ASTR 535 Lecture Notes

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Properties of light, magnitudes, errors, and error analysis

Light

Wavelength regimes:

- gamma rays
- x-rays
- ultraviolent (UV)
  - near: 900–3500 Å
  - far: 100–900 Å

  The 900 Å break is because of the Lyman limit at 912 Å. This is where neutral hydrogen is ionized, so the universe is largely opaque to wavelengths shorter than this.
- visual (V): 4000–7000 Å (note that 'V' is different from 'optical', which is slightly broader: 3500–10000 Å. The 3500 Å cutoff is due to the Earth's atmosphere being opaque to wavelengths shorter than this).
- IR
  - near: 1–5 μ (1–10 μ in online notes)
  - mid: (10–100 μ)
  - far: 5–100 μ (100–1000 μ)
- sub-mm 500–1000 μ
- microwave
- radio

Quantities of light:

- Intensity $I(\theta, \phi)$ [erg s^{-1} ν^{-1} Ω^{-1}]: Encapsulates *direction* light is coming from. Also known as radiance.
- Surface Brightness (SB) [erg s^{-1} cm^{-2} ν^{-1} sterradian^{-1}]: amount of energy *received* in a unit surface element per unit time per unit frequency (or wavelength) from a unit solid angle in the direction (θ, ϕ), where θ is the angle away from the normal to the surface element, and ϕ is the azimuthal angle. To calculate SB, just divide the flux by the angle subtended by the object [rad^2]. At a larger distance, the flux will be smaller, but so will the angle subtended by the object, so SB is independent of distance unless considering cosmological scales, where the curvature of spacetime has an effect.
- Flux (F) [erg s^{-1} cm^{-2} ν^{-1}]: amount of energy passing through a unit surface element in all directions, defined by

$$F_\nu = \int I_\nu \cos(\theta) \mathrm{d}\Omega \tag{1}$$

  where dΩ is the solid angle element, and the integration is over the entire solid angle. The $\cos(\theta)$ factor is important for, e.g., ISM where light is coming from all directions, but for tiny objects, θ is negligibally small and can be dropped. Integrates over *all* directions. Also known as irradiance.
- Luminosity (L) [erg s^{-1}]: *intrinsic* energy emitted by the source per second ($\sim$ power). For an isotropically emitting source,

$$L = 4\pi d^2 F \tag{2}$$

  where d = distance to source (so L can only be calculated if the distance is known). Also known as radiant flux.

What to measure for sources:

- Resolved: directly measure surface brightness (intensity) distribution on the sky, usually over some bandpass or wavelength interval.
- Unresolved: measure the flux. Diffraction is the reason stellar surfaces cannot be resolved. Because of this, we cannot measure SB, so we measure flux, integrated over the entire object.

Questions:

- What are the dimensions of the three quantities: luminosity, surface brightness (intensity), and flux?
- How do the three quantities depend on distance to the source?
- To what quantity is apparent magnitude of a star related?
- To what quantity is the absolute magnitude related?

Amount of light emitted is a function of wavelength, so we are often interested in e.g. flux per unit wavelength (or frequency), also known as *specific* flux. Using $\lambda = \frac{c}{\nu} \rightarrow \frac{\mathrm{d}\lambda}{\mathrm{d}\nu} = \frac{-c}{\nu^2}$

$$\begin{aligned}
\int F_\nu \mathrm{d}\nu &= -\int F_\lambda \mathrm{d}\lambda \\
F_\nu \mathrm{d}\nu &= -F_\lambda \mathrm{d}\lambda \\
F_\nu &= -F_\lambda \frac{\mathrm{d}\lambda}{\mathrm{d}\nu} \\
&= F_\lambda \frac{c}{\nu^2} \\
&= F_\lambda \frac{\lambda^2}{c}
\end{aligned}$$

The negative comes from frequency and wavelength increasing in opposite directions. Note that a constant F_λ implies a *non*-constant F_ν and vice versa. Depending on where you are, a constant chunk of 1 Hz is not the same wavelength range.

Units: often cgs, magnitudes, Jansky (a flux density unit corresponding to 10^{-26} W m^{-2} Hz^{-1})

There are often variations in terminology

Terminology of measurements:

- photometry (broad-band flux measurement): SB or flux, integrated over some wavelength range.
- spectroscopy (*relative* measurement of fluxes at different wavelengths): $f(\lambda)$
- spectrophotometry (*absolute* measurement of fluxes at different wavelengths): $f(\lambda)$
- astrometry: concerned with positions of observed flux, not brightness, but direction.
- morphology: intensity as a function of position; often, absolute measurements are unimportant. Deals with *resolved* objects, intensity as function of position.

Generally, measure *flux* with photometry, and flux *density* (per unit wavelength) with spectroscopy (down to the resolution of the spectrograph). In practice, with most detectors, we measure photon flux [photons cm^{-2} s^{-1}] with a photon counting device, rather than energy flux (which is done with bolometers). The monochromatic photon flux is given by the energy flux (F_λ) divided by the energy per photon ($E_{photon} = \frac{hc}{\lambda}$), or

$$\text{photon flux} = \int F_\lambda \frac{\lambda}{hc} \mathrm{d}\lambda$$

Magnitudes and photometric systems

Magnitudes are related to flux (and SB and L) by:

$$m_1 - m_2 = -2.5 \log \frac{b_1}{b_2}$$

or for a single object:

$$\begin{aligned} m &= -2.5 \log \frac{F}{F_0} \\ &= -2.5 \log F + 2.5 \log F_0 \end{aligned}$$

where the coefficient of proportionality, F_0, depends on the definition of photometric system; the quantity $2.5 \log F_0$ may be referred to as the photometric system zeropoint. Note that this relationship holds regardless of what photometric system you are using. Inverting, one gets:

$$F = F_0 \times 10^{-0.4\mathrm{m}}$$

Just as fluxes can be represented in magnitude units, flux *densities* (flux per unit wavelength/frequency, or *monochromatic* flux, as opposed to integrated flux) can be specified by monochromatic magnitudes:

$$F_\lambda = F_0(\lambda) \times 10^{-0.4\mathrm{m}(\lambda)}$$

although spectra are more often given in flux units than in magnitude units. Note that it is possible that F_0 is a function of wavelength.

Since magnitudes are logarithmic, the $difference$ between magnitudes corresponds to a ratio of fluxes; ratios of magnitudes are generally unphysical. If one is just doing relative measurements of brightness between objects, this can be done without knowledge of F_0 (or, equivalently, the system zeropoint); objects that differ in brightness by ΔM have the same ratio of brightness ($10^{-0.4\Delta M}$) regardless of what photometric system they are in. The photometric system definitions and zeropoints are only needed when converting between calibrated magnitudes and fluxes. Note that this means that if one references the brightness of one object relative to that of another, a magnitude system can be set up relative to the brightness of the reference source. However, the utility of a system when doing astrophysics generally requires an understanding of the actual fluxes.

Monday, January 25

There are three main types of magnitude systems in use in astronomy: STMAG, ABNU, and VEGAMAG. We start by describing the two simpler ones: the STMAG and the ABNU mag system. In these simple systems, the reference flux is just a constant value in F_λ or F_ν. However, these are not always the most widely used systems in astronomy, because no natural source exists with a flat spectrum.

The STMAG (Space Telescope MAGnitude) is defined relative to a source of constant F_λ. In this system, the reference flux is given by

$$F_{0,\lambda} = 3.60 \times 10^{-9} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1}$$

which is equal to the flux of Vega at 5500Å; hence a star of Vega's brightness at 5500Å is defined to have m=0. Alternatively (using $m = -2.5 \log \frac{F}{F_0}$), we can write

$$m_{\mathrm{STMAG}} = -2.5 \log F_\lambda - 21.1$$

for F_λ in cgs units(using cm^{-2} Å^{-1} distinguishes between the collecting area and the wavelength, since both are units of distance. Be careful with units when doing these conversions; as long as the proper flux units come out at the end, the answer should be correct).

The ABNU system is defined relative to a source of constant F_ν and we have

$$F_{0,\nu} = 3.63 \times 10^{-20} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$$

or

$$m_{\text{ABNU}} = -2.5 \log F_\nu - 48.6$$

for F_ν in cgs units. Again, the constant comes from the flux of Vega.

Magnitudes usually refer to the *integrated* flux (over a spectral bandpass, not just a single wavelength). In this case, F and F_0 refer to such integrated fluxes. The STMAG and ABNU integrated systems are defined relative to sources of constant F_λ and F_ν, respectively

$$m_{\text{STMAG}} = -2.5 \log \frac{\int F_\lambda \lambda \text{d}\lambda}{\int 3.6 \times 10^{-9} \lambda \text{d}\lambda}$$
$$m_{\text{ABNU}} = -2.5 \log \frac{\int F_\nu \nu^{-1} \text{d}\nu}{\int 3.6 \times 10^{-20} \nu^{-1} \text{d}\nu}$$

These are defined to be the same at 5500Å. The factors of λ and ν come from the conversion factor hc/λ for photon-counting detectors, where h and c cancel out in the fractions in each equation.

Note that these systems differ by more than a constant, because one is defined by units of F_λ and the other by F_ν, so the difference betwen the systems is a function of wavelength. (Question: what's the relations between m_{STMAG} and m_{ABNU}?)

Note also that, using magnitudes, the measured magnitude is nearly independent of bandpass width, so a broader bandpass does not imply a brighter (smaller) magnitude, which is not the case for fluxes. The reference is being integrated as well, so they cancel.

The standard UBVRI broadband photometric system, as well as several other magnitude systems, however, are not defined for a constant (flat) F_λ or F_ν spectrum; rather, they are defined relative to the spectrum of an A0V star. Most systems are defined (or at least were originally) to have the magnitude of Vega be zero in all bandpasses (VEGAMAGS); if you ever get into this in detail, note that this is not exactly true for the UBVRI system.

For the broadband UBVRI system, we have

$$m_{\text{UBVRI}} \approx -2.5 \log \frac{\int_{UBVRI} F_\lambda(object) \lambda \text{d}\lambda}{\int_{UBVRI} F_\lambda(Vega) \lambda \text{d}\lambda}$$

(as above, the factor of λ comes in for photon counting detectors). This gives the magnitude in U, B, V, R, *or* I, by integrating over that same bandpass. The UBVRI filter set had overlapping bandpasses, so there was a switch to interference filter: the ugriz system used by SDSS (explained below... I think).

Here is a plot to demonstrate the difference between the different systems.

While it seems that STMAG and ABNU systems are more straightforward, in practice it is difficult to measure absolute fluxes, and much easier to measure relative fluxes between objects. Hence, historically observations were tied to observations of Vega (or to stars which themselves were tied to Vega), so VEGAMAGs made sense, and the issue of determining physical fluxes boiled down to measuring the physical flux of Vega. Today, in some cases, it may be more accurate to measure the absolute throughput of an instrumental system, and using STMAG or ABNU makes more sense.

Colors

Working in magnitudes, the difference in magnitudes between different bandpasses (called the color index, or simply, color) is related to the flux ratio between the bandpasses, i.e., the color. In the UBVRI system, the *difference between magnitudes*, e.g. B-V, gives the ratio of the fluxes in different bandpasses *relative to the ratio of the fluxes of an A0V star in the different bandpasses (for VEGAMAG)*. Note the typical colors of astronomical objects, which are different for the different photometric systems. Type 'A' stars have color 0, and have the same SED as Vega. Type 'O' stars have color less than 0, while cooler stars have color greater than 0. In sloan system, have g-r (ugriz). g-r=0 indicates constant F_ν.

$$m = -2.5 \log \frac{\int_B F_\lambda \mathrm{d}\lambda}{\int_V F_\lambda \mathrm{d}\lambda}$$

if B-V=0, then $(\mathrm{B/V})_{object}$ is the same as $(\mathrm{B/V})_{Vega}$, or $\left(\frac{\int F_\nu}{\int F_\lambda}\right)_{obj} = \left(\frac{\int F_\nu}{\int F_\lambda}\right)_{Vega}$. An object with B-V=0 has the same spectral *shape* as Vega. Not necessarily the same flux value at each bandpass (The object could be brighter... but the difference between its B and V magnitudes is the same as the difference between those two magnitudes for Vega... I think). Keep in mind that the UBVRI system is defined relative to spectrum of an A0V star, not simply a flat spectrum.

What would the flux be from an object with some magnitude, x? Need this to know how much observing time I need. E.g., convert the spectrum of an elliptical galaxy to color; if you know F in one filter, you can get F in another filter.

Which is closer to the UBVRI system, STMAG or ABNU?

What would typical colors be in a STMAG or ABNU system?

Wednesday, January 27

UBVRI magnitudes-flux conversion

To convert Vega-based magnitudes to fluxes, look up the flux of Vega at the center of the passband; however, if the spectrum of the object differs from that of Vega, this won't be perfectly accurate. Given UBVRI magnitudes of an object in the desired band, filter profiles (e.g. Bessell 1990, PASP 102,1181), and absolute spectrophotometry of Vega (e.g., Bohlin & Gilliland 2004, AJ 127, 3508, one can determine the flux.

If one wanted to estimate the flux of some object in arbitrary bandpass given just the V magnitude of an object (a common situation used when trying to predict exposures times, see below), this can be done if an estimate of the spectral energy distribution (SED) can be made; given the filter profiles, one can compute the integral of the SED over the V bandpass, determine the scaling by comparing with the integral of the Vega spectrum over the same bandpass, then use the normalized SED to compute the flux in any desired bandpass. Some possibly useful references for SEDs are: Bruzual, Persson, Gunn, & Stryker; Hunter, Christian, & Jacoby; Kurucz).

Things are certainly simpler in the ABNU or STMAG system, and there has been some movement in this direction: the STScI gives STMAG calibrations for HST instruments, and the SDSS photometric system is close to an ABNU system.

Note, however, that even when the systems are conceptually well defined, determining the absolute calibration of any photometric system is very difficult in reality, and determining absolute fluxes to the 1% level is very challenging.

As a separate note on magnitudes themselves, note that some people, in particular, the SDSS, have adopted a modified type of magnitudes, called asinh magnitudes, which behave like normal (also known as Pogson) magnitude for brighter objects, but have different behavior for very faint objects (near the detection threshold); see Lupton, Gunn, & Szalay 1999 AJ 118, 1406 for details.

Observed fluxes and the count equation

What if you are measuring flux with an actual instrument, i.e. counting photons? The intrinsic photon flux from the source is not trivial to determine from the number of photons that you count. To get the number of photons that you count in an observation, you need to take into account

- The area of your photon collector (telescope)
- Photon losses and gains from the Earth's atmosphere (which change with conditions)
- The efficiency of your collection/detection apparatus (which can change with time).

Generally, the astronomical signal (which might be a flux or a surface brightness, depending on whether the object is resolved) can be written in the form of the ***count equation:***

$$S = Tt \int \frac{F_\lambda}{\frac{hc}{\lambda}} q_\lambda a_\lambda \mathrm{d}\lambda \equiv TtS'$$

- S: total number of photons observed (the "signal")
- S' is an observed flux rate, i.e. with all of the real details of the observing system included.
- a_λ: atmospheric transmission, or *throughput*, which is the fraction of photons that make it through. (a typical value is about 0.9; ∼90% of photons)
- q_λ is the system efficiency (which includes telescope efficiency, instrument efficiency, filters, and detector)
- T: telescope collecting area
- t is the integration time (total amount of time spent collecting photons).
- $F_\lambda/\frac{hc}{\lambda}$: flux divided by the energy per photon; gives the number of photons per second per square cm.

T and t are the *only* terms that do not depend on the wavelength (or frequency).

Usually, however, one doesn't use this information to go backward from S to F_λ because it is very difficult to measure all of the terms precisely, and some of them (e.g. a, and perhaps q) are time-variable; a is also spatially variable. Instead, most observations are performed differentially to a set of other stars of well known brightness. If the stars of known brightness are observed in the same observation, then the atmospheric term is (approximately) the same for all stars; this is known as *differential photometry*. From the photon flux of the object with known brightness, one could determine an "exposure efficiency" or an "effective area" for this exposure. Equivalently, and more commonly, one can calculate an *instrumental magnitude*:

$$m = -2.5 \log \frac{S}{t}$$

(i.e., normalize by the exposure time, t, to get counts/sec, although this is not strictly necessary) and then determine the *zeropoint*, z, that needs to be added to give the calibrated magnitude, M (which is still an *apparent* magnitude).

$$M = m + z$$

Note that in the real world, one has to also consider possible differences between a given experimental setup and the setup used to measure the reference brightnesses, so this is only a first approximation (i.e., the zeropoint may be different for different stars with different spectral properties). If using instrumental mags including exposure time normalization, the zeropoint gives the magnitude of a star that will give 1 count/second. The zeropoint describes the *throughput* of the system.

$$M = -2.5 \log \frac{S}{t} + z$$

FOV: usually in arcminute scales. To go from observed to emitted → 2 different observations (know flux of one: SDSS has list of stars with known brightness). A star that is 10 times fainter than one with g=18 has g=20.5.

$$m = -2.5 \log(qF)$$

$$m = -2.5 \log F - 2.5 \log q$$

the quantity '-2.5log q' is the zeropoint.

$$m = -2.5 \log \frac{S}{t}$$

No nearby star... q is still the same, but a changes. If sky is 'well-behaved' atmospheric effects is simple function of distance form zenith; slope of variation can change from one night to the next.

How did standard stars get measured in the first place? Someone had to do some hard work figuring out throughput. Accuracy depends on number of photons, which will determine how much observing time you will need.

If there are no stars of known brightness in the same observation, then calibration must be done against stars in other observations. This then requires that the different effects of the Earth's atmosphere in different locations in the sky be accounted for. This is known as *all-sky*, or absolute, photometry. To do this requires that the sky is "well-behaved", i.e. one can accurately predict the atmospheric throughput as a function of position. This requires that there be no clouds, i.e. *photometric* weather. Differential photometry can be done in non-photometric weather, hence it is much simpler. Of course, it is always possible to obtain differential photometry and then go back later and obtain absolute photometry of the reference stars.

Of course, at some point, someone needs to figure out what the fluxes of the calibrating stars really are, and this requires understanding all of the terms in the count equation. It is challenging, and often, absolute calibration of a system is uncertain to a couple of percent.

It is also common to stop with differential photometry if one is studying variable objects, i.e. where one is just interested in the change in brightness of an object, not the absolute flux level. In this case, one only has to reference the brightness of the target object relative some other object (or ensemble of objects) in the field that are non-variable.

While the count equation isn't usually used for calibration, it is very commonly used for computing the approximate number of photons you will receive from a given source in a given amount of time for a given observational setup. This number is critical to know in order to estimate your expected errors and exposure times in observing proposals, observing runs, etc. Understanding errors in absolutely critical in all sciences, and maybe even more so in astronomy, where objects are faint, photons are scarce, and errors are not at all insignificant. The count equation provides the basis for exposure time calculator (ETC) programs, because it gives an expectation of the number of photons that will be received by a given instrument as a function of exposure time. As we will see shortly, this provides the information we need to calculate the uncertainty in the measurement as a function of exposure time.

Uncertainties in photon rates

Useful reference: Statistics

Required accuracy - depends on the science you're doing. Uncertainties (more descriptive than 'errors') - associated with flux measurements. For given input rate, what is the probability of observing range of all possible input values?

For a given rate of emitted photons, there's a probability function which gives the number of photons we detect, even assuming 100% detection efficiency, because of *statistical* uncertainties. In addition, there may also be *instrumental* uncertainties. Consequently, we now turn to the concepts of probability distributions, with particular interest in the distribution which applies to the detection of photons.

Distributions and characteristics thereof

concept of a distribution: define $p(x)dx$ as probability of event occuring in $x + \mathrm{d}x$:

$$\int p(x)\mathrm{d}x = 1$$

Some definitions relating to values which characterize a distribution:

$$\text{mean} \equiv \mu = \int x p(x)\mathrm{d}x$$
$$\text{variance} \equiv \sigma^2 = \int (x-\mu)^2 p(x)\mathrm{d}x$$
$$\text{standard deviation} \equiv \sigma = \sqrt{\text{variance}}$$

median: mid-point value.

$$\frac{\int_{-\infty}^{x_{median}} p(x)\mathrm{d}x}{\int_{-\infty}^{\infty} p(x)\mathrm{d}x} = \frac{1}{2}$$

mode: most probable value (peak in plot).

Monday, February 1

Note that the geometric interpretation of above quantities depends on the nature of the distribution; although we all carry around the picture of the mean and the variance for a Gaussian distribution, these pictures are not applicable to other distributions, but the quantities are still well-defined.

Also, note that there is a difference between the *sample* mean, variance, etc. and the *population* quantities. The latter apply to the true distribution, while the former are estimates of the latter from some finite sample (N measurements) of the population. The sample quantities are derived from:

$$\text{sample mean: } \bar{x} \equiv \frac{\sum x_i}{N}$$
$$\text{sample variance: } \equiv \frac{\sum (x_i - \bar{x})^2}{N-1} = \frac{\sum x_i^2 - (\sum x_i)^2/N}{N-1}$$

The sample mean and variance approach the true mean and variance as N approaches infinity. But note, especially for small samples, your estimate of the mean and variance may differ from their true (population) values (more below).

The binomial distribution

Given independent events (e.g. when rolling a set of dice, the results of the second roll are independent of the first), two parameters, n and p, where n is the total number of observations and p is the probability of observing x during each observation. Observing photons: For a binomial distribution $P(x, n, p)$, x is the number of photons received, n is the number of photons emitted by the source, and p is the probability of detecting *each* photon. Integrating this actually simplifies it

Now we consider what distribution is appropriate for the detection of photons. The photon distribution can be derived from the *binomial* distribution, which gives the probability of observing the number, x, of some possible event, given a total number of events n, and the probability of observing the particular event (among all other possibilities) in any single event, p, under the assumption that all events are independent of each other:

$$P(x, n, p) = \frac{n! p^x (1-p)^{n-x}}{x!(n-x)!}$$

For the binomial distribution, one can derive:

$$\text{mean} \equiv \mu \equiv \int x p(x) \mathrm{d}x = np$$

$$\text{variance} \equiv \sigma^2 \equiv \int (x-\mu)^2 p(x) \mathrm{d}x = np(1-p)$$

The Poisson distribution

Unlike a gaussian, a poisson distribution is not symmetric about the mean (you can't detect a negative number of photons). It cuts off at zero. THE VARIANCE IS EQUAL TO THE MEAN. THEREFORE, THE STANDARD DEVIATION IS THE SQUARE ROOT OF THE MEAN. This is the 'key super important result'.

In the case of detecting photons, n is the total number of photons emitted, and p is the probability of detecting a photon during our observation out of the total emitted. We don't know either of these numbers. However, we do know that p<<1 and we know, or at least we can estimate, the mean number detected:

$$\mu = np$$

In this limit, the binomial distribution asymtotically approaches the *Poisson* distribution:

$$P(x, \mu) = \frac{\mu^x e^{-\mu}}{x!}$$

From the expressions for the binomial distribution in this limit, the mean of the distribution is μ, and the variance is

$$\text{variance} = \sum_x [(x-\mu)^2 p(x, \mu)]$$

$$\text{variance} = np = \mu$$

This is an *important result*.

Note that the Poisson distribution is generally the appropriate distribution not only for counting photons, but for *any* sort of counting experiment where a series of events occurs with a known average rate, and are independent of time since the last event.

What does the Poisson distribution look like? Plots for $\mu = 2$, $\mu = 10$, $\mu = 10000$.

The normal, or Gaussian, distribution

factor into std. numerical technique (least squares), photons not emitted at perfect constant rate, readout function, applies to lots of stuff.

Note, for large μ, the Poisson distribution is well-approximated around the peak by a *Gaussian*, or *normal* distribution:

$$P(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

This is important because it allows us to use "simple" least squares techniques to fit observational data, because these generally assume normally distributed data. However, in the tails of the distribution, and at low mean rates, the Poisson distribution can differ significantly from a Gaussian distribution. In these cases, least-squares may not be appropriate to model observational data; instead, one might need to consider maximum likelihood techniques.

The normal distribution is also important because many physical variables seem to be distributed accordingly. This may not be an accident because of the *central limit theorem*: if a quantity depends on a number of independent random variables with ANY distribution, the quantity itself will be distributed normally (see statistics texts). In observations, we encounter the normal distribution because *readout* noise is distributed normally. (Readout noise is one important source of *instrumental noise*).

Importance of error distribution analysis

You need to understand the expected uncertainties in your observations in order to:

- predict the amount of observing time you'll need to get uncertainties as small as you need them to do your science
- answer the question: is scatter in observed data consistent with expected uncertainties? If the answer is no, they you know you've either learned some astrophysics or you don't understand something about your observations. This is especially important in astronomy where objects are faint and many projects are pushing down into the noise as far as possible. Really we can usually only answer this probabilistically. Generally, tests compute the probability that the observations are consistent with an expected distribution (the null hypothesis). You can then look to see if this probability is low, and if so, you can reject the null hypothesis.
- interpret your results in the context of a scientific prediction

Confidence levels

For example, say we want to know whether some single point is consistent with expectations, e.g., we see a bright point in multiple measurements of a star, and want to know whether the star flared. Say we have a time sequence with known mean and variance, and we obtain a new point, and want to know whether it is consistent with known distribution?

If the form of the probability distribution is known, then you can calculate the probability of getting a measurement more than some observed distance from the mean. In the case where the observed distribution is Gaussian (or approximately so), this is done using the *error function* (sometimes called $erf(x)$), which is the integral of a gaussian from some starting value.

Some simple guidelines to keep in mind follow (the actual situation often requires more sophisticated statistical tests). First, for Gaussian distributions, you can calculate that 68% of the points should fall within $\pm 1\sigma$ from the mean, and 95.3% should fall within $\pm 2\sigma$ from the mean. Thus, if you have a time line of photon fluxes for a star, with N observed points, and a photon noise σ on each measurement, you can test whether the number of points deviating more than 2σ from the mean is much larger than expected. To decide whether any single point is really significantly different, you might want to use more stringent criterion, e.g., 5σ rather than a 2σ criterion; a 5σ has much higher level of significance. On the other hand, if you have far more points in the range $2-4\sigma$ brighter or fainter than you would expect, you may also have a significant detection of intensity variations (provided you really understand your uncertainties on the measurements).

Also, note that your observed distribution should be consistent with your uncertainty estimates given the above guidelines. If you have a whole set of points, that all fall within 1σ of each other, something is wrong with your uncertainty estimates (or perhaps your measurements are correlated with each other).

For a series of measurements, one can calculate the χ^2 statistic, and determine how probable this value is, given the number of points.

$$\chi^2 = \sum [(observed(i) - model(i))^2 / \sigma_i^2]$$

A quick estimate of the consistency of the model with the observed data points can be made using reduced χ^2, defined as χ^2 divided by the *degrees of freedom* (number of data points minus number of parameters).

$$\chi^2 = \sum_N \frac{(y_i - y(x_i))^2}{\sigma_i^2}$$

for N data points. If every point was 1σ off, $\chi^2 \sim$ number of data points. Can calculate the probability distribution of χ^2. If $\chi^2 = 98$, what's the confidence interval? Will only get 98 one in a million times? Not a good model. Is the data consistent with the model?

Wednesday, February 3

Noise equation: how do we predict expected uncertainties?

Signal-to-noise

Shape parameterized by stddev which $= \sqrt{\mu} \rightarrow$ Fundamental! e.g. source emitting 1000 photons has uncertainty of 100, etc. (telescope, brightness of source, etc. don't matter... only number of photons, no matter how you got them).

$$\frac{S}{N} = \frac{S}{\sqrt{S}} = \sqrt{S}$$

How does S/N change for a given object?

Astronomers often describe uncertainties in terms of the fractional error, e.g. the amplitude of the uncertainty divided by the amplitude of the quantity being measured (the signal); often, the inverse of this, referred to as the signal-to-noise ratio, is used. Given an estimate the number of photons expected from an object in an observation, we can calulate the signal-to-noise ratio:

$$\frac{S}{N} = \frac{S}{\sqrt{\sigma^2}}$$

which is the inverse of the predicted fractional error (N/S).

Consider an object with observed photon flux, S' [cm^{-2} s^{-1}], leading to a signal, $S = S'Tt$. In the simplest case (i.e. perfect instruments), the only noise source is Poisson statistics from the source, in which case:

$$\sigma^2 = S = S'Tt$$
$$\frac{S}{N} = \sqrt{S} = \sqrt{S'Tt}$$

In other words, S/N increases as the square root of the object brightness, telescope area, efficiency, or exposure time. Note that S is directly observable, so one can calculate S/N for an observation without knowing the telescope area or exposure time. We've just broken S down so that you can specifically see the dependence on telescope area and/or exposure time.

Background noise

Rayleigh scattering

A more realistic case includes the noise contributed from Poisson statistics of "background" light, B', which has units of flux per area on the *sky*, not the detector (i.e. a surface brightness); note that this is also usually given in magnitudes (more on the physical nature of this later).

$$B' = \int \frac{B_\lambda}{\frac{hc}{\lambda}} q_\lambda \mathrm{d}\lambda$$

B_λ is the sky brightness at λ...? And B' is the observed flux from the background. So B vs. B' is the same as S vs. S'.

The amount of background in our measurement depends on how much sky area we observe. Say we just use an aperture with area, A, so the total observed background counts is

$$AB = AB'Tt$$

Again, $B'Tt$ is the directly observable quantity, but we split it into the quantities on which it depends to understand what factors are important in determining S/N.

The total number of photons observed, O, is a combination of photons from the source (S), and phtons from the background (AB):

$$O = S + AB = (S' + AB')Tt$$

The variance of the total observed counts, from Poisson statistics, is:

$$\sigma_O^2 = O = S + AB = (S' + AB')Tt$$

To get the desired signal from the object only, we will need to measure separately the total signal and the background signal to estimate:

$$S \equiv S'Tt = O - A < B >$$

where $< B >$ is some estimate we have obtained of the background surface brightness. The noise in the measurement is

$$\sigma_S^2 \approx \sigma_O^2 = S + AB = (S' + AB')Tt$$

where the approximation is accurate if the background is determined to high accuracy, which one can do if one measures the background over a large area, thus getting a large number of background counts (with correspondingly small fractional error in the measurement).

This leads to a common form of the *noise equation*:

$$\frac{S}{N} = \frac{S}{\sqrt{S + AB}}$$

Breaking out the dependence on exposure time and telescope area, this is:

$$\frac{S}{N} = \frac{S'}{\sqrt{S' + AB'}}\sqrt{T}\sqrt{t}$$

Limiting regimes:

- *signal-limited* case: $S' \gg B'A$, the background goes away and we get

$$\frac{S}{N} = \sqrt{S} = \sqrt{S'tT}$$

- *background-limited* case: $B'A \gg S'$ and

$$\frac{S}{N} = \frac{S}{\sqrt{BA}} = \frac{S'}{\sqrt{B'A}}\sqrt{tT}$$

starlight dominates by factor of 1000 for m$_*$ = 12.5 and m$_{BG}$ = 20 ($\Delta m = 7.5 = -2.5 \log \frac{b_{BG}}{b_*} \rightarrow 10^3$)

$\odot$: little circle around star to reduce sky background noise. Depends on how blurry the star is; determined by seeing.

- Dust scattering in the plane of the solar system
- Diffraction limit of space telescope $\sim$ 20th of an arcsecond. ($\sim$ angular resolution).

As one goes to fainter objects, the S/N drops, and it drops faster when you're background limited. This illustrates the importance of dark-sky sites, and also the importance of good image quality.

Consider two telescopes of collecting area, T_1 and T_2. If we observe for the same exposure time on each and want to know how much fainter we can see with the larger telescope at a given S/N, we find this for a signal-limited case:

$$S_2 = \frac{T_1}{T_2} S_1$$

but find *this* for the background-limited case:

$$S_2 = \sqrt{\frac{T_1}{T_2}} S_1$$

Instrumental noise

Most common:readout noise. Gaussian, not poisson. $\mu = 0$, σ = readout noise (depends on detector, e.g. 10 electrons ($\sim$ photons create photoelectrons). Get a smear of 10 no matter how many photons are coming in. Represented by range around zero with no exposure time. Mean is zero, so can't subtract it out, it just blurs what you're looking at. B - background per square arcsecond, A - square arcsecond. Number of pixels over which it's spread:

- sharper image
- 10 arcsec all in one pixel

Can change magnification of optics, but there is a balance. Might have two stars five arcsec apart. Maxiumum resolution is not always the best! readout noise is very important when the background is very low. $N_{pix}\sigma_{rn}^2$ compared to BA. rn more important for spectroscopy applications than imaging.

In addition to the errors from Poisson statistics (statistical noise), there may be additional terms from instrumental errors. A common example of this that is applicable for CCD detectors is readout noise, which is additive *noise* (with zero mean) that comes from the detector and is independent of signal level. For a detector whose readout noise is characterized by σ_{rn},

$$\frac{S}{N} = \frac{S}{\sqrt{S + BA_{pix} + \sigma_{rn}^2}}$$

if a measurement is made in a single pixel. If an object is spread over N_{pix} pixels, then

$$\boxed{\frac{S}{N} = \frac{S}{\sqrt{S + BA + N_{pix}\sigma_{rn}^2}}}$$

This is the key equation! For large σ_{rn}, the behavior is the same as the background-limited case. This makes it clear that if you have readout noise, image quality (and/or proper optics to keep an object from covering too many pixels) is important for maximizing S/N. It is also clear that it is critical to have minimum read-noise for low background applications (e.g., spectroscopy).

There are other possible additional terms in the noise equation, arising from things like dark current, digitization noise, errors in sky determination, errors from photometric technique, etc. (we'll discuss some of these later on), but in most applications, the three sources discussed so far - signal noise, background noise, and readout noise - are the dominant noise sources.

Note the applications where one is likely to be signal dominated, background dominated, and readout noise dominated.

Review

Three sources of uncertainty:

1. Possion statistics from source: $\sigma^2 = S$; $\sigma = \sqrt{S}$. S is the same as μ.
2. Poisson statistics from sky/background: $< B >$ is mean background rate.
3. Non-poisson statistics from instrumentation: aka readout noise. $\mu = 0$ (no exposure time). Smear around μ. $\sigma = \sigma_{rn}$ per pixel. $\sigma^2 = N_{pix}\sigma_{rn}^2$; $\sigma = \sqrt{N_{pix}\sigma_{rn}^2}$

Noise, $N = \sqrt{S + BA + N_{pix}\sigma_{rn}^2}$ (source + sky + instrument).

Error propagation

Monday, February 8

So now we know how to estimate uncertainties of observed count rates. Let's say we want to make some calculations (e.g., calibration, unit conversion, averaging, conversion to magnitudes, calculation of colors, etc.) using these observations: we need to be able to estimate the uncertainties in the calculated quantities that depend on our measured quantities.

Total signal: $S = S + B - < B > + O$. There are uncertainties from all four terms, but $< B > \sim 0$ ($\sigma_{<B>} = 0$), so there are actually only three. $S + BA$: total inside aperture; $< B > A$: total outside aperture (with mean background).

Consider what happens if you have several independently measured known quantities with known error distributions and you combine these into some new quantity: we wish to know what the error is in the new quantity.

$$x = f(u, v, \ldots)$$

The question is, what is σ_x if we know σ_u, σ_v, etc.? In other words, how do the individual uncertainties *propogate* onto the uncertainty of the resultant quantity?

As long as errors are small:

$$x_i - < x > \sim \ (u_i - < u >)\left(\frac{\partial x}{\partial u}\right) + (v_i - < v >)\left(\frac{\partial x}{\partial v}\right) + \ldots$$

where $var_i - < var >$ is the uncertainty, and the derivatives are the amount by which the input quantity depends on other quantities.

Possible class example involving S=100, B=10, A=10, and $\sigma_{rn} = 10 \ldots$? The *sample variance* (by definition, see above notes) is

$$\sigma_x^2 = lim(N \rightarrow \infty)\frac{1}{N}\sum(x_i - < x >)^2$$

$$= \sigma_u^2\left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2\left(\frac{\partial x}{\partial v}\right)^2 + 2\sigma_{uv}^2\frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \ldots$$

The last term is the *covariance*, which relates to whether errors are *correlated*. Covarience is the dgree to which the uncertainty in one quantity might be correlated (or not) with another quantity.

$$\sigma_{uv}^2 = lim(N \rightarrow \infty)\frac{1}{N}\sum(u_i - < u >)(v_i - < v >)$$

If errors are uncorrelated, then $\sigma_{uv} = 0$ because there's equal chance of getting opposite signs on v_i for any given u_i. When working out errors, make sure to consider whether there are correlated errors. If there are, you *may* be able to reformulate quantities so that they have independent errors: this can be very useful.

Understanding these terms is very important!

Examples for *uncorrelated* errrors:

- addition/subtraction:

$$x = u + v$$

$$\sigma_x^2 = \sigma_u^2\left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2\left(\frac{\partial x}{\partial v}\right)^2$$

  Both derivatives = 1, so

$$\sigma_x^2 = \sigma_u^2 + \sigma_v^2$$

  In this case, errors are said to *add in quadrature*.

- multiplication/division:

$$x = uv$$

$$\sigma_x^2 = v^2\sigma_u^2 + u^2\sigma_v^2$$

$$\frac{\sigma_x^2}{v^2u^2} = \frac{\sigma_u^2}{u^2} + \frac{\sigma_v^2}{v^2}$$

- logs:

$$x = \ln u$$
$$\sigma_x^2 = \frac{\sigma_u^2}{u^2}$$

  Note that when dealing with logarithmic quantities, errors in the log correspond to *fractional* errors in the raw quantity.

E.g. $x = 10u \rightarrow \sigma_x = 10\sigma_u$

$N \equiv \sigma_S$

Reasons for adding in quadrature: important! Also don't overestimate errors. This is just as bad as underestimating them.

Distribution of resultant errors

When propagating errors, even though you can calculate the variances in the new variables, the distribution of errors in the new variables is not, in general, the same as the distribution of errors in the original variables, e.g. if errors in individual variables are normally distributed, errors in output variable is not necessarily.

When two variables are added, however, the output is normally distributed.

Determining sample parameters: averaging measurements

We've covered errors in single measurements (σ_i). Next we turn to averaging measurements. Say we have multiple observations and want the best estimate of the mean and variance of the population, e.g. multiple measurements of stellar brightness. Here we'll define the best estimate of the mean as the value which maximizes the likelihood that our estimate equals the true parent population mean.

For equal errors (simplest case), this estimate just gives our normal expression for the sample mean:

$$\bar{x} = \frac{\sum x_i}{N}$$

Using error propagation, the estimate of the error in the sample mean is given by:

$$\sigma_{\bar{x}}^2 = \sum \frac{\sigma_i^2}{N^2} = \frac{\sigma^2}{N}$$

But what if errors on each observation aren't equal, say for example we have observations made with several different exposure times? Then the optimal determination of the mean is using a:

$$\text{weighted mean} = \frac{\sum \frac{x_i}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}}$$

which can be derived from maximum liklihood. large weight means small error. The estimated error in this value is given by:

$$\sigma_\mu^2 = \sum \frac{\frac{\sigma_i^2}{\sigma_i^4}}{(\sum \frac{1}{\sigma_i^2})^2} = \frac{1}{\sum \frac{1}{\sigma_i^2}}$$

where the σ_i's are individual weights/errors (people often talk about the *weight* of an observation, i.e. $\frac{1}{\sigma_i^2}$: large weight means small error.

This is a standard result for determining sample means from a set of observations with different weights.

However, there can sometimes be a subtlety in applying this formula, which has to do with the question: how do we go about choosing the weights/errors, σ_i? We know we can *estimate* σ using Poisson statistics for a given count rate, but remember that this is a sample variance (which may be based on a single observation) not the true population variance. This can lead to biases.

Consider observations of a star made on three nights, with measurements of 40, 50, and 60 photons. It's clear that the mean observation is 50 photons. However, beware of the being trapped by your error *estimates*. From each observation alone, you would estimate errors of $\sqrt{40}$, $\sqrt{50}$, and $\sqrt{60}$. If you plug these error estimates into a computation of the weighted mean, you'll get a mean rate of $< x >= 48.64$.

Imagine 10,000 measurements; 50 is the *true* population mean. σ is really $\sqrt{50}$ for all 3 observations above... <u>estimation</u> of true uncertainties. Avoid by: revised mean: $50 \rightarrow 48.64 \ldots \sqrt{48.64}$. Something other than Poisson stats, some measurements have more weight than others, same uncertainty across all measurements.

Using the individual estimates of the variances, we'll bias values to lower rates, since these will have estimated higher S/N.

Note that it's pretty obvious from this example that you should just weight all observations equally. However, note that this certainly isn't always the right thing to do. For example, consider the situation in which you have three exposures of different exposure times. Here you probably want to give the longer exposures higher weight (at least, if they are signal or background limited). In this case, you again don't want to use the individual error estimates or you'll introduce a bias. There is a simple solution here also: just weight the observations by the exposure time. However, while this works fine for Poisson errors (variances proportional to count rate), it isn't strictly correct if there are instrumental errors as well which don't scale with exposure time. For example, the presence of readout noise can have this effect; if all exposures are readout noise dominated, then one would want to weight them equally, if readout noise dominates the shorter but not the longer exposures, once would want to weight the longer exposures even higher than expected for the exposure time ratios! The only way to properly average measurements in this case is to estimate a sample mean, then use this value scaled to the appropriate exposure times as the input for the Poisson errors.

Another subtlety: averaging counts and converting to magnitudes is not the same as averaging magnitudes.

Wednesday, February 10

Can you split exposures?

Although from S/N considerations, one can determine the required number of counts you need (exposure time) to do your science, when observing, one must also consider the question of whether this time should be collected in single or in multiple exposures, i.e. how long individual exposures should be. There are several reasons why one might imagine that it is nicer to have a sequence of shorter exposures rather than one single longer exposure (e.g., tracking, monitoring of photometric conditions, cosmic ray rejection, saturation issues), so we need to consider whether doing this results in poorer S/N.

Consider the object with photon flux S', background surface brightness B', and detector with readout noise σ_{rn}. A single short exposure of time t has a variance:

$$\sigma_S^2 = S'Tt + B'ATt + N_{pix}\sigma_{rn}^2$$

If N exposures are summed, the resulting variance will be

$$\sigma_N^2 = N\sigma_S^2$$

If a single long exposure of length Nt is taken, we get

$$\sigma_L^2 = S'TNt + B'ATNt + N_{pix}\sigma_{rn}^2$$

The ratio of the noises, or the inverse ratio of the S/N (since the total signal measured is the same in both cases), is

$$\frac{\sigma_N}{\sigma_L} = \sqrt{\frac{S'TNt + B'ATNt + NN_{pix}\sigma_{rn}^2}{S'TNt + B'ATNt + N_{pix}\sigma_{rn}^2}}$$

The only difference is in the readout noise term. In the signal- or background-limited regimes, exposures can be added with no loss of S/N. However, if readout noise is significant, then splitting exposures leads to reduced S/N.

If the desired S/N reguires one hour of exposure, is it better to have, e.g. 3600 1-second exposure, 1800 2-second exposures, 300 12-second exposures. . . Pros of lots of short exposures:

- avoid saturation
- tracking
- cosmic rays
- variable object
- monitor atmospheric throughput variation

cons of lots of exposures:

- data processing

- high data quantity
- can't see object
- readout time penalty

If tracking is bad, image can be blurry, and would get more background. If you're signal-limited though, this doesn't matter. How do you figure out the minimum amount of time to expose? Need to be above the readout noise. If [rn?] $= 10$, then $S_{min} = 100$ (10^2). σ_{rn} is not dependent on t. "Readout noise limited".

Random errors vs systematic errors

So far, we've been discussing *random* errors. There is an additional, usually more trouble-some, type of errors known as *systematic* errors. These don't occur randomly but rather are correlated with some, possibly unknown, variable relating to your observations.

EXAMPLE : flat fielding

EXAMPLE : WFPC2 CTE

Note also that in some cases, systematic errors can masquerade as random errors in your test observations (or be missing altogether if you don't take data in exactly the same way), but actually be systematic in your science observations.

EXAMPLE: flat fielding, subpixel QE variations.

Note that error analysis from expected random errors may be the only clue you get to discovering systematic errors. To discover systematic errors, plot residuals vs. everything

Important to deal with random errors correctly; don't overestimate error bars. Flat fielding: uniform brightness $\rightarrow$ non-uniformity in sensitivity. To what % accuracy is something "flat"?

Monday, February 15

zeropoint, z, bundles count equation into one number for a *particular bandpass*. This is an empirical measurement of the count equation [counts]. Use the count equation to derive the zeropoint. If the error bars are smaller than the scatter of the data points themselves, there's a problem. χ^2 would be much [larger?] than one. A possible issue is the pixel senstivity isn't uniform across a ccd. Stars at the bottom of the chip could be fainter than the ones at the top. CCDs in space telescope. (CCDs are read by moving charge through chip $\rightarrow$ *grating*.) Something about charge-transfer efficiency… observation scatter larger than expected scatter. See handwritten notes for diagrams and more on this.

Important to know the equation for error propagation!

Effects of the earth's atmosphere

The earth's atmosphere has several different effects: it emits light, it absorbs light, it shifts the apparent direction of incoming light, and it degrades the coherence of incoming light, leading to degradation of image quality when collecting light over a large aperture. myBlueaka emission, absorption, refraction, and seeing.

Night sky emission

Sources of sky emission (brightness as function of wavelength):

- air glow: thermally excited molecules (by sunlight). Emission takes time, so this continues into the night. OH lines.
- zodiacal light (outside of our atmosphere), due to dust in the solar system. Units: mag arcsec^2. At 21-22 mag, start becoming background limited. In the optical regime.
- sunlight
- moonlight
- aurorae: line emission
- light pollution
- unresolved stars and galaxies (outside of our solar system)
- thermal emission: IR emission from sky, telescope, and dome. $T_{atm} \rightarrow$ blackbody emission.

Many of these sources are emission line sources, not continuum sources, as shown in this plot. How bright are these sources?

- air glow: strong in lines
- zodiacal light: $m_V \sim$ 22.2 - 23.5 depending on ecliptic latitude (and to a lesser degree on ecliptic longitude) Note that this exists even in Earth orbit.
- sunlight: small away from twilight, depending on distance sun is below horizon. Note different definitions of twilight: civil (6 degrees), nautical (12 degrees; "pretty dang dark"), and astronomical (18 degrees; "dark as it's gonna get"). By astronomical twilight, there is essentially no contribution. However, much useful observing might be done before this.
- moonlight: variable; can be very bright. $\sim$ 10 times brighter at full moon.
- light pollution: strong in distinct lines

Optical:

For broadband work, for example in the V band, $m_{sky} \sim 22$ mag/arcsec at good site so we switch over to background limited around m=22 for good image quality, and switch over around m=20 for poorer image quality. Consequently, image quality matters for faint objects. Moonlight is very significant, hence faint optical imaging requires dark time.

Optical: zodiacal light (m_{sky}) readout noise really important. Biggest contributer? Splitting exposures - be careful.. can become rn limited. Minimum exposure time:?

Optical spectroscopy: sky emission generally not much of a problem except around lines so long as moon is down (or work on bright objects); low dispersion observations can be background-limited for long exposures, but at higher dispersion or shorter exposures, spectroscopy is often readout-noise limited.

IR:

Most ($\sim$ 90%) of the emission in the near-IR is from vibrational and rotational transition emission lines of the OH molecule, the so-called "OH forest." For broadband work in H band, $m \sim 13.5$ mag/arcsec; in K band, $m \sim 12.5$ mag/arcsec. So for all except bright objects, we're background limited. This leads to some fundamental differences in data acquisition and analysis between the near-IR and the optical. For infrared spectra, it's harder to estimate S/N: depends on where your feature is located. Moonlight is not very significant, hence much IR work is done in bright time.

IR: m $\sim$ 13 mag arcsec^{-2} ($\sim$ surface brightness).

Wednesday, February 17

Unsorted notes: IR surface brightness $\sim$ 12.13 mag (1000$\times$ brighter than optical). Optical surface brightness $\sim$ 21-22 mag. Surface brightness varies and fluctuates, esp OH lines. Local conditions in atmosphere scales on the order of degrees. Movie: OH lines "dancing around". 6000Å $\rightarrow$ 9000Å brighter. Floor set by zodiacal light with added stuff by moonlight.

Sky brightness from most sources *varies* with time and position in the sky in an *irregular* fashion. Consqeuently, it's essentially impossible to estimate the sky *a priori*: sky must be determined from your observations, and if your observations don't distinguish object from sky, you'd better measure sky close by in location and in time: especially critical in the IR. See some IR movies (really important!); spectral movie from ESO/Paranal see here.

0.1 Transmission of atmosphere

More unorganized notes: Extinction: mostly from scattering and aerosols. SHOULD REVIEW EXTINCTION FROM ISM! Mean extinction curve (individual observatories) subtracted for optical spectroscopy. A and B bands (nomenclature from Fraunhofer - discovered solar liens $\sim$ 1800s. Absorption lines were labeled alphabetically in ν. A and B bands were actually from O_2. C from Hα, D from sodium (the "sodium D line"), and H and K were two lines of ionized calcium (the CaII H and K lines)). Water vapor is the dominant absorber in the IR. Windows in Earth's atmosphere: YJHK are the filters that fit inside these windows, which are carved out by water absorption. This defines the bandpasses. But water vapor changes too, so absorption strength changes a lot. The bandpass itself changes! High peaks [from some video or plot?] are oqaque; transparent toward the bottom. Still molecular absorption in these windows that could show up in spectrum, e.g. the H-band.

Have to correct for this. More atmosphere = more light lost. How much more? Airmass is defined locally:

$$airmass = \frac{amtofairlookingat}{amtofairthroughzenith}$$

So looking at zenith, no matter where you are, airmass = 1.

Earth's atmosphere doesn't transmit 100% of light. Various things contribute to the absorption of light:

- scattering, e.g., Rayleigh scattering off molecules.
- aerosols: scattering off larger particles (e.g. natural aerosols like fog, forest exudates, and geyser steam, and artificial aerosols like haze, dust, particulate air pollutants and smoke). Particle size $\sim 1\ \mu m$.
- variety of molecules:
  - ozone
  - H_2O
  - O_2
  - CO_2
  - N_2O
  - CH_4

  All of these have a λ dependency.

All are functions of wavelength, time to some extent, and position in sky.

Sources of extinction

In the optical part of the spectrum, extinction is a roughly smooth function of wavelength and arises from a combination of ozone, Rayleigh scattering, and aerosols, as shown in this plot. The optical extinction can vary from night to night or season to season, as shown in this plot. Of course, this is showing the variation over a set of photometric nights; if there are clouds, then the level of variation is much higher. Because of this variation, you must determine the amount of extinction on each night separately if you want accuracy better than a few percent (even for photometric nights). Generally, the *shape* of the extinction curve as a function of wavelength probably varies less than the amplitude at any given wavelength. Because of this, one commonly uses *mean extinction coefficients* when doing spectroscopy where one often only cares about relative fluxes. To first order, the extinction from clouds is "gray", i.e. not a function of wavelength, so relative fluxes can be obtained even with some clouds present.

There is significant molecular absorption in the far-red part of the optical spectrum, in particular, note the A (7600) and B (6800) atmospheric bands from O_2. (Historical note: The 'A' and 'B' nomenclature comes from the Fraunhofer lines, which were labeled alphabetically with increasing (decreasing?) frequency when first discovered in the sun's spectrum in the 1800s. Originally they were not recognized as absorption lines. C is Hα, D is the sodium D line, CaII H and K lines, etc.

In the infrared, the extinction does not vary so smoothly with wavelength because of the effect of molecular absorption. In fact, significant absorption bands define the so-called infrared windows (yJHKLM), as shown in the near IR in this plot. At longer wavelengths, the broad absoprtion band behavior continues, as shown in this plot. In this figure, *transmission* = $f(b_\lambda l)$ where l is path length (units of airmass):

$b_\lambda l$	f
-3	1
-2	0.97
-1	0.83
0	0.5
1	0.111
2	0.000

The L band is at 3.5 μ, M band at 5 μ.

Note that even within the IR "windows", there can still be significant telluric (terrestrial) absorption features, e.g. from $CO_2$2, H_2O, and CH_4. When doing IR spectroscopy, one needs to be aware of these and possibly attempt to correct for them, taking care not to confuse them with real stellar features.

Airmass and zenith distance dependence

Clearly, if the light has to pass through a larger path in the Earth's atmosphere, more light will be scattered/absorbed; hence one expects the least amount of absorption directly overhead (zenith), increasing as one looks down towards the horizon.

Definition of airmass: path length that light takes through atmosphere relative to length at zenith: X $\equiv$ 1 vertically (at $z = 0$). Given the zenith distance, z, which can be computed from:

$$\sec z = (\sin\phi \sin\delta + \cos\phi \cos\delta \cos h)^{-1}$$

where ϕ is the latitude, δ is the declination, and h is the hour angle (h = local sidereal time - α, where α is the right ascenstion), we have

$$X \sim \sec z$$

which is exactly true in the case of a plane parallel atmosphere. Since the earth's atmosphere is not a plane, the plane parallel approximation breaks down for larger airmasses. For $X > 2$, a more precise formula is needed, the following gives a higher order approximation:

$$X = \sec z - 0.0018167(\sec z - 1) - 0.002875(\sec z - 1)^2 - 0.0008083(\sec z - 1)^3$$

How much light is lost going through the atmosphere?

Consider a thin sheet of atmosphere, with incident flux F, and outcoming flux $F + \mathrm{d}F$. Let the thin sheet have opacity $\kappa = N\sigma$, where N is the number density of absorbers/scatterers, and σ is the cross-section/absorber-scatterer.

$$\mathrm{d}F = -\kappa F \mathrm{d}x$$

$$F = F_{\mathrm{top}} e^{-\int \kappa \mathrm{d}x} \equiv F_{\mathrm{top}} e^{-\tau}$$

where τ is the *optical depth* of the atmosphere, which parameterizes how much light is lost. Density and cross-section determine the fraction of light that is lost (opacity). For example, $2\kappa \rightarrow e^{-\tau}$ more absorption (not twice as much). Light lost $\sim e^{-\tau}$. Total τ is increased by the airmass.

If the optical depth through the atmosphere is just proportional to the physical path length (true if same atmospheric structure is sampled in different directions), then

$$\tau(X) \sim \tau_0 X$$

where τ_0 is the optical depth at the zenith.

$$F = F_{\mathrm{top}} e^{-\tau_0 X}$$

Expressing things in magnitudes, we have:

$$m = m_0 + 1.086\tau_0 X$$

Define the *extinction coefficient* k_λ:

$$m_0 = m + k_\lambda X$$

$$k_\lambda \equiv -1.086\tau_0$$

(m_0 - top of atmosphere) so the amount of light lost in magnitudes can be specified by a set of extinction coefficients. Note by this definition, the extinction coefficient will be negative; others may use the opposite sign convention (e.g. defining $m_0 = m - k_\lambda X$). Of course, use of the scaling of τ or k with airmass assumes *photometric weather*.

We will talk later about some details of determining extinction coefficient, but the basic idea is that you can determine the extinction by monitoring the brightness of a star (or a set of stars of known brightness) at a range of different airmasses. This needs to be done as a function of wavelength, i.e., for each filter you observe in.

Instrumental mag - what you measure, given equipment, sky conditions, etc. (as opposed to official, recorded magnitude, I think). k_λ has to be measured in each bandpass as it's λ-dependent.

Random thoughts: what's the difference between error, standard deviation, uncertainty, etc.? Stddev is a way to "describe the uncertainty". Errors are something we "can control", like instrumental stuff.

Atmospheric Refraction

Monday, February 22

Seeing: theory and practice

0.1.1 Other sources of seeing

0.1.2 What does seeing cause the image to look like?