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4 Stars and Stellar populations

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1 Introduction

Galaxies: defined as a gravitationally bound collection of stars, gas, dust, dark matter, and black holes.

Goals:

- Understand general current picture of galaxies and galaxy evolution
- Basic understanding of how galaxies are observed
- Review statistical properties of the galaxy population, and specific properties of different types of galaxies
- Understand physical tools by which we learn and characterize different components of galaxies
- If time permits: what we can learn from the Milky Way

ASTR 555 intended to provide basic overview of galaxies and their components. ASTR 616 to focus on galaxy formation and evolution.

1.1 History

Incomplete!

- 1. 1700's: Messier objects, 39 of which are actually galaxies (total of 110 in the final catalogue).
- 2. 1864: GC, 1888: NGC (William Herschel and son John)
- 3. 1920's: 'Great Debate' between Curtis and Shapley on whether or not galaxies were located within the MW. Resolved by Hubbles discovery of Cepheids in M31.
- 4. 1980's: Importance of environment recognized: **morphology/density relation**; 'Nature vs. Nurture'.
- 5. 1990's: Techniques for finding and confirming high redshift galaxies (z¿2): **Lyman break** galaxies:
 - Lyman limit
 - Rydberg formula: $\frac{1}{\lambda} = R \left(\frac{1}{n_u^2} \frac{1}{n_l^2} \right)$

First large scale surveys, both nearby and at medium redshift. HST imaging of distant galaxies. N-body (dark matter) simulations.

6. 2000's:Precision Cosmology and LCDM, outside optical wavelengths: IR (Spitzer) and sub-mm (JCMT).

7. 2010's: Extended gas halos in galaxies. Possibly detection of DM and dark energy.

1.2 Approaches

Multifaceted approach to studying galaxy formation and evolution:

1.2.1 Nearby galaxies

Galaxy "archaeology": study nearby galaxies in detail, attempt to understand processes that led to their current appearance.

Advantages

can resolve structure, individual stars in nearest galaxies, high S/N observations

Disadvantages

some information may be erased by physical processes (e.g. merging), degeneracies in integrated light

1.2.2 Distant galaxies

Look at galaxy samples at different lookback times, study distribution of properties (galaxy population) as a function of time.

Advantages

direct probe of different stages. Relationship between lookback time and redshift.

Disadvantages

brightness/selection effects, lack of detail, difficulty in associating objects at one redshift to those at another.

1.2.3 Physics of galaxy formation

Advantages

some physics (e.g. gravity) is well understood

Disadvantages

some physics (e.g. star formation) is not well understood. Dynamic range of the problem is huge:

- Dynamic range in distances, from stellar scales to largest scale structure.
- Dynamic range in mass, from stellar scales (1 M_{\odot}) to largest scale structure (10⁶ 10¹⁵ M_{\odot})

1.3 Overview of galaxies and galaxy formation

Cosmological context of galaxy formation and evolution:

• Baryonic matter (and some leptons), non-baryonic matter, dark energy

- Current composition of the universe
- Past composition of the universe

Main components:

Dark Matter (DM); usually non-baryonic, though even some baryonic matter can be hard to see, such as brown dwarfs. DM dominates mass of galaxies, and is currently detectable only by its gravitational effect on motions, or background light (lensing).

Stars Observed properties depend primarily on mass, age, and composition. Variety leads to multiple luminosities and colors in galaxies.

gas molecular, atomic, and ionized gas phases. Low density gas can be hard to detect.

dust mass of ISM varies widely between galaxies. Observed via emisison and absorption.

Central (supermassive) black holes Observed indirectly via gravitational effects, accretion

Formation

How do galaxies come to appear as they do? Important processes (not necessarily in chronological order):

- Gravitational collapse (of dark matter, and later, baryons) in cosmological framework.
 - How big are initial lumps at different size scales?
 - How much angular momentum?
 - How fast do lumps grow?
- Condensation of gas and cooling
 - "hot" vs. "cold" accretion
- Star Formation (not well understood)
 - Under what conditions do stars form?
 - What types (masses) of stars form?
 - Drives chemical evolution, which may impact cooling and future star formation
- Black hole formation
 - Primordial formation vs. formation from early stars
 - How common?
- Feedback/mass loss
 - How much energy? Mechanical or thermal? Does mass escape or just delay accretion?
 - What objects generate it, and how? Possibilities: Winds, supernovae, galactic nucleii.

- Continued accretion from IGM
 - How much?
 - What mode?
 - What composition?
- Merging:
 - halo merging; dynamical friction
 - galaxy merging: gas-rich ("wet") vs. gas-poor ("dry")
- Other environmental processes:
 - Cluster (group?) environment: ram pressure, tides
- Dynamical evolution
 - Dynamical instabilities; e.g. bars and spiral arms
 - Migration
 - Internal vs. external triggers

These processes have *characteristic timescales*, and the relation between them may influence how galaxies form, evolve, and appear.*

Hubble time:

$$t \propto H_0^{-1}$$

Critical density:

Value today:

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} \approx 10^{-29} \text{ [g cm}^{-3]} \approx 5.5 \times 10^{-6} \text{ [cm}^{-3]}$$

 ho_{crit} increases with redshift. "Collapsed" halo has $\overline{
ho} pprox 200
ho_{crit}$.

Gravitational timescales

Orbital time:

$$t_{orb} = \left(\frac{3\pi}{G\overline{\rho}}\right)^{1/2}$$

Freefall time (time for unsupported cloud to collapse...doesn't happen!):

$$t_{ff} = \left(\frac{3\pi}{32G\overline{\rho}}\right) \sim 2.1 \times 10^9 \quad [f^{1/2}n_{-3}^{-1/2} \text{ yr}]$$

- $n_{-3} = \text{gas density } [10^{-3} \text{ cm}^{-3}]$
- f = gas fraction of the cloud

^{*}see a schematic flowchart and some of the complicated links from Mo et al.

Cooling timescale

$$t_{cool} = \frac{\frac{3}{2}nkT}{n^2\Lambda(T)} = 3.3 \times 10^9 \quad \left[\frac{T_6}{n_{-3}\Lambda_{-23}} \text{yr}\right]$$

• $\Lambda_{-23}=$ cooling curve in units of 10^{-23} erg cm $^{-3}$ s $^{-1}$. $\Lambda_{-23}=1$ is roughly the minimum cooling rate for a primordial plasma at $T>10^4$ K. *

What determines gas temperature? A gas cloud is heated during the process of gravitational collapse:

$$T_{vir} \approx 3.6 \times 10^5 \quad \left[K \left(\frac{v_c}{100 \text{km/s}} \right)^2 \right]$$

• $v_c = \sqrt{GM/R}$ characterizes the mass and radius of the halo.

Star-formation/gas consumption time

$$t_{sf} \propto \frac{M_{gas}}{\dot{M}_{sf}}$$

Typical SF rates today are $\sim 1~\text{M}_\odot~\text{yr}^{-1}$, but can be much higher in "starbursts".

Chemical enrichment time

Depends on element, but for many, massive star lifetime, $\sim 10^7$ yr.

Merging time

Orbital timescales between halos.

Dynamical friction time

Only important for massive subhalos.

$$t_{df} \frac{t_{hubble}}{10} M_{main} / M_{sat}$$
 [yr]

(Note error in Moe et al.).

1.4 Toy example

Overly simple, but illustrates the consequences of this scenerio[†].

ullet Galaxies appear to have a maximum size of order 10^{11} - $10^{12} M_{\odot}$, even though there are larger agglomerations of matter, e.g. clusters.

^{*}See Mo, van den Bosch, & White for derivation/discussion of above).

[†]Rees 1995

- Can be crudely explained by understanding of dissipation
- Self-gravitating cloud has two timescales:

1. dynamical, or free-fall:

$$t_{dyn} \sim (G\rho)^{-1/2}$$

2. cooling time:

$$t_{cool} \sim \frac{nkT_g}{n^2\Lambda(T)}$$

where Λ is the cooling function

If $t_{cool} > t_{dyn}$, then a cloud can be in quasi-static equilibrium, i.e. cooling is unimportant. If $t_{cool} < t_{dyn}$ the cloud cools, kinetic energy is converted to radiation, and the cloud collapses. Given a cooling curve for primordial composition, one can calculate the relevant timescales, and find that collapse is unlikely to occur for $M > 10^{12} M_{\odot}$. This implies that dissipation is important, at least for objects we observe as galaxies (e.g. luminous objects).

This argument is really only a suggestion, for a number of reasons: halos are not uniform density, so there's no such thing as a single cooling time for the entire halo.

More Questions: Add these to the ones above, and keep them in mind while studying. Read with a question in mind, connect old information to new information. Add main bullets first, then go through and add details, depending on how much time you have.

1.5 Questions

- When do each of these steps happen and what are their relative importances?
- What sets the masses of galaxies? Sizes? $(10^6 10^{12} M_{\odot})$ Luminosities?
- What sets the distributions of numbers of galaxies as a function of mass/luminosity?
- Does the ratio of baryonic mass/total mass change for different galaxies?
- What triggers star formation in galaxies?
- What is responsible for the rango of galaxy morphology?
- How much of present structure is determined by initial conditions, e.g. initial overdensity, angular momentum (and what are those initial conditions)?
- How much does present appearance depend on basic physics within galaxies, e.g. dynamics and chemical evolution?
- How much depends on environment, e.g. mergers and interactions, background radiation?
- Does the relative importance of these effects (initial conditions, internal evolution, environment) vary for different galaxies?

2 Observing galaxies

2.1 Imaging

2.1.1 Surface brightness

Surface Brightness (SB) is the most fundamental observable for galaxies. It is the basic measured property for imaging of a *resolved* object. SB is independent of distance *until geometry* of the universe becomes important:

$$SB \propto (1+z)^{-4}$$

Units:

- \bullet erg cm⁻² s⁻¹ sterradian⁻¹
- mag arcsec⁻²

When summing SBs, flux units must be used (if you're adding and subtracting magnitude units, you're doing something wrong \odot). Differences in magnitudes correspond to *ratios* in fluxes:

$$m_1 - m_2 = -2.5 \log \frac{F_1}{F_2}$$

In general, flux is a function of wavelength, so SB is measured in different bandpasses, e.g. UBVRI or SDSS (ugriz). The ratio of the flux of the same object in different bandpasses gives an estimate of the **spectral slope** between bandpasses; often expressed as a *color index*, the difference in magnitude between the two bandpasses, such as U-R.

Relevance of SB distribution: How are stars distributed in galaxies? There is not a direct correlation between stellar luminosities and stellar masses, nor is there one between stellar mass and mass in other components.

Typical SB values in galactic centers are:

Spiral $V \sim 20-21~{\rm mag~arcsec^{-2}}$ (not much brighter than typical dark sky)

Elliptical $V \sim 16-17 \text{ mag arcsec}^{-2}$

In principle, SB is measured directly from a 2D detector as an arbitrary function of location. Galaxies are relatively faint; more than half the light from galaxies comes from regions with $SB < SB_{\rm sky}$ so S/N is difficult to obtain. It's challenging to observe anything fainter than 24-25 mag arcsec⁻² in unresolved light (although note recent discovery of "ultra-diffuse galaxies", Subaru (Koda et al.), Dragonfly (van Dokkum et al)). eh? Can observe to significantly lower

surface brightness (30 mag $arcsec^{-2}$) for nearby objects in which you can resolve stars, i.e. add up their individual brightnesses.

Observational challenges:

seeing: affects SB distribution in center, e.g., existence of cores?

sky determination : affects SB distribution in outer regions, see here.

Since SB is the key observable, there can be strong **selection effects** against low SB objects. If all galaxies had the same SB profile (they don't) then low SB galaxies are *strongly* biased against in either magnitude-limited or size-limited catalogs (if mag/size is determined isophotally). The apparent "size" of a galaxy will depend on its SB.

Isophotes are contours of constant SB. They are often well (though not perfectly) represented by ellipses since most galaxies are symmetric at a significant level. Elliptical contours fit spirals and ellipticals for different reasons: *

Spiral viewing angle and disk thickness

Elliptical intrinsic ellipticity (for example, NGC1226 and NGC1316).

Some basics of techniques for ellipse fitting:

- Fourier moments
- Model fitting

In reality, galaxies have more complex features, e.g. asymmetries, departures from elliptical isophotes, bars, spiral arms, jets, etc. (see below).

For a purely axisymmetric object, SB distribution reduces to a 1D (major axis) SB profile. SB profiles are often parameterized, with distributions in the form of a **Sersic Profile**[†] (though there are other forms that can be used as well). The Sersic Profile is given by

$$\Sigma(r) = \Sigma_e \exp\left(-b_n \left[\left(\frac{r}{r_e}\right)^{1/n} - 1\right]\right)$$

- $\Sigma_r = \mathsf{SB}$ at radius r
- r_e = effective radius (aka. half-light radius), the radius enclosing half the total light if the model is extrapolated to infinity.
- $\Sigma_e = \text{SB}$ at $r_e \ (\Sigma_0 \sim 2000\Sigma_e)$ at galaxy center?

^{*}Some basics of techniques for ellipse fitting: Kent, ApJ 266, 562 (1983); Lauer, ApJ 311, 34

[†]plot from MacArthur et al. (2003)

- $b_n \approx 2n 0.324$ and is determined from the definition of r_e (I assume this is talking about where that particular expression came from, including the constants.)
- n depends on the type of galaxy (n = 4 for ellipticals and n = 1 for disks).

SB profile for disk galaxies

Disks (and low luminosity ellipticals) are usually well represented on average by an exponential:

$$\Sigma(r) = \Sigma_s \exp\left(-\frac{r}{r_s}\right)$$

[erg cm $^{-2}s^{-1}sterradian^{-1}$]

$$m(r) = m_s + kr$$
 [magnitudes][mag arcsec⁻²]

SB profile for spheroids

Spheroids are historically characterized by **deVaucouleurs profile**, also known as the " $r^{1/4}$ " law:

$$\Sigma(r) = \Sigma_e \exp\left(-7.67 \left[\left(\frac{r}{r_e}\right)^{1/4} - 1 \right] \right)$$
 [flux units]
$$m(r) = m_0 + kr^{1/4}$$
 [magnitudes]

Many galaxies appear to have SB profiles that are well fit by a combination of multiple components, e.g., a bulge and a disk. The contributions from the different components can be split via bulge-disk decomposition (e.g. MacArthur, Courteau, & Holtzman, ApJ 582, 689 (2003)). Issues:

- covariance between parameters
- 1D vs. 2D
- · validity of models

2.1.2 Sizes of galaxies

Generally, galaxies do not appear to have sharp edges (although SB can drop sharply in some disk galaxies).

- Isophotal (D₂₅ in RC is B=25.0 ????): Beware for low SB galaxies and for galaxies at non-negligible redshift...?????
- Half-light radii (e.g. R_e); requires extrapolation for total brightness
- Petrosian radius: radius at which the *local* SB at that radius drops to some fraction of the *mean* SB within that radius. (in an annulus, SDSS used 0.8r 1.25r, with a fraction of 0.2)

Typical angular sizes:

Nearby galaxies: arcminutesDistant galaxies: arcseconds

Linear sizes can be derived from distance estimate (which depends on cosmology). Typical physical size (of the luminous component) is \sim 1-30 kpc.

2.1.3 Integrated brightness

Issue: galaxies don't have edges. Types of mags:

Metric: brightness within an aperture of fixed angular or physical size. Be careful if galaxies have a range of sizes.

Isophotal: brightness within a specified SB contour, e.g. Holmberg mags, which is the brightness within $\mu_{pg}=26.5$. wtf is μ_{pg} ?? Be careful if galaxies have a range of SB profiles, as sample will be strongly biased against lower SB object.

Model: require assumption about outer SB profile

Petrosian: mag within some fixed number of Petrosian radii (SDSS uses 2). Advantages with regard to SB (distance) and seeing effects. Note relation of Petrosian mag to model mag depends on SB profile. *

Typical integrated brightnesses of galaxies, and star-galaxy separation and crossover in number counts at $V\sim 21$. When determining integrated colors of a galaxy, be sure to use the same region of integration for both bandpasses. For example, use the petrosian radius as determined in the highest S/N bandpass.

Other issues in getting luminosities of galaxies:

- Foreground extinction, which depends strongly on Galactic latitude
- Internal extinction/inclination, which can be significant (up to 1.5 mag in RC3 prescription (???))
- If looking at a population, luminosities need to be compared in the same bandpass. This is
 an issue if looking over a range of redshifts; a fixed observed wavelength range corresponds
 to a different rest frame wavelength range at different redshifts. K-corrections are used
 to account for this at some level[‡]. K-corrections are defined in magnitudes:

$$m_R = M_Q + DM + K_{QR}$$

^{*}http://classic.sdss.org/dr5/algorithms/photometry.html#mag_petro

[†]Burstein & Heiles, ApJS 54,33 (1984); Schlegel, Finkbeiner, & Davis, ApJ 500,525 (1998)

[‡]Hogg et al. astro-ph/0210394; Kinney et al. ApJ 467.38 (1996); Coleman, Wu, Weedman, ApJS 43,393 (1980)

where the object is observed in bandpass R, and you are interested in absolute magnitude in rest-frame bandpass Q (which might be R, the classical K-correction).

For significant redshift, measure in different bandpass, so that K correction is less sensitive to assumptions about spectrum (i.e., Q not the same as R).

If you "know" (make assumptions about) the spectrum of the object, you can do this: Kinney et al, ApJ 467, 38 (1996), Coleman, Wu, Weedman, ApJS 43, 393 (1980). Often, multiple bandpasses (colors) are used to determine best matching template.

2.2 Spectroscopy

The other fundamental observable is the spectral energy distribution (SED), or spectrum, of a galaxy. Spectra are often obtained over some aperture, or long slit; however, note increasing use of integral field spectroscopy (e.g. SDSS MaNGA).

Galaxy spectra are composite:

- Stars
 - multiple velocities
 - luminosity weighted: total number of stars (for each stellar type), times luminosity per star

Generally, individual stellar lines can't be isolated.

- Gas emission (not absorption?)
- Dust absorption/emission

The overall shape and features in SEDs (see also here) provide important information about the nature of the luminous components, i.e. physical properties of the stellar population and the gas component.

2.2.1 Velocities

Spectra allow the study of velocities via the Doppler shift:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

which measures both the bulk velocity of the galaxy and the distribution of stellar velocities within the galaxy. Note that the shift in wavelength is proportional to the emitted wavelength; lines are shifted and broadened more at longer wavelengths. As a result, it is common to work with spectra to that are evenly sampled in $\log \lambda$ rather than λ , because $\Delta \log \lambda \propto \Delta v$.

Bulk velocities

- Mean velocities of galaxies indicate that they are receding, with velocity proportional to distance → expansion of the universe.
- There is a dispersion of velocities around the mean relation; the deviation is called the
 peculiar velocity. In the field, typical peculiar velocities are a few hundred km s⁻¹. In
 clusters, they can be a thousand km s⁻¹.

Internal velocities

Internal velocities can be organized or random.

Organized (e.g. rotation in disk galaxies). Note effects of inclination, which need to be measured to get intrinsic rotation velocity ($v_{obs} = v \sin i$). Also note that populations in disk galaxies are not entirely kinematically "cold"; there is some dispersion around the rotational velocity, typically at the level of 10s of km s⁻¹.

Random broader lines that represent the line-of-sight velocity distribution (LOSVD) of stars within a galaxy. This broadening would be the same at all points in the galaxy.

$$F(\lambda) \propto \int F(v_{los}) S\left(\lambda - \frac{v_{los}\lambda}{c}\right) dv_{los}$$

- S = spectrum of an individual star
- F = fractional distribution of stars at different LOS velocities.

Lines are usually fairly well represented by a Gaussian velocity profile, characterized by the mean velocity:

$$\overline{v} = \int v_{los} F(v_{los}) \mathrm{d}v_{los}$$

and the velocity dispersion:

$$\sigma = \int \left(v_{los} - \overline{v}_{los} \right) F\left(v_{los} \right) dv_{los}$$

e.g., central velocity dispersion or, more generally, the velocity dispersion profile.

Deviations from a Gaussian can be measured (at high S/N), and are usually characterized by higher order moments: *skew* (h_3) and *kurtosis* (h_4):

$$F(v_{los}) \propto \exp{\frac{-\omega^2}{2}} \left[1 + \sum h_k H_k(\omega)\right]$$

where $\omega \equiv (v_{los} - \overline{v})/\sigma$ and H_k are Gauss-Hermite functions.

Extracting information about velocity distribution requires:

 Some model of the underlying composite spectrum: stellar absorption lines are not infinitely narrow, and blends are common: use some template, either from observations of a stellar library or atmospheric models

- An understanding of the instrumental broadening, i.e., the line spread function (LSF)
- for a rough approximation, the instrumental broadening may be close to Gaussian, and if one just wants to characterize the velocity dispersion (i.e. Gaussian profile), one would determine the Gaussian width needed to match template to observed data, and subtract the instrumental Gaussian width in quadrature, since two Gaussian convolutions (LSF and velocity distribution) are equivalent to a single convolution with a Gaussian with width

$$\sigma_{tot}^2 = \sigma_{LSF}^2 + \sigma_{LOSVD}^2$$

Measuring velocity dispersion significantly smaller than the instrumental resolution is challenging, and at some level, becomes impossible.

• sophisticated techniques exist for extracting velocity distribution information, e.g. penalized pixel fitting (pPXF).

Some examples of velocity data. What kind of galaxies are these?

Relevance: Internal velocities can be used to probe mass distribution within galaxies (assuming that gravity drives the motions). For a rotationally supported system:

$$v(r) = \sqrt{\frac{GM(r)}{r}}$$

 \rightarrow Keplerian velocities! For a collisionless system of (independent) particles, the distribution of velocities at any given location characterized by the velocity ellipsoid (with radial, azimuthal, and vertical components), and Jeans equation leads to:

$$\frac{\mathrm{d}\rho\sigma^2}{\rho\mathrm{d}r} + \frac{2\beta\sigma^2}{r} = -g(r)$$

where β is the **velocity anisotropy**:

$$eta \equiv \left(1 - rac{\sigma_{ heta}^2 + \sigma_{\phi}^2}{2\sigma_r^2}
ight)$$

though β is not generally known. So mass modeling of kinematically "hot" systems is not so easy.

Mean velocities of galaxies indicate that they are receding, with velocity proportional to distance \rightarrow expansion of the universe. There is a dispersion of velocities around the mean relation; deviation is called the peculiar velocity. In the field, typical peculiar velocities are a few hundred km/s, in clusters, can be a thousand km/s.

2.3 Distances to galaxies

Some properties of galaxies, such as luminosity and linear size, require knowledge of distances to galaxies:

$$m - M = 5 \log d_L - 5$$

- $d_L = \text{distance [pc] } L = ?$
- M = absolute magnitude (or SB)
- m = apparent magnitude

Various techniques for getting distances*:

Variable stars

Cepheids/RR Lyrae stars through period-luminosity (period-luminosity-color/metallicity) relation.

RGB tip

Nearby galaxies with old stellar populations where individual stars can be resolved.

Geometric techniques

- masers (proper motion compared with radial velocity in central disks)
- exclipsing binaries

SB fluctuations

From imprint of Poisson statistics. Requires objects with a "standard" population (i.e. stellar luminosity function).

Planetary nebulae luminosity function More luminous standard candles:

type-la supernova ("standardizable")

Scaling relations between velocity and luminosity

for relatively nearby galaxies

- Spirals: Tully-Fisher relation between maximum rotational velocity and luminosity
- Ellipticals: fundamental plane relation between velocity dispersion, SB, and physical size $(D_n \sigma)$ relation.

See

Redshift

Galaxies expand with everything else in the cosmic flow.

$$z = rac{\Delta \lambda}{\lambda}$$

$$1 + z = rac{\lambda_{obs}}{\lambda_{emit}}$$

^{*}see Freedman & Madore, ARAA review

- At low $z, z \approx v/c, v = H_0 d$. Determining H_0 requires independent distance measurement; uncertainties are often parameterized by using $H_0 = 100h$ km s⁻¹ Mpc ⁻¹, and distance-dependent quantities given in terms of h. Current best estimate of H_0 is \sim 70 km s⁻¹.
- At high z, need knowledge of cosmological parameters to get distance from redshift (i.e. redshift-distance relation.*)

Note that observed velocity needs to be corrected for earth motion (to heliocentric or barycentric velocity), motion of the Sun (to LSR), rotation of Galaxy (to GSR), and motion of Galaxy within the Local Group.

Peculiar velocities

Galaxies also move around relative to the uniform expansion because of local gravitational attraction from concentrations of mass. As mentioned above, the deviation of the velocity from the smooth flow is called the *peculiar velocity*. Typical peculiar velocities are on the order of several hundred km $\rm s^{-1}$, but depend on environment; c.f., ?? Milky Way and galaxy clusters; if these are ignored, introduces spurious features in distance maps ("fingers of God", redshift space distortions).

Hubble's law and the smooth Hubble flow (at what velocity? what redshift?)

At $z \gtrsim 0.1$, peculiar velocities lead to relatively small distance errors. At lower redshifts, might estimate distances using model of mass distribution and velocity field, but this is likely not well determined.

Spectroscopic redshift

Determining redshift spectroscopically for distant (faint) galaxies is observationally time-consuming. Accurate positions of individual lines require reasonable S/N; emission line galaxies are significantly easier. It is possible to get good redshifts from low S/N observations by taking advantage of multiple spectral features, e.g., by cross-correlation of spectrum against a template in $\log \lambda$ space.

Photometric redshift

Lower accuracy distances are possible from so-called *photometric redshifts* (though can't disperse light.) Use low resolution spectral information, i.e. multiband photometry to constrain redshift.* The different intrinsic SEDs of galaxies, internal reddening, and redshift need to be separated. Accuracy of results obviously depends on quality of photometry and number of bandpasses. Typical accuracies (checked against spectroscopic redshifts) give

$$\frac{\Delta z}{(1+z)} \sim 0.1$$

(sometimes better); note typical $\sim 10\%$ outlier fraction (varies for different types of galaxies).

2.4 Morphological classification

^{*}see, e.g. Hogg, Distance Measures in Cosmology

^{*}e.g. plot (from Niemack et al 2009, ApJ)

Level 5: GALAXY MORPHOLOGY - Ronald J. Buta (2013), also Whittle notes.

Historically, galaxies were considered in terms of their morphology, i.e. the Hubble sequence. However, it is not clear to what extent morphological classification traces underlying physics, and descriptive morphology may be biased by things that are not fundamental. Still, it is widely used, so important to understand a bit.

2.4.1 Morphological systems

Good reference: Sandage in Galaxies and the Universe, 1975. Some pictures:

- ellipticals
- S0s, also known as lenticulars
- spirals
- Tuning fork diagram

2.4.2 Hubble classification

ellipticals: given as En, where

$$n = 10 \left(\frac{1 - b}{a} \right)$$

E0 (sphere) $\rightarrow E7$ (skinny). No distinction between dwarf ellipticals and dwarf spheroidals

spirals: barred or unbarred (SBa,SBb,SBc; or Sa,Sb,Sc; respectively) where a,b,c denote size of bulge (bulge-to-disk ratio: B/D) in decreasing order. Also classified according to tightness of arms and the degree to which arms are resolved into individual HII regions (note that the latter two are essentially impossible to judge for edge-ons). These criteria are not perfectly correlated, and classifiers might emphasize different criteria. Note that spiral arms are generally higher contrast in surface brightness than they are in mass, so they may be somewhat misleading; also note that origin of spiral arms may not be fundamentally related to global galaxy properties.

S0s: (aka lenticulars, or disks); intermediate, no spiral structure but have disk system, split into $S0_1$, $S0_2$, $S0_3$, depending on the amount of dust.

irregulars: Irr I (Magellenic irregulars with lots of distinct HII regions)

Irr II (lack the resolution into distinct HII regions).

Some pictorial examples

2.4.3 deVaucouleurs/RC3 classification

Diagram. See also here.

• Extends Hubble to later spiral types Sd, Sm, and finally Im.

- Extra classes around $S0~(S0^-,S0^+)$
- Allows for intermediate between barred and unbarred:

SA: normal spiralsSB: barred spiralsSAB: transition

- Adds extra distinction for ring vs. s shaped.
- Note that location along spiral dequence may not be the same as in "Hubble classification" (more based on B/D).
- Some pictorial examples
- Numerical galaxy types (T)

Note that morphological classification often depends on multiple characteristics, and therefore it can be somewhat subjective. Quantitative classification schemes have been worked on (see § 2.4.4), but are not widely used for nearby galaxies.

Morphology depends on wavelength (e.g. O'Connell 9609101). Be careful about morphological classification at higher redshift (or at least a direct comparison with lower redshift galaxies) because you may be looking at morphology at a different wavelength (morphological *K-correction:* correction to magnitude (or flux) due to redshift). K-correction is introduced in a previous section... should put in a reference or something.

Based on global characteristics, the Hubble morphological classification of ellipticals is probably not fundamental. Viewing angle must play a part at some level. It may be more meaningful to classify by *isophotal shape** or kinematically by v/σ .

For the spiral sequence, there are correlations of luminosity, surface brightness, rotational velocity and gas fraction with Hubble type. However, each category has a broad range of global observables, and the categories overlap significantly.[†]:

- Considering mean or median values, there is little trend in radius, L_B (Bolometric luminosity?), and total mass for S0-Sc, but later types tend to be smaller, less massive, and less luminous (RH figure 2). However, note that LSB (Low Surface Brightness?) galaxies tend to be later types, and since these are not included, there may be a bias here.
- Typical SB may be lower for latest types, even without inclusion of LSB galaxies; CSB(?????) much higher for large ellipticals than spirals. Mass surface density appears to decrease monotonically with morphological class in spirals (RH figure 3).
- Cold gas is absent in ellipticals, and neutral hydrogen (HI) content appears to increase monotonically with type for spirals. It is possible that molecular gas content decreases with type, but not certain. (RH figure 4).

^{*(}Kormendy and Bender classification)

[†]Some representative data from Roberts and Haynes review (ARAA 1994)

As stated above, all of the aforementioned classification schemes are subjective at some level. Note general problem of human classification on large-scales, e.g. with millions of galaxies, although also note Galaxy Zoo. (????????????)

2.4.4 Quantitative classification schemes

Some of the more quantitative schemes that have been proposed, both parametric and non-parametric, are:

- **Bulge-to-disk ratio**, using B/D decomposition*. Issues: covariance between parameters, 1D vs. 2D, and validity of models.
- Global profile fit, e.g. Sersic index
- Concentration, e.g. SDSS r_{90}/r_{50} , r_{80}/r_{20} , using circular apertures. Elliptical apertures can also be considered, e.g. based on *second order moments*. Related to B/D. Sensitivity to seeing/distance.

2.4.5 Non-symmetric galaxies

Many of the schemes used for nearby galaxies fail for non-symmetric galaxies, for which other indicators have been suggested:

• **Asymmetry:**[†] Rotate about center, self-subtract,

$$A = \frac{0.5|\text{subtracted}|}{\text{total}}$$

Possible indicator of mergers.

- **Clumpiness:** † e.g. S. Subtract smoothed version of image from original image, ratio of flux in subtracted image to flux in original images gives S. Possible indicator of star formation.
- Gini coefficient: Applied to sorted list of pixel values. May be more stable at low resolution, S/N.
- **M20**: second order moment of brightest 20% of galaxy light ¶ May be more stable at low resolution, S/N.

2.5 Galaxy catalogs/resources

2.5.1 Large area/all sky surveys

*MacArthur, Courteau, & Holtzman, ApJ 582, 689 (2003)

[†]Abraham et al, ApJS 107, 1, 1996

[‡]Conselice ApJS 147, 1 (2003)

§Abraham et al, ApJ 588, 218 (2003)

¶Lotz et al., AJ 128, 163 (2004)

- 2.5.2 Smaller region surveys
- 2.5.3 Spectroscopic surveys
- 2.5.4 "Classic catalogs"
- 2.5.5 Modern digital catalogs
- 2.5.6 Web tools

2.5.7 Selection effects

- There is generally no such thing as a catalog that contains "everything" since there will always be objects that fall below the detection criterion for inclusion
- a "complete" catalog is a catalog that has all objects that satisfy well-defined selection criteria, but this still does not mean that you can't get biased results from it, you still have to consider the effects of the selection function
- Consider a plot from a redshift survey of galaxy absolute magnitude vs redshift
- For magnitude limited samples, more luminous objects will be overrepresented: Malmquist bias (but beware that sometimes people refer to different things with this name!)
- Also may need to be careful about systematic biases arising from uncertainties in measured
 magnitudes, distances, if population is not uniform in underlying distribution of absolute
 mags and distances sampled which it isn't! Luminosity function rises towards fainter
 mags, so more galaxies are scattered bright than are scattered faint. In distance, more
 galaxies are located farther than nearer just because of volume element, so more galaxies
 scattered into sample (with corresponding error in absolute mag) than scattered out of it
- Note many biases are possible aside from a simple mag cut, e.g., color bias, surface brightness bias, etc. Whenever you see a relation that shows a correlation, consider selection effects!
- Best approach to selection biases may be to "forward model", i.e. model underlying distribution through selection function and compare to observed properties.

Overview of nearby galaxy population

Consider some of the main quantifiable observable properties of nearby galaxies.

3.1 Statistical properties of galaxies

Consider relative number of galaxies with different parameters, e.g. color or luminosity.

3.1.1 Colors of galaxies

Consider the distribution of SEDs of galaxies, to first order represented by their color. There is a striking bimodality in color*:

- Widley used terminology of red and blue sequences (with "green valley" in between).
- Red sequence is tighter than blue sequence, so latter is sometimes called the "blue cloud".
- Red sequence extends to higher luminosities, blue to lower luminosities, though there is significant overlap.

At some level, this bimodality is nothing more than the bimodality between ellipticals/early-type spirals without much current star formation (red) and later-type spirals with current star formation (blue), although dust, bulges, and metallicity all play a role. The correspondence with morphology is supported by correlation with structural parameters, e.g. multidimensional correlations.[†]

The key point is that color is easily observed and quantified. Given likely differences in stellar populations, the relation between luminosity and stellar mass is probably different for the two different sequences (red and blue). At a rough level, the color allows the stellar mass to be estimated from the luminosity, though some colors have more information about this than others, and some bandpasses for luminosity are more affected by differences in the stellar populations (more later).

Note that the relation shifts when expressed in terms of stellar mass, ‡ and that there appears to be a transistion mass around $10^{10}~\text{M}_{\odot}$ between the two sequences.

3.1.2 Luminosity function

The luminosity function (LF) gives the number density of galaxies as a function of luminosity, and can be expressed in either luminosity or absolute magnitude units.

^{*}quantified from SDSS by Strateva et al., AJ 122, 1861 (2001)

[†]Blanton and Moustakis ARAA 2009

[‡]Baldry et al. 2006

- $\Phi(L)$ = number density of galaxies with luminosity between L and L + dL
- $\Phi(M) =$ number density of galaxies with absolute magnitude between M and M + dM

The integral over the LF gives the total number density of galaxies (at all luminosities). The LF is the first step toward understanding a fundamental question of galaxy formation: What sets the *range* of galaxy luminosities and the relative *numbers* of objects at each luminosity? (though mass may be a more fundamental characteristic than luminosity...) The LF is also an important cosmological probe for evolution of the galaxy population (more later).

Measuring the luminosity function

Measurement of the LF requires distance measurements (to get luminosity), and critically depends on knowledge of the **selection function**. The simplest idea for correction is to weight counts by V_{max} , the largest volume to which the galaxy could have been observed given the selection function. However, there is an issue with large scale structure, so $\frac{V}{V_{max}} \rightarrow$ test for uniformity and understanding of selection function* In general, need to be very careful about understanding selection effects. Model (and verify) them.

For magnitude limited samples, more luminous objects will be overrepresented. This is called the Malmquist bias[†] Also may need to be careful about systematic biases arising from uncertainties in measured magnitudes and distances, if population is not uniform in underlying distribution of absoluate mags and distances sampled (which it isn't). The LF rises toward fainter magnitudes, so more galaxies are scattered bright than are scattered faint (WTF??). In distance, more galaxies are located farther than nearer because of volume element (seriously, WTF), so more galaxies scattered into sample (with corresponding error in absolute magnitude) than scattered out of it. Note that many biases are possible aside from a simple magnitude cut, e.g. color bias, SB bias, etc.

Observed luminosity function, functional fits, and parameters

- Absolute magnitudes (M_V) range from -22 to -8.
- Nomenclature: "dwarf" galaxies vs. "normal" galaxies.
- What objects dominate by number? What objects dominate the total luminosity?
- Different types have clearly different LFs
- Since proportion of a given type depends on environment, so must the general LF.

LFs are usually well characterized by a **Schecter Function**

$$\phi(L) = \frac{\phi_*}{L_*} \left(\frac{L}{L_*}\right)^{\alpha} \exp\left(-\frac{L}{L_*}\right)$$

$$\phi(M) \propto 10^{-0.4(\alpha+1)(M-M^*)} \exp\left[-10^{0.4(M^*-M)}\right]$$

^{*}Significantly more sophisticated methods have been developed; for more details, see Binggelli et al (ARAA 26, 26, 1988)

[†]Be aware that this name is sometimes used to refer to different things.

where $\phi(L)$ is the number of galxies with luminosity between L and L+dL. ϕ_* , L_* , and α (the faint-end slope) are the three parameters. The Schecter function is only an approximation, and is purely empirical.

A possibly useful feature of the Schecter function is that it can be integrated (for $\alpha > -2$) to get *total luminosity density*:

$$j = \phi_* L_* \Gamma (\alpha + 2)$$

 $\Gamma = ?$

Typical "local" parameters* (e.g. SDSS at $z \sim 0.1$) are:

- $\bullet \ \phi^* \sim 0.015 h^3 \ \mathrm{Mpc}^{-3}$
- $M_B^* \sim -19.5$
- $M_R^* \sim -20.5$
- $\alpha = -1$ to -1.5

As noted above, there is a dependence on type.[†]

So what is the implied luminosity density? And (getting ahead of ourselves) what is the implied stellar mass density, c.f. cosmological?

There has been significant discussian over detailed shape and normalization of LF. There are probably both observational issues (e.g. selection functions) and astrophysical ones (e.g. large scale structure, "cosmic variance").

Origin of the luminosity function

relative number of galaxies at different masses, if there is a correlation between mass and luminosity. Related to primordial distribution? compare galaxy LF with dark halo LF, figure from Whittle cooling at highest masses galaxy "efficiency" at higher and lower galaxy masses: feedback? affecting numbers or luminosities?

Evolution of the luminosity function

LF evolution contains a lot of information about galaxy evolution (understand galaxies by "looking back in time"). Some possibilities:

- No evolution: distribution of galaxy luminosities is unchanging.
- Passive evolution: number and internal (stellar) makeup of galaxies is unchanging (no star formation), but luminosity evolves as stars evolve.
- Luminosity evolution: stellar makeup of galaxies changes with time (star formation), leading to luminosity changes.

^{*}parameters for different bandpasses from Montero-Dorta & Prada (2009)

[†]e.g. Driver & Propris, Marzke, figure 3

• Number (density) evolution: number of galaxies changes with time. Galaxies may be either created (formed) or destroyed (e.g. mergers) as a function of luminosity.

There is a large amount of work on this, but the LF definitely shows evolution*. All samples show luminosity evolution, but red galaxies show number density evolution. Schechter parameters: "quenching" of galaxies: moving galaxies from blue to red sequence?

Higher redshift results: the UV luminosity function: (links)

^{*}Faber et al. (2007)

3.2 | Elliptical/Spheriodal galaxies

3.2.1 Surface brightness profiles

Ellipticals are probably better fit by Sersic profiles than by deVaucouleurs*. Previously, stated that *sphericals* are characterized by deVaucouleurs, not ellipticals.

- Correlation of Sersic indices with other parameters suggests that there is something physical going on, but be aware that there may be fit degeneracies.
- Low luminosity Es (and the bulges in spirals) may be better represented by exponentials.
- Correlation of Sersic indices with other parameters suggests that there is something physical
 going on (but be aware of fit degeneracies).
- Slope of SB profile (Sersic *n*) appears to be correlated with luminosity
- SB vs L turns over at intermediate luminosity: two families of shperoidal systems? Or continuous trend?[†] → Cuspy cores.
- Ellipticals exist over wide range of sizes, luminosities, and SBs.

Other profiles that have been used:

- King model (truncated Gaussian for velocity distribution). This is the only profile with theoretical basis; the others are empirical Hubble law doesn't converge.
- Hubble profile:

$$\Sigma(r) = rac{\Sigma_{\scriptscriptstyle S}}{\left(1 + rac{r}{r_{\scriptscriptstyle S}}
ight)^2}$$

where $\Sigma_{\scriptscriptstyle S}=0.25\Sigma_0$

None are perfect matches to data over all scales[‡].

Inner regions of spheroidal galaxies deviate from Sersic profiles fit to outer regions. (Outer regions can also deviate from Sersic profiles, especially notable in central cluster (cD) galaxies.) Inner regions are of particular interest because they may reflect dissipational collapse of low angular momentum material§. Initially, it seemed like galaxies had cores, but this is partly an effect of seeing. Higher spatial resolution (HST) shows that galaxies have both flat and steep (cuspy) inner profiles $^{\$}$. Inner profiles seem to be roughly bimodal (though this bimodality is currently under debate).

 $^{^*}$ Caon et al. 1993

[†]See Kormendy et al. (2009)

[‡]see Burkert 1993 for comparison with deVaucouleurs law

[§]See, e.g. NGC 4472

[¶]e.g. NGC720 vs. NGC4621

Parametric fits to account for these become more complex, e.g. the "nuker law":

$$I(r) = I_b 2^{(\beta - \gamma)/\alpha} \left(\frac{r_b}{r}\right)^{\gamma} \left[1 + \left(\frac{r_b}{r}\right)^{\alpha}\right]^{(\gamma - \beta)/\alpha}$$

where γ is the slope of the inner power law, β is the slope of the outer power law, and α is the sharpness of the break between them.

Profile type is correlated with luminosity. Luminous ellipticals tend to have "cuspy cores" with a break radius and shallower central density profile. Lower luminosity ellipticals have power laws all the way in. Galaxies with cusps may have "extra light", perhaps related to an accretion event; note that at least some of these show distinct kinematic signature in their cores.

The existence of cores may be indicative of a previous merging event, with merging black holes "scouring" the core.

3.2.2 Intrinsic (3D) shapes of ellipticals

- oblate (2 long axes)
- prolate (1 long axis)
- triaxial (all axes different length).

True shapes are determined by by looking at distribution of ellipticities.

- Distribution function is different for fainter and brighter Es.
- For bright giant Es, distribution is *inconsistent* with either prolate or oblate itrinsic shapes: not enough circular galaxies*.
- For fainter Es, distribution is consistent with oblate, prolate, or triaxial.
- Triaxiality is also inferred for some giant Es from observation of isophotal twisting, which you cant get from oblate or prolate shape[†].

3.2.3 Non-axisymmetric features in galaxies

Often described by amplitudes of Fourier moments of intensity distribution as a function of radius, e.g. α_1 , α_2 , α_4

$$R_{iso}(\phi) - R_{ell}(\phi) = a_0 + \sum a_n \cos n\theta + \sum b_n \sin n\theta$$

(ϕ measured from major axis?)

"First even term above ellipses is α_4 term:" (WTF???)

^{*}Tremblay and Merritt Fig 3

[†]de Zeeuw, Fig 1

- "boxy" isophotes: $\alpha_4 < 0$; bright, slow, central cores, strong radio and x-ray.
- "disky" isophotes: $\alpha_4 > 0$; faint, significant rotation-flattening, little radio or x-ray emission, steep cusp.

Good diagram showing the difference: http://astronomy.nmsu.edu/holtz/a555/resources/mofig2.15.gif

Deviations from elliptical fits are seen in elliptical/spheroidal systems, e.g. NGC 821 v. NGC 2300. It is possible that isophotal deviations form a continuous sequence*:

$$S \to S0 \to disky~Es \to true~Es \to boxy~Es$$

It is also possible that true Es are just disky Es viewed edge-on[†], and that there are two distinct classes of Es. Some Es also show non-symmetric deviations from smooth SB distributions, e.g. ripples, shells, and streams, especially in outer regions.

3.2.4 Kinematics

Elliptical galaxies are kinematically *hot*. The random motions of stars is generally large compared to the organized motion (but this is not to say that there is no rotation at all).

The key kinematic quantity is **velocity dispersion**, σ . Galaxies can be characterized by *central* σ , but σ does vary with radius.

Some ellipticals have some rotation, mostly in lower luminosity systems. The relative importance of *organized* motion over *random* motion can be characterized by v_{rot}/σ . Shapes are expected to be influenced by rotation: for an oblate model with isotropic velocity distribution that is flattened by rotation:

$$\frac{v_{rot}}{\sigma} = \sqrt{\frac{\epsilon}{(1 - \epsilon)}}$$

Giant ellipticals have less rotation than this, implying anisotropic velocity dispersions to account for their shape (required for triaxial systems in any case). Low/medium luminosity (high SB) Es, however, *may* be isotropic with flattening caused by rotation [‡].

Low SB Es appear to have anisotropic σ , i.e. more eccentric than expected from rotation; "measured in LG Es 185 (factor of three low in v_{rot}/σ , 147, factor of 10 low)." seriously, wtf.

Significant fraction of Es may have dynamical subcomponents, e.g. velocity and dispersion for some interesting cases.

3.2.5 Relations between different parameters and different families of ellipticals

^{*}e.g. Kormendy and Bender classification

[†]Kormendy and Bender, figure 3

[‡]de Zeeuw, figure 3

Photometric parameters

SB-size relation, between SB and SB-luminosity relations ("Kormendy relations")???. Also, profile *shape* (e.g. Sersic index) with luminosity.

Kinematics vs. luminosity

More luminous galaxies have less rotation, but higher velocity dispersions (**Faber-Jackson relation**, which is roughly $L \sim \sigma^4$, but lots of scatter).

The Fundamental Plane

Faber-Jackson relations and the Kormendy relation (between SB and L/size) are manifestations of the **Fundamental Plane of elliptical galaxies**; there is a correlation between residuals in the Faber-Jackson emphrelation and SB. Galaxies do not populate the entire 3D space of I, r, and σ , but instead populate only a *plane* in this space.

- There is a relation between the **three fundamental global observables**: SB (or luminosity), size, and velocity dispersion*.
- The observed relation given by Virgo ellipticals is:

$$r_e \propto \left(\sigma_0^2\right)^{0.7} I_e^{-0.85}$$

What is the origin of relation? Might expect something like this if the following three assumptions are true:

- 1. Es are in virial equilibrium
- 2. M/L varies systematically with luminosity
- 3. Es form a "homologous" family, all, e.g. with deVaucouleurs profiles.

In this case, one expects:

$$L = c_1 I_c r_c^2$$

$$M = c_2 \frac{\sigma_o^2 r_e}{G}$$

where the first is a definition and the second is related to the **virial theorem** (though this applies to the mean potential energy and mean kinetic energy per unit mass, averaged over the entire system). The constants are related to the shapes and other details (not necessarily constant among the variety of Es).

Combining these, we get

$$r_e = \frac{c_2}{c_1} \left(\frac{M}{L}\right)^{-1} \sigma_0^2 I_c^{-1}$$

If $M/L \propto L^{0.2}$, then the observed fundamental plane is recovered; could arise from stellar populations or from variations in baryon to total mass. Alternatively, if M/L is constant with a structure that varies relative to one or more of the fundamental variables (which we know it does, e.g., systematic variations of Sersic n with luminosity, but not clear if this is entire "explanation").

^{*}original refs: Dressler et al. 1987; Djorgovski and Davis 1987; Bender, Burstein and Faber 199x

However, we still need to understand origin of assumptions; why should parameters, e.g. mass-to-light vary smoothly with luminosity? Recall that this ratio includes dark matter. If ellipticals have dark matter halos, the luminous inner parts are not required to be in virial equilibrium.

There is relatively little scatter around the fundamental plane, implying that the assumptions are reasonably valid over a large range of elliptical properties, which implies some significant regularities in the galaxy formation process.

- Galaxies do not fully populate the entire plane defined by our relation.
 - Consequently, when the plane is projected onto the other two axes, there is a correlation.*
 - In the luminosity (size) σ plane, Luminosity is proportional to velocity dispersion as $L \propto \sigma^4$, which is known as the **Faber-Jackson relation**. However, since the locus of Es isn't perfectly linear and the plane defied by the ellipticals isn't perpendicular to this dimension, the scatter around the Faber-Jackson relation is larger than the scatter around the fundamental plane. A new radius can be defined that incorporates SB such that the new radius vs. σ views the fundamental plane edge-on. Such a size measurement is called D_n , the isophotal diameter of the B=20.75 isophote. This $D_N \sigma$ relation provides a very useful distance estimator (if the fundamental plane really is fundamental).
 - In the SB size plane, relation for *smaller* galaxies:
 - * higher SB for normal small Es
 - * lower SB for diffuse small Es

These are sometimes known as the **Kormendy relations** and are one of the main bases for separating these two types of objects.

- The SB σ plane is presumably related to two underlying physical parameters:
 - 1. density
 - 2. virial temperature

In this plane, one can only form galaxies where *cooling is effective*, i.e. at larger densities and hotter temperatures. This restricts the area in the space in which we can find galaxies.

- Additional features of the galaxy formation process may introduce additional restrictions into allowed locations of galaxies on the fundamental plane. Most luminous ellipticals are located along one line (with some scatter) in the fundamental plane, and most diffuse ellipticals are located along another.
- For more recent descussion, see
- Isophotal deviations vs. luminosity and kinematics
 - Deviations from elliptical shape is correlated with dynamics, such that slower rotators are more likely to be "boxy", and faster rotators are more likely to be "disky".

^{*}Djorgovski figure 2

[†]Kormendy and Bender, figure 2

- There appears to be two kinds of "boxiness", one found in giant Es and one found in bulges, which in fact are relatively rapid rotators. It is likely that these come from different origins: plausibly, mergers in the case of giant Es and bars/disk evolution in the case of bulges.
- The core properties are correlated with the global shapes and dynamics: cuspy core galaxies are boxy and anisotropic, cuspy galaxies are disky and rotating.
- Several types of ellipticals?? Normal and low luminosity Es:
 - significant rotation
 - nearly isotropic
 - oblate spheroids
 - cusps (no cores)
 - disky isophotes

Giant Es:

- non-rotating
- anisotropic (triaxial)
- less flattened
- cuspy cores
- boxy isophotes

Dwarf spheroidals (diffuse ellipticals?):

- lower SB
- off the fundamental plane
- anisotropic (slow rotators for their ellipticity)

Bulges: multiple types? As an aside: globular clusters, which are likely to be significantly different.

• ellipticals as extension of disk galaxies to larger B/D ratio? see, e.g. Cappellari 2013 figure

3.2.6 Spectral energy distributions

Note typical features in ellipticals: stellar absorption lines (4000 Å break, Mg, Fe, etc.) Es are generally red: implications are predominantly old population. Relations:

- Color-luminosity: more luminous Es are redder. However, as wel'll see, variations of color at this level can come from either metallicity or age, so interpretation isn't trivial.
- Mg line strenth-luminosity: more luminous Es have stronger lines. This correlation is even tighter when considering relation between central velocity dispersion and line strength (The Mg- σ relation), over large range of σ . As we'll see, this variation could arise from variations in age or metallicity, or different heavy element abundance ratios. Of equal interest to the Mg- σ relation is the quite small scatter around the relation.

Some Es have signatures of a younger population: E+A (or k+a) galaxies: galaxies with strong Balmer lines. These are relatively rare, but likely are indicative of post-starburst galaxies with significant star formation 1 Gyr ago. These are not associated with a cluster environment. Some show morphological signs of recent interactions.

Stellar populations within ellipticals: Es have gradients in color/line strength, i.e. tend to be redder toward the center (but again, there is some degeneracy between age and metallicity as possible reasons for this).

3.2.7 Interstellar matter in ellipticals

It is now known that Es have significant interstellar gas, seen not in optical, but in X-ray emission \rightarrow hot gas. This makes sense since some gas is expected from stellar evolution, although there is also the possibility of gas from the environment.

Some Es aso show evidence of colder gas and dust. A surprisingly large fraction ($\sim 50\%$) show some evidence for dust their cores* These are typically small components by mass.

3.3 Spiral/Disk galaxies

3.3.1 SB profiles of disk galaxies

Most spiral galaxies are a combination of a disk plus a spheroid. For the SB profile, it is common to "decompose" spiral into two components: exponential disk + Sersic bulge. (e.g. MacArthur, Courteau, & Holtzman, ApJ 582, 689 (2003)).

disk scale lengths are typically several kpc. Different families of disk profiles exist, e.g., Freeman type I and type II profiles, may be related to a bar or dust. Some spirals appear to have truncated disks, easier to see when observing disks edge-on, with truncation around 4 disk scale lengths. Disks are not always perfectly flat, e.g. warps in the outer regions. More luminous galaxies may have thicker disks

bulge Bulges in spirals tend to be better fit with variable Sersic indices, with lower index toward later type, smaller bulges; may be a distinction between "classical bulges" and bulges that may be related to bars and originating from the disk.

Issues with this decomposition:

- covariance between parameters
- 1D vs. 2D
- validity of models (is there anything fundamental about an exponential that guarantees extrapolating it into the center?)

^{*}Lauer et al. 2005

There appears to be some correlation between bulge scale length and disk scale length (Macarthur, Courteau, Holtzman ApJ 582, 689 (2003)), so perhaps components are not independent.

Spiral galaxies have stellar halos: in Milky Way $\rho \propto r^{-3}$ (stellar mass density in halo?); in M31 stars have been found out to 200 kpc. Increasingly, stellar halos are being recognized to have significant substructure, e.g., stellar streams in NGC 891 and M31, Milky Way "field of streams" and some models.

Size-luminosity and surface brightness-luminosity relations: more luminous spirals are larger, but still have higher surface brightness.

Vertical distributions in disk galaxies: disks are not always perfectly flat, nor are they infinitely thin. To first order, disk thickness appears to be independent of radius. Typically, vertical luminosity distribution is characterized by an exponential:

$$I(z) \propto \exp{-|z|/z_s}$$

although sometimes a more general form is used to allow for some flattening toward disk center, characterized by

$$I(z) = \operatorname{sech}^{2/n}\left(\frac{n|z|}{2z_d}\right)$$

Typical scale heights are roughly $0.1 \times \text{disk}$ scale length

In the Milky Way, and some other external galaxies, a single exponential is not a good fit, and a sum of two exponentials has been used, leading to the idea of a thin plus thick disk. Thick disk also visible in velocity and chemical space. Much debate about whether these are distinct components or not, and origin of the thick disk, e.g. recent Bovy et al papers.

3.3.2 Non-axisymmetric features in disk galaxies

Bars

- (Milky way structure, Bland-Hawthorn and Gerhard, 2016 ARAA)
- Many disk galaxies (1/3 to 1/2) have bars, with various morphologies (rings, etc.).
- \bullet Typical axis ratios \sim 2.5 to 5.
- Bars represent significant variations in mass density by factors of 2-3
- Seen edge on, bars may look like bulges, e.g. peanut shaped bulges.
- They likely form as result of dynamical instability in disk, though it's not totally clear whether this instability is caused by an internal or an external trigger.

Spiral arms

- Spiral arms come in a variety of morphologies: "grand-design" vs. multitarm vs. flocculent (from Elmegreen et al, ApJ 2011)
- Arms are more prevalent in blue light, but exist in red light and gas as well, so not entirely from young stars

- Overall mass contrast in spiral arms not so large
- Spiral arms are generally "trailing"; spiral winds behind the rotation

3.3.3 Kinematics

Spiral galaxies are kinematically cold: the random motion of stars is generally small compared to the organized motion (rotation).

Measuring galaxy rotation

Rotation can be measured optically and using HI. HI data can be spatially resolved (e.g. VLA), or unresolved.

While spirals are often characterized by rotation curves (from Vogt el al 2004), velocity fields are 2D:

- several different ways (from Swaters et al. 2002) to look at HI kinematic data
- velocity-coded color (from NRAO)
- Spider diagrams
- PV diagrams: note spread of velocity at any given position: location of peak velocity may not give the maximimum velocity at that position
- From unresolved HI profiles, rotation characterized by line widths, e.g. W_{50} , W_{20} (width at 50, 20 percent of peak line flux)
- Note that curves aren't always perfectly regular, can have features related, e.g., to structure
 in the disk: the nature of these can be clearer in 2D velocity data, e.g. NGC 5383. Note
 that there can also be motions of gas within the disk relative to rotation, and some internal
 velocity dispersion, e.g., UGC8508
- there can be large scale systematic deviations from rotation, e.g., streaming motions in bars Milky Way rotation curve from Klypin et al 2002.

The key kinematic quantity is the *rotation curve*, the *maximum* rotation velocity as a function of radius. However, observed velocities need to be corrected for inclination of galaxy, usually inferred from the observed axis ratio (which can be harder to get for non-axisymmetric disks).

On average, the amplitude and shape of a rotation curve is related to luminosity, but with significant scatter. See some spiral galaxy rotation curves (from (Persic, Salucci, & Stel 1996)).

Extension of rotation curve to large radius is the primary indicator of dark matter.

$$v_{rot} \propto \sqrt{\frac{GM(r)}{r}}$$

This is a proportionality because the geometry is not spherical. Flat rotation curves thus imply $M(r) \propto r$, i.e. significantly more than implied by exponentially declining stellar component

Galaxies are often characterized by the maximum rotational velocity, v_{max} , (which is sometimes hard to measure), or by the velocity at some specified radius, e.g. $2.2R_d$, the radius of maximum velocity for an exponentially declining mass distribution.

Stars have "random" velocities, characterized by their velocity ellipsoid, on top of the rotation field. This is generally small, so hard to observe in external galaxies, but it is known to exist, e.g. in the solar neighborhood (Binney & Merrifield, from Dehnen & Binney 1998). Note that velocity ellipsoid is not isotropic. Velocity dispersion increases for redder (older, on average) populations, leads to decreased rotation velocity for these populations, called "asymmetric drift". Traditional interpretation is that velocity dispersion increases due to gravitational interactions with substructure in disks, e.g. spiral arms, molecular clouds. Alternate interpretation is that older stars are born with higher vertical velocity dispersion, e.g., from a thicker disk: upside-down galaxy formation (although heating would still occur subsequently).

Tully-Fisher relation

Kinematics and luminosity are related by the *Tully-Fisher relation* for spiral/disk galaxies (Courteau et al 2007). Luminosity is proportional to the maximum rotation velocity roughly by $L \propto v^n$, where n=3-4. The slope depends on wavelength, definitions of where velocity is measured, etc. There is relatively small scatter; useful as a distance indicator. Scatter does not appear to be well-correlated with other parameters (e.g. radius or surface brightness); different from ellipticals. Various "explanations" for the scaling relations have been proposed (see e.g., appendices in Courteau et al 2007), along the lines of what we discussed for fundamental plane, with same caveats and limitations. Scaling relations provide a strong constraint for galaxy formation models.

Typical spectra

Disk galaxies show optical emission lines and generally blue continuum emission lines from HII regions, indicating current star formation. Since young stars dominate light, it is more challenging to learn about older stars. Most disks have color gradients: age and/or metallicity gradients; emission lines suggest metallicity gradients. and disk colors. bulges: Peletier and Balcells observations suggest correlation of bulge and disk colors: bulges are not always red. Again, do bulges and disks share some common origin?

Star formation - mass relation

The "main sequence" from Peng et al. (2010), independent of environment; specific SFR (SF per unit mass) nearly independent of stellar mass. Here, SFR from $H\alpha$.

Gas and dust in disk galaxies

Most gas is neutral hydrogen, observed in HI. HI disks are very extended, often well past starlight. Gas and dust are strongly confined to the plane of the galaxy.

Gas mass fractions can be significant, ranging from 5 to 80%. Neutral HI gas fractions (from Catinella et al 2010): higher gas fractions for

- 1. lower stellar mass
- 2. lower stellar mass surface density

- 3. lower concentration
- 4. bluer color

Since gas can be a significant component by mass, especially for lower mass/luminosity galaxies, the *baryonic* Tully-Fisher relation (from Gurovich et al 2010, left is stellar mass, right includes gas mass), has been considered. This seems to show even less scatter than the stellar T-F relation; note that this requires estimate of stellar mass from luminosity (more later).

Gas composition in galaxies also varies from galaxy to galaxy and within galaxies. Nonetheless one can consider "mean", or alternatively present-day, metallicities of galaxies. Find that more luminous galaxies generally are more metal-rich, as manifested by a luminosity-metallicity relation, or a mass-metallicity relation.

3.4 Summary: Galaxy parameter (scaling) relations

Galaxies exhibit some regularities, i.e., systematic correlations of different global properties, all of which need to be reproduced/understood in any model for galaxy formation and evolution.

Structural

luminosity-SB, or luminosity-size $(L - r_e)$

• isophotal shape-luminosity relation for Es (boxy vs. disky).

Kinematic

Faber-Jackson, Tully-Fisher

• luminosity-rotation relation for ellipticals The Faber-Jackson relation also applies to ellipticals, but this is the relation between luminosity and velocity dispersion, not rotational velocity...?

Structural/kinematic

Fundamental Plane

A global generalization to all galaxies: the fundamental manifold*, with an assumption about how to combine rotation and velocity dispersion to get a characteristic "velocity", all galaxies appear to lie in a plane.

Stellar populations

color-magnitude, Mg- σ for Es

Gas

HI-SB, Luminosity-metallicity

Black holes

 $M_{BH} - \sigma$

Central black hold seems to be ubiquitious in galaxies

^{*}Zaritsky et al. (2008)

- BH masses can be estimated (with difficulty; see later). More luminous (spheroid) galaxies appear to have more massive black holes.
- Tighter relation exists with velocity dispersion: $M_{BH} \sigma$ relation (or $M_{spheroid}$)*. Note that black hole mass seems to be more linked to spheroid than to total galaxy mass (although claims to the contrary have been made).

3.5 Environments of galaxies: clusters and cluster galaxies

Galaxies are not homogeneously distributed in space, e.g. SDSS pie diagrams: to z=0.15, to z=0.6, to z=5. Clustering is often described by the *two point correlation function*, which gives the excess number of galaxy pairs over a random distribution as a function of radius.

$$dP = n (1 + \xi(r)) dV$$

Typically, the observed correlation function is well fit by a power law:

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}; \qquad r_0 \sim 5h^{-1} \text{ [Mpc]}$$

so at 5 Mpc separation, there are twice as many galaxies as there are for a random distribution. The amplitude of the correlation function depends on both the absolute magnitude and the color (see, e.g. Zehavi et al 2005, Figs 6 and 15).

Many galaxies reside in groups or clusters of galaxies, systems that are a few Mpc across.

Groups:

- Typically contain 3-30 relatively luminous galaxies (cf., the Local Group). "cf." = "to bring together"...see wikipedia
- Typical sizes are \sim 1-2 Mpc, but note the existence of compact groups, which have several massive galaxies in close proximity (perhaps interacting)

Clusters:

Contain dozens to hundreds of galaxies

Probably best known catalog of local galaxy clusters is Abell cluster, with richness classes 1 to 5, based on number of galaxies in cluster.

Galaxies in galaxy clusters (see e.g.Boselli and Gavazzi, PASP 118, 517 (2006))

• morphology-density relation. (related, color-density relation, Baldry et al 2006) Note this applies to both high and low luminosity galaxies. (See Peng et al, e.g. Fig 6 for more recent summary?)

^{*}Kormendy and Bender, 2001

- S0 galaxies
- Anemic spirals: spirals with significantly lower gas fractions ($\sim 50\%$) than comparable field spirals. Gas deficiency appears largely in HI (not molecular), and preferentially in outer regions of galaxies. However, SFR is also notably lower than in field.

Galaxy clusters contain distributed hot gas, with total mass comparable to that found in galaxies.

- observed with X-ray observations;
- observed with X-ray observations;
- intracluster gas is enriched in heavy elements, so it is not primordial gas
- X-ray observations useful for probing cluster masses, under assumption of hydrostatic equilibrium
 - Most accurate determination requires measurement of density and temperature profiles of X-ray gas
 - Mass estimates can be made from X-ray luminosity and/or temperature (better): the M-kT relation.
 - typical masses are 10^{14} to 10^{15} solar masses; a typical cluster mass function (from Wen et al. 2010)

Galaxy clusters also have dark matter halos beyond those associated with the galaxies.

Shapes of clusters, inhomogeneities: not all clusters are in equilibrium.

3.6 | Some galaxies to be familiar with

- Local group, see census of dwarfs in Mateo annual review 1998. Spirals: MW, M31, M33 (note NGC 604). M32. LMC (note 30 Dor/R136) / SMC. dlrrs, dSphs. See some pictures here, also here; c.f. spheroidal companions of M31, e.g. here. Local morphology-density relation
- Local neighborhood. See various pictures here, e.g., M81 group: M82 (dwarf starburst).
 Sculptor group: NGC 253 (nearest nuclear starburst). Centaurus group: Cen A (NGC 5128).
 Maffei group (Maffei 1). Leo group (NGC 3379/M105).
- Some other famous active galaxies: NGC 1068, NGC 4151, 3C273
- Nearest clusters. Virgo: M87, NGC 4472. Fornax. Coma.
- Large scale structure: Shapley supercluster

Part II: The building blocks of galaxies

4 Stars and Stellar populations

How to relate observables to intrinsic population characteristics:

ion characteristics:

distribution of masses, compositions and ages (star formation history).

SEDs

(or colors) can be observed for individual stars in very nearby galaxies, or integrated starlight in most galaxies.

4.1 Review: stellar evolution

The **Russell-Vogt theorem** states that the internal structure of stars is determined by mass, chemical composition, and age. (This excludes non-spherical symmetric effects, such as rotation, magnetic fields, and binarity).

Luminosity (radius) and effective temperature are derived from equations of stellar structure:

- mass conservation
- hydrostatic equilibrium
- energy equation
- energy transport

along with auxiliary relations:

- equation of state
- opacity
- nuclear reaction rates

4.1.1 Main stages of stellar evolution

- Hydrogen core burning: main sequence (MS)
- Hydrogen shell burning: giant branch (for lower mass stars)
- Helium core burning:
 - horizontal branch (low-mass stars)

- red clump (intermediate-mass stars)
- blue core helium burners (high-mass stars)

Note key transition around 2 M_{\odot} (depends on metallicity): shift to helium flash at lower masses.

- Helium shell burning: Asymtotic Giant Branch (AGB)
- Other nuclear burning for high mass stars
- White dwarf or supernova

Stellar evolution gives **model tracks**, evolution as a function of time for a given mass. For spherical symmetry, calculations are 1D.

4.1.2 Isochrones

An *isochrone* is a cross-section of properties at a fixed time across a range of masses. (Stars that were born from the same collapsing cloud, and are therefore the same age.) Some well-known groups that calculate evolutionary tracks/isochrones:

- Padova
- BASTI (Teramo)
- Dartmouth
- Yale-Yonsei
- Victoria-Regina
- Geneva

Although fairly well-understood, there are still some uncertainties that lead to differences between different calculated isochrones: convective overshoot, diffusion, convection, helium abundance, mass loss, etc. Generally, uncertainties are larger for later stages of evolution. Additionally, there may be missing physics, e.g. rotation and magnetic fields, that would require a full 3D treatment.

Given effective temperature, surface gravity (from mass and radius), and composition, stellar atmospheres give observables: spectral energy distribution/colors. Some model atmospheres:

- Kurucz
- MARCS

Theoretical Color-Magnitude Diagrams (CMDs) \rightarrow observed; need distance and reddening/extinction Age effects (model isochrones) from Yi et al 2001. Metallicity effects: internal (opacity) and atmosphere (line blanketing) effect combine in the same direction to make more metal-rich populations redder: CMDs

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- Metallicity terminology: often given as mass fractions of hydrogen (X), helium (Y), and heavier elements (Z).
- solar abundance: X=0.7, Y=0.28, Z=0.019 (roughly)
- also given by

$$\left\lceil \frac{\text{Fe}}{\text{H}} \right\rceil = \log \left\lceil \frac{(\text{Fe/H})}{(\text{Fe/H})_{\odot}} \right\rceil$$

 Be aware that this is an oversimplification, as Z contains lots of different elements. More later.

Main sequence (MS)

- Very rough scaling relation between luminosity and mass: $L \propto M^{3.5}$
- Main sequence shifts with metallicity: redder for higher metallicity
- Location also depends on helium abundance

Red Giant Branch (RGB)

- note that because of more rapid evolution after MS, RGB stars all have roughly the same mass
- temperature of RGB depends on age/mass: younger and more massive stars are hotter.
 Temperature also depends on metallicity: latter is dominant effect in older populations (greater than 5 Gyr).
- At lower masses/larger ages, tip of RGB is close to constant bolometric luminosity regradless of age or metallicity. In observed plane, leads to roughly fixed tip luminosity if observing at long wavelengths (e.g. I band): basis for Tip of Red Giant Brance (TRGB) distance indicator.
- RGB bump (RGBB from Bono et al 2001), location of which depends on mass/age and metallicity; arises when H burning shell crosses chemical discontinuity.

Horizontal Branch (HB), aka. Red Clump (RC) or more generally He core burning sequence

- High mass stars form blue He core-burning branch (also note red plume)
- Intermediate mass stars form red clump.
- Low mass stars form horizontal branch. Variable mass loss on RGB and at He flash gives a range of masses.
- Horizontal branch morphology depends on metallicity: more metal-poor populations have bluer HB.

- However, there is something else that also affects HB morphology, leading to the so-called second-parameter problem (e.g. M3/M13, from Rey et al 2001). Possibilities: Age? He abundance? Heavy element abundances? Density? Rotation?
- RR Lyrae stars: in instability strip (caused by doubly ionized He, also includes Cepheids, delta Scuti stars, etc.) RR Lyrae stars are indicators of old metal-poor population. Periods of 0.5 days, but multiple groups (Oosterhoff classes) depending on stellar parameters...

Asymtotic Giant Branch (AGB)

- For intermediate masses/ages, AGB is significantly more luminous than the TRGB, hence of potential critical importance to studies of integrated light.
- For lower masses (older populations), AGB tip comparable to TRGB and AGB asymtotically approaches RGB (hence its name).

Potential importance of binaries/interactions

- unresolved (but otherwise non-interacting) binaries: broaden sequences, depending on mass ratios: equal masses give the appearence of a star 0.75 mag brighter.
- interacting binary stars
- blue stragglers: possible stellar merger/interaction products? M3 example from Sandage 1953.
- supernovae type SNIa: arise from binaries, produce different heavy element abundances than core collapse SNe. (e?)

End stages of stellar evolution: WDs, neutron stars, black holes, and supernovae

- projenitor-final mass relation (from Binney and Merrifield)
- White dwarf cooling sequence (from Hansen et al 2007: potential for age dating).
- Supernovae: generate significant fraction of heavy elements (but not all). Significant energy input, thermal and mechanical.

Galactic globular clusters: cornerstone of understanding stellar evolution historically, as apparent examples of a "simple stellar population (SSP)", with all stars of the same age and abundance.

- However, it's not recognized that not all GCs are so simple.
- Some CMDs show clear evidence of multiple components, e.g. NGC2808.
- Long history of evidence of abundance variations between different stars, typified by the Na-O anticorrelation; demonstrated through observations of main sequence stars that this is not a mixing effect.
- Two phenomena have recently been coupled, but not yet a clear understanding of how the multiple populations arise.

4.2 | Simple Stellar Populations and the Initial Mass Function

There is more information in a CMD than is represented by isochrones; one can also consider the relative *numbers* of stars at different locations. CMDs that incorporate number of stars are reffered to as **Hess diagrams** (e.g. Fornax from Battaglia et al., 2006).

For a **simple stellar population (SSP)**, a population with a single age and metallicity, the relative number of stars at each stage is determined by the **initial mass function (IMF)** and the age.

IMF determinations and parameterizations:

- "classical" **Salpeter** IMF (power law with $dN/dM \propto M^{-2.35}$) and others (from Pagel). Note power law form: $dN/dM \propto M^{\alpha}$, or alternatively, $dN/d\log M \propto M^{\Gamma} \propto M^{\alpha+1}$
- Widely used determination of local IMF is by Droupa Tout, and Gilmore 1993, who find

$$dN/dM \propto M^{-2.7}$$
 for $M > 1 M_{\odot}$ $dN/dM \propto M^{-2.2}$ for $0.5 < M < 1 M_{\odot}$ $dN/dM \propto M^{-1.3}$ for $M < 0.5 M_{\odot}$

- Chabrier IMF: log-normal form, $dN/dM \propto \exp(\log M \log M_o)^2$
- Empirically, no strong evidence for variations of the IMF as functions of, e.g., metallicity.
- No well-established theory for predicting the IMF

4.3 Resolved stellar populations

In general, galaxies are not SSPs; their CMDs don't look like those of a cluster. Hess diagrams of resolved stellar populations can be fit with *combinations* of SSPs to derive *constraints* on SFH, e.g. simulated galaxies (from Tolstoy et al.).

Generally speaking, we want the star formation history, represented by SFH(t,Z,M). This gives the number of stars (or stellar mass) at all combinations of age (t), metallicity (Z), and mass (M). Because there has been no variation observed in the IMF, it is usually separated from the rest of the SFH:

$$SFH(t, Z, M) = \xi(M)\psi(t, Z)$$

where ξ is the IMF.

MW neighbors resolved down to oldest MS turnoff ($M\sim5$), because typical distances give distance modulii $\lesssim20$. M31 and neighbors somewhat shallower ($m-M\sim24.5$) without large investment of telescope time.

4.3.1 Results

MW

- solar neighborhood Hipparcos sample: roughly constant SFH, but be aware of dynamical effects and limited volume bias nearby sample to younger populations. When corrected for, star formation history in solar neighborhood likely to have significantly declined (see, e.g. Aumer & Binney 2009).
- Age of oldest stars can be studied by WD sequence, gives $t_{\rm oldest} > 8$ Gyr. Also note presence of disk RR Lyrae stars (t > 10 Gyr??)
- Might consider studying age distribution of clusters, as they are simpler populations, but note problems with disruption of clusters that would lead to a bias to younger clusters.
- Bulge can also be studied, but note foreground population and extinction issues. Predominantly old population, e.g. bulge CMD, and luminosity function (from Ortolani et al 1995).
- Halo (as defined kinematically/spatially) also prdominantly old, both from field population and from globular cluster population.
- Historically, distinction between disk stars/open clusters (population I) and halo stars/globular clusters (population II), with pop I being younger and more metal rich. Note, however, the pop II association with low metallicity is now recognized not to be fundamental; inner halo/bulge significantly more metal rich.
- There is *not* a strong age-metallicity relation in the solar neighborhood (Orion has roughly solar metallicity despite the fact that it is nearly 5 billion years younger). Radial migration, inhomogeneous ISM, inflow. . .?

Local Group (LG) dwarf galaxies

- Carina dSph: striking evidence of episodic star formation, but this is NOT characteristic.
- Others (from Tolstoy et al); note range of SFHs.
- LG dlrrs compilation (from Dolphin et al 2005)
- LG dSphs (from Dolphin et al 2005); possible similarity to dlrrs apart from lack of recent SF?
- Many dwarfs show population gradients, e.g. from RGB/HB morphology (e.g., Harbeck et al 2001), likely metallicity gradients: some show evidence of age gradients, with younger population in center.

M33

(from Holtzman et al 2011); note significant age gradient, and significant component of younger stars.

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M31 halo

(from Brown et al 2006); significantly different from MW bulge, with more of an intermediate-old population?

Can do more distant galaxies if information from advanced stages (RC, HB) is reliable; (e.g. local volume SFH (from Williams et al, in prep), dwarf SFH (from Weisz et al 2011)).

Even farther if info from AGB is reliable.

4.3.2 General conclusions

- Galaxies are not SSPs.
- Galaxies have gradients in their stellar populations
- Significant population of old stars in nearby galaxies?
- More massive galaxies formed stars earlier??

4.4 Unresolved stellar populations

4.4.1 What stars contribute the most light?

• Along the MS, use combination of M-L relation and IMF:

$$L \propto \left(M^{-2.35}M^{3.5}\right) \propto M^{1.15}$$

Massive stars dominate light.

- Compare MSTO with evolved population: stars on the RGB have almost the same mass as those on the MSTO, but they are significantly more luminous. The relative contributions depend on the relative *number* of stars along the giant branch compared to the MSTO stars, but ultimately it is the luminous evolved populations that dominate the total luminosity. (Renzini fig 1.5).
- Since post-MS evolution is fast compared to MS evolution for all masses, it is true that at any given time when the most luminous stars are the most evolved stars, the luminosity is given predominantly by stars of (nearly) a single mass (true for all except youngest ages, see Renzini 1.1) and luminosity vs. lifetime plots.

4.4.2 Integrated brightness

Do SSPs get brighter or fainter as they age?

- At younger ages, have MS "peel-off" effect. As a cluster ages, the MSTO gets redder and fainter, since massive, bright stars (O, B, etc.) evolve off the MS first. Also supergiants...?
- At older ages, have competing effects of IMF and rate of evolution.

 To determine the luminosity of a stellar population, consider the number of stars evolving off the MS (dying) which is given by the evolutionary flux:

$$b(t) = \psi(M_{\text{TO}}) |\dot{M}| \quad [\text{stars yr}^{-1}]$$

where TO stands for turnoff, ψ is the IMF, and $|\dot{M}|$ is the time derivative of the turnoff mass, e.g. Renzini 1.1. The key point is that lower mass stars evolve more slowly.

- The IMF is important, along with age, in determining the luminosity evolution of a galaxy.
- Once most massive stars have died, all stars of lower mass reach comparable luminosity (tip of RGB), hence luminosity of population depends on number of red giants.
- For a typical IMF, the rate at which the number of stars increases toward lower masses is slower than the rate at which the generation of turnoff stars decreases.
- The luminosity of the turnoff also has some smaller effect.

4.4.3 Integrated colors

Do integrated colors depend on the IMF?

- Need to know the relative contributions of each stage of evolution, i.e., from stars of different masses.
- Since all post-MS stars have nearly the same mass, the number of stars in each post-MS stage is determined predominantly by the time spent in each stage:

$$N_i = b(t)t_i$$

So although the total flux is sensitive to the IMF, the relative contributions of each stage are not sensitive to the IMF, at least for older populations (Renzini fig 1.5).

- For all except the youngest population, the later stages of evolution provide a majority of the light. **Evolved stages are nearly all the same mass.**
- Consequently, the integrated spectrum and the relative contributions of various evolutionary stages are nearly independent of the IMF. They do depend on the age and metallicity.

4.4.4 Dependence of color and luminosity on time

See figure 1 from Bruzual and Charlot (1993) for a SSP (single burst) normalized to one solar mass.

- Luminosity variation is a combination of IMF plus rate of evolution for lower masses.
- Color evolution comes from a changing mix of stellar population, but this is just for a single (solar) metallicity.

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- RSG phase, followed by subsequent dimming and reddening.
- Note that this is bandpass dependent, with less dimming at longer wavelengths (at older ages where the RGB is dominant).

Implication: if a range of ages is present, then integrated light is (significantly) weighted toward younger populations.

4.4.5 What stars contribute the most mass?

- For all measured IMFs, the bulk of the mass is in relatively low mass stars ($< 1M_{\odot}$).
- Relative stellar M/L ratios for different IMFs can differ by a factor of two or more. For example, the Salpeter IMF has much more mass the IMFs with low mass turnover or cutoff.

Exercises

- consider four populations with combinations of alpha=-2.35,-1.35, age=3,10 Gyr. How do relative brightnesses compare? How do relative colors compare?
- now consider alpha=-2.35,-1.35 with continuous SF from 3-10 Gyr ago. How do brightnesses and colors compare?
- how do stellar masses compare?
- consider two galaxies with L* and 2 L*, with same IMF and age? How do stellar masses compare? What if IMFs are different? What if ages are different?

4.4.6 The mass-to-light ratio (M/L)

Can you estimate stellar mass from integrated light?

• Consider the *stellar* M/L (not to be confused with *total* M/L, which includes dust, gas, and dark matter, in addition to stars). The stellar M/L is defined in solar terms:

$$\left(\frac{M}{L}\right)_* \equiv \frac{M_*/M_{\odot}}{L_*/L_{\odot}}$$

- For individual stars, M/L decreases as mass increases ($L \propto R^2$). So younger populations will have a lower M/L than older populations.
- The absolute value of M/L of a population depends strongly on the IMF. Relative values characterize different stellar populations for a fixed IMF.
- The stellar M/L depends on luminosity bandpass; there is less variation toward near-IR bandpasses (less sensitivity to younger stars), but still not insigificant. K-band luminosity

is sometimes used as a proxy for stellar mass, but see Bell & deJong results from Bell & deJong (2001).

- More variation in stellar M/L at shorter wavelengths
- Stellar M/L does seem to be well correlated with color, however, this assumes no systematic variations in the IMF.
- Some variations from metallicity
- Some variations from reddening, although reddening and extinction act in opposite directions.
- So, it is possible to get some information about stellar M/L given colors, but the level of uncertainty depends on bandpass and color information available.
 - Variety of different stellar M/L techniques in use, e.g. using different colors, or spectral energy distributions, different assumptions about star formation history
 - $\boldsymbol{-}$ Some uncertainties in relative M/L for different populations even with colors
 - Larger uncertainty in absolute value of M/L because of IMF
 - if IMF were to be variable, then even relative M/L would be challenging

4.4.7 Star formation histories from integrated colors

- Issue: intgrated colors are affected by combination of age distribution, metallicty distribution, and reddening distribution.
- color variations with age: note significant degeneracy between age and metallicity. Both
 older populations and metal-rich populations are redder. However, only young populations
 have very blue colors.
- The situation is somewhat improved with a longer wavelength baseline (e.g. IR colors), but still significant.
- Even if age-metallicity were resolved, there is still an issue of luminosity weighting for non-SSPs.

4.4.8 Integrated spectra

Given age-metallicty degeneracy from colors, measurements of spectral features might help. One can consider a full spectral synthesis to get an ingegrated spectrum for a population. Some existing evolutionary synthesis compilations/codes:

Spectral evolution (BC fig 4) for a variety of SFRs

Dependence of contributor on wavelength

Potential problems with synthetic integrated spectra:

What about spectral features?

For populations

- For older populations
- typical optical spectra

Even with

Main area of application

4.5 | Chemical evolution

Understanding chemical evolution provides another clue about the formation history of galaxies. Since stars generate this evolution, one can in principle learn something about star formation histories by looking at the distribution of compositions. The more we know about metallicities, the better we can constrain ages. The *distribution* of metallicities within galaxies (recall gradients) potentially provides important information about the mode of galaxy formation. We'd like self-consistency between SFH and chemical abundances. Potential information about gas entering and leaving system. Relative abundances of different elements also may provide clues about formation history.

4.5.1 Basic picture of chemical evolution

The basic picture of chemical evolution starts with some initial conditions, then add SFH: (IMF + SFR). This results in chemical enrichment for stars as a function of mass and metallicity. Also may wish to consider input and output of gas from any particular region, with either primordial or modified composition.

- ICs: No heavy elements and pure gas (no stars).
- SFH: birthrate function $\psi(m,t)$ gives number of stars formed with mass m per unit volume, usually split into a separable function:

$$B(m,t) = \xi(M)\psi(t)$$

where $\xi(M)$ is the IMF and $\psi(t)$ is the SFR. This implicitly assumes an IMF which is constant in time, which may or may not be true, but so far, there is no strong observational evidence against it (although note pop III issues).

For any particular star, use stellar evolution to compute the amount and composition of mass returned to the ISM vs. amount locked up in stellar remnants (Pagel fig 2, from Maeder 1992). Note that the *rate* of return depends on stellar mass, especially for the case of Snla, which also depends on the binary fraction.

nucleosynthesis: main element groups and their sources:

- light elements: Big Bang Nucleosynthesis (BBN) plus subsequent destruction (?)
- α elements (even Z from O up...???): massive stars (core collapse SN)

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- Fe-peak elements: type Ia SN and core collapse SN
- elements from slow (s) and rapid r processes (neutron capture, plux beta decay): core collapse SN, AGB stars... (need neutron capture diagram here)

Note that infall might arise from primordial clouds, or from processed material, e.g. mass-loss from halo stars. Outflow might come from SN winds, and in this case, it's possible that the composition of outflow material might be more enriched than the typical composition at any given time. So things can get complicated.

4.5.2 Basic equations of chemical evolution:

Allowing for all sorts of realistic effects makes it difficult to make very simple predictions for the evolution of abundances. However, under some simplifying assumptions, it is possible to do so. A simple chemical evolution model serves as a useful baseline against which observations can be compared to determine where the assumptions may break down. In addition, understanding a simple model allows us to introduce and become aquainted with terminology that is in widespread use.

Consider a total (galaxy) mass M, split into a gas mass g and stellar mass s. For a simple model, consider the gas at any time to be well mixed. At any given time, we wish to know the fraction abundance (Z_i) of element i. Consider an inflow rate into the system, F and outflow (ejection) rate, E. Then we have:

$$M = g + s$$

$$\frac{dM}{dt} = F - E$$

$$\frac{dg}{dt} = F - E + e - \psi' t$$

where ψ' is the SFR in units of mass per time,

$$\psi' = \psi \int \xi \mathrm{d}m \quad [\mathrm{g \ s}^{-1}]$$

(so ξ is normalized to a total of one), and e is the ejection rate of mass from stars,

$$e = \int_{m_t}^{m_U} (m - m_{\text{rem}}) \psi(t - \tau(m)) \xi(m) dm$$

which is a sum over all stellar masses of the product of the SFR at the time of formation of each mass with the mass returned to the ISM, weighted by the IMF. The lower limit of integration is the stellar mass that is dying at time t; lower mass stars don't contribute because they haven't ejected any mass yet. We also have

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \psi' - e$$

since the mass in stars increases by the number of stars formed, but decreases by the amount of mass lost back to the ISM from the previous generation of stars. For the elements, we have

$$\frac{\mathrm{d}(gZ_i)}{\mathrm{d}t} = e_Z(i) - Z_i\psi + Z_FF - Z_EE$$

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(Z_i is the mass fraction of element i; hereafter we'll drop the subscript for simplicity). The mass in each element *increases* by:

- amount of mass released by previous generations
- · amount of mass added by inflow

but *decreases* by:

- amount of mass locked up in new stars
- · amount of mass lost to outflow

The term e_Z is given by:

$$e_{Z} = \int_{m_{\tau}}^{m_{U}} \left[(m - m_{rem}) Z(t - \tau(m)) + m q_{Z} \right] \psi(t - \tau(m)) \xi(m) dm$$

where q_Z represents the fractional mass of element Z synthesized and ejected during stellar evolution (so left term in brackets gives material which returns unprocessed and right term gives newly synthesized contribution). In a simple model, the synthesized masses are independent of the metallicity of the population (although we know this is not true for some elements, more on this later).

To simplify, consider an approximation to these formulae called the *instantaneous recycling* approximation which assumes that all elements are returned instantaneously - good for products of massive stars, but less good for products from lower mass stars, e.g. iron. Then we have

$$e = \psi \int (m - m_{\text{rem}}) \xi(m) dm \equiv (1 - \alpha) \psi$$

where α is the *lock-up fraction*: the fraction of mass "locked up" in stars and remnants.

$$\alpha = 1 - \int_{m_{\tau}}^{m_{U}} m' \xi(m') dm' + \int_{m_{\tau}}^{m_{U}} m_{\text{rem}}(m) \xi(m) dm$$

where the second term is the total mass in stars that have died, and the third term is the mass from those stars that have been locked up in remnants.

Alternatively, one can think of the Return fraction, $R=1-\alpha$. This gives

$$\frac{\mathrm{d}g}{\mathrm{d}t} = F - E + (1 - \alpha)\psi - \psi = F - E - \alpha\psi$$

We also have

$$e_Z = (1 - \alpha)Z_{\psi} + \psi \int_{m_I}^{m_U} mq_Z \xi(m) dm$$

We define the stellar yield, p_Z to be

$$p_Z = \frac{\int_{m_L}^{m_U} m q_Z \xi(m) dm}{\alpha}$$

so the yield gives the fraction of the remnant population synthesized and released in each element. This gives

$$\frac{d(gZ)}{dt} = (1 - \alpha)Z\psi + \alpha\psi p - Z\psi + Z_F F - Z_E E$$

where the RHS terms are recycled material, new production, lock-up in new stars, inflow, and outflow. Simplifying, we have

$$\frac{\mathrm{d}(gZ)}{\mathrm{d}t} = -\alpha Z\psi + \alpha \psi p + Z_F F - Z_E E$$

Using mass locked up in stars, s, as the independent variable:

$$s(t) = \alpha \int_0^t \psi'(t')dt'$$
$$\frac{ds}{dt} = \alpha \psi$$

so we get

$$\frac{dg}{ds} = \frac{F - E}{\alpha \psi} - 1$$

$$\frac{dgZ}{ds} = -Z + p + \frac{Z_F F}{\alpha \psi} - \frac{Z_E E}{\alpha \psi}$$

If we make a further assumption of a uniform wind, $Z_E = Z$, then

$$\frac{\mathrm{d}gZ}{\mathrm{d}s} = -Z\left(1 + \frac{E}{\alpha\psi}\right) + p + \frac{Z_FF}{\alpha\psi}$$

and finally,

$$g\frac{dZ}{ds} = \frac{d(gZ)}{ds} - Z\frac{dg}{ds} = p + (Z_F - Z)\frac{F}{\alpha\psi}$$

This is a basic equation of chemical synthesis. The *simplest* model for chemical evolution assumes no inflow or outflow, a homogeneous system without any spatial differentiation of metallicity, zero initial metallicity, and yields which are independent of composition. This is known as the **Simple**, one-zone model.

In the Simple model, we have

$$\frac{dg}{dt} = -\frac{ds}{dt}$$
$$g\frac{dZ}{ds} = -g\frac{dZ}{dg} = p$$

Solving for Z(g), we get

where μ is the gas fraction

The average abundance in stars is

which tends to p as μ becomes small, i.e. the gas fraction becomes small. For larger gas fractions, the mean < Z > will be lower than the yield. Note that this is the mean metallicity weighted by stellar mass, not by stellar luminosity or by number (but these could easily be derived).

4.6 Measuring stellar abundances

Stellar abundances in individual stars measured from absorption features. Hot stars only allow measurements of very few elements, while cool stars (T<3500 K) are complicated due to very large numbers of lines and prominence of molecular features. When lines are well-separated, abundance determination is possible from measurements of the *equivalent width*. When lines are more crowded, usually approached by spectral synthesis.

In all cases, to measure abundances, need to know:

- atmospheric parameters
 - Temperatures (T_e) may be derived from colors (unless reddening is an issue); also often derived from excitation equilibrium: require lines of different excitation potential to give the same abundance (e.g. Fel lines)
 - Surface gravity ($\log g$) can be derived from luminosities, temperatures and masses, hence distances are required. Alternatively, use ionization equilibrium: require lines of different ionization states to give the same abundance (e.g., Fel vs. Fell).
 - microturbulence
- atomic parameters
 - Astrophysical gf values: Atomic parameters are determined from lab experiments, but not always available at high quality. gf values are derived from measurements of stars in which abundance is presumed to be known from other measurements.

Abundances in unresolved populations are challenging, for the reasons previously discussed (agemetallicity degeneracy, blending of different features, etc.) Present day abundances in galaxies best measured through gas abundances (to be discussed).

4.6.1 Units for measuring abundances

• Stellar abundances: usually measured relative to the sun [X/Y]:

$$\log\left[\frac{(X/Y)}{(X/Y)_{\odot}}\right]$$

- Z_i : mass fraction of element i
- log(X/Y): log of ratio of mass fractions
- $12 + \log(Z_i/H)$: log of ratio of mass fraction to hydrogen, with 12 added (to keep the numbers positive?)

4.6.2 Abundances as function of time

The simple model predicts an increase of the metallicity with time. The rate of increase depends on the SFR. For example, if we assume that the SFR is proportional to the gas mass, $ds/dt = \omega g$, where $\omega = constant$, we get:

$$z = \omega t$$

and the abundance is expected to increase linearly with time.

4.6.3 Relative abundances of different elements

Simple model predicts that, at any time, elements will be found in relative abundances given by the ratio of their yields. This is true to some extent for α elments, e.g. in the solar neighborhood*. Most other elements go in lockstep with O (Fig. 11)

This is not true for Fe. This is plausibly explained for by a significant fraction of Fe coming from Snla, which has a time delay, and so the simple model with instantaneous recycling doesn't hold. This is important to realize, because many direct "metallicity" determinations are make from Fe liens. If Snla is the right explanation, one would expect to a constant ratio of Fe/O for systems in which star formation proceeded rapidly (i.e. finished before Snla go off), but in systems with more extended start formation, Fe/O would increase at higher O abundance (Page I figure 10). Note that α/Fe traces timescales of extended star formation, which is not necessarily the same as age (you could have a short epoch of star formation at a late time to give alpha-enhancement even at younger ages). However, very old populations would be expected to have alpha-enhanced populations, e.g. as observed in globular clusters. With the Snla interpretation, α/Fe provides a clock. Unfortunately, the calibration of the clock is not extremely well known because of the unknown precise nature of SNla projenitors. It is usually considered to be of order 1 Gyr but there is very plausibly a range of SNla ages.

The constancy of element abundance ratio holds for *primary* elements i.e. those that are produced from primordial abundance. In addition some elements are secondary which means that some element must previously exist in order for the secondary element to be created. An example is N, which is produced during the CNO cycle in greater abundance with a larger initial abundance of C. This is observed (Fig. 12)

More generally, the constancy of element abundance ratios could fail if the yields are themselves a function of metallicity. This may be expected to occur, e.g. for elements that are significantly contributed by mass loss, since mass loss may increase with increasing metallicity.

In reality, lots of different elements appear *not* to go in lockstep with one another, presumably because of the origin of different elements from different processes and in different types of stars. These variations may hold important clues for tracing the origin of stellar systems. α/Fe traces duration of star formation (relative to Snla timescale). Other element ratios may trace variations in the IMF, since different mass stars contribute different fractions of heavy elements, or stochastic effects from a given IMF for star formation occurring in small parcels (e.g. clusters). Gives rise to the idea of *chemical tagging*???. Example: Local Group dwarf galaxies and the Galactic halo[†]. Could the halo come to be built up by dwarfs? Caution: just because *present*-

^{*}from Bensby et al. 2003

[†]from Tolstoy et al. 2009

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day dwarfs don't match doesn't mean previously existing ones don't. Note SDSS-III APOGEE survey.

Mass-metallicity relation here? http://arxiv.org/abs/1211.3418

clouds

Intrinsic gas characteristics:

- temperature
- density
- composition
- ionization field
- total gas mass

Gas in galaxies exists in multiple phases:

```
T\lesssim 100\ K
                            T \lesssim 100 \text{ K}
ic) medium (CNM)
                            T\lesssim 8000~K
nic) medium (WNM)
       (WIM/DIG)
                             T \gtrsim 8000 \text{ K (DIG} = \text{diffuse ionized medium)}
dium
                            T > 10^5 \text{ K}
              (HIM)
nedium
                             n \approx x
zed
             gas
```

- HII regions
- AGN
- SN remnants
- PNe

CNM/WNM/WIM/HIM perhaps roughly in pressure equilibrium, but there are other support mechanisms such as turbulence and magnetic fields. If they are in pressure equilibrium, higher temperature components have correspondingly lower densities.

5.1 Observing and characterizing the gas

Two general methods:

1. Emission

Atomic lines

Permitted In low temperature gas (T = 10000 K \sim 1eV), the energy difference to excited states is typically too high for excitations to occur from collisions. An exception is H lines from recombination; these exist from UV through radio wavelengths.

"Forbidden" Low transition probabilities; important in low density gas because collisions are less frequent, allowing time for radiative transitions to occur before collisional de-excitation. Wavelengths are typically in the optical and UV regimes. Note that many of these are doublets because of multiple angular momentum levels. Most prominent in optical: [OIII], [NII], [SII], [OII], [OI]

Fine-structure transitions involving spin alignment with orbital alignment. Typically FIR wavelengths **Hyperfine-structure** transitions involving electron spin alignment with nuclear spin. Radio wavelengths.

Molecular lines

- Vibrational transitions, typically IR wavelengths
- Rotational transitions, typically mm wavelengths. However, rotational transitions only strong for non-symmetric molecules that have a dipole moment (not H₂, although it can have some IR features if shock heated). CO emission is most prominent.
- example spectrum.

Emission in many cases is proportional to density *squared*. If the excitation mechanism is collisional (i.e., requires two particles), the observational quantity is the *emission measure*:

$$EM \propto \int n_e^2 d\ell$$

2. Absorption

Atomic lines

Typically from ground state, e.g. H Lyman transitions, Na 5890, Call most prominent in optical

Molecular lines

Also mostly UV, e.g. H₂ absorption

Absorption is generally more sensitive, but it does require a bright background source, e.g. UV light from quasars. Absorption is *linearly* proportional to density. The observational quantity is *column density*:

$$CD \propto \int n \mathrm{d}\ell$$

5.2 | Heating and cooling

What determines gas temperature?

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5.2.1 Heating

- Radiative heating (both via ionization and via heating through dust)
- Mechanical heating (shocks, jets, etc.)
- Gravitational heating
- Cosmic rays

5.2.2 Cooling

- Bremsstrahlung (free-free)
- Line emission from heavier elements, molecules
- Cooling curves and contributions (Sutherland & Dopita, 1983)

5.3 Cold gas: HI measurements with 21 cm line

- Traces neutral medium
- From spin-flip transition: very rare (10 Myr) but H is very numerous. Relative number in two energy states set by collisional equilibrium and proportional to statistical weights of two levels (3:1); upper levels are populated from (microwave background photons, collisional saturation?)
- HI flux at a given location gives column density (atoms/surface area):

$$n_{HI} \left[\text{cm}^{-2} \right] \approx 1.82 \times 10^{18} \int \left[\frac{T_b(\nu)}{\text{K}} \right] \text{d} \left[\frac{v}{\text{km s}^{-1}} \right]$$

Typical column densities in galaxies are $\sim 10^{20} \ {\rm cm}^{-2}$

• Total HI flux (integrated over entire galaxy), along with distance, gives total HI mass:

$$\frac{M_{HI}}{M_{\odot}} = 2.36 \times 10^5 \left(\frac{D}{\text{Mpc}}\right)^2 F_{HI}$$

for F_{HI} in Jy km s⁻¹.

- 21-cm single dish observations have low spatial resolution. Higher resolution possibly with interferometric observations (e.g. VLA) but information on smooth distribution/total mass can be lost.
- ullet In absorption, HI can be observed to very low column densities ($\sim 10^{13}~{
 m cm}^2$), but only for the Lyman series. Primary tool for studying the IGM:
 - Lyman alpha clouds
 - epoch of reionization

- quasar absorption line systems
- Typical column densities are $10^{13} 10^{21}$ cm⁻²
- Total mass in HI can be a significant fraction of total stellar mass, increasingly important in lower luminosity galaxies.
- Temperature might be inferred from width of line, but need to be careful about turbulence, cloud structure, etc. Estimate can also be made by comparing emission flux with absorption if you have a background source (radio galaxy).

5.4 Molecular gas

- Most dominant molecule is H_2 , but since this has no net dipole moment, it doesn't have strong rotational transition emission. There are some mid-IR lines from shock-heated H_2
- H_2 can be observed in absorption in the far UV
- Molecular gas in emission most often traced by CO. However, relating CO emission strength to total amount of molecular gas is not necessarily trivial. The X (CO) factor relates observed strength of CO to H_2 column density:

$$X \equiv \frac{N(H_2)/\text{cm}^{-2}}{I_{CO}/\text{km s}^{-1}} \sim 2 \times 10^{20}$$

However, there are many issues with this, and almost certainly depends on metallicity.

• Overall mass contribution...less than atomic, but significant?

5.5 Warm ionized gas

Also known as "Diffuse Ionized Gas", or "DIG".

- Visible in Hα
- Heating source likely photoionization, visible through recombination.
- Hard to estimate masses
- Pulsar dispersion gives measurement of $\int n_e d\ell$, which can help a lot.

5.6 Hot gas

• If "very" hot (number please?), detectable in X rays: thermal Bremstrahlung (free-free) cutoff frequency decreases with increasing temperature.

- \bullet At lower temperatures (T $\sim 10^5$ K), very hard to detect: soft X rays are absorbed by neutral hydrogen.
- Detectable in absorption by highly ionized metals, e.g. OVI, CIV, NV.
- Very hard to determine total mass, but mass may be very significant, just from cosmological baryon fraction ???

5.7 Denser ionized gas

• Easiest gas to observe in galaxies.

5.8 Chemical evolution and abundances from gas

5.9 Distribution of gas within spirals

Neutral gas primarily observed in blue sequence galaxies. HI distribution usually extends beyond optical distribution of starlight, while molecular is more centrally concentrated. In some galaxies, CO traces $H\alpha$ well, but not in others. Distribution of HI is very flat compared to distribution of molecular gas (and stars). Extended hot gas is observed in most galaxies.

5.10 Gas as a kinematic tracer

(This section not in online notes anymore...)

Measuring galaxy rotation - both optically and using HI (both spatially resolved, e.g. VLA, and unresolved data).

- Swaters et al. 2002
- Velocity-coded color (from NRAO)
- Spider diagrams
- PV diagrams: note spread of velocity at any given position; location of peak velocity may not give the maximum velocity at that position

Rotation is often characterized by v_{max} : maximum of rotation curve (sometimes hard to measure). From unresolved HI profiles, rotation is characterized by line widths, e.g. W_{50} , W_{20} (width at 50% and 20% of the peak line flux, respectively).

5.11 Interactions between gas and stars

5.11.1 Star formation

Due to the lack of detailed knowledge of star formation, and lack of sufficient resolution, star formation is parameterized within models, e.g. based on stability arguments given gas densities and temperatures. Parameterization is tuned by need to match characteristic observed relations of observed star formation.

Star formation parameterizations: want to relate SFR to some observed property.

- Measure current star formation rate. Three main methods:
 - 1. $H\alpha$ emission (but note possible issues with assumed IMF, leakage)
 - 2. UV as more direct SFR indicator (but associated uncertainty from extinction)
 - 3. IR emission from dust heated by young stars

Typical galaxy SFRs are 0-100 $M_{\odot}~yr^{-1}$. The specific star formation rate (SSFR) is the SFR per unit mass (usually stellar). A typical galaxy has SSFR $\sim 10^{-10}~yr^{-1}$. For nearby resolved galaxies, we can also consider the surface density of star formation.

- Relate star formation with gas properties: Simplest quantity is gas density, observed as gas surface density. Can consider density of neutral gas, molecular gas, or a combination of all phases. Molecular might be considered to be the most relevant, but it is difficult to observe (also, recall profiles of $H\alpha$ vs gas density shown earlier). Also, molecular gas forms from neutral gas.
- Schmidt law:

$$SF \propto \rho^n \propto \Sigma^N$$

(where Σ^N is relevant as it is the observable). Schmidt (1959) suggested $N\sim 2$ based on observations of HI and stars in Milky way. Note that a fixed star formation efficiency would have SFR proportional to gas mass.

- Measured relations show a more complex behavior (Kennicutt Fig 8). Slope in regions of high gas density roughly comparable between galaxies (Schmidt law), but different galaxies have different thresholds for SF. Molecular gas may be more relevant above threshold, see e.g. Mo Fig 9.4
- Kennicutt SF law: has a threshold density, then a power law dependence

several "scenarios" might give observed trends: consider two possible qualitative physical mechanisms behind observed relations.

1. star formation related to growth of instabilities suggests

$$\rho_{SFR} \propto \frac{\rho_{gas}}{(G\rho_{gas})^{-0.5}} \propto \rho^{1.5}$$

we observe surface density, not density. For *fixed* scale height of gas, these are directly proportional, but not for *variable* scale height

2. Alternatively, perhaps SF is related to the local dynamical timescale (e.g., SF triggered by passage through spiral arms). Then

$$\Sigma_{SFR} \propto \frac{\Sigma_{gas}}{(\tau_{dyn})} \propto \Sigma \Omega_{gas}$$

Both give comparable SFRs (Kennicutt Figure 7).

For thresholds, threshold density might be expected from consideration of stability of disks below critical density, disks are stable against large scale perturbations; for a thin isothermal gas disk, $\Sigma_c(r) = \alpha \kappa(r) \sigma/3.36 \text{G}$, where σ is the gas velocity dispersion, and κ is the epicyclic frequency, which is related to the rotation curve. The critical density depends on rotation profile and velocity dispersion. Density of gas in spirals matches reasonably well the profile of critical density (Figs 10 & 11). In this model, the threshold surface density is a function of radius (Fig 14), and varies from galaxy to galaxy. Suggests threshold as f(r) then Schmidt law with $N \sim 1.3$ - with several possible systematic uncertainties (extinction, other factors which vary with r). However, behavior near the threshold is significantly steeper - and most regions of most galaxies appear to be near the threshold.

there are other possible explanations as well, e.g. ability to develop molecular clouds at low densities..

see Leroy et al 2008 for nice discussion of lots of different parameterizations

- Focus on star formation efficiency (SFE), i.e. SFR / gas mass
- Find roughly constant SFE in molecular dominated regions, with declining SFE in outer regions (Figure 1). Note that this suggests there is not a simple threshold.
- Simple SF "laws" do not appear to well match observations (e.g. Figure 5 and Figure 7)
- Perhaps the issue has to do with conditions for molecular cloud formation
- Note issue of observing surface mass density may fall short if spatial resolution is insufficient to resolve real structure! (see, e.g. Leroy et al 2013a)
- Leroy et al 2013b suggest that there are measureable deviations of SFE in molecular dominated regions, that may be related to variations of CO/H

Simulations often adopt some prescription related to one of these mechanisms

global star formation law. Star forming galaxies show range of current star formation rates from ~ 1 Msun/yr to hundreds of Msun/yr; what sets the SFR? Results from a sample of normal galaxies (Kennicutt Fig 2), starburst galaxies(Kennicutt Fig 5), and all galaxies combined (Fig 6). Remarkably tight relation is seen over wide range of densities and star formation rates.

5.11.2 Supernovae and mass loss from stars

supernovae clear out "superbubbles" in the ISM; it is possible that these superbubbles even propagate to the vertical edges of the disks and allow for mass outflow from the disk into the

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halo. If this is the case, one might expect this gas to eventually fall back down onto the disk, leading the the ideas of galactic fountains and chimneys.

- Note active study/debate: extraplanar gas arising from disk (outflow/inflow) or from IGM (inflow), e.g. Milky Way high velocity clouds (HVCs), lagging gas in galaxies
- Note implication for chemical evolution

Significant galactic winds (outflows) are observed in starburst galaxies, e.g. M82. These may be more common at higher redshift.

also possible that the shocks from supernovae act to compress gas clouds, possibly trigering further star formation in nearby regions.

These processes are important for understanding:

- feedback mechanisms by which star formation may be self-regulating
- chemical evolution of galactic disks
- the existence of gas halos around galaxies
- the escape of ionizing radiation from galactic disks.
- overall issue of "gas cycle" in galaxies, e.g. Fig 2 from Lilly et al 2013

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6 Dust

6.1 General importance of dust

- Can obscure light from stars/gas: affects inferences on populations, abundances, etc.
- Emits radiation which can be observed (mostly IR) to probue obscured emission.
- Important for ehemistry of ISM, e.g. created of H₂.
- Important for heating and cooling of ISM
- Probably not important as a significant mass component; likely only a fraction of a percent of ISM by mass

6.2 Extinction

Foreground and internal. Extinction laws and their variation.

Extinction of light is characterized by an **extinction curve**, which gives the relative amount of light lost as a function of wavelength. Note that extinction curves are often in magnitudes (logarithmic). Generally, shorter wavelengths are attenuated more, leading to the phenomenon of reddening. This implies that the dust grain sizes are comparable to the wavelength of the light. Existence of the 2200Å bump. The total amount of extinction is proportional to the amount of reddening, e.g. $R_V \equiv A_V/E(B-V)$. More generally, $R_\lambda \equiv A_\lambda/E(B-V)$.

Variations in the extinction curve exist, both within the Milky Way and in nearby galaxies*. This variation is more significant in the UV. Cardelli, Clayton, and Mathis suggest that extinction curves can be parameterized by a single parameter R_V . Probably related to dust properties, e.g. grain size.

Extinction has both scattering and absorption components. Scattering leads to, e.g., reflection nebula and diffuse light. Measuring scattering suggests optical albedo \sim 0.5, so roughly equal amounts of scattering and absorption. Absorption leads to heating and re-emission.

Discussions of extinction curves refer to the extinction of background light by foreground dust (i.e. a foreground screen). Integrated galaxy light likely does not fall into this scenario (apart from foreground Milky Way extinction). Remember Burstein & Heiles and Sclegel, Fink beiner & Davis for MW maps. Scattering is significant and leads to "saturation".

^{*}Gordon et al. 2003

[†]Witt & Gordon, 2003

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6.2.1 Amount of gas, dust-to-gas ratio, and transparency of galaxies

The amount of extinction seems to be well correlated with the amount of neutral H

$$N(H) = 5.8 \times 10^{21} E(B - V)$$
 $\left[\text{cm}^{-2} \text{ mag}^{-1} \right]$
 $A_V = 5.3 \times 10^{-22} N_H$ $\left[\text{mag cm}^2 \right]$

This suggests that dense (inner) regions of ISM are likely to be optically thick, but not low density ones. Confirmed by observations of "overlapping" galaxies.

6.3 Emission from dust

- Predominant heating is from starlight (some collisional heating in dense clouds)
- Amount of energy absorbed depends on flux, size, and albedo. Amount of energy radiated depends on temperature, size, and emissivity.
- Dust particles are not ideal blackbodies. In addition, they may not be in temperature equilibrium, especially small particles.
- MW dust
- Dust emits in far IR, e.g. MW emission and models. Note sharper emission features from PAHs around 10 microns.
- Dust emission traces star formation; luminous IR galaxies (LIRGs, ULIRGs, 10 to 100x IR emission than typical galaxy).
- Implications for higher redshift: sub-mm observations and the negative K-correction

6.4 Composition

- Solids have broad or non-existent absorption features, so abundances from absorption are difficult or impossible.
- Some inferences from interstellar gas abundances: if stellar abundance ratios are assumed, than observed gas abundances show depletion (from Draine), e.g. of C, Mg, Si, Fe at a level greater than 50%.
- Still can't identify uniquely what molecules the dust is in. Possibilities:
 - silicates
 - oxides
 - carbon solids
 - hydrocarbons, e.g. PAHs (polycyclic aromatic hydrocarbons)
 - carbides

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- metallic Fe
- Some absorption features are observed:
 - 2200Å feature: originally thought to be graphite, now perhaps PAHs
 - 9.7 and 18 microns: silicates? (from Draine)
 - diffuese interstellar bands (DIBs) (from Draine)
- As stated above, emission features are observed in IR from PAHs

Overall, plausible to expect that the dust-to-gas ratio might be a function of metallicity.

6.5 Creation/destruction

Dust is apparently difficult to create in the ISM. It is thought to be created in envelopes of cool stars \rightarrow the amount of dust may also be a function of star formation.

7 Central black holes

7.1

(some references: Peterson, "An Introduction to Active Galactic Nuclei", Ferrarese & Ford (2005)).

Galaxies harbor central supermassive black holes: how do we know?

AGN as indicators of central black holes

Some fraction nearby galaxies who "active" nuclei:

- · Optical emission lines
- Power-law continua spanning most of the EM spectrum.
- Radio sources, often with jets
- AGN review*†
 - Seyfert galaxies: Optical emission lines. Types 1 (broad lines) and 2 (narrow lines).
 Distinguishable from HII regions, etc. from emission line ratios.
 - LINERS: weaker lines, lower ionization
 - BL Lac objects
 - Quasars and QSOs
 - Radio galaxies, e.g. Cen A, M87. Emission is from synchrotron emission. Energy source may be the same, but don't generally directly see nuclear activity. Fanaroff-Riley (FR) types 1 (low radio luminosity, edge-darkened) and 2 (high radio luminosity, edge-brightened).
 - See tables in section 4b of Whittle notes with some spectra.
- Frequency of Seyfert phenomenon 1-2% but LINERS much more common; perhaps tens of percents of local galaxies show activity at some level.

Generally considered to be powered by accretion onto a nuclear black hole. Arguments for the existence of black holes:

ullet Luminosity: can be as high as $\sim 10^{45-58}~{
m erg~s^{-1}}$ for some AGN. **Eddington Luminosity:**

$$L_e = \frac{4\pi G M m_p c}{\sigma_T} = 1.3 \times 10^{46} \frac{M}{10^8 M_{\odot}} \left[{\rm erg \ s^{-1}} \right]$$

 $^{^{}st}$ see book by Peterson, 1997

[†]see Whittle (UVa) lecture notes

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- Non-stellar continua
- Timing arguments. AGN show rapid variability with timescales of days (optical/radio) to hours (X-ray): implies small objects (small fraction of a pc).

Overall physical picture: different active nucleii phenomena all arise from a common physical scenario. Some *intrinsic* differences give rise to different phenomena, whereas some differences arise from viewing angle.

- Unified model: several main regions:
 - Accretion disk
 - Broad line region (BLR)
 - Narrow line regions (NLR)
 - Dust torus
- Alternative model*
- Viewing angle differences: Unified model (and another)
- Some possible intrinsic causes of variation between types
 - Accretion rates and accretion rate differences
 - Differences from variations in gas suppy (quantity and/or efficiency of infall)
 - Differences from different black hole masses?

7.2 Black holes in non-active galaxies

It appears that most galaxies, even inactive ones, harbor central supermassive black holes. This is expected at some level from number density of quasars at higher redshift. Detection of SMBHs^{+‡}:

• **Sphere of influence** (region where velocity generated by central mass is comparable to or larger than velocity determined from overall galaxy potential) is very small:

$$r_h \sim \frac{GM_{BH}}{\sigma^2} \sim 11.2 \frac{M_{BH}}{10^8 M_{\odot}} \left(\frac{\sigma}{200 \text{km s}^{-1}} \right)^{-2} \quad [\text{pc}]$$

- MW galactic center (e.g. UCLA group video)
- Other galaxies: BHs inferred but strictly speaking, not absolutley required§.

If they have BHs, why aren't they active? Gas supply? Advection dominated accretion flow (ADAF)?

^{*}Elvis (2000)

[†]Ferrarese & Ford

[‡]Whittle (UVa) lecture

[§]Ferrarese & Ford 2005

7.3 Measuring black hole masses

- Proper motions
- Stellar and gas kinematics
- Gas kinematics

7.4

- Broad Fe Kα (6.4 KeV X-ray line)
- Reverberation mapping

BH masses have now been made for several dozen galaxies. Initially, correlation seen between BH mass and galaxy luminosity. Relation between BH mass and central velocity dispersion significantly tighter. Origin not well understood. $M_{BH}-\sigma$ relation (or $M_{spheroid}$).

Importance of central black holes

8 Galaxy spectral energy distributions

9 Dark matter and galaxy masses

Dark matter dominates mass in the universe. How much is there? How is it distributed, both among and within galaxies? To what extent does it regulate galaxy properties?

Note from cosmology: we expect

- $\Omega_m \sim 0.27$
- $\Omega_b \sim 0.047$

9.1 Observations of dark matter in galaxies

The amount of dark matter in a galaxy is often characterized by the mass-to-light ratio (M/L). Note that there is a distinction between stellar M/L and total M/L.

9.1.1 Galaxy rotation curves

Rotation curves (RCs) probe dark matter content and profile. The general observation of flat profiles (sometimes reffered to as "isothermal" profiles) implies that $M \propto r$ and $\rho \propto r^{-2}$.

Recall that the shape of the rotation curve is moderately well-correlated with the luminosity, in the sense that one gets steeper RCs for lower L galaxies*. However there is significant variation at each luminosity. Note that not all RCs are flat; there is a variety of shapes.

Comparison of the luminosity profile with the rotation profile shows that the *luminous* mass alone is unable to produce the rotation profile, therefore dark matter is required. The fraction of "dark mass" increases with radius, and also increases with fixed radius with decreasing luminosity.

Dark matter in disk galaxies

Given the luminosity profile, the shape of the RC shows that DM has to be present, because there's no way to get high enough stellar mass-to-light ratios in the outer region. However, determining the amount of dark matter in disk galaxies requires knowledge of the M/L of the stellar population, which is unknown (recall that M/L depends on the SFH, especially the IMF). The stellar M/L can't be derived from the RC because the density distribution of the DM is unknown. It's possible that the presence of dust makes this more challenging to infer.

A common assumption is the "maximum-disk hypothesis", in which the innermost parts of the RC are assumed to be driven by the luminous mass alone. These regions then give a M/L, which is assumed to be constant as a function of radius. Combined with the outer roation curve, mass profiles for the luminous and dark components are derived. However, data is often degenerate in different ratios of disk to DM^{\dagger} . The applicability of this hypothesis is debateable.

^{*}Persic and Salucci, figure 4

[†]see van der Kruit example, from van der Kruit Galaxy Masses 2009 talk

For example, for an exponential disk:

$$V_{max} \propto \sqrt{\frac{M}{r_d}}$$

so we might expect a trend at fixed L (if we extrapolate from M to L) with scale length: not observed in Tully-Fisher relation.

The total M/L in disk galaxies has been found to be \sim 10-50 in regions probed by RCs.

Dark matter in LSB galaxies

The RCs for LSB galaxies confirm a high M/L. Compare two galaxies at identical positions in TF relation and see that the LSB galaxy is much more DM dominated*. This suggests that RCs are correlated not only with luminosity, but also with surface brightness.

LSB RCs are of particular interest because they may provide the most direct probes of DM distribution. The shapes of their RCs are currently under active debate as to what they imply for DM distribution. Numerical DM simulations predict a "cuspy" distribution of DM, with $\rho_{DM} \propto r^{-1}$ in the inner regions, e.g. the Navarro-Frenk-White (NFW) profile:

$$\rho = \frac{\rho_0}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$$

family of profiles characterized by the concentration r_{200}/r_s .

However, there may also be an effect of baryons on DM (adiabatic compression, or expansion?), since in inner regions, gravity from baryons may not be negligible. Some observations[†] suggest more of a core. The interpretation of relatively low velocities in LSB centers can be complicated by a variety of effects, such as projection, beam smearing, non-circular velocities, etc. The "core vs. cusp" debate is still active.

DM halos are expected to extend far beyond what we can see from physical tracers (even HI gas) within the galaxy embedded in their centers. Typical virial radii (from simulations) are expected to be several hundred kpc. So total masses are not well probed by rotation curves.

Dark matter in ellipticals

Theoretically, the presnece of DM in ellipticals is exptected. However, it is harder to observe than in spirals because of the lack of an "easy" dynamical tracer, like a rotating component. They also have higher SB, so there is a larger contribution by baryones to kinematics.

For non-rotating components, the velocity dispersion profiel is insufficient to measure mases without knowledge of *velosity ellipsoid*, parameterized by *velocity anisotropy*:

$$\beta \equiv 1 - \frac{\langle v_{\theta}^2 \rangle}{\langle v_r^2 \rangle}$$

^{*}from de Blok & McGaugh

[†]Swaters et al 2003

[‡]see review by Gerhard 2006

Or more generally, distribution of orbits*. Inclination/projection also plays a role...

Potential tracers:

- Velocity distribution of stars (integrated light). Need to use line profile shapes, 2D distributions (integral field)
- Planetary nebulae
- Globular clusters
- Gas disks (rare, but do exist)

Note that even for non-rotating systems, distribution of mass often described by the *circular velocity*:

$$v_c \equiv \sqrt{\frac{GM(r)}{r}}$$

From stellar velocity distributions, most ellipticals show evidence of flat circular velocity curves, indicating the presence of DM in the inner regions, but not always a lot.

Modelling suggests that a simple estimate[†]

$$M/L \sim 5 \frac{\sigma_e^2 r_e}{GL}$$

gives a reasonable rough estimate; c.f virial relation (???) and relation for central velocity dispersion, e.g. for isothermal sphere. There has been some debate about PN observations at larger radii, with some observations gsuggesting declining circular velocities. However, understanding of velocity anisotropy is important to the conclusion.

Indpendent mass estimate proveded by X-ray halos under the sasumption of hydrostatic equilibrium (requires measuring densities and temperatures), which, when they exist, generally indicate the presence of dark matter halos ‡

$$M(r) = -\frac{k_B T(r) r}{\mu m_p G} \left[\frac{\mathrm{d} \ln \rho}{\mathrm{d} \ln r} + \frac{\mathrm{d} \ln T}{\mathrm{d} \ln r} \right]$$

Bottom line: ellipcals probably have dark matter.

9.1.2 Gravitational lensing

Another way of probing mass. There is a distinction between *strong* and *weak* lensing§. Weak lensing is measured statistically.

^{*}e.g. velocity dispersion profiles, from van der Marel (1994)

[†]Cappellari et al. (2006)

[‡]from Sato et al. 2000

 $[\]S$ both from Treu talk at Galaxy Masses 2009

- 9.2 Total masses of galaxies
- 9.3 Halo abundance matching