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Important things to know:

- Derive the Virial Theorem
- Schechter function
- Sersic relation
- deVouceleurs
- Fundamental Plane
- Tully Fisher relation
- Kormendy relation
- isophotes: boxy vs. disk
- inner SB profiles: cuspy vs. core

I. History

1. 1700's: Messier objects, 39 of which are actually galaxies (total of 110 in the final catalogue).
2. 1864: GC, 1888: NGC (William Herschel and son John)
3. 1920's: 'Great Debate' between Curtis and Shapley on whether or not galaxies were located within the MW. Resolved by Hubbles discovery of Cepheids in M31.
4. 1980's: Importance of environment recognized: **morphology/density relation**; 'Nature vs. Nurture'.
5. 1990's: Techniques for finding and confirming high redshift galaxies ($z \geq 2$): **Lyman break galaxies**:
 - Lyman limit
 - Rydberg formula: $\frac{1}{\lambda} = R \left(\frac{1}{n_u^2} - \frac{1}{n_l^2} \right)$First large scale surveys, both nearby and at medium redshift. HST imaging of distant galaxies. N-body (dark matter) simulations.
6. 2000's: Precision Cosmology and LCDM, outside optical wavelengths: IR (Spitzer) and sub-mm (JCMT).
7. 2010's: Extended gas halos in galaxies. Possibly detection of DM and dark energy.

II. Approaches

Multifaceted approach to studying galaxy formation and evolution:

Galaxy "archaeology"

Study nearby galaxies in detail, attempt to understand processes that led to their current appearance.

- Advantages: can resolve structure, individual stars in nearest galaxies, high S/N observations
- Disadvantages: some information may be erased by physical processes (e.g. merging), degeneracies in integrated light

Distant galaxies

look at galaxy samples at different lookback times, study distribution of properties (galaxy population) as a function of time.

- Advantages: direct probe of different stages. Relationship between lookback time and redshift.
- Disadvantages: brightness/selection effects, lack of detail, difficulty in associating objects at one redshift to those at another.

Physics of galaxy formation

- Advantages: some physics (e.g. gravity) is well understood
- Disadvantages: some physics (e.g. star formation) is not. Dynamic range of the problem is huge.
 - Dynamic range in distances, from stellar scales to largest scale structure.
 - Dynamic range in mass, from stellar scales ($1 M_{\odot}$) to largest scale structure ($10^6 - 10^{15} M_{\odot}$)

III. Overview of galaxies and galaxy formation

Components:

- **Dark Matter:** usually non-baryonic, but some baryonic matter can be hard to see, such as brown dwarfs. Dominates mass of galaxies.
- **Stars:** observed properties depend primarily on mass, age, and composition. Variety leads to multiple luminosities and colors in galaxies.
- **ISM:** molecular, atomic, and ionized gas phases; dust; mass of ISM varies widely between galaxies.
- **Central (supermassive) black holes**

Why do galaxies appear as they do? Important processes (not necessarily in chronological order):

- Gravitational collapse (of dark matter, and later, baryons) in cosmological framework.
 - How big are initial lumps at different size scales?
 - How much angular momentum?
 - How fast do lumps grow?
- Condensation of gas and cooling
 - "hot" vs. "cold" accretion
- Star Formation (not well understood)
 - Under what conditions do stars form?
 - What types (masses) of stars form?
 - Drives chemical evolution, which may impact cooling and future star formation
- Black hole formation
 - Primordial formation vs. formation from early stars
 - How common?
- Feedback/mass loss
 - How much energy? Does mass escape or just delay accretion?

- What objects generate it? Winds, supernovae, galactic nuclei? How?
- Continued accretion from IGM
 - How much?
 - What mode?
 - What composition?
- Merging: Gas-rich vs. gas-poor
- Cluster (group?) environment: ram pressure, tides
- Dynamical evolution
 - Dynamical instabilities
 - Migration
 - Internal vs. external triggers

These processes have *characteristic timescales*, and the relation between them may influence how galaxies form, evolve, and appear.¹

Example

Overly simple, but illustrates the consequences of this scenario².

- Galaxies appear to have a maximum size of order $10^{11} - 10^{12} M_{\odot}$, even though there are larger agglomerations of matter, e.g. clusters.
- Can be crudely explained by understanding of dissipation
- Self-gravitating cloud has two timescales:
 1. dynamical, or free-fall:

$$t_{dyn} \sim (G\rho)^{-1/2}$$

2. cooling time:

$$t_{cool} \sim \frac{nkT_g}{n^2\Lambda(T)}$$

where Λ is the cooling function

If $t_{cool} > t_{dyn}$, then a cloud can be in quasi-static equilibrium, i.e. cooling is unimportant. If $t_{cool} < t_{dyn}$ the cloud cools, kinetic energy is converted to radiation, and the cloud collapses. Given a cooling curve for primordial composition, one can calculate the relevant timescales, and find that collapse is unlikely to occur for $M > 10^{12} M_{\odot}$. This implies that dissipation is important, at least for objects we observe as galaxies (e.g. luminous objects).

This argument is really only a suggestion, for a number of reasons: halos are not uniform density, so there's no such thing as a single cooling time for the entire halo.

More Questions: Add these to the ones above, and keep them in mind while studying. Read with a question in mind, connect old information to new information. Add main bullets first, then go through and add details, depending on how much time you have.

Questions

- When do each of these steps happen and what are their relative importances?
- What sets the masses of galaxies? Sizes? ($10^6 - 10^{12} M_{\odot}$) Luminosities?

¹see Mo et al.

²Rees 1995

- What sets the distributions of numbers of galaxies as a function of mass/luminosity?
- Does the ratio of baryonic mass/total mass change for different galaxies?
- What triggers star formation in galaxies?
- What is responsible for the range of galaxy morphology?
- How much of present structure is determined by initial conditions, e.g. initial overdensity, angular momentum (and what are those initial conditions)?
- How much does present appearance depend on basic physics within galaxies, e.g. dynamics and chemical evolution?
- How much depends on environment, e.g. mergers and interactions, background radiation?
- Does the relative importance of these effects (initial conditions, internal evolution, environment) vary for different galaxies?

IV. Morphological classification

Historically, galaxies were considered in terms of their morphology, i.e. the Hubble sequence. However, it is not totally clear to what extent morphological classification traces underlying physics, and descriptive morphology may be biased by things that may not be fundamental. Still, it is widely used, so important to understand a bit.

Morphological systems

Good reference: Sandage in Galaxies and the Universe, 1975. Some pictures:

- [ellipticals](#)
- [S0s](#), also known as lenticulars
- [spirals](#)
- [Tuning fork diagram](#)

Hubble classification

- **ellipticals:** given as E_n , where

$$n = 10\left(\frac{1-b}{a}\right)$$

E.g., E_0 (sphere) $\rightarrow E_7$ (skinny). No distinction between dwarf ellipticals and dwarf spheroidals

- **spirals:** barred or unbarred (SB_a, SB_b, SB_c ; or Sa, Sb, Sc ; respectively) where a,b,c denote size of bulge (bulge-to-disk ratio: B/D) in decreasing order. Also classified according to tightness of arms and the degree to which arms are resolved into HII regions (note that the latter two are essentially impossible to judge for edge-ons). Later, have SA (normal spirals), SB (barred), and SAB (transition)
- **SOs (aka lenticulars, or disks):** intermediate, no spiral structure but have disk system, split into $S0_1, S0_2, S0_3$, depending on the amount of dust.
- **Irregulars:**
 - Irr I (Magellenic irregulars with lots of distinct HII regions)
 - Irr II (lack the resolution into distinct HII regions).
- Some pictorial examples

deVaucouleurs/RC3 classification

[diagram](#). See also [here](#).

- Extends Hubble to later spiral types Sd, Sm , and finally Im .
- Extra classes around $S0$ ($S0^-, S0^+$)
- Allows for intermediate between barred and unbarred:
 - SA : normal spirals
 - SB : barred spirals
 - SAB : transition
- Adds extra distinction for *ring* vs. *s* shaped.

- Note that location along spiral sequence may not be the same as in “Hubble classification” (more based on B/D).
- Some [pictorial examples](#)
- [Numerical galaxy types \(T\)](#)

Note that morphological classification often depends on multiple characteristics, and therefore it can become somewhat subjective. Quantitative classification schemes have been worked on, but are not widely used.

Wavelength dependence

Morphology depends on wavelength (e.g. [O'Connell 9609101](#)). Be careful about morphological classification at higher redshift (or at least a direct comparison with lower redshift galaxies) because you may be looking at morphology at a different wavelength (morphological **K Correction**: correction to magnitude (or flux) due to redshift).

Classification for ellipticals

Based on global characteristics, the Hubble morphological classification of ellipticals is probably not fundamental. It may be more meaningful to classify by *isophotal shape* (([Kormendy and Bender classification](#))) or kinematically by v/σ .

Classification for spirals

For the spiral sequence, there are correlations of luminosity, surface brightness, rotational velocity and gas fraction with Hubble type. However, each category has a broad range of global observables, and the categories overlap significantly.

Quantitative classification schemes

All of the aforementioned classification schemes are subjective at some level. Some more quantitative schemes have been proposed, both parametric and non-parametric.

- **Bulge-to-disk ratio**, using B/D decomposition³. Issues: covariance between parameters, 1D vs. 2D, and validity of models.
- **Global profile fit**, e.g. Sersic index
- **Concentration**, e.g. SDSS r_{90}/r_{50} , r_{80}/r_{20} , using circular apertures. Elliptical apertures can also be considered, e.g. based on *second order moments*. Related to B/D. Sensitivity to seeing/distance.

Non-symmetric galaxies

Many of the schemes used for nearby galaxies fail for non-symmetric galaxies, for which other indicators have been suggested:

³[MacArthur, Courteau, & Holtzman, ApJ 582, 689 \(2003\)](#)

- **Asymmetry:**⁴ Rotate about center, self-subtract,

$$A = \frac{0.5|\text{subtracted}|}{\text{total}}$$

Possible indicator of mergers.

- **Clumpiness:**⁵ e.g. S . Subtract smoothed version of image from original image, ratio of flux in subtracted image to flux in original images gives S . Possible indicator of star formation.
- **Gini coefficient:**⁶ Applied to sorted list of pixel values. May be more stable at low resolution, S/N .
- **M20:** second order moment of brightest 20% of galaxy light⁷. May be more stable at low resolution, S/N .

Mystery section

Some representative data from [Roberts and Haynes review \(ARAA 1994\)](#):

- Considering mean or median values, there is little trend in radius, $L_B(?)$, and total mass for S0-Sc, but later types tend to be smaller, less massive, and less luminous ([RH figure 2](#)). However, note that LSB(???) galaxies tend to be later types, and, since these are not included, there may be a bias here.
- Typical SB may be lower for latest types, even without inclusion of LSB galaxies; CSB(?????) much higher for large ellipticals than spirals. Mass surface density appears to decrease monotonically with morphological class in spirals ([RH figure 3](#)).
- Cold gas absent in ellipticals, and neutral hydrogen (HI) content appears to increase monotonically with type for spirals. It is possible that molecular gas content decreases with type, but not certain. ([RH figure 4](#)).

V. Basic observational properties of galaxies

Surface Brightness

Surface Brightness (SB) is the basic measured property for imaging of a *resolved* object. **SB is independent of distance until geometry of the universe becomes important:**

$$SB \sim (1 + z)^{-4}$$

Units:

- $\text{erg cm}^{-2} \text{ s}^{-1} \text{ sterradian}^{-1}$
- mag arcsec^{-2}

Magnitudes are often used, but if *summing* multiple SBs, flux units must be used. If you're adding and subtracting magnitudes, you're probably doing something wrong. ☹Differences in

⁴Abraham et al, ApJS 107, 1, 1996

⁵Conselice ApJS 147, 1 (2003)

⁶Abraham et al, ApJ 588, 218 (2003), Lotz et al., AJ 128, 163 (2004)

⁷Lotz et al., AJ 128, 163 (2004)

magnitudes correspond to ratios in fluxes:

$$m_1 - m_2 = -2.5 \log \frac{F_1}{F_2}$$

In general, flux is a function of wavelength, so SB is measured in different bandpasses, e.g. UBVRI or SDSS (ugriz).

The *ratio* of the flux of the same object in different bandpasses gives an estimate of the *spectral slope* between bandpasses; often expressed as a difference in magnitudes, or *color index*, such as U-R.

Relevance of SB distribution: when investigating how stars are distributed in galaxies, be aware that *mass* in stars doesn't necessarily track *light* in stars (nor does it trace mass in other components).

In principle, SB is measured directly from a 2D detector as an arbitrary function of location. However, galaxies are relatively faint. Typical values in center of galaxies:

- Spiral: $V \sim 20 - 21 \text{ mag arcsec}^{-2}$ (not much brighter than typical dark sky)
- Elliptical: $V \sim 16 - 17 \text{ mag arcsec}^{-2}$

More than half the light from galaxies comes from regions with $SB < SB_{\text{sky}}$ so S/N is difficult to obtain.

Problems of seeing and [sky determination](#) for determination of SB distribution in center and outer parts of galaxies. Sky problems are worse with small detectors and/or large galaxies.

Since SB is the key observable, there can be strong *selection effects* against low SB objects. If all galaxies had the same SB profile (they don't) then low SB galaxies are *strongly* biased against in either magnitude-limited or size-limited catalogs (apparent "size" of a galaxy will depend on its SB).

Most galaxies are symmetric at a significant level; isophotes are often well represented by ellipses. Elliptical contours fit spirals and ellipticals for different reasons:

- Spiral: viewing angle/disk thickness
- Elliptical: intrinsic ellipticity

([NGC1226](#), [NGC1316](#)).

Some basics of techniques for ellipse fitting: (insert links!)

In reality, galaxies have more complex features, e.g. asymmetries, departures from elliptical isophotes, bars, spiral arms, jets, etc. (see below).

For a purely axisymmetric object, SB distribution reduces to a 1D SB profile. SB profiles are often parameterized, with distributions of the form of a **Sersic Law**⁸:

$$\Sigma(r) = \Sigma_e \exp \left(-b_n \left[\left(\frac{r}{r_e} \right)^{1/n} - 1 \right] \right)$$

where r_e is the half-light radius (the radius that encloses half the total light if the model is extrapolated to infinity), Σ_e is the SB at the *effective* radius ($\Sigma_0 \sim 2000\Sigma_e$), $b_n \approx 2n - 0.324$ and is determined from the definition of r_e , and n depends on the type of galaxy ($n = 4$ for ellipticals and $n = 1$ for disks).

⁸MacArthur et al. (2003)

- Disks are usually well represented *on average* by an exponential:

$$\Sigma(r) = \Sigma_s \exp\left(-\frac{r}{r_s}\right)$$

in flux units, or

$$m(r) = m_s + kr$$

in magnitudes.

- Spherioids are historically characterized by **deVaucouleurs profile**, otherwise known as the “ $r^{1/4}$ ” law:

$$\Sigma(r) = \Sigma_e \exp\left(-7.67 \left[\left(\frac{r}{r_e}\right)^{1/4} - 1\right]\right)$$

in flux units, or

$$m = m_0 + kr^{1/4}$$

in magnitudes.

There are other forms that can be used as well.

Sizes of galaxies

Generally, galaxies do not appear to have sharp edges (although SB can drop sharply in some disk galaxies).

- Isophotal (D_{25} in RC is B=25.0): Beware for low SB galaxies
- Half-light radii (e.g. R_e)
- Petrosian radius: radius at which SB (in an annulus, SDSS used 0.8 - 1.25r) drops to some fraction of light (SDSS uses 0.2) within this radius.

Typical angular sizes: arcminutes for nearby galaxies, arcseconds for distant galaxies. Can get linear sizes with distance estimate (which depends on cosmology). Typical physical size (of the luminous component) is $\sim 1\text{-}30$ kpc.

Spectral Energy Distribution

Spectra are often obtained over some aperture, or long slit. Note increasing use of integral field spectroscopy.

The overall shape and features in SEDs provide important information about the nature of the luminous components, i.e. physical properties of the stellar population, and the gas component, although these can be challenging to sort out, as we'll discuss later.

Spectra also allow the study of velocities via the Doppler shift:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

Both the bulk velocity of the galaxy and the distribution of velocities of the stars within the galaxy can be measured this way. Mean velocities of galaxies indicate that they are receding, with velocity proportional to distance \rightarrow expansion of the universe. There is a dispersion of velocities around the mean relation; the deviation is called the **peculiar velocity**. This is the

movement of the galaxy *through* space, whereas the mean is the velocity *with* space due to expansion. In the field, typical peculiar velocities are a few hundred km s⁻¹. In clusters, they can be a thousand km s⁻¹.

Internal velocities can be organized, e.g. rotation in disks. They can also be random, leading to broader lines that represent the velocity distribution of stars within a galaxy. This broadening would be the same at all points in the galaxy.

$$F(\lambda) \propto \int N(v_{los}) S\left(\lambda - \frac{v_{los}\lambda}{c}\right) dv_{los}$$

where S represents the spectrum of an individual star and N represents the fractional distribution of stars at different line-of-sight velocities. Lines are usually fairly well represented by a Gaussian velocity profile, characterized by the mean velocity:

$$\int v_{los} F(v_{los}) dv_{los}$$

and the velocity dispersion:

$$\int (v_{los} - \bar{v}_{los}) F(v_{los}) dv_{los}$$

e.g., central velocity dispersion or, more generally, the velocity dispersion profile.

Deviations from a Gaussian can be measured, and are usually characterized by higher order moments: *skew* (h_3) and *kurtosis* (h_4):

$$F(v_{los}) \propto \exp \frac{-\omega^2}{2} \left[1 + \sum h_k H_k(\omega) \right]$$

where $\omega \equiv (v_{los} - \bar{v})/\sigma$ and H_k are Gauss-Hermite functions.

Relevance of internal velocities

Probe mass distribution within galaxies (assuming that gravity drives the motions).

For a rotationally supported system,

$$v(r) = \sqrt{\frac{GM(r)}{r}}$$

For a collisionless system of particles, (*independent particles*) the distribution of velocities at any given location characterized by the velocity ellipsoid, and Jeans equation gives:

$$\frac{d\rho\sigma^2}{\rho dr} + \frac{2\beta\sigma^2}{r} = -g(r)$$

where β is the **velocity anisotropy**:

$$\beta \equiv \left(1 - \frac{\sigma_\theta^2 + \sigma_\phi^2}{2\sigma_r^2} \right)$$

Distances to galaxies

Some properties of galaxies, like luminosities and linear sizes, require knowledge of distances to galaxies:

$$m_M = 5 \log d - 5$$

where d is the distance in parsecs, M is the absolute magnitude (or SB), and m is the apparent magnitude.

Various techniques for getting distances:

- Variable stars: Cepheids/RR Lyrae stars through period-luminosity (period-luminosity-color/metallicity) relation.
- Geometric techniques: masers and eclipsing binaries.
- SB fluctuations: requires objects with a “standard” population
- Planetary nebulae luminosity function
- **Scaling relations between velocity and luminosity**
 - Spirals: **Tully-Fisher relation** between maximum rotational velocity and luminosity
 - Ellipticals: fundamental plane relation between velocity dispersion, SB, and physical size ($D_n - \sigma$) relation.
- Redshift

Regarding redshift: Galaxies expand with everything else in the cosmic flow according to $v = Hd$. At low z , $z \approx v/c$. At high z , need knowledge of cosmological parameters to get distance from redshift (i.e. redshift-distance relation⁹) However, galaxies move around relative to the uniform expansion because of local gravitational attraction from concentrations of mass - the deviation of the velocity from the smooth flow is called the peculiar velocity.

Typical peculiar velocities are on the order of several hundred km s^{-1} , but depend on environment; c.f., Milky Way and galaxy clusters; if these are ignored, introduces spurious features in distance maps (“fingers of God”). At $z \geq 0.1$, peculiar velocities lead to relatively small distance errors. At lower redshifts, might estimate distances using model of mass distribution and velocity field, but this is likely not well determined.

Determining spectroscopic redshift for distant (faint) galaxies is observationally time-consuming. Accurate positions of individual lines require reasonable S/N. It is possible to get good redshifts from low S/N observations by taking advantage of multiple spectral features, e.g., by cross-correlation of spectrum against a template in $\log \lambda$ space.

Lower accuracy distances are possible from so-called *photometric redshifts*. **Can't disperse light.** Use low resolution spectral information, i.e. multiband photometry to constrain redshift. Need to separate different intrinsic SEDs of galaxies, internal reddening, and redshift. Accuracy of results obviously depends on quality of photometry and number of bandpasses. Typical accuracies (checked against spectroscopic redshifts) give

$$\frac{\Delta z}{(1+z)} \sim 0.1$$

(sometimes better); note typical $\sim 10\%$ outlier fraction (varies for different types of galaxies).

⁹see, e.g. Hogg, Distance Measures in Cosmology

Integrated brightness

Issue: galaxies don't have edges! Types of mags:

- **Metric mags:** brightness within an aperture of fixed angular or physical size. Be careful if galaxies have a range of sizes.
- **Isophotal mags:** brightness within a specified SB contour, e.g. Holmberg mags, which is the brightness within $\mu_{pg} = 26.5$. Be aware if galaxies have a range of SB profiles.
- **Model mags:** require assumption about outer SB profile
- **Petrosian mags:** mag within some fixed number of Petrosian radii (SDSS uses 2), where the latter is defined as the radius at which SB (in an annulus, SDSS used 0.8-1.25r) drops to some fraction of light (SDSS uses 0.2) within this radius. Advantages with regard to SB (distance) and seeing effects. Note relation of Petrosian mag to model mag depends on SB profile.

Typical integrated brightnesses of galaxies, and star-galaxy separation and crossover in number counts (... yes?)

Other issues in getting luminosities of galaxies:

- Foreground extinction, which depends strongly of Galactic latitude¹⁰
- Internal extinction/inclination, which can be significant (up to 1.5 mag in RC3 prescription)
- If looking at a population, be sure to compare luminosities in the same bandpass. This is an issue if looking over a range of redshifts; a fixed observed wavelength range corresponds to a different rest frame wavelength range at different redshifts. **K-corrections** are used to account for this at some level¹¹. These require knowledge/assumptions about the spectral energy distribution of galaxies. Alternatively, measure at different observed wavelengths.

VI. Statistical properties of galaxies

Consider relative number of galaxies with different parameters, e.g. color or luminosity.

Luminosity Function

The luminosity function (LF) gives the number density of galaxies as a function of luminosity, which can be expressed either in luminosity or absolute magnitude units.

- $\Phi(L)$ = number density of galaxies with luminosity between L and $L + dL$
- $\Phi(M)$ = number density of galaxies with absolute mag between M and $M + dM$

The integral over the LF gives the total number density of galaxies (at all luminosities). The LF is the first step toward understanding a fundamental question of galaxy formation: What sets the *range* of galaxy luminosities and the relative *numbers* of objects at different luminosities (though mass may be a more fundamental characteristic). It is also an important cosmological probe for evolution of the galaxy population (more later).

¹⁰Burstein & Heiles, ApJS 54,33 (1984); Schlegel, Finkbeiner, & Davis, ApJ 500,525 (1998)

¹¹Hogg et al. astro-ph/0210394; Kinney et al. ApJ 467,38 (1996); Coleman, Wu, Weedman, ApJS 43,393 (1980)

Measuring the LF

Obviously, distance measurements are required to get luminosity. Measurement critically depends on knowledge of the **selection function**. For magnitude limited samples, more luminous objects will be overrepresented. This is called the Malmquist bias¹²

The simplest idea for correction is to weigh counts by V_{max} , the largest volume to which the galaxy could have been observed given the selection function. However, the issue here is large-scale structure. $\rightarrow \frac{V}{V_{max}}$ test for uniformity and understanding of selection function¹³.

Also may need to be careful about systematic biases arising from uncertainties in measured magnitudes and distances, if population is not uniform in underlying distribution of absolute mags and distances sampled (which it isn't). The LF rises toward fainter magnitudes, so more galaxies are scattered bright than are scattered faint (WTF??). In distance, more galaxies are located farther than nearer because of volume element (seriously, WTF), so more galaxies scattered into sample (with corresponding error in absolute magnitude) than scattered out of it. Note that many biases are possible aside from a simple magnitude cut, e.g. color bias, SB bias, etc. In general, need to be very careful about understanding selection effects; model (and verify) them.

Observed LF

- Absolute magnitudes (M_V) range from -22 to -8.
- Nomenclature: “dwarf” galaxies vs. “normal” galaxies.
- What objects dominate by number? What objects dominate the total luminosity?
- Different types have clearly different LFs
- Since proportion of a given type depends on environment, so must the general LF.

Functional fits to the observed LF and parameters of the LF

LFs are usually well characterized by a **Schechter Function**

$$\phi(L) = \frac{\phi_*}{L_*} \left(\frac{L}{L_*} \right)^\alpha \exp \left(-\frac{L}{L_*} \right)$$
$$\phi(M) \propto 10^{-0.4(\alpha+1)(M-M^*)} \exp \left(-10^{0.4(M^*-M)} \right)$$

where $\phi(L)$ is the number of galaxies with luminosity between L and $L+dL$. ϕ_* , L_* , and α (the faint-end slope) are the three parameters. The Schechter function is only an approximation, and is purely empirical.

A possibly useful feature of the Schechter function is that it can be integrated to get total luminosity density:

$$j = \phi_* L_* \Gamma(\alpha + 2)$$

Typical “local” parameters (e.g. SDSS at $z \sim 0.1$) are:

¹²Be aware that this name is sometimes used to refer to different things.

¹³Significantly more sophisticated methods have been developed; see, e.g. Binggelli et al (ARAA 26, 26, 1988) for more details

- $\phi^* \sim 0.015 h^3 \text{ Mpc}^{-3}$
- $M_B^* \sim -19.5$
- $M_R^* \sim -20.5$
- $\alpha = -1 \text{ to } -1.5$

As noted above, there is a dependence on type. So what is the implied luminosity density? And (getting ahead of ourselves) the implied stellar mass density, c.f. cosmological.

There has been significant discussion over detailed shape and normalization of LF. There are probably both observational issues (e.g. selection functions) and astrophysical ones (e.g. large scale structure, “cosmic variance”).

Evolution of the LF

LF evolution contains a lot of information about galaxy evolution. Some possibilities:

- No evolution: distribution of galaxy luminosities is unchanging.
- Passive evolution: number and internal (stellar) makeup of galaxies is unchanging (**no star formation**), but luminosity evolves as stars evolve.
- Luminosity evolution: stellar makeup of galaxies changes with time (*star formation*, leading to luminosity changes).
- Number (density) evolution: number of galaxies changes with time. Galaxies may be either created (formed) or destroyed (e.g. mergers) as a function of luminosity.

There is a large amount of work on this, but certainly the LF shows evolution¹⁴. All samples show luminosity evolution, but red galaxies show number density evolution. Schechter parameters (link?)

Colors of galaxies

Consider the distribution of SEDs of galaxies, to first order represented by their color. There is a striking bimodality in color¹⁵:

- Widley used terminology of red and blue sequences (with “green valley” in between).
- Red sequence is tighter than blue sequence, so latter is sometimes called the “blue cloud”.
- Red sequence extends to higher luminosities, blue to lower luminosities, though there is significant overlap.

At some level, this bimodality is nothing more than the bimodality between ellipticals/early-type spirals without much current star formation (red) and later-type spirals with current star formation (blue), although dust, bulges, and metallicity all play a role. The correspondance with morphology is supported by correlation with structural parameters, e.g. multidimensional correlations¹⁶.

The key point is that color is easily observed and quantified. Given likely differences in stellar populations, the relation between luminosity and stellar mass is probably different for the two different sequences. At a rough level, the color allows one to estimate the stellar mass from the luminosity, though some colors certainly have more information about this than others, and

¹⁴Faber et al. (2007)

¹⁵quantified from SDSS by (Strateva et al., AJ 122, 1861 (2001))

¹⁶Blanton and Moustakis ARAA 2009

some bandpasses for luminosity are more affected by differences in the stellar populations (more later).

Note that the relation shifts when expressed in terms of stellar mass¹⁷, and that there appears to be a transition mass around $10^{10} M_{\odot}$ between the two sequences.

VII. Elliptical/Spheroidal galaxies

Elliptical galaxies are not simple collections of stars that are all similar to one another. They exist over a wide range in luminosity and other quantities.

SB profiles

Ellipticals better fit by Sersic profiles than by deVaucouleurs¹⁸.

- Correlation of Sersic indices with other parameters suggests that there is something physical going on, but be aware that there may be fit degeneracies.
- Spiral bulges, low luminosity Es may be better represented by exponentials.
- Slope of SB profile (Sersic n) appears to be correlated with luminosity
- SB vs L “turns over” (?) at intermediate luminosity: two families of spheroidal systems?
→ **Cuspy and Core**
- Ellipticals exist over wide range of sizes, luminosities, and SBs.

Other profiles that have been used:

- King model (truncated Gaussian for velocity distribution)
- Hubble profile:

$$\Sigma(r) = \frac{\Sigma_s}{\left(1 + \frac{r}{r_s}\right)^2}$$

where $\Sigma_s = 0.25\Sigma_o$

None are perfect matches to data over all scales¹⁹.

Inner regions of spheroidals deviate from Sersic profiles fit to outer regions, (though outer regions can also deviate from Sersic profiles, especially notable in central cluster (cD) galaxies). Inner regions are of particular interest because they may reflect dissipational collapse of low angular momentum material²⁰. Initially, it seemed like galaxies had cores, but this is partly an effect of seeing. Higher spatial resolution (HST) shows that galaxies have both flat and steep (cuspy) inner profiles²¹. Inner profiles seem to be roughly bimodal (though this bimodality is currently under debate).

Parametric fits to account for these become more complex, e.g. the ‘nuker law’:

$$I(r) = I_b 2^{(\beta-\gamma)/\alpha} \left(\frac{r_b}{r}\right)^{\gamma} \left[1 + \left(\frac{r_b}{r}\right)^{\alpha}\right]^{(\gamma-\beta)/\alpha}$$

¹⁷Baldry et al. 2006

¹⁸Caon et al. 1993

¹⁹see Burkert 1993 for comparison with deVaucouleurs law

²⁰See, e.g. NGC 4472

²¹e.g. NGC720 vs. NGC4621

where γ is the slope of the inner power law, β is the slope of the outer power law, and α is the sharpness of the break between them.

Profile type is correlated with luminosity. Luminous ellipticals tend to have “cuspy cores” with a break radius and shallower central density profile. Lower luminosity ellipticals have power laws all the way in.

Galaxies with cusps may have “extra light”, perhaps related to an accretion event; note that at least some of these show distinct kinematic signature in their cores.

Intrinsic (3D) shapes of ellipticals

- oblate (2 long axes)
- prolate (1 long axis)
- triaxial (all axes different length).

True shapes are determined by looking at distribution of ellipticities

- Distribution function is different for fainter and brighter Es.
- For bright giant Es, distribution is *inconsistent* with either prolate or oblate intrinsic shapes: not enough circular galaxies²².
- For fainter Es, distribution is *consistent* with oblate, prolate, or triaxial.
- Triaxiality is also inferred for some giant Es from observation of isophotal twisting, which you can't get from oblate or prolate shape²³.

Non-axisymmetric features in galaxies

Often described by amplitudes of Fourier moments of intensity distribution as a function of radius, e.g. $\alpha_1, \alpha_2, \alpha_4$

$$I(r, \theta) = \sum c_m \cos(m\theta) + \sum s_m \sin(m\theta)$$

$$\alpha_m = \frac{\sqrt{c_m^2 + s_m^2}}{I_o}$$

“First even term above ellipses is α_4 term:” (WTF???)

- “boxy” isophotes: $\alpha_4 < 0$; bright, slow, central cores, strong radio and x-ray.
- “disky” isophotes: $\alpha_4 > 0$; faint, significant rotation-flattening, little radio or x-ray emission, steep cusp.

Deviations from elliptical fits are seen in elliptical/spheroidal systems, e.g. NGC 821 v. NGC 2300. It is possible that isophotal deviations form a continuous sequence²⁴:

$$S \rightarrow S0 \rightarrow \text{disky Es} \rightarrow \text{true Es} \rightarrow \text{boxy Es}.$$

It is also possible that true Es are just diskly Es viewed edge-on²⁵, and that there are two distinct classes of Es. Some Es also show non-symmetric deviations from smooth SB distributions, e.g. ripples, shells, and streams, especially in outer regions.

²²Tremblay and Merritt Fig 3

²³de Zeeuw, Fig 1

²⁴e.g. Kormendy and Bender classification

²⁵Kormendy and Bender, figure 3

Kinematics

Elliptical galaxies are kinematically *hot*. The random motions of stars is generally large compared to the organized motion (but this is not to say that there is *no* rotation).

The key kinematic quantity is **velocity dispersion**, σ . Galaxies can be characterized by *central* σ , but σ does vary with radius.

Some ellipticals do have some rotation, mostly in lower luminosity systems. The relative importance of *organized* motion over *random* motion can be characterized by v_{rot}/σ . Shapes are expected to be influenced by rotation: for an oblate model with isotropic velocity distribution that is flattened by rotation:

$$\frac{v_{rot}}{\sigma} = \sqrt{\frac{\epsilon}{(1 - \epsilon)}}$$

Giant ellipticals have less rotation than this, implying anisotropic velocity dispersions to account for their shape (required for triaxial systems in any case). Low/medium luminosity (high SB) Es, however, *may* be isotropic with flattening caused by rotation²⁶. Low SB Es appear to have anisotropic σ , i.e. more eccentric than expected from rotation; “measured in LG Es 185 (factor of three low in v_{rot}/σ , 147, factor of 10 low).” *seriously, wtf*. Significant fraction of Es may have dynamical subcomponents, e.g. velocity and dispersion for some interesting cases.

Relations between different parameters and different families

Photometric parameters

SB-size relation, between SB and SB-luminosity relations (“Kormendy relations”)???. Also, profile *shape* (e.g. Sérsic index) with luminosity.

Kinematics vs. luminosity

More luminous galaxies have less rotation, but higher velocity dispersions (**Faber-Jackson relation**, which is roughly $L \sim \sigma^4$, but lots of scatter).

The Fundamental Plane

Faber-Jackson relations and the Kormendy relation (between SB and L/size) are manifestations of the **Fundamental Plane of elliptical galaxies**; there is a correlation between residuals in the Faber-Jackson relation and SB. Galaxies do not populate the entire 3D space of I , r , and σ , but instead populate only a *plane* in this space. There is a relation between the **three fundamental global observables**: SB (or luminosity), size, and velocity dispersion²⁷. The observed relation given by Virgo ellipticals is:

$$r_e \propto (\sigma_0^2)^{0.7} I_c^{-0.85}$$

What is the origin of relation? Three assumptions would have to be true:

²⁶de Zeeuw, figure 3

²⁷original refs: Dressler et al. 1987; Djorgovski and Davis 1987; Bender, Burstein and Faber 199x

1. Es are in virial equilibrium
2. M/L varies systematically with luminosity
3. Es form a “homologous” family, all, e.g. with deVaucouleurs profiles.

In this case, one expects:

$$L = c_1 I_c r_c^2$$

$$M = c_2 \frac{\sigma_o^2 r_e}{G}$$

where the first is a definition and the second is related to the **virial theorem** (though this applies to the mean potential energy and mean kinetic energy per unit mass, averaged over the entire system). The constants are related to the shapes and other details (not necessarily constant among the variety of Es).

Combining these, we get

$$r_e = \frac{c_2}{c_1} \left(\frac{M}{L} \right)^{-1} \sigma_o^2 I_c^{-1}$$

If $M/L \propto L^{0.2}$, then the observed fundamental plane is recovered; could arise from stellar populations or from variations in baryon to total mass. Alternatively, if M/L is constant with a structure that varies relative to one or more of the fundamental variables (which we know it does, e.g., systematic variations of Sersic n with luminosity, but not clear if this is entire “explanation”).

However, we still need to understand origin of assumptions; why should parameters, e.g. mass-to-light vary smoothly with luminosity? Recall that this ratio includes dark matter. If ellipticals have dark matter halos, the luminous inner parts are not required to be in virial equilibrium.

There is relatively little scatter around the fundamental plane, implying that the assumptions are reasonably valid over a large range of elliptical properties, which implies some significant regularities in the galaxy formation process.

Galaxies do not fully populate the entire plane defined by our relation. Consequently, when the plane is projected onto the other two axes, there is a correlation²⁸.

Luminosity (size) — σ plane: Luminosity is proportional to velocity dispersion as $L \propto \sigma^4$, which is known as the **Faber-Jackson relation**. However, since the locus of Es isn't perfectly linear and the plane defined by the ellipticals isn't perpendicular to this dimension, the scatter around the Faber-Jackson relation is larger than the scatter around the fundamental plane. A new radius can be defined that incorporates SB such that the new radius vs. σ views the fundamental plane edge-on. Such a size measurement is called D_n , the isophotal *diameter* of the B=20.75 isophote. This $D_N - \sigma$ relation provides a very useful distance estimator (if the fundamental plane really is fundamental).

SB — size plane: Relation for *smaller* galaxies:

- *higher* SB for *normal* small Es
- *lower* SB for *diffuse* small Es

These are sometimes known as the **Kormendy relations** and are one of the main bases for separating these two types of objects.

²⁸Djorgovski figure 2

SB — σ plane: Presumably related to two underlying physical parameters:

1. density
2. virial temperature

In this plane, one can only form galaxies where *cooling is effective*, i.e. at larger densities and hotter temperatures. This restricts the area in the space in which we can find galaxies.

Additional features of the galaxy formation process may introduce additional restrictions into allowed locations of galaxies on the fundamental plane. Most luminous ellipticals are located along one line (with some scatter) in the fundamental plane, and most diffuse ellipticals are located along another.

Isophotal deviations vs. luminosity and kinematics

Deviations from elliptical shape is correlated with dynamics, such that slower rotators are more likely to be “boxy”, and faster rotators are more likely to be “disky”²⁹.

There appears to be two kinds of “boxiness”, one found in giant Es and one found in bulges, which in fact are relatively rapid rotators. It is likely that these come from different origins: plausibly, mergers in the case of giant Es and bars/disk evolution in the case of bulges.

The core properties are correlated with the global shapes and dynamics: cuspy core galaxies are boxy and anisotropic, cuspy galaxies are disk and rotating.

Several types of ellipticals??

Normal and low luminosity Es:

- significant rotation
- nearly isotropic
- oblate spheroids
- cusps (no cores)
- disk isophotes

Giant Es:

- non-rotating
- anisotropic (triaxial)
- less flattened
- cuspy cores
- boxy isophotes

Dwarf spheroidals (diffuse ellipticals?):

- lower SB
- off the fundamental plane
- anisotropic (slow rotators for their ellipticity)

Bulges: multiple types? As an aside: globular clusters, which are likely to be significantly different.

²⁹Kormendy and Bender, figure 2

Spectral energy distributions

Note typical features in ellipticals: stellar absorption lines (4000Å break, Mg, Fe, etc.) Es are generally red: implications are predominantly old population. Relations:

- Color-luminosity: more luminous Es are redder. However, as we'll see, variations of color at this level can come from either metallicity or age, so interpretation isn't trivial.
- Mg line strength-luminosity: more luminous Es have stronger lines. This correlation is even tighter when considering relation between central velocity dispersion and line strength (The Mg- σ relation), over large range of σ . As we'll see, this variation could arise from variations in age or metallicity, or different heavy element abundance ratios. Of equal interest to the Mg- σ relation is the quite small scatter around the relation.

Some Es have signatures of a younger population: E+A (or k+a) galaxies: galaxies with strong Balmer lines. These are relatively rare, but likely are indicative of post-starburst galaxies with significant star formation 1 Gyr ago. These are not associated with a cluster environment. Some show morphological signs of recent interactions.

Stellar populations within ellipticals: Es have gradients in color/line strength, i.e. tend to be redder toward the center (but again, there is some degeneracy between age and metallicity as possible reasons for this).

Interstellar matter in ellipticals

It is now known that Es have significant interstellar gas, seen not in optical, but in X-ray emission \rightarrow hot gas. This makes sense since some gas is expected from stellar evolution, although there is also the possibility of gas from the environment.

Some Es also show evidence of colder gas and dust. A surprisingly large fraction ($\sim 50\%$) show some evidence for dust their cores³⁰ These are typically small components by mass.

VIII. Spiral/Disk galaxies

SB profiles

Size-L/SB-L relations

Vertical distributions in disk galaxies

Non-axisymmetric features in disk galaxies: bars and spiral arms

³⁰Lauer et al. 2005

Kinematics

spirals are kinematically cold, meaning the random motion of stars is small compared to the organized motion (rotation), though there is some dispersion of velocities.

Tully-Fisher relation

: kinematics(maximum rotational velocity)-luminosity relation.

Typical spectra

Gas and dust

IX. Galaxy parameter (scaling) relations

Structural luminosity-SB

- isophotal shape-luminosity relation for Es (boxy vs. disk).

Kinematic Faber-Jackson, Tully-Fisher

- ellipticals luminosity-rotation relation. The Faber-Jackson relation also applies to ellipticals, but this is the relation between luminosity and velocity dispersion, not rotational velocity...?

structural/kinematic Fundamental Plane

Stellar populations

Gas

Black holes

X. Environments of galaxies: clusters and cluster galaxies

- Galaxies are not homogeneously distributed in space... Correlation function
- Groups and Clusters
- Abell cluster
- Galaxies in galaxy clusters (Boselli and Gaazzi, PASP 118, 517, 2006)
- Distributed hot gas in clusters, with mass comparable to that found in galaxies (one individual galaxy?)
 - X-ray observations
 - temperatures are $10^7 - 10^8$ K
 - intracluster gas is enriched in heavy elements
 - X-ray observations useful for probing cluster masses, under assumption of hydrostatic equilibrium.
 - * Need measurement of density and temperature profiles of X-ray gas
 - * Estimates can be made from X-ray luminosity and/or temperature (better): the $M - kT$ relation

* typical masses are $10^{14} - 10^{15} M_{\odot}$; a typical cluster mass function (Wen et al. 2010).

- Shapes of clusters, inhomogeneities: not all clusters are in equilibrium

XI. Some galaxies to be familiar with

- stuff

XII. The building blocks of galaxies

Stars and Stellar populations

Relating observables to intrinsic population characteristics

- **Population characteristics:** distribution of masses, compositions and ages (star formation history).
- **Observe spectral energy distributions (or colors) of stars:** individual stars in very nearby galaxies, integrated starlight for most galaxies.

Stellar evolution

- Internal structure of stars: determined by mass, chemical composition, and age (Russell-Vogt theorem). Excludes non-spherical symmetric effects, e.g. rotation, magnetic fields, binarity, etc.
- Luminosity (radius) and effective temperature derived from equations of stellar structure: mass conservation, hydrostatic equilibrium, energy equation, energy transport, along with auxiliary relations: equation of state, opacity, nuclear reaction rates.
- Main stages of stellar evolution:
 - Hydrogen core burning: main sequence (MS)
 - Hydrogen shell burning: giant branch (for lower mass stars)
 - Helium core burning: horizontal branch, red clump, blue core helium burners. Note key transition around 2 solar masses (depends on metallicity): shift to helium flash at lower masses.
 - Helium shell burning: Asymptotic Giant Branch (AGB)
 - Other nuclear burning for high mass stars
 - White dwarf or supernova
- Model tracks (evolution as a function of time for a given mass); for spherical symmetry, calculations are 1D.
- **Isochrones:** cross-section of properties at a fixed time across a range of masses. Some well-known groups that calculate evolutionary tracks/isochrones:
 - Padova
 - BASTI (Teramo)
 - Dartmouth
 - Yale-Yonsei

- Victoria-Regina
- Geneva
- Uncertainties that lead to some differences between different calculated isochrones: convective overshoot, diffusion, convection, helium abundance, mass loss, etc. Generally, Uncertainties are larger for later stages of evolution. Additionally, there may be missing physics, e.g. rotation and magnetic fields, that would require a full 3D treatment.
- Given effective temperature, surface gravity (from mass and radius), and composition, stellar atmospheres give observables: spectral energy distribution/colors. Some model atmospheres (need links!):
 - [Kurucz](#)
 - [MARCS](#)
- Theoretical Color-Magnitude Diagrams (CMDs) → observed; need distance and reddening/extinction
- Age effects (model isochrones) from Yi et al 2001.
- Metallicity effects: internal (opacity) and atmosphere (line blanketing) effect combine in the same direction to make more metal-rich populations redder: [CMDs](#)
 - Metallicity terminology: often given as mass fractions of hydrogen (X), helium (Y), and heavier elements (Z).
 - solar abundance: X=0.7, Y=0.28, Z=0.019 (roughly)
 - also given by

$$\left[\frac{\text{Fe}}{\text{H}} \right] = \log \left[\frac{(\text{Fe}/\text{H})}{(\text{Fe}/\text{H})_{\odot}} \right]$$
 - Be aware that this is an oversimplification, as Z contains lots of different elements. More later.
- Main sequence (MS)
 - Very rough scaling relation between luminosity and mass: $L \propto M^{3.5}$
 - Main sequence shifts with metallicity: redder for higher metallicity
 - Location also depends on helium abundance
- Red Giant Branch (RGB)
 - note that because of more rapid evolution after MS, RGB stars all have roughly the same mass.
 - temperature of RGB depends on age/mass: younger and more massive stars are hotter. Temperature also depends on metallicity: latter is dominant effect in older populations (greater than 5 Gyr).
 - At lower masses/larger ages, tip of RGB is close to constant bolometric luminosity regardless of age or metallicity. In observed plane, leads to roughly fixed tip luminosity *if* observing at long wavelengths (e.g. I band): basis for Tip of Red Giant Branch (TRGB) distance indicator.
 - RGB bump ([RGBB](#) from [Bono et al 2001](#)), location of which depends on mass/age and metallicity; arises when H burning shell crosses chemical discontinuity.
- Horizontal Branch (HB), aka. Red Clump (RC) or more generally He core burning sequence
 - High mass stars form blue He core-burning branch (also note red plume)
 - Intermediate mass stars form red clump.
 - Low mass stars form horizontal branch. Variable mass loss on RGB and at He flash gives a range of masses.
 - Horizontal branch morphology depends on metallicity: more metal-poor populations have bluer HB.
 - However, there is something else that also affects HB morphology, leading to the so-

called second-parameter problem (e.g. [M3/M13](#), from [Rey et al 2001](#)). Possibilities: Age? He abundance? Heavy element abundances? Density? Rotation?

- RR Lyrae stars: in instability strip (caused by doubly ionized He, also includes Cepheids, delta Scuti stars, etc.) RR Lyrae stars are indicators of old metal-poor population. Periods of 0.5 days, but multiple groups (Oosterhoff classes) depending on stellar parameters...
- Asymptotic Giant Branch (AGB)
 - For intermediate masses/ages, AGB is significantly more luminous than the TRGB, hence of potential critical importance to studies of integrated light.
 - For lower masses (older populations), AGB tip comparable to TRGB and AGB asymptotically approaches RGB (hence its name).
- Potential importance of binaries/interactions
 - unresolved (but otherwise non-interacting) binaries: broaden sequences, depending on mass ratios: equal masses give the appearance of a star 0.75 mag brighter.
 - interacting binary stars
 - blue stragglers: possible stellar merger/interaction products? [M3 example](#) from Sandage 1953.
 - supernovae type SNIa: arise from binaries, produce different heavy element abundances than core collapse SNe. (e?)
- End stages of stellar evolution: WDs, neutron stars, black holes, and supernovae
 - [progenitor-final mass relation](#) (from Binney and Merrifield)
 - [White dwarf cooling sequence](#) (from [Hansen et al 2007](#): potential for age dating).
 - Supernovae: generate significant fraction of heavy elements (but not all). Significant energy input, thermal and mechanical.
- Galactic globular clusters: cornerstone of understanding stellar evolution historically, as apparent examples of a “simple stellar population (SSP)”, with all stars of the same age and abundance.
 - However, it's not recognized that not all GCs are so simple.
 - Some CMDs show clear evidence of multiple components, e.g. [NGC2808](#).
 - Long history of evidence of abundance variations between different stars, typified by the Na-O anticorrelation; demonstrated through observations of main sequence stars that this is not a mixing effect.
 - Two phenomena have recently been coupled, but not yet a clear understanding of how the multiple populations arise.

More information in a CMD than is represented by isochrones

one can also consider relative numbers of stars at different locations: CMDs that incorporate number of stars are referred to as Hess diagrams (e.g. [Fornax](#) from Battaglia et al., 2006).

- For a simple stellar population (SSP, a population with a single age and metallicity), the relative number of stars at each stage is determined by the initial mass function (IMF) and the age.
- IMF determinations and parameterizations:
 - “classical” Salpeter IMF (power law with $dN/dM \propto M^{-2.35}$) and [others](#) (from [Pagel](#)). Note power law form: $dN/dM \propto M^\alpha$, or alternatively, $dN/d\log M \propto M^\Gamma \propto M^{\alpha+1}$
 - Widely used determination of local IMF is by [Drouha Tout, and Gilmore 1993](#), who

find

$$dN/dM \propto M^{-2.7} \text{ for } M > 1M_{\odot}$$

$$dN/dM \propto M^{-2.2} \text{ for } 0.5 < M < 1M_{\odot}$$

$$dN/dM \propto M^{-1.3} \text{ for } M < 0.5M_{\odot}$$

- Chabrier IMF: log-normal form, $dN/dM \propto \exp(\log M - \log M_o)^2$
- Empirically, no strong evidence for variations of the IMF as functions of, e.g., metallicity.
- No well-established theory for predicting the IMF

Star Formation Histories (SFHs) from resolved stellar populations

- In general, galaxies are not SSPs, i.e. their CMDs don't look like those of a cluster. Can fit Hess diagrams of resolved stellar populations with combinations of SSPs to derive constraints on SFH, e.g. [simulated galaxies](#) (from [Tolstoy et al](#)).
- Generally speaking, we want the star formation history: $SFH(t, Z, M)$ gives the number of stars (or stellar mass) at all combinations of age, metallicity, and mass; because of lack of observed IMF variation, usually IMF is separated out:

$$SFH(t, Z, M) = \xi(M)\psi(t, Z)$$

where ξ is the IMF.

- MW neighbors resolved down to oldest MS turnoff ($M \sim 5$), because typical distances give distance moduli $\lesssim 20$.
- M31 and neighbors somewhat shallower ($m - M \sim 24.5$) without large investment of telescope time.
- Results for MW:
 - [solar neighborhood Hipparcos sample](#): roughly constant [SFH](#), but be aware of dynamical effects and limited volume bias nearby sample to younger populations. When corrected for, star formation history in solar neighborhood likely to have significantly declined (see, e.g. [Aumer & Binney 2009](#)).
 - Age of oldest stars can be studied by WD sequence, gives $t_{\text{oldest}} > 8$ Gyr. Also note presence of disk RR Lyrae stars ($t > 10$ Gyr??)
 - Might consider studying age distribution of clusters, as they are simpler populations, but note problems with disruption of clusters that would lead to a bias to younger clusters.
 - Bulge can also be studied, but note foreground population and extinction issues. Predominantly old population, e.g. [bulge CMD](#), and [luminosity function](#) (from [Ortolani et al 1995](#)).
 - Halo (as defined kinematically/spatially) also predominantly old, both from field population and from globular cluster population.
 - Historically, distinction between disk stars/open clusters (population I) and halo stars/globular clusters (population II), with pop I being younger and more metal rich. Note, however, the pop II association with low metallicity is now recognized not to be fundamental; inner halo/bulge significantly more metal rich.
 - For metallicities, note that there is a significant puzzle in that there is *not* a strong age-metallicity relation in the solar neighborhood (e.g. Orion has roughly solar metallicity, but is nearly 5 billion years younger): radial migration, inhomogeneous ISM, inflow ...?

- Results for Local Group (LG) dwarf galaxies
 - Carina dSph: striking evidence of episodic star formation, but this is NOT characteristic.
 - Others (from Tolstoy et al); note range of SFHs.
 - LG dlrrs compilation (from Dolphin et al 2005)
 - LG dSphs (from Dolphin et al 2005); possible similarity to dlrrs apart from lack of recent SF?
 - Many dwarfs show population gradients, e.g. from RGB/HB morphology (e.g., Harbeck et al 2001), likely metallicity gradients: some show evidence of age gradients, with younger population in center.
- Results for M33 (from Holtzman et al 2011); note significant age gradient, and significant component of younger stars.
- M31 halo (from Brown et al 2006); significantly different from MW bulge, with more of an intermediate-old population?
- Can do more distant galaxies if information from advanced stages (RC, HB) is reliable; (e.g. local volume SFH (from Williams et al, in prep), dwarf SFH (from Weisz et al 2011)).
- Even farther if info from AGB is reliable.
- General conclusions:
 - galaxies are not SSPs.
 - galaxies have gradients in their stellar populations
 - significant population of old stars in nearby galaxies?
 - more massive galaxies formed stars earlier??

Integrated light from unresolved stellar populations

- What stars contribute the most light?
 - Along the MS, use combination of M-L relation and IMF:

$$L \propto (M^{-2.35} M^{3.5}) \propto M^{1.15}$$

Massive stars dominate light.

- Compare MSTO with evolved population: RGB has almost same mass as MSTO, but significantly more luminous: relative contributions depend on relative number of stars along the giant branch compared with MSTO stars, but one finds that the luminous evolved populations dominate (Renzini fig 1.5).
- Since post-MS evolution is fast compared to MS evolution for all masses, it is true that at any given time when the most luminous stars are the most evolved stars, the luminosity is given predominantly by stars of (nearly) a single mass (true for all except youngest ages, see Renzini 1.1) and luminosity vs. lifetime plots.
- Integrated brightness: do SSPs get brighter or fainter as they age?
 - At younger ages, have MS “peel-off” effect, but also supergiants! (WTF...)
 - At older ages, have competing effects of IMF and rate of evolution.
 - * To determine the luminosity of a stellar population, consider the number of stars evolving off the MS, or equivalently, dying, which is given by the *evolutionary flux*:

$$b(t) = \psi(M_{\text{TO}}) |\dot{M}| \text{ [stars yr}^{-1}\text{]}$$

where TO stands for turnoff, ψ is the IMF, and $|\dot{M}|$ is the time derivative of the turnoff mass, e.g. Renzini 1.1. The key point is that lower mass stars evolve

more slowly.

- * The IMF is important, along with age, in determining the luminosity evolution of a galaxy.
- * Once one reaches an age where most massive stars have died, all stars of lower mass reach comparable luminosity (tip of RGB), hence luminosity of population depends on number of red giants.
- * For a typical IMF number of stars increases slower toward lower mass than rate of generation of turnoff stars decreases.
- * Luminosity of the turnoff also has some smaller effect.
- Do integrated colors depend on the IMF?
 - * Need to know the relative contributions of each stage of evolution, i.e., from stars of different masses.
 - * Number of stars in each post-MS stage is determined predominately by the time spend in each stage:

$$N_j = b(t)t_j$$

because all post-MS stars have nearly the same mass. So although the total flux is sensitive to the IMF, the relative contributions of each stage are not, at least for older populations (Renzini fig 1.5).

- * For all except the youngest population, the later stages of evolution provide a majority of the light. Evolved stages are nearly all the same mass.
- * Consequently, the integrated spectrum and the relative contributions of various evolutionary stages are nearly independent of the IMF. They *do* depend on the age and metallicity.
- Dependence of color and luminosity on time (from Bruzual and Charlot 199?) for a SSP (single burst) normalized to one solar mass.
 - * luminosity variation is a combination of IMF plus rate of evolution for lower masses.
 - * color evolution comes from changing mix of stellar population, but this is just for a single (solar) metallicity.
 - * note RSG phase, then subsequent dimming and reddening.
 - * note this is bandpass dependent, with less dimming (at older ages where the RGB is dominant) at longer wavelengths.
- Implication: if range of ages is present, integrated light is (significantly) weighted toward younger populations.
- What stars contribute the most *mass*?
 - For all measured
 - relative stellar M/L ratios
- Can you estimate stellar mass from integrated light?
 - Consider the
 - For individual stars
 - Absolute value of
 - stellar M/L ratio depends on
 - So, it is possible
 - * Variety of different
 - * Some uncertainties
 - * Larger
 - * if IMF were to be variable
- Star formation histories from integrated colors

- Issue: integrated colors are affected by combination of age distribution, metallicity distribution, and reddening distribution.
- color variations with age:
 - situation somewhat improved
 - Even if age-metallicity
- Given age-metallicity degeneracy from colors,
- Spectral evolution (BC fig 4) for a variety of SFRs
- Dependence of contributor on wavelength
- Potential problems with synthetic integrated spectra:
- What about spectral features?
 - For populations
 - For older populations
 - typical optical spectra
- Even with
- Main area of application

Chemical evolution

- Understanding
- Start with some initial conditions, then add star formation history: (IMF + SFR). This results in chemical enrichment for stars as a function of mass and metallicity. Also consider input and output of gas from any particular region, with either primordial or modified composition.
 - ICs: No heavy elements and pure gas (no stars).
 - SFH: birthrate function $\psi(m, t)$ gives number of stars formed with mass m per unit volume, usually split into separable function:

$$B(m, t) = \xi(M)\psi(t)$$

where $\xi(M)$ is the IMF and $\psi(t)$ is the SFR. This implicitly assumes an IMF which is constant in time, which may or may not be true, but so far, there is no strong observational evidence against it (although note pop III issues).

- For any particular star, use stellar evolution to compute the amount and composition of mass returned to the ISM vs. amount locked up in stellar remnants (Pagel fig 2, from Maeder 1992). Note that the *rate* of return depends on stellar mass, especially for the case of SNIa, which also depends on the binary fraction.
- nucleosynthesis: main element groups and their sources:
 - * light elements: Big Bang Nucleosynthesis (BBN) plus subsequent destruction (?)
 - * α elements (even Z from O up...??): massive stars (core collapse SN)
 - * Fe-peak elements: type Ia SN and core collapse SN
 - * s and r process (neutron capture, plus beta decay) elements: core collapse SN, AGB stars... (need neutron capture diagram here)
- Note that infall might arise from primordial clouds, or from processed material, e.g. mass-loss from halo stars. Outflow might come from SN winds, and in this case, it's possible that the composition of outflow material might be more enriched than the typical composition at any given time. So things can get complicated.
- Basic equations of chemical evolution

- Allowing for
- Consider a total (galaxy) mass M , split into a gas mass g and stellar mass s . For a simple model, consider the gas at any time to be well mixed. At any given time, we wish to know the fraction abundance (Z_i) of element i . Consider an inflow rate into the system, F and outflow (ejection) rate, E . Then we have:

$$M = g + s$$

$$\frac{dM}{dt} = F - E$$

$$\frac{dg}{dt} = F - E + e - \psi' g$$

where ψ' is the SFR in units of mass per time,

$$\psi' = \psi \int \xi dm$$

(so ξ is normalized to a total of one), and e is the ejection rate of mass from stars,

$$e = \int_{m_t}^{m_U} (m - m_{\text{rem}}) \psi(t - \tau(m)) \xi(m) dm$$

which is a sum over all stellar masses of the product of the SFR at the time of formation of each mass with the mass returned to the ISM, weighted by the IMF. The lower limit of integration is at the stellar mass which is dying at time t ; lower mass stars don't contribute because they haven't ejected any mass yet. We also have

$$\frac{ds}{dt} = \psi' - e$$

since the mass in stars increases by the number of stars formed, but decreases by the amount of mass lost back to the ISM from the previous generation of stars. For the elements, we have

$$\frac{d(gZ_i)}{dt} = e_Z(i) - Z_i \psi + Z_F F - Z_E E$$

(Z_i is the mass fraction of element i ; hereafter we'll drop the subscript for simplicity).

The mass in each element *increases* by:

- * amount of mass released by previous generations
- * amount of mass added by inflow

but *decreases* by:

- * amount of mass locked up in new stars
- * amount of mass lost to outflow

The term e_Z is given by:

$$\psi(t - \tau(m)) \xi(m) dm$$

where q_Z represents the fractional mass of element Z synthesized and ejected during stellar evolution (so left term in brackets gives material which returns unprocessed and right term gives newly synthesized contribution). In a simple model, the synthesized masses are independent of the metallicity of the population (although we know this is not true for some elements, more on this later).

- To simplify, consider an approximation to these formulae called the *instantaneous recycling approximation* which assumes that all elements are returned instantaneously - good for products of massive stars, but less good for products from lower mass stars, e.g. iron. Then we have

$$e = \psi \int (m - m_{\text{rem}}) \xi(m) dm \equiv (1 - \alpha) \psi$$

where α is the *lock-up fraction*: the fraction of mass “locked up” in stars and remnants.

...

...

and finally,

$$g \frac{dZ}{ds} = \frac{d(gZ)}{ds} - Z \frac{dg}{ds} = p + (Z_F - Z) \frac{F}{\alpha \psi}$$

This is a basic equation of chemical synthesis.

- The *simplest* model for chemical evolution assumes no inflow or outflow, a homogeneous system without any spatial differentiation of metallicity, zero initial metallicity, and yields which are independent of composition. This is known as the Simple, one-zone model.

In the simple model, we have

$$\begin{aligned} \frac{dg}{dt} &= -\frac{ds}{dt} \\ g \frac{dZ}{ds} &= -g \frac{dZ}{dg} = p \end{aligned}$$

Solving for $Z(g)$, we get

$$Z$$

Abundances as function of time

Relative abundances of different elements

Gas

Dust

Central black holes

Galaxy spectral energy distributions

Dark matter and galaxy masses