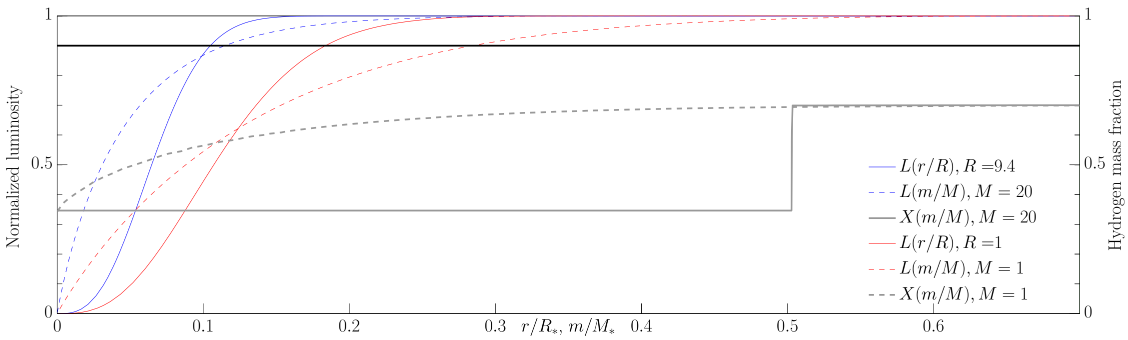

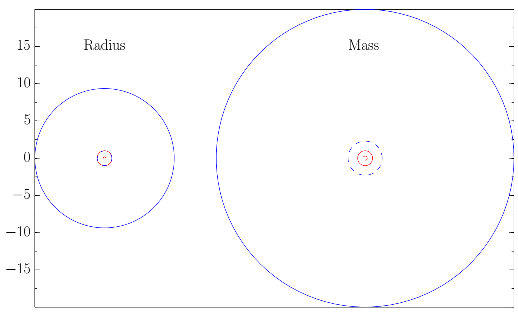

Figure 4.6: Comparison of core sizes for 2 models ($1M_\odot$ and $20M_\odot$) both with $X_c \approx 0.35$ on the main sequence. The top panel shows the normalized luminosity as a function of normalized radius and mass. A horizontal line at $0.9L_{\max}$ will be used to define the approximate core boundary. The blue lines are for the massive model, and red for the less massive one. The low-mass model has a core that is fractionally larger than the core in the high-mass model. The gray solid and dashed lines are the hydrogen mass fraction, given on the right y axis. In the bottom panel, the surface (solid lines) and core (dashed lines) boundaries are shown to scale in absolute masses and radii. On the left for the $9.4R_\odot$ model (blue), we see the core boundary in this space is almost exactly the same size of the entire low-mass star (red). In terms of mass, the core size of the massive star is larger than one solar mass.

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4.3.2 High-mass stars

- The main difference in these stars is the increased temperature in the core, as in Figure 4.7.
- Thus, the CNO cycle is the dominant luminosity source.
- This has the effect of concentrating the the luminosity production in the inner 10% of the mass for a $10M_\odot$ star, compared to about 70% for a $1M_\odot$ star.
- The other effect is a steep temperature gradient in the inner regions due to the high flux. Thus, convection kicks in.
- This region becomes fully mixed chemically, as in Figure 4.4, right.
- The outer regions are radiative, since the ionization regions are very far out in the atmopshere compared to low-mass stars.
- As evolution occurs, the star gets brighter becuase of the strong dependence of L on μ, and μ.
- At the same time, the effective temperature shows a monatonic decrease, as in Figure 4.3, right. This is due to the increasing radius, which increases faster than the luminosity (especially compared to lower-mass stars).

- If the core of the star grows in size due to convective overshoot, it will also extend the MS lifetime and make the star brighter.
- Figure 4.4 (right) shows the core hydrogen mass, and note the shrinking convective core of the higher-mass star over time.
- One of the main reasons for this is the reduced opacity as H is converted into He and the electron scattering processes decreases.
- Also note in the higher-mass star in Figure 4.4 that since hydrogen burning is negligible at the edge of the convective core during the main-sequence phase, the hydrogen profile established during this phase reflects the decrease in the extent of the core. In contrast, the last model is in the hydrogen shell-burning phase, the helium core having grown substantially beyond the smallest extent of the convective core.
- Figure 4.6 shows a comparison of core mass size for a $1M_\odot$ and $20M_\odot$ model in both relative and absolute visualizations.
- Note in this figure that even though the luminosity saturates at a high value very close to the center, the convective region of the core is quite extensive.
- The size of the convective core increases as the mass of the star increases too, due to the higher central temperatures.
- We can estimate the main-sequence lifetime of a star. If $\varepsilon_{\rm H}$ is the energy per unit mass per unit time of hydrogen burning, we know that
$$\tau_{\rm MS} \propto \frac{q_c \varepsilon_{\rm H} M}{L}, \tag{4.23}$$
where q_c is some fraction of the stellar hydrogren mass that actually participates in nuclear burning.
- If $\varepsilon_{\rm H}$ and q_c are roughly independent of total stellar mass, and we assume that $L \propto M^\gamma$ as we showed before, then
$$\tau_{\rm MS} \propto M^{-(\gamma-1)}, \tag{4.24}$$
where we found $\gamma \approx 3-5$, and the relation is written to emphasize that the exponent is always negative, and main-sequence lifetime is inversely related to mass.

4.3.3 A note about very low mass stars

- Stars below about $0.3M_\odot$ are fully convective on the MS.
- They have large opacities due to low temperatures and very high densities.
- The densities are high because the stars need to contract to build up high enough temperatures for nuclear fusion.
- Only the PP-I chain can operate, so helium-3 is never really destroyed.
- But at the lower mass limit (high ρ), electron degeneracy kicks in.
- Conduction is very efficient here, which then cools the core below the minimum ignition temperatures.
- Some lithium or deuterium is burned, but these objects become brown dwarfs and cool down like white dwarfs.
- The difference between them and WDs is that they are fully mixed chemically, and their degenerate electrons do not move relativistically.

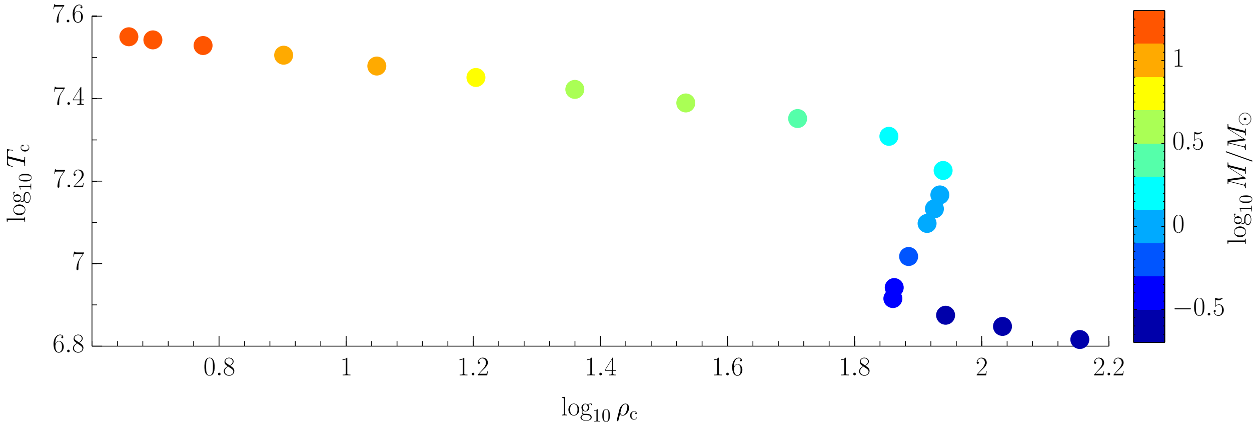

Figure 4.7: The central temperature and density for various `MESA` models of mass given by the colorscale. All models are just on the main sequence when $X_c = 0.68$.

4.4 Summary of main-sequence properties

- In general, the star regulates its nuclear burning rate to maintain hydrostatic equilibrium.
- If the rate increases for some reason, the star expands, thereby decreasing its temperature and density, reestablishing equilibrium.
- Mass rules of thumb:
  - Below about $1.3\,M_\odot$ convective envelopes, above, radiative envelopes
  - Below about $1.2\,M_\odot$ PP chain, above, CNO cycle
  - Below about $1.5\,M_\odot$ late-type stars (F, G, K, M), above, early-type (O, B, A)
- Structure rules of thumb:
  - Low-mass cores
    * PP chain is sufficient to balance gravity
    * Luminosity is not too steep, and energy flux is moderate
    * Radiation is enough to carry out the luminosity from the core
    * Core is radiative
    * Since the PP chain has a low temperature dependence, the region of burning is relatively a large mass fraction of the star, as in Figure 4.4, left.
  - High-mass cores:
    * CNO cycle is necessary to balance gravity
    * High-temperature sensitivity ($\sim T^{20}$) means a very central energy generation region
    * Luminosity is very steep in core with a high flux
    * Temperature gradient is very steep, convection sets in
    * A convective core develops, which is very efficient
    * So efficient that here it equals the adiabatic gradient
    * Core mixing due to convection removes gradients in composition
    * The core temperature and density for different masses is shown in Figure 4.7.
    * Note the strong variations at a little above 1 solar mass stars, where convective cores start to appear.

- Low-mass envelopes:
  * Opacity is rather large because of hydrogen and helium ionization zones and corresponding bound-free transitions
  * Convection is needed to carry the radiative flux through the region; steep temperature gradient
  * Below about $0.3\,M_\odot$ the entire star is convective
- High-mass envelopes:
  * Hydrogen and helium ionized so rather low opacities
  * The radiative flux is carried out by radiation. Radiative envelope.
  * In very massive stars $> 10\,M_\odot$, some opacity peaks due to ionized iron and nickel can cause thin convection zones near the surface

• MS location rules of thumb (as we saw in homology relations):

  - Higher He content results in more luminous and hotter MS tracks.
  - The MS lifetime decreases with increasing He content.
  - Higher metallicity makes the star cooler due to increased opacity.
  - Alpha elements (O, Ne, Mg, Si, S, Ca, Ti, ...) that are enhanced in metal-poor stars produce fainter, cooler MS tracks.
  - Changing the mixing length, or convective efficiency, affects the MS.
  - There is no effect on the luminosity, but an increased effeciency sets up a lower thermal gradient.
  - This increases the effective temperature, and therefore the radius decreases.

• Evolution (on main sequence) rules of thumb:

  - ZAMS to TAMS lifetimes are much shorter for high-mass stars
  - Tracks on HR diagram are vertical for low-mass, and diagonal for high-mass stars
  - Luminosity increases due to increase in molecular weight in core
  - Low-mass stars have abundance changes in core that is smooth
  - High-mass stars have discontinuous changes due to convective mixing (although core shrinking can smooth out the discontinuities)
  - Below about $0.3\,M_\odot$ the core is completely mixed and stars live long
  - Near the end of main-sequence evolution:
    * Nearly isothermal core with zero luminosity
    * Core is very hot from high μ
    * At core boundary temperature is high enough for shell burning to occur
    * The high T and large volume of the burning region leads to high shell L
    * No thermal equilibrium, envelope expands