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Fluid dynamic modeling of the acoustic levitation phenomenon

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Introduction

Levitation is defined as the process in which an object is kept suspended in a way that ensures its stability, by using upward forces that counteract the object's weight, without mechanical contact occurring between the object and the source of the force. There are lines of research that, historically, aim to use electromagnetic waves to obtain experimental data on levitation[1].

Just like electromagnetic waves, acoustic waves can also exert forces on objects. The force produced by acoustic waves is called acoustic radiation force or acoustic radiation pressure [2]. There are several applications that utilize the phenomenon of acoustic radiation force. Among these applications is acoustic levitation [3]. Acoustic wave forces can be generated by progressive or standing acoustic waves.

In this work, the behavior of the system presented in Figure 2 was analyzed. Imposing a vibration with a frequency f on the lower disc, and considering the action of gravity, pressure, and viscous forces due to the existence of a lubricating film between the upper disc and the limiting cylinder, whose lubricant has a specific mass ÿoil = 0.02394 kg/m3 and dynamic viscosity μ_{oil} = 910 P a · s.

The physical domain of the problem presented is a cylinder with an internal diameter of 51 millimeters in which two discs (pistons) with a diameter of 50 millimeters and a mass m are positioned 200 millimeters apart. Furthermore, the fluid between these two cylinders is air and, initially, the internal pressure is equal to atmospheric pressure.

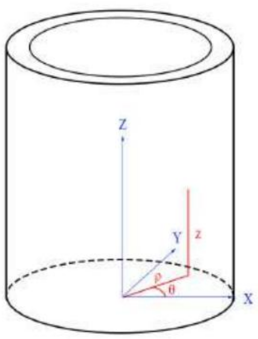

Figure 1: Coordinate system

Figure 2: Physical Model

Mathematical Model

For the purposes of this study, it is assumed that the fluid's density does not vary in the adopted spatial coordinates at the same instant in time, given that the dimensions of the adopted physical system are small and, therefore, propagation is limited.

The change in specific mass occurs almost instantaneously. Therefore, development can begin.
mathematical calculation of the balance of forces acting on the disk, presented in Eq. 1.

$$F = (P ÿ P_{atm})A ÿ W + F_V = m\ddot{L}_s. \tag{1}$$

Therefore, F is the resultant force on the upper piston, P is the absolute pressure inside the cylinder, and Patm is the pressure. atmospheric, A the area of the upper piston, W the weight force, FV the viscous force, determined by Eq. 2 [4], ma mass from the upper piston and L's the acceleration of the upper piston. From this, the following development for the force follows. viscous:

$$F_V = \mu_{oil} \times ÿ \times A_L. \tag{2}$$

$$ÿ = \begin{bmatrix} ÿ_{rr} & ÿ_{rÿ} & ÿ_{rz} \\ ÿ_{ÿr} & ÿ_{ÿÿ} & ÿ_{ÿz} \\ ÿ_{zr} & ÿ_{zÿ} & ÿ_{zz} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{ÿV_s}{ÿr} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{3}$$

where µoil is the dynamic viscosity of the lubricating oil, ÿ the viscous tensor [4], AL the lateral area of the piston superior, Vs the velocity on the Z axis of the upper piston er refers to the radius in the cylindrical coordinate system adopted in this work. Thus, by approximating the velocity profile in r as linear, due to the small space existing between the piston and the cylinder, for the lubricating fluid present:

$$F_V = \mu_{oil} \frac{ÿV_s}{ÿr} A_L = ÿ\mu_{oil} \frac{V_s}{G} A_L. \tag{4}$$

Where G is the space between the piston and the cylinder.

Next, we want to model the absolute pressure in the cylinder. In this way, we know that the pressure can be... modeled using the Universal Gas Law [4], presented in Eq. 5.

$$P = ÿRT\ . \tag{5}$$

In which ÿ is the fluid density, in this case air, R is the gas constant, and T is the fluid temperature. Initially, to model the density, the Continuity Equation [4] was used, presented in Eq. 6. Considering the no-slip hypothesis and, for this model, the linear velocity profile in Z, as shown in
From Fig. 3, we obtain Eq. 7 for the fluid velocity, Vz, as a function of time and space.

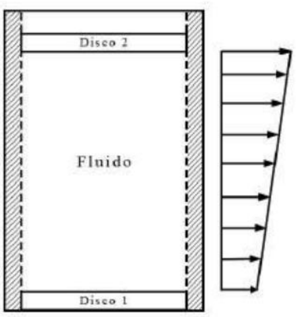

Figure 3: Velocity Distribution

$$\frac{ÿÿ}{ÿt} + \frac{ÿ(ÿV_z)}{ÿz} = 0. \tag{6}$$

$$V_z(t, z) = \frac{V_s(t) ÿ V_i(t)}{L_s(t) ÿ L_i(t)} \times z + V_i(t). \tag{7}$$

Where Ls is the displacement of the upper piston, and Vi and Li are, respectively, the velocity and displacement. imposed on the lower piston. Therefore, by deriving Eq. 7 with respect to Z and implementing it in Eq. 6, we obtain Eq. 8.

$$\frac{ÿÿ}{ÿt} = ÿÿ \frac{V_s(t) ÿ V_i(t)}{L_s(t) ÿ L_i(t)}. \tag{8}$$

At this point, observing Eq. 5, it becomes clear that temperature modeling is still necessary, where... This can be obtained using Eq. 9, developed from the energy balance using the Transport Theorem. Reynolds [4] considering compressible flow terms.

$$\frac{ÿT}{ÿt} = \frac{\frac{4}{3}\mu ar \left(\frac{ÿVz}{ÿz}\right)^{2} - P \left(\frac{ÿVz}{ÿz}\right)}{ÿCp}. \tag{9}$$

In which µar is the dynamic viscosity of air and Cp is the heat capacity of air. Thus, we have a system of equations Ordinary Differentials (ODEs) and their respective initial conditions are presented in Eqs. 10a, 10b, 10c, 11a, 11b, and 12a. and 12b.

$$\begin{cases} \frac{d2Ls}{dt2} = \frac{(ÿ(t)RT\ (t)\ ÿ\ Patm)A\ ÿ\ W + FV}{m}. & (10a) \\ \left.\frac{dLs}{dt}\right|_{t=0} = 0. & (10b) \\ Ls(0) = 0.2. & (10c) \end{cases}$$

$$\begin{cases} \frac{dÿ}{dt} = ÿÿ \frac{Vs(t)\ ÿ\ Vi(t)}{Ls(t)\ ÿ\ Li(t)}. & (11a) \\ ÿ(0) = 1.225. & (11b) \end{cases}$$

$$\begin{cases} \frac{dT}{dt} = \frac{\frac{4}{3}\mu ar \left(\frac{ÿVz}{ÿz}\right)^{2}\ ÿ\ ÿ(t)RT\ (t) \left(\frac{ÿVz}{ÿz}\right)}{ÿCp}. & (12a) \\ T\ (0) = 15. & (12b) \end{cases}$$

Finally, the computationally implemented 4th- order Runge-Kutta method was used to solve this. system.

Results

To obtain the results from the presented modeling, 3 system scenarios were simulated: with frequency f = 10 Hz and mass m = 100 g, with frequency f = 10 Hz and mass m = 200 g and with frequency f = 20 Hz and mass m = 100 g. For the first case, Figures 4 and 5 were obtained; for the second case, Figures 6 and 7. and, for the last case, see Figures 8 and 9.

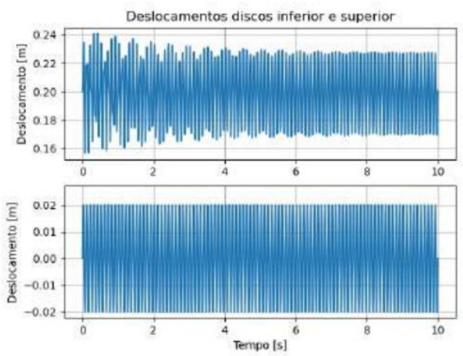

Figure 4: Disk displacement for f = 10 Hz and m = 100 g

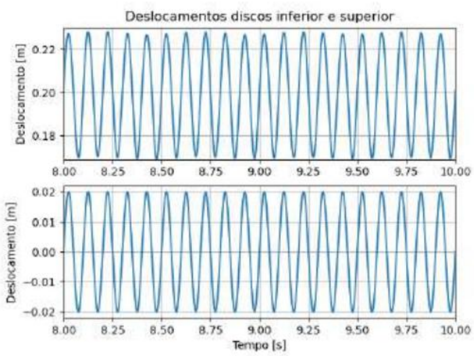

Figure 5: Disk displacement for f = 10 Hz and m = 100 g in steady state

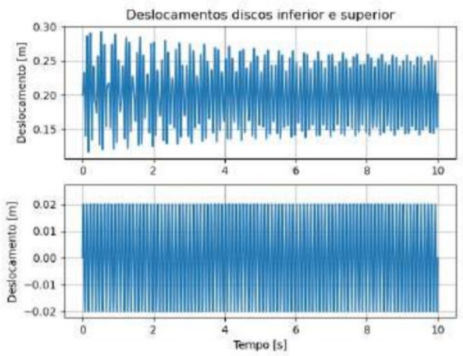

Figure 6: Displacement of the disks for f = 10 Hz and m = 200 g

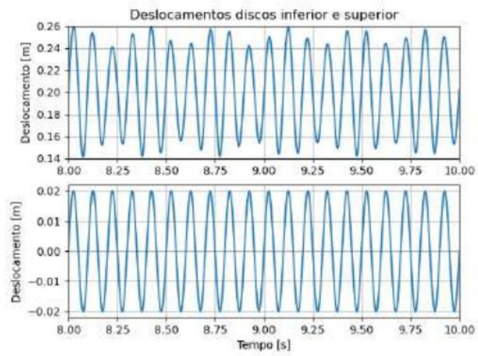

Figure 7: Disk displacement for f = 10 Hz and m = 200 g (Enlarged)

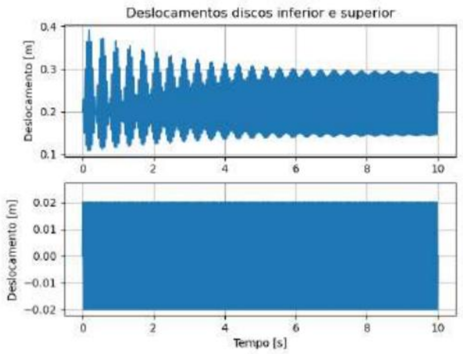

Figure 8: Disk displacement for f = 20 Hz and m = 100 g

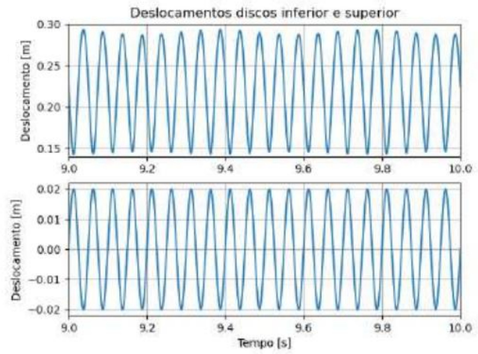

Figure 9: Displacement of the disks for f = 20 Hz and m = 100 g in steady state.

Conclusions

The solution to the one-dimensional problem of acoustic levitation addressed in this work represents a first step towards understanding this complex physical phenomenon. Thus, starting from the presented model, it is possible to analyze the amplitude, phase, and speed of movement of the two discs, considering even the influence of viscous forces between the upper disc and the limiting cylinder.

Two fundamental factors influence the model. Increasing the mass of the upper piston, which influences the weight force, results in a more prolonged transient response due to the increased difficulty in damping viscous forces caused by the increased speed associated with the increased weight force. Furthermore, the clear influence of frequency on the system's phase is evident. At a frequency of 10 Hz, shown in Fig. 5, the lower and upper pistons are in phase (0° phase difference), while at 20 Hz, shown in Fig. 9, the pistons have a 180° phase difference in their respective movements.

References

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