

How I see philosophy  
by F Waismann  
edited by R Harré

## **HOW I SEE PHILOSOPHY**

*By the same author*

INTRODUCTION TO MATHEMATICAL THINKING  
THE PRINCIPLES OF LINGUISTIC PHILOSOPHY

*By the same editor*

AN INTRODUCTION TO THE LOGIC OF THE SCIENCES  
THEORIES AND THINGS  
MATTER AND METHOD  
THE ANTICIPATION OF NATURE

# How I See Philosophy

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Palgrave Macmillan

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F. Waismann, and R. Harré 1968

Softcover reprint of the hardcover 1st edition 1968

*Published by*

MACMILLAN AND CO LTD

*Little Essex Street London WC2*

*and also at Bombay Calcutta and Madras*

*Macmillan South Africa (Publishers) Pty Ltd Johannesburg*

*The Macmillan Company of Australia Pty Ltd Melbourne*

*The Macmillan Company of Canada Ltd Toronto*

*St Martin's Press Inc New York*

Library of Congress catalog card no. 68-12306

ISBN 978-1-349-00104-0 ISBN 978-1-349-00102-6 (eBook)

DOI 10.1007/978-1-349-00102-6

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## PREFACE

IN his *Principles of Linguistic Philosophy* Friedrich Waismann was concerned to expound in as fully explicit and developed a form as possible, the ideas of Wittgenstein. But even though that book was meant as a textbook of Wittgensteinian philosophical method, careful reading brings to light from time to time a hint of new ideas, of a view of philosophy that while sibling to that of Wittgenstein, is Waismann's own. In a series of papers, some of which appeared while he was still working on the text of the *Principles*, Waismann expounded his solutions to the problems which still perplexed him.

In this volume are collected those of his published writings which express his independent approach. Waismann's philosophy has a sufficient unity of theme and method to allow his papers to be put together comfortably as a single book, though they cover a period of more than twenty years. It is hoped that the papers collected here will not only recall him to his many pupils and friends, but will enable those who never knew him to experience something of that combination of richness of thought, subtlety of expression and rigour of argument that was so characteristically Waismann.

The editor and literary executors are grateful to the editors of the following journals for permission to reprint Waismann's papers: *The Proceedings of the Aristotelian Society*, *Synthese*, *Analysis*, and to George Allen & Unwin, for permission to reprint pp. 447-490 from *Contemporary British Philosophy*, III, edited by H. D. Lewis; to Basil Blackwell for permission to reprint pp. 11-31 from *Logic and Language, Second Series*, edited by A. G. N. Flew, and to the North Holland Publishing Company for permission to reprint pp. 84-154 from *Turning Points in Physics*, edited by A. C. Crombie.

Minor corrections have been made to the texts, and, where they exist, English editions of works originally cited under their German or French titles, have been mentioned. My thanks are due to Mr J. Moulder for his valuable assistance.

R. HARRE (Editor)

## SOURCES OF THE MATERIAL

### CHAPTER I

*Contemporary British Philosophy*, III, edited by H. D. Lewis,  
London 1956, pp. 447–490.

### CHAPTER II

*Proceedings of the Aristotelian Society*, Supplementary volume xix  
(1945), pp. 119–150.

### CHAPTER III

*Proceedings of the Aristotelian Society*, volume xlvi (1945–46),  
pp. 77–104.

### CHAPTER IV: Part One

*Synthese*, volume v (1946), pp. 218–219.

#### Part Two

*Logic and Language*, II, edited by A. G. N. Flew, Oxford 1953,  
pp. 11–31.

### CHAPTER V

*Analysis*, 10. 2 (1949), pp. 25–40; 11. 2 (1950), pp. 25–38; 11. 3  
(1951), pp. 49–61; 11. 6 (1951), pp. 115–124; 13. 1 (1952), pp.  
1–14; 13. 4 (1953), pp. 73–89.

### CHAPTER VI

*Turning Points in Physics*, edited by A. C. Crombie, Amsterdam  
1959, pp. 84–154.

The paper ‘The Relevance of Psychology to Logic’, *Proceedings of the Aristotelian Society*, Supplementary volume xvii (1938), pp. 54–67, has not been included in this collection, not forming a natural part of Waismann’s later philosophy.

## CHAPTER I

### HOW I SEE PHILOSOPHY

1. WHAT philosophy is? I don't know, nor have I a set formula to offer. Immediately I sit down to contemplate the question I am flooded with so many ideas, tumbling over one another, that I cannot do justice to all of them. I can merely make an attempt, a very inadequate one, to sketch with a few strokes what the lie of the land seems to me to be, tracing some lines of thought without entering upon a close-knit argument.

It is, perhaps, easier to say what philosophy is not than what it is. The first thing, then, I should like to say is that philosophy, as it is practised today, is very unlike science; and this in three respects: in philosophy there are no proofs; there are no theorems; and there are no questions which can be decided, Yes or No. In saying that there are no proofs I do not mean to say that there are no arguments. Arguments certainly there are, and first-rate philosophers are recognized by the originality of their arguments; only these do not work in the sort of way they do in mathematics or in the sciences.

There are many things beyond proof: the existence of material objects, of other minds, indeed of the external world, the validity of induction, and so on. Gone are the days when philosophers were trying to prove all sorts of things: that the soul is immortal, that this is the best of all possible worlds and the rest, or to refute, by 'irrefutable' argument and with relish, materialism, positivism and what not. Proof, refutation — these are dying words in philosophy, though G. E. Moore still 'proved' to a puzzled world that it exists. What can one say to this — save, perhaps, that he is a great prover before the Lord?

But can it be *proved* that there are no proofs in philosophy? No; for one thing, such a proof, if it were possible, would by its very existence establish what it was meant to confute. But why suppose the philosopher to have an I.Q. so low as to be unable to learn from the past? Just as the constant failure of attempts at constructing a

perpetual motion has in the end led to something positive in physics, so the efforts to construct a philosophical 'system', going on for centuries and going out of fashion fairly recently, tell their tale. This, I think, is part of the reason why philosophers today are getting weaned from casting their ideas into deductive moulds, in the grand style of Spinoza.

What I want to show in this article is that it is quite wrong to look at philosophy as though it had for its aim to provide theorems but had lamentably failed to do so. The whole conception changes when one comes to realize that what philosophers are concerned with is something different — neither discovering new propositions nor refuting false ones nor checking and re-checking them as scientists do. For one thing, proofs require premisses. Whenever such premisses have been set up in the past, even tentatively, the discussion at once challenged them and shifted to a deeper level. Where there are no proofs there are no theorems either. (To write down lists of propositions 'proved' by Plato or Kant: a pastime strongly to be recommended.) Yet the failure to establish a sort of Euclidean system of philosophy based on some suitable 'axioms' is, I submit, neither a mere accident nor a scandal but deeply founded in the nature of philosophy.

Yet there are questions; (and arguments). Indeed, a philosopher is a man who senses as it were hidden crevices in the build of our concepts where others only see the smooth path of commonplace-ness before them.

Questions but no answers? Decidedly odd. The oddness may lessen when we take a look at them at closer range. Consider two famous examples: Achilles and the tortoise, and the astonishment of St Augustine when confronted with the fact of memory. He is amazed, not at some striking feat of memory, but at there being such a thing as memory at all. A sense-impression, say a smell or a taste, floats before us and disappears. One moment it is here and the next it is gone. But in the galleries of the memory pale copies of it are stored up after its death. From there I can drag them out when and as often as I wish, like, and yet strangely unlike, the original — unlike in that they are not perishable like the momen-tary impression: what was transitory has been arrested and has achieved duration. But who can say how this change comes about?

Here the very fact of memory feels mystifying in a way in which

ordinary questions asking for information do not; and *of course* it is not a factual question. What is it?

From Plato to Schopenhauer philosophers are agreed that the source of their philosophizing is wonder. What gives rise to it is nothing recondite and rare but precisely those things which stare us in the face: memory, motion, general ideas. (Plato: What does 'horse' mean? A single particular horse? No, for it may refer to *any* horse; *all* the horses, the total class? No, for we may speak of this or that horse. But if it means neither a single horse nor all horses, what *does* it mean?) The idealist is shaken in just the same way when he comes to reflect that he has, in Schopenhauer's words, 'no knowledge of the sun but only of an eye that sees a sun, and no knowledge of the earth but only of a hand that feels an earth'. Can it be, then, that nothing whatever is known to us except our own consciousness?

In looking at such questions, it seems as if the mind's eye were growing dim and as if everything, even that which ought to be absolutely clear, was becoming oddly puzzling and unlike its usual self. To bring out what seems to be peculiar to these questions one might say that they are not so much questions as tokens of a profound uneasiness of mind. Try for a moment to put yourself into the frame of mind of which Augustine was possessed when he asked: How is it possible to measure time? Time consists of past, present and future. The past can't be measured, it is gone; the future can't be measured, it is not yet here; and the present can't be measured, it has no extension. Augustine knew of course how time is measured and this was not his concern. What puzzled him was how it is *possible* to measure time, seeing that the past hour cannot be lifted out and placed alongside the present hour for comparison. Or look at it this way: what is measured is in the past, the measuring is in the present: how can that be?

The philosopher as he ponders over some such problem has the appearance of a man who is deeply disquieted. He seems to be straining to grasp something which is beyond his powers. The words in which such a question presents itself do not quite bring out into the open the real point — which may, perhaps more aptly, be described as the recoil from the incomprehensible. If, on a straight railway journey, you suddenly come in sight of the very station you have just left behind, there will be terror, accompanied perhaps by slight giddiness. That is exactly how the philosopher

feels when he says to himself, ‘Of course time can be measured; but how *can* it?’ It is as though, up to now, he had been passing heedlessly over the difficulties, and now, all of a sudden, he notices them and asks himself in alarm, ‘But how can that be?’ That is a sort of question which we only ask when it is the very facts themselves which confound us, when something about them strikes us as preposterous.

Kant, I fancy, must have felt something of the sort when he suddenly found the existence of geometry a puzzle. Here we have propositions as clear and transparent as one would wish, prior, it seems, to all experience; at the same time they apply miraculously to the real world. How is that possible? Can the mind, unaided by experience, in some dark manner actually fathom the properties of real things? Looked upon in this way, geometry takes on a disturbing air.

We all have our moments when something quite ordinary suddenly strikes us as queer — for instance, when time appears to us as a curious thing. Not that we are often in this frame of mind; but on some occasions, when we look at things in a certain way, unexpectedly they seem to change as though by magic: they stare at us with a puzzling expression, and we begin to wonder whether they can possibly be the things we have known all our lives.

‘Time flows’ we say — a natural and innocent expression, and yet one pregnant with danger. It flows ‘equably’, in Newton’s phrase, at an even rate. What can this mean? When something moves, it moves with a definite speed (and speed means: rate of change in time). To ask with what speed time moves, i.e. to ask how quickly time changes in time, is to ask the unaskable. It also flows, again in Newton’s phrase, ‘without relation to anything external’. How are we to figure that? Does time flow on irrespective of what happens in the world? Would it flow on even if everything in heaven and on earth came to a sudden standstill as Schopenhauer believed? For if this were not so, he said, time would have to stop with the stopping of the clock and move with the clock’s movement. How odd: time flows at the same rate and yet without speed; and perhaps even without anything to occur in it. The expression is puzzling in another way. ‘I can never catch myself being in the past or in the future’, someone might say; ‘whenever I think or perceive or breathe the word “now”, I am in the present; therefore I am *always* in the present.’ In saying this, he may think

of the present moment as a bridge as it were from which he is looking down at the 'river of time'. Time is gliding along underneath the bridge, but the 'now' does not take part in the motion. What was future passes into the present (is just below the bridge) and then into the past, while the onlooker, the 'self' or the 'I', is always in the present. 'Time flows *through* the "now"', he may feel to be a quite expressive metaphor. Yes, it sounds all right — until he suddenly comes to his senses and, with a start, realizes, 'But surely the moment flies?' (Query: How to succeed in wasting time? Answer: In this way, for instance — by trying, with eyes closed or staring vacantly in front of oneself, to catch the present moment as it is flitting by.) He may come now to look at matters in a different way. He sees himself advancing through time towards the future, and with this goes a suggestion of being active, just as at other times he may see himself floating down the stream whether he likes it or not. 'What exactly is it that is moving — the events in time or the present moment?', he may wonder. In the first case, it looks to him as if time were moving while he stands still; in the second case as if he were moving through time. 'How exactly is it?', he may say in a dubious voice, 'Am I always in the present? Is the present always eluding me?' Both ring true in a way; but they contradict each other. Again, does it make sense to ask, 'At what time is the present moment?' Yes, no doubt; but how *can* it, if the 'now' is but the fixed point from which the dating of any event ultimately receives its sense?

So he is pulled to and fro: 'I am always in the present, yet it slips through my fingers; I am going forward in time — no, I am carried down the stream.' He is using different pictures, each in its way quite appropriate to the occasion; yet when he tries to apply them jointly they clash. 'What a queer thing time must be', he may say to himself with a puzzled look on his face, 'what after all *is* time?' — expecting, half-expecting perhaps, that the answer will reveal to him time's hidden essence. Ranged beyond the intellectual are deeper levels of uneasiness — terror of the inevitability of time's passage, with all the reflections upon life that this forces upon us. Now all these anxious doubts release themselves in the question, 'What is time?' (*En passant* this is a hint that *one* answer will never do — will never remove all these doubts that break out afresh on different levels and yet are expressed in the same form of words.)

As we all know what time is, and yet cannot say what it is, it feels

mystifying; and precisely because of its elusiveness it catches our imagination. The more we look at it the more we are puzzled: it seems charged with paradoxes. 'What is time? What is this being made up of movement only without anything that is moving?' (Schopenhauer.) How funny to have it bottled up! 'I've got here in my hand the most potent, the most enigmatic, the most fleeting of all essences — Time.' (Logan Pearsall Smith of an hour-glass.) For Shelley it is an 'unfathomable sea! whose waves are years', a 'shoreless flood', for Proust — well, why not leave something to the reader?

But isn't the answer to this that what mystifies us lies in the *noun* form 'the time'? Having a notion embodied in the form of a noun almost irresistibly makes us turn round to look for what it is 'the name of'. We are trying to catch the shadows cast by the opacities of speech. A wrong analogy absorbed into the forms of our language produces mental discomfort (and the feeling of discomfort, when it refers to language, is a profound one). 'All sounds, all colours . . . evoke indefinite and yet precise emotions, or, as I prefer to think, call down among us certain disembodied powers whose footsteps over our hearts we call emotions' (W. B. Yeats).

Yet the answer is a prosaic one: don't ask what time is but how the *word* 'time' is being used. Easier said than done; for if the philosopher rectifies the use of language, ordinary language has 'the advantage of being in possession of declensions', to speak with Lichtenberg, and thus renews its spell over him, luring him on into the shadow chase. It is perhaps only when we turn to languages of a widely different grammatical structure that the way towards such possibilities of interpretation is entirely barred. 'It is highly probable that philosophers within the domain of the Ural-Altaic languages (where the subject-concept is least developed) will look differently "into the world" and be found on paths of thought different from those of the Indo-Europeans or Mussulmans' (Nietzsche).

2. It may be well at this point to remind ourselves that the words 'question' and 'answer', 'problem' and 'solution' are not always used in their most trite sense. It is quite obvious that we often have to do something very different to find the way out of a difficulty. A problem of politics is solved by adopting a certain line of action, the problems of novelists perhaps by the invention of devices for

presenting the inmost thoughts and feelings of their characters; there is the painter's problem of how to suggest depth or movement on the canvas, the stylistic problem of expressing things not yet current, not yet turned into cliché; there are a thousand questions of technology which are answered, not by the discovery of some truth, but by a practical achievement; and there is of course the 'social question'. In philosophy, the real problem is not to find the answer to a given question but to find a sense for it.

To see in what the 'solution' of such a 'problem' consists let us start with Achilles who, according to Zeno, is to this day chasing the tortoise. Suppose that Achilles runs twice as fast as the tortoise. If the tortoise's start is 1, Achilles will have to cover successively  $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$ ; this series is endless: so he can never catch the tortoise. 'Nonsense!' (a mathematician's voice), 'the sum of the infinite series is finite, namely 2, and that settles it.' Though perfectly true, his remark is not to the point. It does not remove the sting from the puzzle, the disconcerting idea, namely, that however far we go in the series there is always a next term, that the lead the tortoise has in the race, though naturally getting smaller and smaller, yet never ceases to be: there *can* be no moment when it is strictly zero. It is *this* feature of the case, I suggest, that we do not understand and which throws us into a state of confusion.

But look at it this way. Suppose that we apply the same sort of argument to a minute, then we shall have to argue in some such way as this. Before the minute can be over the first half of it must elapse, then one-quarter of it, then one-eighth of it, and so on *ad infinitum*. This being an endless process, the minute can never come to an end. Immediately we have the argument in this form, the blunder leaps to the eye: we have been confusing two senses of 'never', a temporal and a non-temporal one. While it is quite correct to say that the sequence  $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$  never ends, this sense of the word 'never' has nothing whatever to do with time. All it means is that there is no last term in the series, or (what comes to the same) that to any term, no matter how far out in the sequence, a successor can be constructed according to the simple rule 'halve it': that is what is meant here by 'never'; whereas in saying, for instance, that man will never find out anything to avert death, 'never' is meant in the sense 'at no time'. It is clear that the mathematical assertion concerning the possibility of going on in the sequence by forming new terms according to the rule does not state

anything about actual occurrences in time. The mistake should really be obvious: in saying that, since the start is getting progressively smaller and yet can never cease to be, Achilles can never catch the tortoise, we jump from the mathematical, *non-temporal* to the temporal sense. Had there been two different words in our language to mark these senses the confusion could never have arisen, and the world would be poorer for one of its most attractive paradoxes. But the same word is, as a matter of course, used with different meanings. Result: something like a conjuring trick. While our attention is diverted, while, 'in our mind's eye', we stare fixedly at Achilles as he is speeding along, with each big bound diminishing his distance from the tortoise, the one sense is so innocuously palmed off for the other as to escape notice.

This way of bringing out the fallacy also holds when the other key term is used for presenting the puzzle. As there will 'always' be a next term in the sequence, i.e. a next step in the scheme of subdividing the race-course (the word 'always' looking just as spotless and innocent) we readily fall into the trap of concluding that the tortoise will 'always' be ahead of Achilles, eternally to be chased by his pursuer.

Many are the types of bewilderment: there is the obsessional doubt — can I ever know that other people have experiences, that they see, hear and feel as I do? Can I be sure that memory does not always deceive me? Are there really material objects and not only sense-impressions 'of' them? There is the doubt-like uneasiness — what sort of being is possessed by numbers? There is the anxiety-doubt — are we really free? This doubt has taken many different forms, one of which I shall single out for discussion — the question, namely, whether the law of excluded middle, when it refers to statements in the future tense, forces us into a sort of logical Predestination. A typical argument is this. If it is true now that I shall do a certain thing tomorrow, say, jump into the Thames, then no matter how fiercely I resist, strike out with hands and feet like a madman, when the day comes I cannot help jumping into the water; whereas, if this prediction is false now, then whatever efforts I may make, however many times I may nerve and brace myself, look down at the water and say to myself, 'One, two, three —', it is impossible for me to spring. Yet that the prediction is either true or false is itself a necessary truth, asserted by the law of excluded middle. From this the startling consequence seems to

follow that it is already now decided what I shall do tomorrow, that indeed the entire future is somehow fixed, logically pre-ordained. Whatever I do and whichever way I decide, I am merely moving along lines clearly marked in advance which lead me towards my appointed lot. We are all, in fact, marionettes. If we are not prepared to swallow *that*, then — and there is a glimmer of hope in the ‘then’ — there is an alternative open to us. We need only renounce the law of excluded middle for statements of this kind, and with it the validity of ordinary logic, and all will be well. Descriptions of what will happen are, at present, neither true nor false. (This sort of argument was actually propounded by Lukasiewicz in favour of a three-valued logic with ‘possible’ as a third truth-value alongside ‘true’ and ‘false’.<sup>1</sup>)

The way out is clear enough. The asker of the question has fallen into the error of so many philosophers: of giving an answer before stopping to consider the question. For is he clear what he is asking? He seems to suppose that a statement referring to an event in the future is at present undecided, neither true nor false, but that when the event happens the proposition enters into a sort of new state, that of being true. But how are we to figure the change from ‘undecided’ to ‘true’? Is it sudden or gradual? At what moment does the statement ‘it will rain tomorrow’ begin to be true? When the first drop falls to the ground? And supposing that it will not rain, when will the statement begin to be false? Just at the end of the day, at 12 p.m. sharp? Supposing that the event *has* happened, that the statement *is* true, will it remain so for ever? If so, in what way? Does it remain uninterruptedly true, at every moment of day and night? Even if there were no one about to give it any thought? Or is it true only at the moments when it is being thought of? In that case, how long does it remain true? For the duration of the thought? We wouldn’t know how to answer these questions; this is due not to any particular ignorance or stupidity on our part but to the fact that something has gone wrong with the way the words ‘true’ and ‘false’ are applied here.

If I say, ‘It is true that I was in America’, I am saying that I was in America and no more. That in uttering the words ‘It is true that —’ I take responsibility upon myself is a different matter that does not concern the present argument. The point is that in

<sup>1</sup> Cf. *Polish Logic 1920-39*, edited by Storrs McCall (Oxford, 1967), Papers I, II and III.

making a statement prefaced by the words ‘It is true that’ I do not *add* anything to the factual information I give you. *Saying* that something is true is not *making* it true: cf. the criminal lying in court, yet every time he is telling a lie protesting, his hand on his heart, that he is telling the truth.

What is characteristic of the use of the words ‘true’ and ‘false’ and what the pleader of logical determinism has failed to notice is this. ‘It is true’ and ‘it is false’, while they certainly have the force of asserting and denying, are not descriptive. Suppose that someone says, ‘It is true that the sun will rise tomorrow’ all it means is that the sun will rise tomorrow: he is not regaling us with an extra-description of the trueness of what he says. But supposing that he were to say instead, ‘It is true *now* that the sun will rise tomorrow’, this would boil down to something like ‘The sun will rise tomorrow now’; which is nonsense. To ask, as the puzzle-poser does, ‘Is it true or false *now* that such-and-such will happen in the future?’ is not the sort of question to which an answer can be given: which is the answer.

This sheds light on what has, rather solemnly, been termed the ‘timelessness of truth’. It lies in this: that the clause ‘it is true that —’ does not allow of inserting a date. To say of a proposition like ‘Diamond is pure carbon’ that it is true on Christmas Eve would be just as poor a joke as to say that it is true in Paris and not in Timbuctoo. (This does not mean that we cannot say in certain circumstances, ‘Yes, it was true in those days’ as this can clearly be paraphrased without using the word ‘true’.)

Now it begins to look a bit less paradoxical to say that when a philosopher wants to dispose of a question the one thing he must not do is: to give an answer. A philosophic question is not solved: it *dissolves*. And in what does the ‘dissolving’ consist? In making the meaning of the words used in putting the question so clear to ourselves that we are released from the spell it casts on us. Confusion was removed by calling to mind the use of language or, so far as the use *can* be distilled into rules, the rules: it therefore *was* a confusion about the use of language, or a confusion about rules. It is here that philosophy and grammar meet.

There is one further point that needs elucidation. When we say of a given assertion, e.g. ‘It is raining’, that it is true we can hardly escape the impression that we say something ‘about’ the assertion, namely, that it has the property of trueness. To make such a

statement seems, then, to say *more* than what was asserted originally, namely, that it is raining and that this assertion is true. That, however, leads to queer consequences. For in which sense does it say more? Consider first under which circumstances it would be appropriate to say of two given propositions that the one says 'more' than the other. 'This is red' says more than 'this is coloured' for the obvious reason that anyone can conclude from the first statement to the second but no one reversely; similarly 'today is Tuesday' says more than 'today is a weekday'. The criterion, then, suggests itself that, given two propositions  $p$  and  $q$ ,  $p$  says more than  $q$ , if  $\sim p \cdot q$  is meaningful and  $p \cdot \sim q$  contradictory. The holder of the view that ' $p$  is true' says more than  $p$  ( $p$  standing e.g. for 'It is raining'), may now be challenged to explain what he means by that. Is he using the word 'more' in the sense just explained? If so, the curious consequence ensues that it must *make sense* to assert the conjunction  $\sim p \cdot p$ , that is, in our case, 'It is not true that it is raining and it is raining'. Since this obviously is not what he had in mind, what *does* he mean? We are not contradicting him; we merely remind him of how these words have always been used by him, in non-philosophical contexts that is, and then point out that, if he still wants to use them in this sense, to say what he wanted to say lands him in an absurdity. All we do is to make him aware of his own practice. We abstain from any assertion. It is for him to explain what he means. Not that he cannot do it. In ascribing truth to a given statement, he might say, he wants to express perhaps either (i) that it is 'in accordance with fact' or something of the sort; or (ii) that he *knows* that it is true. In the first case he is faced with the same dilemma, namely, that it must make sense to say, 'It is not in accordance with the facts that it is raining and it is raining'; in the second fresh difficulties are breaking out. For one thing, the words 'it is true that —', when uttered by different people, would then mean different things; for another, and this is more fatal to the advocate of fatalism, in construing the words in this sense, he cuts the ground from under his own feet. No one would then be worried by the question whether, supposing that it is false now that he will write a certain letter tomorrow, it follows that it will really be impossible for him to write that letter, that this line of conduct is barred to him, logically barred. For since 'it is false now' means in the new sense 'he doesn't know yet' nothing follows and the whole question evaporates.

My reason for going into this tangle at some length is that the method applied in unravelling it presents some interesting features. First, we don't *force* our interlocutor. We leave him free to choose, accept or reject any way of using his words. He may depart from ordinary usage — language is not *untouchable* — if it is only in this way that he can explain himself. He may even use an expression one time in this, another time in that, way. The only thing we insist upon is that he should be aware of what he is doing. If we strictly adhere to this method — going over the argument, asking him at each step whether he is willing to use an expression in a certain way, if not, offering him alternatives, but leaving the decisions to him and only pointing out what their consequences are — no dispute can arise. Disputes arise only if certain steps in this procedure are omitted so that it looks as if we had made an assertion, adding to the world's woes a new apple of discord. This would be the true way of doing philosophy undogmatically. The difficulty of this method lies in presenting the subject in a manner which can easily be taken in — in arranging the cases and the ways in which they are connected through intermediate links so that we can gain a clear synoptic view of the whole.

Second, we do not use arguments in order to prove or disprove any 'philosophic view'. As we have no views we can afford to look at things as they are.

Next, we only describe; we do not 'explain'. An explanation, in the sense of a deductive proof, cannot satisfy us because it pushes the question 'Why just these rules and no other ones?' only one stage back. In following that method, we do not *want* to give reasons. All we do is to describe a use or tabulate rules. In doing this, we are not making any discoveries: there is nothing to be discovered in grammar. Grammar is autonomous and not dictated by reality. Giving reasons, bound as it is to come to an end and leading to something which cannot further be explained, *ought* not to satisfy us. In grammar we never ask the question 'why?'

But isn't the result of this that philosophy itself 'dissolves'? Philosophy eliminates those questions which *can* be eliminated by such a treatment. Not all of them, though: the metaphysician's craving that a ray of light may fall on the mystery of the existence of this world, or on the incomprehensible fact that it is comprehensible, or on the 'meaning of life' — even if such questions *could* be shown to lack a clear meaning or to be devoid of meaning

altogether, they are *not silenced*. It does nothing to lessen the dismay they arouse in us. There is something cheap in 'debunking' them. The heart's unrest is not to be stilled by logic. Yet philosophy is not dissolved. It derives its weight, its grandeur, from the significance of the questions it destroys. It overthrows idols, and it is the importance of these idols which gives philosophy its importance.

Now it can perhaps be seen why the search for answers fitting the moulds of the questions fails, is *bound* to fail. They are not real questions asking for information but 'muddles felt as problems' (Wittgenstein) which wither away when the ground is cleared. If philosophy advances, it is not by adding new propositions to its list, but rather by transforming the whole intellectual scene and, as a consequence of this, by reducing the number of questions which befog and bedevil us. Philosophy so construed is one of the great liberating forces. Its task is, in the words of Frege, 'to free the spirit from the tyranny of words by exposing the delusions which arise, almost inevitably, through the use of a word language'.

3. What, only criticism and no meat? The philosopher a fog displayer? If that were all he was capable of I would be sorry for him and leave him to his devices. Fortunately, this is not so. For one thing, a philosophic question, if pursued far enough, may lead to something positive — for instance, to a more profound understanding of language. Take the sceptical doubts as to material objects, other minds, etc. The first reaction is perhaps to say: these doubts are idle. Ordinarily, when I doubt whether I shall finish this article, after a time my doubt comes to an end. I cannot go on doubting for ever. It's the destiny of doubt to die. But the doubts raised by the sceptic never die. Are they doubts? Are they pseudo-questions? They appear so only when judged by the twin standards of common sense and common speech. The real trouble lies deeper: it arises from the sceptic casting doubt on the very facts which underlie the use of language, those permanent features of experience which make concept formation possible, which in fact are precipitated in the use of our most common words. Suppose that you see an object in front of you quite clearly, say, a pipe, and when you are going to pick it up it melts into thin air, then you may feel, 'Lord, I'm going mad' or something of the sort (unless the whole situation is such that you have reason to suspect that it was some clever trick). But what, the sceptic may press now, if such experiences were quite frequent? Would you be prepared

to dissolve the connection between different sense experiences which form the hard core of our idea of a solid object, to undo what language has done — to part with the category of thing-hood? And would you then be living in a phenomenalist's paradise with colour patches and the other paraphernalia of the sense-datum theory, in a disobjected, desubstantialized world? To say in such circumstances, 'Look, it's just tabling now' would be a joke (for even in the weakened verb forms 'tabling', 'chairing' an element of the thing-category lingers on). That is why the sceptic struggles to express himself in a language which is not fit for this purpose. He expresses himself misleadingly when he says that he doubts such-and-such *facts*: his doubts cut so deep that they affect the fabric of language itself. For what he doubts is already embodied in the very forms of speech, e.g. in what is condensed in the use of thing-words. The moment he tries to penetrate those deep-sunken layers, he undermines the language in which he ventilates his qualms — with the result that he seems to be talking nonsense. He is not. But in order to make his doubts fully expressible, language would first have to go into the melting-pot. (We can get a glimmering of what is needed from modern science where all the long-established categories — thinghood, causality, position — had to be revolutionized. This required nothing less than the construction of some new language, not the expression of new facts with the old one.)

If we look at the matter in this way the attitude of the sceptic is seen in a new light. He considers possibilities which lie far outside the domain of our current experience. If his doubts are taken seriously, they turn into observations which cast a new and searching light on the subsoil of language, showing what possibilities are open to our thought (though not to ordinary language), and what paths might have been pursued if the texture of our experience were different from what it is. These problems are not spurious: they make us aware of the vast background in which any current experiences are embedded, and to which language has adapted itself; thus they bring out the unmeasured sum of experience stored up in the use of our words and syntactical forms.

For another thing, a question may decide to go in for another career than dissolving: it may pass into science. Frege, for instance, was prompted to his inquiries by philosophical motives, namely, to

find a definite answer to the question about the nature of arithmetical truths — whether they are analytic or synthetic, *a priori* or *a posteriori*. Starting from this question and pursuing it with all possible rigour, he was led to unearth a whole mine of problems of a scientific nature; and proceeding along these lines, he came to fashion a new instrument, a logic, which in delicacy and range and power far surpassed anything that went by this name before, a subject revealing to this day new and unexpected depths. True, the question from which Frege set out was not too clearly defined owing to the imprecise nature of the Kantian terms in which it was expressed.

A whole chapter might be written on the fate of questions, their curious adventures and transformations — how they change into others and in the process remain, and yet do not remain, the same. The original question may split and multiply almost like a character in a dream play. To mention just a few examples: can logic be characterized completely in a formal way, i.e. without bringing in any extraneous ideas such as the use of language and all that goes with it? Can arithmetic be characterized in any such way, entirely 'from within'? Or will any interpretation include some *Erdenrest* of the empiric? These questions have given rise to extensive research on mathematical interpretation of formal systems. The query how far logical intuition is correct has ramified into a bunch of questions pertaining to the theory of logical types, the axiom of choice, etc., indeed to a far more fundamental issue, namely, whether ordinary logic itself is 'right' as contrasted with the system of inferences evolved by the intuitionists. Or again, are there undecidable questions in mathematics, not in the restricted sense of Gödel, but undecidable in an absolute sense? Are there natural limits to generalization? It is interesting to watch how from a question of this sort, not too precise, somewhat blurred, new and better defined questions detach themselves, the parent question — in Frege's case philosophic *par excellence* — giving rise to a scientist's progeny.

Now something else must be noted — how these questions become, not only precise, but clear (which is not the same thing). To illustrate, can the infinity represented by all natural numbers be compared with the infinity represented by all points in space? That is, can the one be said to be less than, or equal to, the other? When it was first asked, the question had no clear sense — perhaps

no sense at all. Yet it guided G. Cantor in his ingenious search. Before set theory was discovered — or should I rather say ‘invented’? — the question acted as a sort of signpost pointing vaguely to some so far uncharted region of thought. It is perhaps best characterized by saying that it guides our imagination in a given direction, stimulates research along new lines. Such questions do not ‘dissolve’: they are solved, only not in the existing system of thought but rather by constructing a new conceptual system — such as set theory — where the intended and faintly anticipated sense finds its full realization. They are therefore of the nature of incitements to the building of such systems, they point from the not-yet-meaningful to the meaningful.

The question is the first groping step of the mind in its journeys that lead towards new horizons. The genius of the philosopher shows itself nowhere more strikingly than in the new kind of question he brings into the world. What distinguishes him and gives him his place is the passion of questioning. That his questions are at times not so clear is perhaps of not so much moment as one makes of it. There is nothing like clear thinking to protect one from making discoveries. It is all very well to talk of clarity, but when it becomes an obsession it is liable to nip the living thought in the bud. This, I am afraid, is one of the deplorable results of Logical Positivism, not foreseen by its founders, but only too striking in some of its followers. Look at these people, gripped by a clarity neurosis, haunted by fear, tongue-tied, asking themselves continually, ‘Oh dear, now does this make perfectly good sense?’ Imagine the pioneers of science, Kepler, Newton, the discoverers of non-Euclidean geometry, of field physics, the unconscious, matter waves or heaven knows what, imagine them asking themselves this question at every step — this would have been the surest means of sapping any creative power. No great discoverer has acted in accordance with the motto, ‘Everything that can be said can be said clearly’. And some of the greatest discoveries have even emerged from a sort of primordial fog. (Something to be said for the fog. For my part, I’ve always suspected that clarity is the last refuge of those who have nothing to say.)

The great mind is the great questioner. An example in point is Kant’s problem ‘How is geometry possible?’ The way to its solution was only opened up through the rise of the ‘axiomatic method’. Seeing that the axioms of geometry are capable of an

indefinite number of different interpretations and that the particular way they may be interpreted is irrelevant to deductive purposes, Hilbert separated what belongs to the logical form of the axioms from what belongs to their intuitional (or other) content and turned the whole question by saying: a point, a straight line, etc., may be anything that satisfies the axioms. As the business of deduction hinges only on the relations in which the basic terms stand to each other and not on the 'content' we associate with them, and as these relations are fully set out in the axioms, the axioms in their totality determine what a 'point', a 'line', etc., is so far as it is sufficient for deductive needs. Through the rise of this technique it became apparent that the word 'geometry', as understood by Kant, covers, in fact, two totally different sciences, mathematical and physical geometry. It was the failure to distinguish between them that produced Kant's perplexity. 'So far as the laws of mathematics refer to reality, they are not certain; and so far as they are certain, they do not refer to reality' (Einstein). Kant's credit lies in having seen that there is a problem, not in having solved it.

But here a new problem presents itself: How do we know what will satisfy a given question? More generally: How does the answer fit the question? Questions of the current sort ('What is the right time?') show already by their form what sort of answer to expect. They are, so to speak, cheques with a blank to be filled; yet not always so: Augustine's question, 'How is it possible to measure time?' or Kant's question, 'How is geometry possible?' do not trace out the form of the answer. There is no *obvious* link between question and answer, any more than there is in the case of asking 'What is a point?' When Hilbert's idea — that the axioms of geometry jointly provide the 'implicit definition' of the basic terms — was first propounded it came totally unexpected; no one had ever thought of that before; on the contrary, many people had an uneasy feeling, as if this were a way of evading the issue rather than an answer, amongst them no less a man than Frege. He thought the problem still unsolved.

Now is there anything one can do to make a man like Frege see that the axiomatic method provides the correct answer? Can it, for example, be *proved* to him? The point to which attention must now be drawn, though it should really be obvious, is that such a proof cannot be given, and it cannot because he, the asker, has first to be turned round to see the matter differently. What is required

is a change of the entire way of thinking. Indeed, anyone who is puzzled by this problem and yet refuses to accept Hilbert's solution only betrays that he has got stuck in the groove hollowed out by the form in which the question is put. 'A point is —' he begins and then stops. What is to be done to help him to get out of the groove or, better still, to make him shift for himself when he feels 'cramped' in it, is a *discussion*, not a proof.

Frege behaves not so very unlike a man mystified by the question, 'What is time?' We may suggest converting the latter into the question how the word 'time' is being used (which would bring him down to earth). But aren't we cheating him? We seem to be holding out the answer to *one* question, but not to that one which he was asking. He may suspect that we are trying to fob him off with the second best we have in store, his original question still remaining an enigma. Similarly Frege: he considered it a scandal that the questions 'What is a point?', 'What is a number?' were still unanswered.

In either of these cases, the aim of a discussion, in the absence of a proof, can only be to change the asker's attitude. We may, for instance, scrutinize similar, or partially similar, cases, point out that the form of the answer is not always that of the question; by going patiently over such cases, the vast background of analogies against which the question is seen will slowly change. The turning up of a wide field of language loosens the position of certain standards which are so ingrained that we do not see them for what they are; and if we do this in an effective manner, a mind like Frege's will be released from the obsession of seeking strainingly for an answer to fit the mould. Arguments are used in such a discussion, though not as proofs but rather as means to make him see things he had not noticed before: e.g. to dispel wrong analogies, to stress similarities with other cases and in this way to bring about something like a shift of perspective. However, there is no way of proving him wrong or bullying him into mental acceptance of the proposal: when all is said and done the decision is his.

But here more is at stake than loosening a cramped position — it is a question of escaping the domination of linguistic forms. How often are we merely following the channels carved out by numberless repetition of the same modes of expression — as when we say, unsuspectingly, 'Time flows' and are, when confronted (say) with Augustine's paradox, suddenly shocked out of complacency.

Existing language, by offering us only certain stereotyped moulds of expression, creates habits of thought which it is almost impossible to break. Such a mould is, e.g. the actor-action scheme of the Indo-European languages. How deep their influence is can perhaps be surmised from Descartes' conclusion from thinking to the presence of an agent, an ego, different from the thinking, that does the thinking — a conclusion so natural and convincing to us because it is supported by the whole weight of language. Frege's obsession with the question 'What is a number?' is another case. As we can speak of '*the* number five', five, Frege argued, must be the proper name of an entity, a sort of Platonic crystal, indicated by means of the definite article. (A Chinese pupil of mine once informed me that Frege's question is unaskable in Chinese, 'five' being used there only as a numeral in contexts like 'five friends', 'five boats', etc.) Again, when we say of a given statement that it is true, we seem to be saying something 'about' it — evidence of the power of the subject-predicate cliché. Indeed, so strong is the temptation to construe it in this way, namely, as a statement about a statement, that the idea of a different interpretation scarcely occurs to us. It is important to notice that in doing so we assimilate the expression into analogical forms; but it is no less important to notice that none of these analogies needs to be present to our minds: it is enough if they make themselves felt in a dim, inarticulated way. Such patterns have an effect on us like thousands of explicit analogies: they act upon us, one might say, like a field of force, a language field, that draws our mental gaze in a certain direction. And, I venture to add, it is precisely because of the fleeting, half-formed, shadow-like nature of these analogies that it is almost impossible to escape their influence. If we are taken in by them, it is our fault. A philosopher, instead of preaching the righteousness of ordinary speech, should learn to be on his guard against the pitfalls ever present in its forms. To use a picture: just as a good swimmer must be able to swim up-stream, so the philosopher should master the unspeakably difficult art of thinking up-speech, against the current of clichés.

Now for another point. When we dissuade a man like Frege from his search, we seem to be hindering him from reaching the aim he set out to reach. Does our discussion clash, then, with his search? And, if so, in which way? First of all, in no clearly definable way; for he is not yet clearly aware what he is aiming at, and the

discussion brings him gradually to see things in a different light. How is this change brought about? Well, he first saw the question in analogy with other ones, and these analogies are, one by one, destroyed; or rather, in the course of the discussion they are seen to be misleading. In proportion as the whole conceptual background changes, he comes to see that something is wrong with the way he put his question, that the attainment of his object is no longer satisfying. It is not that he gives up because he has tried very hard, but in vain, and has now got tired: no, he gives up because he 'sees' the question differently. And in what does *this* consist? Well, in the fact that he is now well aware of the analogies which were misleading him, that he sees the question against a different linguistic background (a 'figure' sometimes changes when it is seen against a different 'ground'), that a certain strain disappears and that he says, with a sigh of relief, 'Yes, that's it'.

The philosopher contemplates things through the prism of language and, misled (say) by some analogy, suddenly sees things in a new strange light. We can cope with these problems only by digging down to the soil from which they spring. What we do is to light up the mental background from which the question has detached itself; in a clearer perception of some of the crucial concepts the question transforms itself into another one. Not that it has been answered in the current sense. Rather we have removed the factors that prompted the question by a more profound and penetrating analysis. The essence of this process is that it leads the questioner on to some new aspect — and leads him with his spontaneous consent. He agrees to be thus led and therefore ends by abandoning his search. We cannot constrain anyone who is unwilling to follow the new direction of a question; we can only extend the field of vision of the asker, loosen his prejudices, guide his gaze in a new direction: but all this can be achieved only with his consent.

By our critical analysis we try to counteract the influence of the language field, or (what comes to the same) we may help the questioner to gain a deeper insight into the nature of what he is seeking first of all — make him see the build of the concepts and the moulds in which he expresses the question. What matters is more like changing his outlook than proving to him some theorem; or more like increasing his insight. Insight cannot be lodged in a theorem, and this is the deeper reason why the deductive

method is doomed to fail: insight cannot be demonstrated by proof.

What it comes to in the end is that the asker of the question, in the course of the discussion, has to make a number of *decisions*. And this makes the philosophical procedure so unlike a logical one. He compares, for instance, the case before him with analogous ones and has to *judge* how far these analogies hold. That is, it is for him to decide how far he is willing to accept these analogies: he has not, like a slave, to follow blindly in their track.

Science is rich in questions of this type. They are not scientific questions properly and yet they exercise scientists, they are philosophic questions and yet they do not exercise philosophers.

What I have wanted to say in this section and have not said, or only half-said:

- (1) Philosophy is not only criticism of language: so construed, its aim is too narrow. It is criticizing, dissolving and stepping over *all* prejudices, loosening all rigid and constricting moulds of thought, no matter whether they have their origin in language or somewhere else.
- (2) What is essential in philosophy is the breaking through to a *deeper insight* — which is something positive — not merely the dissipation of fog and the exposure of spurious problems.
- (3) Insight cannot be lodged in a theorem, and it can therefore not be demonstrated.
- (4) Philosophic arguments are, none of them, logically *compelling*: they really screen what actually happens — the quiet and patient undermining of categories over the whole field of thought.
- (5) Their purpose is to open our eyes, to bring us to see things in a new way — from a wider standpoint unobstructed by misunderstandings.
- (6) The essential difference between philosophy and logic is that logic *constrains* us while philosophy leaves us free: in a philosophic discussion we are led, step by step, to change our angle of vision, e.g. to pass from one way of putting a question to another, and this with our spontaneous agreement — a thing profoundly different from deducing theorems from a given set of premisses. Misquoting Cantor one might say: the essence of philosophy lies in its freedom.
4. There is a notion that philosophy is an exercise of the intellect

and that philosophic questions can be settled by argument, and conclusively if one only knew how to set about it. What seems to me queer, however, is that I cannot find any really good hard argument; and more than that, the example just discussed must make it doubtful whether any compelling argument *can* be found. Out of this plight I incline to come to a new and somewhat shocking conclusion: that the thing cannot be done. No philosopher has ever proved anything. The whole claim is spurious. What I have to say is simply this. Philosophic arguments are not deductive; therefore they are not rigorous; and therefore they don't prove anything. Yet they have force.

Before going into the matter, I want to show, quite summarily first, how implausible the view is that rigorous arguments are applied in philosophy. A first alarming sign can perhaps already be seen in the notorious fact that the ablest minds disagree, that what is indisputable to the one seems to have no force in the eyes of the other. In a clear system of thought such differences are impossible. That they exist in philosophy is weighty evidence that the arguments have none of the logical rigour they have in mathematics and the exact sciences.

Next, arguments, in the way they are thought of, must contain inferences, and inferences must start somewhere. Now where is the philosopher to look for his premisses? To science? Then he will 'do' science, not philosophy. To statements of everyday life? To particular ones? Then he will never be able to advance a single step beyond them. To general statements? If so, a number of questions raise their ugly heads. By what right does he pass from 'some' to 'all'? ('To Generalize is to be an Idiot', W. Blake.) Can he be sure that his premisses are stated with such clarity and precision that not a ghost of a doubt can creep in? Can he be sure that they contain meat, are not analytic, vacuous, definitions in disguise and the like? Can he be sure that they are true? (How *can* he?) And even supposing, what is not the case, that all these requirements could be met, there is still another task looming before him when it comes to developing the consequences: can he be sure how to operate with the terms? (How *can* he?) I am not letting out a secret when I say that the ordinary rules of logic often break down in natural speech — a fact usually hushed up by logic books. Indeed, the words of common language are so elastic that anyone can stretch their sense to fit his own whims; and with this their 'logic'

is queered. (Plenty of scope for a ‘natural logic’: ‘we know that we are *unhappy*; so we *are* unhappy. We *know* that we are unhappy; so we are *great*.’ (Pascal.) ‘If she had perished, she had perished’: does this entail that she has not perished? If so, by what rule? ‘If I believed that I should be very silly indeed’: does this, or does this not, entail that I don’t believe it? Natural language holds logical problems of its own, lots of them.)

This brings me to another point. Ordinary language simply has not got the ‘hardness’, the logical hardness, to cut axioms in it. It needs something like a metallic substance to carve a deductive system out of it such as Euclid’s. But common speech? If you begin to draw inferences it soon begins to go ‘soft’ and fluffs up somewhere. You may just as well carve cameos on a cheese *soufflé*. (My point is: language is plastic, yielding to the will to express, even at the price of some obscurity. Indeed, how could it ever express anything that does not conform to the cliché? If logicians had their way, language would become as clear and transparent as glass, but also as brittle as glass: and what would be the good of making an axe of glass that breaks the moment you use it?) But language is not hard. And that is why it is dangerous in philosophy to hunt for premisses instead of just going over the ground, standing back and saying: look.

Most philosophic arguments, to ignore constructions *à la* Spinoza, hinge on such points as what ‘can’ and what ‘cannot’ be said or what sort of question it is ‘proper’ and what sort of question it would be ‘inappropriate’ to ask. Much skill and ingenuity has been spent in elucidating such questions as to whether a certain metaphor is ‘natural’, a certain diction ‘fitting’. It would not be right to Burke the point that considerations such as these, while apparently pertaining to matters of style, contribute in fact largely to the forcefulness of an argument, indeed play a very real and decisive part in the way they make us look at the subject. In going over, examining and comparing the various modes of expression that centre around certain key notions, for instance, ‘imagination’, ‘memory’, ‘pleasure’, we catch the first glimpse of what is sometimes called the ‘logic’ of these notions. Now can any of these things be proved? Can it be proved, for example, that a certain diction is ‘fitting’? (Remember, no such thing as a definition of a ‘well-formed formula’.) No philosopher has ever made so much as an attempt. Everyone uses words in this way and he leaves it at

that; and rightly so. For what sort of reasons *could* he give anyway? Here already, at the very threshold, the idea of a philosophic proof begins to ring hollow.

'Ah, but the ordinary use of language.' All right; but even so, it is not that one 'cannot' use language differently. To illustrate: 'frozen music' — does this 'tell' you anything? Perhaps not; yet a saying like 'Architecture is frozen music' (Goethe) drives the point home. To say 'The arms are full of blunted memories' sounds odd, until you come upon it in Proust's contexts. The 'will to understand' does not even flinch before those bogies of the logician, contradictions: it transforms them, wresting a new sense from the apparent nonsense. ('Dark with excess of light', 'the luminous gloom of Plato' — just to remind the reader of two examples of Coleridge.) There are about 303 reasons why we sometimes express ourselves in a contradiction, and understandably so.

Result: it cannot even be proved that a given expression is natural, a metaphor fitting, a question proper (or unaskable), a collocation of words expressive (or devoid of meaning). Nothing of the sort can be demonstrated.

Two other points reinforce what has been said. What we sometimes do in a philosophical discussion is not argue at all but simply raise lots of questions — a method brilliantly employed by Ryle. Indeed, a volley of perplexing questions can certainly not be described as an argument and *a fortiori* not as a logical one, yet it is no less effective in making one turn back in recoil to consider one's views. Lastly, though on the surface the philosopher seems to be engaged in much the same thing as a logician is, for instance, in testing an argument for any loose links in it or in building up an argument, this should not mislead us. For if he were to construct rigorous proofs, where are the theorems established by them? What has he to show as the fruit of his labours?

I have not raised any of these questions wantonly; they force themselves on everyone who tries to arrive at a clear and unbiased view of the matter. Should these difficulties not have their origin in the nature of philosophy itself?

5. I proceed now to consider philosophic arguments, especially those which are regarded as constituting a decisive advance, to see whether they give us any reason for modifying the view advocated here. There are only a few classical cases. One of them is Hume's celebrated argument to show that the relation of cause and effect is

intrinsically different from that of ground and consequence. Now in what does this 'proof' consist? He *reminds* us of what we have always known: that, while it is self-contradictory to assert the ground and deny the consequence, no such contradiction arises in assuming that a certain event, the 'cause', may be followed not by its usual effect but by some other event. If it is asked 'Is this a proof?' what is one to say? It certainly is not the sort of proof to be found in a deductive system. Much the same applies to Berkeley's argument when he tells us that, try as he might, he cannot call up in his mind an abstract idea of a triangle, of just a triangle with no particular shape, any more than he can conceive the idea of a man without qualities. Is this a proof? He points out the obvious. (Only it wants a genius to see it.)

To take my own argument against logical fatalism, it is not strict. The decisive step consists in following a certain analogy with other cases. It is analogical, not logical. Similarly the argument used against Zeno is not conclusive. (I have no space to enlarge upon that.)

Now for two more examples, one of the current sort of argument applied today by philosophers, the other taken from Aristotle.

When we say of someone that he 'sees' or 'hears' an aeroplane, or 'descries' or 'detects' a lark in the sky, or again that he 'tastes' or 'smells' roast pork, we do not ascribe to him an activity. That 'seeing' is not a sort of doing can be illustrated, e.g. by calling attention to the fact that we don't use the continuous present tense. We say 'I see the clock', not 'I am seeing the clock' (save G. E. Moore, who, oddly enough, regularly says that he 'is seeing his right hand'), whereas it is perfectly correct to say 'I am looking at the clock, listening to its ticking', and so in the other cases. Again, while it is proper to say 'I have forgotten to post the letter', no one would say 'I have forgotten to see the letter-box'. There is no sense in asking you, when you look at me, whether your seeing is easy or difficult, quick or slowish, careful or heedless, whether you see me deliberately and whether you have now finished seeing me. So, it is argued, perceiving is not a doing (an argument used by myself in lectures).

The point to be laboured is that this argument is not conclusive. Odd as it sounds, 'I have finished seeing you' *may* be said, though only in very special circumstances. A man with impaired eyesight who, unable to take in the shape as a whole, has perhaps to scan the

face bit by bit in search of some characteristic marks might say, and understandably, 'Now I have finished seeing you'. We too are occasionally in a not much better position, as when, in magnesium light, we look at some scene, and afterwards complain, 'Too quick, I couldn't take it in'. It would seem then that there is no more than a difference in degree between this case and the normal ones. Odd cases, certainly; but what would you think of a mathematician whose theorems collapse when applied to slightly out-of-the-way curves?

For my next example I choose pleasure. Aristotle, in criticizing Plato,<sup>1</sup> pointed out that if pleasure were a process going on in time I could enjoy something swiftly or slowly — an argument which is almost a bombshell in its destructive power. Certainly, to speak in such terms is very odd and sounds absurd. Yet, if I strain my imagination, I can perhaps bring myself to conceive of a set of circumstances under which it would not be entirely unnatural to say such a thing. In listening to music, for example, when I am following a slow and gentle movement, my enjoying it appears in some respects to be different from what I get when listening to an exciting piece of music. The very quality of my enjoyment seems to change as if something of the slow and gentle or of the wild, intoxicating flow of the music had entered into it. If I say, in the one case, that I was enjoying it leisurely like basking in the sun or sipping wine, in the other that I was suddenly carried away, breathlessly following its onrush and enjoying it like a storm at sea — does this sound like sheer nonsense? So there does seem to be a time factor in pleasure.

Amongst the most powerful weapons in the philosopher's armoury are *reductio ad absurdum* and infinite regress arguments. Before proceeding to an appraisal of these forms of reasoning, it will be well to consider how they work in their home land, mathematics.

Let me choose as a typical case the proof that  $\sqrt{2}$  is irrational. If it were a rational number, we could find two integers  $m$  and  $n$  such that

$$m^2 = 2n^2 \quad (1)$$

We may then argue as follows. As  $m^2$  is even,  $m$  must be even; hence  $m = 2m_1$ . Substitution yields

$$2m_1^2 = n^2. \quad (2)$$

<sup>1</sup> Aristotle, *Nicomachean Ethics*, Bk. x, ch. 3.

As  $n^2$  is even,  $n$  must be even; hence  $n = 2n_1$ . Substitution yields

$$m_1^2 = 2n_1^2. \quad (3)$$

If, then, two integers  $m$  and  $n$  exist which stand in the relation (1), they must have halves which stand in exactly the same relation (3), and these must have halves which stand in the same relation, and so on *ad infinitum*; which is plainly impossible,  $m$  and  $n$  being finite. Therefore the tentative assumption (1) cannot hold, and  $\sqrt{2}$  cannot be rational. Q.E.D. This is the prototype of a refutation by infinite regress.

Arguments of this type have been applied outside mathematics. However, when I come to look at them a bit more closely I begin to hesitate. An example will illustrate my doubts. An argument propounded against the use of mechanical models is this. If the elastic properties of matter can be explained as being due to electric forces with which the molecules act on each other, it surely is pointless to explain the action of the electric forces as being due to the elastic properties of a mechanical medium, the 'ether'. To do this is to go round in a circle: elasticity is explained in terms of electric force, and electric force in terms of elasticity; while the attempt to break out of the circle by supposing that the elasticity of the ether is due to 'electric forces' acting between the ether particles and these to the elastic properties of a second-order ether is to be pushed into an infinite series of reduction steps. Thus the mechanistic programme is faced with a dilemma both horns of which are equally fatal.

A formidable argument — or is it? I can well imagine an undaunted champion of the lost cause retort: 'Not a bit of a regress. Yes, the ether is elastic, not, however, in the sense in which a spring is: while elasticity of matter can be reduced to electric force, elasticity of the ether, being an ultimate postulate of the theory, cannot be reduced any further.' And with this the argument falls to the ground.

But this is unconvincing, it will be said. I agree; I am not such an imbecile as to plead for retaining mechanical models and the rest. My point is only to see whether this 'refutation' is compelling. *It isn't.* The advocate of models is not forcibly dislodged from his position. There is, it would seem, always a way of getting out of the dilemma — of wriggling out if you like — which foils the argument. What is shown in it is merely that to cling to models of this sort becomes, in the circumstances, very unnatural. But to say that

something is unnatural is not to say that it is logically impossible: yet this is what the argument should establish. In the mathematical proof cited above no loophole was left for wriggling out. The whole deduction was a 'chain of adamant' — precisely the sort of thing the argument under review is not.

Consider now a similar argument. There cannot be any such thing as volitions, it has been said. Volitions were called in by theorists to provide causes not only for what we (intentionally) do but also for mental processes or operations such as controlling an impulse, paying heed to something, and the like. As a consequence of this, acts of will were supposed to be the sort of thing the presence of which makes an action 'voluntary', or which — somehow, in some unfathomable way — 'gets itself translated' into a bodily or mental act. In fine, volitions were thought of as causes as well as effects of other, mental or physical, occurrences. Now the dilemma: if my pulling of the trigger were the result of a mental act of 'willing to pull the trigger', what of this mental act itself? Was it willed or unwilled? If unwilled, it cannot be called voluntary and therefore not a volition; if willed, then we must suppose, according to the theory, that it results from a prior act, namely, 'willing to will to pull the trigger', and that from another, *ad infinitum*, leaving no possibility for me ever to start.

Brilliant as the argument is, the point to be brought up here is only whether it is logically fatal. Does it really prove that the assumption of acts of willing involves an infinite regress? A believer in such acts need not be cowed into submission. To ask of volitions whether they are themselves voluntary or involuntary acts, he may say, is plain nonsense. Only an *action* can be voluntary or involuntary, not an act of will. It is just the point that an act of will is an act of will and does not issue from any anterior act of will, any more than, in order to recall a thing I must first recall what I want to recall, and before I can even do that I must recall that I want to recall what I want to recall, and so on *ad infinitum*. Just as I can recall a thing without need to call in an act of recalling what I want to recall, so my pulling the trigger may be the direct result of an act of will without the latter issuing from a parent act of will. Thus the whole argument apparently crumbles away.

This is meant not to belittle the argument or detract from its force, but only to get clear as to *what sort* of force it has. If it were conclusive, it would, with its destructive power, do away with a

good many more acts and states of mind, not only with volitions — with intending and desiring, for instance. Indeed, precisely similar arguments can be constructed ‘to deal with them’. Intention: though clearly not the sort of thing to be classed as a simple ‘act’, it yet seems somehow to ‘connect’ with what goes on in us before we carry it into action — such as considering, planning, hesitating, choosing. I may, let us say, intend to find a flaw in a given argument, and when I subsequently turn it over in my mind, this will be the result of my intention. Some mental operations, then, *can* arise from an intention, they are ‘intended’. So what of the intention itself? Is it intended or unintended? If the intention is not intended, it is not the intention, and if it is intended it must be due to another intention, and this to yet another *ad infinitum*. Similarly in the case of desire. Suppose that I feel a desire for a certain thing, is this desire itself desired or undesired? Either answer lands us in absurdities.

If the strength of the argument were to lie in its structure it would, with its devastating effect, apply after the exchange of some of its terms for other ones, e.g. ‘volition’ for ‘intention’ — provided, of course, that certain other circumstances essential to the reasoning are the same. Yet, while the first argument sounds, to say the least, very plausible, no one will be duped by its caricatures. So if it *has* any force it cannot owe it to its structure and consequently cannot be of a logical sort. It is meant to refute the existence of a kind of mental thrust; but then we should remember that to prove the non-existence of something is always a precarious business. ‘No one has ever proved the non-existence of Apollo or Aphrodite’ it has been observed; too much weight, then, need perhaps not be laid on this particular case. What is disturbing, however, is the ease with which arguments can be cast into pseudo-deductive moulds. And it is this fact to which I wish to call attention by examining the argument. As has been shown in the preceding discussion, it is not an isolated case. No philosophic argument ends with a Q.E.D. However forceful, it never forces. There is no bullying in philosophy, neither with the stick of logic nor with the stick of language.

6. In throwing such strong doubts on the power of arguments as used by philosophers I may seem to deny them any value whatever. But such is not my intention. Even if they are lacking in logical rigour this certainly has not prevented an original thinker from

using them successfully, or from bringing out something not seen before or not seen so clearly. So in the case I have discussed: something *is* seen in that argument, something *is* made clear, though perhaps not quite in the sense intended by the arguer. If so, something very important has been left out from the picture.

Perhaps our objections have been doing injustice to philosophic arguments. They were, quite mistakenly as I hope to have shown, supposed to be proofs and refutations in a strict sense. But what the philosopher does is something else. *He builds up a case.* First, he makes you see all the weaknesses, disadvantages, shortcomings of a position; he brings to light inconsistencies in it or points out how unnatural some of the ideas underlying the whole theory are by pushing them to their farthest consequences; and this he does with the strongest weapons in his arsenal, reduction to absurdity and infinite regress. On the other hand, he offers you a new way of looking at things not exposed to those objections. In other words, he submits to you, like a barrister, all the facts of his case, and you are in the position of a judge. You look at them carefully, go into the details, weigh the pros and cons and arrive at a verdict. But in arriving at a verdict you are not following a deductive highway, any more than a judge in the High Court does. Coming to a decision, though a rational process, is very unlike drawing conclusions from given premisses, just as it is very unlike doing sums. A judge has to judge, we say, implying that he has to use discernment in contrast to applying, machine-like, a set of mechanical rules. There are no computing machines for doing the judge's work nor could there be any — a trivial yet significant fact. When the judge reaches a decision this may be, and in fact often is, a rational result, yet not one obtained by deduction; it does not simply follow from such-and-such: what is required is insight, judgement. Now, in arriving at a verdict, you are like a judge in that you are not carrying out a number of formal logical steps: you have to use discernment, e.g. to descry the pivotal point. Considerations such as these make us see what is already apparent in the use of 'rational', that this term has a wider range of application than what can be established deductively. To say that an argument can be rational and yet not deductive is not a sort of contradiction as it would inevitably be in the opposite case, namely, of saying that a deductive argument need not be rational.

This alters the whole picture. The point to be emphasized is that

a philosopher may see an important truth and yet be unable to demonstrate it by formal proof. But the fact that his arguments are not logical does nothing to detract from their rationality. To return to our previous example, the argument used against volition, though it is not what it professes to be, logically destructive, nevertheless has a force difficult to resist. Now to what is this due? It does not need much acumen to find the answer. It is the whole arrangement of so many felicitous examples, preceding the argument, and their masterly analysis, which breathes life into its bare bones; aided greatly by the fact that the connection between a mental thrust and a bodily movement is allowed to remain a mystery. The unsatisfactoriness of this position, together with the amassing of hosts of unanswerable questions and very striking examples — this makes the argument so convincing.

What do you find in reading Ryle or Wittgenstein? Lots of examples with little or no logical bone in between. Why so many examples? They speak for themselves; they are usually more transparent than the trouble maker; each one acts as an analogy; together they light up the whole linguistic background with the effect that the case before us is seen in the light they produce. Indeed, examples aptly arranged are often more convincing and, above all, of a more lasting effect than an argument which is anyhow spidery. Not that the ‘proofs’ proffered are valueless: a *reductio ad absurdum* always points to a knot in thought, and so does an infinite regress. But they *point* only. The real strength lies in the examples. All the proofs, in a good book on philosophy, could be dispensed with, without its losing a whit of its convincingness. To seek, in philosophy, for rigorous proofs is to seek for the shadow of one’s voice.

In order to forestall misinterpretations which will otherwise certainly arise I have to concede one point: arguments on a small scale, containing a few logical steps only, may be rigorous. The substance of my remarks is that the conception of a whole philosophical view — from Heraclitus to Nietzsche or Bradley — is never a matter of logical steps. A *weltanschauung* like any of these or even a new approach like that of Wittgenstein is never ‘arrived at’, in particular it is not deduced, and once found it can neither be proved nor refuted by strictly logical reasoning; though arguments may play a part in making them acceptable. But some authors have disdained even that.

The one remaining question to be asked is this: if the philosopher's views cannot be derived from any premisses, how has he ever arrived at them? How can he get to a place to which no road is leading? This leads to a new and deeper problem.

7. To ask, 'What is your aim in philosophy?' and to reply, 'To show the fly the way out of the fly-bottle' is . . . well, honour where it is due, I suppress what I was going to say; except perhaps this. There is something deeply exciting about philosophy, a fact not intelligible on such a negative account. It is not a matter of 'clarifying thoughts' nor of 'the correct use of language' nor of any other of these damned things. What is it? Philosophy is many things and there is no formula to cover them all. But if I were asked to express in one single word what is its most essential feature I would unhesitatingly say: vision. At the heart of any philosophy worth the name is vision, and it is from there it springs and takes its visible shape. When I say 'vision' I mean it: I do not want to romanticize. What is characteristic of philosophy is the piercing of that dead crust of tradition and convention, the breaking of those fetters which bind us to inherited preconceptions, so as to attain a new and broader way of looking at things. It has always been felt that philosophy should reveal to us what is hidden. (I am not quite insensitive to the dangers of such a view.) Yet from Plato to Moore and Wittgenstein every great philosopher was led by a sense of vision: without it no one could have given a new direction to human thought or opened windows into the not-yet-seen. Though he may be a good technician, he will not leave his marks on the history of ideas. What is decisive is a new way of seeing and, what goes with it, the will to transform the whole intellectual scene. This is the real thing and everything else is subservient to it.

Suppose that a man revolts against accepted opinion, that he feels 'cramped' in its categories; a time may come when he believes, rightly or wrongly, that he has freed himself of these notions; when he has that sense of sudden growth in looking back at the prejudices which held him captive; or a time when he believes, rightly or wrongly, that he has reached a vantage point from which things can be seen to be arranged in clear and orderly patterns while difficulties of long standing dissolve as though by magic. If he is of a philosophic cast of mind he will argue this out with himself and then, perhaps, try to impart what has dawned on him to others. The arguments he will offer, the attacks he will make, the sugges-

tions he will advance are all devised for one end: to win other people over to his own way of looking at things, to change the whole climate of opinion. Though to an outsider he appears to advance all sorts of arguments, this is not the decisive point. What is decisive is that he has seen things from a new angle of vision. Compared to that everything else is secondary. Arguments come only afterwards to lend support to what he has seen. 'Big words, not every philosopher, etc.': but where should one get one's bearings if not from the masters? And besides, once tradition has given way there is always ample scope for specialists to reduce some 'pockets of resistance'. Unpalatable though it may be, behind the arguments so well-planned, so neat and logical, something else is at work, a will to transform the entire way of thinking. In arguing for his view, the philosopher will, almost against his will, have to undermine current categories and clichés of thinking by exposing the fallacies which underly the established views he is attacking; and not only this, he may go so far as to question the canons of satisfactoriness themselves. In this sense, philosophy is the re-testing of the standards. In every philosopher lives something of the reformer. That is the reason why any advance in science when it touches the standards is felt to be of philosophic significance, from Galileo to Einstein and Heisenberg.

If there is any truth in this, the relation of logic and philosophy appears in a new light. What is at issue is not a conflict between a formal and a less formal or informal logic, nor between the behaviour of technical and everyday concepts, but something radically different. It is the difference between drawing a conclusion and seeing, or making one see, a new aspect.

To put the matter in a nutshell, a philosophic argument does more and does less than a logical one: less in that it never establishes anything conclusively; more in that, if successful, it is not content to establish just one isolated point of truth, but effects a change in our whole mental outlook so that, as a result of that, myriads of such little points are brought into view or turned out of sight, as the case may be. Are illustrations necessary? Once Hume had exposed the fallacies of his predecessors when dealing with the notion of causality he had made it impossible for anyone to think along the lines of Spinoza whose world looks to us as strange as the moon. Suppose that you look at a picture-puzzle: at first you can see in it only a maze of lines; then, suddenly, you recognize

a human face. Can you now, having discovered the face, see the lines as before? Clearly not. As with the maze of lines, so with the muddle cleared up by Hume: to recapture the mood of the past, to travel back into the fog has become impossible — one of the big difficulties of understanding the history of philosophy. It is for the same reason that the rise of the linguistic technique in our day has put an end to the great speculative systems of the past.

A philosophy is an attempt to unfreeze habits of thinking, to replace them by less stiff and restricting ones. Of course, these may in time themselves harden, with the result that they clog progress: Kant, the *Alleszermalmer* to his contemporaries, yet proudly upholding his table of categories — which appear to as unduly narrow. The liberator of yesterday may turn into the tyrant of tomorrow.

It can now be seen that the philosopher is not doing what the logician does, only less competently, but doing something altogether different. A philosophic argument is not an *approximation* of a logical one nor is the latter the ideal the philosopher is striving for. Such an account totally misdescribes what really takes place. Philosophy is not an exercise in formal logic, philosophic arguments are not chains of logical inference, only bungled ones, nor can they by any effort be recast into deductive moulds. What is being confused here is the scientist's aim to find new truths and the philosopher's aim to gain insight. As the two things are so entirely out of scale it is small wonder that the philosopher cannot move in the logician's armour. Not even if the logician himself is fighting the battle. The clash over the law of excluded middle in mathematics is a clash between two parties, each in possession of clear and precisely defined concepts. Yet there seems to be no way of settling the dispute by cogent argument. If it were true that philosophical troubles arise from the loose nature of our everyday concepts, why should such conflicts break out in the exactest of the sciences?

There have never been any absolutely cogent reasons for parting with the law of excluded middle, accepting Darwinism, giving up the Ptolemaic system or renouncing the principle of causality. If any of these things could be demonstrated how does it come that there are always partisans of the 'lost causes'? Are they like the unlucky circle-squarers, wasting their time in trying to do what has been shown to be logically impossible? The truth is that

conflicts of this type cannot be resolved, not entirely, either by adducing factual evidence or by logical demonstration. Both sides, of course, bring up arguments in the combat but they are not decisive. These are battles never lost and never won irrevocably. It is a typical situation, a recurrent theme in the history of human thought.

Whenever science arrives at a crucial stage where the fundamental notions become uncertain and are held as it were in solution, disputes of an odd kind are breaking out. The mere fact that leading scientists, in spite of differences in temperament, outlook, etc, take part in them, feel bound to do so, should make us reflect. Now what the protagonists avowedly or unavowedly are trying to do is to win their fellow scientists over to their own way of thinking; and to the degree to which their arguments are attempts at changing the whole intellectual attitude they take on a philosophical character. Is this coincidence?

8. I have so far spoken of 'seeing a new aspect' without making an attempt to explain the term. I hope now to do so, though only perfunctorily, by giving one or two illustrations. There is a sort of paradox connected with the idea of certain discoveries. Descartes, for instance, was the discoverer of analytic geometry. But could he seek for it? To say that he spent years looking for it sounds downright absurd. What we are inclined to say in such a case is: to seek for analytic geometry is not possible — first because it was not seen and then because it was seen. But if he could not seek, how could he find? This leads us straight to the heart of the matter.

Consider first an entirely imaginary case. In the propositional calculus, as it was built up by Frege, two primitive ideas occur, 'not' and 'or'. It was later discovered by Sheffer that the whole calculus can be based on one single idea (his 'stroke' function). Of what kind was this discovery? Suppose that Frege, by a curious chance, had written all his logical axioms in the form

$$\sim(\dots) \vee \sim(\dots)$$

i.e. as a sum of two negations, but had none the less mistakenly believed that *two* symbols were required for expressing these laws, namely ' $\sim$ ' and ' $\vee$ '. Imagine now that someone else looking at these formulae is struck by what, on our assumption, has escaped Frege, namely that they all have one and the same structure and require therefore only one symbol. In what exactly does his

discovery consist? In his *seeing* the formulae in a new way, in his reading a new structure into them. What matters is his apprehension: so long as he does not see the structure of a new system in the old one he has not got it. Anyone may look at the formulae and yet not perceive what Sheffer has perceived, the occurrence of an identical structure. *This* is the discovery, not the introducing of a special symbol for a combination of the old ones. It would have been quite enough, for instance, had Sheffer merely pointed out the constant recurrence of this structure in all the laws without providing his 'stroke'; that is inessential.

This example may illustrate what is meant by the 'seeing of a new aspect'. Seeing such an aspect is often the core of a new discovery. If you look at the formulae, the moment you notice the new structure in them they suddenly seem to change — a phenomenon akin to seeing a figure, say, a drawn cube differently, now as solid and protruding, now as hollow and receding. The one pattern suddenly 'jumps' into the other. Similarly in our case, though there are also differences; thus the new aspect, once it has dawned, can steadily be held in mind and has not that perceptual instability. The apprehension of a new pattern in the formulae seems to hold in it actually more of a visual experience, anyhow to be more closely akin to it than it might at first appear. Seeing and interpreting, looking and thinking seem as it were to fuse here.

If it is now asked whether it is possible for anyone to *seek* for the new aspect, what is one to reply? Well, that something *can* be seen in a new way is seen only when it *is* seen in this way. That an aspect is possible is seen only when the aspect has already flashed and not before: that's why the finding cannot be anticipated, not even by the greatest genius. It always comes unbidden and, as it would seem, in a sudden flash.

To take another case, is the calculation

$$(5 + 3)^2 = 5^2 + 2 \cdot 5 \cdot 3 + 3^2$$

at the same time a proof that

$$(2 + 3)^2 = 2^2 + 2 \cdot 2 \cdot 3 + 3^2?$$

Yes and no — depending on how you look at it. (Does it strike you that the 2 in the middle term is a 'structural' 2, deriving not from the special numbers but from the general form of the operation?) A man, while reckoning with special numbers only,

may yet conceivably do algebra if he sees the special sums in a new way, as the expressions of a general law. (Discovery of algebra as the discovery of an aspect of numerical calculation.)

What goes for these more or less trivial cases goes for Descartes and also for Einstein and Hilbert. They were unable to seek, Einstein for a conceptual gap in the idea of simultaneity, Hilbert for the axiomatic method. Though these discoveries are of a different order altogether, the principle underlying them is the same. None of them has ever 'arrived' at his view because he was never travelling. They did not seek, they found (like Picasso). And that is so wrong with the whole way in which such discoveries are so often presented — as if they were the result of a 'method' or 'procedure', as if the great men arrived at their solutions by drawing logical inferences. This leaves out the most essential thing — the flashing of a new aspect which is *non-inferential*. The moments of seeing cannot be foreseen, any more than they can be planned, forced, controlled, or summoned by will-power.

Is there any truth in what I am saying? I shall not argue. Instead, let me remind you of some observations which will be familiar to you. It is notorious that a philosophy is not made, it grows. You don't choose a puzzle, you are shocked into it. Whoever has pondered some time over some dark problem in philosophy will have noticed that the solution, when it comes, comes with a suddenness. It is not through working very hard towards it that it is found. What happens is rather that he suddenly sees things in a new light — as if a veil had been lifted that screened his view, or as if the scales had fallen from his eyes, leaving him surprised at his own stupidity not to have seen what was there quite plain before him all the time. It is less like finding out something and more like maturing, outgrowing preconceived notions.

To give just one example of vision in philosophy: Wittgenstein saw through a big mistake of his time. It was then held by most philosophers that the nature of such things as hoping and fearing, or intending, meaning and understanding could be discovered through introspection, while others, in particular psychologists, sought to arrive at an answer by experiment, having only obscure notions as to what their results meant. Wittgenstein changed the whole approach by saying: what these words mean shows itself in the way they are used — the nature of understanding reveals itself in grammar, not in experiment. This was at the time

quite a revelation and came to him, so far as I remember, suddenly.

The view advocated here is that at the living centre of every philosophy is a vision and that it should be judged accordingly. The really important questions to be discussed in the history of philosophy are not whether Leibniz or Kant were consistent in arguing as they did but rather what lies behind the systems they have built. And here I want to end with a few words on metaphysics.

To say that metaphysics is nonsense *is* nonsense. It fails to acknowledge the enormous part played at least in the past by those systems. Why this is so, why they should have such a hold over the human mind I shall not undertake here to discuss. Metaphysicians, like artists, are the antennae of their time: they have a flair for feeling which way the spirit is moving. (There is a Rilke poem about it.<sup>1</sup>) There is something visionary about great metaphysicians as if they had the power to see beyond the horizons of their time. Take, for instance, Descartes's work. That it has given rise to endless metaphysical quibbles is certainly a thing to hold against it. Yet if we attend to the spirit rather than to the words I am greatly inclined to say that there is a certain grandeur in it, a prophetic aspect of the comprehensibility of nature, a bold anticipation of what has been achieved in science at a much later date. The true successors of Descartes were those who translated the spirit of this philosophy into deeds, not Spinoza or Malebranche but Newton and the mathematical description of nature. To go on with some hairsplitting as to what substance is and how it should be defined was to miss the message. It was a colossal mistake. A philosophy is there to be lived out. What goes into the word dies, what goes into the work lives.

<sup>1</sup> This probably refers to *Die Sonette an Orpheus*, Part 1, no. 12 (Ed.).

## CHAPTER II

### VERIFIABILITY

Mr MacKinnon's paper<sup>1</sup> raises a number of important questions. In the short time that is at my disposal I can only focus on two or three of them, and this merely in a sketchy way.

1. In the first part of his paper Mr MacKinnon is concerned with representing and criticizing a thesis which he calls 'evidentialism' and which, in his view, is 'the fundamental presupposition' of empiricism. He formulates it thus:

'There is no more to the content of a statement than the total evidence which would warrant its assertion.'

Well, I don't know whether there is any empiricist who has ever expressed such a view; but it should be said clearly that though it seems to me to be some generalized version of phenomenism it is not the view of a more advanced type of empiricism, and I would certainly not accept it as such. As the question involves a point of very great importance, I shall try to state the case as clearly and as briefly as I can.

When we reflect on such a sentence as 'The meaning of a statement is the method of its verification', we should, first of all, be quite clear as to what we mean by the term 'method of verification'. From a logical point of view we are not interested in the various activities that are involved in verifying a statement. What, then, is it we have in mind when we talk of such things? Take an example. Suppose there is a metal ball in front of me, and I have the task of finding out whether the ball is charged with electricity. To do that I connect the ball with an electroscope and watch whether the gold leaves diverge. The statement 'The gold leaves of the instrument diverge' (*s*) describes the verification of the statement 'The ball is charged' (*p*). Now what exactly am I doing when I describe the verification of the statement *p*? I establish a connection between two statements by declaring that the one (*s*) is to follow from the other (*p*). In other words, I lay

<sup>1</sup> D. M. MacKinnon, *Proc. Aris. Soc.*, Suppl. vol. xix (1945), pp. 101-18.

down a *rule of inference* which allows me to pass from the statement 'The ball is charged with electricity' to another that describes an observable situation. By doing this I connect the statement with another one, I make it part of a system of operations, I incorporate it into language, in short, *I determine the way it is to be used*. In this sense giving the verification of a statement is an important part of giving its use, or, to put it differently, explaining its verification is a contribution to its grammar.

In everyday life we understand sentences without bothering much as to the way they are verified. We understand them because we understand the single words which occur in them and grasp the grammatical structure of the sentence as a whole. The question of the verification arises only when we come across a new sort of combination of words. If, for instance, someone were to tell us that he owned a dog that was able to think, we should at first not quite understand what he was talking about and ask him some further questions. Suppose he described to us in detail the dog's behaviour in certain circumstances, then we should say 'Ah, now we understand you, that's what you call thinking'. There is no need to enquire into the verification of such sentences as 'The dog barks', 'He runs', 'He is playful', and so on, as the words are then used as we may say in their *normal* way. But when we say 'The dog thinks', we create a new context, we step outside the boundaries of common speech, and then the question arises as to what is meant by such a word series. In such cases explaining the verification is explaining the meaning, and changing the verification is changing the meaning. Obviously meaning and verification *are connected* — so why say they are not?

But when I say that the statement *p* is connected with the statements *s<sub>1</sub>*, *s<sub>2</sub>*, . . . *s<sub>n</sub>* which describe evidences for it, I do *not* say that *p* is *identical* with *s<sub>1</sub>*, *s<sub>2</sub>*, . . . *s<sub>n</sub>* or their conjunction. To say this would only be true if *s<sub>1</sub>*, *s<sub>2</sub>*, . . . *s<sub>n</sub>* or their conjunction entailed *p*. Now is that so? There *may* be statements which are nothing more than abbreviations for all that which is unfolded in their verification. There are, however, other sorts of statements of which this is certainly not true. Recent discussions on phenomenism, e.g., tend to shew that no conjunction or disjunction of sense datum statements, however complex, entails the existence or the non-existence of a certain material object. If that is so, a material object statement, though it *is* connected with sense datum

statements, is not just an abbreviation for them, rather has it a logical status of its own, and is not equivalent to any truth-function of the latter ones. I think that the result of these discussions is essentially right, and I ask for permission, to make my point quite clear, to add one word more.

The failure of the phenomenalist to translate a material object statement into terms of sense data is not, as has been suggested, due to the poverty of our language which lacks the vocabulary for describing all the minute details of sense experience, nor is it due to the difficulties inherent in producing an *infinite* combination of sense datum statements, though all these things may contribute to it. In the main it is due to a factor which, though it is very important and really quite obvious, has to my knowledge never been noticed — to the 'open texture'<sup>1</sup> of most of our empirical concepts. What I mean is this: Suppose I have to verify a statement such as 'There is a cat next door'; suppose I go over to the next room, open the door, look into it and actually see a cat. Is this enough to prove my statement? Or must I, in addition to it, touch the cat, pat him and induce him to purr? And supposing that I had done all these things, can I then be absolutely certain that my statement was true? Instantly we come up against the well-known battery of sceptical arguments mustered since ancient times. What, for instance, should I say when that creature later on grew to a gigantic size? Or if it showed some queer behaviour usually not to be found with cats, say, if under certain conditions, it could be revived from death whereas normal cats could not? Shall I, in such a case say that a new species has come into being? Or that it was a cat with extraordinary properties? Again, suppose I say 'There is my friend over there'. What if on drawing closer in order to shake hands with him he suddenly disappeared? 'Therefore it was not my friend but some delusion or other'. But suppose a few seconds later I saw him again, could grasp his hand, etc. What then? 'Therefore your friend was nevertheless there and his disappearance was some delusion or other'. But imagine after a while he disappeared again, or seemed to disappear — what shall I say now? Have we rules ready for all imaginable possibilities?

An example of the first sort tends to show that we can think of situations in which we couldn't be certain whether something

<sup>1</sup> I owe this term to Mr Kneale, who suggested it to me as a translation of *Porosität der Begriffe*, a term coined by me in German.

was a cat or some other animal (or a jinni). An example of the second sort tends to show that we can consider circumstances in which we couldn't be certain whether something was real or a delusion. The fact that in many cases, there is no such thing as a conclusive verification is connected with the fact that most of our empirical concepts are not delimited in all possible directions. Suppose I come across a being that looks like a man, speaks like a man, behaves like a man, and is only one span tall — shall I say it *is* a man? Or what about the case of a person who is so old as to remember King Darius? Would you say he is an immortal? Is there anything like an exhaustive definition that, finally and once for all sets our mind at rest? 'But are there not exact definitions at least in science?' Let's see. The notion of gold seems to be defined with absolute precision, say by the spectrum of gold with its characteristic lines. Now what would you say if a substance was discovered that looked like gold, satisfied all the chemical tests for gold, whilst it emitted a new sort of radiation? 'But such things do not happen.' Quite so; but they *might* happen, and that is enough to show that we can never exclude altogether the possibility of some unforeseen situation arising in which we shall have to modify our definition. Try as we may, no concept is limited in such a way that there is no room for any doubt. We introduce a concept and limit it in *some* directions; for instance, we define gold in contrast to some other metals such as alloys. This suffices for our present needs, and we do not probe any farther. We tend to *overlook* the fact that there are always other directions in which the concept has not been defined. And if we did, we could easily imagine conditions which would necessitate new limitations. In short, it is not possible to define a concept like gold with absolute precision, i.e. in such a way that every nook and cranny is blocked against entry of doubt. That is what is meant by the open texture of a concept.

*Vagueness* should be distinguished from *open texture*. A word which is actually used in a fluctuating way (such as 'heap' or 'pink') is said to be vague; a term like 'gold', though its actual use may not be vague, is non-exhaustive or of an open texture in that we can never fill up all the possible gaps through which a doubt may seep in. Open texture, then is something like *possibility of vagueness*. Vagueness can be remedied by giving more accurate rules, open texture cannot. An alternative way of stating this would be to say that definitions of open terms are *always* corrigible or emendable.

Open texture is a very fundamental characteristic of most, though not of all, empirical concepts, and it is this texture which prevents us from verifying conclusively most of our empirical statements. Take any material object statement. The terms which occur in it are non-exhaustive; that means that we cannot foresee completely all possible conditions in which they are to be used; there will always remain a possibility, however faint, that we have not taken into account something or other that may be relevant to their usage; and that means that we cannot foresee completely all the possible circumstances in which the statement is true or in which it is false. There will always remain a margin of uncertainty. Thus the absence of a conclusive verification is directly due to the open texture of the terms concerned.

This has an important consequence. Phenomenalists have tried to translate what we mean by a material object statement into terms of sense experience. Now such a translation would be possible only if the terms of a material object statement were completely definable. For only then could we describe completely all the possible evidences which would make the statement true or false. As this condition is not fulfilled, the programme of phenomenism falls flat, and in consequence the attempts at analysing chairs and tables into patterns of sense data — which has become something of a national sport in this country — are doomed to fail. Similar remarks apply to certain psychological statements such as 'He is an intelligent person'; here again it is due to the open texture of a term like 'intelligent' that the statement cannot be reduced to a conjunction or disjunction of statements which specify the way a man would behave in such-and-such circumstances.

It may have been a dim awareness of this fact that induced Locke to insist on corporeal, and Berkeley on mental substance. Doing away with their metaphysical fog, we may restate what seems to be the grain of truth in their views by saying that a material object statement or a psychological statement has a logic of its own, and for this reason cannot be reduced to the level of other statements.

But there is a deeper reason for all that, and this consists in what I venture to call the *essential incompleteness* of an empirical description. To explain more fully: If I had to describe the right hand of mine which I am now holding up, I may say different things of it: I may state its size, its shape, its colour, its tissue, the chemical compound of its bones, its cells, and perhaps add some

more particulars; but however far I go, I shall never reach a point where my description will be completed: logically speaking, it is always possible to extend the description by adding some detail or other. Every description stretches, as it were, into a horizon of open possibilities: however far I go, I shall always carry this horizon with me. Contrast this case with others in which completeness is attainable. If, in geometry, I describe a triangle, e.g., by giving its three sides, the description is *complete*: nothing can be added to it that is not included in, or at variance with, the data. Again, there is a sense in which it may be said that a melody is described completely in the musical notation (disregarding, for the moment, the question of its interpretation); a figure on a carpet, viewed as an ornament, may be described in some geometrical notation; and in this case, too, there is a sense in which the description may be called complete. (I do not mean the *physical* carpet, but its pattern.) The same applies to a game of chess: it can be described, move by move, from the beginning to the end. Such cases serve merely to set off the nature of an empirical description by the contrast: there is no such thing as completeness in the case in which I describe my right hand, or the character of a person; I can never exhaust all the details nor foresee all possible circumstances which would make me modify or retract my statement. (This was already seen by Leibniz when he said that anything actual is always inexhaustible in its properties and a true image of the Infinite Mind.)

The situation described has a direct bearing on the open texture of concepts. A term is defined when the sort of situation is described in which it is to be used. Suppose for a moment that we were able to describe situations completely without omitting anything (as in chess), then we could produce an exhaustive list of all the circumstances in which the term is to be used so that nothing is left to doubt; in other words, we could construct a *complete definition*, i.e. a thought model which anticipates and settles once for all every possible question of usage. As, in fact, we can never eliminate the possibility of some unforeseen factor emerging, we can never be quite sure that we have included in our definition everything that should be included, and thus the process of defining and refining an idea will go on without ever reaching a final stage. In other words, every definition stretches into an open horizon. Try as we may, the situation will always remain the same: no definition of an empirical term will cover all possibilities. Thus

the result is that the incompleteness of our verification is rooted in the incompleteness of the definition of the terms involved, and the incompleteness of the definition is rooted in the incompleteness of empirical description; that is one of the grounds why a material object statement  $p$  can *not* be verified conclusively, nor be resolved into statements  $s_1, s_2, \dots, s_n$  which describe evidences for it. (In mathematics such a reduction is often possible: thus a statement about rational numbers *can*, without loss of meaning, be translated into statements about integers; but here you have complete description, complete definition and conclusive proof and refutation.)

Why is it, then, that, as a rule, an experiential statement is not verifiable in a conclusive way? Is it because I can never exhaust the description of a material object or of a situation, since I may always add something to it — something that, in principle, can be foreseen? Or is it because something quite new and unforeseen may occur? In the first case, though I know all the tests, I may still be unable to perform them, say, for lack of time. In the second case I can not even be sure that I know all the tests that may be required; in other words, the difficulty is to state completely what a verification would be like in this case. (Can you foresee all circumstances which would turn a putative fact into a delusion?) Now the answer to the question is that *both factors combine* to prevent a verification from being conclusive. *But they play a very different part.* It is due to the first factor that, in verifying a statement, we can never finish the job. But it is the second that is responsible for the open texture of our terms which is so characteristic of all factual knowledge. To see this more clearly, compare the situation in mathematics: here a theorem, say Goldbach's hypothesis, may be undecidable as we cannot go through all the integers in order to try it out. But this in no way detracts from the *closed* texture of the mathematical concepts. If there was no such thing as the (always present) possibility of the emergence of something new, there could be nothing like the open texture of concepts; and if there was no such thing as the open texture of concepts, verification would be incomplete only in the sense that it could never be finished (just as in the case of Goldbach).

To sum up: An experiential statement is, as a rule, not conclusively verifiable for two different reasons:

- (i) because of the existence of an unlimited number of tests;
- (ii) because of the open texture of the terms involved.

These two reasons correspond to two different senses of 'incompleteness'. The first is related to the fact that I can never conclude the description of a material object, or of a situation. I may, for instance, look at my table from always new points in space without ever exhausting all the possibilities. The second (and more exciting one) is due to the fact that our factual knowledge is incomplete in another dimension: there is always a chance that something unforeseen may occur. That again may mean two different things:

- (a) that I should get acquainted with some totally new experience such as, at present, I cannot even imagine;
- (b) that some new discovery was made which would affect our whole interpretation of certain facts.

An illustration of the first sort would be supplied by a man born blind who later obtained the experience of seeing. An illustration of the second sort would be the change brought about by the discovery of a new agent of nature, such as electricity. In this case we perceive that the data of observation are connected in a new and unforeseen way, that, as it were, new lines can now be traced through the field of experience. So we can say more exactly that the open texture of concepts is rooted in that particular incompleteness of our factual knowledge which I have just adumbrated.

What I have said modifies to a greater or lesser extent the account I have given of verification. I said that in giving the method of verification we lay down a rule (or rules) of inference. We should, however, feel grave doubts whether that is so. If a material object statement were to entail a sense datum statement, to entail it in a strictly *logical* sense, then the premiss would be cancelled together with the conclusion: or, to put it differently, a single negative instance would suffice to refute the premiss. Suppose someone told me, 'Look, there is your friend, he is just crossing the street'. Now if I looked in the direction indicated, but failed to perceive the person who is my friend, would I say that the statement was refuted beyond the shadow of a doubt? There may be cases in which I may say that. But there are others in which I would certainly not think that the statement was refuted on the strength of such a single glance; (for instance, when I was led to expect my friend at this hour, or received a letter from him saying that he will arrive at that time, and the like). A discrepancy between a

material object statement and a single sense experience may always be explained away by some accessory assumption: I haven't looked thoroughly, my friend happened in this very second to be behind someone else, he just stepped into a doorway, and so on, not to mention more fanciful theories. I can never exclude the possibility that, though the evidence was against it, the statement may be true.

Whoever considers these facts with unbiassed eyes, will, I trust, assent to the conclusion that a single sense experience, strictly speaking, never excludes a material object statement in the sense in which the negation of  $p$  excludes  $p$ . That means that no sense datum statement  $s$  can ever come into *sharp logical conflict* with a material object statement  $p$ ; in other words:  $p \cdot \sim s$  never represents a *contradiction* in the sense that  $p \cdot \sim p$  does. In the light of this we can no longer adhere to the view that  $p$  entails  $s$ . How, then, should we formulate the 'method of verification' — that is, the connection between a proposition  $p$  and the statements  $s_1, s_2, \dots, s_n$  which are evidences for it? I propose to say that the evidences  $s_1, s_2, \dots, s_n$  speak for or against the proposition  $p$ , that they strengthen or weaken it, which does not mean that they prove or disprove it strictly.

There is a striking analogy to that in the relation that holds between a law of nature  $L$  and certain observational statements  $s_1, s_2, \dots, s_n$ , an analogy which may help to clarify the situation. It is often said that the statements of observation follow from the law (the latter being regarded as a sort of universal premiss). Since an unlimited number of consequences can be derived from a law, the ideal of complete verification is, of course, unattainable; whereas, on the other hand, a single counter-observation seems to suffice to overthrow the law. From this it would follow that, while a law cannot be strictly verified, it can be strictly confuted; or that it can be decided only one way.<sup>1</sup> That is unrealistic. What astronomer would abandon Kepler's laws on the strength of a single observation? If, in fact, some anomaly in a planet's behaviour were detected, the most varied attempts at explaining the phenomenon would first be made (such as the presence of unknown heavy masses, friction with rarefied gases, etc.). Only if the edifice of hypotheses thus erected has too little support in experience, if it

<sup>1</sup> See Karl Popper, *Logik der Forschung*, Vienna, 1935; translated as *The Logic of Scientific Discovery*, London, 1959.

becomes too complex and artificial, if it no longer satisfies our demand for simplicity, or again if a better hypothesis presents itself to us such as Einstein's theory, would we resolve to drop those laws. And even then the refutation would not be valid finally and once for all: it may still turn out that some circumstance had escaped our notice which, when taken into consideration, would cast a different light upon the whole. Indeed, the history of science exhibits cases (Olaf Römer, Leverrier) in which the apparent defeat of a theory later turned into complete victory. Who can say that such a situation will not repeat itself?

Here again the view suggests itself strongly that the relationship between a statement and what serves to verify it was too crudely represented in the past; that it was a mistake to describe it in logical terms such as 'entailment'; that a law is not a sort of universal statement from which particular statements follow; that its logic is still unexplored, and that it may possibly take the form of rules according to which the law's truth-weight — if I am allowed to use such a term — is increased or lessened by the data of observation. Be that as it may, the mere fact that a single counter observation  $\sim s$  can always be reconciled with a general law  $L$  by some accessory assumption shows that the true relation between a law and the experiential evidence for it is much more complicated and only superficially in accord with the customary account.

It will be said that this is due to our representing the case in too simple a manner. In reality the observational statement  $s$  does not follow from  $L$  alone, but from  $L$  plus a number of further premisses which are often not expressly stated. So that, if the observation  $s$  which we expected fails to materialize, we may say that any of the other premisses is false.

Now this would be perfectly correct if the system of premisses could be stated accurately and completely in every single case. But can it? Can we ever be certain of knowing all, really all the conditions on which the result of even the simplest experiment depends? Plainly not; what is stated is only a *part* of the conditions, viz., those which, e.g., can be isolated in experimental technique and subjected to our will, or which can readily be surveyed, etc. The others merge into one indistinct mass: the vague supposition that 'a normal situation subsists', that 'no disturbing factors are present' or in whatever way we may hint at the possibility of

intervention of some unforeseen conditions. The relation between  $L$  and  $s$ , then, when exactly stated, is this: Given such-and-such laws  $L_1, L_2, \dots, L_m$ , given such-and-such initial and boundary conditions  $c_1, c_2 \dots c_n$  and *no other disturbing factors being present*, so-and-so will happen. And here it must be stressed that behind the words italicized a presupposition is concealed which cannot be split up into clear, separate statements. When actually deducing a consequence from a physical law we never make use of this premiss: it never forms part of the body of premisses; it does not enter the process of deduction. But then it should not be termed a premiss at all; what queer sort of premiss that is which is never made use of! What is, in fact, conveyed by these words is only that, in case of a conflict between theory and observation, we shall *search* for disturbing factors whilst considering ourselves free to adhere to the theory. The question at issue is *not* whether a certain system of assumptions is sufficiently comprehensive — that is a question of fact which may be left to the expert; the question in which we take an interest is rather whether there is a *criterion* which assures us that a system of premisses is complete. To this there is no answer; nay, more, we cannot even form any conception of such a criterion; we cannot think of a situation in which a physicist would tell us, 'Well, I have finished the job; now I have discovered the last law of nature, and no more is to be found'. But if this is devoid of meaning, there is no point in insisting, '*If* all the conditions in the universe, and *if* all the laws governing them were known to us, then —'. As the boundary regions of our knowledge are always enveloped in a dust cloud — out of which something new may emerge — we are left with the fact that  $s$  is not a strict logical consequence of  $L$  together with the initial conditions. Saying that the class of premisses is not 'closed' and that *therefore* the conclusion is lacking in stringency comes, in my view, to the same thing as saying that  $s$  is *not* a logical consequence of the premisses as far as they are stated. And that is all I wanted to say.

All this tends to suggest that the relation between a law of nature and the evidences for it, or between a material object statement and a sense datum statement, or again between a psychological statement and the evidence concerning a person's behaviour is a looser one than had been hitherto imagined. If that is correct, the application of logic is limited in an important sense. We may say that the known relations of logic can only hold between statements

which belong to a *homogeneous* domain; or that the deductive nexus never extends beyond the limits of such a domain.

Accordingly we may set ourselves the task of arranging the statements of our language in distinct strata, grouping in the same stratum all those statements linked by clearly apprehended logical relations. It is in this way, for instance, that the theorems of mechanics are organized in a system the elements of which stand in known logical relations with one another and where it is always possible to decide of two theorems in what logical relation they stand — whether one is a consequence of the other, whether they are equivalent, independent of, or in contradiction with each other. In like manner the statements of a physicist in describing certain data of observation (such as the position of a pointer on his gauges) stand in exactly defined relations to one another. Thus a pointer on a scale cannot possibly be opposite 3 and 5 at the same time: here you have a relation of strict exclusion. On the other hand, no statement of mechanics can ever come into sharp logical conflict with a statement of observation, and this implies that between these two kinds of statements there exist no relations of the sort supplied to us by classical logic. So long as we move only among the statements of a single stratum, all the relations provided by logic remain valid. The real problem arises where two such strata make contact, so to speak; it is the problem of these planes of contact which today should claim the attention of the logician. We may, in this context, speak of the *open texture* of the chains of inference (*poröse schlüsse*) which lead from statements of one stratum to those of another; the connection is no longer coercive — owing to the incompleteness of all our data.

You will find that it is this fact to which the rise of philosophical troubles can often be traced. (Think of how confusing it is to assert or to dispute the statement, 'The floor is not solid', as it belongs to two quite distinct strata.) The fracture lines of the strata of language are marked by philosophical problems; the problem of perception, of verification, of induction, the problem of the relation between mind and body, and so on.

You will have noticed that I have used the term 'incompleteness' in very different senses. In one sense we may say of a description of a material object that it is incomplete; in another sense we may say that of our knowledge of the boundary conditions in a field of force. There is a sense in which we say that a list of laws of nature

is always incomplete, and another sense in which even our knowledge of the agents of nature is so; and you may easily find more senses. They all combine, to a varying degree, to create what I have called the open texture of concepts and the looseness of inferences.

Incompleteness, in the senses referred to, is the mark of empirical knowledge as opposed to *a priori* knowledge such as mathematics. In fact, it is the criterion by which we can distinguish perfectly *formalized* languages constructed by logicians from *natural* languages as used in describing reality. In a formalized system the use of each symbol is governed by a definite finite number of rules, and further, all the rules of inference and procedure are stated completely. In view of the incompleteness which permeates empirical knowledge such a demand cannot be fulfilled by any language we may use to express it.

I have tried to give an account of the method of verification, and this has led me to touch on such subjects as open texture, completeness, language strata and logical relations of an open texture. In this account, though I have nowhere departed from the lines of empiricism, I have arrived at an outlook which is rather different from that which Mr MacKinnon presents. I am very puzzled why Mr MacKinnon should think that empiricism is based on the 'fundamental presupposition' that 'there is no more to the content of a statement than the total evidence which would warrant its assertion'. That there is a very close relation between content and verification is an important insight which has been brought to light by empiricists. Only one has to be very careful how to formulate it. Far from identifying the meaning of a statement with the evidences we have for it, the view I tried to sketch leads to a sort of many-level theory of language in which 'every sort of statement has its own sort of logic'.

Mr MacKinnon, like so many other philosophers, talks of the 'verification principle'. I will not let pass this occasion without saying how misleading this mode of expression is. It evokes at once the idea that here we have a principle which is the foundation from which a part or the whole of empiricism can be derived. Now a principle is something that serves as a starting point in a deductive theory. It is in this sense that we speak, e.g., of the Principles of Mechanics. Mr MacKinnon does seem to take it in this sense, for he calls it a 'fundamental presupposition'. It should, however, have

become quite clear that empiricism is not a deductive system based on the verification principle or on some other principle. It is, in fact, not a body of propositions at all, but rather a critical attitude, and what is called the 'verification principle' is no more than a maxim devised to guide our philosophical activity in clarifying the content of a statement.

I shall not enter into a discussion on what Mr MacKinnon says about 'verificatory acts'. I shall confine myself to saying that I entirely fail to see why there should be any difference in kind between a verification made by a physicist in his laboratory and a verification made by an unsophisticated person. Suppose a physicist engaged in directing some intricate experiment is too busy to attend to certain observations; so he trains a young boy to look through a telescope and to tell him in certain intervals whether he sees light or dark. Now when the boy calls out 'Light', why, I ask, is this not as good a verification (or a 'verificatory act') as if Einstein himself had looked through it? We may, indeed, think of the process of verification as divided up amongst different people without impairing in the least the validity of the verification. But perhaps I have not correctly understood Mr MacKinnon.

2. In the second part of his paper Mr MacKinnon is anxious to relate the notions of reality and causality by admitting as real only such objects (or events, or processes) which satisfy the conditions of causality. What he says is 'that the manner of discursive thought . . . reveals itself as an obstinate resolve . . . to admit nothing as real that does not manifest some ground of its occurrence'. That is part of Kant's doctrine according to which nothing can ever become object of our knowledge which did not conform to certain *a priori* forms of our intuition and our understanding. Such an attempt, if it succeeded, would be of tremendous importance. Think how miraculous it would be, using this method, to deduce from it causality, premisses of induction as well as other enjoyable things — I had almost said to produce them out of the conjurer's hat called the Transcendental Argument. How comforting would be the belief that we know the nature of space and time through and through so that we are able to enunciate the principles of geometry without fear of ever being defeated by experience. How reassuring it would be to say that nature *must* obey causal laws — and so on, you know the tune. The question is only whether Nature will conform to Kant. You will realize that

such over-confidence is no longer permissible today, in the age of quantum mechanics. We are told by Mr MacKinnon that we display 'an unwillingness to admit the completely random' (by the bye, what does he mean by that?) 'and discontinuous as objectively real'. But our protest, however strongly worded, would be of no avail if Nature was willing to baffle us. The words Mr MacKinnon has been using state precisely the sort of situation with which we have come face to face in modern physics: things do happen without ground of their occurrence. May I be allowed to say a few words on this subject?

There are people who think that physicists have *just not succeeded* in discovering laws which tell us why things happen in the atomic world, in the cheerful hope that someone some day will have a brain-wave which will enable him to fill the gaps in wave mechanics; on this day the latter will turn into a completely deterministic theory. Let these people realise how wide the cleavage is that separates us from the good old days. The hope they cherish is based on an illusion: it has been proved<sup>1</sup> that the structure of quantum mechanics is such that no further laws can be added to it which would make the whole theory deterministic; for if we could, we should, owing to the uncertainty principle, get entangled in contradictions. (The situation is, in fact, more intricate, but this is not the place to go into it.) So we are faced with the dilemma that quantum mechanics is *either* self-consistent *or* deterministic: you can't have it both ways. The crack in the wall of Determinism is definitive, and there is no way out of the situation.

According to Kant causality is an inescapable form which the nature of our understanding imposes on any given material. If this were so, it would be inconceivable — against the conditions of possible experience — ever to come across any events which did not conform to the principle of causality. Quantum phenomena, however, have forced physicists to depart from this principle, or better, to *restrict* it, whilst a torso of it is retained. Though the fate of a single electron is not governed by causal laws, the particle being free to move about, for instance, to 'jump' in a collision with light waves however it pleases, the behaviour of millions of electrons is statistically predictable. Not exactly that quantum

<sup>1</sup> See, for instance, J. v. Neumann, *Mathematische Grundlagen der Quantenmechanik*, Berlin, 1932; translated as *Mathematical Foundations of Quantum Mechanics*, Princeton, 1955.

mechanics confronts us with a mathematician's dream of chaos come true. For, as I said, there is a causal aspect in the new theory, namely this: there are certain waves connected with the motion of particles, the de Broglie waves, which obey rigorous 'causal' laws. That is, the propagation of these waves is governed by a differential equation of the respectable old type such as you find in the classical physics of fields. Hence we can, given the initial conditions and the values over the boundary of a region during a certain interval of time, predict with absolute precision the propagation of the waves. That is exactly what any causal theory achieves. What is new, however, is the interpretation we must give to these waves: they are a sort of 'probability clouds' the density of which at each point signifies the probability of the occurrence of a particle. So what we can deduce from a theory are only *probability statements* regarding the presence of a particle in a given place at a given time. Such a statement can be tested, not by making a single experiment such as observing a single electron through a microscope, but by repeating the experiment a large number of times, or observing a large number of electrons and forming the mean value of all the data thus obtained. Thus we cannot say where exactly a certain electron will be, but only with what probability, i.e., in what percentage of cases we may expect to find it at a certain place. In other words, the theory can be used only to predict the *average behaviour* of particles. That is the statistical aspect of the theory.

To sum up: quantum mechanics is neither a theory of the causal, deterministic type nor an indeterministic theory, whatever this may be taken to mean. The new physics combines deterministic and indeterministic features. What is deterministic is the law for the propagation of the de Broglie waves. That is, the propagation of these waves is *causally determined* in much the same way as, e.g., the propagation of electromagnetic waves is in the classical theories. What is indeterministic is the *interpretation* of these waves, that is, their connection with the facts of observation, i.e., with those quantities whose value can be specified exactly. Such an interpretation can only be given in statistical terms, and any attempt at interpreting it differently so as to reinstate causality would lead only to conflict with other well-established parts of the theory. Thus we have the curious result that causality holds for the de Broglie waves which are no more than a purely symbolic and

formal representation of certain probabilities whereas the particles themselves obey no causal laws.

To bring home the last point let me add this: If it were possible to repeat exactly the same experiment and to bring about exactly the same conditions, the result would each time be a different one. Therefore the principle 'Like causes — like effects' no longer holds.

But may not quantum mechanics one day be superseded by a better theory that meets our demand for causal explanation? Certainly; no theory is sacrosanct and infallible. This, however, is not the point. What matters is, not whether quantum mechanics draws a true picture of reality, but only whether it draws a *permissible* one. About that there can be little doubt. Kant was of the opinion that if there was no such thing as causality science would simply break down. Now the important thing that has emerged is the *possibility* of constructing a theory along different lines, the *legitimacy* of departing from causality, while science has not died or committed suicide on that account. This suffices to disown any claim on the part of Kant to regard causality as an *indispensable* form of our knowledge of the world. Had he been right, we could not even *entertain* such views as physicists do today; to give up causality, even if in part, would mean to rob us of the very condition for gaining knowledge; which could end in one result only, in complete confusion. But that is not so. Though causality has been severely limited, quantum mechanics is a useful tool. Kant did not foresee the possible forms of physical laws; by laying too much stress on the scheme of causality, by claiming for it an *a priori* status, he unduly narrowed the field of research.

The conclusion to be drawn for the preceding seems to me this: Even if quantum mechanics should one day be found wanting and be superseded by another theory, it still offers a *possible picture* of the material world. This picture is neither self-contradictory nor unintelligible though it may not be the sort of picture to which we are accustomed; anyhow it is a working hypothesis which serves its purpose in that it is fruitful, i.e., that it leads to new discoveries. Whether it contains the ultimate truth we cannot tell (nor can we in the case of the deterministic theories). It's only experience that can bring forward evidence against it. But the very fact that we *can* turn to experience is significant: in doing so we grant that quantum mechanics, and consequently the limits of causality, *can*

be tested in experiment. Hence every attempt at raising the principle of causality to the status of a necessary *a priori* truth is irreconcilable with the situation as it has emerged in science. No matter whether quantum mechanics will stand its ground or will have to undergo some modification or other, the mere fact that the construction of such a theory is legitimate should settle the dispute: it proves that Kant's argument is based on a fallacy.

It was an important step when man learnt to ask the question, Why? But it was also a great step when he learnt to drop this question.

But leaving quantum mechanics and turning to the common world of sense, I still fail to see any ground for accepting Kant's position. True, in order to orient ourselves in the world we must presuppose that there is some sort of order in it so that we may anticipate the course of events and act accordingly. What I fail to see, however, is why this order should be a strictly *causal* one. Suppose, for the sake of argument, that the objects around us were, *on the average*, to display an orderly behaviour, then the world may still be a liveable place. Suppose for instance, the behaviour of chairs and tables, and the support they give us, could be foreseen with much the same accuracy as can the behaviour of Tory and Labour candidates in election times, may we then not make use of them just the same? Or suppose they were to conduct themselves as our best friends do — they won't let us down, no; still you never know — then, as far as I can see, this would be quite enough for all our practical ends. And as to the theoretical ones — well, go to the scientist and you will hear a sorry tale of nature's trickery. I cannot see — nor has Mr MacKinnon's paper helped me on this point — why such a world should not be possible.

This brings me to the topic in which Mr MacKinnon is so much interested — are there any *necessary* conditions which must be fulfilled if we are to attain knowledge of the external world? I propose to drop for the moment the subject of causality and to tackle the problem from a broader angle. Let me begin with some observations on the terms 'reality' and 'knowledge'.

Mr MacKinnon, in his paper, repeatedly speaks of 'the real', 'the reality'; he asks, for instance, whether 'the completely random' can be admitted as 'objectively real'. He blames Berkeley for having omitted 'to face the question of the rules whereby the inclusion in or exclusion from reality was determined in consequence of

which', we are told, 'his theory of knowledge flags'. In another passage he speaks of 'the task of compelling the actual to disclose itself'. My impression is that he talks as if there was a clearly bounded domain called 'the real' or 'the actual' with the implication that it is one of the tasks of the philosopher to define it sharply. Unfortunately the belief that there is such a domain is very slender. Not that I deny for a minute that a word like 'reality' is a blessing; it definitely is. Look at such phrases as 'A tautology doesn't say anything about reality', 'Pure mathematics is not concerned with reality', 'In reality it was not Smith I saw but his brother.' It would be silly to put such a word on an *Index Prohibitorum Verborum* as though it were a sin to use it. It is very handy — if it were not in use, we should have to invent it. On the other hand, when a philosopher looks closely at it, tears it from the context and asks himself, 'Now what *is* reality?' he has successfully manoeuvred himself into a fairly awkward position. For it is surprisingly easy to ask a number of questions which are more or less embarrassing; for instance, 'Is the elastic force present in a spring something real?' I suppose some people would answer 'Yes', some 'No'. The fact is that there simply are no fixed rules that govern the use of the word. To go on — 'Is a magnetic field something real?' 'Is energy? and entropy?' 'Are the de Broglie waves which, in one aspect, constitute the nature of material particles real, or mere symbolic constructions?' Again, I may ask, 'Is the power of my memory real?', 'Is the genius of a people, is the spirit of an age, is the beauty of a spring day real?' Now we begin to see how the idea is lost in indeterminacy. What we must understand is that such a word is used on many different levels and with many different shades of meaning. It has a *systematic ambiguity*. At the same time there is a sort of family likeness between all these uses, and it is that which makes us denote them by one word.

The same applies to a verb like 'to exist'. We use the word in many different senses; we may, for instance, say of a memory picture, an after-image, a mirror image, or again of a material object that it 'exists'; again, we may say of a wave-motion in a space of many dimensions, or of a law of nature, or of a number satisfying certain conditions that it 'exists'; and it is quite obvious that we do use the word in each case according to totally different criteria. So again we have a case of systematic ambiguity.

Next take the term 'knowledge'. Everyone is familiar with the distinction between knowledge by acquaintance and knowledge by description. This division is not fine enough. When I know something by acquaintance, I may know it in very different senses as when I say 'I know sweetness' (meaning 'I am acquainted with this taste'), 'I know misery', 'I know him', 'I know his writings'. In this series we go progressively farther away from simple acquaintance. In a case like 'I know his motives', it is doubtful whether I should say this unless I had experienced some such motive myself. Moreover, there are cases which fall under none of the two groups; as, for instance, when I say 'I know French', 'I know how to deal with that man'. Again, we may speak in different senses of knowledge by description. Compare the case of a reporter who gained knowledge of some hush-hush affair with that of a scientist who claims to possess knowledge of nature. Now ask, Is this knowledge in the same sense? And mark, in the latter case there are again subtle differences. Compare knowledge of the history of certain birds as based on observation with knowledge of the history of our solar system as based on some hypothesis; again knowledge of a natural law of the causal type with knowledge of a statistical law. Quantum mechanics, though it is based on the assumption of a randomness in the behaviour of electrons (and other particles), leads to a lot of predictions. On this ground physicists do not hesitate to honour the newly discovered laws by awarding them the degree of knowledge; whereas Mr MacKinnon thinks 'that we do concede the title unintelligible to any field . . . where such [causal] lines have not been traced'. Well, I shall not argue about that; my sole object is to call attention to the fact that the actual usage is unsettled, that there are many different types of knowledge and that, by talking of knowledge *in general*, we are liable to overlook the very important differences between them. Suppose that someone has a vague awareness of the direction in which history moves — shall or shall I not call this knowledge? Can you draw a clear line to mark where such vague awareness ends and where true knowledge begins? Knowledge as supplied by quantum mechanics was unknown two or three decades ago. Who can tell what forms of knowledge may emerge in the future? Can you anticipate all possible cases in which we may wish to use that term? To say that knowledge is embodied in true propositions does not get you any farther; for there are many different structures that are

called 'propositions' — different, because they are verified in different senses of the word and governed by different sets of logical rules. (Incidentally speaking, the failure to draw a clear line between the meaningful and the meaningless is due to the fact that these terms have themselves a systematic ambiguity, and so has the term 'verifiable').)

There is a group of words such as 'fact', 'event', 'situation', 'case', 'circumstance', which display a queer sort of behaviour. One might say of such words that they serve as pegs: it's marvellous what a lot of things you can put on them ('the fact that —'). So far they are very handy; but as soon as one focuses on them and asks, e.g., 'What is a fact?' they betray a tendency to melt away. The peg-aspect is by far the most important of all. It's just as in the case of the word 'reality': in reality, e.g., 'in reality' is an adverb.

Again, there are many different types of fact; there are many different types of statement which are called 'empirical'; there are many different things which are called 'experience'; and there are many different senses of communication and clarity.

Now if I am to contribute to the main subject of this symposium — that is, to the question whether there are any *necessary conditions* for *gaining knowledge of reality* — what am I to reply? Knowledge of reality! Of *what* sort of reality, and *what* sort of knowledge? As a logician I am bound to say that the notions of reality and knowledge have a systematic ambiguity and, moreover, that they are on each level extremely vague and hazy. I am even not quite clear as to what a condition is, let alone a 'necessary condition'. How questionable all these ideas are! How can I be expected to answer a question which consists only of a series of question marks?

3. So far my criticism has been mainly negative. In conclusion I should like to offer some constructive suggestions. Before doing so, I must warn you that I can't see any ground whatever for renouncing one of the most fundamental rights of man, the right to talk nonsense. And now I suppose I may go on.

People are inclined to think that there is a world of facts as opposed to a world of words which describe these facts. I am not too happy about that. Consider an example. We are accustomed to see colour as a 'quality' of objects. That is, colour cannot subsist by itself, but must inhere in a thing. This conception springs from

the way we express ourselves. When colour is rendered by an adjective, colour is conceived as an attribute of things, i.e., as something that can have no independent existence. That, however, is not the only way of conceiving colour. There are languages such as Russian, German, Italian, which render colour by means of verbs. If we were to imitate this usage in English by allowing some such form as 'The sky blues', we should come face to face with the question, Do I mean the same fact when I say 'The sky blues' as when I say 'The sky is blue'? I don't think so. We say 'The sun shines', 'Jewels glitter', 'The river shimmers', 'Windows gleam', 'Stars twinkle', etc.; that is, in the case of phenomena of lustre we make use of a verbal mode of expression. Now in rendering colour phenomena by verbs we assimilate them more closely to the phenomena of lustre; and in doing so we alter not only our manner of speaking but our entire way of apprehending colour. We *see* the blue differently now — a hint that language affects our whole mode of apprehension. In the word 'blueing' we are clearly aware of an active, verbal element. On that account 'being blue' is not quite equivalent to 'blueing', since it lacks what is peculiar to the verbal mode of expression. The sky which 'blues' is seen as something that continually brings forth blueness — it radiates blueness, so to speak; blue does not inhere in it as a mere quality, rather is it felt as the vital pulse of the sky; there is a faint suggestion of the operating of some force behind the phenomenon. It's hard to get the feel of it in English; perhaps it may help you to liken this mode of expression to the impressionist way of painting which is at bottom a new way of seeing: the impressionist sees in colour an immediate manifestation of reality, a free agent no longer bound up with things.

There are, then, different linguistic means of rendering colour. When this is done by means of adjectives, colour is conceived as an attribute of things. The learning of such a language involves for everyone who speaks it his being habituated to see colour as a 'quality' of objects. This conception becomes thus incorporated into his picture of the world. The verbal mode of expression detaches colour from things: it enables us to see colour as a phenomenon with a life of its own. Adjective and verb thus represent two different worlds of thought.

There is also an adverbial way of rendering colour. Imagine a language with a wealth of expressions for all shades of lustre, but

without adjectives for colours; colours, as a rule, are ignored; when they are expressed, this is done by adding an adverb to the word that specifies the sort of lustre. Thus the people who use this sort of language would say, 'The sea is glittering golden in the sunshine', 'The evening clouds glow redly', 'There in the depth a shadow greenly gleams'. In such phrases colour would lose the last trace of independence and be reduced to a mere modification of lustre. Just as we in our language cannot say 'That's very', but only some such thing as 'That's very brilliant', so in the language considered we could not say 'That's bluish', but only, e.g., 'That's shining bluishly'. There can be little doubt that, owing to this circumstance, the users of such language would find it very hard to see colour as a quality of things. For them it would not be the *things* that are coloured, rather colour would reside in the lustre as it glows and darkens and changes — evidence that they would see the world with different eyes.

'But isn't it still true to say that I have the same experience whenever I look up at the sky?' You would be less happy if you were asked, 'Do you have the same experience when you look at a picture puzzle and see a figure in it as before, when you didn't see it?' You may perhaps say you see the same lines, though each time in a different arrangement. Now what exactly corresponds to this different arrangement in the case when I look up at the sky? One might say: we are aware of the blue, but this awareness is itself tinged and coloured by the whole linguistic background which brings into prominence or weakens and hides certain analogies. In this sense language does affect the whole manner in which we become aware of a fact: the fact articulates itself differently, so to speak. In urging that you *must* have the same experience whenever you look at the sky you forget that the term 'experience' is itself ambiguous: whether it is taken to, e.g., include or to exclude all the various analogies which a certain mode of expression calls up.

Again, consider this case: Suppose there is a number of languages *A*, *B*, *C*, . . . in each of which a proposition is used according to a slightly different logic. Consequently a proposition in the language *A* is not a proposition in exactly the same sense as a proposition in the language *B*, etc. And not only this: what is described by a statement in the language *A*, i.e., if you like, the 'fact', is not a fact in the same sense as a fact described in the

language *B*, etc.; which tends to show that what is called a fact depends on the linguistic medium through which we see it.

I have observed that when the clock strikes in the night and I, already half asleep, am too tired to count the strokes, I am seized by an impression that the sequence will never end — as though it would go on, stroke after stroke, in an unending measureless procession. The whole thing vanishes as soon as I *count*. Counting frees me, as it were, from the dark formlessness impending over me. (Is this not a parable of the rational?) It seems to me that one could say here that counting *alters* the quality of the experience. Now is it the same fact which I perceive when counting and when not counting?

Again suppose there is a tribe whose members count 'one, two, three, a few, many'. Suppose a man of this tribe looking at a flock of birds said 'A few birds' whereas I should say 'Five birds' — is it the same fact for him as it is for me? If in such a case I pass to a language of a different structure, I can no longer describe 'the same' fact, but only another one more or less resembling the first. What, then, is the objective reality supposed to be described by language?

What rebels in us against such a suggestion is the feeling that the fact is there objectively no matter in which way we render it. I perceive something that exists and put it into words. From this it seems to follow that a fact is something that exists independent of and prior to language; language merely serves the end of communication. What we are liable to overlook here is that the way we see a fact — i.e., what we emphasize and what we disregard — is *our* work. 'The sun-beams trembling on the floating tides' (Pope). Here a fact is something that emerges out from, and takes shape against, a background. The background may be, e.g., my visual field; something that rouses my attention detaches itself from this field, is brought into focus and apprehended linguistically; that is what we call a fact. A fact is noticed; and by being noticed it becomes a fact. 'Was it then no fact before you noticed it?' It was, if I *could* have noticed it. In a language in which there is only the number series 'one, two, three, a few, many', a fact such as 'There are five birds' is imperceptible.

To make my meaning still clearer consider a language in which description does not take the form of sentences. Examples of such a description would be supplied by a map, a picture language, a film, the musical notation. A map, for instance, should not be

taken as a conjunction of single statements each of which describes a separate fact. For what, would you say, is the contour of a fact? Where does the one end and the other begin? If we think of such types of description, we are no longer tempted to say that a country, or a story told in a film, or a melody must consist of 'facts'. Here we begin to see how confusing the idea is according to which the world is a cluster of facts — just as if it were a sort of mosaic made up of little coloured stones. Reality is undivided. What we may have in mind is perhaps that *language* contains units, viz. *sentences*. In describing reality, describing it in the form of sentences, we draw, as it were, lines through it, limit a part and call what corresponds with such a sentence a fact. In other words, language is the knife with which we cut out facts. (This account is over simplified as it doesn't take notice of *false* statements.) When we pass to a symbolism of language that admits of no sentences, we are no more inclined to speak of facts.

Reality, then, is not made up of facts in the sense in which a plant is made up of cells, a house of bricks, a stone of molecules; rather, if you want a simile, a fact is present in much the same sense in which a character manifests itself in a face. Not that I invent the character and read it into the face; no, the character is somehow written on the face but no one would on that account say that a face is 'made up' of features symbolic of such-and-such traits. Just as we have to interpret a face, so we have to interpret reality. The elements of such an interpretation, without our being aware of it, are already present in language — for instance, in such moulds as the notion of thinghood, of causality, of number, or again in the way we render colour, etc.

Noticing a fact may be likened to seeing a face in a cloud, or a figure in an arrangement of dots, or suddenly becoming aware of the solution of a picture puzzle: one views a complex of elements as one reads a sort of unity into it, etc. Language supplies us with a means of comprehending and categorizing; and different languages categorize differently.

'But surely noticing a face in a cloud is not inventing it?' Certainly not; only you might not have noticed it unless you had already had the experience of human faces somewhere else. Does this not throw a light on what constitutes the noticing of facts? I would not dream for a moment of saying that I *invent* them; I might, however, be unable to perceive them if I had not certain

moulds of comprehension ready at hand. These forms I borrow from language. Language, then, *contributes to the formation and participates in the constitution* of a fact; which, of course, does not mean that it *produces* the fact.

So far I have dealt with perceptual situations only. This, I am afraid, will not satisfy Mr MacKinnon. What he wants to know is whether there are any *general* conditions of the possibility of factual knowledge. We have seen some of the fallacies involved in putting this question. Still we may ask ourselves whether there are any methodological rules which guide us in gaining knowledge. All I can hope to do here is to throw out some hints.

The empiricist has a let-the-facts-speak-for-themselves attitude. Well, this is his faith; what about his works? Remember, a scientific theory is never a slavish imitation of certain features of reality; a dead, passive replica. It is essentially a *construction* which to a greater or lesser degree reflects our own activity. When, for instance, we represent a number of observations made in a laboratory by a corresponding number of dots and connect them by a graph, we assume, as a rule, that the curve is continuous and analytic. Such an assumption goes far beyond any possible experience. There will always be infinitely many other possible curves which accord with the facts equally well; the totality of these curves is included within a certain narrow strip. The ordinary mathematical treatment substitutes an exact law for the blurred data of observation and deduces from such laws strict mathematical conclusions. This shows that there is an element of convention inherent in the formulation of a law. The way we single out one particular law from infinitely many possible ones shows that in our theoretical construction of reality we are guided by certain principles — *regulative principles* as we may call them. If I were asked what these principles are, I should tentatively list the following:

- (1) Simplicity or economy — the demand that the laws should be as simple as possible.
- (2) Demands suggested by the requirements of the symbolism we use — for instance, that the graph should represent an analytic function so as to lend itself readily to the carrying out of certain mathematical operations such as differentiation.

- (3) Aesthetic principles ('mathematical harmony' as envisaged by Pythagoras, Kepler, Einstein) though it is difficult to say what they are.
- (4) A principle which so regulates the formation of our concepts that as many alternatives as possible become decidable. This tendency is embodied in the structure of Aristotelian logic, especially in the law of excluded middle.<sup>1</sup>
- (5) There is a further factor elusive and most difficult to pin down: a mere tone of thought which, though not explicitly stated, permeates the air of a historical period and inspires its leading figures. It is a sort of field organizing and directing the ideas of an age. (The time from Descartes to Newton, for instance, was animated by an instinctive belief in an Order of Things accessible to the human mind. Though the thinkers of that time have tried to render this tone of thought into a rationalistic system, they failed: for that which is the living spark of rationalism is itself irrational.)

Such, I think, are some of the regulative principles. The formulation of some of them is very vague, and advisedly so: it wouldn't be good policy to reduce mathematical harmony, consonance with the whole background of an age, etc., to fixed rules. It's better to have them elastic. Principle (5) should perhaps better be described as a condition for making — and missing — discoveries.

Now none of these principles is *indispensable*, imposed on us by the nature of our understanding. Kant has tried to condense the tone of thought of the Newtonian age into strict rules — into *necessary conditions* of factual knowledge; with what success can be seen from the subsequent development: the belief in synthetic *a priori* judgments has soon become something of a brake to research, discouraging such lines of approach as non-Euclidean geometry, and later non-causal laws in physics. Let this be a warning.

Writers on the history of philosophy are inclined to attend too exclusively to one aspect only — to the ideas explicitly stated, canvassing their fabric, but disregarding the tone of thought which gives them their impetus. The deeper significance of rationalism,

<sup>1</sup> A more detailed account of this will be given in Chapter III, p. 84, pp. 89–90.

for instance, lies in the fact that it corresponds to what the scientist *does*, strengthening his belief that, if he only tries hard, he *can* get to the bottom of things. But slowly and gradually the mental climate changes, and then a philosophy may find itself out of tune with its time.

I do not think for a minute that what I have said is a conclusive refutation of Kant. On the other hand — you may confute and kill a scientific theory; a philosophy dies only of old age.

## CHAPTER III

### ARE THERE ALTERNATIVE LOGICS?

**I. PRELIMINARY REMARKS.** The writers of text-books of logic commonly take it for granted that there is a single theory, a set of rules embodying what are called the 'laws of thought'; and that these laws are universally the same. This conception seems scarcely to accord with the present level of knowledge. For we do already possess distinct logics — if this term is used to denote precisely elaborated formalized systems: e.g., logics including or excluding a Theory of Types, systems admitting or barring the law of excluded middle, etc. Perhaps one might add that the rise of a conventionalistic mode of thinking — emanating from mathematics — today favours attempts to construct novel logics. Here two ways present themselves.

The one proceeds from science. Already in the discussion of the antinomies it became manifest that there are in principle different ways of eliminating them, and so building up logic in different directions.<sup>1</sup>

More radical is the criticism directed by the intuitionists against the foundations of logic itself, which has led to the development of a propositional calculus that neither contains the law of excluded middle nor admits of its subsequent insertion.<sup>2</sup> Moreover, an impetus to the building up of yet a third and different logic came from quantum mechanics. This logic, for instance, no longer admits the conclusion from the absurdity of  $p \cdot \sim q$  that  $q$

<sup>1</sup> For a symbolic system with a *simple* hierarchy of types see: F. P. Ramsey, 'The Foundations of Mathematics', *Proc. Lond. Math. Soc.*, vol. 25, 1945.

For a symbolic system *without* a hierarchy of types see: A. Church, 'A Set of Postulates for the Foundation of Logic', *Annals of Math.*, 33, 1932. His system has been proved inconsistent by S. C. Koeene and J. B. Rosser, *Annals of Math.*, 36, 1935. The impossibility of a system along such lines has not been proved either. Cf. K. Gödel, 'Russell's Mathematical Logic', in *The Philosophy of Bertrand Russell*, Evanston, 1944, p. 150.

<sup>2</sup> L. E. J. Brouwer, 'Über die Bedeutung des Satzes von ausgeschlossenen Dritten . . .', *J. Math.*, 154, 1925.

A. Heyting, 'Die formalen Regeln der intuitionistischen Logik', *Ber. Akad.*, Berlin, 1930. See *Intuitionism*, Amsterdam, 1956.

is a logical consequence of  $p$ .<sup>1</sup> Further the distributive law breaks down.

These remarks will scarcely impress a philosopher. He might say 'Granted that these systems offer some technical advantage to the special sciences — this doesn't affect our logic in the least: i.e., that logic in accordance with which we actually think and reflect. I know this logic to be valid for the simple reason that its laws are self-evident'. I do not believe that this opinion will bear close investigation. It is too reminiscent of the arguments formerly advanced against non-Euclidean geometries; there too a technical value was the most that was conceded to these geometries, a value which was not to encroach in any way upon the authority of Euclidean geometry; the laws of the latter being 'self evident truths'! It seems to me that the laws of such a 'natural logic', even if they are intuitively evident, can still be apprehended in various ways; and, moreover, that they are essentially moulded on the means of expression and the grammatical structure of existing language. From this viewpoint it would be interesting — and this is the second line of approach — to draw upon some non-European languages and to see whether in the languages of Eskimos, Negroes, Red Indians, the very same 'laws of thought' can in fact be found. It would not be so surprising if there were languages in which, e.g., a distinction were made between negations of different logical significance. A thorough study of exotic languages is likely to reveal a much larger variety of logical structures, and we should thus attain a less biased and more realistic view of these matters.

But leaving aside the question of exotic languages, and turning to our own language, we here find signs which, though concealed beneath the surface, indicate the presence of different logical relations. It is the object of the present paper to call attention to to some phenomena of this kind.

**2. THE CHARACTER OF LOGIC.** But first of all it will be well to go into an objection which may hinder such an inquiry. Are the laws of logic not one and all tautologies, that is, symbols constructed in such a way they cannot possibly be false? If so, how can we speak significantly of more than one logic?

<sup>1</sup> G. Birkhoff and J. v. Neumann, 'The Logic of Quantum Mechanics', *Ann. of Math.*, 37, 1936.

M. Strauss, 'Zur Begründung der statistischen "Transformationstheorie" der Quantenphysik', *Ber. Akad.*, Berlin, 1936.

The doctrine according to which logic is a body of tautologies has cast a flood of light on the status of logic; at the same time it has obscured it: for this view made it appear as if thereby the nature of logic had been grasped finally, once and for all; so that any doubt as to the validity of a logical principle, e.g., of the law of excluded middle, would be unthinkable. But the laws of logic are tautologies only if (1) we confine ourselves to the propositional calculus, disregarding universal and existential propositions, and (2) we *interpret* the logical constants 'not', 'if', etc., as truth-functions. Not that this interpretation is necessary. Intuitionist logic, for instance, is a two-valued logic in which negation, implication, etc., can no longer be defined in terms of truth tables.<sup>1</sup> This shows that certain fundamental assumptions underlie classical logic, assumptions which may well admit of alteration. In this case, as in that of geometry, it would seem advisable to take up an axiomatic standpoint from which alone we can attain full freedom in the treatment of logical systems. The account of logic according to which it consists of tautologies, though in a sense true, does not penetrate to the core of the matter. Taking a broader view, we must say that this doctrine, when not properly understood, has become something of a brake on research, blocking the opening up of new avenues.

This brings me to a second remark. In any discussion concerning the validity of logic one should bear in mind that no proof can be offered for or against a logical principle. Consider, for instance, the circumstances in which we would call a mathematical proof conclusive. This depends essentially on what demonstrative methods we admit. If a formalist recognizes the validity of a proof in which use is made of the '*tertium non datur*' whereas an intuitionist rejects this proof — how can it be shown which of the two is right? It cannot be shown at all, if by this is meant 'be proved'. (For remember that a proof itself is nothing but a transition, or a series of transitions, within a given calculus.) But one *could* decide in favour of the one system or the other; for which decision there may be a number of motives; e.g., the wish to banish from mathematics propositions that cannot be decided in a finite, determinate number of steps; or, more generally, tendencies to build up mathematics in some definite spirit. Such motives,

<sup>1</sup> See K. Gödel, 'Zum intuitionistischen Aussagen Kalkül', in *Ergebnisse eines math. Kolloquiums*, ed. by K. Menger.

however, should not be regarded as *reasons* or *premisses* from which a logical principle can be deduced. It would be truer to say that something like an urge to achieve a definite style of thinking is at the bottom of such a conviction.

'But can we not *see directly* that such a system is *right*, without any formal proof?' No; the appeal to self-evidence supplies us with no unequivocal and precise criterion, as is indicated by the protracted dispute about the law of excluded middle. What we must understand is that the choice between distinct systems of logic is not decided by logic. It may happen that one system is more suitable for this, the other for that purpose; or our decision may be guided by aesthetic preference; but neither has anything to do with any insight into the 'truth' of a system.

**3. A PROPOSITION ALWAYS EITHER TRUE OR FALSE?** 'But surely it cannot be a matter of choice that a proposition can only be either true or false; and this is the basis of the whole of logic'. It is difficult to ignore the impression that the structure of logic — above all the division of propositions into the two classes of the true and the false — is deeply rooted in the nature of existence. The train of thought leading up to such a view might roughly be expressed as follows: a statement describes a possible state of affairs; a state of affairs exists or does not exist; there is no intermediate possibility, and therefore no transition from truth to falsehood. Thus one is led to see in the antithesis of true and false something which is pre-established in reality itself — a mould into which our thought is forced. 'It has to be so' we say; and are now under the impression of having *established* the nature of logic.

It is a striking fact: philosophy seeks to describe the essentials of being, but in its search for them, for propositions to describe them, at the decisive moment it does not arrive at *philosophic* propositions, but at the limit of language. To say 'the event *p* can only either happen or not happen' is to utter an empty tautology. It might be argued 'What belongs to the nature of things cannot indeed be expressed in words'. But the *nature* of language can in a manner reflect the *nature* of the things, and philosophy, as the guardian of grammar, can in fact grasp the nature of things, though not in propositions of language, in rules for the use of language; e.g., in the rule which demands division of propositions into the two classes of the true and the false. This distinction somehow *corresponds* to the nature of reality; and here is the point where logic and reality

meet. Whoever thinks along these lines may well feel convinced that to abandon this principle of classification is impossible.

4. A COUNTER EXAMPLE. But is it really so impossible to imagine a situation in which it would be natural to discard the idea of a two-valued logic? Consider the following example: I say 'If it is fine tomorrow, I shall go out for a walk'. If it is fine and I walk out, my statement is true, if it is fine and I do not walk out, it is false; but suppose it rains the next day and I remain at home? Have I then spoken the truth? No; and yet I have not lied. I have not considered this case, it might be said. I have declared that *if* it is fine I shall go out for a walk; how I am to judge if it is not fine is not determined. The case under consideration is very much like that in which I *promise* to go out for a walk on the condition stated. If it rains the day after, it would be most natural to say: I had *no chance* of fulfilling my promise, but not: I have broken it. We gauge the situation more correctly if we ascribe to language a tendency to drop the alternative 'To keep one's word', 'To break one's word', i.e., not even to allow the question whether I have kept it to arise. And thus it may well be in harmony with the spirit of everyday speech to say that the statement 'If the weather is fine tomorrow, I shall go out for a walk' expresses nothing if it rains, and is neither true nor false.

True, I could lay down special rules for this case, so that to speak of true and false would acquire a meaning. I then fix the use of the statement so that whether it is true or false is determined in every case. But I am not compelled to do so. Indeed I might have declared the statement to be neither true nor false in the case considered; in this I admittedly diverge from one of the principles of logic — but why shouldn't I? — so long as I do not forget that what I am now calling a 'proposition' is a different construction. A proposition is something used according to the rules of a calculus; and in the ordinary calculus, what *is* used according to the rules is always definitely true or definitely false. If this does not apply, we change our idea of a proposition; but we don't make a discovery: we don't find something that is a proposition and yet does not obey such-and-such laws; but we lay down a new convention, construct a new logical space.

It is then like this: the use of such an expression is not yet precisely determined by everyday speech. The use of language leaves open certain gaps. We may fill these gaps in different ways:

e.g., by assimilating the case of the proposition to that of the promise, and so deviating from the laws of logic; or, again, by clinging to the laws of logic and thus departing from the lines of ordinary usage. But these two tendencies cannot be combined. Here one might speak of a conflict of the motives that might guide us in extending our language.

In jurisdiction there are various guiding principles: one lays down that the judge has to decide every case; according to another he is free to leave a case undecided.<sup>1</sup> If we attempt to carry through the analogy into language, we notice that Aristotelian two-valued logic corresponds to the principle of *deciding* every case. But this is not the only possible policy.

There is also the possibility of saying, 'I want *no* decision'. And this tendency would find fulfilment in a language in which it would make sense to say of a given sentence that it is, in certain circumstances, neither true nor false. This would in no way mean that the statement is 'in itself' true or false, and that we are only unable to decide the issue; rather it would mean that we use the sentence according to different rules, in another logical system.

5. PARTIAL NEGATION. We *can*, then, use sentences in such a way that in certain cases they are neither true nor false. And I should now like to show that this mode of application comes even closer to natural thought than classical logic does. Let us consider two examples.

(i) When I say 'Only if  $p$ , then  $q$ ' this is a limitation of  $q$ . One feels that there is an element of negation about this limitation ('I shall go out', 'No, at least only if it is fine'). Is there any justification for this feeling? That is, is it possible to construct a notation in which limitation is treated as a sort of negation? 'Only if  $p$ , then  $q$ ' means  $(p \supset q) \cdot (\sim p \supset \sim q)$ , and that means  $(p \supset q) \cdot (q \supset p)$  i.e.,  $p \equiv q$ . The question should, then, be put like this: How far can  $p \equiv q$  be conceived as a negation of  $q$ ? Is there any sort of operation which transforms  $q$  into  $p \equiv q$ ? To answer this question consider the schema

$p$	$q$	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

<sup>1</sup> The praetor could dismiss a case by saying '*non liquet*'.

The objection against  $q$  contained in the remark 'Only if  $p$ , then  $q$ ' focuses attention on the possibility that  $p$  may be false; in this case not the proposition  $q$  is true, but the proposition  $\sim q$ . One might put it like this: the objection negates  $q$  *only under the condition* that  $p$  is false; the alternative ' $p$  is true' remains unconsidered. Consequently one may interpret the objection as a *partial negation* of  $q$  — partial because it is meant only under the condition ' $p$  is false'. One can make this stand out in the symbolism by attending solely to those truth-conditions which correspond to the possibility ' $p$  is false'. Imagine the rest to be crossed out. The schema then becomes

$p$	$q$	$p \equiv q$
F	T	F
F	F	T

From this it is evident that, looked upon in this way,  $p \equiv q$  has precisely the opposite truth-value of  $q$  and is the negation of  $q$ .

To formulate the matter in a more general way, let us introduce the idea of the  $p$ -aspect of a proposition  $q$ . Let  $T$  stand for a truth-function  $T(p, q, r, \dots)$  of the propositions  $p, q, r, \dots$ . Attend only to those lines of the schema in which  $p$  is true and imagine all the others to be cancelled. The remaining expression is then called the ' $p$ -aspect of  $T$ ', in symbols  $T_p(p, q, r, \dots)$ . We may, for instance, negate a proposition,  $q$ , under the aspect of another,  $p$ . This means that we carry out the operation of negation not in *all* truth-conditions, but only in those which correspond to the possibility ' $p$  is true'. Using this mode of expression we may enunciate the result we have just found thus: the limitation 'Only if  $p$ , then  $q$ ' is the negation of  $q$  under the aspect of  $\sim p$ , or the partial negation of  $q$  with respect to  $\sim p$ ; that is  $p \equiv q$  is  $\approx q$ . This notation makes it clear how far the limitation is at the same time a negation.

(2) What is the relationship between the statements 'If it is fine tomorrow, I shall go out', 'If it is fine tomorrow, I shall not go out'? One feels that there is a *conflict* between them. When the 'if-then'-relation is interpreted as material implication, the logical product of the two propositions is simply 'It is not fine tomorrow', and no conflict arises. But how is it that we still feel some vague

discomfort when in everyday life we come across such a combination of statements? Were someone to say 'If it is fine tomorrow, I shall go out, and if it is fine tomorrow, I shall not go out', we should certainly not understand him as meaning that it will not be fine tomorrow. We should rather say, 'What on earth does he mean? He is surely contradicting himself'.

It is striking that the conventional treatment offers us no basis for bringing out this conflict. From our present point of view it is easy to clear up the situation: the statements  $p \supset q$  and  $p \supset \sim q$  contradict each other *in the p-aspect*; e.g.,  $p \supset \sim q$  is the *partial negation* of  $p \supset q$  with respect to  $p$ ; in symbols

$$p \supset \sim q . \equiv . \sim_p (p \supset q).$$

The logical product of the two statements, interpreted in this way, is a *contradiction in the p-aspect*; and this accords with our feeling that the two statements are 'in conflict' with each other.

The newly introduced symbols obey certain laws; e.g., the law of double negation for partial negation

$$\sim_q \sim_q p \equiv p,$$

the commutative law for partial negations

$$\sim_q \sim_r p \equiv \sim_r \sim_q p,$$

the commutative law for total and partial negation

$$\sim \sim_q p \equiv \sim_q \sim p.$$

It is now possible to approach such tasks as symbolizing the difference between 'Only if  $p$ , then  $q$ ' and 'If  $p$  then  $q$ '. It is simply the difference between  $p \equiv q$  and  $p \supset \sim_p q$ .

It would not be right to say that we are here building up an entirely new logic. It is rather that *within ordinary logic* we have discovered a new system, which, in a fragmentary way was indicated in the contour of natural thought and which has been developed, i.e., we read a new system into the old one. And this system enables us to bring out logical relations which, though more or less clearly indicated in common speech, are not shown in traditional logic. Only we must not lose sight of the fact that we are here concerned with structures that do not quite correspond with our current idea of a proposition, as a proposition is conceived here as something that, in certain cases, is allowed to be neither true nor false.

6. STRONG AND WEAK NEGATION. We can, however, proceeding from ordinary usage, arrive at an entirely new logic.

If we consider how we use language outside the schools, we should be blind if we did not admit that the ideas 'true' and 'false' are often employed in ways running counter to orthodox logic. Thus we say, 'This is nothing but the truth', 'This is not entirely true', 'This is a half-truth', 'This is altogether false'; we say, 'It would be less incorrect to say', 'It would be truer to say' — phrases which strongly suggest that we regard the ideas 'true' and 'false' as capable of gradation.

Now the explanation lies ready at hand: such phrases have 'really' no bearing on the validity of logic; they are rather abbreviated and inaccurate modes of expressing what could be said equally well in a language governed by Aristotelian logic. According to this view, what is really meant by the aforementioned expressions is that a part of what has been said is true, a part false. I admit that conversational English *can* often be interpreted in this way; only I am of the opinion that it *need* not be interpreted thus — and there are cases in which such an interpretation would fail — but that the matter can be looked upon as showing the rudiments of a novel type of logic.

In speaking of different negations I wish first to exclude a misunderstanding: I do not wish to discuss the emotional intensity of negation. Suppose, for instance, that someone is being unjustly accused. When he then protests 'It wasn't me', the tone of voice may express a whole scale of feelings, from dry assertion to violent indignation. With this gradation we are *not* concerned. A logical problem arises only when the purport of the negation varies. This purport shows itself in the manner in which the expression of negation is used.

In describing something that has been said as 'not entirely false' I imply that what has been said comes somehow *near* the truth without attaining it. 'Yellow is not quite the right colour' says not only that what I reject is false, it implies at the same time that it is not far from the truth; I exclude yellow, but admit a whole range of neighbouring colours such as orange, pink, greenish-yellow, etc., whereas if I wish to exclude these also, I should say, 'The colour is not in the least yellow' or something like that. In these two cases the negation has a different *range of application*; and this is a logical difference.

Here something like an idea of ‘truth-distance’ suggests itself, capable of gradation. Indeed language places at our disposal a wealth of expressions of this kind; such as ‘That’s nearly it’, ‘This is far from true’, ‘That is out of the question’. Moreover there are various phrases expressing certain intermediary forms such as ‘That is only half-true’. (Notice that some of these expressions, in another sense, do not refer to the *degree* of truth, but to the *evidence* we have for the truth; thus when we say of a statement that its falsehood stares one in the face. This sense shall not concern us here.)

One could now set oneself the task of reconstruing these conditions in a calculus, e.g. by building up a logic with two distinct negations. A simple example would be the following. Suppose we were to evaluate a statement allowing for three different possibilities: that it is true, not quite true, false, symbolized by 0, 1, 2 respectively. Suppose further that we are given precise rules for such a valuation. We now define two sorts of negation, a ‘weak’ negation symbolized by  $\sim p$ , and a ‘strong’ negation symbolized by  $\approx p$  by means of the tables

$p$	$\sim p$	$\approx p$
0	1	2
1	0	1
2	1	0

Let the reader try to apply these definitions to statements presenting three clear alternatives (as, e.g., in the case in which the catch-words ‘all’, ‘some’, ‘none’ occur, or ‘white’, ‘grey’, ‘black’); he will then find that, though in a more precise form they convey what is meant respectively by the expressions ‘not quite’, ‘not at all’,<sup>1</sup> the two negations are not interchangeable, i.e.  $\sim \approx p$  is different from  $\approx \sim p$ . The strong negation obeys the law of double negation.

$$\approx \approx p \equiv p,$$

<sup>1</sup> Jan Łukasiewicz, has studied a three-valued, Emil Post a many-valued system. These authors were guided by formal (mathematical) considerations only. It is noticeable that none of their definitions of negation lends itself to a natural interpretation. See Łukasiewicz, ‘Philosophische Bemerkungen an aufwertigen Systemen des Aussagen Kalküls’, *Comptes Rendus des séances de la Société des Sciences*, Varsovie, 1930, Classe III. Post, ‘Introduction to a general theory of elementary propositions’ (*Am. Journ. of Math.*, 43, 1921).

whilst for the other one only the weakened relation

$$\sim\sim\sim p \equiv \sim p$$

applies. Moreover, only for the strong negation are the rules of De Morgan valid:

$$\approx(p \cdot q) . \equiv \approx p \vee \approx q^1$$

$$\approx(p \vee q) . \equiv . \approx p \cdot \approx q$$

Two truth-values exclude each other, that is

$$p \cdot \sim p, \quad p \cdot \approx p, \quad \sim p \cdot \approx p$$

are never true. A law of excluded fourth

$$p \vee \sim p \vee \approx p$$

takes the place of the law of excluded middle.

As regards implication it seems natural to regard as relations of implication only those which satisfy the following conditions:

(1) Implication leads from true propositions to true propositions: i.e. if  $p$  is true, and ' $p$  implies  $q$ ' is true, then  $q$  is true. This is the basis for the 'rule of inference' which allows us to pass from  $p$  and ' $p$  implies  $q$ ' to  $q$ .

(2) The relation is reflexive: i.e. ' $p$  implies  $p$ '.

(3) The relation is transitive: i.e. if  $p$  implies  $q$  and  $q$  implies  $r$ , then  $p$  implies  $r$ .

(4) If  $p$  implies  $q$  and  $q$  implies  $p$ , then  $p$  and  $q$  are logically equivalent.

These four conditions are fulfilled when implication is defined in the sense of Łukasiewicz,<sup>2</sup> i.e. by the matrix

$p \supset q$			
	0	1	2
0	0	0	0
1	0	0	0
2	1	0	0

Implication defined in this way cannot, however, be reduced to negation and disjunction.

Again, we may consider the implication  $p \supset q$  under the aspect

<sup>1</sup> It is assumed that conjunction is defined by the rule  $T(p \cdot q) = \max.(Tp, Tq)$  and disjunction by the rule  $T(p \vee q) = \min.(Tp, Tq)$  where  $Tp$  designates the truth-value of  $p$ .

<sup>2</sup> Łukasiewicz, *op. cit.* p. 65.

of  $p$ , for instance  $p \supset, \approx q$ . This would be a mould to render symbolically such phrases as '*If it is fine tomorrow, I shall most certainly not —*'. Here, too, the statements '*If p, then q*' and '*If p, then not-q*' (no matter whether 'not' stands for weak or strong negation) are incompatible: their product yields a sort of contradiction.

These remarks will suffice to substantiate that a propositional calculus — supplying rules of inference — *can* be developed along these lines. Of course, we might as well introduce a logic with a larger number of truth-gradations, for instance a four-valued logic ('true', 'nearly true', 'not quite false', 'entirely false') or a logic with any number of truth-values.

Let no one say that these logics are a mere play with symbols. For there are sub-domains in our language in which a logic with graduated truth-values is quite natural. If the question is, e.g., whether a certain comparison is to the point, in many cases instead of saying 'Yes' or 'No' it is more natural to reply, 'Yes, more or less to the point', 'Not quite to the point', 'Well, so far to the point, but only so far'. But in so doing one *accomplishes the transition to a logic with a graduated scale of truth-values*. The same holds good of most statements describing properties capable of gradations.

One might go even further and introduce something like a *directed* negation, bringing out the difference we have in mind when we say, e.g., 'It is *not yet* morning', 'It is *no longer* morning'. Such phrases indicate the *direction* in which a proposition diverges from the truth. Each of the directed negations may in its turn be graduated. Example: The propositions we are concerned with may be equations within the domain of integers  $a - b = 0$ . One can then fix with precision the 'divergence from the truth' by assigning a positive or negative value: or the value 0, which corresponds to the value 'true'.

One might here apprehend ordinary negation as an equivalent of a sum of operations in a logic with infinitely many truth-values. In other words, ordinary negation can be split up into an infinite disjunction of precise operations; I say 'precise' because each of these operations leaves the communication-value of the original statement unaltered. By extending the disjunction over an increasing number of terms we 'dilute' the original statement more and more; the *limit of this process of diluting is ordinary negation*.

Thus certain features of ordinary logic are fully understood only in the light of many-valued logic.

The last stage in this development would be this. In certain cases propositions can actually be mapped into the elements of a space in which a certain metric is defined. One can then speak, in a manner easily understood, of the 'distance' between two propositions, compare these distances and so transfer the modes of expression of geometry to a system of propositions — to a 'propositional space'. I think that signs of such metrical logics do exist in everyday speech, but only in an indistinct, fragmentary form; and that given goodwill the tendencies of ordinary language could be developed into such a system. But this is not the place to go into it more fully.

**7. OTHER SENSES OF NEGATION.** There are various other grounds for grading negation. Let the reader call to mind for a moment how he would reject a thought that is simply false; or again an assertion which is not only actually false but tendentious as well; a superstitious remark; a statement based on distorted thinking; an utterance which rings wholly false (e.g. full of hollow and empty phrases). In fact the rejection may involve a protest not only against the matter, but also against the manner. In this case we may regard it as combining two negations, one of which is directed against the content, the other against the form of the whole background out of which the statement arises, as when a criminal in court pleads not guilty because 'he has no talent for ethics'. Considering these circumstances, it appears how mistaken it would be to arrange all these negations into a simple graduated scale; rather the multiplicity of negation is as complex as the vocal and stylistic means of expressing it.

If we wish to condense the situation in a formula, we might say: a proposition is always part of a system, and we may either admit the system whilst negating this particular proposition, or we may reject the entire system. This is another sense in which we may speak of 'weak' and 'strong' negation.

Take the phrase 'not only — but —'. In this case the negation is directed against the completeness of what has been said. In saying 'I have not only seen him but —' I resist the impression that the first part is complete and hint that something more is to follow. Here we might speak of a negation directed, not against a statement, but against the completeness of the statement.

Is the term 'hardly' more the expression of a weakened negation or of a weakened assertion, or does it combine both features? Again, traditional logic has no means provided for rendering the contour of such thoughts.

There are many more senses of negation some of which may be reconstrued in a logical calculus.

**8. IDEALIZATION.** It should by now be clear that no definite system of logic is stamped on language; rather is actual usage a loose conglomeration in which fragments of different logical systems are discernible. Common speech is rather like a maze of lines running into one another without yielding a very precise contour. Generally speaking I should say: A logic is always an idealization of the conditions which we meet with in a given language, just as mathematical geometry (e.g. Euclidean geometry of three-dimensional space) is a refinement of the rough data obtained by measuring operations with practically rigid bodies. And just as observations obtained in this way can, in principle, be built into various geometries, so the conditions we find in a given language allow of an idealization in more than one direction. In other words, the process which leads to the crystallizing of a definite logic is not uniquely determined by the actual usage.

From this standpoint the following question arises: how is it that the two-valued system nevertheless occupies such a dominating position? In what does it excel other systems? Perhaps for the following reasons: in most cases the situation does not indicate clearly how many gradations should be assumed — whether three truth-values, or four, or any larger number. The possible choices vary. On that account there is something fluctuating about these systems so far as their practical application is concerned, though they are in embryonic form to be found in actual usage. In this respect they are like gradations of plurality such as: one, two, several, quite a few, a lot, a good many, a tremendous number. Here too the number of the intermediate grades is indeterminate and conditioned by the circumstances of the occasion. That is probably the reason why these concepts have not developed into a rival number-system. And this may also be the answer to the question why none of the many-valued systems has won its way into language.

**9. DOES THE LAW OF EXCLUDED MIDDLE APPLY TO STATEMENTS ABOUT SENSE-IMPRESSIONS?** We come up against

another problem when we ask ourselves: Are there 'vague' or 'indeterminate impressions'? Let me quote some examples to make my meaning clear:

(1) I glance up to the starry sky and I am to say how many stars I see in this moment.

(2) With eyes closed I enter an unfamiliar room, open them for a second and am then to describe what I have seen.

(3) I am in Venice for the first time. I glide along the canals, past ancient palaces, I walk across bridges and squares, I see crowds of people, distant cupolas of churches, etc., all under the vault of a light-blue sky. I am now to put into words this flood of fluctuating impressions.

(4) I enter a room full of groups of people talking in a language familiar to me. A confusion of voices falls upon my ear and I am to describe what I have heard.

(5) I listen for the first time to a piece of music which I cannot quite take in. I am afterwards to hum the tune from memory or to play it on the piano.

(6) I try to recall the appearance of a person; the image has to some extent already faded, and I can no longer clearly visualize his features.

What is common to these (and innumerable other) cases is that I would say, 'my memory is not quite exact'. If I were pressed to put into words my impressions (or memory), I could produce a description in which some vague expressions would occur (e.g. 'I saw many stars'; 'I saw flowers over a bridge, a grave old palace, crowds of people, amongst them for a moment a face that seemed vaguely familiar'; etc.). If I were asked, 'Is that all? Can't you describe it more precisely?' I might perhaps add some further detail, but would finally say, 'I couldn't see it as clearly as all that'.

Suppose now, in example (2), a picture were shown to me and I were asked whether this was the impression I had when entering the room; I might then say 'No, it wasn't like that' or 'Yes, it was roughly like that'. In the latter case, if I were shown a *series* of pictures differing from one another in the finer details but agreeing in other respects, it might well be that about the first I should say, 'Yes, it looked roughly like that' and the same with regard to the second, and so on; that is, that I should recognize the whole series of pictures as 'right', as 'agreeing with the facts' without dis-

tinguishing a particular one as 'the' reproduction. The same applies *mutatis mutandis* to the grasping of a piece of music, etc.

The reason why I lay some stress on this possibility is that our memory often works in the same kind of way. If I try to remember what I saw on entering the room, some memory picture or other will rise before my mind; but if I attempt to recollect more precisely this or that point of detail, I shall perhaps become uncertain and wavering, and this in turn will manifest itself in the fleeting emergence of a number of different details, fluctuating, indistinct, changeful, all of which claim to be 'recollections'. Here we have really something like a competition between different images.

What conclusion are we to draw from this? The usual view is that what I see, the impression I have at the moment I experience it, is quite distinct; it's only afterwards that I cannot remember it so exactly. That is, one has at first a tendency to impute the indeterminacy not to the impression itself, but to memory; but then it seems that the memory image, even if it is not precise, is yet an object which should be describable exactly even in its inexactitude; just as, for instance, a painter is able faithfully to reproduce a landscape shrouded in mist. And yet this is impossible. Once one notices this impossibility, one is readily led to transfer the indeterminacy to *language*. One is inclined to think that my memory picture surely must be of quite a definite character. It is only when I try putting this image into precise words that language fails me. And in like manner what I then saw (or heard) is supposed to be something perfectly determinate.

We follow in this case as in so many others an analogy deriving from the mode of thought of the natural sciences. This leads us to believe in the existence of a metaphysical entity, the impression as I experienced it, an entity supposed to be quite distinct in all its features.

Could we not in this case take up a different position and say that it is precisely the characteristic of such impressions that they cannot be reproduced exactly? The vague form of description ('I saw a fair number of points of light in the sky') is here in fact the more adequate one; for there is here no such thing as perfect precision and distinctness. And we may call an impression 'vague' if it can be described only in vague terms.

Take again the case of memory. Are we justified in saying that though all these memory pictures are nothing more than indistinct

traces left behind by the impression, the impression itself was perfectly definite? But what is the point of assuring us that it was perfectly definite, if we have no means of recapturing it? The truth is that we have nothing but the memory pictures which, if we attempt to render them precise, begin to change. *That is all.* Let us give up hunting for a metaphysical illusion that is always darting on ahead of us and can never be caught. Let us rather say: In fact all we have are the traces in the memory which are indistinct, and any one of them describes the original impression equally well.

Let us put it this way: If I am to reproduce a visual impression in a painted picture, what cannot be expressed is just what may be called the aspect of indeterminacy. For the lines I trace on the surface of the picture (or the notes I hum or play on the piano, etc.) are surely always in themselves determinate. The uncertain colour in the memory, the indistinct shape, the darkly seen, cannot be reproduced in the proper sense of this word. I propose to introduce a new logical principle, e.g., by saying 'Two distinct representations (within certain limits, if they differ but slightly) do not necessarily exclude each other'. This, I think, would clearly show that our attitude towards an impression or a memory image is very different from that towards a material object. Nobody would dream of ascribing two different lengths to a desk and saying that both are right. One description, if it proves true, excludes the other. In the case of an impression, however, this seems not to hold universally. It may be that slightly different pictures are laid before me or slightly different themes are played to me on the piano and that I say of each of them, 'Yes, it was roughly like that'.

That certainly seems a startling consequence. Let me modify this consequence a little by drawing attention to a somewhat similar example.<sup>1</sup> Imagine we are experimenting with a film. If this were exposed for some time, a very large number of dots would appear on it giving a rough impression of a strip if seen from a certain distance. This strip has, of course, nowhere sharp outlines; rather the dots are more thickly concentrated towards the middle and more scattered towards the edges. The question to be discussed is this: what is the width of the strip? True, there is no sharp demarcation; on the other hand I may well say, 'That is certainly

<sup>1</sup> I owe this example to Prof. L. Wittgenstein though he mentioned it to me to illustrate another point.

not the limit', if it is drawn too narrow. 'And neither is that', if it is drawn too wide. What we must understand is that here we have to reproduce the *impression* that the strip makes on us. When one talks of the measurement of the strip's width one thing is clear: this is a measurement in a quite different sense from the usual one; i.e. measurement assumes here a new meaning. This would appear, e.g., in the fact that the statements 'This strip is 13 mm. wide' and 'It is 15 mm. wide' do not necessarily contradict each other whilst in the ordinary sense of 'measurement' they are incompatible. The expression 'The width of the strip' has here altered its grammar.

Similarly with the reproduction of sense-impressions. I cannot describe them *exactly*, or when I use clear-cut concepts to describe them I have to own that representations only slightly differing from each other might equally be admissible. And the same applies to pictures. I do not know whether the curtain I saw for a moment when entering the room was white or grey or had a tinge of yellow; I only know that it was bright. Thus I might say of three distinct pictures that they are equally admissible. What this amounts to ultimately is that we thereby exclude from the picture everything that is too exact and therefore superfluous. What cannot be reproduced by a single picture — the blur, the indeterminacy of the visual impression — can be reproduced by a number of pictures, provided we declare them logically compatible, thereby giving the representation just that sort of vagueness which word language enjoys.

Taking a hint from Brouwer one might say: Since I cannot go back to the original impression to find out what it looked like, the assertion, 'The patch was either white or not white' is undecidable — unless it happens to be decidable in this particular case. What sense could there be in insisting that I must have seen either 735 stars or some other number? What constitutes the value of the principle of excluded middle is the possibility which it implies of *deciding* issues. If this possibility ceases to exist, though one can still persist in saying 'It was either this or not this', it finally becomes *pointless*; to insist on that disjunction means clinging to an empty formula and paying lip-service to the laws of logic.

It should be noticed that it has not been shown by this that the law of excluded middle is false. What we have done was only to draw attention to certain disadvantages which would be involved

in adhering to that law, and thus to prepare for its dismissal. The discarding itself is a matter of free decision.

The result then seems to be: The language in which we describe sense-impressions has a logical structure different from that in which physical objects are described — Aristotelian logic is no longer valid in it. Or, more correctly, it is *characteristic* of sense-impressions that they are not describable in a language of this structure; if we were to force upon them such a form of description, we should do violence to them, and this would betray itself in the emergence of undecidable alternatives.

10. THE LOGIC OF SENSE-IMPRESSIONS. What sort of logic is it then that sets its stamp upon phenomenal language? One might think that the elimination of the law of excluded middle would lead to that system of logic which has been worked out by the intuitionists. That, however, is not so. It will emerge that we have here a logic that is different from both the Aristotelian and the intuitionist. First of all, however, I wish to discuss a difficulty which impedes the construction of such a logic.

If I declare compatible two descriptions such as 'The patch was white', 'The patch was yellowish', I can scarcely avoid allowing also the statement 'The patch was white and not white'; provided that I see in 'yellowish' an instance of 'non-white'. But this brings us in conflict with the law of contradiction. Can I admit an assumption that leads to such absurd consequences?

Let us consider the matter once more. We said that to return to the impression is impossible. I have nothing but the images of memory which, as I try to focus them clearly, begin to vacillate and to take on different forms. It then becomes natural to say: the impression is something which is represented by *all* these images together. And 'by them all together' here means, not by their conjunction or disjunction, but by the *class* of images as a whole. Replacing the images by painted pictures and these in turn by descriptions in words, we can say: the impression I had can adequately be represented only by the entire class of the propositions which are admissible.

Till now we have accepted without questioning the view that the proposition must necessarily be the unit of logic. But what we have said justifies a new scrutiny of this assumption. It points the way towards a novel conception. Suppose when asked to reproduce a past colour impression I am uncertain and hesitate between

different colours without being able to decide conclusively. According to what has been said all those propositions by which I express my recollection are to form a single description. The decisive step consists in comprehending a group of sentences in one independent unit, in other words, in allowing a class of descriptions to be taken as one description.

This surmounts the difficulties with regard to the law of contradiction. For if I allow different answers, I do not mean that I allow their conjunction. If I say the patch I have seen might have been white or grey or yellow, I do not mean to say that it was white *and* grey *and* yellow. I allow the answers only as elements of a class, and it is the class itself we now wish to operate with.

The symbolic treatment of such classes of propositions, though interesting, would lead here too far. I shall limit myself to a few hints only. Suppose  $p, q, \dots, t$  are a number of admissible answers, then we start from the complex

$$\pi = (p, q, \dots, t)$$

We make the following assumptions about  $\pi$ :

(1) If  $\pi$  is part of  $g$  (i.e., if the propositions belonging to  $\pi$  belong also to  $g$ ), we have

$$\pi \supset g$$

In other words, the more precise description implies the less precise one.

(2) In particular  $\pi$  is implied by any of its elements, i.e.,

$$p \supset (p, q, \dots, t)$$

(3) If all elements of  $\pi$  are false,  $\pi$  is false.

The assumptions made so far would determine  $\pi$  in such a way that  $\pi$  would be equivalent to the logical disjunction of its elements,

$$\pi \equiv p \vee q \vee \dots \vee t$$

We have now to express that  $\pi$  is not a truth-function of its elements. To do this we lay it down as a principle:

(4)  $\pi$  may be true even if the truth-values of the single constituents are indeterminate.

Here, then, is the point where the logic of complexes branches off from the logic of truth-functions.

(5) If the law of excluded middle is valid for the single elements, the complex can be decomposed into the disjunction of its members. In symbols

$$(\pi \vee \sim\pi), (\varrho \vee \sim\varrho) : \supset : (\pi, \varrho) . \supset . \pi \vee \varrho$$

The last formula shows that if the law of excluded middle is added to the logic of propositional sets, it goes over into ordinary Aristotelian logic.

I wish to make two further remarks to complete what has been said.

(a) Someone might object: 'All right, what you mean amounts to introducing a certain indeterminacy into your description; that you may simply attain by introducing a free variable with a scope limited to a wider or smaller range. Accordingly the statement "I saw a light patch" would have to be interpreted as meaning "I saw a patch of the colour  $x$ " where  $x$  is limited to the range of the "light colours". The correct symbolic expression of my experience would thus be a proposition containing a free variable'.<sup>1</sup>

Quite so: the logic of sets will in certain circumstances coincide with the logic of propositions containing a free variable — with the logic of *incomplete* descriptions, i.e. of descriptions which leave something open. Only it must be added that an incomplete description cannot always be reduced to a description containing a free variable. If it is a question, e.g., of reproducing a visual impression I had and I say, 'It was an elongated, or again a fringed, or a jagged patch grey in colour', I may perhaps replace 'grey' by a variable which is allowed to vary with a certain range of colours, but not 'elongated' or 'fringed' or 'jagged'; for the various descriptions of shape which are all subsumed under a concept like 'elongated' cannot be reduced to one form. We are, then, left with the fact that my experience is adequately expressed only by the class of all admissible descriptions.

(b) The second remark is that in many cases not even a complete list of such descriptions can be given. I shall not always be able to decide with certainty whether an answer is still admissible or not;

<sup>1</sup> It seems to me that free variables may occur in descriptions too. If the statement was e.g., 'The star is of a light colour' and I were to express it by writing  $(\exists x)fx$ , it cannot be incorrect to form the expression  $\sim(\exists x)\sim fx$ , in words 'The star has all light colours (at the same time)'. As this is nonsense, the formula  $(\exists x)fx$  must be nonsense too — a hint that we may, in certain circumstances, take  $fx$  as symbolizing a statement, not a propositional function.

in certain cases I may hesitate. It will therefore be truer to say, not that  $\pi$  represents a definite group of sentences, but merely that it behaves like such a group; and for which we lay down laws modelled on those for the groups. One might say that  $\pi$  is related to a series of sentences in much the same way as a vague term (e.g. the term 'game') is to a series of instances by which its meaning is first explained to us. I mean by this that the sentences  $q, p, \dots r, \dots$  only have the value of *examples* without exhausting  $\pi$ . Symbolically this may be expressed by writing  $\pi$  as a series of sentences preceded and followed by dots

$$\pi = (\dots, p, q, r, \dots)$$

This is to remind us that the sentences  $p, q, r, \dots$  do not form a closed system.

The indeterminacy of the impression, then, cannot be reproduced by any precise constructions. And so in the end we reach the conclusion that *the logic of sets fully coincides with the logic of vague statements of ordinary language*. For if I say in common speech 'The patch was more or less yellow', 'The patch was yellowish', or, pointing to something yellow, 'Yes, it looked roughly like that', the structure of these propositions corresponds precisely to that of propositional sets which are not sharply delimited. But at the same time we gain a deeper insight into the reasons why the vagueness in this case is not removable and the vague expression is not analysable into a disjunction of precise ones.

To recapitulate: The description of a blurred impression by means of precisely defined terms leads to undecidable questions. We cannot really say that we come into conflict with the laws of logic; it is rather that the principle of excluded middle ceases to apply effectively. To avoid this consequence we frame propositions which pin down reality less precisely. Even if in a particular case I cannot state whether the patch was yellow, I may well say that it was yellowish or light in colour. *Thus it is just the introduction of an imprecise mode of expression that reinstates the laws of logic*. But not completely: for the vagueness of the terms now imperils logic anew. If I say 'The patch was yellowish' and am not fully resolved as to how I am going to use the word, it may be that in certain cases I cannot decide whether or not a colour is yellowish.

In both cases the law of excluded middle loses its validity,

though in different ways. The fault in the one case, it might be said, lies with language, in the other with reality. In the first case the situation is such that what I am to describe (e.g. the colour before me) is clearly given, whereas the doubt is as to whether I should rightly designate the colour by this particular word; we may then speak of a *grammatical* doubt. In the second case, though I possess well-defined concepts, the character of my memory is such that it eludes any exact representation; hence the uncertainty of that law of logic. If the failure of logic is due to language, it is remediable: we have only to redefine our terms. If it is due to reality, it is irremediable.

This sounds as if reality could decide which logic was valid; as though the structure of logic reflected the basic framework of being. Whilst after all the failure of logic in both cases is due to language; namely to the fact that I use in the one too precise, in the other too vague terms. This formulation has the merit of showing us the position of Aristotelian logic in a new light. To explain more fully: Suppose I were to use words in a very vague fashion, i.e. so that their field of application would not at all be clearly defined. This would imply that I should waver in many cases whether to apply the word or not. But other people would then likewise be in the dark as to what I mean. Even I on a later perusal of my notes would find myself in the same position. All this would result in the communication-value of my statement being weakened: others would not quite know whether to agree with me or not; i.e. acceptance and rejection would in comparison to our present situation be reduced to a smaller range. If we wish to conserve the domain of acceptance and rejection, we must sharply delimit our concepts. If, however, we go too far in this direction, if the description becomes over nice, then the other difficulty reappears, namely that a language of such structure is not adequate for describing sense-impressions. In both cases the zone of the decidable becomes smaller, that of the undecidable extends.

Generally speaking, I should like to put it thus: What regulates our formation of concepts is the urge to fashion our statements in such a way that the largest possible range shall be decidable. *Aristotelian logic evinces this tendency.* For it is only another rendering of the same thought when I say: we try to adapt our concepts so as to form statements that will satisfy the law of excluded middle. This aim cannot be attained completely. For if

we sacrifice the precise description in favour of a vague one the difficulty with regard to the law of excluded middle emerges anew. But the situation is not quite the same: in the latter case the difficulties appear only in the margins. On the whole, then, the region of uncertainty, in comparison with the original situation, will be narrowed. Only if we allow the use of terms which are *very* vague will the uncertainty increase again.

From this we may conclude: Aristotelian logic is the logic that meets our desire for decisiveness better than other types of two-valued logic which do not include the law of excluded middle. Behind ordinary logic there is something like a desire for decisiveness.

One might go even further and suggest that this attitude is itself imposed on us by the conditions of our life — e.g. by the necessity of making ourselves understood by other people, of social co-operation, and so forth. On the other hand, we could think of circumstances in which it would seem preferable to leave things in suspense. And we all behave sometimes in such a way as to shun verifying statements. Consider the case of a man who believes that he is marked out for greatness, then we may well imagine that he will try everything in order to avoid exposing himself to the test of experience. Now if such an attitude were to prevail, we might find ourselves using a language in which the factor of indeterminacy would be far more prominent; logic, too, would have to take notice of this circumstance. This seems to point to a relationship between our way of thinking and a pattern of life. It may be that there are different types of behaviour (such as resoluteness, hesitance) which make for different types of logic. The structure of our logic reflects in some way characteristic attitudes of our life. All this would open up a vista of something like a social substructure of logic. But this is not the place to pursue this line of thought.

## CHAPTER IV

### LANGUAGE STRATA<sup>1</sup>

#### *Part One*

In this paper I shall try to sketch a new picture of language which, though still untried, seems to be of wider significance: I want to talk of the way in which language is stratified into 'layers'. In order to approach my subject I can do no better than to delineate some recent stages in the development of philosophy.

The first naive attempt was to find out what things really consist in. Thus Berkeley said: 'I see this *cherry*, I feel it, I taste it, . . . : it is therefore real. Take away the sensations of softness, moisture, redness, tartness, and you have taken away the *cherry*. Since it is not a being distinct from sensations; a *cherry*, I say, is nothing but a congeries of sensible impressions.'<sup>2</sup> Such a view, however, comes up against many difficulties: certainly a cherry doesn't 'consist of' softness, moisture, redness in the same way in which a chair consists of seat, legs and back. And a cherry exists even if there is nobody to perceive it. In the course of the discussion the need arose to go very thoroughly into the meaning of words which denote things, and this has, almost imperceptibly, brought about a change in the whole tone of thought. Indeed, when we look up any modern book on philosophy, what do we find? Discussions on the use of words, words, words. . . . The greatest philosophical problems are compressed into the smallest words — such as 'I will', 'I can', 'I must', 'I ought', 'I am'. What formidable problems each of these phrases harbours! Not that there are many philosophers who would say that philosophy is *meant* to be about words. Far from it! The study of words, according to them, only serves the attainment of philosophical illumination. Still a new technique has sprung up which has revolutionized philosophy.

A more radical view came to be held after the First World War.

<sup>1</sup> In writing the two articles which make up this chapter Waismann was very conscious of exploring new territory and was never satisfied with them. Unfortunately he was not able to carry out the revisions which he wished to make.

<sup>2</sup> From the Third Dialogue; cf. *The Works of George Berkeley*, ed. A. A. Luce and T. E. Jessup (London, 1948), vol. ii, p. 249.

According to it philosophy consists *only* in the study of symbolism. To illustrate this change look again at Berkeley's problem. Instead of saying, 'A cherry *is* nothing but a congeries of impressions', a modern adherent of Berkeley would say that statements about material objects are '*reducible*' to statements about sense experience; in other words, statements about material things can — without loss of meaning — be *translated* into statements which are only about sense contents. Notice the great difference in outlook: the modern formulation makes no attempt at all at stating what a material thing *is*; it is concerned only with the way we *speak of* material things: it is, in actual fact, a thesis about *language*, or, more precisely, about two sub-languages which are supposed to be equivalent. For what is asserted is that for every statement about a material object there can be found another and much more complex statement which is logically equivalent to it and mentions no material objects but only sense experiences. And that is plainly an assertion about language. Thus a second era of philosophy began which may be described as the era of analysing chairs and tables into patterns of sense data. This movement, embodied in Positivism, Phenomenalism, Behaviourism, is connected with philosophers like Bertrand Russell, Whitehead, Carnap — to mention only the most recent group. Everything that is regarded as respectable must now be '*constructed*' in terms of classes and relations, no matter what it is — a chair, the Self, a number, or an instant.

Still more radical is the view held by Wittgenstein. According to his *Tractatus Logico-Philosophicus* every philosophical problem is a 'pseudo-problem' which stands in need, not of being answered, but of being clarified. Such clarification consists in going into the meaning of the single words which are used in the stating of the problem, until you lose any desire to ask this sort of question. Philosophy thus turns into a therapy to cure you from asking silly questions. What were called questions are but 'muddles felt as problems'.

I shall not pause to inquire whether clarification is really all that can be achieved in philosophy. (Incidentally, I myself don't think it is; I suspect that beneath the surface there are problems of a different type; but I shall not go into that.) I would rather call attention to a new stage towards which, it seems to me, the present development moves. Instead of going only into individual cases of

word usage, we may approach the matter in a more general way: we may ask ourselves if philosophical questions arise, so to speak, at random or whether some broad principle can be discovered in the way they are arranged on the language map.

In dealing with this question I want first to introduce a distinction between two ways a logical inquiry may follow. To make my point clear let me use an analogy. In studying the geometry of a curve we may be interested to find out its behaviour in some particular point: whether it has there a tangent, whether it is there continuous, what its measure of curvature there is, and so on. Or we may study the behaviour of the curve *as a whole*: whether it is closed, and if so, whether it is convex, etc. This picture suggests two different sorts of inquiry in logic. The one takes its orientation from the logical relations which hold between certain propositions; a typical question of this sort is whether a given proposition entails another one or contradicts it, or whether the two are equivalent or independent of one another. We are then concerned with the logical nexus *on a small scale*. But now take another question: Suppose we are considering a whole theory based upon certain axioms, and it is asked whether it is 'complete' that is, whether each formula which is constructed according to given rules can be decided in the one way or the other by the means of the theory. Suppose we say 'The theory is complete', then we are making an assertion, not about the relations between two or three or more single propositions, but about the theory *as a whole*. Again, when we are inquiring whether two theories are isomorphous we are concerned with what may be called the '*macrological*' features of these theories, in contrast with questions concerning the '*micrological*' nexus of single statements.

A certain technique has been worked out to deal with problems of this sort. Naturally, these methods can only be applied to deductive systems. Now it seems to me that there is also a sense in talking of macro- and micrological features of a *language*. True, language is not organized in the way a deductive system is; compared with such a system it is of a much more loosely knitted texture. There is nevertheless a difference to be seen when you contrast statements such as: laws of nature, material object statements, sense datum statements, statements describing a dream, a blurred memory picture, sentences which occur in a novel, and so on. It is as if these 'language strata' were constructed

in a different *logical style*. The question which I want to consider now is this: Is it possible to develop this vague feeling into a more precise statement by devising some means for describing the macrological behaviour of such strata? Gauss succeeded in characterizing a curved surface by referring to its inner geometry. Can now a language stratum be similarly characterized from within? Let us see what means there are at our disposal. Well, there are a number of formal ideas upon which we can draw such as: the structure of logic, completeness of a description, 'open' and 'closed texture' of concepts, verifiability, truth, and others.

'I'm afraid I don't quite understand', said Alice. 'It gets easier further on', Humpty Dumpty replied. (*Through the Looking-Glass*.)

*Logic*: The idea that there is only one system of logical rules does not, I think, accord with the present level of knowledge. Intuitionists like Heyting have constructed a logic in which the law of excluded middle is no longer universally valid. Birkhoff and Neumann have suggested that the logical structure of quantum mechanics does not conform to classical logic but that there the distributive law breaks down. The logic of sense impressions<sup>1</sup> seems to be such that not a single proposition but a whole class of propositions is the unit of logic which again points to another system of rules. Or consider this case: If we are engaged in a study of memory pictures, for instance, if I am to describe some half forgotten impression such as I had when I first was in Amsterdam taking in the beauty in glimpses — old patrician palaces, colourful markets, all reflected in the *grachten* and seen as through a light mist — and if I am asked some particular question, e.g., what exactly the palace looked like, I may find that I am unable to decide the issue. Mind, I do not speak of the real Amsterdam (that would involve a material object statement), but of my *impression* of it in which a real and an imaginary town interpenetrate. I cannot go back to the original experience; an impression cannot be preserved and pinned down like a dead beetle. Taking a hint from Brouwer we may say: What is the use of maintaining that my impression must have been either like this or not like this since I have no means of finding out? To insist on the law of excluded middle without the possibility of *deciding* the issue means clinging to an

<sup>1</sup> For a fuller discussion see Chapter III. The material in Part One of this chapter has been included to serve as a summary of Chapters I, II and III.

empty formula and paying lip service to the laws of logic. Though one can still persist in saying 'It was either like this or not like this', it finally becomes *pointless*. We have to put up with the fact that alternatives of this sort are often undecidable. Again, the logic of aphorisms seems to be peculiar: you may say one thing at one time and the opposite thing at another time without being guilty of contradicting yourself. It would also be interesting to investigate the logic of poems. Now all this suggests looking upon a system of logic as a characteristic which sets its stamp upon a particular stratum.

*Completeness*: Suppose I have to describe the building in which this meeting takes place; I may mention many different things — its height, its rooms, the style in which it is built, its age, its history, and so on; but however far I go, I shall never reach a point where my description will be *complete*. Logically speaking, it is always possible to extend the description by adding some detail or other. Every description stretches, as it were, into a horizon of open possibilities. Contrast this case with the following: I describe a figure by saying, 'It is a white square with a black circle in it' (adding the exact size, position and shade of colour), then I have a *complete* picture, and I know that it is complete. Again, a carpet, viewed as a pattern of colour and shape, may be described completely, and so can a game of chess, in some appropriate notation, or a melody. Different is the case in which I describe a triangle, say, by giving its three sides: in this case it is *logically impossible* to add anything to the data that is not entailed by, or at variance with them. Different again is the case in which I describe a dream: my description somehow comes to an end, though not exactly in the way the description of a triangle or of a melody does; nor is it that I just *stop* as in the case in which I say something about this building and think, 'That will do', nor because it is logically impossible to go on, nor because I know for certain that I have told the complete dream. It's rather that I *try* to remember some point of detail and *fail*. Thus the sense of 'completeness' and 'incompleteness' varies with the stratum.

*Open texture*: It is the national sport of English philosophers to 'analyse' chairs and cats into patterns of sense data. Similarly, American behaviourists are eager to 'reduce' psychological statements to statements as to a man's behaviour. In doing this, they have overlooked a most important point — the 'open texture' of

most of our empirical concepts (*Porosität der Begriffe*). What I mean is this: Suppose I have to verify a statement such as 'There is a cat next door'; suppose I go over to the next room, open the door, look into it and actually see a cat. Is this enough to prove my statement? Or must I, in addition to it, touch the cat, pat him and induce him to purr? And supposing that I had done all these things, can I then be absolutely certain that my statement was true? Instantly we come up against the well-known battery of sceptical arguments mustered since ancient times. What, for instance, should I say if that creature later on grew to a gigantic size? Or if it showed some queer behaviour usually not to be found with cats, say, if, under certain conditions, it could be revived from death whereas normal cats could not? Shall I, in such a case, say that a new species has come into being? Or that it was a cat with extraordinary properties? Again, suppose I say 'There is my friend over there'. What if on drawing closer in order to shake hands with him he suddenly disappeared? 'Therefore it was not my friend but some delusion or other'. But suppose a few seconds later I saw him again, could grasp his hand, etc. What then? 'Therefore your friend was nevertheless there and his disappearance was some delusion or other.' But imagine after a while he disappeared again, or seemed to disappear — what shall I say now? Have we rules ready for all imaginable possibilities?

'But are there not exact definitions at least in science?' Let's see. The notion of gold seems to be defined with absolute precision, say by the spectrum of gold with its characteristic lines. Now what would you say if a substance was discovered that looked like gold, satisfied all the chemical tests for gold, while it emitted a new sort of radiation? 'But such things do not happen.' Quite so; but they *might* happen, and that is enough to show that we can never exclude altogether the possibility of some unforeseen situation arising in which we shall have to modify our definition. Try as we may, no concept is limited in such a way that there is no room for any doubt. We introduce a concept and limit it in some directions; for instance, we define 'gold' in contrast to some other metals such as alloys. This suffices for our present needs, and we do not probe any farther. We tend to overlook the fact that there are always other directions in which the concept has not been defined. And if we did, we could easily imagine conditions which would necessitate new limitations. In short, it is not possible to define a concept like

gold with absolute precision, i.e. in such a way that every nook and cranny is blocked against entry of doubt. That is what is meant by the open texture of a concept.

Vagueness should be distinguished from open texture. A word which is actually used in a fluctuating way (such as 'heap' or 'pink') is said to be *vague*; a term like 'gold', though its actual use may not be vague, is non-exhaustive or of an *open texture* in that we can never fill up all the possible gaps through which a doubt may seep in. Open texture, then is something like possibility of vagueness. Vagueness can be remedied by giving more accurate rules, open texture cannot.<sup>1</sup>

Open texture, absent in logical and mathematical concepts, is a very important feature of most of our empirical concepts. That the structure of empirical knowledge is so different from that of *a priori* knowledge may have something to do with the difference between open and closed texture.

*Verifiability*: A relevant question is whether the sentences we consider can be *verified*. Now verification can mean very different things: A physical theory may be verified by observation, though not conclusively; in this case the person who does the observing can in his turn be tested — his eyesight, his reliability, etc., can be scrutinized, and the result of this examination will be a number of new statements, which stand anew in need of verification: we may further examine the expert who examined the observer's eyesight, etc. In following up the threads of verification we nowhere come to an absolute end. Compare with this the case: 'I've got a terrible toothache.' Suppose I go to the dentist, he examines my teeth and says: 'No, there is nothing wrong with them.' Would I then reply 'Oh, I beg your pardon, I *thought* I've got a toothache, but now I see that I was mistaken'? My toothache cannot be argued away by examining my teeth, my nerves etc. If I were asked how I know that I've got a toothache, I should be tempted to reply, 'Because I *feel* it'. That's a very queer reply; for is there anything else I could do with a toothache but feel it? What my reply aimed at, however, was to *shake off* the whole question as irrelevant. How do I know? I've simply got toothache, and that's the end of it. I do not grant that I may be mistaken, I do not recognize a medical examination, or an observation of my eyes. No expert in heaven or earth can refute me. In saying 'I just feel it' I call attention to the

<sup>1</sup> This explanation was given in Chapter II.

fact that the toothache is a datum *given in immediate experience*, not a thing *inferred from something else* on the strength of certain evidences. So long as we move amongst material object statements, verification has no natural end but refers continually to ever new statements; in pursuing these threads, however, we see how secondary lines branch off into other regions: the points where they end abruptly represent 'I-statements'. Thus verification weaves a complicated net, a ramified pattern of lines.

*Truth:* Compare a physical law, a description of this table, of a half forgotten impression, of my present visual field, a statement of my own motives, conjectures as to the motives by which other persons were actuated, quotation of the exact words so-and-so was using, brief summary of the tenor of a political speech, characterization of the *Zeitgeist* of the Renaissance, stating the impression a poem makes on me, ethical judgements, etc. Certainly a précis of a speech cannot be true in the same sense in which an exact quotation is. If you ever try to put some rare and subtle experience, or a half forgotten impression, into words, you'll find that truth is intrinsically tied up with the style of your expression: it needs no less than a poet to render fully and faithfully such fragile states of mind. To go on, a physical law cannot be true in the same sense in which, say, a description of this building is, and the latter description cannot be true in the same sense in which a statement like 'I've got a headache' is. Truth, when applied to a physical law, means roughly speaking: it is well established by observation; it brings widely different things into a close connection; it simplifies our theoretical system; it makes us 'understand' what seemed to be a mystery before; it is fruitful by leading to new predictions and discoveries. (Incidentally, that's why the pragmatist identifies truth with usefulness: he has really got hold of an important aspect, but of *one* aspect only.) Truth in this case, one might say, is not *one* idea but a whole bundle of ideas. None of this applies to truth in the case of simple perception. Suppose you have to make sure that the light is on in your room. Now when you say, 'Yes, it's on', your statement is true *not* because it simplifies matters, *not* because it brings widely different things into connection, *not* because it is fruitful or suggestive — no, nothing of the sort; it's true because it says so-and-so, and so-and-so is as you say it is. And in which sense would you say of an ethical statement that it is true? What a host of problems this one question

raises! Truth: what we must understand is that such a word is used on many different levels and in many different senses. It has a *systematic ambiguity*; and so have 'existence', 'fact', 'statement', 'description', 'knowledge', 'law', 'meaning', 'significant', 'real', 'space', and scores of others.

Thus we see that statements may be *true* in different senses; that they may be *verifiable* in different senses; that they may be *complete* or *incomplete* in different senses; indeed that logic itself may vary with the sort of statement. This suggests grouping in the same stratum all those sentences which are homogeneous, i.e. which logically behave in the same way. And there are quite a number of leitmotifs which combine to impress a certain stamp on such a stratum.

It was hitherto the custom to refer to the single strata by indicating their subject matter such as physical laws, material object statements, descriptions of vague impressions, and the like. What I now suggest doing is to reverse the whole position by saying: Each stratum has a logic of its own; just as the physicist talks of 'eigen values' of a certain equation, so one may talk of the 'eigen logic' of a certain language stratum. If we carefully study the fine structure of such a stratum, that is, the texture of its concepts, the meaning of truth, the web of verification, etc., we may in this way arrive at a characterization of the subject matter; for instance, we may say: a sense impression is something that is describable in a language of such-and-such structure; a material object is something which can be described in such-and-such language; and so on. It is only if we are quite clear as to the logical texture of the language we use that we shall know what we are talking about.

The relations between the different strata are of a most complicated, peculiar and elusive nature. As I cannot go into all that I shall confine myself to giving one example only which may illustrate the situation.

A physical law has no doubt a connection with the observations which support it. But the matter is misrepresented when one says that an observation 'follows' from a law of nature (plus given initial conditions), or that it 'contradicts' it. Remember that there always remains a possibility of reconciling theory and observation by resorting to some accessory assumptions; at least we can never rule out such a possibility *a priori*. We should be well advised, then, to express the relationship more cautiously, e.g., by saying: certain

observations speak *for* or *against* a law, they fit in easily, etc. The point to realize is that a simple observation never excludes a theory in the way in which  $\neg p$  excludes  $p$ . If an observation cannot strictly contradict a theory, it cannot follow from it. What you can deduce from a principle of mechanics, e.g., is some theorem, but never an observational statement. The deductive nexus never extends beyond the limits of a stratum, theoretical physics never passes into experimental physics.

All this tends to show that the relation between a physical law and the evidences we have for it, or between a material object statement and a sense datum statement, or again between a psychological statement and the behaviouristic evidences for it is a looser one than had hitherto been imagined. As a result *logic loses its universal validity*: logic can only be applied to statements which are *homogeneous*. So long as we move among the statements of a single stratum, all the relations provided, say, by classical logic apply. The real problem begins where such strata make contact, so to speak; it is the problem of these planes of contact which today claim the attention of the logician. It may well be that we shall have to introduce such vague terms as 'truthweight', 'favourable', or 'satisfactory', 'evidence', 'strengthening', 'weakening', etc. so as to do full justice to the structure of language.

On account of the systematic ambiguity we cannot even join two sentences of different truth-types by logical particles. Thus our result may be sharpened: not only can a single observation never contradict a law, but it is even inadmissible to form the conjunction of a law  $L$  and an observation statement  $p$ , or to say, 'If  $L$ , then  $p$ '.

Thus language seems to be separated into strata by gaps over which one may jump but which cannot be bridged by logical processes. This fact accounts for many of the traditional problems in philosophy. The core of such a problem often lies in the difficulty of passing from one stratum to another. To give examples: If we start from sense datum statements and ask how we can arrive at material object statements, we are faced with the problem of perception; if we start from material object statements and ask how we can arrive at physical laws, we are studying the problem of induction; if we pursue the relations in the reverse order, i.e. if we travel from physical laws to material object statements and from the latter to sense datum statements, we are embarking on the problem of verification; and so on.

I hope that we have now arrived at a position where we can see that the problem of 'reduction' is mistaken: you can't translate the sentence 'There is a table there' into a very long and very involved combination of sentences saying what one would see and feel and hear when one could perceive the table: you can't, for (1) sense data statements have a truth of their own, and (2) a concept like 'table' is of an open texture which cannot be imitated by any combination of sense datum statements. No conjunction or disjunction of the latter ones exhausts the full meaning of a material object statement. Each such statement is, as it were, a hard core which resists any attempt at breaking it up. Yet, there is a connection between these two sorts of statement (for sense data are evidences) which have urged philosophers to 'reduce' the one to the level of the others, disregarding the deep cleavages which separate them. In short, just as the fracture lines on the earth's surface are marked by geysers and thermal springs, so the fracture lines of language are marked by philosophical problems.

It is due to the lack of insight into the many-level-structure of induction, theories of conclusive verification, logical constructions of language, that many mistaken theories have been formed — the theory of material objects, numbers, and so forth.

These considerations lead to a new picture of language, naturally stratified into layers. According to the older view (presented in Wittgenstein's *Tractatus*) the fabric of language is supposed to be of extraordinary simplicity: all statements are on the same level and constructed according to a simple and uniform plan — one and all of them are truth-functions of atomic propositions. Given the atomic propositions, any other statement can be derived from them. It can be shown, however, that atomic propositions are a myth; that truth functions are by no means the only principle of forming statements; and that different logics apply to different strata. In actual fact the structure of language is of a much higher complexity of which we have just caught a glimpse.

Let me conclude with this remark: We have been suffering from a certain one-sidedness in treating language; most of the efforts were concentrated on illuminating the structure of scientific theories, ideas and methods. But nothing equivalent to that has been done in other spheres which are no less important to human life. The whole world talks of love, but scarcely any serious thinker has given his time to a deep and searching study of the prob-

lems involved in emotion, or has tried to focus on the way in which we argue in every day life, or to give an account of the logic of exotic languages, or of the enormous logical problems involved in literature, poetry, law, ethics — with the result that many of these subjects have been left without anyone to apply the modern technique to such inquiries, and, if I may say so, with disappointing effects on an understanding of human nature. I think that it is only by turning to the whole of language, including all its strata, that we may hope to get a full view of the problems involved. The object of this paper would be attained if it stimulated an inquiry along the lines envisaged towards a neo-humanism.

### *Part Two*

**I. TYPES OF AMBIGUITY.** Both vagueness and 'open texture'<sup>1</sup> must be distinguished from, and likened to, another sort of lack of definition: ambiguity. Of the many types of ambiguity a few examples may be mentioned.

(i) A word may have two altogether different meanings, or better, there may be two words which have the sound in common; thus someone might say, 'How long it is since I have seen the Alps! How I long to see them'. This fact makes possible certain puns — as when a crying child is called the 'Prince of Wails'.

(ii) An extreme case of ambiguity is what is called the *antithetical sense of primal words*. There is evidence that in the oldest languages opposites such as: strong-weak, light-dark, large-small were expressed by the same root word. Thus, in ancient Egyptian *keu* stood for both strong and weak. In Latin *altus* means high and deep, *sacer* both sacred and accursed. Compare further *clamare*, to shout, and *clam*, quietly, secretly, or *siccus*, dry, and *succus*, juice. Nor is it only the ancient languages which have retained as relics words capable of meaning either of two opposites. The same applies to present-day languages. 'To cleave' means to split, but in the phrase 'cleave to' it means to adhere. The word 'without', originally carrying with it both a positive and a negative connotation, is used today in the negative sense only. That 'with' has not only the sense of 'adding to' but also that of 'depriving of' is clear

<sup>1</sup> See pp. 95–97 and Chapter II, pp. 41 ff.

from the compounds 'withdraw' and 'withhold'.

(iii) It is commonly impossible to distinguish between different senses of a word without considering how it is used in context. When used in different contexts, the same word may assume different senses. Take as an example the transitive and intransitive use of a verb: 'I smell the lilac', 'The lilac smells lovely'. But even when a verb is used transitively, it may take on different meanings when connected with words of different types: 'I caught him', 'I caught measles'. We use the word 'like' often in the sense of 'similar'; we say, for instance, 'That man is like his brother', and in this context we may also say 'very like', 'amazingly like', 'so like that one cannot tell them apart'. On the other hand, it would be amazing to learn of two triangles 'so alike that one cannot tell them apart'. In the one case the word admits of degrees of comparison, in the other it does not. Compare the phrase 'Find the key which —' with 'Find the number which —'. In spite of the sameness of the construction the difference in meaning is clearly felt. Thus I might have said 'Compute the number which —', but not 'Compute the key which —', a sign that the word is used according to different substitution rules. Again, compare 'I am trying to solve this equation', 'I am trying to remember a forgotten name', 'I am trying to fall asleep'.

(iv) A word which is used in a quite definite way and in quite definite contexts may be used in a new sort of context; with this change of use often goes a change in meaning. For instance: 'the fruit of a tree', 'the fruit of his labour', 'the fruit of his meditations'; 'to sow seed', 'to sow distrust'. What we use is a picture. If the image becomes a stereotyped figure of speech, we talk of 'figurative meaning'. (This is one of the means by which language grows. A speaker may, on the spur of the moment, place a word in a new collocation, thus giving rise to a new meaning — a process over which there is little control.) Now the point of this is that it is not always possible to say exactly where the metaphor ends and where the word starts having an independent meaning. The phrase 'to sow distrust' is felt to be a metaphor, perhaps also 'the fruit of his meditations', but not 'a fruitless attempt'. Here the pictorial element has faded. Glance through the following list and consider whether you would venture to draw a sharp line between a figure that is still a live image and one which has become a well-worn metaphor: 'The birth of tragedy', 'Drowned in sorrow', 'An

abyss of grief', 'A radiant spirit', 'A flight of phantasy', 'A fiery temper'.

(v) A word may be used in a 'figurative sense'. Remember that almost all terms denoting the mental are derived from words whose primary connotation was sensuous. Thus we speak of an idea 'floating in the mind', we 'call it to mind', we say it is still 'hazy'; an idea is 'engraved upon my memory', it 'makes an impression upon me'; something 'moves us', 'touches us', so that we are 'carried away'; we feel 'stirred', 'beside ourselves'; we talk of a 'brilliant idea', a 'flash of wit'; and so on.

This rising of the meaning of a word from the sphere of the sensuous to that of the mental continues to the present day. Think of expressions such as 'split personality', 'the layers of the subconscious', 'twilight of consciousness', etc. A sensuous element gleams through most of the phrases which denote emotions. We talk of 'shady', 'volcanic', 'unbridled', 'ebullient' characters; 'wooden', 'unpolished', 'crabbed' individuals; of an 'oily', 'smooth' manner, a 'stiff' attitude, a 'lukewarm', 'cool', 'icy' reception; of an 'arid', 'sparkling', 'will-o'-the-wisp' spirit.

The fact that language develops out of the sensuous into the mental produces a peculiar phenomenon: we seem at times to glimpse behind a word another sense, deeper and half hidden, and to hear faintly the entry of another meaning, in and with which others begin to sound, and all accompany the original meaning of the word like the sympathetic chimes of a bell. Hence that deep and sonorous ring in words which is lacking in artificial and invented languages; and hence also the multiplicity of meaning, the indefiniteness, the strange suggestiveness and evasiveness of so much poetry. Hugo von Hofmannsthal once described this phenomenon.

It leads us into the innermost nature of Oriental poetry, into the very mystery and being of language. For this mysteriousness is the deepest element in Eastern language and poetry alike, in so far as everything in it is metaphorical, everything remotely descended from ancient roots. The original root is sensuous, primitive, concise and strong, but the word moves away from it by subtle transitions to new, related meanings, and then to meanings only remotely related: yet in the remotest meaning there is still some echo of the original sound of the world, still some darkly mirrored image of the first sensuous impression . . . In the limitless detail and particularity of description the subject matter itself seems to oppress and overwhelm us: but what would come so close to us

as to hurt us, were we limited to immediate meanings, resolves itself by virtue of the multiplicity of meaning in the words into a magic cloud, and so behind the immediate meaning we divine another which is derived from it. Thus it is that we do not lose sight of the proper and original sense: where, however, this sense was commonplace and mean, it loses its implicit commonplaceness, and often, as we contemplate the word, we hesitate in our perceptive awareness between the particular reality which it symbolizes and a higher reality, and this in a flash leads up to the great and the sublime.

(vi) There are other cases in which the meanings cannot be as clearly separated out as, for instance, in the case of the word 'cold' (where I may say of a day or of a reception that it is cold). Consider a word like 'haughty'. That there is a difference in its use is shown in the fact that the word can be combined with words of very different logical types; thus we may speak of a haughty smile, a haughty tone of voice, a haughty face, a haughty look, a haughty bearing, a haughty speech, a haughty person. So there is a difference in meaning. Yet all these meanings are connected — in saying of somebody that he is haughty, at least part of what we mean is that he has a haughty face, or a haughty bearing, etc. So the meanings interpenetrate, and unite into a larger whole, a sort of cloud in which the several precise conceits are lose. We may say that they *dissolve into vagueness*. Such an example shows how ambiguity may gradually pass into vagueness.

'But shouldn't we still try to distinguish as clearly as possible all the different shades of meaning the word can assume?' Try, and you'll see how puzzling it is. Paul Valéry put this point very well when he said:

You must . . . at some time or another, have noticed this curious fact — that a given word, which may be perfectly obvious when used in the ordinary course of communication, which presents no difficulties whatever when caught up in the give and take of normal conversation, has a way of becoming almost magically embarrassing, strangely resistant and quite unmanageable in definition, as soon as you withdraw it from circulation with the object of examining it closely and apart from its neighbours, as soon, that is, as you try to establish its meaning in isolation from its momentary function. It is almost comic to note the difficulty with which we are confronted when we try to establish the *precise* meaning of a word which, in the ordinary routine of life, we use daily to our complete satisfaction . . . But isolate it, clip its wings and it turns and rends you. You soon become convinced that the number of its meanings is far in excess of its functions. Formerly it was only a

means, but now it is an *end*, the object of a terrible philosophical desire. It is something entirely different from what it was, an enigma, an abyss, a source of mental torment.

(vii) Next, consider a number of statements made by psychologists: 'We perceive the surface of the metal, it is true, but its colour seems to lie *behind* this surface'. 'Lustre-light does not lie *in* the plane of the object to which it belongs, but appears rather either *before* the object or *superimposed on* it.' 'When a shadow moves it moves not *in* the surface of the object but *across* it'. 'When a person is speaking with someone in complete darkness, the voice of the one who answers usually sounds distinctly *behind* the darkness, not *in* the darkness.' 'If you look at a colour disc which turns round quickly, it is better to say that there is a flickering *across* the disc or *before* it in space than to say that the disc *itself* is flickering.' Notice that in all these cases the prepositions which symbolize spatial relations take on a somewhat new sense. In the last example, for instance, to say that there is a flickering *before* the disc in space is to use 'before' in a peculiar way, namely so as to make it *meaningless* to ask exactly what distance, precisely how many millimetres before the surface it lies. Here we have a sense of 'before' or 'in front of' which differs from the ordinary sense. The same holds of the 'behind' in our first example. There are intermediate ones: thus a glowing piece of iron is seen as luminous *throughout* its mass; a rainbow, though extended in space before the observer, does not possess a surface. One feels that one can penetrate more or less deeply *into* the spectral colours, whereas when one looks at the colour of a paper the surface presents a sort of barrier beyond which the gaze cannot pass. The words 'throughout' and 'into' come here closer to, though they have not exactly, the ordinary meaning. Many more examples could be collected, but these will do. 'The English prepositions', says Empson, 'from being used in so many ways and in combination with so many verbs, have acquired not so much a number of meanings as a body of meaning continuous in several directions'. Exactly, there are so many senses and they are so firmly interlocked that they seem to form one continuous body. Thus many words which we wouldn't suspect turn out to be ambiguous. One can hardly make too much of this ambiguity of language through which we often seem to see words like shapes in a mist.

(viii) Then there is such a thing as *systematic ambiguity*. This expression was first coined by Bertrand Russell in connection with his Theory of Types. Without entering into it here we can say that his idea, roughly speaking, is that we must distinguish between different *logical types of symbols*. Beginning with names which stand for 'individuals', we come next to predicates which possibly apply to those names, and then to second-order predicates which possibly apply to the first-order predicates, and so on. We are thus led to consider a hierarchy of symbols which, theoretically, goes on without end. This hierarchy corresponds to a similar hierarchy of *statements*. And statements are divided into different types according to whether they are statements about an individual, or statements about a class of individuals, or statements about a class of classes of individuals, and so on. A statement such as 'Socrates is mortal' is true when there is a corresponding fact, and false when there is no corresponding fact. But take now such a statement as 'All men are mortal'. The truth of it can no longer consist in its correspondence to a single fact, for there are indefinitely many facts such as 'Socrates is mortal', 'Plato is mortal', etc. Now Russell's point is that the meaning of 'truth' which is applicable to the latter sort of proposition is *not the same* as the meaning of 'truth' which is applicable to the proposition 'All men are mortal'; i.e. each type of statement has its own sort of truth.

The main ground for accepting that distinction is that it offers an escape from the paradoxes or antinomies which were a threat to logic.

The imaginary sceptic, who asserts that he knows nothing, and is refuted by being asked if he knows that he knows nothing, has asserted nonsense, and has been fallaciously refuted by an argument which involves a vicious-circle fallacy. In order that the sceptic's assertion may become significant, it is necessary to place some limitation upon the things of which he is asserting his ignorance, because the things of which it is possible to be ignorant form an illegitimate totality. *Principia Mathematica* (Vol. I, Introduction).

Take the case of the Liar, that is of a man who says 'I am lying'; if he is lying he is speaking the truth, and if he is speaking the truth he is lying. We may interpret his statement as saying, 'All propositions which I assert are false'. Is this proposition itself true or false? To clear up the paradox we must distinguish between

elementary propositions which do not refer to a totality of propositions, first-order propositions which do refer to a totality of elementary propositions, second-order propositions which do refer to a totality of first-order propositions, and so on. Now if the liar asserts that all propositions which he asserts are false he is making a first-order statement which does not fall within its own scope, and therefore no contradiction emerges. The decisive point to realize is that the phrase 'all propositions' is an illegitimate totality. As soon as a suitable limitation has been put upon the collection of propositions we are considering, as soon as they are broken up into different orders, the contradiction disappears. We may put it like this: if somebody were to tell us that he is a liar, we could ask him, 'Well, a liar of what order?' If he says he is a liar of the first order he is making a statement of the second order, and this statement may be perfectly true. When he says 'I am a liar of the second order' (including the totality of first-order statements) this would be a statement of the third order; and so on. However far he may extend the scope of propositions to which he is referring, his statement about their falsehood will represent a proposition of higher order. Once we reach this stage, there is no contradiction.

Russell's solution is thus based on the ground that 'true' and 'false' are ambiguous, and that, in order to make them unambiguous, we must specify the order of truth or falsehood which we ascribe to a proposition. Similar considerations apply to negation and disjunction, and indeed to any logical particle. It might seem that they were symbols which had throughout the same meaning. But this is due to a systematic ambiguity in the meanings of 'not', 'or', etc., by which they adjust themselves to propositions of any order.

The ambiguity about which I want to speak is not connected with the Theory of Types but with what may be called the 'many-level structure' of language. I shall first of all explain what I understand by *a language stratum*.

2. LANGUAGE STRATA. Let me begin by introducing a distinction between two paths a logical inquiry may follow. It will perhaps be best to illustrate my point with a picture. In studying the geometry of a curve we may wish to find out its behaviour *at some particular point* — for instance, whether it has a tangent there, whether it is continuous there, what its measure of curvature

is there, and the like. Then we are studying *local* properties of the curve. Or we may wish to study the behaviour of the curve *as a whole* — for instance, whether it is closed or not, and, if it is closed, whether it is convex, etc. Then we are studying its properties *at large*. This picture suggests two different types of investigation in logic. The one takes its orientation from the logical relations which hold between a number of given propositions; a question of this sort is to ask whether a given proposition follows from another one, or contradicts it, or is independent of it, etc. We are then concerned with the logical nexus *on a small scale*, so to speak with *local* relations between propositions. Suppose, on the other hand, considering a certain deductive theory based upon a number of suitable axioms, say Euclidean Geometry or the Theory of Deduction, we ask whether the system under consideration is *free from contradiction*, that is, whether it is ever possible to prove a certain theorem and its contradictory. This is a question of quite a different kind. Suppose we say, 'The theory in question contains no contradiction', then we are making an assertion, not about the relations between two or three or more single propositions, but about the theory *as a whole*. Again, we may inquire whether the deductive theory we are considering is *complete*, i.e. whether any statements that can be constructed in accordance with the given rules (of the theory) can always be decided (in one way or the other) by the means of the theory and decided in a finite number of steps; we may also be investigating whether two given theories are isomorphic (i.e. of the same logical structure so that to each proposition of the one there corresponds precisely one proposition of the other, and vice versa, and that all the logical relations of the propositions in the one are retained in the other). Now in pursuing such questions we are concerned with what may be called the *macrological* features of such theories, in contrast with questions concerning the *micrological* connections of single statements.

A technique has been worked out to deal with problems of that macrological kind. Naturally, these methods — called 'metalogical' and 'metamathematical' — only apply to *deductive systems*. However, it does seem to me that there is also good sense in talking of macro- and micrological features of a *language*. Language, it is true, is not organized in the way a deductive system is; compared with such a system it is of a much more loosely knitted texture. And yet one feels a marked difference when one compares such statements

as: a material object statement, a sense-datum statement, a law of nature, a geometrical proposition, a statement describing national characteristics, a statement describing a half-faded memory picture, a statement describing a dream, a proverb, and so forth. It is as if each of these statements was constructed in a different *logical style*. (I will explain presently what I mean by this.) We may set ourselves the task of grouping statements of our language according to the similarity of their usage in distinct domains, in *language-strata* as I shall venture to call them. Thus laws will form one language stratum, material object statements another one, sense datum statements yet another one, and so on. Now the question which I want to consider is this: Is it possible to develop out of that vague feeling that 'each of them is built in a different logical style' something more precise? Is it possible, say, by characterizing each stratum on the basis of its intrinsic internal fabric or logical texture? To make this clearer let me return to the picture taken from geometry. It was a memorable achievement of mathematical thought when Gauss succeeded in characterizing a curved surface merely 'from within' without any reference to space outside, which amounted to this, that he showed that if two-dimensional beings were living on the surface of a sphere, an egg or a wine-glass, etc., they could, merely through carrying out certain measuring operations within their abode, find out in what sort of surface they were living; in other words, they could learn the 'intrinsic geometry' of their habitation without any reference to three-dimensional space. Now the analogous problem in our case would be this: Can a given language stratum be characterized, not by reference to something outside the subject-matter by dubbing it 'material object', 'memory picture' or the like, but by purely formal motifs? Let us see what means we have at our disposal for such a programme.

We may first investigate the nature of the concepts which a given stratum contains: whether they are absolutely precise and definable with mathematical rigour, or vague, or of an open texture. We may next consider the statements themselves and ask what sort of logic is valid for them. By 'logic' I mean logic in the strict sense, the laws of inference. Aristotelian logic, including the modernized and refined forms of its presentation in *Principia Mathematica*, has gone the same way as Euclidean geometry — a number of different 'logics' have grown up alongside it, more or

less akin to it, just as Euclidean geometry is now surrounded by a number of similar and cognate systems. One effect of this development is the disappearance of that disturbing air of uniqueness that had puzzled philosophers for so long. Birkhoff and von Neumann, for instance, have indicated a system, different from classical logic, which seems to be in better harmony with the structure of quantum mechanics. On the suggestion of Brouwer a logic has been constructed different from classical logic in which mathematical demonstration is actually employed, a logic in which the law of excluded middle is no longer universally true. And, notice, when we pass from the one logic to the other, we get an altogether different mathematics; which goes to show that the sort of logic we apply is an important characteristic ingrained in a certain field of propositions. Change the logic and then the propositions will take on new meanings. Take another example — the logic of half-faded memory pictures. Here the situation is such that we are often unable to call to mind one or the other point of detail, that is, that we are often unable to decide an alternative. What did that bathroom look like I saw the other day on a visit? Was it ivory, was it cream or pale biscuit or maize? Suppose a pattern-book were shown to me, and I was later asked whether *this* was the colour I had seen, perhaps I would not be able to decide. If I were pressed I might have to say, 'I can't remember so distinctly'; if another different shade of yellow were shown to me than I might give the same reply, finally adding, 'all I know is that it was some light yellowish colour'. Notice that, in this case, it is quite natural to use a *vague* term ('light colour') to express the indeterminacy of the impression. If language was such that each and every word was particular and each colour word had a definite, clearly defined meaning, we should find we could not use it. That is, we should come up against alternatives: 'Was it this colour or not?' — which we could not decide. I cannot get back to the impression I had then, it cannot be pinned down and preserved under glass for inspection like a dead beetle. To insist, in these circumstances, on the law of excluded middle, without any means of deciding the issue, is paying lip service to the laws of logic. There are only two alternatives open to us: We must either be prepared to drop the law of excluded middle when we wish to use a language with precisely defined terms; or we shall have to use a language whose words are in one way or another blurred. But we can't have it both ways.

Another way of bringing out this point is to say that, if several colours are shown to me which differ only slightly, they do not necessarily exclude one another. This shows particularly clearly that our attitude towards a half-faded memory image is radically different from that towards a material object. No one would dream of ascribing two different lengths to the table in this room (*a real table*), and saying that both were right. One statement, if it proves true, excludes the other. Whereas it is perfectly correct to say of two slightly different colour statements, when applied to an indeterminate memory picture, that both are compatible; which just shows that the logic of colour words, when applied in this language stratum, is different from their usual logic.

Again, the logic of aphorisms seems to be very peculiar. A man who writes aphorisms may say a thing, and, on another occasion, the very opposite of it without being guilty of a contradiction. For each aphorism, as it stands, is quite complete in itself. Two different aphorisms are not parts of one and the same communication. Suppose you go to a museum where several paintings are hung on the wall. Would you complain that they are not correlated and do not fit into one and the same perspective? Well now, each painting has a pictorial space of its own; what is represented in two paintings, though the paintings may be adjacent, is not in the same pictorial space. It is the first aim of Art, it has been said, to set a frame around Nature. Sometimes the frame is large, sometimes small, but always it is there. An aphorism is Literature and done with ink instead of colours. Of two aphorisms each is in a frame of its own; hence no clash. It would be interesting to penetrate the logic of poems, or of mysticism. Here a contradiction may be a perfectly legitimate means to point to what cannot be said in language. No: seeming contradictions are not always absurd.

To return to our subject: I said that the examples given suggest looking upon a logic as a characteristic which sets its stamp upon a particular language stratum. But there are two further characteristics: truth and verifiability.

**3. SYSTEMATIC AMBIGUITY OF TRUTH AND VERIFIABILITY.** Compare a variety of statements such as: a sense-datum statement, a material object statement, a law of nature, description of something half forgotten, a statement of my own motives, a conjecture as to the motives by which someone else was actuated, quotation of the exact words so-and-so was using, brief summary of the tenor

of a political speech, characterization of the *Zeitgeist* of a certain historical period, a proverb, a poetic metaphor, a mathematical proposition, and so on. Now what I want to emphasize is that the idea of truth varies with the kind of statement; that it has a systematic ambiguity. Take, for instance, a mathematical proposition, say a theorem of geometry. To say that it is true simply means that it can be deduced from such-and-such axioms. As a consequence of this, it may be true in one system of geometry and false in another. And the axioms themselves? They are no concern of the pure mathematician: all he is concerned with is that *if* these and these axioms apply, *then* the theorems apply too. But whether the axioms actually do apply, is not for him to decide. He leaves that to applied mathematics. Hence Russell's definition of mathematics as 'the subject in which we never know what we are talking about, nor whether what we are saying is true'. Here, then, is a very good case for the 'coherence theory of truth'.

Again, a law of nature is never true in the same sense in which, say, 'There is a fire burning in this room' is, nor in the sense in which 'He is an amusing fellow' may be; and the two latter statements are not true in the same sense in which 'I've got a headache' is. Truth, when applied to a physical law, means roughly speaking that it is well established by experimental evidence or other observation, that it brings widely different things into a close connection and makes us 'understand' what seemed a mystery before; that it simplifies our theoretical system, and further, that it is fruitful in leading us to predictions and new discoveries. (That is, incidentally, why the pragmatist identifies truth with usefulness: he has really got hold of one facet, but of one facet only.) Truth, in this case, it may be said, is not *one* idea but a whole bundle of ideas. Nothing of this applies to truth in the case of a simple observation. Suppose you have to make sure that the light is on in your room. Now when you go and look and say 'All right, it's on', your statement is true, *not* because it brings widely different things into connection, *not* because it simplifies I don't know what, *not* because it is fruitful or suggestive — no, nothing of the sort; it is just true because it says so-and-so is as you say it is.

Again, in what sense is one to say of a proverb that it is true? Have you ever tried to put some rare and subtle experience, or some half-forgotten (but strong) impression into words? If you do, you will find that truth, in this case, is inseparably tied up with the

literary quality of your writing: it needs no less than a poet to express fully and faithfully such fragile states of mind. How you say it matters even more than what you say.

Similar remarks apply to verification. A law of nature can be verified by experimental evidence, though not conclusively. Whether a material object statement is capable of conclusive verification is a moot point. Take next a case such as 'I've got a terrible toothache'. Suppose I go to the dentist, he examines my teeth and says, 'All right, there's nothing wrong with them'. Would I then reply, 'Oh, I beg your pardon, I *thought* that I've got a toothache, but now I see that I was mistaken'? My toothache cannot be argued away or refuted by examining my teeth, my nerves, etc. If I were asked how I know that I've got a toothache, I might be tempted to reply, 'Because I *feel* it'. What a queer sort of reply! Is there anything else I can do with a toothache but feel it? What my reply aimed at, however, was something different, namely to *shake off* the whole question as improper, beside the point. How do I know? I've simply got toothache, and that's the end of it. I do not grant that I may have fallen victim to a delusion, I do not recognize a medical examination, an observation of my teeth any psychological tests, a court of experts — no dentist in heaven or earth can refute me. In saying 'I just *feel* it' I am expressing the fact that the toothache is something *given in immediate experience*, not a thing *inferred from something else* on the strength of certain evidences. The first person singular has, amongst other uses, the function to indicate the character of *immediacy* of an experience.

Take the statement, 'There are sea serpents'. How would you verify it? Is it enough that some person has seen them? Perhaps for him; for you the situation is different: you have so far only a man who *says* that he has seen them. So you must check up what he says — you may test his eyesight, go into his past and examine his reliability, and so on. The result of this checking will be a number of statements each of which, in its turn, may again be checked: the expert who examined the man's eyesight may himself be examined, the witnesses who testified may in their turn be scrutinized, etc. In following up the threads of verification we nowhere come to an absolute end, that is, we can never say, 'Now it is conclusively proved that the man was right'. What this particular example shows applies in general. At some point we do

stop, it is true, for practical reasons, when the evidence seems to be sufficient. But theoretically we may go on checking and re-checking our statements as long as we please. So long as we move amongst statements concerning such evidences as illustrated above, verification has no natural end, but refers continually to ever new statements. In pursuing these fibres, however, we see how secondary lines branch off into other regions: the points where they come to a sudden end represent those immediate experiences which an observer has the moment he experiences them, and which, in this moment, cannot be checked against other evidences. These experiences, expressed in 'I'-sentences, are, so to speak, end points of verification — but of verification in a quite different sense. For if we try to use this verification later, it turns to dust. It lives in the moment, and is gone. Still these experiences are the moments of ultimate fulfilment. It is they from which all light of knowledge flows forth. Or, to change the metaphor, they are the points in which knowledge makes direct contact with reality. Without them all our sentences would float in the air cut off from actual facts. What establishes a connection between sentences and reality are these last points of verification, transitory though they may be. Thus a statement may be verified in two quite different senses: either by checking it against other statements, or by appealing to immediate experience. In the case of a material object statement, for instance, some lines refer to other material object statements, i.e. they lead from statement to statement within the same language stratum; some others branch off and penetrate into a different stratum, the 'I'-statements. Thus verification weaves a complicated net, a ramified pattern of lines.

It is easily seen that the term 'meaningful' displays the same ambiguity: its sense varies with the stratum. For instance, a sentence in a novel is meaningful, if (1) it is correct English, i.e. not a broth of words, and (2) it fits in with the other sentences. This meaningfulness has nothing whatever to do with verifiability. (That, by the way, is why Fiction is not false.) This criterion, however, does not apply to experiential statements where verifiability is of some relevance, although it would not be right to equate meaningfulness with verifiability. Again, in which sense is a rule, a definition, a request, a question meaningful? There may even be a sense in which metaphysical statements have a meaning. The trouble with the Logical Positivists was that they attached too

rigid an import to 'meaningfulness' and lost sight of its ambiguity. By virtue of the multiplicity of meaning in this word they lost themselves in a magic cloud out of which they condemned everything that did not conform to their standards. In actual fact they had no machinery, such as they thought they had, by which the senselessness of metaphysics could be *proved*; though it must be admitted that metaphysicians made the greatest efforts to supply them with plausible arguments for such a view. I am afraid that what has been said on this subject was of a profound shallowness.

To sum up this point: Statements may be *true* in different senses, *verifiable* in different senses, *meaningful* in different senses. Therefore the attempts at defining 'truth', or at drawing a sharp line between the meaningful and the meaningless, etc., are doomed to fail.

4. COMPLETENESS. Up till now I have tried to sketch a few leitmotifs which might be used in characterizing a given language stratum. They were: the texture of the concepts together with the sort of logic which obtains and the appropriate senses of 'truth' and 'verifiability' (if the latter applies at all). To these must be added two more factors: the way in which a proposition is integrated into a larger whole; and the relations in which different strata stand to each other.

In order to approach the first point, it will be best to make use again of a geometrical illustration. Suppose you consider a number of statements which are about the same subject *a*. Each of these statements may be represented by a circular area, and the conjunction of two statements by that part of the areas which overlap. The more propositions we take, the smaller will the area become which is common to all the discs; at the same time, the more definite will become the description formed by all these statements. This gives rise to the following problem: Is there anything like a description of *maximum* definiteness, for instance, a description whose geometrical picture is a point? If there was such a thing, this would mean that the description is *complete* in the sense that nothing could be added to it which would make it more definite. Well now, are there language strata in which it is possible to construct something like a 'closed' description? And if so, will such a description contain a finite or an infinite number of single statements?

Geometry provides us with a model in which a complete (closed, perfect) description is attainable and with a finite number of

statements. Thus a triangle is determined when its three sides are given: nothing can be added to these data that is not entailed by, or in contradiction with, them. Here, then, is an example of a description which, on logical grounds, cannot be extended.

A quite different situation seems to hold with regard to experiential statements. However many features I may assert of a thing, say of this chair, or however many relations I may state which hold between it and other things, or however many statements I may make about its life-history, I shall never reach a point where my description can be said to be *exhaustive*, that is, such that no further increment in knowledge is possible. Any real thing is inexhaustible. My knowledge of it is always extensible. There is no maximum description. To use the geometrical illustration: Such a description will always be represented by a whole *region*; a point will be a limit toward which the description tends without ever reaching it. Thus the picture which we make of an experiential statement on our map will never shrink to a point.

There are, however, cases where a complete description *is* attainable. Take, for instance, a game of chess played in a tournament: it can be described, completely, move by move, from the beginning to the end, say in the chess notation. Again, a melody is describable completely in the musical notation (disregarding, of course, questions of interpretation). The same is true of a carpet, viewed as a geometrical ornament of shape and colour.

How curious it is when I describe a dream: when I have narrated my dream, told everything that happened in it, my description is finished. But it comes to an end in a very different way from that in which, e.g., the description of a game of chess comes to an end where there is a natural beginning and a natural end. A dream is fragmentary, enigmatic, and a dream cannot be integrated into a larger whole: you cannot ask, 'What happened before the dream began, or after it was over?' Or rather, *when* you ask such a question, you have already left the dream language and consider the sleeper from outside, from the point of view of a waking man. In this respect a dream has a unity and coherence which makes it nearly akin to a poem, or an aphorism.

The few examples given will suffice to show that statements may be *complete* in very different senses, and that the way they are complete or incomplete is a further important feature of a language stratum.

Finally, we have to investigate in what relations different strata stand to each other. I shall leave this for the moment and confine myself to mentioning one question only: whether the threads of verification, when we follow them up, remain within the given stratum, or lead outside it; in other words, whether a given stratum is *closed* with respect to verification.

5. A NEW PICTURE OF LANGUAGE. We are now in a position to sketch a new picture of language which, though still untried, seems to emerge from all these considerations; a picture of language naturally stratified into layers. This new conception contrasts with such a view as that held by Wittgenstein in his *Tractatus*: according to that view language consists of statements which can, one and all, be derived from atomic propositions by a uniform process. An atomic proposition is one asserting an atomic fact; an atomic fact is a fact which has no parts that are facts; and the uniform method by which any statement can be constructed is that of building up truth-functions of any selection of atomic propositions. This leads to an amazing simplification of the picture we can make for ourselves of the fabric of language. All statements are, so to speak, on a footing, and all are reducible to the same set of atomic propositions. Or better, the totality of propositions is defined by this method of generation. Too good to be true. Apart from the fact that no one has ever succeeded in producing a single atomic proposition, the whole thing is a myth. Moreover, we know for certain that there are many ways of building up statements which have nothing at all to do with truth-functions; such as unfulfilled conditional statements — 'If Hitler had won the war, then . . .' and many others. No: language does not fit this strait-jacket.

There are certain modern trends in Philosophy which seem to have some such background. Phenomenalism, for instance, seems to presuppose that there is one basic language, the sense-datum language, to which any other statement, or at least any material object statement, can be reduced. According to Phenomenalism a material object, say a cat, is a bundle of sense-data tied together and with the edges trimmed off; unless it is a bundle of *sensibilia*, that is the sort of thing which you *would* have seen, if you *had* ever looked; in short, a bundle of highly problematical entities. But no: we have simply to recognize that a statement about a cat is a statement about a cat: and not a truth-function of sense-datum

statements, or an infinite class of perspectives, or an infinite group of *sensibilia*, or heaven knows what. A thing is, so to speak, a hard core that resists any attempt at breaking it up and reducing it to the level of other data, whatever they may be. All this talk about material objects and sense-data is a talk about two language strata, about their relation, about the logic of this relationship. The problem arises along the plane where the two strata make contact, so to speak. The difficulty is to understand in precisely which way a material object statement is related to a sense-datum statement; that is, what sort of relations hold between members of different strata; and that is a problem of logic.

Similarly, Behaviourism is an attempt to reduce psychological statements, e.g., 'What a conceited fellow!' to a very, very long list of statements setting out in which way the person in question would behave under such-and-such circumstances; a very successful way of describing peculiarities of rats which has been transferred to men. The whole thing rests on a *naïveté* — that there is one basic language (suitable for describing the behaviour of rats) into which everything else must be translated. The motto 'Only rats, no men!' overlooks the fact that psychological statements belong to a stratum of their own, with a logic different from that of the language in which you say how a person looks, how he smiles, in short what he has in common with a rat.

We are now in a position to take a further step. It was hitherto the custom to refer to what I have called 'strata' by indicating their subject-matter, using terms such as: 'material object statements', 'descriptions of vague impressions', 'statements of laws of nature', and the like. What I now suggest we do — and this is a programme for the future — is to reverse the whole situation by saying: 'The formal motifs which we have been considering all combine to impress a certain stamp on a stratum; they give us the means to characterize each stratum "from within", that is with no reference to the subject'. If we carefully study the texture of the concepts which occur in a given stratum, the logic of its propositions, the meaning of truth, the web of verification, the senses in which a description may be complete or incomplete — if we consider all that, we may thereby characterize the subject-matter. We may say, for instance: a material object is something that is describable in a language of such-and-such structure; a sense impression is something which can be described in such-and-such

a language; a dream is —, a memory picture is —, and so on. In this way we shall be able to *formalize* these concepts. The analogy with science is obvious. The questions, 'What is a point?' 'What is a straight line?' have been debated for more than 2000 years until the solution was found in a reversal of the problem situation. All the time it was thought that we must first define the meaning of the primitive symbols in geometry before we can see that the axioms are 'self-evident truths' given in intuition. In modern times the terms 'point', 'straight line', 'plane', 'between', 'congruent', etc., are defined as those things and relations which satisfy the axioms of geometry. That is, the axioms in their totality *determine* (within pure mathematics) the meaning of the primitive symbols. In like manner we may say that each stratum has a logic of its own and that this logic determines the meaning of certain basic terms. In some respects this is obvious. Whether a melody is a sequence of air-vibrations, or a succession of musical notes, or a message of the composer, depends entirely on the way you describe it. Similarly, you may look at a game of chess, or on the pattern of a carpet from very different aspects and you will then see in them very different things. Notice how all these words — 'melody', 'game of chess', etc. — take on a systematic ambiguity according to the language stratum in which you talk. The same applies to 'doing a sum', 'writing a letter', or to any action indeed. An action may be viewed as a series of movements caused by some physiological stimuli in the 'Only rats, no men' sense; or as something that has a purpose or a meaning irrespective of the way its single links are produced. An action in the first sense is determined by *causes*, an action in the second sense by *motives* or *reasons*. It is generally believed that an action is determined both by causes and by motives. But if the causes determine the action, no room is left for motives, and if the motives determine the action, no room is left for causes. Either the system of causes is complete, then it is not possible to squeeze in a motive; or the system of motives is complete, then it is not possible to squeeze in a cause. 'Well, now, do you believe that if you are writing a letter you are engaged in two different activities?' No; I mean that there are two different ways of looking at the thing; just as there are two different ways of looking at a sentence: as a series of noises produced by a human agent; or as a vehicle of thought. For a series of noises there may be causes but no reasons; for a series of words expressing thought

there may be reasons but no causes. What we must understand is that the word 'action' has a systematic ambiguity. And yet we are continually invited to regard motives as a special sort of causes; perhaps because we have only the word 'Why?' to ask both for cause and motive. We do not see the ambiguity of the interrogative.

## CHAPTER V

### ANALYTIC—SYNTHETIC

1. WHAT IS ‘ANALYTIC’? Kant says<sup>1</sup> ‘In all judgements in which there is a relation between subject and predicate . . . that relation can be of two kinds. Either the predicate B belongs to the subject A as something contained (though covertly) in the concept A; or B lies outside the sphere of the concept A, though somehow connected with it. In the former case I call the judgement analytical, in the latter synthetical. Analytical judgements (affirmative) are therefore those in which the connection of the predicate with the subject is conceived through identity, while others in which that connection is conceived without identity, may be called synthetical. The former might be called illustrating, the latter expanding judgements, because in the former nothing is added by the predicate to the concept of the subject, but the concept is only divided into its constituent concepts which were always conceived as existing within it, though confusedly; while the latter add to the concept of the subject a predicate not conceived as existing within it, and not to be extracted from it by any process of mere analysis . . . It is clear from this that our knowledge is in no way extended by analytical judgements, but that all they effect is to put the concepts which we possess into better order and render them more intelligible’.

This definition may seem clear enough; yet isn’t it surprising how easy it is to raise questions which are plainly embarrassing? What, for instance, is meant by saying that, in an analytic judgement, the concept of the subject is ‘only *divided into* its constituent concepts’? Is the subject-term to be regarded as a sort of sum total of its constituent concepts, i.e., of all those which, analytically, can be asserted of it? This doesn’t seem to make sense. Suppose we make a number of analytic judgements such as S is P<sub>1</sub>, S is P<sub>2</sub>, . . . , S is P<sub>n</sub>, these being *all* the analytic judgements we can make about this particular subject S. Then, according to Kant, it would seem natural to say that S is thereby ‘divided’, or ‘dissolved’<sup>2</sup>, into the

<sup>1</sup> *Critique of Pure Reason*: Introduction IV.

<sup>2</sup> In German ‘*durch Zergliederung zerfällt*’ (*Critique*), ‘*aufgelöst*’ (*Proleg.*).

constituent concepts  $P_1, P_2, \dots, P_n$ . But what meaning are we to attach to this? Shall we say, e.g., that  $S$  is the collection, or the class, or the totality of all the members  $P_1, P_2, \dots, P_n$ ? To assert of a class any of its member concepts, e.g., to assert of the class  $\{P_1, P_2, \dots, P_n\}$  that it is  $P_1$ , would be absurd. We don't say of the attributes of a thing that *they*, all of them, have any of these attributes. Take the simplest case in which the class consists of one member only, say  $P$ . Then the class consisting of  $P$  will be different from  $P$ , and to say of this class  $\{P\}$  that it is itself  $P$ , i.e. that it has the property expressed by 'P', would be self-stultifying, or more accurately, it would be a type-fallacy, landing us in a lot of well-known logical contradictions. So this way is blocked: the relation of  $S$  to any of its predicates must be different from the relation of a class to any of its members. In what, then, *does* it consist? The same difficulty can be brought out in a slightly different way when we ask — What is meant by saying that the predicate is 'contained in' the concept of the subject, or is 'conceived as existing within it', or that the former can be 'extracted from it' — turns of speech which sound as if they were taken from dentistry? The word 'contain', even if not taken in its strictly spatial sense 'to enclose', is used in a great many different ways as when I say, 'A pound contains 16 ounces', 'This book contains some valuable information', 'The premiss of an inference contains the conclusion', and the like. In *which* of these senses, then, is the predicate to 'be contained in' the notion of the subject? Perhaps in the sense of our last example, that is, in the sense in which a conclusion is often held to 'be contained in' the premiss from which, according to the same view, it can be 'extracted' by inference? That, it seems, is an analogy worth following. But if so, what exactly *is* the relation that holds between the premiss and the conclusion of an inference? Shall I say, e.g., that whenever I think of the premiss, I *coincidentally* think of the conclusion? That would be glaringly untrue: I may consider the premiss *without* noticing some particular conclusion. (How on earth could a mathematical discovery be thrilling?) Saying that the latter is 'contained in' the former, whatever this may mean, can certainly not be taken to refer to any *psychological* relationship between the two such that thinking of the one is accompanied, or followed, by thinking of the other. No; in studying the logical relation between the parts of an inference, we are clearly *not* investigating what is actually going on, or what may be going on,

in someone's mind. Similarly in our case: to say, in the case of an analytic judgement, that the predicate is 'contained in' the concept of the subject can certainly not be taken to mean that whoever thinks of the subject will simultaneously, or a little while later, think of the predicate, or something of this sort. So far our result is mainly negative: we see that the relation can certainly not be a *psychological* one, though we are baffled to say *what* it is. I don't think that these questions are raised wantonly; they present themselves to anyone who tries to understand clearly what an analytic judgement is. Our difficulty is, at least in part, due to the fact that Kant, when he speaks of 'analytic', is unwittingly using nothing but metaphorical terms, which hint at, and at the same time obscure, what the true relation is. Nor is this the only problem with which we are confronted. Another difficulty arises from the fact that Kant's definition is such as to apply, according to his own words, only to statements of the subject-predicate pattern, ignoring other types — such as relational, and existential statements, let alone mathematical formulae like  $7 + 5 = 12$  (which Kant, oddly enough, cites as an example of a synthetic judgement). Subject and predicate are, after all, ideas which are borrowed from the grammar of certain word sentences, but which cannot, without destroying the clarity of our ideas, be applied to forms so fundamentally different. In other words, Kant's definition is *too narrow*, a fact we shall do well to keep in mind too.

Attempts have been made to amend Kant's definition. To quote a recent writer on the subject, A. Pap<sup>1</sup> 'Analytic statements . . . may be roughly characterized as statements whose truth *follows* (my italics) from the very meaning of their terms'. Here I immediately come up against a stumbling block: what can be meant by saying that a statement *follows from the very meaning of its terms*? I should have thought that *one* statement can follow from *another*; but from the meaning — ! Yet, strangely enough, such a view has been taken by no one less than Frege. Criticizing the formalist account of mathematics — the view according to which mathematics is but a sort of game played with inkmarks on paper instead of with chessmen on a board — he very emphatically says: 'If there were any meaning to be considered, the rules [of this game] could not be arbitrarily laid down. On the contrary, the rules follow

<sup>1</sup> 'Indubitable Existential Statements', *Mind*, 1946.

necessarily from the meaning of the marks.<sup>1</sup> Frege's idea seems, roughly, to have been this: If I write down a rule, e.g., the equation ' $2 + 2 = 4$ ', this may be regarded as a configuration in a game not so very unlike a configuration of chessmen on a board. But then the question of truth and falsehood does not arise. As soon as I come to *know*, however, what the marks '2', '+', '=' , '4' mean, I am no longer free to choose any configuration I please to include in my game: the truth of that equation seems rather to be *grounded in* the meaning of those marks. It would lead us far to dig down to the roots from which Frege's mistake springs. One word, however, will not be amiss. Whoever says that a rule, say, an equation, *follows* from the meaning of its terms, is bound to make quite clear what he understands by that. If someone tells me that the rule, ' "John" is spelt with a capital' follows from, or is a consequence of, the rule, 'Any proper name is spelt with a capital', I have no difficulty in understanding what he means. If, on the other hand, he tells me that an equation follows from the meaning of its terms, or that an analytic statement is one whose truth follows from the meaning of its terms, I am absolutely at a loss to make head or tail of it. One thing, at any rate is clear: as the meaning is not a starting point for making deductions, 'follows', in this context, can not mean 'logically follows'; so what *can* it mean? Queer that so subtle a mind as Frege should have failed to see that there is a problem, dropping not the slightest hint as to what he had in mind.

A different approach is made by M. Schlick.<sup>2</sup> The predicate 'is contained in the concept of the subject', he says, 'can only mean that it is part of its definition'. This interpretation has two merits: first, it is not open to the objections we had to raise against Frege; and secondly, it can easily be extended so as to cover other types of judgements. Schlick thus arrives at the following formulation: A judgement is analytic if the ground for its truth lies solely in the *definitions* of the terms which occur in it. 'Consequently, one may say with Kant that analytic judgements rest upon the law of contradiction; they derive from definitions by means of this law'.<sup>3</sup> What Schlick obviously has in mind is that a judgement, or a

<sup>1</sup> *Grundgesetze der Arithmetik*, vol. ii, 1900, p. 158; translated as *The Basic Laws of Arithmetic*, Berkeley, 1964.

<sup>2</sup> *Allgemeine Erkenntnislehre*, 1st ed., 1918, p. 97.

<sup>3</sup> *Loc. cit.*

statement, is analytic if it follows from mere definitions only by logical inference. Similarly Ewing (*A Short Commentary on Kant's Critique of Pure Reason*, 1938): 'An analytic judgement is one which follows (my italics) from the definition of its subject-term.'

Before proceeding to deeper-lying questions let us see whether the last two definitions are satisfactory. It undeniably marks a great advance in them that they refer, not to such elusive and questionable entities as the 'meaning of the terms', but to *definitions*. Before, however, we acquiesce in this view we are bound to ask — Is it really free from any obscurities? Strange as it may seem, when I attend to the question a bit more closely, I become doubtful. If an analytic statement is characterized as one that follows from mere definitions, why is it not itself a definition? A definition behaves in many respects like a rule, e.g., a rule of chess: it is *prescriptive* rather than *descriptive* — it tells us how a word, or a symbol, *is* to be used, not what its actual, or predominant, usage is. If I wish to assert that a definition given is in accord with the actual, or the prevailing, use of language, then I am, truly or falsely, making a *statement*, and no longer laying down a mere definition. There are further analogies between a definition and a rule. Thus a definition may be employed in the *learning* of a language in much the same way in which, say, the rules of chess may be employed in learning chess; a definition may be referred to in order to *justify* a certain use of words in a way similar to that in which a rule of chess may be referred to in order to justify a certain move. A definition, like a rule, can be *set up* at a certain time, and *abandoned*, or *altered*, later on; it may be *recognized*, or it may not; a definition, like a rule, can be *observed*, or be *infringed* in practice; and there are perhaps more such features. In view of this far-reaching analogy it seems very odd that it should break down at one point: what follows from a rule will, generally, be another rule (see the example concerning spelling); why, then, is it that what follows from a definition is, not, as one would expect, a *definition*, but an analytic *judgement*? Why suddenly this difference?

Let us first take a look at one or two examples in order to throw some light on the matter. Suppose I define a dragon as a fabulous winged serpent breathing flame, and a serpent, in its turn, as a scaly reptile, then I can derive from these two definitions the sentence 'A dragon is a fabulous winged and scaly reptile, breathing flame', which, to all appearance, might well pass for an

alternative definition of ‘dragon’; certainly it is not the sort of thing which would be called ‘analytic’. Again, if I define mephitis as poisonous stench, poisonous as a property that causes harm to life, or death, and stench as a foul smell, I am led on to say that mephitis is a foul smell that causes harm to life, or death — a sentence which, in ordinary circumstances, will be taken by most people as a mere re-wording, or a more explicit form, of the definition.

So far we were considering examples culled from word language. Before jumping to conclusions, let us be cautious and look at different sorts of examples, for instance, from symbolic logic, or from arithmetic. Suppose I start with the following three definitions:

$$p \supset q . = . p \vee q \text{ Df} \quad (1)$$

$$p \vee q . = . \sim p \mid \sim q \text{ Df} \quad (2)$$

$$\sim p . = . p \mid p \text{ Df} \quad (3)$$

Add to these the theorem

$$\vdash . \sim \sim p . \equiv . p \quad (4)$$

Before consequences from these formulae can be derived, certain rules of inference must first be laid down; let the following ones be chosen:

- (I) The relation expressed by the sign ‘=’ is symmetrical and transitive.
- (II) Two equivalent expressions may be substituted for each other in any occurrence.
- (III) The statement variables ‘ $p$ ’, ‘ $q$ ’, etc. may be replaced by other, and possibly complex, statement variables. (Rule of substitution.)

Applying these three rules, we may transform  $p \supset q$  as follows:

$$p \supset q . =_{(1)} . \sim p \vee q . =_{(2)} . \sim \sim p \mid \sim q . =_{(4)} . p \mid \sim q . =_{(3)} p \mid q \mid q.$$

Here each step of the transformation is made according to the rules (I)–(III), and, moreover, in accordance with one of the formulae (1)–(4), as indicated by the subscripts. The transformation yields

$$p \supset q . = . p \mid q \mid q \quad (5)$$

which, in view of our three rules of inference, is a *logical consequence* of (1)–(4). As (5) might have been chosen as a *definition* of the symbol ‘ $\supset$ ’ in Sheffer’s notation, we have exactly the same result as before: what can be derived from definitions (and logical truths), is a definition. If, however, we were to rest content with these examples and enunciate it as a general principle that whatever follows from a definition is a definition, we should be mistaken; as can be seen from the example

$$2 = 1 + 1 \text{ Df} \quad (1)$$

$$3 = 2 + 1 \text{ Df} \quad (2)$$

$$4 = 3 + 1 \text{ Df} \quad (3)$$

$$a + (b + 1) = (a + b) + 1 \text{ Df} \quad (4) \text{ (recursive definition of addition)}$$

With the help of these four definitions it is easily proved that  $4 = 2 + 2$ :

$$4 = {}_{(3)}3 + 1 = {}_{(2)}(2 + 1) + 1 = {}_{(4)}2 + (1 + 1) = {}_{(1)}2 + 2.$$

The result

$$4 = 2 + 2$$

is, admittedly, derived from mere definitions; yet to regard it as a *definition* of ‘4’ would be most unnatural. Why not regard  $8 \times 7 = 56$  as a definition of the number 56? Why, indeed, not regard *any* numerical equation as a definition?

In this way we come to see that it is only in *certain* cases true to say that what follows from a definition is a definition, whereas in other cases it is not. Why this should be so is very puzzling, and we shall have to go into this point more fully. However that may be, the examples adduced are, I think, sufficient to dispose of the view that an analytic statement is one that *follows from the definitions* of its terms.

The true state of affairs can be expressed in a somewhat different way. Instead of saying ‘A statement is analytic if it *follows from definitions*’, we shall have to say ‘A statement is analytic if it can, by means of mere definitions, be *turned into a truth of logic*’, i.e., if it is *transformable* into such a truth. Consider an example. Suppose I define a planet as a heavenly body moving round the sun. (This is not quite accurate as it fails to distinguish between planets on the one, and asteroids, comets, meteors, etc. on the

other hand; and further because it ignores the fact that some other fixed stars also have planets. For the sake of simplicity let us, however, disregard these complications and keep to the simple definition.) If I now say, 'All planets move round the sun', I am making an analytic statement. This statement is such that, in virtue of the definition, it can be *turned* into a logical truth. Indeed, replacing 'planet' by its *definiens*, we get 'All heavenly bodies which move round the sun move round the sun', which is precisely the sort of truism one would expect to find in an analytic statement. Although we can see 'with the naked eye' that this is a logical truth, we cannot yet identify the skeleton of the sentence in question with some definite logical formula, i.e., we cannot put a finger on the precise spot in PM, or in any other textbook of logic, saying, '*That's* the logical form of the statement'. We shall rather have to transform our sentence by a number of steps until the logical skeleton of the proposition it expresses can be seen with perfect clarity. And in doing this, I hope to make an incidental gain — to throw more light on the status of a definition. Up till now the behaviour of a definition must have appeared rather erratic; we shall understand the deeper reason for this far better when we come to see a definition in its natural setting of similar and related structures. To this end I shall make use of a somewhat new notation which will help to throw this point into relief.

Let  $p$  stand for the original sentence, 'All planets move round the sun'. The first step is to translate the 'all' idiom into the idiom 'there is no such thing that not'; for instance, 'All men are mortal' can be paraphrased as 'There is no man that is not mortal'. Call this transformation  $T$ . Applying  $T$  to the sentence  $p$  in the way of an operator, we obtain

$Tp$  = There is no planet that does not move round the sun.  
The next step is to put the term 'planet' in the place of a predicate; let  $L$  symbolize this process, and write simply  $LTp$  for  $L(Tp)$ :

$LTp$  = There is no thing such that it is a planet that does not move round the sun. Next eliminate the last restrictive clause and put it conjunctively (operation  $R$ ):

$RLTp$  = There is no thing such that it is a planet and that it does not move round the sun. According to the principle of double negation ( $N$ ), this can be expanded to saying:

$NRLTp$  = There is no thing such that it has not the following property: it is a planet, and it does not move round the sun. Now

translate the idiom 'there is no such thing that not' back into the 'all', or 'whatever' — idiom (transformation  $T^{-1}$ ):

$T^{-1}NRLTp$  = Whatever a thing may be, it has not the following property: it is a planet, and it does not move round the sun.

According to the rule of De Morgan ( $M$ ) this is further equivalent to

$MT^{-1}NRLTp$  = Whatever a thing may be, it has the following property: it is not a planet, or it is not the case that it does not move round the sun.

Apply  $N^{-1}$  (the converse of  $N$ ):

$N^{-1}MT^{-1}NRLTp$  = Whatever a thing may be, it has the following property: it is not a planet, or it moves round the sun.

Now the denial of 'If something is a planet, it moves round the sun' is 'Something is a planet, and it does not move round the sun'; as the denial of the latter, according to  $M$  and  $N^{-1}$ , is 'Something is not a planet, or it does move round the sun', and as the two denials cancel ( $N^{-1}$ ), this must come to the same as the first sentence; which shows that the 'if' idiom can be transformed into the 'not' and 'or' idiom ( $I$ ):

$IN^{-1}MT^{-1}NRLTp$  = Whatever a thing may be, it has the following property: if it is a planet, it does move round the sun.

Let finally  $D$  stand for the definition of 'planet'; applying  $D$  as a sort of operator, we get:

$DIN^{-1}MT^{-1}NRLTp$  = Whatever a thing may be it has the following property: if it is a heavenly body that moves round the sun, then it moves round the sun.

Repeating the  $R$ -step yields:

$RDIN^{-1}MT^{-1}NRLTp$  = Whatever a thing may be, it has the following property: if it is a heavenly body and if it moves round the sun, then it moves round the sun.

Now at last we have reached the stage where the structure of our statement can be seen to coincide with a quite definite formula in PM, namely, with

$$(x) : \phi_x . \psi_x . \supset \psi_x .$$

In words, whatever has the property  $\phi$  and the property  $\psi$ , has the property  $\psi$ . This collects together a range of statements, each of which is of the form  $p . q . \supset . q$ , thus clearly exhibiting its tautologous character. In this way it is finally seen that the statement under consideration is a truth of logic. (In the trans-

formation just carried out only the more important steps have been accounted for; some of the purely idiomatic steps of paraphrasing have been neglected or telescoped.)<sup>1</sup>

It may seem pedantic, indeed excessively so, to go to such lengths to prove a very trivial thing. However, it helps us to see one point which we should not so easily have seen otherwise, viz. how similar the use of a definition is to that of other tools of transformation. Indeed, in looking back on the whole chain of transformations we have carried out, it becomes clear how near a definition comes to any of the other operators — as far as its *function* goes; and how unnatural, for this reason, it would be to separate sharply the concept of a definition from that of other transformers.

A word must here be said on the notion of an *operator*. Suppose we consider any logical equivalence, i.e. an expression of the form

$$(\dots) \equiv (\dots)$$

As we pass along the equivalence, we are led from one expression to another without change of meaning; now there are two directions of doing so, from left to right, or from right to left. Correspondingly, each logical equivalence gives rise to a pair of inverse operators, say  $\Omega$  and  $\Omega^{-1}$ , each of which can be used for transforming an expression into an equivalent one. Conversely, any such operator may be re-written as an equivalence, read from left to right, or from right to left, as the case may be. In this way operators and equivalences are intimately related. Thus we have the following correspondences:

$$\begin{aligned} N: \quad & p \equiv \sim \sim p \\ M: \quad & \sim(p \cdot q) . \equiv . \sim p \vee \sim q \\ I: \quad & p \supset q . \equiv . \sim p \vee q \\ T: \quad & (x) . \phi x . \equiv . \sim(\exists x) . \sim\phi x \\ & \text{etc.} \end{aligned}$$

I do not mean to say that an operator *is* an equivalence; an operator is, as observed, rather the *transition* from one expression to another *in accordance with* an equivalence. But as there is a

<sup>1</sup> Thus we have: there is a thing such that — there is something such that — there is at least one thing such that — there are some things such that — at least one thing exists such that — things (or : objects) exist such that, etc. Or again: there is no such thing that — it is not true (or: it is not the case) that there is a thing such that, etc.

one-one correspondence between pairs of operators on the one hand, and equivalences on the other, we shall in future not always take the trouble to distinguish between the two.

In formal respects it will be observed that two inverse operators, applied successively, yield identity, in symbols  $AA^{-1}=1$ , and further that

$$(A B)^{-1} = B^{-1} A^{-1}.$$

However, it is not the subject of this paper to construct a calculus of operators.

A definition fits into the same scheme on the ground that it, too, can always be written as an equivalence; thus instead of defining the term 'planet' in the usual way, we might have laid it down as an equivalence

$$\chi \text{ is a planet} \equiv \chi \text{ is a heavenly body moving round the sun.}$$

Having said all this, we are now in the position to bring to light the fallacy that lies at the root of Schlick's and Ewing's definitions of the term 'analytic'. Two different concepts have been confused, namely, 'to follow from a definition' and 'to be logically true *in virtue of* a definition'. The point has been made quite clear by W. V. O. Quine in an article published in 1936.<sup>1</sup> 'What is loosely called a logical consequence of definitions is therefore more exactly describable as a logical truth definitionally abbreviated: a statement which becomes a truth of logic when *definienda* are replaced by *definientia*.' The same point, however, was already seen by Frege in 1884 when he wrote:<sup>2</sup> 'When a proposition is called . . . analytic in my sense, this is . . . a judgement about the ultimate ground upon which rests the justification for holding it to be true. This means that the question is . . . assigned, if the truth concerned is a mathematical one, to the sphere of mathematics. The problem thus becomes that of finding the proof of the proposition and of following it up right back to the primitive truths. (*Urvahrheiten*.) If, in carrying out this process, we come only on general logical laws and on definitions, then the truth is an analytic one, with the proviso that we must also take into account all propositions without which any of the definitions would become inadmissible. If, however, it is impossible to give the proof without

<sup>1</sup> 'Truth by Convention': *Philosophical Essays for A. N. Whitehead*, London, 1936, p. 92.

<sup>2</sup> *Die Grundlagen der Arithmetik*, Breslau, 1884, translated as *Foundations of Arithmetic*, J. L. Austin, Oxford, 1950, §3. N.B. The quoted passage is Waismann's own translation.

making use of truths which are not of a general logical character . . . then the proposition is a synthetic one.'

**2. LOGICAL AND IDIOMATIC EQUIVALENCE; DEFINITION AND SUBSTITUTION LICENCE.** In this section we shall consider more closely how logical equivalences, definitions and other operators are related. We have seen how, by means of operators, a certain sentence can be transformed into a truth of logic. In retrospect two points stand out clearly: (1) The sentence is transformable into such a truth *not only* by means of definitions, but with the material aid of certain other operators; (2) the definition *D* behaves, in point of application, *not so differently* from operators such as *N*, *M*, *I*, *T*, *L*, *R* etc. These operators, however, fall into two distinct types which we will call *logical* and *linguistic*, respectively. Thus *N*, *M*, *I*, *T* are of the first, *L* and *R* of the second type. The principle according to which this distinction is made deserves perhaps some attention. The first group consists of those transitions which are valid on *logical* grounds alone. If they are written as equivalences, they become instances of certain logical truths. But the operators of the second group are not purely logical, and cannot be expressed in logical symbols only; rather it is characteristic of them that they are due to *the way word language is used*. To give an example for the first type, the operator *N* is the transition along the equivalence (indicated by the arrow).

$$\xrightarrow{ } p \cdot \equiv \sim \sim p.$$

and this equivalence is true in virtue of its *logical form*; and similarly in the cases of *M*, *I*, and *T*. On the other hand, the operator *L* which enables us to pass from saying 'There is a planet that moves round the sun' to 'There is a thing such that it is a planet that moves round the sun', putting the term 'planet' in the place of a predicate, will be recognized by any user of the English language as idiomatically correct; but it is not a logical truth. In fact, it is used to *prepare* the first sentence for symbolization within the framework of symbolic logic, though it does not go far enough for that; it's only when we apply the further operator *R* (which eliminates the last restrictive clause), that we reach a sentence-form which can be so symbolized. From this it appears that the last two operators have a job very different from that of the former ones: they are transitions *within* word language, used, among other

things, for re-phrasing a sentence such that it can go into symbols. The equivalence:

There is a planet that moves round the sun ≡ There is a thing such that it is a planet that moves round the sun  
 is certainly true; but it is true *neither on empirical nor on logical grounds*; there is no formula in PM which covers such a case. It is true simply because, *according to the idiomatic use of the English language*, the two sentences come to the same. A *logical equivalence* (such as that linked with *M*) is *always* and *universally* true, irrespective of the language to which it is applied; an equivalence of the latter sort, if it is true, is true because it is in accordance with the *particular way* in which a *particular language* (such as English) is used, but it holds no place in a universal system of logic. The two groups of equivalences are therefore not of the same standing. And yet *both* sorts are needed for transforming a certain sentence into a truth of logic. That shows that Frege's and Quine's account of the matter is incomplete in an important respect. According to Frege, only *general logical laws* and *definitions* are permitted in testing the analyticity of a proposition; Quine stresses one thing only, *definitions*. Neither of them seems to have noticed the need for a third kind of processes which are *linguistic* in nature. If we were to limit ourselves to definitions, or to definitions and logical laws, we should never be able to translate the sentence 'All planets move round the sun' into a truth of logic. Frege may have been led to the view he has taken by concentrating on mathematics to the exclusion of word language. A definition which is meant to apply to word language also, must obviously allow for other tools of transformation as well.

All this makes it desirable to modify our definition by saying: A statement is analytic if it can, by means of mere definitions, logical and, further, idiomatic (linguistic) operators, be turned into a truth of logic. We proceed now to consider these means separately.

The idea of a logical operator seems sufficiently clear insofar as it is based on that of a logical equivalence. A definition can always be re-written as an equivalence (see example above). However, there is an important difference in that a definition is valid in virtue of the way a certain term, e.g., the word 'planet', is used in English, without, however, belonging to the body of truths provided once and for all by logic. In this respect a definition is more like an equivalence of the third kind. What is the difference?

If I say that the sentence, 'Some planet moves round the sun' expresses the same fact as the sentence, 'Something is a planet that moves round the sun', I am making a statement about the way two phrases are used, viz., I am stating that the two sentences come to the same. This can, if we like, be construed as a *substitution licence* which gives permission to interchange these two locutions. The difference between a definition and a licence of the last sort is this: whereas a definition refers to *one* term only, and usually provides for its elimination from a context, an idiomatic licence applies to *whole sentences*, or clauses of such, or syntactical constructions, offering them as alternative modes of expression. That the dividing line is not an absolutely precise one, that the two sorts rather shade off into one another, — e.g., what B. Russell calls a 'definition in use' also refers to a whole phrase — lies only in the nature of things and reflects how closely related, at bottom, the two sorts of licences are. Thus, whereas *D* is related to a *definitional* equivalence, and *N*, *M*, *I*, *T* to *logical* equivalences, *L* and *R* are related to *idiomatic* ones. By applying any of these three sorts of means, a statement, if it is analytic, is transformable into a logical truth.

In this way we have arrived at a definition, broad enough to be applicable to word language, and at the same time free from the defects which mar the afore-mentioned attempts. In the next section we shall have to consider some doubtful points to which the definition suggested gives rise. But before we do so it will be well to go into some minor points.

Let us consider the relation between a definition and a substitution licence in more detail, for the moment ignoring equivalences of the idiomatic kind. We shall approach this subject by asking first another sort of question: What is the difference between a substitution and a definition? What, for instance, is the difference between

$$\frac{p \vee q}{p} \text{ and } p \supset q . = . \sim p \vee q \text{ Df?}$$

The former is an instruction saying that  $p \vee q$  is to be put in the place of  $p$  in the context of a *given formula only*; it is *unidirectional*, i.e. it tells us to replace  $p$  by  $p \vee q$ , not  $p \vee q$  by  $p$ , thus making that process *irreversible*; and it *commands* us to carry out this substitution. The latter permits the substitution of  $p \supset q$  by  $\sim p \vee q$  in

any context whatever, not just in that of one particular formula; it allows us to pass *either* from  $p \supset q$  to  $\sim p \vee q$ , or from  $\sim p \vee q$  to  $p \supset q$ , just as we please, making the transition *reversible*; and, finally, it *permits* it merely without actually instructing us to make it. We may express this briefly by saying that a definition is not so much a rule (an instruction) as a *licence for re-writing* a sentence, or a formula, by putting *definiens* for *definiendum* and leaving it to us whether we wish to make use of it. In other words, a definition *paves a way* without forcing us to go it.

Similar remarks apply to *any logical equivalence*: we may pass from the one side of it to the other, in any direction and in any context we please; but we need not do so. Thus every logical equivalence supplies us with a substitution licence for replacing a certain expression by another one. In fact, all the operators which occur in our transformation chain, including the definition, provide us with such licences — further evidence of how closely related they all are.

This way of looking at things has, I submit, the advantage that it makes us see right from the start what matters in logic. It is not so much because of its *truth* that we take an interest in this or that particular equivalence, as because of the *use* we can make of it. What is really of value in a logical equivalence is that it lends itself to a use very similar to that of a definition: it supplies, to say it once more, a substitution licence, and that in actual fact is its more important side. But not *every* substitution licence supplies us with a *definition*; for a definition must be such as to permit the elimination of one symbol by others. Thus I can make use of the equivalence

$$p \supset q . \equiv . \sim p \vee q$$

in order to define the symbol ‘ $\supset$ ’ by putting

$$p \supset q . \equiv . \sim p \vee q \text{ Df}$$

If, however, I were to say

$$\sim \sim p . \equiv . p,$$

I should, it is true, be stating a logical equivalence, but there would be no point in using it as a definition by writing

$$\sim \sim p . = p . \text{Df}$$

For the latter formula may well serve to eliminate the combination

of symbols ‘ $\sim\sim$ ’, but not the symbol ‘ $\sim$ ’ alone, and can, for this reason, not be taken as an explanation of that symbol. Similarly

$$\sim\sim\sim p \cdot \equiv \cdot \sim p,$$

though a perfectly valid equivalence and a substitution licence, would be useless as a definition of the symbol ‘ $\sim$ ’.

Shall we, then, demand of a definition that it should allow the complete elimination of one of the symbols? That won’t do either. Suppose I define addition recursively by writing

$$a + (b + 1) = (a + b) + 1 \text{ Df},$$

then the symbol ‘+’ occurs on both sides of the definition; yet it would be a mistake to reject, on this ground, the definition as circular. For the whole point of the formula is that it reduces the addition of  $a$  and  $b + 1$  to the simpler addition of  $a$  and  $b$ , and, if this process is repeated a suitable number of times, finally to the operation ‘+ 1’; thus we can, step by step, reduce  $7 + 5$  to  $7 + 4$ ,  $7 + 4$  to  $7 + 3$ , and so on, until we reach  $7 + 1$ ; the operation ‘+ 1’ itself is undefinable in arithmetic, being the simple step of forming the successor of a given number. We may bring this out, perhaps somewhat more clearly, by writing

$$a + S(b) = S(a + b) \text{ Df},$$

i.e., the sum of  $a$  and the successor of  $b$  is the successor of  $a + b$ . This formula exhibits the general scheme according to which, in any particular case, the sum of two numbers can be defined in terms of ‘successor’; it is perhaps best characterized as an *instruction for framing particular definitions*, but is commonly itself taken for a definition by recursion. This example shows that a definition *need not* be a licence for eliminating a symbol. And even if a formula *does* permit eliminating a symbol, it need not be a *definition*. Take, for instance, the formula  $4 = 2 + 2$ ; though it permits the elimination of the symbol ‘4’, it would hardly be recognized as a definition of this number. Thus the condition suggested is neither necessary nor sufficient. Whether conditions can be specified which determine, without any doubt, what a definition is, I do not know. Incidentally, in view of the indefiniteness with which nearly all the terms of word language are used, and the need for leaving at least some freedom for adjusting them to new situations which may crop up — would Aristotle have

considered the case of a recursive definition? — I doubt the wisdom of pressing for a hard-and-fast rule, which can lead only to a sort of pseudo-precision. It is perhaps better to keep a term like definition flexible and make a decision, if the need arises, only in individual cases without anticipating the issue. That, of course, applies only to *natural* language; in an artificial, formalized language, the matter may be different.

We may sum up the discussion by saying that definitions are substitution licences of a *particular sort* (leaving the sense of this somewhat open), and that every substitution licence can be re-written as an equivalence. It is only when expressed in this way that we can do logical work with definitions, for instance derive consequences from them.

We can now understand — what we failed to understand before — why it is that definitions behave in such a disorderly manner. The point is that a definition, if it is explicit, can be re-phrased as an equivalence. Now from an equivalence another equivalence may be derived by logical inference; if the latter is such that it can be used as an explanation of one of the symbols involved, it can itself be construed as a definition; thus it happens that the 'logical consequence' of a definition may again be a definition. If, however, the equivalence obtained falls short of this demand, then what follows from a definition will not be a definition. In the former case the procedure by which we 'deduce' a definition from another definition involves, strictly speaking, three separate steps: (1) putting the definition in the form of an equivalence, (2) deriving another equivalence from the given one by logical inference, and (3) re-writing the result obtained in (2) as a definition.

3. MORE ABOUT 'DEFINITION' AND 'ANALYTIC'. I have defined 'analytic' in terms of 'logical truth', and further in terms of certain 'operators' used in transforming a given sentence into a truth of logic. The accuracy of this definition will thus essentially depend on the clarity and precision of the terms used in the definition. If these were precisely bounded concepts, the same would hold of 'analytic'; if, on the other hand, they should turn out to be even ever so slightly ambiguous, blurred or indeterminate, this would affect the concept of analytic with exactly the same degree of inaccuracy. Before proceeding, it will therefore be well to pause and consider these terms more in detail.

I shall first discuss the idea of an operator, and here I shall confine myself to setting out certain difficulties which stand in the way of giving a precise shape to the idea of a definition. Later I must examine the notion of logical truth. I shall try to show that both concepts are more or less blurred, and that in consequence of this the conception of analytic, too, cannot be precisely defined. It is important to make this point quite clear, because it will help us to understand better how analytic statements are applied. To anticipate one result to be established later — it is significant that we do not only 'find out' that a given statement *is* analytic; we more often precisify the use of language, chart the logical force of an expression, by *declaring* such-and-such a statement to be analytic. If 'analytic' was as fixed and settled a term as, say, 'tautology' is, this would be hard to understand: can I, e.g., by decree appoint a given statement to the rank of tautology? It is precisely because, in the case of 'analytic', the boundary is *left open* somewhat that, in a special instance, we may, or may not, recognize a statement as analytic.

What, then, is a *definition*? A definition, it seems, is a licence which permits us to replace a word, or a symbol, by the *definiens*, i.e. to translate an expression into a different idiom. When we say this sort of thing, what we have in mind are perhaps *explicit* definitions, i.e. definitions *per genus et differentiam*, illustrated by such stock examples as 'A planet is a heavenly body revolving round the sun'. And we are perhaps tempted to think that *every* definition conforms to this archetype. We are apt to forget that definitions of this kind are of use only in comparatively simple and trivial cases. The more interesting concepts such as truth and falsity, meaning and purpose, cause and effect, intelligence, time, number, which fascinate theorists, elude our efforts to pin them down in this way and only mock such clumsy attempts at defining.

Indeed, it is easy to see that this scheme is far too narrow. What, for instance, about 'recursive definitions' such as that given for addition?<sup>1</sup> And what about what Russell calls 'definitions in use'? If we are to explain, say, what 'similarity' means (as used in mathematical logic), though we cannot define the word 'similar' in isolation, we can explain its meaning in the context 'One class is similar to another'. Adopting this device, we are able to define terms like 'mass', 'force', 'temperature', and many others.

<sup>1</sup> See p. 137.

What is time? Philosophers since the days of St Augustine have been pondering over this question. The queer thing is that we all seem to know perfectly well 'what time is', and yet if we are asked *what* it is, we are reduced to speechlessness. Indeed, what *should* one say? That time is 'the form of becoming', 'the possibility of change', or some such thing? Wouldn't it be extraordinary if someone, instead of saying, 'Don't hurry, still plenty of time' were to say, 'Don't hurry, still plenty of form of becoming'? Would it be any better to declare, 'Time is measurable duration'? Needless to say this will not do either. So what? We can't help feeling *puzzled* by the question, and we are apt to express this puzzlement in Augustine's words: 'What is time? If I am not asked, I know; if I am asked, I don't.'<sup>1</sup> But what exactly is the difficulty? We know what the word 'time' means in the sense that we are able (1) to understand it in various contexts ('He has come just in time', 'What is the right time?' 'My time is up, I must go', etc.) and (2) to use it on the proper sort of occasions in the right sort of contexts. But it would not be right to say that we know the meaning of the word in the further sense (3) that we are able to *reduce* its whole immensely variegated use to a simple formula. Knowing how to apply the word in the right sort of way is one, condensing its usage into a single formula, a very different thing.

Here it might be asked: *can* the word be defined? But *why* should I try to find a definition? A definition would enable me to eliminate the word 'time' from any given context and replace it by its *definiens*. But it is *just the point* that there exists in English no other word, nor any combination of words which does the job the noun 'time' does. There is but one word, and no other, to express what I want to express, and that is just the word 'time'. In vain do I look for some paraphrase, or circumlocution, or roundabout mode of expression which can be used *in place* of the word 'time', though, in *particular* cases, such rewording *is* possible. To give some examples: by this time = now; a long time ago = long ago; in times to come = in future; in time = in good season, early enough; behind time = late; at the same time = simultaneously, however; from time to time = now and then, occasionally; time after time = repeatedly; out of due time = prematurely, too late, not in proper season; against time = with all possible speed or haste; in the nick

<sup>1</sup> *Confessiones*, Liber XI, Cap. XV, translated as *The Confessions of St Augustine*, most recently by R. Warner, New York, 1963.

of time = just at the exact moment; in no time = as quick as a wink; time enough = sufficiently long, no need for haste; now is your time = now is your opportunity; had a good time = enjoyed myself; mark time = make no progress; time-honoured = venerable by antiquity; etc. Thus the word 'time' can in fact be eliminated from a great many phrases; but the difficulty is to go on with *one* translation *consistently* for all the phrases in which the word occurs, i.e. to make the translation hold *throughout*. The fact is that there is no standard translation that can serve to eliminate the word from any context whatsoever and replace it by the *definiens*.

So to answer the question we look into diverse phrases in which the word occurs, spread out before us, as it were, the whole tortuous usage. We connect this word with others, we put it into various contexts, we trace over the lines of its use, and by doing this we convey its meaning. Indeed, if anyone is able to use the word correctly, in all sort of contexts and on the right sort of occasions, he knows 'what time is', and no formula in the world can make him wiser.

It should be noticed that this is only a *very rough* account of the matter. There is no such thing as a *standard* test to decide infallibly whether, e.g., a child really does grasp the meaning of that word. If the question 'What is the right time?' only elicits a blank from him, if he shows not the slightest sign of understanding the most common phrases, we should certainly not be satisfied. On the other hand, we do not require him to know *all* the idioms of speech such as 'in the nick of time', 'the time of one's life', 'hell of a time'. But to ask, 'In what moment does he catch the meaning?' is like asking in which moment a man who is learning to play chess turns into a chess player. These are not the right questions to ask.

What, then, are we to say in reply to the question whether a word like 'time' *can* be defined? We are inclined to say that it can not, and the reason for this, I suppose, is that we think of a definition as a *concise* formula covering the word's use. But we have already seen the infiltration of other types of definitions alongside the explicit ones by which the use of a word is explained in certain specified contexts. As there is no limit to the number of these contexts — a good example of this is the definition of irrational numbers in mathematics — there is no limit to the complexity of the pattern woven by this sort of definition. If such

accretions are admitted, why not take a bolder step and include the case we are considering too? Admittedly, that would be a departure from the ordinary (traditional) use of the word 'definition'; but why not stretch this usage?

My object in dealing with this sort of question was not to recommend a way of speaking in which spreading out a word's use may be called a definition, but rather make you see that 'definitions in use' gradually shade off into more and more complicated patterns, and that it would be unnatural to say, 'So far it is a definition and from there on it is no longer'. On the other hand, if a person feels unhappy about extending the use so far, we can also see that there is a point in this. What troubles him is perhaps this that the use of the word *cannot be formalized*. Indeed, the usage of a word like 'time' is *not only* far more intricate than that of, let us say, 'similar' (in the theory of classes), but also *irregular, loose*, and above all, *incomplete*, the latter in the sense that new figures of speech may come, and in fact have come, into being (such as 'time is money'), without apparently altering the sense of the word. It is therefore not only *difficult*, but *next to impossible* to tabulate all the phrases in which it occurs. *Its use can no longer be distilled into rules*. The more we attend to this peculiarity, the less satisfied we feel with calling such a procedure a definition. Having reached this stage, we are perhaps more inclined to say something like this; it is true, we can make a person understand the word 'time' by producing examples of its use, characteristic examples: but what we cannot do is to present a fixed formula comprising as in a magic crystal the whole often so infinitely complicated and elusive meaning of the word. Accordingly, one may perhaps wish to distinguish between 'definable' in the strict sense, and 'teachable' (or 'learnable'), thus bringing out the difference just considered.

Incidentally, the fact that one can perfectly well know 'what time is' without knowing all the idioms of language is a pointer which suggests that all is not well with the doctrine that 'the meaning of a word is the way in which it is used'. Indeed, if that was correct, it would only be natural to expect that the adding of *any* new phrase, when it occurs for the first time, such as 'time is money', alters something in the meaning of that word; which is obviously far from the truth. Ask yourself whether you are prepared to say that in learning what a number is one has also to learn the use of such phrases as 'a number of people', 'a small

number', 'a round number', etc.? Would it be right to say that, if a child is unfamiliar with such expressions, he does not know what a number is? Such examples should make us hesitate to accept the formula 'meaning = use'.

Let us now go back to the point where we spoke of the variety of definitions, and consider some more examples. What about dictionary definitions? Here some qualms may be felt in letting them pass as definitions. For if you follow up the references in a dictionary, you will sooner or later be turned back to the point from where you started, having described some odd serpentine course through a whole range of related expressions. Such circular definitions are of course inadmissible where logical strictness is required (as in geometry), but they are no blemish in the case of a dictionary; for it is not so much meant to give definitions as to *remind* the user of a language, who is more or less familiar with a given word, of its more subtle implications, to place it in its natural setting of root and derivatives, and show, where this is possible, its origin in other words and languages, and the stages through which it has passed. Now, going into the etymology of a word is certainly not considered to be the business of the logician who wants to *define* it — evidence that the objects of a lexicographer and of a logician are not the same. Another point which deserves mentioning is that a definition, as we find it in a dictionary, must not be confused with the *assertion* that it is in accord with the actual use of language; it is rather the fact of its being printed in a *dictionary* which indicates, or claims, that the meaning thus explained is actually the *current* or *prevailing* one. Further, a dictionary only rarely gives the precise equivalent of a word, for even synonyms (as masculine, manly, virile; leap, jump; assist, help, aid; festive, festival, festal; saintly, holy; polychrome, multi-coloured) usually differ either in context, or in suggestion and emotional overtone. More often a dictionary will give expressions which come as *near as possible* to the meaning of a word without coinciding with it completely; sometimes the dictionary will content itself with producing a few characteristic examples of the word's use; in yet others it will merely comment on the usage. Finally, and this is a most important point, a dictionary supplies lots of *factual information*; i.e. by consulting a dictionary we can learn many things about the actual word. Thus a dictionary conveys at least four widely different things:

- (1) A declaration as to the meaning of a word, or at least a comment on the way it is used;
- (2) An indication that the meaning given actually attaches to the word in present-day English;
- (3) Observations concerning stem and derivative, the history and etymology of the word;
- (4) Some extra-linguistic information.

Considerations such as these will show that *in praxi* we are using the term 'definition' much less strictly than in deductive systems such as geometry, or symbolic logic. A very vague usage, however, will destroy nearly the whole idea of analytic. For instance, if dictionary definitions were admitted, almost *every* sentence might be turned into an analytic one, and we should be left with hardly any criterion. What would be the good of doing this? That Aedes are mosquitoes which transmit the virus of yellow fever to mankind, that the Rubicon was crossed by Julius Caesar in 49 b.c., that sulphur is used in the medical treatment of skin-diseases, that the Tübingen school is a school of rationalistic, theological criticism, that Esperanto was invented by Zamenhof, that Emma Albani was a Canadian soprano, and that there are nine grades of mandarins — all these might with some justice claim to be analytic, since they could be learnt from merely looking up dictionaries. It is plain that at least *some* restriction of the term's use will have to be adopted, or else any distinction will become lost in a haze of indeterminacy. For this reason we shall have to preclude dictionary definitions. (Further supporting grounds for this will be given later.) But how exactly are we to regulate the use of the word 'definition'? May we, for instance, include in the concept what is sometimes called 'implicit definitions' such as the axioms of geometry? May we also include *non-verbal* types of explanation — such as pointing to an object and pronouncing its name? If not, why not?

Consider for a moment the last point. Suppose you say that 'ostensive definitions' are not to be admitted, because they are not definitions in the proper sense. Very well then; there is, unless I am mistaken, no argument to force you to accept them, nor would I attempt to give one. All I intend to do is to invite you to look carefully at such acts of pointing and notice in which respects they are like and in which respects they are unlike verbal

definitions. Obviously, pointing to something, say, a sample of beige, *helps* you to understand what 'beige' means; in this respect, the act of pointing at an object, together with pronouncing the word, is no doubt very similar to a definition, similar to the *job* it performs: it enables you to *apply* the word on later occasions; for *without* such previous explanation you would not have been able to apply the words as you do. So one cannot dispute that there is a connection between the understanding of the word and the act of pointing. Further, an ostensive definition is like a verbal one in that it is a *precept* which in application can be *followed*, or *broken*, and that it may be referred to in order to *justify* the word's use. In short, like an ordinary definition it has an explanatory, prescriptive and justifying force. On the other hand, pointing does not go far enough: precisely the *same* act of pointing may be used in explaining words of different meaning: if I explain, say, the name 'Venus' by pointing to a certain star, the same gesture might *also* be used in explaining the name 'morning star', and yet the two names, though they refer to one and the same object, are by no means synonymous. Thus I may point to a certain bright star in the evening sky with the words 'That's Venus', but if I were to say instead 'That's the morning star', circumstances being the same, I should be mistaken, for this is contrary to the use of language. Another example: 'That is blue' (pointing to the sky) and 'that blues' (imitating German); in spite of the same act of pointing, 'is blue' and 'blues' do not have exactly the same meaning. What such examples show is that an ostensive definition, unlike an ordinary one, leaves part of the use unsettled; it explains *something*, not *everything*. Nor is this the only difference. In explaining a word like 'beige' by pointing at a certain colour sample, we pass beyond the bounds of word language and connect the word with something in reality, in contrast to a verbal definition which remains entirely within word language. Besides, a sample is very unlike a word: I may, say in painting, copy from a *sample* of beige, not from the *word* 'beige'. The *word* is an adjective, and it is borrowed from French: but to say of the *sample* that it is an adjective or borrowed from French would plainly be absurd. But the most important difference is perhaps this that an ostensive definition can no longer be used as a *substitution licence*. If someone explains to me the name 'John' by pointing to a certain man, and later writes in a letter 'John has died', I cannot re-write this sentence by taking

the word 'John' out of it and putting the real John there. (But remember that a recursive definition, too, cannot be used as a substitution licence). It is a consequence of this fact that ostensive definitions cannot be transformed into equivalences and handled in making inferences.

In comparing the two kinds of definition, verbal and ostensive, we thus come to see the features in which they agree and those in which they disagree. If we look at *some* of them only, we are inclined to assimilate ostensive definitions to verbal ones; if we attend to the others, we are disposed to declare against assimilating them. But the way out is clear. In asking this question — are ostensive definitions really definitions? — it looked as if we were confronted with a formidable problem. But once we see that the question presents a *conflict*, two sets of forces pulling in opposite directions, we are no longer tempted to seek after 'the right answer'. All we do is to show anyone who is puzzled by this question that he must make a choice. At first he was *ignorant*, finally, after the features of the case have been set out as fully and clearly as possible, he may be *irresolute about what to say*. We may then help him to come to a decision by drawing his attention to the advantages and disadvantages that are likely to accrue from either choice, but *we don't decide for him*. That is the sense in which it may be said that our method of doing philosophy is *entirely undogmatic*. No-one can agree or disagree with us, because we abstain from making any assertion.

Suppose, however, for the sake of argument, that you declare *for* including ostensive definitions in your concept, then the next step is to direct your attention to the enormous variety this form of explanation can assume. You may point to a person saying, 'That's John'; but can you in the same way point to a colour, to a length, to a cardinal point, to a tone? Yet there are obvious similarities between these examples. Thus you may span a length with two fingers and say 'That is an inch'; or you may, while a tone is sounding, or standing forth against a background of others, call attention to it with a gesture, e.g. motion for silence, and say, 'Listen, that is top C'. You may even point to a group of things whose number can be taken in at a glance, or make a ring round them with your hand, and say 'Look, two apples', 'Three nuts', etc. (This is a way in which children may in fact come to learn the first few numerals.) Again, you may say 'Give me the apple *and* the

pear!', accompanying the words with a sort of collecting gesture or 'Will you have the apple *or* the pear?' and present the other with a choice in a gesture. Do these gestures explain the meaning of 'and' and 'or'? Difficult to tell; they *help*, anyhow in part, the other person to understand these words. So far may one speak even of an ostensive definition of a number (if it is small), or of a connective.

Pointing to a red piece of paper and pronouncing the word 'red', can be misunderstood in various ways — another may take the word to mean 'red', 'brightly coloured', 'oblong', 'paper', 'smooth', 'shining', 'thin', or even 'this direction' and so on. But if I say 'This colour is red', the mention of the type of word, 'colour', prevents such misunderstandings. Another point: compare the expressions 'Red', 'That's red', 'This colour is called red', each expression being accompanied by a corresponding gesture. The first is the sort of thing we may say to a child in teaching him the rudiments of language; and indeed, the explanation is rudimentary. Only the last is the articulate ostensive definition we give to adults. The first is perhaps more properly called a step in a process of training, or drilling the child, i.e. in generating in him a certain language habit, not so different from a conditioned reflex. We feel that pointing at an object and uttering the word 'red' is somehow incomplete. We are inclined to say that this act leaves out the essential feature of a definition. What is essential seems to be the *semantic* relation of the word to the object which, in the fully developed form of the definition, is expressed, e.g. by the words '*is called* red', or 'red' '*means*', it '*designates*', '*indicates*', '*denotes*', '*stands for*', '*refers to*', '*is the name of*' this colour. These words, it seems, have a definite meaning, and it is *this meaning* which makes the connection between the word and the object. If we ask what inclines us to take such a view, one possible reply is as follows. If I say to a Chinaman 'That is blue' and make a corresponding gesture of the hand, he may understand 'Destroy this object!' So the sound of the spoken words, the gesture and the object are not enough to express the ostensive definition. What is still missing is the *sort of connection* between the word 'blue' and the object signified. This connection is expressed by the word 'means', 'designates', etc. as listed above. It seems that the other person has first to *understand* these words in order to grasp that the whole process is a *definition*. In the crude form

which we give to a child these words are omitted. We simply say 'That's blue' without explaining that this is meant as *definition*. That's what makes the procedure look incomplete. We may perhaps say that the child must first be trained mechanically without understanding the process, and that from this indistinct haze a clear idea of what an ostensive definition is will later develop. Accordingly, we must distinguish between more primitive and more developed forms of ostensive definitions, without however, being able always to specify precisely the point where 'training' ceases and 'definition' begins. This, then, is a further indeterminacy to be found in the idea of an ostensive definition.

Nor is that the end of the matter. What a 'tickle' is cannot be explained by pointing to one, and a verbal definition is hardly the sort of thing that is wanted. What we do in such a case is to tickle a child and say 'Now you feel what a tickle is like'. As Peirce says in regard to the word 'lithium':<sup>1</sup> 'The peculiarity of this definition — or rather this precept that is more serviceable than a definition — is that it tells you what the word "lithium" denotes by prescribing what you are to do in order to gain a perceptual acquaintance with the object of the word.' Similarly, I can't point to an after-image, but I may instruct a person what to do in order to obtain one. The same goes for a 'musty smell', a 'sensation of giddiness' or 'having a lump in one's throat', a 'singing in one's ears', 'pins and needles in one's legs', etc. And what sort of explanation is it, if a doctor invites you to look through a microscope, saying 'Look, that's the germ which causes tuberculosis'? It is not a verbal definition, and it is not quite the sort of thing usually called an ostensive definition. He offers you a chance to have a look for yourself. There is a vast variety of procedures more or less similar to the last one — use of illustrations, photos, models, films; use of projected stereoscopic images in natural colour for anatomical demonstrations; use of geographical and astronomical maps and globes; clinical demonstrations of patients; tuning forks; producing of musical themes to illustrate some technical term; use of lantern slides in lecturing on architecture or plastic arts; and so forth.

Different again is the case of what is called *déjà vu*, i.e. that sudden feeling which sometimes sweeps over us of 'having been here before', as if at some indefinite past time in just this place,

<sup>1</sup> *Collected Papers*, Cambridge (Mass.), 1931–58, 2.330.

we were saying just these words. The description in words does not quite convey the strange, unreal quality of that experience, and yet it is not in my power to evoke it at will, or to instruct someone else what he is to do in order to procure it for himself.

Again, consider this case. A traveller who has been to some South Sea Island wants to explain a term used there by the natives. He will first try to translate it; but suppose there is no word in English, or in any European language, to express precisely the meaning of that native word; what then? He will have to proceed differently: he may first introduce us to the specific customs, traditions, ceremonies, magical rites of those islanders, and initiate us in their social order and their beliefs. He may then proceed to explain the meaning of that word by giving us a detailed account of the way the word is used there by the natives, embedding this usage in the whole culture of those people, e.g. presenting the role it plays in their ceremonies, festivals, etc., in short, making us see it against the background of their tribal customs and beliefs. Explaining the meaning of such a word can no longer be severed from depicting the life of a primitive community. (Something of that is also true of our own language, only it is we ourselves who supply most of the background.)

Thus we see that there are a good many procedures, similar in some respects, very different in others, which may be called verbal definitions, and a great variety of others which may be called non-verbal definitions (or explanations). Each of these groups is made up of widely different types. Even if we single out one special type such as ostensive definition, on closer inspection it seems to lose its unity and tends to fray out into different individual strands. In calling attention to such widely different cases, it is not my object to show that we do *not yet* know where the boundary around them runs, but on the contrary to show that it is pointless to seek for one. True, we *could* draw the line somewhere, but *any* drawing will in the end be arbitrary.

Before leaving this subject, it will be well to consider a further question. Do ostensive definitions (if they are admitted) give rise to analytic statements? We may be tempted to answer that they do. When we ask ourselves what inclines us to take such a view, we may perhaps think of some such example as this. A metre is defined as the length of the standard metre in Paris. Consider now the sentence, 'The standard metre is 1 metre long'. Owing to the

definition laid down that means ‘The standard metre has that length which is possessed by the standard metre’, i.e. ‘The standard metre has the length it has’. This statement, we are inclined to say, is *true by ostensive definition*. To cite a somewhat similar case, suppose that the colour red was extremely rare and that a piece of standard red was on that account always kept under glass and exhibited in a museum. We can imagine that people on Sundays go to the museum, point to that treasure behind the glass and say reverently, ‘Look, that’s pure red’. That may be taken in two senses — as an ostensive definition of ‘red’, or as the statement that the object is *in fact* red. Now if the word ‘red’ is *defined* by reference to the standard, the latter statement is to be construed as saying, ‘This object has the colour red, and red is defined as the colour of this same object’, which boils down to saying ‘This object has the colour it has’. Here again it might seem natural to say that this statement was true by ostensive definition.

Mr A. Pap, in an article previously quoted,<sup>1</sup> seems to take this very view. ‘An existential statement’, he writes ‘may be regarded as a condensed version of a logical sum of elementary statements . . . that is “( $\exists x$ ) F( $x$ )” is short for “F( $a$ ) or F( $b$ ) . . . , or F( $n$ )”, where  $a, b, \dots n$  are objects that may have the property F. Now, each of the logical summands (alternatives) is a merely probable statement, since there are empirical data . . . which might falsify it . . . If, on the other hand, at least one of the summands should turn out to have the character of an ostensive definition, the existential statement would not be a dubitable empirical hypothesis, but would be analytic in the sense of being *true by ostensive definition* . . . Consider . . . the statement “there exists at least one rod which has exactly the length of 1 metre”. This statement is equivalent to the logical sum “either  $a$  is one metre long or  $b$  has that property . . . or  $n$  has that property”, where the series  $a \dots n$  includes all the rods in the universe. Now, with respect to  $n - 1$  rods in the universe it is, indeed, a question of *fact* whether their length is or is not exactly 1 metre. But if the expression “1 metre” is to have any meaning at all, one rod in the universe must have the predicated property by ostensive definition (this is, of course, the standard metre); hence the above existential statement is analytic in the special sense of being both significant and true by ostensive definition.’ Similar considerations lead him to take an analogous

<sup>1</sup> ‘Indubitable Existential Statements’, *Mind*, 1946.

view with respect to saying 'there are red surfaces'. As the word 'red' is significant, there must be at least one red surface (or patch), by pointing to which the meaning of 'red' can be explained. Now to say of *this* surface (or patch) that it is red must again be analytic, i.e. true by ostensive definition; hence the existential statement 'there are red surfaces' must be analytic too in the same special sense.

The crucial point in this argument is that in both cases a certain statement is claimed to be true, not on empirical evidence, but in virtue of ostensive definition. Now there is obviously something very queer about this account. First, and this is to be no more than a hint, the logic of the argument can plainly be followed even by a blind man who is entirely unacquainted with the colour red and therefore in no position to grasp the content of any ostensive definition of this sort. That tends to show that the ostensive definition is quite irrelevant, that it does not really enter into the argument. This is borne out if we consider how the ostensive definition is supposedly applied. A definition, a *verbal* one, is an operator which can be used to transform a given sentence. But how exactly is the *ostensive* definition used? Where does its *transforming* function come into play? There is plainly nothing in any of our examples which corresponds to such a performance. Indeed, what matters is *not* the connection between the *word* 'metre' and the *actual* standard in Paris (which is made use of, for instance, in adjusting measures), or between the *word* 'red' and the actual *specimen* of the colour (which is made use of in applying the word in speech), but solely the *verbal description* of this connection. It is only the latter which is needed to transform any of these sentences into truisms. That is why a person who has never come across a metre or never seen the colour red can still argue that the standard metre has the length it has, or that the sample has the colour it has. So what is really needed in transforming any such sentence into a truism is no more than a *verbalization* of the actual process of the ostensive definition. This verbal description however, *cannot take the place* of the definition itself, i.e. it cannot act *in lieu* of the definition. If that were so, it would be the easiest thing in the world to make a blind man understand what white is simply by telling him that it is the colour of snow; which is too good to be true.

What fosters the confusion is the ambiguous way we use such

words as 'explain', 'define', etc. Suppose, for instance, I want to explain what 'musty' means. This I may do by telling someone, 'A "musty" smell is one which you experience in damp and long-shut cupboards'. Now, what is this, *a verbal explanation* which permits replacing 'musty' by any such circumlocution, or a *precept* which instructs you what you are to do in order to gain such an experience? Plainly both can be meant, i.e. either is the sort of thing which would normally be *called* an explanation of the illusion that there are analytic statements which are true *in virtue* of ostensive definition. If Pap claims that certain of his statements are 'analytic in the sense of being *true by ostensive definition*', and observes that this 'kind of analyticity is just as deserving of the philosopher's attention as the more familiar analyticity that is possessed by statements which are true by *verbal definition*', he is clearly wrong. For what he says makes it look as if *ostensive definitions and verbal definitions were used in a precisely parallel way*, a blunder the origin of which has just been exposed.

I think that I have now given some of the reasons why we need not concern ourselves with ostensive definitions. I do not mean to prejudge the question whether they *are* definitions, or deprecate their value. For certain purposes they should, perhaps, be recognized as definitions. All I want to say is that, as they do not give rise to a specific class of analytic statements, we may, in what follows, safely disregard them and concentrate on verbal definitions only.

After this digression we may now resume the main line of the argument. In forming a concept like 'definition' we are grouping together a cluster of unequal things, held together only by a sort of family likeness. It is important to notice that this family likeness is itself of a *vague* kind, consisting of all sorts of similarities which need have nothing in common, just as, to use an example given by Wittgenstein, members of the same family may be alike in many different ways, e.g. may have the same abrupt manner of talking, the same way of smiling with the eyes, of bending the head, of knitting the brows, or the same expression of defiance. Not that all these features need be present: but now this, now that, and at another time some combination of them, thus giving rise to a boundless variety of possible ways of 'looking alike'. As with people, so with definitions. If we group cases of definition together according to their similarity, it is easy to see that the similarity

resolves itself into a bundle of relations, holding at times only between the forms (expressions) of the definitions, at other times between the various jobs they perform. And it is also easy to see that many different ways of grouping can be adopted. Thus we have seen some of the reasons for excluding dictionary definitions, or for not classing ostensive definitions with others, etc. There is no *specific* feature to be found that is peculiar to all the situations in which the term 'definition' might be used. It should be noticed that much the same indeterminacy is characteristic of words such as 'explaining', 'describing', 'introducing' (into a notation), 'stating precisely', 'declaring the meaning', 'making intelligible', and scores of others.

My purpose in calling attention to these indeterminacies can now be seen. Any inexactitude in the idea of a definition will be reflected in a similar inexactitude in the conception of analytic: according as the boundary of what is called 'definition' is drawn more broadly or more narrowly, the class of expressions to be included in the range of analytic will become larger or smaller. For example, if *definitions in use* are precluded, it can not be proved that the statement 'If  $\alpha$  is similar to  $\beta$ ,  $\beta$  is similar to  $\alpha$ ' is analytic; if *recursive* definitions are precluded, it cannot be proved that a formula like ' $7 + 5 = 12$ ' is analytic (in the sense of being valid on the ground of definitions only); without *implicit* definitions such as those referring to the concept of betweenness it cannot be proved that saying, e.g., 'If A, B and C are points on a straight line, and B is between A and C, then B is also between C and A' is analytic. If, on the other hand, *dictionary definitions* are permitted, almost *every* statement might be proved to be analytic. Our trouble is not with the last sort of definition which, for the purposes of the present paper, may be excluded; nor is it with any of the other types of definition just mentioned. Our trouble is with the sort of definition, or explanation, instanced by the word 'time'. Suppose one is asked whether 'Time is measurable' is analytic — what ought one to reply? We are, perhaps, first inclined to answer, yes. What tempts us to do this is that it seems to be *part of the meaning* of 'time' that time should be measurable. Yet this claim can hardly be substantiated, i.e. there seems to be hardly any way of transforming the given expression step by step into a truth of logic. What we could do is, at the most, to point out some of the uses (such as 'timing', 'timepiece', 'what is the right time?' etc.) which

seem to indicate that time is measurable. This, however, will lead only to a scarcely enviable position since there is no sharp line which separates those uses which, as one would say, are *characteristic* of the concept, from those which are not. Here all sorts of questions may be missed: Are you willing to say that the word 'time', before sand-glasses, water-clocks, or sun-dials had come into use, had a meaning different from that which it has now? Would you say that 'mythological time' must be something quite different, because, though Uranus was deposed by Cronus, and Cronus by Zeus, it is not possible to *date* these 'events', any more than it makes sense to ask whether the reign of Uranus lasted as long as that of Cronus? Would you be prepared to say that, in case the world was such that time could not be measured — say, because of the absence of sequences of recurrent events — time would not be what it is now? Here, I suppose, you may be inclined to say that it lies *in the nature* of time that it can be measured. But what do you mean by the expression 'it lies in the nature of time'? That this is part of the *definition* of the word 'time'? But as there is no definition to refer to, but only a use, forming a vast maze of lines, as it were, you will feel that this argument loses its point. On what, then, rests your assurance? Some people may turn to intuition, and take a more or less Kantian point of view. In order to see whether intuition can show the *necessity* of our statement ('time is measurable'), we have to consider some of the facts which underlie the measurement of time. One such fact is the existence of sequences of recurrent events, such as the tides, the seasons, the phases of the moon, the revolution of the starry sky, the oscillations of a pendulum, the vibrations of a spring, etc. That is by no means enough. It is easy to imagine the following case: when I compare two such sequences, I find that they stand in a certain constant ratio to each other which, for a given pair, e.g. moon phases and sidereal days, is always the same. But there is nothing impossible in the idea that, if I started from a different series, e.g., the swings of a weather cock, and compared them with the events of another series such as thunderstorms or night frosts, I might *also* find a constant ratio between them, whereas comparing a member of the first group with a member of the second would lead to *no* constant relation. In other words, it makes sense to suppose that all the sequences in the universe should fall into two distinct classes, the members of the same class being correlated by well-defined

constant ratios, whereas between members of different classes no such relation holds. Then time would, as it were, bifurcate into two distinct rivers (say, 'star time' and 'storm time'), each flowing at an even rate, and yet irregular when viewed from the other: it would have become impossible to establish a common all-embracing time scale for the whole universe. Experience teaches that all those sequences which are linked by constant ratios in fact fit only in one extensive system; those which fit badly or not at all to the one system, do not fit together in a rival system. But even that must not be taken as an irrefutable fact; there are certain doubts, connected with the moon's irregularity, as to whether the ratio-linked sequences really fit together *precisely* in a coherent system, or whether in the long run this may not turn out to be illusory. We can, indeed, imagine that within historical periods 'irregularities' are scarcely noticeable which, on an astronomical scale, may pile up to such an extent as to make a chronology of the universe impossible. Why not, indeed, imagine that events of high frequency (vibration of strings, tuning forks, etc.) should be ratio-linked within short periods of observation, a few minutes or hours only, but incommensurable for longer periods, a few weeks or months? In such a case the 'uniformity' of time's flow would be a very transient thing. A time of this sort may be likened to a space in which no measurement is definable, what is called a 'non-metrical', 'topological' space (i.e. the type of space jelly-fish would likely be familiar with if they were doing geometry and living in a world of liquids or jellified objects). Incidentally, time may be non-metric without being disorderly. Recitals of poems would always bring forth words in precisely the same order, though it would not be a proper question to ask whether one stanza lasts *as long* as another. The attempt to measure a period between two events would be like trying to measure the Reign of Terror in terms of thunderstorms or earthquakes.

The fact that only one system exists in which all correlated sequences are linked now assumes a profound significance. It is no less significant that the time scale, based on this system, is in keeping with our natural estimation of the lapse of time (for short intervals anyhow), and further that it leads to the formulation of *simple* laws of nature. That this should be so does not follow from the mere idea of a time scale; it is conceivable that the time scale of the clock should not lead to simple and transparent laws, and

that it should first have to undergo some transformation before it could serve the purpose of rendering 'the equations of mechanics as simple as possible', to use Poincaré's words.<sup>1</sup> That shows what an enormous variety of cases can be thought of in which time wouldn't be measurable in the way it actually is. I say all this, because looking at things in this light may loosen a position which prevents one from seeing the importance of certain facts. Once you are aware of the vast mass of evidence which is relevant to the case, you will be less inclined to think that time 'must' be measurable.

Consider a somewhat analogous case: can pain be measured? There is a temptation to say just the opposite here, that it lies in the nature of pain that it *cannot* be measured. But suppose that psychologists had devised a technique for measuring pain, or at least for assigning numbers to pains in roughly the same way in which hardnesses are ordered according to Mohs' scale. We may imagine that an idiom has come into use in which the doctor asks the nurse 'What pain had the patient last night?' and she replies 'Pain number 3'. Ask yourself whether you would say in such a case that the word 'pain' has changed in meaning? Compare this question with the following: Supposing that certain astronomical speculations regarding departures from the constant ratios for the cycles of events had proved true (so that no universal and precise time scale could be set up), would you say that the word 'time' had now a new meaning? Now all this obviously hinges on the absence of a definition of 'time', or, more generally, on the absence of a *precise grammar* of this word. That brings me to the crucial point. When we were asking this sort of question, namely, whether the meaning of 'time' or 'pain' changes when a method of measuring is introduced, we were thinking of the meaning of a word as *clear-cut*. What we were not aware of was that there are no *precise rules* governing the use of words like 'time', 'pain', etc., and that consequently to speak of the 'meaning' of a word and to ask whether it has or has not changed in meaning, is to operate with too blurred an expression.

4. CONTINGENT AND NECESSARY. You may feel the same sort of difficulty if asked whether it is just a *contingent fact* that time is measurable? You may be dissatisfied, indeed very reluctant, to

<sup>1</sup> *Le Valeur de la Science*, Paris, 1914; translated as *The Value of Science*, New York, 1958, ch. ii.

call that mere chance; one of the reasons for your reluctance, I suppose, is that you picture time as a straight line which can be divided, and subdivided, into equal intervals; this picture is so deeply engraved upon you that you feel unable to part with it. It is like trying to persuade a geometer to give up his diagrams. Indeed the grammar of temporal relations is the same as the grammar of the relations between points on a straight line. There is an isomorphism between the two systems. Once you have noticed that, you may be inclined to say: ‘Though “time is measurable” and “rock salt is cleavable”, sound superficially alike, they are very different: the one is accidental, the other is not.’ And you say that perhaps in the tone of a man who is calling attention to a notorious fact. But in saying this you do not want to object to any of the facts which make it possible to measure time. You do not want to state, or deny, any fact at all. What you don’t see is that you are *irresistibly urged* to use a certain mode of representation which means a lot to you, in fact *that* mode which enables you to visualize with the greatest ease all sorts of temporal relations. As so often in philosophy, a statement appears so convincing precisely because it is no statement, but the obscure expression of a desire to use certain images, or a certain pictorial representation, to satisfy certain needs.

All that, I suppose, makes you reject the idea that it is just an accidental fact and no more that time is measurable. On the other hand, you cannot maintain that what this sentence expresses is a *necessary truth*. For you do not want to deny that under certain specifiable conditions it would be untrue. This puts you in a peculiar position. It is as if two forces were acting upon you in opposite directions, and you were attempting to *follow them both at the same time*. So you feel on the one hand a strong reluctance to say that what the sentence expresses is a *contingent fact*, but on the other hand you are equally disinclined to say that it is *necessary* in the sense that its opposite is unthinkable. Or you may feel a tendency to say — if it was not too absurd — that what the sentence expresses is contingent *and* necessary at the same time. Of course, you don’t really want to say it, and yet you may feel a slight temptation to say this very thing.

The reason for this wavering between opposite poles is that this is a case in which the philosophical antithesis ‘contingent-necessary’ loses its edge. The sentence ‘time is measurable’

exemplifies a wide class of statements which, so to speak, are on the borderline between necessary and contingent, the *a priori* and the empirical. Other such statements are : I see with my eyes and hear with my ears; every experience is *someone's* experience, i.e. any experience, when it occurs, belongs to someone who *has* this experience; two people cannot share a pain, though it may well be that they have pains which are in every respect *alike*; you cannot look directly into another man's mind to see what is going on there: experience is eternally *private*; I cannot feel pain outside my own body (except, perhaps, in an amputated limb); I have never migrated from my body into another one, nor have I ever had two bodies; I do not choose the mouth to speak with; a material object has a certain *permanency*, it doesn't suddenly vanish into nothingness; space has no more than three dimensions.

Now the first thing that strikes one about all these expressions is that each of them can be used in two senses, a *trivial* or everyday sense, and in an *important* or exciting sense. If, for instance, I were to tell you that time is measurable, you may well reply, '*Of course* it is; but what of it?' In this reply you seem to dismiss my remark as a sort of platitude. Indeed, viewed in a commonsense way the expression presents nothing remarkable. Suppose now I draw your attention to the possibility that recurrent events may *not* be linked by constant ratios in a single extensive system, but either in several such systems (each giving rise to a time scale of its own with no correlation between them), or *in no system at all*. Indeed, why should the vibrations of a balance-wheel keep so constant with the phenomena of the starry heavens such as the eclipses of the moon? Why should the periodic changes in brightness observed in the Cepheids in the utmost depths of space be commensurable with terrestrial events such as the swings of a pendulum? Why should not such sorts of events, or indeed *any* others, behave capriciously like gusts of wind, showers or volcanic eruptions? Nature, for all we know, might never have presented to us *any correlated* systems, and then the measuring of time would be an impossibility. There is no *a priori* reason that this should be so. The fact that sequences *can* be correlated is established by observation; its mystery is not dispelled by saying that time flows 'evenly' (Newton) or that time and its properties are *necessary* conditions for the possibility of knowledge (Kant): for it is certainly possible to think of such experiences as would frustrate

any attempt at setting up a time scale. Not only that, the time scale obtained in the ordinary way (grounded on correlated sequences), is precisely that which lends itself to a very simple formulation of physical laws: Newton's laws of motion, Maxwell's field equations, etc., hold for ordinary time, not for time lapses defined in terms of frosts, showers, or veerings of a weathercock. That reveals a very remarkable fact: that ordinary time leads to the discovery of clear and simple rules, and thus helps us to unveil the secrets of nature, to penetrate to an underlying timeless Order of Things. Now would you say that this is just a strange coincidence, an 'accidental fact' like that that my hair is turning grey? If you look at the question in this light, you can hardly escape the impression that there is something profound, almost mysterious, behind the fact that time is measurable. You may now feel that you have got hold of something that has a deep, *metaphysical* significance, pointing to some 'rationality' inherent in existence; i.e. you lose a certain commonsense complacency, and you are led to wonder. (A philosopher is a man who is able to wonder at the obvious.) And now notice how you may, at one time, pronounce the words 'time is measurable' in a dry, matter-of-fact tone, and at another time in a solemn tone as if you were saying something charged with hidden meaning.

Or consider for a moment the fact that there are material objects, solid, permanent, hardly changing in figure and size, in short 'practically rigid' bodies, and that geometry is connected with the existence of this sort of objects. Would you say that this is just an accidental circumstance? You may, and you may not. There is obviously no reason whatever, you may argue, why the world shouldn't be otherwise; for instance, if it comprised liquids and gases only, metrical geometry could not be applied. Looked at in this way, the existence of such bodies seems to be a contingent fact. From another aspect, however, geometry seems to be the *very basis* of what may be called a *rational understanding* of the material world, at least of an important side of it. (Think of the rôle played by geometry in the conceptions of Descartes, Spinoza, Kant, or again in the shaping of scientific ideas from Newton to H. Hertz and Einstein). Anyhow, geometry seems to have set the example to all research. Considering all this, you may now ask in a puzzled tone, 'How *can that be accidental* which leads to a rational insight into the nature of space?' It is as if certain *obvious* features of the

everyday world over which, up to now, we have been passing heedlessly, suddenly lost their commonplaceness and took on a new, hidden significance.

The same shift of meaning may occur when you consider an expression like 'Every experience is the experience of a person, i.e. it occurs in somebody's mind'. At first this sounds like a truism. Yet, if you ponder over it a while, you may say to yourself in a puzzled tone, 'Surely that can't be all, there is something more to it than this. There are no such things as ownerless experiences floating in mid-air, they must be experiences of someone who *has* them'. The very fact that experiences do not occur by themselves seems now to take on a tremendous significance.

In all these cases you notice that expressions which have a trivial use in daily life, when made part of certain trains of thought, lose their triviality, become, as it were transcendentalized, and acquire metaphysical stature. And with this goes an equally peculiar shift in the senses of most of the words used in the new context, such as 'time', 'reality', 'necessary', 'rational', and many others. A word can on one interpretation have a solemn resonance, although in another context it seems trivial, shallow. That makes intelligible that peculiar hovering of words between triviality and 'significance' and the tendency to turn from the one into the other which is so characteristic of metaphysics. Incidentally, something of the same kind can occur outside philosophy in common speech, as when we speak of a 'vast panorama' seen from a high mountain, and, on the other hand, of the 'vastness' of space, just as if 'vast' meant the same in both cases. It is as if, once having noticed the relativity of all size, we become no longer overwhelmed with the expanse before us; and so the peculiar feeling-tone, originally associated with the sense of vastness which the *sensuous* impression conveys to us, is, so to speak, spirited away to some other realm, e.g., to space. The word 'vast' now becomes, in a way, meaningless; but the point is that its *aura* has been transferred elsewhere; it is as if the word, detached or disembodied as it is, preserved its weight, its soul without its substance.

Sometimes you will observe just the opposite phenomenon, that some idea loses the peculiar fascination it exerts on our minds. For instance, so long as we think that the earth is unique among the heavenly bodies, we are tempted to attribute to that a deep, metaphysical significance, which, however, evaporates, once the

earth is seen to be just one planet amongst many others. Similarly, Euclidean geometry may become a source of bewilderment when it appears to us as remarkable, mysterious, unlike anything else, and we get fascinated by — or shall I say obsessed with? — the idea that this geometry must be of transcendental importance, until the spell is broken, a broader view opens up before us, and a whole cluster of geometries is seen, grouped around Euclidean geometry according to similarities in their logical structure. Once the latter is collated with its natural environment, it loses its disturbing air of uniqueness and becomes, in a way, trivialized. And the same goes for logic. What we do in such cases is that we try to give a *synoptic presentation*: we silence the questionings which resemble a problem by setting a number of similar cases side by side. And it is characteristic that the mere juxtaposition of cases suffices to dispel our disquietude. For that shows that our puzzlement springs from a failure to see things in their true perspective.

After having shown in these examples the peculiar tendency of certain statements to turn from the trivial, everyday sense to a profound, metaphysical one, or to hover between the two, I want to bring out a further point. All these examples are puzzling expressions in that neither of the labels ‘contingent’, ‘necessary’ is exactly to the point. I shall do this by discussing two examples.

Consider the sentence ‘I see with my eyes’. At first sight it seems to state a contingent fact; and there is an obvious truth in this, for if I close my eyes I lose sight of everything I see. But that it is different from an ordinary statement of fact can be seen if it is compared with such a statement as ‘I eat with a knife and fork’. It is easy for me to imagine that I do without them (e.g. when eating asparagus); but can I imagine that I see without eyes? If someone were to tell you ‘I see you quite distinctly with my eyes shut’, what would you make of it? That this is glaringly untrue, or that it is a contradiction in terms? How *can* he see without eyes? What could ‘see’ mean here? Plainly not what it normally means. Is it not *part of the meaning* of ‘I see X’ that I should have my eyes open, and not only have them open, but also look in the direction of X (disregarding for a moment such things as mirror reflections, etc.)? If so, the sentence in question must clearly be self-contradictory.

It would be no argument to say that one can see an after-image

with one's eyes shut; for it is plain that in this case 'seeing' is used in a different sense. But let us see in what this difference consists. When I say, 'I see the clock on the mantelpiece', this implies that there is a clock there, i.e. it implies that what I see is something real, existing independent of myself; *that is the way we use* the verb 'see' in normal (perceptual) situations. But when I say, 'I am seeing an after-image', I do not mean to say that there actually are, independent of myself, certain funny things, called 'after-images', which can be seen by me and by no one else. What is true is only that in order to describe this particular experience, I can make use of the perception verb 'see'. On the other hand, as I don't want to say that I 'see' after-images in the sense in which I see the clock when I look at it, I am perhaps inclined to say some such thing as 'It is *as if* I see after-images', thus making it quite clear that I do not perceive any actual objects. Instead of this, I may use the more idiomatic phrase 'I am seeing after-images'. The uncommon construction is here quite apposite as an indication that I am also using the verb 'see' in an uncommon sense. For normally we don't say 'I am seeing the clock', i.e. we don't use the continuous present tense of the verb 'see' (except in such phrases as 'I am seeing the sights'). Another way of bringing out the difference is to say something like this. It is characteristic of the one usage that I may say, 'I see you when I look at you; if I don't look at you, I don't see you'; but it would be ridiculous to say, 'I see after-images when I look at them; if I don't look at them, I don't see them'. In fact, we all know that no such condition attaches to seeing after-images, dark spots, etc. On the contrary, they have a way of being there no matter where I turn my head or where I look. I can neither look *at* them nor look *away* from them, because there is nothing for me to do which is analogous to looking at a thing, except perhaps (what would be highly misleading) *paying heed* to them. Unless you are willing to accept so strained an interpretation, you have to admit that it makes *no sense* to speak of 'looking' in the latter case. This suggests that the peculiar sense of 'seeing' that interests us here might be characterized in the following way: 'I am seeing X = I see X though I cannot look at X'; 'cannot' means here: 'it makes no sense to talk of "looking"'. Now the whole point of the argument is that there are two senses of the verb 'see' corresponding to two distinct patterns of usage, but not two senses of the verb 'eat', corresponding to 'eating with a knife and fork',

and 'eating without a knife and fork'. We may, of course, draw as many distinctions as we like; we may, for instance, insist that in the last case, too, 'eating' is used in different senses; what is wrong is only to think that we can justify this distinction as a natural one.

Let us now resume the main thread of the discussion. I said that it is *part of the meaning* of 'seeing' that I can see a thing only when my eyes are open, and when I look at it, it will, I hope, have become clear that this applies to one sense of the verb only. In *this* sense, then, saying 'I see you with my eyes shut' must be some sort of contradiction; if so, saying, 'When I see you, my eyes are open', must be some sort of *non-factual a priori* statement. That we are inclined to say, 'When I see you, my eyes *must be* open', stressing the 'must be', in fact indicates that we feel the connection between the seeing and the eyes not to be an accidental one. I might also have said that the mention of the eyes in 'I see you with my eyes' is *redundant* as all it means is that I see you; or again that saying 'Whenever I see, I see with my eyes' scarcely conveys any information; indeed, it has a truistic ring like saying 'Whenever I have a toothache, I feel it'. No one ever says such a thing, except when he wants to *remind* another person of the meaning of a word, to drive home to him its implications; but apart from that, it doesn't tell us anything about facts. This, then, should clinch the argument that saying 'I see you with my eyes shut' must be meaningless, viz., self-contradictory.

But suppose it was discovered that, in certain rare and strange circumstances, people can see with their eyes shut? Can I not imagine that, say, by rubbing my eyelids with a certain mixture, they suddenly become transparent like glass? Wouldn't you at least *understand* what I mean? Couldn't you, for instance, follow such a story from the Arabian Nights? Certainly; but what if I now ask you, 'How can you even *imagine* such a thing, considering that it is self-contradictory? How can you give a *sense* to these words?' It seems that you *ought* not to be able to understand them. On the other hand, is it not quite easy to imagine such a situation? In calling your attention to such, if only fictitious cases, my object is to make you see to what an extent non-linguistic, experiential factors come into play here. Indeed, you may think up even more fanciful situations in which a person may be said to see, for instance, with his skin. In case you feel uneasy about that, it is well to remember that something remotely similar to that can

actually occur in blindness. I mean what is known as the 'facial vision' of the blind. Thus a blind man seems to perceive objects through the skin of his face; for instance, he can, to some extent anyhow, tell the shape of objects unknown to him, and that without using the sense of touch; some who are totally blind, it is said, even have the power of discerning a cloud on the horizon. That only adds to your reluctance to regard such sentences as meaningless. In fact, after you have vividly imagined a number of such possibilities, you may feel that there is *no point* in arguing from the use of language only; instead you may be prepared to say 'Of course, it is *experience* which teaches that I cannot see without eyes'. Yet at the same time you can't help being worried by this account, because you feel that it would be most unnatural to say that 'When I see you, my eyes are open' is an empirical statement in just the same sense in which, 'When I write, I hold a fountain pen in my hand', is. In this situation you may feel a strong need to use some such expression as this. The first statement, you will perhaps say, *if* it is empirical, is at any rate *less empirical, more a priori, or less contingent and more necessary* than the second; again you may say that the first statement is *very nearly, or almost analytic*, whereas its opposite, i.e. the statement 'I need not have my eyes open when I see you' or 'I can see you though my eyes are shut' comes *very close* to a contradiction without being one.

Again, consider the sentence, 'Space has three dimensions'. You will hardly be tempted to regard it as a contingent statement of fact, and yet you may hesitate to call it a necessary truth. The reason for this, I suppose, is that you fail to see *why* space should have three dimensions, and not, for instance, four. There is nothing like a *rational insight* into its truth. The statement, you will be inclined to say, is certainly not *as necessary* as a truth of logic such as  $p \Rightarrow p$ . In the latter case, I see not only *that* it is true, but also *why* it is true: it is a tautology, i.e. a symbol constructed in such a way that it cannot be false. I *understand* what it is that makes it a necessary truth. And the same goes for  $2 + 2 = 4$ . But to the assertion that space has three dimensions there seems to be nothing which corresponds to an insight, a deeper understanding. In logic, or pure mathematics, to speak with Frege, 'we are not concerned with objects which we come to know as something alien from without through the medium of the senses, but with objects

given directly to our reason, and as its nearest kin, utterly transparent to it'. But with space having three dimensions, you feel that you have just to *accept* it, to take it lying down, like any other fact which, as a fact, is *opaque* to reason. Following this line of thought, you may readily say to yourself: I see, it's like a contingent fact; no, it isn't like that because . . . , and here you produce a list of reasons. So, you go on arguing, it is more like a necessary truth; no, it can't be like that because . . . , and now you produce a list of counter-reasons. You are in conflict about what to say.

What you really want to say is perhaps, *not* so much that the statement is necessary, as that it is *non-contingent*. That you prefer to stress the absence of contingency rather than necessity, or again that you are inclined to speak of one statement being *less* necessary than another, betrays a tendency to blur that nice distinction. If you look more closely at the examples — which, by the way, are themselves very unlike — it will strike you that each of them is *far more obvious* and, perhaps, more *certain* than a statement of particular, current fact, and yet has not quite the force of a logical necessity. To give some illustrations: even if you had been immured all your life in a prison cell with impenetrable walls and without windows, you would know *for certain* that space goes on outside; you know *for certain* that space is behind your back etc.; such statements, it seems, are not quite contingent, and not quite necessary, not quite empirical, and not quite *logical*, they are half way between. What lies behind this curious impression?

The words 'contingent' and 'necessary' are used like this. We say that it is a *contingent fact* that I am sitting on this chair, that it is raining outside, and the like, but that it is a *necessary truth* that a half of a half is a quarter, that every husband is male, etc. The difference between the two types of statement, it might be said, is that one of the first type describes a *fact*, whereas any one of the second is not used *descriptively* at all, but, say, in calculating, or in drawing inferences. And one is perhaps inclined to say that, although there are two kinds of statement, there are not two *kinds of fact*, contingent and necessary. There is only *one* kind of fact. Facts are what they are, contingent. Everything that is could be otherwise. Contrary to appearance, a necessary truth does not *say* that so-and-so 'must be' such-and-such. It does not say anything at all, it is non-informative; although, and that is another part of the story, it *can* be construed — and should perhaps best be

construed — as a statement about the use of certain *words* or *symbols*. This is the sense in which it has been said that 'necessary propositions are really verbal'. (Cf. an article by N. Malcolm, *Mind*, 1940.) If we accept this account, it seems to remove the impression that there are facts of a *peculiar kind*, inherent in the nature of things, and the uneasiness which this idea is likely to awaken.

But there is something queer about this view. For one may well ask a holder of it: If, as you say, necessary propositions are not descriptive, how can they be construed as statements about *the use of language*? Should one not rather say that they are like tautologies, completely devoid of content? This difficulty will be cleared up later. Let us, however, for the moment, concentrate on another point. To say that necessary statements are not descriptive is quite right; but it does not follow from this that all non-necessary statements are on the same logical level, contingent; nor that all facts are on the same level. For there is a class of statements which are puzzling in that none of these labels seems to fit. Would you call it just an ordinary *fact* that I see with my eyes, that I do not feel your pain, that I have only one body, that I am the same person I was yesterday, that time is measurable, and that space has three dimensions? Are these statements about *facts* or about *words*? If you were pressed, you wouldn't be too sure of the answer; and anyhow, you wouldn't be happy about saying either. You may be unwilling to commit yourselves. And you would be shy of calling them contingent, and shy of calling them necessary. Why? One of the reasons which makes you hesitate can be brought out by asking: What is a contingent statement? Are we really quite clear what we mean by it? Suppose someone says that a statement is contingent if it refers to a fact. Does this help us? Of course not; for what is a fact? 'Every experience presupposes someone who has it' — is this just a fact like any other? Is there not something more to it — does it not bring out the *form* in which an experience presents itself, a form which it shares with anything that *is* experience? And yet, is it not a *fact* that experiences do occur in persons? If one says, 'I see with my eyes', is one, or is one not, stating a fact? What about such things as: Space goes on in every direction; the future is yet undetermined; there is an intelligible Order of Nature. Are they facts? Are they non-facts? Are they non-contingent facts?

Let us once more look at the statements. What is so remarkable about them is that they are obviously not the sort of ordinary report on things and situations which one associates with the idea of a particular current fact. So far they are clearly different from contingent statements; and yet one feels, or seems to feel, that they tell us something about the word. What inclines one to take the latter view is presumably this. One wants to say, But look! Don't you see that *in fact* I do not feel your pain, that I *do* see with my eyes, that space is *in fact* three-dimensional? Can you deny it? This makes us wish to say that sentences like these are about facts, or that they state, express facts. But how *can* they if they are not contingent? The difficulty that confronts us here can be brought out as follows. When we speak of a fact, we seem to have in mind something that is asserted in an empirical statement the truth of which depends on experience. So long as we have not turned to experience we do not know whether that statement is true or not; it may conceivably be either. And even if we know that it is true, it must be possible for us to specify what it would be like if it were false instead; otherwise we would be said not to *understand* the statement. This has led philosophers to say that a fact is what has alternatives; or that a fact is *contingent*. But if we follow this line, we get into difficulties. For we are inclined to say that the statements we are considering are clearly about facts, yet we feel that, if they are about facts, it should not be so difficult for us to imagine situations in which they would be untrue. But that's precisely what we cannot do. If we try to deny any of these statements — i.e. if we try to imagine a situation in which I *do* feel your pain, in which I don't see with my eyes, etc., — we are up against a very odd state of affairs. The impression one gets is that we can *say* such things, but cannot think them; or that, in the attempt to think them, we get 'lost and embangled' in inextricable absurdities, to use Berkeley's phrase. Indeed, what would it mean to think of an alternative to 'I have only one body'? Would you understand me if I called that in question? Can I for my part, by the boldest stroke of imagination, picture to myself what it would feel like to possess *two*? Can I really, not in abstract, lifeless terms, but concretely, vigorously depict such a situation with all the small details which would give colour to it? Can we visualize, or conceive in any way that actual space is four-dimensional? That it is not unbounded? That time has a beginning and an end? Would not

our whole thinking be plunged in confusion if we made any such attempt?

If these statements are not contingent, ought we to say that they are necessary? What seems to foster such a view is this. We feel that these statements are *unique* in the sense that they have no alternatives; if they are unique, if no other state of affairs is conceivable, then they are necessary. For necessary just means that which admits of no alternatives. Have we thereby *proved* our point? Let us see. When we ask whether these statements are necessary it is important to bear in mind that a word like 'necessary' is used in a great many different ways. What has just been said brings out only one of the facets of the word's meaning. Another sense, closely allied to it, is the idea that a necessary statement is absolutely *certain* — that its authority cannot be shaken by any empirical evidence to the contrary; or rather that no such evidence is *relevant* to it, either to its truth or to its falsity. Once we look upon it in this way we begin to waver. After all, we say to ourselves, how do we *know* that space is three-dimensional *if not by experience*? Yet if this item of knowledge is *based on experience*, how can it be *necessary*? And if it *was* necessary, how could it tell us anything about experience? Or again, if it is empirical in the sense that it *rests* on experience, it seems very odd that it should not be *refutable* by it. Normally, what is derived from experience can be annulled by experience. Yet, one would say, it is utterly inconceivable even to think of observations which would make us give up saying that space is three-dimensional. Is this not very puzzling? We may feel now that a statement like 'Space has three dimensions' is in certain respects like a necessary, and yet in certain other respects like an empirical truth. Is it, then, empirical and *a priori* at the same time? Or is this an absurd thing to say? But let us, for the time being, shelve the question. There is yet another sense of 'necessary', that which we have in mind when we say of something proved in mathematics or in logic that it is necessarily true. What we mean by this, anyhow in part, seems to be that any such statement is through and through *rational*; we can reach an *insight* into its truth such as is denied us in the sphere of brute facts. Not that all parts of mathematics are alike in this respect. One feels that geometry — the science of actual space — is in this regard very different from *pure* mathematics — a point already seen by Gauss when he wrote (in a letter to Olbers, 1817): 'I keep coming closer to the

conviction that the necessary truth of our geometry cannot be proved, at least *by* the human intellect *for* the human intellect. Perhaps in another life we shall arrive at other insights into the nature of space which at present we cannot reach. Until then we must place geometry on an equal basis, not with arithmetic, which has a purely *a priori* foundation, but with mechanics.' The difference we all feel can be put as follows. A man may be puzzled why space should have just three dimensions, and not, let us say, four, but he will not be puzzled about why  $2 + 2 = 4$ . Or if he is he will be confused. In the latter case, as indeed in the case of any purely mathematical truth, we can demonstrate it with all the rigour we may desire, and once we comprehend the proof, no obscurity is left, nothing that makes us ask the question 'Why?' — everything is perfectly clear and transparent. This question worries us only so long as we have not attained to a *complete* understanding. Where such complete understanding is within our reach, we stop asking the question — a very significant sign that full insight *has* been attained. This is possible in the domain of numbers which are given us not from outside as something foreign to us, but as constructions entirely clear and rational which, therefore, can hold nothing mysterious for us. And the same goes for logic. However, when we turn to things we know only through the medium of the senses the situation is radically different. In the natural sciences we think that we have explained a phenomenon if we are able to deduce it from certain general laws of nature together with certain data of observation. These laws can sometimes be traced back to other more general ones, and these to what is termed ultimate principles, or else they may be derived from certain hypotheses such as the atomic theory — but in the end we have to stop short at them as something we have to submit to. We can only bow before them, as it were, accept them humbly as the given Order of Nature, unable to gain an insight into them. There is, we are apt to say, something opaque and impenetrable to reason about them which resists rational understanding. Why just these laws and not others, we keep wondering. Yet however far we may go in reducing them to deeper ones, we feel that we shall never reach the point where our mind, no longer worried, will cease to ask this question. There is nothing that answers to that full comprehension, or insight, that is within our reach in a logically perfect science. On the contrary, some dark residuum will always remain; something

that cannot be penetrated completely. As with laws of nature, so with the three-dimensionality of space. Here, too, we feel ourselves to be in the presence of a most important feature of reality — yet of a feature which, as we can't look through it, seems to oppress and overwhelm us with its factuality. It appears dark, mysterious, almost as if decreed by fate — inscrutable. It is in this sort of situation that the question 'Why?' assumes that disquieting metaphysical ring. And it is precisely this which rebels in us against regarding this statement (about the dimensions of space) as necessary. In fact, it now seems to us decidedly *non-necessary*, much more like an empirical statement of fact.

Thus the mystery seems only to deepen. The statements so far considered, we persuade ourselves, are about facts — don't we understand quite clearly what they say? Is it not a *fact* that space is three-dimensional, that I see with my eyes, that I feel only *my* pain, and so in the other cases? And yet they cannot be about *ordinary*, empirical facts; if they were, why the trouble about alternatives? And if they are unique, non-contingent, how can they describe anything at all? Is not the very idea of a description tied up with the notion of true and false, and hence with that of alternatives? To hint at a paradox — we say that something, though it is not the case, is possible. No one is sitting on this chair, but someone might be sitting on it. It is as though something was not the case, and yet was more the case than if it *could* not be the case. When we say this we have perhaps at the back of our minds a picture of reality as a sort of island surrounded by a sea of possibilities. Not that we ever explicitly entertain such an idea. But to admit the use of a crude picture is always better than to use this same picture unadmittedly. To say that the soul is a little man in one's head that peeps out from one's eyes is less harmful than to use the same picture in a covert or 'sublimated' way so that it might escape censorship in Freud's sense. Similarly, it is better to confess to ourselves that we are tempted to look upon reality as if it was set in a surrounding of mere possibilities. The queer thing, however, is that saying 'I feel your pain', 'I have got two bodies', etc., is not describing something which just *happens* not to be the case. What one wants to say is rather that it *makes no sense* to say such a thing. In other words, what the *undenied* statements express, or purport to express, is no longer *contrasted* with other possible states of affairs — in the language of our simile, the

island is no longer marked off from the sea. The 'facts' we have been considering are not opposed to anything else. *Are* they facts?

Thus we waver to and fro. It is as if we said — Surely, all these are facts; only to add after a moment's reflection — But how can they *be* facts? This conflict gives rise to a peculiar mental uneasiness. It is as though, up to now, we had been passing heedlessly over the difficulties, and now, all of a sudden, we noticed them and asked ourselves in alarm — But look, how can that be? That is a sort of question which occurs over and over again in philosophy. It seems as if two forces were acting upon us, pulling us in opposite directions. We are alternatively tempted to say, 'Facts of course; no, impossible'. We are torn between doubts, and don't know what to say. We have reached a point where the very idea of a fact appears in a mysterious new light.

Queer — getting confused about things which we all understand in daily life is often the first sign that we have made progress in philosophy. So we are worried now about the idea of a fact. It seems as if the mind's eye were growing dim, as if everything, even things which ought to be perfectly clear and simple, were becoming oddly puzzling and unlike their usual selves. Indeed, our bewilderment may be summed up in the question: What is a fact? What troubles us is the *character* of those statements, their apparent likeness, and again their unlikeness, to current reports and descriptions. One way of bringing out this feature is to ask — Is the fact that I am mortal as *contingent* as the fact that I have a scar on my left thumb? Or are there *degrees* of being contingent? Should one perhaps say that statements can be arranged according to this degree? And does the character of the statements which puzzle us not lie just in this: that they are a sort of limiting case to which ordinary statements may approach, only less contingent and more certain than these? Are they to be likened, it might be asked, to laws of nature? Or are they different from them in kind? Another question worth mentioning is this: Is not the difficulty of deciding between contingent and necessary in the last analysis due to some deficiency of ordinary language? If so, can it be remedied — for instance, can a 'more logical' language be conceived in which every statement can be put neatly into one of the two baskets? Or is the deeper reason for our predicament this, that the distinction 'contingent — necessary' is not exhaustive? Is there

room for a third class of statements as yet overlooked by the theorists who have drawn this distinction?

These are some of the questions we have to treat. They cluster round the notion of a fact, and its linguistic counterpart, that of an empirical statement. Before, however, grappling with these notions, we must turn our gaze into another direction. It has already been noticed that in an attempt to deny any of those queer statements we are faced with insurmountable difficulties. One may well ask, is this perhaps due to certain limitations of our natural language? If so, why not extend language so as to make it fit to do the job? That brings us to a new subject, the part played by natural language in this discussion.

5. ORDINARY LANGUAGE. The appeal to 'the ordinary use of language' has become a feature of philosophical technique of the very first importance. If a philosopher talks in a queer way — using words in very special, very wide, very uncommon senses, or indulging in highfalutin' phrases — it is well to remind him of the *ordinary* way we use these words in plain talk on simple matters. To bring him down to earth from his flight of thought was, no doubt, the right move at a time when people got strangely intoxicated with all sorts of metaphysical moonshine. When an idealist, quite seriously, said, 'Time is unreal' and Moore retorted, 'Do you mean to say that you had no breakfast this morning?' this had a sobering effect, and I am not going for a minute to dispute that, or detract a whit from its therapeutic efficacy. So long as people are liable to be duped by words, Moore's technique will continue to perform its wholesome purgative function. On the other hand, one should not mistake purgatives for food. If philosophy is not to die of sheer inanition the philosopher has to *say* something, whether he gives shape and expression to pregnant, yet only half-consciously formed ideas, or — as some Renaissance philosophers or Francis Bacon did — hits upon one of those watchwords which lighten up vast new aspects of thought and feeling, or serve as seeds for subsequent developments, or — as the rationalists did — gives a powerful impetus to scientific thought, or energizes new ways of looking, kindling the imagination — as Herder and Hegel did — for the unsounded depths of history, or what not. He may do a thousand things; reformulate results previously obtained so as to make us see them in a new setting and with a new significance, awaken an analytic spirit,

forge powerful weapons of analysis, make us sensitive to new aspects; he may discover new types of questions, cast doubt on accepted answers, deepen our understanding, heighten our critical powers, penetrate to deeper insights: but whatever he does, he is first and foremost an agent of ferment. If then, he is to fulfill his mission, the battle cry 'ordinary use only!' is likely to have a shackling or, indeed, a paralyzing effect.

If a philosopher goes 'too wild', it is well to apply a dose of this panacea, the recall to 'ordinary language'. But should one always and on principle do so? I wish to say provocatively that there is positive *danger* in this notion. For one thing, to speak of *the* ordinary use of language is already questionable, implying as it does, that *there is* such a thing, and a unique one, and that one can find out what it is. But how ought one to determine what this ordinary use is, e.g. in a case of doubt? What ought one to do — to ask people? *Any* people? Or only the competent ones? And who is to decide who is 'competent' — the leading circles of society, the experts of language, the writers just in vogue? And supposing there are people generally considered competent — what if they disagree? Is perhaps the right way of tackling the question just this — to write down a long list of actually observed uses, taking note of the frequency of each use, and distilling the whole into a statistical table? But is this the sort of thing a philosopher wants to do? Is he interested in the random fluctuations of speech, that sea with its endless waves and ripples?

Simply thus to refer to 'the' ordinary use is naive. There are *uses*, differing from one another in many ways, e.g. according to geography, taste, social standing, special purpose to be served, and so forth. This has long been recognized by linguists who distinguish between use in writing and use in talking, or (in a different sense) between speech and language, between poetic and prose expression, elevated and every-day diction, literary style and more colloquial speech, slang and idiom, or again between jargon, cant and argot; not to mention shop-talk, college lingo, etc. All these are particular ways of using language, loosely revolving around a — not too clearly defined — central body, the standard speech. Thus one may, not without a show of reason, speak of a *prevailing* use of language, a use, however, which by degrees shades into less established ones. And what is right, appropriate, in the one may be slightly wrong, wrong, or out of place in the others. And this

whole picture is in a state of flux. One must indeed be blind not to see that there is something unsettled about language; that it is a living and growing thing, adapting itself to new sorts of situations, groping for new means of expression, for ever changing. True, there are words, often dating back to time immemorial, which are fairly definite and hardly change in the course of centuries. Through age-long and persistent use they have become smooth and polished like pebbles on a beach. In them, and the way they are used, we find stored up the results of the experience of many generations. They represent the *conservative* element in language. Others change perceptibly. If a new way of looking at things is growing up, a word may acquire new values, or be completely transformed and loaded with new meanings. An instance of this is the word 'romantic'.<sup>1</sup> When it first came into use, in the middle of the 17th century, it was a term of depreciation, implying 'false', 'imaginary', 'unnatural', 'chimerical', 'ridiculous'; it meant 'like the old romances' with their fabulous world and their false, high-flown sentiments, and was an expression of a newly-awakened critical spirit. The word soon began to mean something which, though absurd, was attractive to the imagination. In this sense it was, already with some appreciation, applied to the scenes depicted in the old romances with their castles, forests and moonlight. With the gradual shift of feeling which began in the 18th century in this country the word acquired a specific modern sense: it became a term for a *literary* emotion, applied to nature *seen through the medium* of poetry and fiction, like 'picturesque', also a word emerging at this time, which was used to describe scenes that were 'like pictures', seen through the coloured glass of painting. The English word was borrowed into German, and there, under the influence of philosophers and critics, was brought into opposition with what was called 'classical' and imbued with vague metaphysical notions. From Germany it came to France, and from France it returned to its homeland, deeply changed by its journey and holding a world of new meanings. Thus the history of European thought is curiously mirrored in the usage of a single word. Numberless examples can be given to illustrate similar changes. All of us have witnessed in our time a significant change of meaning in words like 'axiom', 'geometry', 'tautology', 'meta-

<sup>1</sup> See Grimm's *Wörterbuch*, Leipzig, 1854, and an article by Logan Pearsall Smith in S.P.E. Tract, No. 17.

physics'; the word 'proposition' almost looks as if it was dying; and the word 'meaningless', formerly a term of abuse, or at least a fighting word, has now very nearly come to be used as a descriptive term.

But beside these changes, recorded by lexicographers, there is another current, slower and deeper, not yet understood, which flowing on through many centuries leads to a reshaping of language, and, as a result of this, to a change of the world picture embodied in it. This suggests a theme so vast that it might be the subject of a long treatise; the way, namely, in which language moulds our apprehension of qualities, processes, human actions, etc.

Language is always changing. That is a commonplace, yet, oddly enough, one not enough heeded by those who are clamouring for 'the ordinary use of language', quite prepared, it seems, to damn everything out of hand — in philosophy — if it fails to conform to its standards. While appreciating the service done to clear thinking by the insistence on the normal use, I feel that the time has come to say a word of warning against the cult of it, for such it has almost become. Like any cult, while it is likely to protect its votaries from certain dangers — getting trapped in the vagaries of speech — it is apt to make them blind to the obvious narrowness of such a view, particularly when it is just on the point of becoming one of the major influences of our time. It tends to instil, in the faithful and in the not-so-faithful alike, a belief, a complacent one, in the adequacy of language which is far from the truth. In actual fact, language is a deficient instrument, and treacherous in many ways. As this opens a subject of vast dimensions I shall confine myself to a few scattered observations.

First, I shall try to argue that a departure from the beaten track need not only not be anathematized, but may be the *very thing* to be striven for — in poetry, science and in philosophy. My second point is that language, far from serving merely to report facts, is a collective instrument of thought that enters experience itself, shaping and moulding the whole apprehension of phenomena (such as colour and lustre, e.g.) in a certain definite way, and, who knows, giving to them just that subtle bias which makes all the difference. How curiously different, for instance, must the world of colour have appeared to the Romans who had in their language no word for grey, brown, nor any *generic* word for blue (though they had a number of words to denote particular shades of this

colour). How curiously different, it would seem, must human action appear when seen through the filter of Eskimo language where, owing to the lack of transitive verbs, it is likely to be perceived as a sort of happening without an active element in it. (In Greenlandic one cannot say 'I kill him', 'I shoot the arrow', but only 'He dies to me', 'The arrow is flying away from me', just as 'I hear' is expressed by 'me-sound-is'.) Eskimo philosophers, if there were any, would be likely to say that what we call action is 'really' a pattern, or Gestalt, of succeeding impressions. Just as Greenlandic assimilates action to impression — which strikes us as strange — so our language tends to bias us in just the opposite way: it makes us assimilate perception to action. We say not only 'I cut the tree', but also 'I see the tree': the use of the same construction makes it appear as if the 'I' was the *subject* from which issued the seeing, and as if the seeing was a sort of action directed at the tree; nor are we any better off if we use the passive voice 'The tree is seen by me' — for now it almost looks as if something *happened* to the tree, as if it had to undergo or suffer my seeing it. Following the clues of speech, we are led to interpret the world of experience one-sidedly, just as 'owing to the common philosophy of grammar', as Nietzsche put it, i.e. 'owing to the unconscious domination and guidance of similar grammatical functions' the way seems barred against certain other possibilities of world-interpretation. In other words, every language contains, deep-sunken in it, certain moulds, designs, forms to apprehend phenomena, human action, etc. It is hardly going too far to say that a whole world picture is wedded to the use of the transitive verb and the actor-action scheme that goes with it — that if we spoke a different language we would perceive a different world. By growing up in a certain language, by thinking in its semantic and syntactical grooves, we acquire a certain more or less uniform outlook on the world — an outlook we are scarcely aware of until (say) by coming across a language of a totally different structure we are shocked into seeing the oddity of the obvious, or what seemed to be obvious. Finally, I want to say that philosophy *begins* with distrusting language — that medium that pervades, and warps, our very thought. But this is perhaps too strong an expression. I do not mean to say that language *falsifies* experience, twists it into something else; the point is that it supplies us with certain categorial forms without which the formation of a coherent system

of experience, a world picture, would be impossible. In this sense, language shapes and fashions the frame in which experience is set, and different languages achieve this in different ways. A philosopher, more than others, should be sensitive to this sort of influence, alive to the dangers that lie dormant in the forms of expression — the very thing, that is, which, so misguidedly, has been raised to the standard in philosophical controversy.

When I spoke of the change of language I was not thinking of those cases which delight the heart of a philologist — *umlaut*, *ablaut*, and the like. Nor was I referring to changes in meaning and vocabulary — what was originally stupid, wanton, Latin *nescius*, becomes ‘nice’; a horse that is well-fed and grows a smooth, shiny coat is ‘glad’ — *glatt* in German; what is now silly was formerly ‘sely’ corresponding to German *selig* — happy, blessed; for while such changes are instructive in many ways, they are hardly such as to deserve the philosopher’s attention. Neither was I thinking of those more subtle changes in the *valeurs* of a word which — as in the case of ‘romantic’ — are significant of a change in the tone of thought of a whole period — of a half-conscious awakening of new ways of feeling and responses to nature, so elusive and yet, to the historian, so important. What I had in mind were cases which are best illustrated by a few examples.

Nothing is so opposed as day and night; yet there is a sense, as when we speak of a ‘three days’ journey’, in which ‘day’ includes night. ‘Man’ is used in contrast to woman, but occasionally as a term including woman; and a similar shift of sense is perceptible in ‘he’ and ‘she’ — as an arguer, woman is also ‘he’. We say of a child that he is two years ‘old’, not two years ‘young’, just as we inquire ‘How *long* (not how *short*) will you stay?’ or ‘How *far* (not how *near*) is it from here to the station?’ The word ‘quality’, while for the most part used indifferently, is sometimes uttered in a peculiar tone — as when we say ‘He has quality’. White and black are commonly contrasted with colours in the strict and proper sense (‘illustrations in colour’ *versus* ‘illustrations in black and white’), yet in certain contexts we are inclined to reckon them amongst the colours; as when we say ‘Look round you — everything you see has some colour or other’, thinking, perhaps, that even air and vapour, or glass and water are possessed by some very pale, some very pearly tone. Thus ‘colour’ tends to absorb into its meaning all shades, even black and white, the otherwise ‘colourless’

hues. But these are instances betraying a deeper drift. In the ordinary sense, motion is opposed to rest, speed to slowness, size to littleness, numerous to a few, depth to shallowness, strength to weakness, value to worthlessness, just as far is opposed to near, hot to cold, dry to wet, dark to bright, heavy to light, and true to false. And this was, roughly, the way in which Greek philosophers regarded such contrasts. 'Up' for them was simply 'not-down', 'soft' 'not-hard', 'dry' 'not wet', and so on.<sup>1</sup> The fact that two polar terms were in use may have played a role in underpinning the belief that things which are hot and cold, or hard and soft, etc., are different, not in degree, but in kind — a fateful belief, for on it hinged their understanding — no, their lack of understanding of change. They signally failed to penetrate it. The Greeks never mastered the problem of motion — which is but the simplest case — they never evolved a science of dynamics, which is surprising enough in view of their genius for mathematics. They give the impression that they somehow got started on the wrong track — for them heavenly and terrestrial motion were entirely different, the one governed by law, eternal and unchanging, the other lawless, corrupt, confused; if faced with a change, such as a thing getting heated, they thought that one quality must be destroyed to let the opposite quality take its place. Thus they were, perhaps as a consequence of their quaint ideas, mightily impeded in coming to grips with the problem of change.

In science a language has come into use in which those contrasted terms are looked upon as degrees of one and the same quality — darkness as light intensity of illumination, slowness as the lower range of speed, rest as the limiting case of motion; there is a scale only of hardness, not of softness, only a physical theory of heat, not a theory of coldness; what we measure is the strength of a rope, a current, etc., not its weakness, what we count is number, not fewness; the air has a degree of moisture, not of dryness; and everything has weight and mass, even an electron. Again, we speak of health irrespective of whether it is good or bad health, and of the value of things which are of no value. Under the influence of such examples, it would seem, a term like 'truth-value' has been coined to cover both truth and falsity of a statement, just as 'verification' is, prevalently, used to include falsification. 'Distance', 'width', 'wealth', 'intelligence' are further nouns which

<sup>1</sup> See, e.g., A. P. Rossiter, *The Growth of Science*, Cambridge, 1939.

had the same career; though the same is not so true of the adjectives — 'distant', 'wealthy', 'intelligent' are not yet relativized, any more than 'hard', 'hot', 'speedy', 'weighty' are, or 'healthy', 'valuable' and 'worthy'; on the contrary, they retain the original sense. Adjectives, it would appear, have a much tougher life than nouns, and not only in English. But that only in passing.

Here we see a whole array of terms shifting in a parallel way, and in a way which is of far-reaching consequence: for the construction of modern science is bound up with it and would not have been possible without it. The change-over from the static view — where the adjective is seen as the expression of a permanent quality — to a dynamical which apprehends quality as a variable degree within a certain scale made possible 'functional thinking' (I use the word as mathematicians do), the kind of thinking that can cope with change and the conceptual difficulties it presents. What happened was obviously this: one term of a pair of contraries had a tendency to swallow up the other and stand for the whole range of variation. Whether this tendency can be traced to the rationalizing influence of science, or whether it is prior to science and has itself given an impetus to that revolution of thought, is a question still undecided.<sup>1</sup> It is, in this context, perhaps not without significance that Latin and Greek were lacking in all the finer means to express continuous change and functional dependence: in Latin, for instance, there are no *general* terms to express the relation 'the more — the less'; the phrases used for 'the more — the more' are confined to simple *proportionality*, the analogue to *statics*.<sup>2</sup> Nor has any classical language an equivalent for 'to become' (*devenir* in French, *devenire* in medieval Latin) so essential to our way of describing a change in quality, for neither *fieri* nor *γίγνεσθαι* can be used in the same way to express the idea of *continuous* change. There are no uses of intransitive verbs such as 'to soften' (*rubesco* is inceptive), etc.

The new idiom, which sprang up first in the vernacular about the fourteenth century, has not entirely displaced the older one (as can still be seen from the adjectives cited above). Both exist side by side. Though the use of 'moisture' for dry as well as wet

<sup>1</sup> My attention has been drawn to this aspect of the matter by my former pupil J. L. Hevesi.

<sup>2</sup> Cf. Ettmayer, *Analytische Syntax der Französischen Sprache*, vol. ii, Vienna, 1936, pp. 935 ff.

(as in meteorology), or of 'truth-value' in logic still has the ring of jargon, in other instances the new idiom has become completely naturalized — as with 'distance' for near and far, 'age' for young and old, 'size' for big and small, 'density' for thick and thin. Yet even so, we can use any such term in two distinct ways — we may ask '*Is he old?*' or '*How old is he?*'; and so in the other cases.

At the time of Nicole Oresme, Bishop of Lisieux, when a new way of looking at change was growing up, and with it a new way of speaking of qualities, this must have been felt as a shocking departure from the ordinary use, supported and sanctioned as it was by old tradition. How the cloisters of the schoolmen must have resounded with '*intensio et remissio formarum*' — the disputes as to whether a quality might have degrees and, in changing, could yet remain the same, or whether this was patent nonsense. One may imagine the indignant outcries of the purists of the time, their loathing of what must have appeared to them as 'new-fangled ways of speaking' and as a 'complete perversion' of grammar. The latter, more even than the vocabulary, embodies a good deal of the conservatism of mankind, and progress had often to be made in the teeth of the enormous resistance offered by its structure to ways of thinking which do not, or not smoothly, fit its grooves. (See what has been said in the foregoing on Greek language and absence of dynamics.) Grammar draws a *cordon sanitaire* against any rebellious ideas that dare to crop up.

The importance of functional correlation can, moreover, be seen in a different domain: in perspective, and the enthusiasm with which it was universally greeted when it was discovered — another coincidence? — at the very time when new aspects of thought and feeling were just about to take shape: Duccio's Maesta and Giotto's wall paintings in the Capella degli Scrovegni in Padua both belong to the early fourteenth century. The 'strange fascination which perspective had for the Renaissance mind cannot be accounted for exclusively by a craving for verisimilitude', as Panofsky<sup>1</sup> observes. A sensibility to functional relation is apparent in this, and the interest in perspective — so alien to the Greeks — is almost symbolic of the time. A reflex of it can still be caught from the writings of Leonardo da Vinci and Dürer. As perspective rests essentially on a clear understanding of the way in which two variables, the apparent size of an object and its distance from the

<sup>1</sup> E. Panofsky, *Albrecht Dürer*, Princeton, 1943, vol. i, p. 260.

beholder, are connected, Leonardo saw in painting a 'science'. He certainly must have been struck by the affinity between this 'science' and the philosophical speculations on dynamics of the schoolmen of which he was fully aware (he even employed their ideas in his theory of painting).

If those pedantic schoolmen and -masters had had their way, there would today be no science and no dynamics; but, for consolation, 'correct' grammar. To look at any departure from the norm as a crime is nothing but a blind prejudice; and a fateful one at that as it tends to drain the life-blood of any independent inquiry. Language is an instrument that must, as occasion requires, be bent to one's purpose. To stick to language as it is can only lead to a sort of Philistinism which insists on the observance of the *cliché* and will end up with a *hara-kiri* of living thought. Indeed, the guardian of language who jealously watches over its 'correctness' is in the long run bound to turn into a reactionary who looks askance at any innovation. Correctness is a useful, but a negative virtue. Follow those prophets, and you will soon find yourself imprisoned in a language cage, clean, disinfected, and unpleasant like a sanatorium room.

Understandably enough, there is an instinctive prejudice against neologisms, in part springing from a wholesome fear that novelty of speech may screen poverty of thought. We all dislike new words. And yet there is another and perfectly proper urge to give expression to meanings so far unexpressed, or, in the present language, indeed inexpressible. When Freud, for instance, says *der Patient erinnert den Vorfall* he is using the verb *erinnern* in a novel manner; in the ordinary way, the verb is used reflexively, *sich an etwas erinnern*. Why has Freud (who wrote a very good style) diverged at this point? There is a queer way in which a neurotic person who is under treatment may suddenly remember long-forgotten scenes of his early life which, as Freud puts it, have been 'repressed' and are now being re-lived. What has been inaccessible to the patient, however hard he may have tried, breaks, in a violent storm of emotion, through to consciousness. In order to set apart this kind of remembrance from the ordinary one where we remember at will, Freud uses the verb transitively, in a way no one has done before; and with this syntactical innovation goes a semantic change. By this use Freud has enriched the German language. Such stray deviations, hit upon in a lucky hour and

accepted by custom, these little, yet expressive departures from the beaten track, have not only a vividness, a sparkle of their own, but they sharpen the tools of thought and keep language from going blunt. So why cavil at them?

What those sticklers for correctness prefer not to see is that we are living in a *changing world*, and that language is always lagging behind these changes. To cite only one sort of example out of a great many parallel ones — in psychological experiments one constantly comes across situations which call for new ways of describing. If Maxwell discs, for instance, are rotated one sees, so long as the movement is slow, several colour sectors, and when the disc is spinning rapidly, a uniform colour, the result of fusion, but in between there is a certain point where a flicker is seen. There are cases in which the colour itself is seen flickering, and others, as when the disc is watched through a small screen-hole, which are more aptly described by saying that there is a flickering *across* the disc or *before* it in space, or again that the disc's surface is seen *behind* the flicker. These modes of expression, though perfectly natural and instantly understood by every one, yet digress from the norm. For 'before' and 'behind', while clearly denoting spatial relations, are used in such a way that it no longer makes sense to ask, 'Exactly how many millimetres before the disc is the flickering?' Here we have a sense of 'before' which admits of no distance. To cite a few similar cases — if we look at a metal its colour seems to lie *behind* its surface, just as its glitter appears *in front of*, or *superimposed* on it; the glow of a piece of red-hot iron is seen not simply as colour that lies on its surface but as *extending back* into the object. Again, it has been said that, when a person is speaking with someone in complete darkness, the voice of the other sounds distinctly *behind* the darkness, not *in* the darkness. In some cases an object is seen as 'desurfaced', with a filmy, fluffy sort of outline, a bit unreal perhaps. Queer idioms which say what cannot quite be said by anything else: but condemn them on account of that? Notice with what unerring instinct language contrives to say, at the cost of a slight departure, what would be unsayable if we moved along the rigid grooves of speech. Indeed, how should one describe such phenomena if not by breaking away from the *clichés*? Is there anything objectionable in that? If so, language could never keep pace with life. Yet new situations, unforeseen, arise, and with them the need of describing them; it

can only be met by adjusting language — either by coining new words, or, as the word-creating faculty is scanty, by pressing old ones into new services, in this way cutting through the dead mass of convention. It is precisely because speech runs so much in ready-made moulds that an occasional anomaly, a happy flouting of the laws of grammar, an uncommon phrasing, arouses our attention and lends lustre to the point we want to bring out. It is in this way, by *transgressing*, that language manages to achieve what it is meant to achieve, and that it grows. Why, then, the squeamishness?

Not only should the scientist be free to deviate from common language, where the need arises, but he is bound to do so if he is to convey a new insight not in conformity with the ideas dominant of the time, with ideas, moreover, precipitated in language. The classical example of this is Einstein. When he was groping his way, there was, in his own words, ‘a feeling of direction’, of going towards something he didn’t quite know — which centred more and more on a suspicion that all was not well with the idea of simultaneity. He could at first not say what was wrong with it, and yet felt that here, if anywhere, was the key to all the dark puzzles that troubled the physicists at that time. Had he been brought up as a pupil of G. E. Moore, imbued with a belief in the infallibility of the ordinary modes of expression, he could never have made his discovery, clogged as he would have been by the dead weight of usage. As it was, he paid no respect to common sense, let alone the common speech. He insisted on asking himself, ‘Do I *really* understand what I mean when I say that two events are simultaneous?’ Once the question was brought into sharp focus, he came to see, gradually perhaps and to his surprise, that there was a gap in his understanding. For the sense in which we speak of two events happening at the same time, when they are in the same place, or nearby, cannot be applied to events in distant places. It would be *blind*, he felt, to apply the familiar meaning of ‘simultaneous’ to these other cases — it would only land us in perplexities beyond resolve. Einstein saw that the term ‘simultaneous’ had first to be *defined* for the case of distant events, and defined in such a way that the definition supplies us with a method to decide experimentally whether or not two events are simultaneous. This ‘seeing’ of a crucial point in the meaning of ‘simultaneous’ has *absolutely nothing* to do with the way the word is actually used in language. It

is as well to remind you that in 1905, when Einstein's first essay appeared, there was only *one* use, not two uses of 'simultaneous', and that it would be absurd to pretend that, when Einstein found a difference in meaning, he was making a *linguistic* discovery. (A side-light on how wrong the philosophical equation meaning = use is.) On the contrary, anyone who had taken ordinary language, or commonsense, for his guide, and had been asked whether he understood what 'simultaneous' meant, would have replied with a decided 'Yes' — no matter whether he could, or could not, specify a method for finding out. He would have said that the meaning of the word is clear in itself and needs no further explanation. In fact, no one before Einstein, whether a plain man, a scientist, or a philosopher, doubted for a minute that the concept was clear to him, so clear that he need not trouble. That's precisely what made people slur over the decisive point. Einstein *saw*: that is how he freed himself from the thought-habits imposed on us by speech, radically so. By following the lead of language, or of the common-sense philosophers, one would have barred oneself from the spark of insight which was to be the dawn of a new era in physics.

These facts speak for themselves. That science cannot live under the tutelage of any ideas on 'correctness', will perhaps be conceded. But this is true not only of science. Poetry is forever groping along the borders of the unspeakable, wresting new land from the vast void of the unexpressed. It is its mission to break through the wall of conventional views that encloses us, to startle us into seeing the world through fresh eyes. This is what all the great poets from Dante to Baudelaire have performed, and that is their glory. However, it is a large subject, too large to be treated here. I shall pick out only one tiny point, and one, moreover, that concerns prose — Flaubert's style, which, in Proust's phrase, has 'renewed our vision of things'. In a work of fiction, nature is usually treated as background to men; against this background stand out the main characters of the story, the way they act, think, speak, feel and behave. The contrast between the uniformity of nature and the uniqueness of the human world is, in French, expressed by the use of two tenses — the imperfect for things and processes, and the perfect for men and actions. But with Flaubert, what men do is, in essence, always the same — it is like the succession of rain and sunshine, spring and summer, the ripening of the corn, and the

phases of the moon. There is something dull and repetitive about them which pervades them with a sort of dispassionate sadness. There is a passage in *Madame Bovary* where Flaubert speaks of 'the eternal monotony of passion which has ever the same forms and the same language'. A revealing passage; for what he has tried to do and has done is to bring about something like a shift in our way of seeing people and things; and this he achieves, simply, by his relentless use of the imperfect, assimilating, in language, his apprehension of men to that of things (remember Greenlandic!). Everything, including human action, is resolved into a perpetual and monotonous flux, revealing the melancholy essence of human existence. Describing people in the forms appropriate to things produces a peculiar effect indeed — 'what, up to the time of Flaubert, has been merely action, has become impression', as Proust puts it. As we read over the pages of his novels, we are made to feel in what people say that they would always say precisely the same thing, that their whole life can be poured into a phrase as into a little vial. And when the perfect is used — on rare occasions only, as when the narrative changes direction — it is again with a queer effect: it gives to a thing (when it occupies the place of a subject) a character of activity, it is as if a furtive ray of sunlight was falling on it, imparting to it, for a fleeting instant, a life of its own: change suddenly turns into action. And from this arises that unique Flaubertian vision of things which, like any artist's vision, can only be communicated through his style. Besides the tenses, the conjunction 'and' is used in an entirely new way. It hardly ever binds phrase to phrase, but has a more musical function — to mark a pause in the beat of the rhythm, to indicate that the moving wave we have been following has spent itself, and that another is about to build itself up. To this must be added a novel use of the present participles, of adverbs, and of certain pronouns and prepositions — grammatical peculiarities which all contribute to give shape to a world picture in which life is seen as a smooth change of one state passing into another without the persons taking any active part in the action — a picture that reminds one of some huge escalator which goes on and on, never stopping, never breaking its monotony. But where an 'action' does intervene in the flow of events, its protagonists are, in general, *things*, acting on a plane of non-human drama. What a vision! And yet a vision attained by distorting syntax. This, I think, should

be enough to instil a drop of scepticism into the belief that all is well with ordinary language; it makes one wonder whether there is not, after all, a hard atom of truth in the view that ordinary speech is only good for saying things that are no longer worth saying.

By giving such copious examples my aim was to drive home the point defended here — that the ideal of correctness is a deadening one, that it is in vain to set up a language police to stem living developments. (I have always suspected that correctness is the last refuge of those who have nothing to say.)

Poets and literary critics feel, perhaps more keenly today than ever before, that there is something disquieting about language. If I correctly read the signs, there is a susceptibility to the perils of words, a growing one, and a suspicion that language comes between us and the things we want to say. 'In speaking one always says more than one intends to' observes Sartre; and T. S. Eliot, having noticed the vanity of words to express what is unique in experience, says 'The particular has no language'. Philosophers, on the other hand, are on the whole more likely to be found in the opposite camp — 'debunking' all this talk as 'pseudo-complaints which masquerade as genuine'.<sup>1</sup> I think that this is a mistaken attitude for a number of reasons, and this is perhaps the place to set out some of them.

First, to talk of *the* ordinary use of language is, as I have already hinted,<sup>2</sup> unrealistic. Though I would not go so far as Ezra Pound in saying that our whole speech is 'churning and chugging' today, the fact remains that language is in a state of flux. But, it will be said, that is the concern of the historian of language, not of the philosopher. All the philosopher needs to know is the *stock*-use of a word or phrase, as it is employed at present, in contrast with its non-stock uses. This answer is unsatisfactory. Though it would be silly to pretend that one did not know the stock-use of 'cat' or 'shut the door', there are other cases where one would feel less sure. Is a 'taste of onions' the stock-use and a 'taste for history' derived, secondary, figurative? (But it is not *felt* as a metaphor!) Is only a 'brilliant sunshine' standard-use and a 'brilliant style'

<sup>1</sup> Alice Ambrose, 'The Problems of Linguistic Inadequacy', in *Philosophical Analysis*, ed. Max Black, Cornell University Press, 1950.

<sup>2</sup> See pp. 172 ff.

non-standard? Is 'day' as opposed to night, or as including night, the norm? What about speaking of a 'wild laughter', a 'brooding silence', or saying that a 'recollection of this experience moved in his eyes'? It is easy to see that the 'stock-use' shifts with the context, and shifts in time. What was stock-use may become obsolescent and fall into the limbo of silence, just as new uses may spring up and may, in their turn, become standard language; but where is one to draw the line? It is well to remember that almost all expressions which refer to the mental are derived from others whose primary sense was sensuous and that this is a process which goes on to the present day; just as a good many words, under the influence of science, philosophy, or something still more elusive, have only in fairly recent times undergone a change in meaning — e.g. 'organic', 'nervous', 'unconscious', 'original', 'creative', 'objective', 'curiosity', 'to entail', etc. This is continuous change and continuous creation in language. Finally, there is such a thing as ambiguity which — except in exceptional cases — mars any attempt to single out one use as the stock one. Exactly how many standard-uses has 'nature'? What about 'in', 'on', 'about' etc.? 'The English prepositions', says Empson, 'from being used in so many ways and in combination with so many verbs, have acquired not so much a number of meanings as a body of meaning continuous in several dimensions'.<sup>1</sup> If so, or if the uses shade off into one another imperceptibly, how can one peel off and throw away all the non-stock uses and retain the stock ones? Yes, this view is unrealistic.

Next, and this raises a bigger issue, even if there was such a thing as a stock-use, it need not matter much to the philosopher. I mean, he need not be *bound* to this use; I should even go further and say that, sooner or later, he is bound to commit the crime and depart from it — that is, if he has something new to say. In this respect, his position is not altogether different from that of the poet or the scientist, and for similar reasons. He, too, may have come to see something which, in the ordinary way, cannot quite be said. I shall argue later that this is a characteristic feature of some philosophizing. To mention here just one small point, the English language has been enriched by many words coined by philosophers who were sensitive to gaps in our vocabulary. 'Optimism', for

<sup>1</sup> *Seven Types of Ambiguity* (2nd ed., London, 1947), p. 5.

instance, is due to Leibniz, and was borrowed from him by Voltaire. 'Impression' in its modern sense was introduced by Hume, 'intuition' by De Quincey, 'intuitionism' by Sidgwick, 'intuitionist' by H. Spencer. 'Scientist' is an invention of Whewell, 'aesthetic' one of Baumgarten, and so on. That even the laws of grammar can be flouted with salubrious effect can be seen from Lichtenberg's remark that one should say 'It thinks in me'.

My third point is that certain features of one's own language are noticed and appreciated in their full significance only when it is compared with other languages — with German (verbal way of expressing colour), Greenlandic (dominance of the impression verb), Latin (absence of words for blue, grey, and brown), etc. Is, then, the philosopher to go to the Eskimos to learn his trade? Not exactly; yet the mere *awareness* of other possibilities is, philosophically, of the utmost importance: it makes us see in a flash other ways of world-interpretation of which we are unaware, and thus drives home what is conventional in our outlook. The technique of the ordinary-use philosophers has suffered from the fact that they restricted themselves to the study of one language to the exclusion of any other — with the result that they became blind to those ubiquitous features of their own language on which their whole mode of thinking, indeed their world picture, depends.

Connected with this is another large point — the misleadingness of our speech-forms. That language, 'the embodied and articulated Spirit of the Race', as Coleridge put it, is in many ways inadequate can, I take it, by no one be doubted. In particular, it is the syntax and the field of analogies embedded in language which, unperceived, hold our thought in thrall, or push it along perilous lines. We shall soon have occasion to substantiate this point.

But there are still more reasons for guarding against this official doctrine. The one is that its champions pay heed only to the actual use of language, not to its gaps, revealing as they are. Suppose, for instance, that I say 'I ought to do so-and-so'; when I say that it is obvious that the I is here only a pseudo-subject from which the ought seems to proceed, whereas in fact it is more a *point d'appui* to which it is directed. We regard a rule of ethics, politeness, etc., as something outside ourselves which applies to us as objects. We are rather in a passive (obedient) frame of mind, and what is active is, at most, the consent we give to that duty. 'I am under an

obligation', 'it is my duty' are therefore phrases which are more appropriate. That 'ought' does not refer to an occult activity betrays itself in a number of features; thus we do not say 'I will ought', 'I choose (decide) to ought', any more than we say 'I ought to ought', or 'I am resolved upon oughting'. There is no such thing as a 'will to ought'. The complete absence of these idioms is revealing. That philosophers have concentrated on the use, and neglected the non-use, of expressions is a further weakness of their technique.

But there is another and more serious danger lurking in this approach which is best expressed in a comparison. Just as Hume has tried to make association do the work of reason and failed, so a similar danger is involved in the teachings of Moore, and still more in that of his followers — in that they rely on the machinery of *language* to do the thinking for them, without exercising judgement. In both cases the mistake is to trust in something blind, mechanical, to ignore what is understanding, insight. Only this can explain such a monstrous utterance as that of N. Malcolm<sup>1</sup> (approved by J. Wisdom) that one learns necessary truths by 'the eyes and the ears' — by attending, namely, to the way in which people use words. Surely, what we learn, and all we can learn in this way, are language *habits*. The story told is somewhat like this. A child watches on Monday two people speaking and the one of them saying 'A camel is larger than a flea'; on Tuesday he overhears the other chap saying 'A flea is smaller than a camel'; on Wednesday —. So after weeks of strenuous observation, checking and cross-checking, he arrives at the necessary truth that B is smaller than A follows from A is larger than B, and *vice versa*.

It should be obvious that this story misses the point. What has gone wrong? Suppose I say that if fleas are smaller than camels, camels must be larger than fleas — am I really merely recording *how people use* the expressions 'smaller than', 'larger than' and following their use? I may and I may not be. Suppose that this sentence was printed in a book for learning English, then I might use it to learn the sense of these two expressions. But surely this is not the sort of situation a philosopher has in mind when he is worried about the status of necessary propositions. Is not the point rather this — that once I *have* acquired the use of these expressions and understand what they mean, I cannot help acknowledging

<sup>1</sup> 'Are Necessary Propositions Really Verbal?', *Mind*, 1940.

that the antecedent entails the consequent? Suppose a child who, let us say, is already familiar with these phrases is asked, 'A camel is larger than a horse; now would you say that a horse is larger or smaller, than a camel?', he will reply, 'Don't be absurd — *of course* it's smaller'. The '*of course*' is here pertinent, it brings out that he has *seen the point* of the argument. Those words *have* already a meaning for him, and it is in virtue of this meaning that he recognizes the necessary truth. When you say, 'A is larger than B, so *of course* B is smaller than A', the '*of course*' is not a mere embellishment of speech, it answers to a click of comprehension — it betrays that you have grasped the relation between 'larger than' and 'smaller than', not that you have noticed or picked up certain language habits. There is a force in the argument. To make the point still clearer, let me contrast this case with the following one. Suppose that a child has learnt to form the plural of a noun by adding *-s* to the singular as in *boot*, *boots*, or *root*, *roots*; when he first comes across the word *foot* he forms, by analogy, the plural *foots*. You may then put him right; in teaching him that one says *feet* you are stating a fact which, as such, has nothing to do with insight. English, for all we know, may well have developed along lines so that such a '*childish mistake*' would have become the correct form; why shouldn't it? There is nothing *rational* about it, any more than that *cum* in Latin takes the ablative and not e.g., the genitive; whereas when you say 'If A is larger than B, B *must* be smaller than A' there *is* something rational about it — and it is precisely this which makes you say that you have grasped the point. If this was not so, things would be very queer indeed. Suppose, for instance, that a person, though he is perfectly aware of the use of those two expressions, yet refused to follow it and insisted instead that two things A and B may both be larger than one another — what then? Surely, if he has *only* to learn a use, and no more, he is free to depart from it? Yet we would not treat this departure as we would treat that of a child, namely as a childish error, nor as we treat the departure of a Lichtenberg, namely as a flash of genius revealing the familiar in a sudden light, but we would soon begin to suspect that he was not quite right in his head. But why? The difference in our reaction, the applying of two so different standards would be most unfair — *if* Malcolm was right, if we did in fact learn necessary truths by 'the eyes and the ears'. Saying 'one foot, two fooots' and saying 'A and B are larger than

each other' would now be exactly on a par — namely offences against the use of language. Very queer.

This brings up a further point. The appeal to the actual use of language is *not the ultimate test* we apply when we want to establish a necessary truth. Suppose the question is whether the relation 'equal', as applied to length, is transitive, and necessarily so. Imagine three rods  $a$ ,  $b$ ,  $c$ , one of silver, the other of ivory, the third of wood. For the purpose of comparison they are placed side by side in pairs, one pair of ends being adjusted till they coincide; inspection of the position of the other pair of ends will then decide whether they are equal. Suppose now that inspection shows  $a$  and  $b$  to be equal, and also  $b$  and  $c$ : does this, or does this not, entail that  $a$  and  $c$  are equal? There is a tendency to say that it does, and its supporters will have recourse to the use of language. Don't you see, they will say, that the word 'equal' is actually used in this way? But how do we know that language is *right*? Is it a sort of Supreme Court from which there is no appeal? Or is it really impossible to give a sense to a non-transitive use? What would you say, we might ask a champion of the ordinary use, if it was found that, though  $a$  and  $b$ , and  $b$  and  $c$  had the same length, namely judged by the criterion (placing them side by side), yet  $a$ , when directly compared with  $c$ , was a bit shorter? That 'there must have been some mistake somewhere'? But what if it was *always* like this? May not the mere fact of juxtaposing two rods have some unknown effect on their length — make, for instance, the silver rod shrink when brought in contact with wood? Suppose now that he replies something like this: If rods were ever to behave in this curious sort of way, this only shows that some distorting influence was at work; but what has this to do with their length? The question was whether  $a=b$  and  $b=c$  entail  $a=c$ . To this the answer is 'Yes', no matter how queerly material rods behave. Our concern is with *spatial intervals*, not with sticks. Though there is some truth in this, it is not the whole truth. First of all, what exactly is he doing when he says this sort of thing? He obviously wants to make the inference *independent of experience* so that he can stick to it whatever may happen to the physical rods. That is to say, he insists on using a language in which ' $a=b$ ,  $b=c$ , so  $a=c$ ' is an *inference licence*, not an empirical statement, or again, in which this relation is adopted as a *convention*. But as a convention, emptied of content, it does not say anything about the actual

world, and in particular it does not help us to infer, or predict, the results of experiments with actual rods. And this makes us see the drawback of this view — namely that it offers no guarantee that the rule adopted will be applicable. Look here, we might say to him, if we were living in a sort of Lewis Carroll world where things expand and shrink unaccountably, what will become of your rule? You may cling to it, yes; and you may insist that any deviation observed must be due to some distorting force, blaming physics for the discrepancy. Yet the fact remains that your rule cannot be relied on. So what is the good of having it? Wouldn't you do better without it?

But these are fancies, it will be said, so why care for them? Even if they are fancies — which is not too sure — it is enough to show that the rule must answer to something in reality, have some empirical backing if it is not to be worthless. But they are not fancies. Consider this case: in making visual comparison of a number of lines, drawn parallel on a blackboard, you find that two adjacent lines are the same length, the first and the second, the second and the third, and so on. However, if the middle ground is covered up and the first is compared with the last, a difference is noticed. In this sort of situation it is tempting to say that, although two adjacent lines *appear* to be equal, they are not in reality. That is a dangerous distinction. While talking of the actual chalk marks, it makes perfectly good sense to say, 'Yes, they look equal; but let's make sure' and apply a foot-rule. But can we do the same in the case of the visual lines? If the criterion for comparing is to be our impression, the contrast of appearance and reality loses its sense; surely in the realm of the given what appears is? Poincaré, on a similar occasion, asks whether such an experience can be expressed by saying  $a=b$ ,  $b=c$ ,  $a>c$ , or whether this would bring us into conflict with the law of contradiction. 'As we cannot believe that two quantities equal to a third are not equal to one another', he concludes that  $a$  is different from  $b$  and  $b$  from  $c$ , but that 'the imperfection of our senses' does not permit us to distinguish between them. And this, the need to avoid clashing with logic, 'has compelled us to invent the mathematical continuum'.<sup>1</sup> But all this shows is that he does not want to give up the transitive use of 'equal'; and it lands us in the same difficulty as before, namely, that it seems to make no sense to speak of 'the imperfection of our

<sup>1</sup> *The Foundations of Science*, p. 46.

senses' unless there is something — e.g. actual physical objects — against which they can be checked.

'Imperfection of the senses' — this phrase only provokes the question, What exactly would perfection be like? Can it be described? Suppose that I glance up at the night sky for a moment and am to tell how many stars I have seen; would it be right to reply 'I have seen  $n$  points of light, but I don't know what number  $n$  is?' That is, must I have seen some *definite* number though I can't tell how many? (It should be noticed that this is a question of describing *what I see*, not of the number of physical stars which could be counted out on a photographic plate.) Again, suppose that I look out into the rain. If I am to put into words what I see I am faced with much the same difficulty. The picture of the rain is typically blurred. Perhaps in the foreground I see a few drops (which I can even count) and then a streaky background fading away into grey. The idea of a 'perfect perception' now seems to be such that we can take in in an instant the number of light points or drops we see, and say precisely how many there are. But this is a mistake. My uncertainty is not due to my not having looked carefully enough; for however carefully I may look, or scan over the picture of rain, my impression will always remain the same. The blur is just as essential a feature of sense perception as other features are, e.g. the limitedness of the visual field; i.e. without the blur it would no longer be what it is. It would turn into something radically different, something of which we have hardly an idea, anyhow not a clear one.

If we like, we may of course speak of the 'inexactitude' of sense perception; only this inexactitude differs entirely from that exemplified (say) by 'This rod is about 15 inches long'. While in the latter case we can say 'Let's measure and see whether it is *exactly* 15 inches long or not', there is no sense in speaking of the exact number of the points of light or rain drops, just as there is no sense in speaking of a rod that is *exactly*  $\sqrt{2}$  inches long. 'Inexact' is opposed to 'exact' and, like the latter, can only be used where there is some way of attaining exactitude. But that's precisely what is not possible in our case. I can take in, perhaps, five or six stars simultaneously, but not a thousand. There is no method by which they can be counted in the twinkling of an eye; and the same applies to the raindrops. Curious though it may seem, the idea of a cardinal number ceases to apply in such cases.

That's why we speak of 'countlessly many' stars, hinting at the inappropriateness of counting.

The point is that sense perception is inexact in a very different sense from that in which, e.g., a painted picture, or a map, is inexact. I may say, for instance, that one map is more accurate than another map, in the sense that it is more faithful, more detailed, on a larger scale, etc. But when I say that a map is more accurate than a description given in words, the sense of 'accurate' has shifted. A verbal description is less accurate in a more fundamental sense. Consider the accuracies involved in a measurement, an estimation, the naming of a colour, in the description of a pain, or someone's face. In each of these cases, there is an accurate and an inaccurate, and, perhaps, a scale of accuracy; by discussing in each case what would constitute 'greater' or 'less' accuracy we come to see that the standards of judging differ on each level and that it would be unwise to reduce them to the same denominator. That's why it is pointless to apply the standard of mathematical accuracy ( $\sqrt{2}$ ) to physical measurement, or the standard of physical measurement to visual experience — to the lines in my field of view. And this, I hope, also shows what is wrong with Poincaré's attempt to blame 'the imperfection of our senses' for our not seeing the inequality of two adjacent lines. 'Imperfection of our senses' is here used like the 'blurredness of the picture of rain', namely without contrast to anything else.

Having cleared up this confusion, we may now resume the thread of the argument. The alleged difficulty was that, if we say  $a = b$ ,  $b = c$ ,  $a > c$ , we get involved in absurdities. But do we? Let us see what would go wrong if we adopted this new way of speaking. Would a contradiction ensue? By no means; all that would happen is that the word 'equal', or the symbol '=' would no longer be used transitively. On the other hand, the new idiom would have definite merits — for instance, we should no longer be driven to say, in the case considered, that the senses 'delude' us. As a result of this, we may now be prepared to concede that the transitivity of 'equal' may be dropped, or that stating ' $a$  and  $b$ , and  $b$  and  $c$  are equal, but  $a$  and  $c$  are not' is not in itself a contradiction, but only feels like a contradiction when the transitive use is already taken for granted. Instead, we may be disposed to say that the transitive use is very suitable when it is a question of talking of physical rods or other measurable quantities, but less suitable so far as

impressions (visual, but also tactile, auditory, etc.) are concerned — a clear hint that we are *not talking the same language* when we talk about sense impressions and when we talk about physical objects.

But here an objection is likely to raise its head — are we *entitled* to speak of equality in the cases of our lines, or in similar ones? Should we not rather say of two adjacent lines that they are ‘indistinguishable in length’, or anything that is less committing? But why should we? Is anything wrong with that expression? If two (visual) lines are ‘indistinguishable in length’, they give us the impression of being equal. So why not say so — if we want to record the impression? But what one feels when one raises this sort of objection is perhaps this. One feels that the word ‘equal’ takes on now a slightly different sense. It is, for example, no longer certain that the three relations ‘greater’, ‘equal’, ‘less’ form an exhaustive disjunction. May not two lines, some distance apart, be incomparable — i.e. stand in a relation completely foreign to ordinary geometry? But this is only part of the story; for all terms, when applied to what may be called ‘visual geometry’, assume a somewhat different meaning. Thus it makes sense to say, in this curious, almost unexplored field, that a tiny portion of a circle is straight, that a circle and a tangent drawn to it have a small stretch in common, or again (an observation of Hume’s) that two straight lines, if they converge under a very acute angle, coalesce near their meeting point, etc. What is new in this visual geometry and what has to be added to the ‘ideal’ geometry of mathematics, is a peculiar factor of inexactitude, or blur, which it is difficult to symbolize.<sup>1</sup>

But in all this there is no *logical* absurdity. So we can, after all, give up transitivity (which has now become rather a hindrance) and adopt a different convention — the very thing to which Poincaré takes exception. And if we do this, we can talk of visual phenomena, or other impressions in an idiom better suited to their peculiar nature.

Now what all this shows, and this brings me back to the point, is that we are not *slaves* of the existing language. On the contrary, we are free to criticize it, discuss its merits or demerits in certain respects and *in the light of reasoning*, and then decide whether to

<sup>1</sup> Cf. J. Hjelmslev, *Die natürliche Geometrie. (Abhandlungen aus dem Math. Sem. der Hamburgischen Universität, vol. 2, 1923.)*

stick to it, or else to modify or redesign its use. Take once more the use of 'equal'. Is it necessary that it should be used transitively? If 'necessary' means 'inevitable', this is not necessary; it is only necessary relative to a language which we are agreed to apply. But we can decide otherwise. The very fact that we can influence a person, can make him change his mind by force of an *argument* is sufficient proof that it is not the language *habits* on which the issue hinges.

And this makes us see the fundamental blunder committed in saying that we learn necessary truths by 'the eyes and the ears'. Whilst it is quite true that we have to learn the use of language, this does not prove that necessary truths are learned in the same way. For we may well notice 'how people use expressions in certain circumstances', for instance, that the relation referred to is transitive, and yet decide *against* regarding it as a necessary truth. But then it is not the use of language, nor any empirical fact discoverable by Mass Observation which settles the matter, but something else — namely insight, understanding. What is to be wondered at is that so many philosophers today trust in the turning of the wheels of the language machine rather than in reason, showing such a prejudice for the blind.

6. THE I — AND IT — ASPECT. In the last chapter a number of examples were given to drive home the point that language is subject to changes. But, it will be said, such changes are the concern of the literary critic and the historian of language, not of the philosopher. The latter is concerned with the *ordinary*, i.e. stock use of words and phrases. While I am not denying that this is often important, it seems to me insufficient for two reasons. First, the philosopher need not be *bound* to the actual (prevalent) use of language; I should even go further and say that he is *bound*, sooner or later, to commit the crime and depart from it — if he has something new to say, that is. In this respect, his position is not so different from that of the poet or the scientist, and for similar reasons. He too, may have come to see something which, in the ordinary way, can not quite be said. The other reason springs from an even deeper source, the misleadingness of ordinary language. When Lichtenberg said, 'One should not say "I think", but rather "it thinks", just as one says "it thunders"', he was of course departing from the ordinary use, and of course perfectly aware of it. All right. He has done it. And in doing it he has

outraged the German grammarians no less than the English, French, and the rest. Yet at the same time there is something illuminating about this saying. It is one of those rare flashes of lucidity which light up and throw into prominence a feature of our language that is deeply questionable. Like a meteor, it first startles us with its piercing paradoxicality, leaving behind it a trail of golden lights; or, to speak without metaphor, as the meaning gradually sinks into us, a new horizon discloses itself. Am I the maker of my thoughts, do they not rather occur to me, we wonder? 'A man cannot say, "I will compose poetry". The greatest poet even cannot say it; for the mind in creation is as a fading coal, which some invisible influence, like an inconstant wind, awakens to transitory brightness.' Are these words of Shelley not also true of thinking — of making mathematical discoveries, for instance, or of gaining a new insight in philosophy? There are many mathematicians who bear witness to the sudden and involuntary flashing forth of a solution at the most unexpected moments; Gauss, for instance. One morning, when he was 19 years old, before getting up, he saw clearly and distinctly that the construction of the regular polygon with 17 sides follows from one of his ideas on algebra. He did not seek for it — the discovery came 'of itself'. Poincaré, in a study on *Mathematical Creation*, narrates how one morning, walking on the cliffs, an idea came to him — the germ of a great discovery — with 'the characteristics of brevity, suddenness and immediate certainty'; or how, another time, while on a geological excursion, having forgotten all about his mathematical work, at the moment when he put his foot on the step of a bus, he had a sudden illumination; he had no time to verify the idea as he was just in the middle of a conversation, but again he felt a perfect certainty. Similar experiences are reported by F. Klein. In the night which, because of asthma, he spent sitting on the sofa, almost in a state of exhaustion, there suddenly stood before his eyes a certain diagram embodying the central theorem he had been in search of without having been able to find it. In the morning he had only to write it down. It is a well-known fact that 'when one works at a hard question nothing good is accomplished at the first attack. Then one takes a rest, longer or shorter, and sits down anew to the work. During the first half-hour, as before, nothing is found, and then all of a sudden the decisive idea presents itself to the mind'. (Poincaré.) Such a sudden inspiration,

he adds, 'is only fruitful, if it is on the one hand preceded and on the other hand followed by a period of conscious work'. All that lends poignancy to the question — How far am *I* the maker of my thoughts, and how far do they occur to me? And if they just 'occur', what is there *I-ish* about them? On the other hand, if there is nothing *I-ish* about them, why do they choose just *me* for their rendezvous?

And are Shelley's words not also true on a much lower plane? Who does not know how a forgotten name may mock us, being on the tip of the tongue, beckoning us to fill the rhythm we sense with sound, or to complete its initial letter, yet escaping us continually? Perhaps we no longer take an interest in it, we do not *want* to remember, and yet we cannot lay the ghost. Who has not been tantalized by a tune one vainly tries to put out of one's head? On the other hand, when we are groping for the right words to express something, they seem to resent it — 'All things resist being written down', says Kafka somewhere. In the night, when I wake up, drowsy and sleep-heavy, numberless thoughts prey on me, turning me into a battlefield between their wild desire to be thought through me and my brain's longing for sleep. And this, this fantastic situation, is described by the use of the active voice, namely, by simply saying 'I think'. But who can tell how much of 'me' and how much of 'it' (or 'them') is involved in that? 'Thoughts', noticed Coleridge, 'are their own masters'. True, they respond to our efforts, but only up to some point. We seem to catch sight here of a strange *intermundium*, governed jointly by the *I* and the *It*, of a sort of *condominium*. They are my thoughts, and yet they aren't.

My present point, however, is this. Lichtenberg could never have staggered us, holding out a new vision, and at the same time making us see the onesidedness of our current expressions, had he slavishly adhered to the ordinary use of language. As it is, he was not only justified in flouting the rules of grammar, but bound to do so, as only in this way, by breaking through the dead crust of language, was he able to fulfil his mission. But of course not everyone is a Lichtenberg, and not everyone has something to say to surprise us out of all complacency.

It is, I think, time to distinguish between philosophers who for showing off's sake commit the sin of saying things in a stilted unsayable jargon, and those who have seen something new and in

the attempt to give expression to it are driven to hammer out their own language. What lies at the root of this is something of great significance, the fact, namely, that language is never complete for the expression of all ideas, on the contrary, that it has an essential *openness*. A philosopher, as he muses on some item or other, may notice a subtle, yet elusive difference never noticed before, but when he wants to draw attention to it language is wanting. There ought to be a word, yet there is none. To take a trivial case, you may feel a slight ambiguity in the ‘nots’ when you say ‘It is not raining’, ‘If it is not raining —’, ‘Would it were not raining’, etc. You may feel that it makes a difference whether you deny a fact, a mere possibility, or whether you use the word ‘not’ in the forming of a supposal, etc., and yet if you try to bring out these differences you may find yourself suffering from a sort of speech suffocation as there is only one word to mark these different shades. Again, you may sometimes feel a need for having two ways of describing a state of affairs, one for the case when you speak about something *absent*, another when you want to say something about what is just *in front of* you. And the reason for this may be your having noticed that in the second case, when you refer to something present, what you are saying is neither ‘true’ nor ‘false’ in the *same* sense as when you speak about something absent (but rather ‘correct’ or ‘incorrect’), that you cannot possibly ‘express a view’, ‘hold an opinion’, or ‘have a belief’ about things which stare you in the face (though, perhaps, about ‘what they mean’, or how they are to be named), that you do not ‘describe a fact’ at all (but rather ‘choose the proper words’), and that accordingly the idea of a mistake, and with it that of a statement, undergoes a parallel inflection of sense. Yet existing language denies you the possibility of stating such conceptual differences, for lack of the requisite words. One last example. Suppose an astronomer were to tell you ‘All planets of Algol are ring-shaped’; you would be surprised; and, perhaps, even more so if he told you that, as there are not any, his statement is perfectly true (the astronomer turning now into a logician, symbolizing his statement by writing

$$(x) : x \in \text{pl} . \supset x \in \text{ri}.$$

But that is a trick. What he said led you to expect that, in actual fact, there are such planets. If there are none, you feel somehow cheated. Yet to say ‘Algol has no planets, and all of them are

ring-shaped', though it sounds very odd, is not the sort of thing we call a logical contradiction. So the latter statement, strictly speaking, does not entail that there are planets. Though it does not entail it, the relation between the two statements comes pretty *close* to that of entailment, just as the above conjunction is very *nearly* a contradiction. In such a case you may feel a strong need to have a word for denoting a relation weaker than entailment, yet stronger than implication, or a word to express something that is *almost* a contradiction. That is the sort of situation which occurs time and again in philosophy. We become aware of some subtle difference, or of an unnamed sort of relationship, but in the attempt to impart this knowledge we are hampered by existing language, its paucity of words. Nor is this the only reason for the inadequacy of ordinary language, as will be seen before long.

Wittgenstein said something very important on this score when he pointed out (in unpublished lecture notes) that a philosopher may at times feel uneasy in ordinary language and have a longing for a novel way of expression. (The examples discussed by him are whether one should not say 'I wish a pale red for this paper' instead of 'I wish this paper were red' — as what one imagines seems to be hazier than the actuality of the paper being red; and further, whether one should not, for similar reasons, say 'This paper is not rode' as the negation of 'This paper is red'. Such a mode of expression, he adds, would fulfil a certain craving of the philosopher which our ordinary language does not fulfil and which, so long as it is unfulfilled, may produce a mental cramp — which, however, is loosened when he is shown a way of expression which satisfies that need.) That is important; at the same time, I think, it strikes the wrong note: it makes it appear as if it was all a matter of *emotion* — as if a philosopher were just seized with an uncontrollable and, as his examples seem to suggest, unreasonable hankering after a new way of speaking. That is far from the truth. The real point is that we see an important difference, but cannot express it so long as we are tied to the ordinary way of expression. Seeing something in a new light is often the core of a philosophical discovery, or of a new insight. Such an insight *may* be expressible in the simplest words of language (Hume); or it may not (Lichtenberg). In the latter case, we are driven, not by a deliberate search for novelty, but by a sort of inner necessity, to find the form of expression which will bring out the point — but this is not in the

least a matter of ‘craving’, nor of a ‘mental cramp’ (‘discomfort’), and its ‘loosening’. To speak of it in such terms gives, quite unduly, an impression as if it was a question of sentiment and feeling — whereas the point is that so long as we remain within the bounds of the conventional we cannot express what we are anxious to express: a perfectly legitimate need remains unsatisfied. And there is a good reason why we are dissatisfied with existing language: it is the seeing of a new point, and the impossibility of putting it into words which goads a philosopher on in his search for new means of expression. There is a *rational* element in it, the fight of thought with the obtuseness of speech. It is thanks to this, to the unremitting efforts of philosophers (and poets) to express the inexpressible, that language becomes an increasingly subtle instrument of thought. In what Wittgenstein said, the most essential thing is left out — that breaking away from the norm is often the *only way* of making oneself understood.

Suppose that a philosopher has seen a point never seen before which, however, owing to the recalcitrance of language, cannot be expressed in the customary ways, what will he do? He may cast about for some old word which has fallen into desuetude, but which, for our benefit, may be revived; failing this, he may turn to foreign languages — we are so often forced to eke out our vocabulary with terms borrowed from French, German, etc. — or to that ever-active mint for coining new expressions, popular idiom. But the supply of words for the finer purposes of thought is deficient everywhere; and, moreover, it may be that what is lacking is not the words but the grammatical forms. What will he do in this case? Well, he will, perhaps, use a peculiar mode of expression — words placed in new and strange combinations such as have never been employed before, a bold simile, or something that looks at first sight like a mishmash of different logical categories; again he may use a startling construction in which the normal bond of syntax is wilfully broken. All that is open to him. Suppose now that a philosopher, with a flair for language, hits upon a suggestive collocation of words, novel and yet instantly intelligible which expresses to a nicety what he wants to say — ought he to be ashamed of this? I think we should be grateful to him for enriching our language. ‘But it is not correct.’ But, hang it all, how should it be expressed if not like this? If it says what he wants to say, he is perfectly within his rights — he has achieved

what he meant to achieve, and it would be silly to find fault with him. Expressiveness, not correctness, is the life and spirit of language. Philosophers are, therefore, quite justified when they, in order to bring into the focus of attention some novel point, put together words in a combination never used before. That is just one of the things that makes philosophy so exciting.

To return to Lichtenberg, when he proposed that one should say 'It thinks' — is this the ideal way of putting it? The point is that neither 'I think' nor 'It thinks' is quite apposite. The truth is somewhere between the two. Language, in constraining us to ascribe thinking to a grammatical subject, subtly shifts the emphasis and draws a somewhat misleading picture of the true state of affairs: it creates the illusion that thinking is a doing that requires a doer. It is, and it is not. In actual fact, there is an interplay between what I do, and what comes to me. To leave no misunderstandings on this point, there are certain cases where one feels that 'I think' is the right expression — as when one says 'I will consider the matter, think it over', 'I will concentrate on the point', 'I am all attention', etc. It is right, because it is within my will-power to consider it, to think it over, generally, to pay heed to it. There are other cases where '*it* thinks' comes nearer the truth — as when a new idea 'occurs' to me, or when the solution of a problem, long sought after, suddenly and seemingly without effort flashes upon me. (Remember what mathematicians have to say on this score.) In such cases, thinking is not something planned, or controlled. Similarly, getting clear about some point in philosophy increases the rationality in the world, and yet it is not itself a rational process. But ordinarily, thinking is neither quite ascribable to my waking self — whatever that may mean — nor to a mysterious entity called 'It'. Language has not quite got the forms to express this. It cannot help making use of customary forms as precipitated in the scheme of verbal inflexion — a fact which tends to assimilate the case in question to other similar, and slightly similar ones without taking into account the differences there are. Thus the use of the first person pronoun in 'I walk', 'I take umbrage', 'I grow a beard', 'I sleep', 'I dream', 'I die', 'I suffer', 'I must', 'I ought to', 'I think' tends to make the cases appear more alike than they are; and this is perhaps unavoidable. After all, language has of necessity to make use of a limited number of forms of inflexion which are applied to all sorts of things no

matter how disparate; with the result that they are pressed into the same moulds and thus present themselves in an analogous fashion. Or to change the metaphor, we trace the form of our expression and believe ourselves to follow the contour of the object; by some optical illusion we seem to see in the things what, in fact, is engraved on our spectacles. To quote Lichtenberg once more: 'Our wrong philosophy is embodied in the whole of language; we cannot, so to speak, reason without reasoning wrongly. Everyone who speaks German is a philosopher of the people. Our whole philosophy is a rectification of the use of language, therefore a rectification of a philosophy, and of the commonest one. However, the common philosophy has the advantage of being in possession of declensions and conjugations. True philosophy will, thus, always be taught with the words of the wrong one. To give verbal explanations is of no avail; for with such explanations I do not yet alter the pronouns and their declension.'

Consider a bit more closely the case of a man who is writing poetry. The very expression 'to write poetry' is apt to call up the wrong sort of association — it makes it look as if, like writing a letter, it referred to an *activity*, expressed in the transitive verb 'to write'. Yet these are surface resemblances which veil a world of difference. Suppose that I have to write a letter. I may remind myself of this in the morning, thinking perhaps that I shall be free after tea to do it. At 5.30 I say to myself, 'Now it's really time to write it'; I sit down, have perhaps some idea of what I want to say, and say it. And when the letter is finished it is *my* work, from the beginning to the end. But would I say, looking at my watch, 'Now it's really time to write that poem'? Perhaps — if I was a poetaster. Otherwise Shelley's warning stands; and supported by the consensus of almost all those who have ever been 'visited by the Muse', to use this hackneyed picture. (I could quote endlessly.) Poetry — 'it won't come when called — you may as well whistle for the wind', as Byron said. Indeed a poem has a way of welling up at the most unexpected moments — when one is fatigued, just before falling asleep, while shaving, when out on a winter walk with the fingers half-frozen, while taking a bath, and so on *con dolce variazione*. In short a poem comes when *it* wills, not when I will. Varying a saw, one may say '*poema nascitur, non fit*'. True, one senses, perhaps, the first stir of inspiration, but it seems to lie outside one's control. Absence of effort, receptiveness is more

likely to succeed than exertion or conscious drive. Housman, in his *Name and Nature of Poetry*, confesses that the fountain would sometimes run dry in the middle of a poem so that he would have to write it himself. And this is the sort of thing that happens so often in writing poetry — the onrush of words and ideas is arrested, what came effortlessly comes haltingly, with strain and anguish, or not at all, and the words no longer have that strange luminous power they had so long as the vision lasted.

Thus a poem, when it comes, comes of itself. One often feels its approach in a curious sort of way as if something from deep down were to rise up and become a word. Something as yet unknown, faceless, begins to stir, setting up vibrations in the deeper layer of one's being, as it were, and these vibrations pass to the lips in a rhythmic movement. A number of floating words begin to solidify, and as the poet murmurs the verses over he feels a sonnet, or some other poem take shape. So long as this enchanted state lasts, he may get the poem finished; or else the impulse spends itself, the rhythm dies down: the cry is broken. The experience varies a good deal. Some poets find themselves suddenly in the grip of a particular rhythm out of which words gradually precipitate themselves, others have the impression as if some inner voice dictated the poem to them, as if they were eavesdropping rapturously on themselves, spellbound not to miss a single word.

Related to this is the poet's impression that he is not so much the originator of the poem as its receiver. 'I have been surprised at the observations made by some of my characters', we are told by Thackeray. 'It seems as if an occult Power was moving the pen. The personage does or says something, and I ask, How the dickens did he come to think of that?' Keats avowed that he had often not been aware of the beauty of some thought or expression until after he had written it down; it had then struck him with astonishment and seemed rather the production of another person than his own. Writers of automatic script are often certain that they are operated by some other power. This queer impression of being worked upon from outside may be heightened to an abnormal degree. (Paul Valéry relates a case of mistaken identity when he fell a victim to a musical composition of which he could make no possible use. 'Inspiration must have been guilty of a faulty choice', he observed.) Is this not the ground why so many poets disclaim authorship for their work? They feel that a poem comes unsought as a gift from

the blue without any sort of effort on the part of the recipient. And this marks a further difference between writing poetry and writing a letter. In the second case, it is I who write the letter, and I am responsible for every word I use in it. In the first case, the writer is under the overwhelming impression that something 'not quite himself' has taken possession of him, and that he does not *quite* deserve praise or blame for the results.

What I want to show by making this digression into poetry is how *inadequate* language is to deal with such things. Only a few pronouns and very few forms of verbal inflexion exist in language, and none of them is quite appropriate. Thus it would not be quite true to say of a poem that *I* have been writing it, since this would assign too active a part to me, nor that it got itself written *through me* as though it was endowed with a life of its own, nor that it has been written by a mysterious third one who was at work inside me, making use of the mechanism of my body. None of these expressions seems to fit. In order to describe such things more adequately one needs, perhaps, a language in which the boundary between 'I' and 'it' is fluid — of notions, anyhow of terms, that are less definite and held, as it were, in solution.

If it is asked whether this process, namely writing a poem, is active or passive, or predominantly active or predominantly passive, what ought one to reply? Well, consider first another case — day-dreaming. Is it active or passive? Most of us, I presume, would not be able to tell straight off. Why? The state of mind referred to is such as to contain, interspersed amongst itself, portions that are within our will-power, and others that are not. To see what is voluntary in it, we may ask whether we can stop it, or promote it. Suppose that I indulge in such fancies, this is in part *my* doing, and in part it is not. It is not my doing insofar as I have no control over the course my daydreams take, any more than I control my dreams in the night-time; rather I am in the role of a spectator who watches, perhaps helplessly, a play going on and on that inner stage. What is *unwilled* is not describable as an *act*, nor ascribable to 'me' as the *subject*: it *happens* to me, I don't do it. Sometimes, e.g. in a state of deep emotion, I cannot help falling into such fancies — 'I was lost in reverie' we say significantly, thus bringing out the passive aspect of the matter. Yet I am not *entirely* passive as I am in sleep, for I consciously indulge in vain fancies; true, I *might* stop them at any moment, yet I don't — 'I

give rein to my imagination' as the phrase goes, i.e., I give myself up to the pleasure of building castles in the air. Insofar as I abstain from interfering a *voluntary* element is present, not to be found in a real dream, a vision, a state of trance, and the like. Thus active and passive, or better receptive, frames of mind alternate, indeed interpenetrate, in this experience; and accordingly language shows a wavering between active and passive modes of expression — 'I am daydreaming', 'I am lost in dreams', 'I fall a prey to fancies', etc.

Again, when something very exciting has happened to us, we cannot shake off the memory of it — mechanically we go over the last scene, repeat the same phrases, re-live, enact the past hour time and again, moving in a circle as it were, until it becomes an obsession with us. *Are we masters in our own house?*

Such examples make us notice how artificial and doubtful the whole distinction between active and passive often is. 'To be born', for instance, is passive, 'to come into the world' active in voice, yet both have exactly the same signification, just as 'to fall ill' is but a variant of 'to be taken ill'. Now is the 'real' sense of these idioms an active or a passive one? Or is this not a proper question to ask? What about 'falling in love'? Would you say that it is (predominantly) active, or (predominantly) passive, or neither, or that it contains components of both? What about such expressions as 'to stumble', 'to sink', 'to tremble with fear', 'to stand trial for murder', 'to despair'? Would it be sensible to ask, Could you have helped despairing? And what about hoping? Is it voluntary, something that you do? To describe it in this way would mean to apply to it predicates such as 'intentional', 'resolute', 'premeditated' — which is obviously wide of the mark; we do not hope on purpose, nor do we say 'I surprised him in the act of hoping'. Yet hoping is not something 'mechanical', 'automatic', 'instinctive' either, any more than it is something 'habitual' or a 'reflex' (whether conditioned or unconditioned). None of these dichotomies applies. You may feel here a need for a more impersonal mode of expression — e.g. 'there is hope in my heart', '*die Hoffnung keimt*' etc. Nor does the matter end here, for quite similar considerations apply, e.g. to the idea of speaking. If I am very angry, for instance, I do not choose my words, any more than I choose the tone of my voice. They may, indeed, escape my lips against my will, just as I may be surprised at the hoarseness of my own voice. And yet it

would be psychologically wrong to say that I am in a 'passive' frame of mind, merely. No, there is a strong impulse to action in me which may or may not, discharge itself.<sup>1</sup>

<sup>1</sup> This series of articles was never completed.

## CHAPTER VI

### THE DECLINE AND FALL OF CAUSALITY

1. The year 1927 is a landmark in the evolution of physics — the year which saw the obsequies of the notion of causality. To avoid misconceptions, it should not be thought that the concept fell a victim to the unbridled antipathy of certain physicists or their indulgence in fancies. The truth is that men of science came, very reluctantly and almost against their will, to recognize the impossibility of giving a coherent causal description of the happenings on the atomic scale, though some of them — curiously enough, amongst them Planck, Einstein, de Broglie, Schrödinger — could never bring themselves to accept wholeheartedly so drastic a renunciation of classical ideals.

'But how can one find out such a thing? You mean physicists have not yet discovered the true causes?' No; I mean causality has definitely come to an end: atomic science has penetrated to a depth where an entirely new orientation is called for. As Niels Bohr put it: 'In the general problem of quantum theory, one is faced not with a modification of the theories describable in terms of usual physical concepts, but with an essential failure of the pictures in space and time on which the description of natural phenomena has hitherto been based.'<sup>1</sup>

The vast shift of perspective and, what goes with it, the change in our whole intellectual climate — that is the subject we must contemplate if we wish to reach a deeper understanding of the new outlook. In dealing with a matter of such dimensions, it is always good to choose a historical approach. This I shall do. I shall first touch upon the classical conception of causality as it gradually took shape in the work of Newton and his predecessors in the seventeenth century, and then discuss some of the reasons which led to its abandonment.

Attempts made by philosophers to clarify the notions of cause and effect, to formulate more precisely the content of the law of causality and, if possible, to throw light on the sort of validity

<sup>1</sup> *Atomic Theory and the Description of Nature*, Cambridge, 1934.

which one may claim for it were not too successful. A much clearer picture of causality emerges from science.

2. Man has learnt mechanics from the stars. Indeed, the first great feat of the dawning age of science was the construction of celestial mechanics. Given Newton's laws of motion and the inverse square law, the problem of planetary motion can be condensed into a set of differential equations. Their solution has the following property: given the configuration of the solar system (the positions and velocities of the planets) at some initial time, the configuration at any other time can be calculated with any degree of accuracy without further recourse to observation. In other words, the solar system is such that its initial state *determines* its entire future; and 'determines' here means: makes computable on the basis of laws stated in precise mathematical form.

This, in essence, is the scheme as it emerged from the *Principia*. In Einstein's words, 'Newton's theory of gravitation is probably the greatest stride ever made in the effort towards the causal nexus of natural phenomena.'<sup>1</sup> It should be noticed that this scheme departs from the pre-scientific idea of causation. The popular view is openly animistic: the cause brings forth the effect, produces it. On a more enlightened level — say, on that of Hume or J. S. Mill — we meet with the idea that two events, *C* and *E*, are related as cause and effect if they fulfil certain conditions, namely if (1) they are contiguous in space and time (2) *C* precedes *E*, and (3) *C* is unfailingly followed by *E*. A fourth trait that seems to cling to the idea, necessity or compulsion, was submitted to a destructive analysis by Hume: he did not 'deny' causation as one so often hears it said, he tried to purge the concept from extraneous elements. Now what our example shows is that this notion, whether the naive or the purged one, does not apply here: it would be most arbitrary to pick out one configuration of the solar system and call it the 'cause', and another such configuration and call it the 'effect'. Indeed, in the symbolic language evolved in science these terms drop out altogether. What takes their place? The concepts of *functional dependence*. We find phenomena so correlated in nature that, when one measurable quantity, characterizing a certain physical state (say, the Earth's distance from the sun), varies, this corresponds to a change of another (say,

<sup>1</sup> *Sidelights on Relativity*: I, *Ether and Relativity*. II, *Geometry and Experience* (trans. G. B. Jeffrey and W. Perrett), London, 1922.

the Earth's acceleration towards the sun), according to a simple mathematical rule. The one quantity (acceleration) is then said to depend upon, or to be a 'function' of, the other (distance). A well-known example is the way the pitch of a note (expressed by the frequency of a vibrating string) varies with the length of the string, is a function of it.<sup>1</sup> It should be noticed that, if one variable quantity is a function of another, this does not imply the existence of any *causal* relation between the two, but merely a correspondence between the values they may take. Thus the pressure of a gas contained in a closed vessel depends on its temperature and its volume. This dependence is expressed in the law

$$pv = RT,$$

where  $p$  denotes the pressure,  $v$  the volume,  $T$  the temperature, while  $R$  is a constant. This equation may be solved for *any* of the three variables as a function of the two other ones,

$$p = RT/v \text{ or } v = RT/p \text{ or } T = pv/R,$$

without suggesting that a change in volume or temperature is the 'cause' of a change in pressure, or *vice versa*.

The knowledge of such laws enables us to derive, and thus predict, the future from the present, generally the unknown from the known.<sup>2</sup> For, to return to the solar system, information about its present state can similarly be used to calculate the condition in which it was at any past moment of its history, the reason being that the equations of dynamics are insensitive to a change in the direction of time — i.e. remain invariant under a change from  $t$  to  $-t$ .

It would lead us too far to relate in detail how a system of dynamics was built up capable of accounting for the phenomena in the heavens and on earth. The decisive step was the introduction of the quantitative method through the work of Galileo. On its basis, a rational formulating of the laws of nature became possible, culminating in Newton's *Principia*. With the rise of the quantitative method, causality began to take on a new aspect. Briefly, the

<sup>1</sup> A discovery ascribed to Pythagoras.

<sup>2</sup> 'The most direct, and in a sense the most important, problem which our conscious knowledge of nature should enable us to solve is the anticipation of future events, so that we may arrange our present affairs in accordance with such anticipations.' H. Hertz, *Introduction to the Principles of Mechanics* (translated by D. E. Jones), London, 1899.

question 'why' came to be superseded by the question 'how'. Indeed, when Galileo turned to inquire *how* bodies fall and not *why* they fall,<sup>1</sup> this must have looked to his contemporaries rather trivial, a side-stepping of the deeper issues involved. Yet it was this 'side-stepping' more than anything else that started science on its career.

One may, of course, still think of gravity as the true cause underlying free fall and all the rest: but then gravity, or force in general, is not an *event*, while only events are supposed to stand in causal relationship. What the scientist has in mind when he expresses himself in some such way is that Newton's law of attraction is part of the formulae which govern planetary motion. However, and this is the point, in these formulae there does not occur anything that answers to the popular idea of cause and effect. They describe what happens under given conditions, they do not state the cause (read: the reason) *why* things happen in the way they do. I do not mean to say that there is *no* causal nexus: there *is*, only not in the sense envisioned by philosophers, namely as a relation between two events: this scheme, lacking the right sort of flexibility, must be dropped. The point had not escaped Newton when he declined to go in search for a cause of gravity ('*hypotheses non fingo*', General Scholium of the *Principia*).

Extending the result to other domains, we may say: the laws of classical physics are such that, if the state of an isolated system defined at a certain point of time is known, it can be computed for any other instant so that the whole course of events can be plotted in advance. That is the essence of determinism. Physicists were successful in establishing laws of such a type also outside celestial mechanics, though for certain limited fields only — e.g. for the phenomena of heat conduction or those of electromagnetism in free space. For cases of the latter sort — where processes are supposed to take place in a continuum and propagate by nearby action in contrast with Newton's gravitation presumed to act instantaneously across empty space — the principle of causality must be reformulated, e.g. as follows. What takes place at a certain point at a given time depends entirely upon what has been

<sup>1</sup> 'It does not seem expedient to me now to investigate what may be the cause of acceleration' (*Discorsi*, translated as *Dialogue concerning the two chief World Systems*, Chicago, 1953, third day), his main concern being to find a law according to which it takes place.

happening in the immediate neighbourhood 'just before'. (That is a crude way of putting it: technically speaking, ordinary differential equations have to be replaced by partial ones and initial by boundary conditions — a further step towards the shaping of the concept of causality.) However, it would be wrong to close our eyes to the fact that the situation is different in other fields: there are, for example, no similarly precise laws in biology — no one could have foreseen the exact ways of evolution; and even in the science of inanimate nature, for so long considered the stronghold of determinism, there are kinds of phenomena — e.g. Brownian movement, turbulent flow — which resist causal analysis.

What, then, are we to think of the dictum 'Every event has a cause', i.e. is determined by laws — often referred to as the 'Law of Causation'? For some ranges of facts it seems true, for others doubtful. Actual evidence is far too meagre to substantiate such a sweeping statement. Even in those cases which may seem most favourable to such an interpretation, the validity of a law can never be established with mathematical precision. Nonetheless belief in a universe governed by laws which allow of no breaks was steadily growing until it became almost an article of faith. Thus according to Helmholtz, 'The causal law bears the character of a purely logical law even in that the consequences derived from it do not really concern experience itself but the understanding thereof, and that therefore it could never be refuted by any possible experience.'<sup>1</sup> On this view, it is nothing but the demand to understand. In another passage, however, he is more outspoken, declaring that 'the final aim of all natural science is to resolve itself into mechanics', at a time when the very idea of mechanics was still closely linked to that of an iron chain of relentless necessity. 'The great abstract law of mechanical causality', wrote the biologist E. Haeckel,<sup>2</sup> 'now rules the entire universe as it does the mind of man', and so on. This was written one year before the discovery of quanta which was to knock over all our ideas on the subject.

3. Before turning to the new ideas, a word on the rise of the notion of causality. Strange as it sounds, belief in a firm Causal Order of Things has not always held sway in the dominion of science. It rather is the result of a long historical development

<sup>1</sup> *Physiologische Optik*, III, translated as *Helmholtz's Treatise on Physiological Optics*, New York, 1924.

<sup>2</sup> *The Riddle of the Universe*, London, 1929.

which began with Kepler and Galileo and reached its height with Laplace. Growing out of the new science, that conviction may, in its career, have absorbed into its meaning traces of an earlier outlook, which still deeply colour its meaning.<sup>1</sup> Be that as it may, in Newton's time, at any rate, determinism had yet to prove its worth. Nothing is more characteristic of this than the attitude taken up towards a problem which then began to arouse the interest of mathematicians and astronomers alike — is our solar system *stable*? Will Saturn wander off into space as a consequence of its slowing down and receding from the sun, or will it remain a member of our system? Will the acceleration of Jupiter cause it to come nearer and nearer to the sun in ever narrowing spirals until it crashes into the sun? And is a similar fate in store for our moon — to come smashing down to earth? Or will these heavenly bodies reverse their present trends before that happens? In other words, will the perturbations increase with time, piling up to disastrous dimensions, or will their effects merely oscillate between certain definite limits? Such questions are part of the problem of the stability of the solar system. Newton himself was of the opinion that divine intervention may be necessary from time to time to put the solar system back to order and save it from destruction, either through collisions of its members or by the planets scattering into space<sup>2</sup> — an echo, perhaps, of the medieval view that God participates in the day-to-day running of the universe? Even Euler had his doubts: there were too many forces involved, with interactions too complicated to subject them to mathematical analysis. Newton and Euler represent, between them, the temper of an age when the idea of causal determination had not yet attained the rank of an undisputed principle, and when scientists did not wriggle away in horror at the suggestion of God's possible intervention in the universe. It was only later when Laplace came to attack the 'grand problem', that belief in an unbroken causal chain was vindicated. He was able to show that notwithstanding perturbations and fluctuations in the planets' motions caused by their interaction the solar system will retain its inner structure. In

<sup>1</sup> See section 5.

<sup>2</sup> Newton's reason for holding such a view is the existence of 'some inconsiderable irregularities' in the otherwise concentric orbits of the planets, 'which may have arisen from the mutual actions of comets and planets upon one another, and which will be apt to increase, till this system wants a reformation'. (*Opticks*, Query 31.)

particular, the length of the Earth's year is invariable — a fact of obvious significance for the continuance of organic life on this planet.<sup>1</sup>

With Laplace the principle of causality began to take on its present form, in which it was to reign supreme for more than 150 years. In the solar system everything operates with clockwork precision. There are no hidden conditions, and hence no possibilities of surprise. As with the sun and its family, so with any self-contained physical system. Determinism, with its belief in a gap-less chain of causes, came to be regarded not so much as a heuristic principle, a lodestar in the light of which certain domains of fact can be ordered, as an inherent feature of nature herself. Any account that failed to conform to these standards — e.g. in biology — was deemed unsatisfactory and dismissed as 'unscientific' (see Haeckel).

It was this success that led Laplace to proclaim determinism in a most rigorous and uncompromising form. In his *Philosophical Essay on Probabilities*<sup>2</sup> he says, in words justly celebrated for their verve and vision: 'We may regard the present state of the universe as the effect of its past and the cause of its future. An intelligence which at a given moment knew all the forces that animate nature, and the respective positions of the beings that compose it, and further possessing the scope to analyse these data, could condense into a single formula the movement of the greatest bodies of the universe and that of the least atom: for such an intelligence nothing could be uncertain, and past and future alike would be before its eyes.' You see, the problem of the solar system all over again, only magnified to comprise the whole universe! The world an immense machine moving with absolute precision. The essence of this view is that all events in nature are predetermined by law, and that this determinism permits of no flaws in the chain of causes.

It was, however, for Lamettrie to carry this view to its utmost logical consequence, declaring (in his book *L'homme machine*)<sup>3</sup> that man is a machine. Every atom in the world obeys laws, whether it

<sup>1</sup> Proved by him in 1773. It may be remarked, in passing, that his solution holds only of a strictly mechanical system (the 'ideal' case), leaving out of account such things as tidal friction (partly dissipating into heat), light pressure, and so on.

<sup>2</sup> Translated by Truscott and Emsey, New York, 1902.

<sup>3</sup> Critical edition by A. Vartarian, Princeton, 1960.

is part of a lifeless substance or of a living organism. Every atom of the human body must therefore move according to the same mechanical laws which govern the motion of the planets: its behaviour is uniquely determined by the configuration and state of motion of all the other atoms in the universe. But if so, there cannot be such a thing as freedom of choice: what a man will do, the words he will utter, even how he will feel and think, all this is the necessary outcome of the events preceding it. Man, then, acts like a machine, a clock, following the same blind mechanical necessity as everything else. A formidable argument. The idea of complete and unbroken determinism has always bewitched thinkers since the time of Democritus. Everyone, of course, feels that there is something wrong with it: human behaviour is not as exactly foreseeable as a lunar or solar eclipse. Yet that does not answer the argument. Indeed, if one holds on to the Laplacian program, according to which the fate of every single atom is predetermined, there seems no way out from the conclusion that every limb of our body must move with necessity in the way it does, and that our persuasion of being able to move our limbs freely as we please must be due to some illusion. Given the premisses, the conclusion seems unavoidable. No one can act against the laws of nature: and to act in accordance with them wouldn't *be* acting — I would not be the doer of my deeds. Nature would rather act *through* me, and I, a mere onlooker, would passively follow the movements of my own mouth when I speak, of my own hand when I write, . . . That is the dilemma. Theorists were haunted by it for centuries. It took scientists a long time to find out what was wrong with it. In fact, it was not before the rise of a non-causal mode of thinking, expressed in the uncertainty principle, that the ground was cut from beneath this outlook: at least one weak link of the underlying view of nature was laid bare and disposed of. (Some disagree, e.g. Schrödinger.)

4. Now for a more philosophical point. What is regarded as a rational explanation, as 'satisfactory', 'natural', 'acceptable' and what not is, in part, tied to the whole climate of opinion of a time and apt to change with it, and in part, and this seems to me the more important aspect, tied to something unhistorical: the rationality of a certain way of looking at the world. To illustrate what I mean, predilection for mechanical explanation, quite

foreign to Aristotle but typical of the period from Newton<sup>1</sup> to Einstein, or belief in the continuity of nature as expressed in the adage *natura non facit saltus*<sup>2</sup> — these represent not so much objective truths as attitudes of mind which, for a time, hold thought in a firm position. Another such attitude, only much more deep-rooted and hence much more difficult to dislodge, is belief in an immutable causal order. Above all, however, the scientist is guided in his research by a deep faith in the comprehensibility of nature, by the conviction that notwithstanding the kaleidoscopic character of phenomena, there lies some simple and clear pattern that can be fully comprehended — a faith not arrived at on the strength of evidence but prior to it and, in this sense, not rational.<sup>3</sup> As Planck says: 'Faith points the way and sharpens the senses.'

When a 'new attitude became firmly established, men sought to make it seem objective and necessary by giving it a philosophical setting . . . The people who are under the influence of a 'Weltanschauung' want to *fix* it, to make it seem not so much a particular *attitude* as a *necessary fact*. They then endeavour, by expressing it in a metaphysics, to give it a universal validity.'<sup>4</sup> This is one of the great rôles played by metaphysics in history, though not the only one. To put it differently, an attitude has a propensity to crystallize into a metaphysics, just as, conversely, a metaphysics is, in part anyhow, an intellectualization of certain attitudes.

A conspicuous instance of this process is Kant. He was writing at a time when the ideas of Newton had gained currency and the principle of causality began to dominate the scientific scene. Curiously enough, the same laws which, not so long ago, because they failed to conform to *their* standards of satisfactoriness, were spurned by the schoolmen as absurd, at the most as superficial, evading the real issue, were now hailed as rational, transparent to the mind like geometry, and, like geometry, they were raised to the Olympian heights of the *a priori*. The idea of causality (which after all was not yet so firmly implanted in Newton's mind) was now

<sup>1</sup> 'I wish we could derive the rest of the phenomena of nature by the same kind of reasoning from mechanical principles.' (Preface to the 1st ed. of *Principia*.)

<sup>2</sup> According to Buffon, all classifications in biology are an 'error in metaphysics', in so far as we see discontinuous species, genera, etc. in nature where, in actual fact, there is only a great chain of beings.

<sup>3</sup> 'The eternal mystery of the world is its comprehensibility.' (Einstein, 'Physics and Reality', *J. Franklin Institute*, vol. 221, pp. 349–382).

<sup>4</sup> T. E. Hulme, *Speculations*, London, 1924.

given the status of a category or of a rigid scheme without which an understanding of reality would be impossible. The Law of Causation came thus to be turned into a presupposition for the possibility of experience so that it cannot be criticized on the basis of experience.<sup>1</sup> In the *Critique of Pure Reason* Kant said that whenever something happens it 'presupposes something upon which it follows according to a rule'. Between cause and effect, moreover, there exists a 'necessary connection', and as this has no empirically verifiable meaning (Hume), Kant concluded that its origin must be sought in the nature of our intellect which impresses its own mark on the things.<sup>2</sup> In the *Critique* the principle of inertia, of action and reaction, and of conservation of matter is claimed to have *a priori* validity, while in the *Metaphysical Foundations* even the law of gravitation, and in the *Opus Postumum* the whole body of principles of Newtonian mechanics is presented as *a priori* truths independent of experience.

It should be noticed that Kant published these ideas in 1781 (first ed. of the *Critique*), that is 8 years after Laplace had shown the solar system to be governed by laws which leave no loop-hole for supernatural interference. Kant, who took a deep interest in astronomy (and even made a contribution to it), must have been aware of Laplace's work and the stir it made at the time. This may illustrate what I have been saying — that new ways of thinking evolved in science, when they have become firmly established, tend to harden into metaphysics so that they are regarded as necessary, as a sort of inescapable grooves within which thought is bound to run. The historical tracing of the rise of such ideas is then a means of loosening up such a cramped attitude and regaining intellectual freedom.

It was only C. Maxwell who raised a warning voice against the taken-for-granted manner in which determinism was accepted by his contemporaries. 'The promotion of natural knowledge', he wrote, 'may tend to remove that prejudice in favour of determinism which seems to arise from assuming that the physical science of the future is a mere magnified image of that of the past'. Prophetic words, but of course unheeded at the time.

5. As shown in the foregoing, the idea of causation, at first

<sup>1</sup> Cf. Helmholtz.

<sup>2</sup> 'The understanding does not draw its laws from nature but prescribes them to nature.' (*Prolegomena*, § 36.)

pretty vague and shot through with animistic elements, became narrowed down and refined into a differential law, expressing the idea that the course of natural phenomena is uniquely determined by certain conditions. This by no means exhausts the account of the matter. To mention only one question, if laws of *any* degree of complexity are permitted it is *always* possible (as shown by Fourier<sup>1</sup>) to subsume a given sequence of events, however irregular, under a mathematical law.<sup>2</sup> The principle of causality would, in this case, become devoid of factual content and turn into a triviality. But physics is not trivial. Two ways offer themselves to escape that consequence. One, suggested already by Maxwell, is to demand that the run of events should be completely independent of place and time, that is, such that a causal connection is *universally* valid. Against this it may be said that the 'constants of nature' (e.g. the constant of gravitation) need perhaps not be absolutely constant but may slowly change with the age of the universe, and with them the laws of nature, in which case a historical element would enter into their formulation. In point of fact, several recent cosmological theories envisage just such a state of things. It seems therefore wiser not to commit ourselves to a definition which rules out such possibilities from the start. The other possible way is to demand that a law should be (a) simple and (b) such that it can serve as a basis for making predictions. The queer and rather surprising thing is that the traits (a) and (b) seem to go together — a fact often adduced as a sign of a 'rationality' of nature. Without going into this point, perhaps only this much may be said. As there are no precise rules for determining what is and what is not 'simple' (the concept of simplicity being half aesthetic, half pragmatic), the notion of causality, so far as it is based on it, seems also to lack precision so that it is not possible to state exactly what is and what is not subject to causal laws — a discouraging result as 'we shall certainly not be prepared to formulate the difference between chaos and order by saying that the former is accessible to an eminent mathematician only, the latter to a mediocre.'<sup>3</sup>

<sup>1</sup> *Théorie Analytique de la Chaleur*, translated as *The Analytical Theory of Heat*, New York, 1955.

<sup>2</sup> Mathematically speaking, any curve, however haphazard and bizarre — provided it is continuous within a certain interval or has only a finite number of discontinuities — can always be covered by a formula (Fourier series).

<sup>3</sup> M. Schlick, 'Die Kausalität in der gegenwärtigen Physik', *Ges. Aufsätze*, Vienna, 1938.

Now for another aspect. It is true that the terms 'cause' and 'effect', and the observation of the regularity with which they follow each other, are very useful in practical life and in the infancy of science. Thus it is often cited as an instance of a causal relation that a body, when deprived of support, falls to the ground. Here we have apparently all the characteristic traits of causality as enumerated by Hume or Mill: the causing event is 'depriving the body of its support', the effect 'falling to the ground', and the two are connected by regularity. As J. S. Mill says, 'The Law of Causation, the recognition of which is the main pillar of inductive science, is but the familiar truth, that invariability of succession is found by observation to obtain between every fact in nature and some other fact which has preceded it.'<sup>1</sup> But to say such a thing does not penetrate to the heart of the matter. What is overlooked here is that science, instead of establishing connections between facts *as such*, rather analyses them. This analysis consists in decomposing (in thought) the unique course of actual events into elements which are simpler, recurrent and capable of measurement. The 'uniform' relations of which philosophers are so fond are to be found, not between the events themselves, but rather between the elements into which they have been dissected. To give an example, the astronomer decomposes the acceleration of our moon into a part due to the attraction of the earth and into a part due to that of the sun; and he does so despite the fact that none of these partial accelerations has ever been the subject of observation: they are theoretical constructions. What can be observed is the phenomenon as a whole. Yet it is only when it is so dissected into components that clear and simple laws emerge. The event as such is without interest to the scientist. There is no science which deals with an event such as 'the sun rising above a sea of purple'. Far from aiming at producing a faithful and detailed copy of reality, science is an endeavour to comprehend it. And this it does, firstly by analysing it into component parts, then seeking for laws to connect them and finally reconstructing reality out of these elements — or, in short, by analysis and synthesis. One tries therefore in vain to set up laws which, as Mill puts it, connect 'every fact in nature' with 'some other fact which has preceded it'. This is not and has never been the program of the natural sciences. At best, it is a beginning towards describing the methods of research.

<sup>1</sup> *A System of Logic*, Bk. III, chap. v.

It will by now, I hope, have become clear what is wrong with the accounts of causality offered by most philosophical writers. Science is concerned with the relations holding between the simplified elements into which the facts of observation are dissected. It is when these elements are represented by quantities that functional dependencies between them may be recognized, leading to the discovery of physical laws. The *law of nature* thus takes the place of the relation between cause and effect and makes it redundant to analyse this relation more precisely. At the same time, the difficulties encountered prevent the concept of determinism from assuming sharp outlines.

The expression 'law of nature' is, in its use, intimately linked to that of 'explanation'. In the history of it we shall find stored up — like layers of rock deposited one on top of the other — different senses, reflecting differing modes of thought some of which have survived to the present day and still live on as overtones of the expression or as half-conscious associations. The idea of a law of nature is essentially *modern* in origin and belongs to a whole cluster of ideas which came to be expressed at about the same time — indicating an important change in human thought, when the need was felt to have names or words to stand for various activities and ways of looking at things. Such words as 'arrange', 'classify', 'organize', 'category', 'method', 'system', 'systematic', 'regular' appeared with their modern meaning in language. Only two of them are earlier than the seventeenth, and most of them are not found till the eighteenth century. Thus 'arrange' was a military term like 'array', and 'regular' was used only of monastic orders until the close of the sixteenth century.

The Latin *lex* was first applied to material phenomena by Bacon. Later in the seventeenth century 'law' came to be used in the same sense, though it did then not quite mean what it does today. The 'laws of nature' were conceived of at that time as commands of God. It is not without significance that the expression is found first in the hands of theologians — of those, for instance, who followed the teachings of John Calvin. This protestant leader was working towards a conception of God as the Absolute Ruler of the universe, governing it by laws decided upon at the beginning of creation. This theological use has left traces upon our language — we still speak today of nature 'obeying' laws, although we no longer

think of them as being imposed upon nature by the will of the Divine Legislator.

It was only by a slow and not always smooth process of emancipation that the idea threw off its more obvious theological associations and acquired a new significance. Galileo, for instance, called his quantitative rules ‘principles’, ‘ratios’ or ‘proportions’. His ‘principle of inertia’, restricted to terrestrial objects, is, apart from this difference, the same as Newton’s ‘first law of motion’, Newton using the term ‘law of nature’ freely as it had become current at his time, though not unopposed by some, such as Robert Boyle, who thought the term ‘an improper and figurative expression’. When an arrow is shot from a bow, he wrote, ‘none will say that it moves by a law, but by an external impulse’. Thus, ‘law of nature’ was at first construed as a command of God and then as something inherent in nature. ‘Nature is constrained by the rational order of her law which lives infused in her’ — this dictum of Leonardo da Vinci marks, perhaps, the point where the one conception turns into the other.

The change of meaning is itself indicative of the more rational view of life which was beginning to dominate men’s minds. It was the time when the miraculous was falling into disrepute and Hobbes protested against the element of the supernatural in the medieval ‘romances’ — the same time when the word ‘romantic’ made its appearance as a derogatory term to brand what is false, hollow and unnatural.<sup>1</sup> Hobbe’s writing is characteristic of the Age of Reason which rose in the later part of the seventeenth century (the time of Newton’s manhood). Intellectually, men’s minds seem to have been influenced above all by that conception of impersonal law and order governing the universe — a conception scarcely entertained in the preceding century. Poets and philosophers alike were delighted by the perfect *order* in which, in their view, the cosmos was arranged. The appreciation of nature’s regularity, rather trite to us, was for those men a source of poetic inspiration — anyhow filled them with enthusiasm. (‘Enthusiastic’ like ‘fanatic’, which originally meant ‘possessed by a god or demon’, underwent a similar inflection of sense about the middle of the seventeenth century.) For us, breathing in a different air, it is almost impossible to recapture that mood, yet it forms the background against which the rise of the term ‘law of nature’ in its

<sup>1</sup> See, for instance, L. P. Smith, *Four Words* (S.P.E. Tract 17).

modern sense must be seen. It is but one term of a whole group whose more or less parallel changes of meaning curiously reflect that movement of thought.

The great watchword of the time, however, was *Reason*. It must have exerted a deep fascination as it was turning up time and again in the writings of the period — occupying, e.g., the central place in the systems of Spinoza, and generally in the philosophy of the rationalists. In Spinoza particularly, we seem to catch a note of that majestic harmony of celestial mechanics which was just growing up towards the end of his life. Indeed, the program of this school of thought was to apply the mathematical mode of reasoning to *all* problems, whether metaphysical, moral or scientific — to exalt the powers of reason and rational method at the expense of blind faith, revelation and Fancy, just as this power was glorified by Milton —

whence the soul  
*Reason receives, and Reason is her being (Paradise Lost, V).*

The cult of Reason suggested belief in the rationality of being and thus created the atmosphere in which science could thrive. Without such belief there would be no science, though it need not be expressed in words. Those who did give expression to it, the rationalist thinkers, paid only lip-service to the spirit — today their systems are petrified and belong to the curios of history, while the faith that prompted them lives on in science, opening ever new horizons.

I shall not endeavour to trace the origins of the idea of a law of nature as that would mean to go back to the Greeks and even to their predecessors in the East — a job for which I don't feel qualified. What I have in mind was no more than to cast a sidelight on its chequered career. For in the light of that history we begin to understand why words like 'law' or 'cause' tend to change their meaning with every context and let us see each time other facets. The idea of fate or destiny standing above men and gods, as we find with the Greek tragedians, the aesthetic conception of mathematical harmony as expressed in the Pythagorean 'cosmos', the biblical idea of God as the law-giver of the universe associated with that of rationality — these are some of the sources which have contributed to deepen the term with their significations. Growing out of its long history, the word has absorbed into its meaning ideas

and modes of thinking inherited from many different ages, races and civilizations. Part of its meaning has been shaped by the Pythagorean brotherhoods who led a life of mathematical and religious contemplation. It has travelled from Babylon to the Holy Land where it was in the hands of the priests, and from there to Rome, to the Stoics, taking up some of the teachings of Democritus. Then, at the time of the Renaissance, it came to the northern countries until, with Newton's *Principia*, it became the cornerstone of science. In the course of this Odyssey, the term underwent a series of strange transformations until, with the birth of modern science, it definitely took shape in the form of a mathematical equation. Yet, in the interpretation given to the modern formulae by physicists and philosophers, much of the older signification is still alive or at least dormant — like the pagan gods who, masked as demons, were living on into the Christian era.

6. To return to Laplace: if there *were* a being that *had* the power —. If —! It all seemed a question of efficiency. In *principle*, it was thought, all the information as regards the particles in the universe *could* be gathered; in principle, the equations for them could be written down; in principle, they could be solved; in principle, the entire future could be foreseen. That to accomplish such a task far surpasses our faculties seemed no objection as this was entirely due to our limitations. If, faced with a system of enormous mechanical complexity — such as a roulette wheel — we resort to probability laws, we avail ourselves of a makeshift device that only reflects our ignorance but could, in principle, be dispensed with. We are in the position of a millionaire who, instead of insisting on a detailed account of the state of his finances, acquiesces in being given a rough estimate.

Laplace's calculator represents the high-water mark of the tide of determinism as it was rising steadily since the beginnings of modern science. However, the words in which he proclaims the triumph of determinism at the same time reveals its weakness (though this was not noticed for a long time). For what is requisite for his program is detailed, i.e. *complete and precise* information as to the state of all the particles in the universe at a given instant of time. Hence the question arises: is this possible? Disregarding completeness for the moment and considering precision, it was of course always recognized that *absolute* precision is out of reach, only this was minimized: the accuracy of any measurement, it was

supposed — rather light-heartedly as it would seem now — could be increased to any degree by improved technique. Irrespective of whether absolute precision will ever be attainable with our blunt instruments, we can at any rate go on refining our measuring methods, it was assumed, and proportionally our predictions

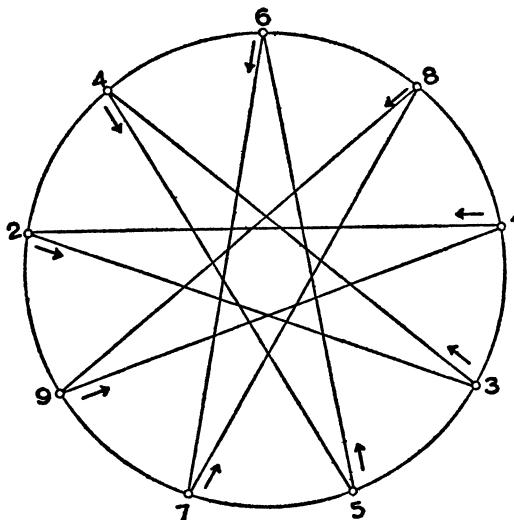


FIG. 1

concerning the future will become more and more reliable. That there is *no limit* to this approach, this was, ultimately, the tacit assumption underlying classical physics, and one so brilliantly vindicated by the successes in astronomy.

To come from the heavens down to earth, is it really quite certain that mechanics permits accurate prediction under all conditions?<sup>1</sup> Let me consider a simple case to throw light on the question.<sup>2</sup> Suppose an elastic ball moves freely on a surface, say, a round board at the edge of which it is elastically reflected. Both ball and surface are supposed to be perfectly smooth and elastic so that no friction occurs. Once set in motion, the ball will, under the idealized assumption, move on for ever, rebounding from the

<sup>1</sup> Doubts as to that were first raised by R. V. Mises (*Probability, Statistics, and Truth*) 2nd ed., London, 1957, and even before in an article in *Die Naturwissenschaften*, 1922.

<sup>2</sup> For a similar example cf. M. Born, *Physics in My Generation*, London, 1956.

wall, thus providing us with a sort of perpetual billiards. Given the initial conditions (position and velocity), what will be the track described by the ball? There are two widely different possibilities: the ball may describe a closed track of finite length so that the motion will be a periodic one, or it may start on a zigzag path never to return to a position occupied once before. (Figures 1 and 2). What sort of track will be followed depends on the initial state. What is peculiar about this case is that small causes will have big effects. Indeed, the slightest change in the initial state, say, in

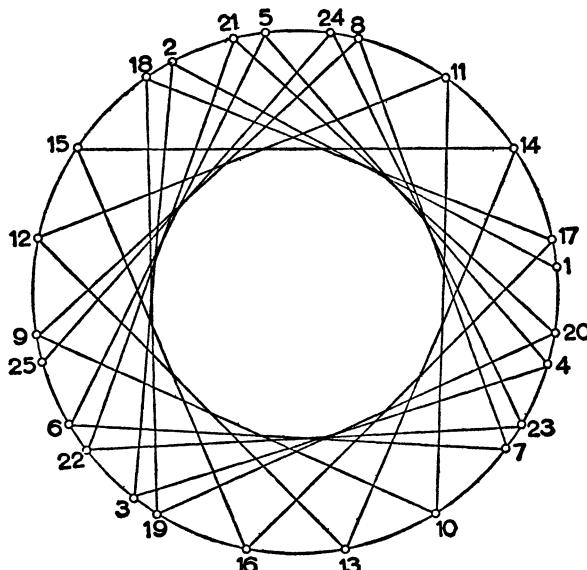


FIG. 2

position or direction of motion may be enough to turn a closed path into an open one. Can we in this case really predict the path with certainty? Only under one condition: that position and direction at the start are known with *complete accuracy*. The least error in these data would foil any attempt on our part at predicting the future. The simplest case of a periodic motion is that the ball just rolls across the board from end to end, traversing a diameter. What will happen if the ball starts not exactly in the line of the diameter but turns aside from it by a very small angle  $\phi$ ? In this case, the ball will arrive at the point  $2'$  instead of at  $2$ . (Fig. 3.)

Suppose the line  $1-2'$  subtends an angle  $\alpha$  at the centre of the circle, then the sort of track described by the ball will depend upon the ratio  $\alpha/2\pi$ : if this ratio is a *rational* number, say,  $p/q$ , the ball will, having been reflected at the wall exactly  $q$  times, return to its original place, while in the opposite case (that the ratio is *irrational*), it will never return. Suppose now that we do not know *precisely* the initial angle of divergence, allowing for a latitude  $\Delta\phi$ , then we cannot decide to what category the track will belong, whether it

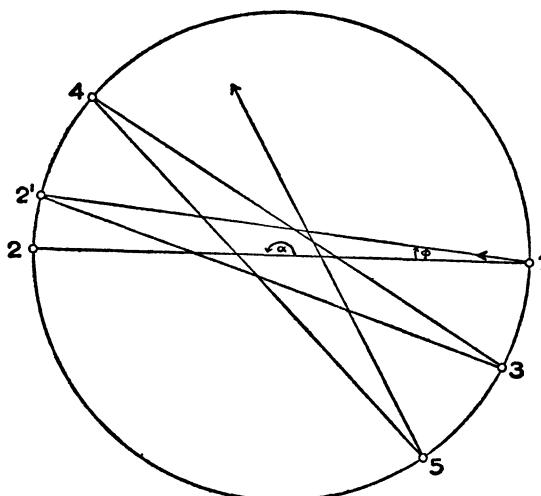


FIG. 3

will be an open or a closed one. Conditions may even be such that the uncertainty  $\Delta\phi$ , however small at the beginning, may increase with time (Fig. 4) so that, if we only wait long enough, the ball may at a given moment be found *anywhere* on the board. In such a case, we should know absolutely nothing about its position, and determinism has turned into complete indeterminism.

To avoid such a consequence one has to demand mathematical precision regarding the initial state. That, however, raises a serious question — namely, is there any *sense* in speaking of such a thing? Suppose the distance of the ball's surface to the wall is measured at a given instant, does it make sense to say that it amounts to two centimeters? Owing to the coarse-grainedness of matter, the ball's surface is not a mathematical surface, any more

than that of the wall. Seen from close by, such a surface dissolves into a cloud of molecules, blurred and ever-changing. The same is true for a measuring rod — the particles at its ends perform a wild, irregular dance, blurring its length. In view of this, one comes to recognize how utterly Utopian the idea of absolute precision is.

The point of our example will now be seen. Even in classical mechanics the causal scheme does not always work, not under all circumstances. Whether it works or not hinges on one condition — that measurements can be made with unlimited accuracy. Causality stands and falls with this requirement. Should for some reason or

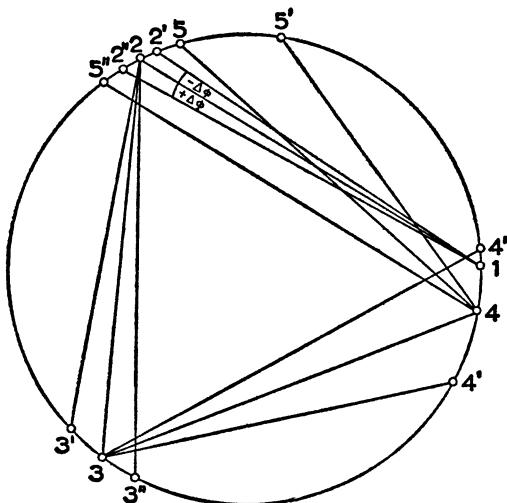


FIG. 4

other its fulfilment be impossible, we should be prevented from making predictions, and the principle of causality, so far as its criterion is predictability, would break down: though not 'false' in the current sense, it would become inapplicable since its pre-supposition is unrealized.

A last point: it should not be concluded that the example discussed provides a case of *acausal* motion. One would rather be inclined to say that there exist circumstances (such as a slight unevenness on the ground or the surface of the ball) which are too small to be detected in ordinary observation but which none the less

determine the path in the normal causal way. Taking into account such 'hidden parameters', there may still be a gleam of hope of maintaining the idea of determinism. In the *initial* phase, anyhow, the course *can* be predicted, and it is only for the more distant future that it becomes uncertain. Thus the case presents a curious half-way house, showing not so much the fall as the decline of causality — the point, that is, where the principle begins to lose its applicability. The uncertainties we shall meet on the atomic level are of a much more radical sort, with the consequence that we shall be forced to give up entirely the idea of deterministic laws in favour of statistical ones.

To sum up: determinism is an *idealization* rather than a statement of fact, valid only under the assumption that unlimited accuracy is within our reach, an assumption which in view of the atomic structure of our measuring instruments is anything but realistic. It is chiefly in astronomy, the foremost domain of precision, that we can at least *approach* the classical ideal.

7. The story of the mechanical conception of nature, with its stress on mechanical models, stands in marked contrast with that of the causal conception. Both have come to an end, though in different ways: for while the former was never really refuted in a strict sense but, suffering decline, was allowed to die quietly of sheer inanition, the end of causality came dramatically, with a bang, not a whimper. I am speaking of Heisenberg's uncertainty principle.

First a remark. As we descend to the atomic level, we are entering a strange world, stranger than anything out of the Arabian nights. Indeed, we should not expect, in a field which lies so far outside the reach of our senses, to find the same sort of relations and laws as those which hold in our large-scale world. Of course, all new experience makes its appearance within the frame of our perceptions, and is describable with the words of plain language: otherwise communication would break down. But it would be an illusion to think that the behaviour of the things in the world around us and their properties can be extrapolated into the atomic domain. On the contrary, as we cross the boundary we must be prepared to find that our common notions and ideas desert us: and that's exactly what has happened in quantum theory.

To return to a point mentioned previously, Newton's scheme of mechanics is an idealization characterised by the fact that it draws

a picture of nature, if only a quite schematic one without filling in the details,<sup>1</sup> that is independent of the means of observing. Indeed, classical physics may be described as that idealization in which we can study natural phenomena without referring to ourselves. This assumption, however deeply rooted in our whole way of thinking, indeed in the forms of our language, is no longer true of the two great achievements of this century, relativity and quantum mechanics. In both cases, the whole aspect was changed with the recognition of the part played by the observer. In particular in the case of quantum mechanics, the widening of the horizon and the emergence of new ranges of fact have shaken the presuppositions on which classical physics was built, leading to a revision not only of our customary concepts ('particle', 'path', 'wave', etc.) but even of the demands deemed indispensable for rational explanation. That it was an idealization became clear only as soon as one came across phenomena which defy any account in terms of classical theory. Thus arose the need to analyse the conditions underlying the shaping of our concepts — a circumstance of significance far beyond the special theory under consideration.

The difficulty for the non-scientist, or philosopher, is to free himself of the bondage of the customary outlook. A much deeper insight into the phenomena of nature has in fact been gained as a result of such emancipation, but at the cost of a far-reaching renunciation with regard to describing and comprehending the behaviour of atomic particles within the compass of our ordinary notions. Thus it is not possible to trace the motion of such a tiny object in space and time; it is not possible to order the events on this scale in such a way that they form causal chains; it is not even possible to arrange them in a coherent manner within the frame of space and time; after all, the only forms in which we can visualize them. The usual demands for visualization and causal order are unfulfillable.

Owing to lack of time to go into all that, I shall confine myself to one example, just to illustrate the sort of situation we are here confronted with. The ideal of a complete scientific description, it was thought, consists in this: to describe precisely what happens at any point in space at any instant of time. It may be called the

<sup>1</sup> These are the laws of force, stated by Newton only for the case of gravitation. It is these laws which put content into the otherwise empty frame of Newtonian mechanics.

Laplacian ideal. It reigned supreme from the beginnings of modern science in the seventeenth century till 1927. At this time, however, it became clear that the idea of such a continuous description is incompatible with experimental facts. Though it is very tempting to think that, if we could only have kept a particle under steady observation we could have traced its path, we shall see before long why this is not possible. We are forced to give up the whole space-time picture of events on the atomic scale — and with it the demand for visualization — and resign ourselves to an unpicturable state of affairs. This situation has important consequences. One of them — as stressed before — concerns causality. If the idea of an unbroken description has to be abandoned the principle of causality cannot be maintained either: for this principle is in science chained to the possibility of such a description. If a continuous description is no longer possible the very foundation of the causal principle melts away; indeed, the principle assumes continuous description as a necessary condition: the breakdown of the latter, therefore, entails the breakdown of the former. So much for the general background against which the fall of causality must be judged.

Let us now see in a bit more detail why such a description is not possible. When we want to 'see' a particle (an electron, proton, nucleon, . . .) we must illuminate it. Ordinary light will not do: its wave-length being about a hundred million times larger<sup>1</sup> than an electron, it will simply pass around it. We shall therefore have to use radiation of extremely short wave-length (hard X-rays,  $\gamma$ -rays). The shorter the wave-length, the more energetic the radiation, according to the Einstein law  $E = h\nu$ . To observe a particle means, then, to shoot at it with high-energy photons which, after they have hit the target, rebound from it with diminished energy (Compton effect) and are reflected into the eye of the observer, or onto a photographic plate. So far so good. But now for the crucial point: the disturbance suffered by our particle in its collision with a photon, it may be thought, is calculable and so predictable. Strangely enough, this is not so: the collision process is unsurveyable in its finer details. Try as we may, each time we attempt to 'see' the electron it gets a kick — and is somewhere else. While ordinary objects such as we handle in daily life can be observed

<sup>1</sup> The dimensions of an electron are of the order of  $10^{-12}$  cm, the wave-length of visible light lies between 4,000 and 8,000 Å, 1 Å =  $10^{-8}$  cm (1 Ångström unit).

without disturbing them, the situation on this scale is entirely different. Observing an electron is, unavoidably, interfering with it. In the act of observation it is pushed by a photon, and this must alter its velocity. The situation is sometimes described by saying that such a minute object, if observed, i.e. interfered with, is taking a zigzag course, being tossed about under the impact of the photons like a boat in a heaving sea. In reality, it is far worse: for we cannot even speak of 'the same' particle. Suppose we observe an atomic object and an instant later a similar one near-by, then we can't even be sure that it is 'the same'. Owing to the interaction between the object and the process of observing, which cannot be controlled, it is not possible to follow its course continuously. Two observations, even if following one another very shortly, should rather be regarded as disconnected events, and it is not possible to combine them unambiguously into a single comprehensive picture. Nor is there any way of telling what 'happens' between one observation and the next. In other words, any picture of what is 'really' going on contains gaps which cannot be filled in. That is why any attempt at tracing the path of an atomic particle is doomed to fail. As a consequence, the question as to whether a particle, really and truly, is the same is not only undecidable but devoid of meaning. (The reader is referred to the mystifying experiment in which electrons pass an opaque screen with two holes in it and where one can't tell afterwards through which of them an electron has gone: not only is the question undecidable, it is meaningless on account of the fact that the electrons, in passing, suffer diffraction, i.e. behave like waves and thus are lost in the wave pattern.) All the evidence points to the need to revise drastically the notion of particles in the classical sense — itself taken over from common experience, such as motes in a sunbeam, specks of dust — to give up the idea of a precise path along with identifiability, permanence (i.e. continuous existence in space) — in short, the attributes of thinghood. An electron, in contrast (say) to the pen before me, is not objectifiable in a manner independent of the way it is observed: any observation is made at the expense of breaking the connection between the past and the future, owing to the uncontrollable disturbance it creates. This circumstance, the unavoidable interference with the run of events together with its unsurveyability, introduces an element of uncertainty into any possible observation and thus blocks the way to a causal analysis.

Moreover, the interaction between the object and the device used in observing it frustrates any clear line being drawn between a property of the object — e.g. the location of an electron — and the agency through which it is observed. Owing to the arbitrariness of this distinction, atomic particles can no longer be described in the same way as ordinary objects of sense perception.<sup>1</sup>

Summarizing, it may be said: a new class of facts connected with the existence of the quantum of action imposes upon us not only, as in relativity, a certain limitation of the concepts hitherto employed, but a radical renunciation in regard of any attempt to describe, or pictorially represent, what is going on in the atomic world within the framework of space, time and causality. To use Heisenberg's words, 'Atoms . . . possess geometrical qualities in no higher degree than colour, taste, . . . The atom of modern physics can only be symbolized by a partial differential equation in an abstract multi-dimensional space . . . Every type of visual conception we might wish to design is, *eo ipso*, faulty.'<sup>2</sup>

Strange consequences, you may think, but after all only to be expected when one attempts to look at the unlookable.

8. Before going into the reasons which have forced physicists to forgo the ideal of determinism, one point should be made perfectly clear. All our knowledge concerning the inner structure of atoms is ultimately derived from experiments in which atoms emit energy or collide with one another. In any such experiment causality is already *presupposed* so far as the large-scale apparatus is concerned. If this were not so, physicists could not even understand their experiments nor draw any conclusions from them. Thus the blackening of a photographic plate, or a flash on a scintillation screen is the effect of a particle impinging on it. In other words, we assume the existence of causal chains which lead from an event on the atomic scale to the effect observed in experiment. What Kant did not and could not foresee was that, while causality is thus indispensable for an interpretation of an experiment, it does not follow that it must also apply to the hidden reality which manifests itself in the experiment. The existence of causality on the macroscopic level together with acausality on the microscopic presents an inner tension which could only be released when it was shown

<sup>1</sup> It is not even feasible to speak of a *definite number* of particles within a given volume of space.

<sup>2</sup> *Philosophic Problems in Nuclear Science*, London, 1952.

that ordinary mechanics is included within quantum mechanics as a limiting case.

After this preliminary remark, let us turn to the main question. In which way does the manner in which phenomena are observed enter the picture of the physical world? Suppose we let a particle fly through a small hole in a diaphragm behind which a photographic plate or a scintillation screen registers its arrival, then we can make sure of its location with as much accuracy as we please. Such an experiment, however, tells us nothing about the velocity of the particle, or its momentum. If we want to find out the latter we have to make use of some movable part of the apparatus — say, a screen that can freely swing round a hinge. A particle shot at it will, owing to the law of conservation of momentum, transmit its impulse to this part of the apparatus, thus permitting us to infer from the observed motion the particle's momentum. In this case, we know next to nothing about its location: the particle may have hit the screen *anywhere*, and this introduces an unavoidable latitude into the result of the experiment. Let us call such experiments *complementary* as what we learn from the one cannot be learnt from the other. Such experiments stand in a relation of mutual exclusion to one another: no experiment can be devised that would inform us accurately both of location and momentum. The study of this relationship has led to the formulation of what must be regarded as one of the dominating principles of atomic physics — the uncertainty relation.

To say that all measurement is imprecise is a platitude. The fact brought to light by Heisenberg, however, goes much further than that: not only is there no absolute precision but a definite *limit* to it, depending on the experimental situation. More precisely, if one devises an experiment to trace the motion of an atomic particle by measuring its initial position and velocity it turns out that these two measurements stand in a relation of mutual exclusion. Any experiment that aims at measuring both will — owing to the unavoidable interaction of the object with the measuring instrument — only lead to a blurring of the results. The essentially new aspect of the matter is that the latitudes left in determining the two quantities are coupled according to the rule

$$(uncertainty \text{ as to position})$$

$$\times (uncertainty \text{ as to momentum}) = \text{const.}$$

In symbols

$$\Delta x \cdot \Delta p \approx h,$$

where  $h$  is Planck's constant ( $h = 6.625 \times 10^{-27}$  erg sec).  $h$ , though exceedingly small, is not zero, with the consequence that the two uncertainties cannot dwindle to nil: the ideal of absolute precision is strictly unattainable. Indeed, the smaller  $\Delta x$ , i.e. the better we succeed in measuring the particle's *position*, the larger  $\Delta p$ , i.e. the less accurate will be the *momentum*, and *vice versa*. Precise information about the one implies *total ignorance* of the other. (If  $\Delta x \rightarrow 0$ ,  $\Delta p \rightarrow \infty$ .) It is as if we had purchased knowledge of the one at the price of uncertainty as to the other, *complete* knowledge at the price of *complete* ignorance. Hence the name 'uncertainty relation', 'relation of indeterminacy' (*Unschärferelation*).

The departure from classical theory jumps to the eye: while in the latter it has always been taken for granted — if tacitly — that the inaccuracies involved in measurement are (1) negligible and (2) independent of one another, the new principle asserts that they are related in the way described. Its significance lies in this: that Planck's *quantum of action sets a limit to the possible accuracy of any measurement*, depending on the experimental arrangement, a limit which, in favourable circumstances, can be reached but never surpassed (or 'underpassed', as I should rather have said). It thus introduces a new and fundamental feature into the picture of the physical world, connected with the existence of an indivisible quantum  $h$  which, from the classical standpoint, really is an irrational element. There seems to be some sort of connection between two kinds of discontinuity in nature — the quantum of action and the atomic structure of matter, though one cannot yet clearly see what the relation is.

As the breakdown of causality is an immediate consequence of the principle, it is of importance to understand clearly what it amounts to. But first an objection. Is the principle not in conflict with the notorious fact that position and velocity of an ordinary object can be measured without ado? That this is possible is due to the extreme smallness of Planck's constant. Writing  $mv$  instead of  $p$ , we may re-state the principle in the form

$$\Delta x \cdot \Delta v \approx h/m.$$

This shows that, as the mass  $m$  increases, the right-hand side of the relation tends to zero. For heavy bodies,  $h/m$  is too small to

have an appreciable effect: hence the impression that there is no limit to the possible accuracy of measurement. Given a body weighing one gramme, we can in principle fix its position within  $2 \times 10^{-13}$  cm (a distance of the order of the dimensions of an electron) and its velocity within  $2 \times 10^{-13}$  cm/sec — or 6 microns per century. But replace the body by an electron, and you have an altogether different situation. Suppose the electron's position is to be determined within  $10^{-5}$  cm (wave-length of ultraviolet rays), then the uncertainty in speed will be 500 km/sec. Considering that such an accuracy is not much to boast of — it would be like locating a grain of shot within a margin of 100 km — let us try to do better and fix the position within  $10^{-10}$  cm, then the indeterminacy in speed will be 50,000 km/sec. Conversely, if the velocity is to be measured accurately, say, within a micron per second, the electron may be found anywhere within a distance of 50 km — its location will be quite 'unsharp'. So much to illustrate the Heisenberg relation.

To guard against misconceptions, it should perhaps be stressed that it is not because our instruments are too blunt that these quantities, position and momentum, cannot be measured simultaneously: this would still be a contingent feature. The root lies deeper: even if the particle is in no way physically interfered with by our observing it so that its physical state is not altered, we are still prevented from acquiring information about it which goes beyond the limits laid down by the uncertainty relation. As the same relation can also be derived on the basis of very general ideas — e.g. when one thinks of a particle as a wave-group — it certainly can have nothing to do with the bluntness of our instruments.

But this only raises another and more radical question. Is there any *sense* in ascribing such properties to a particle? If one says, 'It is not possible to measure exactly both position and momentum', this is a misleading way of putting the principle: for it now looks as if the particle, before it had been observed, did in fact possess some definite properties which, however, are disturbed by the act of observation. God, if I may say so, looks down and sees where the particle is and how it moves: *He knows*, while we — owing to a conspiracy of nature? — are kept in the dark: a situation reminiscent of views held before relativity. The earth, it was thought at the time, like a ship, sails through an ether-sea, carrying with it the

physicists and their instruments. However, the instant an interferometer is placed in position to ascertain the earth's motion something strange happens: the arms of the instrument experience a shortening in the line of motion so that the attempt is baffled. There *is* motion, there *is* contraction, only the two things are so precisely adjusted that they cancel out — that despite motion the effect is nil. So in our case: the object *has* position and momentum, unfortunately, however —. Is nature really so spiteful? But here we are falling just into the sort of error we have to avoid, namely, conceiving of such objects in analogy with the things around us. In imputing current physical properties to them, we are treating them as if they were just grains of sand, only smaller: which is a mistake. At least we cannot do so without sinning against the first commandment in science: never make statements that cannot be checked.<sup>1</sup> Great care should therefore be taken over the manner of formulating the Heisenberg relation. A more correct expression would be to say: there is no physical law in which reference is made to the exact position and momentum of a particle. Each of these quantities can be determined with any degree of accuracy, only not under the same experimental conditions — it is just the point that their study requires mutually exclusive arrangements. We can thus obtain 'contrasting pictures, each referring to an essential aspect of empirical evidence'.<sup>2</sup> The mistake, then, is to ascribe to an atomic object properties such as position and momentum *independent of observation* — as if such properties were inherent in the atom itself, regardless whether it is observed or not. In taking such an attitude, one fails to see that, in contrast to ordinary physics, such terms have meaning only insofar as the attributes they refer to can be observed in experiment. In speaking of location or velocity of an atom one must, therefore, not lose sight of the whole experimental situation of which they form part: divorced from it they no longer have a physical meaning.

This mistake, however, is only the consequence of another and still deeper one — that we think in the categories of ordinary language and use, along with it, the forms of pictorial visualization. Following this habit of thought, we are naturally tempted to apply

<sup>1</sup> Thus the orbits of the electrons within the atom in Bohr's first theory have about as much claim to reality as the hell-circles of Dante.

<sup>2</sup> N. Bohr, *Atomic Physics and Human Knowledge*, New York, 1958.

common notions, such as space and time, motion and path, thing and property, cause and effect in an uncritical way to a domain, far beyond the range of ordinary experience, to which they are not adapted. Or, looked upon the other side, a description of atomic events cannot be squeezed into the ready-made moulds of ordinary language, fitted as they are for describing our familiar world. What is needed is the shaping of a set of new concepts, permitting us to deal with what is entirely outside the compass of ordinary language. The latter, having been moulded under the influence of certain features which the outside world constantly presents to its users, bears still the stamp of them, in the vocabulary and the logical forms which have gone into its shaping. Classical physics is essentially an extension — and refinement — of the ways of thinking, observing and describing which we all employ in every-day life. Therefore it has not given rise to the sort of philosophic questions which are apt to confound the student of quantum theory. The novel thing about the latter is this: the phenomena studied in this part of physics present features which cannot be accounted for within the framework of classical concepts. To give a rational account of them, it was first of all necessary to become aware of certain presuppositions underlying the application of these concepts so that they may be modified and, if possible, adjusted to the new conditions. But the moulding of a system of concepts means nothing less than the creation of a new language, a new mode of thinking. (Uncommon sense, not common sense is required for such a feat.) It is the recognition of the inadequacy of ordinary concepts and ordinary language which may be of some interest to the philosopher.

The state of affairs sharply formulated in the Heisenberg principle has two consequences. The first concerns the mode of being of atomic particles. Here it must be said that they are never completely objectifiable, i.e. as completely describable in classical terms as objects on the ordinary scale are: their description always contains an uncertainty which is in part objective, due to the uncertainty relation, in part subjective, due to our incomplete knowledge. (The latter can in some circumstances be reduced to nought.) As Heisenberg says: 'they form a world of potentialities or possibilities rather than one of things and facts.'<sup>1</sup>

The other consequence closely connected with the first one is

<sup>1</sup> *Physics and Philosophy: The Revolution in Modern Science*, New York, 1958.

that it is *impossible to predict* the result of an observation with certainty; what can be predicted is only the *probability* of any such result. And that means that causality ceases to operate on this level. Not that this amounts to a 'refutation' of the causal principle in a strictly logical sense of the word. However, it cuts the ground from beneath the Laplacian program, removing the presupposition on which it rests, namely, that the state of the particles in the universe can, even in principle, ever be known with sufficient accuracy. Indeed, what has turned out is that the *present is unknowable* (not completely knowable) so that from the data available no conclusive inferences can be drawn with regard to the future. Whether one should speak here of a breakdown of causality or rather of the category of substance is a matter of taste.

9. I do not share the view that mathematics is such a terrific obstacle that it must be avoided at any price. In fact, it is the only means of making the situation really clear. In what follows, I shall confine myself to quite simple elementary mathematics such as any schoolboy will be able to follow.

Wave mechanics, the work of de Broglie and Schrödinger, has been modelled after Einstein's theory of light quanta ('needle radiation'). The uncertainty principle may be illustrated first for photons so that whatever we can learn about *their* behaviour will, within the frame of this article, equally well apply to that of material particles. Einstein's theory (light possessing an atomic structure) is not exactly a revival of Newton's corpuscular hypothesis. For photons are not corpuscles which move according to Newton's law of motion. How, then, do they move? The answer to this question holds a surprise: 'the path of a photon' is an expression which has no physical meaning. We say, for instance, that the sun emits photons and that, if a screen is placed in their way, they will not hit the ground. Moreover, we can, by using the laws of wave optics (superposition of waves), calculate the amplitudes in a given experimental setting; as the amplitudes are related to the average number of photons we can find out, for any region in space, the average number of photons which are present there. All this we can do, except one thing: describe the path of a single photon on its way from the sun to the Earth.

Let us see why this is not possible. We are accustomed to speak loosely of a light ray in empty space and picture it as a straight line — a precise line without thickness. However, if we want to

give substance to this idea we run into difficulties. How should such a ray be produced? A straight line, in geometry, is determined, say, by a point and its direction. If we wish to produce a light ray which travels along a straight line we have first of all to make sure that it passes through some definite point in space. This we can do by letting light pass through a small hole in a screen. The smaller the hole, the more precisely marked will be the point through which the ray has to pass. However, if we make the hole very small, e.g. so small that its width is but the wavelength of the light used, diffraction occurs: the light having passed the hole bends round, producing on the far side of the screen a characteristic pattern of light and dark rings. This shows that light, considered as a shower of photons, does not continue in its original direction (if it had one), but suffers deflection. Result: we have fixed a *point*, but at the price of making the direction *indeterminate*; i.e. we have succeeded in making photons pass fairly precisely through a point in space but failed to let them have a definite direction.

Let's try again, this time by making the *direction* definite. To this end, we may use two screens, each with a hole in it, and let light pass through the holes. Then the direction of the beam between the two holes will be pretty precise. If, however, the holes become very small diffraction will take place with its undesired consequences and thus foil our plan. To avoid this, we have to make sure that the holes are large in comparison with the wavelength. In this case, however, the beam of light will be of appreciable thickness. Result: we have fixed the *direction* of the ray, but at the price of leaving the point *poorly defined*; we have failed again.

We are thus faced with a dilemma. If we succeed in making light pass through a well-defined point it will be scattered; and if we succeed in giving it a precise direction it will no longer pass through a point. We can't have it both ways. What is at the bottom of this dilemma? The answer is: the physical existence of such a precise rectilinear ray is at variance with wave optics. If the laws of wave optics hold — as, in fact, they do — there cannot be such a thing as a 'precise path' of a light ray or a photon; or more correctly, one *may* speak of such a path, in a loose sense though, meaning a ray of some thickness, small for us, yet large compared with a wavelength.

Let us now consider a bit more in detail what happens in an

experiment of the sort just described. Imagine a screen with a hole in it. Let  $\Delta x$  denote its width. Suppose (monochromatic) light passes the hole in a direction perpendicular to the screen (Figure 5). According to a simple calculation — which you find in any textbook — only *part* of the incoming light moves on in the original direction; another part is deflected. The bulk of the latter is bent aside by a certain angle  $\alpha$  ('the first diffraction maximum'). The fact that concentric light and dark fringes appear on the wall opposite or on a photographic film placed there allows

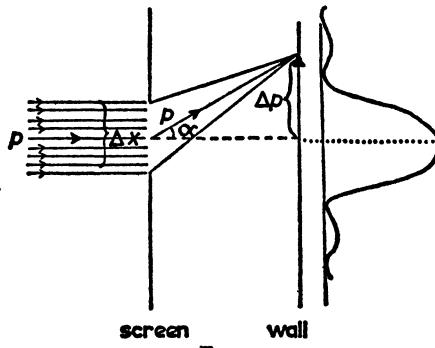


FIG. 5

us to assert that photons have passed through; precisely at what spot they passed remains so far unknown. The width  $\Delta x$ , therefore, represents the latitude allowed to the location of a photon on its flight through the screen, or, as one usually says, the 'uncertainty' of its position. As a swarm of photons passes through, some of them will undergo deflection — in the diagram upwards or downwards as the case may be — while others continue in their line of motion. To be deflected means to acquire an additional momentum in the direction parallel to the screen, the total momentum remaining constant. If  $p$  denotes the original momentum, and  $\Delta p$  the newly acquired one, a glance at Figure 5 shows that  $p$ ,  $\Delta p$  and  $\alpha$  are related, roughly, by the equation

$$\sin \alpha \approx \Delta p / p$$

or

$$\Delta p \approx p \sin \alpha. \quad (1)$$

On the other hand, wave theory asserts that the three quantities  $\Delta x$ ,  $\alpha$  and  $\lambda$  (the wave-length) are connected by

$$\Delta x \cdot \sin \alpha \approx \lambda. \quad (2)$$

Before going on, let us see what the last equation means.  $\lambda$  is constant for a given sort of light. The meaning of (2) is that the product of the two factors on the left-hand side is constant: the smaller  $\Delta x$ , the larger will be  $\sin \alpha$ , and therefore  $\alpha$ . That is to say, the better you succeed in reducing the hole to a point, the stronger will be the scattering. If  $\Delta x = \lambda$ ,  $\sin \alpha = 1$ , and  $\alpha = 90^\circ$ : when the width shrinks to just one wavelength, the direction of the light moving behind the hole becomes *completely uncertain*: a photon may now fly on to any place, and light will be scattered in all directions. To avoid this, to hold a photon (so far as possible) in its track, you have to keep down the second factor of the product: but if so,  $\Delta x$  will go up.

We can now see why any attempt at producing such a ray is doomed to failure. Indeed, what is it we are trying to do? To send light through a point-like hole ( $\Delta x = 0$ ) and, at the same time, preserve its direction ( $\alpha = 0$ ). The fulfilment of these conditions, however, is incompatible with equation (2) according to which the product of the deviations must be finite. The impossibility of producing such a light ray is therefore due to the wave nature of light and not to any shortcomings of the experimenter. The best we can do, by way of approach, is to reduce both  $\Delta x$  and  $\alpha$ , for example, by putting  $\Delta x = \sin \alpha = \sqrt{\lambda}$ . Suppose the experiment is performed with ultraviolet light ( $\lambda = 10^{-5}$  cm), then equation (2) leads, roughly, to  $\Delta x = 3 \times 10^{-3}$  cm and  $\alpha = \frac{1}{5}$  degree: that is, to a beam 30 microns thick and of so little scattering that, if you 'aim' with it at a spot 1 metre away you may be 3 millimetres beside the mark. The shorter the wave-length, the better the approximation to the 'ideal' case of a ray without thickness. Whatever the technique used, the actual path will always be unsharp.<sup>1</sup>

To turn to a more general question, is it possible to observe a photon that has (a) a definite position (b) a definite momentum? Let us look once more at our experiment. If there were no such thing as diffraction, if photons were small hard pellets obeying Newton's laws of motion, all would be well. A photon would simply fly through the hole with unchanged velocity till it hit the wall behind. By making the hole smaller and smaller, we should succeed in forcing the photon to traverse an almost precise straight line. As it is, photons represent only one aspect of light, its wave

<sup>1</sup> That's why it is not possible to give a physical meaning to the phrase 'the precise path of a photon'.

nature being the other. Wave nature means diffraction, diffraction means change of momentum, and so departure from classical ideas is unavoidable.

To see how the two things, position and momentum, are related let us go back to our two equations. From (2) follows

$$\sin \alpha \approx \lambda / \Delta x;$$

if this is substituted in (1) we obtain

$$\Delta p \approx p \lambda / \Delta x$$

or

$$\Delta x \cdot \Delta p \approx p \lambda. \quad (3)$$

$p$ , the momentum of a photon, is by definition

$$p = h v / c. \quad (4)$$

As frequency  $v$  and wave-length  $\lambda$  stand in the simple relation

$$v \lambda = c \quad (c = \text{velocity of light})$$

we have

$$v = c / \lambda.$$

If this is written in the place of  $v$  in (4), we obtain

$$p = h c / \lambda c = h / \lambda;$$

substitution of this expression in (3) yields

$$\Delta x \cdot \Delta p \approx h. \quad (5)$$

That is the celebrated Heisenberg relation. What does it mean? It means that it is impossible to get hold of a photon which possesses (a) a sharply defined position (b) a sharply defined momentum. For in this 'ideal' case we should have  $\Delta x = 0$  and  $\Delta p = 0$ , and therefore  $\Delta x \cdot \Delta p = 0$ , in contradiction to (5).

As momentum is related to space as energy is to time (energy being the fourth component of the energy-momentum vector), the uncertainty relation can also be transferred to energy and time:

$$\Delta t \cdot \Delta E \approx h. \quad (6)$$

Generally speaking, two variables, a kinematic and a dynamic one, are always linked together by a law such that the accuracy in the one restricts the possible accuracy in the other.

One more point: a photon, for all we know, *may* describe a precise path; it's only when we try to *observe* it that the difficulty arises. As Heisenberg says, 'Natural science does not simply describe and explain nature; it is a part in the interplay between nature and ourselves; it describes nature as exposed to our method of questioning.'<sup>1</sup>

10. What has been said of photons, applies word for word to electrons, or for that matter to any material particles. For exactly the same sort of experiment can be carried out with particles. Suppose a shower of electrons falls on a thin crystal; the atoms in the crystal act as a lattice of diffracting centres. If electrons behave like waves, similar effects are to be expected. In fact, when the experiment was performed by Davisson and Germer (in 1927, only three years after de Broglie had introduced the idea of matter waves), light and dark rings appeared on a photographic plate where it had been hit by the electrons after their passage through the crystal. Just as in the case of the photons, we can reason: the blackening of the plate shows that electrons have impinged on it, the arrangement of the dark spots in the form of concentric rings, i.e. of a diffraction pattern, that the impinging electrons do not continue in their original direction before the crystal was placed in their way — as they ought to do if they were to obey Newton's first law of motion. Only a certain part of them do so; another part is deflected towards the first ring, a smaller percentage towards the second ring, and so on. In other words, some particles are not deflected at all, some are to a certain degree, some to a higher degree, and so on. Can this curious behaviour, perhaps, be explained by assuming that the electrons, as they fly past the atoms in the crystal, interact with them? To take the parallel case of the photons, can *their* behaviour be explained by supposing that they interact with the rim of the hole? What such interaction could, at most, explain is that those near the rim are deflected: but then we should expect a gradual falling off of the intensity of light on the far side of the screen, not periodic zones of light and darkness. The inevitable conclusion is that diffraction is due to the wave nature of light, and that photons do not obey Newton's laws. As with photons, so with material particles: they do not move according to classical laws — the phenomenon of diffraction rather points to a sort of wave pattern underlying their behaviour.

<sup>1</sup> *Loc. cit.*

Applying the same sort of mathematical analysis to atomic particles, we are led to the relation

$$\Delta x \cdot \Delta p \approx h,$$

which expresses the uncertainty principle for such particles. Although this result is obtained from an analysis of the processes involved in measuring, it is neither limited to the special circumstances of the experiment nor is its content a purely negative one. Indeed, the very fact that it sets a limit to the accuracy of any particle description and renders it incomplete points to what is complementary to it, a wave description. Both modes of description would be contradictory within the framework of classical concepts, while the conflict is evaded by what Bohr calls the complementarity principle. As this touches on an important point a few words may be added.

There is a trend to explain physical reality in terms of particles: this was the ideal of classical physics since the times of Descartes or even Democritus. Opposed to it is a trend to account for the phenomena in terms of waves only: that was Schrödinger's original conception to which he still seems attracted. However, the curious thing is that none of these descriptions exhausts the possibilities presented in experiment: each of them leaves out something that can only be supplied by the other. A complete description, or as this is impossible, a maximum description must take into account both aspects. 'By playing with both pictures (particle and wave), by going from one picture to the other and back again, we finally get the right impression of the strange kind of reality behind our atomic experiments.'<sup>1</sup> That this reality should be such that two different and mutually exclusive sets of concepts are needed to describe it is certainly a surprising and at the same time a very remarkable result. There is nothing absurd in the notion of a wave-particle duality: it appears so only when one attempts to combine both aspects into a single comprehensive picture. From the new vantage point classical physics must appear one-sided — treating matter as composed of particles and light as composed of waves, whereas each of them presents both aspects: only there is no conflict between these aspects since they never manifest themselves in the same experimental setting.

11. The uncertainty principle marks the final break with the

<sup>1</sup> W. Heisenberg, *loc. cit.*

past. It does not merely state that, though there may be causal laws governing atomic events, they are at present unknown: it flatly denies the existence of such laws. This is the Copenhagen interpretation (Bohr and Heisenberg), accepted today by the great majority of physicists. In view of such far-reaching consequences for our whole world picture, it is understandable that a wave of doubt should have been created by the principle when it was first formulated. Many were the attempts to get round it. To mention only one, Einstein, always a non-believer, suggested an ingenious thought experiment to outwit the uncertainty relations. In a discussion (at the Solvay Congress in Brussels, 1930) at which Bohr was present, he pointed out a possible way to determine time and change in energy of an atomic event without any uncertainty. Consider, he said, a box capable of holding radiant energy (e.g. lined with perfect mirrors). Weigh the box. Now release one photon from it (by means of a clockwork inside the box that works a shutter), at a moment fixed with as much precision as you want. Weigh the box again. The change in mass tells you the energy lost, according to the formula  $E=mc^2$ . In this way, concluded Einstein, one could, in principle at least, measure the energy of the photon emitted and the time of its escape with any desired degree of accuracy, in contradiction to the Heisenberg relation. Bohr spent a sleepless night over the argument. The next morning, however, submitting the conditions of the experiment to a searching analysis, he was able to clear up the discrepancy. It would lead us too far to go into details here. Only so much may perhaps be said: Einstein had failed to apply his own theory of general relativity to the case in point. The box, hung e.g. on a spring-balance, moves in recoil from the escaping photon, upwards or downwards as the case may be; its position is changed in the gravitational field of the earth, and this, according to general relativity, changes the rate of the clock rigidly connected with the box: in the case of a downward movement, the rate will be accelerated, in the opposite case decelerated. This gives rise to an uncertainty in regard to the exact time of the photon's flight. On the other hand, there will also be an uncertainty in measuring the weight of the box, and therefore the change in energy. Calculation shows that the two uncertainties, referring to time and energy, stand in exactly the relation as is demanded by the uncertainty principle. Einstein was defeated, not convinced. And if a man of his imagination did not succeed,

who will? There seems to be no escape from the uncertainty principle.<sup>1</sup>

12. Let us pause for a moment and see where we have got to. We had to give up many things thought indispensable for a rational explanation — the postulate of continuity of description and consequently a tracing of the motion of atomic particles, thus foregoing to connect their past and future behaviour; next, a sharp distinction being made between the phenomena and the means chosen for their observation, based on the impossibility of getting to know their detailed interaction; further, permanence, individual existence and recognizability of such particles along with ascribing well-defined states to them, i.e. renouncing the demands for visualization by the use of space-time pictures; and finally, a causal understanding of the phenomena. We had to give up so much: what do we stand to gain?

With regard to the experiment with photons which pass through a hole, it was pointed out that any attempt at tracing their course in detail must end in failure, with the consequence that the behaviour of a single photon is unpredictable. The same goes for electrons, protons, . . . Are we, then, to give up physical science altogether as a bad job? Far from it. Though it is perfectly true that we cannot follow the behaviour of a single particle on its journey in space and time, we may try something else. Suppose, for example, we shoot an electron from a sort of gun, aiming always in the same direction, say, at a spot on a photographic plate where its impact is recorded. Let us repeat the experiment a large number of times, it will then be found on inspection that the points of impact lie scattered about — a fact which defies the time-honoured rule ‘same cause, same effect’. Indeed, if exactly the same experiment is repeated (supposing this is possible), the result will each time be different, showing fluctuations. My present point, however, is this: although each electron hits the target in a different place, these places are arranged in an orderly way, within concentric circles forming a perfectly regular pattern. *Order is thus born out of randomness.* For the result can be explained by saying: light rings appear where many electrons impinge on the plate, dark ones where none or only few fall. What cannot be predicted of one individual electron, namely where it will arrive, *can* be predicted

<sup>1</sup> The reader will find many other examples discussed in Bohr's Gifford Lectures.

of a large number of them, and very accurately. If we, then, cease to care for the fate of a single particle and turn instead to that of a crowd we regain the apparently lost faculty to formulate laws to predict the future. Suppose that a million electrons are sent in the same direction, quantum mechanics allows us to foretell how many of them are likely to hit the centre, what fraction to fall on the first ring, the second ring, and so on; predictions the more accurately confirmed, the larger the crowd.

What emerges is a *statistical* theory unconcerned with individuals and taking account only of large assemblages. Its structure is such that statements derived from it, when translated into terms of observation, describe the average behaviour, or the probability with which an electron will be found at a certain place: where it will in fact be, thereof the laws keep silent.<sup>1</sup>

In Laplace's view, nature is predetermined, a gigantic clock-work, each atom describing a path according to unswerving mechanical necessity. In the modern view, the universe may still be a machine — a misleading metaphor — a machine, however, more in the nature of a roulette wheel than of a clock. The behaviour of a wheel, though unpredictable in detail, on the average exhibits a marked regularity. That is the aspect stressed in the study of quantum phenomena. While with Laplace chance was nothing but the name of ignorance, it now forms a fundamental and irreducible element in any description of nature.

Let me illustrate this with an example. If we have before us, say, a milligram of radium comprising a vast number of atoms, within 1,600 years half of it will have disintegrated. Supposing now it were possible to pick out one single atom, no one can tell whether it will erupt within the next 10 seconds or go on existing in its present state for the next 10,000 years. There is absolutely no clue to foretell what will happen. And that is not due to our human ignorance: it is *objectively uncertain* when the atom will disintegrate. Natural law sets down merely statistical decrees. Like in the case of men and women in London over the age of 60 who are going to die within the next year, only laws of a statistical kind can be formulated. (Are we, then, to look upon nature from the viewpoint of an Insurance Company? Not exactly: for while the

<sup>1</sup> This view — that underlying the regularity we observe there is complete randomness on the atomic scale — was first expounded by F. Exner, an experimental physicist in Vienna, in 1919.

question as to why a single atom behaves in the way it does is unanswerable, the parallel question with regard to a human being permits of a wide range of answers — one of the reasons why physics cannot and should not serve as a model for the social sciences.)

After reading this account you may be inclined to say: surely there must be some difference in the state of two such atoms; since the one is just on the point of undergoing disintegration while the other is not, they cannot be alike. So you turn the Law of Causation into a definition: you *choose* to call two states 'the same' if they are followed by the same consequents, while in the contrary case they are the same only in appearance, not in reality. Very well; but this distinction is of use only under one condition — that it can be substantiated by really finding out what the difference in question is. Otherwise it is an empty verbal distinction. Now one of the queer things brought to light by atomic physics is that two atoms may be wholly and exactly alike — I don't mean approximately alike as two eggs or two raindrops but alike as two whole numbers, i.e. absolutely indistinguishable. And this bars the way to turning the Law of Causation into a mere definition. The fact remains that of two atoms in all respects perfectly alike the one is doomed, the other is not. All one can say is that an atom erupts whenever it likes to: and if it does, this is, quite literally, an *uncaused* event.

But may it not be that there are still some undiscovered laws which, if taken into account, would permit us to foretell the precise instant of eruption? Surely we are not omniscient? Such an assumption can be disproved by a number of arguments. To mention only one: it is known that the nucleus is surrounded by a 'mountain of potential' sloping down at the inside, and an  $\alpha$ -particle is imprisoned within this 'crater'. To get out of it, it must have sufficient energy to climb over the mountain wall. However, it is found that its energy is scarcely half that value. According to classical laws, therefore, it could never escape. Quantum theory, on the other hand, provides us with means to calculate with what probability such a particle, considered as a wave, may escape, even if its energy is too small to surmount the wall — a result well-confirmed by experiment.<sup>1</sup> Now if there were deterministic laws

<sup>1</sup> This theory was developed by the Russian physicist G. Gamow, who lives in the U.S., and independently by Condon and Gurney.

permitting us to make precise predictions, an  $\alpha$ -particle could not also be regarded as a wave ('representing probability') and these laws could not agree so closely with the results obtained from probability considerations. In addition, there are other reasons equally strong to dispose of such a possibility. When all is said and done, radium disintegration goes on spontaneously undetermined by causes — a reminder of how remote we here are from the world of ordinary experience.

To sum up the case of causality: in the world as we perceive it there is not only no sufficient evidence in support of it, but, what is worse, there cannot be any. For what appears on this level as the causal order of the world can always be regarded as the result of a huge number of elementary processes which come into play here, and their levelling influence — irrespective of the question whether the 'true laws', those governing the behaviour of photons, electrons, . . . are, or are not, of a causal type. Each particle may behave in a haphazard way, straying hither and thither as chance would have it, and yet, owing to the enormous number involved, the irregularities will in the end be smoothed out, presenting a perfectly regular picture. The common experience 'same cause, same effect', so often taken for the principle of causality, has in fact little to do with the deeper problem. Considering that two states which go under the name of 'the same cause' are, from a strict point of view, alike only in appearance, while the true states, determined by the particles involved, their arrangement in space and state of motion, hardly ever really agree, one comes to see how little weight such an observation carries. Indeed, any large-scale observation informs us only of the mean value of the quantities involved while leaving the details undetermined.

To add one point as regards the inaccuracy to be expected in any such law — if we check a statement, say, about Brownian movement and observe under the microscope just  $n$  granules we shall find it inaccurate within a margin of  $\sqrt{n}$ . That is to say, if we meet with 100 particles we must be prepared to find fluctuations about the value predicted by theory of about 10 — which is 10% of the whole. If the number is much larger, say,  $10^8$  the departure to be expected will be of the order  $10^4$ , that is one hundredth of 1%. This illustrates how the degree of accuracy tends to increase with the total number of grains of pollen, molecules or atoms by whose interplay the effect is brought about. To give an idea of the order

of number involved in a large-scale observation, 32 grams of oxygen contain  $6 \times 10^{23}$  O<sub>2</sub> molecules representing a mechanical system with  $2 \times 3 \times 6 \times 10^{23} = 36 \times 10^{23}$  free parameters (neglecting the inner structure of the O-atoms). A causal analysis of the behaviour of such a system requires twice as many data, in contrast to gas theory that makes use only of two, pressure and temperature. On account of this, the latter is only capable of making probability assertions to be tested by means of statistical methods. That these assertions are *almost* causal, i.e. have probabilities very close to 0 or 1, does in no way change the situation. As in any large-scale observation  $10^{24}$  or more molecules are involved there is extremely little departure from, and hence extremely high accuracy of, any law empirically found and tested. But no law is absolutely exact. For this reason, recourse to ordinary experience will never be able to prove the existence of strictly causal laws.

It is only when we descend to the atomic level that the question of causality can be put to the test; and here all the facts speak decidedly against it. The only theory known at present capable of connecting and unifying an enormously wide range of phenomena, quantum theory, is in sharp logical contradiction with it. It goes without saying that this theory, on account of the openness of experience, may have to be revised in the future: will it perhaps be so revised as to lead back to determinism? If anyone cherishes such hopes I am afraid I will disappoint him. For according to a theorem proved by von Neumann<sup>1</sup> such wishes are unfulfillable. What von Neumann has proved is this: given quantum mechanics in its present shape, it is not possible to modify, complete or extend it — say, by introducing hidden parameters — such as to transform it into a deterministic theory; for any such extension would render the theory so modified self-contradictory. Hence only two courses are open to us — either to abolish quantum theory in its entirety and start afresh from scratch, or retain the present foundation while radically renouncing a return to determinism. But there is no room for tinkering with the theory. In view of its undeniable great successes, it is difficult to believe that it should be entirely false. And if it is on the right track causality does not stand a chance.

As for the renunciation of deterministic ideals, let me remind

<sup>1</sup> *Mathematical Foundation of Quantum Mechanics.*

you of some precedents in the history of science. While Kepler laboured all his life to find the key to the riddle of the universe — why the planets are arranged at just the distances from the sun as they are — Newton's celestial mechanics not only left the question unanswered but eliminated it from science. Again, it should be remembered that classical mechanics was achieved at the price of foregoing the search for a cause of uniform motion. Until the time of Galileo it was believed that a body could move with constant speed only if it was pushed by force; the turning point came when it was realized that uniform motion, like rest, is a state that demands no explanation. Similarly, the physicists succeeding Maxwell, after many unsuccessful attempts to find a mechanical substructure for his theory, had to resign themselves to the fact that the field concepts can not be further reduced to mechanical terms.<sup>1</sup> Today, it seems, we are passing through a similar period of readjustment of thought as that in which Galileo was laying the groundwork of modern science. In particular, just as people at that time had to accustom themselves to the idea that uniform motion *has no cause* — which, at least at the time, entailed a certain renunciation as regards the demands for a causal understanding — so today we have to shift to a position from which things emerge in a new perspective. Modern physics has led to the insight that causal analysis is, after all, only *one* way of understanding nature alongside of which e.g. invariance, equivalence, symmetry, but also complementarity, take their place as categories of rational explanation. I mean, just as Kepler's search for a fourth law, in the light of our present knowledge, was illusory and had to be re-interpreted as a historical question concerning the evolution of the planetary system, so the demand for a 'direct representation of physical reality in space and time' may well prove just as ill-starred — with all due respect for Einstein.

13. It is hardly to be expected that the revolutionary change in our concepts thus inaugurated will remain confined to the study of quantum phenomena. For one thing, the notions of space and time

<sup>1</sup> First expressed by H. Hertz. In contrast with this, Lord Kelvin avowed in 1884: 'I am never content until I have constructed a mechanical model of the object I am studying. If I succeed in making one, I understand; otherwise I do not. Hence I cannot grasp the electromagnetic theory of light. I wish to understand light as fully as possible, without introducing things that I understand still less.' The mood of renunciation is clearly perceptible in such utterances.

will probably have to go into the melting pot. For the uncertainty relations reveal an unsuspected connection between geometrical and dynamic properties of elementary particles, i.e. between momentum and energy on the one hand and the possibility of locating them in the frame of space and time on the other. Our intuitive ideas, appropriate as they are for ordinary experience, cannot, it seems, be transferred to the atomic realm. The continuity of space and time stands indeed in strange contrast to the discontinuity of matter, energy and action which has become so prominent a feature of our picture of the world.

Even logic, to all appearance *a priori par excellence*, is not unaffected by the general drift, and new ideas based on Bohr's principle of complementarity begin to take shape.<sup>1</sup> To conclude with a few words on this situation. Suppose that the momentum of an electron has been determined with a high degree of accuracy, then its location will be indeterminate. Suppose, however, that someone none the less asserts that the particle is, at that instant, in a certain place — is his assertion true or false? If it were true this would mean that the electron can certainly be found in the place specified, if false that it cannot possibly be found there. To insist that it must be either true or false, even if it may be impossible for us to decide the issue only brings us into conflict with the Heisenberg relation (complete knowledge of momentum entails total ignorance as to location). On the other hand, to say that the assertion is neither true nor false is incompatible with the law of excluded middle. So what?

What we have before us is a conflict between logic and physics. If logic is right quantum theory must be wrong, and if quantum theory is right (in particular the uncertainty relation), then classical logic must be wrong (in particular the law of excluded middle). According to the Copenhagen interpretation, the assertion is undecidable, neither true nor false. Let us consider once more what the law of excluded middle really boils down to. Guided by it, we should have to say that only one alternative holds — that our particle is here, in this place, or is not here, namely somewhere else; and that this is true on purely logical grounds. If so, we are

<sup>1</sup> G. Birkhoff and J. v. Neumann, 'The Logic of Quantum Mechanics' (*Annals of Mathematics* 37 (1936)); C. F. v. Weizsäcker, 'Komplementärität und Logik' (*Die Naturwissenschaften*, 1955). Even Heisenberg has given his blessing to this enterprise.

constrained to ascribe a definite position to the particle, whereas quantum theory says that this whole idea must be discarded. That is, roughly, the clash between logic and physics.

It wouldn't be surprising if someone, turning over the situation in his mind, were to say, 'Even if there is no possible experiment to decide the issue the assertion taken in itself must be true or false. It certainly is understandable and hence meaningful: if it is neither true nor false what *can* it be?' In such an argument, however, due regard is not paid to the fact that an assertion concerning the electron's location has physical meaning only insofar as it is connected with a certain experimental arrangement. But the point is that, once the momentum is measured in experiment, this *destroys* the possibility of making, at the same time, another arrangement for determining the location and thus does away with the very precondition under which the assertion makes sense. Indeed, one gets into hopeless difficulties when one ascribes properties to particles independent of the means of observation. To put it in another way, the trouble with insisting that the assertion 'must be true or false, only we don't know which' is that it creates the impression that the electron *did* occupy a quite definite place, unfortunately one not determinable in experiment — whereas what one really ought to have said is that the whole idea of describing an electron in particle terms breaks down here. In saying, 'But a particle must be *somewhere*' one is overstepping the bounds laid down by the complementarity principle, i.e. one is trying to stretch one aspect beyond the limits of its application to the exclusion of the other. To uphold the law of excluded middle is, in this case, tantamount to disregarding the wave aspect, while the real point is the inadequacy of any one mode of description which at best tells only half the story.

14. To mention just one feature of the sort of logic — some call it 'complementarity logic'<sup>1</sup> — that is emerging from the discussion, in atomic theory we meet with statements which *exclude* one another, such as a description of momentum and of position — 'complementary' statements as we may call them. What is characteristic of them is that there is an embargo against using them in the same context. This feature is not an entirely new one, for it has a parallel in the case of classical logic: giving the content of a proposition  $p$ , we can do two things with it, assert it or deny

<sup>1</sup> So von Weizsäcker, *loc. cit.*

it; though the result of each operation makes perfectly good sense, this no longer holds good of their conjunction. Any two such propositions behave like complementary statements in quantum theory. It is this feature of ordinary logic which is generalized in complementarity logic. Given a statement  $s$ , there will be other statements (in general infinitely many)

$$s_1^*, s_2^*, s_3^*, \dots$$

which are complementary to the given one. (For example,  $s$  being the statement of the momentum of a particle, complementary statements will be: that the particle is at a distance of 1 cm, 2 cm, 3 cm, ..., say, from the zero point of a certain scale.) To a given statement  $s$  there is, then, a set of statements complementary to it such that, when  $s$  is decided in experiment (is true or false), any member  $s^*$  of that set is *undecided* (neither true nor false). And just as in ordinary logic the joining of  $p$  and not  $p$  is taboo, so is in the new logic the combining of  $s$  and  $s^*$  into a conjunction. But while in ordinary logic it is only in exceptional cases that the conjunction of two meaningful statements is meaningless, in the new logic that holds to a much wider extent. In this due recognition is paid to the peculiar logical situation encountered before in connection with the law of excluded middle. Indeed, classical logic is tied to the presupposition that alternatives, at least in principle, are decidable and that statements — so far as they assert or deny something — can be divided into the two classes ‘true’ and ‘false’. Once the assumption underlying this dichotomy is shaken it will be difficult to keep up that law. But that is exactly the situation that confronts us in quantum theory. For here it is a *law of nature* which, given the precise momentum, prevents us from deciding issues as to the location: undecidability is a direct consequence of the uncertainty relation. It is of course perfectly true that in describing experimental evidence we have to make use of plain language and common logic. Our present concern, however, is in the logical relations within that theory, and here ordinary language can no longer serve as guide.

To mention another difference, while in ordinary logic the statements  $s$  and ‘ $s$  is true’ (or not  $s$  and ‘ $s$  is false’) are equivalent, i.e. have always the same truth-value, in the new logic they are not. Indeed the truth or falsity of  $s$  entails the truth or falsity of ‘ $s$  is true’ but not conversely. For if the assertion ‘ $s$  is true’ is false it

does not follow that  $s$  is false — it may be undecided. Supposing, on the other hand, that the foregoing assertion is true, then  $s$  is true. For this reason the classical equivalences hold good only for the case of truth, not of falsity, thus revealing a curious asymmetry between these concepts. It is because  $s$  and ‘ $s$  is true’ are on a different logical level and therefore not translations of one another that Frege’s method of defining connectives like ‘and’, ‘or’, ‘if’ in terms of truth-tables is no longer usable in the new field. It would lead us too far to explain how these terms can be defined. Only so much can be said: in the system of Birkhoff and von Neumann the formula

$$a \vee \bar{a}$$

is always valid. Considering, however, that the symbols which occur in it are defined in a different way and take on a different sense, it will be seen that it is no longer the expression of the law of excluded middle. The difference comes out when it is noticed that, while  $a \vee \bar{a}$  is always true, ‘ $a$  is true or  $\bar{a}$  is true’ is not: in this sense the classical law breaks down. What emerges is a non-truth-functional (and hence non-tautologous) logic in which all the classical rules are formally preserved except the two distributive laws. However, the interpretation given to the formalism has to be changed so that, in fact, we have a new logic embodying new ways of thought.

The philosophical lesson to be drawn from this development is that even logic is not exempt from the pressure of facts. Needless to say that logic does not describe the real world and can therefore not be refuted: but it may be well- or not well-adapted to prevailing conditions. What underlies ordinary logic, the dichotomy of statements, no doubt reflects the character of certain deep-seated features of our world, in particular the fact that issues, on the whole, are decidable.<sup>1</sup> However, on closer scrutiny of these principles, including their application in the realm of science, their limitation became manifest. Brought face to face with the sort of facts revealed in the study of quantum phenomena, these principles prove ill-adapted and have to be modified. The account given is in contrast with views commonly held on logic, according to which

<sup>1</sup> Not always: counter-factuals (‘If Hitler had been liquidated at the time before Munich, then etc.’) are as a rule undecidable, neither true nor false, at best plausible.

it has nothing to fear from discoveries regarding the actual world. The underlying idea seems to be that physics, being based on experiment, may any day be overthrown by new facts coming to light, while logic, being of infinite hardness, cannot — so that, in the case of a collision, physics will get all the bumps. To use Brouwer's words, 'Admittedly, the application of the logical principles to natural phenomena sometimes gave the wrong results, but in that case the blame was always laid at the door of the axioms of the conceptual system, never at the door of logic.'<sup>1</sup> Quantum physics, I submit, presents a strong case against traditional logic, and belief in it as the only possible one has become a form of provincialism.

Lastly, what about the assertion — put forward, for instance, by the distinguished physicist P. Jordan<sup>2</sup> — that the gaps in the causal chains open the way to moral freedom? I think it is a mistake; but time does not permit me to go into the question.

<sup>1</sup> 'Mathematik, Wissenschaft und Sprache' (*Monatshefte f. Math. u. Phys.* 1929).

<sup>2</sup> *Science and the Course of History*, New Haven, 1955.

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