October 20

COMPUTER PROBLEM 3.1: [80 pts]: This assignment deals with convection and radiation zones in main-sequence stars over a range of different masses. You are going to quantitatively illustrate where we expect such zones to exist in stars to provide an overall understanding of stellar structure during the period of core-hydrogen burning.

What to do

Your overall goal is to prepare a main plot (described in more detail below) showing where these zones exist. For this purpose, you need to run a bunch of `MESA` models for different mass stars. We will use these models for future studies so make sure to keep the directory(ies) neat and handy.

1. The models should be within the stellar mass range of $M_* = 0.3\,M_\odot - 20\,M_\odot$. The number of models you compute is up to you, but the "mass grid" should be sufficiently populated so you don't miss any important details, and so that you can make the required plots with enough number of points and precision. One of the models should be $M_* = 1\,M_\odot$, of course.
2. The models are to be of solar hydrogen mass fraction and metallicity. For each mass, you will run the code until the core looks like the core of today's Sun, $X \approx 0.35$, which is the model you will want to use for what follows. You will also want the history data (really just $T_{\rm eff}$ and L) for the ZAMS (zero-age main sequence) point, where $X \approx 0.7$. You shouldn't need to run pre-main sequence models.
3. That's all you need to do with `MESA`. Now you need to carefully explore the resulting data and determine the locations of all the convection and radiation zones for each star mass. Use the criteria discussed in class.

What to hand in

After reading the below information you will find you need to hand in at least 7 plots, as well as text pertaining to items 4 and 6.

1. A nice H-R diagram of your stars. The symbol size should represent the star's radius, and the color the star's mass at the time you stopped your models. Also give an indication of age somehow discretely so we have an idea of the range there. Finally, plot a small symbol for where each star began on the ZAMS, so we see how much "evolution" occured. These symbols do NOT need to show the mass or radius information as in the more evolved points. [15 pts]
2. A plot that shows where the internal convection and radiation zones are for each star. [25 pts]
   - This should be a publication-quality plot (clearly labeled, easy to read, useful for future reference, etc.).
   - The plot is to be as follows: the x axis is $\log M_*/M_\odot$ and the y axis is $m(r)/M_*$.
   - The interior radiation and convection zones should be clearly demarcated by however you wish to do it.
   - Finally, somehow denote the dominant H-burning process for everywhere that nuclear burning is taking place. In many cases, you will have some regions where PP is dominant and some where CNO is (in the same star).
3. A second plot of exactly the same information but instead where r/R_* is the ordinate. These should be on separate pieces of paper. [5pts]
4. A summary paragraph or two of how you obtained the parameters in your plot, i.e., how you determined where the zones are from the stellar profiles. Mention here any issues you ran into or anomalous cases (if any). You don't need to do much interpretation here, just write down what you did to gather the information that went into these plots. [10pts]
5. Pick two of your models. One should be $M_* \leq 1.1 M_\odot$ and one $M_* \geq 3 M_\odot$. For each of these 2 models you will make 2 more plots, so 4 in total here. One of the plots for each mass should have the three quantities ∇, $\nabla_{\rm ad}$, and $\nabla_{\rm rad}$ plotted as a function of interior $\log T$, but reversed, so $\log T$ decreases to the right. The other plot for each mass will be the Brunt-Väisälä frequency (squared), or N^2 in appropriate units as a function too of reversed $\log T$. Make sure it's scaled properly as to be visible. [10pts]

6. Write another page or two on the above 4 plots. Describe what is happening and connect what you observe in these figures to the findings shown in your **main** plot. Compare and contrast the behavior for the 2 different masses. Try to describe and decipher any unusual "features" you see, such as bumps or spikes, in each case. Connect them to things we may have discussed in class, not forgetting, for example, our study of the opacities. Indeed, you may find it necessary to generate other supplementary plots of other quantities in the models to get your point across here. Feel free to draw on and label these plots by hand to point things out. Be thorough. Also in this part, be sure to answer the following question for each of these 2 stellar mass models:

   - In the interior regions where $\nabla_{\rm rad}$ is large, what is your idea as to the physical reasons causing this?

   Use Equation (3.65) from the notes as a guide if needed. [15 pts]

Things to keep in mind

1. Your main plot is going to be judged by the faculty to determine who made the best publication-quality image (oh, and it should be correct too). Therefore the plot should be full page and nice, and on a separate page from your other stuff! You also may want to keep it secret from your classmates so they don't steal your ideas. An emailed .pdf or .eps of your plot would also be good and I can collect them all into one document to show in high quality on a computer screen. The one judged best will receive a special prize.
2. You may want to consider your initial stellar mass grid spaced out in terms of $\log M_*/M_\odot$, rather than in linear $M_*/M_\odot$. But do what you want.
3. You may need to consider adding quantities to what `MESA` saves in the profiles. This is easily done using the .list files.
4. You don't need to create any pre-main-sequence models.
5. It may or may not be reasonable to interpolate certain quantities to make your mass grid denser for your plots. It's up to you.
6. There are very fast and easy, or long and complicated ways of obtaining the quantities you need for the plots. Hopefully you find the fast/easy ways.

Part II

Stellar Evolution

Unit 4

The Main Sequence

4.1 Summary of stellar structure

- Mass and radius relationship

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho$$

- Hydrostatic equilibrium

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -g\rho$$

- Energy

$$\frac{\mathrm{d}L}{\mathrm{d}r} = 4\pi r^2 \rho\varepsilon$$

- Energy transport

$$L = 4\pi r^2 (F_{\mathrm{rad}} + F_{\mathrm{conv}})$$

  - Radiation

$$F_{\mathrm{rad}} = -\frac{4acT^3}{3\kappa_{\mathrm{R}}\rho}\frac{\mathrm{d}T}{\mathrm{d}r}$$

  - Convection

$$F_{\mathrm{conv}} = \rho C_p T (g\delta)^{1/2} \frac{\ell^2}{4} H_p^{-3/2} (\nabla - \nabla_{\mathrm{ad}})^{3/2}.$$

- Equation of state

$$P = \frac{\rho k_{\mathrm{B}} T}{\mu m_{\mathrm{u}}} + \frac{a}{3} T^4$$

- Rosseland opacity

$$\kappa_{\mathrm{R}} = \kappa_{\mathrm{R}}(\rho, T, \mu)$$

- Energy generation

$$\varepsilon = \varepsilon(\rho, T, \mu) = \varepsilon_{\mathrm{nuc}} + \varepsilon_{\mathrm{grav}}$$

  where

$$\varepsilon_{\mathrm{grav}} = -T\frac{\mathrm{d}S}{\mathrm{d}t}$$

  is related to Kelvin-Helmholtz contraction (and note the time dependence).

- Abundance changes

$$\begin{aligned}\frac{\mathrm{d}X}{\mathrm{d}t} &= -\frac{\varepsilon_{\mathrm{nuc}}}{26.7\,\mathrm{MeV}}\\ \frac{\mathrm{d}Y}{\mathrm{d}t} &= -\frac{\mathrm{d}X}{\mathrm{d}t}\end{aligned}$$

- Boundary conditions

$$\begin{array}{cccc} r & \longrightarrow & 0; & m(r) \longrightarrow 0 \\ r & \longrightarrow & 0; & L(r) \longrightarrow 0 \\ r & \longrightarrow & R; & m(r) \longrightarrow M \\ r & \longrightarrow & R; & \rho(r) \longrightarrow 0 \\ r & \longrightarrow & R; & T(r) \longrightarrow T_{\mathrm{eff}} \end{array}$$

- Vogt-Russell Theorem
  - There are only 2 free parameters in the equations needing to be solved: the total stellar mass M and the chemical composition.
  - The theorem states that just these 2 parameters uniquely determine the structure. Evolution is only based on the changing of the composition (mainly) and mass (sometimes) due to nuclear burning.