Basic Principles of Solar Acoustic Holography

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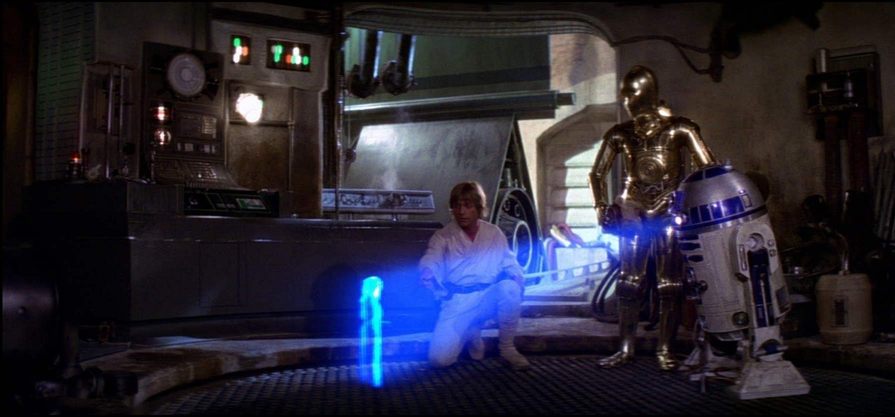


Heliospheric Holography

What is it?

- Computational seismic holography is accomplished by the *phase-coherent* wave-mechanical reconstruction of the *p*-mode acoustic field into the solar interior based on helioseismic observations at the solar surface
  1. Observe disturbance on surface
  2. Reverse time to identify source

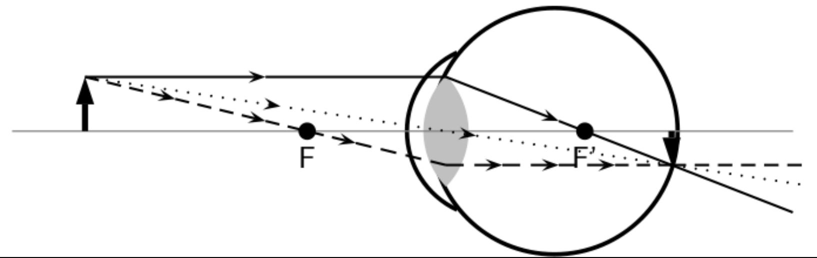
Why use it?

- Space-weather forecasting:
  predict flares before they face Earth!

Comparison with optics

- Submerged sources in sun $\sim$ things we see
- Photosphere $\sim$ surface of cornea (front of eye)

Both involve refocusing radiation to render stigmatic images that can be sampled over focal surface at any desired depth.



Historical Note

- Concept proposed in 1975 by Roddier
- Developed over the 1990s by Lindsey and Braun
  → Key to locating and examining fine structure in the interior and far side of the sun.

Seismic holography was first applied to helioseismic data from the SOlar Heliospheric Observatory (SOHO).
"New" (1998 - 1999) solar acoustic phenomena:

- 'acoustic moats' surrounding sunspots
- 'acoustic condensations' 1020 Mm beneath active regions
- 'acoustic glories' surrounding complex active regions
- first helioseismic images of a flare

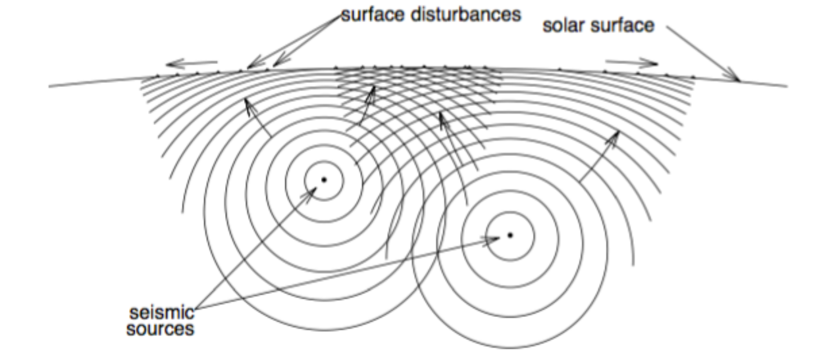
“Heismic holography should *not* be indentified as a method for physical *modelling* of solar interior structure.”

“Basic Principles of Solar Acoustic Holography”

C. Lindsey and D. C. Braun (2000)

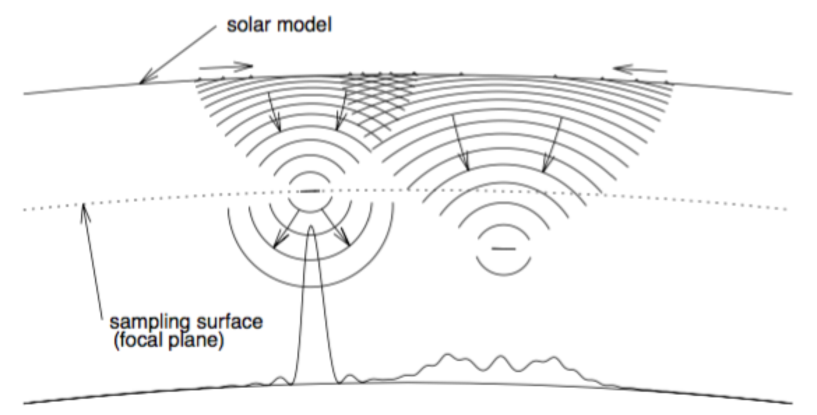
- Compare simple acoustic-power to phase-sensitive
- Propose “simple computational principles” to produce images from high quality helioseismic observations.

Step 1: Helioseismic observations



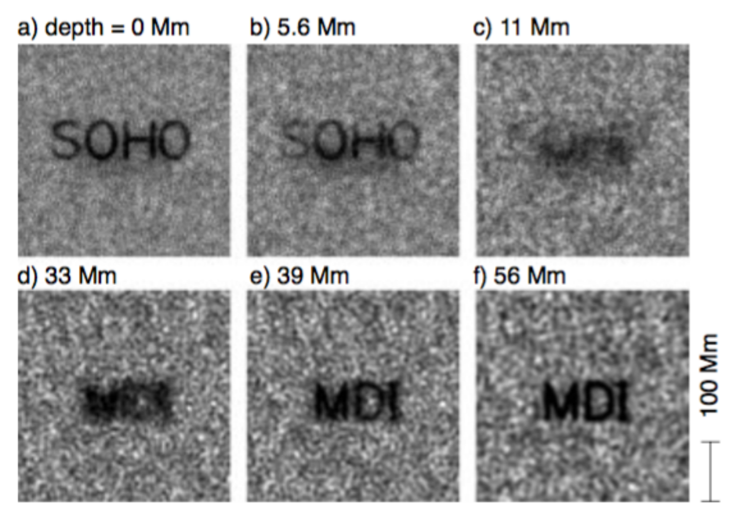
- Acoustic waves are produced in the interior and travel *upward*.
- Pattern of ripples appears on surface directly *above* the sources (what we observe).
- The waves are *absorbed* upon reaching the surface.

Step 2: Apply observations to a model



- Disturbance now propagating back down.
- "Focal plane" lies in the interior, at a depth z_{plane}.
  - $z_{\text{plane}} = z_{\text{source}}$: diffraction-limited signature
  - $z_{\text{plane}} \neq z_{\text{source}}$: unfocused, diffuse profile

3-D perspective



- $z_{\mathrm{SOHO}} = 0$ Mm
- $z_{\mathrm{MDI}} = 56$ Mm ($\sim \frac{1}{10}$ $\mathrm{R}_{\odot}$).

- (b) & (c) acoustic *stalactites*
- (d) & (e) acoustic *stalagmites*

Seismic holography is *not* in any sense a representation or approximation of solar acoustics in terms of ray optics.

The Computational Task

Two perspectives:

1. "spectral"
   - wavenumber-frequency (k, ν)
   - (computationally advantageous)
2. "time distance"
   - space-time (x, t)
   - (more intuitive)

Can extrapolate the (incomplete) acoustic field in the interior.

The space-time perspective

Acoustic *egression*

- $\psi(\mathbf{r}', t')$: *Actual* acoustic field
- $H_+(\mathbf{r}, z, t)$: *incomplete regression* of the acoustic field
- $(\mathbf{r}', 0, t') \longrightarrow$ surface
- $(\mathbf{r}, z, t) \longrightarrow$ focal point

$$H_+(\mathbf{r}, z, t) = \int \mathrm{d}t' \int_{a<|\mathbf{r}-\mathbf{r}'|<b} \mathrm{d}^2 r' G_+(|\mathbf{r}-\mathbf{r}'|, z, t-t')\psi(\mathbf{r}', t')$$

$$H_+ = G_+ * \psi'$$

$G_+ \longrightarrow$ Green's function that expresses how a single transient point disturbance propagates forward or backward in time between $(\mathbf{r}', 0, t')$ and $(\mathbf{r}, z, t)$.

The space-time perspective

Acoustic *ingression*

H_-: "acoustic ingression", the time reversal of H_+; waves converge into the focal point and contribute to the disturbance.

Green's function:

$$G_-(|\mathbf{r}-\mathbf{r}'|, z, t-t') = G_+(|\mathbf{r}-\mathbf{r}'|, z, t'-t)$$

The wavenumber-frequency perspective

- $\hat{\psi}(\mathbf{k}, \nu)$: Fourier transform of $\psi(\mathbf{r}, t)$
- $\hat{G}_+(|\mathbf{k}|, z, \nu)$: Fourier transform of $G_+(|\mathbf{r}|, z, t)$

From the convolution theorem:

$$\hat{H}_+(\mathbf{k}, z, \nu) = \hat{G}_+(|\mathbf{k}|, z, \nu)\hat{\psi}(\mathbf{k}, \nu)$$

$$\hat{H}_+ = \hat{G}_+ \times \hat{\psi}'$$

Multiplication is computationally *faster* than convolution, so this is the preferred method.

Temporal Fourier Transform

Need large pupils to image deeper focal planes, but this produces coma, primary astigmatism, and higher-order aberrations.

$$(\mathbf{r}, t) \rightarrow (\mathbf{r}, \nu)$$

$$\check{H}_+(\mathbf{r}, z, \nu) = \int_{a<|\mathbf{r}-\mathbf{r}'|<b} \mathrm{d}^2 r' \check{G}_+(|\mathbf{r}-\mathbf{r}'|, z, \nu)\check{\psi}(\mathbf{r}', \nu)$$

$$\check{H}_+ = \check{G}_+ * \check{\psi}'$$

Subjacent vs. Superjacent Vantage Holography

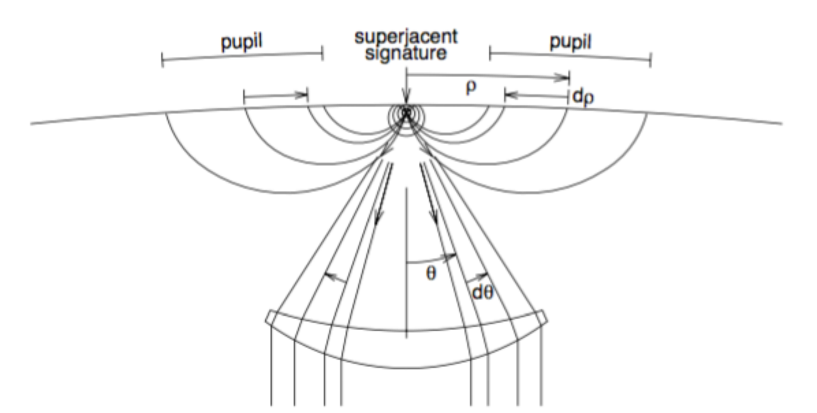
Superjacent:

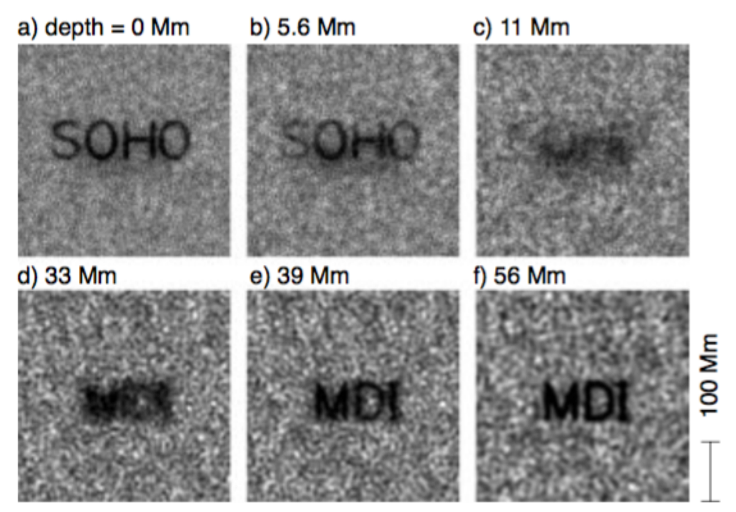
- Wave propagated directly *upward* from the source to surface
- applies to active regions.

Subjacent:

- inner radius, a, of the pupil annulus is much greater than the depth of the focal plane;
- applies to quiet sun (often the practical choice)

Subjacent Vantage Holography





- $a = 15$ Mm, $b = 45$ Mm
- (a) & (b): superjacent
- (c) mixed perspective
- (d), (e), & (f): predominantly subjacent

Egression power maps
Lindsey and Braun (1998)

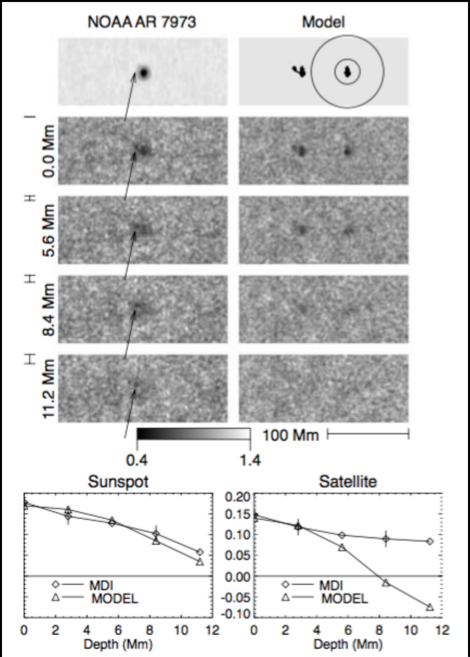
Top panels

- $\nu = 6$ mHz ($\Delta\nu = 1$ mHz)
- model annulus around "Rorschach" splotches

Bottom panels

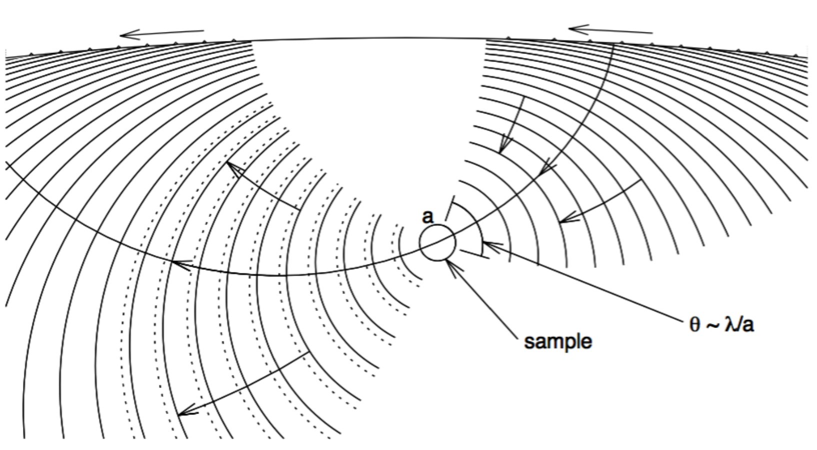
- Contrasts between object and surroundings

All subjacent vantage!



Phase-Sensitive Holography

A *gedanken* experiment



- Produce waves that travel from right to left, into the refractive sample.
- a is the characteristic diameter of the sample.

Phase-Sensitive Holography

A *gedanken* experiment

- $\Delta n = 0 \longrightarrow$ No phase-shift
- $\Delta n \neq 0 \longrightarrow$ Phase-shift
  - $\Delta n = \Delta c_s / c_s \longrightarrow$ refractive perturbation
  - $\Delta t \sim a \Delta n / c \longrightarrow$ time delay
  - $\Delta \phi \sim 2\pi\nu a \Delta n / c \longrightarrow$ phase shift

To relate $\Delta\phi$ to H_+ and H_-, define the *temporal* Fourier transforms

- $\check{H}_+(\mathbf{r}, z, \nu) \leftrightarrow H_+(\mathbf{r}, z, t)$
- $\check{H}_-(\mathbf{r}, z, \nu) \leftrightarrow H_-(\mathbf{r}, z, t)$.

Then

$$\Delta\phi = \arg\left(\left\langle \check{H}_+(\mathbf{r}, z, \nu) \check{H}_-^*(\mathbf{r}, z, \nu) \right\rangle_{\Delta\nu}\right)$$

Green's Functions

$$G_{\pm}(|\mathbf{r}-\mathbf{r}'|, z, t-t')$$

Characterizes the propagation of acoustic disturbances between a surface point, $(\mathbf{r}', 0, t')$ and source point, $(\mathbf{r}, z, t)$

These propagations take place in the solar *model* to which helioseismic observations, $\psi(\mathbf{r}', t')$. are applied.

Computational seismic holography is intended as a braod and flexible diagnostic generality, not to be confined to any particular model.

Dispersionless acoustics

- Pulse propagates in the form of an infintely thin *wavefront*.
- $\mathbf{r}'$ responds with ripple characterized by the same infinitely sharp temporal profile as the source.
- The Green's function is invarient with respect to both time and horizontal translation.

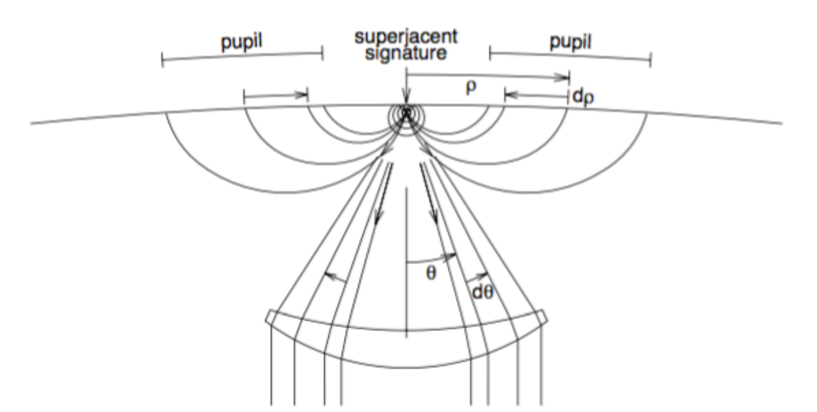
$$G_+(|\mathbf{r}-\mathbf{r}'|, z, t-t') = \delta\left(t - t' - T\left(|\mathbf{r}-\mathbf{r}'|, z\right)\right) f\left(|\mathbf{r}-\mathbf{r}'|, z\right)$$

- T: travel time
- f: amplitude of pulse

Dispersionless acoustics
Reflected waves

- $\nu > 5.5$ mHz absorbed by photosphere.
- $\nu < 4.5$ mHz reflected from photosphere.
  Green's function is characterized by a sum of n components, where each n is a "skip", or reflection from the photosphere.

1. subjacent: ρ decreases as θ increases
2. superjacent: ρ increases after reaching a minimum as θ continues to increase toward $180°$.

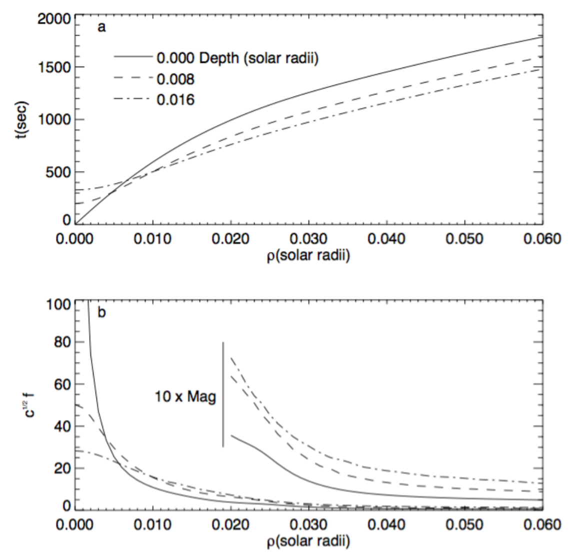


pupil
superjacent
signature
pupil
ρ
dρ
θ
dθ

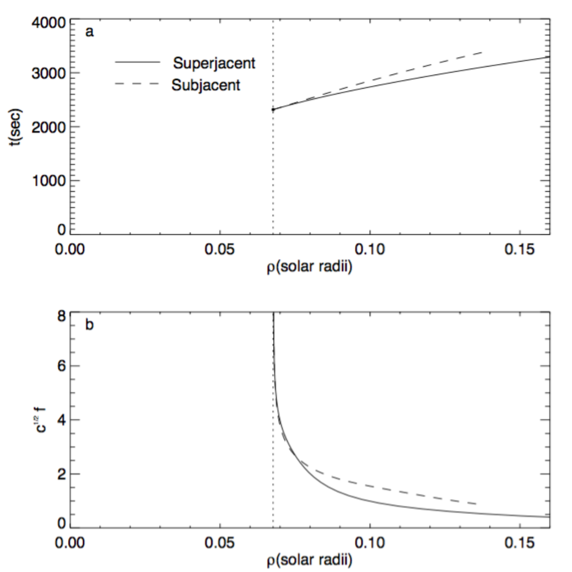
Single-skip holography

T (travel time [s])

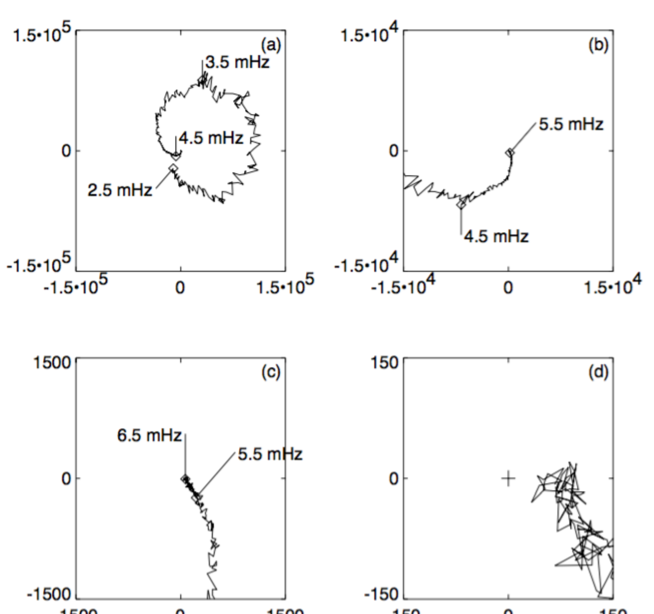
f (amplitude)



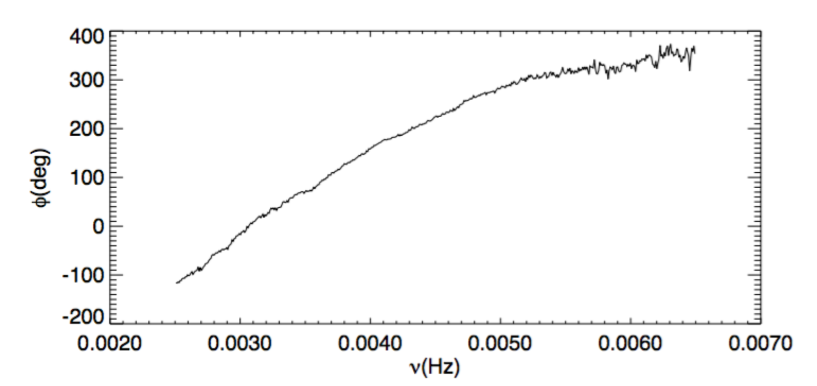
Two-skip holography



Phase correlation between dispersionless egression computation $\check{H}_+$ and surface amplitude $\check{\psi}$



The phase of the correlation



Comparison to other techniques

- time-distance helioseismology (aka. tomography)
- ring diagram analysis