APPENDIX

I

DIRECTION COSINES AS FUNCTIONS OF ORIENTATION ANGLES

In this appendix, the direction cosines associated with each of 24 sets of angles describing the orientation of rigid body B in a reference frame A are tabulated. To use the tables, proceed as follows: let $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ be a dextral set of mutually perpendicular unit vectors fixed in the reference frame A, and let $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$ be a similar such set fixed in the body B. Regard $\mathbf{b}_i$ as initially aligned with $\mathbf{a}_i$ ($i = 1, 2, 3$); select the type of rotation sequence of interest (that is, body-three, body-two, space-three, or space-two); letting θ_1, θ_2, θ_3 denote the amounts of the first, the second, and the third rotation, respectively, pick the rotation sequence of interest [for example, 3-1-2 (corresponding to a $\theta_1\mathbf{b}_3$, $\theta_2\mathbf{b}_1$, $\theta_3\mathbf{b}_2$ body-three sequence or a $\theta_1\mathbf{a}_3$, $\theta_2\mathbf{a}_1$, $\theta_3\mathbf{a}_2$ space-three sequence)]; finally, locate the table corresponding to the rotation sequence chosen. The nine entries in the table [with s_i and c_i standing, respectively, for $\sin\theta_i$ and $\cos\theta_i$ ($i = 1, 2, 3$)] are the elements C_{ij} of the associated direction cosine matrix, which elements are defined as $C_{ij} \triangleq \mathbf{a}_i \cdot \mathbf{b}_j$ ($i, j = 1, 2, 3$). Moreover, by reading a row or column of the table, one can determine how to express any of $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ in terms of $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$, or any of $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$ in terms of $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$. For example, in the table corresponding to the body-three 3-1-2 sequence, the third row reveals that $\mathbf{a}_3 = -c_2s_3\mathbf{b}_1 + s_2\mathbf{b}_2 + c_2c_3\mathbf{b}_3$, while the second column indicates that $\mathbf{b}_2 = -s_1c_2\mathbf{a}_1 + c_1c_2\mathbf{a}_2 + s_2\mathbf{a}_3$.

Body-three: 1-2-3

	b_1	b_2	b_3
a_1	C_2C_3	$-C_2S_3$	S_2
a_2	$S_1S_2C_3 + S_3C_1$	$-S_1S_2S_3 + C_3C_1$	$-S_1C_2$
a_3	$-C_1S_2C_3 + S_3S_1$	$C_1S_2S_3 + C_3S_1$	C_1C_2

Body-three: 2-3-1

	b_1	b_2	b_3
a_1	C_1C_2	$-C_1S_2C_3 + S_3S_1$	$C_1S_2S_3 + C_3S_1$
a_2	S_2	C_2C_3	$-C_2S_3$
a_3	$-S_1C_2$	$S_1S_2C_3 + S_3C_1$	$-S_1S_2S_3 + C_3C_1$

Body-three: 3-1-2

	b_1	b_2	b_3
a_1	$-S_1S_2S_3 + C_3C_1$	$-S_1C_2$	$S_1S_2C_3 + S_3C_1$
a_2	$C_1S_2S_3 + C_3S_1$	C_1C_2	$-C_1S_2C_3 + S_3S_1$
a_3	$-C_2S_3$	S_2	C_2C_3

Body-three: 1-3-2

	b_1	b_2	b_3
a_1	C_2C_3	$-S_2$	C_2S_3
a_2	$C_1S_2C_3 + S_3S_1$	C_1C_2	$C_1S_2S_3 - C_3S_1$
a_3	$S_1S_2C_3 - S_3C_1$	S_1C_2	$S_1S_2S_3 + C_3C_1$

Body-three: 2-1-3

	b_1	b_2	b_3
a_1	$S_1S_2S_3 + C_3C_1$	$S_1S_2C_3 - S_3C_1$	S_1C_2
a_2	C_2S_3	C_2C_3	$-S_2$
a_3	$C_1S_2S_3 - C_3S_1$	$C_1S_2C_3 + S_3S_1$	C_1C_2

Body-three: 3-2-1

	b_1	b_2	b_3
a_1	C_1C_2	$C_1S_2S_3 - C_3S_1$	$C_1S_2C_3 + S_3S_1$
a_2	S_1C_2	$S_1S_2S_3 + C_3C_1$	$S_1S_2C_3 - S_3C_1$
a_3	$-S_2$	C_2S_3	C_2C_3

Body-two: 1-2-1

	b_1	b_2	b_3
a_1	C_2	S_2S_3	S_2C_3
a_2	S_1S_2	$-S_1C_2S_3 + C_3C_1$	$-S_1C_2C_3 - S_3C_1$
a_3	$-C_1S_2$	$C_1C_2S_3 + C_3S_1$	$C_1C_2C_3 - S_3S_1$

Body-two: 1-3-1

	b_1	b_2	b_3
a_1	C_2	$-S_2C_3$	S_2S_3
a_2	C_1S_2	$C_1C_2C_3 - S_3S_1$	$-C_1C_2S_3 - C_3S_1$
a_3	S_1S_2	$S_1C_2C_3 + S_3C_1$	$-S_1C_2S_3 + C_3C_1$

Body-two: 2-1-2

	b_1	b_2	b_3
a_1	$-S_1C_2S_3 + C_3C_1$	S_1S_2	$S_1C_2C_3 + S_3C_1$
a_2	S_2S_3	C_2	$-S_2C_3$
a_3	$-C_1C_2S_3 - C_3S_1$	C_1S_2	$C_1C_2C_3 - S_3S_1$

Body-two: 2-3-2

	b_1	b_2	b_3
a_1	$C_1C_2C_3 - S_3S_1$	$-C_1S_2$	$C_1C_2S_3 + C_3S_1$
a_2	S_2C_3	C_2	S_2S_3
a_3	$-S_1C_2C_3 - S_3C_1$	S_1S_2	$-S_1C_2S_3 + C_3C_1$

Body-two: 3-1-3

	b_1	b_2	b_3
a_1	$-S_1C_2S_3 + C_3C_1$	$-S_1C_2C_3 - S_3C_1$	S_1S_2
a_2	$C_1C_2S_3 + C_3S_1$	$C_1C_2C_3 - S_3S_1$	$-C_1S_2$
a_3	S_2S_3	S_2C_3	C_2

Body-two: 3-2-3

	b_1	b_2	b_3
a_1	$C_1C_2C_3 - S_3S_1$	$-C_1C_2S_3 - C_3S_1$	C_1S_2
a_2	$S_1C_2C_3 + S_3C_1$	$-S_1C_2S_3 + C_3C_1$	S_1S_2
a_3	$-S_2C_3$	S_2S_3	C_2

Space-three: 1-2-3

	b_1	b_2	b_3
a_1	C_2C_3	$S_1S_2C_3 - S_3C_1$	$C_1S_2C_3 + S_3S_1$
a_2	C_2S_3	$S_1S_2S_3 + C_3C_1$	$C_1S_2S_3 - C_3S_1$
a_3	$-S_2$	S_1C_2	C_1C_2

Space-three: 2-3-1

	b_1	b_2	b_3
a_1	C_1C_2	$-S_2$	S_1C_2
a_2	$C_1S_2C_3 + S_3S_1$	C_2C_3	$S_1S_2C_3 - S_3C_1$
a_3	$C_1S_2S_3 - C_3S_1$	C_2S_3	$S_1S_2S_3 + C_3C_1$

Space-three: 3-1-2

	b_1	b_2	b_3
a_1	$S_1S_2S_3 + C_3C_1$	$C_1S_2S_3 - C_3S_1$	C_2S_3
a_2	S_1C_2	C_1C_2	$-S_2$
a_3	$S_1S_2C_3 - S_3C_1$	$C_1S_2C_3 + S_3S_1$	C_2C_3

Space-three: 1-3-2

	b_1	b_2	b_3
a_1	C_2C_3	$-C_1S_2C_3 + S_3S_1$	$S_1S_2C_3 + S_3C_1$
a_2	S_2	C_1C_2	$-S_1C_2$
a_3	$-C_2S_3$	$C_1S_2S_3 + C_3S_1$	$-S_1S_2S_3 + C_3C_1$

Space-three: 2-1-3

	b_1	b_2	b_3
a_1	$-S_1S_2S_3 + C_3C_1$	$-C_2S_3$	$C_1S_2S_3 + C_3S_1$
a_2	$S_1S_2C_3 + S_3C_1$	C_2C_3	$-C_1S_2C_3 + S_3S_1$
a_3	$-S_1C_2$	S_2	C_1C_2

Space-three: 3-2-1

	b_1	b_2	b_3
a_1	C_1C_2	$-S_1C_2$	S_2
a_2	$C_1S_2S_3 + C_3S_1$	$-S_1S_2S_3 + C_3C_1$	$-C_2S_3$
a_3	$-C_1S_2C_3 + S_3S_1$	$S_1S_2C_3 + S_3C_1$	C_2C_3

Space-two: 1-2-1

	b_1	b_2	b_3
a_1	c_2	s_1s_2	c_1s_2
a_2	s_2s_3	$-s_1c_2s_3 + c_3c_1$	$-c_1c_2s_3 - c_3s_1$
a_3	$-s_2c_3$	$s_1c_2c_3 + s_3c_1$	$c_1c_2c_3 - s_3s_1$

Space-two: 1-3-1

	b_1	b_2	b_3
a_1	c_2	$-c_1s_2$	s_1s_2
a_2	s_2c_3	$c_1c_2c_3 - s_3s_1$	$-s_1c_2c_3 - s_3c_1$
a_3	s_2s_3	$c_1c_2s_3 + c_3s_1$	$-s_1c_2s_3 + c_3c_1$

Space-two: 2-1-2

	b_1	b_2	b_3
a_1	$-s_1c_2s_3 + c_3c_1$	s_2s_3	$c_1c_2s_3 + c_3s_1$
a_2	s_1s_2	c_2	$-c_1s_2$
a_3	$-s_1c_2c_3 - s_3c_1$	s_2c_3	$c_1c_2c_3 - s_3s_1$

Space-two: 2-3-2

	b_1	b_2	b_3
a_1	$c_1c_2c_3 - s_3s_1$	$-s_2c_3$	$s_1c_2c_3 + s_3c_1$
a_2	c_1s_2	c_2	s_1s_2
a_3	$-c_1c_2s_3 - c_3s_1$	s_2s_3	$-s_1c_2s_3 + c_3c_1$

Space-two: 3-1-3

	b_1	b_2	b_3
a_1	$-s_1c_2s_3 + c_3c_1$	$-c_1c_2s_3 - c_3s_1$	s_2s_3
a_2	$s_1c_2c_3 + s_3c_1$	$c_1c_2c_3 - s_3s_1$	$-s_2c_3$
a_3	s_1s_2	c_1s_2	c_2

Space-two: 3-2-3

	b_1	b_2	b_3
a_1	$c_1c_2c_3 - s_3s_1$	$-s_1c_2c_3 - s_3c_1$	s_2c_3
a_2	$c_1c_2s_3 + c_3s_1$	$-s_1c_2s_3 + c_3c_1$	s_2s_3
a_3	$-c_1s_2$	s_1s_2	c_2