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| ESEIAAT - UPC |
| Study for the computational resolution of conservation equations of mass, momentum and energy. Possible application to different aeronautical and industrial engineering problems: Case 1B |
| Report |
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| **[Seleccione la fecha]** |

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Contenido

[1 Introduction 4](#_Toc478301721)

[2 State of the art 5](#_Toc478301722)

[2.1 Numerical methods 5](#_Toc478301723)

[3 Four materials 6](#_Toc478301724)

[3.1 Conduction equation 7](#_Toc478301725)

[3.2 Discretization 7](#_Toc478301726)

[4 Smith-Hutton problem 8](#_Toc478301727)

[4.1 General convection-diffusion equation 8](#_Toc478301728)

[4.2 Discretization 9](#_Toc478301729)

[4.3 Boundary conditions 13](#_Toc478301730)

[4.4 Results 14](#_Toc478301731)

[Figure 1: General scheme of the four materials problem 6](#_Toc478300847)

[Figure 2: General scheme of the Smith-Hutton problem 8](#_Toc478300848)

[Figure 3: Control volumen (2D) 9](#_Toc478300849)

[Table 1: Problem coordinates 6](#_Toc478300857)

[Table 2: Physical properties of the materials 6](#_Toc478300858)

[Table 3: Boundary conditions 6](#_Toc478300859)

[Table 4: Particular cases of the convection-diffusion equation 9](#_Toc478300860)

[Table 5: Function for different schemes 13](#_Toc478300861)

# Introduction

# State of the art

The laws governing the processes of heat transfer and fluid flow are usually expressed in terms of differential equations. Some of these equations are the momentum equation, the energy equation and the mass conservation equation, among others. However, these expressions usually don’t have an analytical solution except for some simple cases. To solve complex problems it is necessary to use numerical methods.

## Numerical methods

Numerical methods are based in dividing the domain that is going to be studied in different pieces. This process is called discretization.

# Four materials

The four materials problem is a two-dimensional transient conduction problem. It consists in a long rod composed of four different materials with different properties. The general scheme of the problem is represented in Figure 1:

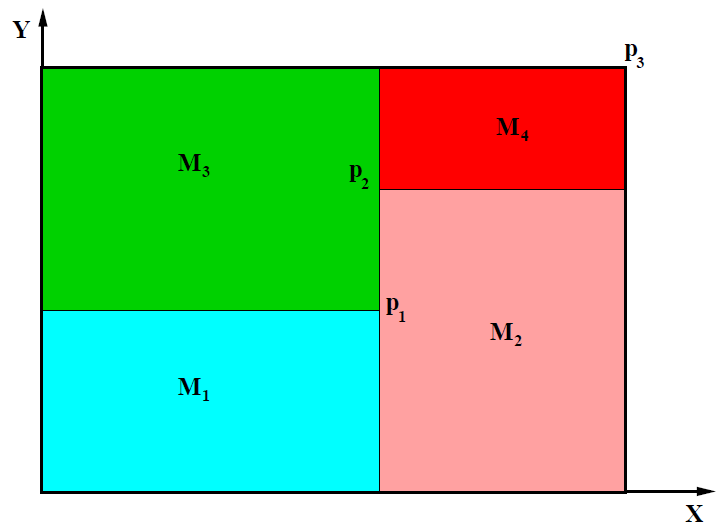


Figure 1: General scheme of the four materials problem

|  |  |  |
| --- | --- | --- |
|  | x [m] | y [m] |
| p1 | 0.50 | 0.40 |
| p2 | 0.50 | 0.70 |
| p3 | 1.10 | 0.80 |

Table : Problem coordinates

|  |  |  |  |
| --- | --- | --- | --- |
|  | ρ [kg/m3] | cP [J/kgK] | λ [W/mK] |
| M1 | 1500.00 | 750.00 | 170.00 |
| M2 | 1600.00 | 770.00 | 140.00 |
| M3 | 1900.00 | 810.00 | 200.00 |
| M4 | 2500.00 | 930.00 | 140.00 |

Table 2: Physical properties of the materials

|  |  |
| --- | --- |
| Cavity wall | Boundary condition |
| Bottom | Isotherm at |
| Top | Uniform length |
| Left | In contact with a fluid at and heat transfer coefficient |
| Right | Uniform temperature (where is the time in seconds) |

Table : Boundary conditions

The initial temperature field is .

## Conduction equation

The conduction heat transfer is described by Equation 1:

Equation 1: Conduction equation

Where is the density, the temperature, the conductivity of the material, the specific heat of the material and the inner heat of the material (source term).

## Discretization

The domain is discretized using the node centred distribution, to avoid having conflictive control volumes between the different materials. The discretization of the equation begins with the following procedure:

# Smith-Hutton problem

The following problem is a two-dimensional steady convection-diffusion problem, represented in Figure 2:

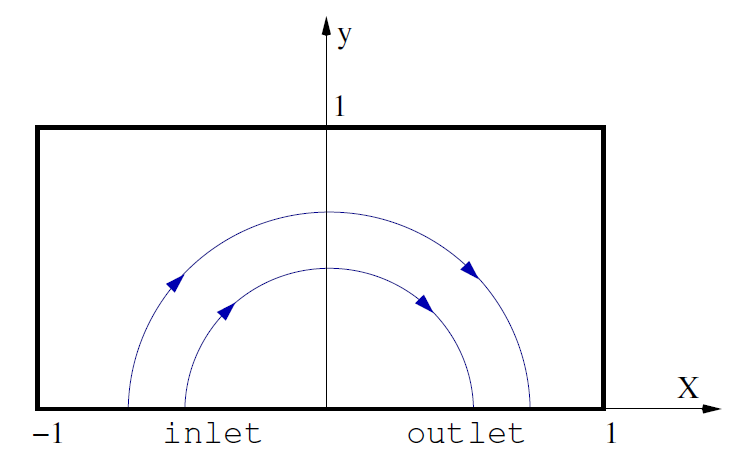


Figure 2: General scheme of the Smith-Hutton problem

The velocity field is given by:

Equation : Prescribed velocity

And the boundary conditions for the variable are:

Equation 3: Boundary conditions of the Smith-Hutton problem

Where .

## General convection-diffusion equation

The mathematical formulation needed to solve this problem is the convection diffusion-equation:

Equation 4: Generic convection-diffusion equation

Where is the density, the velocity, the diffusion coefficient, the source term, and the general variable that is going to be studied.

The conservation equations can be seen as particular cases of the convection-diffusion equation. For example, in Table 1:

|  |  |  |  |
| --- | --- | --- | --- |
| Equation | φ | Γ | Sφ |
| Mass conservation | 1 | 0 | 0 |
| Momentum |  | μ |  |
| Energy (for a semi perfect gas) | u | λ/cv |  |

Table 4: Particular cases of the convection-diffusion equation

## Discretization

It is necessary to discretize Equation 2 in space and time. The control volume used to discretize the problem is specified in Figure 1:

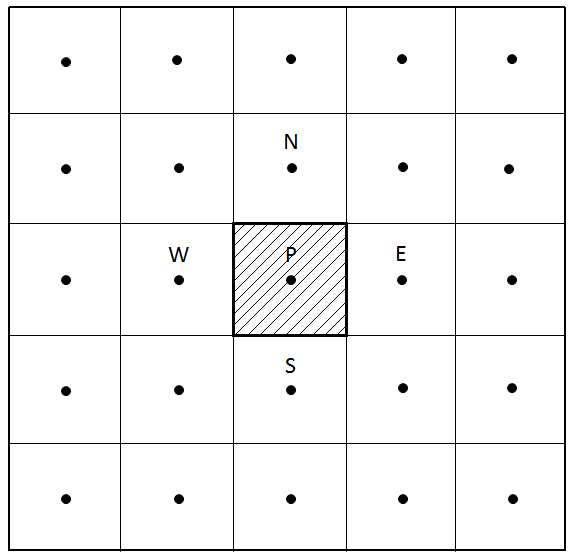


Figure 3: Control volumen (2D)

To do so, it is easier to start with the discretization of the mass equation:

Equation 5: Mass conservation equation

The first step is to integrate Equation 3 over time and space. Taking only the first term of the equation:

Using the Divergence Theorem, the second term of the equation transforms into a surface integral:

In the second term, to integrate over time an implicit scheme is used. The final discretized mass equation is:

Equation 6: Discretized mass equation (2D)

The discretization of the convection-diffusion equation is very similar to that of the mass equation. Integrating over the volume the transport term of Equation 2 and applying the Divergence Theorem:

The same procedure is used for the diffusion term:

Where:

To simplify the source term, it is linearized:

So that the resulting equation is:

Multiplying by the discretized mass equation (Equation 4) and subtracting the result in this equation, the discretized convection-diffusion equation is obtained:

To simplify the problem, a new variable is introduced to the problem: the total flux. But this variable is split in the two dimensions of the problem, the flux in the x-direction and the flux in the y-direction.

Equation 7: Total (convection plus diffusion) fluxes

Introducing the expressions of the total flux, the discretized convection diffusion equation becomes:

Equation 8: Final discretization of the convection diffusion equation

Where the flow rates are:

However, it is necessary to know how the fluxes are going to be evaluated. In order to use adimensional numbers, a new variable is defined:

Where is the Peclet number and is the distance between the point that is going to be studied *i*, and the point next to it, *i+1*. The value of φ and the value of the gradient are a combination of the and , so that can be expressed as:

The coefficients and are dimensionless and depend on the Peclet number. However, since B is a combination of A and the Peclet number, and both coefficients are symmetric, it can be deduced that:

Equation 9: Dimensionless coefficients A and B

Where is an operator that denotes the greater of A and B.

Combining Equation 6 and Equation 7, the following formulation is obtained:

Where

Equation 10: Discretized coefficients of the Smith-Hutton problem

And the Peclet numbers are:

The only operation that should be defined is the value of the coefficient . This value depends on the integration scheme that is going to be used. Some of its values are in Table 2:

|  |  |
| --- | --- |
| Scheme | Formula for |
| Central difference (CDS) |  |
| Upwind (UDS) |  |
| Hybrid (HDS) |  |
| Power law (PLDS) |  |
| Exponential (EDS) |  |

Table 5: Function for different schemes

For a better explanation of the different schemes used, search PATANKAR.

## Boundary conditions

To apply the boundary conditions to the problem, some modifications on the coefficients defined by Equation 10 have to be done. The easiest one is for the points in the left, top and right sides of the domain, in which the value of is defined.

Equation 11: Discretized coefficients for the boundary points

In the bottom it is necessary to distinguish between the inlet and the outlet. A similar approach to that of the left, top and right side is used to determine the coefficients of the points in the inlet:

Equation 12: Discretized coefficients for the inlet

However, the most difficult point is that of the outlet. The only condition is that the derivative of in the vertical direction is equal to zero. Following this reasoning, the following coefficients are obtained:

Equation 13: Discretized coefficients for the outlet

## Results