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ESEIAAT - UPC

Study for the computational resolution of conservation equations of mass, momentum and energy. Possible application to different aeronautical and industrial engineering problems: Case 1B

Attachment A - Results

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Degree: Grau en Enginyeria en Tecnologies Aeroespacials
Delivery date: 10-06-2017(Spring 2016-2017)

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1 Smith-Hutton problem

In the following sections there are the results of the Smith-Hutton problem for all the resolution schemes that have been coded. The purpose of this chapter is to demonstrate that all the schemes produce a similar result.

1.1 $\rho/\Gamma = 10$

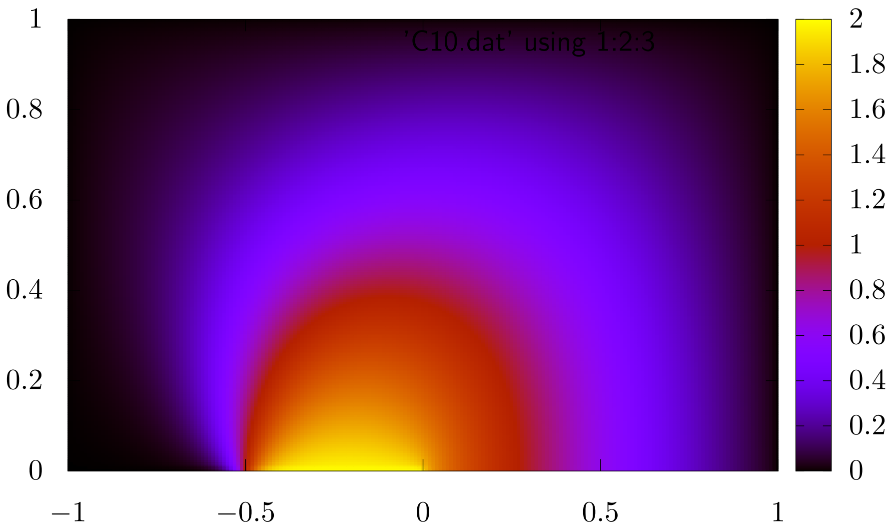


Figure 1.1: Representation of the whole domain for $\rho/\Gamma = 10$ (CDS)

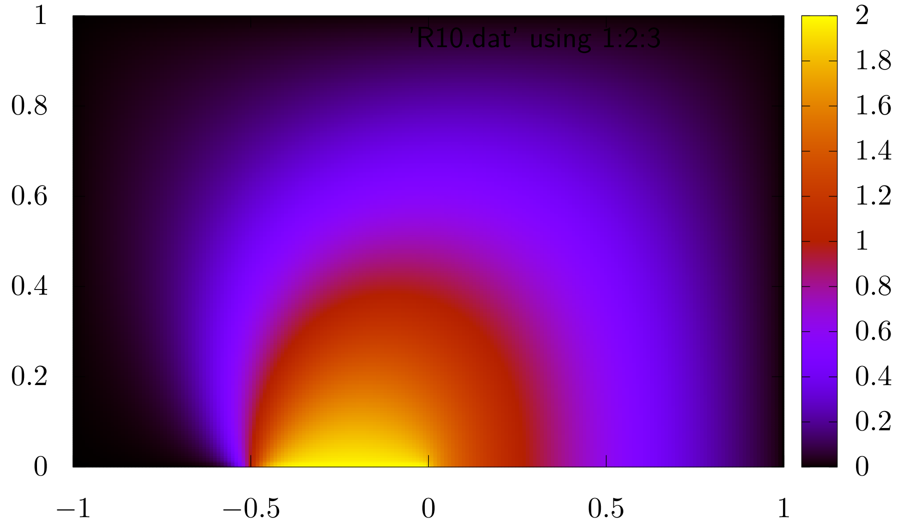


Figure 1.2: Representation of the whole domain for $\rho/\Gamma = 10$ (UDS)

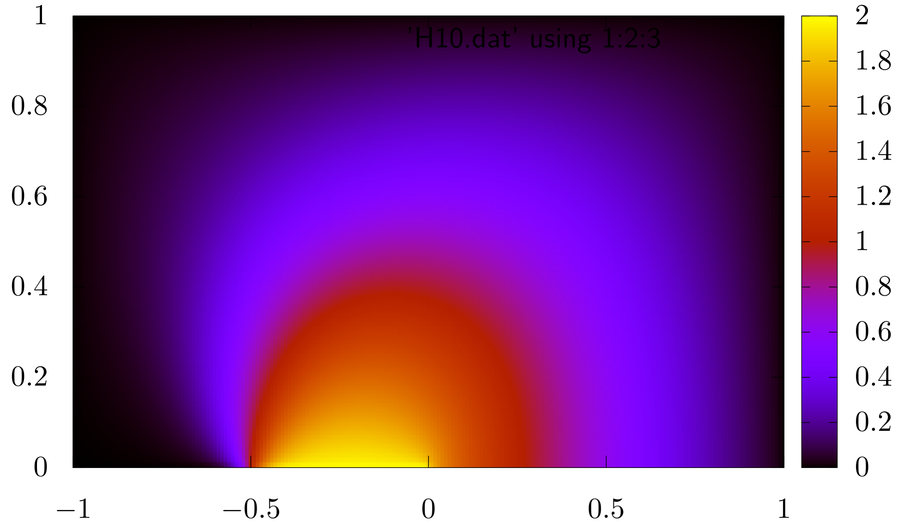


Figure 1.3: Representation of the whole domain for $\rho/\Gamma = 10$ (HDS)

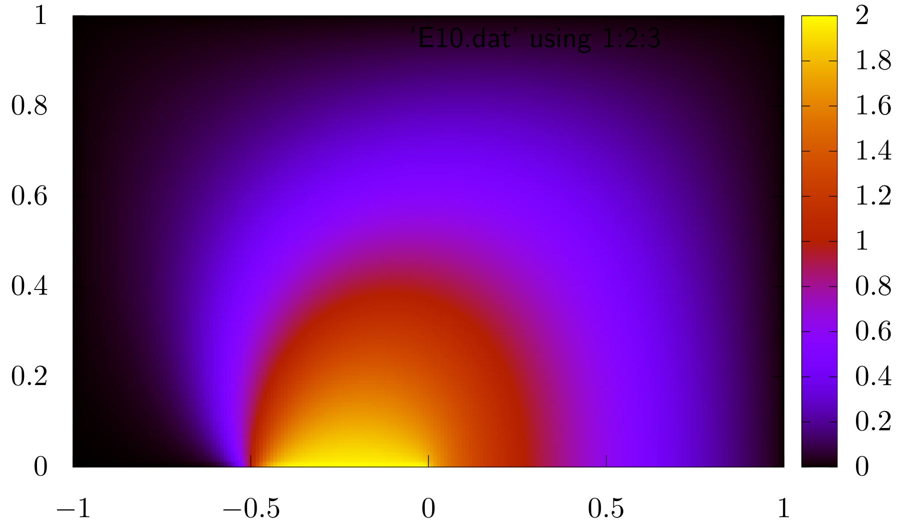


Figure 1.4: Representation of the whole domain for $\rho/\Gamma = 10$ (EDS)

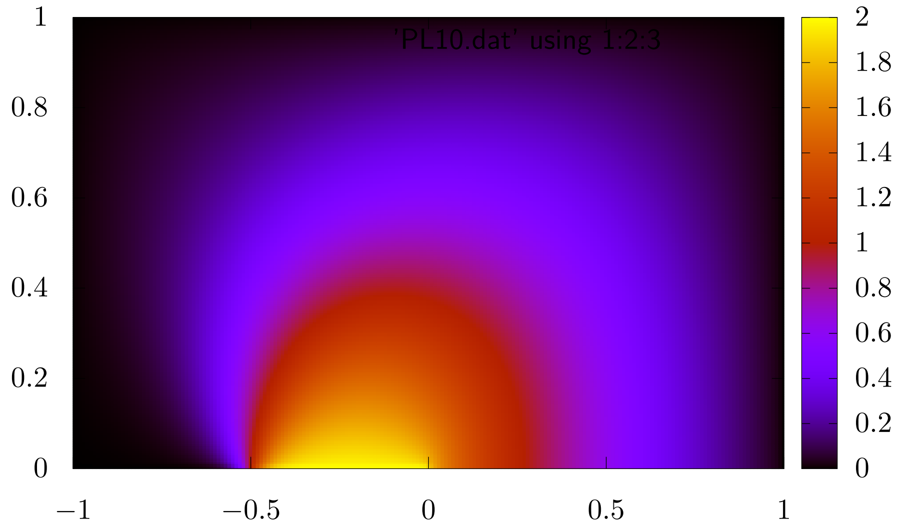


Figure 1.5: Representation of the whole domain for $\rho/\Gamma = 10$ (PLDS)

1.2 $\rho/\Gamma = 10^3$

In this section there is not a solution for the central differencing scheme because it diverges.

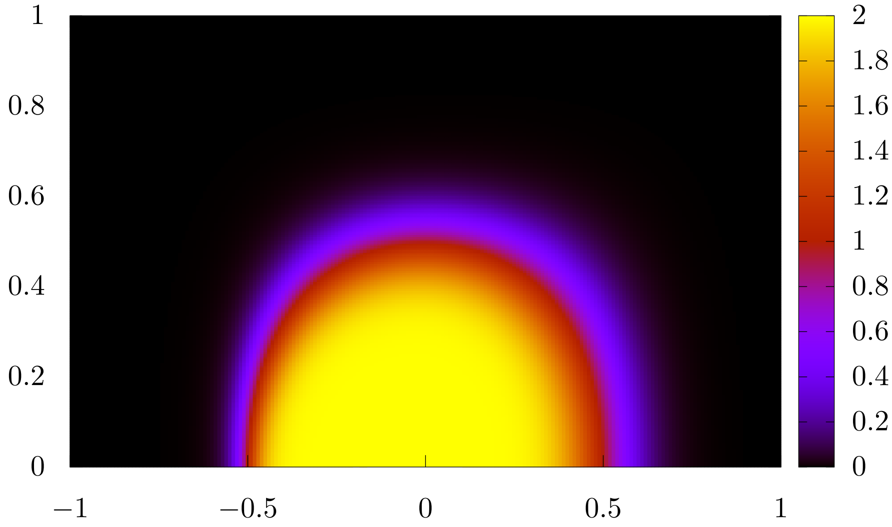


Figure 1.6: Representation of the whole domain for $\rho/\Gamma = 10^3$ (UDS)

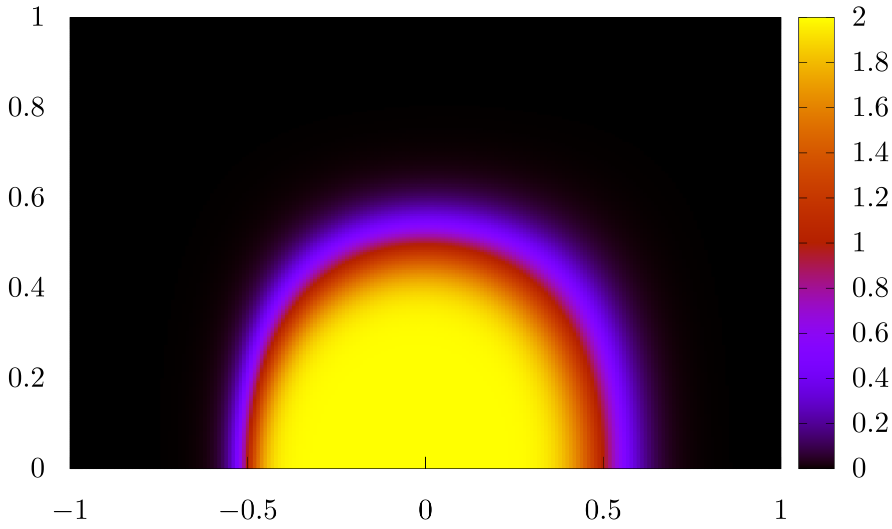


Figure 1.7: Representation of the whole domain for $\rho/\Gamma = 10^3$ (HDS)

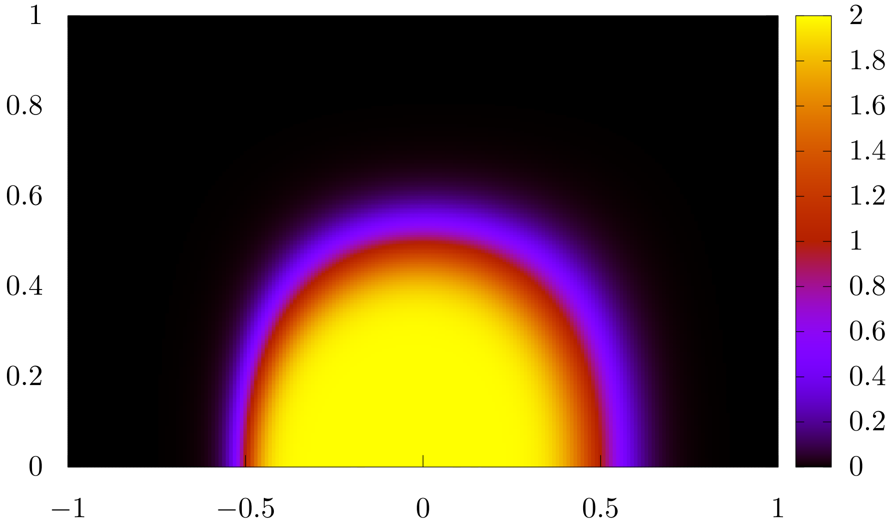


Figure 1.8: Representation of the whole domain for $\rho/\Gamma = 10^3$ (EDS)

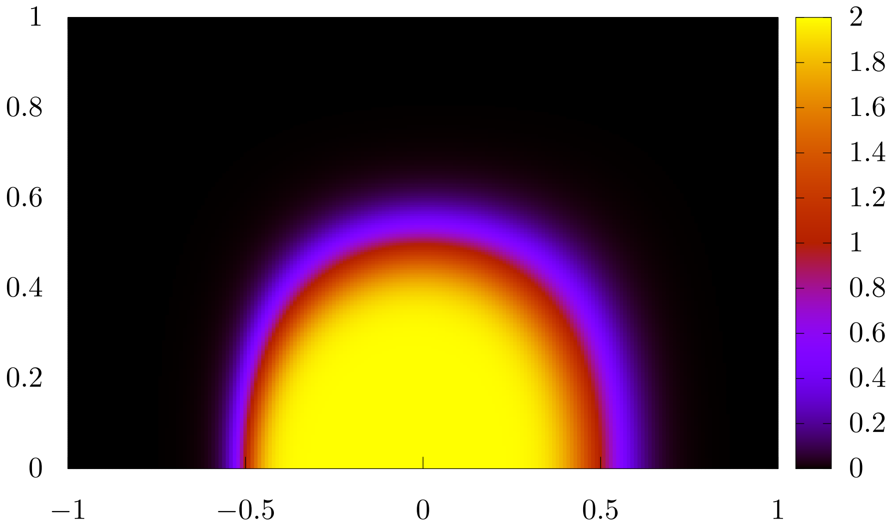


Figure 1.9: Representation of the whole domain for $\rho/\Gamma = 10^3$ (PLDS)

1.3 $\rho/\Gamma = 10^6$

Like in the previous section, in this case there is no results for the central differencing scheme because of its divergence.

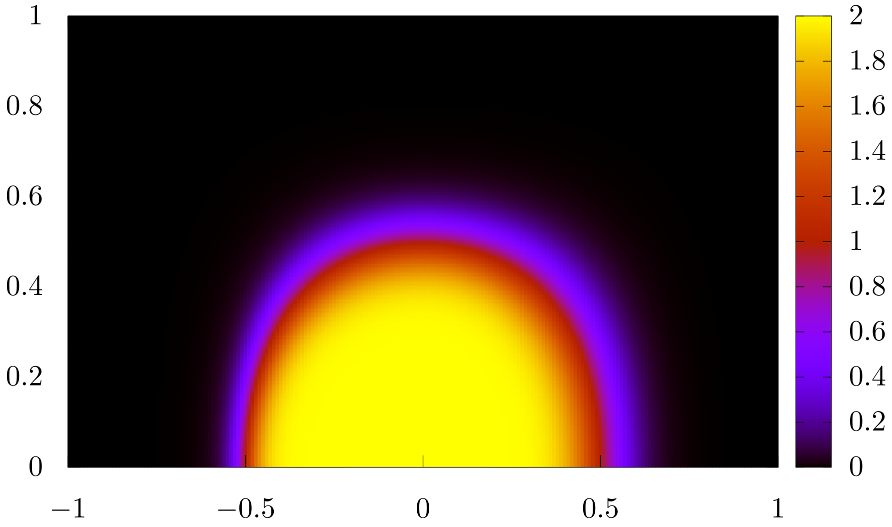


Figure 1.10: Representation of the whole domain for $\rho/\Gamma = 10^6$ (UDS)

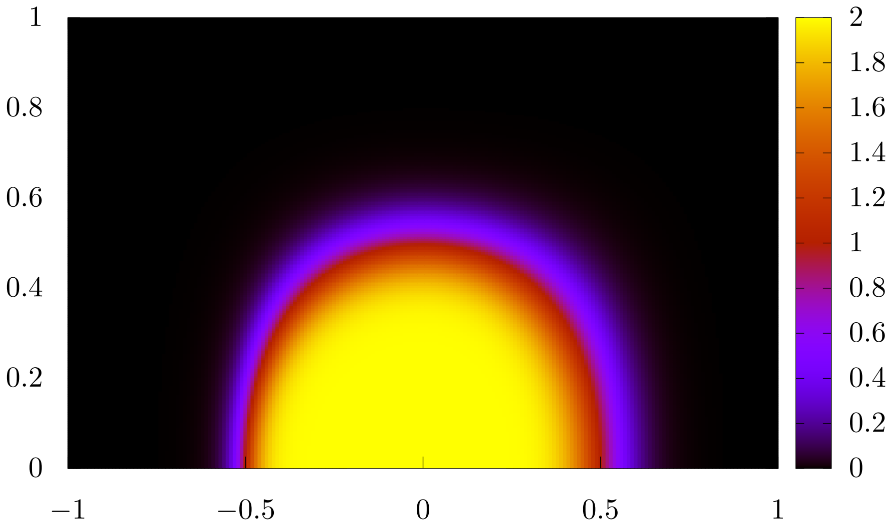


Figure 1.11: Representation of the whole domain for $\rho/\Gamma = 10^6$ (HDS)

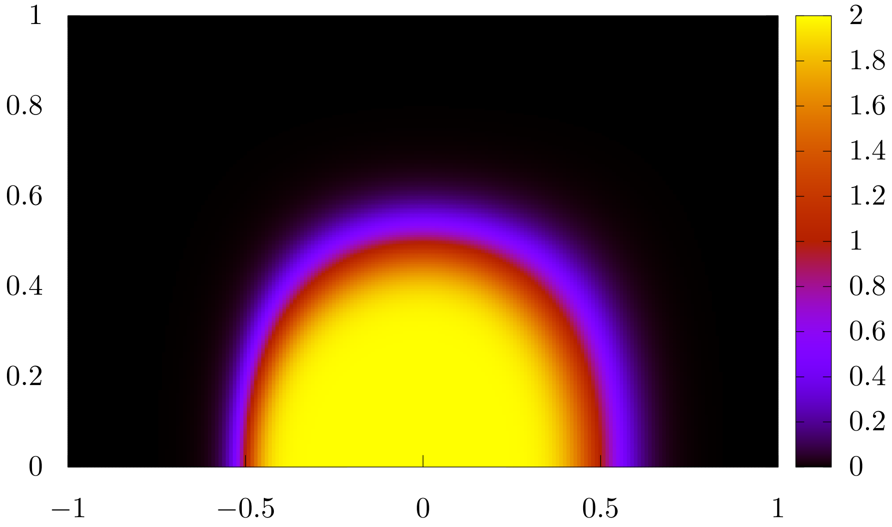


Figure 1.12: Representation of the whole domain for $\rho/\Gamma = 10^6$ (EDS)

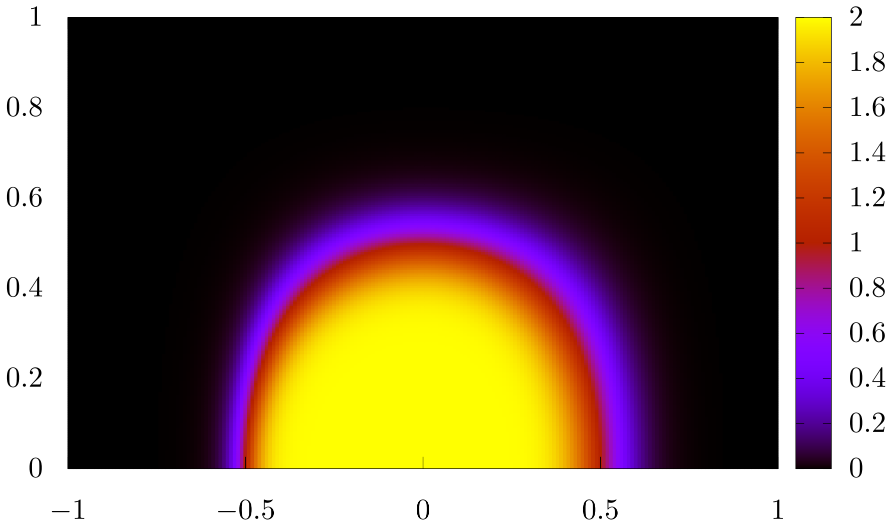
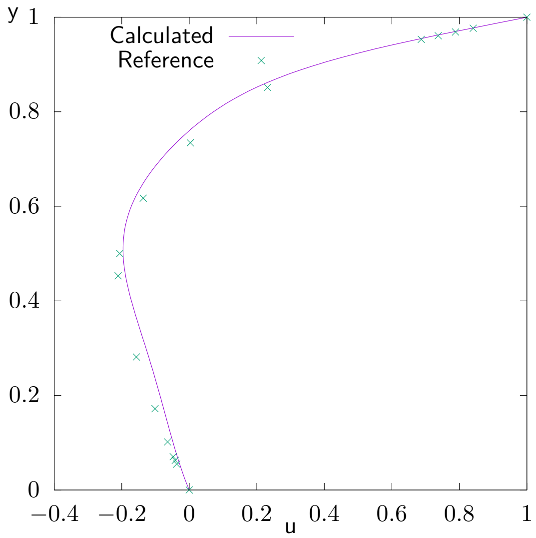


Figure 1.13: Representation of the whole domain for $\rho/\Gamma = 10^6$ (PLDS)

2 | Driven cavity problem



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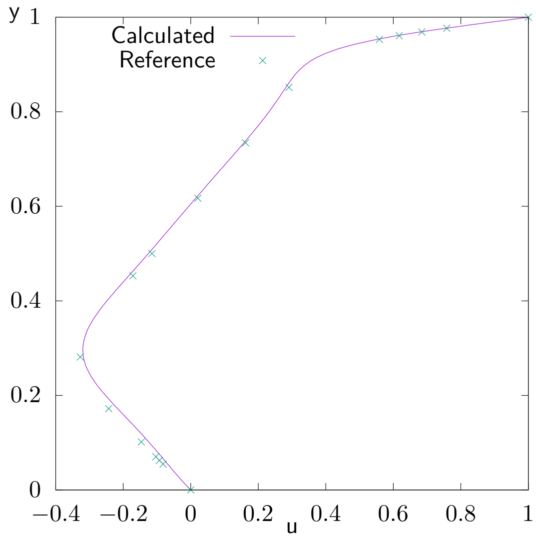
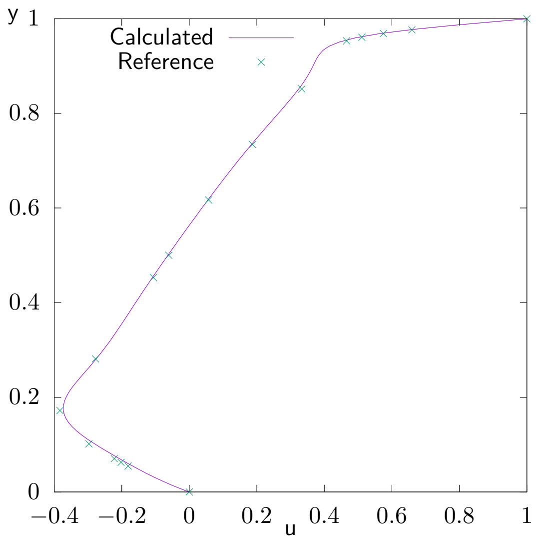
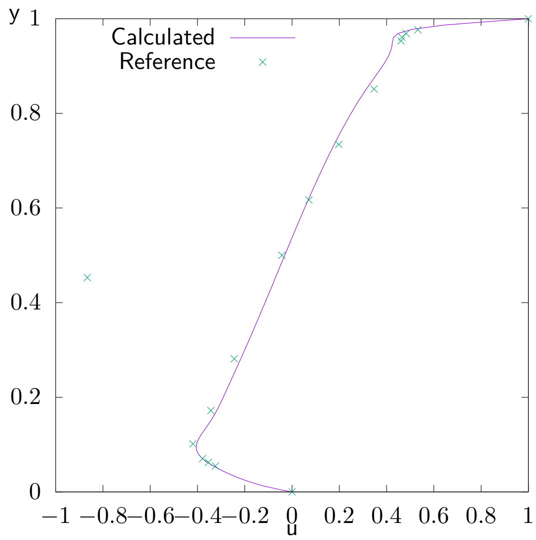


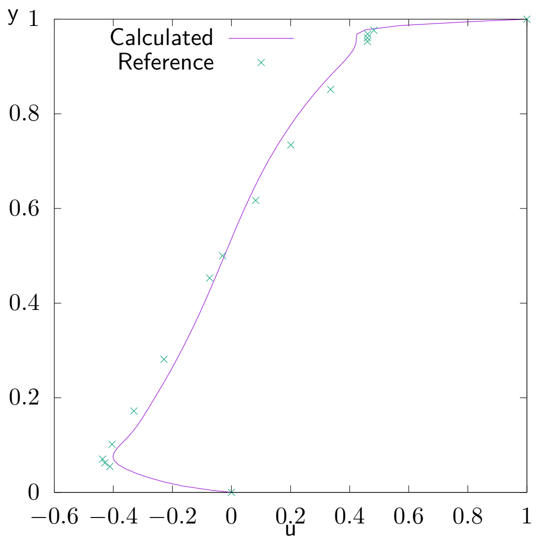
(b) $Re = 400$



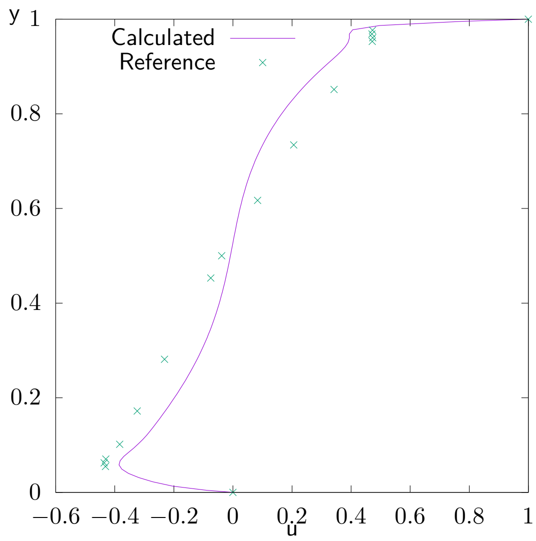
(c) $Re = 1000$



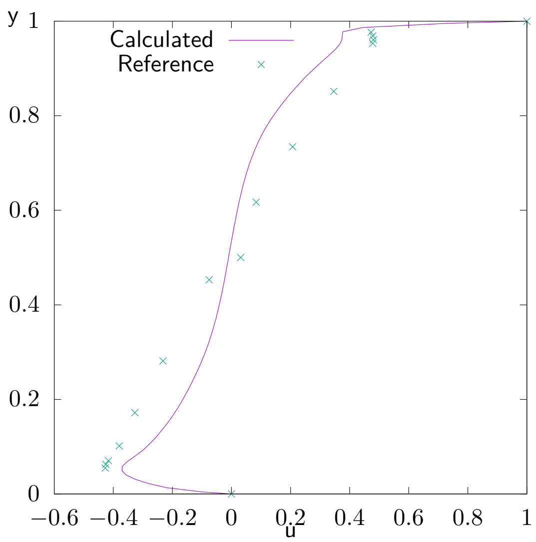
(d) $Re = 3200$



(e) $Re = 5000$

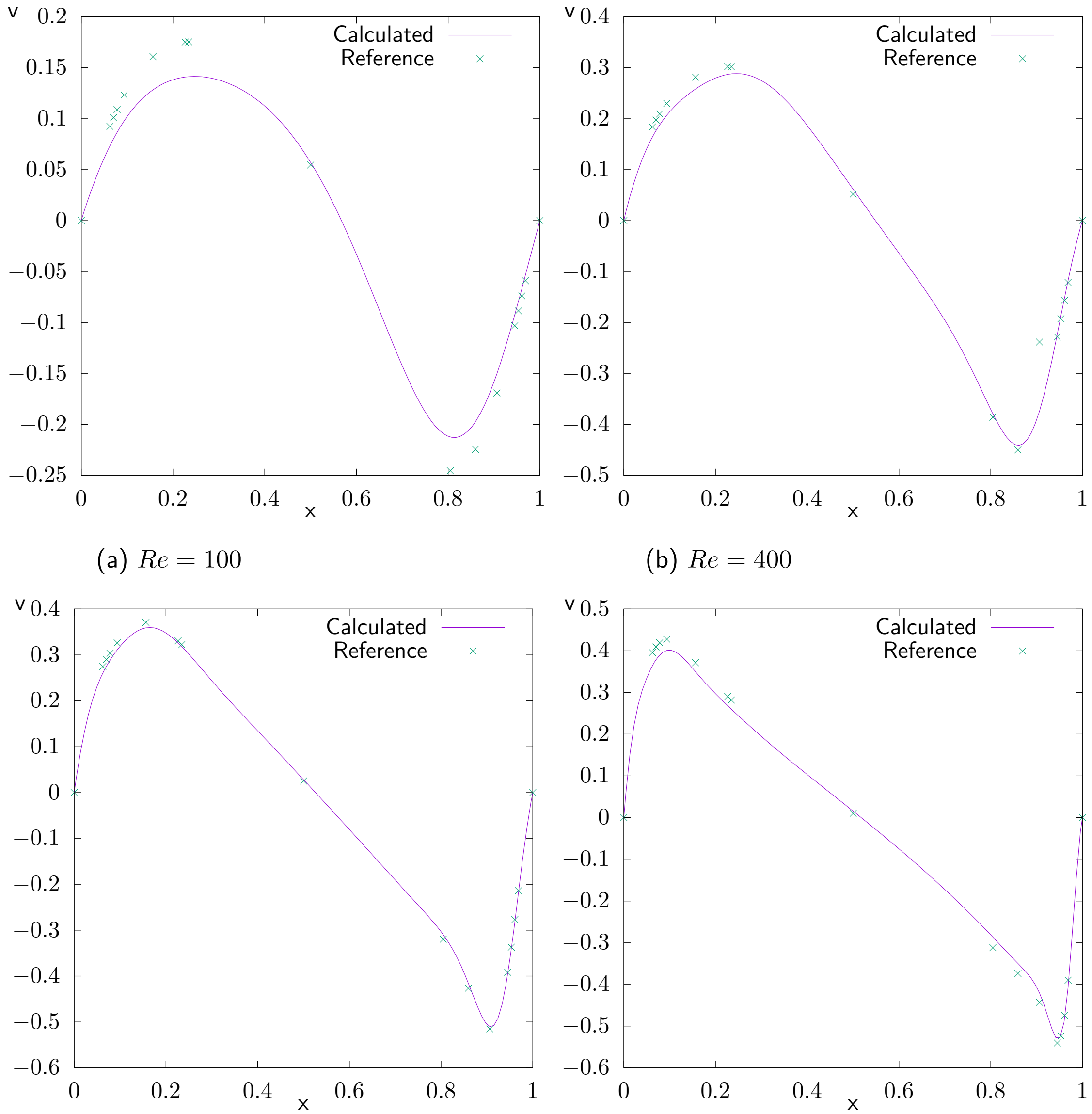


(f) $Re = 7500$



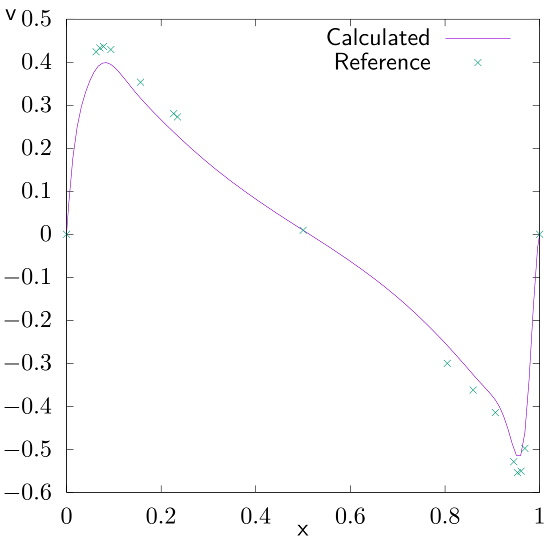
(g) $Re = 10000$

Figure 2.1: Comparison between the reference solution and the calculated one of the horizontal velocity along the vertical line in the geometric centre of the cavity [1]

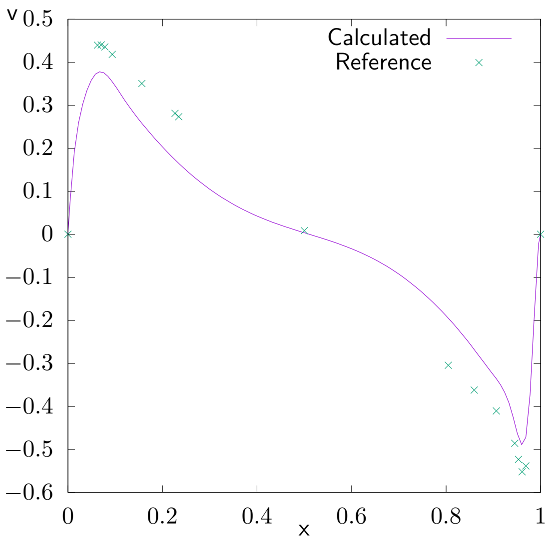


(a) $Re = 100$ (b) $Re = 400$

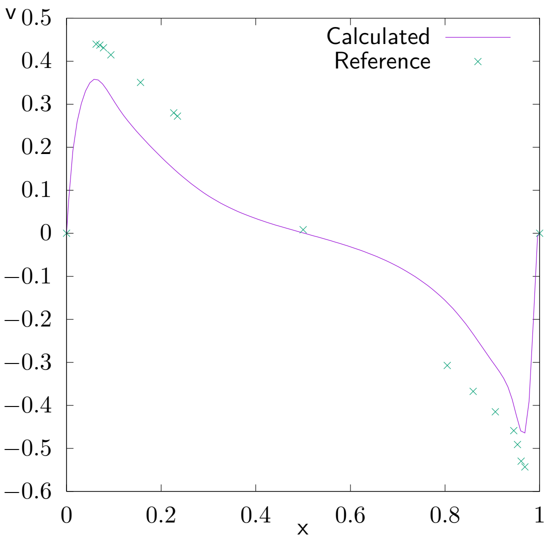
(c) $Re = 1000$ (d) $Re = 3200$



(e) $Re = 5000$



(f) $Re = 7500$



(g) $Re = 10000$

Figure 2.2: Comparison between the reference solution and the calculated one of the vertical velocity along the horizontal line in the geometric centre of the cavity [1]

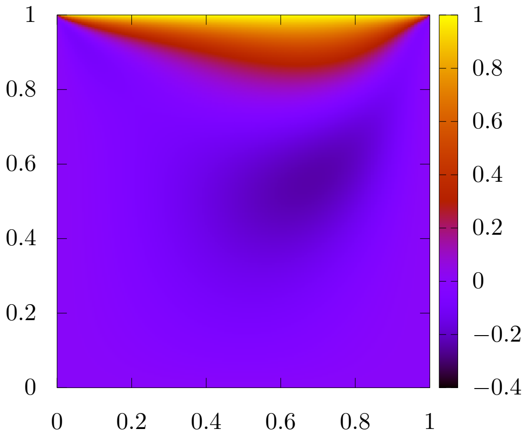
x	Re=100	Re=400	Re=1000	Re=3200	Re=5000	Re=7500	Re=10000
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
0.9766	0.53%	0.41%	1.07%	5.91%	7.13%	15.08%	28.11%
0.9688	0.72%	0.49%	1.14%	6.57%	8.95%	17.61%	32.03%
0.9609	1.04%	0.77%	1.69%	7.74%	10.25%	18.46%	33.60%
0.9531	1.32%	1.06%	2.40%	8.22%	10.50%	18.68%	34.58%
0.8516	11.04%	2.76%	3.10%	8.88%	15.28%	32.24%	52.58%
0.7344	1020.07%	2.66%	2.01%	10.68%	24.81%	50.19%	73.70%
0.6172	-15.41%	25.44%	3.47%	10.85%	31.55%	61.89%	73.63%
0.5	3.19%	-7.42%	-4.01%	18.68%	30.72%	74.06%	34.89%
0.4531	7.98%	-5.56%	-1.91%	91.66%	30.59%	64.80%	112.56%
0.2813	19.18%	2.53%	1.24%	12.55%	21.26%	46.49%	55.93%
0.1719	21.37%	9.01%	3.10%	8.44%	14.73%	29.52%	49.33%
0.1016	20.90%	12.06%	5.98%	5.22%	8.95%	17.16%	36.85%
0.0703	22.33%	13.08%	7.10%	1.78%	10.75%	14.22%	25.13%
0.0625	20.17%	13.27%	7.29%	0.43%	11.22%	14.20%	22.76%
0.0547	19.93%	13.41%	7.39%	-1.15%	11.26%	13.84%	20.37%
0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 2.1: Error of the horizontal velocities in the vertical central plane

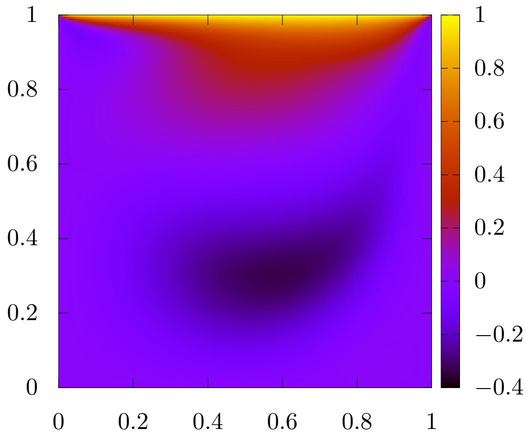
y	Re=100	Re=400	Re=1000	Re=3200	Re=5000	Re=7500	Re=10000
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
0.9688	7.43%	2.41%	-1.00%	-1.16%	11.69%	17.15%	20.24%
0.9609	7.29%	2.02%	-0.95%	0.95%	10.81%	14.42%	17.89%
0.9531	7.43%	1.98%	-0.68%	2.17%	9.08%	11.53%	16.78%
0.9453	7.52%	1.87%	-0.57%	3.11%	7.55%	10.48%	18.96%
0.9063	8.21%	-57.88%	1.57%	5.68%	8.89%	18.94%	37.44%
0.8594	9.09%	1.84%	2.81%	7.39%	12.11%	26.30%	47.77%
0.8047	10.32%	1.77%	3.34%	9.04%	16.06%	35.75%	53.28%
0.5	-6.69%	-14.99%	-10.95%	-40.59%	3.41%	61.67%	331.52%
0.2344	15.90%	3.89%	1.56%	8.54%	18.07%	39.01%	46.30%
0.2266	16.04%	4.24%	1.74%	8.44%	17.50%	37.78%	45.79%
0.1563	16.84%	6.79%	3.61%	6.87%	12.28%	26.49%	44.39%
0.0938	17.15%	8.22%	4.96%	7.54%	9.99%	16.36%	32.74%
0.0781	17.29%	8.58%	5.25%	8.26%	10.81%	15.88%	29.86%
0.0703	17.31%	8.73%	5.40%	8.62%	11.32%	16.21%	29.30%
0.0625	17.29%	8.82%	5.51%	8.93%	11.70%	16.66%	29.23%
0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 2.2: Error of the vertical velocities in the horizontal central plane

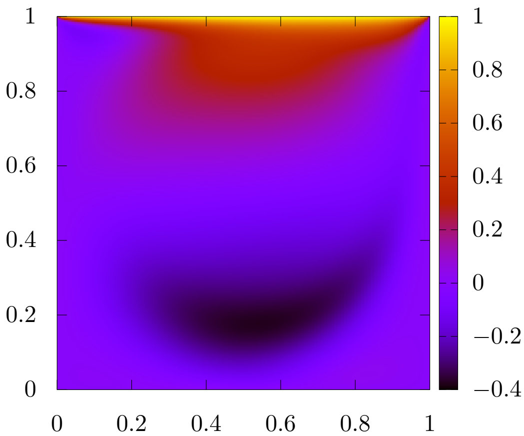
2.1 Horizontal velocities



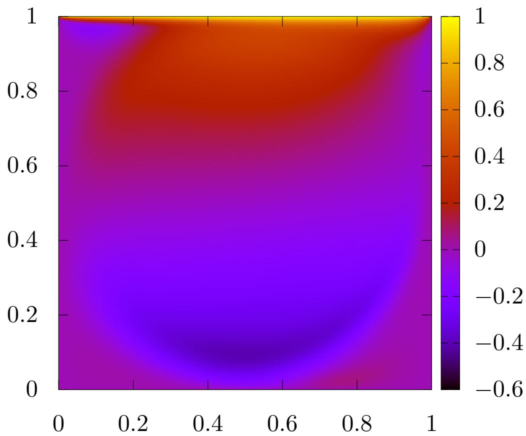
(a) $Re = 100$



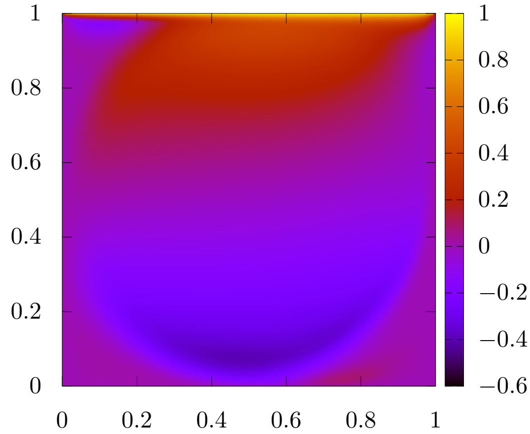
(b) $Re = 400$



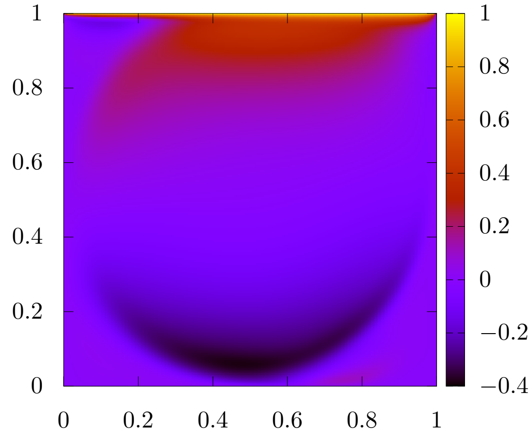
(c) $Re = 1000$



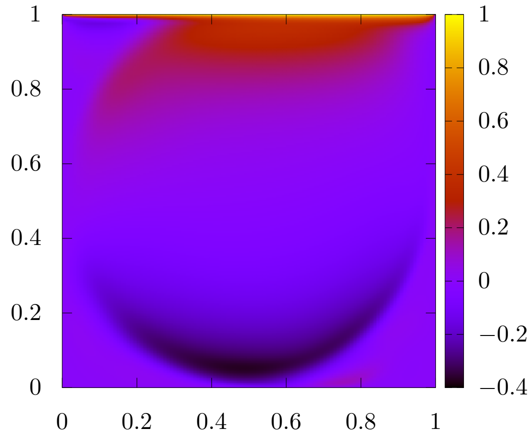
(d) $Re = 3200$



(e) $Re = 5000$



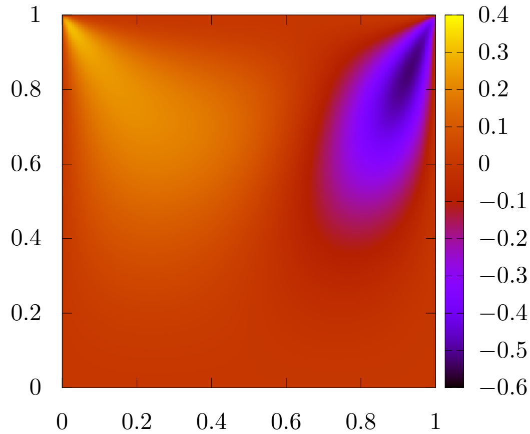
(f) $Re = 7500$



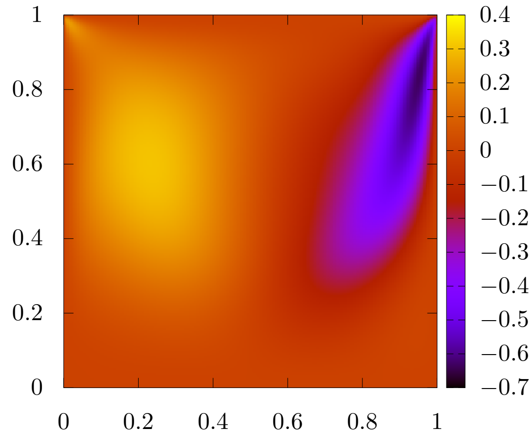
(g) $Re = 10000$

Figure 2.3: Horizontal velocity inside the cavity

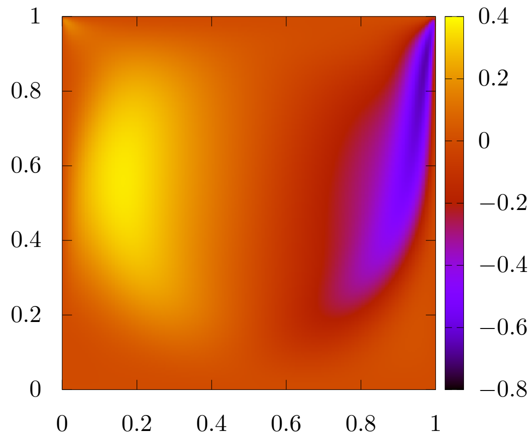
2.2 Vertical velocities



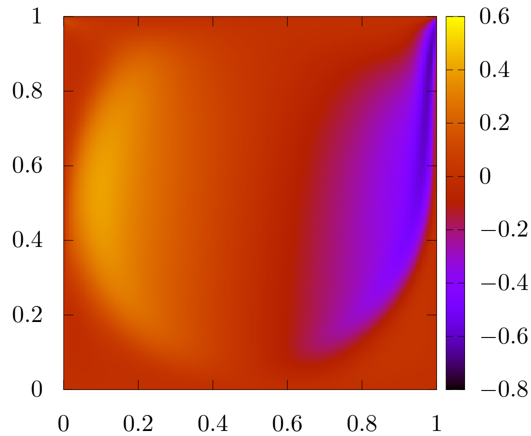
(a) $Re = 100$



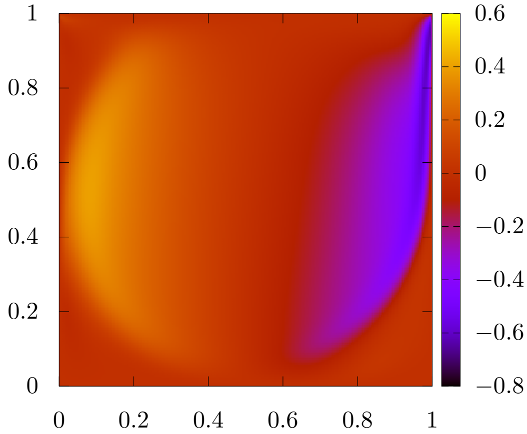
(b) $Re = 400$



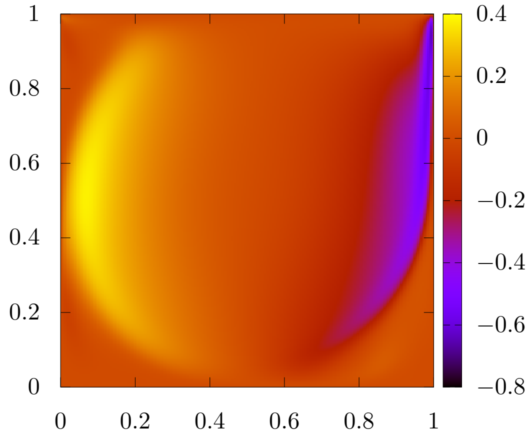
(c) $Re = 1000$



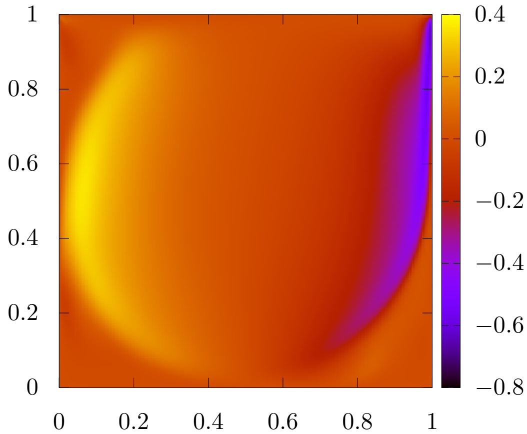
(d) $Re = 3200$



(e) $Re = 5000$



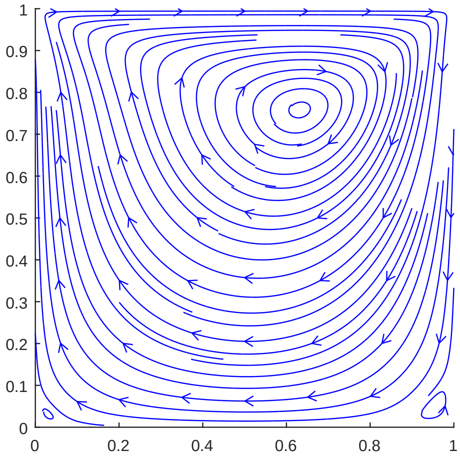
(f) $Re = 7500$



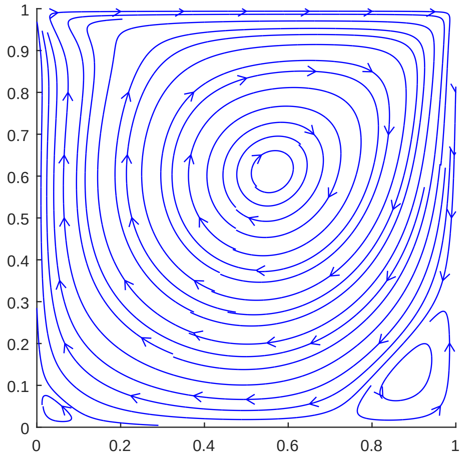
(g) $Re = 10000$

Figure 2.4: Vertical velocity inside the cavity

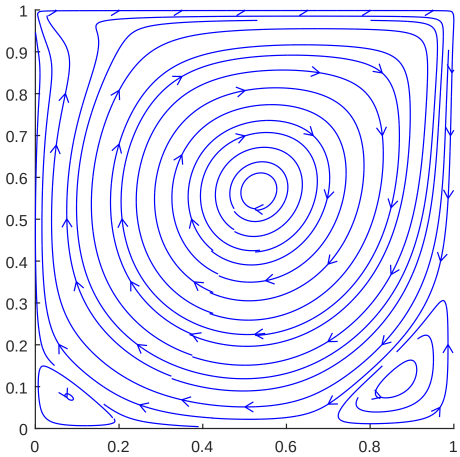
2.3 Streamlines



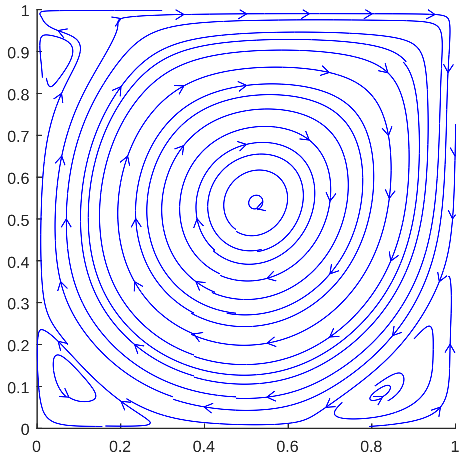
(a) $Re = 100$



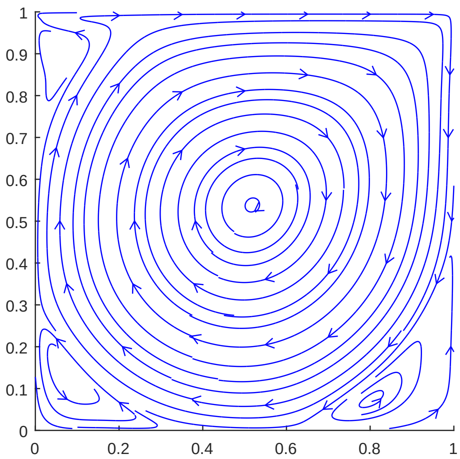
(b) $Re = 400$



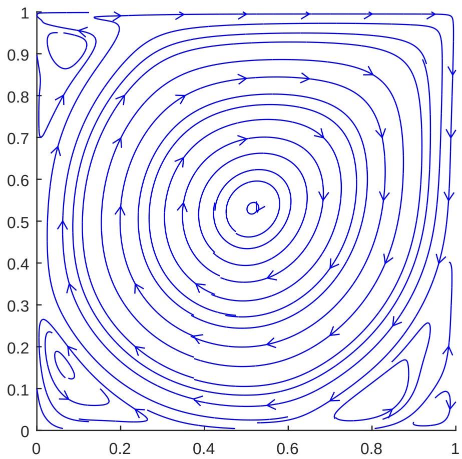
(c) $Re = 1000$



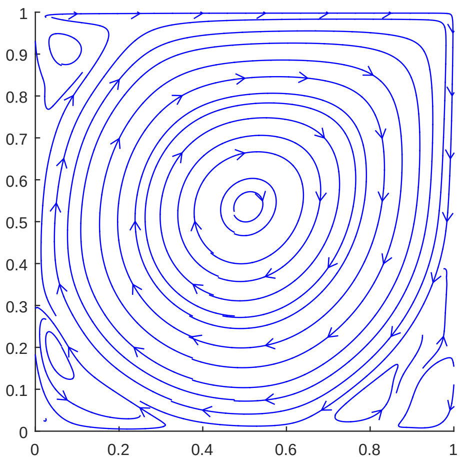
(d) $Re = 3200$



(e) $Re = 5000$



(f) $Re = 7500$



(g) $Re = 10000$

Figure 2.5: Streamlines of the flow inside the cavity

3 Buoyancy

	$Ra = 10^3$		$Ra = 10^4$		$Ra = 10^5$		$Ra = 10^6$	
	Calculated	Bench	Calculated	Bench	Calculated	Bench	Calculated	Bench
u_{max}	3.708	3.649	16.198	16.178	34.508	34.73	66.192	64.63
y	0.81	0.813	0.83	0.823	0.85	0.855	0.85	0.85
v_{max}	3.908	3.697	19.648	19.617	68.663	68.59	220.23	219.36
x	0.17	0.178	0.11	0.119	0.07	0.066	0.03	0.038
$\bar{Nu}$	1.122	1.118	2.244	2.243	4.535	4.519	9.031	8.8
$Nu_{1/2}$	1.095	1.118	2.228	2.243	4.536	4.519	9.110	8.799
Nu_0	1.367	1.117	2.331	2.238	4.669	4.509	9.419	8.817
Nu_{max}	1.795	1.505	3.626	3.528	8.045	7.717	20.019	17.925
y	0.09	0.092	0.15	0.143	0.07	0.081	0.03	0.038
Nu_{min}	0.873	0.692	0.616	0.586	0.764	0.729	1.062	0.989
y	1	1	1	1	1	1	1	1

Table 3.1: Comparison between the benchmark solution and the calculated one [2]

	$Ra = 10^3$	$Ra = 10^4$	$Ra = 10^5$	$Ra = 10^6$
u_{max}	-1.627%	-0.126%	0.640%	-2.417%
y	0.369%	-0.851%	0.585%	0.000%
v_{max}	-5.715%	-0.159%	-0.106%	-0.397%
x	4.494%	7.563%	-6.061%	20.844%
$\bar{Nu}$	-0.358%	-0.046%	-0.355%	-2.620%
$Nu_{1/2}$	2.049%	0.654%	-0.375%	-3.540%
Nu_0	-22.343%	-4.158%	-3.547%	-6.830%
Nu_{max}	-19.282%	-2.787%	-4.251%	-11.681%
y	2.174%	-4.895%	13.580%	20.635%
Nu_{min}	-26.116%	-5.128%	-4.742%	-7.386%
y	0.000%	0.000%	0.000%	0.000%

Table 3.2: Percentage of error between the benchmark solution and the calculated one

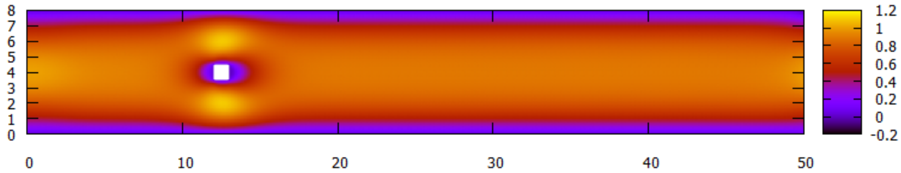
4 Square cylinder

4.1 Recirculation length

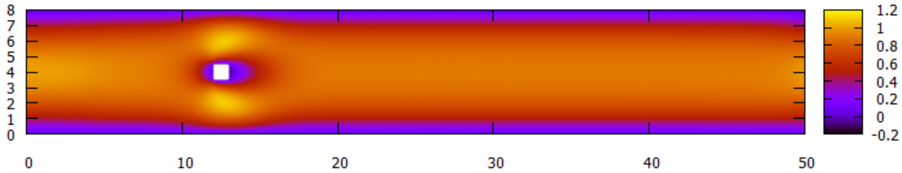
Re	Reference	Calculated
1	-0.0096	0.1067
3	0.1012	0.1865
5	0.212	0.2650
10	0.489	0.4474
30	1.597	1.4971
50	2.705	2.7702

Table 4.1: Comparison of the recirculation length (Reference value vs. Calculated value) [3]

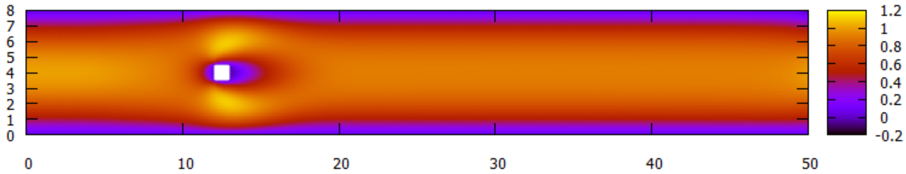
4.2 Horizontal velocity



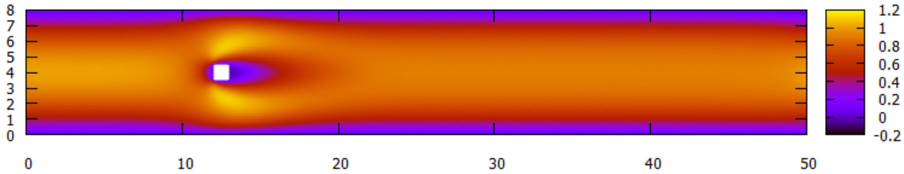
(a) $Re = 1$



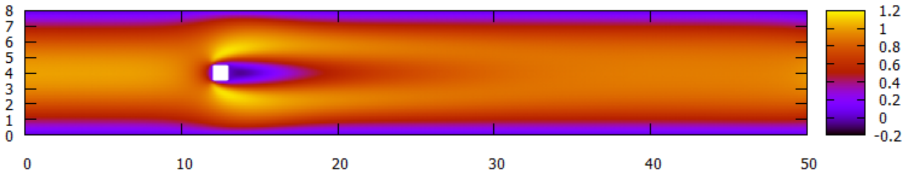
(b) $Re = 3$



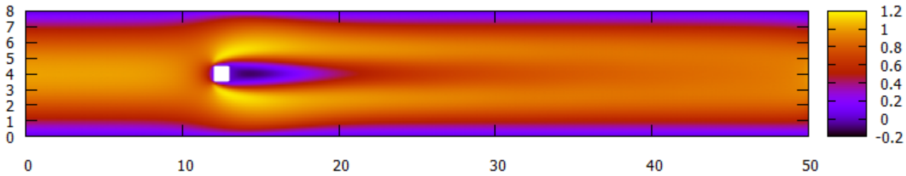
(c) $Re = 5$



(d) $Re = 10$



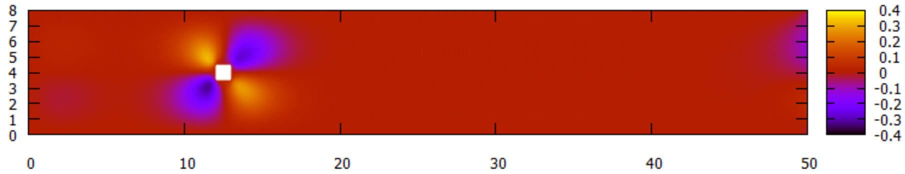
(e) $Re = 30$



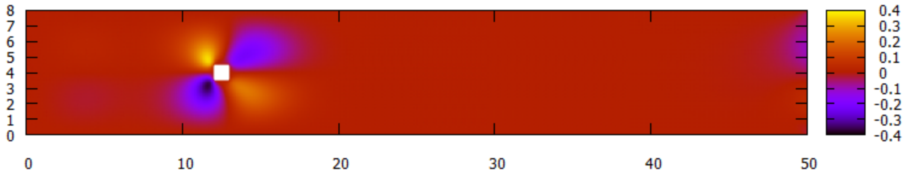
(f) $Re = 50$

Figure 4.1: Horizontal velocity in the channel

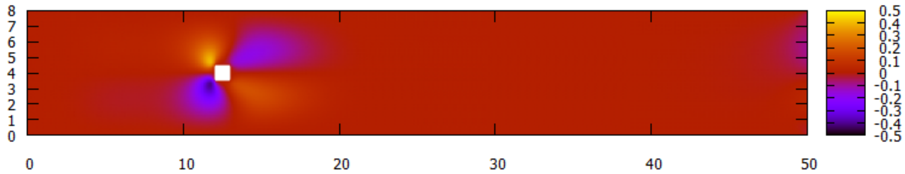
4.3 Vertical velocity



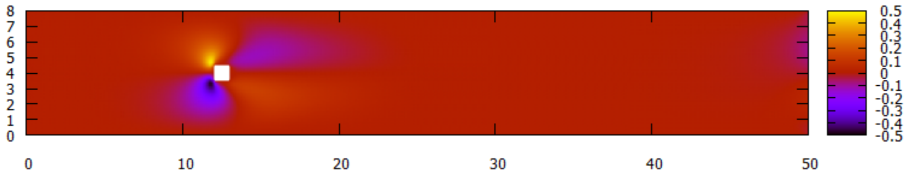
(a) $Re = 1$



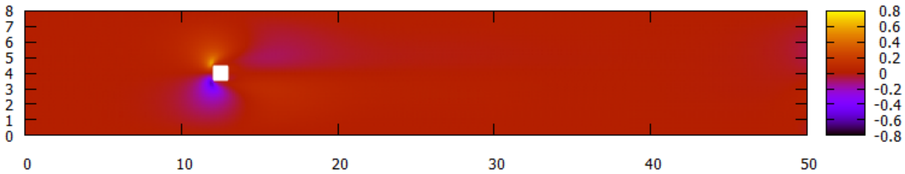
(b) $Re = 3$



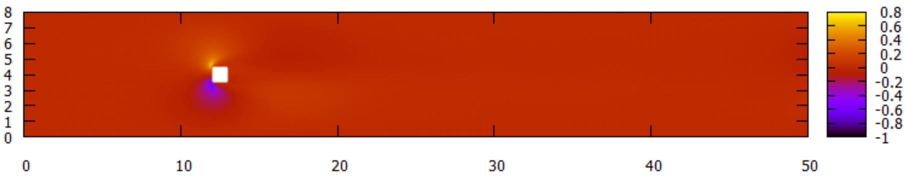
(c) $Re = 5$



(d) $Re = 10$



(e) $Re = 30$



(f) $Re = 50$

Figure 4.2: Vertical velocity in the channel

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