

# Árvores de Decisão do Zero: ID3, C4.5 e CART (Titanic) · Documento Único

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## Observações do enunciado

1. Discussões, código e resultados **\*\*neste único PDF\*\***.
2. O código está em pacote instalável ``dtree_lab``.
3. Execução com `*runner*` (opcional) e scripts descritos neste documento.

## Seção 1 · Preparação dos dados (Titanic)

**\*\*Atributos:\*\*** ``Pclass``, ``Sex``, ``Age``, ``SibSp``, ``Parch``, ``Fare``, ``Embarked``.

**\*\*Tratamento:\*\*** imputação (mediana/moda), `*split*` 80/20 estratificado, discretização (``Age``, ``Fare``) por quantis para ID3.

**###** Código · utilitários (imputação, split, discretização)

```

from __future__ import annotations
from typing import Tuple
import numpy as np
import pandas as pd

def train_test_split_stratified(y: np.ndarray, test_size: float = 0.2, seed: int = 42) -> Tuple:
    rng = np.random.default_rng(seed)
    idx = np.arange(len(y))
    test_idx = []
    for c in np.unique(y):
        class_idx = idx[y == c]
        rng.shuffle(class_idx)
        n_test = max(1, int(round(test_size * len(class_idx))))
        test_idx.extend(class_idx[:n_test])
    test_idx = np.array(sorted(test_idx))
    train_idx = np.array([i for i in idx if i not in set(test_idx)])
    return train_idx, test_idx

def discretize_equal_frequency(series: pd.Series, bins: int = 4, labels: bool = True) -> pd.Series:
    """Discretiza por quantis (~mesmo número de amostras por faixa)."""
    q = np.linspace(0, 1, bins + 1)
    edges = np.unique(series.quantile(q).values)
    edges[0] = -np.inf
    edges[-1] = np.inf
    cats = pd.cut(series, bins=edges, include_lowest=True)
    return cats.astype(str) if labels else cats

def discretize_equal_width(series: pd.Series, bins: int = 4, labels: bool = True) -> pd.Series:
    """Discretiza por largura fixa (intervalos iguais)."""
    cats = pd.cut(series, bins=bins, include_lowest=True)
    return cats.astype(str) if labels else cats

def impute_simple(df: pd.DataFrame) -> pd.DataFrame:
    """Imputa NaNs: numéricos -> mediana; categóricos/objeto -> moda."""
    out = df.copy()
    for col in out.columns:
        if out[col].dtype == object:
            if out[col].isna().any():
                out[col] = out[col].fillna(out[col].mode().iloc[0])
        else:
            if out[col].isna().any():
                out[col] = out[col].fillna(out[col].median())
    return out

```

### Métricas auxiliares

```
from __future__ import annotations
import numpy as np

def accuracy(y_true, y_pred) -> float:
    y_true = np.asarray(y_true)
    y_pred = np.asarray(y_pred)
    return float((y_true == y_pred).mean())

def confusion_matrix(y_true, y_pred):
    y_true = np.asarray(y_true)
    y_pred = np.asarray(y_pred)
    labels = sorted(list(set(y_true) | set(y_pred)))
    L = len(labels)
    lab2i = {lab: i for i, lab in enumerate(labels)}
    m = np.zeros((L, L), dtype=int)
    for t, p in zip(y_true, y_pred):
        m[lab2i[t], lab2i[p]] += 1
    return labels, m
```

## **Seção 2 · Implementações do zero**

### **2.1 ID3 (ganho de informação; atributos categóricos)**

```

from __future__ import annotations
from dataclasses import dataclass
from typing import Any, Dict, List, Optional
import numpy as np
import pandas as pd
from collections import Counter

def _entropy(y: np.ndarray) -> float:
    vals, cnt = np.unique(y, return_counts=True)
    if cnt.sum() == 0:
        return 0.0
    p = cnt / cnt.sum()
    return float(-(p * np.log2(p)).sum())

def _info_gain_categorical(x: np.ndarray, y: np.ndarray) -> float:
    ent_total = _entropy(y)
    vals, cnt = np.unique(x, return_counts=True)
    ent_cond = 0.0
    for v, c in zip(vals, cnt):
        ent_cond += (c / len(x)) * _entropy(y[x == v])
    return ent_total - ent_cond

@dataclass
class ID3Node:
    atributo: Optional[str] = None
    filhos: Optional[Dict[Any, 'ID3Node']] = None
    rotulo: Optional[Any] = None
    is_leaf: bool = False
    classe_majoritaria: Optional[Any] = None

    def __str__(self, nivel: int = 0) -> str:
        ident = " " * nivel
        if self.is_leaf:
            return f""{ident}Folha: {self.rotulo}
        """
        s = f""{ident}[{self.atributo}]
        """
        for v, fchild in (self.filhos or {}).items():
            s += f""{ident} -> {v}:
{fchild.__str__(nivel+2)}""
        return s

class ID3:
    def __init__(self, max_depth: Optional[int] = None, min_samples_split: int = 2):
        self.max_depth = max_depth
        self.min_samples_split = min_samples_split
        self.tree_: Optional[ID3Node] = None
        self.features_: List[str] = []

    def _best_attribute(self, X: pd.DataFrame, y: np.ndarray, attrs: List[str]) -> Optional[str]:
        gains = []
        for col in attrs:
            gains.append((_info_gain_categorical(X[col].values, y), col))
        gains.sort(key=lambda t: (-t[0], X[t[1]].nunique(), t[1]))
        return gains[0][1] if gains else None

```

## 2.2 C4.5 (razão de ganho; contínuos por limiar; categórico multi-ramo)

```

from __future__ import annotations
from dataclasses import dataclass
from typing import Any, Dict, List, Optional, Tuple
from collections import Counter
import numpy as np
import pandas as pd

def _entropy_list(y_list: List[Any]) -> float:
    cnt = Counter(y_list); n = len(y_list)
    return -sum((c/n)*np.log2(c/n) for c in cnt.values()) if n else 0.0

def _gain_ratio_numeric(col: List[float], y: List[Any], thr: float) -> Tuple[float, float]:
    left_y = [y[i] for i in range(len(y)) if col[i] < thr]
    right_y = [y[i] for i in range(len(y)) if col[i] >= thr]
    if not left_y or not right_y:
        return 0.0, 0.0
    ent_total = _entropy_list(y)
    pL = len(left_y)/len(y); pR = 1.0 - pL
    ent_div = pL * _entropy_list(left_y) + pR * _entropy_list(right_y)
    info_gain = ent_total - ent_div
    split_info = -sum(p * np.log2(p) for p in [pL, pR] if p > 0)
    return info_gain, (info_gain/split_info if split_info != 0 else 0.0)

def _best_split_numeric(col: List[float], y: List[Any]) -> Tuple[Optional[float], float]:
    uniq = sorted(set(col))
    if len(uniq) < 2:
        return None, -1.0
    cands = [(uniq[i] + uniq[i+1]) / 2 for i in range(len(uniq) - 1)]
    best_thr, best_gr = None, -1.0
    for t in cands:
        _, gr = _gain_ratio_numeric(col, y, t)
        if gr > best_gr:
            best_gr, best_thr = gr, t
    return best_thr, best_gr

@dataclass
class C45Node:
    atributo: Optional[str] = None
    limiar: Optional[float] = None
    filhos: Optional[Dict[Any, 'C45Node']] = None
    rotulo: Optional[Any] = None
    is_leaf: bool = False

    def __str__(self, nivel: int = 0) -> str:
        ident = " " * nivel
        if self.is_leaf:
            return f""{ident}Folha: {self.rotulo}
"""
        if self.limiar is not None:
            s = f""{ident}[{self.atributo} < {self.limiar:.6g}]
"""
            s += f""{ident} -> < :
{self.filhos['<'].__str__(nivel+2)}""
            s += f""{ident} -> >=:

```

## 2.3 CART (índice de Gini; divisões binárias; categórico por subconjunto)

```

from __future__ import annotations
from dataclasses import dataclass
from typing import Any, Dict, List, Optional, Set, Tuple
from collections import Counter
import numpy as np
import pandas as pd

```

```

def _gini(y) -> float:
    y = np.asarray(y); n = len(y)
    if n == 0: return 0.0
    cnt = Counter(y)
    return 1.0 - sum((c/n)**2 for c in cnt.values())

```

```

@dataclass

```

```

class CARTNode:

```

```

    atributo: Optional[str] = None
    limiar: Optional[float] = None
    cats_esq: Optional[Set[Any]] = None
    is_categorical: bool = False
    filhos: Optional[Dict[str, 'CARTNode']] = None
    rotulo: Optional[Any] = None
    is_leaf: bool = False

```

```

    def __str__(self, nivel: int = 0) -> str:

```

```

        ident = " " * nivel

```

```

        if self.is_leaf:

```

```

            return f""{ident}Folha: {self.rotulo}

```

```

    """

```

```

        if self.is_categorical:

```

```

            s = f""{ident}[{self.atributo} . {sorted(list(self.cats_esq or []))}]

```

```

    """

```

```

            s += f""{ident} -> . :

```

```

{self.filhos['in'].__str__(nivel+2)}""

```

```

            s += f""{ident} -> . :

```

```

{self.filhos['out'].__str__(nivel+2)}""

```

```

            return s

```

```

        s = f""{ident}[{self.atributo} < {self.limiar:.6g}]

```

```

    """

```

```

        s += f""{ident} -> < :

```

```

{self.filhos['<'].__str__(nivel+2)}""

```

```

        s += f""{ident} -> >=:

```

```

{self.filhos['>='].__str__(nivel+2)}""

```

```

        return s

```

```

class CART:

```

```

    def __init__(self, max_depth: Optional[int] = None, min_samples_split: int = 2, min_samples_leaf: int = 1):

```

```

        self.max_depth = max_depth

```

```

        self.min_samples_split = min_samples_split

```

```

        self.min_samples_leaf = min_samples_leaf

```

```

        self.tree_: Optional[CARTNode] = None

```

```

        self.features_: List[str] = []

```

```

        self.cat_features_: Set[str] = set()

```

```

    def _best_split_numeric(self, x: np.ndarray, y: np.ndarray) -> Tuple[Optional[float], float]:

```

```

        uniq = np.unique(x)

```

```

        if uniq.size < 2: return None, np.inf

```



### Seção 3 · Resultados com o `train.csv` enviado

Configuração: \*split\* estratificado 80/20 (`seed=42`), `max\_depth=6`.

#### ID3

Acurácia (treino): 0.8808 \nAcurácia (teste): 0.8034

Matriz de confusão (teste):

labels: 0, 1

||0|1|

|-|-|-|

|0|99|11|

|1|24|44|

**\*\*Árvore (ID3)\*\***

```

[Sex]
-> female:
[Pclass]
-> 1:
[Fare]
-> (14.454, 30.5]:
[Embarked]
-> C:
[Age]
-> (35.0, inf]:
[Parch]
-> 0:
Folha: 0
-> S:
Folha: 1
-> (30.5, inf]:
Folha: 1
-> 2:
[Age]
-> (-inf, 22.0]:
Folha: 1
-> (22.0, 28.0]:
[Parch]
-> 0:
[Embarked]
-> C:
Folha: 1
-> S:
[Fare]
-> (14.454, 30.5]:
Folha: 1
-> (30.5, inf]:
Folha: 1
-> (7.896, 14.454]:
Folha: 1
-> 1:
[SibSp]
-> 0:
Folha: 1
-> 1:
[Embarked]
-> S:
Folha: 0
-> 2:
Folha: 1
-> 2:
Folha: 1
-> 3:
Folha: 1
-> (28.0, 35.0]:
Folha: 1
-> (35.0, inf]:
[Parch]
-> 0:
[Fare]
-> (14.454, 30.5]:
[Embarked]
-> S:

```

**\*\*Regras (ID3)\*\***



## C45

Acurácia (treino): 0.8219 \nAcurácia (teste): 0.8202

Matriz de confusão (teste):

labels: 0, 1

||0|1|

|-|-|-|

|0|103|7|

|1|25|43|

**\*\*Árvore (C45)\*\***

```

[Sex]
-> female:
[SibSp < 6]
-> < :
[Pclass < 2.5]
-> < :
[Fare < 28.8562]
-> < :
[Age < 53.5]
-> < :
[Parch < 1.5]
-> < :
Folha: 1
-> >=:
Folha: 1
-> >=:
[Embarked]
-> S:
Folha: 1
-> >=:
Folha: 1
-> >=:
[Fare < 32.8813]
-> < :
[Age < 1.5]
-> < :
Folha: 1
-> >=:
[Embarked]
-> C:
Folha: 1
-> Q:
Folha: 1
-> S:
Folha: 0
-> >=:
Folha: 0
-> >=:
Folha: 0
-> male:
[Age < 1.5]
-> < :
Folha: 1
-> >=:
[Fare < 387.665]
-> < :
[SibSp < 4.5]
-> < :
[Parch < 2.5]
-> < :
[Pclass < 1.5]
-> < :
Folha: 0
-> >=:
Folha: 0
-> >=:
Folha: 0
-> >=:
Folha: 0
-> >=:

```

## **\*\*Regras (C45)\*\***

```
Sex == female AND SibSp < 6 AND Pclass < 2.5 AND Fare < 28.8562 AND Age < 53.5 AND Parch < 1.
Sex == female AND SibSp < 6 AND Pclass < 2.5 AND Fare < 28.8562 AND Age < 53.5 AND Parch >= 1
Sex == female AND SibSp < 6 AND Pclass < 2.5 AND Fare < 28.8562 AND Age >= 53.5 AND Embarked
Sex == female AND SibSp < 6 AND Pclass < 2.5 AND Fare >= 28.8562 => predict 1
Sex == female AND SibSp < 6 AND Pclass >= 2.5 AND Fare < 32.8813 AND Age < 1.5 => predict 1
Sex == female AND SibSp < 6 AND Pclass >= 2.5 AND Fare < 32.8813 AND Age >= 1.5 AND Embarked
Sex == female AND SibSp < 6 AND Pclass >= 2.5 AND Fare < 32.8813 AND Age >= 1.5 AND Embarked
Sex == female AND SibSp < 6 AND Pclass >= 2.5 AND Fare < 32.8813 AND Age >= 1.5 AND Embarked
Sex == female AND SibSp < 6 AND Pclass >= 2.5 AND Fare >= 32.8813 => predict 0
Sex == female AND SibSp >= 6 => predict 0
Sex == male AND Age < 1.5 => predict 1
Sex == male AND Age >= 1.5 AND Fare < 387.665 AND SibSp < 4.5 AND Parch < 2.5 AND Pclass < 1.
Sex == male AND Age >= 1.5 AND Fare < 387.665 AND SibSp < 4.5 AND Parch < 2.5 AND Pclass >= 1
Sex == male AND Age >= 1.5 AND Fare < 387.665 AND SibSp < 4.5 AND Parch >= 2.5 => predict 0
Sex == male AND Age >= 1.5 AND Fare < 387.665 AND SibSp >= 4.5 => predict 0
Sex == male AND Age >= 1.5 AND Fare >= 387.665 => predict 1
```

## **CART**

Acurácia (treino): 0.8682 \nAcurácia (teste): 0.8258

Matriz de confusão (teste):

labels: 0, 1

||0|1|

|-|-|

|0|100|10|

|1|21|47|

## **\*\*Árvore (CART)\*\***

```

[Sex · ['female']]
-> . :
[Pclass < 2.5]
-> < :
[Fare < 28.8562]
-> < :
[Fare < 28.2312]
-> < :
[Age < 53.5]
-> < :
[SibSp < 0.5]
-> < :
Folha: 1
-> >=:
Folha: 1
-> >=:
[Pclass < 1.5]
-> < :
Folha: 1
-> >=:
Folha: 0
-> >=:
Folha: 0
-> >=:
Folha: 1
-> >=:
[Fare < 20.6625]
-> < :
[Age < 7]
-> < :
Folha: 1
-> >=:
[Fare < 8.0396]
-> < :
[Age < 29.25]
-> < :
Folha: 1
-> >=:
Folha: 0
-> >=:
[Fare < 15.8]
-> < :
Folha: 0
-> >=:
Folha: 1
-> >=:
[Parch < 0.5]
-> < :
Folha: 1
-> >=:
[Age < 5.5]
-> < :
[Age < 3.5]
-> < :
Folha: 0
-> >=:
Folha: 1
-> >=:

```



**\*\*Regras (CART)\*\***

```
Sex · ['female'] AND Pclass < 2.5 AND Fare < 28.8562 AND Fare < 28.2312 AND Age < 53.5 AND SibSp < 2.5 => predict 0
Sex · ['female'] AND Pclass < 2.5 AND Fare < 28.8562 AND Fare < 28.2312 AND Age < 53.5 AND SibSp < 2.5 => predict 0
Sex · ['female'] AND Pclass < 2.5 AND Fare < 28.8562 AND Fare < 28.2312 AND Age >= 53.5 AND SibSp < 2.5 => predict 0
Sex · ['female'] AND Pclass < 2.5 AND Fare < 28.8562 AND Fare < 28.2312 AND Age >= 53.5 AND SibSp < 2.5 => predict 0
Sex · ['female'] AND Pclass < 2.5 AND Fare < 28.8562 AND Fare >= 28.2312 => predict 0
Sex · ['female'] AND Pclass < 2.5 AND Fare >= 28.8562 => predict 1
Sex · ['female'] AND Pclass >= 2.5 AND Fare < 20.6625 AND Age < 7 => predict 1
Sex · ['female'] AND Pclass >= 2.5 AND Fare < 20.6625 AND Age >= 7 AND Fare < 8.0396 AND Age < 53.5 => predict 1
Sex · ['female'] AND Pclass >= 2.5 AND Fare < 20.6625 AND Age >= 7 AND Fare < 8.0396 AND Age < 53.5 => predict 1
Sex · ['female'] AND Pclass >= 2.5 AND Fare < 20.6625 AND Age >= 7 AND Fare >= 8.0396 AND Age < 53.5 => predict 1
Sex · ['female'] AND Pclass >= 2.5 AND Fare < 20.6625 AND Age >= 7 AND Fare >= 8.0396 AND Age < 53.5 => predict 1
Sex · ['female'] AND Pclass >= 2.5 AND Fare >= 20.6625 AND Parch < 0.5 => predict 1
Sex · ['female'] AND Pclass >= 2.5 AND Fare >= 20.6625 AND Parch >= 0.5 AND Age < 5.5 AND Age < 53.5 => predict 1
Sex · ['female'] AND Pclass >= 2.5 AND Fare >= 20.6625 AND Parch >= 0.5 AND Age < 5.5 AND Age < 53.5 => predict 1
Sex · ['female'] AND Pclass >= 2.5 AND Fare >= 20.6625 AND Parch >= 0.5 AND Age >= 5.5 AND Age < 53.5 => predict 1
Sex · ['female'] AND Pclass >= 2.5 AND Fare >= 20.6625 AND Parch >= 0.5 AND Age >= 5.5 AND Age < 53.5 => predict 1
Sex · ['female'] AND Age < 6.5 AND SibSp < 2.5 => predict 1
Sex · ['female'] AND Age < 6.5 AND SibSp >= 2.5 AND Parch < 1.5 => predict 0
Sex · ['female'] AND Age < 6.5 AND SibSp >= 2.5 AND Parch >= 1.5 AND Age < 3.5 => predict 1
Sex · ['female'] AND Age < 6.5 AND SibSp >= 2.5 AND Parch >= 1.5 AND Age >= 3.5 => predict 0
Sex · ['female'] AND Age >= 6.5 AND Pclass < 1.5 AND Age < 53 AND Fare < 26.1438 => predict 0
Sex · ['female'] AND Age >= 6.5 AND Pclass < 1.5 AND Age < 53 AND Fare >= 26.1438 AND Fare < 51.6979 => predict 0
Sex · ['female'] AND Age >= 6.5 AND Pclass < 1.5 AND Age < 53 AND Fare >= 26.1438 AND Fare >= 51.6979 => predict 0
Sex · ['female'] AND Age >= 6.5 AND Pclass < 1.5 AND Age >= 53 AND SibSp < 0.5 => predict 0
Sex · ['female'] AND Age >= 6.5 AND Pclass < 1.5 AND Age >= 53 AND SibSp >= 0.5 AND Age < 62.5 => predict 0
Sex · ['female'] AND Age >= 6.5 AND Pclass < 1.5 AND Age >= 53 AND SibSp >= 0.5 AND Age >= 62.5 => predict 0
Sex · ['female'] AND Age >= 6.5 AND Pclass >= 1.5 AND Fare < 51.6979 AND Embarked · ['C'] AND Age < 62.5 => predict 0
Sex · ['female'] AND Age >= 6.5 AND Pclass >= 1.5 AND Fare < 51.6979 AND Embarked · ['C'] AND Age < 62.5 => predict 0
Sex · ['female'] AND Age >= 6.5 AND Pclass >= 1.5 AND Fare < 51.6979 AND Embarked · ['C'] AND Age < 62.5 => predict 0
Sex · ['female'] AND Age >= 6.5 AND Pclass >= 1.5 AND Fare < 51.6979 AND Embarked · ['C'] AND Age < 62.5 => predict 0
Sex · ['female'] AND Age >= 6.5 AND Pclass >= 1.5 AND Fare >= 51.6979 AND Fare < 63.0229 AND Age < 62.5 => predict 0
Sex · ['female'] AND Age >= 6.5 AND Pclass >= 1.5 AND Fare >= 51.6979 AND Fare < 63.0229 AND Age < 62.5 => predict 0
Sex · ['female'] AND Age >= 6.5 AND Pclass >= 1.5 AND Fare >= 51.6979 AND Fare >= 63.0229 => predict 0
```