Homework 8

(1/29, 30 minutes)A color of a star gives the ratio of fluxes in two different wavelength bandpasses. When expressed in magnitude units, colors are given by a difference in magnitude between two different wavelength bandpasses and are often defined relative to the colors of some other objects. For example, in the VEGAMAG (e.g., UBVRI) system, all colors are relative to the color of the star Vega, i.e. in this system, the color of Vega is defined to be zero. The spectrum (F_λ) of Vega is (very!) roughly proportional to λ^{-2} (in the optical), i.e. F_λ(Vega) $\approx$ 3.6$\times 10^{-9} \left(\frac{\lambda}{5500}\right)^{-2}$erg cm^{-2} s^{-1} Å^{-1}.

1 If you approximate the B bandpass as a square bandpass between 4000 and 5000 Å, and the V bandpass as a square bandpass between 5000 and 6000 Å, what is the flux of Vega in both B and V

$$F = \int F_\lambda \mathrm{d}\lambda$$
$$F_B = \int_{4000}^{5000} 3.6 \times 10^{-9} \left(\frac{\lambda}{5500}\right)^{-2} \lambda \mathrm{d}\lambda =\sim 2.43 \times 10^{-18}$$
$$F_V = \int_{5000}^{6000} 3.6 \times 10^{-9} \left(\frac{\lambda}{5500}\right)^{-2} \lambda \mathrm{d}\lambda =\sim 1.99 \times 10^{-18}$$

so F_B and F_V are about 2.43×10^{-18} and 1.99×10^{-18} erg s^{-1} cm^{-2}, respectively.

2 If you observe a star that has B-V = 1 and V = 20, what is its flux in the B and V bandpasses?

The flux of a star of magnitude m_V can be calculated using the relation:

$$m_V = -2.5 \log \frac{F_V}{F_{V,0}}$$

The flux of Vega ($F_{V,0}$) in the V bandpass is about 3.63×10^{-9} erg s^{-1} cm^{-2} Å^{-1}. For a star with m_V=20:

$$20 = -2.5 \log \frac{F_V}{3.63 \times 10^{-9}}$$
$$F_V = [10^{\frac{20}{-2.5}}] 3.63 \times 10^{-9}$$
$$=\sim 6.63 \times 10^{-17}$$

Since B-V = 1, F_B for the star can be calculated using the relation:

$$\begin{aligned} B - V &= -2.5 \log \frac{F_B}{F_V} \\ 1 &= -2.5 \log \frac{F_B}{6.63 \times 10^{-17}} \\ F_B &= [10^{\frac{1}{-2.5}}]6.63 \times 10^{-17} \\ &=\sim 2.64 \times 10^{-17} \end{aligned}$$

3 What does it mean, quantitatively, for an object to have B-V = 1.0 ?

The object's B-magnitude is higher than its V-magnitude, so it is a cooler (redder) star (relative to an A0V star, like Vega). This difference in magnitudes corresponds to a flux ratio of about 2.51, so the star is about two and a half times brighter through the V filter than it is through the B filter.

4 If a star with a spectral energy distribution like Vega has a magnitude of 21 at 4500 Å in the VEGAMAG system (i.e. spectrum same shape as Vega, but normalized differently), what would its magnitude (m(4500)) be in the STMAG system? the ABNU system? What would the m(4500) - m(5500) color be in each of the three systems?

$$\begin{aligned} m_{\lambda=4500} &= -2.5 \log \frac{F_{\lambda=4500}}{F_{\lambda=4500,0}} \\ 21 &= -2.5 \log \frac{F_{\lambda=4500}}{3.6 \times 10^{-9} \left(\frac{4500}{5500}\right)^{-2}} \\ F_{\lambda=4500} &= 2.14 \times 10^{-17} \end{aligned}$$