

tlu4_#11

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AP Service

AP is a shipping service that guarantees overnight delivery of packages in the continental US. The company has various hubs at major cities and airports across the country. Packages are received at hubs, and then shipped to intermediate hubs or to their final destination.

The manager of the AP hub in Cleveland is concerned about labor costs, and is interested in determining the most effective way to schedule workers. The hub operates seven days a week, and the number of packages it handles varies from one day to another. The table below provides an estimate of the number of workers needed each day of the week.

Day of the Week	Workers Required
Sunday	18
Monday	27
Tuesday	22
Wednesday	26
Thursday	25
Friday	21
Saturday	19

Package handlers at AP are guaranteed a five-day work week with two consecutive days off. The base wage for the handlers is \$750 per week. Workers working on Saturday or Sunday receive an additional \$25 per day. The possible shifts and salaries for package handlers are:

Shift	Days Off	Wage
1	Sunday and Monday	\$775



2	Monday and Tuesday	\$800
3	Tuesday and Wednesday	\$800
4	Wednesday and Thursday	\$800
5	Thursday and Friday	\$800
6	Friday and Saturday	\$775
7	Saturday and Sunday	\$750

Set x_1 as the first day start work(Sunday with five consecutive days), x_2 the second day(Monday with five consecutive days), x_3 as the third day start work(Tuesday with five consecutive days), x_4 the fourth day start work(Wednesday with five consecutive days), x_5 as the fifth day start work(Thursday with five consecutive days), x_6 the sixth day start work(Friday with five consecutive days), x_7 as the seventh day start work(Saturday with five consecutive days).

Objective function

min: $775 X_1 + 750 X_2 + 775 X_3 + 800 X_4 + 800 X_5 + 800 X_6 + 800 X_7$

Subject to

$$X1 + X4 + X5 + X6 + X7 \geq 18;$$

$$X1 + X2 + X5 + X6 + X7 \geq 27;$$

$$X1 + X2 + X3 + X6 + X7 \geq 22;$$

$$X1 + X2 + X3 + X4 + X7 \geq 26;$$

$$X1 + X2 + X3 + X4 + X5 \geq 25;$$

$$X2 + X3 + X4 + X5 + X6 \geq 21;$$

$$X3 + X4 + X5 + X6 + X7 \geq 19;$$

$$x1 \geq 0;$$

$$x2 \geq 0;$$

$$x3 \geq 0;$$

$$x4 \geq 0;$$

$$x5 \geq 0;$$

$$x6 \geq 0;$$

$$x7 \geq 0;$$

and x_j is integer, for $j=1,2,3,4,5,6,7$.

```
library(lpSolve)
f.obj <- c(775,750,775,800,800,800,800)
f.con <- matrix(c(1,0,0,1,1,1,1,
                  1,1,0,0,1,1,1,
                  1,1,1,0,0,1,1,
                  1,1,1,1,0,0,1,
                  1,1,1,1,0,0,1,
                  1,1,1,1,1,0,0,
                  0,1,1,1,1,1,0,
                  0,0,1,1,1,1,1), nrow=7,byrow=TRUE)
f.dir <- c('>=', '>=', '>=', '>=', '>=', '>=', '>=')
f.rhs <- c(18,27,22,26,25,21,19)
lp('min', f.obj, f.con,f.dir,f.rhs,all.int=TRUE)
## Success: the objective function is 25675.
```

The total cost is \$25675.

Then solve the 7 decision variables (x1, x2, x3, x4, x5, x6, x7).

```
lp7<-lp('min', f.obj, f.con,f.dir,f.rhs,all.int=TRUE)
lp7$solution
## [1] 1 13 2 2 7 0 8
```

I get 18 available workers on Sunday;

(available workers on Sunday: $X_1 + X_4 + X_5 + X_6 + X_7 = 1 + 2 + 7 + 0 + 8 = 18$)

I get 29 available workers on Monday;

(available workers on Monday: $X_1 + X_2 + X_5 + X_6 + X_7 = 1 + 13 + 7 + 0 + 8 = 29$)

I get 24 available workers on Tuesday;

(available workers on Tuesday: $X_1 + X_2 + X_3 + X_6 + X_7 = 1 + 13 + 2 + 0 + 8 = 24$)

I get 26 available workers on Wednesday;

(available workers on Wednesday: $X_1 + X_2 + X_3 + X_4 + X_7 = 1 + 13 + 2 + 2 + 8 = 26$)

I get 25 available workers on Thursday;

(available workers on Thursday: $X_1 + X_2 + X_3 + X_4 + X_5 = 1 + 13 + 2 + 2 + 7 = 25$)

I get 24 available workers on Friday;

(available workers on Friday: $X_2 + X_3 + X_4 + X_5 + X_6 = 13 + 2 + 2 + 7 + 0 = 24$)

I get 19 available workers on Saturday;

(available workers on Saturday: $X_3 + X_4 + X_5 + X_6 + X_7 = 2 + 2 + 7 + 0 + 8 = 19$)

To sum up, I get 18 available workers on Sunday, 29 available workers on Monday, 24 available workers on Tuesday, 26 available workers on Wednesday, 25 available workers on Thursday, 24 available workers on Friday, and 19 available workers on Saturday.