ASTR 535 Lecture Notes

Jon Holtzman

Spring 2016

course website: http://astronomy.nmsu.edu/holtz/a535

Friday, January 22

Properties of light, magnitudes, errors, and error analysis

Light

Wavelength regimes:

- gamma rays
- x-rays
- ultraviolent (UV)
  - near: 900–3500 Å
  - far: 100–900 Å

  The 900 Å break is because of the Lyman limit at 912 Å. This is where neutral hydrogen is ionized, so the universe is largely opaque to wavelengths shorter than this.
- visual (V): 4000–7000 Å (note that 'V' is different from 'optical', which is slightly broader: 3500–10000 Å. The 3500 Å cutoff is due to the Earth's atmosphere being opaque to wavelengths shorter than this).
- IR
  - near: 1–5 μ (1–10 μ in online notes)
  - mid: (10–100 μ)
  - far: 5–100 μ (100–1000 μ)
- sub-mm 500–1000 μ
- microwave
- radio

Quantities of light:

- Intensity $I(\theta, \phi)$ [erg s^{-1} Ω^{-1} ν^{-1}] Encapsulates *direction* light is coming from. Also known as radiance.

- Surface Brightness (SB) [erg s^{-1} cm^{-2} ν^{-1} sterradian^{-1}] amount of energy *received* in a unit surface element per unit time per unit frequency (or wavelength) from a unit solid angle in the direction (θ, ϕ), where θ is the angle away from the normal to the surface element, and ϕ is the azimuthal angle. SB is independent of distance unless considering cosmological scales, where the curvature of spacetime has an effect.

- Flux (F): amount of energy passing through a unit surface element in all directions, defined by

$$F_\nu = \int I_\nu \cos(\theta) \mathrm{d}\Omega \tag{1}$$

  where dΩ is the solid angle element, and the integration is over the entire solid angle. The $\cos(\theta)$ factor is important for, e.g., ISM where light is coming from all directions, but for tiny objects, θ is negligibally small and can be dropped. Integrates over *all* directions. Also known as irradiance.

- Luminosity (L): [erg s^{-1}] *intrinsic* energy emitted by the source per second ($\sim$ power). For an isotropically emitting source,

$$L = 4\pi d^2 F \tag{2}$$

  where d = distance to source. Also known as radiant flux.

What to measure for sources:

- Resolved: directly measure surface brightness (intensity) distribution on the sky, usually over some bandpass or wavelength interval.

- Unresolved: measure the flux. Diffraction is the reason stellar surfaces cannot be resolved. Because of this, we cannot measure SB, so we measure flux, integrated over the entire object.

Note that Luminosity can only be calculated if the distance is known.

Questions:

- What are the dimensions of the three quantities: luminosity, surface brightness (intensity), and flux?

- How do the three quantities depend on distance to the source?

- To what quantity is apparent magnitude of a star related?

- To what quantity is the absolute magnitude related?

Amount of light emitted is a function of wavelength, so we are often interested in e.g. flux per unit wavelength or frequency, also known as *specific* flux. Using $\lambda = \frac{c}{\nu} \rightarrow \frac{d\lambda}{d\nu} = \frac{-c}{\nu^2}$

$$\begin{aligned}
\int F_\nu d\nu &= \int F_\lambda d\lambda \\
F_\nu d\nu &= F_\lambda d\lambda \\
F_\nu &= F_\lambda \frac{d\lambda}{d\nu} \\
&= -F_\lambda \frac{c}{\nu^2} \\
&= -F_\lambda \frac{\lambda^2}{c}
\end{aligned}$$

Note that a constant F_λ implies a *non*-constant F_ν and vice versa. Depending on where you are, a constant chunk of 1 Hz is not the same wavelength range, depending on where you are.

Units: often cgs, magnitudes, Jansky (a flux density unit corresponding to 10^{-26} W m^{-2} Hz^{-1})

There are often variations in terminology

Terminology of measurements:

- photometry (broad-band flux measurement) SB or flux, integrated over some wavelength range.
- spectroscopy (relative measurement of fluxes at different wavelengths) $f(\lambda)$
- Spectrophotometry (absolute measuremnet of fluxes at different wavelengths) $f(\lambda)$
- astrometry: concerned with positions of observed flux, not brightness, but direction.
- morphology: intensity as a function of position; often, absolute measurements are unimportant. Deals with *resolved* objects, intensity as function of position.

Generally, measure *flux* with photometry, and flux *density* with spectroscopy (down to the resolution of the spectrograph). In practice, with most detectors, we measure photon flux [photons cm^{-2} s^{-1}] with a photon counting device, rather than energy flux (which is done with bolometers). The monochromatic photon flux is given by the energy flux (F_λ) divided by the energy per photon ($E_{photon} = \frac{hc}{\lambda}$), or

$$\text{photon flux} = \int F_\lambda \frac{\lambda}{hc} d\lambda$$

Magnitudes and photometric systems

Magnitudes are related to flux (and SB and L) by:

$$m_1 - m_2 = -2.5 \log \frac{b_1}{b_2}$$

or for a single object:

$$\begin{aligned} m &= -2.5 \log \frac{F}{F_0} \\ &= -2.5 \log F + 2.5 \log F_0 \end{aligned}$$

where the coefficient of proportionality, F_0, depends on the definition of photometric system; the quantity $-2.5 \log F_0$ may be referred to as the photometric system zeropoint. Note that this relationship holds regardless of what photometric system you are using. Inverting, one gets:

$$F = F_0 \times 10^{-0.4\mathrm{m}}$$

Just as fluxes can be represented in magnitude units, flux densities can be specified by monochromatic magnitudes:

$$F_\lambda = F_0(\lambda) \times 10^{-0.4\mathrm{m}(\lambda)}$$

although spectra are more often given in flux units than in magnitude units. Note that it is possible that F_0 is a function of wavelength.

Since magnitudes are logarithmic, the $difference$ between magnitudes corresponds to a ratio of fluxes; ratios of magnitudes are generally unphysical. If one is just doing relative measurements of brightness between objects, this can be done without knowledge of F_0 (or, equivalently, the system zeropoint); objects that differ in brightness by ΔM have the same ratio of brightness ($10^{-0.4\Delta M}$) regardless of what photometric system they are in. The photometric system definitions and zeropoints are only needed when converting between calibrated magnitudes and fluxes. Note that this means that if one references the brightness of one object relative to that of another, a magnitude system can be set up relative the brightness of the reference source. However, the utility of a system when doing astrophysics generally requires an understanding of the actual fluxes.

Monday, January 25

There are three main types of magnitude systems in use in astronomy. We start by describing the two simpler ones: the STMAG and the ABNU mag system. In these simple systems, the reference flux is just a constant value in F_λ or F_ν. However, these are not always the most widely used systems in astronomy, because no natural source exists with a flat spectrum.

The STMAG (Space Telescope Mag) is relative to a source of constant F_λ. In this system, $F_{0,\lambda} = 3.60 \times 10^{-9}$ erg cm^{-2} s^{-1} Å^{-1}, which is the flux of Vega at 5500 Å; hence a star of Vega's brightness at 5500 Å is defined to have m=0. Alternatively, we can write

$$m_{STMAG} = -2.5 \log F_\lambda - 21.1$$

for F_λ in cgs units.

In the ABNU system, things are defined relative to a source of constant F_ν and we have

$$F_{0,\nu} = 3.63 \times 10^{-20} erg\ s^{-1}\ cm^{-2}\ Hz^{-1}\ 10^{-0.4\mathrm{m}_\nu}$$

or

$$m_{ABNU} = -2.5 \log F_{nu} - 48.6$$

for F_ν in cgs units. Again, the constant comes from the flux of Vega.

Magnitudes usually refer to the flux integrated over a spectral bandpass. In this case, F and F_0 refer to fluxes integrated over the bandpass. The STMAG and ABMAG integrated systems are defined relative to sources of constant F_λ and F_ν systems, respectively.

$$m_{STMAG} = -2.5 \log \frac{\int F_\lambda \lambda \mathrm{d}\lambda}{\int 3.6 \times 10^{-9} \lambda \mathrm{d}\lambda}$$

$$m_{ABNU} = -2.5 \log \frac{\int F_\nu / \nu \mathrm{d}\nu}{\int 3.6 \times 10^{-20} / \nu \mathrm{d}\nu}$$

These should have the same value at 5500 Å. The factors of λ and ν come from the conversion factor hc/λ, where the constants, h and c, cancel out in the fractions in each equation.

Note that these systems differ by more than a constant, because one is defined by units of F_λ and the other by F_ν, so the difference betwen the systems is a function of wavelength. They are defined to be the same at 5500 Å. (Question: what's the relations between m_{STMAG} and m_{ABNU}?)

Note also that, using magnitudes, the measured magnitude is nearly independent of bandpass width, so a broader bandpass does not imply a brighter (smaller) magnitude, which is not the case for fluxes. The reference is being integrated as well, so they cancel.

The standard UBVRI broadband photometric system, as well as several other magnitude systems, however, are not defined for a constant (flat) F_λ or F_ν spectrum; rather, they are defined relative to the spectrum of an A0V star. Most systems are defined (or at least were originally) to have the magnitude of Vega be zero in all bandpasses (VEGAMAGS); if you ever get into this in detail, note that this is not exactly true for the UBVRI system.

For the broadband UBVRI system, we have

$$m_{UBVRI} \approx -2.5 \log \frac{\int_{UBVRI} F_\lambda(object) \lambda \mathrm{d}\lambda}{\int_{UBVRI} F_\lambda(Vega) \lambda \mathrm{d}\lambda}$$

(as above, the factor of λ comes in for photon counting detectors). This gives the magnitude in U, B, V, R, *or* I, by integrating over that same bandpass. The UBVRI filter set had overlapping bandpasses, so there was a switch to interference filter: the ugriz system used by SDSS (explained below... I think).

Here is a plot to demonstrate the difference between the different systems.

While it seems that STMAG and ABNU systems are more straightforward, in practice it is difficult to measure absolute fluxes, and much easier to measure relative fluxes between objects. Hence, historically observations were tied to observations of Vega (or to stars which themselves were tied to Vega), so VEGAMAGs made sense, and the issue of determining physical fluxes boiled down to measuring the physical flux of Vega. Today, in some cases, it may be more accurate to measure the absolute throughput of an instrumental system, and using STMAG or ABNU makes more sense.

Colors

Working in magnitudes, the difference in magnitudes between different bandpasses (called the color index, or simply, color) is related to the flux ratio between the bandpasses, i.e., the color. In the UBVRI system, the *difference between magnitudes*, e.g. B-V, gives the ratio of the fluxes in different bandpasses *relative to the ratio of the fluxes of an A0V star in the different bandpasses (for VEGAMAG)*. Note the typical colors of astronomical objects, which are different for the different photometric systems.
A stars have color 0, have same SED as Vega. O stars have color less than 0, while coolor stars have color greater than 0. In sloan system, have g-r (ugriz). g-r=0 indicates constant F_ν.

$$m = -2.5 \log \frac{\int_B F_\lambda \mathrm{d}\lambda}{\int_V F_\lambda \mathrm{d}\lambda}$$

if B-V=0, then $(\mathrm{B/V})_{object}$ is the same as $(\mathrm{B/V})_{Vega}$.

What would the flux be from an object with some magnitude, x? Need this to know how much observing time I need. E.g., convert the spectrum of an elliptical galaxy to color; if you know F in one filter, you can get F in another filter.
Which is closer to the UBVRI system, STMAG or ABNU?
What would typical colors be in a STMAG or ABNU system?

Wednesday, January 27

UBVRI magnitudes-flux conversion

To convert Vega-based magnitudes to fluxes, look up the flux of Vega at the center of the passband; however, if the spectrum of the object differes from that of Vega, this won't be perfectly accurate. Given UBVRI magnitudes of an object in the desired band, filter profiles (e.g. Bessell 1990, PASP 102,1181), and absolute spectrophotometry of Vega (e.g., Bohlin & Gilliland 2004, AJ 127, 3508, one can determine the flux.

If one wanted to estimate the flux of some object in arbitrary bandpass given just the V magnitude of an object (a common situation used when trying to predict exposures times, see below), this can be done if an estimate of the spectral energy distribution (SED) can be made; given the filter profiles, one can compute the integral of the SED over the V bandpass, determine the scaling by comparing with the integral of the Vega spectrum over the same bandpass, then use the normalized SED to compute the flux in any desired bandpass. Some possibly useful references for SEDs are: Bruzual, Persson, Gunn, & Stryker; Hunter, Christian, & Jacoby; Kurucz).

Things are certainly simpler in the ABNU or STMAG system, and there has been some movement in this direction: the STScI gives STMAG calibrations for HST instruments, and the SDSS photometric system is close to an ABNU system.

Note, however, that even when the systems are conceptually well defined, determining the absolute calibration of any photometric system is very difficult in reality, and determining absolute fluxes to the 1% level is very challenging.

As a separate note on magnitudes themselves, note that some people, in particular, the SDSS, have adopted a modified type of magnitudes, called asinh magnitudes, which behave like normal (also known as Pogson) magnitude for brighter objects, but have different behavior for very faint objects (near the detection threshold); see Lupton, Gunn, & Szalay 1999 AJ 118, 1406 for details.

Observed fluxes and the count equation

What if you are measuring flux with an actual instrument, i.e. counting photons? The intrinsic photon flux from the source is not trivial to determine from the number of photons that you count. To get the number of photons that you count in an observation, you need to take into account the area of your photon collector (telescope), photon losses and gains from the Earth's atmosphere (which changes with conditions), and the efficiency of your collection/detection apparatus (which can change with time). Generally, the astronomical signal (which might be a flux or a surface brightness, depending on whether the object is resolved) can be written

$$S = Tt \int \frac{F_\lambda}{\frac{hc}{\lambda}} q_\lambda a_\lambda \mathrm{d}\lambda \equiv TtS'$$

where S is the observed photon flux (the "signal"), T is the telescope collecting area, t is the integration time, a_λ is the atmospheric transmission (more later) and q_λ is the system efficiency (which includes telescope, filters, optics, detector, etc.); S' is an observed flux rate, i.e. with all of the real details of the observing system included. I refer to this as the *count equation*.

Usually, however, one doesn't use this information to go backward from S to F_λ because it is very difficult to measure all of the terms precisely, and some of them (e.g. a, and perhaps q) are time-variable; a is also spatially variable. Instead, most observations are performed differentially to a set of other stars of known brightness. If the stars of known brightness are observed in the same observation, then the atmospheric term is (approximately) the same for all stars; this is known as *differential photometry*. From the photon flux of the object with known brightness, one could determine an "exposure efficiency" for this exposure. Equivalently, and more commonly, one can calculate an *instrumental magnitude*:

$$m = -2.5 \log \frac{S}{t}$$

(i.e., normalize by the exposure time to get counts/sec, although this is not strictly necessary) and then determine the *zeropoint* that needs to be added to give the calibrated magnitude (M, make sure you recognize that this is still an apparent magnitude):

$$M = m + z$$

Note that in the real world, one has to also consider possible differences between a given experimental setup and the setup used to measure the reference brightnesses, so this is only a first approximation (i.e., the zeropoint may be different for different stars with different spectral properties)! If using instrumental mags including exposure time normalization, the zeropoint gives the magnitude of a star that will give 1 count/second.

If there are no stars of known brightness in the same observation, then calibration must be done against stars in other observations. This then requires that the different effects of the Earth's atmosphere in different locations in the sky be accounted for. This is known as all-sky, or absolute, photometry. To do this requires that the sky is "well-behaved", i.e. one can accurately predict the atmospheric throughput as a function of position. This requires that there be no clouds, i.e. photometric weather. Differential photometry can be done in non-photometric weather, hence it is much simpler! Of course, it is always possible to obtain differential photometry and then go back later and obtain absolute photometry of the reference stars. It is also common to stop with differential photometry if one is studying variable objects, i.e. where one is just interested in the change in brightness of an object, not the absolute flux level.

Of course, at some point, someone needs to figure out what the fluxes of the calibrating stars really are, and this requires understanding all of the terms in the count equation. It is challenging, and often, absolute calibration of a system is uncertain to a couple of percent.

While the count equation isn't usually used for calibration, it is very commonly used for computing the approximate number of photons you will receive from a given source in a given amount of time for a given observational setup. This number is critical to know in order to estimate your expected errors and exposure times in observing proposals, observing runs, etc. Understanding errors in absolutely critical in all sciences, and maybe even more so in astronomy, where objects are faint, photons are scarce, and errors are not at all insignificant. The count equation provides the basis for exposure time calculator (ETC) programs, because it gives an expectation of the number of photons that will be received by a given instrument as a function of exposure time. As we will see shortly, this provides the information we need to calculate the uncertainty in the measurement as a function of exposure time.

Errors in photon rates

For a given rate of emitted photons, there's a probability function which gives the number of photons we detect, even assuming 100% detection efficiency, because of statistical uncertainties. In addition, there may also be instrumental uncertainties. Consequently, we now turn to the concepts of probability distributions, with particular interest in the distribution which applies to the detection of photons.

Distributions and characteristics thereof

- concept of a distribution: define $p(x)dx$ as probability of event occuring in $x + dx$:

$$\int p(x)\mathrm{d}x = 1$$

Some definitions relating to values which characterize a distribution:

$$\begin{aligned} mean \equiv \mu &= \int x p(x)\mathrm{d}x \\ variance \equiv \sigma^2 &= \int (x-\mu)^2 p(x)\mathrm{d}x \\ standard\ deviation \equiv \sigma &= \sqrt{variance} \end{aligned}$$

Noise equation: how do we predict expected errors?

Error propagation

Determining sample parameters: averaging measurements

Random errors vs systematic errors