Econometrics 3

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Assignment 3

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Question 1

You are interested in the response of output growth $y_{1,t}$ to fiscal policy shocks $e_{2,t}$. The structural model is as follows:

$$\begin{aligned} y_{1,t} &= \gamma_1 y_{2,t} + \alpha_{11} y_{1,t-1} + \alpha_{12} y_{2,t-1} + e_{1,t} \\ y_{2,t} &= \gamma_2 y_{1,t} + \alpha_{21} y_{1,t-1} + \alpha_{22} y_{2,t-1} + e_{2,t} \end{aligned} \tag{1}$$

Question 1.a

a) Under what conditions can this structural model be transformed in structural VAR form? Write down the conditions in terms of the parameters above and explain.

The conditions that we need to transform this structural model to structural VAR form are that the parameters do not change with the time. In other words γ_1, α_{11}, α_{12}, γ_2, α_{21} and α_{22} should be the same for all periods of time in the model to estimate them. Another assumptions are $E(e_t) = 0$, $E(e_t e_t^{'}) = \Sigma_e$ and Γ is invertible where:

$$\begin{aligned} \Gamma &= \begin{bmatrix} 1 & -\gamma_1 \\ -\gamma_2 & 1 \end{bmatrix} \\ e_t &= \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix} \\ \Sigma_e &= \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \end{aligned} \tag{2}$$

Question 1.b

b) Assuming the model can be written in a structural VAR form, write down the structural VAR formulation.

First we rewrite in lag terms:

$$\begin{aligned} y_{1,t} &= \gamma_1 y_{2,t} + \alpha_{11} y_{1,t-1} + \alpha_{12} y_{2,t-1} + e_{1,t} = \gamma_1 y_{2,t} + \alpha_{11}(L) y_{1,t} + \alpha_{12}(L) y_{2,t} + e_{1,t} \\ y_{2,t} &= \gamma_2 y_{1,t} + \alpha_{21} y_{1,t-1} + \alpha_{22} y_{2,t-1} + e_{2,t} = \gamma_2 y_{1,t} + \alpha_{21}(L) y_{1,t} + \alpha_{22}(L) y_{2,t} + e_{2,t} \end{aligned} \tag{3}$$

Rewrite as:

$$\underbrace{\begin{bmatrix} 1 & -\gamma_1 \\ -\gamma_2 & 1 \end{bmatrix}}_{\Gamma}\underbrace{\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} \alpha_{11}(L) & \alpha_{12}(L) \\ \alpha_{21}(L) & \alpha_{22}(L) \end{bmatrix}}_{A(L)}\underbrace{\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix}}_{Y_t} + e_t$$
$$\Gamma Y_t = AY_{t-1} + e_t \tag{4}$$

And the reduce form is:

$$\begin{aligned} \Gamma Y_t &= A(L)Y_t + e_t \\ Y_t &= \underbrace{\Gamma^{-1}A(L)}_{A^*(L)}Y_t + \underbrace{\Gamma^{-1}e_t}_{u_t} \\ Y_t &= A^*(L)Y_t + u_t \end{aligned} \tag{5}$$

Question 1.c

c) Based on b), use backward substitution to derive the expression for the IRF of output growth to a fiscal policy shock at horizons 0, 1 (i.e. $\frac{\partial y_{1,t+h}}{\partial e_{2,t}}$,h = 0, 1).
For derive IRF we should rewrite the expression of b):

$$\begin{aligned} Y_t &= A^*(L)Y_t + u_t \\ Y_t - A^*(L)Y_t &= +u_t \\ (I_2 - A^*(L))Y_t &= u_t \\ Y_t &= (I_2 - A^*(L))^{-1}u_t \\ Y_t &= (I_2 - A^*(L))^{-1}\Gamma^{-1}e_t \end{aligned} \tag{6}$$

using backward substitution, first we found the matriz Gamma inverse

$$\Gamma^{-1} = \begin{bmatrix} 1 & -\gamma_1 \\ -\gamma_2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{1-\gamma_1\gamma_2} & \frac{\gamma_1}{1-\gamma_1\gamma_2} \\ \frac{\gamma_2}{1-\gamma_1\gamma_2} & \frac{1}{1-\gamma_1\gamma_2} \end{bmatrix} \tag{7}$$

With this we found $A^*(L)$

$$\begin{aligned} A^*(L) = \Gamma^{-1}A(L) &= \begin{bmatrix} \frac{1}{1-\gamma_1\gamma_2} & \frac{\gamma_1}{1-\gamma_1\gamma_2} \\ \frac{\gamma_2}{1-\gamma_1\gamma_2} & \frac{1}{1-\gamma_1\gamma_2} \end{bmatrix}\begin{bmatrix} \alpha_{11}(L) & \alpha_{12}(L) \\ \alpha_{21}(L) & \alpha_{22}(L) \end{bmatrix} \\ A^*(L) &= \begin{bmatrix} \frac{\alpha_{11}(L)+\alpha_{21}(L)\gamma_1}{1-\gamma_1\gamma_2} & \frac{\alpha_{12}(L)+\gamma_1\alpha_{22}(L)}{1-\gamma_1\gamma_2} \\ \frac{\gamma_2\alpha_{11}(L)+\alpha_{21}(L)}{1-\gamma_1\gamma_2} & \frac{\gamma_2\alpha_{12}(L)+\alpha_{22}(L)}{1-\gamma_1\gamma_2} \end{bmatrix} \end{aligned} \tag{8}$$

Using $A^*(L)$ we can calculate $I_2 - A^*(L)$:

$$\begin{aligned} I_2 - A^*(L) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{\alpha_{11}(L)+\alpha_{21}(L)\gamma_1}{1-\gamma_1\gamma_2} & \frac{\alpha_{12}(L)+\gamma_1\alpha_{22}(L)}{1-\gamma_1\gamma_2} \\ \frac{\gamma_2\alpha_{11}(L)+\alpha_{21}(L)}{1-\gamma_1\gamma_2} & \frac{\gamma_2\alpha_{12}(L)+\alpha_{22}(L)}{1-\gamma_1\gamma_2} \end{bmatrix} \\ I_2 - A^*(L) &= \begin{bmatrix} \frac{1-\gamma_1\gamma_2-\alpha_{11}(L)-\alpha_{21}(L)\gamma_1}{1-\gamma_1\gamma_2} & -\frac{\alpha_{12}(L)+\gamma_1\alpha_{22}(L)}{1-\gamma_1\gamma_2} \\ -\frac{\gamma_2\alpha_{11}(L)+\alpha_{21}(L)}{1-\gamma_1\gamma_2} & \frac{1-\gamma_1\gamma_2-\gamma_2\alpha_{12}(L)-\alpha_{22}(L)}{1-\gamma_1\gamma_2} \end{bmatrix} \end{aligned} \tag{9}$$

The inverse of $I_2 - A^*(L)$ is:

$$[I_2 - A^*(L)]^{-1} = \begin{bmatrix} \frac{1-\gamma_1\gamma_2-\gamma_2\alpha_{12}(L)-\alpha_{22}(L)}{\theta} & \frac{\alpha_{12}(L)+\gamma_1\alpha_{22}(L)}{\theta} \\ \frac{\gamma_2\alpha_{11}(L)+\alpha_{21}(L)}{\theta} & \frac{1-\gamma_1\gamma_2-\alpha_{11}(L)-\alpha_{21}(L)\gamma_1}{\theta} \end{bmatrix} \tag{10}$$

Where $\theta = 1-\gamma_1\gamma_2-\gamma_2\alpha_{12}(L)-\alpha_{22}(L)-\alpha_{11}(L)-\alpha_{21}(L)\gamma_1+\alpha_{11}(L)\alpha_{22}(L)-\alpha_{21}(L)\alpha_{12}(L)$. For last we need to find $[I_2 - A^*(L)]^{-1}\Gamma^{-1}$, we have both matrix so:

$$\begin{aligned} [I_2 - A^*(L)]^{-1}\Gamma^{-1} &= \begin{bmatrix} \frac{1-\gamma_1\gamma_2-\gamma_2\alpha_{12}(L)-\alpha_{22}(L)}{\theta} & \frac{\alpha_{12}(L)+\gamma_1\alpha_{22}(L)}{\theta} \\ \frac{\gamma_2\alpha_{11}(L)+\alpha_{21}(L)}{\theta} & \frac{1-\gamma_1\gamma_2-\alpha_{11}(L)-\alpha_{21}(L)\gamma_1}{\theta} \end{bmatrix} \begin{bmatrix} \frac{1}{1-\gamma_1\gamma_2} & \frac{\gamma_1}{1-\gamma_1\gamma_2} \\ \frac{\gamma_2}{1-\gamma_1\gamma_2} & \frac{1}{1-\gamma_1\gamma_2} \end{bmatrix} \\ [I_2 - A^*(L)]^{-1}\Gamma^{-1} &= \begin{bmatrix} \frac{1-\alpha_{22}(L)}{\theta} & \frac{\gamma_1+\alpha_{12}(L)}{\theta} \\ \frac{\gamma_2+\alpha_{21}(L)}{\theta} & \frac{1-\alpha_{11}(L)}{\theta} \end{bmatrix} \end{aligned} \tag{11}$$

The effect of e_2 on y_1 is $\frac{\gamma_1+\alpha_{12}(L)}{\theta}$, the $\frac{\partial y_{1,t+1}}{\partial e_{2,t}} = \frac{\alpha_{12}}{\theta}$ and $\frac{\partial y_{1,t}}{\partial e_{2,t}} = \frac{\gamma_1}{\theta}$

Question 1.d

d) Are there any identification problems in estimating the structural parameters γ_1 and γ_2 from the structural VAR formulation? Explain, and if there are such problems, give in words a solution to these. We do have identification problem because we have eight parameters to estimate (γ_1, α_{11}, α_{12}, γ_2, α_{21}, α_{22}, σ_1 and σ_2) but in the empirical model (the reduce form), we can estimate only seven estimates and hence there is underidentification problem in the model.

The solution for this problem implies to find a instrumental variable or make another restriction. But IV are hard to find and very unlikely to be truely exogenous. Exist others ways to make another restriction such as 1) The short-run restrictions, 2) The long-run restrictions, 3) The informal identification and 4) The sign restrictions to use the theory to make restrictions about the behaviour we expect of our model. The first one try to reduce the problem assuming an implicit ordering in the Model, with this order we can make one contemporaneous parameters (gamma one or gamma two) equal to zero, however the estimates about the error depends on the parameter equal to zero. The second one use the economic theory to impose impose a restriction on parameters but based on long-term relationships. For example, the change in monetary policy on GDP, we know that there is neutrality of money, so in the long term it must be annulled. But this condition is a non-linear constraint, so it complicates the problem and, depending on the problem, we need to identify whether or not this type of constraint can be applied.The third one model means to move the model until the impulse response function resembles what the investigator expects. This is an informal and very subjective methodology depending on what the researcher thinks. And the last one, a very similar to the third one, is the sign restriction in which the form in which it should be is not directly stated, but we must say for example that the effect of fiscal policy on GDP must be positive and the coefficients must be reduced to positive values, and with that it gives me a restriction that can be solved. For this particular case, we can use the sign restrictions to impose the effect of fiscal policy or the other alternative is to look for an instrument such as the announcement of fiscal policy.

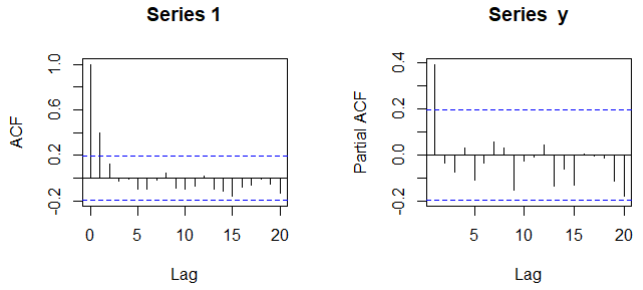
Question 2

a) Let the true model be $y_t = \alpha + u_t$, where $u_t = \epsilon_t + \beta\epsilon_{t1}$ with $\beta = 0.7$, and ϵ_t i.i.d.N (0, 1), and $\alpha = 0$. Let T be the sample size. This is a model unknown the econometrician, who is interested in testing whether $E(y_t) = 0$, and knows there may be some autocorrelation in the errors.

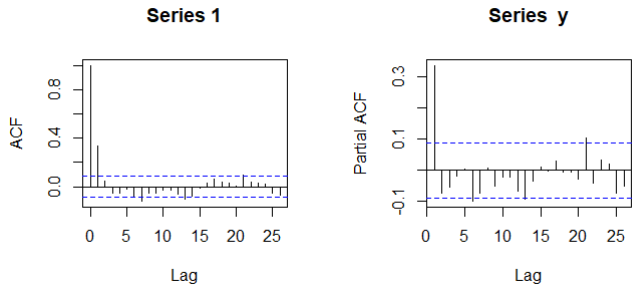
Question 2.a

Simulate one sample from this model letting T = 100, 500, 1000. Plot the ACF and PACF of this model, and explain whether they make sense.

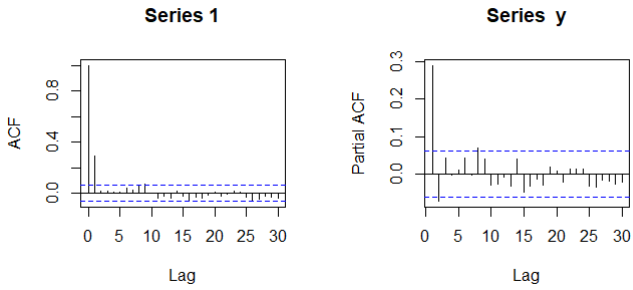
After simulate the moving average model (MA1), we can see that as we increase the T from 100 to 1000, the interval of confidence shrinks. Therefore, it makes possible more accurate results. Also, as the model is MA(1), the autocorrelation function has just one significant lag. It makes sense with the model and the observation we added to the simulation.



T=100



T=500



T=1000

Question 2.b

Simulate S = 500 replications from this model for T = 100, 500, 1000 with burn-in B = 100 observations each time (so you should have 500 datasets, each with T observations). Calculate for each simulation the t-statistic for the null hypothesis that $\alpha = 0$ $t = \overline{y}/\hat{\sigma y}$, where $\overline{y} = PT$ t=1 y_t, and $\hat{\sigma}_{y]}$ is the Newey-West HAC estimator of the standard error of $\overline{y}$ using the quadratic spectral kernel. Calculate the fraction of times in S = 500 simulations that this test rejects, for each sample size, and significance level 5% (using standard normal critical values), and compare it the nominal level 5%. Are they close or not to this nominal 5% level? If yes, explain why.

After the simulation S=500 for T=100, 500 and 1000 was done, it was calculated for each simulation the t-statistic t $=\overline{y}/\sigma_y$ where σ_y was estimated using Newey-West HAC estimator. After that, the fraction rejection time was calculated. As a result for T=100, 500 and 1000 the fraction rejection was of 0.082, 0.072 and 0.06, respectively. It can be concluded that the rejection rate get closer to 5% as we increase the sample size because as the sample increase, it resembles to a normal distribution.

Question 2.c

Let the true model now by

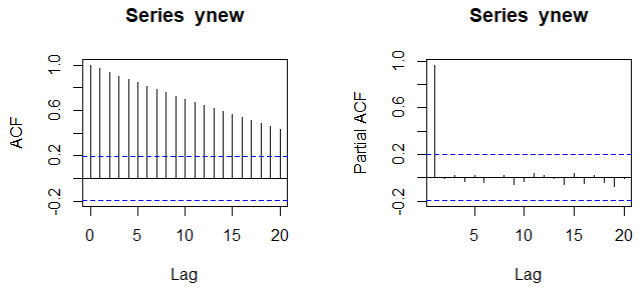
$$y_t = \alpha + 0.7y_{t1} + u_t, \text{ where } u_{1-t} \text{ i.i.d.N (0, 1), and } \alpha = 0$$

Let T be The sample size. This is again a model unknown the econometrician, who is interested in testing whether $E(yt) = 0$, and knows there may be some autocorrelation in the errors. Simulate one sample from this model letting T = 100, 500, 1000, with B = 100 burn-in observations (so compute the model forward with T + B observations, and discard the first B observations) Explain why we need to do this "burn-in".

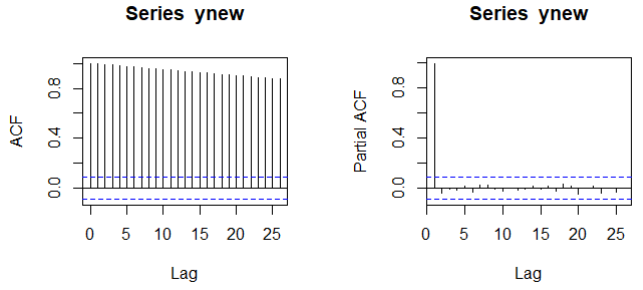
The simulation was done for an AR(1) model with T = 100, 500, 1000, B = 100 and we added a burn-in equal to T+B and then the first observation of B was deleted. The burn-in method is a good way to find a good starting point when the one we have is bias. This is because, unlike MA(1), AR(1) is not always a stationary model. Therefore if we include initial values the estimates would be bias. Therefore, using burn-in we guarantee that the model will be stationary and unbiased.

Question 2.d

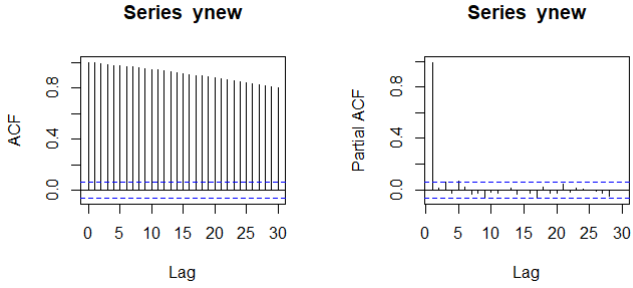
Based on c), plot the ACF and PACF of this model, and explain whether they make sense



T=100



T=500



T=1000

The plots of PACF for each sample show a significant lag of one because as we know the model is a AR(1). The plots of ACF for each sample shows several significant lags (that increase with the sample size) decaying. It represent an AR(1) that is increasing with the sample size. Finally, we can notice that the confidence values get shrink as the sample size increase. It allows us more precision in the final results.

Question 2.e

Simulate S = 500 replications from this model for T = 100, 500, 1000 with burn-in B = 100 observations each time (so you should have 500 datasets, each with T observations). Calculate for each simulation the t-statistic for the null hypothesis that $\alpha = 0$ $t = \overline{y}/\hat{\sigma}y$, where $\overline{y} = PT$ t=1 y_t, and $\hat{\sigma}_{y]}$ is the Newey-West HAC estimator of the standard error of $\overline{y}$ using the quadratic spectral kernel. Calculate the fraction of times in S = 500 simulations that this test rejects, for each sample size, and significance level 5% (using standard normal critical values), and compare it the nominal level 5%. Are they close or not to this nominal 5% level? If yes, explain why. If not, explain and investigate what is the source of distortions, keeping in mind that the simulated (also called)

"empirical" rejection frequency should be close to the nominal level as the sample size increases, because asymptotically, the t test is normally distributed.
After the simulation S=500 for T=100, 500 and 1000 was done, it was calculated for each simulation the t-statistic t $=\overline{y}/\sigma_y$ where σ_y was estimated using Newey-West HAC estimator. After that, the fraction rejection time was calculated. As a result for T=100, 500 and 1000 the fraction rejection was of 0.168, 0.088 and 0.08, respectively. It can be concluded that the rejection rate decays as we increase the sample size. However, in comparison with MA(1), the rejection rate do not get close to nominal 5% level even for higher sample sizes. One reason of this is because the Newey-West HAC estimator could be biased therefore the rejection come out to be more than 5%. In addition, the AR(1) model is much more sensitive for this bias than MA process therefore the difference between the rejection rates as the sample size increase.

Appendix

```
install.packages("lmtest")
library(lmtest)
install.packages("forecast")
library(forecast)
install.packages("tseries")
library(tseries)
install.packages("dplyr")
library(dplyr)
library(sandwich)

# Solution 2(a) ------------------------------------------------------------

#Sample 100#
Ti = 100
beta = 0.7
alpha = 0

e = matrix(0,nrow = Ti,ncol = 1)
e[1,1] = rnorm(1, mean=0, sd = 1)
for (i in 2:Ti){
  e[i,1] =  rnorm(1, mean = 0, sd = 1)
}

plot(e)

u = matrix(0, nrow = Ti, ncol = 1)
u[1,1] = e[1,1]
for (i in 2:Ti){
  u[i,1] =  e[i,1] + beta*e[i-1,1] + rnorm(1,sd = 1)
}

y = matrix(0, nrow = Ti, ncol = 1)
y = alpha + u

acf(y)
pacf(y)

#Sample 500#
Ti = 500
beta = 0.7
alpha = 0

e = matrix(0,nrow = Ti,ncol = 1)
e[1,1] = rnorm(1, mean=0, sd = 1)
```

```r
for (i in 2:Ti){
  e[i,1] =  rnorm(1, mean = 0, sd = 1)
}

plot(e)

u = matrix(0, nrow = Ti, ncol = 1)
u[1,1] = e[1,1]
for (i in 2:Ti){
  u[i,1] =  e[i,1] + beta*e[i-1,1] + rnorm(1,sd = 1)
}

y = matrix(0, nrow = Ti, ncol = 1)
y = alpha + u

acf(y)
pacf(y)

#Sample 1000
Ti = 1000
beta = 0.7
alpha = 0

e = matrix(0,nrow = Ti,ncol = 1)
e[1,1] = rnorm(1, mean=0, sd = 1)
for (i in 2:Ti){
  e[i,1] =  rnorm(1, mean = 0, sd = 1)
}

plot(e)

u = matrix(0, nrow = Ti, ncol = 1)
u[1,1] = e[1,1]
for (i in 2:Ti){
  u[i,1] =  e[i,1] + beta*e[i-1,1] + rnorm(1,sd = 1)
}

y = matrix(0, nrow = Ti, ncol = 1)
y = alpha + u

acf(y)
pacf(y)

# Solution 2(b) ------------------------------------------------------------
#Sample=100
rm(list = ls())
set.seed(4534938432)

S = 500
B = 100
Ti = 100
beta = 0.7
alpha = 0
e = matrix(0, nrow = Ti, ncol = S)
for (i in 1:S){
  e[1,i] = rnorm(1, mean = 0, sd = 1)
  for (j in 2:Ti){
    e[j,i] =  rnorm(1, mean = 0, sd = 1)
  }
}
u = matrix(0, nrow = Ti, ncol = S)
for (i in 1:S){
  u[1,i] = e[1,i]
  for (j in 2:Ti){
    u[j,i] = e[j,i] + beta*e[j-1,i]
  }
}
y = matrix(0, nrow = Ti, ncol = S)
```

```
y = alpha + u

ymean = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  ymean[,i] = mean(y[,i])
}
sigmahat = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  sigmahat[,i] = sqrt(kernHAC(lm(y[,i] ~ 1), bw = bwNeweyWest, prewhite = 0))
}
tstat = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  tstat[,i] = ymean[,i]/sigmahat[,i]
}
reject = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  reject[,i] = isTRUE(tstat[,i]>1.96) | isTRUE(tstat[,i]< (-1.96))
}
table(reject)
hist(tstat)
plot(density(tstat))

#Sample 500#

S = 500
B = 100
Ti = 500
beta = 0.7
alpha = 0
e = matrix(0, nrow = Ti, ncol = S)
for (i in 1:S){
  e[1,i] = rnorm(1, mean = 0, sd = 1)
  for (j in 2:Ti){
    e[j,i] =  rnorm(1, mean = 0, sd = 1)
  }
}
u = matrix(0, nrow = Ti, ncol = S)
for (i in 1:S){
  u[1,i] = e[1,i]
  for (j in 2:Ti){
    u[j,i] = e[j,i] + beta*e[j-1,i]
  }
}
y = matrix(0, nrow = Ti, ncol = S)
y = alpha + u

ymean = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  ymean[,i] = mean(y[,i])
}
sigmahat = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  sigmahat[,i] = sqrt(kernHAC(lm(y[,i] ~ 1), bw = bwNeweyWest, prewhite = 0))
}
tstat = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  tstat[,i] = ymean[,i]/sigmahat[,i]
}
reject = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  reject[,i] = isTRUE(tstat[,i]>1.96) | isTRUE(tstat[,i]< (-1.96))
}
table(reject)
hist(tstat)
plot(density(tstat))

#Sample 1000#

```

```r

S = 500
B = 100
Ti = 1000
beta = 0.7
alpha = 0
e = matrix(0, nrow = Ti, ncol = S)
for (i in 1:S){
  e[1,i] = rnorm(1, mean = 0, sd = 1)
  for (j in 2:Ti){
    e[j,i] =  rnorm(1, mean = 0, sd = 1)
  }
}
u = matrix(0, nrow = Ti, ncol = S)
for (i in 1:S){
  u[1,i] = e[1,i]
  for (j in 2:Ti){
    u[j,i] = e[j,i] + beta*e[j-1,i]
  }
}
y = matrix(0, nrow = Ti, ncol = S)
y = alpha + u

ymean = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  ymean[,i] = mean(y[,i])
}
sigmahat = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  sigmahat[,i] = sqrt(kernHAC(lm(y[,i] ~ 1), bw = bwNeweyWest, prewhite = 0))
}
tstat = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  tstat[,i] = ymean[,i]/sigmahat[,i]
}
reject = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  reject[,i] = isTRUE(tstat[,i]>1.96) | isTRUE(tstat[,i]< (-1.96))
}

table(reject)
hist(tstat)
plot(density(tstat))




# Solution 2(c, d) ------------------------------------------------------------


# If T (sample size) = 100
rm(list = ls())
set.seed(600000)


Ti = 100
B = 100
beta = 0.7
alpha = 0
burnin = Ti+B

y = matrix(0, nrow = burnin, ncol = 1)
for (i in 2:burnin){
  y[i,1] =  alpha + y[i-1,1] + rnorm(1, mean = 0, sd = 1)
}

ynew = tail(y,-B)

```

```r

acf(ynew)
pacf(ynew)

##########################################
# If T (sample size) = 500
Ti= 500
B = 100
beta = 0.7
alpha = 0
burnin = Ti+B

y = matrix(0, nrow = burnin, ncol = 1)
for (i in 2:burnin){
  y[i,1] =  alpha + y[i-1,1] + rnorm(1, mean = 0, sd = 1)
}

ynew = tail(y,-B)

acf(ynew)
pacf(ynew)

##########################################
# sample 1000
Ti = 1000
B = 100
beta = 0.7
alpha = 0
burnin = Ti+B

y = matrix(0, nrow = burnin, ncol = 1)
for (i in 2:burnin){
  y[i,1] =  alpha + y[i-1,1] + rnorm(1, mean = 0, sd = 1)
}

ynew = tail(y,-B)


acf(ynew)
pacf(ynew)

# Solution 2(e) ------------------------------------------------------------

rm(list = ls())
set.seed(600000000)

# sample 100
S = 500
B = 100
Ti = 100
beta = 0.7
alpha = 0
burnin = Ti+B

y = matrix(0, nrow = burnin, ncol = S)
for (i in 1:S){
  for (j in 2:burnin){
    y[j,i] =  alpha + beta*y[j-1,i] + rnorm(1, mean = 0, sd = 1)
  }
}

ynew = tail(y,-B)

ymean = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  ymean[,i] = mean(y_new[,i])
}
sigmahat = matrix(0, nrow = 1, ncol = S)
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for (i in 1:S){
  sigmahat[,i] = sqrt(kernHAC(lm(y_new[,i] ~ 1), bw = bwNeweyWest, prewhite = 0))
}
tstat = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  tstat[,i] = ymean[,i]/sigmahat[,i]
}
reject = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  reject[,i] = isTRUE(tstat[,i]>1.96) | isTRUE(tstat[,i]< (-1.96))
}
table(reject)
hist(tstat)
plot(density(tstat))

##############################################

# sample 500
S = 500
B = 100
Ti = 500
beta = 0.7
alpha = 0
burnin = Ti+B

y = matrix(0, nrow = burnin, ncol = S)
for (i in 1:S){
  for (j in 2:burnin){
    y[j,i] =  alpha + beta*y[j-1,i] + rnorm(1, mean = 0, sd = 1)
  }
}

ynew = tail(y,-B)

ymean = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  ymean[,i] = mean(y_new[,i])
}
sigmahat = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  sigmahat[,i] = sqrt(kernHAC(lm(y_new[,i] ~ 1), bw = bwNeweyWest, prewhite = 0))
}
tstat = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  tstat[,i] = ymean[,i]/sigmahat[,i]
}
reject = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  reject[,i] = isTRUE(tstat[,i]>1.96) | isTRUE(tstat[,i]< (-1.96))
}
table(reject)
hist(tstat)
plot(density(tstat))

##############################################

# If T (sample size) = 1000
S = 500
B = 100
Ti = 1000
beta = 0.7
alpha = 0
burnin = Ti+B

y = matrix(0, nrow = burnin, ncol = S)
for (i in 1:S){
  for (j in 2:burnin){
    y[j,i] =  alpha + beta*y[j-1,i] + rnorm(1, mean = 0, sd = 1)
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```
  }
}

ynew = tail(y,-B)

ymean = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  ymean[,i] = mean(y_new[,i])
}
sigmahat = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  sigmahat[,i] = sqrt(kernHAC(lm(y_new[,i] ~ 1), bw = bwNeweyWest, prewhite = 0))
}
tstat = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  tstat[,i] = ymean[,i]/sigmahat[,i]
}
reject = matrix(0, nrow = 1, ncol = S)
for (i in 1:S){
  reject[,i] = isTRUE(tstat[,i]>1.96) | isTRUE(tstat[,i]< (-1.96))
}
table(reject)
hist(tstat)
plot(density(tstat))
```