Basic Principles of Solar Acoustic Holography

ASTR 500

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Outline

"Basic Principles of Solar Acoustic Holography"
C. Lindsey and D. C. Braun
2000

The Basic Principle

Define **helioseismic holography** as the *phase-coherent* computational reconstruction of the acoustic field in the solar interior. Information can be extracted from the p-mode power spectrum, and models can be devolped based on these holgraphic signatures.

HOMOGRAPHY $\neq$ MODEL!

In this paper:

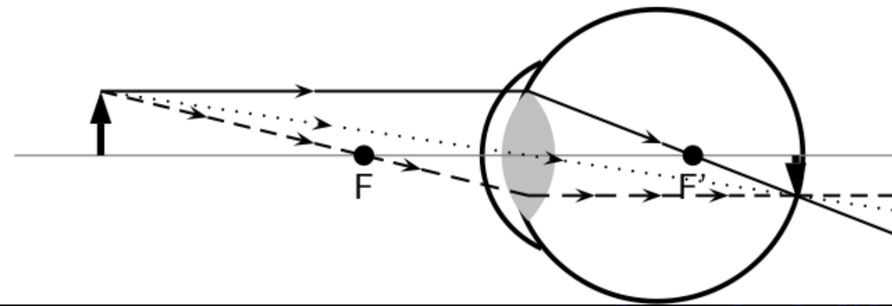
- Compare simple acoustic-power to phase-sensitive
- Propose "simple computational principles" to produce images from high quality helioseismic observations.

Drawing on principles in optics and optical holography

- Submerged sources in sun $\sim$ things we see
- Photosphere $\sim$ surface of cornea (front of eye)

Both involve refocusing radiation to render stigmatic images that can be sampled over focal surface at any desired depth.

Producing *stigmatic images* of the source of the disturbance.



Historical Note

- Concept proposed in 1975 by Roddier
- Developed over the 1990s by Lindsey and Braun (current authors)
- $\rightarrow$ Key to locating and examining fine structure as deep as possible.

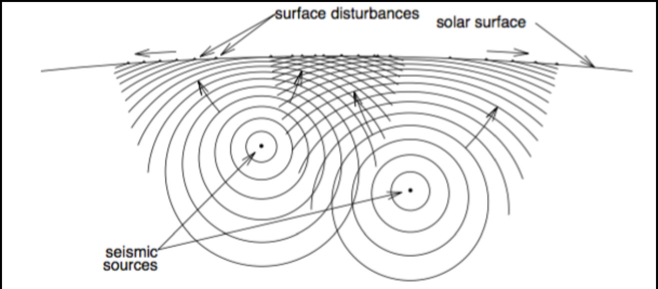
Seismic holography was first applied to helioseismic data from SOHO. “New” (1998-1999) solar acoustic phenomena:

- ‘acoustic moats’ surrounding sunspots
- ‘acoustic condensations’ 10-20 Mm beneath active regions
- ‘acoustic glories’ surrounding complex active regions
- first helioseismic images of a flare

$\rightarrow$ solar cycle dependence of global p-modes! (which is … ?)

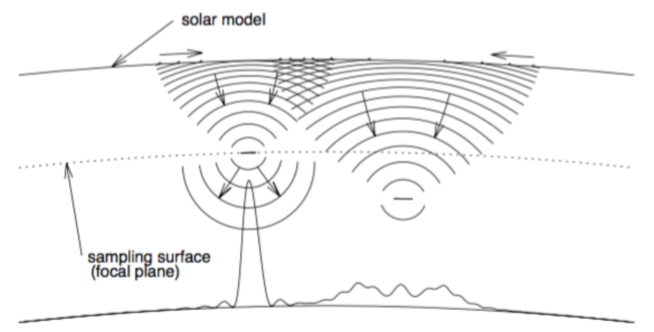
Part 2: Basic Principles of Computational Seismic Holography

2.1; Figure 1



- Well-defined acoustic sources
- All we see is the pattern of ripples at the surface, propagating from points directly *above* the sources.
- The waves are absorbed upon reaching the surface (accurate for $\nu > \sim 5.5$ mHz (what's the significance of this??)

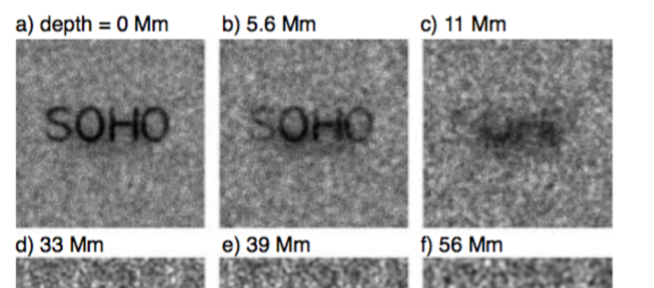
2.2; Figure 2



- Apply time-series of observations to model with no sources, sinks, or scattering (of what?)
- The observances are "seen" at the "pupil".
- Place the "focal plane" at the location of the sources, and get a diffraction-limited signature (left side of figure).
- If focal plane is above or below the source, we get an unfocused, diffuse profile (right side of figure).

2.3; Figure 3

- Simulation: random acoustic noise in model that contains alphanumeric absorbers at six different locations, from just below the surface to a depth of 56 Mm ($\sim \frac{1}{10}$ R$_\odot$).
- ‘acoustic stalactite’ of the aborber the de-focused plume.
- a diffuse ‘stalagmite’ appears closer to the absorber
- sharp, diffraction-limited silhouette at 56 Mm.
- depth diagnostics accomplished by focusing and de-focusing, rather than the appearance or disappearance that would be used in realistic phyiscal models.



2.4

Seismic holography is most certainly not a representation of solar acoustics in terms of ray optics. These are mechanical waves, not electromagnetic ones, though they have similiar behavior, such as interference and diffraction. Thus, it suffers from the same limitations as other helioseismological observations, and the same kind of optimization techniques used to extract information from coherent electromagnetic radiation can also be used here.

Part 3: The Computational Task

3.1

Two perspectives:

1. the "spectral": the disturbance in terms of the normal modes of the medium
2. the "time distance": closer to that of time-distance helioseismology

Terms used:

- space-time
- wavenumber-frequency

3.2

Given

- acoustic amplitude
- its derivative

can extrapolate the acoustic field anywhere in the interior! Although it's incomplete; we're getting a significant fraction, but not the entire interior. Future terminology:

- H - incomplete regression of the acoustic field
- ψ - Actual acoustic field

3.3: The space-time perspective

$\psi(\mathbf{r}', t')$ - acoustic field, secured at time t' and horizontal location $\mathbf{r}'$ regression is expressed by a formalism called the "acoustic egression", $H_+(\mathbf{r}, z, t)$, an imcomplete, but coherent assessment of the local acoustic disturbance that has emanated from the "focal point", $(\mathbf{r}, z)$, of the computation at time t based on its succeeding emergence at the overlying surface over the range of locations and time expressed by $(\mathbf{r}', 0, t')$. This can be expressed by an integral of the form:

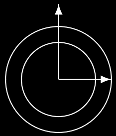
$$H_+(\mathbf{r}, z, t) = \int \mathrm{d}t' \int_{a<|\mathbf{r}-\mathbf{r}'|<b} \mathrm{d}^2 r' G_+(|\mathbf{r}-\mathbf{r}'|, z, t-t')\psi(\mathbf{r}', t')$$

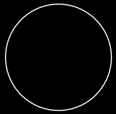
G_+ - Green's function that expresses how a single transient point disturbance propagates forward or backward in time between $(\mathbf{r}', 0, t')$ and $(\mathbf{r}, z, t)$.

3.4

H_- - acoustic ingression, time reverese of H_+. Describes waves coherently converging into a point, rather than from it. Replace G_+ by its time reverse:

$$G_-(|\mathbf{r}-\mathbf{r}'|, z, t-t') = G_+(|\mathbf{r}-\mathbf{r}'|, z, t'-t)$$





3.5

After computing H_+, square and integrate it to produce an egression power map over the time period in desired range. p-mode absorption in sunspots has already been confirmed this way!

3.6: The wavenumber-frequency perspective

- $\hat{\psi}(\mathbf{k}, \nu)$ - Fourier transform of $\psi(\mathbf{r}, t)$
- $\hat{G}_+(|\mathbf{k}|, z, \nu)$ - Fourier transform of $G_+(|\mathbf{r}|, z, t)$

From the convolution theorem:

$$\hat{H}_+(\mathbf{k}, z, \nu) = \hat{G}_+(|\mathbf{k}|, z, \nu), \hat{\psi}(\mathbf{k}, \nu)$$

Multiplication is computationally *faster* than convolution, so this perspective is used henceforth.

3.7

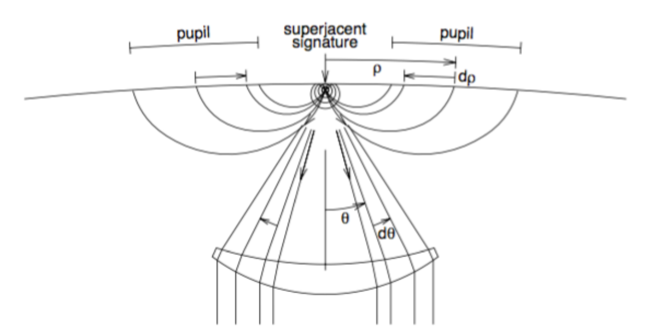
Start getting aberrations; some are easily corrected (e.g. spherical aberration, distortion, and curvature of field). Some however, are not (coma, primary astigmatism, and higher order aberrations) for large pupils that are needed to form deep focal planes (deep sources), or for imaging the far side of the sun. In this case, the aforementationed wavenumber perspective cannot be used.

3.8

$$\check{H}_{+}(\mathbf{r}, z, \nu) = \int\limits_{a<|\mathbf{r}-\mathbf{r}'|<b} \mathrm{d}^2 r' \check{G}_{+}(|\mathbf{r}-\mathbf{r}'|, z, \nu) \check{\psi}(\mathbf{r}', \nu)$$

Part 4: Subjacent Vantage Holography

Figure 4



4.1

Previously: "*Super*jacent vantage Holography". *Sub*jacent vantage holography is when the inner radius, a, of the pupil annulus is much greater than the depth of the focal plane (where the source is). This usually applies to quiet sun areas, whereas the superjacent vantage applies to active regions.

4.2

In contrast to normal optics, the diffraction limit is set by how compact the inner radius is.

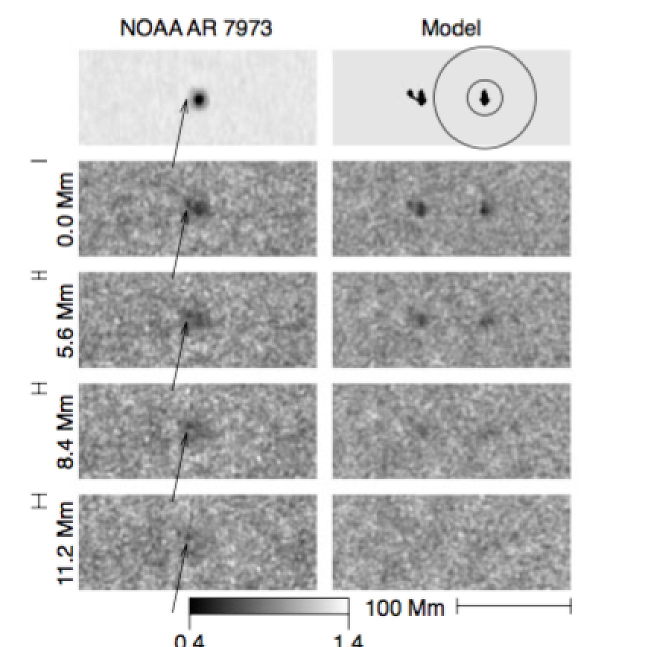
4.3

(Explains which panels in figure 3 are sub or superjacent)

Part 5: An Example

5.1

Figure 5



5.2

5.3

5.4

5.5

Fundamental limitation: As sound speed increases with depth, wavelength *increases*, which results in a coarser diffraction limit at any frequency.

Part 6: Acoustic Modeling Based on Holographic Images

6.1

Applying phase-sensitive holography to models: Flexible procedures, such as inversions, would characterize the acoustic environment in pysical terms such as:

- acoustic emissivity
- acoustic opacity
- refractivity
- flow velocity

6.2

$$\langle |H_+(\mathbf{r}, z)|^2 \rangle = \int \mathrm{d}^2\mathbf{r}' \int \mathrm{d}z' g^{-1}(|\mathbf{r} - \mathbf{r}'|, z, z') S(\mathbf{r}', z')$$

Part 7: Phase-Sensitive Holography

7.1

7.2

The need for phase-sensitive holography is two-fold:

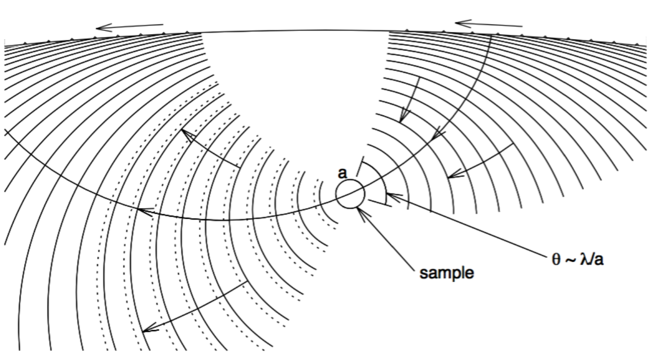
1. straight-forward quantitative probe of refractive anomalies that we expect from thermal perturbations
2. ?

7.3

Visualize phase-sensitive holography in terms of a *gedanken experiment*.

- No phase-shift
- Phase-shift:
  - $\Delta n = \Delta c/c \rightarrow$ refractive perturbation
  - $\Delta t \sim a\Delta n/c \rightarrow$ time delay
  - $\Delta\phi \sim 2\pi\nu a\Delta n/c \rightarrow$ phase shift

Figure 6



Part 8: Green's Functions

8.1

Green's function:

$$G_{\pm}(|\mathbf{r}-\mathbf{r}'|, z, t-t')$$

Characterizes the acoustics of the solar *model* to which helioseismic observations, $\psi(\mathbf{r}', t')$, are applied to accomplish acoustic regressions.

Computational is a broad, flexible diagnostic, not intended for any one particular model.

Outlining intuitive concepts used to fashion Green's Functions appropriate for practical diagnostic applications.

8.2

Acoustic formalism:

- Field, ψ is normalized wrt energy flux
- Solar inteior acoustics (in the absense of sources and sinks) is *time-reversal invariant*

Given these two conditions, the same Green's functions characterize the propagation both forward and backward in time between surface $(\mathbf{r}', 0, t')$ and source $(\mathbf{r}, z, t)$

8.3: Dispersionless acoustics

Pulse propagates in the form of a *wavefront*. Surface location, $\mathbf{r}'$ responds with ripple characterized by the same infinitely sharp temporal profile as the source, but properly attenuated. The Green's function is invarient with respect to both time and horizontal translation.

$$G_+(|\mathbf{r}-\mathbf{r}'|, z, t-t') = \delta\left(t - t' - T\left(|\mathbf{r}-\mathbf{r}'|, z\right)\right) f\left(|\mathbf{r}-\mathbf{r}'|, z\right)$$

8.4

8.5

T (travel time) and f (amplitude of pulse) both depend strongly on the sound speed variation with depth. Derive Γ (optimal optical path) that satisfies Fermat's principle.

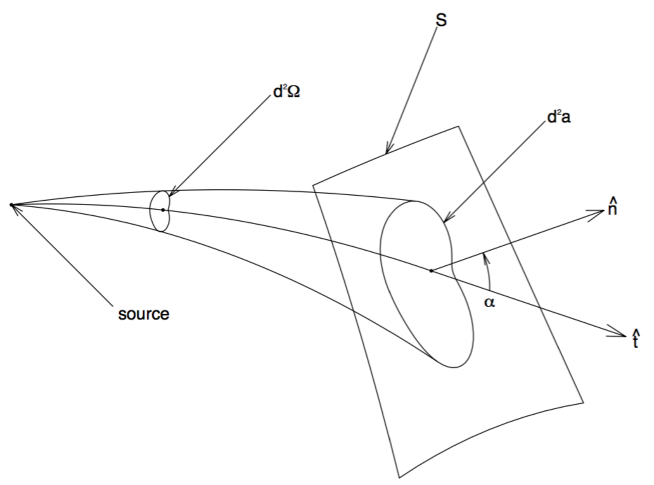
Obtain T!

8.6

Obtain f; acoustic flux must be in proportion to solid angle subtended by the optical paths leading to the boundary of the surface element.

Acoustic energy flux density: cf^2

Figure 7



8.7

- $\nu > 5.5$ mHz absorbed by photosphere upon first encounter after leaving source.
- $\nu < 4.5$ mHz reflected from the specular ("mirror-like"). In this case the Green's function is characterized by a sum of n components, where each n is a "skip", or reflection from the photosphere (include diagram)

8.8

Multiple-skip holography: Two branches

1. ρ decreases as θ increases
2. ρ increases after reaching a minimum as θ continues to increase toward $180°$.

Figure 8

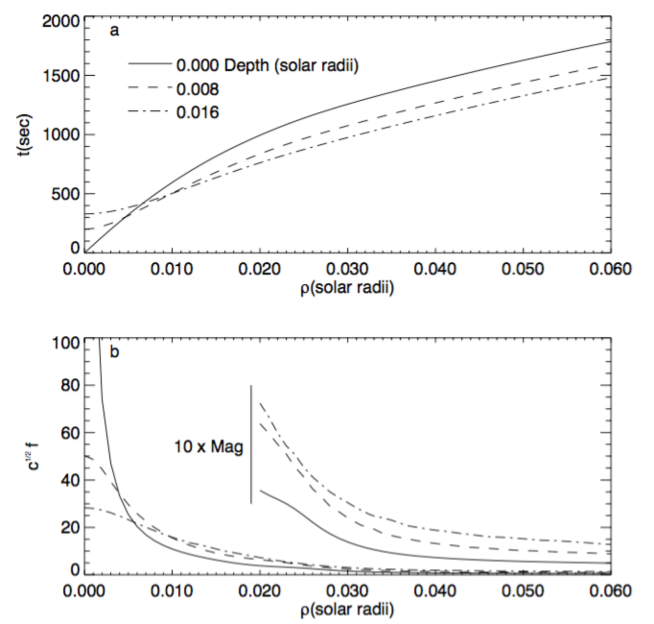
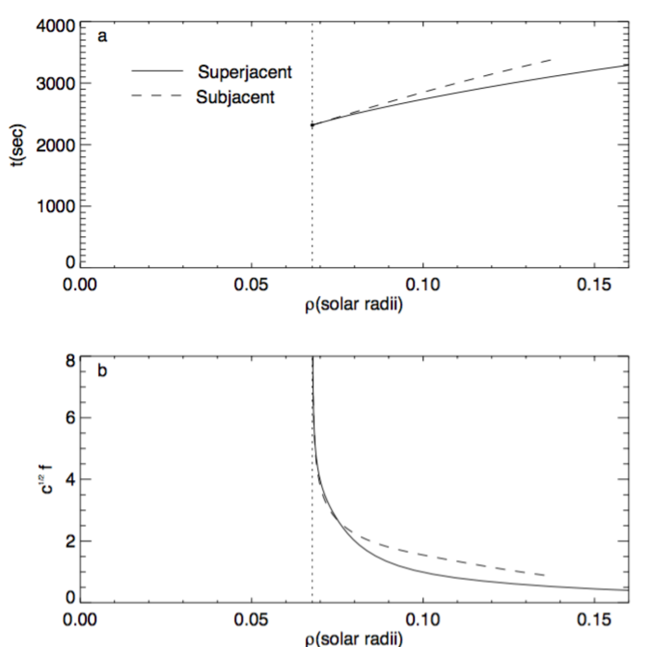

a
0.000 Depth (solar radii)
0.008
0.016
t(sec)
2000
1500
1000
500
0
0.000
0.010
0.020
0.030
0.040
0.050
0.060
ρ(solar radii)
b
10 x Mag
$c^{1/2} f$
100
80
60
40
20
0
0.000
0.010
0.020
0.030
0.040
0.050
0.060
ρ(solar radii)

Figure 9



a
Superjacent
Subjacent
t(sec)
4000
3000
2000
1000
0
0.00
0.05
0.10
0.15
ρ(solar radii)
b
$c^{1/2}$ f
8
6
4
2
0
0.00
0.05
0.10
0.15
ρ(solar radii)

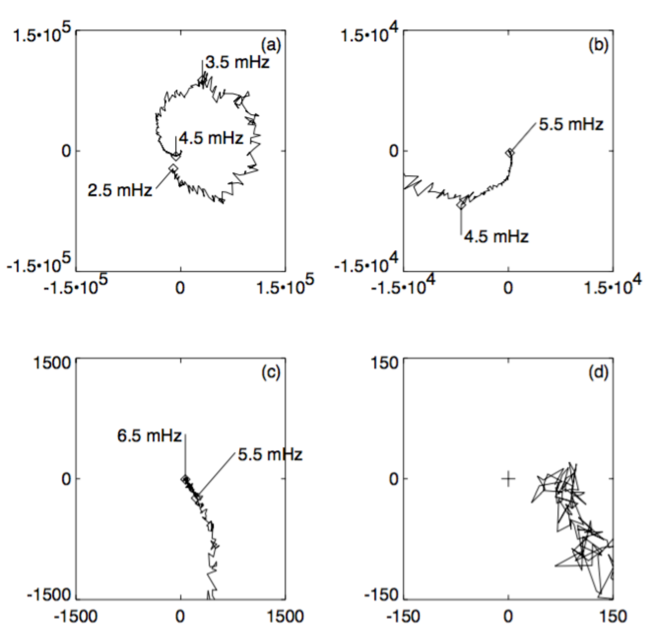
8.10: Dispersion

In reality, acoustic waves are *significantly* dispersed near the photosphere.

8.9

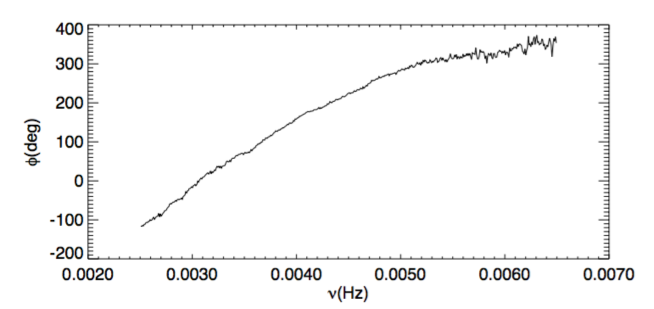
8.11

Figure 10



(a)
1.5•10^5
0
-1.5•10^5
-1.5•10^5
0
1.5•10^5
3.5 mHz
4.5 mHz
2.5 mHz
(b)
1.5•10^4
0
-1.5•10^4
-1.5•10^4
0
1.5•10^4
5.5 mHz
4.5 mHz
(c)
1500
0
-1500
-1500
0
1500
6.5 mHz
5.5 mHz
(d)
150
0
-150
-150
0
150

Figure 11


400
300
200
100
0
-100
-200
φ(deg)
0.0020
0.0030
0.0040
0.0050
0.0060
0.0070
ν(Hz)

8.12

8.13

8.14

Part 9: Summary