GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LINEAR REGRESSION

I. LINEAR REGRESSION
II. POLYNOMIAL REGRESSION

EXERCISES:
III. LINEAR REGRESSION IN R
IV. PREDICTING BASEBALL SALARIES

I. LINEAR REGRESSION

	continuous	*categorical*
supervised	???	???
unsupervised	???	???

	continuous	*categorical*
supervised	regression	classification
unsupervised	dimension reduction	clustering

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ε = **residual** (the prediction error)

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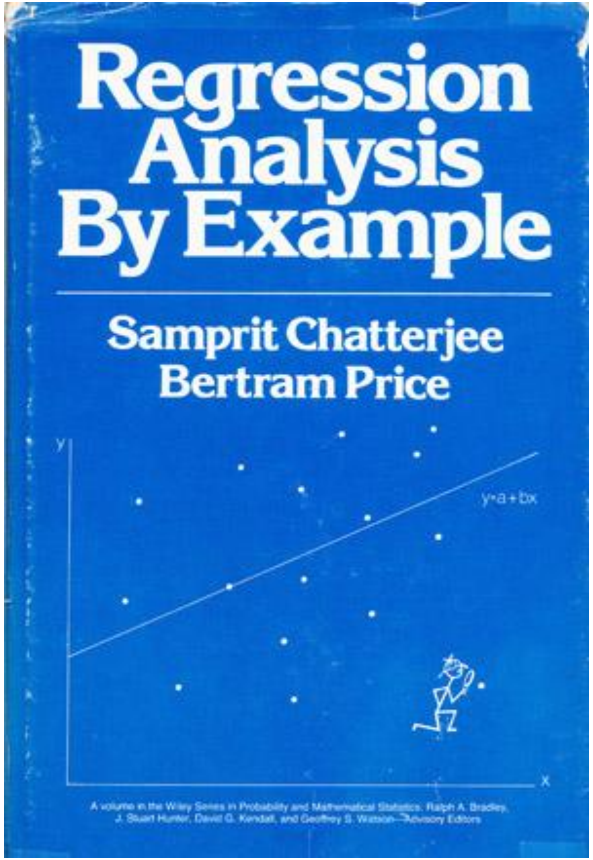
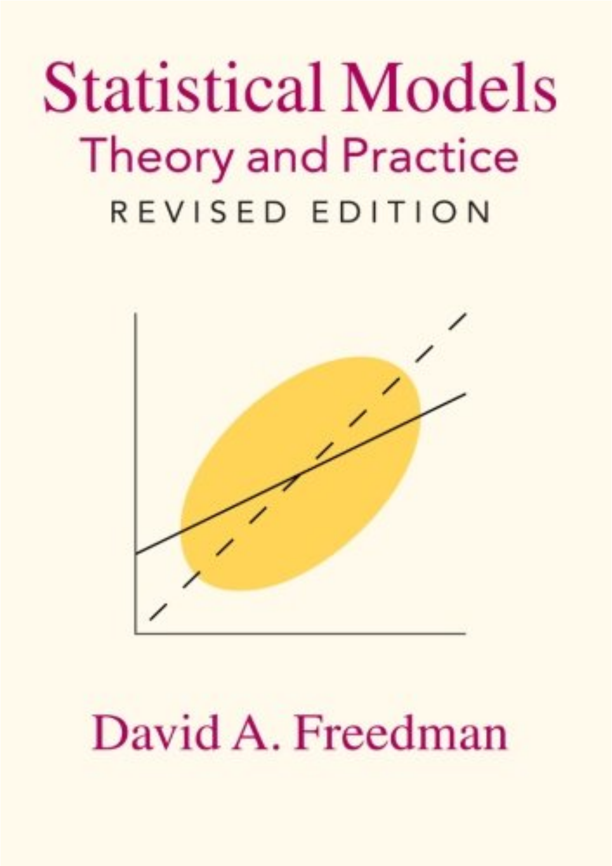
$$y = \alpha + \beta_1 x_1 + \ldots + \beta_n x_n + \varepsilon$$

Linear regression involves several technical assumptions and is often presented with lots of mathematical formality.

The details are not very important for our purposes, but you can check them out if you're interested.



Regression Analysis By Example
Samprit Chatterjee
Bertram Price
y
y=a+bx
x
A volume in the Wiley Series in Probability and Mathematical Statistics, Ralph A. Bradley, J. Stuart Hunter, David G. Kendall, and Geoffrey S. Watson—Advisory Editors



Statistical Models
Theory and Practice
REVISED EDITION
David A. Freedman

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And there are other ways.

And software implements them as well.

II: POLYNOMIAL REGRESSION

Consider the following **polynomial regression** model:

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"Although polynomial regression fits a *nonlinear* model to the data, as a statistical estimation problem it is *linear*, in the sense that the regression function E(y|x) is linear in the unknown parameters that are estimated from the data. For this reason, polynomial regression is considered to be a special case of multiple linear regression." -- Wikipedia

Polynomial regression allows us to fit very complex curves to data.

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But there is a problem with the model we've written down so far.

This model displays **collinearity**, which means the predictor variables are highly correlated with each other.

$$y = \alpha + \beta_1 x + \beta_2 x^2 + \ldots + \beta_n x^n + \varepsilon$$

```
> x <- seq(1, 10, 0.1)
> cor(x^9, x^10)
[1] 0.9987608
```

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NOTE

For identical features, this results in a *singularity*. We will see an example of this in just a minute!

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OPTIONAL NOTE

These polynomial functions form an *orthogonal basis* of the function space.

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  - More

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- Techniques for evaluating designs

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- Techniques for evaluating designs
  - Training metrics

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- Techniques for producing designs
  - Theory
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  - More
- Techniques for evaluating designs
  - Training metrics
  - Test set performance / Cross-validation

EXERCISES