Penalized Power-Generalized Weibull Distributional Regression

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1. Background

- Covariates usually enter the hazard function, $\lambda(t)$, through the scale parameter only. This is the case for the popular proportional hazards (PH) model.
- $\lambda(t)$ is used to express the risk of a particular event occurring at time t.
- Multi-parameter regression (MPR) models, where multiple distributional parameters dependent on covariates, lead to increased flexibility.

2. Power-Generalized Weibull (PGW)

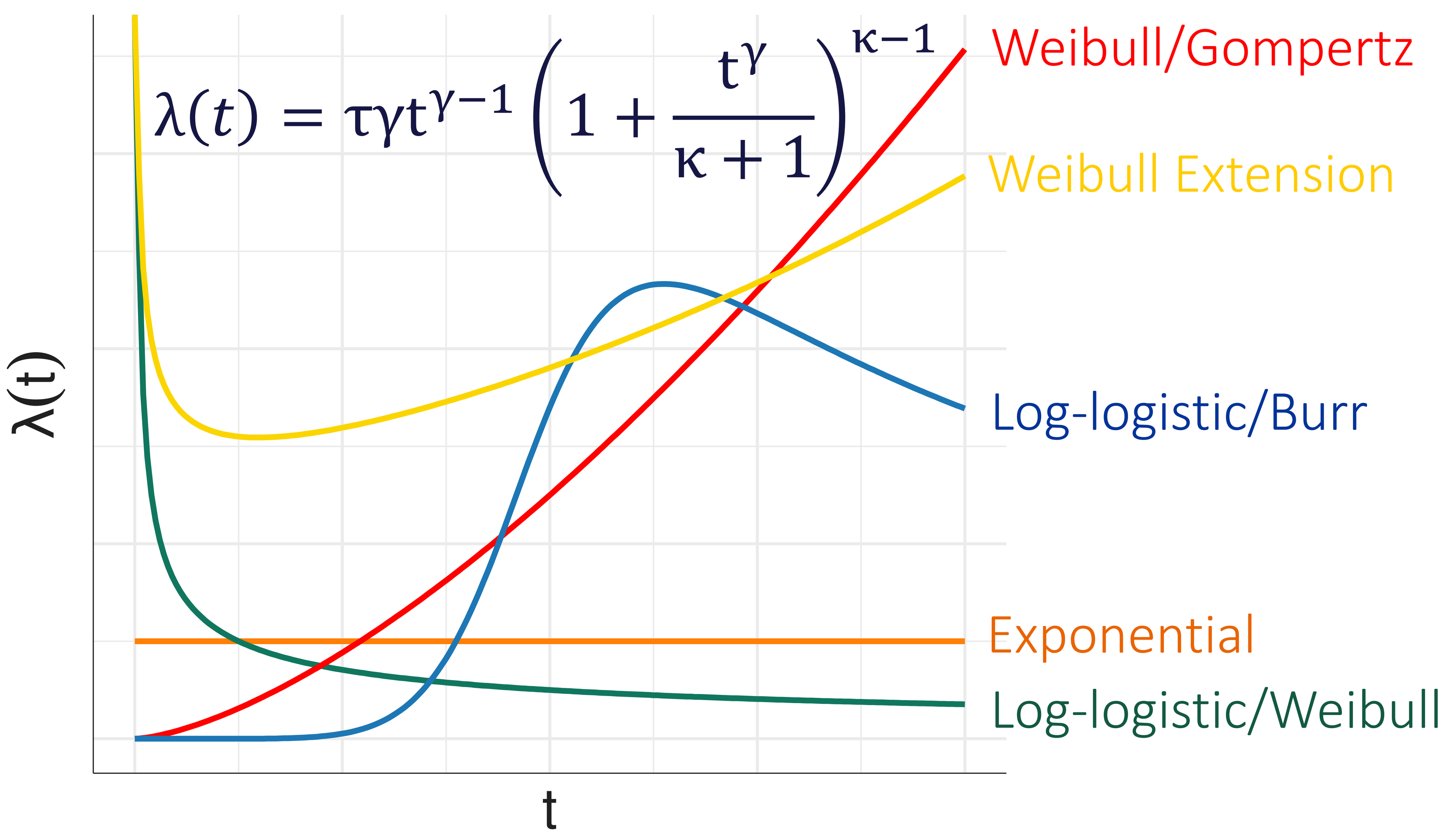

Figure 1. PGW hazard shapes.

3. Distributional Regression

Our model allows covariates to enter the PGW hazard through the scale (τ) and shape (γ) parameters where the additional shape (κ) parameter is independent of covariates:

$$log(\tau) = x^T\beta \qquad log(\gamma) = x^T\alpha \qquad log(\kappa+1) = \omega$$

where x is the covariate vector and β, α and ω are the regression coefficients associated with τ, γ and κ.

Scale: $\tau > 0 \Rightarrow$ Magnitude of the hazard

Shape: $\gamma > 0 \Rightarrow$ Time evolution of the hazard

Shape: $\kappa > -1 \Rightarrow$ Specifies the baseline distribution

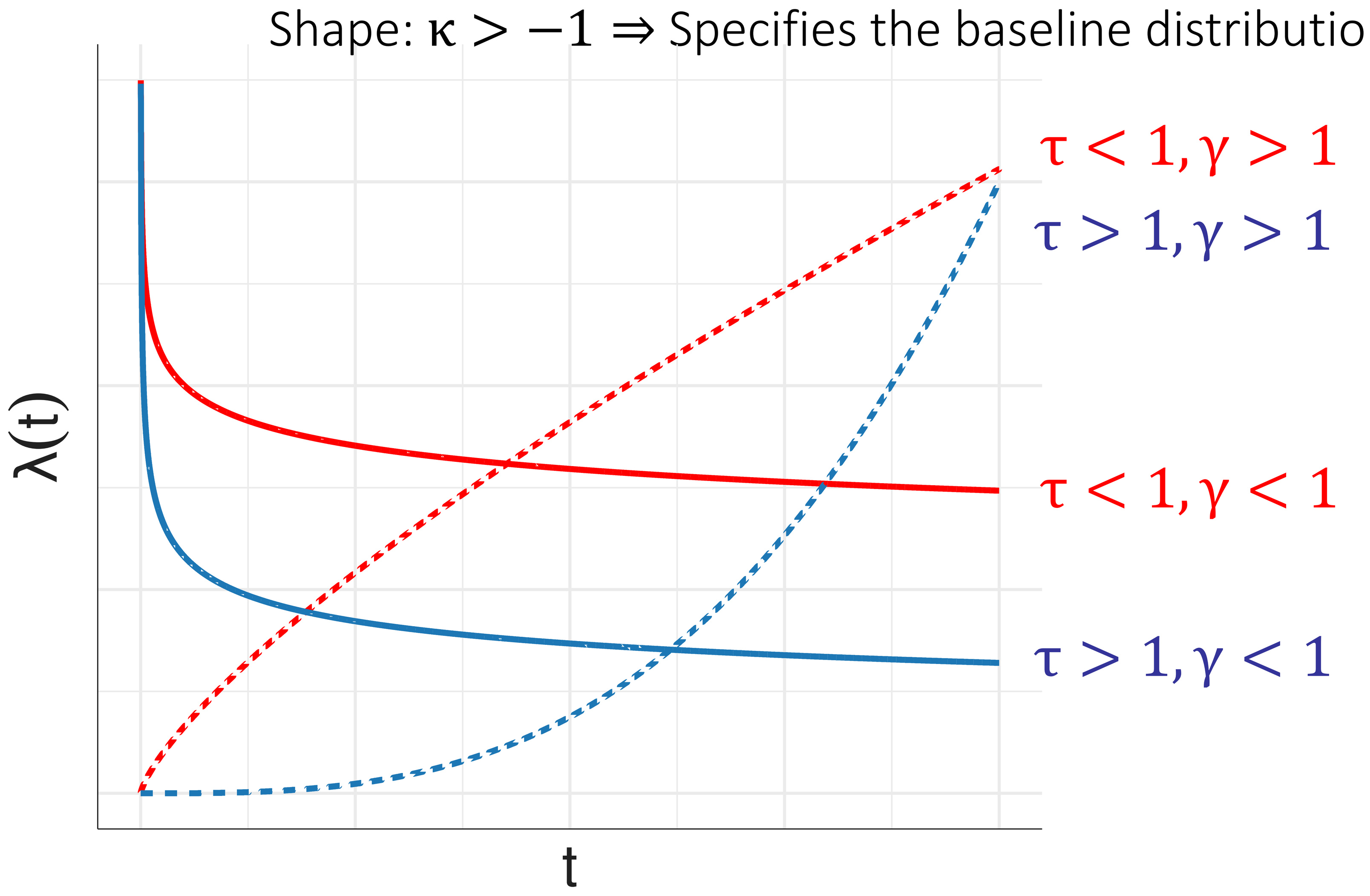

Figure 2. Impact of the shape (γ) and scale (τ) on the hazard.

- Variable selection and parameter estimation are carried out using penalized regression (via the adaptive lasso penalty).

4. Simulation Study

- A simulation study was carried out to assess the performance of the penalized PGW MPR model.
- Variable selection and inference were of interest.
- Good performance across all metrics.

	Scale (β)			Shape (α)		
n	C(7)	MSE	PT	C(7)	MSE	PT
500	6.86	0.02	0.87	6.78	0.00	0.81
2000	6.95	0.00	0.96	6.96	0.00	0.96

Table 1. C, average correct zeros; MSE, average mean squared error; PT, probability of choosing the true model – Variable selection results.

5. Data Application

- Veteran dataset – Survival package in R
- Randomized trial of two treatment groups for lung cancer

	PH	MPR
cell type: squamous	β	α
cell type: large	β	β
karno	β	α, β
Tuning Parameter	0.01706318	0.02833773
BIC	1466.42	1463.72

Table 2. β = "selected in scale", α = "selected in shape", and those which are non-significant (at the 5% level) are shown in gray.

	Intercept	Large	Squamous	Karno
β	-2.38	-0.42	-	-0.06
α	0.59	-	-0.13	0.01
ω	0.35	-	-	-

Table 4. Coefficient estimation from MPR output where those which are non-significant (at the 5% level) are shown in gray.

- Unlike PH models, MPR models allow for time-varying hazard ratios.
- This is the case for the squamous cell which has a smaller hazard, relative to the reference category, and this hazard is also decreasing over time.

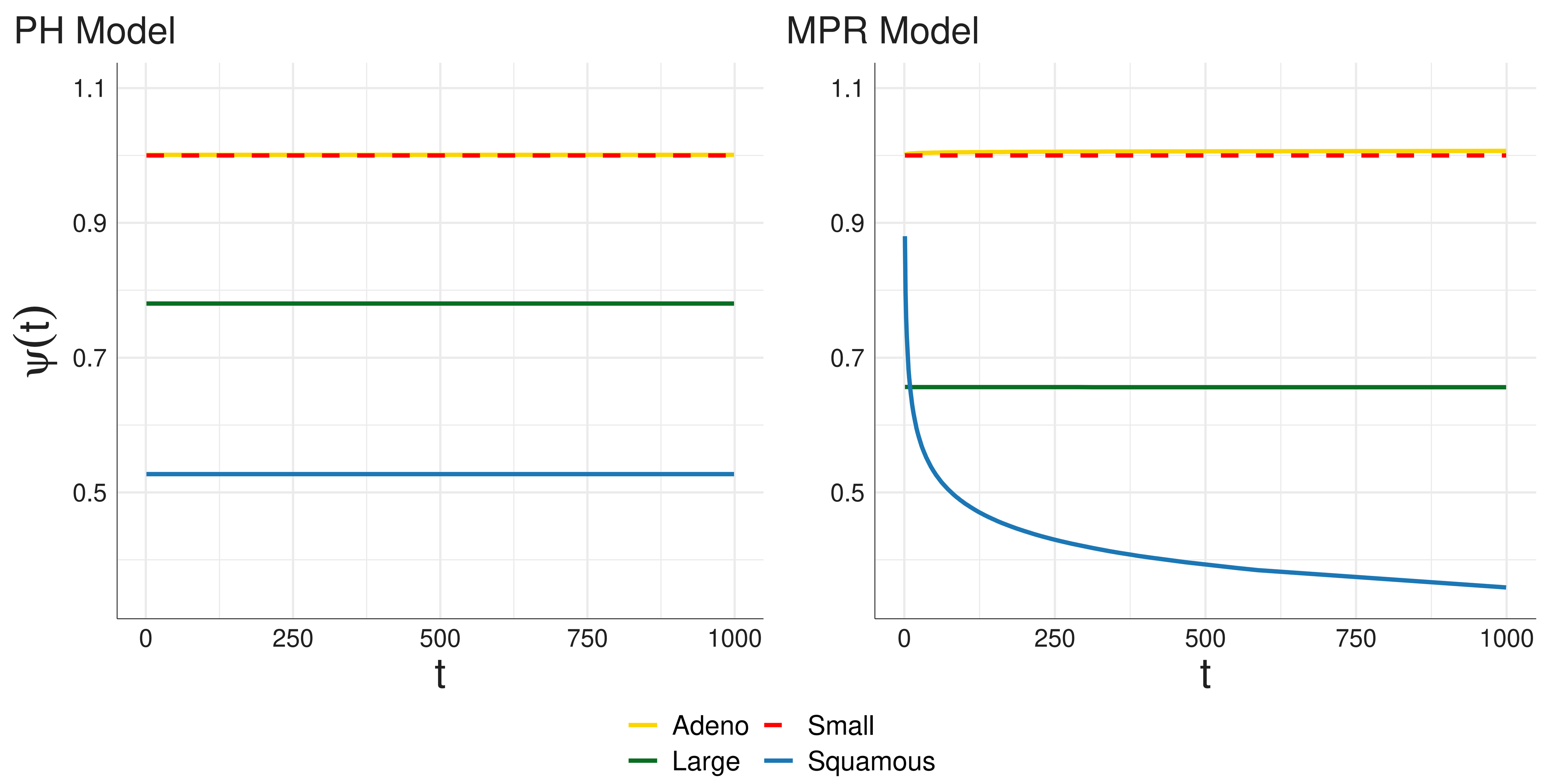

Figure 4. Cell type hazard ratios.

6. Conclusions

- Time-varying hazard ratios occur naturally by allowing the shape parameter to depend on covariates.
- Increased understanding of real-world data.

Contact Details and References

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- Jaouimaa et al. (2019). Penalized Variable Selection in Multi-Parameter Regression Survival Modelling. arXiv.