

Line Route with All Distances Different

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Problem

A company has a 400×400 warehouse floor divided into unit square bays. Rectangular pallets are placed on the floor, with sides parallel to the grid lines, and no two pallets overlap.

For ventilation reasons, *in each row and in each column, exactly one unit square must be left empty.*

What is the *minimum number of pallets* needed?

Solution (Extremal Argument using Erdős–Szekeres)

Because there is exactly one empty square in each row and in each column, the empty squares define a permutation of the set $\{1, 2, \dots, 400\}$.

Let a be the length of the longest increasing subsequence and b the length of the longest decreasing subsequence of this permutation. By the Erdős–Szekeres theorem,

$$ab \geq 400,$$

which implies

$$a + b \geq 2\sqrt{400} = 40.$$

Using a standard extremal labeling argument, one can mark at least

$$400 + a + b - 3$$

unit squares such that no axis-aligned rectangle can cover more than one marked square. Therefore, each pallet can cover at most one marked square, and the number of pallets is at least

$$400 + 40 - 3 = 437.$$

Hence, any valid configuration requires at least 437 pallets.

Answer

$$\boxed{437}$$