_{21}(L)} & \frac{1-\gamma_1\gamma_2-\alpha_{11}(L)-\alpha_{21}(L)\gamma_1}{\theta} \end{bmatrix} \quad (10)$$

Where $\theta = 1 - \gamma_1\gamma_2 - \gamma_2\alpha_{12}(L) - \alpha_{22}(L) - \alpha_{11}(L) - \alpha_{21}(L)\gamma_1 + \alpha_{11}(L)\alpha_{22}(L) - \alpha_{21}(L)\alpha_{12}(L)$. For last we need to find $[I_2 - A^*(L)]^{-1}\Gamma^{-1}$, we have both matrix so:

$$[I_2 - A^*(L)]^{-1}\Gamma^{-1} = \begin{bmatrix} \frac{1-\gamma_1\gamma_2-\gamma_2\alpha_{12}(L)-\alpha_{22}(L)}{\theta} & \frac{\alpha_{12}(L)+\gamma_1\alpha_{22}(L)}{\theta} \\ \frac{\theta}{\gamma_2\alpha_{11}(L)+\alpha_{21}(L)} & \frac{1-\gamma_1\gamma_2-\alpha_{11}(L)-\alpha_{21}(L)\gamma_1}{\theta} \end{bmatrix} \begin{bmatrix} \frac{1}{1-\gamma_1\gamma_2} & \frac{\gamma_1}{1-\gamma_1\gamma_2} \\ \frac{\gamma_2}{1-\gamma_1\gamma_2} & \frac{1}{1-\gamma_1\gamma_2} \end{bmatrix} \quad (11)$$

$$[I_2 - A^*(L)]^{-1}\Gamma^{-1} = \begin{bmatrix} \frac{1-\alpha_{22}(L)}{\theta} & \frac{\gamma_1+\alpha_{12}(L)}{\theta} \\ \frac{\gamma_2+\alpha_{21}(L)}{\theta} & \frac{1-\alpha_{11}(L)}{\theta} \end{bmatrix}$$

The effect of e_2 on y_1 is $\frac{\gamma_1+\alpha_{12}(L)}{\theta}$, the $\frac{\partial y_{1,t+1}}{\partial e_{2,t}} = \frac{\alpha_{12}}{\theta}$ and $\frac{\partial y_{1,t}}{\partial e_{2,t}} = \frac{\gamma_1}{\theta}$

Question 1.d

d) Are there any identification problems in estimating the structural parameters γ_1 and γ_2 from the structural VAR formulation? Explain, and if there are such problems, give in words a solution to these. We do have identification problem because we have eight parameters to estimate (γ_1 , α_{11} , α_{12} , γ_2 , α_{21} , α_{22} , σ_1 and σ_2) but in the empirical model (the reduce form), we can estimate only seven estimates and hence there is underidentification problem in the model.

The solution for this problem implies to find a instrumental variable or make another restriction. But IV are hard to find and very unlikely to be truly exogenous. Exist others ways to make another restriction such as 1) The short-run restrictions, 2) The long-run restrictions, 3) The informal identification and 4) The sign restrictions to use the theory to make restrictions about the behaviour we expect of our model. The first one try to reduce the problem assuming an implicit ordering in the Model, with this order we can make one contemporaneous parameters (gamma one or gamma two) equal to zero, however the estimates about the error depends on the parameter equal to zero. The second one use the economic theory to impose impose a restriction on parameters but based on long-term relationships. For example, the change in monetary policy on GDP, we know that there is neutrality of money, so in the long term it must be annulled. But this condition is a non-linear constraint, so it complicates the problem and, depending on the problem, we need to identify whether or not this type of constraint can be applied. The third one model means to move the model until the impulse response function resembles what the investigator expects. This is an informal and very subjective methodology depending on what the researcher thinks. And the last one, a very similar to the third one, is the sign restriction in which the form in which it should be is not directly stated, but we must say for example that the effect of fiscal policy on GDP must be positive and the coefficients must be reduced to positive values, and with that it gives me a restriction that can be solved. For this particular case, we can use the sign restrictions to impose the effect of fiscal policy or the other alternative is to look for an instrument such as the announcement of fiscal policy.

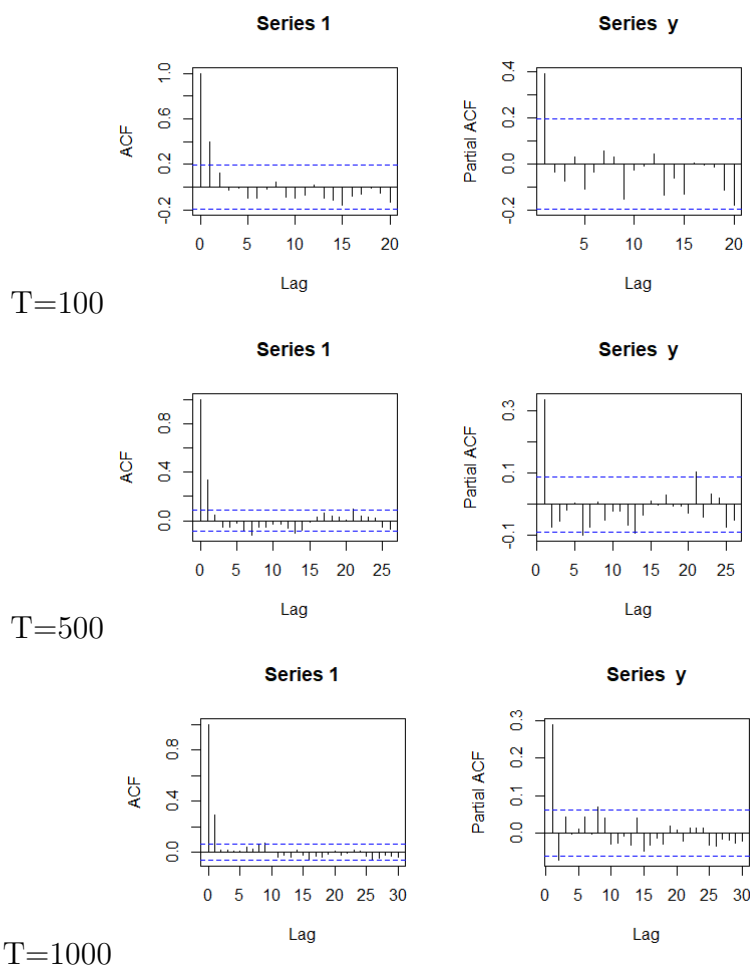
Question 2

a) Let the true model be $y_t = \alpha + u_t$, where $u_t = \epsilon_t + \beta\epsilon_{t-1}$ with $\beta = 0.7$, and ϵ_t i.i.d.N (0, 1), and $\alpha = 0$. Let T be the sample size. This is a model unknown the econometrician, who is interested in testing whether $E(y_t) = 0$, and knows there may be some autocorrelation in the errors.

Question 2.a

Simulate one sample from this model letting $T = 100, 500, 1000$. Plot the ACF and PACF of this model, and explain whether they make sense.

After simulate the moving average model (MA1), we can see that as we increase the T from 100 to 1000, the interval of confidence shrinks. Therefore, it makes possible more accurate results. Also, as the model is MA(1), the autocorrelation function has just one significant lag. It makes sense with the model and the observation we added to the simulation.



Question 2.b

Simulate $S = 500$ replications from this model for $T = 100, 500, 1000$ with burn-in $B = 100$ observations each time (so you should have 500 datasets, each with T observations). Calculate for each simulation the t-statistic for the null hypothesis that $\alpha = 0$ $t = \bar{y}/\hat{\sigma}_y$, where $\bar{y} = \frac{1}{T} \sum_{t=1}^T y_t$, and $\hat{\sigma}_y$ is the Newey-West HAC estimator of the standard error of \bar{y} using the quadratic spectral kernel. Calculate the fraction of times in $S = 500$ simulations that this test rejects, for each sample size, and significance level 5% (using standard normal critical values), and compare it the nominal level 5%. Are they close or not to this nominal 5% level? If yes, explain why.

After the simulation $S=500$ for $T=100, 500$ and 1000 was done, it was calculated for each simulation the t-statistic $t = \bar{y}/\sigma_y$ where σ_y was estimated using Newey-West HAC estimator. After that, the fraction rejection time was calculated. As a result for $T=100, 500$ and 1000 the fraction rejection was of 0.082, 0.072 and 0.06, respectively. It can be concluded that the rejection rate get closer to 5% as we increase the sample size because as the sample increase, it resembles to a normal distribution.

Question 2.c

Let the true model now by

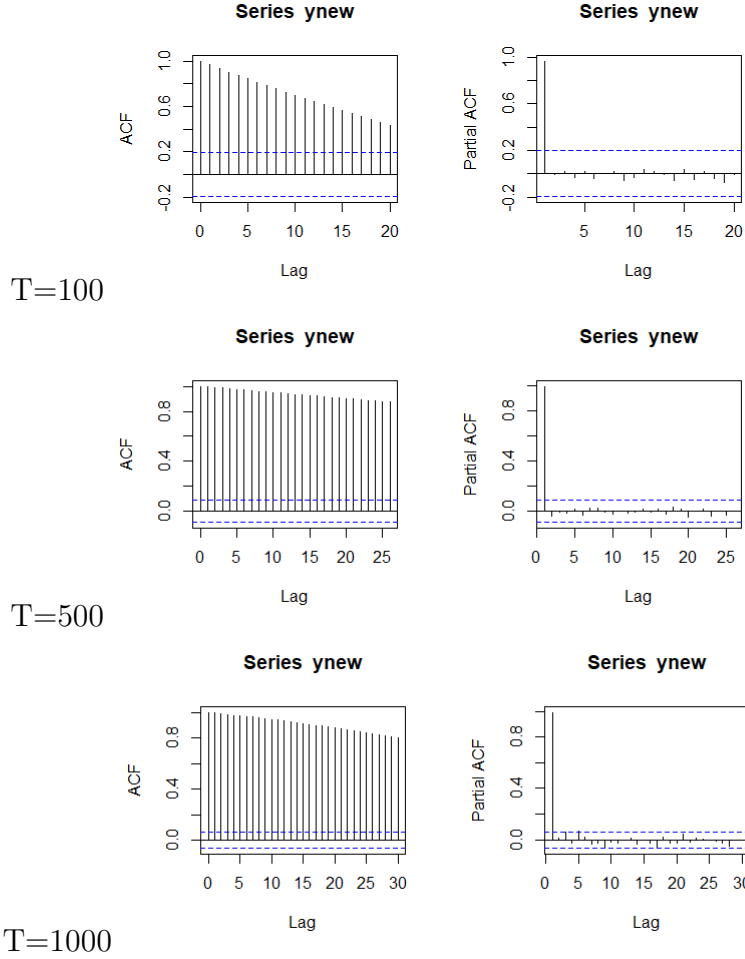
$$y_t = \alpha + 0.7y_{t-1} + u_t, \text{ where } u_{1-t} \text{ i.i.d.N } (0, 1), \text{ and } \alpha = 0$$

Let T be The sample size. This is again a model unknown the econometrician, who is interested in testing whether $E(y_t) = 0$, and knows there may be some autocorrelation in the errors. Simulate one sample from this model letting $T = 100, 500, 1000$, with $B = 100$ burn-in observations (so compute the model forward with $T + B$ observations, and discard the first B observations) Explain why we need to do this "burn-in".

The simulation was done for an AR(1) model with $T = 100, 500, 1000$, $B = 100$ and we added a burn-in equal to $T+B$ and then the first observation of B was deleted. The burn-in method is a good way to find a good starting point when the one we have is bias. This is because, unlike MA(1), AR(1) is not always a stationary model. Therefore if we include initial values the estimates would be bias. Therefore, using burn-in we guarantee that the model will be stationary and unbiased.

Question 2.d

Based on c), plot the ACF and PACF of this model, and explain whether they make sense



The plots of PACF for each sample show a significant lag of one because as we know the model is a AR(1). The plots of ACF for each sample shows several significant lags (that increase with the sample size) decaying. It represent an AR(1) that is increasing with the sample size. Finally, we can notice that the confidence values get shrink as the sample size increase. It allows us more precision in the final results.

Question 2.e

Simulate $S = 500$ replications from this model for $T = 100, 500, 1000$ with burn-in $B = 100$ observations each time (so you should have 500 datasets, each with T observations). Calculate for each simulation the t-statistic for the null hypothesis that $\alpha = 0$ $t = \bar{y} / \hat{\sigma}_y$, where $\bar{y} = \frac{1}{T} \sum_{t=1}^T y_t$, and $\hat{\sigma}_y$ is the Newey-West HAC estimator of the standard error of \bar{y} using the quadratic spectral kernel. Calculate the fraction of times in $S = 500$ simulations that this test rejects, for each sample size, and significance level 5% (using standard normal critical values), and compare it the nominal level 5%. Are they close or not to this nominal 5% level? If yes, explain why. If not, explain and investigate what is the source of distortions, keeping in mind that the simulated (also called)

”empirical” rejection frequency should be close to the nominal level as the sample size increases, because asymptotically, the t test is normally distributed.

After the simulation $S=500$ for $T=100, 500$ and 1000 was done, it was calculated for each simulation the t-statistic $t = \bar{y}/\sigma_y$ where σ_y was estimated using Newey-West HAC estimator. After that, the fraction rejection time was calculated. As a result for $T=100, 500$ and 1000 the fraction rejection was of $0.168, 0.088$ and 0.08 , respectively. It can be concluded that the rejection rate decays as we increase the sample size. However, in comparison with $MA(1)$, the rejection rate do not get close to nominal 5% level even for higher sample sizes. One reason of this is because the Newey-West HAC estimator could be biased therefore the rejection come out to be more than 5%. In addition, the $AR(1)$ model is much more sensitive for this bias than MA process therefore the difference between the rejection rates as the sample size increase.

Appendix

```
1 install.packages("lmtest")
2 library(lmtest)
3 install.packages("forecast")
4 library(forecast)
5 install.packages("tseries")
6 library(tseries)
7 install.packages("dplyr")
8 library(dplyr)
9 library(sandwich)
10
11 # Solution 2(a) -----
12
13 #Sample 100#
14 Ti = 100
15 beta = 0.7
16 alpha = 0
17
18 e = matrix(0,nrow = Ti,ncol = 1)
19 e[1,1] = rnorm(1, mean=0, sd = 1)
20 for (i in 2:Ti){
21   e[i,1] = rnorm(1, mean = 0, sd = 1)
22 }
23
24 plot(e)
25
26 u = matrix(0, nrow = Ti, ncol = 1)
27 u[1,1] = e[1,1]
28 for (i in 2:Ti){
29   u[i,1] = e[i,1] + beta*e[i-1,1] + rnorm(1,sd = 1)
30 }
31
32 y = matrix(0, nrow = Ti, ncol = 1)
33 y = alpha + u
34
35 acf(y)
36 pacf(y)
37
38 #Sample 500#
39 Ti = 500
40 beta = 0.7
41 alpha = 0
42
43 e = matrix(0,nrow = Ti,ncol = 1)
44 e[1,1] = rnorm(1, mean=0, sd = 1)
```

```

45 for (i in 2:Ti){
46   e[i,1] = rnorm(1, mean = 0, sd = 1)
47 }
48
49 plot(e)
50
51 u = matrix(0, nrow = Ti, ncol = 1)
52 u[1,1] = e[1,1]
53 for (i in 2:Ti){
54   u[i,1] = e[i,1] + beta*e[i-1,1] + rnorm(1,sd = 1)
55 }
56
57 y = matrix(0, nrow = Ti, ncol = 1)
58 y = alpha + u
59
60 acf(y)
61 pacf(y)
62
63 #Sample 1000
64 Ti = 1000
65 beta = 0.7
66 alpha = 0
67
68 e = matrix(0,nrow = Ti,ncol = 1)
69 e[1,1] = rnorm(1, mean=0, sd = 1)
70 for (i in 2:Ti){
71   e[i,1] = rnorm(1, mean = 0, sd = 1)
72 }
73
74 plot(e)
75
76 u = matrix(0, nrow = Ti, ncol = 1)
77 u[1,1] = e[1,1]
78 for (i in 2:Ti){
79   u[i,1] = e[i,1] + beta*e[i-1,1] + rnorm(1,sd = 1)
80 }
81
82 y = matrix(0, nrow = Ti, ncol = 1)
83 y = alpha + u
84
85 acf(y)
86 pacf(y)
87
88 # Solution 2(b) -----
89 #Sample=100
90 rm(list = ls())
91 set.seed(4534938432)
92
93 S = 500
94 B = 100
95 Ti = 100
96 beta = 0.7
97 alpha = 0
98 e = matrix(0, nrow = Ti, ncol = S)
99 for (i in 1:S){
100   e[1,i] = rnorm(1, mean = 0, sd = 1)
101   for (j in 2:Ti){
102     e[j,i] = rnorm(1, mean = 0, sd = 1)
103   }
104 }
105 u = matrix(0, nrow = Ti, ncol = S)
106 for (i in 1:S){
107   u[1,i] = e[1,i]
108   for (j in 2:Ti){
109     u[j,i] = e[j,i] + beta*e[j-1,i]
110   }
111 }
112 y = matrix(0, nrow = Ti, ncol = S)

```



```

113 y = alpha + u
114
115 ymean = matrix(0, nrow = 1, ncol = S)
116 for (i in 1:S){
117   ymean[,i] = mean(y[,i])
118 }
119 sigmahat = matrix(0, nrow = 1, ncol = S)
120 for (i in 1:S){
121   sigmahat[,i] = sqrt(kernHAC(lm(y[,i] ~ 1), bw = bwNeweyWest, prewhite = 0))
122 }
123 tstat = matrix(0, nrow = 1, ncol = S)
124 for (i in 1:S){
125   tstat[,i] = ymean[,i]/sigmahat[,i]
126 }
127 reject = matrix(0, nrow = 1, ncol = S)
128 for (i in 1:S){
129   reject[,i] = isTRUE(tstat[,i]>1.96) | isTRUE(tstat[,i]< (-1.96))
130 }
131 table(reject)
132 hist(tstat)
133 plot(density(tstat))
134
135 #Sample 500#
136
137 S = 500
138 B = 100
139 Ti = 500
140 beta = 0.7
141 alpha = 0
142 e = matrix(0, nrow = Ti, ncol = S)
143 for (i in 1:S){
144   e[1,i] = rnorm(1, mean = 0, sd = 1)
145   for (j in 2:Ti){
146     e[j,i] = rnorm(1, mean = 0, sd = 1)
147   }
148 }
149 u = matrix(0, nrow = Ti, ncol = S)
150 for (i in 1:S){
151   u[1,i] = e[1,i]
152   for (j in 2:Ti){
153     u[j,i] = e[j,i] + beta*e[j-1,i]
154   }
155 }
156 y = matrix(0, nrow = Ti, ncol = S)
157 y = alpha + u
158
159 ymean = matrix(0, nrow = 1, ncol = S)
160 for (i in 1:S){
161   ymean[,i] = mean(y[,i])
162 }
163 sigmahat = matrix(0, nrow = 1, ncol = S)
164 for (i in 1:S){
165   sigmahat[,i] = sqrt(kernHAC(lm(y[,i] ~ 1), bw = bwNeweyWest, prewhite = 0))
166 }
167 tstat = matrix(0, nrow = 1, ncol = S)
168 for (i in 1:S){
169   tstat[,i] = ymean[,i]/sigmahat[,i]
170 }
171 reject = matrix(0, nrow = 1, ncol = S)
172 for (i in 1:S){
173   reject[,i] = isTRUE(tstat[,i]>1.96) | isTRUE(tstat[,i]< (-1.96))
174 }
175 table(reject)
176 hist(tstat)
177 plot(density(tstat))
178
179 #Sample 1000#
180

```

```

181
182 S = 500
183 B = 100
184 Ti = 1000
185 beta = 0.7
186 alpha = 0
187 e = matrix(0, nrow = Ti, ncol = S)
188 for (i in 1:S){
189   e[1,i] = rnorm(1, mean = 0, sd = 1)
190   for (j in 2:Ti){
191     e[j,i] = rnorm(1, mean = 0, sd = 1)
192   }
193 }
194 u = matrix(0, nrow = Ti, ncol = S)
195 for (i in 1:S){
196   u[1,i] = e[1,i]
197   for (j in 2:Ti){
198     u[j,i] = e[j,i] + beta*e[j-1,i]
199   }
200 }
201 y = matrix(0, nrow = Ti, ncol = S)
202 y = alpha + u
203
204 ymean = matrix(0, nrow = 1, ncol = S)
205 for (i in 1:S){
206   ymean[,i] = mean(y[,i])
207 }
208 sigmahat = matrix(0, nrow = 1, ncol = S)
209 for (i in 1:S){
210   sigmahat[,i] = sqrt(kernHAC(lm(y[,i] ~ 1), bw = bwNeweyWest, prewhite = 0))
211 }
212 tstat = matrix(0, nrow = 1, ncol = S)
213 for (i in 1:S){
214   tstat[,i] = ymean[,i]/sigmahat[,i]
215 }
216 reject = matrix(0, nrow = 1, ncol = S)
217 for (i in 1:S){
218   reject[,i] = isTRUE(tstat[,i]>1.96) | isTRUE(tstat[,i]< (-1.96))
219 }
220
221 table(reject)
222 hist(tstat)
223 plot(density(tstat))
224
225
226
227
228 # Solution 2(c, d) -----
229
230
231 # If T (sample size) = 100
232 rm(list = ls())
233 set.seed(6000000)
234
235
236 Ti = 100
237 B = 100
238 beta = 0.7
239 alpha = 0
240 burnin = Ti+B
241
242 y = matrix(0, nrow = burnin, ncol = 1)
243 for (i in 2:burnin){
244   y[i,1] = alpha + y[i-1,1] + rnorm(1, mean = 0, sd = 1)
245 }
246
247 ynew = tail(y,-B)
248

```

```

249
250 acf(ynew)
251 pacf(ynew)
252
253 #####
254 # If T (sample size) = 500
255 Ti= 500
256 B = 100
257 beta = 0.7
258 alpha = 0
259 burnin = Ti+B
260
261 y = matrix(0, nrow = burnin, ncol = 1)
262 for (i in 2:burnin){
263   y[i,1] = alpha + y[i-1,1] + rnorm(1, mean = 0, sd = 1)
264 }
265
266 ynew = tail(y,-B)
267
268 acf(ynew)
269 pacf(ynew)
270
271 #####
272 # sample 1000
273 Ti = 1000
274 B = 100
275 beta = 0.7
276 alpha = 0
277 burnin = Ti+B
278
279 y = matrix(0, nrow = burnin, ncol = 1)
280 for (i in 2:burnin){
281   y[i,1] = alpha + y[i-1,1] + rnorm(1, mean = 0, sd = 1)
282 }
283
284 ynew = tail(y,-B)
285
286
287 acf(ynew)
288 pacf(ynew)
289
290 # Solution 2(e) -----
291
292 rm(list = ls())
293 set.seed(6000000000)
294
295 # sample 100
296 S = 500
297 B = 100
298 Ti = 100
299 beta = 0.7
300 alpha = 0
301 burnin = Ti+B
302
303 y = matrix(0, nrow = burnin, ncol = S)
304 for (i in 1:S){
305   for (j in 2:burnin){
306     y[j,i] = alpha + beta*y[j-1,i] + rnorm(1, mean = 0, sd = 1)
307   }
308 }
309
310 ynew = tail(y,-B)
311
312 ymean = matrix(0, nrow = 1, ncol = S)
313 for (i in 1:S){
314   ymean[,i] = mean(y_new[,i])
315 }
316 sigmahat = matrix(0, nrow = 1, ncol = S)

```

```

317 for (i in 1:S){
318   sigmahat[,i] = sqrt(kernHAC(lm(y_new[,i] ~ 1), bw = bwNeweyWest, prewhite = 0))
319 }
320 tstat = matrix(0, nrow = 1, ncol = S)
321 for (i in 1:S){
322   tstat[,i] = ymean[,i]/sigmahat[,i]
323 }
324 reject = matrix(0, nrow = 1, ncol = S)
325 for (i in 1:S){
326   reject[,i] = isTRUE(tstat[,i]>1.96) | isTRUE(tstat[,i]< (-1.96))
327 }
328 table(reject)
329 hist(tstat)
330 plot(density(tstat))
331
332 #####
333
334 # sample 500
335 S = 500
336 B = 100
337 Ti = 500
338 beta = 0.7
339 alpha = 0
340 burnin = Ti+B
341
342 y = matrix(0, nrow = burnin, ncol = S)
343 for (i in 1:S){
344   for (j in 2:burnin){
345     y[j,i] = alpha + beta*y[j-1,i] + rnorm(1, mean = 0, sd = 1)
346   }
347 }
348
349 ynew = tail(y,-B)
350
351 ymean = matrix(0, nrow = 1, ncol = S)
352 for (i in 1:S){
353   ymean[,i] = mean(y_new[,i])
354 }
355 sigmahat = matrix(0, nrow = 1, ncol = S)
356 for (i in 1:S){
357   sigmahat[,i] = sqrt(kernHAC(lm(y_new[,i] ~ 1), bw = bwNeweyWest, prewhite = 0))
358 }
359 tstat = matrix(0, nrow = 1, ncol = S)
360 for (i in 1:S){
361   tstat[,i] = ymean[,i]/sigmahat[,i]
362 }
363 reject = matrix(0, nrow = 1, ncol = S)
364 for (i in 1:S){
365   reject[,i] = isTRUE(tstat[,i]>1.96) | isTRUE(tstat[,i]< (-1.96))
366 }
367 table(reject)
368 hist(tstat)
369 plot(density(tstat))
370
371 #####
372
373 # If T (sample size) = 1000
374 S = 500
375 B = 100
376 Ti = 1000
377 beta = 0.7
378 alpha = 0
379 burnin = Ti+B
380
381 y = matrix(0, nrow = burnin, ncol = S)
382 for (i in 1:S){
383   for (j in 2:burnin){
384     y[j,i] = alpha + beta*y[j-1,i] + rnorm(1, mean = 0, sd = 1)

```

```

385   }
386 }
387
388 ynew = tail(y,-B)
389
390 ymean = matrix(0, nrow = 1, ncol = S)
391 for (i in 1:S){
392   ymean[,i] = mean(y_new[,i])
393 }
394 sigmahat = matrix(0, nrow = 1, ncol = S)
395 for (i in 1:S){
396   sigmahat[,i] = sqrt(kernHAC(lm(y_new[,i] ~ 1), bw = bwNeweyWest, prewhite = 0))
397 }
398 tstat = matrix(0, nrow = 1, ncol = S)
399 for (i in 1:S){
400   tstat[,i] = ymean[,i]/sigmahat[,i]
401 }
402 reject = matrix(0, nrow = 1, ncol = S)
403 for (i in 1:S){
404   reject[,i] = isTRUE(tstat[,i]>1.96) | isTRUE(tstat[,i]< (-1.96))
405 }
406 table(reject)
407 hist(tstat)
408 plot(density(tstat))

```