GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LOGISTIC REGRESSION

I. LOGISTIC REGRESSION
II. OUTCOME VARIABLES
III. ERROR TERMS
IV. INTERPRETING RESULTS

EXERCISES:
IMPLEMENTING A LOGISTIC FIT IN R

I. LOGISTIC REGRESSION

	continuous	*categorical*
supervised	???	???
unsupervised	???	???

	continuous	*categorical*
supervised	regression	classification
unsupervised	dimension reduction	clustering

Q: What is **logistic regression**?

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A: A generalization of the linear regression model to *classification* problems.

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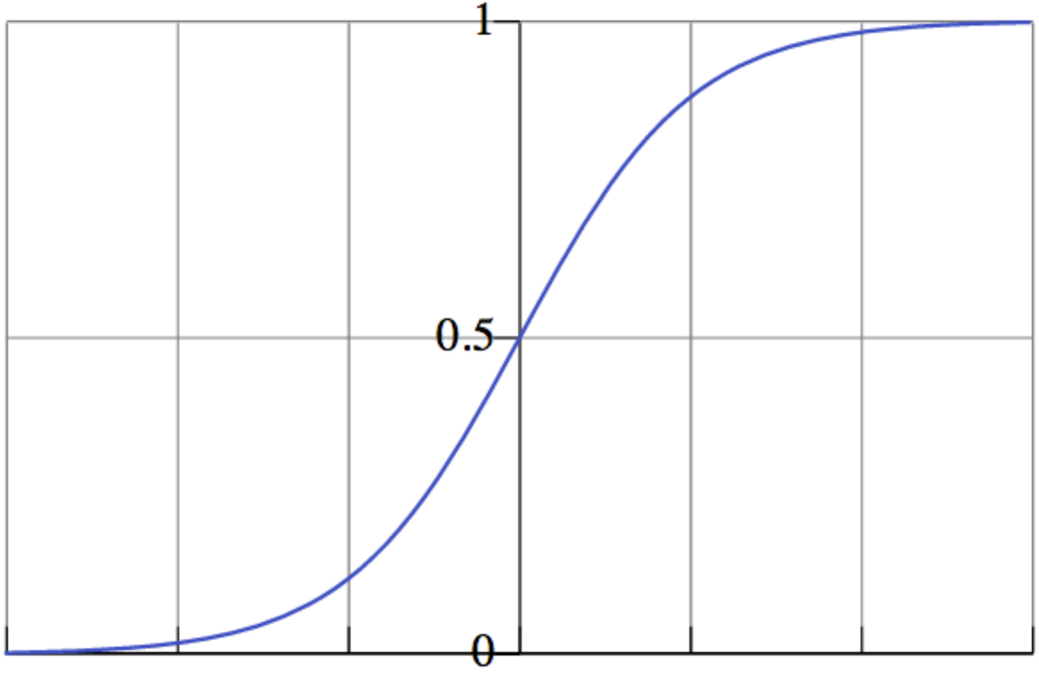
In logistic regression, we use a set of covariates to predict *probabilities* of (binary) class membership.

These probabilities are then mapped to *class labels*, thus solving the classification problem.

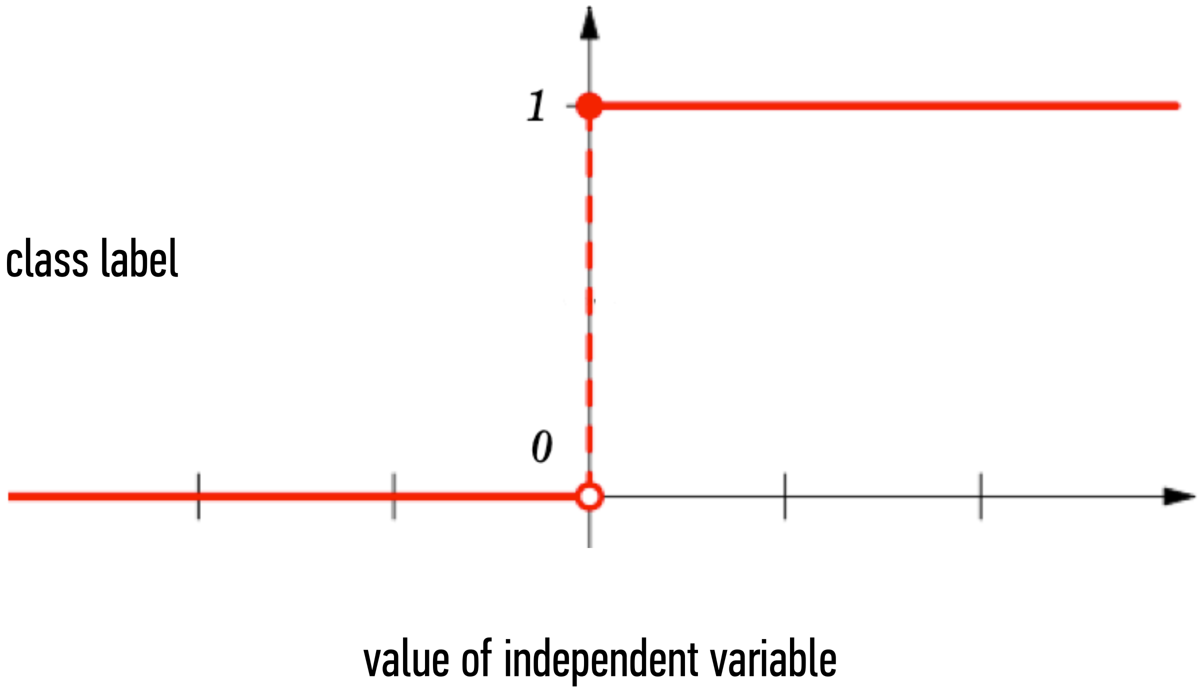
probability of belonging to class



value of independent variable

NOTE

Probability predictions look like this.



NOTE

Probabilities are "snapped" to class labels (e.g., by threshholding at 50%).

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The second difference is in the error term.

II. OUTCOME VARIABLES

The key variable in any regression problem is the **conditional mean** of the outcome variable *y* given the value of the covariate *x*:

$$E(y|x)$$

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In linear regression, we assume that this conditional mean is a linear function taking values in *(-∞, +∞)*:

$$E(y|x) = \alpha + \beta x$$

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The first step in extending the linear regression model to logistic regression is to map the outcome variable $E(y|x)$ into the unit interval.

Q: How do we do this?

A: By using a transformation called the **logistic function**:

$$E(y|x) = \pi(x) = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$$

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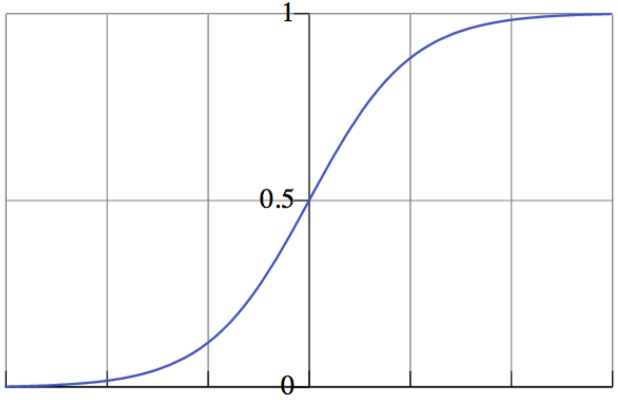

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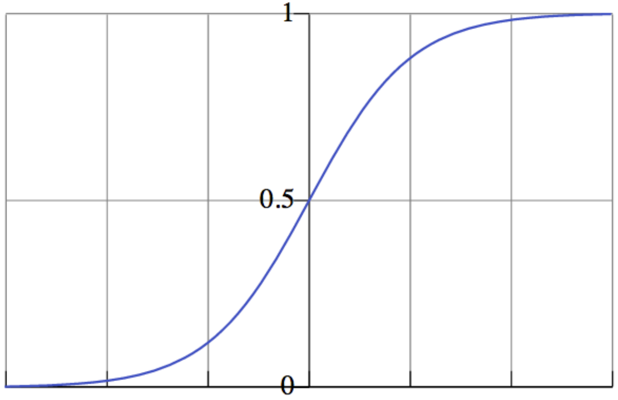

NOTE

For any value of x, y is in the interval [0, 1]

This is a nonlinear transformation!

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NOTE

This name hints at its usefulness in interpreting our results.

We will see why shortly.

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III. ERROR TERMS

The second difference between linear regression and the logistic regression model is in the error term.

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One of the key assumptions of linear regression is that the error terms follow independent Gaussian distributions with zero mean and constant variance:

$$\epsilon \sim N(0, \sigma^2)$$

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NOTE

This is the same distribution followed by a coin toss.

Think about why this makes sense!

These two key differences define the logistic regression model, and they also lead us to a kind of unification of regression techniques called **generalized linear models**.

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Briefly, GLMs generalize the distribution of the error term, and allow the conditional mean of the response variable to be related to the linear model by a **link function**.

In the present case, the error term follows a Bernoulli distribution, and the logit is the link function that connects us to the linear predictor.

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The logit is the **canonical link function** for the Bernoulli distribution; there are some other usable link functions (probit, tobit) but the logit simplifies things nicely and is most commonly used.

IV. INTERPRETING RESULTS

In linear regression, the parameter β represents the change in the response variable for a unit change in the covariate.

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In logistic regression, β represents the change in the logit function for a unit change in the covariate.

Interpreting this change in the logit function requires another definition first.

The **odds** of an event are given by the ratio of the probability of the event by its complement:

$$O(x=1) = \frac{\pi(1)}{(1-\pi(1))}$$

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The **odds ratio** of a binary event is given by the odds of the event divided by the odds of its complement:

$$OR = \frac{O(x=1)}{O(x=0)} = \frac{\pi(1)/[1-\pi(1)]}{\pi(0)/[1-\pi(0)]}$$

Substituting the definition of $\pi(x)$ into this equation yields (after some algebra),

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This simple relationship between the odds ratio and the parameter β is what makes logistic regression such a powerful tool.

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A: The odds ratio of a binary event gives the increase in likelihood of an outcome if the event occurs.

Suppose we are interested in mobile purchase behavior. Let y be a class label denoting purchase/no purchase, and let x denote a mobile OS (for example, iOS).

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In this case, an odds ratio of 2 (eg, $\beta = \log(2)$) indicates that the odds of purchase is twice as high for an iOS user as for a non-iOS user.

EXERCISE