Wave Stuff

A

acoustic waves (aka sound waves)

- Longitudinal
- Isotropic
- Compressible (propagate by means of adiabatic compression and decompression)
- travel at sound speed (determined by the medium in which they were *created*, and maintain this speed even if they travel into medium with a different characteristic sound speed).
- Important quantities:
  - sound pressure
  - particle velocity
  - particle displacement
  - sound intensity
- exist because of a pressure restoring force: local compression (or rarefaction) sets up a *pressure gradient* in opposition to the motion
- carry energy away from source
- large enough amplitude → shock wave, but usually small; ambient gas slightly disturbed
- exist in medium with low or non-existent magnetic field
- isotropic propagate equally in all directions. The phase and group velocities are both equal to the sound speed (hence, the part where acoustic waves are sound waves).

Alfvén waves

Alfvén waves are a type of MHD wave in which ions oscillate in response to a restoring force provided by an effective tension on the magentic field lines. Low frequency (compared to the ion cyclotron frequency) traveling oscillation of the ion and the magnetic field. The ion mass density provides the inertia and the magnetic field line tension provides the restoring force. They propagate in the direction of the magnetic field, although waves exist at *oblique* incidence and smoothly change into the *magnetoacoustic* wave when the propagation is *perpendicular* to the magnetic field. The motion of the ions and the perturbation of the magnetic field are in the same direction and transverse to the direction of propagation. The wave is dispersionless.

Locally supported, phase and group velocities with magnitude equal to local Alfvén spped, directed along the magnetic field. Alfvén waves are easily excited by various dynamical perturbations of magnetic field lines, and are weakily dissipative (can propagate long distances, deposit energy and momentum far from source).

Properties:

- m=0 (Axisymmetric, or azimuthally symmetric)
- transverse (shear) perturbations (perpendicular to $\vec{F}_{res}$); plasma has characteristic elasticity.
- parallel to $\vec{B}$ (the *group* velocity is strictly along $\vec{B}$, but the *phase* velocity doesn't have to be. The energy also propagates along the field lines, probably because the wave envelope is what carries all the information).
- (only) driving force: magnetic tension, which "snaps" the field back into a straight line, producing an Alfvén wave.
- Purely magnetic and incompressible in nature (in untwisted straight cylinder... twist may cause some compression).
- Displacement of plasma together with magnetic field frozen into it.
- velocity: $v_A = \frac{B}{\mu_o \rho}$; ~ 1000 km s^{-1} in the corona.

How to observe:

- Only get Doppler shifts from *long*-period waves (> a few minutes).
- Measure additional (i.e. non-thermal) broadening of coronal emission lines; indirect way to observe short-period waves.
- Spatial variation in Doppler shift for long periods. Gyrosynchrotron emission in radio regime.
- V_A: temporal resolution?

Effects of twisting (or *torsion*):

- Coupling of various MHD modes

(From *Priest*): Types of Alfvén waves:

- Shear or Torsional: No accompanying pressure or density changes (plasma).
- Compressional or Fast-Mode: Becomes a fast magnetoacoutsic or fast-mode wave when pressure gradients are included (Note: these are often the preferred names, even when the pressure gradient is unimportant, so as to avoid confusion with shear Alfvén waves).

B

ballooning modes

- m > 1
- Role not established yet

Bessel's equations

(From Boas p. 587): Bessel functions are damped sines and cosines. Solutions of differential equations can be represented by power series. Graphs, formulas; electricty, heat, hydrodynamics, elasticity, wave motion, quantum..., cylindrical symmetry...

Bessel's differential equation:

$$x^2 \frac{d^y}{dx2} + x\frac{dy}{dx} + (x^2 - p^2)y = 0$$

where p is the *order* of the Bessel function y and is a constant, and y is the solution. a = integer → cylindrical, a = half-integer → spherical.

$$x^2 y'' + xy' + (x^2 - p^2)y = 0$$

$$x(xy') + (x^2 - p^2)y' = 0$$

body modes

C

coronal loops

- Main observational feature of the magnetic structure in the upper solar atmosphere.

coronal seismology

cusp speed

See *tube speed*

D

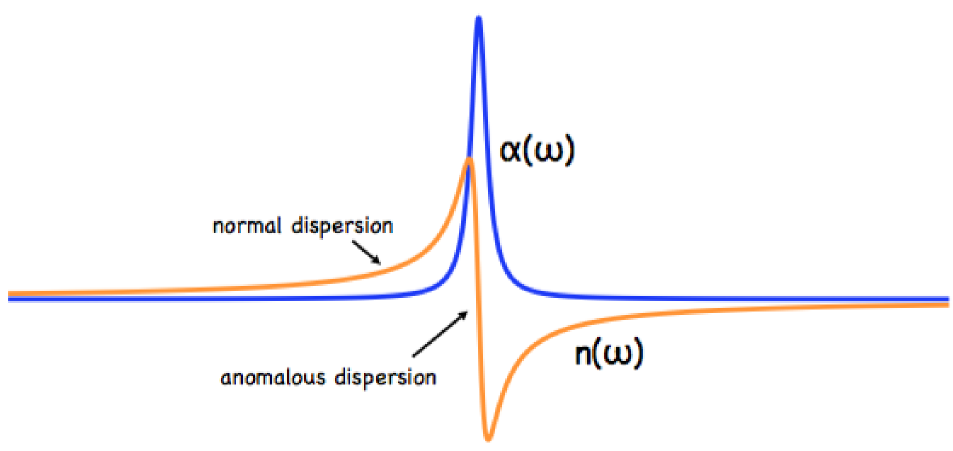
dispersion

Dispersion is when the distinct phase velocities of the components of the envelope cause the wave packet to "spread out" over time. The components of the wave packet (or envelope) move apart to the degree where they no longer combine to complete the envelope.

Causes different components of the wave to have different phase velocities.

Different kinds of dispersion:

- Normal dispersion: strictly increasing Re $\epsilon(\omega)$ with increasing ω.
- Anomalous dispersion: decreasing Re $\epsilon(\omega)$ with increasing ω.
- Resonant absorption: occurs in regions where Im $\epsilon(\omega)$ is large.



No dispersion: $v_{ph} = v_{gr}$; dispersion: $v_{ph} \neq v_{gr}$. A medium that is free from dispersion has index of refraction that is constant as a function of frequency, so all wavelengths are similarly affected. Permittivity and permeability are functions of frequency. Non-dispersive waves have phase speed (v_{ph}, speed that wave actually travels through medium?) independent of wavenumber, k. All waves of *any* k propagate at the same speed. Uniform dissipation, resonant mode conversion, physical mechanisms vs. ?

Dispersion *relation*:

- Relates the wavelength (or wavenumber) of a wave to its frequency.
- Describe the effect of dispersion in a medium on the properties of a wave traveling through that medium.

E

F

fast waves

- $C_{A_0} < C_{fast} < C_{A_e}$
- highly dispersive.
- Kink modes
- Sausage modes
- propagate faster than both V_A and C_s

flute modes

See *ballooning modes*

fundamental modes

See *global modes*

G

global modes

- $y(x, t)$ governed by some PDE with no *explicit* time dependence.

- A global mode is a solution of the form $y(x,t) = \hat{y}(x)e^{i\omega t}$
- PDE-dynamical system of infinitely many equations coupled together.

gravity waves

Generated in fluid medium or interface between two media when the force of gravity or buoyency tries to restore equilibrium e.g. “wind waves” from between atmosphere and ocean. P-modes are global acoustic oscillations. (Note: *gravitational* waves are not the same thing; they have something to do with relativity).

group velocity

gyrosynchrotron radiation

Electromagnetic emission emitted by mildly relativistic electrons moving in a magnetic field (as opposed to synchrotron, with *ultra*relativistic particles).

H

I

instabilities

A disturbance that is not stabilized by the resulting forces.

J

K

kink modes/waves/oscillations/instabilities

Fast kink waves:

- transverse (general property of fast waves)
- $v_{ph} = c_k = \sqrt{\frac{\rho_o V_{Ao}^2 + \rho_e V_{Ae}^2}{\rho_o + \rho_e}} \approx V_A \sqrt{\frac{2}{1+\frac{\rho_e}{\rho_o}}}$ in the low-β plasma.
- Period $P = \frac{2\ell}{V_A}\sqrt{\frac{1+\rho_e/\rho_o}{2}}$ where $\lambda = 2\ell$ (ℓ is the loop length). Typically, $\ell \approx 60 - 600$ Mm in the corona.
- Period of global kink mode, $P = \frac{2\ell}{c_K}$
- Important observation from which magnetic field strength can be derived.
- slab: phase speed = V_{A_e}
- tube: kink speed:

$$c_K = \frac{B_i^2 + B_e^2}{\sqrt{\mu(\rho_i + \rho_e)}}$$

  (mean Alfvénic speed). Slab (or tube) is moved laterally; little variation in cross-sec, density, or intensity.

Standing and propagating both rapidly damped.

Slow kink waves:

- longitudinal (velocity directed along magnetic field)
- compressible (variations in density and intensity)

Other (or simply unorganized):

- oblique (inclined with respect to the flow direction)
- *weakly compressible*, but could nevertheless be observed with imaging instruments as *periodic standing* or *propagating* displacements of coronal structures, e.g. coronal loops. The frequency of transverse or “kink” modes is given by the following expression:

$$w_K = \sqrt{\frac{2k_z B^2}{\mu(\rho_i + \rho_e)}}$$

  In a cylindrical model of a loop, the parameter *azimuthal wave number*, m, is equal to 1 for kink modes.
- In the long wavelength limit, the phase speed of all but sausage fast modes tends to the so-called kink speed, which corresponds to the density weighed average Alfvén speed.
- Two instabilities of axisymmetric, current-carrying plasmas

    - sawtooth relaxations
    - fishbone oscillations

  associated with instability of internal kink modes
- Both standing and propagating, T = $\sim$ seconds-minutes.
- lowest spatial harmonics along field; global (fundamental) modes of coronal loops nodes of displacement at footpoints, maximum at apex.

L

leaky modes

- Waves are allowed to radiate into the external medium, i.e. the condition of mode localization is relaxed.
- complex eigenfrequencies
- Bessel functions are replaced by Hankel functions in the dispersion relation. Can be the fundamental harmonic.
- Wavenumbers below cutoff value?
- Has electric field that decays monotonically for a finite distance in the transverse direction but becomes oscillatory everywhere beyond that finite distance.
- Mode "leaks" out of the waveguide as it travels down it, producing attenuation.
- Relative amplitude of oscillatory part (leakage rate) must be sufficiently small that the mode maintains its shape as it decays, in order to be called a mode at all.

longitudinal waves

waves in which the displacement of the *medium* is in the same direction as, or the opposite direction to, the direction of travel of the wave.

M

magnetoacoustic waves

- A magnetosonic wave (also magnetoacoustic wave) is a longitudinal wave of ions (and electrons) in a magnetized plasma propagating perpendicular to the stationary magnetic field.
- compressible
- slow MHD wave; slow MA waves only have 1-3 oscillations before damping out, observed oscillations are manifestation of rapid damping due to radiative energy losses. Reduced $\vec{B}$ regions = increased $\rho \rightarrow$ rapid radiative losses. *Fast* MH waves radiate little because they're damped too slowly.
- collectively supported by the plasma environment, i.e., the wave mode acts across neighbouring magnetic field lines and across transverse plasma inhomogeneities.
- observed as distrubances of EUV (and possibly X-ray) emission

magnetohydrodynamic (MHD) waves

- study of electrically conducting fluids (plasma)
- Theoretical foundation:
  - dispersion relation of MHD modes of a plasma cylinder
  - models: loops, prominence fibrils, plumes, various filaments
  - evolutionary equations
- Considerations of observed waves:
  - geometry: simple (slab or tube)? or more complex?
  - mode
    - longitudinal vs. transverse
    - compressible vs. incompressible
    - oscillating vs. propagating
    - fast vs. slow
    - propagating vs. standing
    - isotropic vs. anisotropic
    - phase differences?
- Interpreting observations: distinguish between modes that are pressure driven (acoustic/slow magnetoacoustic) and magnetically driven (Alfvén)

- Relations between the (internal and external) characteristic speeds (Alfvén, sound, and tube speeds) *determine properties of MHD modes guided by the tube.*
- The behavior of linear perturbations of the form

  $$\delta P_{tot}(r)\mathrm{exp}\left[i(k_z z + m\phi - \omega t)\right]$$

  is governed by the following system of first order differential equations and algebraic equations:

  $$D\frac{d}{dr}(r\xi_r) = (C_A^2 + C_s^2)\ldots$$

  Dissipation and Damping:
- Dissipation of MHD waves is manifold:
  - Couple with each other
  - interact non-linearly
  - resonantly interact with the closed waveguide
  - devolop non-linearly (e.g. solitons or shock waves can form)
- Inhomogeneous and magnetized plasma has two particular dissipation mechanisms of MHD waves:
  - Resonant absorption
  - Phase mixing

modes

A wave may be a superposition of lots of other waves. Each of those waves is a "mode" of the resultant wave (think of the foundation of Fourier Analysis: sums of sines and cosines). Modes with the lowest wave number are *global*, or *fundamental* modes.

- Different modes are driven by *different restoring forces.*

Moreton wave

Chromospheric signature of a large-scale coronal shock wave. Generated by flares, $\sim$ fast-mode MHD waves.

N

Normal modes

Vibrational state of an oscillatory *system* where the frequency is the same for all elements. E.g. resonant frequencies: equally spaced multiples of the fundamental.

O

oscillations

Three types:

1. un-damped
2. damped
3. forced

P

phase mixing

- Large gradients in Alfvén velocity.
- Alfvén waves suffer intense phase mixing
- Cause decay of Alfvén waves.
- *Not* likely to operate in closed magnetic structures (e.g. coronal loops)

Phase speed

$$v_p = \frac{\lambda}{T} = \frac{\omega}{k}$$

$$k = \frac{2\pi}{\lambda}, \omega = \frac{2\pi}{T}$$

polarization

- Doesn't apply to longitudinal waves, e.g. sound.
- Linear waves oscillate (transversely) in a single direction.

pressure waves

Longitudinal, fast, generated by turbulence near the photosphere, observed by measureing Doppler shift of absorption lines in the photosphere, spherical harmonics:

- n: radial order, number of nodes in radial direction
- l: angular degree, or harmonic degree, number of node lines on surface, $\sim$ total number of planes slicing the sun
- m: azimuthal number; number of surface nodal lines that cross the equator, number of planes slicing longitudinally. $-l \leq m \leq +l$

propagating acoustic waves (slow)

- $v_{ph} < 150$ km s^{-1} $\rightarrow$ slow
- longitudinal, compressive, anisotropic
- Parallel to $\vec{B}$, perturbation of $\vec{B}$ is negligible.
- Generated impulsively at one end of a footpoint.
- Only penetrate $\sim$ 10% into loop before damped by thermal conduction
- weak dispersion in coronal conditions ($V_A \gg c_s$)
- 3 phases: periodic, QP, decay
- period = 3, 5, 10 minutes? Or 2–22 seconds? (see kink_1),
- velocity: 50–200 km s^{-1}
- $c_T = \sqrt{\frac{c_s^2 v_A^2}{c_s^2 + v_A^2}}$ propagate sub-sonically at c_T, which is less than c_s
- "large" amplitude, max in top of chromosphere
- Observed using spectroscopy (intensity variations in EUV emission and Doppler shifts)

propagating acoustic waves (fast)

- $v_{ph} > 150$ km s^{-1} → fast (*or* transverse standing waves).
- Quasi-isotropic
- Driven by magnetic forces + plasma pressure forces
- Compressive (magnetic sound wave)
- Speed: $c_F = \sqrt{c_s^2 + v_A^2}$
- Moreton waves in the chromosphere
- Fast EUV waves in the corona

R

resonance

Periodic driving force frequency matches the wave frequency → large amplitude.

resonant absorption

- Mechanism of wave heating
- could damp kink mode oscillations.
- Loss of acoustic power in sunspots. Time scales:
  1. damping: collective mode → local mode, *independent* of dissipation.
  2. dissipative damping of small scale perturbations of local mode.

  $\tau_1 \ll \tau_2$
- inherently non-linear

S

sausage modes

- m = 0

- The fast magnetoacoustic sausage mode is another type of localized, modified fast magnetoacoustic wave.
- Mainly transverse.
- Standing fast sausage modes have symmetric tube modes
- Has a long-wavelength cutoff (trapped sausage modes do not exist at longer wavelengths). Approaches a cut-off at the external Alfvén speed. (Under condition of *mode localization*). *Effects* of the sausage mode are easiest to observe in the radio regime (not the wave itself), where more of a "point" is observed, rather than extended...
- Main feature is the periodic fluctuation of the cross-sectional area of the waveguide. This change is also associated with periodic fluctuations in density and temperature within the waveguide.
- Distinct sign of sausage oscillations is when periodic phenomena in cross-section and intensity are almost 180° out of phase → strong signal. (Less distinct signal is when periodicities in pore size don't match any intensity variations).
- Produce perturbations in density and magnetic field strength, and the corresponding plasma motions cause pulsations in the tube cross-section.
- Associated with perturbations of the loop cross-section and plasma concentration.
- Perturbations of plasma in the radial direction are stronger than perturbations along the field.
- Mode conversion and absorption through Alfvén resonance cannot take place (slow resonance can still operate).
- Phase speed is in the range between the Alfvén speed inside and outside the loop.

slow waves

- $C_{T_0} < C_{slow} < C_{s_0}$
- longitudinal
- acoustic(?)
- propagate slower than both V_A and C_s

speeds

Characteristic speeds of MHD:

1. sound

$$C_s = \sqrt{\frac{\gamma p_0}{\rho_0}}$$

$$C_s \approx 166 T_o^{1/2} ms^{-1} = 200 kms^{-1}$$

2. Alfvén

$$C_A = \frac{B_0}{\sqrt{\mu_0 \rho_0}}$$

$$V_A = 2.18 \times 10^{12} \frac{B_o}{\sqrt{n_o}} \text{m s}^{-1} = 3000 \text{km s}^{-1}$$

($B_o \sim 100$ G; $n_o \sim 10^{16}$ m^{-3})

spherical harmonics

3 kinds of resonant modes of oscillation:

- p (pressure)
- g (gravity)
- f (?)

Numbers:

- n - *order*; Number of nodes in radial direction
- l - *harmonic degree*; number of node lines on surface ∼ total number of planes slicing the sun.
- m - *something*; number of surface nodal lines that cross the equator; phase
  $-l \leq m \leq l$ (direction of waves is important); number of planes slicing longitudinally.

standing acoustic oscillations

Characteristics:

- Pressure forces in opposition

- Period = 7–31 minutes (20 minutes from another source)

- Decay times = 5.7–36.8 minutes
- Peak velocity = 200 km/sec

Standing oscillations vs. propagating waves

- In loops, propagating waves damp before reaching opposite footpoint.
- Velocity and intensity are 90° out of phase for standing oscillations, and are in phase for propagating acoustic waves.
- Frequencies less than the cutoff are standing oscillations, waves with frequency greater than the cutoff propagate into the chromosphere.
- no loop shape change or displacement
- near footpoints.

standing waves

Confined to a finite region of space → quantifies the frequency

- Fundamental (2 nodes, one on each end):

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

  where L is the string length, F is the tension, μ is the mass density
- 3 nodes: $f_2 = 2f_1$
- 4 nodes: $f_3 = 3f_1$

surface modes

evanescent behavior in both media (inside and outside cylinder).

T

torsional vibration

- angular vibration of an object ∼ shaft along the axis of vibration

transverse waves

(from wikipedia:) moving waves that *consist* of oscillations occurring perpendicular to the direction of energy transfer. If a *transverse* wave is moving in the positive x-direction, its *oscillations* are in up and down directions that lie in the yz plane. Light is an example of a transverse wave. ($\vec{B}$ and $\vec{E}$ oscillate in directions perpendicular to the direction in which the light is actually traveling). With regard to transverse waves in matter, the *displacement of the medium* is perpendicular to the direction of propagation of the wave. Examples: A ripple in a pond and a wave on a string.

trapped modes

For a specified geometry, uniqueness of the solution to a forcing problem at a particular frequency is equivalent to the non-existence of a trapped mode at that frequency. A trapped mode is a solution of the corresponding homogeneous problem and represents a free oscillation with finite energy of the fluid surrounding the fixed structure. For a given structure, trapped modes may exist only at discrete frequencies. Mathematically, a trapped mode corresponds to an eigenvalue embedded in the continuous spectrum of the relevant operator.

tube speed

Also known as *cusp speed*, the tube speed is the combination of sound and Alfvén speeds.

$$C_T = C_s C_A / (C_A^2 + C_s^2)^{1/2}$$

U

W

waves

- Caused by any turbulence in a medium.
- Observable properties

- Period
- wavelength
- amplitude
- temporal/spatial signatures (shape of perturbation)
- characteristic scenerios of evolution (e.g. damped?)

waveguide

- Width $\sim$ same order of magnitude as the wavelength of the guided wave.
- Pores extending up from the photosphere into the solar atmosphere can act as an MHD waveguide.
- Flux tubes are excellent waveguides

wave number

Number of waves in a unit distance. $k = 2\pi$

X

Y

Z