Probing axion-like particles and dilatons with pulsar timing

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We use the extreme precision of pulsar timing to constrain the coupling of axions or axion-like particles (ALPs) to photons and electrons. We do this through two different effects: first, the oscillation of photons into ALPs within and near the pulsar magnetosphere modifies the time of arrival of the signals in a frequency-dependent way non-degenerate with other effects considered in pulsar timing. Second, if the ALP has coherent oscillations, those can be detected as a time dependent birrefringence or time dependent modifications of the DM. The bounds found are...

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I. INTRODUCTION

db:Axions as DM candidate; ALPs in general

The use of high-precision data to look for particles weakly interacting with the standard model (SM) is currently living a golden era []. Among these data sets, one having a particularly exquisite accuracy are the measurements of radio signals from pulsars []. The data typically includes of time of arrival (TOA) at different frequencies and polarization and for different pulsars. The purpose of this work is showing how these data can be used to put constraints on possible couplings between photons and electrons and new light particles that may (or may not) have a non-trivial expectation value. We do this by deriving the difference in the arrival time of photons of different frequencies and changes in the polarization.

II. EQUATIONS OF MOTION IN MAGNETIC AND ALPINE BACKGROUND

We use the convention $\{+,-,-,-\}$ for the metric and the rest of conventions from Ref. [1]. We will only consider the cases where the SM is supplemented by an extra spin-0 field that can be a scalar or a pseudoscalar. The Lagrangian density in $c = \hbar = 1$ units reads (cf., e.g., Ref. [2])

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \mathcal{L}_{eA} + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m_\phi^2}{2}\phi^2 + \mathcal{L}_{\phi\, SM}, \quad (1)$$

where $\mathcal{L}_{eA}$ represents the SM term for the electrons (including the coupling to photons A_μ), $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and

$$\mathcal{L}_{\phi\, SM} = -\kappa\phi\left[\frac{d_e}{4}F_{\mu\nu}F^{\mu\nu} + d_{m_e} m_e \bar{e}e\right], \quad (2)$$

in the case of scalars [3] and

$$\mathcal{L}_{\phi\, SM} = -\frac{g^0_{a\gamma\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}\,, \quad (3)$$

with $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$, in the case of pseudo-scalars [1]. We also introduced $\kappa = \sqrt{4\pi}/M_{Pl}$. A particularly well-motivated case is that of the QCD axion [4, 5], for which [6]

$$g^0_{a\gamma\gamma} := \frac{\alpha}{2\pi f_a}\frac{E}{N}\,, \quad (4)$$

with $\alpha = 1/137$ and E/N is the ratio of the electromagnetic and the color anomaly ($E/N = 8/3$ for complete $SU(5)$ representations and $E/N = \mathcal{O}(1)$ in general). Here f_a is the axion decay constant whose phenomenological viable range is [2]

$$10^9\,\mathrm{GeV} \lesssim f_a \lesssim 10^{13}\,\mathrm{GeV}\,. \quad (5)$$

The QCD axion mass is related to f_a by

$$m \approx 5.7\mu\mathrm{eV}\left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right)\,. \quad (6)$$

From Eq. (5), the viable axion mass range is $10^{-6}\,\mathrm{eV} \lesssim m \lesssim 10^{-2}\,\mathrm{eV}$. On the other hand, for ALPs $g^0_{a\gamma\gamma}$ and m are unrelated (see, e.g., Ref. [7] for a review). In any case, we will only consider light candidates in this work. We will define $g \equiv g^0_{a\gamma\gamma}$ in the following.

Concerning dilaton coupling, they are also ubiquitous in extensions of the SM [3, 8] ***db:More on bounds***.

We will be interested in the physical situation where an electromagnetic wave is emitted in a pulsar magnetosphere, propagates through the interstellar medium (ISM) and is eventually detected on Earth. In this physical situation, the fields present in Eq. (1) have different backgrounds over which the signal propagates. The effect of the background of electrons will be taken into account by the effects in the dispersion relations of photons[1]. For

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[1] If we were interested in the propagation of ϕ when $d_{m_e} \neq 0$,

the electromagnetic field, we will only consider the possibility of background magnetic fields[2]. Finally, it is particularly interesting to consider the situations where the (pseudo) scalar field ϕ is responsible for the dark matter in the universe. We will focus in the cases with small masses for which the occupation numbers in the field are so high that it can be treated as a condensate with profile

$$\bar{\phi}(t, \vec{x}) \ = \phi_0 \cos(m_\phi t - \vec{k}_\phi \cdot \vec{x}), \tag{7}$$

which one can connect to the dark matter (DMa) abundance[3] as

$$\rho_{\rm DMa} = \frac{1}{2} m_\phi^2 \phi^2 \approx 0.4\,{\rm GeV/cm^3}. \tag{8}$$

In this paradigm, one assumes the wave vector to be related to the virial velocity as $|k_\phi| \approx m_\phi v_{\rm vir} \approx 10^{-3} m_\phi$ [8]. Thus, we see that space derivatives are typically subdominant with respect to time derivatives. This also immediately means

from rho dm?

$$\kappa\phi_0 \approx \left(\frac{10^{-29}\ {\rm eV}}{m_\phi}\right). \tag{9}$$

The scalar and Maxwell equations in the axionic case read

$$(\Box - m^2)\phi = -\frac{g}{4} F_{\mu\nu}\tilde{F}^{\mu\nu}\,, \tag{10}$$

$$\partial_\mu F^{\mu\nu} = -\frac{g}{2}\epsilon^{\mu\rho\lambda\nu} F_{\mu\rho}\partial_\lambda\phi\,, \tag{11}$$

while for the scalar case are

$$(\Box - m^2)\phi = -\frac{d_e\kappa}{4} F_{\mu\nu}F^{\mu\nu}\,, \tag{12}$$

$$\partial_\mu((1 + d_e\kappa\,\phi))F^{\mu\nu}) = 0. \tag{13}$$

Equation (13) already tells us that the effect of the background (7) for oscillating frequencies below the typical frequency of the wave (radio) is simply modifying the electric charge by

$$\frac{1}{\alpha} \equiv \frac{4\pi}{e^2} \rightarrow \frac{4\pi(1 + d_e\kappa\,\phi)}{e^2}. \tag{14}$$

Similarly, the corrections coming from d_{m_e} will be taking into account by a considering a time dependent electron mass

$$m_e \rightarrow (1 + d_{m_e}\kappa\,\phi)m_e. \tag{15}$$

in the relevant calculations (again, we are always in the limit of very slowly evolving background). Any test of the variations of the fundamental constants α or m_e can then be translated into a test on the couplings of the (pseudo) scalar fields to the SM. Notice that one may still get constraints compatible with other observations ***db:Atomic clocks, EXPRESSO [10]*** due to the different time scales. The effect in the propagation due to the oscillations in the DMa potentials (their effect in geodesic propagation) have been already discussed in [11] which is a nice paper to get inspiration.

These equations should be supplemented by the corrections due to vacuum polarization in the case of very strong magnetic fields $B \gtrsim 10^{13}$ G [12]. They amount to a correction in the Lagrangian

$$\mathcal{L}_{QED} = \frac{\alpha^2}{90 m_e^4}\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right] \tag{16}$$

Here α and m_e are in principle time dependent (though it is not clear yet if this will be observable). Normally (cf. [12]), this term breaks the degeneracy of photons and scalars, and makes the bounds weaker.

III. EFFECTS ON TIMING

Let us write the standard formula of timing of the different polarizations $\pm$ as

$$t(\pm, \omega) = t_e(\pm, \omega) + t_p(\pm, \omega) + t_d(\pm, \omega), \tag{17}$$

where $(\pm, \omega)$ refers to the TOA of different polarizations and frequency, t_e refers to the contribution from emission (e.g. the physics close to neutron star or binary system), t_p refers to the effects due to interstellar propagation and t_d represents the terms having to do with detection (e.g. comparison with an atomic clock, Solar system physics,...). All of these contributions may be affected by the presence of the axionic 'oscillating' field. Let us mention some possibilities:

- For t_e
  1. At emission, the presence of the magnetic field of the pulsar will change the dispersion relation of the radio waves. As a result, one expects extra contribution to the residual of the TOA at different frequencies (sec. IV). Maybe the only effects are on the shape and polarization (the influence of the magnetosphere is never discussed in timing).
  2. One may try to look for the oscillating redshift effect, due to the attraction of the pulsar to the Galactic halo (whose gravitational field is sourced by (7), and thus oscillates).
  3. One can try to look for changes in the dynamics of the binary interacting with the axionic field. Change in acceleration due to the

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we should also consider the background of electrons as a source for the field ϕ. These effects are ignored in the present analysis. They would impact the propagation in the magnetosphere (where we are indeed interested in the propagation of ϕ) but it is not clear now how strong the effect is. So far we only consider the ISM.

[2] The effect of background electric fields is ignored since they are very small next to the pulsar [9], and not coherent and small in the ISM ***db:Here I improvised a bit***.

[3] We warn the reader that we will reserved the acronym DM for *dispersion measurement*, to be introduced later.

Galactic gravitational field. Dynamical friction. Friction due to the interaction of the form (2) (a sort of fifth force). There will be a conservative and a radiative component. The ideal would be to find some resonant effect.

4. Finally, ω_B and ω_p will oscillate in this theory. I wonder if there is a way to look for this (maybe in more refined spectral properties).

- For t_p
  1. There is an effect on TOA from the oscillatory gravitational field sourced by (7). This is similar to GWs and already studied in [11].
  2. The interstellar magnetic field also generates some effects, which are well studied in the case of light from far-away objects []. For TOA they are smaller than the one close to the pulsar (sec. IV).
  3. The interstellar axion field (7) generates an extra contribution to Faraday rotation that one can try to look for in data (sec. V).
  4. The standard DM column depends on the fundamental constants e and m_e. Both oscillate with time (due to the couplings (14) and (15)), so one can look for oscillations in DM (recall that DM(t) is known for several pulsars).
- For t_d
  1. The physics of the Solar system may also be affected by the oscillating axion.
  2. The physics of the atomic clock may be affected due to the couplings (14) and (15) (this is already studied in [13]).

The ideal would be to study all these effects. Notice that they correspond to different time scales , which means that they are eventually sensitive to different masses (if the oscillations are too fast they may average out in certain observables. If they are too slow they may be at scales beyond reach). One can also try to constrain the inhomogeneity in the field (7). In this note we will focus in few of the effects: For t_e, first point. For t_p, second, third and forth point.

IV. EFFECTS DUE TO A BACKGROUND MAGNETIC FIELD

The ALP-photon oscillations are described by the coupled system (10)–(11). In the short-wavelength approximation[4], the wave equations will have a contribution from the plasma. There are three different regimes we will consider: the interior of the magnetosphere, the exterior of the magnetosphere and the interstellar medium.

A. Effects in the magnetosphere

db:I'm ignoring the QED part here. It can be easily implemented, but it seems important only for $g \lesssim 10^{-13}\,\mathrm{GeV}$. We work in the limit where the gyrofrequency $\omega_B = eB/m = 10^{19} B_{12}/s$ is large as compared with other frequencies. As the waves propagate outwards, this changes. Still $\omega_B \gg \omega_p$. The typical effects from dispersion are studied in intensity, cyclotron absorption, shape and polarization, e.g. [14–22]. In principle the corrections for large enough gB are of the same order, so one expects observable effects. But nobody discusses timing... The separation between interior and exterior regions is characterized by the quantity

$$A_p \equiv \frac{\omega_p^2}{\omega^2}\gamma_c, \tag{18}$$

being larger or smaller than one [23]. In this formula, $\omega_p \equiv \left(\frac{4\pi e^2 n_e}{m_e}\right)^{1/2} \approx 2\cdot 10^5 \sqrt{n_e/\mathrm{cm}}\,\mathrm{Hz}$, and γ_c is the boost factor of the electrons in the plasma, $\gamma_c \gtrsim 10^3$. For n_e we consider $n_e = n_{GJ}\lambda \approx 10^{11} B_{12}/P_1 (R/r)^3 \lambda\,\mathrm{cm}^{-3}$, with a multiplicity factor $\lambda = 10^2 - 10^6$ and $R \equiv R_{star}$. We take $\lambda = 10^3$. Normalized by the values above, the quantity A_p is of order

$$A_p \sim 5\times 10^6 \frac{B_{12}\gamma_3\lambda_3}{P_1\nu_1^2}\left(\frac{R}{r}\right)^3 , \tag{19}$$

and therefore typically $A_p \gg 1$ near the stellar surface.

The equations of motion for the coupled system are[5] (we choose z in the direction of the external magnetic field $\bar{B}$)

$$\begin{aligned} \left(k^2\delta_{\alpha\beta} - k_\alpha k_\beta - \omega^2\epsilon_{\alpha\beta}\right)E_\beta + g\bar{B}\omega\phi\delta_{\alpha 3} &= 0, \\ \bar{B}E_z g\omega + (k^2+m^2-\omega^2)\phi &= 0. \end{aligned} \tag{20}$$

The reason why we keep the three polarizations of $\vec{E}$ is that for $A_p \gg 1$, the mode that eventually leaves the magnetosphere is not always transverse [15]. In the magnetosphere, the plasma is basically moving along the magnetic field, which generates the dielectric permittivity tensor

$$\epsilon_{\alpha\beta} = \delta_{\alpha\beta} - \left\langle \frac{\omega_p^2}{\gamma_c^3(\omega - k_z v)^2}\right\rangle \delta_{\alpha 3}\delta_{\beta 3} \tag{21}$$

[4] In our case, the photon/ALP wavelength, $\lambda \sim 30(\mathrm{GHz}/\nu)\,\mathrm{cm}$, is much shorter than the typical length scale of the magnetic field of the pulsar.

[5] For the propagation in a neutron star atmosphere see [24, 25].

where v is the velocity along z and $\gamma_c = (1-v^2)^{-1/2}$. To find the dispersion relation we will work in the small angle approximation with $k_z = k\cos(\theta)$, and look for solutions for the EM modes eventually leaving the magnetosphere. One easily finds that there are modes with $n = k/\omega = 1$ which propagate in a straight line. One expects that these modes will not be amplified enough to provide a strong radio emission. There is another mode which becomes transverse as it propagates outwards in the magnetosphere, but it is a plasma mode from the point of generation (at $r_{em} \sim 10-100\ km$) till the place where $\theta \sim \theta_* \equiv \left(\frac{\omega_p}{\omega \gamma_c^{1/2}}\right)^{1/2}$. From there on, the wave keeps its transverse nature. One must be careful when arriving to $r_A \sim 10^2 R$ where $A_p \sim 1$, since our approximations change there. Finally, the wave arrives at the 'escape' point at $r_{esc} \sim 10^3 R$ [15]. From r_{esc} the wave propagates in a straight line (no more magnetosphere).

We start at $r \ll r_A$. We take an approximation where θ, θ_* and $g\bar{B}/\omega \sim 5\times 10^{-5} B_{12} g_{-11} (R/r)^3 \nu_1^{-1}$, $m/\omega \sim 2\times 10^{-5} m_{-10}/\nu_1$ are smaller than unity. The dispersion relation will be close to $n = 1$. One knows that the relevant dispersion relation is [15, 23]

$$n = 1 + \frac{\theta^2}{4} - \left(\theta_*^4 + \frac{\theta^4}{16}\right)^{1/2} + F(g,\omega,\theta,\theta^*,m), \quad (22)$$

where $F(g=0,\omega,\theta,\theta^*,m) = 0$. If one assumes that the correction F is of the same order as θ^2, one needs to solve a cubic equation. One can show that the solutions to the equation are real (*casus irreducibilis*), and can be found in a complicated form (exact to this order). [Paolo: Checking this case]

One can also assume that F is smaller than the other small parameters, and still larger than θ^4 (assuming a small mass does not change anything substantially). In that case, the extra equation can be solved linearly, yielding

$$F = \frac{g^2B^2\left(\theta^4\left(G(\theta,\theta_*) - \theta^2\right) + 4\theta_*^4\left(G(\theta,\theta_*) - 3\theta^2\right)\right)}{2\left(m^2\left(\theta^4 - \theta^2 G(\theta,\theta_*) + 16\theta_*^4\right) + 8\theta_*^4\omega^2 G(\theta,\theta_*)\right)} \quad (23)$$

with $G(\theta,\theta_*) = \sqrt{\theta^4 + 16\theta_*^4}$. [Paolo: In this case, F is much smaller than unity, $F \sim 10^{-5}$ for the the extreme value $B(r=R) \sim 10^{15}$ Gauss. Furthermore, in the relevant parameter space, the dependence of F on the frequency is $F \sim 1/\omega^2$. Therefore it's a small effect which would produce a difference in the TOA that is degenerate with the DM of the ISM, $n \sim 1 + \omega_p^2/(2\omega^2)$.]

For $r > r_A$, the case $g = 0$ is very easy: two transverse modes with dispersion relation satisfying $n = 1$ and

$$n = 1 + \frac{\omega_p^2 \gamma_c}{2\omega^2}\theta^2. \quad (24)$$

Adding the g coupling produces a correction which is larger[6] than the second term in the previous expression. Indeed

$$n = 1 + \frac{B^2g^2 + m^2 - \sqrt{B^4g^4 + 2B^2g^2\left(2\theta^2\omega^2 + m^2\right) + m^4}}{4\omega^2} \quad (25)$$

db:I don't know if this formula should agree with some of the roots of the case (32). It seems that the plasma effects are different because they are multiplied by θ^2 in the case where the plasma flows along B

To lowest order in θ, the expression above reduces to

$$n = 1 - \frac{B^2g^2}{2(B^2g^2+m^2)}\theta^2\,, \quad (26)$$

which does not depend on ω.

B. Plasma effects outside the magnestosphere

The propagation along the z direction is described by the first-order system [12]

$$i\frac{\partial}{\partial z}\left|\psi(z)\right\rangle = \mathcal{M}\left|\psi(z)\right\rangle\,, \quad (27)$$

where the three-vector $|\psi(z)\rangle$ is a linear combination of the two photon polarizations along the x and y directions and of the ALP state. The 3×3 mixing matrix M reads [26]

$$\mathcal{M} := \begin{pmatrix} \omega + \Delta_{xx} & \Delta_{xy} & gB_x/2 \\ \Delta_{yx} & \omega + \Delta_{yy} & gB_y/2 \\ gB_x/2 & gB_y/2 & \omega - m^2/(2\omega) \end{pmatrix}\,, \quad (28)$$

where ω is the photon/ALP energy. For simplicity, in the following we assume that $\vec{B}$ is polarized along the y direction, $B_x = 0$, $B_y \equiv B$.

The off-diagonal terms Δ_{xy} and Δ_{yx} give rise to Faraday rotation by mixing the photon polarizations and do not play a role in our analysis. The diagonal terms Δ_{xx} and Δ_{yy} contain two independent contributions: QED vacuum polarization effects and plasma effects [12, 26]. Neglecting beam polarization effects,

$$\Delta_{xx}^{\rm QED} \sim \Delta_{xx}^{\rm QED} \sim \omega\zeta := \omega\frac{\alpha}{45\pi}\left(\frac{B}{B_c}\right)^2 \quad (29)$$

where $\alpha = 1/137$ and $B_c = m_e^2/e \approx 4.41\times 10^{13}$ Gauss [27]. We shall consider magnetic fields such that $\zeta \ll 1$, so that higher-order quantum corrections can be safely neglected, although this term is not necessarily smaller than other dimensionless quantities in the problem. On the other hand, plasma effects arise from the presence of free charges in the magnetosphere and yield

$$\Delta_{xx}^{\rm plasma} \sim \Delta_{xx}^{\rm plasma} \sim -\frac{\omega_p^2}{2\omega}\,, \quad (30)$$

[6] [Paolo: I get that the second term is much larger than the axion correction, am I doing something wrong?]

where $\omega_p = \sqrt{4\pi\alpha n_e/m_e}$ is the plasma frequency, n_e and m_e being the electron density of the plasma and the electron mass, respectively.

[Paolo: Off-diagonal QED effects negligible?]

[Paolo: To self: change the discussion to active plasma with cyclotron frequency larger than ω.]

In momentum space, the eigenvalues of Eq. (27) are given by $\det(\mathcal{M} - k\mathbb{1}) = 0$. The latter equation has three roots

$$k_0 = \omega(1+\zeta) - \frac{\omega_p^2}{2\omega}\,, \tag{31}$$

$$k_\pm = \frac{4\omega^2 - \omega_p^2 - m^2 + 2\zeta\omega^2 \mp \sqrt{m^4 + 4B^2g^2\omega^2 + \left(\omega_p^2 - 2\zeta\omega^2\right)\left(\omega_p^2 - 2\zeta\omega^2 - 2m^2\right)}}{4\omega}\,, \tag{32}$$

and the group velocity simply reads $v = \partial\omega/\partial k$.

The time of arrival of a signal travelling at the speed v across a distance d is simply

$$T = \int_0^d \frac{dx}{v} = \int_0^d dx \frac{\partial k}{\partial \omega}\,, \tag{33}$$

where d is the distance of the source and the integral is performed along the line of sight.

To simplify the following expressions, we consider $\omega_p \ll \omega$ and $\zeta \ll 1$ and we keep terms up to $\mathcal{O}(\omega_p^2/\omega^2)$, $\mathcal{O}(\zeta)$ while neglecting $\mathcal{O}(\zeta\omega_p^2/\omega^2)$ terms. In this limit,

$$v_0^{-1} = 1 + \zeta + \frac{\omega_p^2}{2\omega^2}\,, \tag{34}$$

$$\begin{aligned} v_\pm^{-1} = 1 + \frac{\zeta}{2} + \frac{m^2+\omega_p^2}{4\omega^2} \\ \pm\frac{1}{2}\left(Y^3\zeta + \frac{\omega_p^2}{2\omega^2}(2Y - Y^3) - \frac{m^2 Y}{2\omega^2}\right)\,, \end{aligned} \tag{35}$$

where we defined the dimensionless quantity

$$Y := \frac{1}{\sqrt{1 + \frac{4B^2g^2\omega^2}{m^4}}} \equiv \frac{1}{\sqrt{1+\eta^2}}\,, \tag{36}$$

with $\eta := 2gB\omega/m^2$. Note that $0 \leq Y \leq 1$. We shall neglect vacuum polarization effects ($\zeta = 0$) and consider two regimes separately, namely $Y \sim 0$ and $Y \sim 1$.

The case $Y \sim 1$ corresponds to $\eta \ll 1$, i.e. $m \gg \sqrt{2Bg\omega} \approx 10^{-15}\,\mathrm{eV}\sqrt{B_{-3}g_{-11}\nu_1}$. In this case, we get

$$v_0^{-1} = 1 + \frac{\omega_p^2}{2\omega^2}\,, \tag{37}$$

$$v_+^{-1} = 1 + \frac{m^2}{2\omega^2} - \frac{m^2+\omega_p^2}{8\omega^2}\eta^2 + \frac{3}{16}\frac{\omega_p^2}{\omega^2}\eta^4\,, \tag{38}$$

$$v_-^{-1} = 1 + \frac{\omega_p^2}{2\omega^2} + \frac{m^2+\omega_p^2}{8\omega^2}\eta^2 - \frac{3}{16}\frac{\omega_p^2}{\omega^2}\eta^4\,, \tag{39}$$

Note that, because $\eta^2 \sim \omega^2$, the $\mathcal{O}(eta^2)$ correction in the above dispersion relations gives a term which is independent of ω and will therefore equally change the TOA of all frequencies. However, the $\mathcal{O}(eta^4)$ term gives a correction which scales as ω^2. The corresponding delay in the TOA is

$$\Delta T_{\rm axion} = \mathrm{AC}_1\nu^2 \qquad m \gg \sqrt{2Bg\omega}\,, \tag{40}$$

where we have defined

$$\mathrm{AC}_1 = \int_0^d dx \frac{12\pi^2 B^4 g^4 \omega_p^2}{m^8}\,. \tag{41}$$

Let us look at the other regime, $Y \sim 0$, which corresponds to $\eta \gg 1$, i.e. $m \ll \sqrt{2Bg\omega} \approx 10^{-15}\,\mathrm{eV}\sqrt{B_{-3}g_{-11}\nu_1}$. In this case, we get

$$v_0^{-1} = 1 + \frac{\omega_p^2}{2\omega^2}\,, \tag{42}$$

$$v_\pm^{-1} = 1 + \frac{m^2+\omega_p^2}{4\omega^2} \pm \frac{m^2 - 2\omega_p^2}{4\omega^2}\frac{1}{\eta}\,, \tag{43}$$

This case is more interesting, since it gives a correction which scales as $1/\omega^3$. The corresponding delay in the TOA is

$$\Delta T_{\rm axion} = \frac{\mathrm{AC}_2}{\nu^3} \qquad m \ll \sqrt{2Bg\omega}\,, \tag{44}$$

where we have defined

$$\mathrm{AC}_2 = \int_0^d dx \frac{m^2(m^2 - 2\omega_p^2)}{64\pi^3 Bg}\,. \tag{45}$$

This correction should be compared to the ordinary delay due to the dispersion measure, $\Delta T_{\rm DM} = K\frac{\rm DM}{\nu^2}$, where $K := e^2/(2\pi m_e) \approx 4.149 \times 10^{-3}\,\mathrm{GHz^2pc^{-1}cm^3s}$, and we have defined the dispersion measure

$$\mathrm{DM} := \int_0^d dx n_e\,. \tag{46}$$

Let us put some numbers. We consider $d = 1\,\mathrm{kpc}$, $B = 10^{-3}\,\mathrm{Gauss}$, $g = 10^{-11}\,\mathrm{GeV^{-1}}$, $m = 10^{-15}\,\mathrm{eV}$ (which saturates the regime of validity) and $n_e = 0.01\,\mathrm{cm^{-3}}$. With the latter value, $\Delta T_{\rm DM} \sim 0.04\,\mathrm{s}$ and $DM \approx 10\,\mathrm{pc/cm^3}$, which is a standard number. Interestingly, for the same values $\Delta T_{\rm axion} \approx -0.02\,$s, i.e. it is of the same order of magnitude. Note that typically $\omega_p \gg m$ in this regime, so that AC $\sim m^2$. For example, $\Delta T_{\rm axion} \approx -0.0002\,$s for $m = 10^{-16}\,$eV.

Note that the effect might be detectable also for smaller magnetic fields. If $B \approx 10^{-6}\,$Gauss, the regime of validity imposes $m \ll \sqrt{2Bg\omega} \approx 4 \times 10^{-17}\,{\rm eV}\sqrt{B_{-6}g_{-11}\nu_1}$. In this case, for $m = 10^{-17}\,$eV, we get $\Delta T_{\rm axion} \approx -0.002\,$s, only one order of magnitude smaller than the DM term. Note also that the effect of the axion coupling in this regime is stronger at lower frequencies, which is good for experiments.

[Paolo: The rest was commented in the tex file.]

V. EFFECTS ON POLARIZATION

The idea of looking for oscillations in the polarization angle of arriving signals is very easy. The maximum difference in the polarization seems to be [28]

$$\Delta\theta = \frac{1}{2} g \Delta\phi \tag{47}$$

If this we can compute as (which is the case if the phase changes constantly)

$$\Delta\theta = \frac{1}{2} g \dot{\phi} L \sim 10^4 \frac{g}{10^{-10}\,{\rm GeV}^{-1}} \sqrt{\frac{\rho_{DM}}{0.4\,{\rm GeV}/cm^3}} \frac{L}{kpc} \tag{48}$$

db:I don't know if the previous or the following is relevant From (7), that the maximum change in the angle is

$$\frac{1}{2} g \Delta\phi \sim 10^{-11} \frac{g}{10^{-10}\,{\rm GeV}^{-1}} \sqrt{\frac{\rho_{DM}}{0.4\,{\rm GeV}/cm^3}} \frac{10^{-11}\,{\rm eV}}{m}. \tag{49}$$

Thus, for masses of $O(10^{-22}\,{\rm eV})$ this effect is $O(1)$. In this case, the oscillations would be of $O(years)$, so one can put a tight constraint in g for these masses. Unfortunately this goes far from the axion line...

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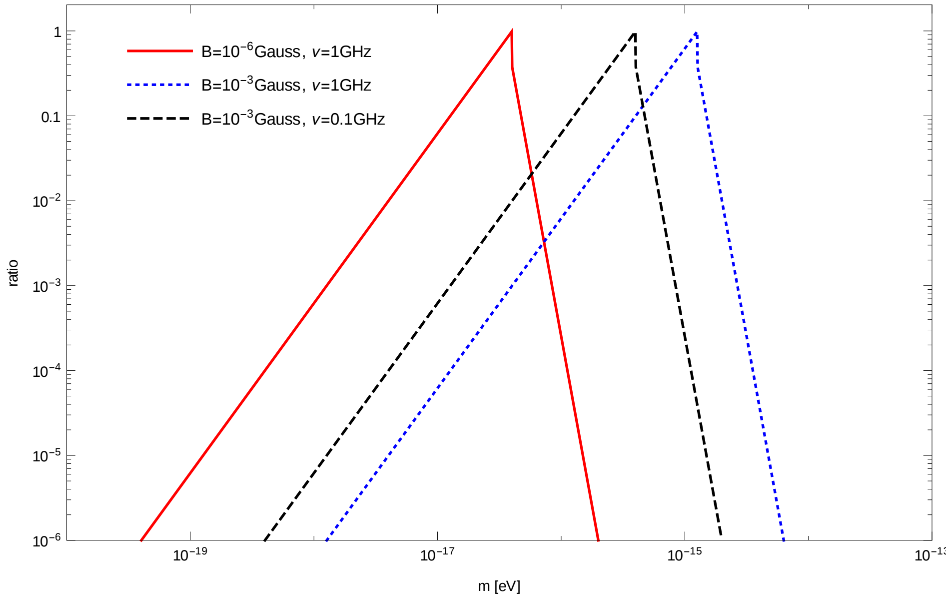

FIG. 1. The ratio $\Delta T_{\rm axion}/\Delta T_{\rm DM}$ for different values of B and ν. We consider $d = 1\,{\rm kpc}$ and $n_e = 0.01\,{\rm cm}^{-3}$. For these values, $\Delta T_{\rm DM} \sim 0.04\,{\rm s}$ and $DM \approx 10\,{\rm pc/cm}^3$. Note that the curves are obtained using Eq. (44) when $m \ll \sqrt{2Bg\omega}$ and using Eq. (40) when $m \gg \sqrt{2Bg\omega}$.