ASTR 535 Lecture Notes

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course website: http://astronomy.nmsu.edu/holtz/a535

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Properties of light, magnitudes, errors, and error analysis

Light

Wavelength regimes:

- gamma rays
- x-rays
- ultraviolent (UV)
  - near: 900–3500 Å
  - far: 100–900 Å

  The 900 Å break is because of the Lyman limit at 912 Å. This is where neutral hydrogen is ionized, so the universe is largely opaque to wavelengths shorter than this.
- visual (V): 4000–7000 Å (note that 'V' is different from 'optical', which is slightly broader: 3500–10000 Å. The 3500 Å cutoff is due to the Earth's atmosphere being opaque to wavelengths shorter than this).
- IR
  - near: 1–5 μ (1–10 μ in online notes)
  - mid: (10–100 μ)
  - far: 5–100 μ (100–1000 μ)
- sub-mm 500–1000 μ
- microwave
- radio

Quantities of light:

- Intensity $I(\theta, \phi)$ [erg s^{-1} ν^{-1} Ω^{-1}]: Encapsulates *direction* light is coming from. Also known as radiance.

- Surface Brightness (SB) [erg s^{-1} cm^{-2} ν^{-1} sterradian^{-1}]: amount of energy *received* in a unit surface element per unit time per unit frequency (or wavelength) from a unit solid angle in the direction (θ, ϕ), where θ is the angle away from the normal to the surface element, and ϕ is the azimuthal angle. To calculate SB, just divide the flux by the angle subtended by the object [rad^2]. At a larger distance, the flux will be smaller, but so will the angle subtended by the object, so SB is independent of distance unless considering cosmological scales, where the curvature of spacetime has an effect. If SB is per unit *area*, how is that independent of distance?
- Flux (F) [erg s^{-1} cm^{-2} ν^{-1}]: amount of energy passing through a unit surface element in all directions, defined by

$$F_\nu = \int I_\nu \cos(\theta) \mathrm{d}\Omega \tag{1}$$

  where $\mathrm{d}\Omega$ is the solid angle element, and the integration is over the entire solid angle. Intensity is per solid angle. All the solid angles that make up a sphere add up (integrate) to get the total flux through that surface. The $\cos(\theta)$ factor is important for, e.g., ISM where light is coming from all directions, but for tiny objects, θ is negligibally small and can be dropped. Integrates over *all* directions. Also known as irradiance. Can also be per λ, obviously.
- Luminosity (L) [erg s^{-1}]: *intrinsic* energy emitted by the source per second ($\sim$ power). For an isotropically emitting source,

$$L = 4\pi d^2 F \tag{2}$$

  where d = distance to source (so L can only be calculated if the distance is known). Also known as radiant flux.

What to measure for sources:

- Resolved: directly measure surface brightness (intensity) distribution on the sky, usually over some bandpass or wavelength interval.
- Unresolved: measure the flux. Diffraction is the reason stellar surfaces cannot be resolved. Because of this, we cannot measure SB, so we measure flux, integrated over the entire object.

Questions:

- What are the dimensions of the three quantities: luminosity, surface brightness (intensity), and flux?
- How do the three quantities depend on distance to the source?
- To what quantity is apparent magnitude of a star related?
- To what quantity is the absolute magnitude related?

specific flux: per unit wavelength (or frequency). Using $\lambda = \frac{c}{\nu} \rightarrow \frac{d\lambda}{d\nu} = \frac{-c}{\nu^2}$

$$\begin{aligned}
\int F_\nu \mathrm{d}\nu &= -\int F_\lambda \mathrm{d}\lambda \\
F_\nu \mathrm{d}\nu &= -F_\lambda \mathrm{d}\lambda \\
F_\nu &= -F_\lambda \frac{\mathrm{d}\lambda}{\mathrm{d}\nu} \\
&= F_\lambda \frac{c}{\nu^2} \\
&= F_\lambda \frac{\lambda^2}{c}
\end{aligned}$$

The negative comes from frequency and wavelength increasing in opposite directions. Note that a constant F_λ implies a *non*-constant F_ν and vice versa. For example, a wavelength difference of 100 Å around 1215 Å and 6563 Å corresponds to a frequency difference of 2200 GHz and 71 GHz, respectively.

Units: cgs, magnitudes, Jansky (a flux density unit: 1 Jy = 10^{-26} W m^{-2} Hz^{-1})

Terminology of measurements:

- photometry (broad-band flux measurement): SB or flux, integrated over some wavelength range.
- spectroscopy (*relative* measurement of fluxes at different wavelengths): $f(\lambda)$
- spectrophotometry (*absolute* measurement of fluxes at different wavelengths): $f(\lambda)$
- astrometry: concerned with positions of observed flux, not brightness, but direction.
- morphology: intensity as a function of position; often, absolute measurements are unimportant. Deals with *resolved* objects, intensity as function of position.

In general:

- photometry: measure *flux*
- spectroscopy: flux *density* (per unit wavelength, down to the resolution of the spectrograph)

In practice, measure *photon* flux [photons cm^{-2} s^{-1}] with a "photon counting device", (rather than energy flux, which is done with bolometers). The monochromatic photon flux is given by the energy flux (F_λ) divided by the energy per photon ($E_{photon} = \frac{hc}{\lambda}$), or

$$\text{photon flux} = \int F_\lambda \frac{\lambda}{hc} \mathrm{d}\lambda$$

Have a complete understanding of the difference between intensity, flux, and luminosity and their units. Recognize and understand that these can be specified per unit wavelength or per unit frequency and how to convert between the two.

Magnitudes and photometric systems

Magnitudes are related to flux (or SB or L) by:

$$m_1 - m_2 = -2.5 \log \frac{b_1}{b_2}$$

or for a single object:

$$\begin{aligned} m &= -2.5 \log \frac{F}{F_0} \\ &= -2.5 \log F + 2.5 \log F_0 \end{aligned}$$

where the coefficient of proportionality, F_0, depends on the definition of the photometric system, and the quantity $2.5 \log F_0$ may be referred to as the photometric system *zeropoint*. (Note that this relationship holds *regardless of what photometric system you are using*. Inverting, we get:

$$F = F_0 \times 10^{-0.4\mathrm{m}}$$

Flux *density*: flux per unit wavelength/frequency (aka *monochromatic* flux as opposed to integrated flux). This can be specified by monochromatic magnitudes:

$$F_\lambda = F_0(\lambda) \times 10^{-0.4\mathrm{m}(\lambda)}$$

although spectra are more often given in flux units than in magnitude units. Note that it is possible that F_0 is a function of wavelength.

Since magnitudes are logarithmic, the *difference* between magnitudes corresponds to a *ratio* of fluxes; ratios of magnitudes are generally unphysical. If one is just doing relative measurements of brightness between objects, this can be done without knowledge of F_0 (or, equivalently, the system zeropoint); objects that differ in brightness by ΔM have the same ratio of brightness ($10^{-0.4\Delta M}$) regardless of what photometric system they are in. The photometric system definitions and zeropoints are only needed when converting between calibrated magnitudes and fluxes. In other words, if you reference the brightness of A relative to the brightness of B, a magnitude system can be set up with B as the reference source. However, the utility of a system when doing astrophysics generally requires an understanding of the actual fluxes.

Know how magnitudes are defined, and that relative fluxes can be represented as magnitudes independent of the magnitude system.

Monday, January 25

Three main types of magnitude systems in use in astronomy:

1. STMAG
2. ABNU
3. UBVRI

The STMAG and ABNU magnitude systems are the simplest. In these systems, the reference flux is just some *constant* value in F_λ or F_ν. However, these are not always the most widely used systems in astronomy, because no natural source exists with a flat spectrum.

STMAG (Space Telescope MAGnitude) system: defined relative to F_λ. The reference flux is given by

$$F_{0,\lambda} = 3.60 \times 10^{-9} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1}$$

which is equal to the flux of Vega at $\lambda = 5500$ Å; hence a star of Vega's brightness at 5500 Å is defined to have m=0 (i.e., if $F = F_0$, then $m = \log(F/F_0) = \log(1) = 0$). Alternatively (using $m = -2.5 \log \frac{F}{F_0}$):

$$m_{\text{STMAG}} = -2.5 \log F_\lambda - 21.1$$

for F_λ in cgs units (using cm^{-2} Å^{-1} distinguishes between the collecting area and the wavelength, since both are units of distance. Be careful with units when doing these conversions; as long as the proper flux units come out at the end, the answer should be correct).

ABNU system: defined relative to F_ν. The reference flux is given by

$$F_{0,\nu} = 3.63 \times 10^{-20} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} = F_{\nu,Vega}$$

or

$$m_{\text{ABNU}} = -2.5 \log F_\nu - 48.6$$

for F_ν in cgs units.

Magnitudes usually refer to the *integrated* flux (over a spectral bandpass, not just a single wavelength). The STMAG and ABNU integrated systems are defined relative to sources of *constant* F_λ and F_ν, respectively

$$m_{\text{STMAG}} = -2.5 \log \frac{\int F_\lambda \lambda \mathrm{d}\lambda}{\int 3.6 \times 10^{-9} \lambda \mathrm{d}\lambda}$$

$$m_{\text{ABNU}} = -2.5 \log \frac{\int F_\nu \nu^{-1} \mathrm{d}\nu}{\int 3.6 \times 10^{-20} \nu^{-1} \mathrm{d}\nu}$$

These are defined to be the same at 5500 Å. The factors of λ and ν come from the conversion factor hc/λ for photon-counting detectors, where h and c cancel in each flux ratio.

Note that these systems differ by more than a constant, because one is defined by units of F_λ and the other by F_ν, so the difference betwen the systems is a function of wavelength. (Question: what's the relations between m_{STMAG} and m_{ABNU}?)

Note also that, using magnitudes, the measured magnitude is nearly independent of bandpass width, so a broader bandpass does not imply a brighter (smaller) magnitude, which is not the case for fluxes. The reference is being integrated as well, so they cancel.

The standard UBVRI broadband photometric system, as well as several other magnitude systems, however, are not defined for a constant (flat) F_λ or F_ν spectrum; rather, they are defined relative to the spectrum of an A0V star. Most systems are defined (or at least were originally) to have the magnitude of Vega be zero in all bandpasses (VEGAMAGS); if you ever get into this in detail, note that this is not exactly true for the UBVRI system.

For the broadband UBVRI system, we have

$$m_{\mathrm{UBVRI}} \approx -2.5 \log \frac{\int_{UBVRI} F_\lambda(object)\lambda \mathrm{d}\lambda}{\int_{UBVRI} F_\lambda(Vega)\lambda \mathrm{d}\lambda}$$

(as above, the factor of λ comes in for photon counting detectors). This gives the magnitude in U, B, V, R, *or* I, by integrating over that same bandpass. The UBVRI filter set had overlapping bandpasses, so there was a switch to interference filter: the ugriz system used by SDSS.

Here is a plot to demonstrate the difference between the different systems.

While it seems that STMAG and ABNU systems are more straightforward, in practice it is difficult to measure absolute fluxes, and much easier to measure relative fluxes between objects. Historically, observations were tied to observations of Vega (or to stars which themselves were tied to Vega), so VEGAMAGs made sense, and the issue of determining physical fluxes boiled down to measuring the physical flux of Vega. Today, in some cases, it may be more accurate to measure the absolute throughput of an instrumental system, and use STMAG or ABNU.

Know that there are several different magnitude systems in use, and understand how they differ. Know when it is important to know what the magitude system is, and when it isn't.

Colors

In the UBVRI system, the *difference* between magnitudes (e.g. B-V) gives the ratio of the fluxes in different bandpasses (F_B/F_V) *relative to the ratio of the fluxes of an A0V star in the same two bandpasses* (for VEGAMAG). Note the typical colors of astronomical objects, which are different for the different photometric systems. Type 'A' stars have color = 0, and have the same SED as Vega. Type 'O' stars have color < 0, while cooler stars have color > 0.

Sloan system: ugriz (g-r=0 indicates constant F_ν).

$$m = -2.5 \log \frac{\int_B F_\lambda \mathrm{d}\lambda}{\int_V F_\lambda \mathrm{d}\lambda}$$

if B-V=0, then $(\mathrm{B/V})_{object}$ is the same as $(\mathrm{B/V})_{Vega}$, or $\left(\frac{\int F_\nu}{\int F_\lambda}\right)_{obj} = \left(\frac{\int F_\nu}{\int F_\lambda}\right)_{Vega}$. An object with B-V=0 has the same spectral *shape* as Vega. Not necessarily the same flux value at each bandpass Keep in mind that the UBVRI system is defined relative to spectrum of an A0V star (not simply a flat spectrum, like the STMAG and ABNU systems).

To know how much observing time is needed, you need to know the flux for object with some magnitude, x. E.g., convert the spectrum of an elliptical galaxy to color; if you know F in one filter, you can get F in another filter.

Questions:

- Which is closer to the UBVRI system, STMAG or ABNU?

- What would typical colors be in a STMAG or ABNU system?

Understand how colors are represented by a difference in magnitude, and recognize how colors expressed in magnitude are related to the shape of the underlying spectrum, with differences for different magnitude systems.

UBVRI magnitudes-flux conversion

Wednesday, January 27

To convert Vega-based magnitudes to fluxes, just look up the flux of Vega at the center of the passband. However, if the spectrum of the object differs from that of Vega, this won't be perfectly accurate. Given UBVRI magnitudes of an object in the desired band, filter profiles (e.g. Bessell 1990, PASP 102,1181), and absolute spectrophotometry of Vega (e.g.,), Bohlin & Gilliland 2004, AJ 127, 3508, one can determine the flux.

To estimate the flux of some object in an arbitrary bandpass given just the V magnitude of an object (a common situation used when trying to predict exposures times, see below), estimate the spectral energy distribution (SED). The integral of the SED can be computed over the V bandpass if the filter profiles are known. The scaling is determined by comparing this integral with that of the spectrum of Vega over the same bandpass. The normalized SED is then used to compute the flux in any desired bandpass.

(Some possibly useful references for SEDs are: Bruzual, Persson, Gunn, & Stryker; Hunter, Christian, & Jacoby; Kurucz). Things are certainly simpler in the ABNU or STMAG system, and there has been some movement in this direction: the STScI gives STMAG calibrations for HST instruments, and the SDSS photometric system is close to an ABNU system. However, even when the systems are conceptually well defined, determining the absolute calibration of any photometric system is very difficult in reality, and determining absolute fluxes to the 1% level is very challenging.

As a separate note on magnitudes themselves, note that some people, in particular, the SDSS, have adopted a modified type of magnitudes, called *asinh* magnitudes, which behave like normal (also known as Pogson) magnitudes for brighter objects, but have different behavior for very faint objects (near the detection threshold). See Lupton, Gunn, & Szalay 1999 AJ 118, 1406 for details.

Observed fluxes and the count equation

If you are measuring flux with an actual instrument, the *intrinsic* photon flux from the source is not trivial to determine from the number of photons that you count. To get the number of photons that you count in an observation, you need to take into account

- The area of your photon collector (telescope)
- Photon losses and gains from the Earth's atmosphere (which change with conditions)
- The efficiency of your collection/detection apparatus (which can change with time).

Generally, the astronomical signal (which might be a flux or a surface brightness, depending on whether the object is resolved) can be written in the form of the ***count equation:***

$$S = Tt\int \frac{F_\lambda}{\left(\frac{hc}{\lambda}\right)} q_\lambda a_\lambda \mathrm{d}\lambda \equiv TtS'$$

- S: total number of photons observed (the "signal")
- S': observed flux rate [photons s^{-1} cm^{-2}], with all of the real details of the observing system included.
- a_λ: atmospheric transmission (or *throughput*), the fraction of photons that make it to the detector. A typical value is about 0.9, or ∼90% of the photons. Variable in both space and time.
- q_λ: system efficiency (telescope, instrument, filters, and detector). Can be time-variable.
- T: telescope collecting area
- t: integration time (total amount of time spent collecting photons).
- $\frac{F_\lambda}{(hc/\lambda)}$: flux divided by the energy per photon; gives the number of photons per second per square cm.

T and t are the *only* terms that do not depend on the wavelength (or frequency).

Most of this information isn't needed to go backward from S to F_λ, since the terms can be very difficult to measure precisely. Instead, most observations are performed differentially to a set of other stars of well-known brightness. If the stars of known brightness are observed in the same observation, then the atmospheric term is (approximately) the same for all stars. This is known as *differential photometry.* From the photon flux of the object with known brightness, an "exposure efficiency" or an "effective area" for this exposure can be determined. Equivalently, and more commonly, an *instrumental magnitude* can be calculated:

$$m = -2.5\log\frac{S}{t}$$

Normalize by the exposure time, t, to get counts s^{-1}, (although this is not strictly necessary). Then determine the *zeropoint*, z (which describes the throughput of the system). Adding z to the instrumental magnitude gives the *calibrated* magnitude, M (which is still an *apparent* magnitude).

$$M = m + z$$

Note that in the real world, one has to also consider possible differences between a given experimental setup and the setup used to measure the reference brightnesses, so this is only a first approximation (i.e., the zeropoint may be different for different stars with different spectral properties). If using instrumental mags including exposure time normalization, the zeropoint gives the magnitude of a star that will give 1 count s^{-1}.

$$M = -2.5\log\frac{S}{t} + z$$

Example: to go from observed to emitted →, take an additional observation of an object with known brightness (SDSS has list of these). A star that is 10 times fainter than one with g=18 has g=20.5.

$$m = -2.5 \log(qF)$$

$$m = -2.5 \log F - 2.5 \log q$$

the quantity '$-2.5 \log q$' is the zeropoint.

If there is no nearby star, q is still the same, but changes in a, effects of the atmosphere, need to be calibrated. This is known as *all-sky*, or absolute, photometry. This requires that the sky is "well-behaved", i.e., the atmospheric throughput as a function of position can be accurately predicted, which requires *photometric* weather (no clouds). Differential photometry can be done in non-photometric weather, hence it is much simpler. It is always possible to obtain differential photometry, then later obtain absolute photometry of the reference stars.

To figure out what the fluxes of the calibrating stars actually are requires understanding all of the terms in the count equation. This is challenging, and often absolute calibration of a system is uncertain to a couple of percent.

It is common to stop with differential photometry when studying variable objects, where only the *change* in brightness is important. In this case, the brightness of the target needs to be referenced relative to another object (or ensemble of objects) in the field that are *non*-variable.

While the count equation isn't usually used for calibration, it is very commonly used for computing the approximate number of photons you will receive from a given source in a given amount of time for a given observational setup. This number is critical to know in order to estimate your expected errors and exposure times in observing proposals, observing runs, etc. Understanding errors in absolutely critical in all sciences, and maybe even more so in astronomy, where objects are faint, photons are scarce, and errors are not at all insignificant. The count equation provides the basis for exposure time calculator (ETC) programs, because it gives $N_{\mathrm{photons}}(t)$. As we will see shortly, this provides the information we need to calculate the *uncertainty* in the measurement as a function of exposure time.

Random note: The field of view (FOV) is usually in arcminute scales.

Understand the count equation and the terms in it. Understand the distinction between estimating count rates from an understanding of all the terms in the count equation vs. measuring the overall throughput (zeropoint) by observing stars of known brightness. Know instrumental magnitudes and zeropoints are.

Uncertainties in photon rates

Useful reference: Statistics

For a given rate of *emitted* photons, there's a probability function which gives the rate of *detected* photons. Even assuming 100% detection efficiency, there are still *statistical* uncertainties, and possibly *instrumental* uncertainties.

General probability distributions and their characteristics

$p(x)dx \equiv$ probability of event occuring in $x + \mathrm{d}x$:

$$\int p(x)\mathrm{d}x = 1$$

Some definitions relating to values that characterize a distribution:

$$\text{mean} \equiv \mu = \int xp(x)\mathrm{d}x$$

$$\text{variance} \equiv \sigma^2 = \int (x-\mu)^2 p(x)\mathrm{d}x$$

$$\text{standard deviation} \equiv \sigma = \sqrt{\text{variance}}$$

$$\frac{\int_{-\infty}^{x_{median}} p(x)\mathrm{d}x}{\int_{-\infty}^{\infty} p(x)\mathrm{d}x} = \frac{1}{2}$$

mode: most probable value (peak in plot).

Monday, February 1

Note that the geometric interpretation of the above (well-defined) quantities depends on the nature of the distribution. There is a difference between the *sample* quantities and the *population* quantities. The latter apply to the true distribution, while the former are *estimates* of the latter from some finite sample (N measurements) of the population. The sample values approach the true ones as N approaches infinity, but for small samples, they may differ.

The sample quantities are derived from:

$$\text{sample mean: } \bar{x} \equiv \frac{\sum x_i}{N}$$

$$\text{sample variance: } \equiv \frac{\sum (x_i - \bar{x})^2}{N-1} = \frac{\sum x_i^2 - (\sum x_i)^2/N}{N-1}$$

the binomial distribution

For astronomical observations, the photon distribution can be derived from the *binomial* distribution:

$$P(x,n,p) = \frac{n! p^x (1-p)^{n-x}}{x!(n-x)!}$$

where

- x: number of photons in a single event
- n: total number of events
- p: probability of observing x

under the assumption that all events are independent of each other (e.g., when rolling dice, the result of the second roll is independent of the result of the first). For the binomial distribution:

$$\text{mean} \equiv \mu \equiv \int x p(x) \mathrm{d}x = np$$

$$\text{variance} \equiv \sigma^2 \equiv \int (x-\mu)^2 p(x) \mathrm{d}x = np(1-p)$$

Understand the concept of probability distribution functions and basic quantities used to describe them: mean, variance, standard deviation, median, and mode. Understand the difference between population quantities and sample quantities.

The Poisson distribution

Unlike a gaussian, a poisson distribution is not symmetric about the mean (you can't detect a negative number of photons); it cuts off at zero.

For photon detection, n is the total number of photons emitted, and p is the probability of detecting a single photon out of the total emitted during the observation. We don't know either of these numbers, but we do know that p<<1 and we can estimate the mean number detected:

$$\mu = np$$

In this limit, the binomial distribution asymtotically approaches the *Poisson* distribution:

$$P(x, \mu) = \frac{\mu^x e^{-\mu}}{x!}$$

From the expressions for the binomial distribution in this limit, the mean of the distribution is μ, and the variance is given by:

$$\sigma^2 = \sum_x [(x-\mu)^2 p(x, \mu)]$$

$$\sigma^2 = np = \mu$$

In other words, the standard deviation, σ, is the square root of the mean. **This is an *important result***.

Note that the Poisson distribution is generally the appropriate distribution for *any* sort of counting experiment where a series of events occurs with a known average rate, and are independent of time since the last event.

Plots of what the Poisson distribution looks like for $\mu = 2$, $\mu = 10$, $\mu = 10000$.

Understand the Poisson distribution and when it applies. Know how the variance/standard deviation of the Poisson distribution is related to the mean of the distribution.

The normal, or Gaussian, distribution

For large μ, the Poisson distribution is well-approximated around the peak by a Gaussian distribution:

$$P(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

This allows us to use "simple" least squares techniques to fit observational data, which is normally distributed. However, in the *tails of the distribution*, and at *low mean rates*, the Poisson can differ significantly from a Gaussian. In these cases, least-squares may not be appropriate to model observational data, maximum likelihood techniques need to be considered.

central limit theorem: if a quantity depends on a number of independent random variables with ANY distribution, the quantity itself will be distributed normally (see statistics texts). In observations, we encounter the normal distribution because *readout* noise is distributed normally. Normal distributions occur often in nature.

Know what a Gaussian is, including the full functional form. Understand under what circumstances the Poisson distribution is similar to a normal distribution.

Importance of error distribution analysis

The expected uncertainties in observations need to be understood in order to:

- predict the amount of observing time needed to get uncertainties as small as they need to be.
- Figure out if the scatter in the observed data is consistent with expected uncertainties. If not, then you've either learned some astrophysics or you don't understand something about your observations. This is especially important in astronomy where objects are faint and many projects are pushing down into the noise as far as possible. This can usually only be answered probabilistically. Generally, tests compute the probability that the observations are consistent with an expected distribution (the null hypothesis). If it is low, the null hypothesis can be rejected.
- interpret your results in the context of a scientific prediction

Confidence levels

For example, say we want to know whether some single point is consistent with expectations, e.g., we see a bright point in multiple measurements of a star, and want to know whether the star flared. Say we have a time sequence with known mean and variance, and we obtain a new point, and want to know whether it is consistent with known distribution?

If the form of the probability distribution is known, then you can calculate the probability of getting a measurement more than some observed distance from the mean. In the case where the observed distribution is Gaussian (or approximately so), this is done using the *error function* (sometimes called $erf(x)$), which is the integral of a gaussian from some starting value.

Some simple guidelines to keep in mind follow (the actual situation often requires more sophisticated statistical tests). First, for Gaussian distributions, you can calculate that 68% of the points should fall within $\pm 1\sigma$ from the mean, and 95.3% should fall within $\pm 2\sigma$ from the mean. Thus, if you have a time line of photon fluxes for a star, with N observed points, and a photon noise σ on each measurement, you can test whether the number of points deviating more than 2σ from the mean is much larger than expected. To decide whether any single point is really significantly different, you might want to use more stringent criterion, e.g., 5σ rather than a 2σ criterion; a 5σ has much higher level of significance. On the other hand, if you have far more points in the range $2-4\sigma$ brighter or fainter than you would expect, you may also have a significant detection of intensity variations (provided you really understand your uncertainties on the measurements).

Also, note that your observed distribution should be consistent with your uncertainty estimates given the above guidelines. If you have a whole set of points, that all fall within 1σ of each other, something is wrong with your uncertainty estimates (or perhaps your measurements are correlated with each other).

For a series of measurements, one can calculate the χ^2 statistic, and determine how probable this value is, given the number of points.

$$\chi^2 = \sum[(observed(i) - model(i))^2/\sigma_i^2]$$

A quick estimate of the consistency of the model with the observed data points can be made using reduced χ^2, defined as χ^2 divided by the *degrees of freedom* (number of data points minus number of parameters).

$$\chi^2 = \sum_N \frac{(y_i - y(x_i))^2}{\sigma_i^2}$$

for N data points. If every point was 1σ off, $\chi^2 \sim$ number of data points. Can calculate the probability distribution of χ^2. If $\chi^2 = 98$, what's the confidence interval? Will only get 98 one in a million times? Not a good model. Is the data consistent with the model?

Wednesday, February 3

Noise equation: how do we predict expected uncertainties?

Signal-to-noise

Shape parameterized by stddev which $= \sqrt{\mu} \to$ Fundamental! E.g. source emitting 1000 photons has uncertainty of 100, etc. (telescope, brightness of source, etc. don't matter... only number of photons, no matter how you got them).

$$\frac{S}{N} = \frac{S}{\sqrt{S}} = \sqrt{S}$$

How does S/N change for a given object?

Astronomers often describe uncertainties in terms of the fractional error, e.g. the amplitude of the uncertainty divided by the amplitude of the quantity being measured (the signal); often, the inverse of this, referred to as the signal-to-noise ratio, is used. Given an estimate the number of photons expected from an object in an observation, we can calulate the signal-to-noise ratio:

$$\frac{S}{N} = \frac{S}{\sqrt{\sigma^2}}$$

which is the inverse of the predicted fractional error (N/S).

Consider an object with observed photon flux, S' [cm^{-2} s^{-1}], leading to a signal, $S = S'Tt$. In the simplest case (i.e. perfect instruments), the only noise source is Poisson statistics from the source, in which case:

$$\sigma^2 = S = S'Tt$$
$$\frac{S}{N} = \sqrt{S} = \sqrt{S'Tt}$$

In other words, S/N increases as the square root of the object brightness, telescope area, efficiency, or exposure time. Note that S is directly observable, so one can calculate S/N for an observation without knowing the telescope area or exposure time. We've just broken S down so that you can specifically see the dependence on telescope area and/or exposure time.

Background noise

Rayleigh scattering

A more realistic case includes the noise contributed from Poisson statistics of "background" light, B', which has units of flux per area on the *sky*, not the detector (i.e. a surface brightness); note that this is also usually given in magnitudes (more on the physical nature of this later).

$$B' = \int \frac{B_\lambda}{\frac{hc}{\lambda}} q_\lambda \mathrm{d}\lambda$$

B_λ is the sky brightness at $\lambda \ldots$? And B' is the observed flux from the background. So B vs. B' is the same as S vs. S'.

The amount of background in our measurement depends on how much sky area we observe. Say we just use an aperture with area, A, so the total observed background counts is

$$AB = AB'Tt$$

Again, $B'Tt$ is the directly observable quantity, but we split it into the quantities on which it depends to understand what factors are important in determining S/N.

The total number of photons observed, O, is a combination of photons from the source (S), and phtons from the background (AB):

$$O = S + AB = (S' + AB')Tt$$

The variance of the total observed counts, from Poisson statistics, is:

$$\sigma_O^2 = O = S + AB = (S' + AB')Tt$$

To get the desired signal from the object only, we will need to measure separately the total signal and the background signal to estimate:

$$S \equiv S'Tt = O - A < B >$$

where $< B >$ is some estimate we have obtained of the background surface brightness. The noise in the measurement is

$$\sigma_S^2 \approx \sigma_O^2 = S + AB = (S' + AB')Tt$$

where the approximation is accurate if the background is determined to high accuracy, which one can do if one measures the background over a large area, thus getting a large number of background counts (with correspondingly small fractional error in the measurement).

This leads to a common form of the *noise equation*:

$$\frac{S}{N} = \frac{S}{\sqrt{S + AB}}$$

Breaking out the dependence on exposure time and telescope area, this is:

$$\frac{S}{N} = \frac{S'}{\sqrt{S' + AB'}}\sqrt{T}\sqrt{t}$$

Limiting regimes:

- *signal-limited* case: $S' \gg B'A$, the background goes away and we get

$$\frac{S}{N} = \sqrt{S} = \sqrt{S'tT}$$

- *background-limited* case: $B'A \gg S'$ and

$$\frac{S}{N} = \frac{S}{\sqrt{BA}} = \frac{S'}{\sqrt{B'A}}\sqrt{tT}$$

starlight dominates by factor of 1000 for $m_* = 12.5$ and $m_{BG} = 20$ ($\Delta m = 7.5 = -2.5 \log \frac{b_{BG}}{b_*} \rightarrow 10^3$)

$\odot$: little circle around star to reduce sky background noise. Depends on how blurry the star is; determined by seeing.

- Dust scattering in the plane of the solar system
- Diffraction limit of space telescope $\sim$ 20th of an arcsecond. ($\sim$ angular resolution).

As one goes to fainter objects, the S/N drops, and it drops faster when you're background limited. This illustrates the importance of dark-sky sites, and also the importance of good image quality.

Consider two telescopes of collecting area, T_1 and T_2. If we observe for the same exposure time on each and want to know how much fainter we can see with the larger telescope at a given S/N, we find this for a signal-limited case:

$$S_2 = \frac{T_1}{T_2} S_1$$

but find *this* for the background-limited case:

$$S_2 = \sqrt{\frac{T_1}{T_2}} S_1$$

Instrumental noise

Most common:readout noise. Gaussian, not poisson. $\mu = 0$, σ = readout noise (depends on detector, e.g. 10 electrons ($\sim$ photons create photoelectrons). Get a smear of 10 no matter how many photons are coming in. Represented by range around zero with no exposure time. Mean is zero, so can't subtract it out, it just blurs what you're looking at. B - background per square arcsecond, A - square arcsecond. Number of pixels over which it's spread:

- sharper image
- 10 arcsec all in one pixel

Can change magnification of optics, but there is a balance. Might have two stars five arcsec apart. Maxiumum resolution is not always the best! readout noise is very important when the background is very low. $N_{pix}\sigma_{rn}^2$ compared to BA. rn more important for spectroscopy applications than imaging.

In addition to the errors from Poisson statistics (statistical noise), there may be additional terms from instrumental errors. A common example of this that is applicable for CCD detectors is readout noise, which is additive *noise* (with zero mean) that comes from the detector and is independent of signal level. For a detector whose readout noise is characterized by σ_{rn},

$$\frac{S}{N} = \frac{S}{\sqrt{S + BA_{pix} + \sigma_{rn}^2}}$$

if a measurement is made in a single pixel. If an object is spread over N_{pix} pixels, then

$$\boxed{\frac{S}{N} = \frac{S}{\sqrt{S + BA + N_{pix}\sigma_{rn}^2}}}$$

This is the key equation! For large σ_{rn}, the behavior is the same as the background-limited case. This makes it clear that if you have readout noise, image quality (and/or proper optics to keep an object from covering too many pixels) is important for maximizing S/N. It is also clear that it is critical to have minimum read-noise for low background applications (e.g., spectroscopy).

There are other possible additional terms in the noise equation, arising from things like dark current, digitization noise, errors in sky determination, errors from photometric technique, etc. (we'll discuss some of these later on), but in most applications, the three sources discussed so far - signal noise, background noise, and readout noise - are the dominant noise sources.

Note the applications where one is likely to be signal dominated, background dominated, and readout noise dominated.

Review

Three sources of uncertainty:

1. Possion statistics from source: $\sigma^2 = S$; $\sigma = \sqrt{S}$. S is the same as μ.
2. Poisson statistics from sky/background: $< B >$ is mean background rate.
3. Non-poisson statistics from instrumentation: aka readout noise. $\mu = 0$ (no exposure time). Smear around μ. $\sigma = \sigma_{rn}$ per pixel. $\sigma^2 = N_{pix}\sigma_{rn}^2$; $\sigma = \sqrt{N_{pix}\sigma_{rn}^2}$

Noise, $N = \sqrt{S + BA + N_{pix}\sigma_{rn}^2}$ (source + sky + instrument).

Error propagation

Monday, February 8

So now we know how to estimate uncertainties of observed count rates. Let's say we want to make some calculations (e.g., calibration, unit conversion, averaging, conversion to magnitudes, calculation of colors, etc.) using these observations: we need to be able to estimate the uncertainties in the calculated quantities that depend on our measured quantities.

Total signal: $S = S + B - < B > + O$. There are uncertainties from all four terms, but $< B > \sim 0$ ($\sigma_{<B>} = 0$), so there are actually only three. $S + BA$: total inside aperture; $< B > A$: total outside aperture (with mean background).

Consider what happens if you have several independently measured known quantities with known error distributions and you combine these into some new quantity: we wish to know what the error is in the new quantity.

$$x = f(u, v, \ldots)$$

The question is, what is σ_x if we know σ_u, σ_v, etc.? In other words, how do the individual uncertainties *propogate* onto the uncertainty of the resultant quantity?

As long as errors are small:

$$x_i - < x > \sim \ (u_i - < u >) \left(\frac{\partial x}{\partial u}\right) + (v_i - < v >) \left(\frac{\partial x}{\partial v}\right) + \ldots$$

where $var_i - < var >$ is the uncertainty, and the derivatives are the amount by which the input quantity depends on other quantities.

Possible class example involving S=100, B=10, A=10, and $\sigma_{rn} = 10\ldots$? The *sample variance* (by definition, see above notes) is

$$\sigma_x^2 = lim(N \to \infty) \frac{1}{N} \sum (x_i - < x >)^2$$

$$= \sigma_u^2 \left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v}\right)^2 + 2\sigma_{uv}^2 \frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \dots$$

The last term is the *covariance*, which relates to whether errors are *correlated*. Covarience is the dgree to which the uncertainty in one quantity might be correlated (or not) with another quantity.

$$\sigma_{uv}^2 = lim(N \to \infty)\frac{1}{N}\sum(u_i - < u >)(v_i - < v >)$$

If errors are uncorrelated, then $\sigma_{uv} = 0$ because there's equal chance of getting opposite signs on v_i for any given u_i. When working out errors, make sure to consider whether there are correlated errors. If there are, you *may* be able to reformulate quantities so that they have independent errors: this can be very useful.

Understanding these terms is very important!

Examples for *uncorrelated* errrors:

- addition/subtraction:

$$x = u + v$$

$$\sigma_x^2 = \sigma_u^2 \left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v}\right)^2$$

  Both derivatives = 1, so

$$\sigma_x^2 = \sigma_u^2 + \sigma_v^2$$

  In this case, errors are said to *add in quadrature*.
- multiplication/division:

$$x = uv$$

$$\sigma_x^2 = v^2\sigma_u^2 + u^2\sigma_v^2$$

$$\frac{\sigma_x^2}{v^2u^2} = \frac{\sigma_u^2}{u^2} + \frac{\sigma_v^2}{v^2}$$

- logs:

$$x = \ln u$$

$$\sigma_x^2 = \frac{\sigma_u^2}{u^2}$$

  Note that when dealing with logarithmic quantities, errors in the log correspond to *fractional* errors in the raw quantity.

E.g. $x = 10u \to \sigma_x = 10\sigma_u$

$N \equiv \sigma_S$

Reasons for adding in quadrature: important! Also don't overestimate errors. This is just as bad as underestimating them.

Distribution of resultant errors

When propagating errors, even though you can calculate the variances in the new variables, the distribution of errors in the new variables is not, in general, the same as the distribution of errors in the original variables, e.g. if errors in individual variables are normally distributed, errors in output variable is not necessarily.

When two variables are added, however, the output is normally distributed.

Determining sample parameters: averaging measurements

We've covered errors in single measurements (σ_i). Next we turn to averaging measurements. Say we have multiple observations and want the best estimate of the mean and variance of the population, e.g. multiple measurements of stellar brightness. Here we'll define the best estimate of the mean as the value which maximizes the likelihood that our estimate equals the true parent population mean.

For equal errors (simplest case), this estimate just gives our normal expression for the sample mean:

$$\bar{x} = \frac{\sum x_i}{N}$$

Using error propagation, the estimate of the error in the sample mean is given by:

$$\sigma_{\bar{x}}^2 = \sum \frac{\sigma_i^2}{N^2} = \frac{\sigma^2}{N}$$

But what if errors on each observation aren't equal, say for example we have observations made with several different exposure times? Then the optimal determination of the mean is using a:

$$\text{weighted mean} = \frac{\sum \frac{x_i}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}}$$

which can be derived from maximum liklihood. Large weight means small error. The estimated error in this value is given by:

$$\sigma_\mu^2 = \sum \frac{\frac{\sigma_i^2}{\sigma_i^4}}{(\sum \frac{1}{\sigma_i^2})^2} = \frac{1}{\sum \frac{1}{\sigma_i^2}}$$

where the σ_i's are individual weights/errors (people often talk about the *weight* of an observation, i.e. $\frac{1}{\sigma_i^2}$: large weight means small error.

This is a standard result for determining sample means from a set of observations with different weights.

However, there can sometimes be a subtlety in applying this formula, which has to do with the question: how do we go about choosing the weights/errors, σ_i? We know we can *estimate* σ using Poisson statistics for a given count rate, but remember that this is a sample variance (which may be based on a single observation) not the true population variance. This can lead to biases.

Consider observations of a star made on three nights, with measurements of 40, 50, and 60 photons. It's clear that the mean observation is 50 photons. However, beware of the being trapped by your error *estimates*. From each observation alone, you would estimate errors of $\sqrt{40}$, $\sqrt{50}$, and $\sqrt{60}$. If you plug these error estimates into a computation of the weighted mean, you'll get a mean rate of $< x >= 48.64$.

Imagine 10,000 measurements; 50 is the *true* population mean. σ is really $\sqrt{50}$ for all 3 observations above… estimation of true uncertainties. Avoid by: revised mean: $50 \rightarrow 48.64 \ldots \sqrt{48.64}$. Something other than Poisson stats, some measurements have more weight than others, same uncertainty across all measurements.

Using the individual estimates of the variances, we'll bias values to lower rates, since these will have estimated higher S/N.

Note that it's pretty obvious from this example that you should just weight all observations equally. However, note that this certainly isn't always the right thing to do. For example, consider the situation in which you have three exposures of different exposure times. Here you probably want to give the longer exposures higher weight (at least, if they are signal or background limited). In this case, you again don't want to use the individual error estimates or you'll introduce a bias. There is a simple solution here also: just weight the observations by the exposure time. However, while this works fine for Poisson errors (variances proportional to count rate), it isn't strictly correct if there are instrumental errors as well which don't scale with exposure time. For example, the presence of readout noise can have this effect; if all exposures are readout noise dominated, then one would want to weight them equally, if readout noise dominates the shorter but not the longer exposures, once would want to weight the longer exposures even higher than expected for the exposure time ratios! The only way to properly average measurements in this case is to estimate a sample mean, then use this value scaled to the appropriate exposure times as the input for the Poisson errors.

Another subtlety: averaging counts and converting to magnitudes is not the same as averaging magnitudes.

Wednesday, February 10

Can you split exposures?

Although from S/N considerations, one can determine the required number of counts you need (exposure time) to do your science, when observing, one must also consider the question of whether this time should be collected in single or in multiple exposures, i.e. how long individual exposures should be. There are several reasons why one might imagine that it is nicer to have a sequence of shorter exposures rather than one single longer exposure (e.g., tracking, monitoring of photometric conditions, cosmic ray rejection, saturation issues), so we need to consider whether doing this results in poorer S/N.

Consider the object with photon flux S', background surface brightness B', and detector with readout noise σ_{rn}. A single short exposure of time t has a variance:

$$\sigma_S^2 = S'Tt + B'ATt + N_{pix}\sigma_{rn}^2$$

If N exposures are summed, the resulting variance will be

$$\sigma_N^2 = N\sigma_S^2$$

If a single long exposure of length Nt is taken, we get

$$\sigma_L^2 = S'TNt + B'ATNt + N_{pix}\sigma_{rn}^2$$

The ratio of the noises, or the inverse ratio of the S/N (since the total signal measured is the same in both cases), is

$$\frac{\sigma_N}{\sigma_L} = \sqrt{\frac{S'TNt + B'ATNt + NN_{pix}\sigma_{rn}^2}{S'TNt + B'ATNt + N_{pix}\sigma_{rn}^2}}$$

The only difference is in the readout noise term. In the signal- or background-limited regimes, exposures can be added with no loss of S/N. However, if readout noise is significant, then splitting exposures leads to reduced S/N.

If the desired S/N reguires one hour of exposure, is it better to have, e.g. 3600 1-second exposure, 1800 2-second exposures, 300 12-second exposures... Pros of lots of short exposures:

- avoid saturation
- tracking
- cosmic rays
- variable object
- monitor atmospheric throughput variation

cons of lots of exposures:

- data processing
- high data quantity
- can't see object
- readout time penalty

If tracking is bad, image can be blurry, and would get more background. If you're signal-limited though, this doesn't matter. How do you figure out the minimum amount of time to expose? Need to be above the readout noise. If [rn?] $= 10$, then $S_{min} = 100$ (10^2). σ_{rn} is not dependent on t. "Readout noise limited".

Random errors vs systematic errors

So far, we've been discussing *random* errors. There is an additional, usually more troublesome, type of errors known as *systematic* errors. These don't occur randomly but rather are correlated with some, possibly unknown, variable relating to your observations.

EXAMPLE : flat fielding

EXAMPLE : WFPC2 CTE

Note also that in some cases, systematic errors can masquerade as random errors in your test observations (or be missing altogether if you don't take data in exactly the same way), but actually be systematic in your science observations.

EXAMPLE: flat fielding, subpixel QE variations.

Note that error analysis from expected random errors may be the only clue you get to discovering systematic errors. To discover systematic errors, plot residuals vs. everything

Important to deal with random errors correctly; don't overestimate error bars. Flat fielding: uniform brightness → non-uniformity in sensitivity. To what % accuracy is something "flat"?

Monday, February 15

zeropoint, z, bundles count equation into one number for a *particular bandpass*. This is an empirical measurement of the count equation [counts]. Use the count equation to derive the zeropoint. If the error bars are smaller than the scatter of the data points themselves, there's a problem. χ^2 would be much [larger?] than one. A possible issue is the pixel senstivity isn't uniform across a ccd. Stars at the bottom of the chip could be fainter than the ones at the top. CCDs in space telescope. (CCDs are read by moving charge through chip → *grating*.) Something about charge-transfer efficiency… observation scatter larger than expected scatter. See handwritten notes for diagrams and more on this.

Important to know the equation for error propagation!

Effects of the earth's atmosphere

Four effects:

1. Emission
2. Absorption
3. Refraction (shifting apparent direction of incoming light)
4. Seeing (degrading the coherence of incoming light, leading to degradation of image quality when collecting light over a large aperture)

Night sky emission

(brightness as function of wavelength)

Sources (many of which are emission line sources, not continuum sources, as shown in this plot):

- **air glow:** thermally excited molecules (by sunlight). Emission takes time, so this continues into the night. OH lines. Lines are pretty bright.
- **zodiacal light:** (outside of our atmosphere), caused by dust in the solar system, and emits in the optical regime. Brightness: $m_V \sim 22.2$ - 23.5 [magnitudes arcsec^{-2}], depending on ecliptic latitude (and to a lesser degree on ecliptic longitude) Note that this exists even in Earth orbit. When brightness of observed objects drop to 21-22 mag, go from signal-limited to background-limited.
- **sunlight:** Brightness is small away from twilight, depending on distance sun is below horizon. Note that there are Different definitions of twilight:
  - civil (6 degrees)
  - nautical (12 degrees; "pretty dang dark"),
  - astronomical (18 degrees; "dark as it's gonna get").

  By astronomical twilight, there is essentially no contribution from sunlight, but much useful observing can be done before this.
- **moonlight:** Brightness is variable; can be very bright. $\sim$ 10 times brighter at full moon (compared to what?).
- **aurorae:** line emission
- **light pollution:** Strong (bright) in distinct lines
- **unresolved stars and galaxies:** (outside of our solar system)
- **thermal emission:** IR emission from sky, telescope, and dome. T$_{atm}$ $\rightarrow$ blackbody emission.

Optical:

For broadband work, for example in the V band, $m_{sky} \sim 22$ mag/arcsec at good site, so S/N becomes background limited around m=22 for good image quality, and m=20 for poorer image quality. Consequently, image quality matters for faint objects. Moonlight is very significant, hence faint optical imaging requires dark time. Readout noise of zodiacal light (m_{sky}) really important. Be careful when splitting exposures; can become readoutnoise limited. Consider the minimum exposure time needed (context?)

Optical spectroscopy: sky emission is generally not much of a problem (except around lines) so long as moon is down (or work on bright objects); low dispersion observations can be background-limited for long exposures, but at higher dispersion or shorter exposures, spectroscopy is often readout-noise limited.

IR:

Most ($\sim 90\%$) of the emission in the near-IR is from vibrational and rotational transition emission lines of the OH molecule, the so-called "OH forest." For broadband work:

- H band: $m \sim 13.5$ mag arcsec^{-1}
- K band: $m \sim 12.5$ mag arcsec^{-1}.

($\sim$ surface brightness); So for all except bright objects, we're background limited. This leads to some fundamental differences in data acquisition and analysis between the near-IR and the optical. For infrared spectra, it's harder to estimate S/N: depends on where your feature is located. Moonlight is not very significant, hence much IR work is done in bright time.

Wednesday, February 17

Unsorted notes: IR surface brightness $\sim$ 12.13 mag (1000$\times$ brighter than optical). Optical surface brightness $\sim$ 21-22 mag. Surface brightness varies and fluctuates, esp OH lines. Local conditions in atmosphere scales on the order of degrees. Movie: OH lines "dancing around". 6000Å $\rightarrow$ 9000Å brighter. Floor set by zodiacal light with added stuff by moonlight.

Farther in the IR (5 μ +), thermal emission from the sky dominates and is extremely bright. In fact, when working at wavelengths with thermal background, the exposure time is often limited by the time it takes to saturate the detector with background (sky) light, in seconds or less.

Sky brightness from most sources *varies* with time and position in the sky in an *irregular* fashion. Consqeuently, it's essentially impossible to estimate the sky *a priori*: sky must be determined from your observations, and if your observations don't distinguish object from sky, you'd better measure sky close by in location and in time: especially critical in the IR. See some IR movies (really important!); spectral movie from ESO/Paranal see here.

Transmission of atmosphere

More unorganized notes: Extinction: mostly from scattering and aerosols. SHOULD REVIEW EXTINCTION FROM ISM! Mean extinction curve (individual observatories) subtracted for optical spectroscopy. A and B bands (nomenclature from Fraunhofer - discovered solar liens $\sim$ 1800s. Absorption lines were labeled alphabetically in ν. A and B bands were actually from O_2. C from Hα, D from sodium (the "sodium D line"), and H and

K were two lines of ionized calcium (the CaII H and K lines)). Water vapor is the dominant absorber in the IR. Windows in Earth's atmosphere: YJHK are the filters that fit inside these windows, which are carved out by water absorption. This defines the bandpasses. But water vapor changes too, so absorption strength changes a lot. The bandpass itself changes! High peaks [from some video or plot?] are oqaque; transparent toward the bottom. Still molecular absorption in these windows that could show up in spectrum, e.g. the H-band. Have to correct for this. More atmosphere = more light lost. How much more? Airmass is defined locally:

$$\text{airmass} = \frac{\text{amt of air looking at}}{\text{amt of air through zenith}}$$

So looking at zenith, no matter where you are, airmass = 1.

Earth's atmosphere doesn't transmit 100% of light. Various things contribute to the absorption of light:

- scattering, e.g., Rayleigh scattering off molecules.
- aerosols: scattering off larger particles (e.g. natural aerosols like fog, forest exudates, and geyser steam, and artificial aerosols like haze, dust, particulate air pollutants and smoke). Particle size $\sim 1\ \mu m$.
- variety of molecules:
  - ozone
  - H_2O
  - O_2
  - CO_2
  - N_2O
  - CH_4

  All of these have a λ dependency.

All are functions of wavelength, time to some extent, and position in sky.

Sources of extinction

In the optical part of the spectrum, extinction is a roughly smooth function of wavelength and arises from a combination of ozone, Rayleigh scattering, and aerosols, as shown in this plot. The optical extinction can vary from night to night or season to season, as shown in this plot. Of course, this is showing the variation over a set of photometric nights; if there are clouds, then the level of variation is much higher. Because of this variation, you must determine the amount of extinction on each night separately if you want accuracy better than a few percent (even for photometric nights). Generally, the *shape* of the extinction curve as a function of wavelength probably varies less than the amplitude at any given wavelength. Because of this, one commonly uses *mean extinction coefficients* when doing spectroscopy where one often only cares about *relative* fluxes. To first order, the extinction from clouds is "gray", i.e. not a function of wavelength, so relative fluxes can be obtained even with some clouds present.

There is significant molecular absorption in the far-red part of the optical spectrum, in particular, note the A (7600) and B (6800) atmospheric bands from O_2. (Historical note: The 'A' and 'B' nomenclature comes from the Fraunhofer lines, which were labeled alphabetically with increasing (decreasing?) frequency when first discovered in the sun's spectrum in the 1800s. Originally they were not recognized as absorption lines. C is Hα, D is the sodium D line, CaII H and K lines, etc.

In the infrared, the extinction does not vary so smoothly with wavelength because of the effect of molecular absorption. In fact, significant absorption bands define the so-called infrared windows (yJHKLM), as shown in the near IR in this plot. At longer wavelengths, the broad absoprtion band behavior continues, as shown in this plot. In this figure, *transmission* $= f(b_\lambda l)$ where l is path length (units of airmass):

$b_\lambda l$	f
-3	1
-2	0.97
-1	0.83
0	0.5
1	0.111
2	0.000

The L band is at 3.5 μ, M band at 5 μ.

Note that even within the IR "windows", there can still be significant telluric (terrestrial) absorption features, e.g. from CO_2, H_2O, and CH_4. When doing IR spectroscopy, one needs to be aware of these and possibly attempt to correct for them, taking care not to confuse them with real stellar features.

Airmass and zenith distance dependence

Clearly, if the light has to pass through a larger path in the Earth's atmosphere, more light will be scattered/absorbed; hence one expects the least amount of absorption directly overhead (zenith), increasing as one looks down towards the horizon.

Definition of airmass: path length that light takes through atmosphere relative to length at zenith: X $\equiv$ 1 vertically (at $z = 0$). Given the zenith distance, z, which can be computed from:

$$\sec z = (\sin\phi \sin\delta + \cos\phi \cos\delta \cos h)^{-1}$$

where ϕ is the latitude of the observatory location, δ is the declination, and h is the hour angle (h = local sidereal time - α, where α is the right ascenstion), we have

$$X \sim \sec z$$

which is exactly true in the case of a plane parallel atmosphere. Since the earth's atmosphere is not a plane, the plane parallel approximation breaks down for larger airmasses. For $X > 2$, a more precise formula is needed, the following gives a higher order approximation:

$$X = \sec z - 0.0018167(\sec z - 1) - 0.002875(\sec z - 1)^2 - 0.0008083(\sec z - 1)^3$$

How much light is lost going through the atmosphere?

Consider a thin sheet of atmosphere, with incident flux F, and outgoing flux $F + \mathrm{d}F$. Let the thin sheet have opacity $\kappa = N\sigma$, where N is the number density of absorbers/scatterers, and σ is the cross-section/absorber-scatterer.

$$\mathrm{d}F = -\kappa F \mathrm{d}x$$

$$F = F_{\mathrm{top}} e^{-\int \kappa \mathrm{d}x} \equiv F_{\mathrm{top}} e^{-\tau}$$

where τ is the *optical depth* of the atmosphere, which parameterizes how much light is lost. Density and cross-section determine the fraction of light that is lost (opacity). For example, $2\kappa \to e^{-\tau}$ more absorption (not twice as much). Light lost $\sim e^{-\tau}$. Total τ is increased by the airmass.

If the optical depth through the atmosphere is just proportional to the physical path length (true if same atmospheric structure is sampled in different directions), then

$$\tau(X) \sim \tau_0 X$$

where τ_0 is the optical depth at the zenith.

$$F = F_{\mathrm{top}} e^{-\tau_0 X}$$

Expressing things in magnitudes, we have:

$$m = m_0 + 1.086 \tau_0 X$$

Define the *extinction coefficient* k_λ [magnitudes of absorption per airmass, I think]:

$$m_0 = m + k_\lambda X$$

$$k_\lambda \equiv -1.086 \tau_0$$

(m_0 - top of atmosphere) so the amount of light lost in magnitudes can be specified by a set of extinction coefficients. Note by this definition, the extinction coefficient will be negative; others may use the opposite sign convention (e.g. defining $m_0 = m - k_\lambda X$). Of course, use of the scaling of τ or k with airmass assumes *photometric weather*.

We will talk later about some details of determining extinction coefficient, but the basic idea is that you can determine the extinction by monitoring the brightness of a star (or a set of stars of known brightness) at a range of different airmasses. This needs to be done as a function of wavelength, i.e., for each filter you observe in.

Instrumental mag - what you measure, given equipment, sky conditions, etc. (as opposed to official, recorded magnitude, I think). k_λ has to be measured in each bandpass as it's λ-dependent.

Random thoughts: what's the difference between error, standard deviation, uncertainty, etc.? Stddev is a way to "describe the uncertainty". Errors are something we "can control", like instrumental stuff.

Atmospheric Refraction

Monday, February 22

Amount of refraction depends on where you're looking zenith: = 0.

The direction of light as it passes through the atmosphere is also changed because of refraction since the index of refraction changes through the atmosphere. The amount of change is characterized by Snell's law:

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

Let z_0 be the true zenith distance (relative to how much star moves), z be the observed zenith distance, z_n by the observed zenith distance at layer n in the atmosphere, μ be the index of refraction at the surface, and μ_n be the index of refraction at layer n. At the top of the atmosphere:

$$\frac{\sin z_0}{\sin z_N} = \frac{\mu_N}{1}$$

At each infinitessimal layer:

$$\frac{\sin z_n}{\sin z_{n-1}} = \frac{\mu_{n-1}}{\mu_n}$$

and so on for each layer down to the lowest layer:

$$\frac{\sin z_1}{\sin z} = \frac{\mu}{\mu_1}$$

Multiply these to get:

$$\sin z_0 = \mu \sin z$$

from which we can see that the refraction depends *only on the index of refraction near the earth's surface*.

We define astronomical refraction, r, to be the angular amount that the object is displaced by the refraction of the Earth's atmosphere:

$$\sin(z + r) = \mu \sin z$$

In cases where r is small (pretty much always):

$$\sin z + r \cos z = \mu \sin z$$

$$r = (\mu - 1) \tan z$$

$$\approx R \tan z$$

where we have defined R, known as the "constant of refraction".

A typical value of the index of refraction is $\mu \sim 1.00029$, which gives $R = 60$ arcsec (red light).

The direction of refraction is that a star apparently moves *towards* the zenith. Consequently in most cases, star moves in both RA and DEC:

$$r_\alpha = r \sin q$$

$$r_\delta = r \cos q$$

where q is the *parallactic* angle, the angle between N and the zenith:

$$\sin q = \cos\phi \frac{\sin h}{\sin z}$$

Note that the expression for r is only accurate for small zenith distances ($z <\sim 45$). At larger z, can't use plane parallel approximation. Observers have empirically found:

$$r = A \tan z + B \tan^3 z$$

$$A = (\mu - 1) + B$$

$$B \sim -0.07''$$

but these vary with time, so for precise measurements, you would have to determine A and B on your specific night of observations.

Why is it important to understand refraction? Clearly, it's relevant for pointing a telescope, but this is generally always automatically handled in the telescope pointing software. If you're just taking images, then the stars are just a tiny bit moved relative to each other, but who really cares? One key issue today is the use of multiobject spectrographs, where slits or fibers need to be placed on objects to accuracies of a fraction of an arcsec. For small fields, refraction isn't too much of an issue, but for large fields, it can be . . . note SDSS plates.

The other *extremely* important effect of refraction arises because the index of refraction varies with wavelength, so the astronomical refraction also depends on wavelength: This

λ	R
3000	63.4
4000	61.4
5000	60.6
6000	60.2
7000	59.9
10000	59.6
40000	59.3

gives rise to the phenomenon of *atmospheric dispersion*, or *differential refraction*. Because of the variation of index of refraction with wavelenth, every object actually appears as a little spectrum with the blue end towards the zenith. The spread in object position is proportional to tan z.

Measuring rotation curve of galaxy. Not at parallactic angle? Not getting all the light, just at wavelength used to position the slit (?) Instrument: atmospheric dispersion corrector. No rainbow straight up above gets bigger as you move down. <u>Two</u> prisms that move relative to each other (APO doesn't have this.)

1. emission, 2. transmission, 3. refraction, 4. seeing:theory and practice. Adaptive optics - deformable mirrors that move really fast.

This effect is critical to understand for spectroscopy when using a slit or a fiber, since the location of an object in the sky depends on the wavelength. If you point to the location at one wavelength, you can miss another wavelength (depending on how big your fiber is) and the relative amount of flux you collect will be a function of wavelength, something you may want to take into account if you're interested in the relative flux (continuum shape) over a broad wavelength range. Note the consequent importance of the relation between the orientation slit orientation and the parallactic angle: a slit aligned with the parallactic angle will not lose light as a function of wavelength, but otherwise it will. However, for a slit at the parallactic angle, be careful about matching up flux at different wavelengths for extended objects.

Seeing: theory and practice

References: Coulson, ARAA 23,19; Beckers, ARAA 31, 13; Schroeder 16.II.

Generally, a perfect astronomical optical system will make a perfect (diffraction-limited) image for an incoming plane wavefront of light. The plane will focus to a nice point. The Earth's atmosphere is turbulent and variations in the index of refraction cause the plane wavefront from distant objects to be distorted. These cause several astronomical effects:

- scintillation, which is amplitude variations, which typically vary over scales of cm: generally very small for larger apertures. "twinkling", small aperture (e.g. eye) prevents from getting below a fraction of a % in brightness... something. Planets don't twinkle because they're bigger
- seeing: positional changes and image quality changes. The effect of seeing depends on aperture size: for small apertures, one sees a diffraction pattern moving around, while for large apertures, one sees a set of diffraction patterns (speckles) moving around on scale of $\sim$ 1 arcsec. These observations imply:
  - local wavefront curvatures flat on scales of small apertures
  - instantaneous slopes vary by $\sim$ an arcsec

The time variation scales are several milliseconds and up.

star actually moves around: A/B, where A is the aperture size and B is the size of wiggles in the wavefront. Comparing 8" telescope to 4-m telescope (See handwritten diagrams). Both telescopes produced the same end resutl, but how it got there is different for each one. Difference? Aperture size compared to length of [curve] $\sim$ 6-8 inches (size scale of wavefront pieces) approx. same as waveplane front.

Wednesday, February 24

Cells in the atmosphere each have a different index of refraction. All we have are statistical descriptions of turbulence. Difference of squares: average over all points. E.g. at 1 cm rms [something] = x, at 0.5 cm rms = y, etc.

The effect of seeing can be derived from theories of atmospheric turbulence, worked out originally by Kolmogorov, Tatarski, Fried. Here are some pertinent results, without derivation:

A turbulent field can be described statistically by a *structure function*:

$$D_N(x) == < |N(r+x) - N(r)|^2 >$$

where x is the separation of points, N is any variable (e.g. temperature, index of refraction, etc.), and r is position. This describes how the "quantity" changes as a function of separation between the two points. Quantity of... turbulence? Translate N to phase change of wave (determine image quality).

The structure function of a velocity field can be described as the difference between the values of the flow velocity between two points with coordinates r and $r+x$. See this paper.

Komogorov turbulence gives:

$$D_n(x) = C_n^2 x^{2/3}$$

where C_n is the refractive index structure constant. From this, one can derive the phase structure function at the telescope aperture:

$$D_\phi(x) = 6.88\left(\frac{x}{r_0}\right)^{5/3}$$

where the coherence length r_0 (also known as the Fried parameter) is:

$$r_0 = 0.185\left(\lambda^{6/5}\right)\left(\cos^{3/5} z\right)\left[\int \left(C_n^2 \mathrm{d}h\right)\right]^{-3/5}$$

where z is the zenith angle and λ is the wavelength. Using optics theory, one can convert D_ϕ into an image shape. Bigger r_0 is better; integral is over the height of the atmosphere. r_0 same size as telescope...?

Physically, r_0 is (roughly) proportional to the image size from seeing:

$$d \sim \lambda/r_0$$

as compared with the image size from diffraction-limited images:

$$d \sim \lambda/D$$

Seeing dominates when $r_0 < D$; a larger r_0 means better seeing.

Seeing is more important than diffraction at shorter wavelengths (and for larger apertures); diffraction is more important at longer wavelengths (and for smaller apertures); the effects of diffraction and seeing cross over in the IR for most astronomical-sized telescopes ($\sim$5 microns for 4m); the crossover falls at a shorter wavelength for smaller telescope or better seeing.

Image size from seeing is almost the same, regardless of λ (*little* better toward the IR, but not much). Diffraction swamps seeing at *really* long wavelengths. Bigger telescopes are seeing-limited. r_0 has λ in it, hence the lack of λ dependence. Crossover in the micron regime.

The meat of r_0 is in $\int (C_n^2 \mathrm{d}h)$; as you might expect, this varies from site to site and also in time. At most sites, there seems to be three regimes of "surface layer" (wind-surface interactions and manmade seeing), "planetary boundary layer" (influenced by diurnal heating), and "free atmosphere" (10 km is tropopause: high wind shears), as seen in this plot. A typical astronomical site has $r_0 \sim 10$ cm at 5000Å.

We also want to consider the coherence of the same turbulence pattern over the sky: this coherence angle is called the *isoplanatic angle*, and the region over which the turbulence pattern is the same is called the *isoplanatic patch*. This is relevant to adaptive optics, where we will try to correct for the differences across the telescope aperture; if we do a single correction, how large a field of view will be corrected?

$$\theta \sim 0.314 r_0/H$$

where H is the average distance of the seeing layer:

$$H = \sec z \left[\frac{\int \left(C_n^2 h^{5/3} \mathrm{d}h\right)}{\int (C_n^2 \mathrm{d}h)} \right]^{3/5}$$

For $r_0 = 10$ cm, $H \sim 5000$ m, $\theta \sim 1.3$ arcsec. In the IR, $r_0 = 70$ cm, $H \sim 5000$ m, $\theta \sim 9$ arcsec.

Note however, that the "isoplanatic patch for image motion" (not wavefront) is $\sim 0.3D/H$. For $D = 4$ m, $H \sim 5000$ m, $\theta_{kin} = 50$ arcsec. This is relevant for low-order atmosphereic correction, i.e. tip-tilt, where one is doing *partial* correction of the effect of the atmosphere.

Want the isoplanatic patch to be big. Area where wiggles, wavefronts are the same. Smaller patch if H (height in atmosphere) is bigger. H is bigger at top of atm than at bottom. Take out seeing entirely: correction in one area doesn't work in another area. Space telescopes get wider FOV. Adaptive optics: multiple corrections for different angles (complicated). Better in IR (bigger r_0). 1/2' or 1' patch. Most adaptive optics in NIR. Partial correction: coherent over larger angular scale, but not down to diffraction-limited. Wobbly wavelength front; can correct *some* spatial area. Measure *mean* tilt and correct for that → *active* optics (or tip-tilt system).

Other sources of seeing

Although the Earth's atmosphere provides a limit on the quality of images that can be obtained, at many observatories, there are other factors that can dominate the image quality budget. These have been recognized over the past several decades to be significant effects.

- Dome seeing: arises from turbulence pattern around the dome and the interface between inside the dome and outside the dome. Even small temperature differences can lead to significant image degradataion. $T_{out} = T$, $T_{in} = T + \Delta T$. Could open dome early, climate control during the day, get rid of it completely (roll-off roof!). This tends to be worse for smaller domes.

- Mirror seeing: arises from turbulence right above the surface of the mirror, which can arise if the mirror temperature differs from that of the air above it. Thinner mirrors are better; less mass, or thermal something-or-other.
- Wind shake of the telescope. Maybe getting rid of the dome isn't such a good idea... Good idea to put telescopes on mountain on edge facing the wind. As the wind goes up over where the telescope is, the turbulence happens later, on the other side, as it flows down the less-steep side of the mountain. Minimizeing turbulence from wind laminar flow, or something like that.
- Poor telescope tracking.
- Design, quality, and alignment of the telescope optics: (In general, however, telescope design is done such that the image degradation from telescope design is significantly smaller than that arising from seeing).
- (lack of) effect of clouds. Cause decreased *transparency*, but don't actually make the images blurry. The seeing can atually be better with thin haze.

What does seeing cause the image to look like?

Monday, February 29

The "quality" of an image can be described in many different ways. The overall shape of the distribution of light from a point source is specified by the *point spread function* (PSF). Diffraction gives a basic limit to the quality of the PSF, but seeing, aberrations, or image motion add to structure/broadening of the PSF. For a good ground-based telescope, seeing is generally the dominant component of the PSF.

There is no equation for seeing; the most common way of describing the seeing is by specifying the full-width-half-maximum (FWHM) of the image, which may be estimated either by direct inspection or by fitting a function (usually a Gaussian, where $FWHM = 2\sigma\sqrt{2\ln 2} \approx 2.354\sigma$) However, the sampled PSF is integrated over pixels; not the PSF. The FWHM doesn't fully specify a PSF, so always consider how applicable the quantity is.

Another way of describing the quality of an image is to specify its *modulation transfer function* (MTF). The MTF and PSF are a Fourier transform pair, thin gaussian ←FT→ fat gaussian so the MTF gives the *power* in an image on various *spatial* scales. Turbulence theory makes a prediction for the MTF from seeing:

$$MTF(\nu) = \exp\left[-3.44\left(\frac{\lambda\nu}{r_o}\right)^{5/3}\right]$$

where ν is the spatial frequency, $\xi = \frac{1}{\lambda}$ [cycles per cm]. (Aside: wavenumber $k = \frac{2\pi}{\lambda} = 2\pi\xi$)

- MTF: power as function of spatial frequency
- PSF: intensity as function of position

Note that a gaussian goes as $e^{-\nu^2}$, so this is close to a gaussian. The shape of seeing-limited images is roughly Gaussian in core but has more extended wings. This is relevant because the seeing is often described by fitting a Gaussian to a stellar profile. Note that the stellar profile is the same for all stars, and doesn't depend on the brightness of the star. They all get lost in the noise eventually… sooner for fainter stars, which look smaller. In case the wings are needed, fitting to a Gaussian won't work; a potentially better empirical fitting function is a Moffat function:

$$I = p_1 \left[1 + \frac{(x - p_2)^2}{p_4^2} + \frac{(y - p_3)^2}{p_5^2} \right]^{-p_6}$$

"Nothing special, six parameters, more accurate."

Ways of characterizing the PSF:

- *encircled energy* as a function of radius, or at some specified radius. The encircled energy is just the cumulative integral of the PSF. Encircled energy requirements are often used for specifying optical tolerances.
- Strehl ratio: more commonly used in adaptive optics applications not for "normal observing". The Strehl ratio is the ratio between the peak amplitude of the PSF and the peak amplitude expected in the presence of diffraction only. With normal atmospheric seeing, the Strehl ratio is *very* low. However, the Strehl ratio is often used when discussing the performance of adaptive optics systems. Ratio of how good the actual image is to the best that could be obtained (pure diffraction-limited). A really good adaptive optics system would have a Strehl ratio of about 1/2. Seeing of a big telescope is nowhere close to diffraction-limited… fraction of a %. From Nancy's solutions: the Strehl ratio is the ratio of the amoutn of light actually delivered by the optical system into the Airy disk to the theoretical maximum at the diffraction limit.

1 Astronomical Optics

A 20th magnitude star gives ~ 0.01 photons s^{-1} cm^{-2} at 5000 Å through a 1000 Å filter, which gives 1200 photons s^{-1} at a 4-meter telescope.

1.1 Single surface optics and definitions

By definition, an optical system collects light and (usually) makes images. This requires the bending of light rays, which is accomplished using *curved surfaces*:

- lenses (refraction)
- mirrors (reflection)

Building large telescopes with lenses is difficult, but the instruments often have lenses.

The operation of refractive optical systems is given by Snell's law of refraction:

$$n \sin i = n' \sin i'$$

where n and n' are the indices of refraction and i and i' are the angles of incidence relative to the normal to the surface. For reflection:

$$i' = -i$$

An optical element takes a source at s and makes an image at s'. The image can be *real* or *virtual*. A real image exists at some point in space; a virtual image is formed where light rays apparently emanate from or converge to, but at a location where no light actually appears. For example, in a Cassegrain telescope, the image formed by the primary is virtual, because the secondary intercepts the light and redirects it before light gets to the focus of the primary (see figure 1). The image will not necessarily be a perfect image: all rays regardless

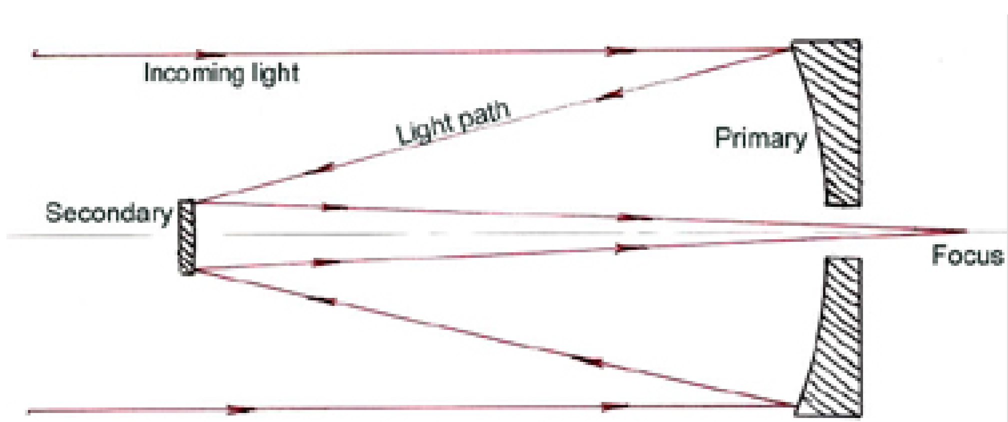

Figure 1: Diagram showing the optical arrangement of the Cassegrain.

of height y at the surface ("surface" refers to the lens or mirror), might not cross at the same point. This is the subject of *aberrations* (see § 1.3). For a "smooth" surface, the amount of aberration will depend on how much the different rays differ in y, which depends on the shape of the surface:

- *paraxial* rays are near the center of the aperture.
- *marginal* rays are on the edge of the aperture.
- the *chief* ray passes through the center of the aperture.

To define nominal (unaberrated) quantities, we consider the *paraxial regime*, a small region near the optical axis surrounding the chief ray. In this regime, all angles are small, aberrations vanish (everything converges to a point), and a surface can be wholly specified by its radius of curvature, R (where the image is at the center of a "circle" with radius R).

The *field angle* is formed when receiving light off-axis by some angular amount. It is not necessarily zero in the paraxial regime.

For the time being, we are ignoring *diffraction* and considering *geometric* optics, where spatial scales are much larger than the wavelength ($x >\sim \lambda$).

The basic relation between the object location at s and the image location at s' can be derived as a function of a surface where the index of refraction changes (Schroeder, chapter 2).

$$\frac{n'}{s'} - \frac{n}{s} = \frac{(n'-n)}{R}$$

where n and n' are the index of refraction where the light is coming from and in the lens, respectively. The points at s and s' are called *conjugate* (if one is known, the other can be calculated). If either s or s' is at infinity (true for astronomical sources at s), the other distance is defined as the *focal length*, f, of the optical element, and is where the light is focused. For $s = \infty$, $f = s'$.

The quantity on the right side of the equation, which depends only on the surface parameters (not the image or object locations), is defined as the *power*, P, of the surface:

$$P \equiv \frac{(n'-n)}{R} = \frac{n'}{f'} = \frac{n}{f}$$

A similar derivation can be made for the case of reflection:

$$\frac{1}{s'} + \frac{1}{s} = \frac{2}{R}$$

This shows that $f = R/2$ for a mirror.

The same result is obtained when considering reflection as refraction with $n' = -n$ (same amplitude, opposite sign):

$$\frac{n'}{s'} + \frac{n'}{s} = \frac{(n'+n')}{R}$$

The *focal ratio* (or *F-number*) is defined as

$$f/ = \frac{f}{A}$$

where A is the aperture diameter and $f/$ denotes the focal ratio. For example, a 3.5-m telescope with a focal ratio of $f/10$ has a focal length of 35 meters. The focal ratio gives the beam "width"; systems with a small focal ratio have a short focal length compared with A, and hence the imcoming beam to the image is wide. Systems with small focal ratios are called "fast" systems; systems with large focal ratios are called "slow" systems.

Wednesday, March 2

The *magnification* of a system gives the ratio of the image height to the object height:

$$\frac{h'}{h} = \frac{s' - R}{s - R} = \frac{ns'}{n's}$$

The magnification is negative for both inverted images and for reflection ($n' = -n$). Magnification is an important quantity for *multi-element* systems (see § **??**).

The *plate scale* is defined as the "motion" of an image for given beam from infinity. From a consideration of the chief rays for objects on-axis and at field angle α:

$$\tan\alpha \approx \alpha = \frac{x}{f}$$

or

$$\text{scale} \equiv \frac{\alpha}{x} = \frac{1}{f}$$

In other words, the scale, in units of angular motion (arcsec) per physical motion in the focal plane (x), is given by $1/f$ (important!) For a fixed aperture diameter, systems with a small $f/$ (smaller f) have a *larger* scale, i.e. more light in a patch of fixed physical size. These are "faster" systems.

Example: $\alpha = \frac{1}{35m} = \frac{1}{35\times10^3} = 2.86\times10^{-5}$ radians $= 5.98''/\text{mm}$ (there are 206265 arcseconds per radian).

1.2 Multi-surface systems

To combine surfaces, the image from the first surface becomes the source for the second surface, and so on for each surface in the system. The basic parameters of multi-surface systems can generally be described by equivalent single-surface parameters. For example, the *effective* focal length (the focal length of the first element multiplied by the magnification of each subsequent element) of a multi-surface system can be defined as the focal length of some equivalent single-surface system. The two systems (single and multi) are equivalent in the paraxial approximation ONLY.

1.2.1 A lens (has two surfaces)

Consider a lens in air ($n \sim 1$). The first surface gives:

$$\frac{n}{s_1'} - \frac{1}{s_1} = \frac{n-1}{R_1} = P_1$$

The second surface gives:

$$\frac{1}{s_2'} - \frac{n}{s_2} = \frac{n-1}{R_2} = P_2$$

but we have $s_2 = s_1' - d$ (remember we have to use the plane of the second surface to measure distances for the second surface).

The effective focal length from the center of the lens is given (after some algebra) by:

$$P = \frac{1}{f'} = P_1 + P_2 - \frac{d}{n} P_1 P_2$$

$$P = \frac{(n-1)}{R_1} + \frac{(1-n)}{R_2} - \frac{d}{n} \frac{(n-1)(1-n)}{R_1 R_2}$$

From this, we derive the *thin lens* formula:

$$P = \frac{1}{f'} = \frac{(n-1)}{R_1} + \frac{(1-n)}{R_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f'} = \frac{1}{f_1} + \frac{1}{f_2}$$

1.2.2 plane-parallel plate

Zero power, but moves image laterally:

$$\Delta = d\left[1 - \left(\frac{1}{n}\right)\right]$$

Application to filters: variation of focus. If the focal length changes, the scale changes; need to be careful when changing filters. The focus may need to be shifted a bit. The filter width doesn't matter but the index of refraction does. Holographic grating.

1.2.3 Two-mirror telescopes

In astronomy, most telescopes are two-mirror telescopes of Newtonian, Cassegrain, or Gregorian design. All 3 types have a concave primary. The Newtonian has a flat secondary, the Cassegrain a convex secondary, and the Gregorian a concave secondary. The Cassegrain is the most common for research; it is more compact than a Gregorian and allows for magnification by the secondary. Basic parameters are outlined here. Each of these telescope types defines a *family* of telescopes with different first-order performances. From the usage/instrumentation point of view, important quantities are:

- the diameter of the primary, which defines the light collecting power

- the scale of the telescope, which is related to the focal length of the primary and the magnification of the secondary:
$$f_{\text{eff}} = f_1 m$$
(alternatively, the focal ratio of the telescope, which gives the effective focal length with the diameter)
- the back focal distance, which is the distance of the focal plane behind the telescope.

From the design point of view, we need to specify:

- the radii of curvature of the mirrors
- the separation between the mirrors

The relation between the usage and design parameters can be derived from simple geometry. Some basic definitions:

- ratio of focal lengths, ρ:
$$\rho = \frac{R_2}{R_1} = \frac{f_2}{f_1}$$
- magnification of the secondary, m (be aware that s_2' is *negative* for a Cassegrain):
$$m = -\frac{s_2'}{s_2}$$
- *back focal distance*, the distance from the primary vertex to the focal plane (often expressed in units of the primary focal length, or primary diameter):
$$f_1 \beta = D\eta$$
- primary focal ratio, F_1:
$$F_1 = \frac{f_1}{D}$$
- ratio of marginal ray heights, k:
$$k = \frac{y_2}{y_1}$$

Using some geometry, some basic relations between these quantities can be derived, in particular:
$$\rho = \frac{mk}{(m-1)}$$
and
$$(1+\beta) = k(m+1)$$
β in units of primary mirror focal length. Usually, f_1 is limited by technology/cost, in which case m can be chosen to match the desired scale. k is directly related to separation of mirrors, and is a compromise between a shorter telescope and blocking out more light vs. a longer

telescope and blocking less light. In either case, the focal plane has to be kept behind the primary.

Want telescope to match detector: how many arcseconds per pixel? Straightforward in paraxial regime. Need to tune magnification of secondary. How far apart do the mirrors need to be…

One final thing to note is how to focus a Cassegrain telescope. Most instruments are placed at a fixed location behind the primary. Ideally, this will be at the back focal distance, and everything should be set as designed. However, sometimes the instrument may not be exactly at the correct back focal distance, or it might move slightly because of thermal expansion/contraction. In this case, focussing is usually then done by moving the secondary mirror (not the detector).

The amount of image motion for a given secondary motion is given by:

$$\frac{\mathrm{d}\beta}{\mathrm{d}k} = \frac{\mathrm{d}}{\mathrm{d}k} k\,(m+1) - 1$$

Working through the relations above, this gives:

$$\frac{\mathrm{d}\beta}{\mathrm{d}k} = m^2 + 1$$

where m^2 is the magnification of the secondary mirror. So the amount of focal plane motion ($f_1 \mathrm{d}\beta$) for a given amount of secondary motion ($f_1 dk$) depends on the magnification of the system.

If the secondary is moved, k is changed. Since ρ is fixed by the mirror shapes, the magnification can be changed as the secondary is moved. This is expected since the system focal length is being changed: $f = m f_1$. So it's possible that a given instrument could have a slightly varying scale if its position is not perfectly fixed relative to the primary. Alternatively, if you need to independently focus and set the scale (SDSS for example), then two things need to be moved.

Note that even if the instrument is right at the back focal distance, movement of the secondary is required to account for mechanical changing of spacing between the primary and secondary as a result of thermal expansion/contraction.

Monday, March 7, 2016

1.2.4 Definitions for multi-surface system: stops and pupils

- aperture stop: determines the amount of light reaching an image (usually the primary mirror).
- field stop: determines the angular size of the field. This is usually the detector, but for a large enough detector, it could be the secondary.
- pupil: location where rays from all field angles fill the same aperture. Image of the primary mirror (not formed by the primary mirror). Number of elements in system = number of pupils.

- entrance pupil: image of aperture stop as seen from source object (usually the primary).
- exit pupil: image of aperture stop formed by all subsequent optical elements.

Stops are relevent for correcting aberrations. In a two-mirror telescope, the location of the exit pupil is where the image of the primary is formed by the secondary. This can be calculated using $s = d$ as the object distance (where d is the separation of the mirrors), then with the reflection equation, we can solve for s' which gives the location of the exit pupil relative to the secondary mirror. If one defines the quantity δ, such that $f_1\delta$ is the distance between the exit pupil and the focal plane, then (algebra not shown):

$$\delta = \frac{m^2 k}{m + k - 1} = \frac{m^2(1+\beta)}{m^2 + \beta}$$

This pupil is generally not accessible, so access to a pupil is needed, additional optics are used.

The exit pupil is an important concept. For aberrations, it is the total wavefront error at the exit pupil which gives the system aberration. Pupils are important for aberration compensation. They can also be used to put light at a location that is independent of pointing errors.

1.3 Aberrations

1.3.1 Surface requirements for unaberrated images

Next consider *non*-paraxial rays. The surface required to make an unaberrated image can be derived using *Fermat's principle*, which states that light travels in the path such that infinitesimally small variations in the path doesn't change the travel time to first order: $\mathrm{d}t/\mathrm{d}l$ is a minimum. For a single surface, this reduces to the statement that light travels the path which takes the least time. An alternate way of stating Fermat's principle is that the *optical path length* (OPL) is unchanged to first order for a small change in path. The OPL is given by:

$$OPL = \int c\mathrm{d}t = \int \frac{c}{v} v\mathrm{d}t = \int n\mathrm{d}s$$

Fermat's principle has a physical interpretation when one considers the wave nature of light. It is clear that around a stationary point of the optical path light, the maximum amount of light can be accumulated over different paths with a minimum of destructive interference. By the wave theory, light travels over all possible paths, but the light coming over the "wrong" paths destructively interferes, and only the light coming over the "right" path constructively interferes.

Fermat's principle can be used to derive the basic laws of reflection and refraction (Snell's law).

Consider a perfect imaging system that takes all rays from an object and converges them to an image. Since Fermat's principle says the only paths taken will be those for which the

OPL is minimally changed for small changes in path, the only way a perfect image will be formed is when all optical path lengths along a surface between an image and object point are the same - otherwise the light doesn't get to this point.

Instead of using Fermat's principle, we could solve for the parameters of a perfect surface using analytic geometry, but this would require an inspired guess for the correct functional form of the surface.

The perfect surface depends on whether the light comes from a source at finite or infinite distance, and whether the mirror is concave or convex. The z-axis is the optical axis, perpendicular to the y-axis. The goal is to figure out the shape of the surface, $y(z)$, that gives a perfect image.

Concave mirror with one conjugate at infinity: Parabola

Sample application: primary mirror of telescope looking at stars. Fermat's principle gives:

$$y^2 = 2R_z$$

where $R = 2f$, the radius of curvature at the mirror vertex. Note, however, that a prabola makes a perfect image only for *on*-axis images (field angle = 0), which is why we don't always use this shape.

Concave mirror with both conjugates at finite distances: Ellipse

Sample application: Gregorian secondary looking at image formed by primary. Again, this is perfect only for field angle = 0.

$$\frac{(z-a)^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y^2 - \frac{2zb^2}{a} + \frac{z^2b^2}{a^2} = 0$$

where

$$a = \frac{s+s'}{2}$$

$$b = \sqrt{(ss')}$$

$$R = \frac{ss'}{s+s'} = \frac{2b^2}{a}$$

Convex mirror with both conjugates at finite distance: Hyperbola

Sample application: Cassegrain secondary looking at image formed by primary:

$$\frac{(z-a)^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$y^2 + \frac{2zb^2}{a} - \frac{z^2b^2}{a^2} = 0$$

where

$$a = \frac{s+s'}{2}$$

$$b^2 = \sqrt{(-ss')}$$

(s is negative)

$$R = -\frac{2b^2}{a}$$

Let secondary "relax" to get perfect image. Most telescopes adjust both surfaces, which allows for more freedom to get perfect images on and off-axis. ***Convex mirror with one conjugate at infinity: Parabola***

...but this has no astronomical applications.

2D to 3D

So far all cases have considered a one-dimensional surface. Generalize to 2D surfaces by rotating around the z-axis. For the equations, simply replace y^2 with $(x^2 + y^2)$.

Conic sections

All of these figures are *conic sections*, described by a single equation:

$$\rho^2 - 2Rz + (1 + K)z^2 = 0$$

where $\rho^2 = x^2 + y^2$ and R is the radius of curvature at the mirror vertex, K is called the conic constant ($K = -e^2$, where e is the eccentricity for an ellipse, $e(b, a)$).

$K > 0$ prolate ellipsoid
$K = 0$ sphere
$-1 < K < 0$ oblate ellipsoid
$K = -1$ paraboloid
$K < -1$ hyperboloid

1.3.2 Aberrations: general description and low-order aberrations

Wednesday, March 23

Now consider what happens for surfaces that are *not* perfect, e.g. for a field angle $\neq 0$ in the cases discussed above (since only a sphere is symmetric for all field angles), or for field angle $= 0$ for a conic surface that doesn't give a perfect image.

This is where *aberrations* arise; the light from all locations in the aperture does not land at any common point.

Aberrations can be considered in either of two ways:

- All rays don't land at a common point.
- Wavefront deviates from a spherical wavefront.

These two descriptions are equivalent. The former refers to *transverse* aberrations (the distance by which the rays miss the paraxial focus), or *angular* aberrations (the angle by which the rays deviate from the perfect ray which will hit paraxial focus). The latter refers to the wavefront error (the deviation of the wavefront from a spherical wavefront as a function of location in the exit pupil).

In general, the angular and transverse aberrations can be determined from the optical path difference between a given ray and that of a spherical wavefront. The relations are given by:

$$\text{angular aberration} = \frac{\mathrm{d}\,(2\Delta z)}{\mathrm{d}\rho}$$

$$\text{transverse aberration} = s'\frac{\mathrm{d}\,(\Delta z)}{\mathrm{d}\rho}$$

s' is the focal length. If the aberrations are not symmetric in the pupil, then define angular and transverse x and y aberrations separately by taking derivatives with respect to x or y instead of ρ.

Spherical aberration

First, consider the axisymmetric case of looking at an object on axis (field angle = zero) with an optical element that is a conic section. Consider where rays land as $f(\rho)$, and derive the effective focal length, $f_e(\rho)$, for an arbitrary conic section [figure here]:

$$z_0 = \frac{\rho}{\tan(2\phi)} = \frac{\rho(1-(\tan\phi)^2)}{2\tan\phi}$$

$$\tan\phi = \frac{\mathrm{d}z}{\mathrm{d}\rho}$$

from the conic equation:

$$\rho^2 - 2Rz + (1+K)z^2 = 0$$

$$z = \frac{R}{(1+K)}\left[1-\left(1-\frac{\rho^2}{R^2}(1+K)\right)^{1/2}\right]$$

$$z \approx \frac{\rho^2}{2R} + (1+K)\frac{\rho^4}{8R^3} + (1+K)^2\frac{\rho^6}{16R^5} + \ldots$$

$$\frac{\mathrm{d}z}{\mathrm{d}\rho} = \frac{\rho}{(R-(1-K)\,z)}$$

$$z_0 = \frac{\rho}{2}\left[\frac{R-(1+K)z}{\rho} - \frac{\rho}{R-(1+K)z}\right]$$

$$f_e = z + z_0$$

$$f_e = \frac{R}{2} + \frac{(1-K)z}{2} - \frac{\rho^2}{2(R-(1+K)z)}$$

$$f_e = \frac{R}{2} - (1+K)\frac{\rho^2}{4R} - (1+K)(3+K)\frac{\rho^4}{16R^3} - \ldots$$

$$\Delta f = f_e - \frac{R}{2}$$

Note that f_e is independent of z only for $K = -1$, a parabola. Also note that Δf is symmetric with respect to ρ. $R/2$: paraxial focus. Spherical aberration is different from focusing…can't just move focal plane around to get a perfect image. Where's the best place to focus to minimize aberrations? "Circle of least confusion". Deviations are symmetric in pupil, but not symmetric for focal position. Can solve TSA equations for conic section equations.

Spherical aberration is defined as the aberration resulting from $K \neq -1$ ($K + 1 = 0$ for parabola). Rays from different radial positions in the entrance aperture focus at different locations. It is an aberration which is present on axis as seen here. Spherical aberration is symmetric in the pupil. There is no location in space where all rays focus at a point. Note that the behavior (image size) as a function of focal position is not symmetric. One can define several criteria for where the "best focus" might be, leading to the terminology paraxial focus, marginal focus, diffraction focus, and the circle of least confusion.

The asymmetric nature of spherical aberration as a function of focal position distinguishes it from other aberrations and is a useful diagnostic for whether a system has this aberration. This is shown in this figure which shows a sequence of images at different focal positions in the presence of spherical aberration. A *transverse spherical aberration* (TSA) is the image size at paraxial focus. This is not the location of the minimum image size.
(figure here)

$$\frac{\mathrm{TSA}}{\Delta f} = \frac{\rho}{f - z(\rho)}$$

$$\mathrm{TSA} = -(1+K)\frac{\rho^3}{2R^2} - 3(1+K)(3+K)\frac{\rho^5}{8R^4} + \ldots$$

The difference in angle between the "perfect" ray from the parabola and the actual ray is called the *angular aberration*, in this case *angular spherical aberration* (ASA).
(figure here)

$$\mathrm{ASA} = 2(\phi_P - \phi) \approx \frac{\mathrm{d}}{\mathrm{d}\rho}(2\Delta z) \approx -(1+K)\frac{\rho^3}{R^3}$$

where $2\Delta z$ gives the optical path difference between the two rays.

This is simply related to the transverse aberration:

$$\mathrm{TSA} = \frac{R}{2}\mathrm{ASA}$$

We can also consider aberration as the difference between our wavefront and a spherical wavefront, which in this case is the wavefront given by a parabolic surface.
(figure here)

$$\Delta z = z_{\mathrm{parabola}} - z\left(K\right) = -\frac{\rho^4}{8R^3}\left(1+K\right) + \ldots$$

More stuff here, but pretty sure it's a repeat.

General aberration description

Surface described by polynomial, optical path difference, low order…something, OPD (last term doesn't have field angle dependence). Restrict to third-order aberrations (fourth order in OPD). Order of pupil backed down by 1. θ - "amount off axis that you go". (Reveiw "order" for maths). $\theta^2 y$ - θ is squared so direction doesn't matter (astigmatism), θ - coma (linear), $y\rho^2$ - spherical aberration (no θ).

We can describe deviations from a spherical wavefront generally. Since all we care about are optical path differences, we write an expression for the optical path difference between an arbitrary ray and the chief ray, and in doing this, we can also include the possibility of an off-axis image, and get

$$OPD = OPL - OPL(chiefray)$$

$$OPD = A_0 y + A_1 y^2 + A_1' x^2 + A_2 y^3 + A_2' x^2 y + A_3 \rho^4$$

where we've kept terms only to fourth order and chosen our coordinate system such that the object lies in the y-z plane. The coefficients, A, depend on lots of things, such as (θ, K, n, R, s, s').

Note that rays along the y-axis are called *tangential* rays, while rays along the x-axis are called *sagittal* rays.

Analytically, generally restricted to *third-order* aberrations, which are fourth-order (in powers of x, y, ρ, or θ) in the optical path difference, because of the derivative we take to get transverse or angular aberrations. In the third-order limit, $A2 = A2'$, and $A1 = -A1'$. Working out the geometry, we find for a mirror that:

$$A_0 = 0$$

$$A_1 = \frac{n\theta^2}{R}$$

$$A_2 = -\frac{n\theta}{R^2}\left(\frac{m+1}{m-1}\right)$$

$$A_3 = \frac{n}{4R^3}\left[K + \left(\frac{m+1}{m-1}\right)^2\right]$$

From the general expression, we can derive the angular or the transverse aberrations in either the x or y direction. Considering the aberrations in the two separate directions, we find:

$$AA_y = 2A_1 y + A_2(x^2 + 3y^2) + 4A_3 y\rho^2$$

$$AA_x = 2A_1' x + 2A_2 xy + 4A_3 x\rho^2$$

The first term is proportional to $\theta^2 y$ and is called *astigmatism*. The second term is proportional to $\theta(x^2 + 3y^2)$ and is called *coma*. The final term, proportional to $y\rho^2$ is *spherical aberration*, which we've already discussed (note for spherical, $AA_x = AA_y$ and in fact the AA in any direction is equal, hence the aberration is circularly symmetric).

Astigmatism

For astigmatism, rays from opposite sides of the pupil focus in different locations relative to the paraxial rays. At the paraxial focus, we end up with a circular image. As you move away from this image location, you move towards the tangential focus in one direction and the sagittal focus in the other direction. At either of these locations, the astigmatic image looks like a elongated ellipse. Astigmatism goes as θ^2, and consequently looks the same for opposite field angles. Astigmatism is characterized in the image plane by the *transverse* or *angular* astigmatism (TAS or AAS), which refer to the height of the marginal rays at the paraxial focus. Astigmatism is symmetric around zero field angle. elliptical shape $\rightarrow$ circle $\rightarrow$ ellipse again. Need to consider tangential rays and sagittal rays. "Useful": look for where T changes to S. This figure shows the rays in the presence of astigmatism. This figure shows the behavior of astigmatism as one passes through paraxial focus.

Coma

Looks like a comet! Same distance from pupil, not same distance from focal plane. Can't tell which side of (?) you're on. For coma, rays from opposite sides of the pupil focus at the same focal distance. However, the tangential rays focus at a different location than the sagittal rays, and neither of these focus at the paraxial focus. The net effect is to make an image that vaguely looks like a comet, hence the name coma. Coma goes as θ, so the direction of the comet flips sign for opposite field angles. Coma is characterized by either the *tangential* or *sagittal transverse/angular coma* (TTC, TSC, ATC, ASC) which describe the height/angle of either the tangential or sagittal marginal rays at the paraxial focus: $TTC = 3TSC$.

This figure shows the rays in the presence of coma. This figure shows the behavior of coma as one passes through paraxial focus. In fact, there are two more third-order aberrations: *distortion* and *field curvature*. Neither affects image quality, only location (unless you are forced to use a flat image plane). Field curvature gives a curved focal plane: if imaging onto a flat detector, this will lead to focus deviations as one goes off-axis. Distortion affects the location of images in the focal plance, and goes as θ^3. The amount of field curvature and distortion can be derived from the aberration coefficients and the mirror parameters.

We can also determine the relevant coefficients for a surface with a displaced stop (Schroeder p 77), or for a surface with a decentered pupil (Schroeder p89-90); it's just more geometry and algebra. With all these realtions, we can determine the optical path differences for an entire system: for a multi-surface system, we just add the OPD's as we go from surface to surface. The final aberrations can be determined from the system OPD.

Wednesday, March 30

1.3.3 Aberration compensation and different telescope types

Using the techniques above, we can write expressions for the system aberrations as a function of the surface figures (and field angles). If we give ourselves the freedom to choose surface figures, we can eliminate one (or more) aberrations.

For example, given a conic constant of the primary mirror, we can use the aberration relations to determine K_2 such that spherical aberration is zero; this will give us perfect images on-axis. We find that:

$$K_2 = \left(\frac{m+1}{m-1}\right)^2 + \frac{m^3}{k\,(m-1)^3}\,(K_1+1)$$

satisfies this criterion. If we set the primary to be a parabola ($K_1 = -1$), this gives the conic constant of the secondary we must use to avoid spherical aberration. This type of telescope is called a *classical* telescope. Using the aberration relations, we can determine the amount of astigmatism and coma for such telescopes, and we find that coma gives significantly larger aberrations than astigmatism.

If we allow ourselves the freedom to choose both K_1 and K_2, we can eliminate both spherical aberration and coma. Designs of this sort are called *aplanatic*. The relevant expression, in terms of the magnification and back focal distance (we could use the relations discussed earlier to present these in terms of other paraxial parameters), is:

$$K_1 = -1 - \frac{2(1+\beta)}{m^2(m-\beta)}$$

We can only eliminate two aberrations with two mirrors, so even this telescope will be left with astigmatism.

There are two different classes of two-mirror telescopes that allow for freedom in the shape of both mirrors: Cassegrain telescopes and Gregorian telescopes (Newtonians have a flat secondary). For the classical telescope with a parabolic primary, the Cassegrain secondary is hyperbolic, whereas for a Gregorian it is ellipsoidal (because of the appropriate conic sections derived above for convex and concave mirrors with finite conjugates). For the aplanatic design, the Cassegrain telescope has two hyperbolic mirrors, while the Gregorian telescope has two ellipsoidal mirrors. An aplanatic Cassegrain telescope is called a *Ritchey-Chretien* telescope.

The following table gives some characteristics of “typical” telescopes. Aberrations are given at a field angle of 18 arc-min in units of arc-seconds. Coma is given in terms of tangential coma.

Parameter	CC	CG	RC	AG
m	4.00	-4.00	4.00	-4.00
k	0.25	-0.417	0.25	-0.417
1-k	0.75	1.417	0.75	1.417
mk	1.000	1.667	1.000	1.667
ATC	2.03	2.03	0.00	0.00
AAS	0.92	0.92	1.03	0.80
ADI	0.079	0.061	0.075	0.056
$\kappa_m R_1$	7.25	-4.75	7.625	-5.175
$\kappa_P R_1$	4.00	-8.00	4.00	-8.00

Table 1: Characteristics of Two-Mirror Telescopes

The image quality is clearly better for the aplanatic designs than for the classical designs, as expected because coma dominates off-axis in the classical design. In the aplanatic design, the

Gregorian is slightly better. However, when considerations other than just optical quality are considered, the Cassegrain usually is favored: for the same primary mirror, the Cassegrain is considerably shorter and thus it is less costly to build an enclosure and telescope structure. To keep the physical length the same, the Gregorian would have to have a faster primary mirror, which are more difficult (i.e. costly) to fabricate, and which will result in a greater sensitivity to alignment errors. Both types of telescopes have a *curved* focal plane.

1.4 Sources of aberrations

So far, we have been discussing aberrations which arise from the optical design of a system when we have a limited number of elements. However, it is important to realize that aberrations can arise from other sources as well. These other sources can give additional third-order aberrations, as well as higher order aberrations. Some possible sources include:

- misfigured (a slightly harsh term) or imperfectly figured optics: rarely is an element made exactly to specification.
- misalignments. If the mirrors in a multiple-element system are not perfectly aligned, aberrations will result. The centers must be lined up. These can be derived (third-order) from the aberration expressions for decentered elements. For two mirror systems, decentering or tilting the secondary introduces a constant amount of coma over the field. Coma dominates astigmatism for a misaligned telescope. The Ritchey-Chretian has a "high" tolerance for this (not much wiggle room).
- mechanical/support problems. When the mirrors are mounted in mirror cells the weight of the mirror is distributed over some support structures. Because the mirrors are not infinitely stiff, some distortion of the mirror shape will occur. Generally, such distortion will probably change as a function of which way the telescope is pointing. Separate from this, becuase the telescope structure itself is not perfectly stiff, one expects some flexure which gives a different secondary (mis)alignment as a function of where one is pointing. Finally, one might expect the spacing between the primary and secondary to vary with temperature, if the telescope structure is made of materials which have non-zero coefficients of expansion.
- chromatic aberration. Generally, we've only been discussing mirrors since this is what is used in telescopes. However, astronomers often put additional optics (e.g., cameras or spectrographs) behind telescopes which may use refractive elements rather than mirrors. There are aberration relations for refractive elements just as we've discussed, but these have additional dependences on the indices of refraction of the optical elements. For most refractive elements, the index of refraction varies with wavelength, so one will get wavelength-dependent aberrations, called chromatic aberrations. These can be minimized by good choices of materials or by using combinations of different materials for different elements; however, it is an additional source of aberration.
- seeing. This is the only natural source of aberration (the one we can't control). The earth's atmosphere introduces optical path differences between the rays across the aperture of the telescope. This is generally the **dominant** source of image degradation from a ground-based telescope. Consequently, one builds telescopes in good sites, and

as far as design and other sources of image degradation are concerned, one is generally only interested in getting these errors small when compared with the smallest expected seeing errors.

1.5 Ray tracing

For a fully general calculation of image quality, one does not wish to be limited to third-order aberrations, nor does one often wish to work out all of the relations for the complex set of aberrations which result from all of the sources of aberration mentioned above. Real world situations also have to deal with *vignetting* in optical systems, in which certain rays may be blocked by something and never reach the image plane (e.g., in a two-mirror telescope, the central rays are blocked by the secondary).

Because of these and other considerations, analysis of optical systems is usually done using *ray tracing*, in which the parameters of an optical system are entered into a computer, and the computer calculates the expected images on the basis of geometric optics. Many programs exist with many features: one can produce *spot* diagrams which show the location of rays from across the aperture at an image plane (or any other location), plots of transverse aberrations, plots of optical path differences, etc. `zmacs` is a ray-tracing program.

(Demo ray trace program. Start with on-axis object, single mirror. Where is focus? What will image look like with spherical mirror? What do we need to do to make it perfect? How does it depend on aperture size? Now how do off-axis images look like? spot diagrams, through focus, ray fan, opd plots, etc. Now introduce second mirror. What determines where focus will be? Magnification? What shape to make a perfect on-axis image? What do off-axis images look like? How do we make them better? Now how is performance? Real 3.5m and 1m prescriptions. Issue: guider.)

1.6 Physical (diffraction) optics

Up until now, we have avoided considering the wave nature of light which introduces *diffraction* from interference of light coming from different parts of the aperture. Because of diffraction, images of a point source will be slightly blurred. From simple geometric arguments, we can estimate the size of the blur introduced from diffraction: (figure here)

We find that:

$$\theta \sim \frac{\lambda}{D}$$

Using this, we find that the diffraction blur is smaller than the blur introduced by seeing for $D > 0.2$ meters at 5500 Å, even for the excellent seeing conditions of 0.5 arcsecond images. However, the study of diffraction has become important recently because of several reasons: 1) the existence of the Hubble Space Telescope, which is diffraction limited (no seeing), 2) the increasing use of infrared observations, where diffraction is more important than in the optical, and 3) the development of adaptive optics, which attempts to remove some of the distortions caused by seeing. Consequently, it's now worthwhile to understand some details about diffraction.

To work out in detail the shape of the images formed from diffraction involves understanding wave propagation. Basically, one integrates over all of the source points in the aperture (or exit pupil for an optical system), determining the contribution of each point at each place in the image plane. The contributions are all summed taking into account phase differences at each image point, which causes reinforcment at some points and cancellation at others. The expression which sums all of the individual source points is called the *diffraction integral*. When the details are worked out, the intensity in the image plane is related to the intensity and phase at the exit pupil. In fact the wavefront is described at any plane by the *optical transfer function*, which gives the intensity and phase of the wave at all locations in that plane. The OTF at the pupil plane and at the image plane are a Fourier transform pair. Consequently, we can determine the light distribution in the image plane by taking the Fourier transform of the pupil plane; the light distribution, or point spread function, is just the modulus-squared of the OTF at the image plane. Symbolically, we have

$$PSF = |FT(OTF(pupil))|^2$$

where FT represents a Fourier transform, and

$$OTF(pupil) = P(x,y)e^{ik\phi(x,y)}$$

$P(x,y)$ is the *pupil function*, which gives the transmission properties of the pupil, and usually consists of ones and zeros for locations where light is either transmitted or blocked (e.g., for a circular lens, the pupil function is unity within the radius of lens, and zero outside; for a typical telescope the pupil function includes obscuration by the secondary and secondary support structure). ϕ is the phase in the pupil. More relevantly, ϕ can be taken to be the optical path difference in the pupil with some fiducial phase, since only OPDs matter, not the absolute phase. Finally the wavenumber k is just $\frac{2\pi}{\lambda}$.

For the simple case of a plane wave with no phase errors, the diffraction integral can be solved analytically. The result for a circular aperture with a central obscuration, when the fractional radius of the obscuration is given by ϵ, the expression for the PSF is:

$$PSF \propto \left[\frac{2J_1(v)}{v} - \epsilon^2\frac{2J_1(\epsilon v)}{\epsilon v}\right]^2$$

$$v = \frac{\pi r}{\lambda F}$$

where J_1 is a first order Bessel function, r is the distance in the image plane, λ is the wavelength, and F is the focal ratio ($F = f/D$).

This expression gives the so-called *Airy pattern* (the solution?) which has a central disk surrounded by concentric dark and bright rings. The radius of the first dark ring is at the physical distance $r = 1.22\lambda F$, or alternatively, the angular distance $\alpha = 1.22\lambda/D$. This gives the size of the *Airy disk*.

For more complex cases, the diffraction integral is solved numerically by doing a Fourier transform. The pupil function is often more complex than a simple circle, because there are often additional items which block light in the pupil, such as the support structures for the secondary mirror.

This figure shows the Airy pattern, both without obscurations, and with a central obscuration and spiders in a setup typical of a telescope.

In addition, there may be phase errors in the exit pupil, because of the existence of any one of the sources of aberration discussed above. For general use, ϕ is often expressed as an series, where the expansion is over a set of orthogonal polynomials for the aperture which is being used. For circular apertures with (or without) a central obscuration (the case most often found in astronomy), the appropriate polynomials are called *Zernike* polynomials. The lowest order terms are just uniform slopes of phase across the pupil, called tilt, and simply correspond to motion in the image plane. The next terms correspond to the expressions for the OPD which we found above for focus, astigmatism, coma, and spherical aberration, generalized to allow any orientation of the phase errors in the pupil. Higher order terms correspond to higher order aberrations.

This figure shows the form of some of the low order Zernike terms: the first corresponds to focus aberration, the next two to astigmatism, the next two to coma, the next two to trefoil aberration, and the last to spherical aberration.

A wonderful example of the application of all of this stuff was in the diagnosis of spherical aberration in the Hubble Space Telescope, which has been corrected in subsequent instruments in the telescope, which introduce spherical aberration of the opposite sign. To perform this correction, however, required and accurate understanding of the amplitude of the aberration. This was derived from analysis of on-orbit images, as shown in this figure. Note that it is possible in some cases to try to recover the phase errors from analysis of images. This is called *phase retrieval*. There are several ways of trying to do this, some of which are complex, so we won't go into them, but it's good to know that it is possible. But an accurate amplitude of spherical aberration was derived from these images. This derived value was later found to correspond almost exactly to the error expected from an error which was made in the testing facility for the HST primary mirror, and the agreement of these two values allowed the construction of new corrective optics to proceed. . .

1.7 Adaptive optics

Monday, April 4

The goal of *adaptive optics* is to partially or entirely remove the effects of atmospheric seeing. This is different from *active* optics, which works at lower frequencies ($\ll 1$ Hz) and has the main goal of removing aberrations from the change in telescope configuration as the telescope moves (for example, small changes in alignment from flexure (?) or sag (??) of the primary mirror surface as the telescope moves). Adaptive optics must work at 10 to 1000 Hz. At low frequencies, the active optics can be done with actuators on the primary and secondary mirrors themselves. At the high frequencies required for adaptive optics, however, these large mirrors cannot respond fast enough, so it is required to form a pupil on a smaller mirror which can be rapidly adjusted; hence adaptive optics systems are really separate astronomical instruments.

Many adaptive optics systems functioning and/or under development: (lots of links here).

The basic idea of an adaptive optics system is to rapidly sense the wavefront errors and then correct for them on timescales faster than those at which the atmosphere changes. Consequently, there are three parts to an adaptive optics system:

1. A component that senses wavefront errors
2. A control system that figures out how to correct these errors
3. An optical element that receives the signals from the control system and implements wavefront corrections

There are several methods used for wavefront sensing. Two that are in fairly common use today are

1. Shack-Hartmann sensors
2. wavefront curvature sensing devices

In a Shack-Hartmann sensor, an array of lenslets is put in a pupil plane and each lenslet images a small part of the pupil. Measuring image shifts between each of the images gives a measure of the local wavefront tilts. Wavefront curvature devices look at the intensity distribution in out-of focus images. Other wavefront sensing techniques include pyramid wavefront sensors and phase diversity techniques. Usually, a star is used as the source, but this is not required for some wavefront sensors (in other words, an extended source can be used).

To correct wavefront errors, a deformable mirror is used. These can be generically split into two categories: segmented and continuous faceplate mirrors, where the latter are more common. A deformable mirror is characterized by the number of adjustable elements: the more elements there are, the more correction can be done. LCD arrays have also been used for wavefront correction.

In general, it is very difficult to achieve complete correction even for ideal performance, and the effectiveness of different adaptive optics systems needs to be considered. This effectiveness depends on the size of the aperture, the wavelength, the number of resolution elements on the deformable mirror, and the quality of the site. Clearly, more resolution elements are needed for larger apertures. Equivalently, the effectiveness of a system will decrease as the aperture is increased for a fixed number of resolution elements. The return can be considered as a function of Zernike order corrected and aperture size. (uh, do hwut?) For large telescopes, you'll only get partial correction unless a very large number of resolution elements on the deformable mirror are available. The following table gives the mean square amplitude, Δ_j, for Kolmogorov turbulence after removal of the first j terms; the rms phase variation is just $\frac{\sqrt{\Delta_j}}{2\pi}$ For small apertures, you can make significant gains with removal of just low order terms, but for large apertures you need very high order terms. Note various criteria for quality of imaging, e.g. $\lambda/4$, etc.

Z_j	n	m	Expression	Description	Δ_j	$\Delta_j - \Delta_{j-1}$
Z1	0	0	1	constant	1.030 s	n/a
Z1	0	0	1	constant	1.030 s	n/a
Z1	0	0	1	constant	1.030 s	n/a
Z1	0	0	1	constant	1.030 s	n/a
Z1	0	0	1	constant	1.030 s	n/a
Z1	0	0	1	constant	1.030 s	n/a
Z1	0	0	1	constant	1.030 s	n/a
Z1	0	0	1	constant	1.030 s	n/a
Z1	0	0	1	constant	1.030 s	n/a
Z1	0	0	1	constant	1.030 s	n/a
Z1	0	0	1	constant	1.030 s	n/a

r = distance from center circle; ϕ = azimuth angle; $S = (D/r_0)^{5/3}$.

Another important limitation is that an object on which you can derive the wavefront is needed. Measurements of wavefront are subject to noise just like any other photon detection, so bright sources may be required. This is even more evident considering a source that is within the same isoplanatic patch as the desired object is needed. Recall that the wavefront changes on time scales of milliseconds. These requirements place limitations on the amount of sky over which it is possible to get good corrections. It also places limitations on the sorts of detectors that are needed in the wavefront sensors (fast readout and little to no readout noise).

band	λ	r_0	τ_0	τ_{det}	V_{lim}	θ_0	Coverage(%)
U	0.365	9.0	0.009	0.0027	7.4	1.2	1.8E-5
B	0.44	11.4	0.011	0.0034	8.2	1.5	1.8E-5
V	0.365	9.0	0.009	0.0027	7.4	1.9	1.8E-5
R	0.365	9.0	0.009	0.0027	7.4	2.6	1.8E-5
I	0.365	9.0	0.009	0.0027	7.4	3.5	1.8E-5
J	0.365	9.0	0.009	0.0027	7.4	5.1	1.8E-5
H	0.365	9.0	0.009	0.0027	7.4	7.0	1.8E-5
K	0.365	9.0	0.009	0.0027	7.4	10.1	1.8E-5
L	0.365	9.0	0.009	0.0027	7.4	17.0	1.8E-5
M	0.365	9.0	0.009	0.0027	7.4	27.0	1.8E-5
N	0.365	9.0	0.009	0.0027	7.4	64	1.8E-5

Conditions are: 0.75 arcsec seeing at 0.5 μ; $\tau_{det} \sim 0.3$, $\tau_0 = 0.3\, r/V_{wind}$; $V_{wind} = 10$ mIsec; $H = 5000$; photon detection efficiency (including transmission and QE) = 20%; spectral bandwidth = 300 nm; $SNR = 100$ per Hartmann-Shack image; detector noise = $5e^-$.

The isoplanatic patch limitation is severe. In many cases, we might expect non-opticmal performance if the reference object is not as close as it should be ideally.

In most cases, both because of lack of higher order correction and because of reference star vs. target wavefront differences, adaptive optics works in the partially correcting regime. This typically gives PSFs with a sharp core, but still with extended wings.

The problem of sky coverage can be avoided by using so-called *laser guide stars*. The idea is to create a "star" by shining a laser up into the atmosphere. To date, two generic classes of lasers have been used: *Rayleigh beacons* and *sodium beacons*. The Rayleigh beacons scatter off a layer roughly 30 km above the Earth's surface; the sodium beacons scatter off a layer roughly 90 km above the Earth's surface. Laser guide stars still have some limitations: the path through the atmosphere that the laser traverses does not exactly correspond to the path that light from a star traverses, because the latter comes from an essentially infinite distance; this leads to the effect called *focal anoisoplanatism*. In addition, laser guide stars cannot generally be used to track image motion since the laser passes up and down through the same atmosphere and image motion is cancelled out. To correct for image motion, separate tip-tilt tracking is required. Note that even with perfect correction, one is still limited by the isoplanatic patch size. As one moves further and further away from the reference object, the correction will gradually degrade.

In principle, correction over a wider field of view is possible with *multiple* deformable mirrors and multiple reference objects, giving rise to the concept of *multi-conjugate* adaptive optics (MCAO) systems.

Systems with single laser guide stars have certainly been tested and appear to work; but remember, only over an isoplanatic patch, and often with partially corrected images. Several implementations of system with multiple guide stars actually exist (at VLT and Keck?) to allow sampling of a larger cylinder/cone through the atmosphere; some of these are designed to correct at particular layers to maximize FOV, e.g. ground layer adaptive optics (GLAO). The bulk of adaptive optics work has been done in the near-IR.

Extreme (high-contrast) AO.

A variant on adaptive optics: lucky imaging.

Science with adaptive optics. Typical AO PSFs. Morphology vs. photometry.

1.8 AO Examples

Lots of links.

Wednesday, April 13, 2016

2 Telescopes

One real-world issue for large telescopes is the technology of how to build a large mirror which will not be so heavy that it will sag under its own weight. Additionally, since it has been recognized that good image quality requires that the mirrors be at the same temperature as the outside air, the mirror technology must be such that the mirror has a short thermal time constant, or, in other words, it must be able to change temperature to match the outside air fairly quickly. If necessary, one can consider thermally controlling the mirror, e.g., with heating or air conditioning.

In the large mirror regime, there are currently three leading technologies.

- Borosilicate honeycomb - large thermal mass, need to keep daytime temp as close to observing temp as possible. University of Arizona (Tucson) doing this under a football stadium. The first is the construction of a single large mirror (monolithic) made from borosilicate glass, but having large hollowed out regions to keep the weight down. This borosilicate honeycomb design has been pioneered by Roger Angel at the Mirror Lab of the University of Arizona. This type of mirror has been successfully cast in a 3.5m size (used in the ARC 3.5m (APO), WIYN 3.5m (KPNO), and the Starfire Optical Range Telescope near Albuquerque), and in a 6.5m format for the MMT conversion (Mt. Hopkins, AZ) and the Magellan (Las Campanas Observatory, Chile) telescopes; they have also been made in an 8m format (x2) for the Large Binocular Telescope (Mt. Graham, AZ).
- Thin mirrors - lightweight, better for TE, but not as stiff, uses an active support system (changing gravity load during tracking, timescales of minutes or hours, not like adaptive optics). *Active* optics - operates on longer timescales, structural mechanical stuff. Gemini was a big deal. VLT (Europe). The second design is also monolithic but has a mirror which is significantly thinner than the borosilicate mirror. These thin mirrors are being built primarly by two companies, Corning (USA) and Schott (Germany). They use materials with good thermal properties, ULE (Corning) and Zerodur (Schott). Thin mirrors are being used in ESO's 3.5m New Technology Telescope (La Silla, Chile), Japan's 8m Subaru telescope (Mauna Kea, Hawaii), the two 8m Gemini telescopes (Mauna Kea and Cerro Pachon, Chile), and ESO's Very Large Telescopes (4 8m's on Cerro Paranal).
- Segmented - lots of little mirrors, each element needs its own correct shape (overall shape is hyperbola). Finally, the third design make use of segmented mirrors, in which a large mirror is made by combining many small mirrors. This design is currently operational in the 10m Keck telescope (Mauna Kea), the 11m Hobby-Eberly Telescope, the 11m SALT telescope, and the 10m Gran Telescopio de las Canarias. Future 30m class telescopes: TMT, GMT, and E-ELT.

Tabular summary of the world's telescopes!

The borosilicate mirrors have the advantage that they are stiffer than the other designs, so the mirror support is less complicated. For thin mirrors, the support system must be activated to allow for changing shape as a function of telescope pointing. For segmented mirrors, each segment must be controlled to make sure the entire surface is smooth. The thick mirror is also less susceptible to wind shake, which can adversely affect image quality. The thin and segmented mirrors have the advantage of better thermal properties since they contain less total material.

The choice of a primary mirror technology can be complicated. In designing a large telescope, one generally first decides on an optical prescription which is chosen considering the main scientific goals for the project (e.g., large field, IR, good image quality, etc.). The primary mirror choice is made considering the choice of site (e.g, are there large temperature changes, lots of wind, etc.), availability, issues of engineering complexity, and, especially, cost (and politics). The choice of a mount and control system to use is basically a cost and operations issue.

2.1 Mirror coatings

Mercury - highly reflective liquid :) Aluminum usually highest choice. Silver and gold better at longer wavelengths, but go back at shorter ones; mostly used for telescopes that work primarily in the IR.

Aluminum, silver, gold most commonly used. See, e.g. here for relative reflectances as a function of wavelength, also here.

Issues with mirror cleaning and recoating.

2.2 Telescope mounts

We've talked about the optics that go into telescopes. However, it's clear that these optics need to be supported in some structure and kept in alignment with each other. The support structures needed are really an engineering issue (and a challenging one for large telescopes), and we won't disucss it here. In addition to supporting the optics, the structure also needs to be capable of tracking astronomical objects as they move across the sky because of the rotation of the earth.

There are two main different sorts of telescope mounts found in observatories: the *equatorial* mount and the *altitude-azimuth (alt-az)* mount. The equatorial mount is by far the most common for older telescopes, but the alt-az design is being used more frequently for newer, especially larger, telescopes. In the equatorial design, the telescope move along axes which are parallel and perpendicular to the polar axis, which is the direction parallel to the earth's rotation axis. In such a mount, tracking the earth's rotation only requires motion along one axis, the one perpendicular to the polar axis, and the tracking motion is at a uniform rate. In the alt-az mount, the telescope moves along axes which are perpendicular and parallel to the local vertical axis. With this mount, however, tracking of celestial objects requires motions of variable speed along both axes. An additional complication of an alt-az mount is the fact that, for a detector which is fixed to the back of the telescope, the image field rotates as the telescope tracks an object. Note, however, that the telescope pupil does not rotate with the object.

An equatorial mount is much easier to control for pointing and tracking. However, from an engineering point of view, it is much more demanding to construct, especially for large telescopes which have significant weight. The engineering complications generally result in a significantly larger cost (for large telescopes) than for an alt-az design. An alt-az telescope, however, has a significantly more complex control system, and must have an image rotator for the instruments.

Regardless of mount type, the mount is never built absolutely perfectly, i.e. with axes exactly perpendicular, exactly aligned as they should be, totally round surfaces, optics aligned with mechanics, etc. As a result, a telescope does not generally point perfectly. However, many effects of an imperfect telescope are quite repeatable, so they can be corrected for. This corrrection is done by something called a pointing model, which records the difference in true position from prediction position over the sky, and, once derived, the pointing model can be implemented to significantly improve pointing. A good telescope points to within a few arcseconds after implementation of a good pointing model.

Related to pointing is tracking performance. The issue here is how long the telescope can stay pointed at a given target. You can consider this question as how well the telescope can point over the area of the sky through which your object will drift. Since your required pointing stability should be significantly less than one arcsec, so that tracking does not degrade the image quality significantly, almost no telescopes have sufficiently good pointing to track to within an arcsecond for an arbitrarily long time. Most telescopes can track sucessfully for several minutes, but will give significant image degradation for exposures longer than this. Consequently, most telescopes/instruments are equipped with *guide cameras*, which are used to continually correct the pointing by observing an object somewhere in the field of view of the telescope (possibly the object you are interested in, but usually not, since that's where your detector is looking). These days, most guiders are *autoguiders*, meaning that they automatically find the position of the guide object, compute the pointing offsets needed to keep this object in one position, and send these offsets as commands to the telescope. The observer generally just has to choose a guide object for the autoguider to use, though they also may have to adjust the guide camera sensitivity or gain to insure that the guide star has a strong signal. These days, many autoguiders can automatically find guide stars in the field or from some on-line catalog (e.g., the HST Guide Star Catalog, which catalogs stars down to V 14). However, if one is taking long exposures and knows that they'll need to use guide stars, make sure to find out whether such a facility is available ; if not, it may still be possible to find guide stars in advance of your observing run, e.g., from the sky survey. If so, you should seriously consider doing so, as it can take a frustratingly long amount of time to search for a guide star at the telescope in real time. Since telescope time is heavily oversubscribed at most facilities, you really want to maximize your efficiency, and doing so is a large part of what will make you a "expert" observer.

Note guiding in spectrographs is often done off of the slit with a slit-viewing camera.

Axes of machine…

- equatorial: simpler, don't have to track dec, just RA. Rotation rate of Earth is used, so it's fixed.
- alt-ax: More complicated, have to track in both axes. Get constant speed (vector-wise) but each individual speed varies. From control-system point of view is more complex, but is the choice for large telescopes (cheaper). Can't point to zenith, 3.5 m has ~ 520 degree range of "wrap", telescope has to swing all the way around once passing zenith, too much swiveling can wrap up cables (don't want that).

Picture orientation: mirrors that of the telescope axes. N/E vs. zenith/horizon. N/S changes for alt-az telescope, aka entire field rotates. Need third axis to compensate when taking long images. For single point, doesn't matter. Rotating point is still a point. Assuming perfect system, build up table of offsets, which will be repeatable for a well-built telescope. Called *pointing models*.

2.3 Using telescopes

(skipped over this in class)

2.4 Planning observing

(skipped over this in class)

Monday, April 18, 2016

3 Instrumentation

Review: Different angles have different wavefronts. Multi-conjugate $\rightarrow$ multiple deformable mirror.

Often, astronomers use additional optics between the telescope and their detector. These, in conjunction with a detector, make up an *instrument*.

3.1 Location of optics

Before going into specifics, consider the effect of placing optics at different locations within an optical system, like a telescope.

Optics placed in or near a focal plane will affect images at different field angles differently. Optics in a focal plane will not affect the image quality at any given field angle; however, such optics might be used to control the location of an image of the *pupil* of the telescope.

Optics placed in or near a pupil plane will affect images at all field angles similarly, and will have an effect on the image quality.

3.2 Refractive optics and chromatic aberration

In many instruments, lenses are used rather than mirrors; they can be cheaper and lead to more compact designs. Recall, however, that when lenses are used, chromatic effects will arise because the index of refraction of glasses changes with wavelength. While they can often be minimized by the use of use of multiple elements to make achromatic combinations, they are not always negligible. In particular, if an instrument is used at multiple wavelengths, some refocussing may be required.

3.3 Field Flatteners

As we've discussed, all standard two-mirror telescopes have curved focal planes, so the image locations are in a curved plane. It is possible to make a simple lens to correct the field curvature. We know that a plane-parallel plate will shift an image laterally, depending on the thickness of the plate. If we don't want to affect the image quality, only the location, we want the correcting element to be located near the focal plane.

Consequently, we can put a lens right near the telescope focal plane to flatten the field. For a field which curves towards the secondary mirror, the correct shape to flatten the field is just a plano-concave lens with the curved side towards the secondary. Often, the field flattener is incorporated into a detector dewar as the dewar window.

3.4 Focal plane reimagers

A focal reimager is a reimaging system which demagnifies/magnifies the telescope focal plane. In a simple form, it consists of two lenses: a collimator and a camera lens. The collimator lens is placed such that the telescope focal plane is put at the focal length of the collimator, so that it converts the telescope beam into a collimated beam (note that the focal ratio of the collimating lens itself will be larger than that of the telescope so that the beam underfills the lens to allow for off-axis light as well). The camera lens then refocuses the light with the desired focal ratio. The magnification of the system is given by:

$$m = \frac{f_{camera}}{f_{collimator}}$$

Consequently, the scale in the image plane of the focal reimager is just the scale in the telescope focal plane multiplied by the ratio of the focal ratio of the camera to that of the telescope.

Note that with a focal plane reimager, one does not necessarily get a new scale "for free". The focal reimaging system may introduce additional aberrations giving reduced image quality. In addition, there is always some light lost at each additional optical surface from reflection and/or scattering, so the more optics there are in a system, the lower the total throughput is.

Note that it is possible to do focal reduction/expansion *without* reimaging, i.e., by putting optics in the converging beam.

3.5 Pupil reimagers

Often, an additional lens, called a *field lens* is placed in or near the telescope focal plane. This does not affect the focal reduction but is used to reimage the telescope pupil somewhere in the reimager. One reason this may be done is to minimize the size that the collimator lens needs to be to get off-axis images. The size of the field lens itself depends on the desired size of the field that one wishes to reimage.

Another use of reimaging the pupil is when building a *coronagraph*, an imaging system designed to observe faint sources near bright ones. Scattered light, diffraction, and sometimes detector effects (e.g., charge bleeding in a CCD) can all contribute to the difficulty in seeing the faint source. A partial solution is to put an occulting spot in the telescope focal plane which removes most of the light from the bright object. However, the diffraction structure is still a problem. It turns out you can remove this by reimaging the pupil after the occulting spot and putting a mask in around the edges which are the source of the diffraction; this mask is called a *Lyot stop*. The resulting image in the focal plane of the focal reducer is free of both bright source and diffraction structure.

Note that for really high contrast imaging, you also need to consider other sources of far-field light including light scattered from small-scale features on optical elements, and far-field light from seeing. Minimizing the former required very smooth optics, while minimizing the latter requires high-performance adaptive optics (e.g. "extreme-AO").

Wednesday, April 20

Pupil reimagers are also widely used in IR systems to reduce emission via cold pupil stops. The issue here is that the telescope itself contributes infrared emission which acts as additional background in the observations. There is little that can be done about emission from the primary, since you need to see light from the primary to see your object. However, emission can be blocked from regions of the pupils that are obscured already, such as the secondary and/or secondary support structures, by putting a mask in the pupil plane. However, the mask needs to be colder than the telescope itself or else the mask would contribute the background, so it is usually placed within the dewar that contains the detector and camera optics.

3.6 Filters

Filters are used in optical systems (usually imaging systems) to restrict the observed wavelength range. Using multiple filters thus provides color information on the object being studied. Generally, filters are *loosely* classified with widths of:

- broadband ($>\sim$ 1000 Å),
- medium band (100 $<\sim$ 1000 Å), or
- narrow band (1 $<\sim$ 100 Å).

Perhaps a better distinction between different filters is by how they work. Many broad band filters use *colored glass*, which has pigments that absorb certain wavelengths of light and let others pass. Bandpasses can be constructed by using multiple types of colored glass. These are generally the most inexpensive filters.

Interference filters use the principle of *interference*, and are made by using two partially reflecting plates separated by a distance d. The principle is fairly simple: when light from the different paths combines constructively, light is transmitted; when it combines destructively, it is not. Simple geometry gives:

$$m\lambda = 2dn\cos\theta$$

It is clear from this expression that the passband of the filter will depend on the angle of incidence. Consequently, narrowband filters will have variable bandpasses across the field if they are located in a collimated beam; this can cause great difficulties in interpretation. However, if the filter is located in a focal plane or a converging beam, the mix of incident angles will broaden the filter bandpass. This can be a serious effect in a fast beam. Bandpasses of interference filters can also be affected by the temperature.

Interference filter diagram

Since interference filters will pass light at integer multiples of the wavelength, the extra orders often must be blocked. This can be done fairly easily with colored glass.

The width of the bandpass of a narrowband filter is determined by the amount of reflection at each surface. Both the wavelength center and the width can be tuned by using multiple cavities and/or multiple reflecting layers, and most filters in use in astronomy are of this more complex type.

The same principles by which interference filters are made are used to make antireflection coatings.

Note filters can introduce aberrations, dust spots, reflections, etc; one needs to consider these issues when deciding on the location of filters in an optical system.

3.7 Fabry-Perot Interferometer

A *Fabry-Perot* system makes use of a tunable interference filter. The filter is tuned in wavelength by adjusting one of the:

- spacing
- index of refraction (usually by changing the pressure)
- tilt of the interference filter

A tunable interference filter is called an *etalon*. Often, etalons are made to provide very narrow bandpasses, on the order of 1 Å.

A picture taken with a Fabry-Perot system covers multiple wavelengths because the etalon is located in the collimated beam between the two elements of the focal reducer. At each etalon setting, one observes an image which has rings of constant wavelength. By tuning the etalon to give different wavelengths at each location, one build up a "data cube", through which observations at a constant wavelength carve some surface. Consequently, to extract constant wavelength information from the Fabry-Perot takes some reasonably sophisticated reduction techniques. It is further complicated by the fact that to get accurate quantitative

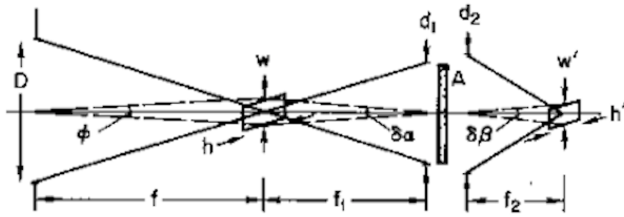

Fig. 12.4. Schematic layout of slit spectrometer with dispersing element *A*. See text, Section 12.II, for definitions of parameters.

information, one requires that the atmospheric conditions be stable over the entire time when the data cube is being taken.

3.8 Spectrographs

A spectrograph is an instrument which separates different wavelengths of light so they can be measured independently. Most spectrographs work by using a *dispersive* element, which directs light of different wavelengths in different directions. A conventional spectrograph has a collimator, a dispersive element, a camera to refocus the light, and a detector. The performance of a spectrograph is characterized by the *dispersion*, which gives the amount that different wavelengths are separated, and the *resolution*, which gives the smallest difference in wavelength that two different monochromatic sources can be separated. There are different sorts of dispersive elements with different characteristics; two common ones are prisms and diffraction gratings, with the latter the most commonly in use in astronomy.

Monday, April 25

The dispersion depends on the characteristic of the dispersing element. Various elements can be characterized by the angular dispersion, $\mathrm{d}\theta/\mathrm{d}\lambda$, or alternatively, the reciprocal angular dispersion, $\mathrm{d}\lambda/\mathrm{d}\theta$. In practice, we are often interested in the linear dispersion, $\mathrm{d}x/\mathrm{d}\lambda = f_2 d\theta/\mathrm{d}\lambda$ or the reciprocal linear dispersion, $\mathrm{d}\lambda/\mathrm{d}x = \frac{1}{f_2}\mathrm{d}\lambda/\mathrm{d}\theta$ where the latter is often referred to simply as the dispersion in astronomical contexts, and is usually specified in Å/mm or Å/pixel.

If the source being viewed is extended, it is clear that any light which comes from regions parallel to the dispersion direction will overlap in wavelength with other light, leading to a very confused image to interpret. For this reason, spectrographs are usually used with slits or apertures in the focal plane to restrict the incoming light. Note that one dimension of spatial information can be retained, leading to so-called *long-slit* spectroscopy. If there is a single dominant point source in the image plane, or if they are spaced far enough (usually in combination with a low dispersion) that spectra will not overlap, spectroscopy can be done in *slitless* mode. However, note that in slitless mode, there can be a significant impact from sky emission.

The resolution depends on the width of the slit or on the size of the image in slitless mode, because all a spectrograph does is create an image of the focal plane after dispersing the light. The "width" of a spectral line will be given by the width of the slit or the image, whichever is smaller. In reality, the spectral line width is a convolution of the slit/image profile with diffraction. The spatial resolution of the detector may also be important.

Note that *throughput* may also depend on the slit width, depending on the seeing, so maximizing resolution may come at the expense of throughput. Throughput = seeing + slit width.

Given the width, ω, and height, h, of a linear slit, we can get an *image* of the slit with width ω' and height h':

$$h' = h\frac{f_2}{f_1}$$

Or for an angular width, $\phi = \omega/f$, (where f is the focal length of the telescope) and height: $\phi' = h/f$:

$$\omega' = r\omega\frac{f_2}{f_1}$$

where we have allowed that the dispersing element might magnify/demagnify the image in the direction of dispersion by a factor r, which is called the anamorphic magnification.

Using this, we can derive the difference in wavelength between two monochromatic sources which are separable by the system.

$$\delta\lambda = \omega'\frac{\mathrm{d}\lambda}{\mathrm{d}x}$$

$$\delta\lambda = r\omega\frac{f_2}{f_1}\frac{\mathrm{d}\lambda}{\mathrm{d}x}$$

The bigger the slit, the lower the resolving power. The resolution is often characterized in dimensionless form by

$$R \equiv \frac{\lambda}{\delta\lambda} = \frac{\lambda f_1}{r\omega f_2\,(\mathrm{d}\lambda/\mathrm{d}x)}$$

Note that there is a maximum resolution allowed by diffraction. This resolution is given aproximately by noting that minimum angles which can be separated is given by approximately λ/d_2, where d_2 is the width of the beam at the camera lens, from which the minimum distance which can be separated is:

$$\omega_{min} = f_2\frac{\lambda}{d_2}$$

The slit width which corresponds to this limit is given by:

$$\omega' = r\omega\frac{f_2}{f_1} = f_2\frac{\lambda}{d_2}$$

or

$$\omega = \frac{f_1}{r}\frac{\lambda}{d_2}$$

and the maximum resolution is

$$R_{max} = \frac{d_2}{f_2\,(\mathrm{d}\lambda/\mathrm{d}x)} = d_2\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}$$

3.9 Astronomical spectrographs

- Slitless spectographs.
- Long slit spectrographs
- Image slicers: preserving resolution and flux.
- Fiber spectrographs: multiobject data.
- Slitlets: multiobject data.
- Integral field spectrographs.

3.10 Dispersing elements

3.10.1 Prisms

Perhaps the simplest conceptual dispersing element is a prism, which disperses light because the index of refraction of many glasses is a function of wavelength. From Snell's law, one finds that:

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = \frac{t}{d}\frac{\mathrm{d}n}{\mathrm{d}\lambda}$$

where t is the base length, and d is the beamwidth. Note that prisms do not have anamorphic magnification ($r = 1$). The limiting resolution of a prism, from above is:

$$R_{max} = \frac{d_2}{f_2(\mathrm{d}\lambda/\mathrm{d}x)} = d_2\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}$$

$$R_{max} = t\frac{\mathrm{d}n}{\mathrm{d}\lambda}$$

For many glasses, $\mathrm{d}n/\mathrm{d}\lambda \propto \lambda^{-3}$. So dispersion and resolution are a function of wavelength for a prism. In addition, the resolution offered by a prism is relatively low compared with other dispersive elements (e.g. gratings) of the same size. Typically, prisms have $R < 1000$. Consequently, prisms are rarely used as the primary dispersive element in astronomical spectrographs. They are occasionally used as cross-dispersing elements.

3.10.2 Gratings

Diffraction gratings work using the principle of multi-slit interference. A diffraction grating is just an optical element with multiple grooves, or slits (not to be confused with the slit in the spectrograph). Diffraction gratings may be either transmissive or reflective. Bright regions are formed where light of a given wavelength from the different grooves constructively interferes. The location of bright images is given by the *grating equation*:

$$m\lambda = \sigma\left(\sin\theta + \sin\alpha\right)$$

for a reflection grating, where

- σ is the groove spacing

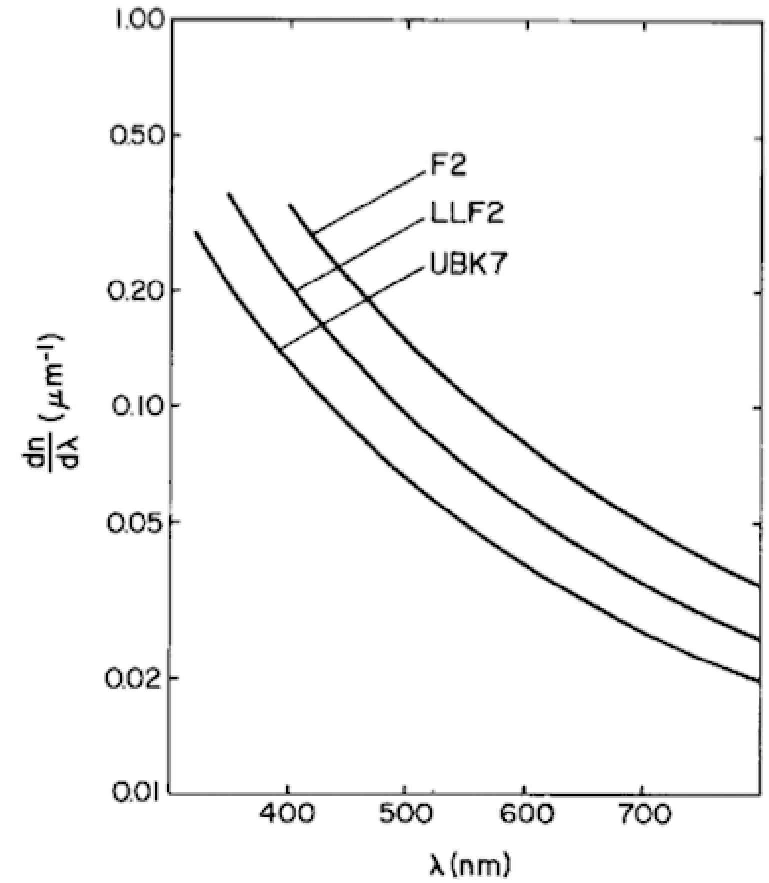

Fig. 13.1. Dispersion curves for three glasses from the Schott glass catalog.

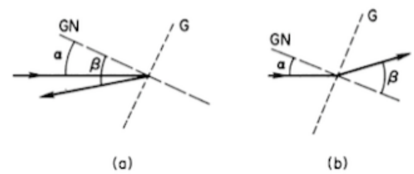

Fig. 13.2. Schematic showing angles of incidence α and diffraction β for (a) reflection grating and (b) transmission grating. See the discussion following Eq. (13.2.1) for the sign convention.

- m is the order
- α is the angle of incidence
- θ (β in picture above) is the angle of diffraction

α and θ are measured relative to the normal to the grating surface.

The dispersion of a grating can then be derived:

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = \frac{m}{\sigma \cos\theta}$$

The dispersion is larger at higher order, and for a finer ruled grating. The equation can be rewritten as

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = \frac{\sin\theta + \sin\alpha}{\lambda \cos\theta}$$

from which it can be seen that high dispersion can also be achieved by operating at large values of α and θ. This is the principle of an echelle grating, which has large σ, and operates at high m, α and θ, and gives high dispersion and resolution. An advantage of this is that one can get a large fraction of the light over a broad bandpass in a series of adjacent orders.

Typical gratings have groove densities between 300 and 1200 lines/mm. Echelle gratings have groove densities between 30 and 300 lines/mm.

The anamorphic magnification for a grating can be derived by looking at how θ changes with α at fixed λ:

$$r = \frac{\mathrm{d}\theta}{\mathrm{d}\alpha} = \frac{\cos\alpha}{\cos\theta} = \frac{d_1}{d_2}$$

where the d's are the beam diameters. Note that higher resolution occurs when $r < 1$, or $\theta < \alpha$.

(insert figure here)

The limiting resolution can be derived:

$$R_{max} = \frac{d_2}{f_2\,(\mathrm{d}\lambda/\mathrm{d}x)} = d_2 \frac{\mathrm{d}\theta}{\mathrm{d}\lambda}$$

$$R_{max} = \frac{d_2 m}{\sigma \cos\theta} = \frac{mW}{\sigma} = mN$$

where W is the width of the grating ($= d_2/\cos\theta$), and N is the total number of lines in the grating.

Note that light from different orders can fall at the same location, which can be confusing. This occurs when:

$$m\lambda' = (m_1)\lambda$$

or

$$\lambda' - \lambda = \frac{\lambda}{m}$$

The order overlap can be avoided using either an *order-blocking filter* or by using a cross-disperser. The former is more common for small m, the latter for large m.

Comparison between gratings operating in low order, gratings operating in high order, and prisms shows that higher resolution is available from gratings, and that echelles offer higher resolution than typical low-order gratings.

Grating efficiency is the fraction of incident light directed into a given diffracted order. For a simple grating, less light is diffracted into higher orders. However, a grating that maximizes the light put into any desired order can be constructed by *blazing* the grating, which involves tilting each facet of the grating by some blaze angle. The blaze angle is chosen to maximize the efficiency at some particular wavelength in some particular order; it is set so that the angle of diffraction for this order and wavelength is equal to the angle of reflection from the grating surface. The *blaze function* gives the efficiency as a function of wavelength.

A special case of high efficiency is when the angle of incidence equals the angle of diffraction, i.e. the diffracted light at the desired wavelength comes back to the same direction of in the incoming light. This is called the Littrow configuration; high efficiency spectrographs often try to work close to this configuration.

(insert figure here)

Typical peak efficiencies of reflective diffraction gratings are of order 50-80%. Recently, a new technology for making diffraction gratings, volume phase holographic (VPH) gratings, as been developed, and these are attractive because they offer the possibility of very high efficiencies ($>$ 90% peak efficiency).

3.10.3 Grisms

A grism is a combination of a prism and a diffraction grating. These are combined such that light is dispersed, but light at a chosen central wavelength passed through the grism with direction unchanged. This feature allows grisms to be placed in an imaging system (e.g., in a filter wheel) to provide a spectroscopic (usually low resolution) capability.

3.11 Operational items: using a spectrograph

Date?

Choice of dispersion: wavelength coverage vs. dispersion/resolution, available gratings, etc. Using grating tilt to select wavelength range.

Choice of slit width (science, seeing).

How to put object in slit. Imaging the slit. Slit viewing cameras.

(DEFER FOLLOWING TO SECTION ON DATA REDUCTION???)

Spectrograph calibration (not including basic detector calibration, to be discussed soon).

Wavelength calibration: correspondance between pixel position (in wavelength dimension) and wavelength. Arc lamps, wavelength solutions. Subtleties: extrapolation, line curvature, flexure (using skylines to calibrate).

Flux calibration: relative fluxes at different wavelengths. Spectrophotometric standards. Subtleties: differential refraction

Spectral extraction: object extraction and sky subtraction. Subtleties: S-distortion, differential refraction: spectral traces. Issues: variation of focus along slit and implications for sky line subtraction, scattered light.

Relative fluxes along slit: slit width variations.

Examples of typical spectra: line lamps, flat fields, stellar spectra, galaxy spectra. Night sky emission.

3.12 Non-dispersive spectroscopy

It is also possible to use interference effects to measure spectral energy distributions instead of a dispersing element. The Fabry-Perot is an example of such a type of instrument, although it does not record all wavelengths simultaneously.

(insert figure here)

Another instrument which uses interference to infer spectroscopy information is the Fourier Transform Spectrometer (FTS), which is basically a scanning Michaelson interferometer. The light from the source is split into two parts using a beamsplitter. One part of light is reflected off a fixed flat mirror and the other is reflected off a mirror which can be moved laterally. The two images are combined to form fringes. The fringe pattern changes as the path length of the second beam is changed. The intensity modulation for a given wavelength (λ) or wavenumber ($k = 2\pi/\lambda$) is given by:

$$T\left(k, \Delta x\right) = \frac{T_{max}}{2}\left[a + \cos\left(2k\Delta x\right)\right]$$

and the flux after integrating over all wavelengths is:

$$F\left(\Delta x\right) = C\int I(k)T(k, \Delta x)\mathrm{d}k = C\int I(k)\cos(2k\Delta x)\mathrm{d}k$$

where $I(k)$ is the input spectrum. Consequently it is possible to recover the input spectrum by taking the Fourier cosine transform of the recorded intensity. In practice, a discrete Fourier transform is used.

The FTS requires scanning in path spacing. But unlike the Fabry-Perot, it yields information on intensity at all wavelengths simultaneously.

4 Detectors

Monday, May 2, 2016

4.1 Basic Principles and Properties

Detectors work because they are made of material that interacts with photons. Three general types, based on what is generated by the photon:

Photographic detector chemical reaction

Photomultiplier/photon counter event, usually detected in real time. Electrons are rapidly accelerated toward anode by photon, then more electrons are excited and accelerate, and so on. High voltage. These are *not* imagers.

Photon collecter photoelectron, usually (electronically) stored for subsequent readout. CCDs are a subset of this type.

In a photon counting device, some fraction of incident photons hit a photosensitive material and eject a photoelectron. This electron is amplified numerous times to create a large "swarm" of electrons which is detected as a pulse. Thus, photons are "counted" as they come in. Simple photomultipliers do not retain any information about the location on the detector where the photon hits. There are some modern devices, however, called microchannel plates, which are essentially arrays of small photomultipliers where positional information can be obtained; one of the more common of these is called a MAMA (Multi-Anode…something), and exists in several instruments on the Hubble Space Telescope. Traditional photomultipliers were the workhorse of photometry from the 50's to the late 70's. More recently, a more sensitive type of photon counter, called an avalanche photo-diode, has been used.

Photon collecting array detectors are in more common usage today. In these devices, incoming photons create photoelectrons which are trapped in local potential wells. The amount of energy needed to eject a photoelectron depends on the type of material used. In the optical, silicon provides a good choice, but the excitation energy for silicon is too high to be used in the infrared. In the IR, various different substances are used, including HgCdTe, InSb, and PtSi. After a specified amount of time, the photoelectrons are "counted". The method by which this is done differs between different types of arrays. In CCDs, the charge is physically clocked down columns of the device, a single row at a time (a parallel transfer) then read out of serial register; CCDs are inherently asymmetric in rows and columns. In IR devices, each pixel is read individually, in sequence. Material: semiconductor. In optical, pure silicon is used, which isn't sensitive at wavelengths greater than $\sim 1\ \mu$m. At 1μm, "barely" enough energy. Other materials used in IR…First two are sensitive out to about 2.5 μm, last one to $\sim 5\ \mu$m (see book in library about the physics here!)

Detectors are characterized by a variety of different important quantities that should be considered when buying them…what do you need for your science? The "Biggies":

Quantum efficiency fraction of photons detected, usually a function of wavelength. Typical values:

- photographic: ~ 0.1 %
- photomultiplier: $\sim$ 10-20 %
- array detector: $\sim$ 20-90 %

Size and resolution elements (given that optics can be used to change the scale, the number of resolution elements may be more critical):

- photographic: large (many inches), good resolution
- photomultiplier: several inches, no resolution (though there are arrays of photomultipliers, e.g. MAMA). APDs (?) have very small collecting areas.
- Array detector: individual detectors started out small but are continually growing. The largest current CCDs are around 2.5 in^2, wheras the largest IR arrays are roughly 1 in^2. Larger effective sizes are now available (for CCDs) because modern devices are made to be "buttable", i.e. several can be placed side by side with only very small gaps between them. Pixel sizes of array detectors are typically 15-25 microns (although arrays with smaller pixels are very common, e.g., for digital camera, etc, applications, they are less frequently used in astronomy because they generally don't provide a good match to telescope scales).

Discovery efficiency product of sensitivity and area.

Readout effects The process of counting electrons is never totally exact, so noise is introduced. Generally this is at a level of 1-100 electrons rms. Readout effects can come in two different forms:

1. pattern noise: introduced in the readout process, but has some spatial correlation. *Fixed* pattern noise has the same spatial pattern over the detector from exposure to exposure, and thus can be corrected. If the pattern noise is not fixed, it changes every time exposure is read out.
2. random noise: what people usually refer to as readout noise. If, instead of photoelectrons, photon *events* are counted (photomultiplier), there is no readout noise; this is the key advantage of photon counters.

Wednesday, May 4, 2016 Should add some margins to these notes...

Dark current Electrons in a given substance will be moving around at speeds correlated with the operating temperature of the device. If there is enough thermal motion, an electron can be liberated from the substance and then counted as a (spurious) photon detection. This is called dark current. Devices with lower photoelectric threshholds (used at longer wavelengths) are more susceptible to dark current, thus they must be operated at a colder temperature. CCDs are typically operated between -70°C and -120°C, IR arrays are colder (77K for liquid N_2, 4K for liquid He). Dark current can be subtracted using calibration data, but note that there is still the Poisson *noise* associated with the dark current, so even with dark subtraction, additional noise is generated.

Linearity relation between number of output electrons and input photons. In a fully linear detector, the slope of the relation is given by the quantum effeciency, which can be considered as a function of the count level for *non*linearity.

- photographic: nonlinear (characteristic curve)
- photoelectric: linear except for dead-time correction. A dead-time correction applies for bright sources; if two photons arrive essentially simultaneously, they will only be counted as a single photon.
- array detector: CCDs *usually* linear up to 50-90% of full well. IR arrays are usually slightly nonlinear over their entire range, but repeatably so.

Full well maximum number of photons possible to detect.

- photographic: limited by number of gains
- photomultiplier: unlimited
- array detector: $\sim$ 100,000 photons

Other possible effects/issues:

- Defects. Most detectors are not perfect; there are often small regions which are unusable because of very low quantum efficiency, blocked columns, etc. Many devices are rejected for astronomical use because of too many defects. dead pixels - no response at all. sensitivity changes from pixel to pixel (reason for flats).
- Modulation transfer function. In some detectors at some wavelengths, charge deposited at one location can be spread over a wider region, leading to degradation of image quality. This is often most notable at long wavelengths in CCDs, where one can see moderately large halos around point sources. This is usually due to the penetration of long wavelength photons to the substrate of the device from which they can be reflected back. characterizing the *size* of an image.
  A related effect is fringing on chips, which results from interference of monochromatic (not broadband) incident light that is reflected from the substrate, which is relevant because the night sky spectrum contains strong monochromatic features. Since the substrate surface is not perfectly flat, this can lead to irregular patterns in the background. Examples: DIS red fringing, 1m i band imaging. The details of fringing depend on specifics of the chip construction, and fringing effects can be mitigated in some cases. what back of device looks like, hard to deal with this, don't see it when the moon is up, amount of fringing is proportional... also depends on what direction you're facing.
- Readout speed: typically 25 microsec/pixel for array detectors. However, the chips can in many cases be read at a variety of speeds. In general, when chips are read faster, the readout noise increases. Larger chips now often have multiple readout channels, so several regions of the chip can be read simultaneously, decreasing the total readout time. ccd: charge transferred down, then across to *amplifier*, split into four parts (quad-channel readout vs. single channel readout.) More expensive though...
- Saturation behavior. When full well is reached, the chip is said to be saturated. In some array detectors, especially CCDs, when a given pixel is saturated, additional charge often leaks into adjacent pixels, usually into adjacent rows rather than columns because of the construction of the CCD. Separately from detector saturation, one occasionally sees saturation of the readout electrons which can have the effect of a saturated pixel affecting the counts in subsequent pixels.

- Hysteresis generally refers to processes in which the prior exposure history of a detector affects subsequent exposures. A common example is *residual image*, in which an area of a detector which was subject to a very bright source in a previous exposure will continue to "glow" in subsequent exposures. Another form of hysteresis is so-called quantum efficiency hysteresis (QEH) in which the quantum efficiency of the chip changes as a function of previous exposure history. One manifestation of this effect has been used by astronomers, mostly with older CCDs; for some reason, when these CCDs were subjected to an extended influx of ultraviolet light, their optical quantum efficiency was found to be increased, and to stay increased at a stable level as long as the chip was kept cold. This led to the practive of "UV-flooding" CCDs as they were being cooled.
- Reciprocity: sensitivity changes as a function of photon incidence *rate*. Exists in photographic plates, *maybe* in some array detectors? Dead-time correction is a reciprocity failure.

4.1.1 Digitization

In array detectors, after the charge is collected and read out, it is sent through a chain of electrons which *digitizes*[1] the signal, often after amplifying it. The digitization is made by a device known as an A/D convertor; these work by comparing an input signal with a set of reference voltages which successively differ by factors of two. Thus an input signal is translated into a series of bits depending on whether the input voltage exceeds a series of reference voltages. Typical A/D correctors in use in astronomy consider 16-bits. The digital signal which comes out of the CCDs is variously referred to as counts, digital numbers (DN), or analog-to-digital units (ADU). The number of output counts is related to the number of input counts by a constant which depends on the amplification in the electronics. The amplification factor is known by most people as the gain, but astronomers define the gain of a device by the number of input electrons divided by the number of output counts (i.e., the inverse gain); this "astronomers" gain is specified in units of e-/DN. Because the number which we receive from the electronics chain differs from the number of input electrons (i.e, the number of detected photons), the calculation of noise must take this into account. The photon counting noise (rms) is given by the square root of the number of detected photons. The number of detected photons is given by GC, where G is the (inverse) gain and C is the number of detected counts. Consequently, the noise in electrons is $\sqrt{GC}$, and in units of counts is given by $\sqrt{C/G}$. This is apart from readout noise; the latter is usually specified in units of electrons, giving a total noise in electrons of $\sqrt{GC + \sigma_{rn}^2}$, or, in units of counts, by $\sqrt{C/G + \sigma_{rn}^2/G^2}$.

A/D converters can only measure a positive incoming signal. At low light levels, the true input signal can be negative in the presence of readout noise. To avoid trucation of the negative signals, a constant voltage, called the bias, is added to the signal before it passes through the A/D. This bias must later be removed to preserve the correct count ratios between different sources; this is generally accomplished in CCDs by using the overscan

[1] Online definition: convert into a digital form that can be processed by a computer

region of the image.

A/D convertors can introduce small systematic errors in recorded count rates if the reference voltages are not carefully controlled.

4.1.2 Dynamic range

4.1.3 Determining gain and readout noise

4.2 CCDs

4.3 IR detectors

4.4 UV and other detectors

5 Data reduction: details and subtleties

6 Photometry

(All lecture notes from this section copied from website. Not edited at all yet).

Consider techniques for stellar photometry, surface photometry, calibration of broad band and emission line photometry.

6.1 Aperture photometry

In aperture photometry, we simply identify stars and add up all of the light in the surrounding pixels. The light is spread over several pixels in a form given by the point spread function PSF(i, j). We must also measure the background and subtract its contribution to the sum.

Error estimation: we're going to sum over pixels. First just consider the object aperture:

where the sum is over N pixels, and B is the background per pixel. To get the true signal S, N would need to approach infinity since the stellar profile continues out to great distances. In practice, however, we choose N such that a repeatable fraction of light falls within the N pixels independent of exposure. Next, we want to determine the error in our measurement of S. The noise is

which is just

Now we have to consider how we will estimate B. In the simplest case, we just go away from the star and take then mean level. Generally, we want to minimize effects of any nonuniform background, so we consider an annulus around the star. If this annulus contains Na pixels, then

and

From this we see that errors in determining the background are small contributors to the total error in the star if Na ¿ ¿ N, and generally that is the case. If the errors in determining the background are negligible, then

which is an important result that we have seen before, but here the area is explictly given in terms of a number of pixels. Using this, one can consider the optimal choice of aperture. As the aperture size increases, you get a increase in total signal, but also an increase in background, so the change in S/N depends on how bright the star is relative to background. The optimal choice of aperture will depend on the brightness of the star: a larger aperture will be better for brighter stars, a smaller aperture for fainter stars. But also recall that we don't want to use a too small aperture or else we won't be able to compare results from different frames because of changes in the PSF.

This leads to the commonly used technique of using small apertures for all of the stars on the frame, but large apertures for a few bright stars. The few bright stars are assumed to have representative PSFs for this frame, so all of the small aperture measurements can be aperture corrected to large aperture measurements without the increase in noise you would get if you actually used a large aperture. Note that you can't go arbitrarily small as you will eventually run into the problem of PSF variations across the frame if for no other reason than any small dependences on pixel centering.

In fact, you can do better for S/N if you use additional information. If you know the shape of the PSF, you can use this information to fit your stellar image, increasing the S/N in the process. Simple linear least squares argument, if you know PSF and position accurately, leads to

where

Note that you'll improve S/N, but only if your assumption that your knowledge of the PSF is good is valid. This naturally leads into the next area of stellar photometry in crowding fields, when you are forced into fitting the PSF whether you like it or not.

It is customary to express the observed number of counts in a frame in terms of an instrumental magnitude:

6.2 Crowding

Clearly, the above technique breaks down as you have more than a few stars in your frame for two reasons: the stars may have overlapping light, and there may be stars in your sky annuli. This leads to techniques for crowded field photometry, a complicated subject which we'll just review quickly.

In crowded field photometry, the idea to that you have to solve for the brightnesses of overlapping stars while considering the contribution of neighboring stars. To do this requires that you have information about the point spread function. There are several possibilities for how you might consider accounting for neighbors:

- Fit all stars that have an effect on each other simultaneously. Unfortunately, in very crowded fields, this can lead to huge groups of stars to be considered simultaneously, which can be computationally difficult, if not impossible. (DAOPHOT NSTAR)
- Fit each star independently, but iterate the fit: at each iteration, subtract the contribution for all neighbors before fitting each star. (DOPHOT)
- Use some combination of the above. (DAOPHOT ALLSTAR)

Simply, the technique consists of: finding stars, finding the PSF, grouping the stars, and simultaneously fitting for stellar brightnesses and positions. You almost certainly have to do positions because your initial estimates will probably be biased by neighbors. You also might consider fitting for the background as well. We'll consider each one of the steps in order:

6.2.1 Finding stars

Need to consider automation because of completeness issues, not to mention tedium.

Look for peaks; clearly need to consider background noise, so look for peaks above some noise threshold. More sublety: look for peaks which look like they have the right shape to be stars. Matched detector algorithm. Shape parameters, e.g. sharpness, roundness for additional spurious object rejection.

6.2.2 Finding the PSF

Find bright isolated stars. Tabulate the PSF. However, rememeber that you're going to have to use this PSF to estimate brightnesses for other stars, so you'll need to be able to interpolate it accurately. Consider using a functional fit to PSF, possibly carry along residuals as well.

6.2.3 Grouping stars

As discussed above, either consider all stars that overlap, or iterate each star individually with improved neighbor subtraction at each iteration.

6.2.4 Fitting stars

For all pixels under consideration, compare observed values with first guess values. Use residuals to refine your guesses (nonlinear least squares). Continue iterating until parameters converge.

6.2.5 Subleties

Sky estimation. Mode approx: 3*median - 2 * mean.

Multiple iterations of entire procedure to improve PSF by subtracting neighbors, also to find new stars under wings of other stars.

Multiple colors/frames simultaneously.

Completeness. Function of crowding. Spurious detections as well as misses.

Averaging measurements. Averaging mags not the same as averaging counts. Beware of weighted means if you use estimated errors, because these can be biased.

6.3 Atmospheric extinction

f you want to compare brightnesses of stars in different observations, you need to correct for the different absorption of the earth's atmosphere as a function of airmass. Note that if you are just comparing brightnesses within individual frames (differential photometry), correcting for atmospheric absorption isn't necessary, to the extent that the absorption is the same for all objects in the frame.

We've already desribed the absorption/scattering effects of earth's atmosphere:

F = F0exp(- τ_0X) or m = m0 +1.086τ_0X m = m0 + kX m0 = m - kX where k is the extinction coefficient. As we've discussed, this can vary from night to night, so if you're interested in accurate photometry (better than a couple percent), you need to measure it on your night. Also remember that the extinction coefficient is wavelength dependent, so you need a separate number for each filter. It's also critical to measure this to check for photometric weather: even if you don't require a few percent, if there are clouds you might get a lot worse! So you need to check. The extinction coefficient can be determined by making multiple observations of a star at different airmasses. Then you can solve for k and m0 using least squares. Note that you need to sample a good range of airmasses to get good leverage on the fit, and you should bracket the airmasses of all of your program objects.

When doing broad-band work, however, there's an additional subtlety because of the wavelength dependence. Two different stars might have different extinction coefficients in the same filter because the stars might have very different colors, which has an effect if the filter bandpass is broad. This problem can be solved by using second order extinction coefficients, where the extinction is a function not only of the airmass, but also of the stellar color:

$$m = m_0 + kX + k_2(color)X$$

where color can be either an observed or a known color for the star. Second order effects will have more importance in the limits of broader bandpasses, working with objects of extreme colors, and/or working with bandpasses in which the extinction coefficient varies rapidly within the bandpass (e.g., the near-UV). Another important thing to remember is that although we correct for the effects of the Earth's atmosphere using extinction coefficients, the actual response function of an imaging system includes the wavelength-dependent response of the Earth's atmosphere, which varies depending on airmass. If one wants to determine an "average" wavelength response of a system, one probably wants to include the effects of the atmosphere at some "typical" airmass, and then correct the photometry to this airmass; for example, this is what is done by the SDSS photometric pipeline, where all photometry is corrected to an airmass of 1.3.

Using the above formalism, you have to solve for extinction coefficients plus a magnitude for every star you observe. This requires a fair bit of observing to constrain all of the parameters. Clearly, if you can observe stars of known brightnesses, you will have better constraints on the extinction coefficients. These leads us into the discussion of standard stars.

6.4 Standard stars

Standard stars are required so that different observers are able to compare results with each other. The reason this is true is because every observational setup is likely to have different response functions, so the same stars will not be observed to have the same brightnesses (even relative brightnesses!) with each separate setup. Differences in response come from many factors: size and condition of the telescope optics, number and type of optics in the system, bandpass and quality of the filter, response function of the CCD, etc. In addition, it is very difficult to measure all of these responses in any absolute sense, so it is almost impossible to go straight from observations to inferences about physical fluxes in absolute units.

To get around these problems, systems of standard stars have been set up so observers can calibrate any new observations against the known brightnesses of the standard stars. A standard system is just a more-or-less arbitrary choice of one particular instrumental setup, but is generally one which is very stable and which someone has spent a lot of observing time establishing. Generally, the standard system is tied to absolute flux observations of some primary standard star so that observations on the standard systems can be converted to physical fluxes.

Let's consider some examples. First consider two identical systems. Clearly these will give the same observed flux as each other. Now let the systems differ only by a multiplicative factor in throughput. Now the observed fluxes will differ by a constant factor, or a constant additive factor in magnitudes. With observations of standard stars, it is straightforward to find the magnitude difference between the observed magnitude and the standard magnitude to calibrate out this constant factor. The additive constant in magnitudes is called the zeropoint.

A slight divergence is called for here to mention that different packages apply some "software" zeropoints to the calculation of instrumental magnitude just to make the numbers (instrumental magnitudes) look "reasonable" (i.e., familiar). This is no problem since we determine a zeropoint on top of this to calibrate, but caution is required if talking about instrumental magnitudes measured with different software systems!

OK, back to differing instrumental systems. Now let's consider the more realistic case where the shape of the response curve as well as the absolute throughput differs from the standard system. Let's consider a system which has slightly different filters with a different wavelength cutoff. Now you observe a different number of counts from the standard system, and the difference depends on the spectrum of the object you are looking at. Now spectra can differ for lots of reasons, but the biggest effect is just from spectral slope differences. If your filter has a shorter wavelength cutoff (on the red side), then you'll observe less counts than the standard system, and preferentially less counts for a redder star. To first order, this can be calibrated out by solving for an additive constant which depends on the spectrum of the object being looked at. Since we don't in general know the spectrum, we have to parameterize it by something we can observe, and the best choice here is the stellar color, as inferred from observations taken through more than one filter.

Transformation coefficients.

$$
\begin{aligned}
m &= -2.5\log(counts/s) + z_{soft} \\
m_0 &= m - kX \\
M &= m_0 + t(color) + z
\end{aligned}
$$

where capital letters are the magnitude on the standard system, z is the zeropoint, and t is the transformation coefficent. The color is generally parameterized by the ratio of the flux at two different wavelengths, or, in magnitudes, the difference between the magnitudes. The two wavelengths should be measured near in wavelength to the wavelength of the filter being corrected; generally, one uses the bandpass being corrected as one of the wavelenghts and an adjacent bandpass as the other. For example, when correcting V magnitudes, people usually use B - V, V - R, or V - I for the color term.

There are two ways to define the color, either in terms of the observational system or in terms of the standard system. The latter is slightly preferred for using least-sqaures (small errors on the independent variable), and also because it allows observations from different nights to be combined. Note that this formulation does not require you to know the colors of your objects a priori, it's just algebra to figure them out as long as you have observations in both filters.

The use of these first-order transformation coefficients is accurate as long as your filter system does not differ much from the standard system, and additionally, that the spectrum of your program objects does not differ significantly from the spectrum of the standard objects. The more these conditions are not met, the less accurate the results. Some additional accuracy in the case of differing systems can be achieved by using higher order transformation coefficients. However, even in this case, it is always important to remember that if the spectrum of the program object differs significantly from the standards, derived fluxes can be significantly in error.

Certainly, you get to a point when the response of one system is so different than the response of another system that no transformation can be determined. In this case, you have two different photometric systems. In fact, there are several different photometric systems at use in astronomy today, and each has advantages and disadvantages.

It is common practice to combine the equations for extinction and transformation coefficients into a single set of equations:

$$M_i = m_i + k_i X + t_i(M_i - M_j) + z$$

where the subscripts refer to measurements in different filters, and where I have ignored second order coefficients. The advantage of combining the equations is that you can use the information about the known magnitudes of the standard stars for the extinction term, so you can combine observations of different standards at different airmasses to derive the extinction coefficient and do not need to observe the same star at multiple airmasses. In practice, one observes a set of standards of different colors at different airmasses. Then one uses least squares techniques to solve for the values of k, t, and z which minimize the difference between the observed, transformed, magnitudes and the standard magnitudes. Then one applies these coefficients to measurements of your program objects to derive their magnitudes. Clearly, to do so, you must measure the colors of your program objects by making observations in more than one bandpass since, unlike the standard stars, the colors of your program objects are not known a priori.

Note that for calibration of broad-band observations, we do not require knowledge of the transmission function. Of course, if you have such knowledge, you could derive synthetic transformations, if you also know the transmission functions of the standard system. Beware, however, that getting accurate transmission functions can be very difficult.

6.5 Photometric systems

What are some of the considerations for choosing a photometric system? One general approach is to consider a broadband system to give a low resolution approximation to the spectra of objects, regardless of the details of what objects you might be looking at.

For broadband systems, the most common has been the UBVRI system, originally established by Johnson in the 1960s. However, even the UBVRI system has undergone some revisions, and Johnson's original system is not quite the one used predominantly today.

UBVRI. Cousins RI. Subtleties of Johnson vs. Cousins vs. Landolt.

Gunn system, uvgriz. Additional consideration: night sky brightness.

HST system. Additional consideration: maximum throughput.

Sloan system. Additional consideration: "clean" bandpasses.

Choice of photometric system for studies of stars: determination of temperatures, reddenings, surface gravities, metallicites.

Medium band: Stromgren system, uvbyβ. Others: DDO, Washington.

What goes into a choice of filters to use often involves a tradeoff of better spectral information vs. S/N considerations. Note that S/N considerations must include both the errors in the bandpasses being considered, and also the degree to which different objects are separated in color space. (possible problem to determine best filter choice given some sensitivities, astrophysical colors of stars).

Narrow band filters most often used to study emission line objects.