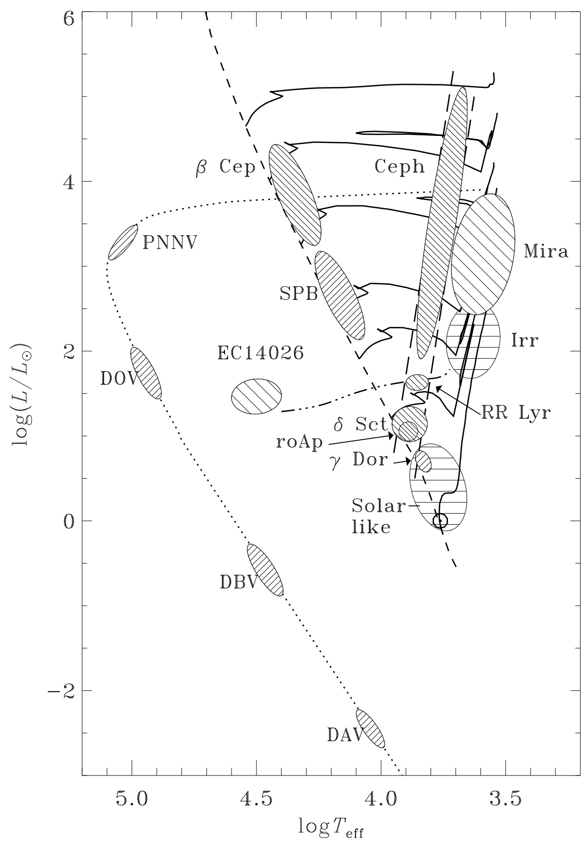

Figure 5.16: Schematic Hertzsprung-Russell diagram illustrating the location of several classes of pulsating stars. The dashed line shows the zero-age main sequence, the continuous curves are selected evolution tracks, at masses 1, 2, 3, 4, 7, 12 and $20 M_\odot$, the dot-dashed line is the horizontal branch and the dotted curve is the white-dwarf cooling curve. From Christensen-Dalsgaard [2003].

December 1 . . . .

5.6 Instability Strip and Pulsations

5.6.1 Background

- We first consider the types of stars that display pulsations.
- Figure 5.16 shows where many of these types of stars lie on the H-R diagram.
- The Cepheid instability strip is the main thing to pay attention to, which spans from high luminosity down to the main sequence.
- Here, there are Cepheids, RR Lyrae, and δ-Scuti stars, etc.
- The instability strip is very narrow, and we've learned that it's hard to "catch" stars in the Hertzsprung Gap.
- The higher luminosity stars must therefore be more massive, going through their "loops," which occur on a much longer timescale.
- These stars have pulsations excited by an opacity mechanism, explained later.

- Typically only one mode is observed for the more luminous stars (Cepheids and RR Lyrae), a radial mode.
- For the stars near the main sequence, such as the δ Scuti, γ Dor, slowly pulsating B stars (SPB) and β Cephei, multiple modes are observed.
- δ Scutis have masses around $1.5 - 2.5 M_\odot$ (convective cores, F stars), and have periods of an hour or so.
- This makes their observation a bit difficult.
- They are expected to have acoustic pulsations.
- The Mira variables are related to the AGB stage of stars.
- The "EC14026" stars are subdwarf B variables (SdB), believed to also pulsate from an opacity mechanism (related to iron).
- These interesting stars are on the horizontal branch as core helium burning is proceeding.
- However, at 35,000K they are much bluer than most HB stars.
- They have somehow lost most of their H envelope beforehand.
- The "Irr" irregular pulsators show strange amplitude variations, but are now known as red giant solar-like pulsators.
- White dwarfs pulsate all along the cooling phase (ZZ Ceti as they are sometimes known).
- They are pulsating with periods up to 10 min, much longer than their dynamical timescales.
- Likely pulsating with internal gravity modes.
- Finally, solar-like oscillations are expected in stars < 7000K, due to convective mechanisms operating near the surface.
- Pulsations can be observed by intensity fluctuations, doppler-velocity observations, and line-profile variations (for nonradial modes).

5.6.2 Pulsation mechanisms

- The adiabatic sound speed of stellar interiors is given as

$$c_s^2 = \frac{\gamma p}{\rho}, \tag{5.32}$$

where again

$$\gamma = \frac{\mathrm{d}\ln p}{\mathrm{d}\ln \rho}.$$

- The most fundamental period of pulsations is inversely proportional to the mean density of a star

$$\Pi \propto \overline{\rho}^{-1/2}. \tag{5.33}$$

- This can be obtained from hydrostatic equilibrium and some basic principles, but can more easily be seen just from the dynamical timescale, Eq. (2.3):

$$t_{\mathrm{dyn}} = \left(\frac{R^3}{GM}\right)^{1/2} \propto \overline{\rho}^{-1/2},$$

which is its direct counterpart.

- A rigorous treatment can provide an exact equation for Eq. (5.33).
- Radial modes are standing waves, and the fundamental mode has only a node at the center and the surface of the star
- After that, we have overtones of the fundamental mode, with nodes placed internally to the star
- Cepheids are observed to pulsate in fundamental radial modes
- Their magnitude, temperature, radius, and surface velocity change as a function of time
- The temperature varies by about 1000 K
- The radius by about 10%
- The star is brightest when it is *expanding* after its minimum radius was reached
- There is a phase lag between the radius and the luminosity
- We do know that as stars pulsate, energy is lost in each pulsation cycle, as most of the volume of the star damps it
- These pulsations can only continue of there is a driving mechanism that is feeding energy into the pulsation, at least as much that has been damped out
- The first proposition to explain such pulsations was that of a heat engine (convert thermal to mechanical energy)
- Pulsations are driven if positive work is done, and damped if negative work is done (on a layer, and throughout the star)
- Eddington postulated a valve mechanism, whereby a layer of a star could block energy that would then push the layer outward.
- After expansion, the blocked region would allow energy to pass through and the cycle would start again
- So, simplistically, the driving mechanism is this
  - The increase in opacity due to ionization halts the energy flow
  - The absorbed energy ionizes and eventually heats the gas and the pressure increases, pushing the local layer outward past its equilibrium point (expansion)
  - This ionization process reduces the opacity since now the density decreases (expansion) and the temperature does not decrease too rapidly because of partial recombination ($\kappa \propto \rho T^{-3.5}$)
  - Radiation can now flow freely, the gas cools and can no longer support the overlying weight
  - The star contracts and compresses and the density increases
  - The opacity is thus raised again and the whole process stars over
- This is the κ mechanism
- The other main driving mechanism is stochastic driving due to convection near the surface
- These modes are typically stable, but there is a lot of acoustic energy in outer convection zones such that this noise is transferred to the energy of global oscillations

5.6.3 Ionization zones

- The ionization zones being considered are hydrogen and the first and second ionization levels of helium
- These occur at about 10,000-20,000K
- To fully ionize helium requires about 40,000K
- For hot stars, with a $T_{\text{eff}} \geq 7500$K, these ionization zones are near the surface
- There is not much mass here and oscillations cannot be driven strongly
- In cooler stars with $T_{\text{eff}} \lesssim 5500$K, these zones occur deeper
- However, convection is occuring in these envelopes
- Convection is efficient and does not allow a sufficient "blocking" of the radiation or energy flux
- Pulsations may not therefore occur from this mechanism, and the red edge of the instability strip is defined
- Most of the driving is due to ionizing He zones from modeling
- The H ionization zones are interesting and since they lie above the He zones, they "carry" the emergent luminosity outward
- Thus the phase lag in the luminosity-minimum radius relations
- Note that these ionization zones alter the value of Γ_1, and lower the adiabatic gradient.
- These regions can often be unstable to convection which can also drive pulsations through convective flux
- For β Cephei stars, it is Fe-group elements doing the driving

5.6.4 Asteroseismology

- The surface of a star oscillates with displacements in the radial and horizontal directions as:

$$\boldsymbol{\delta r} = \xi_r \hat{\boldsymbol{r}} + \boldsymbol{\xi}_{\text{h}},$$

- The diplacements are solutions to the equations of motion expressed as

$$\xi_r(r, \theta, \phi, t) = \sqrt{4\pi}\, \tilde{\xi}_r(r) Y_\ell^m(\theta, \phi) \exp(-i\omega t) \tag{5.34}$$

  and

$$\boldsymbol{\xi}_{\text{h}} = \sqrt{4\pi}\tilde{\xi}_{\text{h}}(r) \left(\frac{\partial Y_\ell^m}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{\sin\theta} \frac{\partial Y_\ell^m}{\partial \phi} \hat{\boldsymbol{\phi}} \right) \exp(-i\omega t). \tag{5.35}$$

- The r dependence terms are the (as of yet undetermined) eignenfunctions of the star, while ω is the angular eigenfrequency of a mode
- The Y_ℓ^m are spherical harmonics
- Three "quantum" numbers describe each mode: n gives the overtone, or number of nodes in the radial direction
- ℓ is the degree and specifies the number of surface nodes
- m is the azimuthal order and $|m|$ gives the number of nodal lines of longitude

- The equations of motion are derived from hydrodynamics (continuity, momentum, energy, Poisson, etc.) and approximations to them
- Namely 2: adiabatic small-amplitude (linear) changes
- After the approximations and plugging the spherically symmetric expansions into them, a set of coupled differential equations are derived
- These are then solved for the eigenfrequencies and eigenfunctions
- The pulsations can be classified in various ways
- **Radial modes** are the simplest with $\ell = 0$, these are the breathing modes.
- The star expands and contracts, heats and cools. The fundamental mode is what is foundin Cepheid and RR Lyrae stars.
- The center is a node and the surface is an antinode
- The first overtone $n = 2$ has 1 node away from the center.
- The motions above and below that region move in antiphase
- When stars exhibit both fundamental and first overtone pulsations, the ratio of their frequencies can be used to determine interior properties
- Unlike musical instruments, their ratios vary, which at least tells us that stars do not have constant temperatures or uniform sound speed
- **Nonradial modes** have $\ell > 0$ and $n \geq 1$, thus, a surface node
- These modes are degenerate in m as there are $2\ell + 1$ modes with the same frequency
- Rotation and possibly other effects can break this degeneracy
- The simplest is the dipole $\ell = 1$ mode
- The equator is a node, so the northern hemisphere will expand while the southern contracts, and vice versa
- Depending on n there are internal radial nodes as well
- $\ell = 2$ are quadrupolar, while $\ell = 3$ are octupolar modes
- Beyond this, except for the Sun, we cannot detect higher-degree modes, as the surface averaging partially cancels neighboring regions of hot/cold (intensity) or up/down (Doppler velocity)
- Nodal regions beyond about $\ell = 3$ are practically unattainable in disc-integrated observations (including for the Sun)