Eve and Frank's Chessboard Trails

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Problem

Eve and Frank are observing a square chessboard divided into 3600×3600 unit squares arranged in rows and columns. Each square contains either a *knight* or a *bishop*, arranged in a checkerboard pattern so that any two squares sharing a side contain different pieces.

Solution

Step 1: Balance is automatic

Along any allowed trail, each step moves to a neighboring plot. Because the chessboard is colored in a checkerboard pattern, each step switches the type of piece encountered. Thus the pieces along a trail strictly alternate.

If a trail contains an even number of squares, it visits the same number of knights and bishops. If it contains an odd number of plots, the difference is exactly one. Hence every allowed trail is automatically balanced, and the balance condition imposes no additional restriction.

Therefore, the problem reduces to determining the minimum number of allowed trails needed to cover all plots.

Step 2: An explicit construction

Eve assigns one trail to each column of the field. Each trail starts at the top plot of its column and proceeds straight downward through all 3600 plots in that column.

These 3600 trails:

- obey the movement rules,
- are balanced by Step 1, and
- together cover every plot exactly once.

Thus $M \leq 3600$.

Step 3: Why fewer trails are impossible

To show that Bob cannot do better, Frank argues as follows.

Each plot is divided along its diagonal from the upper-left corner to the lower-right corner, producing two triangular half-plots. Think of these diagonals as mirrors.

Light beams are sent into the field through the midpoints of all boundary edges, perpendicular to the boundary. There are exactly $4 \cdot 3600$ such entry points. Each beam reflects off the diagonal mirrors and eventually exits the boundary, thereby pairing the boundary edges into $2 \cdot 3600$ disjoint pairs.

Consider how these beams interact with Bob's trails. Whenever a beam passes through a region corresponding to a segment of a trail, it exits that region on the same type of boundary edge (horizontal or vertical) on which it entered. A beam can change from a horizontal boundary to a vertical boundary, or vice versa, only by passing through a triangular half-plot that is not paired with another half-plot by the trail structure.

Each trail has exactly two endpoints. Each endpoint creates exactly one such unpaired half-plot. Therefore the total number of unpaired half-plots is exactly $2M$.

On the other hand, among the $2 \cdot 3600$ boundary pairs, at least 3600 pairs must connect a horizontal boundary edge with a vertical boundary edge. Each such mixed pair forces a beam to pass through at least one unpaired half-plot.

Hence

$$3600 \leq 2M.$$

Repeating the same argument after shifting all beam entry points by half a unit along the boundary produces an independent pairing and yields the same inequality again. Adding the two inequalities gives

$$2 \cdot 3600 \leq 2M,$$

so

$$M \geq 3600.$$

Conclusion

Bob has exhibited a covering using 3600 trails, and Marley has shown that no covering can use fewer. Therefore the minimum possible number of trails is

$$M = 3600.$$

Answer

$$\boxed{3600}$$