Galí, Jordi. 2014. "Monetary Policy and Rational Asset Price Bubbles."

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Roadmap

- Theme: Optimal monetary policy in the presence of rational asset price bubbles
  - A general equilibrium framework that calls into question the canonical "leaning against the wind" philosophy.
- 2 stages:
  1. Galì's partial equilibrium example of a bubbly asset and it's relationship with the real interest rate.
  2. Galì's general equilibrium model of the endogenous relationship between bubbles and monetary policy.

Rational bubbles: An overview

1. What is a rational bubble?
    - The component of an asset's price that exceeds its fundamental value.
    - Agents have rational expectations.
2. How are rational bubbles supported in equilibrium?
    - Many models: Bubbles improve dynamic efficiency because they serve as a savings vehicle.
3. Are bubbles good or bad?
    - Maybe bubbles improve welfare.
        - Reduce dynamic inefficiency.
        - Relax borrowing constraints and increase investment.
    - Maybe bubbles are undesirable.
        - Contribute to market volatility.
        - Crowd out investment.
        - Redistribute an asset sub-optimally.
4. What is their relationship with monetary policy?

Galì's partial equilibrium

- Galì unpacks the relationship between the real interest rate and an asset price bubble in partial equilibrium.
  - An increase in the real interest rate permanently increases the bubble's size.
- Up next:
  1. Statement and setup of partial equilibrium example.
  2. Proof that interest rates permanently increase bubble size.
  3. Discussion of possible reconciliations with empirical evidence and the "leaning against the wind" philosophy.

Partial equilibrium: Setup

- We will explore types of two channels through which interest rates may be related to bubbles.
  - An arbitrage channel: A larger real interest rate contributes to bubble price appreciation.
  - Other potential channels: An interest rate shock may have arbitrary additional effects on bubble price
- Consider an asset with price Q_t and that returns dividends D_t.
  - The asset is traded by a risk neutral agent who has the opportunity to earn the risk free interest rate, R_t.

Partial equilibrium: The components of price

- The asset's discounted price equals its expected future value.

$$Q_t = \frac{E_t\{D_{t+1} + Q_{t+1}\}}{R_t} \tag{1}$$

- Split the asset's price into a **fundamental** and a **bubbly** component.

$$Q_t = Q_t^F + Q_t^B \tag{2}$$

- The fundamental component of price has a unique solution.
  - It is defined by its future dividends.

$$Q_t^F = E_t\left[\sum_{k=1}^{\infty}\left(\prod_{j=0}^{k-1}\frac{1}{R_{t+j}}\right)D_{t+k}\right] \tag{3}$$

Partial equilibrium: Bubble dynamics

- The bubble component of price is the difference between overall price and fundamental price.

$$Q_t^B = Q_t - Q_t^F \tag{4}$$

- Bubble price is restricted by the "no arbitrage" condition.
  - In equilibrium, return on the bubble equals return on saving.

$$R_t = \frac{E_t\{Q_{t+1}^B\}}{Q_t} \tag{5}$$

- Larger $R_t \implies$ larger bubble price appreciation.

Partial equilibrium: Bubble dynamics

- Log linearize (5) and introduce a bubble shock term.

$$E_t\{q^B_{t+1}\} = q^B_t + r^B_t \tag{6}$$
$$q^B_{t+1} = q^B_t + r^B_t + \xi_{t+1} \tag{7}$$

- How does an interest rate shock affect bubble growth?

$$r_t = c + \rho r_{t-1} + \epsilon_t \tag{8}$$

- The unexpected real interest rate shock term, ϵ_t, is related to the size of the future bubble.

$$q^B_{t+1} = q^B_t + \rho r_{t-1} + \epsilon_t + c + \xi_{t+1} \tag{9}$$

Partial equilibrium: Bubble dynamics

- Iterate the equation for bubble size out to k periods in the future, and differentiate with respect to ϵ_t.

$$\begin{aligned}\frac{\partial q^B_{t+k}}{\partial \epsilon_t} &= 1 + \rho + \rho^2 + ... + \rho^k \\ &= \frac{1-\rho^{k+1}}{1-\rho},\ \rho < 1 \end{aligned} \tag{10}$$

- Take the limit as $k \to \infty$ and recover the permanent effect of an interest rate shock on bubble price.

$$\lim_{k\to\infty} \frac{\partial q^B_{t+k}}{\partial \epsilon_t} = \frac{1}{1-\rho} > 0 \tag{11}$$

- Unexpected interest rate growth permanently augments the bubble's size.

Partial equilibrium: Reconciliation with conventional wisdom

- Could this model be compatible with the "leaning against the wind" philosophy?
  - The fundamental component of asset price is decreasing in the interest rate.
    - We may observe a decline in overall asset prices even though the bubbly components of price have grown.
  - We might not have captured the complete relationship between interest rates and bubble price.
- Other interest rate channels:
  - Suppose that interest rates and bubbles are related in other ways.
  - What would this mean for the arbitrage channel?
  - What would be the overall direction of the relationship?

Partial equilibrium: Other potential channels

- Let ψ_r capture an arbitrary contemporaneous correlation between interest rate shocks and bubble shocks.

$$\xi_t = \psi_r(r_t - E_{t-1}[r_t]) + \xi_t^* \tag{12}$$

- Repeat the previous steps to compute the permanent effect of an interest rate shock on the bubble's price.

$$\lim_{k\to\infty} \frac{\partial q_{t+k}^B}{\partial \epsilon_t} = \psi_r + \frac{1}{1-\rho} \tag{13}$$

- If ψ_r is sufficiently negative, the permanent effect of a positive interest rate shock may be to decrease bubble prices.

General Equilibrium: Overview

- Consider an infinite horizon overlapping generations model.
- This model describes the behavior of three key actors.
  1. Consumers
  2. Firms
  3. Monetary Policy
- Features:
  1. A market for an infinitely lived bubbly asset.
  2. A market for differentiated consumption goods.
  3. Nominal rigidities in the form of sticky prices.
  4. Absence of capital.
  5. Fixed labor supply.
- Our job: Solve for a bubbly equilibrium. Take a first order approximation. Determine the optimal monetary policy response to fluctuations in the size of the bubble.

General Equilibrium: Life cycle of bubbles

- Bubbles are intrinsically worthless assets that have a positive price.
- Bubbly assets are differentiated based when they were born.
  - In each period, a quantity, δ, of bubbles is born and the same quantity dies.
  - The aggregate size of the bubble is constant.
  - When bubbles are born, they adhere themselves to the wealth of the young agents.
- For all t and $k < t$, there exist bubbles that were born in period $t - k$ that have not yet died by period t.
  - Let $Z_{t|t-k}$ denote this quantity.

General Equilibrium: Consumers

- Agents live for two generations; a young and an old.
- Lifetime utility of an agent is the discounted sum of utility earned in each period.

$$\log C_{1,t} + \beta E_t\{\log C_{2,t+1}\} \tag{14}$$

  - Utility is a function of consumption across the spectrum of goods.

$$C_{j,t} = \left(\int_0^1 C_{j,t}(i)^{1-\frac{1}{\epsilon}}\, di\right)^{\frac{\epsilon}{\epsilon-1}} \tag{15}$$

General Equilibrium: Consumers

- The young **consume**, buy **bonds**, and buy **old bubbles** using their wages and an endowment of **new bubbles**.

$$\int_0^1 \frac{P_t(i)C_{1,t}(i)}{P_t}di + \frac{Z_t^m}{P_t} + (1-\delta)\sum_{k=0}^{\infty} Q_{t|t-k}^B Z_{t|t-k}^B = W_t + \delta Q_{t|t}^B \tag{16}$$

- The old use their firm's **dividends** and the return on **bonds** and **bubbles** to finance consumption.

$$\int_0^1 \frac{P_{t+1}(i)C_{2,t}(i)}{P_{t+1}}di = D_{t+1} + \frac{Z_{t+1}^m(1+i_t)}{P_{t+1}} + (1-\delta)\sum_{k=0}^{\infty} Q_{t+1|t-k}^B Z_{t|t-k}^B \tag{17}$$

- $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon}di\right)^{\frac{1}{1-\epsilon}}$ is the aggregate price index.

General Equilibrium: Consumer first order conditions

- Consumers maximize utility by choosing consumption bundles such that (18) holds.

$$C_{j,t}(i) = C_{j,t} \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \tag{18}$$

- The Euler equation links overall consumption growth with the real interest rate.

$$1 = \beta(1 + i_t) E_t \left\{ \frac{P_t}{P_{t+1}} \frac{C_{1,t}}{C_{2,t+1}} \right\} \tag{19}$$

General Equilibrium: Firms

- Firms produce differentiated consumption goods.
  - Firms are owned by the old generation.
- Firms use labor as an input.
  - Labor is inelastically supplied by the young generation.
- Capital is excluded.
  - Without capital, there are no aggregate means of saving; the economy is dynamically inefficient.
  - Bubbles can serve to improve dynamic efficiency.
- Prices are sticky.
  - Nominal rigidites allow for monetary policy to have real effects by impacting wage and dividends.

General Equilibrium: Firms

- Firm i produces the quantity $Y_t(i)$ of consumption good i.
- Labor, denoted $N_t(i)$, is the only firm input.

$$Y_t(i) = N_t(i) \tag{20}$$

- At the end of period $t-1$, firms set prices for the period t in order to maximize expected discounted profits.

$$P_t^* = \text{argmax}_P \left\{ E_{t-1} \left\{ \beta \frac{C_{1,t-1}}{C_{2,t}} Y_t(i) \left(\frac{P}{P_t} - \frac{\epsilon}{1-\epsilon} W_t \right) \right\} \right\} \tag{21}$$

- Under price flexibility or in the absence of uncertainty, firms choose a constant real wage.

General Equilibrium: Monetary policy

- Nominal interest rates are set according to the interest rate rule (22).

$$1 + i_t = RE_t \left[\Pi_{t+1}\right] \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Q_t^B}{Q^B}\right)^{\phi_b} \tag{22}$$

  - Inflation is denoted by Π_{t+1} and the inflation target by Π.
  - Definitions of Q_t^B and Q^B are forthcoming.
- A positive ϕ_b is in accordance with the "leaning against the wind" philosophy.
- Up next: Decomposing the endogenous interplay between the monetary policy rule, the evolution of the bubble, and welfare.

Bubble evolution: An overview

- Each cohort receives a bubble endowment.
  - The bubbly endowment is equal to a fraction δ of the aggregate bubble.
  - A fraction δ of all preexisting bubbles evaporates.
    - The size of the aggregate bubble is constant across time.
- The young generation purchases bubbles from the old generation.
- Bubbles that are born in different periods are treated like different assets.
  - The price of a bubbly asset depends on when it was born.
  - The price of a bubbly asset may deviate throughout time,.

Bubble evolution: Bubbly syntax

- The old bubble index captures bubbles that were "born" in previous periods.

$$B_t = \delta \sum_{k=1}^{\infty} (1-\delta)^k Q^B_{t|t-k} \tag{23}$$

- The new bubble index captures bubbles that were "born" today.

$$U_t = \delta Q^B_{t|t} \tag{24}$$

- The aggregate bubble captures the value of all bubbles.

$$\begin{aligned} Q^B_t &= U_t + B_t \\ &= \delta \sum_{k=0}^{\infty} (1-\delta)^k Q^t_{t|t-k} \end{aligned} \tag{25}$$

Bubble evolution: First order conditions

- There is a market for each bubbly asset.
  - Bubbles return no dividends; their price reflects their discounted capital gain.

$$Q^B_{t|t-k} = (1-\delta)\beta E_t \left\{ \left(\frac{C_{1,t}}{C_{2,t+1}} \right) Q^B_{t+1|t-k} \right\} \tag{26}$$

- The price of the aggregate bubble today depends on the expectation of its price tomorrow.

$$Q^B_t = \beta E_t \left[\frac{C_{1,t}}{C_{2,t+1}} B_{t+1} \right] \tag{27}$$

- In equilibrium, the stochastic discount factor is linked to the real interest rate.

$$\beta E_t \left[\frac{C_{1,t}}{C_{2,t+1}} \right] = \frac{1}{R_t} \tag{28}$$

Bubble evolution: Sources of randomness

- The appearance of new bubbles is driven by random shocks.
  - U_t is an exogenous i.i.d. random process with mean $U > 0$.

$$U_t = U + \epsilon_t \tag{29}$$

- Unexpected deviations in the old bubble's price are an alternate source of randomness.
  - Let $B_t - E_{t-1}\{B_t\}$, be exogenous and independent from shocks to the new bubble.
- Galì's mechanical choices here are reflective of modeling challenges posed by bubbles.
  1. Why incorporate multiple sources of randomness?
  2. Why decompose the old bubble into different assets?

Bubble evolution: Assessing Galì's modeling choices

- Many models of rational bubbles involve a "new bubble" component that is driven by a sunspot shock.

$$\boxed{\text{Size of the bubble today}} = \boxed{\text{Sunspot shock today}} + \boxed{\text{Size of the bubble yesterday}}$$

$$\boxed{\text{Real interest rate}} = \boxed{\text{Size of the bubble today}} \div \boxed{\text{Size of the bubble yesterday}}$$

- The real interest rate must follow the lead of the shocks.
  - We **cannot** explore an endogenous relationship between bubble dynamics and monetary policy.
- We **can** answer questions about under what conditions is it possible to sustain an bubbly equilibrium.

Bubble evolution: Assessing Galì's modeling choices

- Imagine that Galì had make old bubble prices be constant.

$$Q_t^B = \frac{1}{R_t}\delta\left[(1-\delta)U + \sum_{k=2}^{\infty}(1-\delta)^k U_{t-k}\right] \tag{30}$$

- Randomness of the old bubble provides flexibility and a structure that allows us to endogenize monetary policy.

$$Q_t^B = \beta\frac{1}{R_t}E_t\left[B_{t+1}\right] \tag{31}$$

- The old bubble's expectation fluctuates in accordance with the monetary policy rule and exogenous shocks to the new bubble.
  - $E_t[B_{t+1}]$ provides an additional degree of freedom.

Equilibrium dynamics: The deterministic case

- We will solve for a bubbly steady state solution.
  - We will take linear approximations in the neighborhood of this steady state to study the stochastic case.
- Of note:
  - The existence conditions for a bubbly steady state are:
    1. Independent of monetary policy.
    2. Identical to the stochastic case.
  - The bubble-free steady state is dominated by the bubbly steady state.

Equilibrium dynamics: The deterministic case

- Fix bubble growth so that $U_t = U$ and $B_t = E_{t-1}[B_{t+1}]$.
- Firms choose a constant real wage.

$$W = \frac{\epsilon - 1}{\epsilon} \tag{32}$$

- Firm dividends are tied to the real wage.

$$D = 1 - \frac{\epsilon - 1}{\epsilon} \tag{33}$$

- The young splits its wage between consumption and bubbly assets. The old consumes dividends and bubble sales.

$$C_{1,t} = W - B_t \tag{34}$$
$$C_{2,t+1} = 1 - W + B_{t+1} \tag{35}$$

Equilibrium dynamics: The deterministic case

- Exploit the FOC linking the real interest rate to consumption growth and rearrange.

$$R(B_t, B_{t+1}) = \frac{1}{\beta}\frac{C_{2,t+1}}{C_{1,t}} \tag{36}$$
$$= \frac{1}{\beta}\frac{1 - W + B_{t+1}}{W - B_t}$$

- Rearrange the FOCSs associated with the bubble to recover a complete solution for the old bubble's growth.

$$B_{t+1} = \frac{(1 - W)(B_t + U)}{\beta W - (1 - \beta)B_t - U} \tag{37}$$

- A bubble-free deterministic steady state is suboptimal.
  - $R(0,0) < 1 \implies \frac{1}{C_1} < \frac{\beta}{C_2}$
  - Bubbles improve dynamic efficiency.

Equilibrium dynamics: The stochastic case

- Prices are sticky and monetary policy has real effects.
- Two monetary policy channels:
  1. The standard channel.
  2. The bubbly channel.
- The larger the bubble, the lower the central bank should set nominal interest rates.
- Coming up:
  1. Quantifying the relationship between monetary policy and bubble stability.
  2. Solving for the optimal monetary policy rule, ϕ_b.

Monetary policy and bubbles: The standard channel

- Intuition underlying the standard channel:
  - Suppose that $\phi_b > 0$. In response to an increase in the bubble's price, the nominal interest rate grows.
    1. Sticky prices $\implies$ R_t increases.
    2. In equilibrium, the real wage declines.
    3. The young generation's marginal utility of consumption decreases.
    4. The young saves less; buys less bubbles.
  - The choice of ϕ_b affects the volatility of wages and dividends.
- Result: Minimizing the volatility of wages and dividends calls for a $\phi_b > 0$.
  - In accordance with "leaning against the wind".

Monetary policy and bubbles: The bubbly channel

- Intuition underlying the bubbly channel:
  - Suppose that $\phi_b > 0$. In response to an increase in the bubble's price, the nominal interest rate grows.
    1. Sticky prices $\implies$ R_t increases.
    2. Expected value of the old bubble increases.
    3. Future bubble sizes are larger.
  - Higher nominal interest rates increase future bubble size.
    - Monetary policy does not have immediate affects on bubble dynamics.
- The bubble's size is related to welfare in that bubbles affect relative consumption across generations.

Monetary policy and bubbles: The bubbly channel

- Remember the old bubble's equilibrium condition:

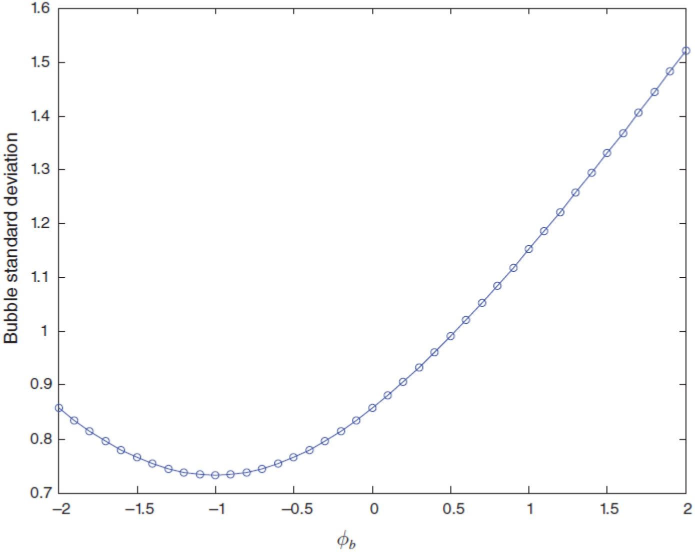
$$Q_t^B = \beta \frac{1}{R_t} E_t \left[B_{t+1} \right] \tag{38}$$

- To illustrate, log-linearize.

$$E_t[b_{t+1}] = q_t - r_t + \beta \tag{39}$$
$$b_{t+1} = q_t - r_t + \xi_t + \beta \tag{40}$$

- Result: Minimizing future bubble volatility calls for a $\phi_b = -1$.
    - Opposite of leaning against the wind.

Monetary policy and bubble fluctuations



Optimal monetary policy

- What is the optimal monetary policy response to bubble price fluctuations?
- What value of ϕ_b minimizes welfare loss?

Optimal monetary policy

- Choose the second order approximation of lifetime utility as the measure of welfare to optimize.

$$\max\left\{\log(C_1)+\beta\log(C_2)-\frac{1}{2}(\mathrm{var}\{\hat{c}_{1,t}\}+\beta\mathrm{var}\{\hat{c}_{2,t}\})\right\}$$

- Maximizing welfare corresponds to minimizing $\mathrm{var}\{\hat{c}_{2,t}\}$.

$$\mathrm{var}\{\hat{c}_{2,t}\}=(1-\Gamma)\hat{d}_t+\Gamma\hat{b}_t \tag{41}$$

- The welfare maximizing choice of ϕ_b depends on the relative strength of ϕ_b in the different channels.
  - Relative strength depends on the steady state size of the bubble, B.
    - Larger $B \implies$ larger relative strength of bubbly channel.

Optimal monetary policy