DYNAMICAL MODELING AND ANALYSIS OF EPIDEMICS: EBOLA AS A CASE STUDY

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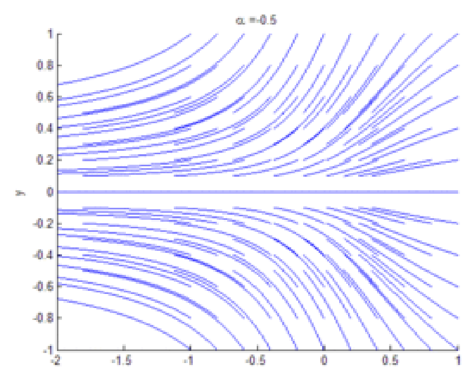
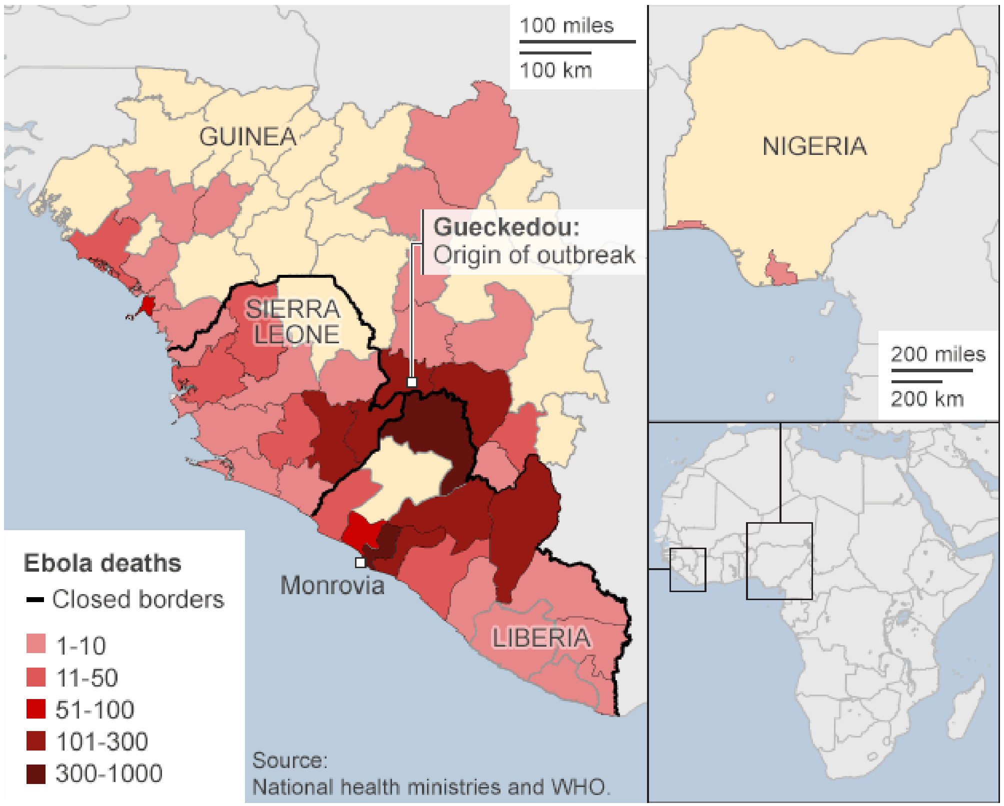
MOTIVATION & BACKGROUND

- Future of Medicine and Biotechnology
- Dynamical Systems and Their Bifurcations



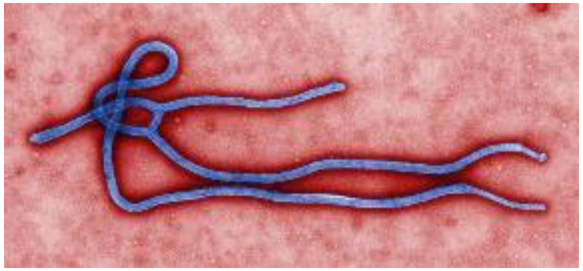
A SNAPSHOT OF THE 2014 EBOLA OUTBREAK IN WEST AFRICA



Source: http://www.bbc.com/news/world-africa-28755033

THE 2014 EBOLA OUTBREAK IN WEST AFRICA

- Geography:
  - Majority of cases in Liberia, Guinea, and Sierra Leone
  - Transported to Nigeria and Senegal
- Since 22 March 2014:
  - Over 3,000 deaths
  - Over 6,500 cases
  - 50~90% mortality rate
  - No FDA-approved treatments, cures or vaccines



OBAMA: EBOLA' IS SPIRALING OUT OF CONTROL'

"Here's the hard truth: In West Africa, Ebola is now an epidemic of the likes we have not seen before. It is spiraling out of control. It is getting worse. It is spreading faster and exponentially. If the outbreak is not stopped now, we could ***be looking at hundreds of thousands of people infected****."*

– President Obama, September 16th, 2014

Source: http://www.whitehouse.gov/the-press-office/2014/09/16/remarks-president-ebola-outbreak

A MATHEMATICAL APPROACH TO STUDY THE OUTBREAKS OF EBOLA

- Objective:
  - Better understand the mathematical dynamics of a population infected by Ebola
  - Model the progress of an epidemic in a large population
- Deterministic Compartmental Models:
  - The SIR model

A SIMPLE MODEL OF AN EPIDEMIC: SIR MODEL

- THE SIR MODEL (1927: W. O. Kermack and A. G. McKendrick)
  - Three compartments:
    - Susceptible, S
    - Infected, I
    - Removed, R
  - Uses a fixed population: $N = S(t) + I(t) + R(t)$

A SIMPLE MODEL OF AN EPIDEMIC: SIR MODEL (CONT'D)

- Please see the board for the process of the derivation:

A SIMPLE MODEL OF AN EPIDEMIC: SIR MODEL (CONT'D)

$$\frac{dS}{dt} = -\beta S I$$
$$\frac{dI}{dt} = \beta S I - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

A SIMPLE MODEL OF AN EPIDEMIC: SIR MODEL (CONT'D)

$R_0 < 1$	infection will die out in the long run
$R_0 > 1$	infection will be able to spread in a population

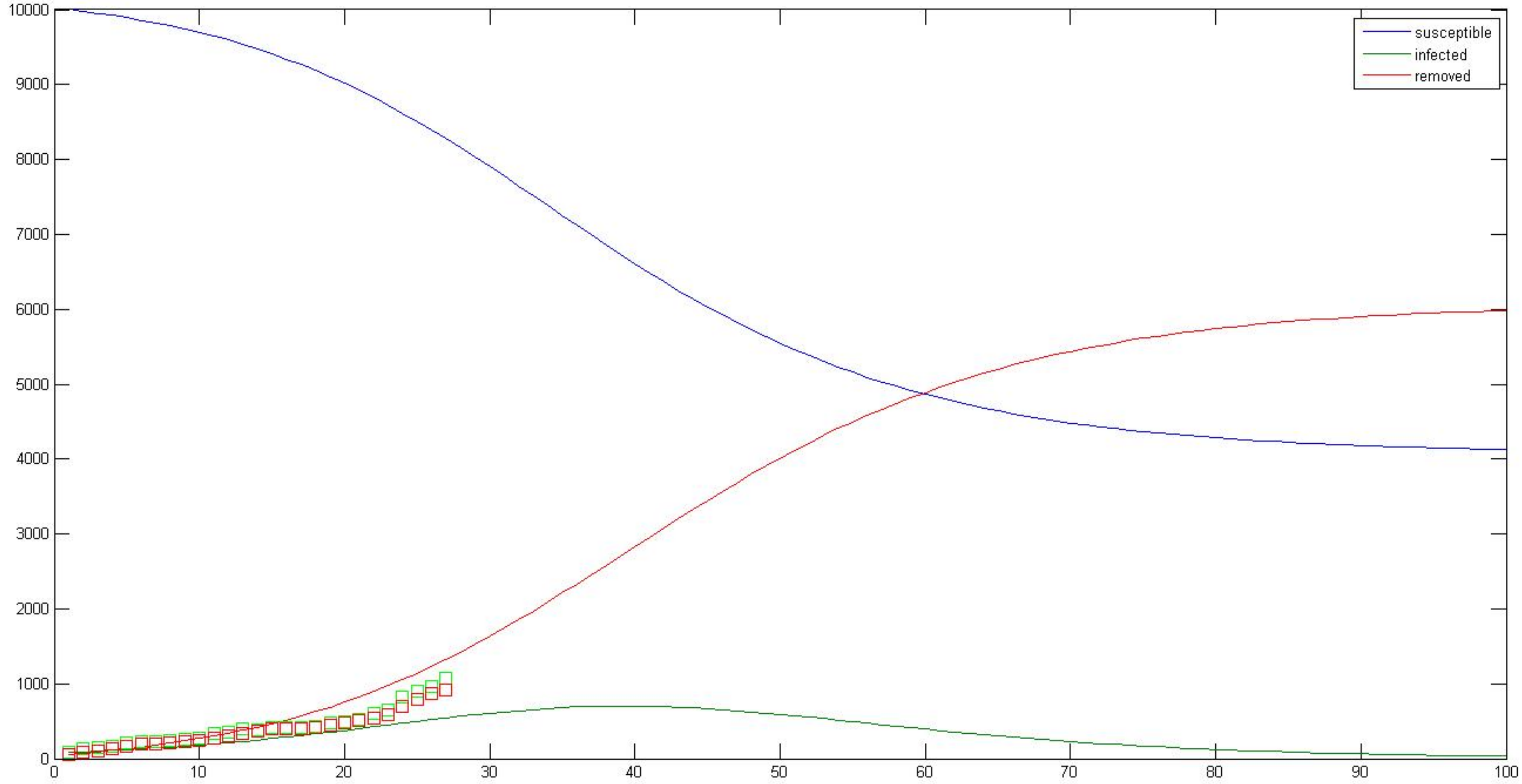
MATLAB: SOLVING THE ODE

```
% sir_ode.m
%
% Imlements a SIR infection model
%   dS/dt = -beta SI/N
%   dI/dt = beta SI/N - gamma I
%   dR/dt = gamma I
%
% Inputs:
%   t - Time variable
%   x - Variable containing 3 populations (S, I, and R)
%   p - Parameters (transmission rate: beta, infection rate: gamma)
% Output:
%   dx - First derivative: the rate of change of the populations

function dx = sir_ode(t,x,p)
    beta = p(1);
    gamma = p(2);

    S = x(1);
    I = x(2);
    R = x(3);
    N = S + I + R;

    dS = -beta * S * I / N;
    dI = beta * S * I / N - gamma * I;
    dR = gamma * I;

    dx = [dS; dI; dR];

end
```

OUTLOOK: AN OMINOUS EBOLA FORECAST IN GUINEA

$\beta = 0.27$, $\gamma = 0.18$
$R_0 = 1.51$

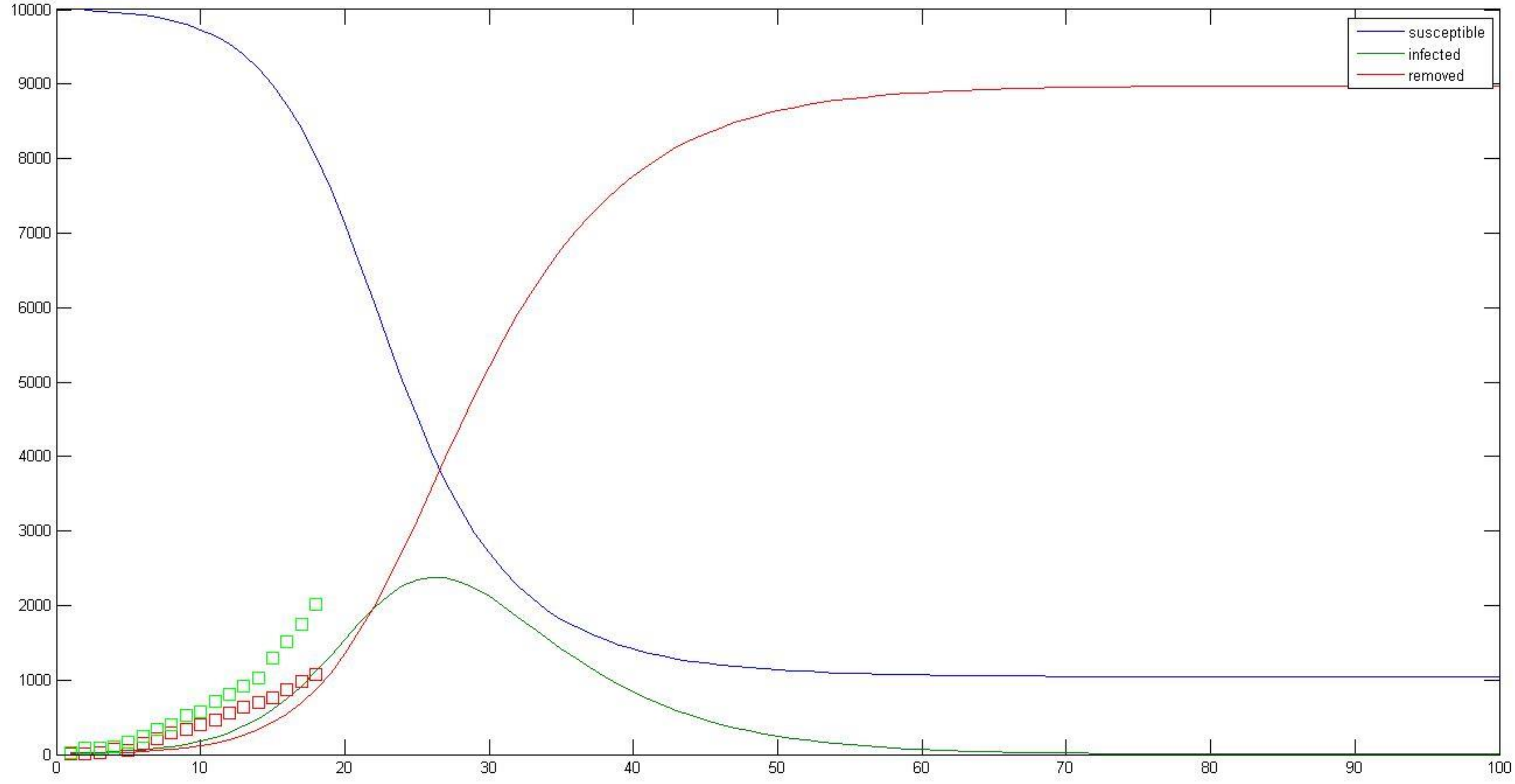
S = 10,000, I = 86, R = 60



OUTLOOK: AN OMINOUS EBOLA FORECAST IN SIERRA LEONE

$\beta = 0.45$, $\gamma = 0.18$
$R_0 = 2.52$

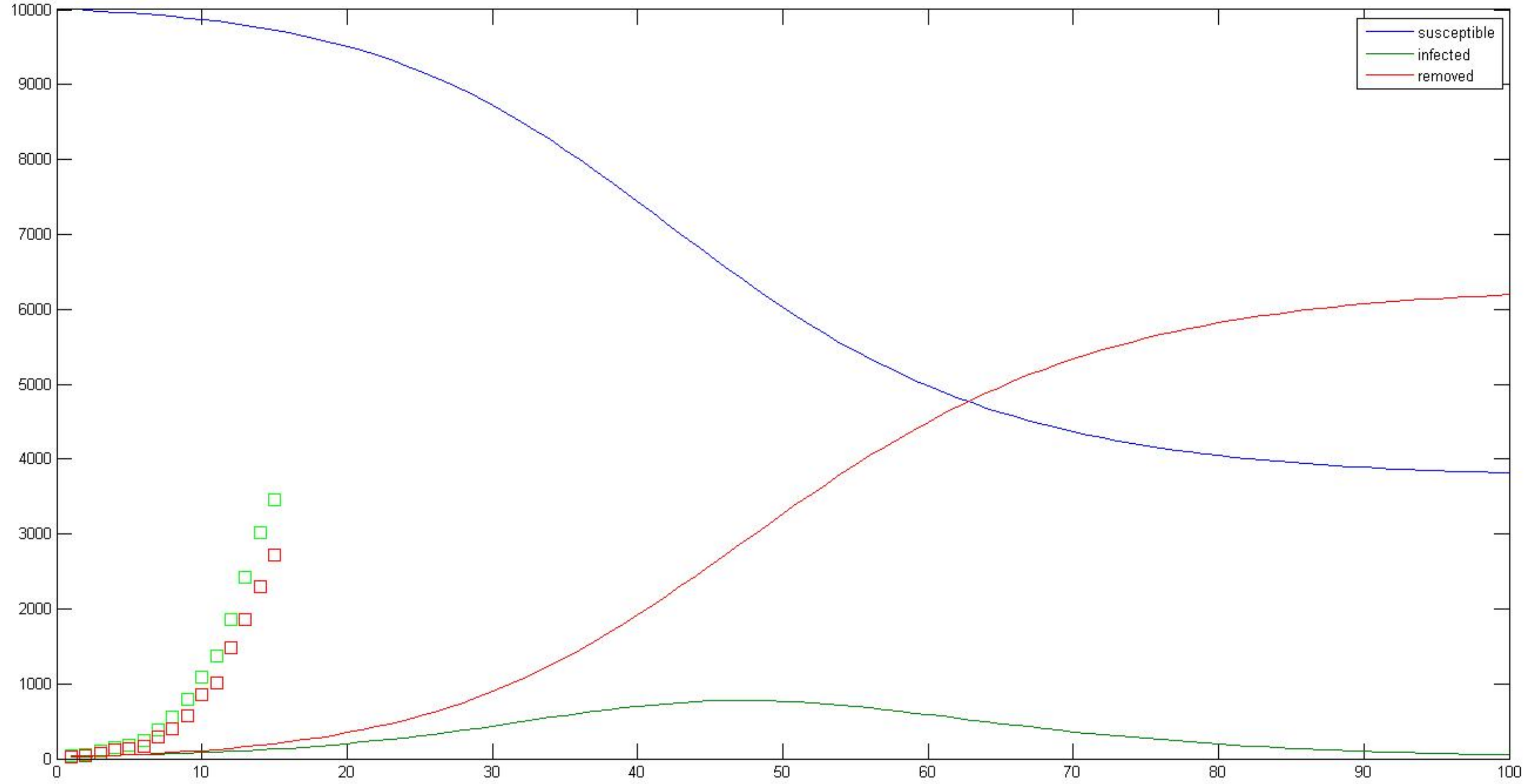
S = 10,000, I = 16, R = 5



OUTLOOK: AN OMINOUS EBOLA FORECAST IN LIBERIA

$\beta = 0.28$, $\gamma = 0.18$
$R_0 = 1.57$

S = 10,000, I = 33, R = 24



REGRESSION ANALYSIS:
WE NEED CONTAINMENT

MATLAB: FINDING THE SUMS OF THE RESIDUALS

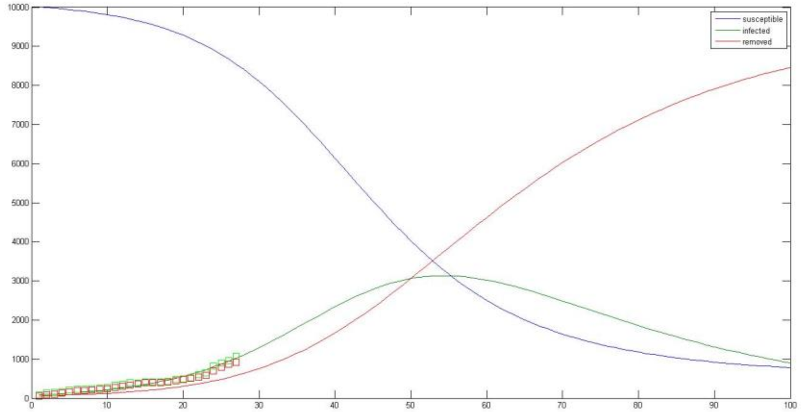
```
% sir_discrepancy.m
%
% Finds the sum of squares of residuals
% Measures the discrepancy between the data and estimation from SIR model
%
% Inputs:
%   p - Parameters (transmission rate: beta, infection rate: gamma)
%   data - Actual data points
%   tspan - Time span for which the ODEs are solved
%   x0 - Initial conditions
% Output:
%   disc - Sum of squares of the discrepancy

function disc = sir_discrepancy(p, data, tspan, x0)

[t,x] = ode45(@sir_ode,tspan,x0,[],p);
I = x(:,2);
I = I(1:27);
disc = sum((I-data').^2);

end
```

MATLAB: OPTIMIZING THE PARAMETERS

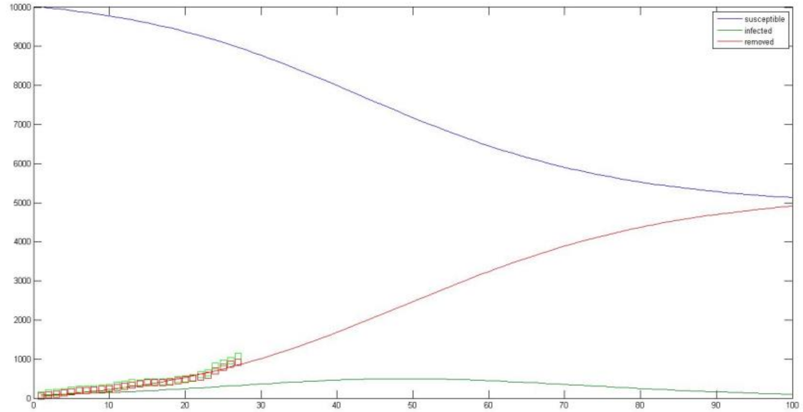
```
% sir_optimize.m
%
% Find the optimized parameters using fminsearch function.
% Starts at p0 and attempts to find a local minimizer p_opt
% which would give the smallest discrepancy.
%
% Inputs:
%   data - Actual data points
%   tspan - Time span for which the ODEs are solved
%   x0 - Initial conditions
%   p0 - Parameters (transmission rate: beta, infection rate: gamma)
% Output:
%   p_opt - Optimal values for beta and gamma

function p_opt = sir_optimize(data, tspan, x0, p0)

p_opt = fminsearch(@sir_disc_nested, p0);
    function disc = sir_disc_nested(p)
        disc = sir_discrepancy(p, data, tspan, x0);
    end

end
```

FITTING THE SIR MODEL – OPTIMIZE THE PARAMETERS: GUINEA

$\beta = 0.16$, $\gamma = 0.05$
$R_0 = 3.08$

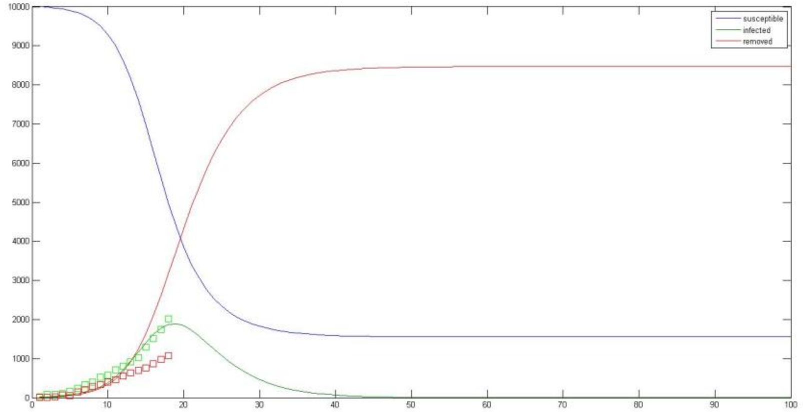


$\beta = 0.22$, $\gamma = 0.16$
$R_0 = 1.40$

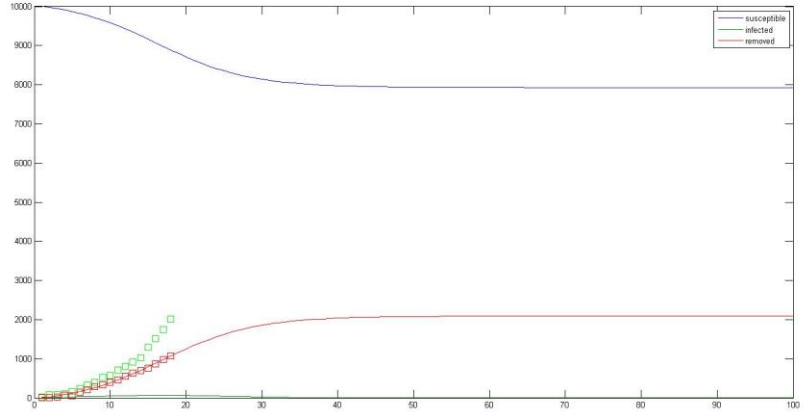


FITTING THE SIR MODEL – OPTIMIZE THE PARAMETERS: SIERRA LEONE

$\beta = 0.68$, $\gamma = 0.31$
$R_0 = 2.21$

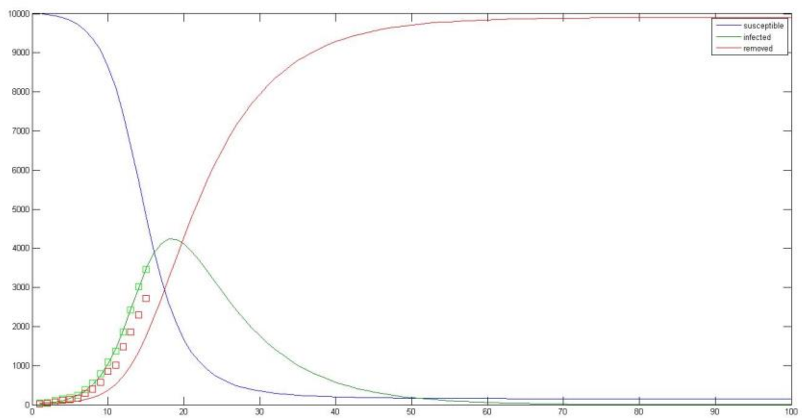


$\beta = 1.52$, $\gamma = 1.36$
$R_0 = 1.11$

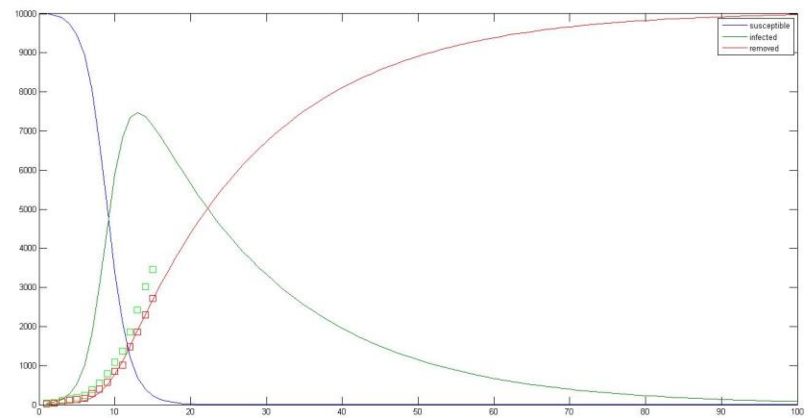


FITTING THE SIR MODEL – OPTIMIZE THE PARAMETERS: LIBERIA

$\beta = 0.53$, $\gamma = 0.13$
$R_0 = 4.24$



$\beta = 0.76$, $\gamma = 0.05$
$R_0 = 14.32$



OUTLOOK: U.S. INTERVENTION

U.S. military will lead $750 million fight against Ebola in West Africa

– The Washington Post, September 16th, 2014

Source: http://www.washingtonpost.com/national/health-science/us-military-to-lead-ebola-fight/2014/09/15/69db3da0-3d32-11e4-b0ea-8141703bbf6f_story.html

OUTLOOK: U.S. INTERVENTION (CONT'D)

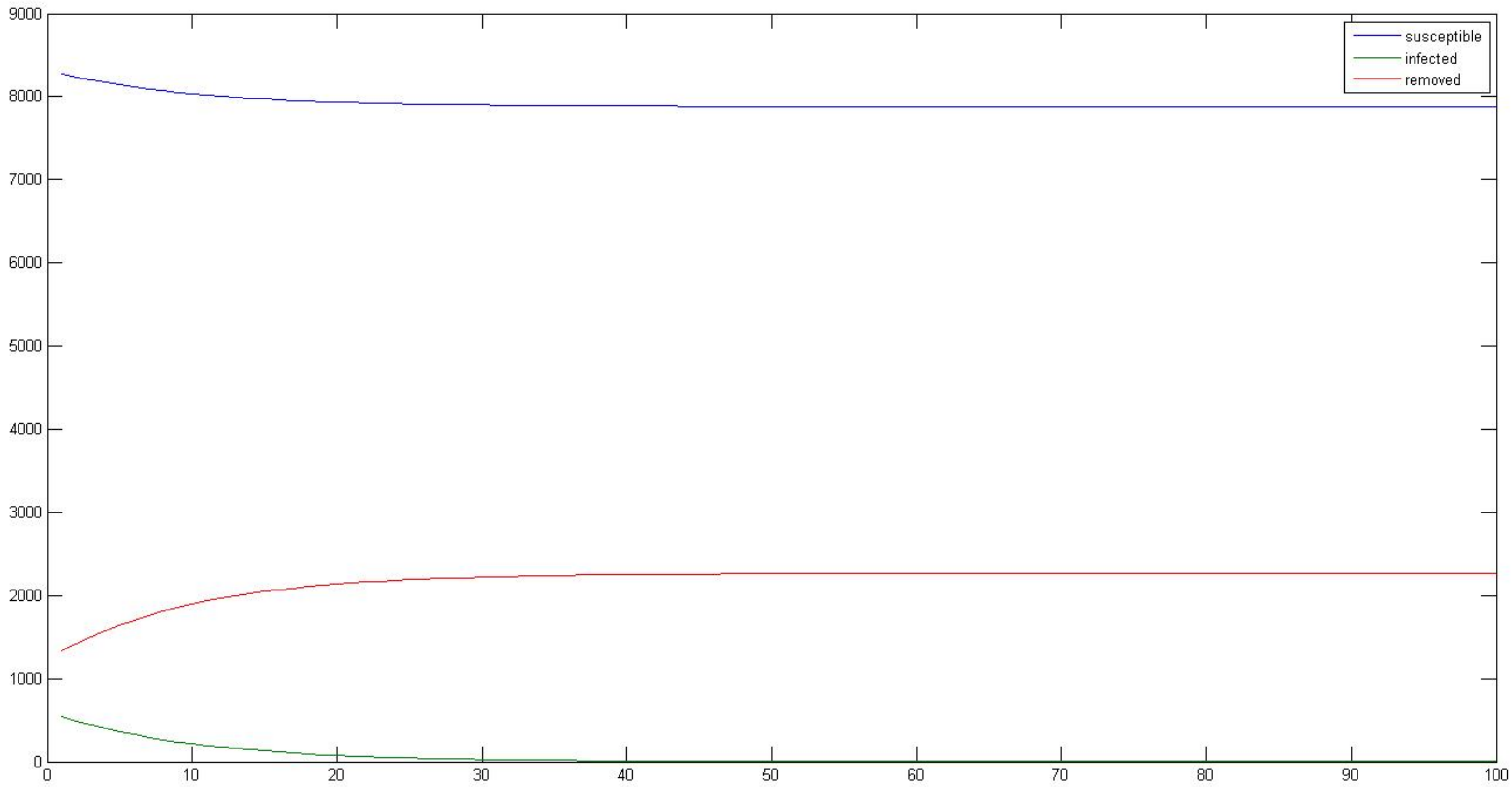
Benchmark Analysis - Pre-Obama Intervention	Guinea	Liberia	Sierra Leone
Transmission rate, ß (per day)	0.270	0.280	0.450
US Effort	$ 175	$ 175	$ 175

Benchmark Analysis - Post-Obama Intervention	Guinea	Liberia	Sierra Leone
Transmission rate, ß (per day)	0.063	0.065	0.105
Transmission rate, ß (per day) - Discounted	0.095	0.098	0.158
US Effort	$ 750	$ 750	$ 750

OUTLOOK: A PROMISING EBOLA FORECAST IN GUINEA POST-US INTERVENTION

$\beta = 0.095$, $\gamma = 0.18$
$R_f = 0.53$

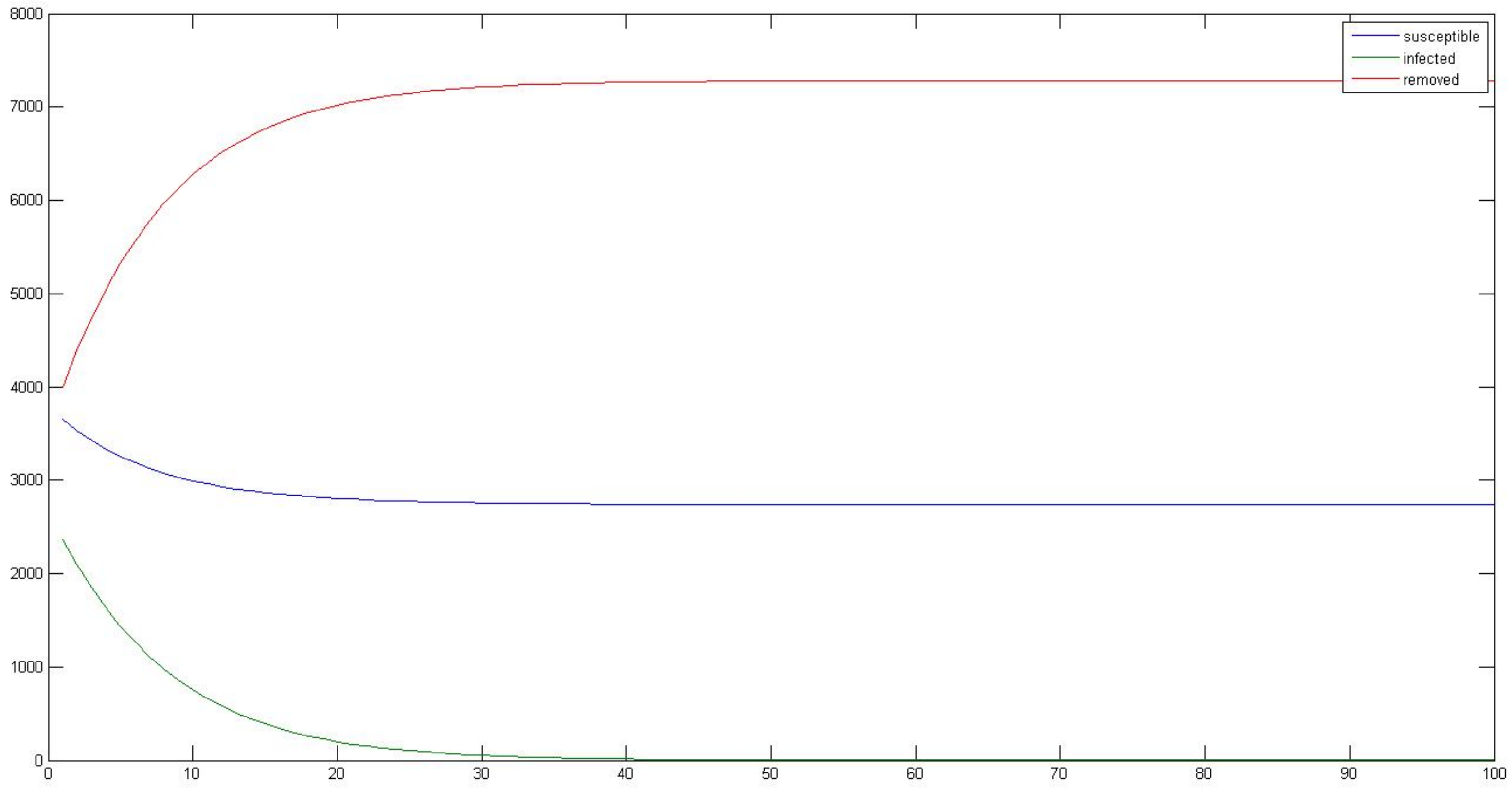
S = 8,277, I = 542, R = 1,326



OUTLOOK: A PROMISING EBOLA FORECAST IN SIERRA LEONE POST-US INTERVENTION

β = 0.016, γ = 0.18
R_f = 0.88

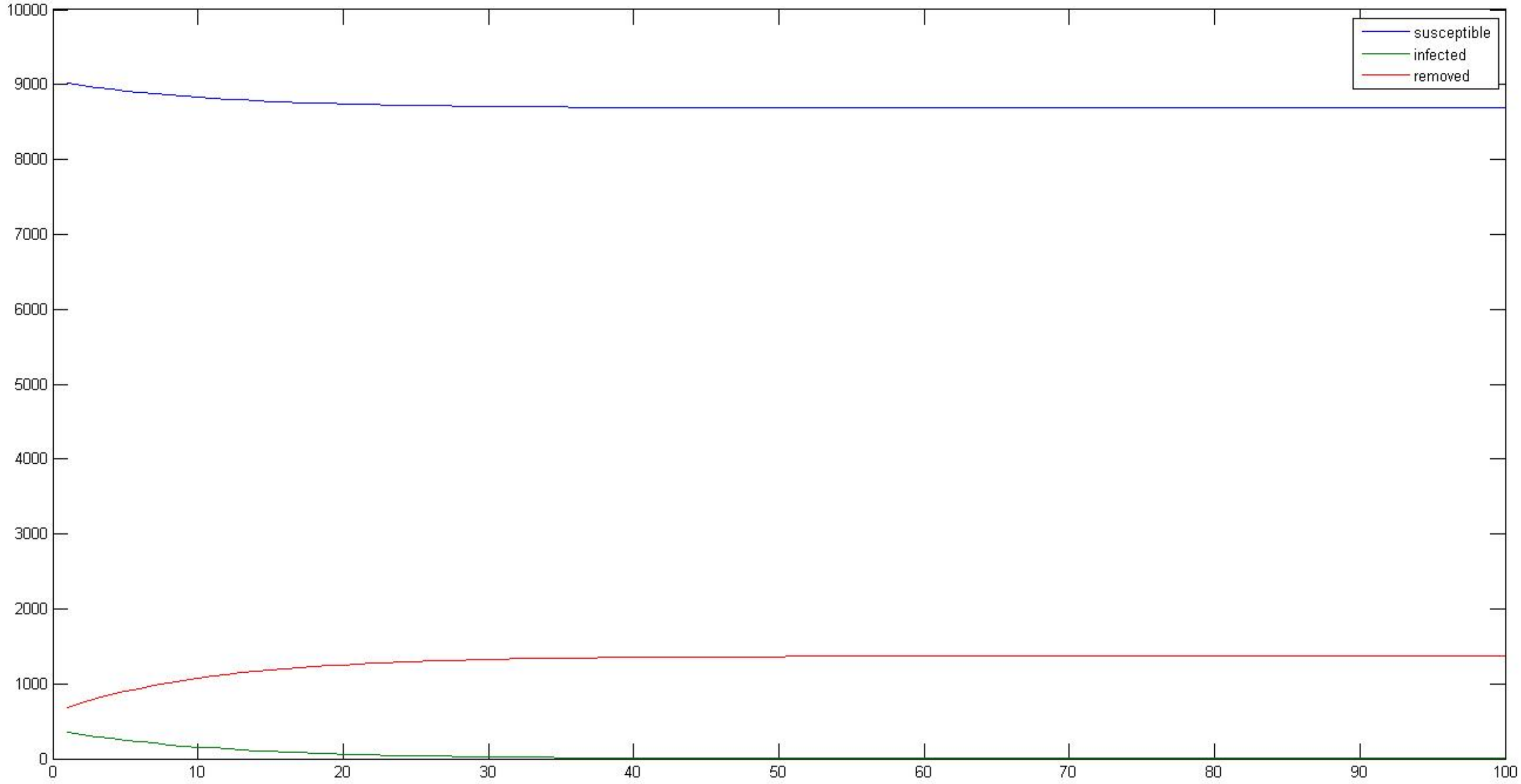
S = 3,661, I = 2,367, R = 3,992



OUTLOOK: A PROMISING EBOLA FORECAST IN LIBERIA POST-US INTERVENTION

$\beta = 0.098$, $\gamma = 0.18$
$R_f = 0.55$

S = 9,018, I = 353, R = 686



FURTHER RESEARCH

- Extension of the SIR model:
  - Factors such as a quarantine, immunity being passed from mother to child and even a zombie apocalypse
  - e.g. SEIR MODEL (Exposed)
- What we can do as engineers / applied scientists:
  - Equipment Decontamination
  - Personnel Decontamination
  - Point of Care Diagnostic Devices
  - Improved Patient and Body Transport
  - Data Collection and Management

QUESTIONS?

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THE END