Basic Principles of Solar Acoustic Holography

Laurel Farris
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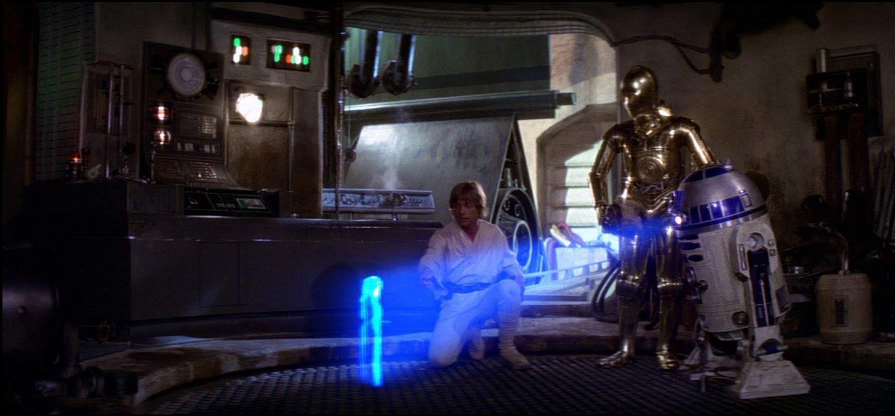


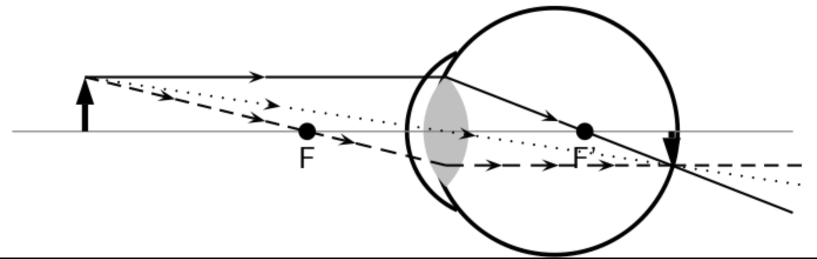
The Basic Principle of Heliospheric Holography

Defined as the *phase-coherent* computational reconstruction of the acoustic field in the solar interior or far side of the sun.

Comparison with optics

- Submerged sources in sun $\sim$ things we see
- Photosphere $\sim$ surface of cornea (front of eye)

Both involve refocusing radiation to render stigmatic images that can be sampled over focal surface at any desired depth.



Historical Note

- Concept proposed in 1975 by Roddier
- Developed over the 1990s by Lindsey and Braun
  $\rightarrow$ Key to locating and examining fine structure in the interior and far side of the sun.

Seismic holography was first applied to helioseismic data from the SOlar Heliospheric Observatory (SOHO).
"New" (1998 - 1999) solar acoustic phenomena:

- 'acoustic moats' surrounding sunspots
- 'acoustic condensations' 1020 Mm beneath active regions
- 'acoustic glories' surrounding complex active regions
- first helioseismic images of a flare

Motivation

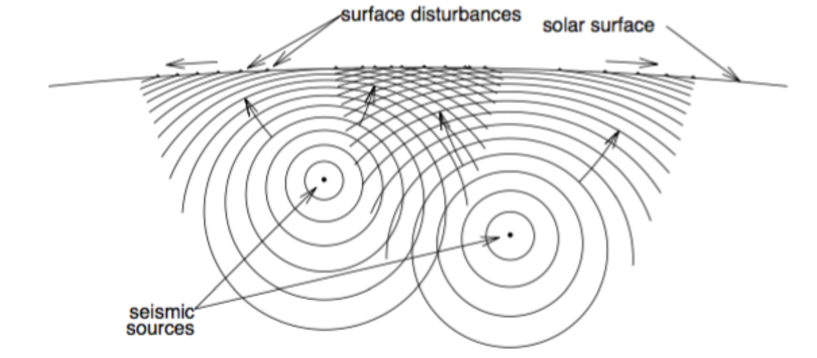
Space weather application $\rightarrow$ See flares before they turn to face Earth!

“Basic Principles of Solar Acoustic Holography”

C. Lindsey and D. C. Braun; 2000

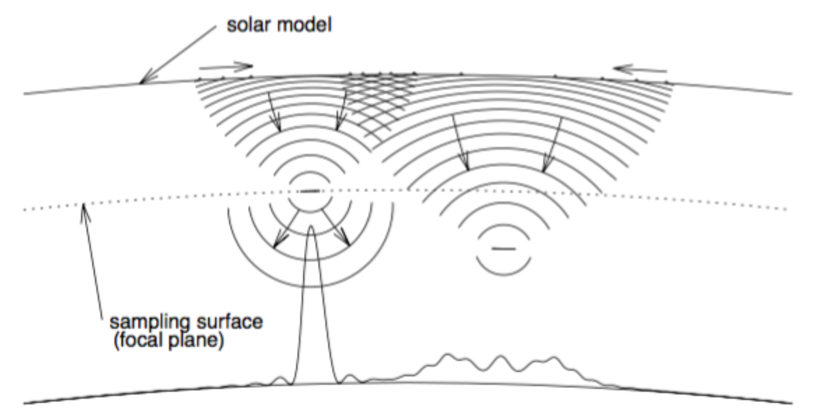
- Compare simple acoustic-power to phase-sensitive
- Propose “simple computational principles” to produce images from high quality helioseismic observations.

Step 1: Helioseismic observations



- Interior sources of acoustic waves
- Pattern of ripples on surface directly *above* the sources.
- The waves are *absorbed* upon reaching the surface, not reflected back down or propagated into atmosphere. (accurate for $\nu > \sim 5.5$ mHz)

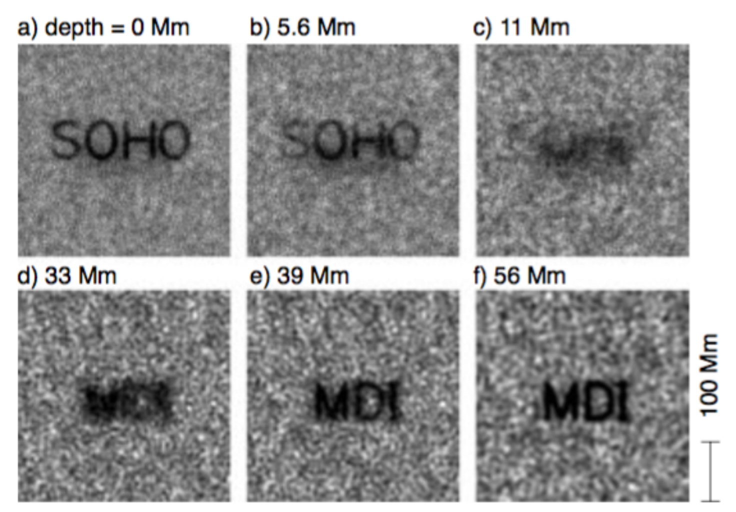
Step 2: Apply observations to a model



Disturbance now propagating back down.
Consider a "focal plane" in the interior, at a depth z_{plane}.

- $z_{\text{plane}} = z_{\text{source}}$: diffraction-limited signature
- $z_{\text{plane}} \neq z_{\text{source}}$: unfocused, diffuse profile

Absorbers confined to infinitely thin sheets



- $z_{\mathrm{SOHO}} = 0$ Mm
- $z_{\mathrm{MDI}} = 56$ Mm ($\sim \frac{1}{10}$ R$_\odot$).
- (b) & (c) acoustic *stalactites*
- (d) & (e) acoustic *stalagmites*

The Computational Task

Two perspectives:

1. “spectral”
   - wavenumber-frequency (k, ν)
   - (computationally advantageous)
2. “time distance”
   - space-time (x, t)
   - (more intuitive)

Given

- acoustic amplitude
- derivative normal to surface surrounding medium that is free of sources, sinks, or scatterers

can extrapolate the (incomplete) acoustic field in the interior.

The space-time perspective

Acoustic *egression*

- $\psi(\mathbf{r}', t')$: *Actual* acoustic field
- H: *incomplete regression* of the acoustic field
  - $H_+(\mathbf{r}, z, t)$ "acoustic egression"
    "focal point" $(\mathbf{r}, z, t) \rightarrow$ surface $(\mathbf{r}', 0, t')$.
  - H_- "acoustic ingression" (next slide)

$$H_+(\mathbf{r}, z, t) = \int \mathrm{d}t' \int_{a<|\mathbf{r}-\mathbf{r}'|<b} \mathrm{d}^2 r' G_+(|\mathbf{r}-\mathbf{r}'|, z, t-t') \psi(\mathbf{r}', t')$$

$G_+ \rightarrow$ Green's function that expresses how a single transient point disturbance propagates forward or backward in time between $(\mathbf{r}', 0, t')$ and $(\mathbf{r}, z, t)$.

After computing H_+, square and integrate it to produce an egression power map over the time period in desired range.

The space-time perspective

Acoustic *ingression*

H_-: "acoustic ingression", the time reversal of H_+; waves converge into the focal point and contribute to the disturbance.

Green's function:

$$G_-(|\mathbf{r}-\mathbf{r}'|, z, t-t') = G_+(|\mathbf{r}-\mathbf{r}'|, z, t'-t)$$

The wavenumber-frequency perspective

- $\hat{\psi}(\mathbf{k}, \nu)$: Fourier transform of $\psi(\mathbf{r}, t)$
- $\hat{G}_+(|\mathbf{k}|, z, \nu)$: Fourier transform of $G_+(|\mathbf{r}|, z, t)$

Fourier transform of the egression:

$$\hat{H}_+(\mathbf{k}, z, \nu) = \hat{G}_+(|\mathbf{k}|, z, \nu)\hat{\psi}(\mathbf{k}, \nu)$$

from the convolution theorem. Multiplication is computationally *faster* than convolution, so this is the preferred method.

Temporal Fourier Transform

Need large pupils to image deeper focal planes, but this produces coma, primary astigmatism, and higher-order aberrations.

$$(\mathbf{r}, t) \to (\mathbf{r}, \nu)$$

$$\check{H}_+(\mathbf{r}, z, \nu) = \int\limits_{a<|\mathbf{r}-\mathbf{r}'|<b} \mathrm{d}^2 r' \check{G}_+(|\mathbf{r} - \mathbf{r}'|, z, \nu) \check{\psi}(\mathbf{r}', \nu)$$

Subjacent vs. Superjacent Vantage Holography

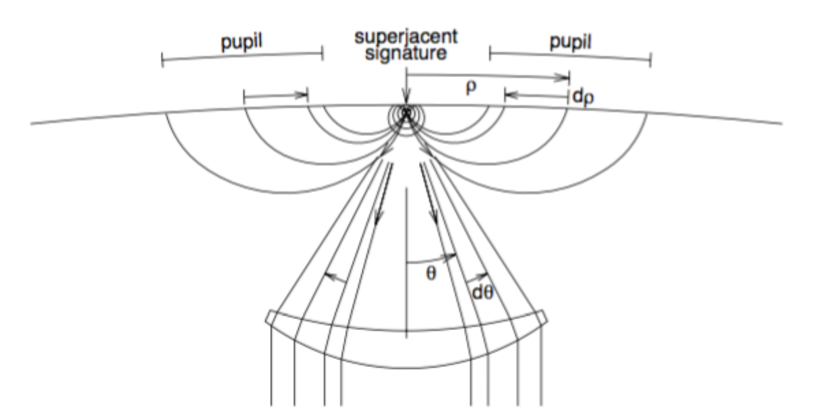
Superjacent:

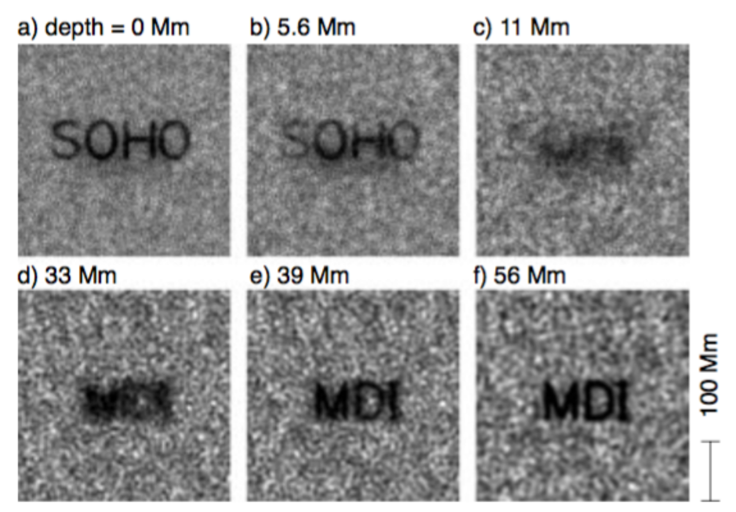
- Wave propagated directly *upward* from the source to surface
- applies to active regions.

Subjacent:

- inner radius, a, of the pupil annulus is much greater than the depth of the focal plane;
- applies to quiet sun (often the practical choice)

Subjacent Vantage Holography





- $a = 15$ Mm, $b = 45$ Mm
- (a) & (b): superjacent
- (c) mixed perspective
- (d), (e), & (f): predominantly subjacent

Egression power maps

Lindsey and Braun (1998)

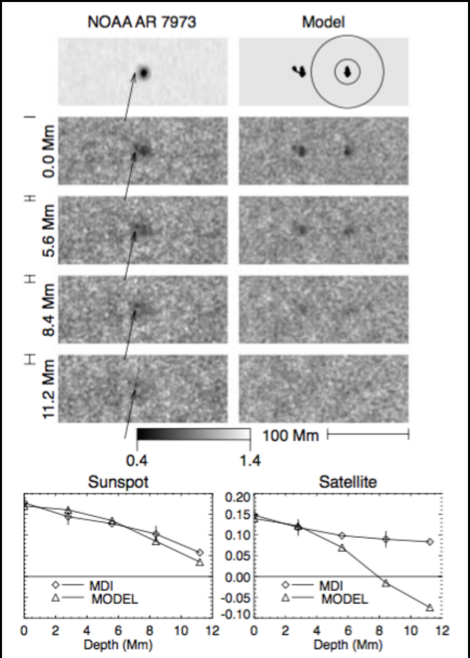
Top panels

- $\nu = 6$ mHz ($\Delta\nu = 1$ mHz)
- model annulus around "Rorschach" splotches

Bottom panels

- Contrasts between object and surroundings

All subjacent vantage!



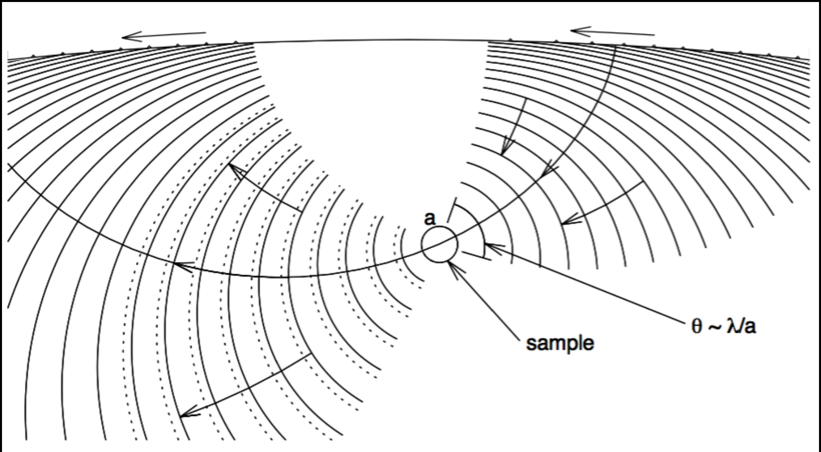
Acoustic Modeling Based on Holographic Images

Flexible procedures, such as inversions, would characterize the acoustic environment in pysical terms such as:

- acoustic emissivity
- acoustic opacity
- refractivity
- flow velocity

where the last two result from the application of phase-sensitive holography

gedanken experiment



- Produce waves that travel from right to left, into the refractive sample.

gedanken experiment

- No phase-shift: $\Delta n = 0$
- Phase-shift:
  - $\Delta n = \Delta c_s / c_s \rightarrow$ refractive perturbation
  - $\Delta t \sim a \Delta n / c \rightarrow$ time delay
  - $\Delta \phi \sim 2\pi \nu a \Delta n / c \rightarrow$ phase shift

  where a is the characteristic diameter of the sample.

To relate $\Delta\phi$ to H_+ and H_-, define the *temporal* Fourier transforms

- $\check{H}_+(\mathbf{r}, z, \nu) \leftrightarrow H_+(\mathbf{r}, z, t)$
- $\check{H}_-(\mathbf{r}, z, \nu) \leftrightarrow H_-(\mathbf{r}, z, t)$.

Then

$$\Delta\phi = \arg\left(\left\langle \check{H}_+(\mathbf{r}, z, \nu) \check{H}_-^*(\mathbf{r}, z, \nu) \right\rangle_{\Delta\nu}\right)$$

Green's Functions

$$G_{\pm}(|\mathbf{r}-\mathbf{r}'|, z, t-t')$$

Characterizes the propagation of acoustic disturbances between a surface point, $(\mathbf{r}', 0, t')$ and source point, $(\mathbf{r}, z, t)$

These propagations take place in the solar *model* to which helioseismic observations, $\psi(\mathbf{r}', t')$. are applied.

Dispersionless acoustics

- Pulse propagates in the form of an infintely thin *wavefront*.
- $\mathbf{r}'$ responds with ripple characterized by the same infinitely sharp temporal profile as the source, but properly attenuated.
- The Green's function is invarient with respect to both time and horizontal translation.

$$G_+(|\mathbf{r}-\mathbf{r}'|, z, t-t') = \delta\left(t-t'-T\left(|\mathbf{r}-\mathbf{r}'|, z\right)\right) f\left(|\mathbf{r}-\mathbf{r}'|, z\right)$$

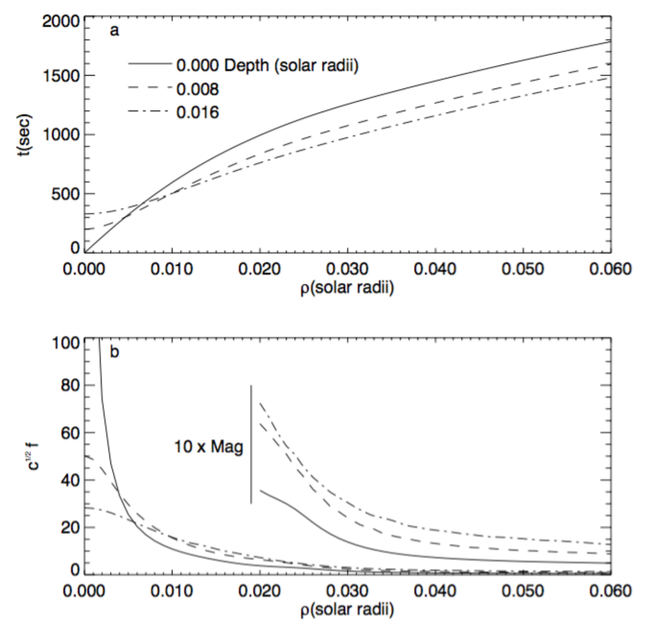
- T: travel time
- f: amplitude of pulse

Dispersionless acoustics
Reflected waves

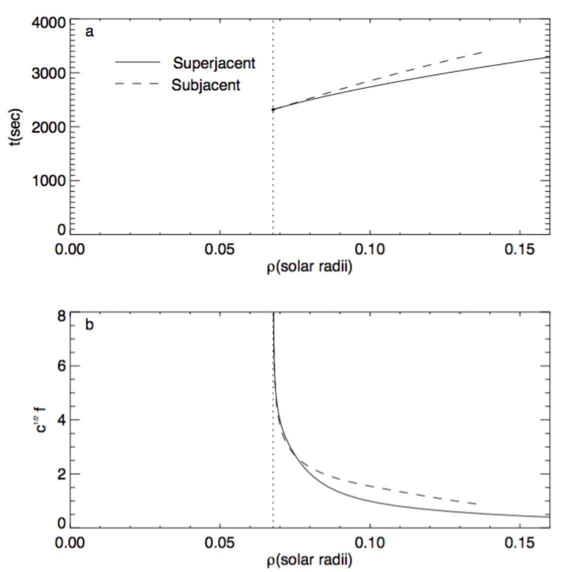
- $\nu > 5.5$ mHz absorbed by photosphere.
- $\nu < 4.5$ mHz reflected from photosphere.
  Green's function is characterized by a sum of n components, where each n is a "skip", or reflection from the photosphere.

1. subjacent: ρ decreases as θ increases
2. superjacent: ρ increases after reaching a minimum as θ continues to increase toward $180°$.

Single-skip holography

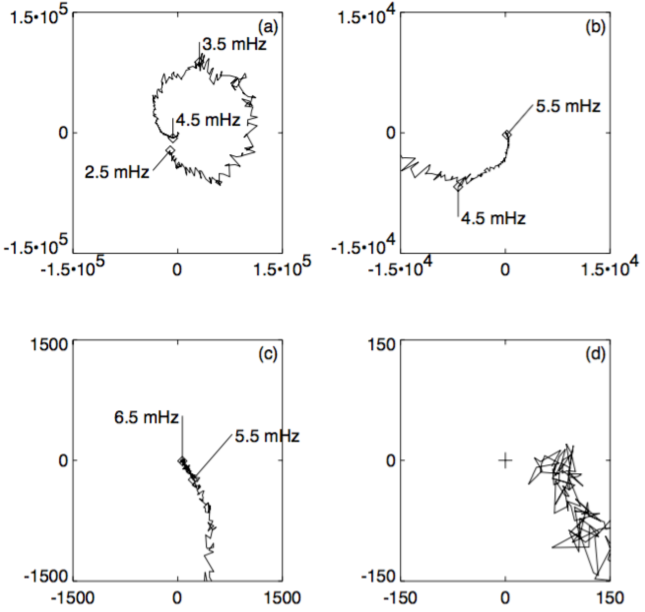


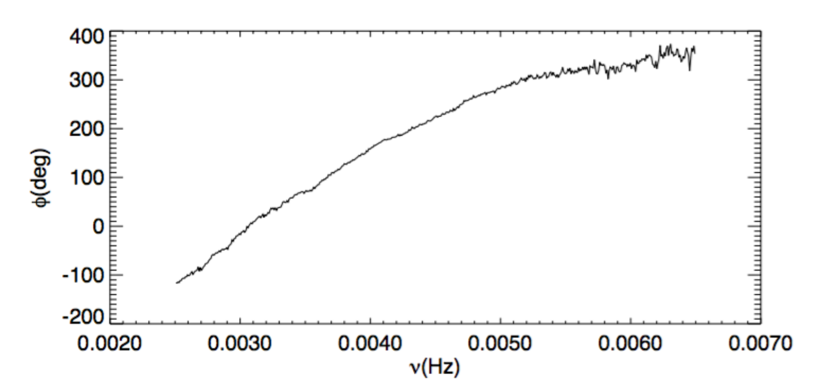
Two-skip holography



Dispersion

In reality, acoustic waves are *significantly* dispersed near the photosphere.


(a)
3.5 mHz
4.5 mHz
2.5 mHz
1.5•10^5
0
-1.5•10^5
-1.5•10^5
0
1.5•10^5
(b)
5.5 mHz
4.5 mHz
1.5•10^4
0
-1.5•10^4
-1.5•10^4
0
1.5•10^4
(c)
6.5 mHz
5.5 mHz
1500
0
-1500
-1500
0
1500
(d)
150
0
-150
-150
0
150



400
300
200
100
0
-100
-200
φ(deg)
0.0020
0.0030
0.0040
0.0050
0.0060
0.0070
ν(Hz)

Comparison to other techniques

- time-distance helioseismology (aka. tomography)
- ring diagram analysis

Applications

- acoustic glories
- acoustic moats and/or
- confirming p-mode absorption in sunspots (p. 267)

Movie!

Take-home points