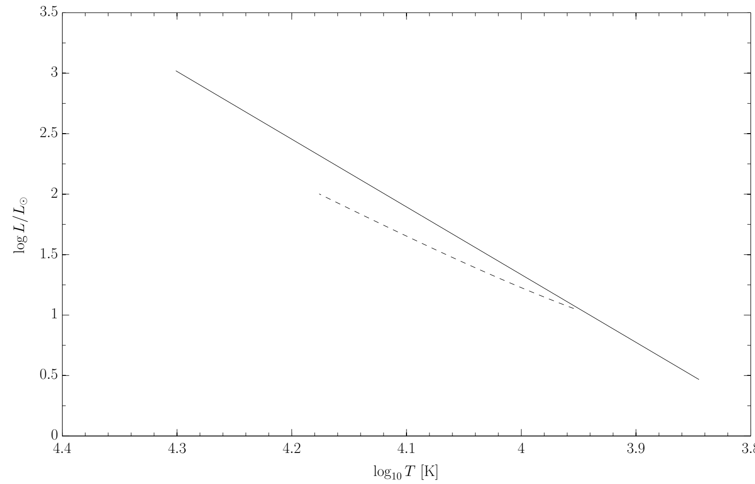

Figure 4.1: The solid line represents the main sequence for stars of various mass but fixed μ in the CNO cycle, so $L \propto M^{5.6}$. The dashed line is the evolutionary track for a star with a given mass but increasing μ, so $L \propto \mu^{4.8}$.

October 27 ....

4.2.5 Dependence on heavy metal abundances

- Let's look at lower-mass stars and their dependence on the metal mass fraction Z.
- This is only approximate because the depth of the outer convection zone depends on Z, but for homologous stars this will still give the right trend.
- The metals appear mainly through the κ_0 term in the bound-free processes in deep interiors where only metals still have electrons.
- Furthermore there is no dependence of ε_0 on Z in the PP-chain reactions.
- So considering $\beta = 3.5$ and $\alpha = 1$ we have for the product CD

$$CD = \text{const}\,\frac{ZR^{0.5}M^{\nu+2}}{R^{\nu+3}M^{5.5}}. \tag{4.16}$$

Kramers, $\nu = 5$	$R \propto Z^{0.13}$	$L \propto Z^{-1.1}$	$T_{\text{eff}} \propto Z^{-0.34}$	$L \propto T_{\text{eff}}^{3.24}$

- For stars of a given mass the luminosity increases with decreasing metals.
- The lifetime of a metal-poor star is shorter than for a metal-rich star: it burns through its fuel faster.
- The opacity decreases with smaller Z and radiation escapes more easily.
- For stars with the same mass but different Z, $L \propto T_{\text{eff}}^{3.24}$, which is less steep than stars for the same Z but different mass, $L \propto T_{\text{eff}}^{4.3}$, and less steep than one for changing mean molecular weight too.
- Stars with few metals, like Population II ones, would have a main sequence **below** the solar abundance one (for low mass).

4.2.6 Contracting stars in radiative equilibrium

- Consider a star contracting from gravitational energy release in its young life.
- Its temperature is not high enough for nuclear reactions to take place, so the quantity D is not applicable here.
- But we can use C. During contraction, the mass stays relatively constant, as does composition and μ.
- For Kramer's we find

Kramers	$L \propto R^{-0.5}$	$T_{\text{eff}} \propto R^{-0.63}$	$T_{\text{eff}} \propto L^{1.1}$	$L \propto T_{\text{eff}}^{0.95}$

- But is this what we observe? Kind of for high-mass stars. As they contract they slightly increase in luminosity.
- And the temperature increases across the HR diagram
- But not at all for low-mass stars, where they contract at constant temperature along almost vertical lines.
- Why? Convection needs to be taken into account in low-mass stars.

4.2.7 Convective stars

- Convection zones in stars change them in 2 main ways
  - The radius becomes smaller.
  - Energy transport becomes more effective than radiation (because of the large absorption coefficients) and the temperature gradient gets reduced.
- As energy transport is increased, more energy is lost, gas pressure decreases and gravity starts to dominate ...
- The star contracts, thus increasing the internal temperature and the energy generation ..
- This balances the energy loss and thermal equilibrium is restored, albeit at a smaller radius ...
- The luminosity is larger, thus the effective temperature must be larger ...
- The star moves up and to the left on the HR diagram
- We can treat convective stars similarly with just a change to the equation of state.
- In convection zones
$$\frac{\mathrm{d}\ln P}{\mathrm{d}\ln\rho} = \Gamma_1.$$
- We assume $\Gamma_1 = \gamma = 5/3$ is constant .
- Thus we have the polytropic relation we found before by integrating the above equation
$$P = K\rho^{\gamma}.$$
- We can assume for simplicity that a convection zone extends all the way to the surface, and we match to the photosphere by
$$K = \frac{P_{\text{ph}}}{\rho_{\text{ph}}^{\gamma}}. \tag{4.17}$$

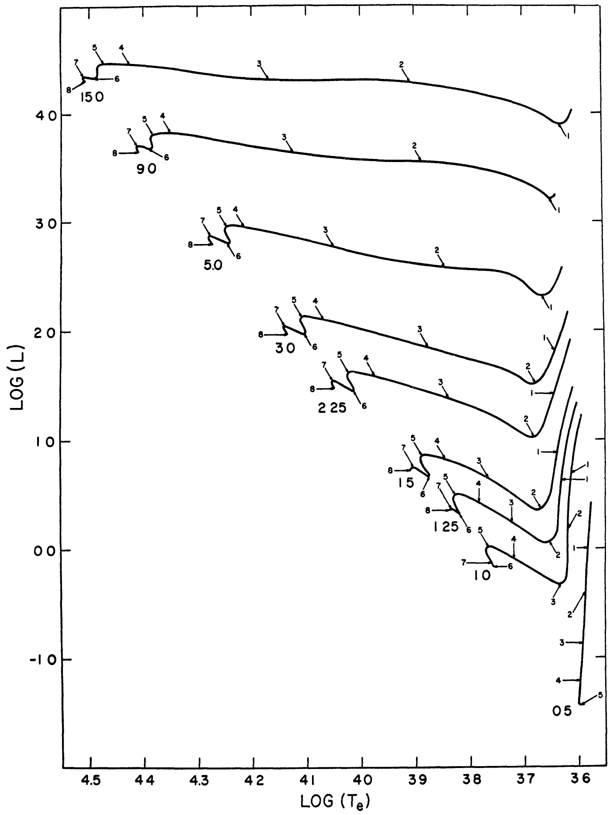

Figure 4.2: Evolutionary tracks for pre-main sequence stellar models of the given masses. Composition is $X = 0.708$ and $Z = 0.02$. From Iben [1965].

- Using radiative transfer at the photosphere (which we have NOT done), one can show that the pressure and density in the above equation give

$$K = \left[\frac{GM(\alpha+1)}{R^2\kappa_0^{\mathrm{ph}}}\right]^{-\eta}\left(\frac{k_\mathrm{B}}{\mu m_\mathrm{u}}\right)^{1+\eta} T_\mathrm{eff}^{1+\eta(-\beta+1)}, \tag{4.18}$$

  where $\eta = (\gamma-1)/(\alpha+1)$ and α and β have their usual definitions in the opacity expressions.

- There are a few questionable assumptions so far
  - We assume that convection extends all the way to the surface, but it becomes very inefficient there because of the density fall off.
  - We also assume γ remains constant in the convection zone so that the polytrope relation holds, but in ionization zones it can change quite a bit.

- Anyway, we continue with a polytrope $n = 1.5$ where from before

$$K = K_\mathrm{poly} = N_{3/2}GRM^{1/3}. \tag{4.19}$$

- Matching these two Ks and solving for the effective temperature gives

$$T_\mathrm{eff} = N_{3/2}^{\xi}\left(\frac{G\mu}{k_\mathrm{B}}\right)^{\xi(\eta+1)}\left(\frac{\kappa_0^{\mathrm{ph}}}{\alpha+1}\right)^{-\xi\eta} R^{\xi(1-2\eta)}M^{\xi(\eta+1/3)}, \tag{4.20}$$

  where $\xi = 1/[1+\eta(\beta+1)]$.

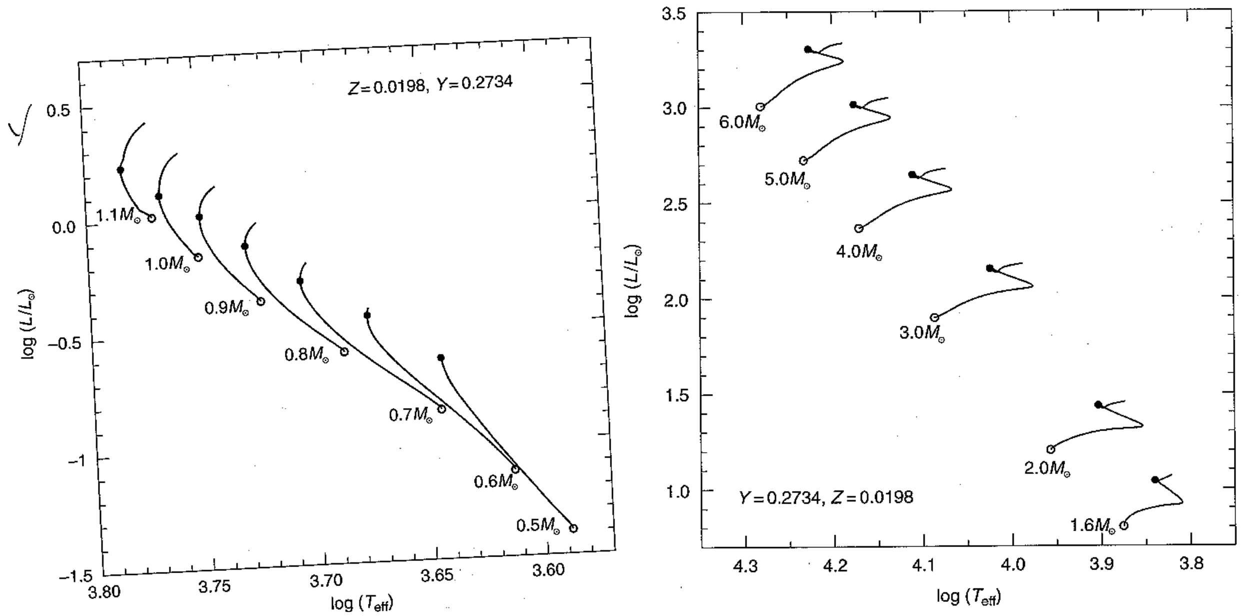

Figure 4.3: (Left): Low-mass star evolution on the main sequence. (Right): The same but higher-mass stars.From Salaris and Cassisi [2006].

- Solving for the radius R and using $L = 4\pi\sigma R^2 T_{\rm eff}^4$ gives an expression for the luminosity, as well as other similar-type relations we've computed before. The results are

$$\begin{aligned}
T_{\rm eff} &\simeq 2400\,\mathrm{K}\left(\frac{M}{M_\odot}\right)^{0.2}\left(\frac{R}{R_\odot}\right)^{0.06}, \\
\frac{L}{L_\odot} &\simeq 0.03\left(\frac{M}{M_\odot}\right)^{0.8}\left(\frac{R}{R_\odot}\right)^{2.2}, \\
\frac{L}{L_\odot} &\simeq 0.03\left(\frac{M}{M_\odot}\right)^{-7.0}\left(\frac{T_{\rm eff}}{2400\,\mathrm{K}}\right)^{40.0}.
\end{aligned}$$

- Note that
  - The dependence of the temperature on parameters is very weak: these stars all have similar temperatures.
  - This describes the contraction of convective stars as they form.
  - This is known as the *Hayashi track*.
  - The slope of the luminosity (the sign) is wrong a bit, but it's pretty good.
  - There cannot be stars to the right of the Hayashi track (with a lower effective temperature).
- A glance at the approach to the main sequence along the Hayashi track is shown in Figure 4.2.

4.3 Evolution on the main sequence

4.3.1 Low-mass stars

- The time of arrival on the main sequence is known as the ZAMS - zero-age main sequence.
- "Where" it ends up depends only on mass and chemical mixture.

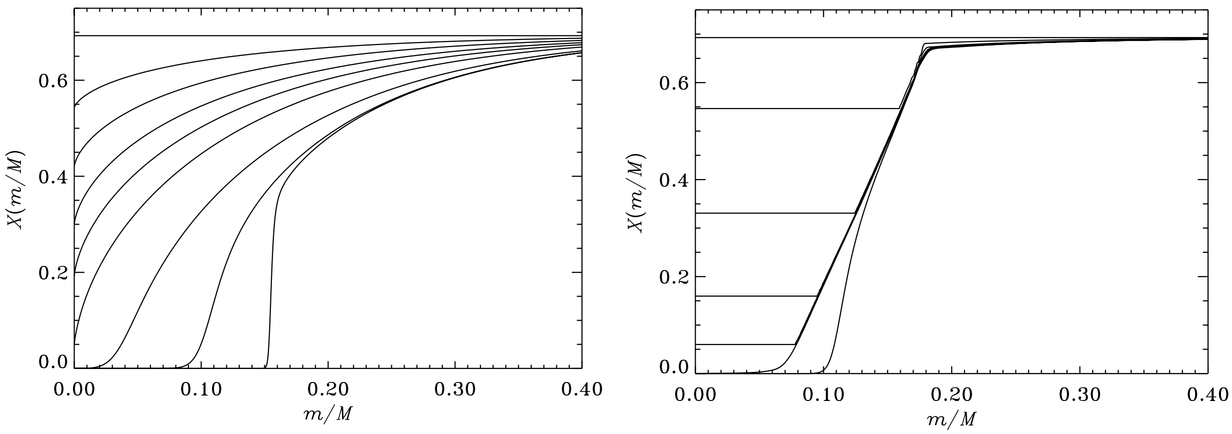

Figure 4.4: (Left): Hydrogen profiles showing the gradual exhaustion of hydrogen in a $1M_\odot$ star. The homogeneous initial model consists of a mixture with a hydrogen abundance by mass of 0.699. X as a function of the mass fraction $m/M_\odot$ is plotted for nine models which correspond to ages of 0, 2.0, 3.6, 5.0, 6.2, 7.5, 9.6, 11.0 and 11.6 times 10^9 years, after the onset of hydrogen burning. The model at 5.0×10^9 years corresponds roughly to the present Sun, whereas the last two models are in the shell hydrogen burning phase. (Right): The same but for a $2.5M_\odot$ star. The lines show the hydrogen profiles for models of age 0, 1.5, 3.1, 4.0, 4.4, 4.6, and 4.8 times 10^8 years. From Christensen-Dalsgaard [2008].

- The lower mass limit is roughly about $0.1M_\odot$.
- The upper mass limit is about $100M_\odot$.
- The mean molecular weight changes a lot. Consider fully ionized H in core at beginning at ZAMS (see Equation (2.22)):
$$\mu = \frac{4}{3 + 5X - Z} \simeq 0.61. \tag{4.21}$$
- As all of it gets converted to He, we then have
$$\mu = \frac{4}{3 + 5(0) - Z} \simeq 1.3. \tag{4.22}$$

  It more than doubles!
- This change (increase) in mean molecular weight causes changes in other things. Note that the opacity is reduced, as He is less opaque than H.
- The number of free particles also decreases, as does the pressure.
- A low-mass star slightly contracts its core and heats up. Firstly, ρ increases by the core contracting. As this happens gravitational energy gets released according to the Virial theorem, which partly goes to increasing the thermal energy of the core - increased T.
- This must increase the pressure to account for the "heavier" material. Indeed, according to the equation of state, if P increases, and μ increases, then ρT must certainly increase.
- The nuclear energy generation increases and then so does the luminosity.
- This causes a slow increase of the star's luminosity over the whole MS phase.
- Also, as the core contracts, the surface radius increases, slightly for low-mass stars, more rapidly for high-mass stars. This can affect the effective temperature (see below).
- See Figure 4.3 for the MS evolution.

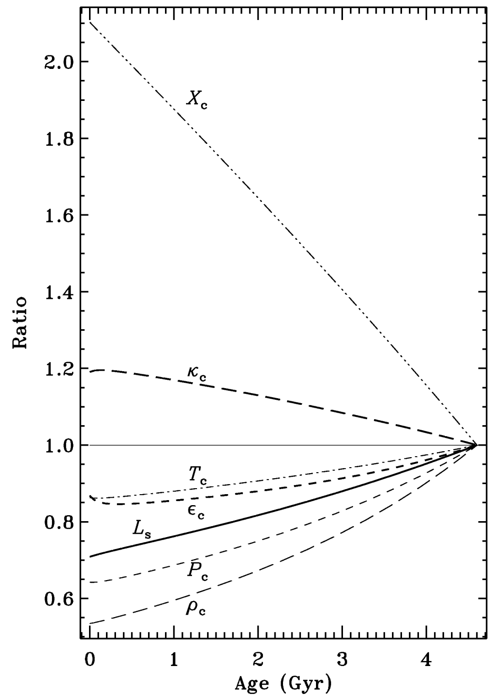

Figure 4.5: The changes in the solar properties at the center of a solar model. All variables are normalized with respect to the present Sun. From Christensen-Dalsgaard [2008].

- Figure 4.4 (left) shows the core hydrogen mass fraction as a function of time. The burning region extends out to a significant radial distance.
- How much has the solar luminosity changes over time? Calculations (including homology ones) show that the Sun's luminosity compared to its ZAMS luminosity L_0 is $L \simeq 1.26L_0$. This means it was about 25% less luminous than it is today, which has/had implications for the Earth.
- All of the main properties of the Sun from its ZAMS point until today are shown in Figure 4.5.
- The important things to note are the increase in density and decrease in X. One would think the temperature increase too would really increase the nuclear energy generation rate ϵ_c.
- But as we saw, it not only depends on T but also on X^2, so it is somewhat halted by the decreasing hydrogen amount over time.