FRIEDRICH-ALEXANDER
UNIVERSITÄT
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FACULTY OF ENGINEERING

Computeranimation

Lesson 4 – Dynamics

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Motivation

Topics

- Rigid Transformation
- Animation
- Collision
- **Dynamic**
- Mass-Spring Simulation
- Rigging and Skeletal Animation

Introduction

Kinematics vs. Dynamics

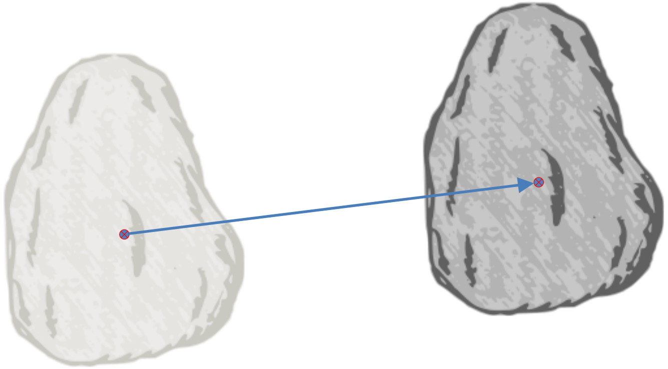
- **Kinematic**: Description of movement without considering accelerating forces
- **Dynamic**: Description of movement as a consequence of forces
- Both are areas of mechanics

- Until now every movement was described as a simple position-orientation-shift per time
- Now we want to include forces

Introduction

Linear Dynamic Movement

- The description of movements that contain only translation
- No rotation is considered here
- Movements always are triggered by forces at the center of mass



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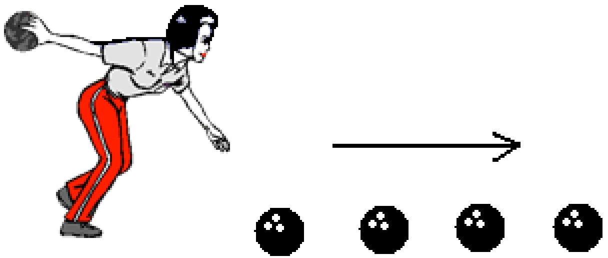
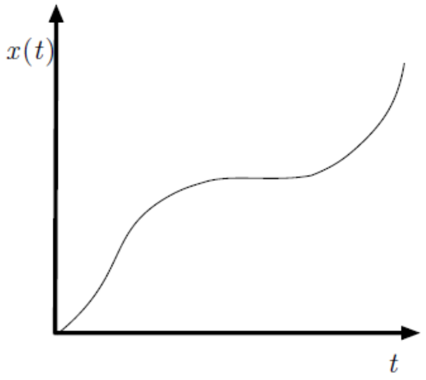
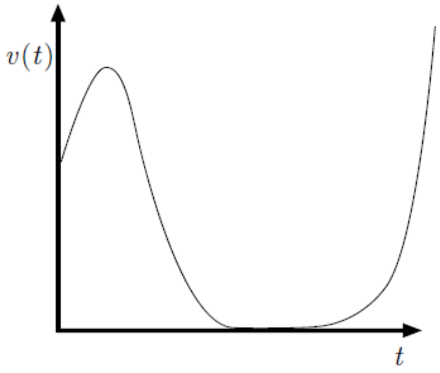
Introduction

Linear Dynamic Movement

- The ***position*** of an object at time t is defined as $x(t)$.
- The ***velocity*** of an object at time t is $v(t) = \dot{x}(t)$.
- The ***acceleration*** of an object at time t *is* $a(t) = \dot{v}(t) = \ddot{x}(t)$.



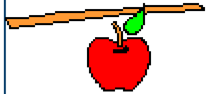




Introduction

Linear Dynamic Movement

- The ***impulse*** of an object at time t is $p(t) = m \cdot v(t)$.
- The ***change of the impulse*** is $\dot{p}(t) = m \cdot \dot{v}(t) = m \cdot a(t)$.



Newton‘s 2nd law of motion:

$$\boldsymbol{F = m \cdot a}$$

Introduction

Linear Dynamic Movement - Conclusion

position	$x(t)$
velocity	$v(t) = \dot{x}(t)$
acceleration	$a(t) = \dot{v}(t) = \ddot{x}(t)$
impulse	$p(t) = m \cdot v(t)$
change of the impulse	$\dot{p}(t) = m \cdot \dot{v}(t) = m \cdot a(t)$

Introduction

Angular Dynamic Movement

- We describe *linear* motion in the forementioned context
- What about force induced rotations?

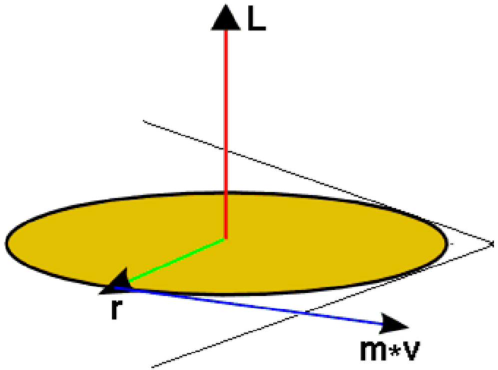
- The ***angular oriantation*** at time t is defined as $R(t)$.
- The ***angular velocity*** at time t is $\omega(t)$ (unit is $\frac{rad}{s}$).

- Warning: we cannot derive $R(t)$ to get $\omega(t)$.
- Use $q(t)$ (the quaternion-representation of $R(t)$) instead.
- $\dot{q}(t) = \frac{\partial q(t)}{\partial t} = \frac{1}{2} w(t) \cdot q(t)$ ($w(t)$ is the quaternion-representation of $\omega(t)$).

Introduction

Angular Quantities

- The ***angular momentum***: $\vec{L}(t) = \vec{r} \times p(t) = \vec{r} \times m \cdot v(t)$.
- The ***torque of an object***: $\tau = \dot{L}(t) = \frac{\delta L}{\delta t} = \frac{\delta(r \times p)}{\delta t} = r \times F(t)$.
- The ***inertia tensor***: $\boldsymbol{I} = \int r^{2} \mathrm{d}m$.
- The ***angular velocity***: $\omega = \boldsymbol{I}^{-1} \cdot \vec{L}(t)$.



Rigid Body Simulation

State of an Object

- To simulate an objects movement over time in a physical correct way
- We need to solve ***Ordinary Differential Equations*** (ODE)
- This way we integrate the path and the rotations of objects

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Rigid Body Simulation

State of an Object

- To simulate an objects movement over time in a physical correct way
- We need to solve ***Ordinary Differential Equations*** (ODE)
- This way we integrate the path and the rotations of objects

Constant State:

Mass:	m
Inertia tensor:	$\mathbf{I}$

Dynamic State:

Position/Orientation:	$x(t)$ and $q(t)$
Lin./Ang. velocity:	$\mathrm{v}(t)$ and $w(t)$
External Forces:	$\mathrm{F}_{\mathrm{ext}}(t)$

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Rigid Body Simulation

State of an Object

- We have to discretize the time domain into equally distant steps Δt
- Each step we search for $x(t + \Delta t)$ and $q(t + \Delta t)$
- Considering the ODEs‘ left sides are known at time t
- To solve for the next time step we have to ***integrate*** the ODEs

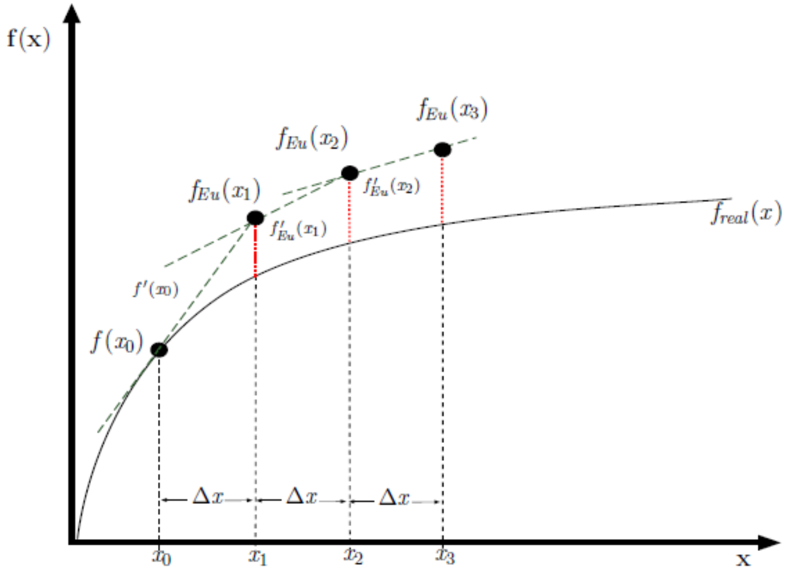
$$\boxed{\begin{gathered} v(t) = \dot{x}(t) \\ \frac{1}{2} w(t) q(t) = \dot{q}(t) \\ a(t) = \ddot{x}(t) \\ F(t) = \dot{p}(t) \\ \tau(t) = \dot{L}(t) \end{gathered}}$$

Rigid Body Simulation

Simple Euler Integrator

- given an analytic Function $f(x)$
- integrate the value $f(x + \Delta x)$
  by a linear approximation:

$$f(x_{i+1}) = f(x_i) + \Delta x \cdot f'(x_i)$$



Now we bring this in the context of rigid body simulation!

Rigid Body Simulation

Integrate Object‘s State

Linear

$$\mathrm{F}_{\mathrm{ext}}(t) = \sum_i F_i(t)$$

$$p(t+\Delta t) = p(t) + \Delta t \cdot F_{ext}(t)$$

$$v(t+\Delta t) = \frac{p(t+\Delta t)}{m}$$

$$x(t+\Delta t) = x(t) + \Delta t \cdot v(t+\Delta t)$$

Angular

$$\tau(t) = \sum_i r_i \times F_i(t)$$

$$L(t+\Delta t) = L(t) + \Delta t \cdot \tau(t)$$

$$\omega(t+\Delta t) = I^{-1} \cdot L(t+\Delta t)$$

$$q(t+\Delta t) = q(t) + \Delta t \cdot w(\omega(t+\Delta t))$$