

# 07 | Linear Regression

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# Learning Objectives

After this lesson, you should be able to:

- Define simple linear regression
- Build a linear regression model using *statsmodels*
- Evaluate model fit using statistical analysis (t-tests, p-values, t-values, confidence intervals)

# Here's what's happening today:

- Simple Linear Regression
  - Interpreting the regression's coefficients
  - Are these coefficients significant?
  - Common Regression Assumptions
  - Model Fit and  $R^2$
- The Normal Distribution
  - The 68 – 90 – 95 – 99.7 Rule
- Hypothesis Testing
  - Two-Tail Hypothesis Testing
  - t-values
  - p-values
  - Confidence Intervals
- The Student's t-distribution

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# Simple Linear Regression

# Simple Linear Regression

- The simple linear regression model captures a linear relationship between a single feature variable  $x$  and a response variable  $y$

$$y = \beta_0 + \beta_1 \cdot x + \varepsilon$$

- $y$  is the **response** variable (what we want to predict); also called *dependent* variable, *endogenous* variable, or *regressand*
- $x$  is the **feature** variable (what we use to train the model); also called *explanatory* variable, *independent* variable, *exogenous* variable, or *regressor*
- $\beta_0$  and  $\beta_1$  are the **regression's coefficients**; also called the *model's parameters*
  - $\beta_0$  is the line's intercept;  $\beta_1$  is the line's slope
- $\varepsilon$  is the **error** term; also called the residual

# Simple Linear Regression (cont.)

- Given  $x = (x^{(1)}, x^{(2)}, \dots, x^{(n)})$ ,  $y = (y^{(1)}, y^{(2)}, \dots, y^{(n)})$ , and  $\varepsilon = (\varepsilon^{(1)}, \varepsilon^{(2)}, \dots, \varepsilon^{(n)})$ , we can formulate the linear model as

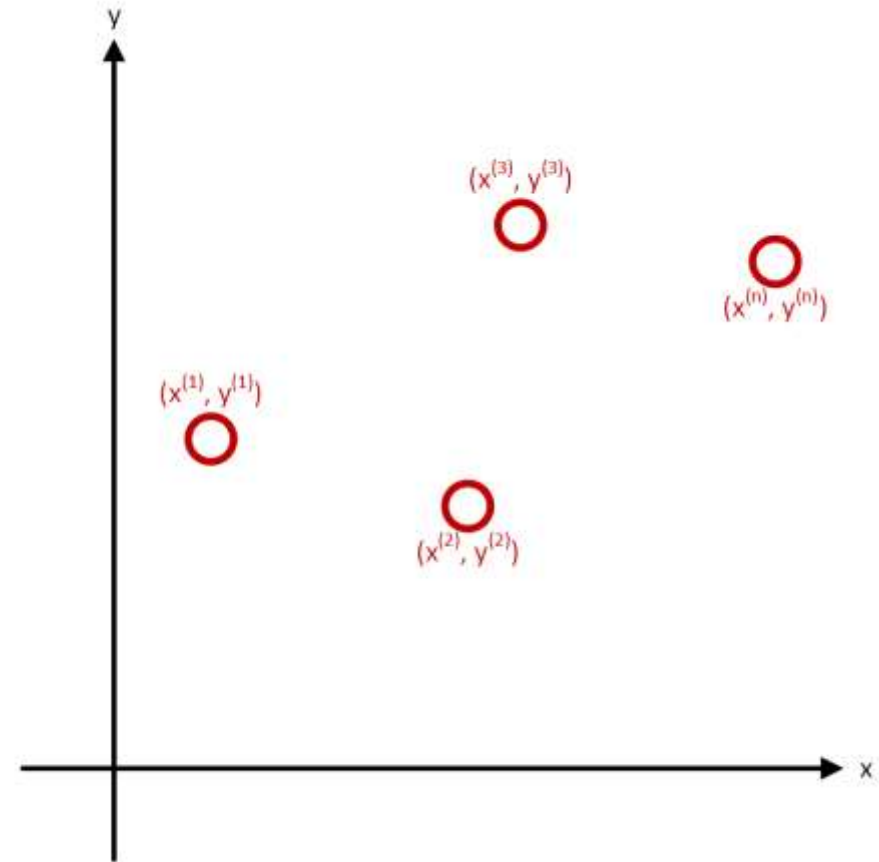
$$y^{(i)} = \beta_0 + \beta_1 \cdot x^{(i)} + \varepsilon^{(i)}$$

- In words, this equation says that for each sample  $i$ ,  $y^{(i)}$  can be explained by  $\beta_0 + \beta_1 \cdot x^{(i)}$

- In our Python environment,  $x$  and  $y$  represent *pandas Series* and  $x^{(i)}$  and  $y^{(i)}$  their values at index  $i$
- E.g. (SF housing dataset),
  - $x$  is the property's size (`df.Size`)
  - $y$  is the property's sale price (`df.SalePrice`)

# Simple Linear Regression (cont.)

- $\varepsilon$  is a “white noise” disturbance which we **do not observe**
  - $\varepsilon$  models how the observations deviate from the exact slope-intercept relation
- We **do not observe** the constants  $\beta_0$  or  $\beta_1$  either, so we have to estimate them



# Simple Linear Regression (cont.)

- E.g. (SF housing dataset),

$$\widehat{SalePrice} = \hat{\beta}_0 + \hat{\beta}_1 \cdot Size$$



# How to interpret *statsmodels* report?

Dep. Variable:	SalePrice	R-squared:	0.236
Model:	OLS	Adj. R-squared:	0.235
Method:	Least Squares	F-statistic:	297.4
Date:		Prob (F-statistic):	2.67e-58
Time:		Log-Likelihood:	-1687.9
No. Observations:	967	AIC:	3380.
Df Residuals:	965	BIC:	3390.
Df Model:	1		
Covariance Type:	nonrobust		

The model's fit

Is the model's fit significant?

The estimated coefficients  
 $\hat{\beta}_0$  (the intercept) and  
 $\hat{\beta}_1$  (the slope; "size")

	coef	std err	t	P> t	[95.0% Conf. Int.]
Intercept	0.1551	0.084	1.842	0.066	-0.010 0.320
Size	0.749	0.043	17.246	0.000	0.664 0.835

Are these estimated significant?  
(i.e., are they meaningful?; do  
they make sense?)

Omnibus:	1842.865	Durbin-Watson:	1.704
Prob(Omnibus):	0.000	Jarque-Bera (JB):	3398350.943
Skew:	13.502	Prob(JB):	0.00
Kurtosis:	292.162	Cond. No.	4.40

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# Simple Linear Regression

*Interpreting the regression's coefficients  $\hat{\beta}$*

# Interpreting the regression's coefficients

$$\hat{\beta}_0 = .155$$

- What's the unit of  $\hat{\beta}_0$ ?
  - $[\hat{\beta}_0] = [\text{Sale Price}] = \$M$
- How to interpret  $\hat{\beta}_0$ ?
  - $\hat{\beta}_0 = .155 (\$M) = \$155k$
  - $\text{Sale Price} (\text{Size} = 0) = \hat{\beta}_0 + \hat{\beta}_1 \cdot 0 = \hat{\beta}_0$
  - The model predicts that a property of 0 sqft would cost \$155k

$$\hat{\beta}_1 = .750$$

- What's the unit of  $\hat{\beta}_1$ ?
  - $[\hat{\beta}_1] = \frac{[\text{Sale Price}]}{[\text{Size}]} = \frac{\$M}{1,000 \text{ sqft}}$
- How to interpret  $\hat{\beta}_1$ ?
  - $\hat{\beta}_1 = .750 (\$M / 1,000 \text{ sqft})$   
 $= \$750k / 1,000 \text{ sqft}$
  - The model predicts that each additional 1,000 sqft costs \$750k

# Simple Linear Regression

*Are the regression's coefficients  $\hat{\beta}$  significant?*

# Are the regression's coefficients $\hat{\beta}$ significant?

The  $\beta$  coefficients follow a normal distribution:

$$\mu_{\beta} \sim N(\beta, (X^T X)^{-1} \sigma^2)$$

(or)

$$\mu_{\beta_j} \sim N(\beta_j, v_j \sigma^2)$$

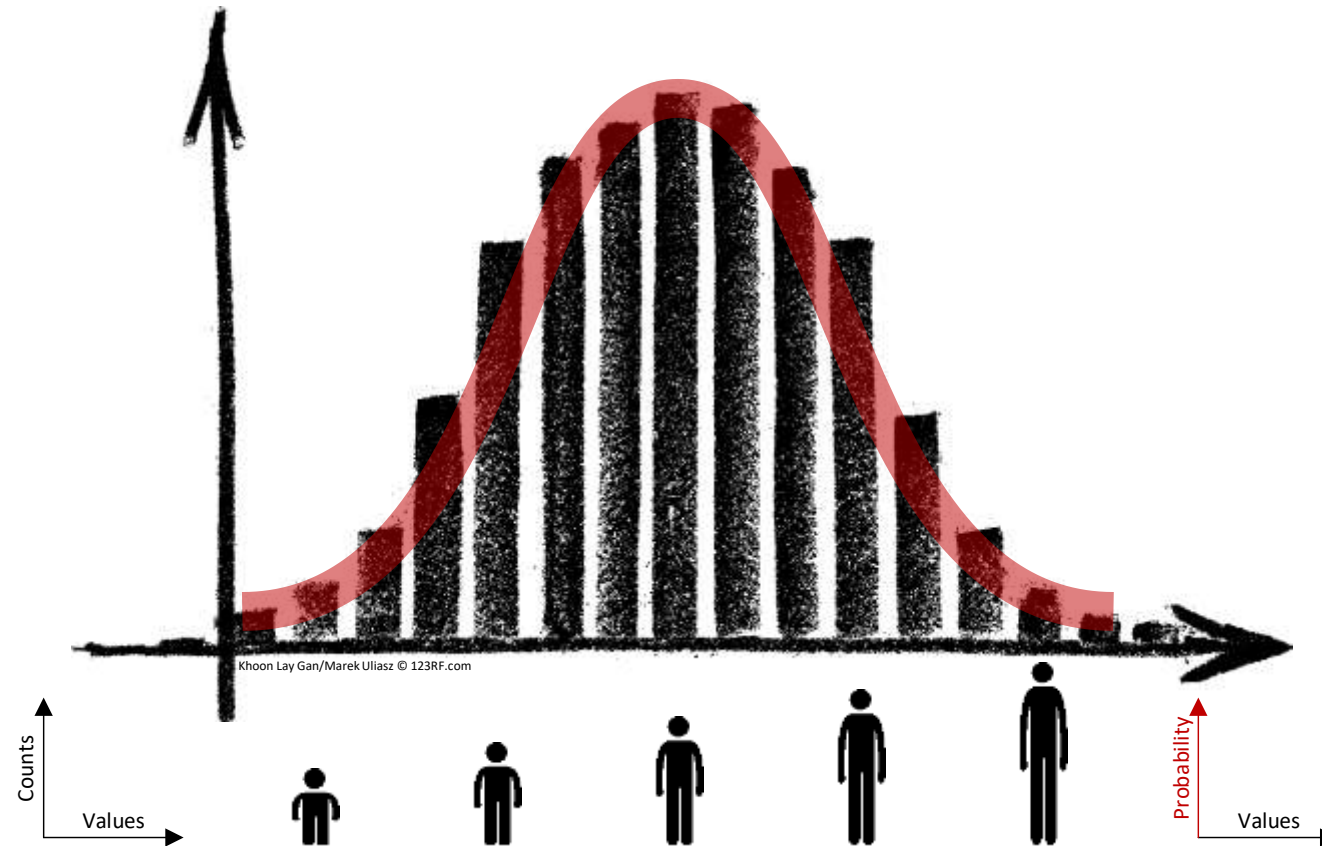
$$(X^T X)^{-1} = \begin{pmatrix} v_0 = v_{0,0} & \cdots & v_{0,j} & \cdots \\ \vdots & \ddots & & \\ v_{j,0} & & v_j = v_{j,j} & \\ \vdots & & & \ddots \end{pmatrix}$$

*( $v_j$  is the  $j^{th}$  diagonal element of  $(X^T X)^{-1}$ )*

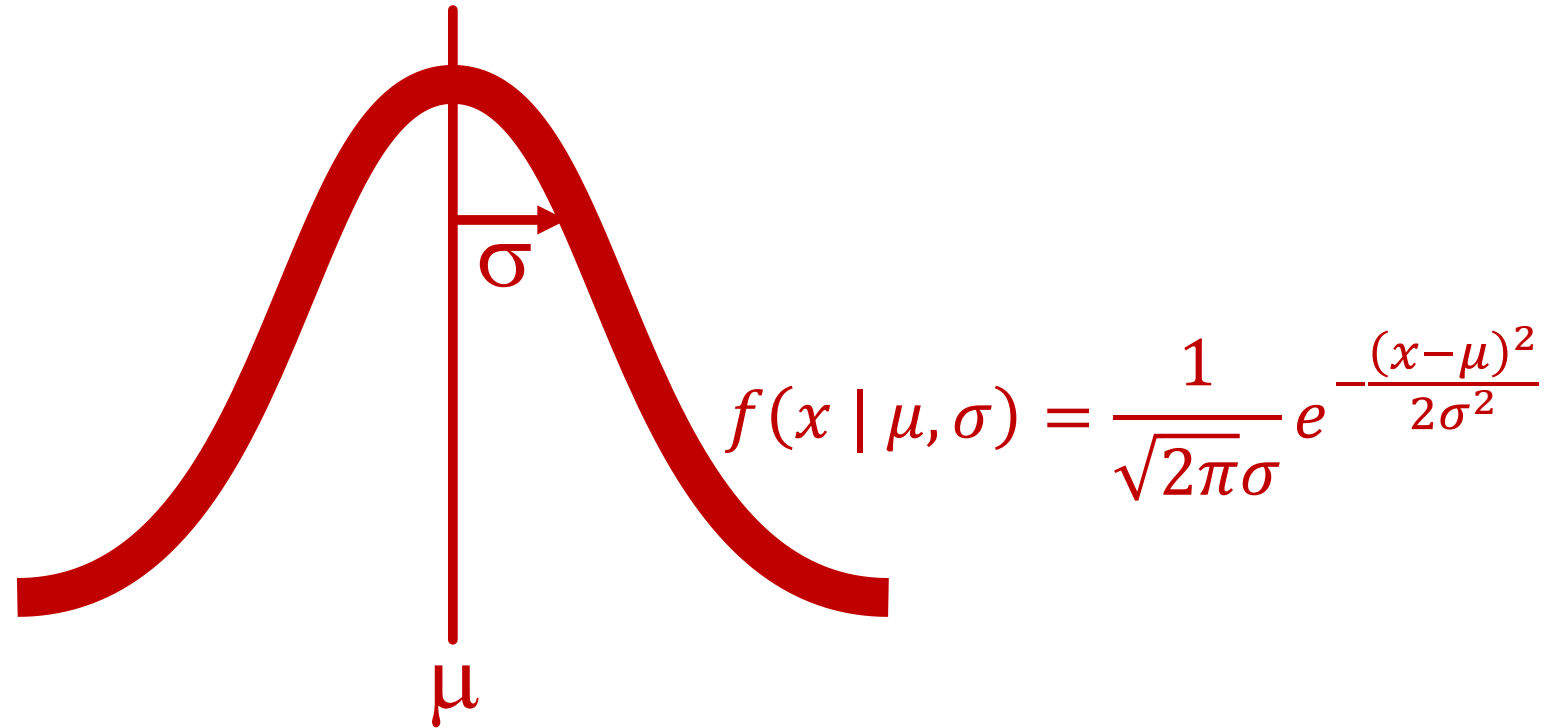
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# The Normal Distribution

People's height follows a bell shape distribution. (For men in the US, the average height is around 70 inches (5'10) with a standard deviation of 4 inches; few people are shorter than 67 inches; few are as tall as 73 inches)

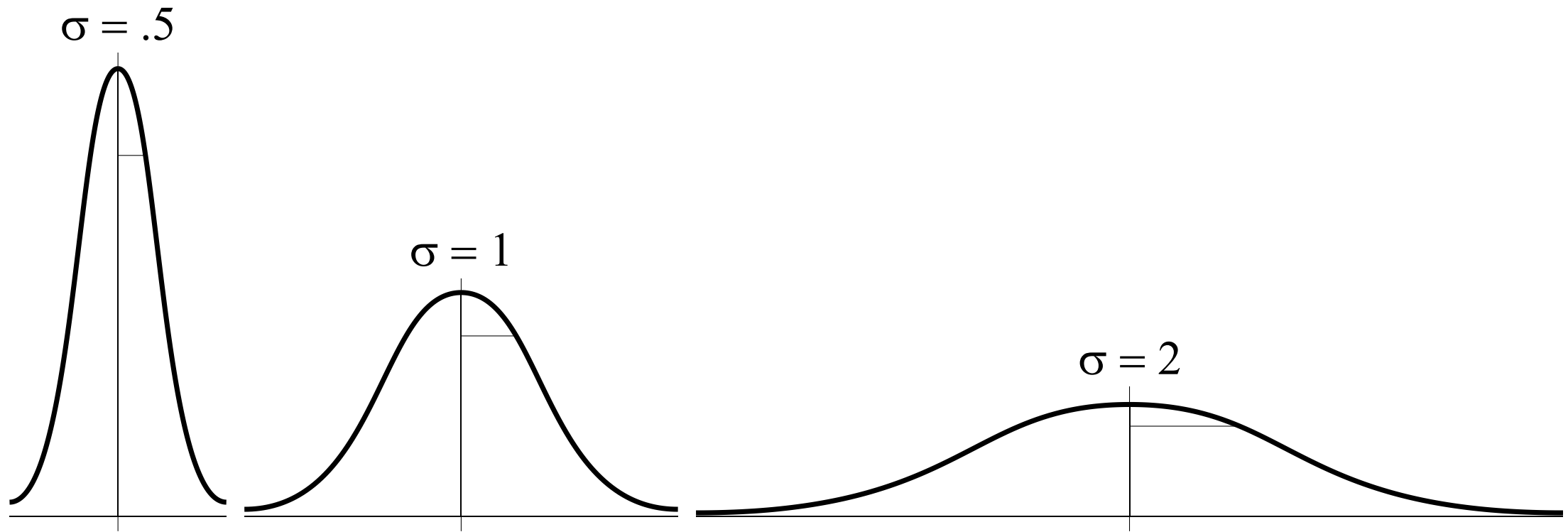


# The Normal Distribution

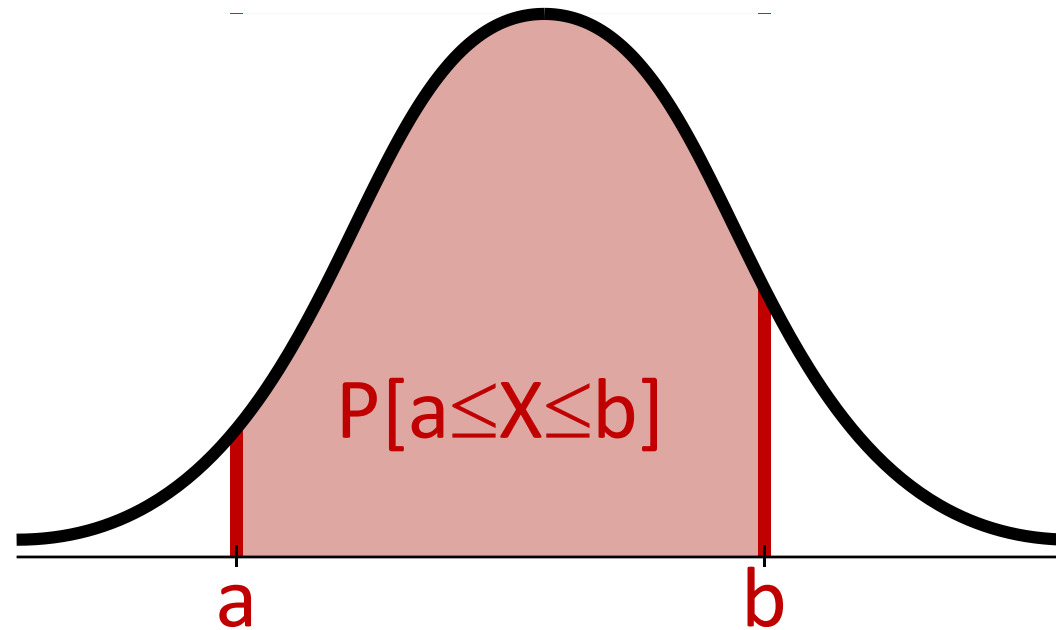




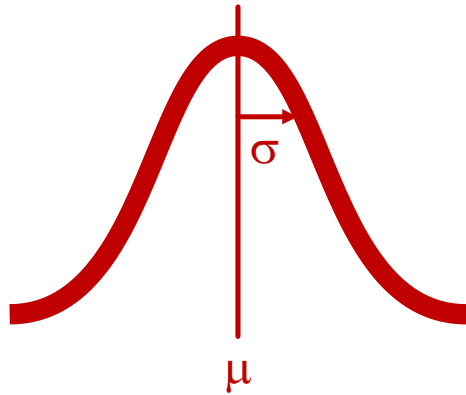
This bell-shaped curve is a probability density function (PDF):  
The area under the curve is always 1 (for any  $\sigma$ ) (cont.)



The area under the curve is called a Cumulative Distribution Function (CDF)

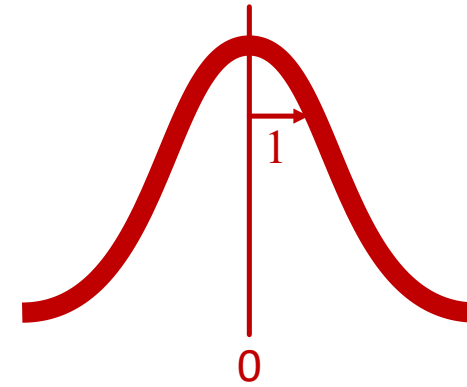


# The Standard Normal Distribution ( $\mu = 0; \sigma = 1$ )



$$X \sim N(\mu, \sigma)$$

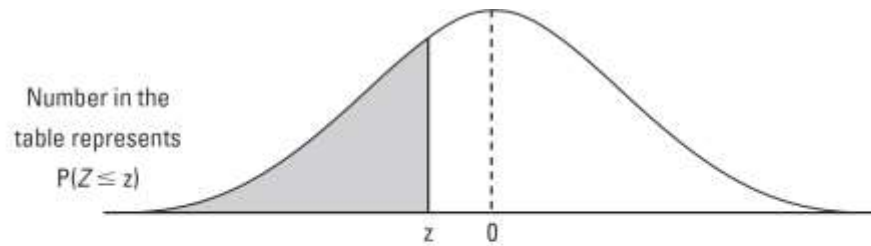
$$X = \mu + \sigma \cdot Z$$



$$Z = \frac{X - \mu}{\sigma}$$

$$Z \sim N(0, 1)$$

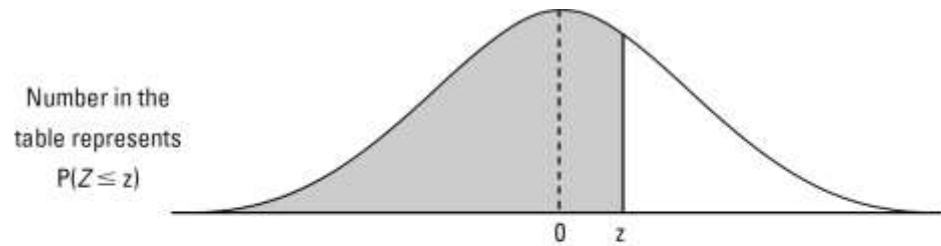
# The Standard Normal Distribution Table



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0003	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110

-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

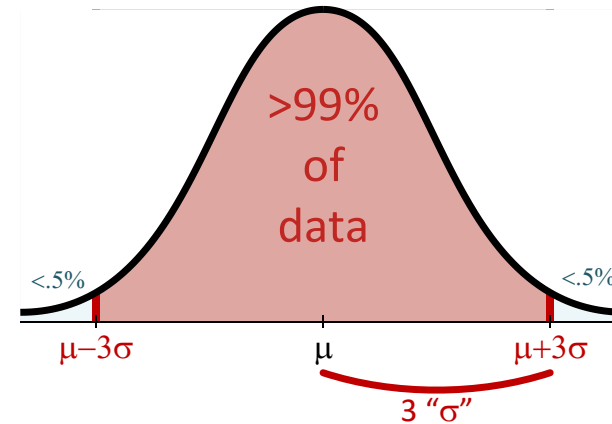
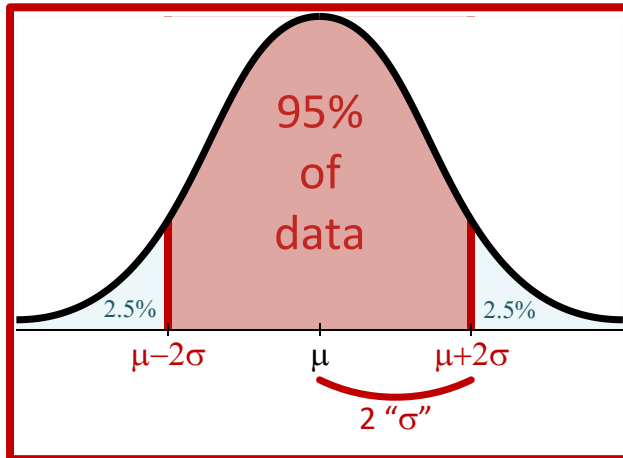
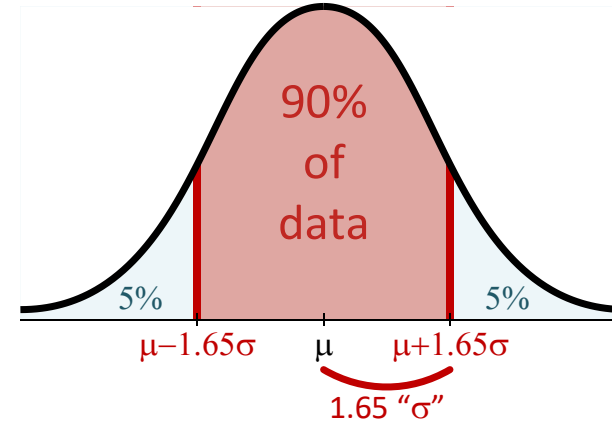
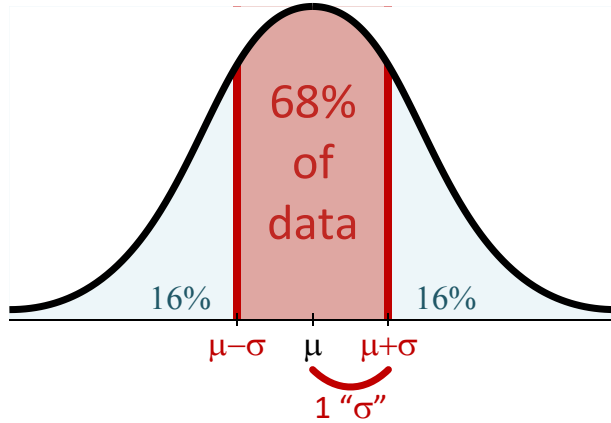
# The Standard Normal Distribution Table (cont.)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

# The 68 – 90 – 95 – 99.7 Rule (cont.)



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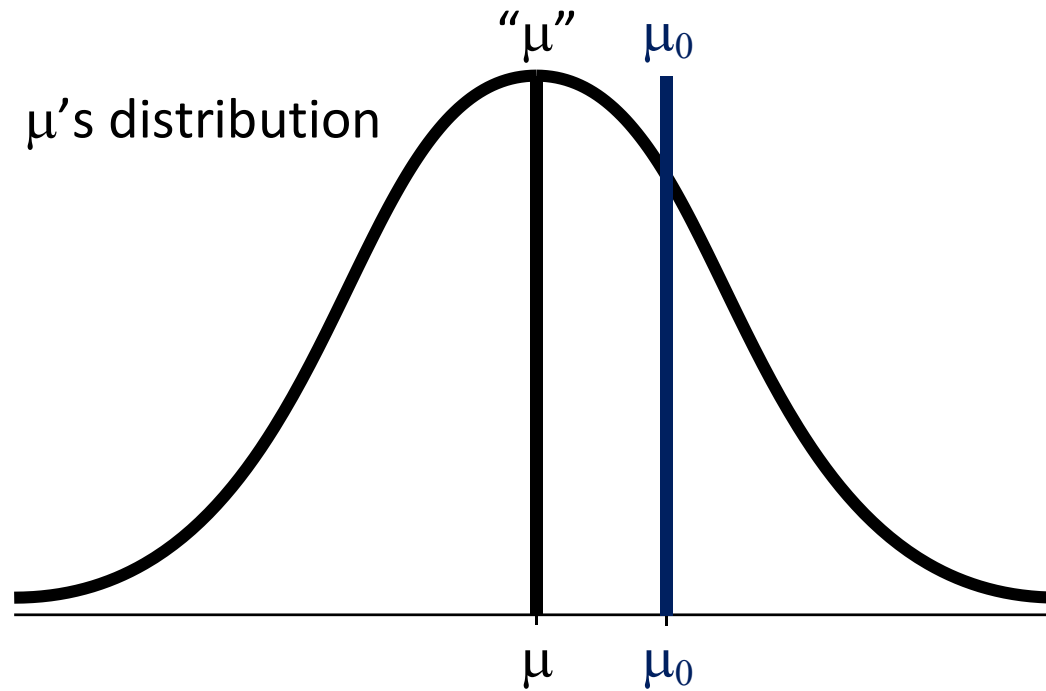
# Hypothesis Testing

# Hypothesis Testing

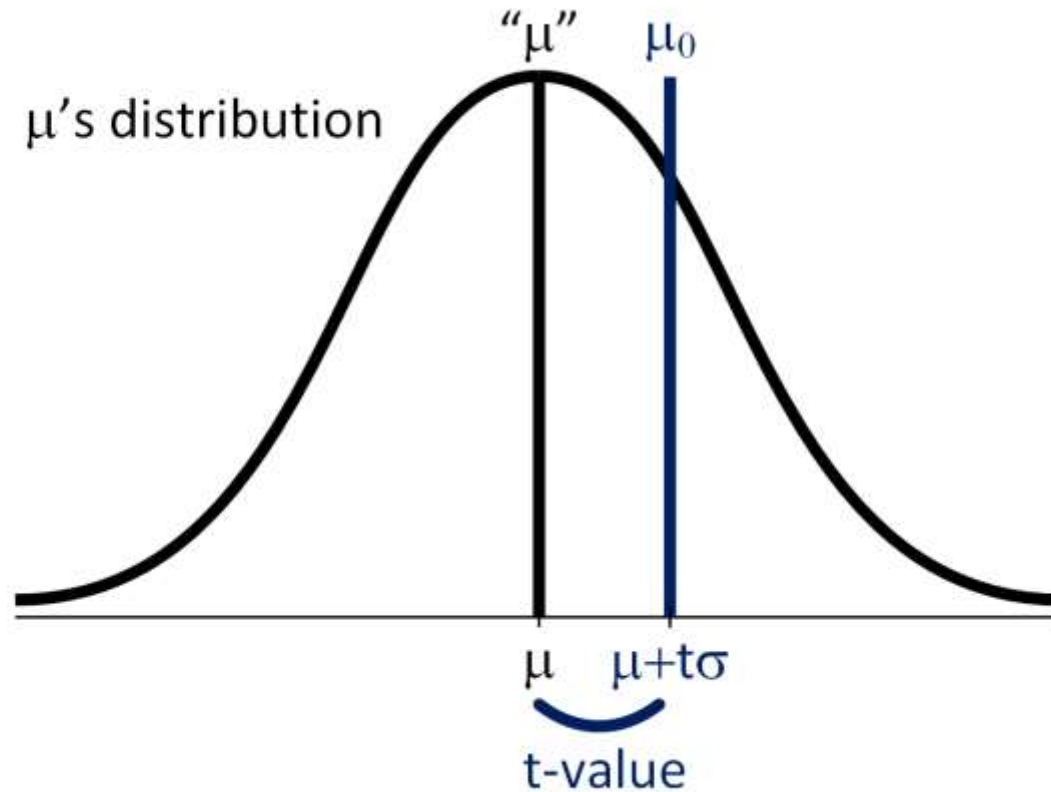
- A hypothesis is an assumption about the a population parameter. E.g.,
  - $\mu_{\beta_0} = \text{<a specific value, e.g. 0.155>}$
  - $\mu_{\beta_1} = \text{<a specific value, e.g. 0.750>}$
- In both cases, we made a statement about a population parameter that may or may not be true
- The purpose of hypothesis testing is to make a statistical conclusion about **rejecting** or **failing to reject** such statement



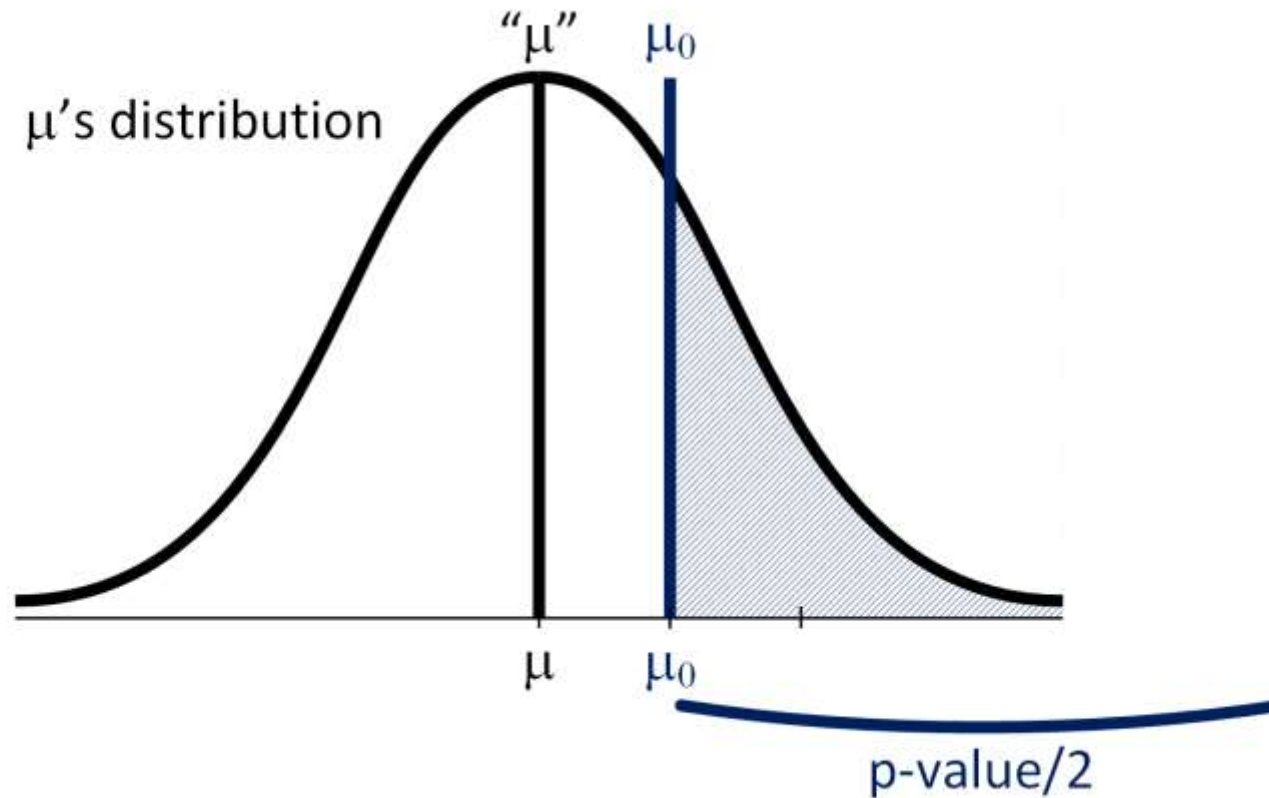
# Two-Tail Hypothesis Testing (cont.)



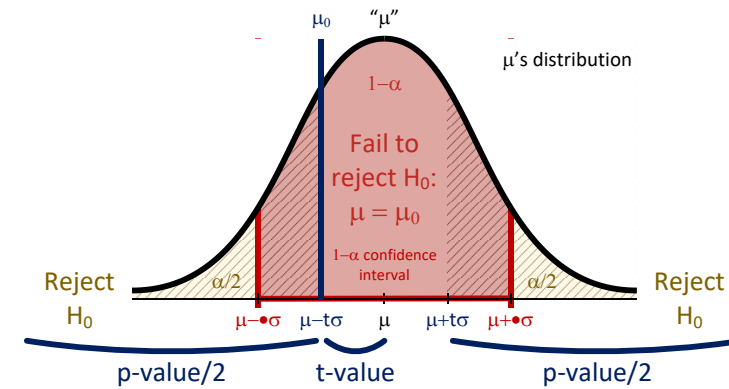
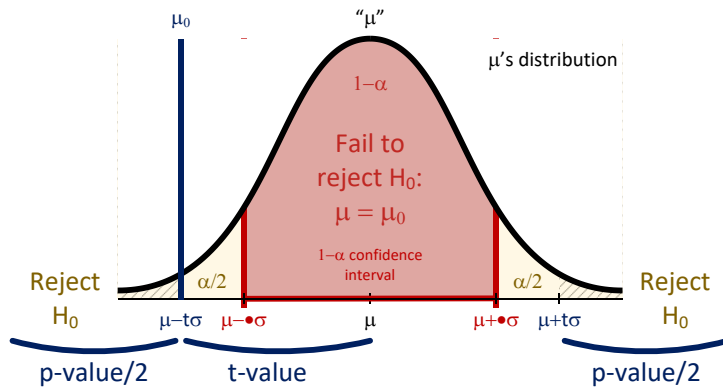
*t-value* measures the difference to  $\mu_0$  in  $\sigma$ . *t-values* of large magnitudes (either negative or positive) are less likely. The far left and right “tails” of the distribution curve represent instances of obtaining extreme values of  $t$ , far from  $\mu$



*p-value* determines the probability (assuming  $H_0$  is true) of observing a more extreme test statistic in the direction of  $H_a$  than the one observed

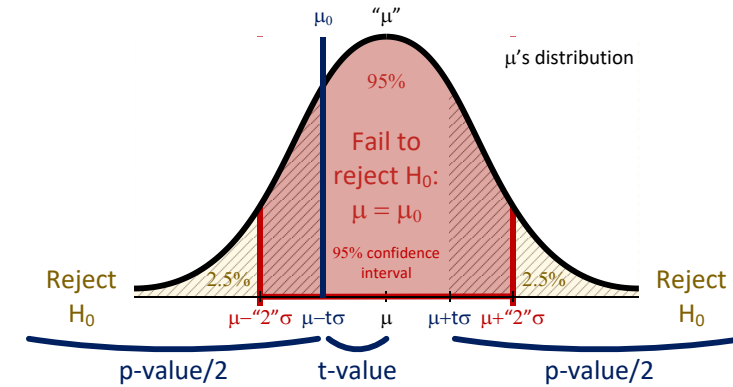
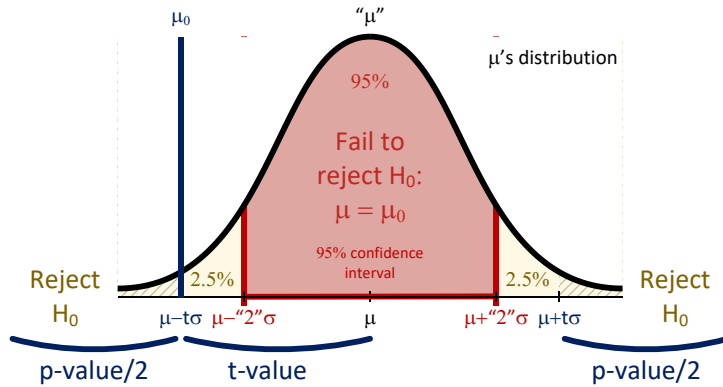


# Two-Tail Hypothesis Testing (*simplified*) (cont.)



$ t\text{-value} $	p-value	$1 - \alpha$ Confidence Interval ( $[\mu - \sigma, \mu + \sigma]$ )	$H_0 / H_a$	Outcome
$< \cdot$	$> \alpha$	$\mu_0$ is inside	Did not find evidence that $\mu \neq \mu_0$ : Fail to reject $H_0$	$\mu = \mu_0$
$\geq \cdot$	$\leq \alpha$	$\mu_0$ is outside	Found evidence that $\mu \neq$ $\mu_0$ : Reject $H_0$	$\mu \neq \mu_0$

# Two-Tail Hypothesis Testing (*simplified*) ( $\alpha = .05$ ) (cont.)



$ t\text{-value} $	p-value	95% Confidence Interval ( $[\mu - 2\sigma, \mu + 2\sigma]$ )	$H_0 / H_a$	Outcome
$< \sim 2^{(*)}$ (*) (check t-table)	$> .05$	$\mu_0$ is inside	Did not find that $\mu \neq \mu_0$ : Fail to reject $H_0$	$\mu = \mu_0$
$\geq \sim 2$	$\leq .05$	$\mu_0$ is outside	Found evidence that $\mu \neq \mu_0$ : Reject $H_0$	$\mu \neq \mu_0$

# Simple Linear Regression

*Are the regression's coefficients  $\hat{\beta}$  significant? (cont.)*

What  $\beta_1$  would make our multiple linear regression model useless?

- (the simple linear regression model again, without intercept to keep things simple)

$$y = \beta_1 \cdot x + \varepsilon$$

- Answer: If  $\beta_1 = 0$ , we don't have a linear model
  - ( $y = 0$  isn't very exciting, is it?)

# Is the regression's coefficient $\hat{\beta}$ significant?

- The *null hypothesis* ( $H_0$ ) represents the status quo; that the mean of the regression's coefficient  $\beta$  is equal to 0, i.e. that  $\beta$  is not significant:

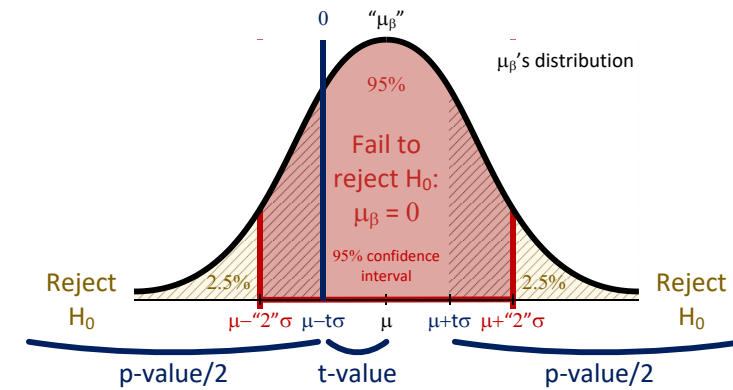
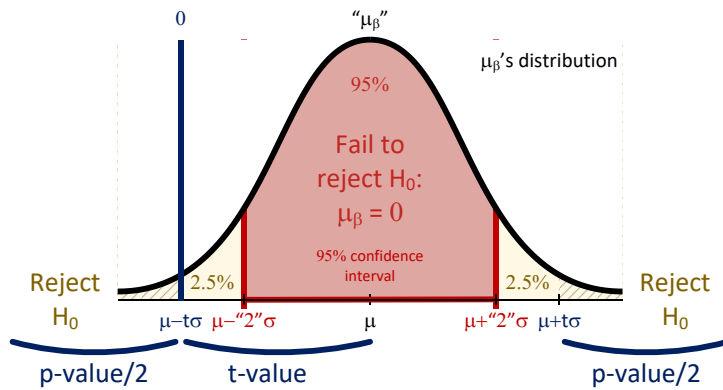
$$H_0: \mu_{\beta} = 0$$

- The *alternate hypothesis* ( $H_a$ ) represents the opposite of the null hypothesis and holds true if the *null hypothesis* is found to be false; that the mean of the regression's coefficient  $\beta$  is not equal to 0, i.e. that  $\beta$  is significant:

$$H_a: \mu_{\beta} \neq 0$$



# Is the regression's coefficient $\hat{\beta}$ significant? (at the 5% significance level)



$ t\text{-value} $	p-value	95% Confidence Interval ( $[\mu_{\beta} - 2\sigma, \mu_{\beta} + 2\sigma]$ )	$H_0 / H_a$	Outcome
$< \sim 2^{(*)}$ (*) (check t-table)	$> .05$	0 is inside	Did not find that $\mu_{\beta} \neq 0$ : Fail to reject $H_0$	$\mu_{\beta} = 0$ ; the coefficient $\beta$ is not significant
$\geq \sim 2$	$\leq .05$	0 is outside	Found evidence that $\mu_{\beta} \neq 0$ : Reject $H_0$	$\mu_{\beta} \neq 0$ ; the coefficient $\beta$ is significant

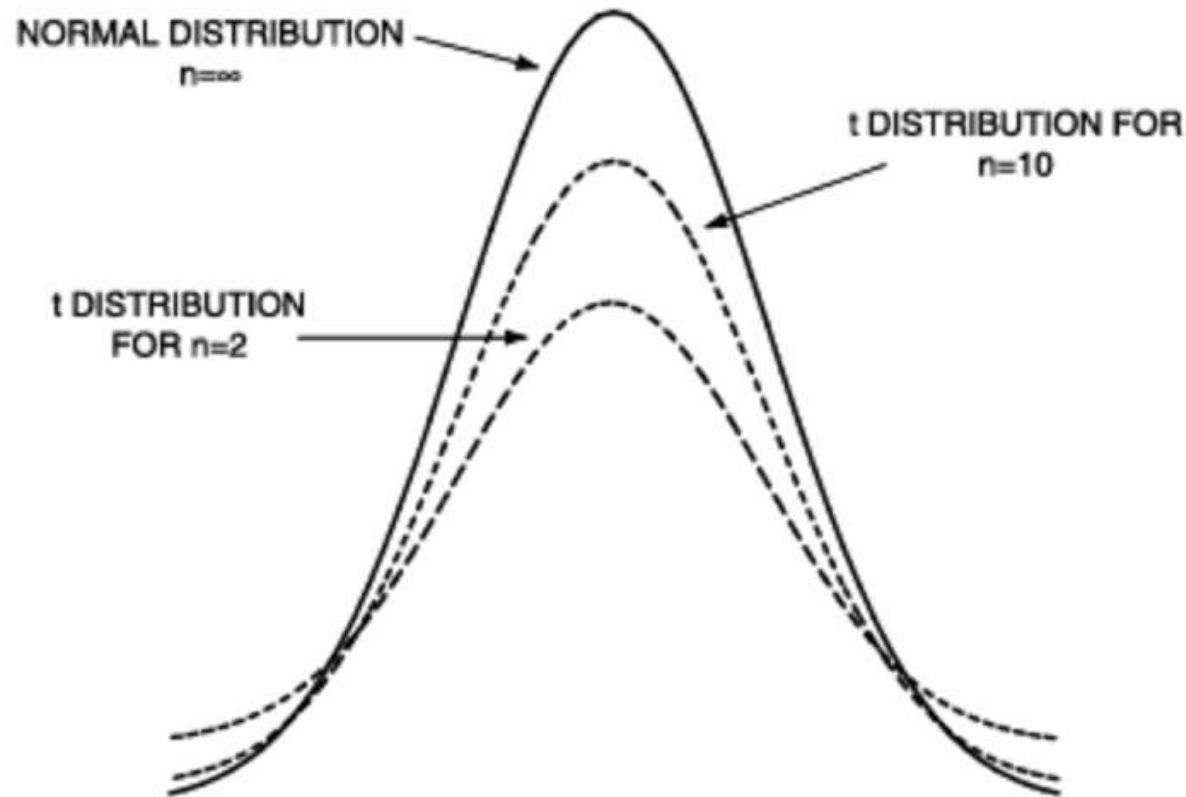
# Details of *statsmodels*' coefficients table

	coef	std err	t	P> t	[95.0% Conf. Int.]
Feature variable j, e.g., "Intercept" or "Size"	$\hat{\beta}_j$	$\sqrt{v_j} \cdot \hat{\sigma}$	$z_j = \frac{\hat{\beta}_j}{\sqrt{v_j} \cdot \hat{\sigma}}$ <p>(or)</p> $\frac{coef}{std\ err}$	2 × area under the curve from the Student's t-distribution between $ t $ and $+\infty$	$\hat{\beta}_j \pm z_{\alpha=.025} \cdot \hat{\sigma}$ <p>(the value reported in the Student-t distribution table under the 5<sup>th</sup> column for <math>\alpha = .025</math>)</p>

DS

# The Student's t-distribution

FYI | We simplified things a bit... t-values refer to the Student's t-distribution, not the normal distribution; the reason behind this is that we substituted  $\hat{\sigma}$  for  $\sigma$  (and  $\hat{\beta}$  for  $\beta$ )



FYI | We simplified things a bit... t-values refer to the Student's t-distribution, not the normal distribution; the reason behind this is that we substituted  $\hat{\sigma}$  for  $\sigma$  (and  $\hat{\beta}$  for  $\beta$ ) (cont.)

$\sigma^2$  is estimated by  $\hat{\sigma}^2$

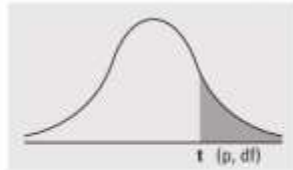
$$\hat{\sigma}^2 = \frac{1}{df} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$df = \underbrace{n}_{\text{number of samples}} - \underbrace{k}_{\text{number of parameters (intercept included)}}$$

with

# Student's t-distribution table: as the sample size grows, the Student's t-distribution converges to a normal distribution

Numbers in each row of the table are values on a  $t$ -distribution with ( $df$ ) degrees of freedom for selected right-tail (greater-than) probabilities ( $p$ ).



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208

14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697071	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
CI	———	———	80%	90%	95%	98%	99%	99.9%

# Simple Linear Regression

*Are the regression's coefficients  $\hat{\beta}$  significant? (cont.)*

# SalePrice as a function of Size (cont.)

<b>Dep. Variable:</b>	SalePrice	<b>R-squared:</b>	0.236
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.235
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	297.4
<b>Date:</b>		<b>Prob (F-statistic):</b>	2.67e-58
<b>Time:</b>		<b>Log-Likelihood:</b>	-1687.9
<b>No. Observations:</b>	967	<b>AIC:</b>	3380.
<b>Df Residuals:</b>	965	<b>BIC:</b>	3390.
<b>Df Model:</b>	1		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P> t	[95.0% Conf. Int.]
<b>Intercept</b>	0.1551	0.084	1.842	0.066	-0.010 0.320
<b>Size</b>	0.7497	0.043	17.246	0.000	0.664 0.835

<b>Omnibus:</b>	1842.865	<b>Durbin-Watson:</b>	1.704
<b>Prob(Omnibus):</b>	0.000	<b>Jarque-Bera (JB):</b>	3398350.943
<b>Skew:</b>	13.502	<b>Prob(JB):</b>	0.00
<b>Kurtosis:</b>	292.162	<b>Cond. No.</b>	4.40

$$SalePrice \text{ [\$M]} = \underbrace{.155}_{\hat{\beta}_0} + \underbrace{.750}_{\hat{\beta}_1} \times Size \text{ [1,000 sqft]}$$

(the slope is significant but not the intercept)



$\text{SalePrice} \sim 0 + \text{Size}$  ('0' meaning the intercept is forced to 0) (cont.)

<b>Dep. Variable:</b>	SalePrice	<b>R-squared:</b>	0.565
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.565
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	1255.
<b>Date:</b>		<b>Prob (F-statistic):</b>	7.83e-177
<b>Time:</b>		<b>Log-Likelihood:</b>	-1689.6
<b>No. Observations:</b>	967	<b>AIC:</b>	3381.
<b>Df Residuals:</b>	966	<b>BIC:</b>	3386.
<b>Df Model:</b>	1		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P> t	[95.0% Conf. Int.]
<b>Size</b>	0.8176	0.023	35.426	0.000	0.772 0.863

<b>Omnibus:</b>	1830.896	<b>Durbin-Watson:</b>	1.722
<b>Prob(Omnibus):</b>	0.000	<b>Jarque-Bera (JB):</b>	3370566.094
<b>Skew:</b>	13.300	<b>Prob(JB):</b>	0.00
<b>Kurtosis:</b>	291.005	<b>Cond. No.</b>	1.00

$$\text{SalePrice } [\$M] = \underbrace{0.}_{\hat{\beta}_0} + \underbrace{.810}_{\hat{\beta}_1} \times \text{Size } [1,000 \text{ sqft}]$$

(the slope is significant)

# SalePrice ~ Size (with outliers removed) (cont.)

<b>Dep. Variable:</b>	SalePrice	<b>R-squared:</b>	0.200
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.199
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	225.0
<b>Date:</b>		<b>Prob (F-statistic):</b>	1.41e-45
<b>Time:</b>		<b>Log-Likelihood:</b>	-560.34
<b>No. Observations:</b>	903	<b>AIC:</b>	1125.
<b>Df Residuals:</b>	901	<b>BIC:</b>	1134.
<b>Df Model:</b>	1		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P> t	[95.0% Conf. Int.]
<b>Intercept</b>	0.7082	0.032	22.152	0.000	0.645 0.771
<b>Size</b>	0.2784	0.019	15.002	0.000	0.242 0.315

<b>Omnibus:</b>	24.647	<b>Durbin-Watson:</b>	1.625
<b>Prob(Omnibus):</b>	0.000	<b>Jarque-Bera (JB):</b>	53.865
<b>Skew:</b>	0.054	<b>Prob(JB):</b>	2.01e-12
<b>Kurtosis:</b>	4.192	<b>Cond. No.</b>	4.70

*SalePrice [\$M] =*

$$\underbrace{.708}_{(was .155)} + \underbrace{.278}_{(was .750)} \times Size [1,000 sqft]$$

(both intercept and slope are now significant)

DS

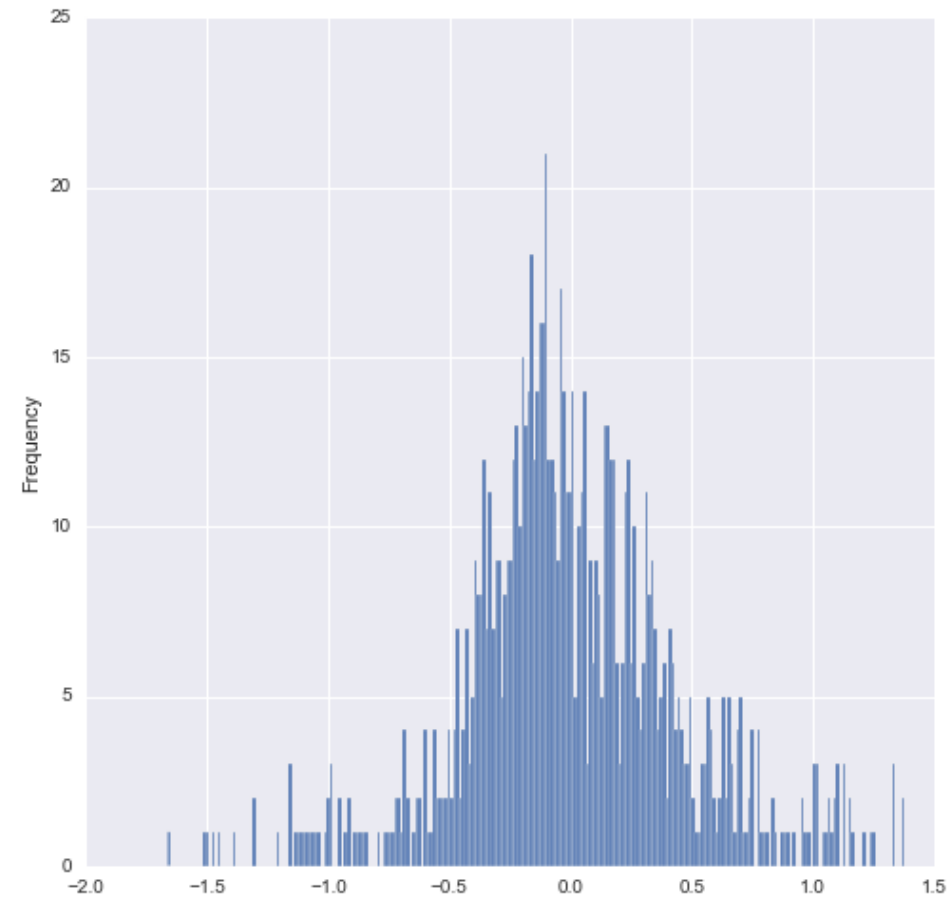
# Simple Linear Regression

*Common Regression Assumptions*

# Common Regression Assumptions (Part 1)

- The model is linear
  - $x$  significantly explains  $y$
- $\varepsilon \sim N(0, \cdot)$ 
  - Specifically, we expect  $\varepsilon$  to be 0 on average, i.e.,  $\mu_\varepsilon = 0$
- $x$  and  $\varepsilon$  are independent
  - $\rho(x, \varepsilon) = 0$

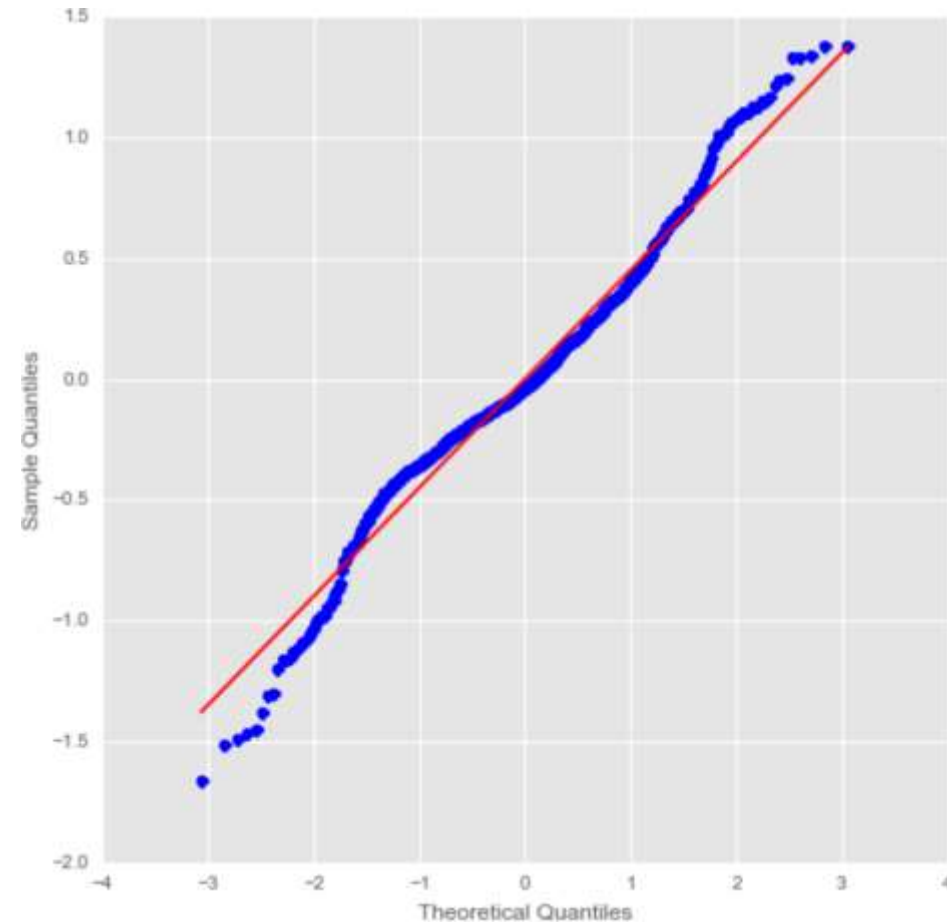
Is  $\varepsilon \sim N(0, \cdot)$ ?



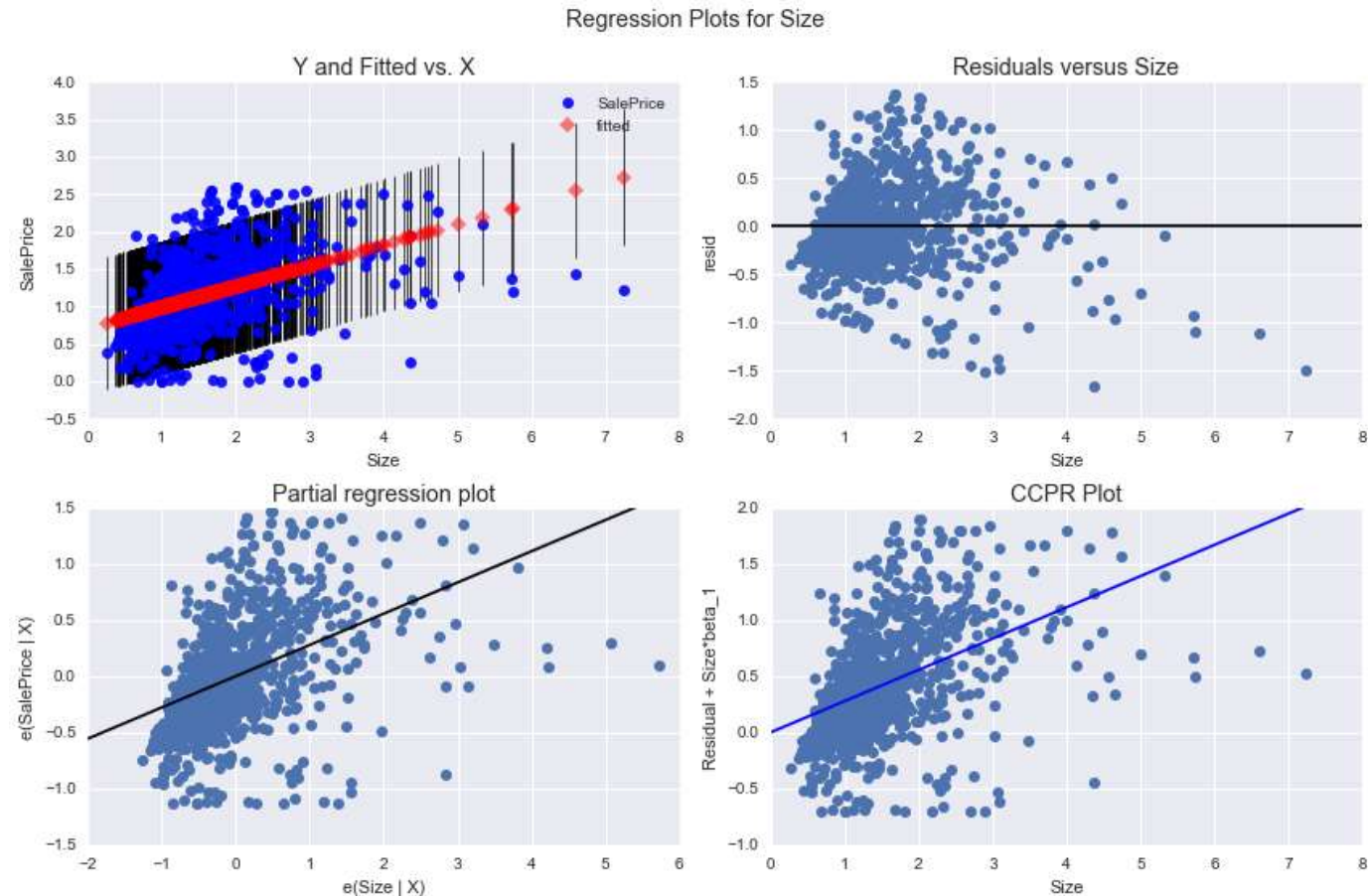
$\varepsilon \sim N(0, \cdot) : \text{.qqplot()}$

- “Quantile-Quantile (q-q) Plot”
- Graphical technique for determining if two datasets come from populations with a common distribution
- Plot of the quantiles of the first dataset (vertically) against the quantiles of the second’s (horizontally)
- If unspecified, the second dataset will default to  $N(0, 1)$
- If the two datasets come from a population with the same distribution, the points should fall approximately along a 45-degree reference line
- The greater the departure from this reference line, the greater the evidence for the conclusion that the datasets have come from populations with different distributions

$\varepsilon \sim N(0, \cdot)$ : `.qqplot()` (with `line = 's'`) (cont.)



$x$  and  $\varepsilon$  are independent: `.plot_regress_exog()`





# $x$ and $\varepsilon$ are independent: `.plot_regress_exog()` (cont.)

- Scatterplot of observed values ( $y$ ) compared to fitted values ( $\hat{y}$ ) with confidence intervals against the regressor ( $x$ )

- `.plot_fit()`

## ▸ “Residual Plot”

- Scatterplot of the model’s residuals ( $\hat{\varepsilon}$ ) against the regressor ( $x$ )

## ▸ “Partial Regression Plot” and “CCPR Plot (Component and Component-Plus-Residual)”

- (useful for multiple regression)

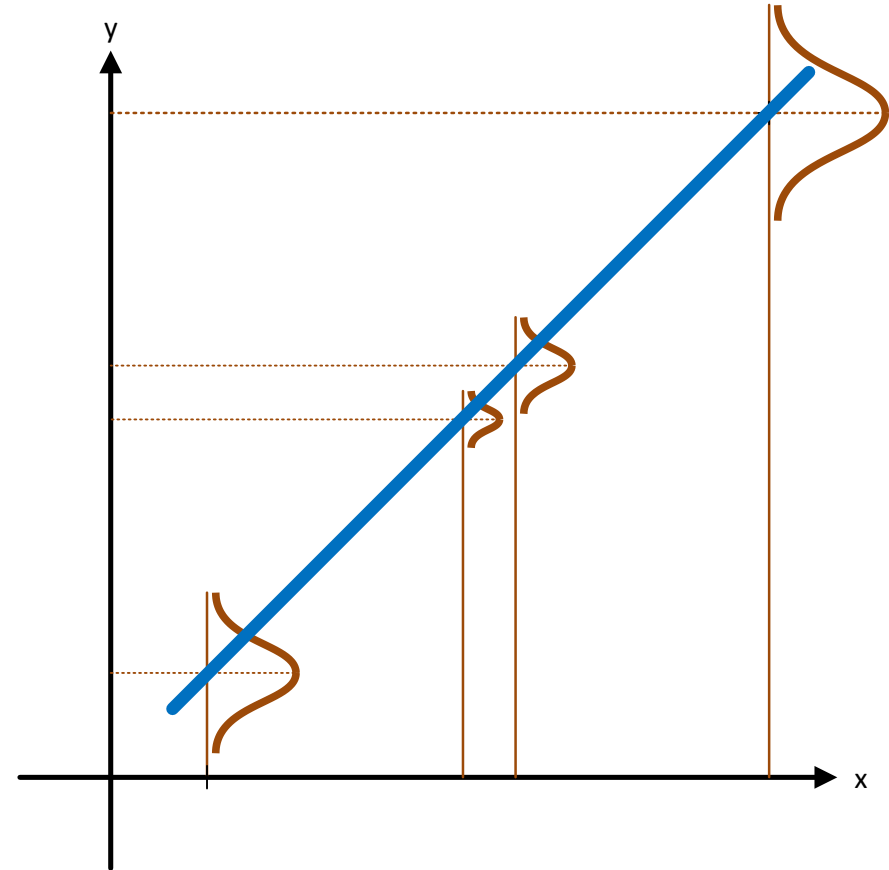
DS

# Simple Linear Regression

*Assessing the model's fit with  $R^2$*

# Fit and Inference

- The deviations of the data from the best fitting line are normally distributed about the line. Since  $\mu_{\varepsilon} = 0$ , we “expect” that on average, the line will be correct
- How confident we are about how well the relationship holds depends on  $\sigma_{\varepsilon}^2$



# Assessing the model's fit with $R^2$

- When a measure of how much of the total variation in  $y$ ,  $\sigma_y^2 = \beta^2 \sigma_x^2 + \sigma_\varepsilon^2$ , is explained by the portion associated with the explanatory variable  $x$ ,  $\sigma_{\hat{y}}^2 = \beta^2 \sigma_x^2$ ; also called systematic variation (the variation explained by your model)

$$R^2 = \frac{\sigma_{\hat{y}}^2}{\sigma_y^2} = \frac{\beta^2 \sigma_x^2}{\beta^2 \sigma_x^2 + \sigma_\varepsilon^2}$$

- $0 \leq R^2 \leq 1$  (since  $-1 \leq \rho_{xy} \leq 1$ )
- $1 - R^2 = \frac{\sigma_\varepsilon^2}{\beta^2 \sigma_x^2 + \sigma_\varepsilon^2}$  is the idiosyncratic variation (the variation left unexplained by your model)

# Assessing the model's fit with $R^2$ (cont.)

When x significantly explains y	When x does not significantly explains y
<input type="checkbox"/> The fit is <b>better</b>	<input type="checkbox"/> The fit is <b>worse</b>
<input type="checkbox"/> The <b>explained</b> systematic variation dominates	<input type="checkbox"/> The <b>unexplained</b> idiosyncratic variation dominates
<input type="checkbox"/> $\sigma_\varepsilon^2$ is low (and/or $\beta^2 \sigma_x^2$ is high)	<input type="checkbox"/> $\sigma_\varepsilon^2$ is high (and/or $\beta^2 \sigma_x^2$ is low)
<input type="checkbox"/> $R^2 = \frac{1}{1 + \underbrace{\frac{\sigma_\varepsilon^2}{\beta^2 \sigma_x^2}}_{\cong 0}}$ is closer to 1	<input type="checkbox"/> $R^2 = \frac{1}{1 + \underbrace{\frac{\sigma_\varepsilon^2}{\beta^2 \sigma_x^2}}_{\gg 1}}$ is closer to 0

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