Table 5-1 Reactions of the PP chains

Reaction	*Q value, Mev*	*Average ν loss, Mev*	S_0, *kev barns*	$\frac{dS}{dE}$, *barns*	B	τ_{12}, *years*†
$H^1(p,\beta^+\nu)D^2$	1.442	0.263	3.78×10^{-22}	4.2×10^{-24}	33.81	7.9×10^{9}
$D^2(p,\gamma)He^3$	5.493		2.5×10^{-4}	7.9×10^{-6}	37.21	4.4×10^{-8}
$He^3(He^3,2p)He^4$	12.859		5.0×10^{3}		122.77	2.4×10^{5}
$He^3(\alpha,\gamma)Be^7$	1.586		4.7×10^{-1}	-2.8×10^{-4}	122.28	9.7×10^{5}
$Be^7(e^-,\nu)Li^7$	0.861	0.80				3.9×10^{-1}
$Li^7(p,\alpha)He^4$	17.347		1.2×10^{2}		84.73	1.8×10^{-5}
$Be^7(p,\gamma)B^8$	0.135		4.0×10^{-2}		102.65	6.6×10^{1}
$B^8(\beta^+\nu)Be^{8*}(\alpha)He^4$	18.074	7.2				3×10^{-8}

† Computed for $X = Y = 0.5$, $\rho = 100$, $T_6 = 15$ (sun).

Figure 1.3: The properties of the relevant chains of the proton-proton reaction. From Clayton [1983].

September 1 . . . .

1.2.1 PP-I chain

- The proton-proton reaction is

$$\begin{aligned}
{}^1\mathrm{H} + {}^1\mathrm{H} &\longrightarrow {}^2\mathrm{D} + \mathrm{e}^+ + \nu = 2(7.289) - 13.136 - 0.263 = 1.18\,\mathrm{MeV};\ (10^9\,\mathrm{yr}) \\
{}^2\mathrm{D} + {}^1\mathrm{H} &\longrightarrow {}^3\mathrm{He} + \gamma = 13.136 + 7.289 - 14.93 = 5.49\,\mathrm{MeV};\ (6\,\mathrm{s}), \\
{}^3\mathrm{He} + {}^3\mathrm{He} &\longrightarrow {}^4\mathrm{He} + {}^1\mathrm{H} + {}^1\mathrm{H} = 2(14.93) - 2.42 - 2(7.289) = 12.86\,\mathrm{MeV};\ (10^6\,\mathrm{yr})
\end{aligned}$$

  Noting that the first two reactions have to happen twice to produce 2 ^{3}He nuclei, the total energy is 2(1.18)+2(5.49)+12.86 = 26.2 MeV, as we saw before.

- Another way to write this is

$$ {}^1\mathrm{H}({}^1\mathrm{H}, \mathrm{e}^+\nu_\mathrm{e})^2\mathrm{D}({}^1\mathrm{H}, \gamma)^3\mathrm{He}({}^3\mathrm{He}, 2{}^1\mathrm{H})^4\mathrm{He}, \tag{1.51}$$

  where everything to the left of the comma is an ingredient of the reaction, and everything to the right is a product.

- See the table in Figure 1.3 for quantities, particularly the neutrino loss contribution. The times given apply to a single nucleus in the stellar interior environment.

- Neutrino production is the main reason why we know these processes are taking place in stellar interiors.

- The onset of H burning through this channel can occur at about 5-10 MK.

- The first part of the chain is by far the slowest, because a proton is converting into a neutron through the weak force (beta decay), and this is quite rare.

- Note this chain can occur in a pure hydrogen gas.

- The first reaction in the chain goes the slowest and so the rate of energy generation is controlled by it.

- The deuterium burning reaction is very fast, and stars should destroy all of it. The large abundance on Earth is an interesting problem.

PROBLEM 1.7: [5 pts]: The Sun's luminosity is $L_\odot = 3.9 \times 10^{33}\,\mathrm{erg\,s^{-1}}$. Assume that the energy for this luminosity is provided solely by the PP-I chain, and that neutrinos carry off 3% of the energy liberated. How many neutrinos are produced per second? What is the neutrino flux at earth?

PROBLEM 1.8: [5 pts]: Show that at a temperature of $T = 15$ MK the temperature exponent (Equation (1.33)) in the first reaction of the PP-I chain is $n \simeq 4$.

1.2.2 PP-II and PP-III chains

- After the second step of the PP-I chain, the ^{3}He that was produced has a choice, which is dictated mainly by temperature.
- Helium 4 can also be produced from hydrogen in 2 other ways.
- PP-II chain:
$$^{3}\mathrm{He}(^{4}\mathrm{He},\gamma)^{7}\mathrm{Be}(\mathrm{e}^-,\nu_\mathrm{e})^{7}\mathrm{Li}(^{1}\mathrm{H},^{4}\mathrm{He})^{4}\mathrm{He} \tag{1.52}$$
- PP-III chain:
$$^{7}\mathrm{Be}(^{1}\mathrm{H},\gamma)^{8}\mathrm{B}(,\mathrm{e}^+\nu_\mathrm{e})^{8}\mathrm{Be}(,^{4}\mathrm{He})^{4}\mathrm{He} \tag{1.53}$$
- Note that the berrylium 7 nucleus has a choice to react with an electron (to form lithium 7) or a proton (to form berrylium 8).
- For temperatures above about 15 million K, helium 3 likes to react with helium 4 (rather than itself as in the last reaction of PP-I) and so the PP-I chain is not as dominant at hotter conditions.
- The average neutrino energy loss from PP-II in the berrylium electron capture is 0.8 MeV.
- The average neutrino energy loss from PP-III in the positron decay of boron is 7.2 MeV, which is large.
- These neutrinos coming from the Sun can be detected and have been critical to understanding fusion processes. They are the dominant ones observed in the water tank experiments and the cleaning fluid experiment: $\nu_\mathrm{e}(^{37}\mathrm{Cl},\mathrm{e}^-)^{37}\mathrm{Ar}$.
- The branching ratio among these three chains depends quite sensitively on interior conditions.

PROBLEM 1.9: [10 pts]: Compute the *full* Q values for the PP-II and PP-III chains (don't forget to include any necessary contributions from previous chains in the computation).

1.2.3 CNO cycle

- Another way to turn hydrogen into helium is by producing (and burning) carbon, nitrogren, and oxygen, as long as such species exist. This typically happens in higher-mass stars with higher internal temperatures.
- There are two possible ways this happens, each involving 6 reactions. This bi-cycle is written as

$$^{12}\mathrm{C}(^{1}\mathrm{H},\gamma)^{13}\mathrm{N}(,\mathrm{e}^+\nu_\mathrm{e})^{13}\mathrm{C}(^{1}\mathrm{H},\gamma)^{14}\mathrm{N}(^{1}\mathrm{H},\gamma)^{15}\mathrm{O}(,\mathrm{e}^+\nu_\mathrm{e})^{15}\mathrm{N}(^{1}\mathrm{H},^{4}\mathrm{He})^{12}\mathrm{C}, \tag{1.54}$$

$$^{14}\mathrm{N}(^{1}\mathrm{H},\gamma)^{15}\mathrm{O}(,\mathrm{e}^+\nu)^{15}\mathrm{N}(^{1}\mathrm{H},\gamma)^{16}\mathrm{O}(^{1}\mathrm{H},\gamma)^{17}\mathrm{F}(,\mathrm{e}^+\nu)^{17}\mathrm{O}(^{1}\mathrm{H},^{4}\mathrm{He})^{14}\mathrm{N}. \tag{1.55}$$

Table 5-2 The CNO reactions

Reaction	Q value, Mev	Average ν loss, Mev	S(E = 0), kev barns	$\frac{dS}{dE}$, barns	B
$C^{12}(p,\gamma)N^{13}$	1.944		1.40	4.26×10^{-3}	136.93
$N^{13}(\beta^+\nu)C^{13}$	2.221	0.710			
$C^{13}(p,\gamma)N^{14}$	7.550		5.50	1.34×10^{-2}	137.20
$N^{14}(p,\gamma)O^{15}$	7.293		2.75		152.31
$O^{15}(\beta^+,\nu)N^{15}$	2.761	1.00			
$N^{15}(p,\alpha)C^{12}$	4.965		5.34×10^{4}	8.22×10^{2}	152.54
$N^{15}(p,\gamma)O^{16}$	12.126		2.74×10^{1}	1.86×10^{-1}	152.54
$O^{16}(p,\gamma)F^{17}$	0.601		1.03×10^{1}	-2.81×10^{-2}	166.96
$F^{17}(\beta^+\nu)O^{17}$	2.762	0.94			
$O^{17}(p,\alpha)N^{14}$	1.193		Resonant reaction		167.15

Figure 1.4: The properties of the relevant chains of the CNO cycle. From Clayton [1983].

- The nucleon making 2 choices is ^{15}N upon interaction with a proton.
- The energy released is $Q = 25.02$ MeV.
- For Pop. I stars, CNO material makes up about 75% of metals, and O is about 70% of CNO.
- The slowest reaction and the one that determines the overall reaction rate is the proton capture $^{14}\mathrm{N}(^{1}\mathrm{H},\gamma)^{15}\mathrm{O}$. This reaction is highly temperature dependent (Problem 1.10).
- See Figure 1.5 for the crossover regime between PP and CNO channels.
- As this is the slowest reaction, the most abundant species in the CNO cycle is ^{14}N.
- Generally, through these channels, the final abundances of C and O decrease, while that of N increases.
- We'll come to He burning later in the post main-sequence unit

PROBLEM 1.10: [5 pts]: Show that at a temperature of $T = 15$ MK the temperature exponent in the slowest reaction of the CNO cycle is $n \simeq 20$.

COMPUTER PROBLEM 1.1: [20 pts]: Here you will look at the effects of "turning off" nuclear reactions at the main sequence to see how stellar evolution changes. `MESA` allows one full control of nuclear energy generation. We will explore how this changes the star right after the time it formed and is getting to the main sequence.
Hand in your answers to the questions below with figures. You may also prepare a document with all answers and figures and send as a .pdf (only).

What to do

1. Copy the `WORK_DIR` to wherever you will be running MESA, rename it to something sensible.
2. Edit the `inlist_project` file. In `&star_job`, make sure `pgstar_flag=.true.` and we don't need to create a pre-main sequence track yet, so you can add `create_pre_main_sequence_model=.false.` In the `&controls` section, we want the `initial_mass=1.0`. Run the model until about 10 billion years, so set `max_age=1d10`. Most importantly, for this first run, turn off the nuclear reaction rates by setting `eps_nuc_factor=0` and `dxdt_nuc_factor=0`.
3. You shouldn't need to run more than about 1000 models (timesteps) to reach that age based on the default dt. All the data gets saved in `LOGS/`.

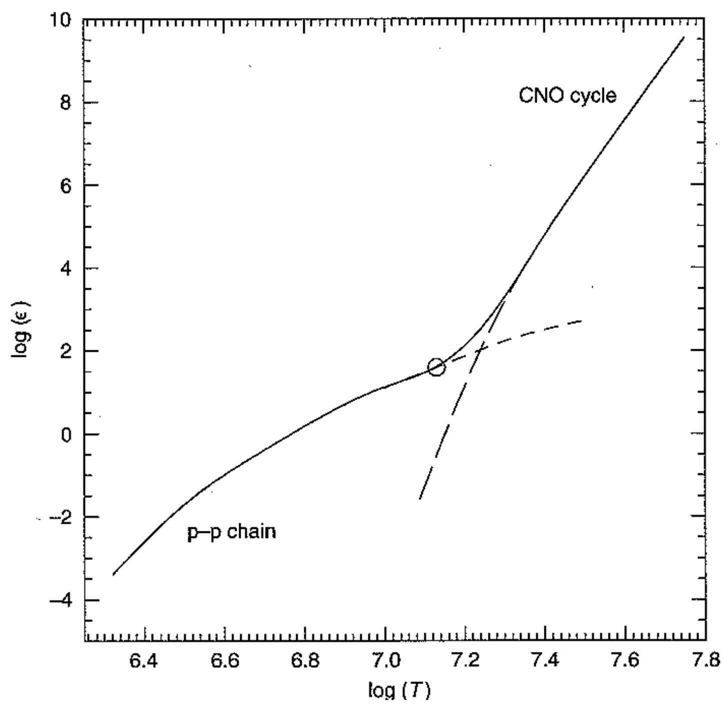

Figure 1.5: The nuclear energy rate as a function of temperature. The Sun is marked as a circle. From Salaris and Cassisi [2006].

4. Now copy a new working directory and maybe copy the inlist you just used into it and change the following: Turn on the reactions by setting that variable to 1. We need a stopping criterion, because 10 billion years would take us to the terminal age main sequence, and so we'll use the onset of hyrdrogen burning and set an abundance criterion. So in `&controls` add `xa_central_lower_limit_species(1)='h1'` and then `xa_central_lower_limit(1)=0.69` (the default initial H abundance is 0.7). So right after a little bit of central hydrogen is burned (depleted), the simulation will stop.

Questions

1. How old was the star with nuclear burning when the hydrogen abundance dropped below 0.7? Is that reasonable?
2. Plot a proper HR diagram with the "tracks" of both stars on it (luminosity vs effective temperature). Try to give some indication of age on the plot.
3. Describe the two tracks qualitatively.
4. Explain why or how the star with no nuclear burning gets so much hotter than the star with nuclear burning. What is physically happening? Then show a plot that should confirm your explanation. What happens for the star with no burning at later times, explain its track on the HR diagram? (Look around for the appropriate quantities to plot in the evolution variables, it's up to you).

1.3 List of things not discussed

These are things that belong in this unit that won't be covered or will be later on.

- Equilibrium abundances. How the species change with time as material gets depleted or created. There are good discussions of this in other texts.

Table 4-1 Atomic mass excesses†

Z	*Element*	A	$M - A$, *Mev*
0	n	1	8.07144
1	H	1	7.28899
	D	2	13.13591
	T	3	14.94995
	H	4	28.22000
		5	31.09000
2	He	3	14.93134
		4	2.42475
		5	11.45400
		6	17.59820
		7	26.03000
		8	32.00000
3	Li	5	11.67900
		6	14.08840
		7	14.90730
		8	20.94620
		9	24.96500
4	Be	6	18.37560
		7	15.76890
		8	4.94420
		9	11.35050
		10	12.60700
		11	20.18100
5	B	7	27.99000
		8	22.92310
		9	12.41860
		10	12.05220
		11	8.66768
		12	13.37020
		13	16.56160
6	C	9	28.99000
		10	15.65800
		11	10.64840
		12	0
		13	3.12460
		14	3.01982
		15	9.87320
7	N	12	17.36400
		13	5.34520
		14	2.86373
		15	0.10040
		16	5.68510
		17	7.87100
8	O	14	8.00800
		15	2.85990
		16	−4.73655
		17	−0.80770
		18	−0.78243
		19	3.33270
		20	3.79900
9	F	16	10.90400
		17	1.95190
		18	0.87240
		19	−1.48600
		20	−0.01190
		21	−0.04600
10	Ne	18	5.31930
		19	1.75200
		20	−7.04150
		21	−5.72990
		22	−8.02490
		23	−5.14830
		24	−5.94900
11	Na	20	8.28000
		21	−2.18500
		22	−5.18220
		23	−9.52830
		24	−8.41840
		25	−9.35600
		26	−7.69000
12	Mg	22	−0.14000
		23	−5.47240
		24	−13.93330
		25	−13.19070
		26	−16.21420
		27	−14.58260
		28	−15.02000
13	Al	24	0.1000
		25	−8.9310
		26	−12.2108
		27	−17.1961
		28	−16.8554
		29	−18.2180
		30	−17.1500
14	Si	26	−7.1320
		27	−12.3860
		28	−21.4899
		29	−21.8936
		30	−24.4394
		31	−22.9620
		32	−24.2000
15	P	28	−7.6600
		29	−16.9450
		30	−20.1970
		31	−24.4376
		32	−24.3027

Figure 1.6: Mass excesses from Clayton [1983].

- Heavier element burning chains. Helium burning up to silicon burning will be discussed in later sections when post-main sequence evolution is covered.
- Gravity also provides an energy source, and will be discussed soon.