October 1 . . . .

3. Bound-free absorption

   - Here an ion absorbs a photon which frees a bound electron.
   - Causes continuum opacity at wavelengths bluer than the ionizing photon.
   - It can be shown that the opacity

   $$\kappa_{\mathrm{b-f}} \approx 10^{25} Z(1+X)\rho T^{-3.5}. \tag{3.23}$$

     This also has a Kramers form.
   - At temperatures below about 10^4K, this should be used with caution since photons are not energetic enough to ionize a gas.
   - This source of opacity can be larger than free-free absorption by a few orders of magnitude.

4. Bound-bound absorption

   - This is photon-induced transitions of electrons between bound levels in an atom or an ion.
   - At large temperatures, most photons are ionizing so this contribution is smaller to the total opacity (maybe 10%).
   - Very difficult computations since there are millions of absorption lines possible. Need oscillator strengths as well as equation of state.

5. H^-

   - We just note that in stellar atmospheres (like our Sun) contributions from negative hydrogen and molecules and grains dominate the opacity.
   - Low energy photons starting in the infrared can be absorbed in this bound-free transition.
   - What is needed for the H^- opacity are free electrons, which at low temperatures might not be completely abundant from ionization of H.
   - Other sources of these electrons are the outer ones from metals, so this is sensitive to metallicity.

3.1.5 Consequences

- Figure 3.1 shows the contributions from different opacity sources in a stellar interior.
- At low temperatures and partial ionization, b-b and b-f absorption dominates from the bound electrons.
- As ionization occurs at higher temperatures, free-free opacity takes over, but as T continues to increase, $\kappa_{\mathrm{f-f}}$ decreases and scattering from free electrons dominates.
- In reality, all of these processes are occuring at any one time and place, so contributions must be added correctly.
- Note that the sum of the Rosseland mean opacities of each component is not the same as the Rosseland mean of the sum
- Figure 3.2 shows opacity data from the OPAL project.[1] We see that the Sun's opacity can reach 10^5 or so near the surface where atoms recombine.
- Figure 3.2 can be understood in a few limiting cases:

[1] http://opalopacity.llnl.gov/

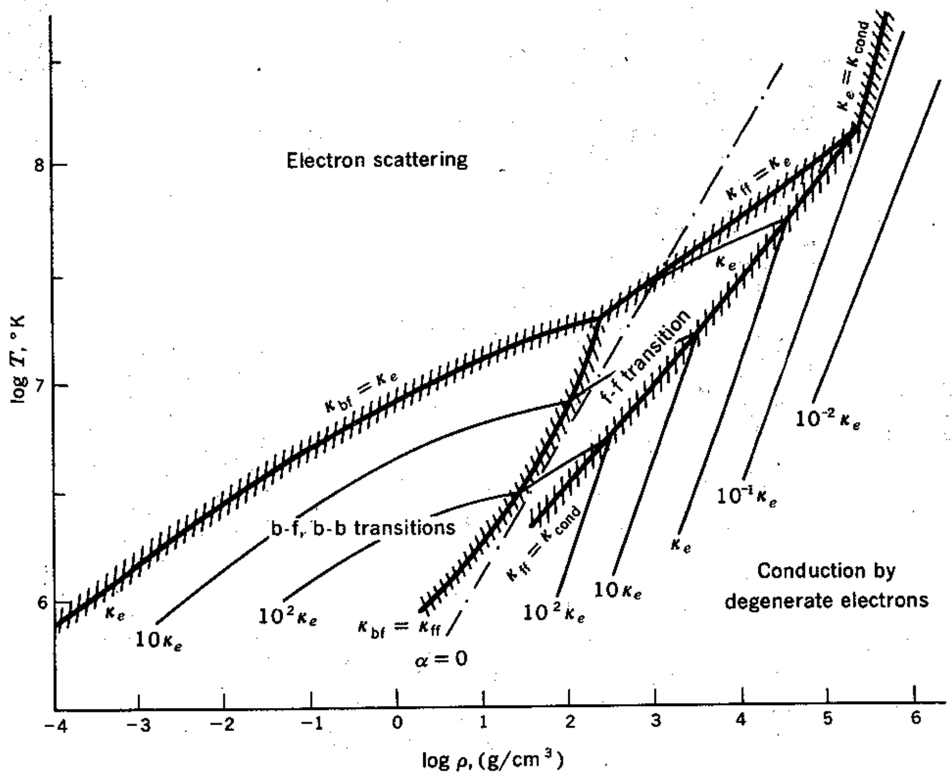

Figure 3.1: Opacity contributions in the $\rho - T$ plane. The values are given in units of electron scattering opacity κ_e. Conduction is not a radiative opacity, and will be discussed in Sec. 3.2. From Clayton [1983] after Hayashi et al. [1962].

- For the low density, high temperature case (small R), and for the high temperature regime at any density, all H and He is ionized and the electrons are free particles. Here we expect electron scattering, where Equation (3.21) is dominant. Indeed, $\kappa_{\rm e} \approx 0.34\,{\rm cm}^2\,{\rm g}^{-1}$ for an $X = 0.7$ composition, which agrees with the values over most of the temperature range.
- Below 10,000 K the opacities all converge to small values. Here, all electrons go to the ground state of H and He. Incoming photons can only cause a bound free transition if their wavelengths are shorter than $912\,\AA$ (for H) or $228\,\AA$ (for HeII). In fact, the whole Lyman series (edge) only extends to $1216\,\AA$ (for H). But at this temperature, the blackbody curve is peaked at about $2880\,\AA$, and there just aren't many photons with such high energies available. The stellar region in which this applies is near the surface, but the Sun does not have such conditions as the plots show. Also see Figure 3.3.
- At intermediate temperatures we see increased opacity, particularly as the density increases (larger R). Even for the lowest density curve we see a bump around temperatures of 5.0-5.5. Wien's law tells us the wavelengths are in the $100 - 300\,\AA$ range. This suggests the HeII Lyman edge, even though there aren't many He ions. The bump at about 4.5-4.75 at larger densities could be due to a bound-free absorption of HeI. In any case, these large radiative opacities are found in the H and He ionization zones in stars.
- Note the Kramers shape to the opacities, which decrease at higher temperatures.
- Keep in mind however that these are mean opacities, independent of individual wavelengths.

3.1.6 Eddington Luminosity

- Let's take a quick stop and look at an interesting consequence of diffusive radiation.

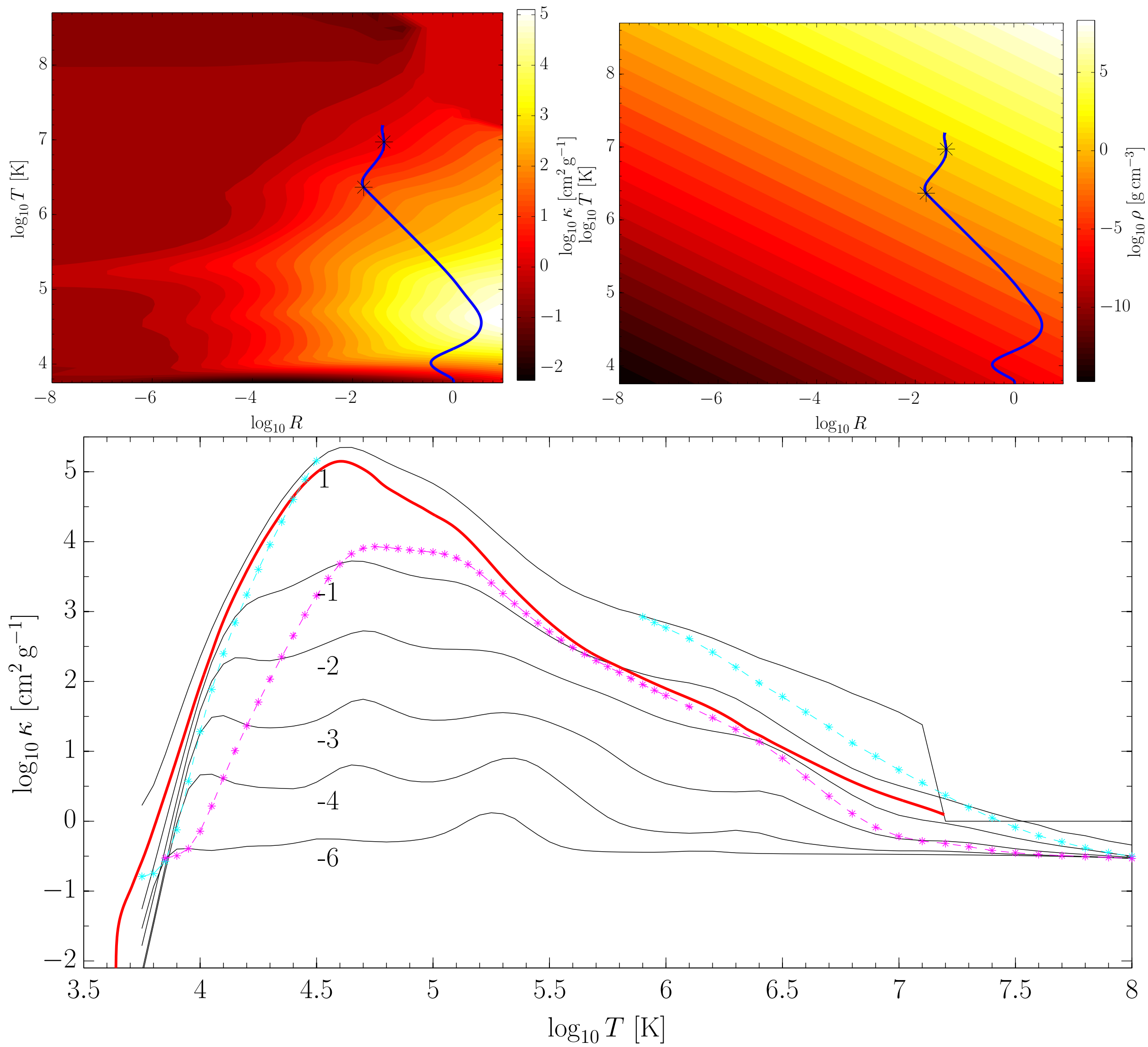

Figure 3.2: (Left top): OPAL Rosseland mean opacity data for a solar composition mixture. The x-axis quantity, R, is defined as $R = \rho T_6^{-3}$, where $T_6 = T \times 10^{-6}$ as usual. Since R can be multivalued for any ρ, T pair, on the right panel is plotted the density, for reference. In each case the blue curve are the values for a standard interior solar model, and the 2 black stars represent the boundaries of the core and convection zone ($0.2R_\odot$ and $0.7R_\odot$, respectively). The direction of increasing T is toward the stellar center. (Bottom): Cuts through the opacity values for specific values of $\log R$, given by the labeled curves. The red curve is the opacity from a current solar model. The cyan and the magenta symbols represent the 0.5 and 0.99 ionization curves for pure hydrogen, respectively, as in Figure 2.3. (Specifically the data are from computation 200503120009, Table 73. The calculations consider about 20 species.)

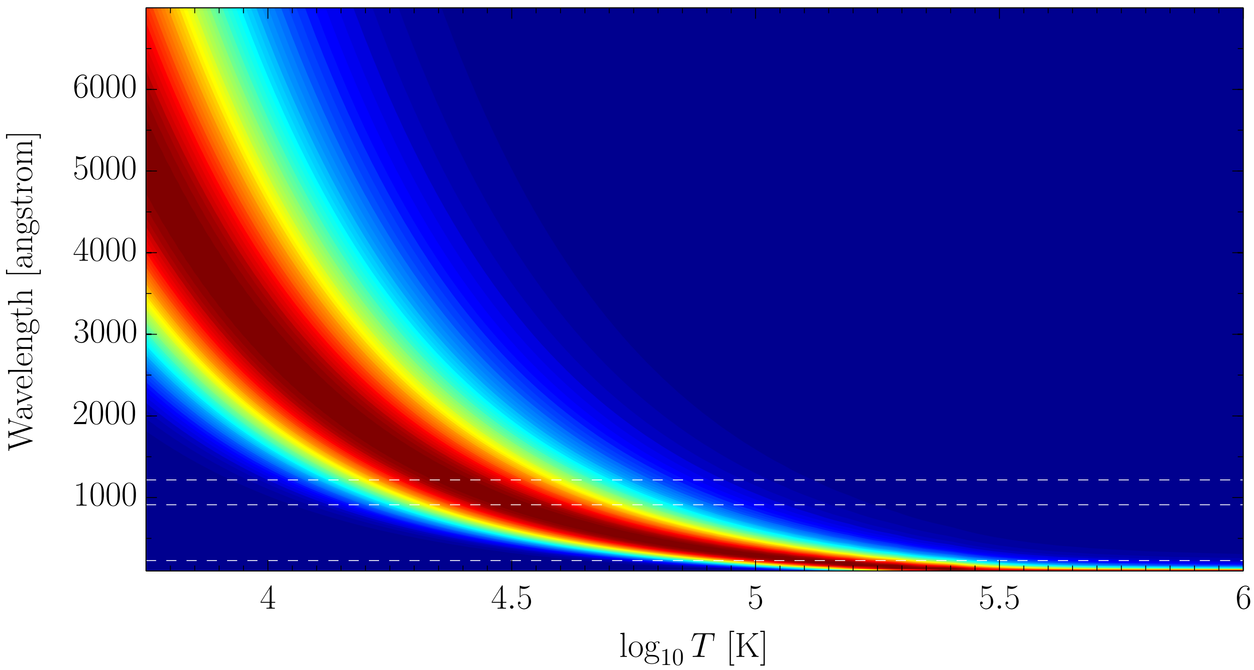

Figure 3.3: Blackbody curves for a range of wavelengths and temperatures. The curve at each temperature is normalized to its maximum value across wavelengths. Wavelength guides from top to bottom are at $1216\,\mathring{A}$, $912\,\mathring{A}$, and $228\,\mathring{A}$.

- Consider the near-surface of a star where radiation into space dominates over the gas pressure
- We found before that
$$P_{\rm rad} = \frac{1}{3}aT^4,$$
so
$$\frac{{\rm d}P_{\rm rad}}{{\rm d}T} = \frac{4}{3}aT^3\frac{{\rm d}T}{{\rm d}r}. \tag{3.24}$$
- The radiative flux can then be expressed as (see Equation (3.6))
$$F_{\rm rad} = -\frac{c}{\kappa_{\rm R}\rho}\frac{{\rm d}P_{\rm rad}}{{\rm d}r}. \tag{3.25}$$
- Let's assume that it might be possible that the radiation pressure overcomes gravity.
- In that case consider hydrostatic (non)equilibrium to occur when
$$-\frac{{\rm d}P_{\rm rad}}{{\rm d}r} > \rho g. \tag{3.26}$$
- Using Equation (3.25) this becomes
$$\frac{F_{\rm rad}\kappa_{\rm R}}{c} > g. \tag{3.27}$$
- If we just consider the area near the surface and look and consider $L = 4\pi R^2 F_{\rm rad}$, then
$$\frac{\kappa_{\rm R}L}{4\pi cGM} > 1. \tag{3.28}$$
- We can write the quantity on the left as
$$\frac{\kappa_{\rm R}L}{4\pi cGM} = 7.8\times10^{-5}\kappa_{\rm R}\left(\frac{L}{L_\odot}\right)\left(\frac{M}{M_\odot}\right)^{-1}, \tag{3.29}$$
which shows that this number is typically small, much less than 1.

- For massive stars, however, the luminosities can get quite large.
- So if we define the Eddington luminosity at the point where this number is unity, then

$$\frac{L_{\rm Edd}}{L_\odot} \approx 3.7 \times 10^4 \left(\frac{M}{M_\odot}\right), \tag{3.30}$$

  where we used $\kappa = 0.34$.
- If the Eddington luminosity starts to approach this a good fraction of this value, then equilibrium is lost and severe mass loss occurs.

3.1.7 Final tools

- There are several manipulations we can carry out to make the expressions we derived more useful for later.
- For future use we will need different forms of Equation (3.7). Take hyrdrostatic equilibrium and use logarithmic derivatives:

$$\frac{\mathrm{d}\ln P}{\mathrm{d}\ln r} = -\frac{Gm\rho}{rP}. \tag{3.31}$$

- Dividing both sides by $\mathrm{d}\ln T/\mathrm{d}\ln r$ gives a new quantity we'll call "del"

$$\nabla \equiv \frac{\mathrm{d}\ln T}{\mathrm{d}\ln P} = -\frac{r^2 P}{Gm\rho}\frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}r}, \tag{3.32}$$

  which is the true driving gradient in the star.
- If we now consider that the luminosity L is carried ONLY by radiation, then we can define "delrad"

$$\nabla_{\rm rad} \equiv \left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_{\rm rad} = \frac{3}{16\pi acG}\frac{P\kappa_{\rm R}}{T^4}\frac{L}{m}, \tag{3.33}$$

  where we used Equation (3.7).
- So if $\nabla = \nabla_{\rm rad}$, the all the luminosity is radiative. If $\nabla_{\rm rad} > \nabla$, there is some other transport mechanism of the energy in addition to radiation.
- This quantity is the local slope which is required if all the luminosity were carried by radiation through diffusion.
- In fact, we will use this as a comparison in this unit to a similar quantity we've already introduced in Equation (3.3.1),

$$\nabla_{\rm ad} \equiv \left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_{\rm ad} = \frac{\Gamma_2 - 1}{\Gamma_2}. \tag{3.34}$$

  where this is defined in an "adiabatic" sense, or, i.e., at constant entropy.
- The value of 0.4 comes when considering an ideal gas:

$$\nabla_{\rm ad} = \frac{\frac{5}{3} - 1}{\frac{5}{3}} = \frac{2}{5}. \tag{3.35}$$