Basic Principles of Solar Acoustic Holography

ASTR 500

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Outline

"Basic Principles of Solar Acoustic Holography"
C. Lindsey and D. C. Braun
2000

The Basic Principle

Define **helioseismic holography** as the *phase-coherent* computational reconstruction of the acoustic field in the solar interior. Information can be extracted from the p-mode power spectrum, and models can be devolped based on these holgraphic signatures.

HOMOGRAPHY $\neq$ MODEL!

In this paper:

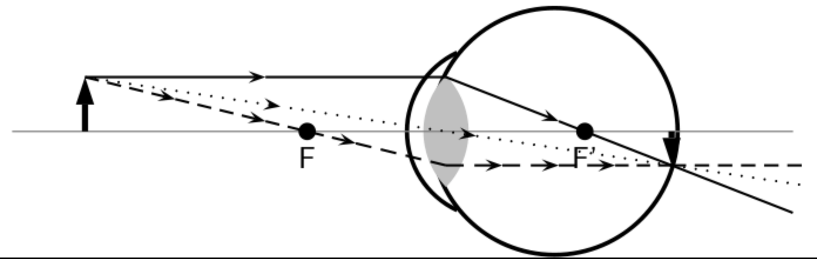
- Compare simple acoustic-power to phase-sensitive
- Propose "simple computational principles" to produce images from high quality helioseismic observations.

Drawing on principles in optics and optical holography

- Submerged sources in sun $\sim$ things we see
- Photosphere $\sim$ surface of cornea (front of eye)

Both involve refocusing radiation to render stigmatic images that can be sampled over focal surface at any desired depth.

Producing *stigmatic images* of the source of the disturbance.



Historical Note

- Concept proposed in 1975 by Roddier
- Developed over the 1990s by Lindsey and Braun (current authors)
- $\rightarrow$ Key to locating and examining fine structure as deep as possible.

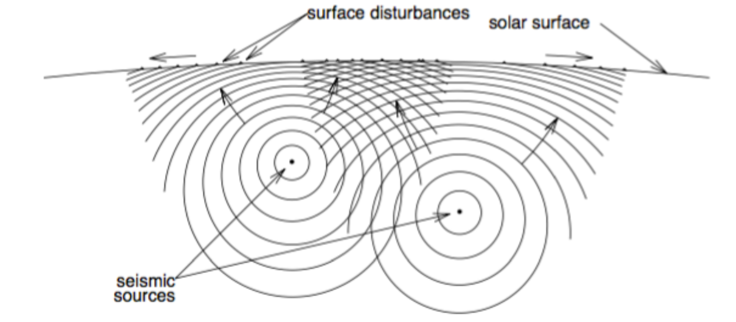
Seismic holography was first applied to helioseismic data from SOHO. "New" (1998-1999) solar acoustic phenomena:

- 'acoustic moats' surrounding sunspots
- 'acoustic condensations' 10-20 Mm beneath active regions
- 'acoustic glories' surrounding complex active regions
- first helioseismic images of a flare

$\rightarrow$ solar cycle dependence of global p-modes! (which is ... ?)

Computational Seismic Holography

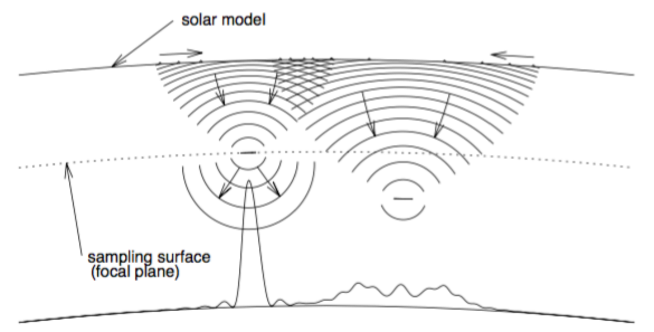
A simple example



- Interior sources of acoustic waves
- Pattern of ripples produced on surface directly *above* the sources.
- The waves are *absorbed* upon reaching the surface, not reflected back down or propagated into atmosphere. (accurate for $\nu > \sim 5.5$ mHz)

Go back in time!

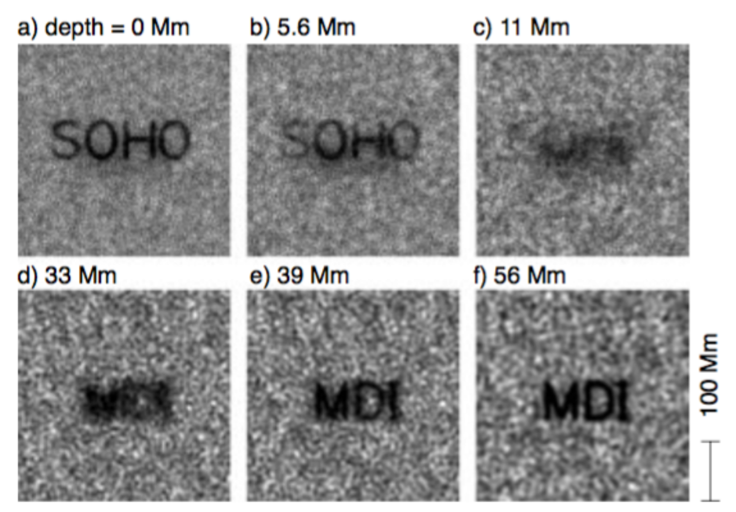
Apply observations to a model that does *not* have any sources (or sinks, or scatters) → this region is the “pupil.”



Consider a “focal plane” in the interior, at a depth z_{plane}.

- $z_{\text{plane}} = z_{\text{source}}$: diffraction-limited signature
- $z_{\text{plane}} \neq z_{\text{source}}$: unfocused, diffuse profile

Absorbers confined to infinitely thin sheets



- $z_{\mathrm{SOHO}} = 0$ Mm
- $z_{\mathrm{MDI}} = 56$ Mm ($\sim \frac{1}{10}$ $\mathrm{R}_\odot$).

- c) acoustic *stalactite*
- d) acoustic *stalagmite*

Words to accompany previous slide

- Simulation: random acoustic noise in model
- Absorbers confined to infinitely then sheets
- depth diagnostics accomplished by focusing and de-focusing, rather than the appearance or disappearance that would be used in realistic physical models.

The Computational Task

Two perspectives:

1. "spectral": wavenumber-frequency
2. "time distance": space-time (more intuitive)

Given

- acoustic amplitude
- derivative normal to surface surrounding medium that is free of sources, sinks, or scatterers

can extrapolate the (incomplete) acoustic field in the interior.

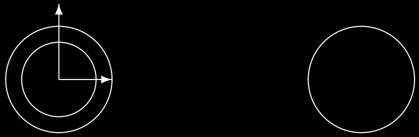
The space-time perspective

- $\psi(\mathbf{r}', t')$: Actual acoustic field at time t' and horizontal location $\mathbf{r}'$.
- H: incomplete regression of the acoustic field
  - $H_+(\mathbf{r}, z, t)$: "acoustic egression" disturbance that emanated from the "focal point" at $(\mathbf{r}, z, t)$ and emerged at the overlying surface at $(\mathbf{r}', 0, t')$.
  - H_-: "acoustic ingression", the time reversal of H_+; waves converge into the focal point and contribute to the disturbance.

$$H_+(\mathbf{r}, z, t) = \int \mathrm{d}t' \int_{a<|\mathbf{r}-\mathbf{r}'|<b} \mathrm{d}^2 r' G_+(|\mathbf{r}-\mathbf{r}'|, z, t-t')\psi(\mathbf{r}', t')$$

G_+ - Green's function that expresses how a single transient point disturbance propagates forward or backward in time between $(\mathbf{r}', 0, t')$ and $(\mathbf{r}, z, t)$. For ingression, replace G_+ by its time reverse:

$$G_-(|\mathbf{r}-\mathbf{r}'|, z, t-t') = G_+(|\mathbf{r}-\mathbf{r}'|, z, t'-t)$$



3.5

After computing H_+, square and integrate it to produce an egression power map over the time period in desired range. p-mode absorption in sunspots has already been confirmed this way! Also other “new” features mentioned earlier.

The wavenumber-frequency perspective

- $\hat{\psi}(\mathbf{k}, \nu)$: Fourier transform of $\psi(\mathbf{r}, t)$
- $\hat{G}_+(|\mathbf{k}|, z, \nu)$: Fourier transform of $G_+(|\mathbf{r}|, z, t)$

Fourier transform of the egression:

$$\hat{H}_+(\mathbf{k}, z, \nu) = \hat{G}_+(|\mathbf{k}|, z, \nu), \hat{\psi}(\mathbf{k}, \nu)$$

from the convolution theorem. Multiplication is computationally *faster* than convolution, so this is the preferred method.

Can probably leave this out

Start getting aberrations; some are easily corrected (e.g. spherical aberration, distortion, and curvature of field). Some however, are not (coma, primary astigmatism, and higher order aberrations) for large pupils that are needed to form deep focal planes (deep sources), or for imaging the far side of the sun. In this case, the aforementationed wavenumber perspective cannot be used.

Another type of Fourier Transform

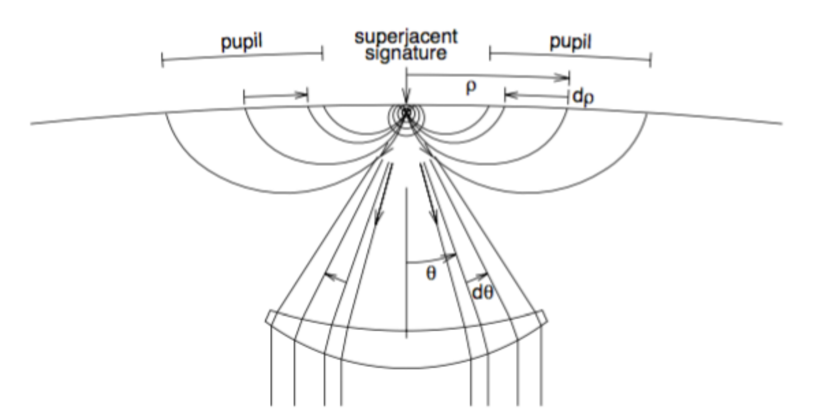
Temporal transform only:

$$\check{H}_+(\mathbf{r}, z, \nu) = \int_{a<|\mathbf{r}-\mathbf{r}'|<b} \mathrm{d}^2 r' \check{G}_+(|\mathbf{r}-\mathbf{r}'|, z, \nu) \check{\psi}(\mathbf{r}', \nu)$$

Subjacent Vantage Holography

Previously: "*Super*jacent vantage Holography". *Sub*jacent vantage holography is when the inner radius, a, of the pupil annulus is much greater than the depth of the focal plane (where the source is). This usually applies to quiet sun areas, whereas the superjacent vantage applies to active regions.

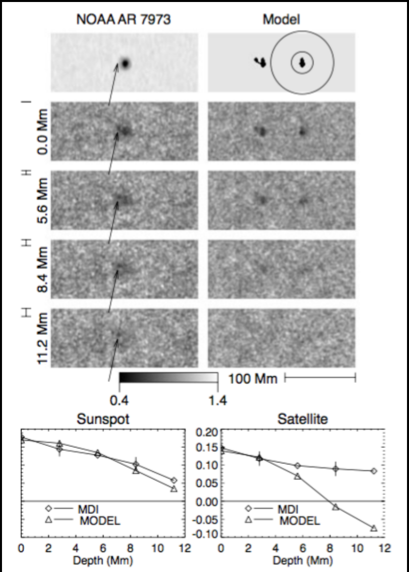
Subjacent Vantage Holography



Needs work

4.2 In contrast to normal optics, the diffraction limit is set by how compact the inner radius is. Get finest diffraction limit by securing waves with the highest spherical harmonic degree, l.
4.3 (Explains which panels in figure 3 are sub or superjacent)
Need to re-order this.

Egression power maps



stuff

Fundamental limitation

As sound speed increases with depth, wavelength *increases*, which results in a coarser diffraction limit at any frequency.

Acoustic Modeling Based on Holographic Images

Applying phase-sensitive holography to models: Flexible procedures, such as inversions, would characterize the acoustic environment in pysical terms such as:

- acoustic emissivity
- acoustic opacity
- refractivity
- flow velocity

Inversions

$$\langle |H_+(\mathbf{r}, z)|^2 \rangle = \int \mathrm{d}^2\mathbf{r}' \int \mathrm{d}z' g^{-1}(|\mathbf{r} - \mathbf{r}'|, z, z') S(\mathbf{r}', z')$$

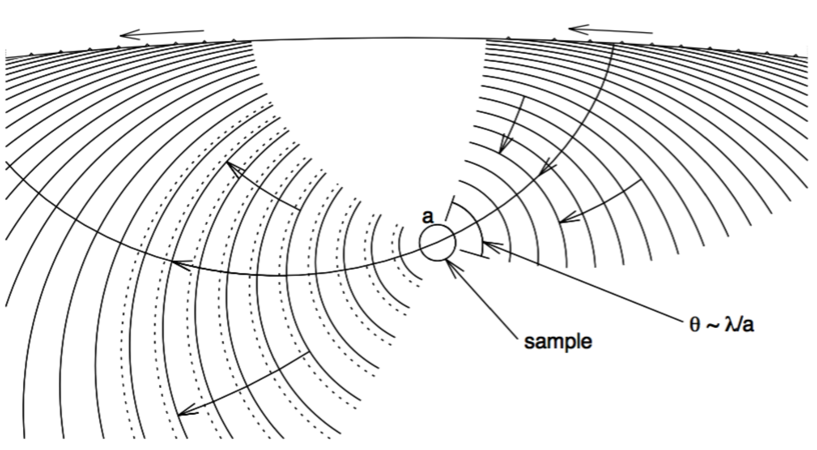
$$S(\mathbf{r}, z) = \int \mathrm{d}^2\mathbf{r}' \int \mathrm{d}z' g^{-1}(|\mathbf{r} - \mathbf{r}'|, z, z') \langle |H_+(\mathbf{r}', z')|^2 \rangle$$

Phase-Sensitive Holography

Purpose is to incorporate the basic utiilities of optical interferometry into solar interior diagnostics. The need for phase-sensitive holography is two-fold:

1. straight-forward quantitative probe of refractive anomalies that we expect from thermal perturbations
2. Something else

Visualize in terms of a *gedanken experiment* (aka “thought experiment”).



- Produce waves that travel from right to left, into the refractive sample.

- No phase-shift (refractive index of sample is similar to that of the surrounding medium).
- Phase-shift:
  - $\Delta n = \Delta c_s / c_s \rightarrow$ refractive perturbation
  - $\Delta t \sim a \Delta n / c \rightarrow$ time delay (a - characteristic diameter of the sample)
  - $\Delta \phi \sim 2\pi \nu a \Delta n / c \rightarrow$ phase shift

To relate $\Delta\phi$ to H_+ and H_-, define the *temporal* Fourier transforms, $\check{H}_+(\mathbf{r}, z, \nu)$ and $\check{H}_-(\mathbf{r}, z, \nu)$ of $H_+(\mathbf{r}, z, t)$ and $H_-(\mathbf{r}, z, t)$. Then

$$\Delta\phi = \arg\ldots$$

Green's Functions

$$G_{\pm}(|\mathbf{r} - \mathbf{r}'|, z, t - t')$$

Characterizes the acoustics of the solar *model* to which helioseismic observations, $\psi(\mathbf{r}', t')$, are applied to accomplish acoustic regressions.

Computational is a broad, flexible diagnostic, not intended for any one particular model.

Outlining intuitive concepts used to fashion Green's Functions appropriate for practical diagnostic applications.

8.2

Acoustic formalism:

- Field, ψ is normalized wrt energy flux
- Solar inteior acoustics (in the absense of sources and sinks) is *time-reversal invariant*

Given these two conditions, the same Green's functions characterize the propagation both forward and backward in time between surface $(\mathbf{r}', 0, t')$ and source $(\mathbf{r}, z, t)$

8.3: Dispersionless acoustics

Pulse propagates in the form of a *wavefront*. Surface location, $\mathbf{r}'$ responds with ripple characterized by the same infinitely sharp temporal profile as the source, but properly attenuated. The Green's function is invarient with respect to both time and horizontal translation.

$$G_+(|\mathbf{r}-\mathbf{r}'|, z, t-t') = \delta\left(t-t'-T\left(|\mathbf{r}-\mathbf{r}'|, z\right)\right) f\left(|\mathbf{r}-\mathbf{r}'|, z\right)$$

8.4

8.5

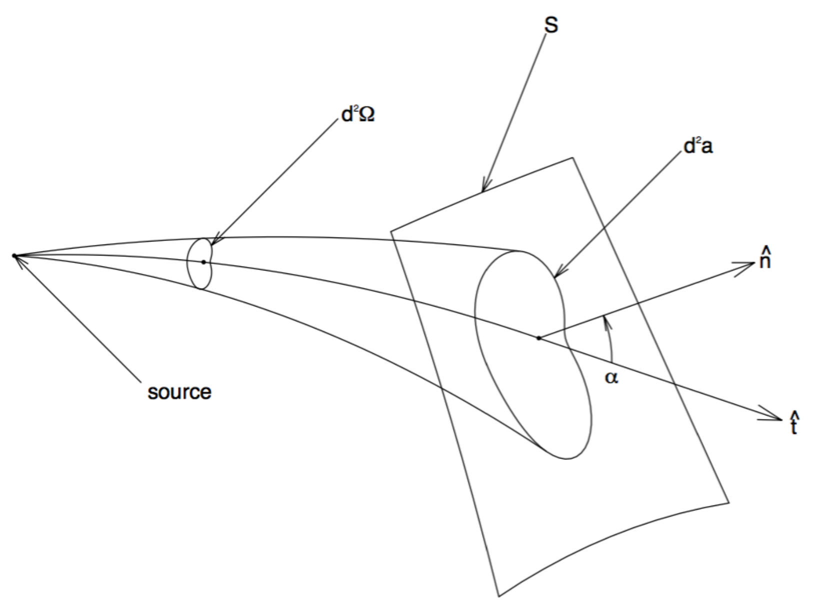
T (travel time) and f (amplitude of pulse) both depend strongly on the sound speed variation with depth. Derive Γ (optimal optical path) that satisfies Fermat's principle.

Obtain T!

8.6

Obtain f; acoustic flux must be in proportion to solid angle subtended by the optical paths leading to the boundary of the surface element.

Acoustic energy flux density: cf^2


S
d²Ω
d²a
n̂
source
α
t̂

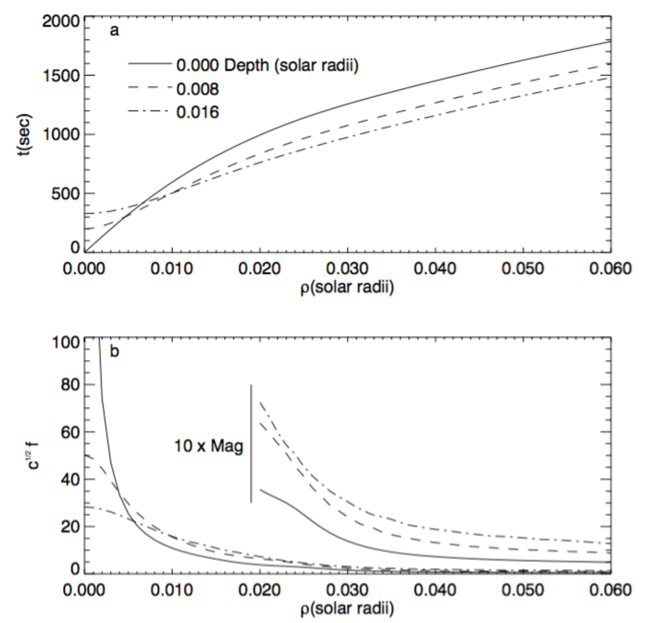
8.7

- $\nu > 5.5$ mHz absorbed by photosphere upon first encounter after leaving source.
- $\nu < 4.5$ mHz reflected from the specular (“mirror-like”). In this case the Green’s function is characterized by a sum of n components, where each n is a “skip”, or reflection from the photosphere (include diagram)

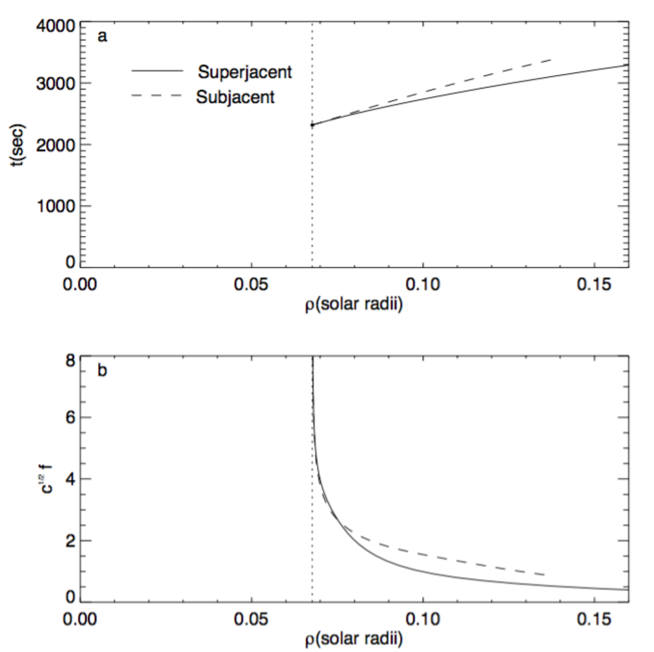
8.8

Multiple-skip holography: Two branches

1. ρ decreases as θ increases
2. ρ increases after reaching a minimum as θ continues to increase toward $180°$.



a
0.000 Depth (solar radii)
0.008
0.016
t(sec)
2000
1500
1000
500
0
0.000
0.010
0.020
0.030
0.040
0.050
0.060
ρ(solar radii)
b
10 x Mag
$c^{1/2} f$
100
80
60
40
20
0
0.000
0.010
0.020
0.030
0.040
0.050
0.060
ρ(solar radii)

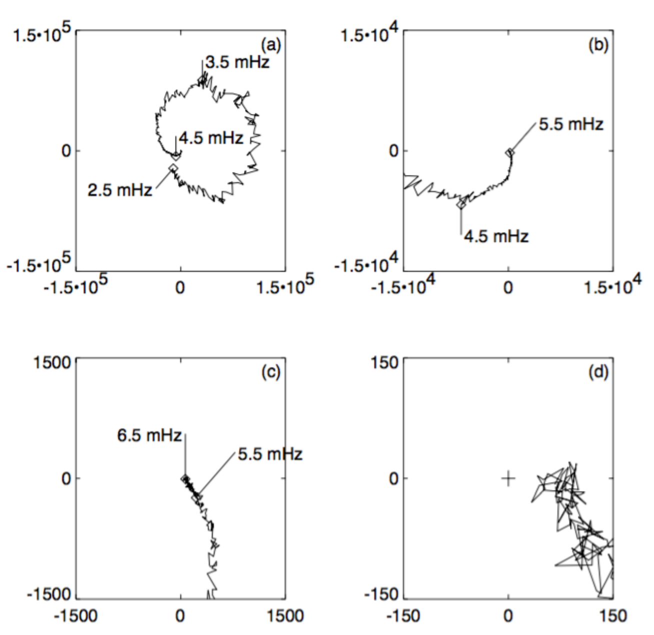

a
Superjacent
Subjacent
t(sec)
4000
3000
2000
1000
0
0.00
0.05
0.10
0.15
ρ(solar radii)
b
c^{1/2} f
8
6
4
2
0
0.00
0.05
0.10
0.15
ρ(solar radii)

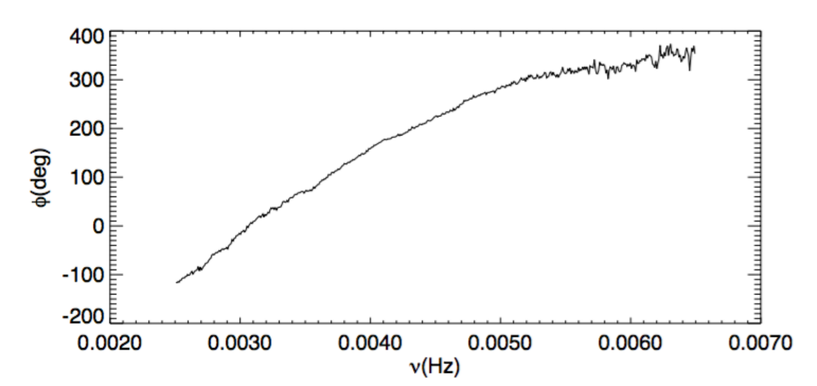
Dispersion

In reality, acoustic waves are *significantly* dispersed near the photosphere.

8.9

8.11


(a)
3.5 mHz
4.5 mHz
2.5 mHz
1.5•10^5
0
-1.5•10^5
(b)
5.5 mHz
4.5 mHz
1.5•10^4
0
-1.5•10^4
(c)
6.5 mHz
5.5 mHz
1500
0
-1500
(d)
150
0
-150


400
300
200
100
0
-100
-200
φ(deg)
0.0020
0.0030
0.0040
0.0050
0.0060
0.0070
ν(Hz)

8.12

8.13

8.14

Summary