MISSION REQUIRMENTS

Parameter	Value
Orbit Altitude	$h = 500000\ m$
Semi-major axis	$a = h + R_e = 6878137\ m$
Kourou Latitude	$\varphi = 5.2°$
Payload Mass	$M_{PL} = 300\ kg$

DESIGN CONCEPTS AND LAUNCHER ARCHITECTURE

2 stages launcher

- Liquid Oxygen (LOX)
- Kerosene (RP1)

Parameter	Symbol	1° STAGE	2° STAGE
Structural Coefficient	ε	0.11	0.14
Vacuum Specific Impulse	$I_{sp}\ [s]$	310	330
Maximum Axial Acceleration	$a_{max}\ \left[\frac{m}{s^2}\right]$	$7g_0$	$4g_0$
Reference Aerodynamic Area	$A_a\ [m^2]$	1	$-$
Nozzle Exit Area	$A_e\ [m^2]$	0.3	$-$
Nozzle Exit Pressure	$P_e\ [Pa]$	40000	$-$
Maximum Dynamic Pressure	$q_{max}\ [Pa]$	45000	$-$

STAGING OPTIMIZATION 1

Minimize the Gross Lift Off Weight (GLOW) for a certain Payload

Use `@Staging` to get the best distribution among the stages using the Lagrange multiplier method.

Starting from: $u = \frac{\Delta V_{required}}{c}$

- Total Payload Ratio: $\lambda_{tot} = \frac{e^{-\frac{u}{N}} - \varepsilon}{(1-\varepsilon)^N}$
- Stage Payload Ratio: $\lambda_i = \lambda_{tot}^{\frac{1}{N}}$
- Stage Mass Ratio: $\Lambda_i = \frac{1}{\varepsilon(1-\lambda_i)+\lambda_i}$

Starting from a first guess Lagrange multiplier: $p_0 = \frac{1}{\Lambda_i c\varepsilon - c}$

The goal is to find the actual Lagrange multiplier that minimize the function:

$$f = \Delta V_{required} + \sum_{i=1}^{2} c_i \ln\left(\frac{1 + pc_i}{pc_i\varepsilon_i}\right)$$

Matlab `@fZero` works very well when providing a good initial guess.

STAGING OPTIMIZATION 2

Check:

- final ΔV:

$$\begin{cases} \Delta V_i = c_i \ln(\varepsilon_i (1-\lambda_i) + \lambda_i) \\ \left|\sum_{i=1}^{2} \Delta V_i - \Delta V_{required}\right| < 10^{-5} \end{cases}$$

- Lagrange multiplier is a minimum:

$$\frac{\partial^2 f}{\partial \Lambda_i^2} = -\frac{1+pc_i}{\Lambda_i^2} + \left(\frac{\varepsilon_i}{1-\varepsilon_i \Lambda_i}\right)^2 > 0$$

Using the Lagrange multiplier:

- Mass Ratio for each stage: $\Lambda_i = \frac{1+pc_i}{pc_i \varepsilon_i}$
- Payload Ratio for each stage: $\lambda_i = \frac{1-\Lambda_i \varepsilon_i}{(1-\varepsilon_i)\Lambda_i}$
- Total Payload Ratio: $\lambda_{tot} = \prod_{i=1}^{2} \lambda_i$
- Initial Mass: $M_0 = M_{PL}/\lambda_{tot}$
- Sub Rocket mass: $m_{subR_i} = m_{subR_{i+1}}/\lambda_i$
- Stage Mass: $m_{stage_i} = m_{subR_i} - m_{subR_{i+1}}$
- Structural Mass: $m_{str_i} = m_{stag\ i} \varepsilon_i$
- Fuel Mass: $m_{prop_i} = m_{stage_i} - m_{str\,i}$
- Mass at I stage Burn Out: $m_{f1} = M_0 - m_{prop_1}$
- Mass at II stage Ignition: $m_{i_2} = M_0 - m_{stage_1}$
- Mass at II stage Burn Out: $m_{f_2} = m_{i_2} - m_{prop_2}$
- Mass at I stage Ignition: $m_{i_1} = m_{f_1} + m_{prop_1}$

Parameter	Symbol	1°STAGE	2°STAGE
Mass Ratio	Λ_i	5.1151	4.2083
Payload Ratio	λ_i	0.0961	0.1135
Stage Mass	$m_{stag\ i}\ [kg]$	$2.4867 \cdot 10^4$	$0.233 \cdot 10^4$
Mass at ignition	$m_{i_i}\ [kg]$	$2.7510 \cdot 10^4$	$2.6428 \cdot 10^3$
Mass at Burn Out	$m_{f\,i}\ [kg]$	$5.3782 \cdot 10^3$	627.9939
Structural mass	$m_{str_i}\ [kg]$	$2.7354 \cdot 10^3$	$0.3280 \cdot 10^3$
Propellant Mass	$m_{prop_i}\ [kg]$	$2.2132 \cdot 10^4$	$0.2015 \cdot 10^4$

THRUST AND MASS FLOW RATE

Thrust: $T_i = m_{f_i} a_{max_i}$

Mass Flow Rate: $\dot{m}_i = \frac{T_i}{c_i}$

Stage Burning Time: $t_{bo_i} = \frac{m_{p_i}}{\dot{m}_i}$

Parameter	Symbol	1° STAGE	2°STAGE
Thrust	$T_i\ [N]$	$3.6920 \cdot 10^5$	$2.4634 \cdot 10^4$
Mass Flow Rate	$\dot{m}_i\ \left[\frac{kg}{s}\right]$	121.4441	7.6120
Burning Time	$t_{bo_i}\ [s]$	182.2408	264.6883

LAUNCHER DYNAMICS

Knowing:

- Earth radius:
  $R_e = 6378137\ m$
- Earth Angular Velocity:
  $\vec{\omega}_e = [0 \quad 0 \quad \frac{2\pi}{86164}]$

Initial Position Vector: $\vec{r}_0 = [R_e \cos(\varphi) \quad 0 \quad R_e \sin(\varphi)]$

Initial Velocity Vector: $\vec{v}_0 = \vec{\omega}_e \times \vec{r}_0$

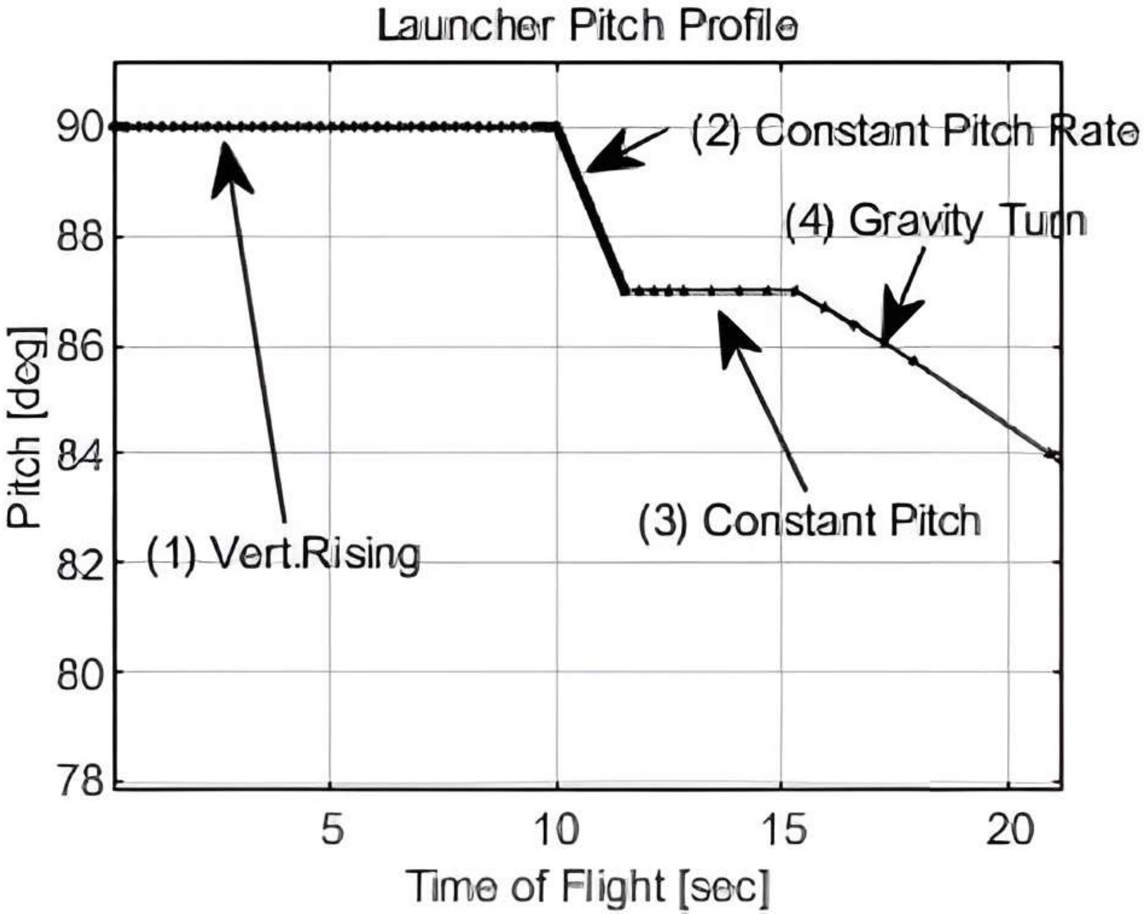
After propagating the state through the dynamical model:

Altitude: $h = r - R_e$

Relative Velocity Vector evolution in time: $\vec{v}_{rel} = \vec{v} - \vec{\omega}_e \times \vec{r}_0$

Thrust: $T = c\dot{m} + (P_e - P_a)A_e$

Mass: $m = m_0 - \dot{m}t$



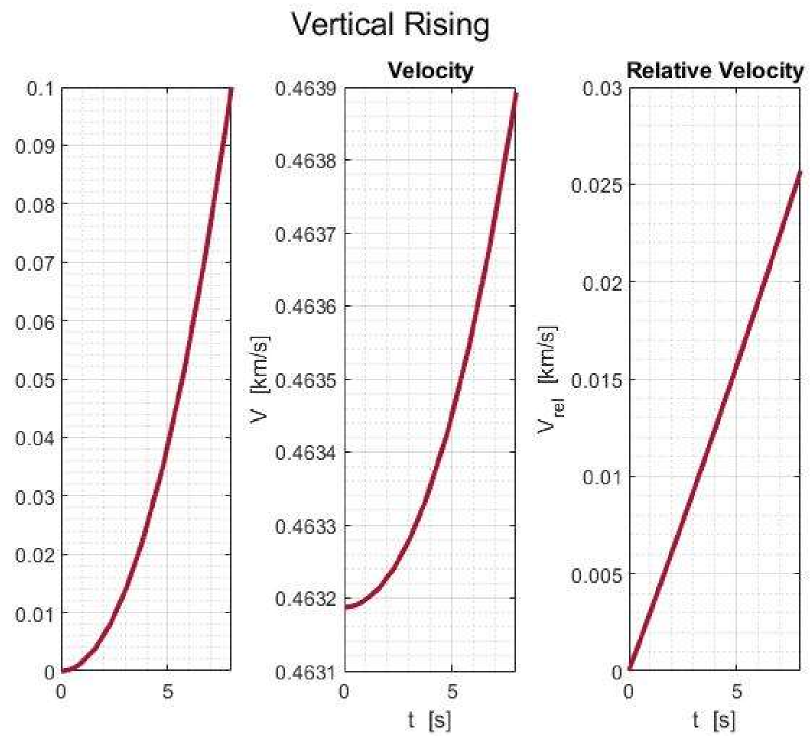
PHASES of Endo atmospheric flight

1) Vertical Rising - **VR**
2) Constant Pitch Rate - **CPR**
3) Constant Pitch - **CP**
4) Gravity Turn - **GT**

ENDO ANTMOSPHERIC FLIGHT

Vertical Rising VR ①

During the phase of Vertical Rising the launcher keeps 90 degrees of pitch angle until it reaches 100 m of altitude



DIRECTION OF THE THRUST

$$\vec{u} = \vec{r}$$

DYNAMICAL MODEL

$$\dot{\vec{r}} = \vec{v}$$

$$\dot{\vec{v}} = -\frac{\mu}{r^3}\vec{r} + \frac{T}{m}\frac{\vec{r}}{r} - \frac{1}{2}\rho\frac{A_a}{m}C_D\vec{v}_{rel}v_{rel}$$

ode stop function **`@EventFcnVR`**

ENDO ANTMOSPHERIC FLIGHT

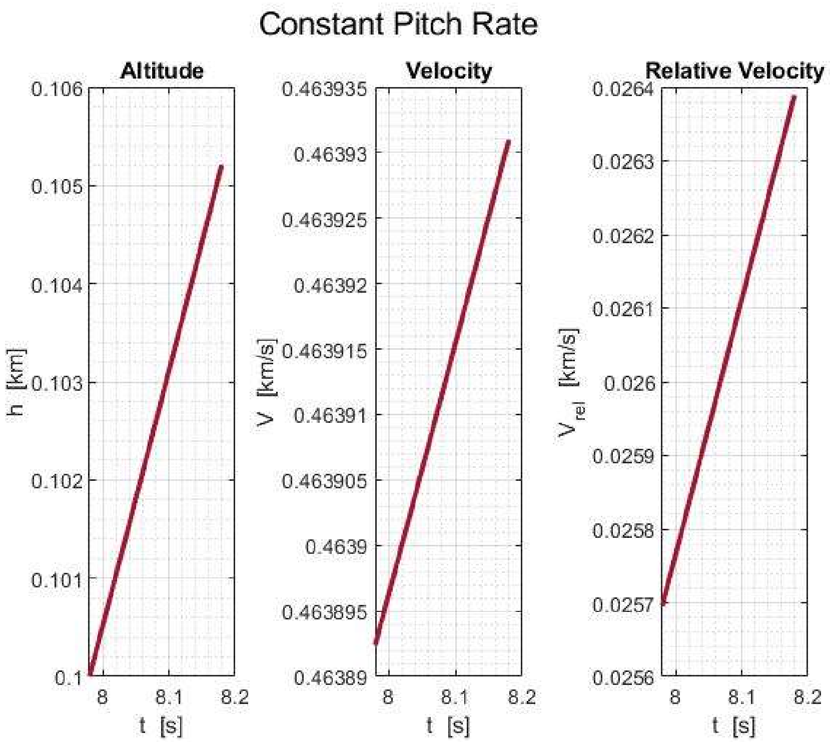
Constant Pitch Rate CPR

②

$$\begin{cases} \omega_{pitch} = 1°/s \\ \alpha = 0.2\,° \\ kick\ angle \end{cases}$$

$$\Delta t_{CPR} = \alpha / \omega_{pitch}$$

$$\theta = \omega_{pitch}(t - t_{VR})$$



DIRECTION OF THE THRUST

$$\vec{u} = \cos(\omega_{pitch} t)\,\vec{r} + \sin(\omega_{pitch} t)\,\vec{e}$$

$$\text{with } \vec{e} = [0\ 0\ 1] \times \hat{r}$$

DYNAMICAL MODEL

$$\dot{\vec{r}} = \vec{v}$$

$$\dot{\vec{v}} = -\frac{\mu}{r^3}\vec{r} + \frac{T}{m}\frac{\vec{u}}{u} - \frac{1}{2}\rho\frac{A_a}{m}C_D\vec{v}_{rel}v_{rel}$$

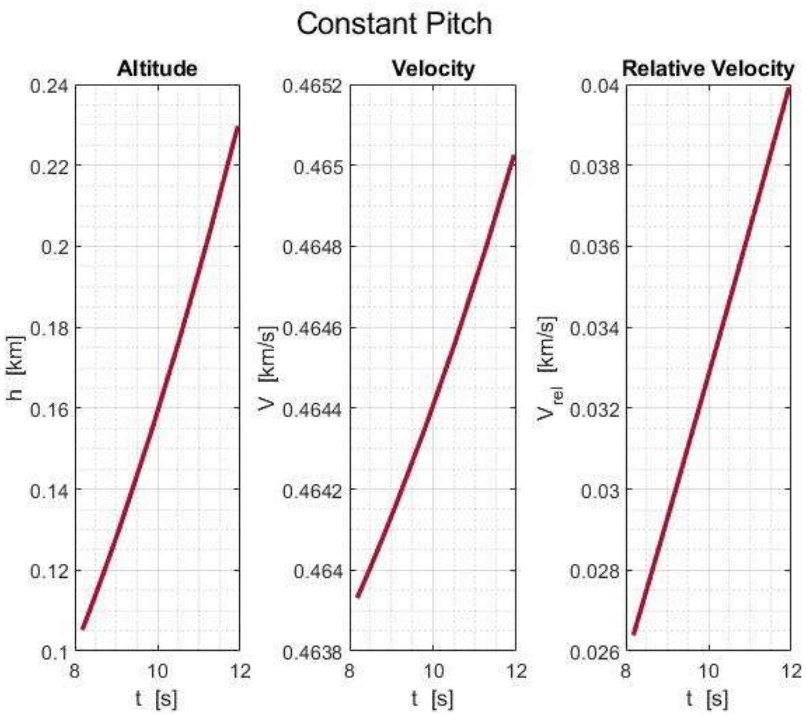
ENDO ANTMOSPHERIC FLIGHT

Constant Pitch CP

③

$$\gamma = \arccos\left(\frac{\vec{v}_{rel} \cdot \vec{e}}{|\vec{v}_{rel}||\vec{e}|}\right) \qquad \gamma - \theta = 0$$

$AoA = 0$
Get a final pitch equal to the kick angle
$\alpha = 90° - \theta_{end} = 0.2\ °$
keep this angle constant till the end of the integration



DIRECTION OF THE THRUST

$$\vec{u} = \cos(\alpha)\,\vec{r} + \sin(\alpha)\,\vec{e}$$

DYNAMICAL MODEL

$$\dot{\vec{r}} = \vec{v}$$

$$\dot{\vec{v}} = -\frac{\mu}{r^3}\vec{r} + \frac{T}{m}\frac{\vec{u}}{u} - \frac{1}{2}\rho\frac{A_a}{m}C_D\vec{v}_{rel}v_{rel}$$

ode stop function **`@EventFcnCP`**

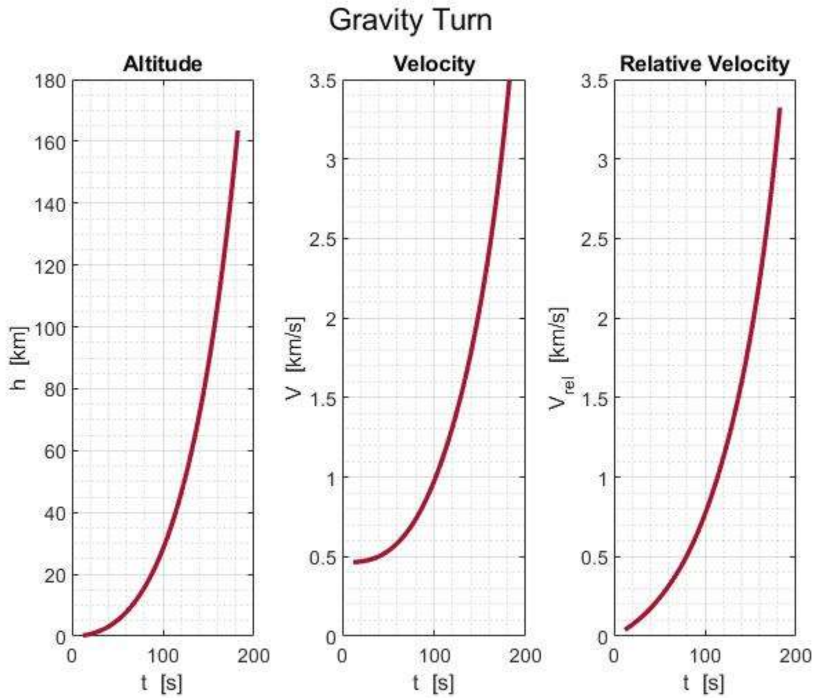
ENDO ANTMOSPHERIC FLIGHT

Gravity Turn GT

④

The integration time is dictated by the time of burn out of the first stage:

$$t_{bo1} = \frac{m_{prop_1}}{\dot{m}_1}$$



DIRECTION OF THE THRUST

$$\vec{u} = \vec{v}_{rel}$$

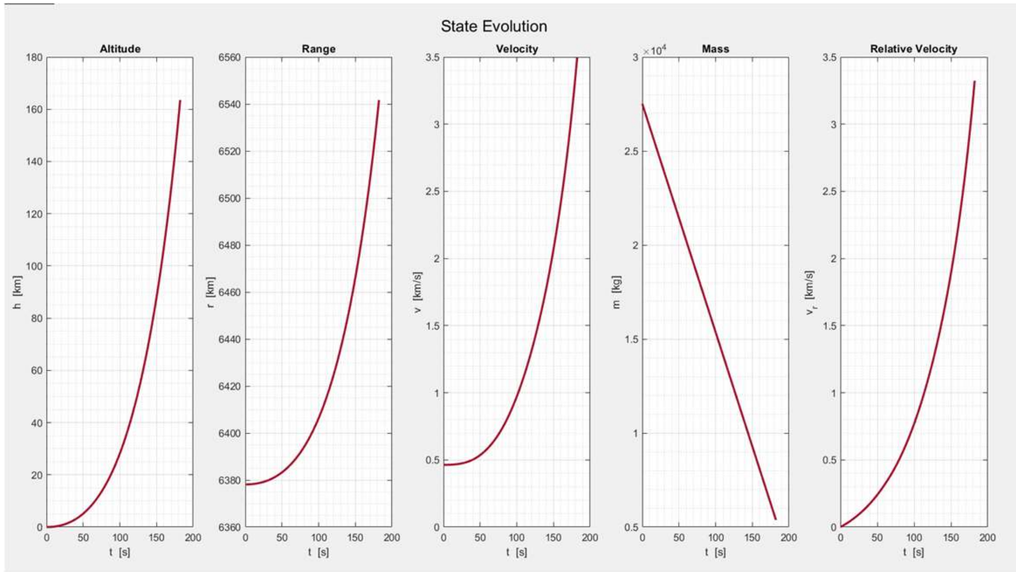
DYNAMICAL MODEL

$$\dot{\vec{r}} = \vec{v}$$

$$\dot{\vec{v}} = -\frac{\mu}{r^3}\vec{r} + \frac{T}{m}\frac{\vec{r}}{r} - \frac{1}{2}\rho\frac{A_a}{m}C_D\vec{v}_{rel}v_{rel}$$

STATE EVOLUTION

Phase	Final Altitude	Δt
Vertical Rising	$100\ m$	$7.9797\ s$
Constant Pitch Rate	$105.2084\ m$	$0.2\ s$
Constant Pitch	$229.7433\ m$	$3.7646\ s$
Gravity Turn	$163.54\ km$	$170.2965\ s$
		TOTAL $= 182.2408\ s = t_{bo_1}$

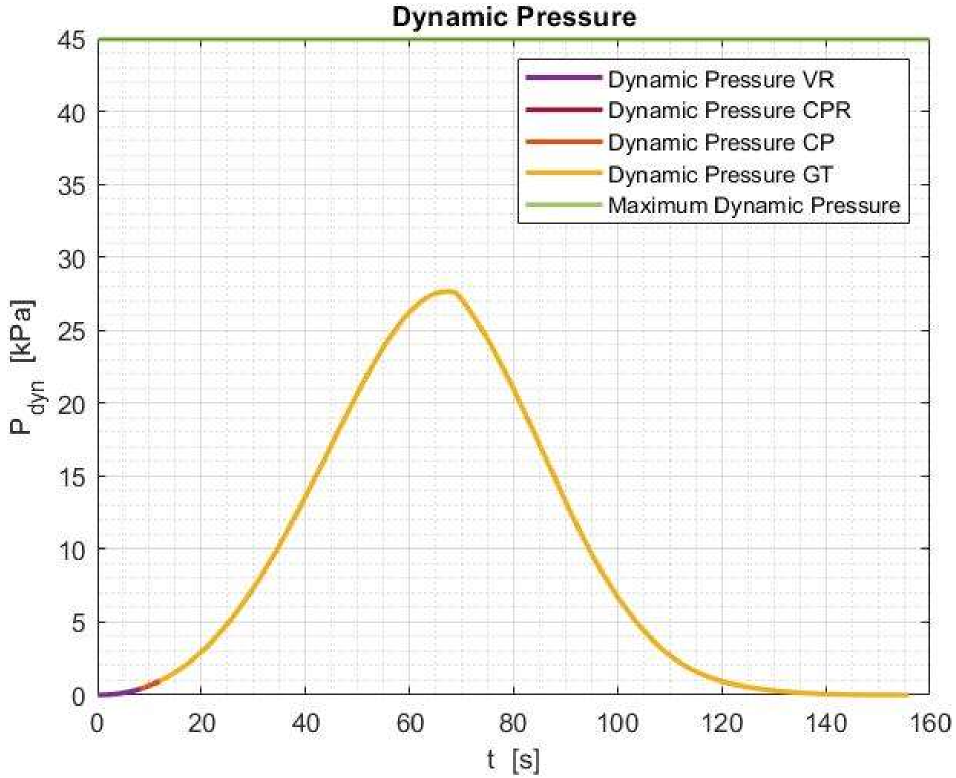


DYNAMICAL PRESSURE CHECK

to determine:

- density ρ, the
- atmospheric pressure P_a and
- speed of sound a

for each of the four phases by entering the altitude



Atmospheric model:
`@expEarthAtm`

COSTRAINT:

$$q_{max} = 45000\ Pa$$

$$q_j = \frac{1}{2}\rho_j v_{rel_j}^2$$

with $j = 1,2,3,4$
number of phases

DELTA V LOSSES

by **`trapz`** integration

Parameter	Value
Gravity Losses	$1.7276 \left[\frac{km}{s}\right]$
Aerodynamic Losses	$8.9101 \cdot 10^{-2} \left[\frac{km}{s}\right]$
Pressure Losses	$3.2370 \cdot 10^{-1} \left[\frac{km}{s}\right]$

ΔV_{TOT} Losses	$2.1404 \left[\frac{km}{s}\right]$

1) Gravity Losses

$$\gamma = cos^{-1}\left(\frac{\vec{v}_{rel} \cdot \vec{e}}{|\vec{v}_{rel}||\vec{e}|}\right) \longrightarrow a_{grav} = \frac{\mu}{|\vec{r}|^2} \sin\gamma$$

2) Aerodynamic Losses

$$\begin{cases} M_j = \dfrac{v_{rel_j}}{a_j} \\ C_{D_j} = interp1\left(M_0, C_{D0}, M_j, linear\right) \end{cases} \longrightarrow a_{drag_j} = \frac{\frac{1}{2} A_a \rho_j v_{rel_j}^2 C_{D_j}}{m_j}$$

3) Pressure Losses

$$a_{press_j} = \frac{P_{a_j} A_e}{m_j}$$

ATTITUDE DETERMINATION

FROM $\vec{b}$ AND $\vec{\imath}$ TO COSINE DIRECTORS

Triad of versors for the Body Reference Frame:

$$\begin{cases} \vec{b}_x = \vec{u}/|\vec{u}| \\ \vec{b}_y = \vec{e}/|\vec{e}| \\ \vec{b}_z = \vec{b}_x \times \vec{b}_y \end{cases}$$

Triad of versors for the Inertial Reference Frame:

$$\begin{cases} \vec{\imath}_x = [1\ 0\ 0] \\ \vec{\imath}_y = [0\ 1\ 0] \\ \vec{\imath}_z = [0\ 0\ 1] \end{cases}$$

Matrix of Cosine Directors as:

$$C_{ij} = \vec{b}_i \cdot \vec{\imath}_j$$

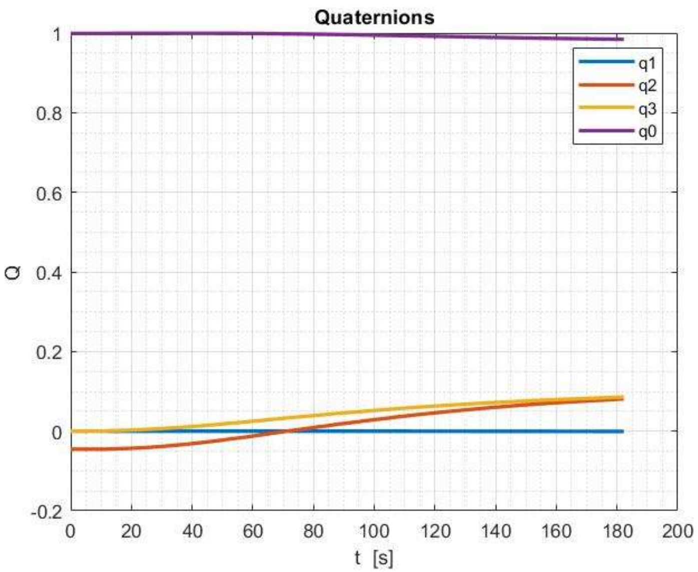
QUATERNIONS

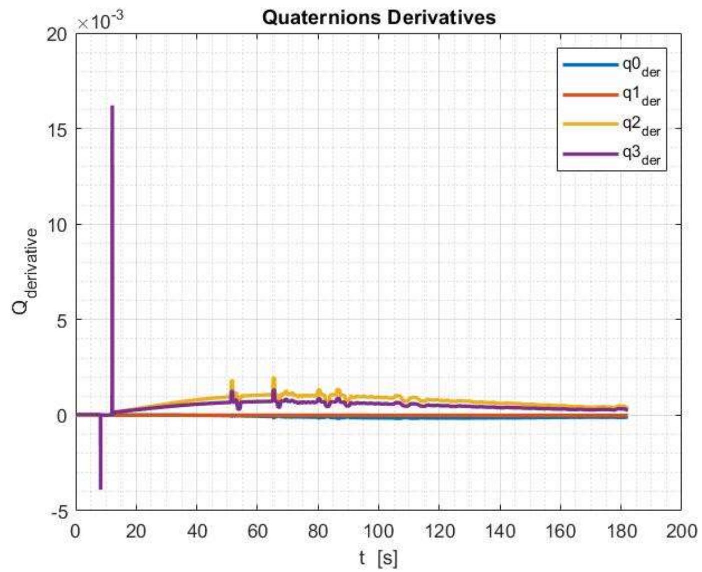
$$q_0 = \frac{1}{2}\sqrt{1 + C_{11} + C_{22} + C_{33}} \qquad \vec{q} = \frac{1}{4q_0}\begin{bmatrix} C_{23} - C_{32} \\ C_{31} - C_{13} \\ C_{12} - C_{21} \end{bmatrix}$$

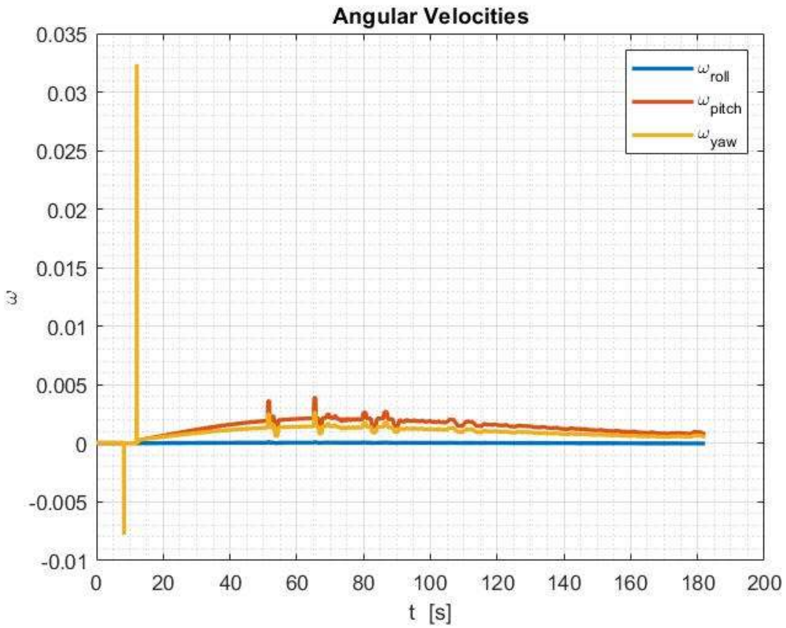
ANGULAR VELOCITY

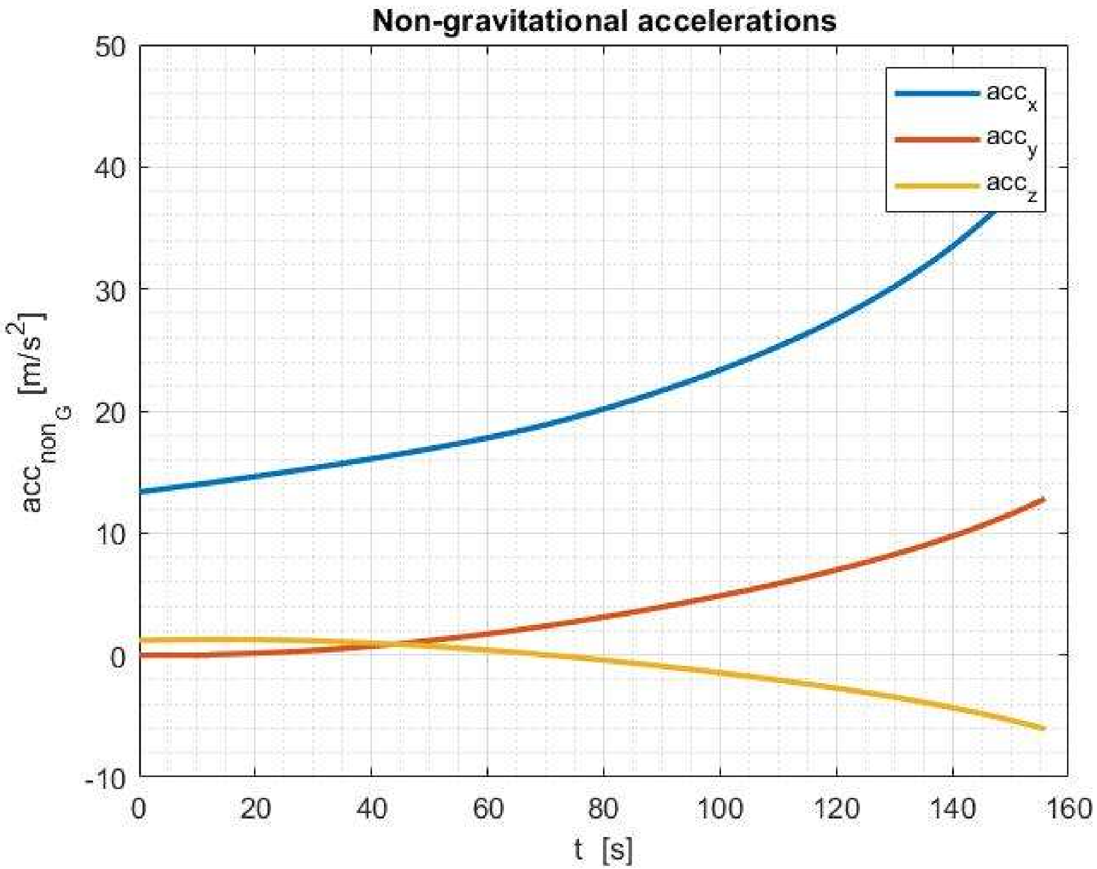
$$\dot{Q} = \frac{1}{2}\vec{q}\vec{\omega}$$

$$\begin{bmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = 2 \begin{bmatrix} q_0 & q_1 & q_2 & q_3 \\ -q_1 & q_0 & q_3 & -q_2 \\ -q_2 & -q_3 & q_0 & q_1 \\ -q_3 & q_2 & -q_1 & q_0 \end{bmatrix} \begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$









NON-GRAVITATIONAL ACCELERATION

$$\vec{a} = \frac{T}{m}\frac{\vec{u}}{u} - \frac{1}{2}\rho\frac{A_a}{m}C_D\vec{v}_{rel}v_{rel}$$

IMU MODEL

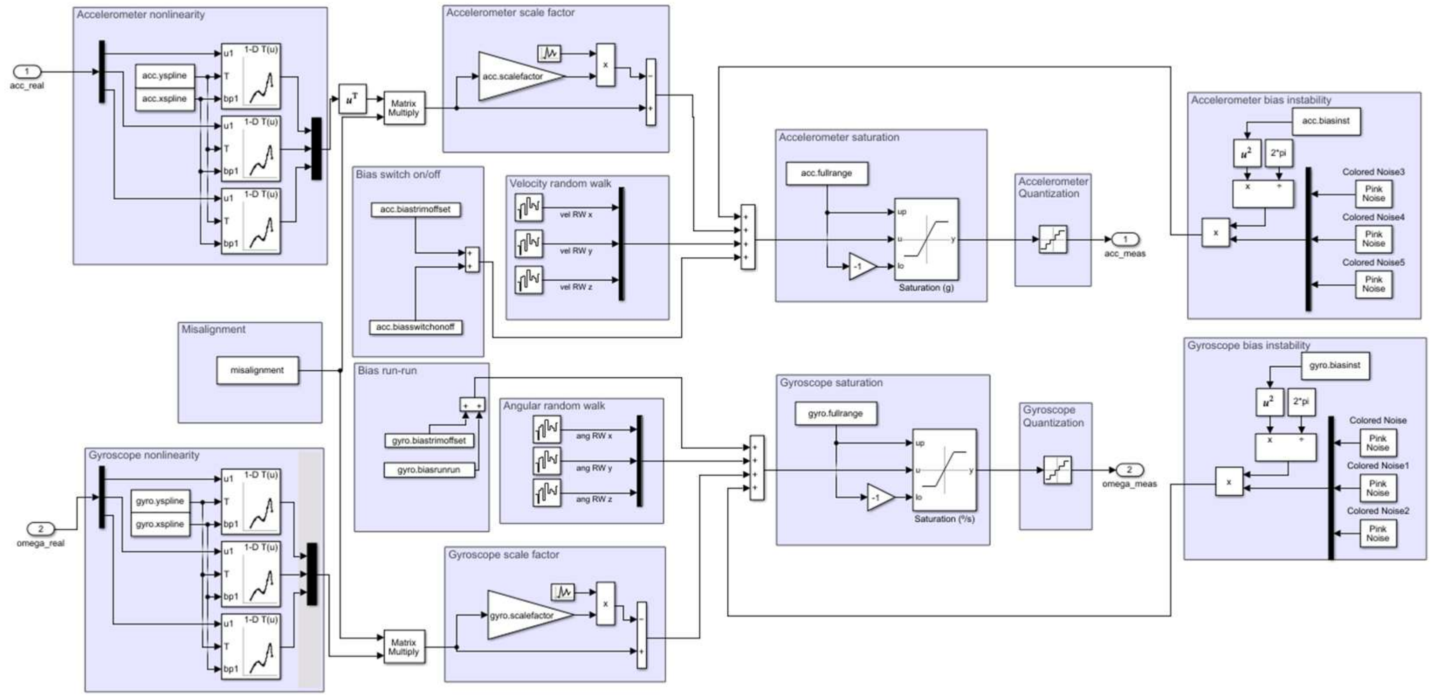

ELECTRONICS & DEFENSE
STIM300
Inertial
Measurement
Unit
SAFRAN

Parameters

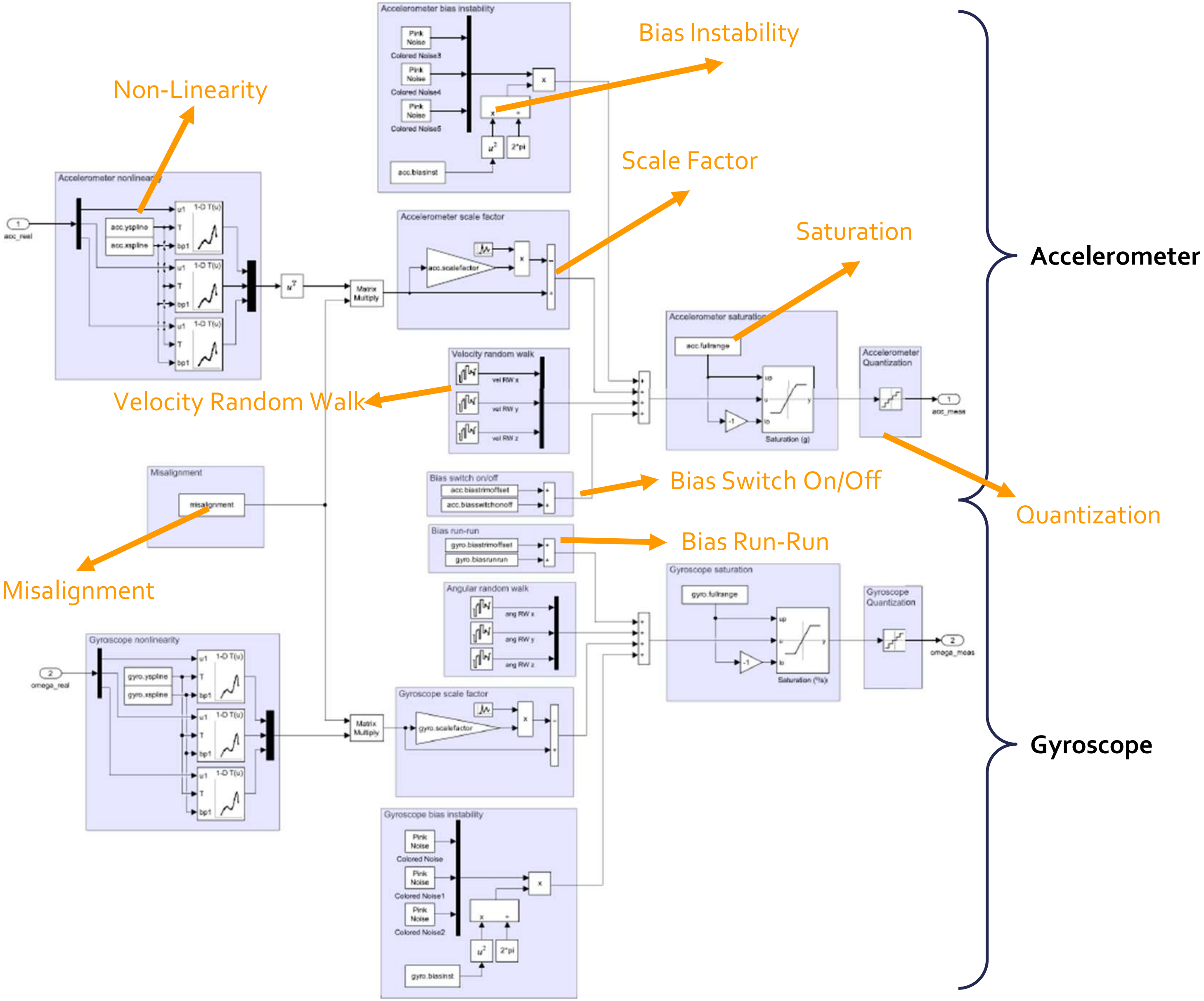
Extracted from the data sheet of the STIM300

Parameter	Gyroscope	Accelerometer
Scale Factor Accuracy	±400 °/s ⟶ ±500 ppm	±5 g ⟶ ±200 ppm
Non-Linearity	±200 °/s ⟶ 15 ppm ±400 °/s ⟶ 20 ppm	±5 g ⟶ 100 ppm
Bias Run-Run /	4 °/h	
Bias switch on/off repeatability		±0.38mg
Bias Instability	0.3 °/h	0.02mg
Angular / Velocity Random Walk	0.15 °/√h	0.03 m/s/√h
Misalignment	1 mrad	1 mrad

IMU Simulink Model



IMU Simulink Model



Non-Linearity

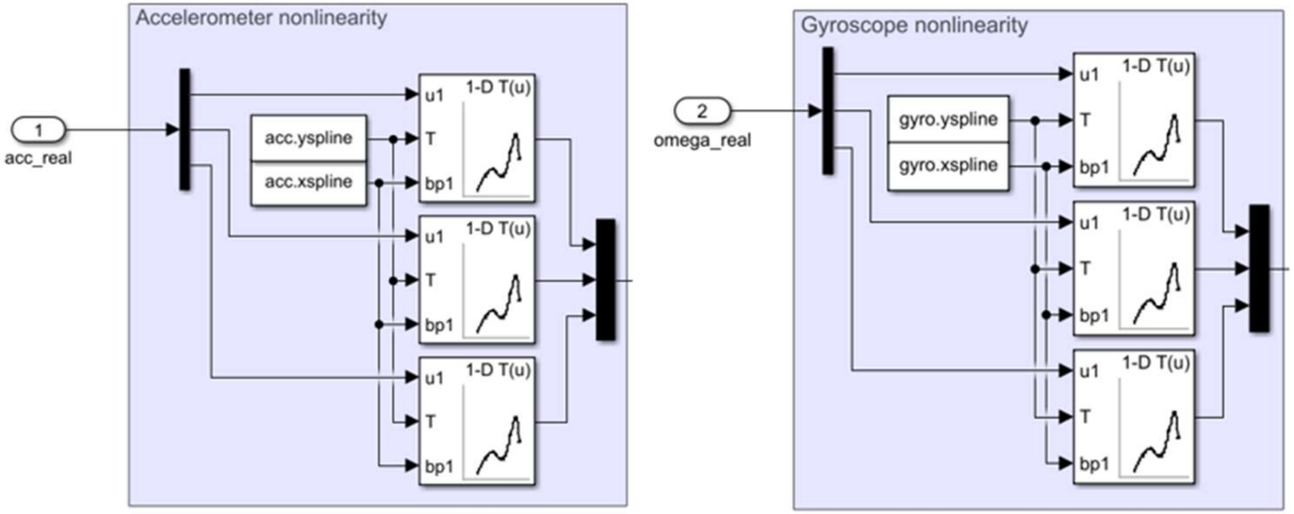
Spline cubic model used with Simulink `Look up with Akima spline Interpolation´ block

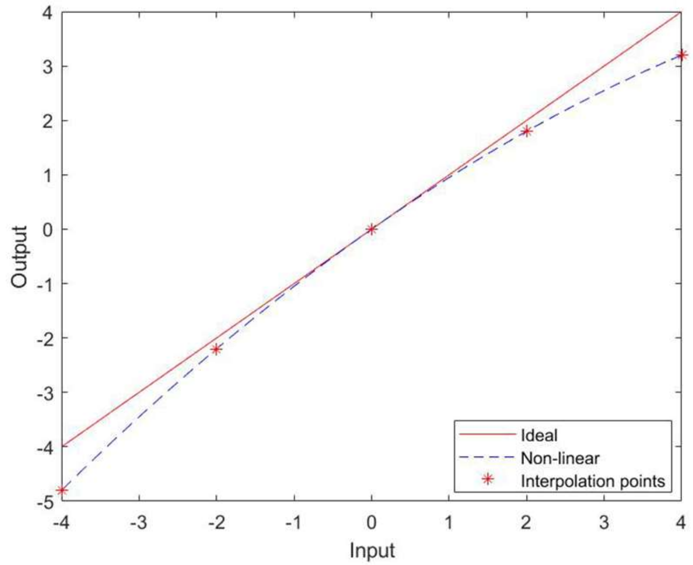
Input $[deg/s]$	**Output** $[deg/s]$
-400	-400 - 20/1000000 · 400
-200	-200 - 15/1000000 · 400
0	0
200	200 - 15/1000000 · 400
400	400 - 20/1000000 · 400

Table 2: Gyroscope spline points

Input $[g]$	**Output** $[g]$
-5	-5 - 100/1000000 · 5
0	0
5	5 - 100/1000000 · 5

Table 3: Accelerometer spline points



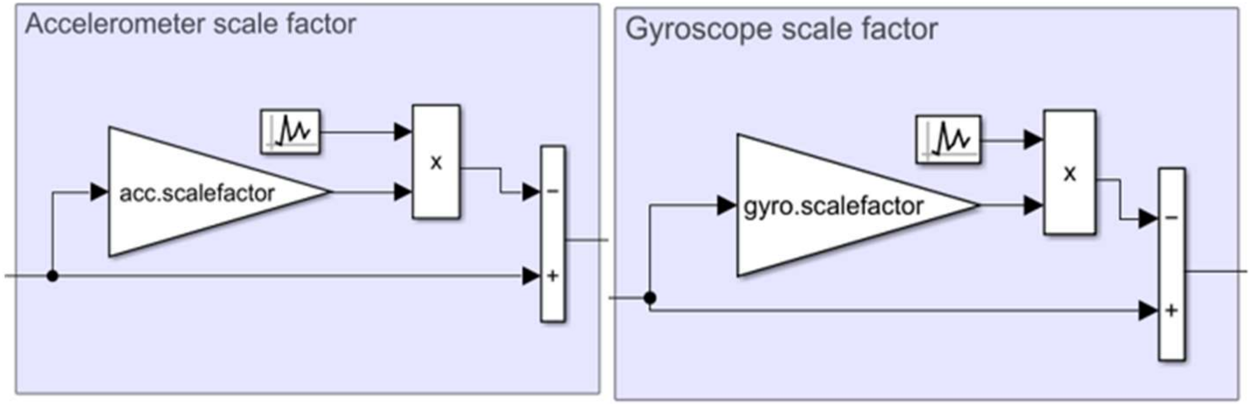


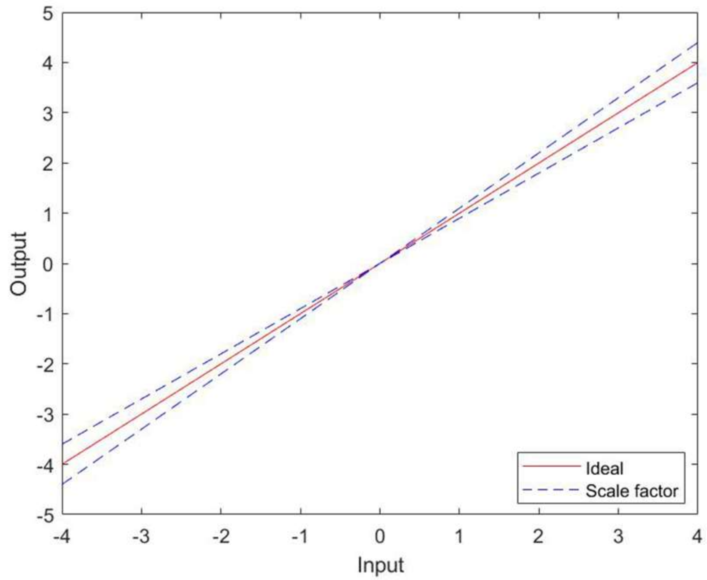
Scale Factor Accuracy

Model used:

y = x − SF · x

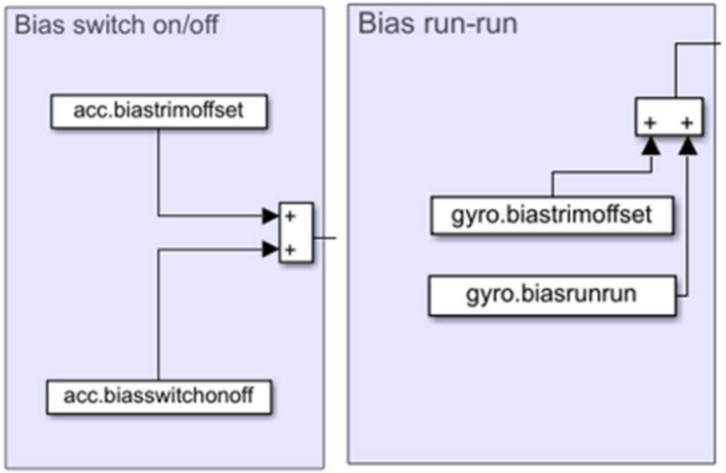
where
x=input
y=output
SF=Scale factor





Bias Run-Run / Bias switch on/off

Average change in a bias between successive turns-on in the gyroscope/ accelerometer.



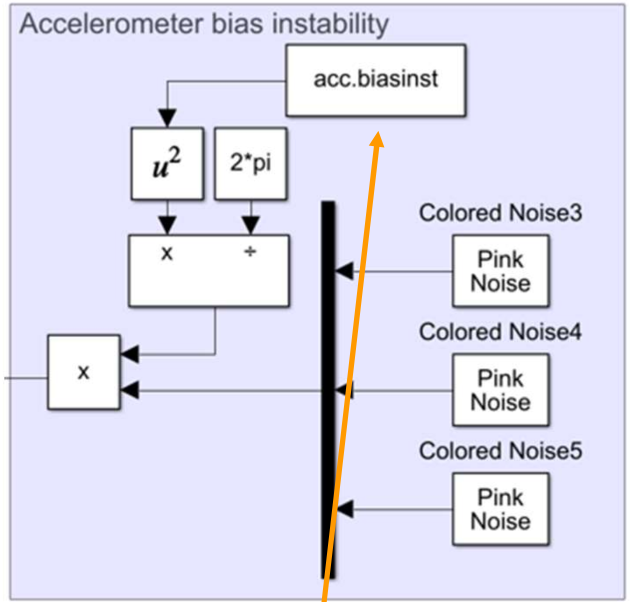
Accelerometer Bias switch on/off	Gyro Bias run-run
Bias Trim Offset range = ±50 mg (offset)	Bias Trim Offset range = ±1 º/s (offset)
Bias switch on/off repeatability = ±0.38 mg	Bias run run = 4º/h
Overall bias switch on/off: Randomized summation.	Overall bias run-run: Randomized summation.

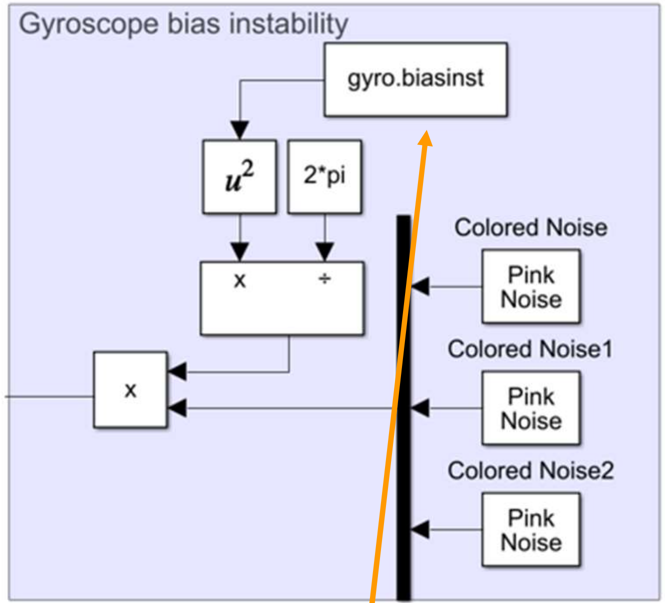
Bias Instability

Measure of how the bias will drift during operation over time at a constant temperature

Modeled as pink noise scaled with the power

$$S_\Omega(f) = \begin{cases} \left(\frac{B^2}{2\pi}\right)\frac{1}{f}, & \text{if } f \leq f_0. \\ 0, & \text{otherwise.} \end{cases}$$





Accelerometer Bias Instability	**Gyro Bias Instability**
0.02 mg	0.3°/h

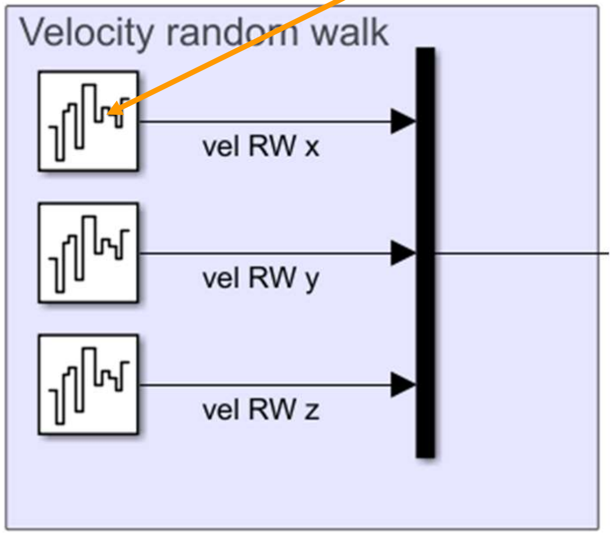
Sample time for the pink noise generation is 2000 samples per second(Hz): **0.5 ms of sample period.**

Angular/ Velocity Random Walk

Modeled as white noise taking the power spectral density found in the datasheet.

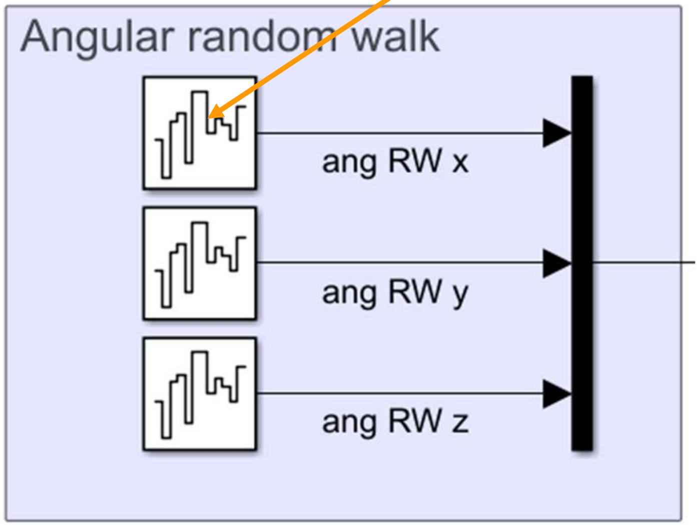
	Accelerometer
Velocity Random Walk	$0.03\ m/s/\sqrt{h}$

$$\left(0.03\,\frac{m}{s}\,\frac{1}{\sqrt{h}}\cdot\frac{\sqrt{1h}}{60\,\sqrt{s}}\cdot\frac{G}{9.81\,\frac{m}{s^2}}\right)^2 = 2.598\cdot 10^{-7}\,\frac{G^2}{Hz}$$



	Gyroscope
Angular Random Walk	$0.15\ ^{\circ}/\sqrt{h}$

$$\left(0.15\,^{\circ}\,\frac{1}{\sqrt{h}}\cdot\frac{\sqrt{1h}}{60\,\sqrt{s}}\right)^2 = 6.250\cdot 10^{-6}\left[\frac{^{\circ 2}}{s^2}\,\frac{1}{Hz}\right]$$

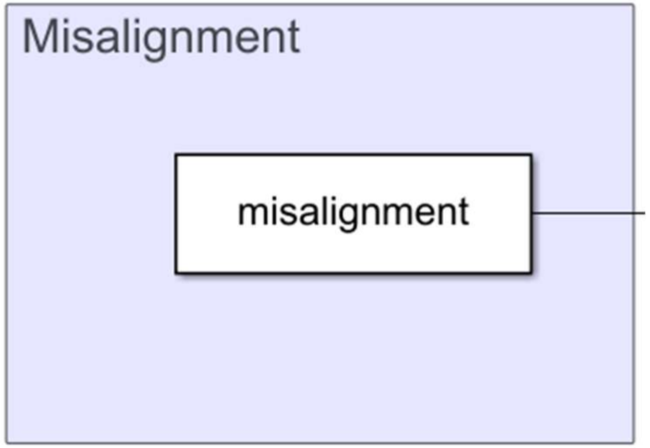


Misalignment

Measure of the deviation between the internal sensing axes and the ones marked on the case of the IMU (the installation ones).

Modeled as a "Misalignment matrix" multiplied with the real input.

It is a constant value:
1 mrad per axis (α)

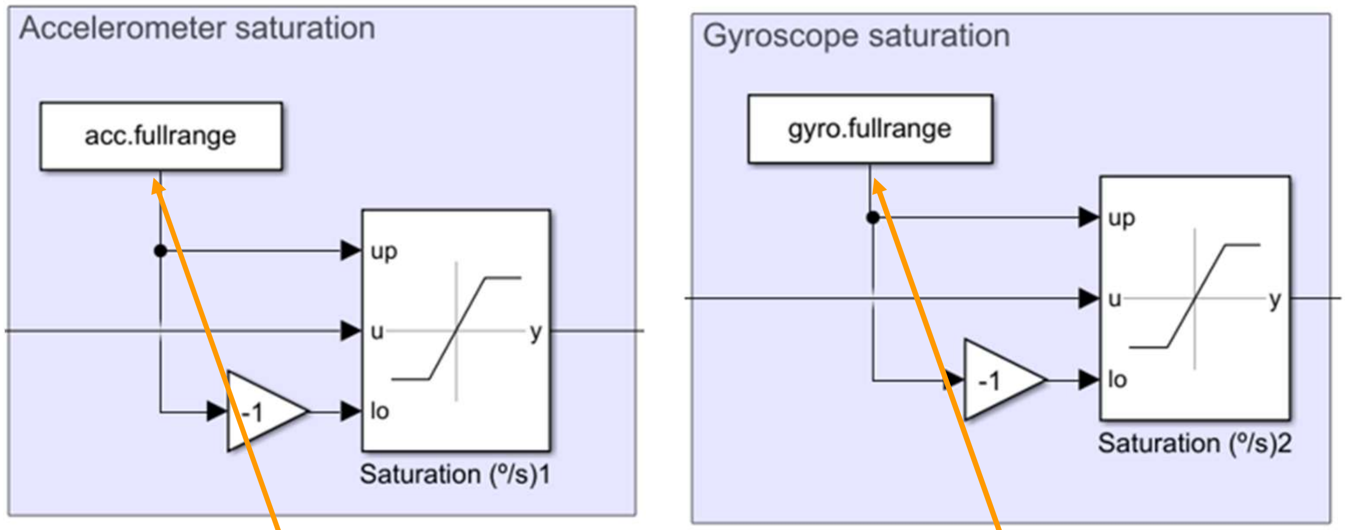


$$[Misaligment] = \underbrace{\begin{bmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix}}_{\textbf{R\_pitch}} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}}_{\textbf{R\_roll}} \underbrace{\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\textbf{R\_yaw}}$$

Saturation

Full scale range limits the max. and min. measurements that the accelerometer and gyroscope can obtain.

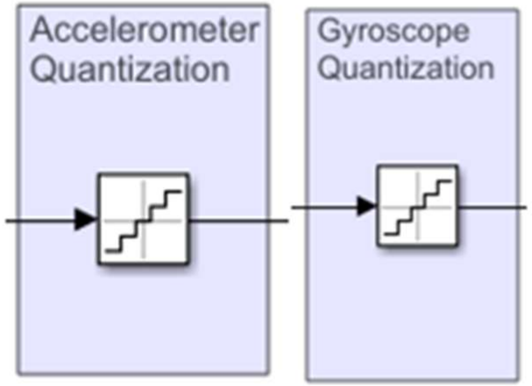
Added as the last step to ensure no additional contributions exceeds the saturation limits.



Accelerometer full scale range	Gyro Bias Instability
± 5 g	± 400 °/s

Quantization

Added to simulate digitalization error for the final result



This block uses a round-to-nearest method to change a smooth input signal into a stair-step shape.

Equation for round-to-nearest:

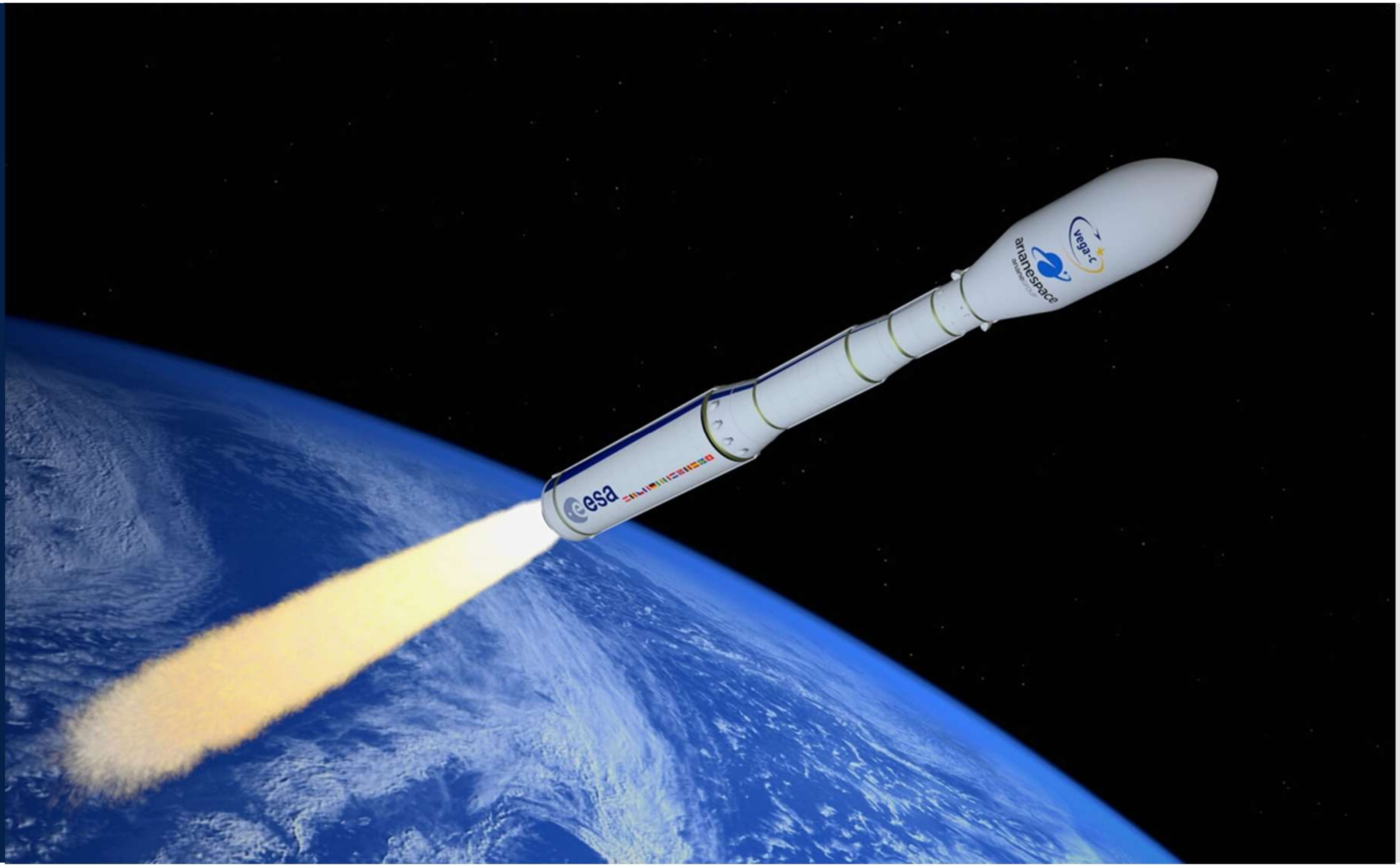
$$y = q \cdot round\left(\frac{u}{q}\right)$$

where:

y=quantized output

q=quantization interval $q = 2\frac{full\ range}{2^{24}}$

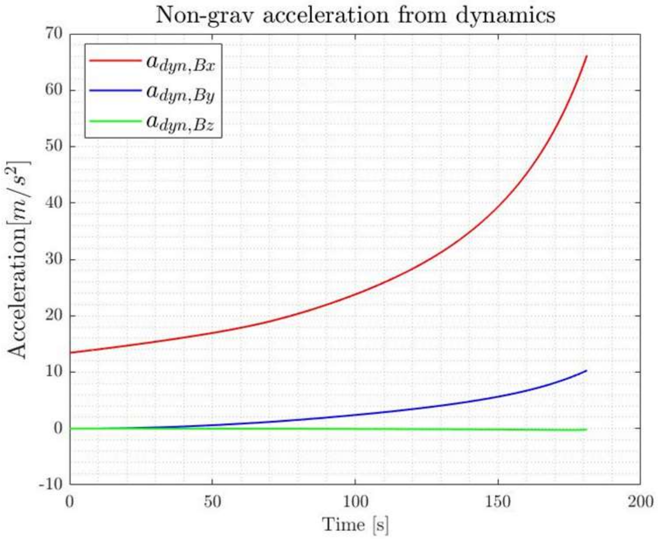
TRAJECTORY RECONSTRUCTION

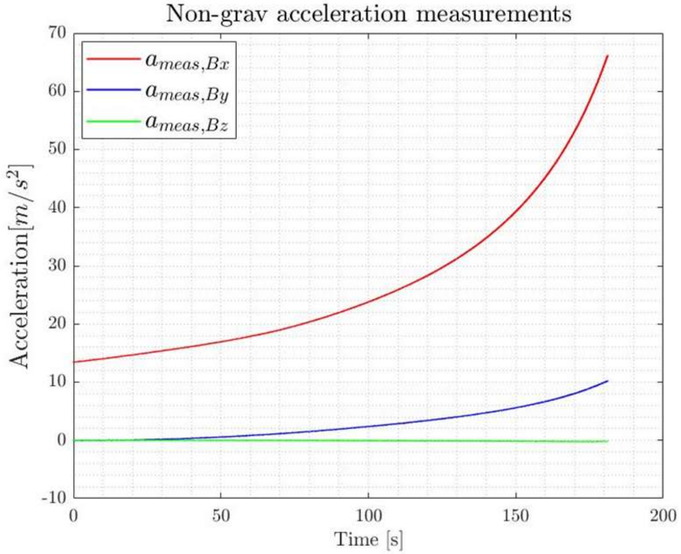


Trajectory Measurements

Acceleration

Comparison between the theoretical trajectory and the actual measurements





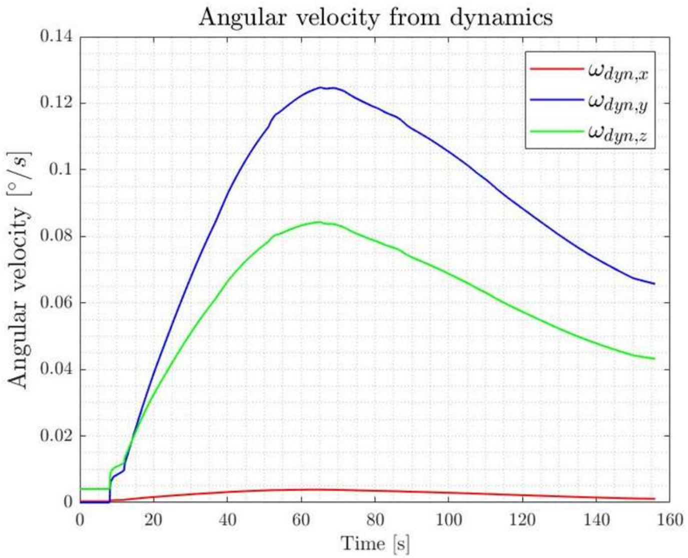
$$a^B_{non-grav} = q^{-1} \otimes a^N_{non-grav} \otimes q$$

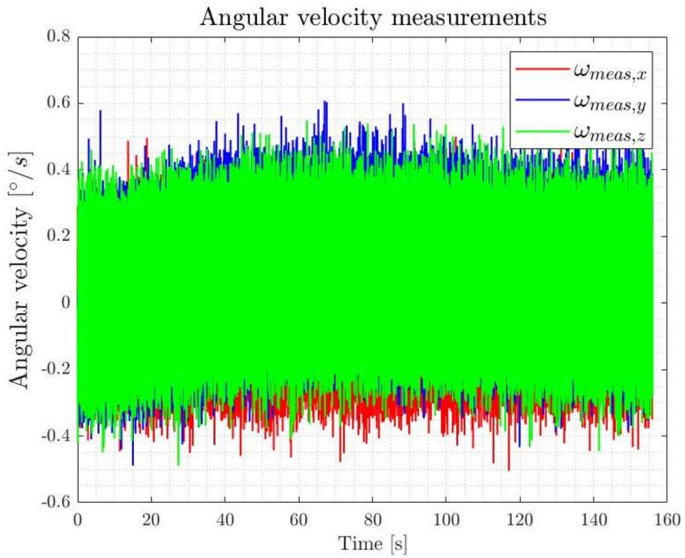
$$\sigma_{acc} = \sqrt{P_n \cdot f} = \sqrt{0.0224\frac{g^2}{Hz} \cdot 2000Hz} \cdot 9.81\frac{m/s^2}{g} = 0.0224\ m/s^2$$

Trajectory Measurements

Angular velocity

Comparison between the theoretical trajectory and the actual measurements





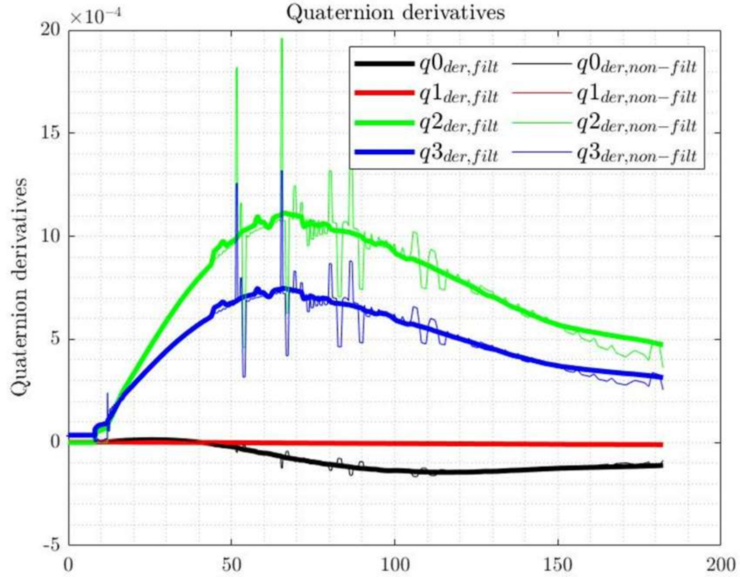
With respect to part 1-A trajectory:

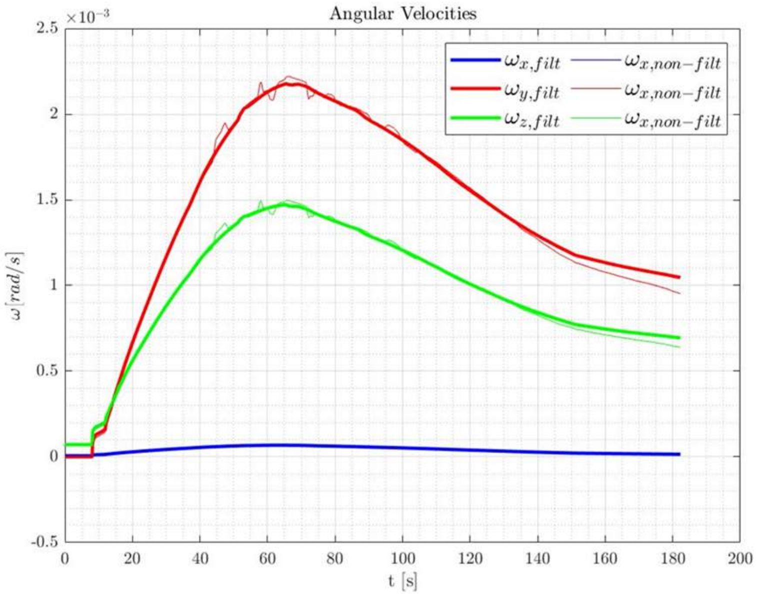
- Moving average filter for **quaternion derivatives**: sliding window of 50 samples
- Moving average filter for **angular velocity of the trajectory**: sliding window of 50 samples

$$\sigma_{ang} = \sqrt{P_n \cdot f} = \sqrt{0.625 \cdot 10^{-5} \, \frac{(°/s)^2}{Hz} \cdot 2000 \, Hz} = 0.1118 \, °/s$$

DATA PRE-PROCESSING

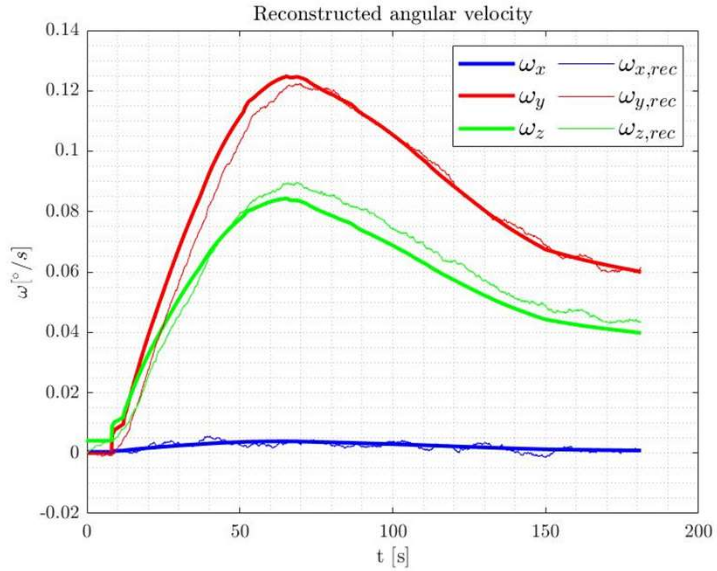
Correcting numerical integration issues

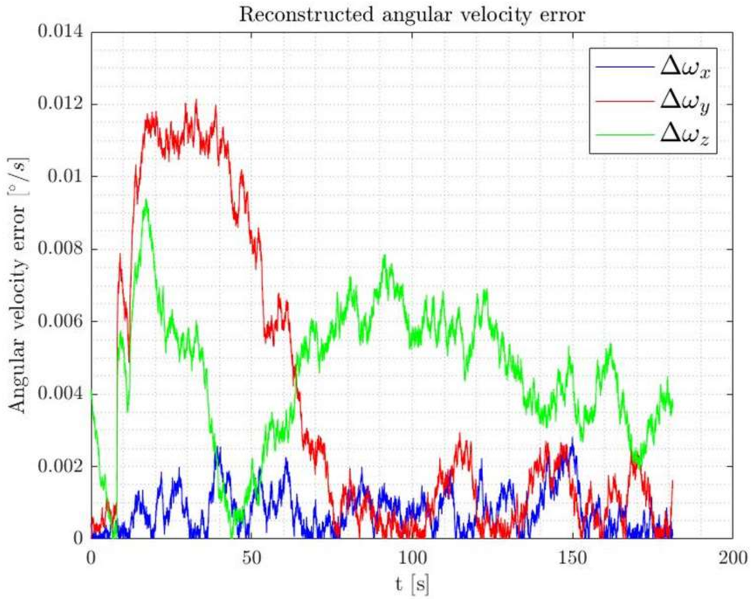




- A low-pass filter does not remove completely white noise
- Moving average filter parameters were tuned manually -> better approach could have been to compute FFT

ANGULAR VELOCITY RECONSTRUCTION





- Moving average filter with a sliding window of 14000 samples -> 7 seconds in measurements
- Rest of IMU errors remain:
  - Bias run-run
  - Bias switch on/off
  - Bias instability
  - Misalignments
- Angular velocity absolute error up to 0.012 deg/s

MISALIGNMENT MODELING

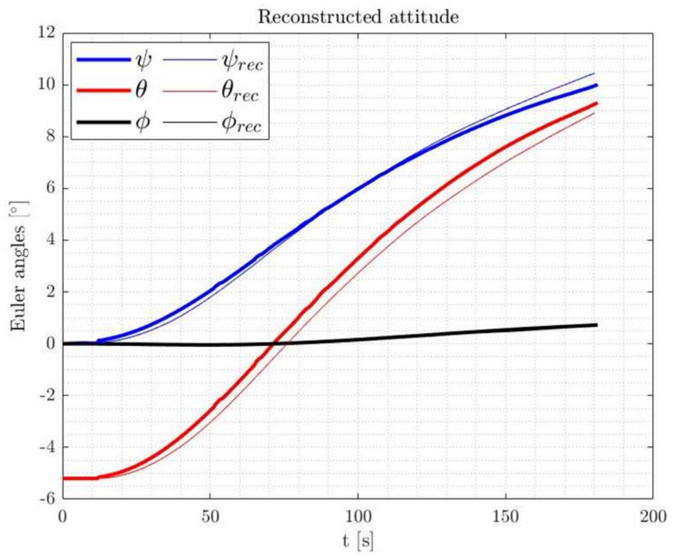
- Misalignment among local vertical and local horizontal reference system (R_ref) -> 2 deg
- Misalignment among IMU platform and launcher body axis (R_axes) -> 2 deg
- Misalignment of MEMS sensor axes (R_IMU)

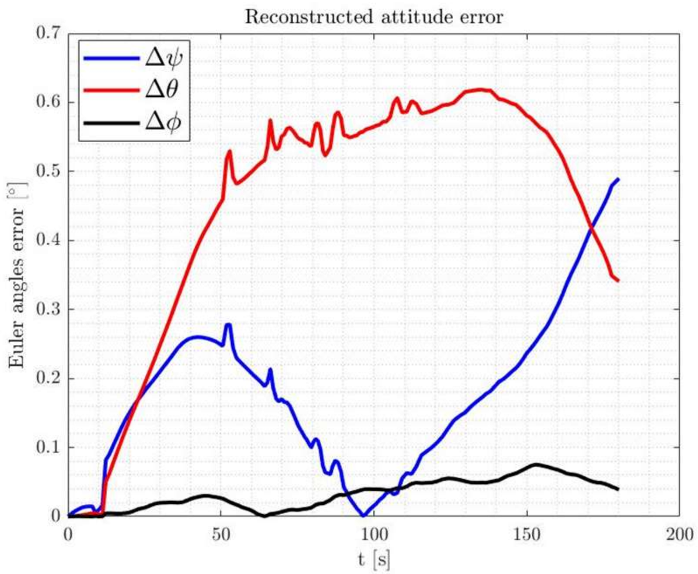
$$\mathbf{R_x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \mathbf{R_y} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \mathbf{R_z} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{mis} = R_z \cdot R_y \cdot R_x$$

$$R_{mis,total} = R_{ref} \cdot R_{axes} \cdot R_{IMU}$$

ATTITUDE RECONSTRUCTION





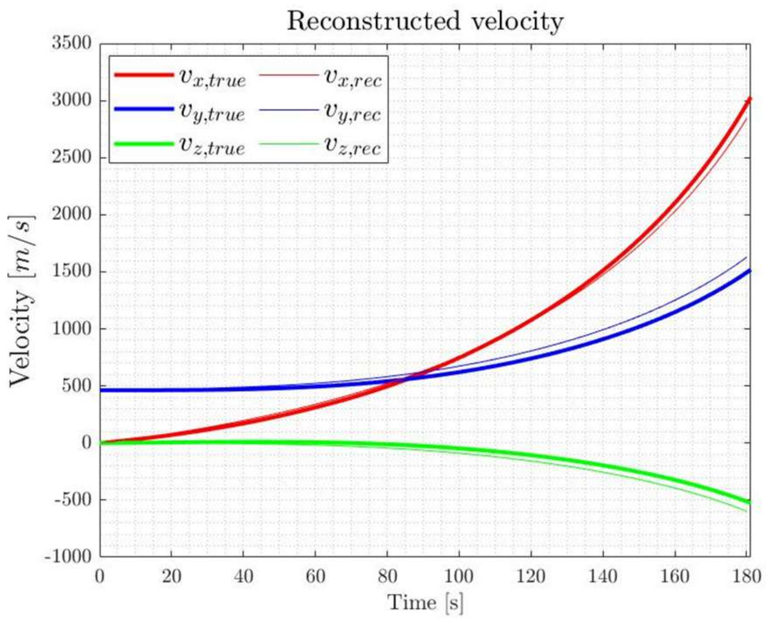
$$\dot{q} = \frac{1}{2} \cdot q \otimes w_{meas,filt}$$

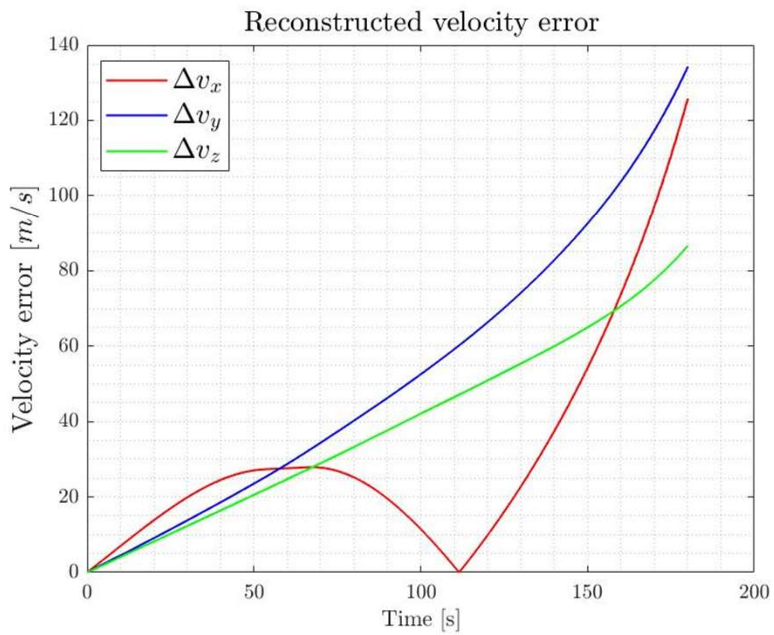
- Provide physical meaning: convert to Euler angles

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \mathrm{atan2}(2(q_0 q_1 + q_2 q_3), 1 - 2(q_1^2 + q_2^2)) \\ \mathrm{asin}(2(q_0 q_2 - q_3 q_1)) \\ \mathrm{atan2}(2(q_0 q_3 + q_1 q_2), 1 - 2(q_2^2 + q_3^2)) \end{bmatrix}$$

- Up to 0.6 deg absolute error

VELOCITY RECONSTRUCTION





$$a^N_{non-grav} = q \otimes [0, a_{non-grav}]^B \otimes q^{-1}$$

$$g^N = -\mu \frac{r_{i-1}}{|r_{i-1}|^3}$$

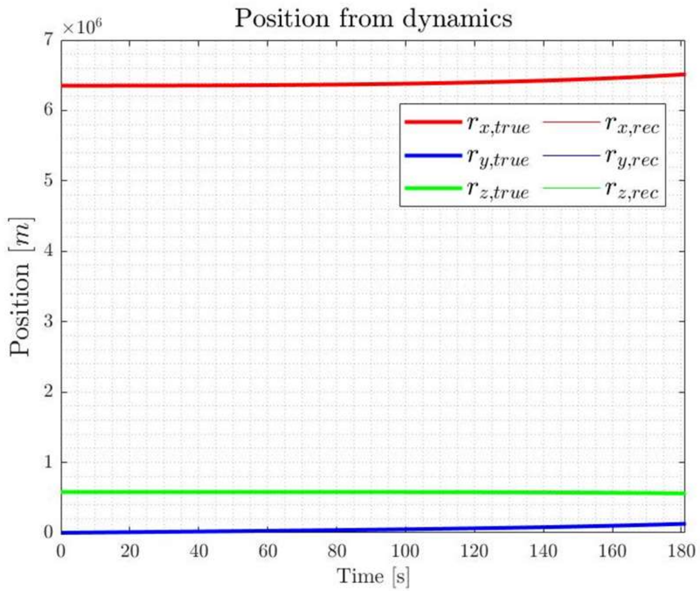
$$a^N = a^N_{non-grav} + g^N$$

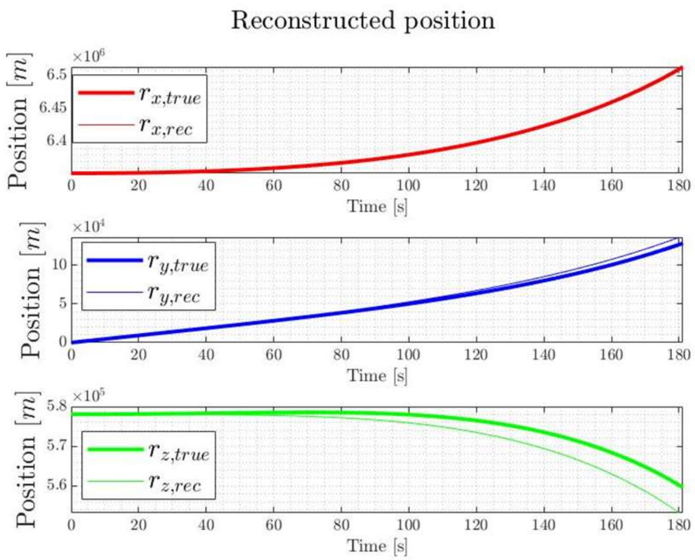
$$v^N_i = v_{i-1} + a^N_i \cdot \Delta t$$

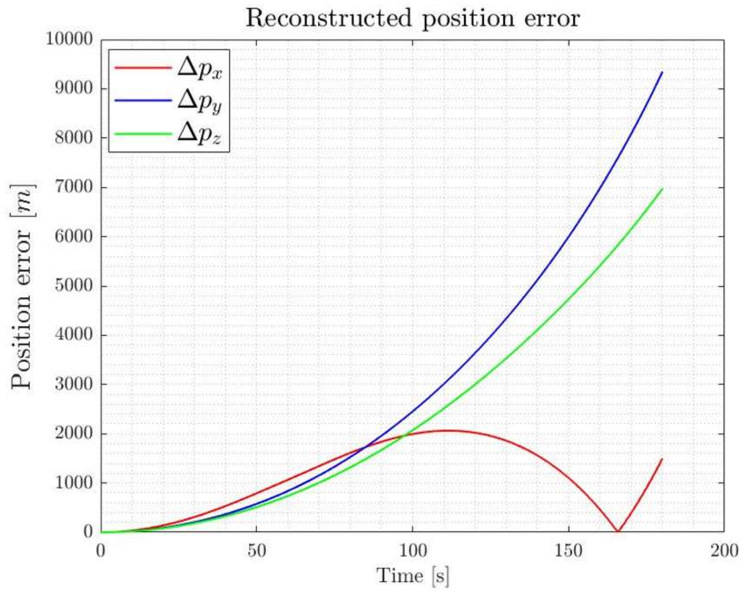
$$r^N_i = r_{i-1} + v^N_i \cdot \Delta t$$

- Up to 140 m/s of absolute error

POSITION RECONSTRUCTION

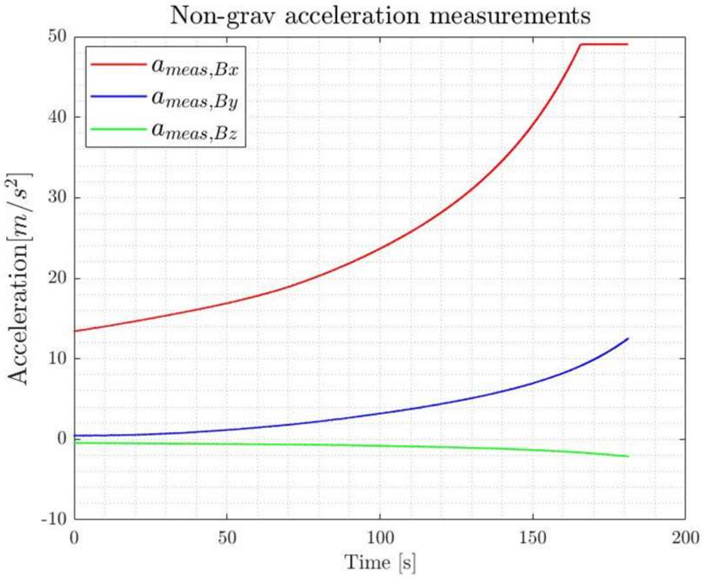


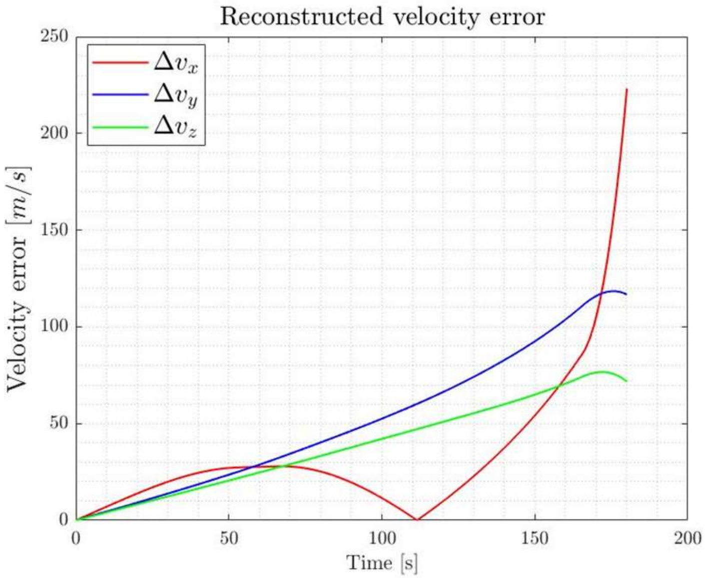


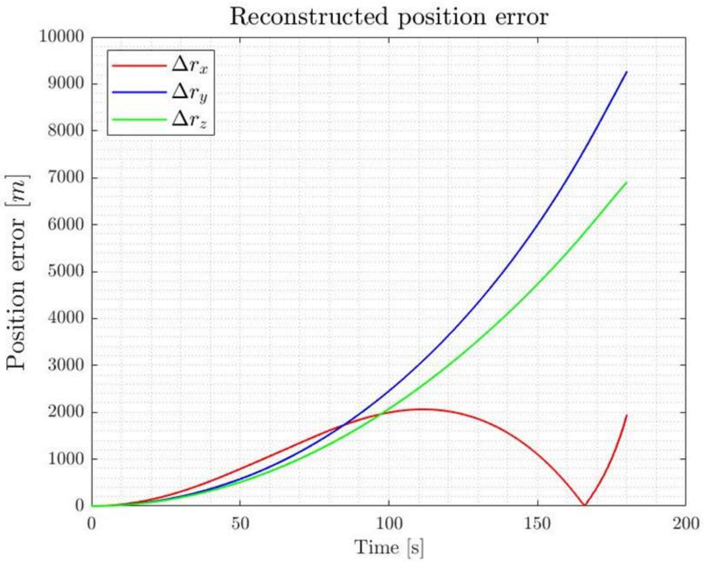


- Up to 9,000 m of absolute error

ACCELEROMETER SATURATION







- Accelerometers saturate at 5 g
  - Velocity error up to 230 m/s
  - Similar error on position

IMPORTANCE OF MISALIGNMENT

- If misalignment error on reference system and IMU platform with respect to launcher axis are not considered ...

	Error in position	Error in velocity
misalignment considered	9,000 meters	230 m/s
misalignment not considered	2,500 meters	70 m/s