Empirical Template Lightcurve Fitting for PS1 Photometry of Cepheids

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1 Goal

This documents sketches the approach to fitting empirical light curve templates to the PS1 photometry to Cepheids in the Galactic disk. We need to account for the non-simultaneous photometry, the sparse sampling, and the severe dust reddening, A_λ, where the extinction curve shape may or may not be known; excxept for the dust reddening these issues are also present for RRLyrae in PS1, where the problem has been solved (Sesar et al 2016).

2 Towards an empirical model

LI has derived empirical light curve templates for Cepheids, as a function of period. In a fiducial band, say the r-band, the model for the Cepheids absolute magnitude, $M_r^m(\varphi \mid P)$ is:

$$M_r^m(\varphi \mid P, [Fe/H]) \equiv \overline{M}_r(P) + f_A \times \hat{M}_r(\varphi \mid P), \tag{1}$$

where P is the period, φ the pulsation phase, $\hat{M}_r(\varphi \mid P)$ the lightcurve template (with $\langle \hat{M}_r(\varphi \mid P) \rangle \equiv 0$), and $f_A \sim 1$ is an amplitude scaling factor. **HW**: The pulsation phase at time t_{obs} is given by $\varphi = \varphi_0 - (t_{obs} - t_{fiducial})/P$, with $\varphi_0 \equiv \varphi(t_{fiducial})$. The period-averaged mean absolute magnitude, $\overline{M}_r(P)$ is presumed to be known (and have negligible scatter). **HW**: What is the role of [Fe/H] here? Do $\overline{M}_r(P)$ and $\hat{M}_r(\varphi \mid P)$ depend on it, or not?

HW: Add the following sentence(?) Analogously, there is a template for the r-λ color of the Cepheids, which depends on phase, $C_\lambda(\phi)$, as multi-color templates imply to reasonable approximation that $C_\lambda(\phi \mid P, [\mathrm{Fe/H}]) \approx C_\lambda(\phi)$. Such a 'color template', $C_\lambda(\phi)$, would need to be constructed empirically. **HW**: @LI: you have constructed such a template, right?

For a Cepheid with distance modulus DM and A_λ, the predicted apparent magnitude in band λ at pulsation phase φ is then:

$$m_\lambda^m(\varphi \mid P) = M_r^m(\varphi \mid P) + C_\lambda(\varphi) + A_\lambda + DM. \tag{2}$$

For any one Cepheid the parameter vector, **HW**: $P, \varphi_0, f_\lambda, DM, A_\lambda$, can then be constrained by the light-curve data, $\{m_\lambda(t_k), \delta m_\lambda(t_k)\}$ (with $\lambda = \{g, r, i, z, y\}$ and $k = 1, N_{epoch}$).

If there are function calls for $\hat{M}_r(\varphi \mid P)$, $\overline{M}_r(P)$ and $C_\lambda(\phi)$ were available, this could be fed into BS's machinery.

There are two more pieces of information available, the apparent magnitudes in the K and W1 band, at presumably unknown/unknowable period; fortunately Cepheids have amplitudes of only 0.2 mag in the infrared. This provides an additional constraint on A_λ, if we know $\overline{M}_r - \overline{M}_{K/W1}$ from unreddened Cepheids. Then the data likelihood should have a term that $m_{K/W1}$ must match $\overline{M}_{K/W1}(P) + A_{K/W1} + DM$ to within $\sqrt{\sigma^2_{phot}(K/W1) + \sigma^2_{variability}(K/W1)}$.

HW: @LI: at some point, please make sure in facts and terminology, this introductory Section agrees and is consistent with what is written below, and what you do.

3 What's next?

- Provide the fiducial template light curve: $\hat{M}_r(\varphi \mid P)$

- Construct and provide the color term (for gri?), $C_\lambda(\phi)$
- Feed into Brani's machinery

4 Update: empirical templates

HW: How is this Section related to the previous section; there is quite a bit of overlap, but slightly different terminology; please consolidate this at some point. For a given Cepheid in PS1, we have the following observed quantities: $m^o_\lambda(t_k)$,$\sigma^o_\lambda(t_k)$, with $\lambda = g...z$ and $k = 1$, N_{epoch}. We can write the observed magnitudes as follows:

$$m^o_\lambda(t_k) = \Delta_\lambda * T^m_\lambda(t_k \mid P) + \overline{M}_\lambda(P) + DM + c_\lambda * E(B-V) \tag{3}$$

where the quantities P, Δ_λ, DM and $E(B-V)$ are unknown parameters that we want to constrain, c_λ is known (assuming a reddening law) and $T^m_\lambda(t_k \mid P)$ is an empirical model defined to have $< T^m_\lambda >_P = 0$ and $< \Delta t^m_\lambda >_P = 1$.

4.1 Update on how to construct the empirical templates

LI suggested (in conversation with HWR) the following possible path towards constructing empirical templates. Let's presume for a set of $i = 1, I$ fiducial calibration ("c") Cepheids there is a set of photometry $m^c_{\lambda,i}(\phi_k)$; for historical reasons these may only exist in Johnson photometry bands, BVRI.. Fourier fitting can turn the lightcurve of each calibration object into a smooth function in each band: $m^c_{BVRI,i}(\phi)$. Then for each object at each pulsation phase we have their instantaneous Johnson colors; this allows us a color transformation to the PS1 system (with a presumed uncertainty of 0.03 (?)), $m^c_{griz,i}(\phi)$. We can either view the I calibration templates as independent templates (and try them all in the subsequent optimization); or we can construct their ensemble average $T^m_{griz}(\phi_k)\langle m^c_{griz,i}(\phi) - \overline{m^c}_{griz,i}\rangle_i$, where $\overline{m^c}$ is the phase-average for one object, and $\langle..\rangle_i$ is the ensemble average. We may also construct separate mean template lightcurves in different period regimes, i.e. $T^m_{griz}(\phi_k \mid P)$.

4.1.1 Approach A for constructing the empirical templates

We constructed separate amplitude-scaled mean template lightcurves in 10 different period regimes, $T^m_{griz}(\phi_k \mid P)\langle \frac{m^c_{griz,i}(\phi) - \overline{m^c}_{griz,i}}{\Delta_{griz,i}} \rangle$.

4.2 Update: Model Fitting

In order to take full advantage of the multi-band observations, and given that the quantities DM and $E(B-V)$ do not depend on λ, we can write:

$$m^o_g(\phi_k \mid P) = \Delta_g * F * T^m_g(\phi_k \mid P) + \overline{M}_g(P) + DM + c_g * E(B-V) + (a_{0,g} + b_g * (\log P - 1)) \tag{4}$$

$$m^o_r(\phi_k \mid P) = \Delta_r * F * T^m_r(\phi_k \mid P) + \overline{M}_r(P) + DM + c_r * E(B-V) + (a_{0,r} + b_r * (\log P - 1)) \tag{5}$$

$$m^o_i(\phi_k \mid P) = \Delta_i * F * T^m_i(\phi_k \mid P) + \overline{M}_i(P) + DM + c_i * E(B-V) + (a_{0,i} + b_i * (\log P - 1)); \tag{6}$$

where $F(P)$ is a scaling factor *empirically calibrated and we phases the observations by using the equation:*

$$\phi_k = \frac{t_k - tmax}{P} \mod 1, \tag{7}$$

−> F now is a fitting parameter. with $tmax$ epoch of the maximum in the g-band Thus, we want to determine

$$p(\{m^o_\lambda(t_k), \sigma^o_\lambda(t_k), t_k\} \mid P, tmax, DM, E(B-V), \Delta_g).$$

This means that with an externally determined set of $\overline{M}_{grizy}(P)$, $T^m_{grizy}(\phi_k \mid P)$ and extinction curves c_{grizy}, The above expression is the likelihood of all photometric data, given five parameters.

Note that $T^m_{grizy}(\phi_k \mid P)$ don't require that there is no phase lag, i.e. to the. extent that $T^m_{zy}(\phi_k \mid P)$ can be constructed from fiducial RIJ lightcurves, it does not make a difference that gri are in phase and zy are not.

At this point, the question arises how to best optimize the 5 parameters; the RRL work may provide guidance.

Alternatively, the quantities Δ_g, F_r and F_g could be included into the models: $T^m_g(t_k \mid P, \Delta_g)$, $T^m_r(t_k \mid P, \Delta_g, F_r)$ and $T^m_i(t_k \mid P, \Delta_g, F_i)$.

4.2.1 Approach A for fitting

We determine

$$p(\{m^o_\lambda(t_k), \sigma^o_\lambda(t_k), t_k\} \mid P, tmax, DM, E(B-V), \Delta_g).$$

by using Equations 4,5, and 6, and by using values of $F_r(P)$ and $F_i(P)$ empirically calibrated on the calibration sample. Thus, the fit is optimised on the free parameters: $(DM, E(B-V), \Delta_g)$, while the period is optimised on a grid with steps of 8 secs. $tmax$ is set by using the epoch of the maximum g magnitude, if there are more than 7 observations in the g, the epoch of the maximum r magnitude if there are less than 7 epochs in g and at least 7 in the r. $tmax$ may also be set as a free parameters, but this would increase the computational time. This could be done in a second interaction, where we could optimise the period search around the first guess.

4.3 Update on 07/04/17: Model Fitting with Differential Evolution optimization method

4.3.1 Building the empirical templates

I downloaded the CCD photometry for ~ 576 Cepheids from the Berdnikov 2014 catalog [1]. Then I select only light curves for which there are at least 25 observations in each of the three bands: B,V and Ic. This cut reduces the light curve templates to 89 objects. The physical parameters for these Cepheids (Period, and possibly distance and extinction) are take from the Fernie catalog at http://www.astro.utoronto.ca/DDO/research/cepheids/ Currently, I am using a third order Fourier series to model the empirical light curves, but I plan to use a cross validation approach in the near future either with periodic splines or with Fourier series. Finally, I produce templates files with: phases, g-band magnitude - mean(g-band mag), r-band magnitude - mean(r-band mag), i-band magnitude - mean(i-band mag). The plots of the light curves used are hosted at https://github.com/laurainno/Cepheids/tree/master/Templates

4.3.2 Fitting procedure

In order to take full advantage of the multi-band observations, and given that the quantities DM and $E(B-V)$ do not depend on λ, we can write:

$$m^o_g(\phi_k \mid P) = F_g * T^m_g(\phi_k \mid P) + DM + c_g * E(B-V) + (a_{0,g} + b_g * (\log P - 1)) \tag{8}$$
$$m^o_r(\phi_k \mid P) = F_r * T^m_r(\phi_k \mid P) + DM + c_r * E(B-V) + (a_{0,r} + b_r * (\log P - 1)) \tag{9}$$
$$m^o_i(\phi_k \mid P) = F_i * T^m_i(\phi_k \mid P) + DM + c_i * E(B-V) + (a_{0,i} + b_i * (\log P - 1)); \tag{10}$$

where $F_{g,r,i}(P)$ are scaling factors which can vary in the range [0.6, 1.7]. Because the models T^m_{gri} are constructed by using the Epoch of maximum light as the $\phi = 0$ point, we also allow for a rigid shift in phase of the model, which is described by the phase shift: $\delta\phi$.

Summarizing, we want to determine

$$p(\{m^o_\lambda(t_k), \sigma^o_\lambda(t_k), t_k\} \mid P, \delta\phi, DM, E(B-V), F_g, F_r, F_i).$$

This means that with an externally determined set of a_{grizy}, b_{grizy}, $T^m_{grizy}(\phi_k \mid P)$ and extinction curves c_{grizy}aaz<aaz<azz<azz<aaza«ZAA<A<ZAza<ZA«zazaa<za<za<zaZAAZZAZAAA<ZAzaza<, The above expression is the likelihood of all photometric data, given seven parameters.

The optimization process is done by adopting differential evolution methods, which are able to optimize on the seven parameters at the same time for each trial template model ($sim90$) in a reasonable amount of time ($sim20$ minutes per object).

[1] http://adsabs.harvard.edu/abs/2014AstL...40..125B