Generate random points and place in a list (or array) by assigning index:

Index	x-coordinate	y-coordinate
0	0	0
1	3.19	7.56
2	6.92	5.42
3	9.47	3.33
4	9.98	5.86
5	7.86	10.00
6	-1.40	3.33
7	-4.08	7.12
8	-7.00	4.57
9	-8.95	3.33
10	-8.12	8.26
11	-2.00	-3.81
12	-9.93	-2.89
13	-2.72	-6.39
14	-5.85	-7.16
15	-10.80	-7.16
16	4.60	-1.03
17	5.72	-3.7
18	10.04	-4.57
19	3.69	-7.63
20	8.62	-7.63

Calculate distance from origin (P_0) to find the closest point:

Index	x-coordinate	y-coordinate	Distance from origin
1	3.19	7.56	8.21
2	6.92	5.42	8.79
3	9.47	3.33	10.04
4	9.98	5.86	11.57
5	7.86	10.00	12.72
6	-1.40	3.33	3.61
7	-4.08	7.12	8.21
8	-7.00	4.57	8.36
9	-8.95	3.33	9.55
10	-8.12	8.26	11.58
11	-2.00	-3.81	4.30
12	-9.93	-2.89	10.34
13	-2.72	-6.39	6.95
14	-5.85	-7.16	7.66
15	-10.80	-7.16	12.96
16	4.60	-1.03	4.71
17	5.72	-3.7	6.81
18	10.04	-4.57	11.03
19	3.69	-7.63	8.47
20	8.62	-7.63	11.52

Point with index number 6 (P_6) is closest to origin (0^{th} index point)

Find unit vector $\mathbf{n}_{0/6}$ from point $\mathbf{P}_6$ to origin ($\mathbf{P}_0$):

$$\mathbf{n}_{0/6} = \frac{\mathbf{P}_0 - \mathbf{P}_6}{|\mathbf{P}_0 - \mathbf{P}_6|} = \frac{[0.00, 0.00] - [-1.40, 3.33]}{\sqrt{(0.00 - (-1.40))^2 + (0.00 - 3.33)^2}} = [0.39, -0.92]$$

Calculate unit vectors from point P_6 to points P_1-P_5 and P_7-P_{20} and dot products:

Example calculation between $\mathbf{P_6}$ and $\mathbf{P_1}$:

$$\mathbf{n}_{1/6} = \frac{\mathbf{P}_1 - \mathbf{P}_6}{|\mathbf{P}_1 - \mathbf{P}_6|} = \frac{[3.19, 7.56] - [-1.40, 3.33]}{\sqrt{(3.19 - (-1.40))^2 + (7.56 - 3.33)^2}} = [0.74, 0.68]$$

$$\mathbf{n}_{0/6} \cdot \mathbf{n}_{1/6} = \cos(\theta) = -0.337$$

because $\mathbf{n}_{0/6} \cdot \mathbf{n}_{1/6}$ is negative, point $\mathbf{P}_1$ will be removed.

Unit vectors with respect to P_6 and dot products:

Index	x-coordinate	y-coordinate	Unit vector	Dot product
1	3.19	7.56	[0.735, 0.678]	-0.337
2	6.92	5.42	[0.970, 0.244]	+0.154
3	9.47	3.33	[1.000, 0.000]	+0.390
4	9.98	5.86	[0.976, 0.217]	+0.181
5	7.86	10.00	[0.811, 0.584]	-0.221
7	-4.08	7.12	[-0.577, 0.816]	-0.976
8	-7.00	4.57	[-0.976, 0.216]	-0.580
9	-8.95	3.33	[-1.000, 0.000]	-0.390
10	-8.12	8.26	[-0.806, 0.591]	-0.859
11	-2.00	-3.81	[-0.084, -0.996]	+0.884
12	-9.93	-2.89	[-0.808, -0.589]	+0.227
13	-2.72	-6.39	[-0.134, -0.991]	+0.859
14	-5.85	-7.16	[-0.390, -0.920]	+0.695
15	-10.80	-7.16	[-0.667, -0.745]	+0.425
16	4.60	-1.03	[0.809, -0.588]	+0.856
17	5.72	-3.7	[0.711, -0.702]	+0.924
18	10.04	-4.57	[0.823, -0.568]	+0.844
19	3.69	-7.63	[0.421, -0.907]	+0.999
20	8.62	-7.63	[0.675, -0.738]	+0.942

Remove the points which result in negative dot product.

Remaining points:

Index	x-coordinate	y-coordinate	Distance from origin
2	6.92	5.42	8.79
3	9.47	3.33	10.04
4	9.98	5.86	11.57
11	-2.00	-3.81	4.30
12	-9.93	-2.89	10.34
13	-2.72	-6.39	6.95
14	-5.85	-7.16	7.66
15	-10.80	-7.16	12.96
16	4.60	-1.03	4.71
17	5.72	-3.7	6.81
18	10.04	-4.57	11.03
19	3.69	-7.63	8.47
20	8.62	-7.63	11.52

Point with index number 11 (P_{11}) is closest to origin (0^{th} index point)

Find unit vector $\mathbf{n}_{0/11}$ from point $\mathbf{P}_{11}$ to origin ($\mathbf{P}_0$):

$$\mathbf{n}_{0/11} = \frac{\mathbf{P}_0 - \mathbf{P}_{11}}{|\mathbf{P}_0 - \mathbf{P}_{11}|} = \frac{[0.00, 0.00] - [-2.00, -3.81]}{\sqrt{(0.00 - (-2.00))^2 + (0.00 - (-3.81))^2}} = [0.465, 0.886]$$

Calculate unit vectors from point P_{11} to points P_2-P_4 and P_{12}-P_{20} and dot products:

Example calculation between $\mathbf{P}_{11}$ and $\mathbf{P}_{12}$:

$$\mathbf{n}_{12/11} = \frac{\mathbf{P}_{12} - \mathbf{P}_{11}}{|\mathbf{P}_{12} - \mathbf{P}_{11}|} = \frac{[-9.93, -2.89] - [-2.00, -3.81]}{\sqrt{(-9.93 - (-2.00))^2 + (-2.89 - (-3.81))^2}} = [-0.993, 0.115]$$

$$\mathbf{n}_{0/11} \cdot \mathbf{n}_{12/11} = \cos(\theta) = -0.360$$

because $\mathbf{n}_{0/11} \cdot \mathbf{n}_{12/11}$ is negative, point $\mathbf{P}_{12}$ will be removed.

Unit vectors with respect to P_{11} and dot products:

Index	x-coordinate	y-coordinate	Unit vector	Dot product
2	6.92	5.42	[0.695, 0.719]	+0.960
3	9.47	3.33	[0.849, 0.528]	+0.863
4	9.98	5.86	[0.778, 0.628]	+0.918
12	-9.93	-2.89	[-0.993, 0.115]	-0.360
13	-2.72	-6.39	[-0.269, -0.963]	-0.978
14	-5.85	-7.16	[-0.754, -0.656]	-0.932
15	-10.80	-7.16	[-0.934, -0.356]	-0.750
16	4.60	-1.03	[0.921, 0.388]	+0.772
17	5.72	-3.7	[1.000 0.014]	+0.478
18	10.04	-4.57	[0.998, -0.063]	+0.408
19	3.69	-7.63	[0.830, -0.557]	-0.108
20	8.62	-7.63	[0.941, -0.338]	0.138

Remove the points which result in negative dot product.

Remaining points:

Index	x-coordinate	y-coordinate	Distance from origin
2	6.92	5.42	8.79
3	9.47	3.33	10.04
4	9.98	5.86	11.57
16	4.60	-1.03	4.71
17	5.72	-3.7	6.81
18	10.04	-4.57	11.03
20	8.62	-7.63	11.52

Point with index number 16 (P_{11}) is closest to origin (0^{th} index point)

Find unit vector $\mathbf{n_{0/11}}$ from point $\mathbf{P_{16}}$ to origin ($\mathbf{P_0}$):

$$\mathbf{n}_{0/16} = \frac{\mathrm{P}_0 - \mathrm{P}_{16}}{|\mathrm{P}_0 - \mathrm{P}_{16}|} = \frac{[0.00, 0.00] - [4.60, -1.03]}{\sqrt{(0.00 - 4.60)^2 + (0.00 - (-1.03))^2}} = [-0.977, 0.219]$$

Example calculation between **P_{16}** and **P_2**:

$$\mathbf{n}_{2/16} = \frac{\mathrm{P}_2 - \mathrm{P}_{16}}{|\mathrm{P}_2 - \mathrm{P}_{16}|} = \frac{[6.92, -5.42] - [4.60, -1.03]}{\sqrt{(6.92 - 4.6)^2 + (5.42 - (-1.03))^2}} = [0.338, 0.941]$$

$$\mathbf{n}_{0/16} \cdot \mathbf{n}_{2/16} = \cos(\theta) = -0.173$$

because $\mathbf{n}_{0/16} \cdot \mathbf{n}_{2/16}$ is negative, point P_2 will be removed.

Unit vectors with respect to P_{16} and dot products:

Index	x-coordinate	y-coordinate	Unit vector	Dot product
2	6.92	5.42	[0.338, 0.941]	-0.173
3	9.47	3.33	[0.745, 0.667]	-0.612
4	9.98	5.86	[0.615, 0.788]	-0.464
17	5.72	-3.7	[0.389, -0.922]	-0.352
18	10.04	-4.57	[0.838, -0.545]	-1.000
20	8.62	-7.63	[0.520, -0.854]	-0.6516

Remove the points which result in negative dot product.

Since all points are removed, P_5, P_{11} and P_{16} will be used to find the Voronoi cell associated with origin (P_0)

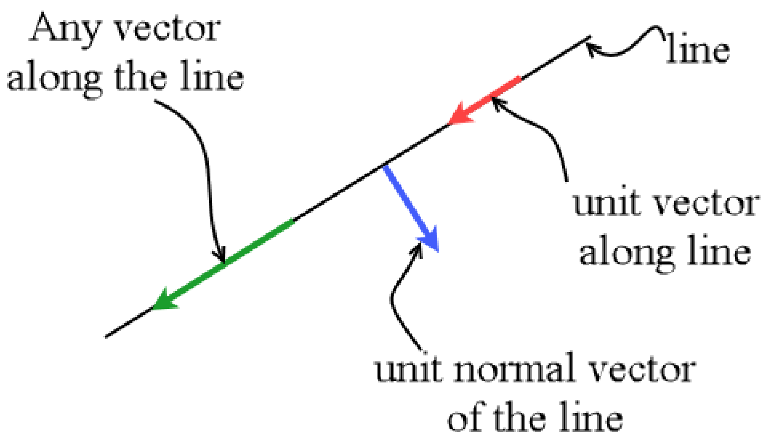
Let's find coordinate of the midpoints:

$$\mathrm{mP}_{6-0} = \frac{\mathrm{P}_6 + \mathrm{P}_0}{2} = [-0.700, 1.665]$$

$$\mathrm{mP}_{11-0} = \frac{\mathrm{P}_{11} + \mathrm{P}_0}{2} = [-1.000, -1.905]$$

$$\mathrm{mP}_{16-0} = \frac{\mathrm{P}_{16} + \mathrm{P}_0}{2} = [2.300, -0.515]$$

Finding the unit vector of a line from its normal vector:



Normal vector **n** will be perpendicular to any vector **V** along the line:

$$\mathbf{V} \cdot \mathbf{n} = V_x \cdot n_x + V_y \cdot n_y = 0$$

select $V_x = 1$ and find V_y: $V_y = -\dfrac{n_x}{n_y}$

calculate unit vector $\mathbf{m}$ along line: $m_x = \dfrac{1}{\sqrt{1^2 + V_y^2}}$ and $m_y = \dfrac{V_y}{\sqrt{1^2 + V_y^2}}$

Parametric equation of a line:

$$l = \mathbf{P} + \lambda \mathbf{m} \quad -\infty < \lambda < \infty$$

where $\mathbf{P}$ is any point on the line

Cyclic ordering of lines passing through mP_{0-6}, mP_{0-11} and mP_{0-15}:

Line Index	Passing through	Unit normal
L1	**mP_{0-6}**=[-0.700,1.665]	**$n_{0/6}$**=[0.39,-0.92]
L2	**mP_{0-11}**=[-1.000,-1.905]	**$n_{0/11}$**=[0.465,0.886]
L3	**mP_{0-16}**=[2.300,-0.515]	**n_{016}**=[-0.977,0.219]

Select line with index **L1** as the first line and calculate angles between remaining lines:

Angle between line **L1** and **L2**:

$$\cos\theta = \mathbf{n}_{0/6} \cdot \mathbf{n}_{0/11} = -0.634$$

$$\sin\theta\mathbf{k} = \mathbf{n}_{0/6} \times \mathbf{n}_{0/11} = 0.773\mathbf{k}$$

$\mathbf{k}$ is the unit vector along z-axis

$$\theta = \arctan 2\left(\frac{0.773}{-0.634}\right) = 129.358^\circ$$

Angle between line **L1** and **L3**:

$$\cos\theta = \mathbf{n}_{0/6} \cdot \mathbf{n}_{0/16} = -0.582$$

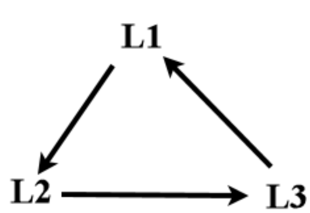
$$\sin\theta\mathbf{k} = \mathbf{n}_{0/6} \times \mathbf{n}_{0/16} = -0.848\mathbf{k}$$

$\mathbf{k}$ is the unit vector along z-axis

$$\theta = \arctan 2\left(\frac{-0.848}{-0.582}\right) = -145.554^\circ$$

Since θ is negative, add 360°: $\theta = 214.446^\circ$

Cyclic order of lines:



Unit vectors along lines:

Line **L1:**

$$V_{L_1}^x = 1 \text{ and } V_y = -\frac{n_{0/6}^x}{n_{0/6}^y} = 0.424$$

$$m_{L_1}^x = \frac{1}{\sqrt{1^2 + V_y^2}} = 0.921 \text{ and } m_{L_1}^y = \frac{V_y}{\sqrt{1^2 + V_y^2}} = 0.390$$

$$\mathbf{m}_{L_1} = [0.921, 0.39]$$

Line **L2:**

$$V_{L_2}^x = 1 \text{ and } V_y = -\frac{n_{0/11}^x}{n_{0/11}^y} = -0.525$$

$$m_{L_2}^x = \frac{1}{\sqrt{1^2 + V_y^2}} = 0.885 \text{ and } m_{L_2}^y = \frac{V_y}{\sqrt{1^2 + V_y^2}} = -0.465$$

$$\mathbf{m}_{L_2} = [0.885, -0.465]$$

Line **L3:**

$$V_{L_3}^x = 1 \text{ and } V_y = -\frac{n_{0/16}^x}{n_{0/16}^y} = 4.461$$

$$m_{L_3}^x = \frac{1}{\sqrt{1^2 + V_y^2}} = 0.219 \text{ and } m_{L_3}^y = \frac{V_y}{\sqrt{1^2 + V_y^2}} = 0.976$$

$$\mathbf{m}_{L_3} = [0.219, 0.976]$$

Calculate intersection points of lines:

Intersection of lines L1 and L2:

Parametric equation of line $\mathbf{L1} \Rightarrow \mathbf{L1} = \mathbf{mP}_{6-0} + \lambda_1 \mathbf{m}_{L_1}$

Parametric equation of line $\mathbf{L2} \Rightarrow \mathbf{L2} = \mathbf{mP}_{11-0} + \lambda_2 \mathbf{m}_{L_2}$

$$\mathbf{L1} = \mathbf{L2} \Rightarrow \begin{bmatrix} m^x_{L_1} & -m^x_{L_2} \\ m^y_{L_1} & -m^y_{L_2} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} mP^x_{11-0} - mP^x_{6-0} \\ mP^y_{11-0} - mP^y_{6-0} \end{bmatrix} \Rightarrow \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} -4.265 \\ -4.100 \end{bmatrix}$$

x-coordinate of intersection between $\mathbf{L1}$ and $\mathbf{L2}$:

$$L1^x = mP^x_{6-0} + \lambda_1 \cdot m^x_{L_1} = -4.6285$$

$$L2^x = mP^x_{11-0} + \lambda_2 \cdot m^x_{L_2} = -4.6285$$

y-coordinate of intersection between $\mathbf{L1}$ and $\mathbf{L2}$:

$$L1^y = mP^y_{6-0} + \lambda_1 \cdot m^y_{L_1} = 0.0015$$

$$L2^y = mP^y_{11-0} + \lambda_2 \cdot m^y_{L_2} = 0.0015$$

Intersection of lines L2 and L3:

Parametric equation of line $\mathbf{L2} \Rightarrow \mathbf{L2} = \mathbf{mP}_{11-0} + \lambda_1 \mathbf{m}_{L_2}$

Parametric equation of line $\mathbf{L3} \Rightarrow \mathbf{L3} = \mathbf{mP}_{16-0} + \lambda_2 \mathbf{m}_{L_3}$

$$\mathbf{L2} = \mathbf{L3} \Rightarrow \begin{bmatrix} m^x_{L_2} & -m^x_{L_3} \\ m^y_{L_2} & -m^y_{L_3} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} mP^x_{16-0} - mP^x_{11-0} \\ mP^y_{16-0} - mP^y_{11-0} \end{bmatrix} \Rightarrow \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 3.020 \\ -2.863 \end{bmatrix}$$

x-coordinate of intersection between $\mathbf{L1}$ and $\mathbf{L2}$:

$$L2^x = mP^x_{11-0} + \lambda_1 \cdot m^x_{L_2} = 1.673$$

$$L3^x = mP^x_{16-0} + \lambda_2 \cdot m^x_{L_3} = 1.673$$

y-coordinate of intersection between $\mathbf{L1}$ and $\mathbf{L2}$:

$$L2^y = mP^y_{11-0} + \lambda_1 \cdot m^y_{L_2} = -3.309$$

$$L3^y = mP^y_{16-0} + \lambda_2 \cdot m^y_{L_3} = -3.309$$

Intersection of lines L3 and L1:

Parametric equation of line $\mathbf{L3} \Rightarrow \mathbf{L3} = \mathbf{mP}_{16-0} + \lambda_1 \mathbf{m}_{L_3}$

Parametric equation of line $\mathbf{L1} \Rightarrow \mathbf{L1} = \mathbf{mP}_{6-0} + \lambda_2 \mathbf{m}_{L_1}$

$$\mathbf{L3} = \mathbf{L1} \Rightarrow \begin{bmatrix} m^x_{L_3} & -m^x_{L_1} \\ m^y_{L_3} & -m^y_{L_1} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} mP^x_{6-0} - mP^x_{16-0} \\ mP^y_{6-0} - mP^y_{16-0} \end{bmatrix} \Rightarrow \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 3.906 \\ 4.186 \end{bmatrix}$$

x-coordinate of intersection between $\mathbf{L1}$ and $\mathbf{L2}$:

$$L3^x = mP^x_{16-0} + \lambda_1 \cdot m^x_{L_3} = 3.155$$

$$L1^x = mP^x_{6-0} + \lambda_2 \cdot m^x_{L_1} = 3.155$$

y-coordinate of intersection between $\mathbf{L1}$ and $\mathbf{L2}$:

$$L3^y = mP^y_{16-0} + \lambda_1 \cdot m^y_{L_3} = 3.298$$

$$L1^y = mP^y_{6-0} + \lambda_2 \cdot m^y_{L_1} = 3.298$$

Program Outline:

Import allowed modules

```
import random
import math as m
import matplotlib.pyplot as plt
import numpy as np
```

Generate Random points

Example of randomly generated points

[Index, x_cord, y_cord, distance to origin]

```
P=[[1,3.19,7.56,8.21],
    [2,6.92,5.42,8.79],
    [3,9.47,3.33,10.04],
    [4,9.98,5.86,   11.57],
    [5, 7.86,   10.00,  12.72],
    [6, -1.40,  3.33,   3.61],
    [7, -4.08,  7.12,   8.21],
    [8, -7.00,  4.57,   8.36],
    [9, -8.95,  3.33,   9.55],
    [10,-8.12,  8.26,11.58],
    [11,    -2.00,  -3.81,  4.30],
    [12 ,-9.93, -2.89,  10.34],
    [13 ,-2.72, -6.39,  6.95],
    [14 ,-5.85, -7.16,  7.66],
    [15 ,-10.80,    -7.16,  12.96],
    [16,    4.60,   -1.03,  4.71],
    [17 ,5.72,  -3.7,   6.81],
    [18 ,10.04, -4.57,  11.03],
    [19,    3.69,   -7.63,  8.47],
    [20 ,8.62,  -7.63,  11.52]]
```

Perform the necessary calculation to determine the Voronoi cell

Plot the result: (example)

```
#Intersection points
IP=[[-4.6285,0.0015],[1.673,-3.309],[3.155,3.298],[-4.6285,0.0015]]

P_matrix=np.array(P)
IP_matrix=np.array(IP)

#Plot generated point
plt.plot(P_matrix[:,1],P_matrix[:,2],"*")

#plot the cell
plt.plot(IP_matrix[:,0],IP_matrix[:,1])

#plot the origin
plt.plot(0,0,"rx")
```

Screen shot of the example