FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG
FACULTY OF ENGINEERING

Computeranimation

Lesson 1 –Transformations

COMPUTER GRAPHICS • ERLANGEN

Motivation

Topics

- **Rigid Transformation**
- Animation
- Collision
- Dynamic
- Mass-Spring Simulation
- Rigging and Skeletal Animation
- Motion Capturing using RGB-D Sensor

Rigid Bodies

Math Primer

- Positions
- Spaces
- Matrices

Transformations

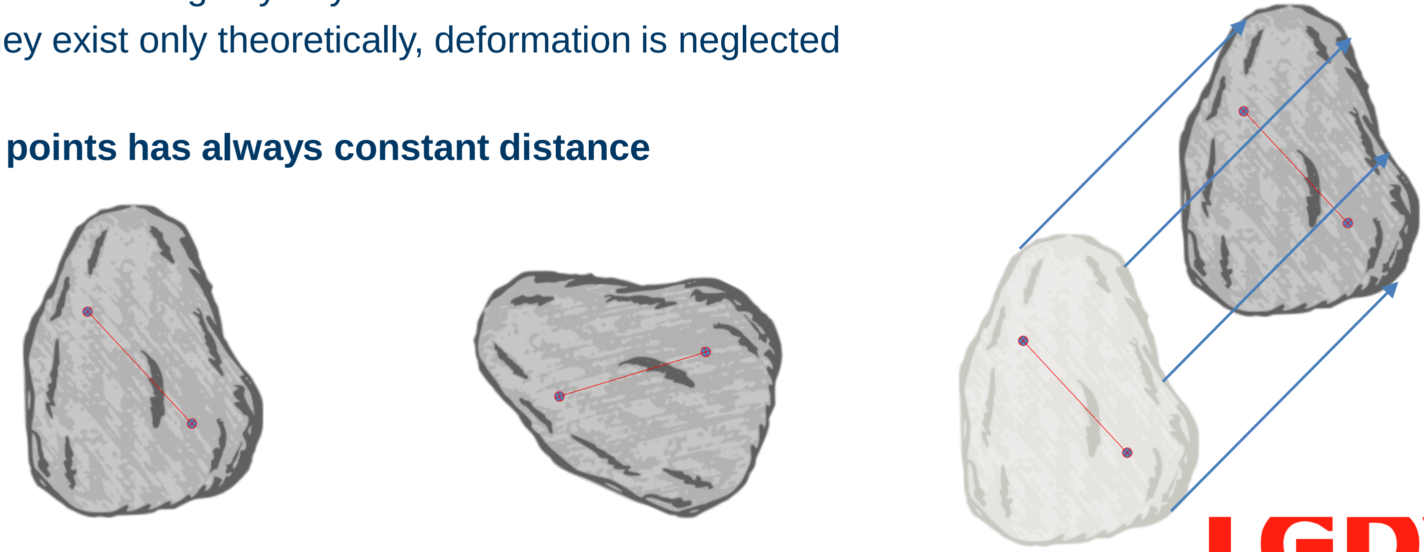
Application

Rigid Bodies

Definition:

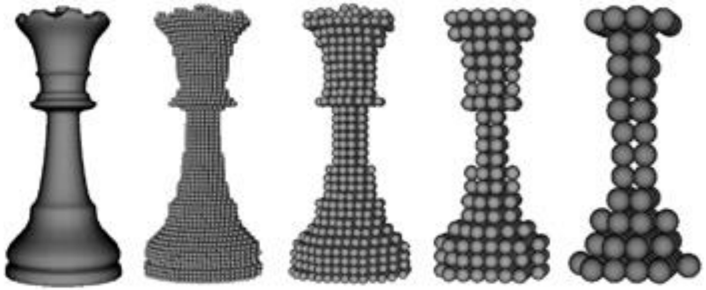
- Bodies that do not change by any influences
- In physics they exist only theoretically, deformation is neglected

→ Each pair of points has always constant distance



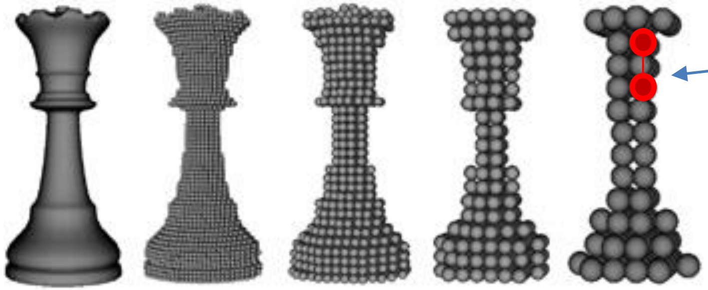
Rigid Bodies

- Rigid bodies usually are modelled as a set of discrete points



Rigid Bodies

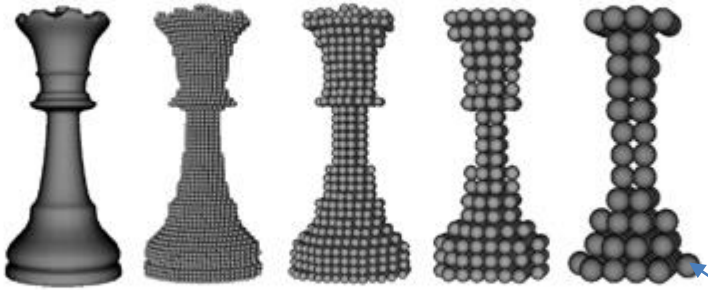
- Rigid bodies usually are modelled as a set of discrete points
- The points have constant relative positions



Pose to each other is constant

Rigid Bodies

- Rigid bodies usually are modelled as a set of discrete points
- The points have constant relative positions
- The single points' masses sum up to the rigid body's total mass



$$M_{total} = \sum_{i=0}^{N-1} m_i$$

rigid body with total mass M_{total}

point mass with single mass m_i

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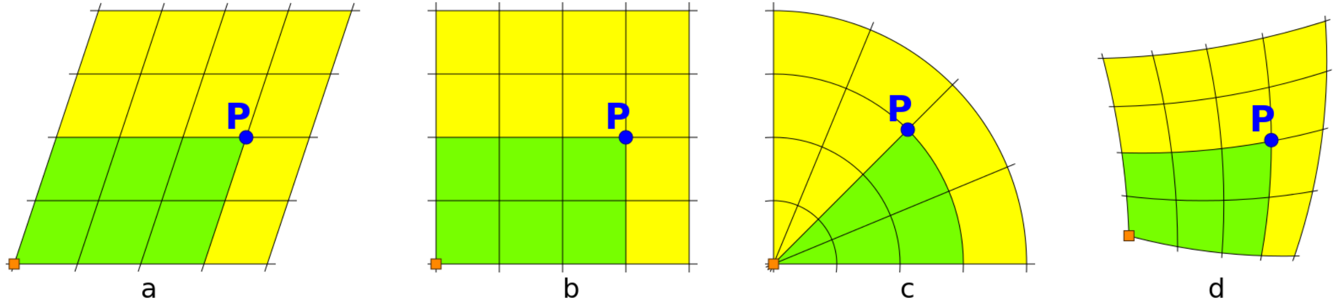
Math Primer

Position

- A **unique** location within a **geometrical space**
- This space is defined by a **coordinate system**
- A Coordinate System consists of an **origin** ($\vec{0}$) and …
- … spanning vectors (e.g. *unit vectors* in *Cartesian Systems*)

(a) rectilinear systems
(b) rectilinear & orthogonal systems
(c) curvilinear orthogonal grid
(d) curvilinear grid



P Position within the coordinate system

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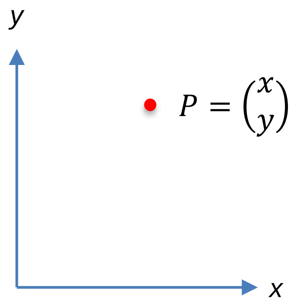
Math Primer

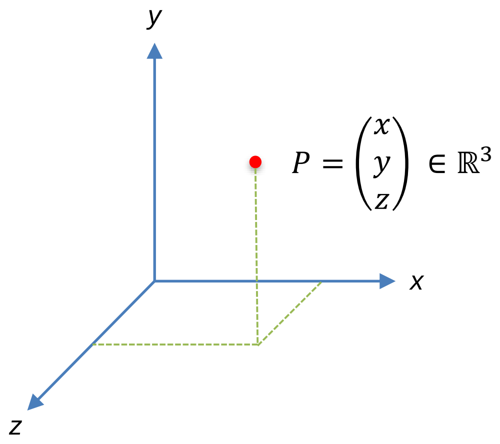
Position

- Points in a space usually are defined via coordinates $\in \mathbb{R}^2$ or $\in \mathbb{R}^3$

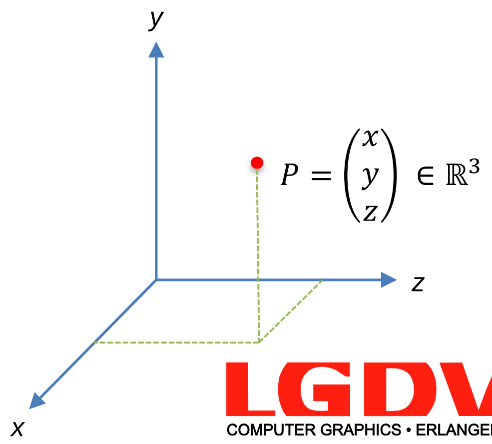
$\mathbb{R}^2$



$\mathbb{R}^3 \; - \textit{right handed}$



$\mathbb{R}^3 \; - \textit{left handed}$



Rigid Bodies

Math Primer

Positions

Spaces

Matrices

Transformations

Application

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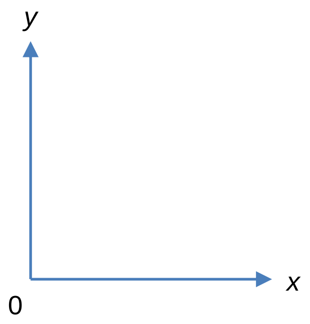
Math Primer

Spaces

- Within a single Coordinate System a point is *uniquely* defined by its **coordinates**

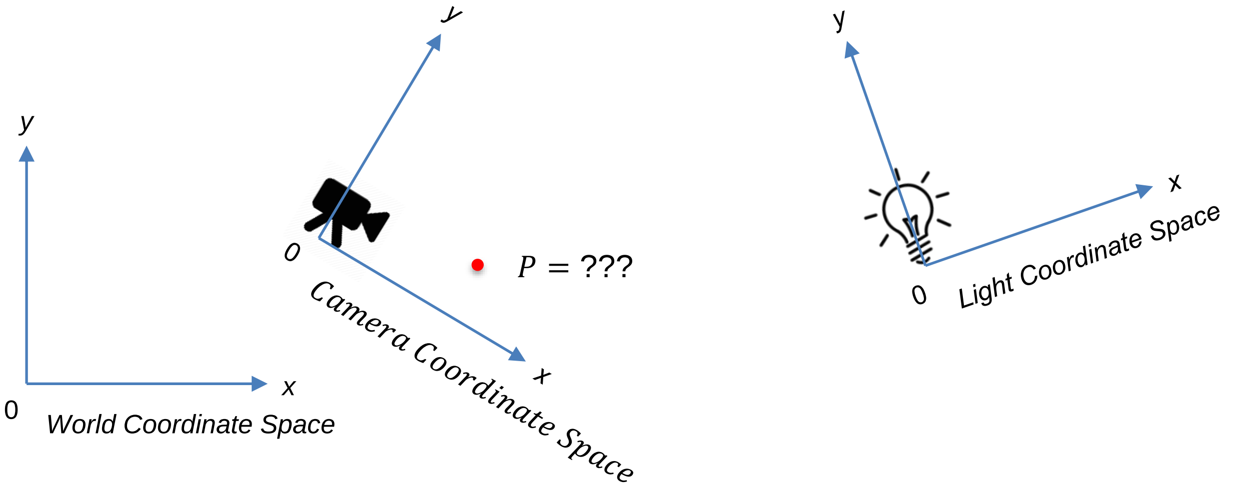


$$P = \begin{pmatrix} x_P \\ y_P \end{pmatrix}$$

Math Primer

Spaces

- Within a single Coordinate System a point is uniquely defined by its **coordinates**
- Things become complicated when more point of views are needed
- The position of P **depends** on the point of view

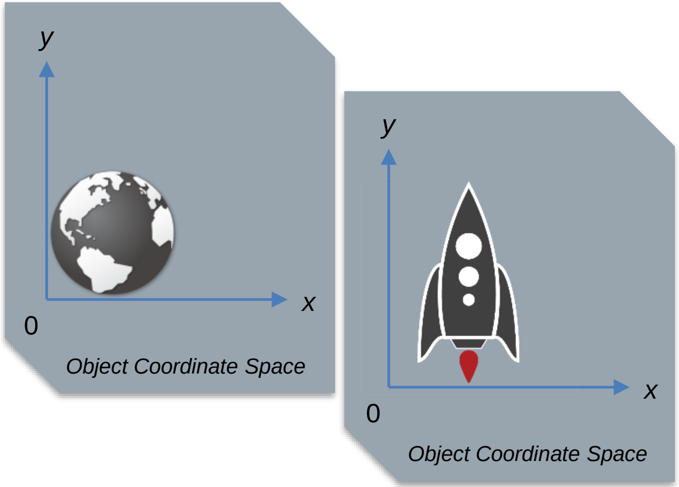


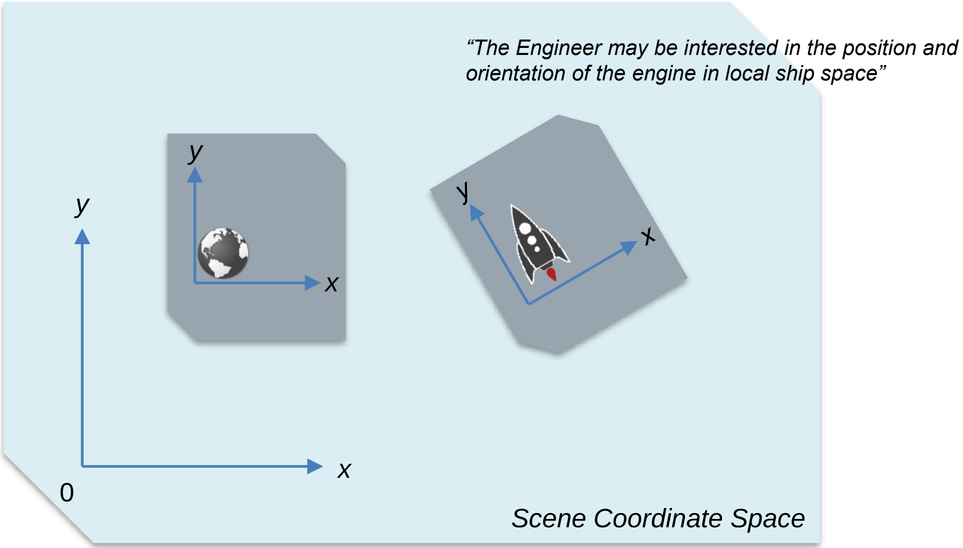
Math Primer

Spaces

- Positions often need to be expressed in different spaces

"The Astronaut wants to know the position of the landing place in world's local space"





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Math Primer

Matrices

- A Table of Elements ($\boldsymbol{a}_{ij}$) arranged in a rectangular structure of $\boldsymbol{M}$ **rows** and $\boldsymbol{N}$ **columns**

$$\begin{bmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{M1} & \cdots & a_{MN} \end{bmatrix}$$

Math Primer

Special Matrices

Symmetric Matrices

$$A = A^T = \begin{bmatrix} 1 & 10 & 30 \\ 10 & 2 & 64 \\ 30 & 64 & 3 \end{bmatrix}$$

Quadratic Matrices

$$M = N \rightarrow \begin{bmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{M1} & \cdots & a_{MM} \end{bmatrix}$$

Orthogonal Matrices

$$|\det(A)| = 1$$

Math Primer

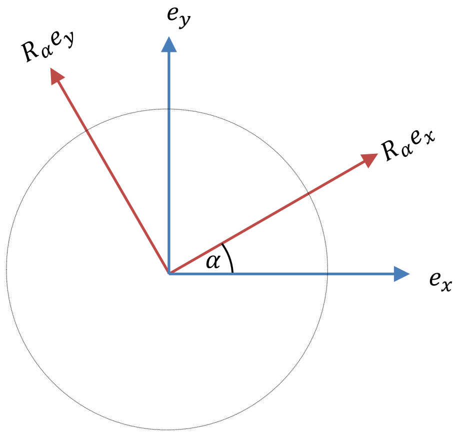
Orthogonal Matrices

- Of special interest for computer graphics (spaces, rotations, …)
- Most important property: $A^{-1} = \mathrm{A}^{\mathrm{T}} \rightarrow A^T A = I$

Rotation Matrix

- A simple 2D matrix that rotates a point about the angle α

$$R_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$



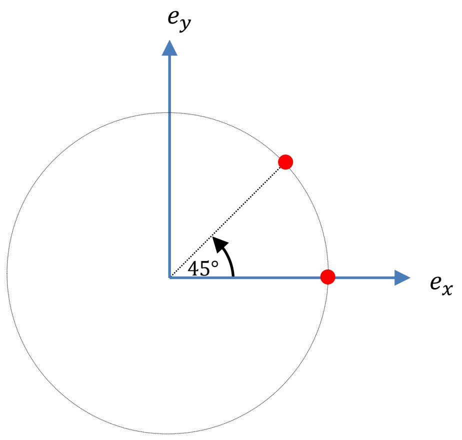
Math Primer

Matrix-Vector-Multiplication

- Represents a transformation to a vector.
- Example (rotation about 45°):

$$R_{45°} = \begin{bmatrix} \cos 45° & -\sin 45° \\ \sin 45° & \cos 45° \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \cdot 1 - \frac{1}{\sqrt{2}} \cdot 0 \\ \frac{1}{\sqrt{2}} \cdot 1 + \frac{1}{\sqrt{2}} \cdot 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



Transformations

Spaces

- Expressing a single Point (defined in one Coordinate System) in another system is called **Transformation**

Transformations

Affine Transformations

Linear Transformations | Rigid Transformations

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Transformations

Affine Transformation

$$f: X \rightarrow Y \qquad x \rightarrow \boldsymbol{A} \cdot \boldsymbol{x} + \boldsymbol{b}$$

$\boldsymbol{A}$: linear Transformation
$\boldsymbol{b}$: vector in Y

preserve…
… points
… preserve lines
… preserve planes
… parallel lines remain parallel

not necessarily preserve…
… angles
… ratios of distances between points on a straight line

Transformations

Affine Transformation

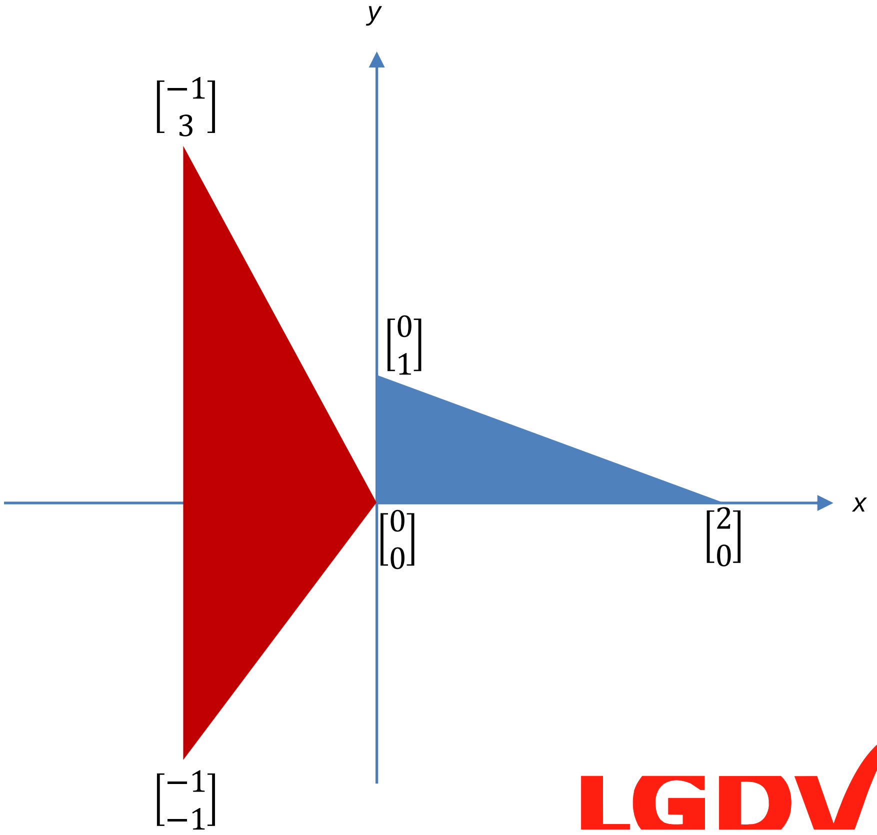
Example:

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \cdot 0 + 1 \cdot 1 - 1 \\ 2 \cdot 0 + 1 \cdot 1 - 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \cdot 0 + 1 \cdot 0 - 1 \\ 2 \cdot 0 + 1 \cdot 0 - 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

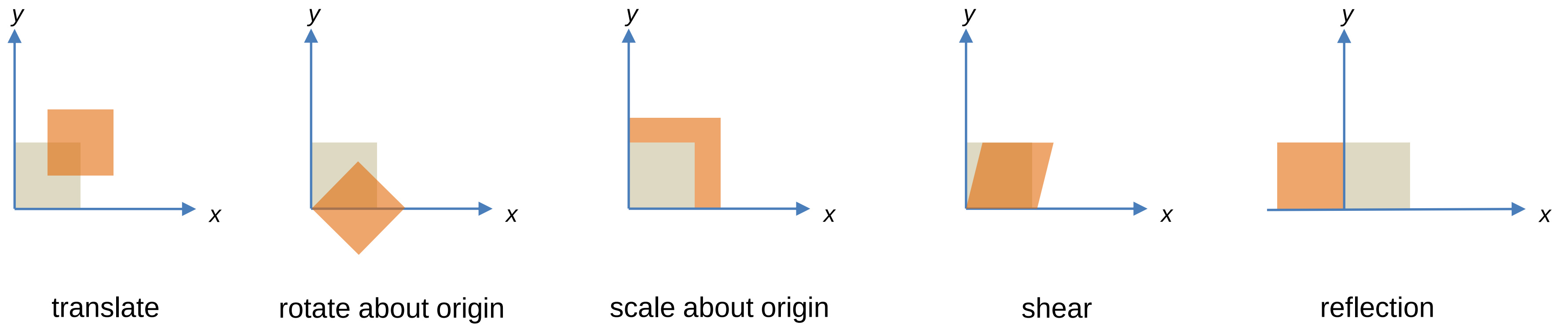
$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \cdot 2 + 1 \cdot 0 - 1 \\ 2 \cdot 2 + 1 \cdot 0 - 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$



Transformations

Affine Transformation

Classes of Affine Transformations



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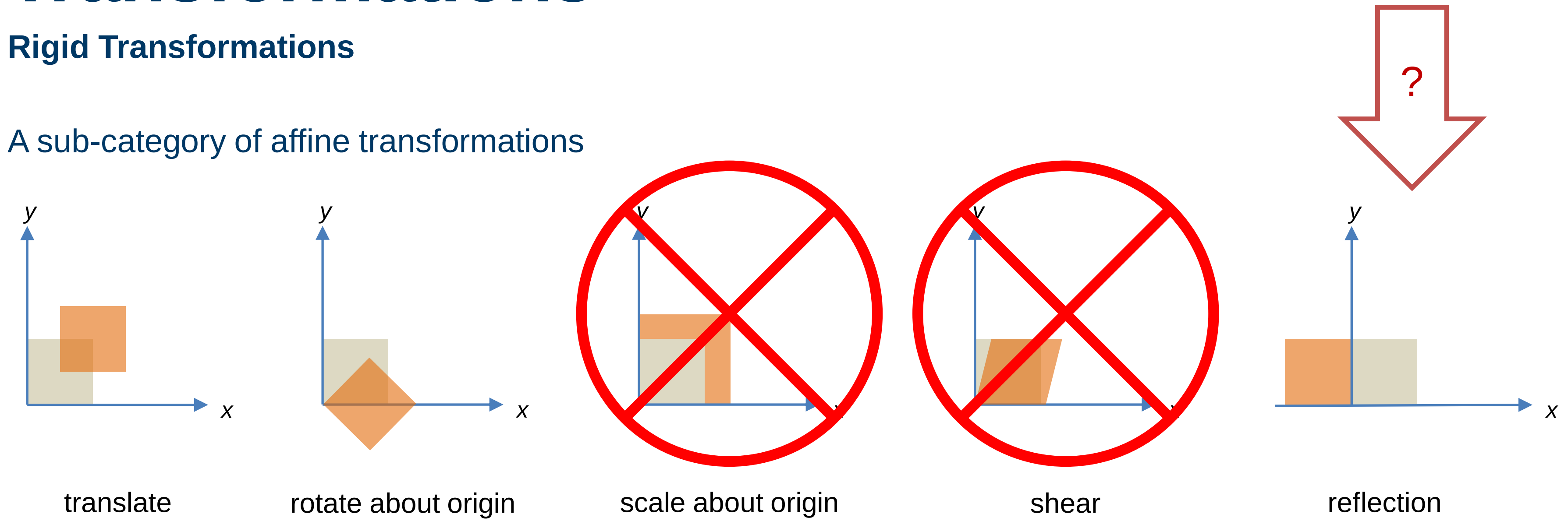
Transformations

Rigid Transformations

A sub-category of affine transformations



preserves the distance between every pair of points

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Transformations

Rigid Transformation Pipeline

- Since rigid transformations (amongst others) are linear, they can be put together

only rotation: $\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \boldsymbol{R}_{-45°} \cdot (\boldsymbol{R}_{+45°} \cdot \begin{bmatrix} x \\ y \end{bmatrix}) \rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \cdot (\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix})$

only translation: $\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \left(\begin{bmatrix} x \\ y \end{bmatrix} + \overrightarrow{t_0} \right) + \overrightarrow{t_1} \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} + (\overrightarrow{t_0} + \overrightarrow{t_1})$

both: $\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \boldsymbol{R}_1 \cdot \left(\boldsymbol{R}_0 \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \overrightarrow{t_0} \right) + \overrightarrow{t_1}$

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