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5.4 Last stages of evolution: low-mass stars

Here we discuss the late stages of evolution after the thermal pulses to the white dwarf.

5.4.1 Production of s elements

- As mentioned earlier, AGB stars are spectroscopically enriched in s-process elements.
- The s refers to *slow* neutron captures (compared to β decay).
- About half of the elements heavier than Fe are created by this process.
- You need neutrons to make these heavier elements, where the neutron eventually decays into a proton to make a stabler isotope.
- The neutrons densities needed are lowish, of order $10v$ per unit volume.
- Since at this stage there is a large abundance of N-14 in the intershell region, it gets converted to Ne by
$$^{14}\mathrm{N}(\alpha,\gamma)^{18}\mathrm{F}(\beta^+,\nu)^{18}\mathrm{O}(\alpha,\gamma)^{22}\mathrm{Ne}. \tag{5.14}$$
- If the temperatures at the base of the intershell region get to a few hundreds of MK, then $^{22}\mathrm{Ne}(\alpha,\mathrm{n})^{25}\mathrm{Mg}$ can provide a source of neutrons.
- For stars of initial mass $< 3M_\odot$, it probably does not get hot enough for this reaction to occur.
- There may be some channels through carbon 13 that provide a high neutron density for low-mass AGB stars.
- However, this is still a very active area of research due to many physical uncertainties.

5.4.2 Planetary Nebula

- AGB stars of low mass continute to brighten as their (outer) H shell approaches the surface.
- The thermal pulse number is set by the mass of the H envelope and the mass of the CO core.
- If the CO core exceeds about $1.4M_\odot$, non-degenerate, non-explosive carbon burning sets in and the AGB phase is over.
- Due to mass-loss processes during the AGB pulses, no star is really able to reach that core mass, because the H-burning shell stops when it's at about $10^{-3}M_\odot$ below the surface.
- The mass loss can be quite large, even $10^{-5}\,\mathrm{M_\odot\,yr^{-1}}$
- These superwinds that are created speed off at $10\,\mathrm{km\,s^{-1}}$
- The mass loss could be explained by pulsations (Mira variables).
- The gas compression, and subsequent cooling and formation of molecules and dust grains, can trap the outgoing radiation and get carried away.
- Anyway, however it happens, when the pulses stop, and the star evolves to hotter effective temperatures.
- The maximal luminosity depends on the star's initial mass (and that of its envelope) and how much mass it has lost.

- The H shell burning region approaches the surface and the effective temperature increases.
- An even faster stellar wind is produced, up to $2000\,\mathrm{km\,s^{-1}}$, which bumps up against the previous shell ejecta - this produces interesting features in the planetary nebula.
- The shell is dusty and optically thick (masers).
- The envelope is irradiated by UV radiation from remaining hot central star (core).
- The gas gets ionized and recombines quickly, giving distinct emission lines when the stellar remnant reaches about $30,000$K.
- A thin H-burning shell continues until the bluest point on the evolutionary track.
- Then, the H-rich envelope and He-rich layer contract quickly. A few scenarios are now possible:
  1. All nuclear burning shuts off and the star cools as a WD.
  2. The heating of the He from contraction leads to a thermal runaway, and the star goes back near to the AGB (born again). Then does the same stuff and cools as a WD.
  3. The heating of the envelope causes a H-burning runaway and the star is a *self-induced nova*. The process can be dynamic and blow off all H layers to become a DB white dwarf. Or, the process can be quiescent and it will burn H and start to cool down, possibly leading to another nova event.

5.4.3 White Dwarfs

- Recall all the discussion in Sec. 2.6.4.
- Degenerate matter obeys polytropic relations $P \sim \rho^\gamma$.
- For non-relativistic particles, $\gamma = 5/3$.
- For relativistic particles, $\gamma = 4/3$.
- The cores of evolved degenerate stars like white dwarfs are dominated by electron pressure rather than ion pressure.
- That's because $\mu_e \approx 2$ and $\mu_{\text{ion}} \approx 12$, and $P \propto \mu^{-1}$.
- Why are white dwarfs special?
- Let's simply consider approximations to equilibrium with averages over the star, so

$$\frac{P}{M} = \frac{GM}{4\pi R^4}. \tag{5.15}$$

- For the polytrope, replacing density by its average value

$$P \sim \left(\frac{M}{R^3}\right)^\gamma. \tag{5.16}$$

- The "pressure" term f_p from equilibrium and the EOS, and the "gravity" term f_g from equilibrium, are

$$f_p \sim \frac{M^{\gamma-1}}{R^{3\gamma}}; \qquad f_g \sim \frac{M}{R^4}. \tag{5.17}$$

- Their ratio must be 1 for equilibrium

$$f = \frac{f_g}{f_p} \sim M^{2-\gamma} R^{3\gamma-4}. \tag{5.18}$$

- This is $M^{1/3}R$ for $\gamma = 5/3$, and $M^{2/3}$ for $\gamma = 4/3$.
- So consider a star less than some critical mass $M < M_{\rm crit}$ and non-relativistic electrons. The star can get into an equilibrium by just adjusting R so that $f = 1$.
- If we increase M so that $f > 1$ (more gravity), R must decrease to regain equilibrium (hence more massive WDs are smaller)
- Now consider relativistic electrons.
- We can only get equilibrium by setting the mass to a certain value $M = M_{\rm crit}$.
- If $M < M_{\rm crit}$, $f < 1$, and the pressure term is dominant and so the star expands so that the electrons become non relativistic.
- But if $M > M_{\rm crit}$ and $f > 1$, the gravity term forces the star to contract. But this does not help, because F is independent of R!
- The star collapses without "finding" an equilibrium.
- Clearly, $M_{\rm crit}$ is some limit.
- So again, consider a total degenerate equation of state then (recall Eq. 2.51)

$$P \approx \frac{R}{\mu_e}\rho T + K_\gamma \left(\frac{\rho}{\mu_e}\right)^\gamma . \tag{5.19}$$

- γ depends on density and relativistic effects, being $\gamma = 5/3$ for $\rho \ll 10^6$ and $\gamma = 4/3$ for $\rho \gg 10^6$.
- Using polytropic relationships once can derive a critical mass that governs the future behavior of the core of these dense stars

$$M_{\rm crit} = \left(\frac{K_{4/3}}{fG}\right)^{3/2} \mu_e^{-2}, \tag{5.20}$$

  where f is the ratio of the mean density to the central density.
- The cricital mass is then identified as the Chandrasekhar mass (Equation (2.168)):

$$\frac{M_{\rm Ch}}{M_\odot} = \frac{5.836}{\mu_e^2} = 1.456 \left(\frac{2}{\mu_{\rm e}}\right)^2 . \tag{5.21}$$

- It's also important to see how the central temperature and density depend on this critical mass:

$$\frac{\rho_c}{\mu_e} = \frac{1}{8}\left(\frac{K_{4/3}}{K_{5/3}}\right)^3 \left(\frac{M_c}{M_{\rm crit}}\right)^2 \approx 2.4\times 10^5\,{\rm g\,cm^{-3}} \left(\frac{{\rm M_c}}{{\rm M_{crit}}}\right)^2 , \tag{5.22}$$

$$T_c = \frac{1}{R}\frac{K_{4/3}^2}{K_{5/3}}\left(\frac{M_c}{M_{\rm crit}}\right)^{4/3} \approx 0.5\times 10^9 \left(\frac{M_c}{M_{\rm crit}}\right)^{4/3} {\rm K}. \tag{5.23}$$

- For core masses below critical, maximum temperatures cannot exceed about 500 million K.
- In white dwarfs it is believed that the electrons are relativistic in the central part, but non-relativistic in the outer part.
- This changes the above results quantitatively, but not qualitatively.
- The mass of the core compared to the critical (Chandrasekhar) mass can be distinguised by 4 cases:

Case 1: If $M_c < M_{\text{crit}} \approx M_{\text{Ch}}$ and if there is no significant envelope (from mass loss or just a small original mass), so that M_c will NOT approach M_{Ch} during shell burning, then the core becomes degenerate, will cool, and the star becomes a white dwarf. T_c peaks. If it is a member of a binary system, then it can accrete enough mass to ignite carbon, which will detonate He and destroy the star in a runaway, producing at Type I supernova

Case 2: Initially if $M_c < M_{\text{crit}}$ but there remains an envelope such that shell burning M_c can grow to M_{Ch}, the core becomes degenerate and cools. However, ρ_c increases with M_c and carbon will be ignited. This will happen for $4 \lesssim M/M_{\odot} < 8$, and the stars will likely become white dwarfs as well, but C-O white dwarfs.

Case 3: If $M_{\text{crit}} < M \lesssim 40M_{\odot}$, degeneracy does not happen. The core can heat up even more (because of no degeneracy) and further nuclear reactions can occur. Eventually the core collapses leading to formation of a neutron star and ejection of the envelope. This is a Type II supernova. See next section.

Case 4: If $M_c \geq 40M_{\odot}$, the core also will burn C non degenerately. Black hole. See next section.

- In reality, almost all stars born with about $8M_{\odot}$ or less will lose much mass and will not reach the interior conditions to ignite carbon.
- Most WDs are observed at $0.6M\odot$ with very little variation.
- The higher-mass stars will lose all of their envelope and become CO WDs.