GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

NAIVE BAYES CLASSIFICATION

I. INTRO TO PROBABILITY
II. NAÏVE BAYES CLASSIFICATION

LAB:
III. A SPAM FILTER

I. INTRO TO PROBABILITY

Q: What is a **probability**?

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The probability of event A is denoted $P(A)$.

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A: This set is called the **sample space** Ω. Event A is a member of the sample space, as is every other event.

The total probability of the sample space $P(\Omega)$ is 1.

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NOTE

The symbol ∩ is often used for intersection. For example, “$P(A \cap B)$”.

Q: Is $P(AB)$ equal to $P(A)P(B)$?

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A: Maybe, maybe not. More later...

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This information about B *transforms* the sample space.

Take a moment to convince yourself of this!

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This is called the **conditional probability** of A given B, written $P(A|B) = P(AB) / P(B)$.

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Notice, with this we can also write $P(AB) = P(A|B) * P(B)$.

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This can be written as $P(A|B) = P(A)$.

Using the definition of the conditional probability, we can also write:

$$P(A|B) = P(AB) / P(B) = P(A) \rightarrow P(AB) = P(A) * P(B)$$

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→ $P(A|B) = P(B|A) * P(A) / P(B)$ by rearranging last step

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This result is called **Bayes' theorem**. Here it is again:

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Some facts:
- This is a simple algebraic relationship using elementary definitions.
- It's interesting because it's kind of a "wormhole" between two different "interpretations" of probability.
- It's a very powerful computational tool.

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The *frequentist interpretation* regards an event’s probability as its limiting frequency across a very large number of trials.

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The *frequentist interpretation* regards an event’s probability as its limiting frequency across a very large number of trials.

The *Bayesian interpretation* regards an event’s probability as a “degree of belief,” which can apply even to events that have not yet occurred.

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This a good direction to head if you like math and/or if you're interested in learning about cutting-edge data science techniques.

II. NAÏVE BAYES CLASSIFICATION

Suppose we have a dataset with features $x_1, \ldots, x_n$ and a class label c. What can we say about classification using Bayes' theorem?

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$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Bayes' theorem can help us to determine the probability of a record belonging to a class, *given* the data we observe.

source: Data Analysis with Open Source Tools, by Philipp K. Janert. O'Reilly Media, 2011.

Each term in this relationship has a name, and each plays a distinct role in any Bayesian calculation (including ours).

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **likelihood function**. It represents the joint probability of observing features $\{x_i\}$ given that that record belongs to class C.

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We can approximate the value of the likelihood function from the training data.

This term is the **prior probability** of c. It represents the probability of a record belonging to class c before the data is taken into account.

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The value of the prior is also observed from the data.

This term is the **normalization constant.** It doesn't depend on c, and is generally ignored until the end of the computation.

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The normalization constant doesn't tell us much.

This term is the **posterior probability** of c. It represents the probability of a record belonging to class c after the data is taken into account.

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The goal of any Bayesian computation is to find ("learn") the posterior distribution of a particular variable.

The idea of Bayesian inference, then, is to **update** our beliefs about the distribution of c using the data ("evidence") at our disposal.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Then we can use the posterior for prediction.

Methods	Predictions
“classical” (frequentist)	point estimates
Bayesian	distributions

Q: What piece of the puzzle we've seen so far looks like it could intractably difficult in practice?

Remember the likelihood function?

$$P(\{x_i\}|C) = P(\{x_1, x_2, \ldots, x_n\})|C)$$

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Observing this exactly would require us to have enough data for every possible combination of features to make a reasonable estimate.

Q: What piece of the puzzle we've seen so far looks like it could intractably difficult in practice?

A: Estimating the full likelihood function.

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Q: So what can we do about it?

A: Make a simplifying assumption. In particular, we assume that the features x_i are conditionally independent from each other:

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This "naïve" assumption simplifies the likelihood function to make it tractable.

III. SPAM FILTER

EXERCISE – SPAM FILTER (DOCUMENT CLASSIFICATION)

KEY OBJECTIVES

- preprocess data
- perform naïve Bayes classification

TOOLS

- tm, R