Comments and Corrections

Corrections to "Time-Step Considerations in Particle Simulation Algorithms for Coulomb Collisions in Plasmas"

Bruce I. Cohen, Andris M. Dimits, Alex Friedman, and Russel E. Caflisch

***Abstract*—An error in (17) of B. I. Cohen, A. M. Dimits, A. Friedman, and R. E. Caflisch, IEEE Trans. Plasma Sci. 38, 2394 (2010), is corrected. The consequences of this error, which are minor, are discussed.**

***Index Terms*—Algorithms, collision processes, computer applications, numerical analysis, particle collisions, plasmas.**

In [1, eqs. (12)–(17)], a next higher order correction in powers of $\Delta t^{1/2}$ to the Langevin equations for Coulomb collisions was derived. In [1, eq. (16)], the 1-D drag–diffusion equation is given as

$$y_{n+1} = y_n + a(y_n)\Delta t + b(y_n)\Delta W + \frac{1}{2}\, b\frac{db}{dy}\bigg|_{y_n} (\Delta W^2 - \Delta t) \tag{1}$$

where a is the drag coefficient, b is the diffusion coefficient, y_n represents a velocity variable at time step with index n, and ΔW is a Gaussian random variable with zero mean and variance Δt. The last term in (1) is the so-called Milstein correction [2]. The ΔW^2 in the Milstein term is the square of the ΔW in the diffusion term.

In [1, eq. (17)], an error was made by us in the Milstein term: Namely, the same random number N_1 should have been used rather than a new random number N_4 in keeping with (1) given here. The corrected version of [1, eq. (17)] is thus

$$\begin{aligned} v_z^{t+\Delta t} &= v_z^t + F_d\Delta t + g\Delta t^{1/2} N_1 + \frac{1}{2} g\frac{dg}{dv}\Delta t \left(N_1^2 - 1\right) \\ v_{\perp 1,2}^{t+\Delta t} &= \Delta v_{\perp 1,2}. \end{aligned} \tag{2}$$

Furthermore, N_4^2 should be replaced in the text of [1] with N_1^2 wherever N_4^2 appears after (17). The error in [1, eq. (17)] propagated into the simulations whose results are reported in [1, Fig. 7]. We have corrected the simulation algorithm and repeated the simulations shown in [1, Fig. 7]. The new numerical results differ very little from the previous results ($< 1\%$ differences) and are not detectable to the eye in the revised [1, Fig. 7]. This is a consequence of the fact that the Milstein correction has little effect on the moments of the velocity distribution owing to the statistical average $\langle N_1^2 - 1\rangle = 0$ and the smallness of the coefficient dg/dv in the Milstein term in (2) over most of the velocity distribution as discussed in [1, Sec. III].

References

[1] B. I. Cohen, A. M. Dimits, and R. E. Caflisch, "Time-step considerations in particle simulation algorithms for Coulomb collisions in plasmas," *IEEE Trans. Plasma Sci.*, vol. 38, no. 9, pp. 2394–2406, Sep. 2010.

[2] G. N. Milstein, "Approximate integration of stochastic differential equations," *Theory Probab. Appl.*, vol. 19, pp. 552–562, 1974.

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B. I. Cohen, A. M. Dimits, and A. Friedman are with the Lawrence Livermore National Laboratory, Livermore, CA 94551-0808 USA (e-mail: cohen1@llnl.gov; dimits1@llnl.gov; afriedman@lbl.gov).

R. E. Caflisch is with the Department of Mathematics, University of California at Los Angeles, Los Angeles, CA 90095-1555 USA (e-mail: caflisch@math.ucla.edu).

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