Rational Housing Bubbles

Laurel Britt

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Roadmap

- Chapter 1: Literature review and general overview of rational bubbles
  - Broad discussion of topics
  - "The Macroeconomics of Rational Bubbles: A User's Guide" (Martin and Ventura, 2018)
    - A blueprint model and its mechanics
    - Areas to build on and improve
- Chapter 2: Extending to housing bubbles
  - "Housing Bubbles" (Arce and Salido, 2011)
    - Model mechanics
    - Fruitful interpretations

Rational Bubbles

- What makes these bubbles rational?
  - An interpretation in which bubbles arise in perfect information.
  - Agents are rational and informed but bubbles still arise.
- Why study bubbles?
  - Model "boom-bust cycles that seem to be driven by **random and capricious** changes to **market psychology**".
    - Asset price volatility that cannot be explained by "**market fundamentals**".
  - Identify conditions in which bubbles arise.
    - Designing optimal policy responses.
  - Modeling trick to engineer price volatility that is informed by model conditions.

Up Next: Martin and Ventura's Baseline Setup

- Where are we heading?
  - An overlapping generations model with capital in the production function.
  - This model allows two equilibrium outcomes:
    - A deterministic outcome with equilibrium conditions preventing the emergence of bubbles.
    - A stochastic outcome that attaches bubbles to firm value thorough an exogenous process.
  - Bubbles happen when capital is valued above its depreciated level.
  - Crucial to this model's mechanisms is an occasionally binding borrowing constraint.
    - Constraint may be relaxed by a wealth effect from inflated asset values (bubbles).

Summary of Blueprint Model

Overlapping generations model in which agents live for two periods; there is a young generation and an old generation. The generations interact with each other in a goods market and act as borrowers in a foreign credit market in order to maximize old-age consumption.

- Capital goods are attached to firms.
  - All firms are owned by the old generation.
  - Market value of firms is a key determinant in old-age consumption.
    - **Fundamental Equilibrium:** Firms are valued for their undepreciated capital.

      $$V_t = (1 - \delta)K_t \tag{1}$$

    - **Bubbly Equilibrium:** Value of firms may contain a bubbly element that pushes firm value above their level of capital.

      $$V_t = (1 - \delta)K_t + B_t \tag{2}$$

    - We will examine the properties of B_t after we cover the fundamental equilibrium.

Summary of Blueprint Model:

- The young use wages and borrowing to purchase capital from the old in order to maximize consumption in the subsequent period.

$$W_t + F_t = K_{t+1} - (1-\delta)K_t + V_t$$

  - Labor supply fixed at 1.
    - **Equilibrium Condition**: Labor supply equals labor demand
  - Borrowing is subject to a constraint that restricts debt repayment to the expected value of firms in the old-age.
    - **Equilibrium condition:** Interest rate fixed at R.

$$RF_t \leq E_t\left[V_{t+1}\right]$$

- The old finance consumption through rental income from capital and by selling capital to the young generation. They must also repay debt.

$$C^o_{t+1} = r_{t+1}K_{t+1} + V_{t+1} - RF_t$$

Fundamental Equilibrium: Solution

Optimization Problem:

$$\max_{K_{t+1},F_t}\{K_{t+1}r_t+(1-\delta)K_{t+1}-RF_t\} \tag{3}$$

Such that:

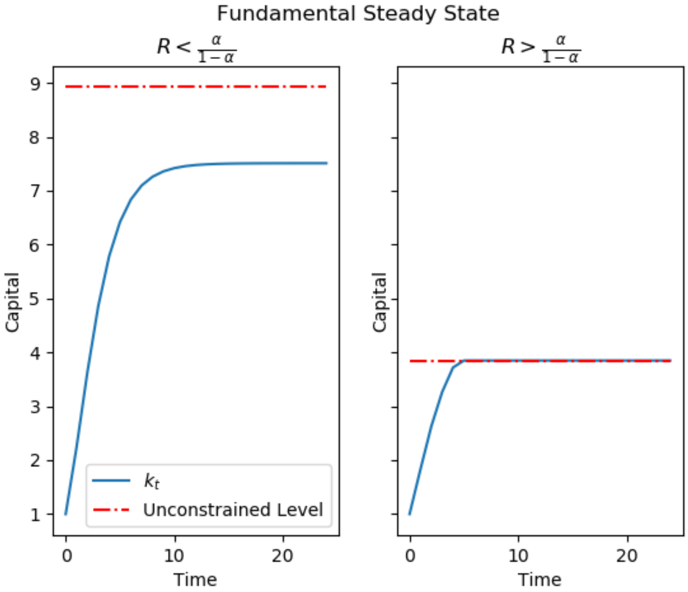
$$W_t+F_t=K_{t+1} \tag{4}$$
$$RF_t\le(1-\delta)K_{t+1} \tag{5}$$

Law of motion of capital:

$$K_{t+1}=\quad \min\left\{\frac{R}{R+\delta-1}(1-\alpha)AK_t^{\alpha},\left(\frac{\alpha A}{R+\delta-1}\right)^{\frac{1}{1-\alpha}}\right\} \tag{6}$$

- Path contains two regions, differentiated by whether the borrowing constraint is binding.
  - For low levels of K_t, the borrowing constraint binds.
  - Wage: $W_t=(1-\alpha)AK_t^{\alpha}$
- Path converges monotonically to a steady state from any initial condition.
  - The borrowing limit is binding in the steady state if the interest rate is below a certain threshold, $\bar{R}=\frac{\alpha}{1-\alpha}$

Path of Capital, Low Initial Capital



Bubbly Equilibrium: A Stochastic Model

The study of bubbles in this simple model involves expanding the set of equilibria to include cases in which the price of firms does not equal the value of their capital stock:

$$V_t = (1-\delta)K_t + B_t$$

A few caveats:

- Recall that a number of firms own all of the capital and that the old generation is the owner of these firms. Each generation firms are passed down from the old to the young.
- Additionally, in each period the young purchases the new capital that was created in that period.

$$W_t + F_t = K_{t+1} - (1-\delta)K_t + V_t$$

From here on, we distinguish between old firms and new firms. Old firms are those whose value is captured in the term V_t. They are carried over from previous periods. New firms are those which derive their value from investment in the current period.

Evolution of the Bubbly Component

- Each period, some firms arrive with a bubbly component attached to their value.
  - Let B_t be the size of the aggregate bubble at the beginning of period t.
- During each period, a quantity of new bubbles is created and adhere to the value of new firms in the economy.
  - Let N_t be the aggregate size of new bubbles that arose in period t.
  - New bubbles only appear in new firms
- Between each period and the next, the size of the bubbly component of the economy is augmented by a stochastic growth factor.
  - Let g_{t+1} be the growth rate of bubbles from period t to period $t+1$.
- Then the size of the aggregate bubble at the beginning of period $t+1$ is denoted:

$$B_{t+1} = g_{t+1}(B_t + N_t) \tag{7}$$

Market Psychology

Bubble dynamics are determined by the emergence of new bubbles, an exogenous stochastic process chosen by the modeler.

- Imagine that there are two states of the world, a bubbly state and a fundamental state.
  - In the bubbly state, market sentiment is such that investors are willing to invest in new bubbles.
  - In the fundamental state, the value of new firms is not endowed with a bubbly component.
- For the sake of this presentation, let's consider a straightforward example:

$$N_t = \begin{cases} \eta & z_t = B \\ 0 & z_t = F \end{cases}$$

Choice Behavior for a Representative Agent

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- Our agent makes four distinct choices:
  1. What quantity of **old firms** to invest in/purchase.
  2. What portion of **old bubbles** to invest in/purchase.
  3. What quantity of **new firms** to invest in.
  4. What quantity to **borrow.**
- An Immediate Question: Why does he get to choose what portion of the bubble to invest in?
  - Firms are freely and seamlessly merged, separated, bought, and sold.
  - Ultimately, all that matters is the amount of capital purchased and the quantity of the bubble invested in.

Bubbly Equilibrium: Optimization Problem

Update the young's optimization problem in period t.

- Free disposal of bubbles drives wedge between bubble investment and capital investment.
  - Introduce X_t, the fraction of the bubble demanded.
  - **Equilibrium Condition:** $X_t = 1$
- Maximize expected old-age consumption by choosing capital and bubble investment and borrowing:

$$\max_{K_{t+1},F_t,X_t}\{E_t[C^o_{t+1}]\} = K_{t+1}(r_{t+1} + 1 - \delta) + E_t[g_{t+1}](X_t B_t + N_t) - RF_t \quad (8)$$

  - Subject to a budget constraint:

$$W_t + F_t = K_{t+1} + X_t B_t$$

  - And a borrowing constraint that limits debt to the expected value of firms in the next period:

$$RF_t \leq K_{t+1}(1 - \delta) + E_t[g_{t+1}](X_t B_t + N_t)$$

Bubbly Equilibrium: Solution

Solve for the steady state level of capital per effective worker.
Optimization Problem:

$$\max\{K_{t+1}(r_{t+1}+1-\delta)+E_t[g_{t+1}(B_t+N_t)]-RF_t\} \tag{9}$$

Such that

$$W_t+F_t=K_{t+1}+X_tB_t \tag{10}$$
$$RF_t\leq(1-\delta)K_{t+1}+E[g_{t+1}](B_t+N_t) \tag{11}$$

- Crucial First Order Condition: Expected bubble growth is equal to the interest rate (precludes arbitrage opportunities)

$$E_t[g_t]=R \tag{12}$$

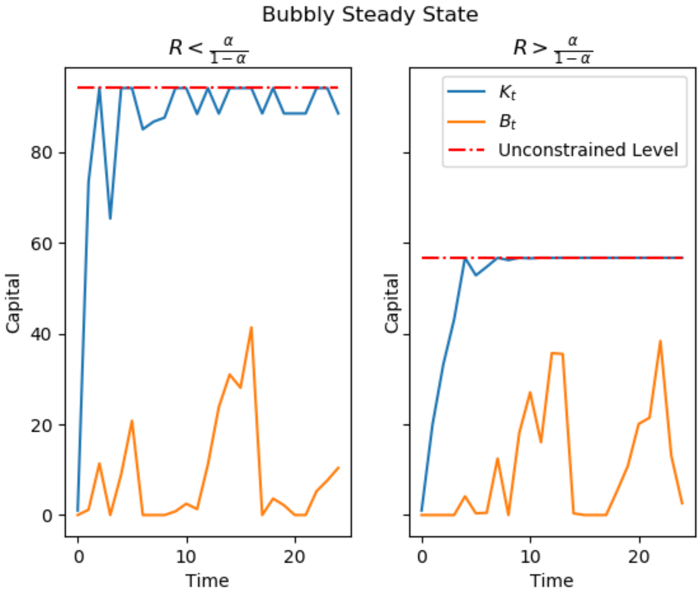
Bubbly Equilibrium: Solution

Law of Motion of Capital:

$$K_{t+1} = \min\left\{ \frac{R}{R+\delta-1}[(1-\alpha)AK_t^{\alpha} + N_t], \left(\frac{\alpha A}{R+\delta-1}\right)^{\frac{1}{1-\alpha}} \right\} \tag{13}$$

- Path of capital accumulation has two branches.
  - Constrained branch depends on K_t and N_t.
    - Low interest rates
  - Unconstrained branch is constant across time.
    - High interest rates
- Old bubbles do not impact investment.
- New bubbles only impact investment along the constrained path.
  - Bubbles will not destabilize economy for sufficiently high interest rates.

Path of Capital in Bubbly Equilibrium, Low Level of Initial Capital



Where do we go from here?

- What do we like about the blueprint model?
  - A mechanism to engineer random asset price volatility into a simple macro model.
    - Source of movements is market psychology.
  - A mechanism to study the effects of bubbles in our models.
    - Welfare Implication: Bubbles are good!
    - Policy Implication: Don't burst the bubble.
- Avenues to purse:
  - Complicating the market psychology.
    - η and transition probabilities
  - Endogenizing the interest rate.
- What does the blueprint model leave to be desired?
  - Where does the borrowing constraint come from?
  - Why do bubbles emerge and contract? What is the source of their dynamics?
  - How might things change in a multi-good economy? Does the location of bubbles matter?

Housing Bubbles and a Bubbly Philosophy

Óscar Arce and David López-Salido present another simple overlapping generations model with bubbles.

- Agents live for three periods and maximize lifetime utility.
  - Young agents consume, borrow, and make housing investments.
  - Middle-aged agents consume and save.
  - Old agents consume remaining wealth.
- The Role of Bubbles:
  - "...we put forward a theory of housing bubbles that is consistent with the view of rational bubbles as market-based mechanisms that alleviate situations of asset and collateral scarcity..."

Multiple Equilibria: A Sunspot Model

- This model is not quite a sunspot model.
  - Fundamentals may preclude the feasibility of some steady states
  - Physical shocks can toggle between steady states.
- Our Goals:
  - Identify and characterize steady states.
  - Describe transition mechanisms between steady states.
  - Consider welfare implications and optimal monetary policy responses.

The Optimization Problem

- Utility

$$U_t = \log(c_t^y) + \beta[\log(h_{t+1}^y) + \log(c_{t+1}^m)] + \beta^2 \log(c_{t+2}^o) \tag{1}$$

- The Young
  - Resource Constraint:

$$c_t^y + P_t h_{t+1}^y - d_t \leq 1 \tag{2}$$

  - Borrowing Constraint:

$$d_t \leq (1 - \theta) P_t h_{t+1}^y \tag{3}$$

    - θ measures tightness of the borrowing constraint.

The Optimization Problem: Constraints

- The Middle-Aged:
  - Resource Constraint

$$c^m_{t+1} + R_t d_t + a_{t+1} \leq P_{t+1} h^y_{t+1} \tag{4}$$

    - Middle-aged wealth is equal to the value of the housing stock.
    - R_t is the as the interest rate on borrowing at time t.
  - The middle-aged is confronted with multiple savings vehicles.
    1. Make loans.
    2. Purchase pure bubble assets.
    3. Purchase unoccupied housing.
- The Old
  - Resource Constraint

$$c^o_{t+1} \leq S_{t+1} a_{t+1} \tag{5}$$

Solving the Model: Big Picture

Our Ultimate Goal: Identify and compare steady states

1. First order conditions.
   - Extract schedules for supply and demand of loanable funds.
2. Aggregate and apply loans and housing market clearing conditions.
3. Comparative statics.

First Order Conditions

- Tightness of the borrowing constraint (Kuhn-Tucker)

$$\mu_t[(1-\theta)P_t h^y_{t+1} - d_t] = 0 \tag{6}$$

  - Tightness depends on ratio of return on housing investment to the cost of borrowing: $\frac{P_{t+1}}{P_t R_t}$.
  - For sufficiently low R_t the borrowing constraint will bind.
- Relaxed Constraint
  - Housing expenditure: $P_t h^y_{t+1} = \gamma\beta \frac{1}{1-\frac{P_{t+1}}{P_t R_t}}$
    - γ is young consumption.
  - Demand is decreasing in R_t.

$$d_t = \gamma\beta\left[\left(1-\frac{P_{t+1}}{P_t R_t}\right)^{-1} - (2+\beta)\right] \tag{7}$$

  - Saving is increasing in R_t.

$$a_{t+1} = \gamma\beta^2 R_t \tag{8}$$

First Order Conditions

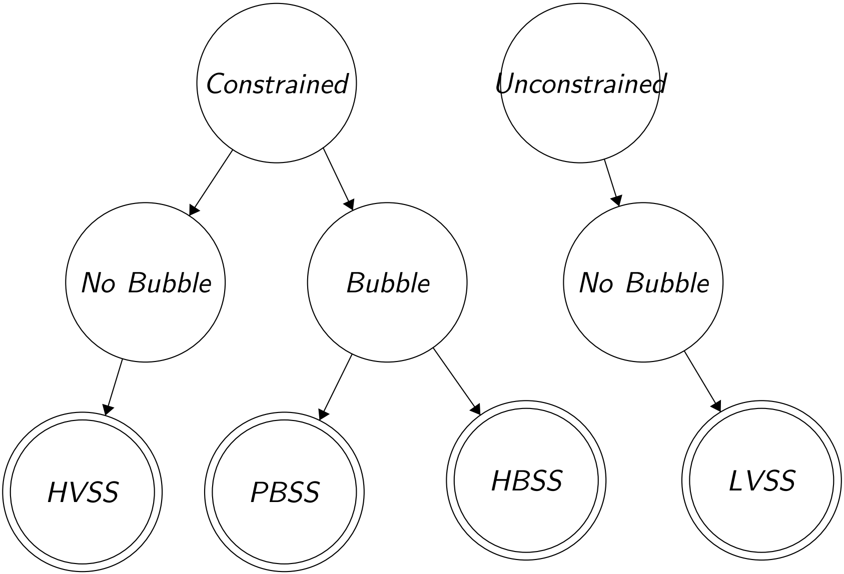
- Binding Constraint
  - Housing expenditure: $P_t h^y_{t+1} = \frac{1-\gamma}{\theta}$.
  - Demand is constant in R_t and decreasing in θ.

$$d_t = \frac{(1-\gamma)(1-\theta)}{\theta} \tag{9}$$

  - Saving is decreasing in R_t and increasing in θ.

$$a_{t+1} = (1-\gamma)\frac{\beta}{1+\beta}\frac{\frac{P_{t+1}}{P_t} - (1-\theta)R_t}{\theta} \tag{10}$$

Multiple Equilibria: Diagram



Fundamental Steady States (No Bubble)

In a fundamental steady state, saving is synonymous with loan-making.

- Market Clearing Conditions
  - Loans Market: Total saving must equal total borrowing.
    - $A = D$
  - Housing Market: Entire housing stock must be purchased by the young for occupancy.
    - $H^y_{t+1} = H$

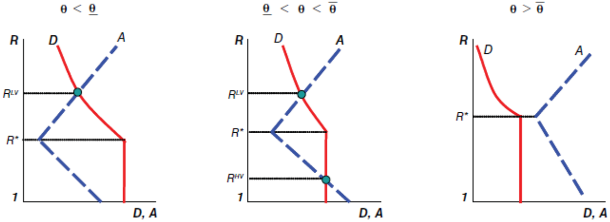


FIGURE 1. LOANS SUPPLY AND DEMAND: MULTIPLE STEADY STATES

Introducing Bubbles

Definition

A pure bubble equilibrium is one in which a fixed stock of freely disposable, intrinsically unproductive assets has a positive per unit price, Q_t.

- New Equilibrium Conditions:

$$A^{PB} = D^{PB} + B^{PB} \tag{11}$$

Definition

A housing bubble equilibrium is one in which a non-zero fraction of the total housing stock is purchased by the middle-aged as a store of wealth.

- New Equilibrium Conditions:

$$A^{HB} = D^{HB} + B^{HB} \tag{12}$$

$$B^{HB} = (H - H^{y})P^{HB} \tag{13}$$

Bubbly Existence Conditions

Theorem

A bubbly steady state only exists if $R = 1$.

- No arbitrage: $\frac{Q_{t+1}}{Q_t} = R_t$
- Steady state : $Q_{t+1} = Q_t$.

Theorem

A bubbly steady state is only feasible if $\theta > \hat{\theta}$

- See diagram!

Welfare Considerations

Compare aggregate lifetime utility across the steady states.

$$U = \log(C^y) + \beta[\log(H^y) + \log(C^m_{t+1})] + \beta^2 \log(C^o_{t+2})$$

Theorem

$$U^{HB} < U^{PB} = U^{HV} = U^{LV}$$

- $H^y = H - \frac{B^{HB}}{P}$

Theorem

$$\frac{d(U^{PB} - U^{HB})}{d\theta} > 0$$

- A tighter borrowing constraint grows the bubble.

Tendency Towards Bubbles

Theorem

Suppose that at $t = 0$, the economy is in an unconstrained steady state. At $t = 1$, a one-period positive loan supply shock of size $F > 0$ takes place. Then, if

$$F \begin{cases} < \hat{F}, \textit{the unique possible steady state is the LVSS} \\ = \hat{F}, \textit{the unique possible steady state is the HVSS} \\ > \hat{F}, \textit{the unique possible steady state is bubbly} \end{cases}$$

where $\hat{F} = \frac{D^{HV} - D^{LV}}{(1+\beta)}$.

- Multiplier effect on savings.
- Role of θ.
  - Tighter constraint means more prone to bubble formation.

Bubble Fragility

Theorem

Suppose that at $t = 0$, the economy is in the pure bubble steady state. At $t = 1$, a one-period negative loan supply shock of size $F > 0$ takes place. Then, if $F > \hat{F}^{PB}$, the only possible steady state is the LVSS where $\hat{F}^{PB} = B^{PB}$.

Theorem

Suppose that at $t = 0$, the economy is in the housing bubble steady state. At $t = 1$, a one-period negative loan supply shock of size $F > 0$ takes place. Then, if $F > \hat{F}^{HB}$, the only possible steady state is the LVSS where $\hat{F}^{HB} < \hat{F}^{PB}$.

- Fragility depends on the size of the bubble (negatively on θ).
- Housing bubbles are more fragile than pure bubbles.
  - Reduction in housing price reduces middle-aged wealth.

Takeaways

- Bubbly steady states are incompatible with high interest rates.
  - Borrowers must be constrained.
- Pure bubbles are neutral but housing bubbles are bad because they waste productive resources.
  - Compare to argument that bubbles crowd out productive investment.
- Preference shocks that increase (decrease) relative preference for housing vs. consumption may lead to the bursting (creation) of a bubble.
- Preference shocks that increase relative preference for consumption today vs tomorrow may lead to creation (bursting) of bubble.
- Housing bubbles are more easily burst than pure bubbles.

Questions?