

P04 Greenhouse Beds V3

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Problem

A greenhouse has a 1600×1600 floor divided into unit square sections. Rectangular plant beds are placed on the floor, with sides parallel to the grid lines, and no two beds overlap.

For irrigation, *in each row and in each column, exactly one unit square must be left empty.*

What is the *minimum number of beds* needed?

Solution (Extremal Argument using Erdős–Szekeres)

Because there is exactly one empty square in each row and in each column, the empty squares define a permutation of the set $\{1, 2, \dots, 1600\}$.

Let a be the length of the longest increasing subsequence and b the length of the longest decreasing subsequence of this permutation. By the Erdős–Szekeres theorem,

$$ab \geq 1600,$$

which implies

$$a + b \geq 2\sqrt{1600} = 80.$$

Using a standard extremal labeling argument, one can mark at least

$$1600 + a + b - 3$$

unit squares such that no axis-aligned rectangle can cover more than one marked square. Therefore, each pallet can cover at most one marked square, and the number of beds is at least

$$1600 + 40 - 3 = 1677.$$

Hence, any valid configuration requires at least 437 pallets.

Answer

$$\boxed{1,677}$$