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Segundo Parcial:

2.1 Encuentre la expresión del espectro de fourier (de forma exponencial y trigonométrica) para la señal.

$$x(t) = |6 \operatorname{sen}(3t + (\pi/4))|^2$$

solución

$$x(t) = |6 \operatorname{sen}(3t + (\pi/4))|^2$$
$$= 36 \operatorname{sen}^2 (6t + (\pi/2))$$
$$= 36 \left(\frac{1 - \cos(6t + (\pi/2))}{2} \right)$$

$$= 18 - 18 \cos(6t + (\pi/2))$$
$$= 18 - 18 \cos(6t)\cos(\pi/2) - \operatorname{sen}(6t)\operatorname{sen}(\pi/2)$$
$$= 18 - 18 (0 - \operatorname{sen}(6t))$$
$$= 18 + 18 \operatorname{sen}(6t)$$

$$a_0 = \frac{1}{t_f - t_i} \int_{t_f}^{t_i} x(t)\, dt =$$

$$a_0 = \frac{1}{\pi - (-\pi)} \int_{-\pi}^{\pi} 18 + 18 \operatorname{sen}(6t)\, dt$$

$$a_0 = \frac{1}{2\pi} \left[\int_{-\pi}^{\pi} 18\, dt + \int_{-\pi}^{\pi} 18 \operatorname{sen}(6t)\, dt \right]$$

$$a_0 = \frac{1}{2\pi} \left[18t \Big|_{-\pi}^{\pi} - 3\cos(6t) \Big|_{-\pi}^{\pi} \right]$$

$$a_0 = \frac{1}{2\pi} \left[(18(\pi) - 18(-\pi)) - (3\cos(6\pi) - 3\cos(-6\pi)) \right]$$

$$a_0 = \frac{1}{2\pi} \left[36\pi - 0 \right]$$

$$a_n = \frac{2}{T_f - T_i} \int_{\pi}^{\pi} x(t) \cos(n\omega_0 t)\, dt$$

sabemos que $\omega_0 = 1$

$$a_n = \frac{\cancel{2}}{\cancel{2}\pi} \int_{-\pi}^{\pi} (18 + 18 \operatorname{sen}(6t)) \cos(nt)\, dt$$

$$a_n = \frac{1}{\pi} \left[\int_{-\pi}^{\pi} (18 \cos(nt)\, dt + \int_{-\pi}^{\pi} \underbrace{18 \operatorname{sen}(6t)}_{\text{impar}} \underbrace{\cos(nt)}_{\text{par}}\, dt \right]$$

$$a_n = \frac{1}{\pi} \left[\frac{18 \operatorname{sen}}{n} (nt) \Big|_{-\pi}^{\pi} + 0 \right] = 0$$

si se quiere comprobar, se tiene que:

$$a_n = \frac{1}{\pi} \left[\int_{-\pi}^{\pi} 18 \cos(nt)\, dt + \int_{-\pi}^{\pi} 18 \operatorname{sen}(6t) \cos(nt)\, dt \right]$$

$$a_n = \frac{18}{\pi} \left[\int_{-\pi}^{\pi} 18 \cos(nt\, dt + \int_{-\pi}^{\pi} 18 \operatorname{sen}(6t) \cos(nt)\, dt \right]$$

$$a_n = \frac{18}{\pi} \left[\int_{-\pi}^{\pi} \cos(nt)\, dt + \int_{-\pi}^{\pi} \left(\frac{\operatorname{sen}(6t + nt) + \operatorname{sen}(6t + nt)}{2} \right) \right]$$

- $u = 6t + nt$
  $du = (6 + n)\, dt$
  $dt = \frac{du}{6+n}$
- $z = 6t - nt$
  $dz = (6 - n)\, dt$
  $dt = \frac{dz}{6-n}$

$$a_n = \frac{18}{n} \left[\frac{1}{n} \operatorname{sen}(nt) \Big|_{-\pi}^{\pi} + \frac{1}{2} \int_{-\pi}^{\pi} \frac{\operatorname{sen}(u)\, du}{6+n} + \frac{1}{2} \int_{\pi}^{\pi} \frac{\operatorname{sen}(z)\, dz}{6-n} \right]$$

$$a_n = \frac{18}{n} \left[0 + \frac{1}{12+2n} (-\cos(6t + nt)) \Big|_{-\pi}^{\pi} + \frac{1}{12+2n} (-\cos(6t - nt)) \Big|_{-\pi}^{\pi} \right]$$

$$a_n = \frac{18}{n} \left[\frac{1}{12+2n} (-\cos(6\pi + n\pi) + \cos(-6\pi - n\pi)) + \right.$$
$$\left. \frac{1}{12+2n} (-\cos(6\pi - n\pi) + \cos(6\pi - n\pi) \right]$$

$$\cos(6\pi + n\pi) = \cos(6\pi)\cdot\cos(n\pi) - \text{sen}(6\pi)\,\text{sen}(n\pi)$$
$$= 1\cdot\cos(n\pi) - 0$$

$$\cos(6\pi - n\pi) = \cos(6\pi)\cos(n\pi) + \text{sen}(6\pi)\,\text{sen}(n\pi)$$
$$= 1\cdot\cos(n\pi) + 0.$$

$$a_n = \frac{18}{\pi}\left[\frac{1}{12+2n}\left(\cancelto{0}{-\cos(n\pi) + \cos(n\pi)}\right) + \frac{1}{12+2n}\left(\cancelto{0}{-\cos(n\pi) + \cos(n\pi)}\right)\right]$$

$$a_n = 0.$$

Ahora, se calcula b_n.

$$b_n = \frac{2}{\pi-(-\pi)}\int_{-\pi}^{\pi}(18 + 18\,\text{sen}(6t))\,\text{sen}(n\omega_0 t)\,dt$$

$$\omega_0 = 1.$$

$$b_n = \frac{18}{\pi}\int_{-\pi}^{\pi}\text{sen}(nt)\,dt + \frac{18}{\pi}\int_{-\pi}^{\pi}\text{sen}(6t)\,\text{sen}(nt)\,dt$$

Sabiendo que:

$$\text{sen}(\theta)\,\text{sen}(\alpha) = \frac{\cos(\theta-\alpha) - \cos(\theta+\alpha)}{2}$$

Entonces:

$$b_n = \frac{18}{\pi}\int_{-\pi}^{\pi}\text{sen}(nt)\,dt + \frac{18}{2\pi}\int_{-\pi}^{\pi}\cos(6t-nt)\,dt + \frac{18}{2\pi}\int_{-\pi}^{\pi}\cos(6t+nt)\,dt$$

$$b_n = \frac{18}{\pi}\left(-\cos(nt)\right)\Big|_{-\pi}^{\pi} + \frac{18}{2\pi(6t-nt)}\left(\text{sen}(6t-nt)\right)\Big|_{-\pi}^{\pi}$$
$$+ \frac{18}{2\pi(6t+nt)}\left(\text{sen}(6t+nt)\right)$$

$$b_n = \frac{18}{\pi}\left(\cos(n\pi) - \cos(-n\pi)\right) + \frac{18\;\text{sen}((6-n)\pi) - \text{sen}((6-n)\pi)}{2\pi\,(6t-nt)}$$
$$+ \frac{18\;\text{sen}((6+n)\pi) - \text{sen}((6-n)\pi)}{2\pi\,(6+n)}$$

Decimos que para $n \neq 6$; $b_n = 0$. Sin embargo para $n = 6$ debemos calcular el límite.

$$b_6 = 18 \lim_{n \to 6} \frac{d/dn \left[sen((6-n)\pi) - sen((6-n)-\pi) \right]}{d/dn \left[2\pi (6-n) \right]}$$

$$b_6 = 18 \lim_{n \to 6} \frac{\cos((6-n)\pi)(-\pi) - \cos((6-n)\pi)\pi}{-2\pi}$$

$$b_6 = \frac{18 \quad \cos(0)(-\pi) - \cos(0)\pi}{-2\pi} = \frac{-36\pi}{-2\pi} = 18$$

$b_6 = 18$ y suponemos $b_{-6} = -18$

Por lo tanto:

$$b_n = \begin{cases} 18 & n = 6 \\ -18 & n = -6 \\ 0 & \forall \setminus \{0, 6\} \end{cases}$$

Para la serie exponencial compleja, decimos que:

$$C_n = \frac{a_n - j b_n}{2}$$

$$C_n(6) = \frac{0 - j(18)}{2} = -j9$$

$$C_n(-6) = \frac{0 - j(-18)}{2} = j9$$

Por lo tanto:

$$C_n = \begin{cases} 18 & n = 0 \\ -9j & n = \{6\} \\ 9j & n = \{-6\} \\ 0 & \forall n \setminus \{0, 6, -6\} \end{cases}$$

Para construir la señal:

$$x(t) = \sum_{n=-N}^{N} C_n e^{jnt}$$

$$x(t) = C_{-6} e^{-j6t} + C_0 e + C_6 e^{j6t}$$

$$= [9j(\cos(6t) - 9j^2 \operatorname{sen}(6t) + 18 - 9j \cos(6t) - 9j^2 \operatorname{sen}(6t)]$$

$$x(t) = 18 + 18 \operatorname{sen}(6t)$$

El error relativo se calcula según:

$$E_r[\%] = \left[1 - \frac{1}{P_x} \sum_{n=-N}^{N} |C_n|^2\right] . 100\%$$

y para este caso:

$$P_x = \frac{1}{T} \int_{ti}^{tf} |x(t)|^2 dt$$

$$P_x = \frac{1}{2\pi} \int_{-\pi}^{\pi} |18 + 18 \operatorname{sen}(6t)|^2 dt$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{\pi} 18^2 dt + \int_{-\pi}^{\pi} 2(18)^2 \operatorname{sen}(6t) dt + \int_{-\pi}^{\pi} 18^2 \operatorname{sen}^2(6t) dt\right]$$

$$= \frac{1}{2\pi} \left[18^2 t \Big|_{-\pi}^{\pi} - \frac{2(18)^2 \cos(6t) dt}{6} \Big|_{-\pi}^{\pi} + 18^2 \int_{-\pi}^{\pi} \frac{1}{2} (1 - \cos(12t)) dt\right]$$

$$= \frac{1}{2\pi} \left[18^2 t \Big|_{-\pi}^{\pi} - \frac{18^2}{3} \cos(6t) \Big|_{-\pi}^{\pi} + \frac{18^2}{2} t \Big|_{-\pi}^{\pi} - \frac{1}{12} \operatorname{sen}(12t) \Big|_{-\pi}^{\pi}\right]$$

$$= \frac{162}{\pi}(2\pi) - 54[\cos(6\pi) - \cos(-6\pi)] - \frac{1}{2\pi}[\cancel{\operatorname{sen}(12\pi)}^{0} - \cancel{\operatorname{sen}(-12\pi)}^{0}]$$

$$= 324 - 0 + 162 - 0 = 486$$

$$E_r[\%] = \left[1 - \frac{(-9)^2 + (18)^2 + (9)^2}{486}\right] \times 100\%$$

2.2.

$$e(t) = A_c \cos(2\pi f_c(t)) \qquad A_c,\ f_c \in \mathbb{R}$$

$$y \qquad y(t) = \frac{e(t) + m(t)\,c(t)}{A_c}$$

$$y(w) = F\left\{ e(t) + \frac{m(t)\,e(t)}{A_c} \right\}$$

$$F\{A_c \cos(2\pi f_c t)\} = A_c\, F\{\cos(2\pi f_c t)\} = F\{e(t)\}$$

$$\cos(2\pi f_c t) = \frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}$$

$$A_c\left[F\left\{\frac{e^{j2\pi f_c t}}{2}\right\} + F\left\{\frac{e^{j2\pi f_c t}}{2}\right\}\right] = \frac{A_c}{2}\left[2\pi\delta(w - 2\pi f_c)\right.$$

$$\left. + 2\pi\delta(w + 2\pi f_c)\right]$$

$$F\{e(t)\} = A_c \pi \delta\left[(w - 2\pi f_c) + (w + 2\pi f_c)\right]$$

$$F\left\{\frac{m(t)\,A_c \cos(2\pi f_c t)}{A_c}\right\} = F\left\{\frac{m(t)\,e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\right\}$$

$$F\left\{\frac{m(t)\,e(t)}{A_c}\right\} = \frac{1}{2}\left[F\left\{m(t)\,e^{j2\pi f_c t}\right\} + F\left\{m(t)\,e^{-j2\pi f_c t}\right\}\right]$$

$$F\left\{\frac{m(t)\,e(t)}{A_c}\right\} = \frac{1}{2} M\left[(w - 2\pi f_c t) + (w + 2\pi f_c t)\right]$$

$$y(w) = A_c \pi \delta\left[(w - 2\pi f_c t) + (w + 2\pi f_c t) + \frac{1}{2} M\left[(w - 2\pi f_c t) + (w + 2\pi f_c t)\right]\right]$$