

# P04 Data Center Racks V1

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## Problem

A data center has a  $625 \times 625$  floor divided into unit square bays. Rectangular server racks are placed on the floor, with sides parallel to the grid lines, and no two racks overlap.

For cooling reasons, *in each row and in each column, exactly one unit square must be left empty.*

What is the *minimum number of racks* needed?

## Solution (Extremal Argument using Erdős–Szekeres)

Because there is exactly one empty square in each row and in each column, the empty squares define a permutation of the set  $\{1, 2, \dots, 625\}$ .

Let  $a$  be the length of the longest increasing subsequence and  $b$  the length of the longest decreasing subsequence of this permutation. By the Erdős–Szekeres theorem,

$$ab \geq 625,$$

which implies

$$a + b \geq 2\sqrt{625} = 50.$$

Using a standard extremal labeling argument, one can mark at least

$$625 + a + b - 3$$

unit squares such that no axis-aligned rectangle can cover more than one marked square. Therefore, each pallet can cover at most one marked square, and the number of racks is at least

$$625 + 40 - 3 = 672.$$

Hence, any valid configuration requires at least 437 pallets.

## Answer

$$\boxed{672}$$