

# Wave Stuff

## A

### acoustic waves (aka sound waves)

- Longitudinal
- propagate by means of adiabatic compression and decompression
- travel at sound speed (determined by medium)
- Important quantities:
  - sound pressure
  - particle velocity
  - particle displacement
  - sound intensity
- exist because of a pressure restoring force: local compression (or rarefaction) sets up a *pressure gradient* in opposition to the motion
- carry energy away from source
- large enough amplitude  $\rightarrow$  shock wave, but usually small; ambient gas slightly disturbed
- exist in medium with low or non-existent magnetic field
- isotropic

### Alfvén waves

- *Purely magnetic* and *incompressible* in nature
- transverse (perpendicular to  $\vec{F}_{res}$ )
- torsional
- incompressible (under conditions of a straight cylinder (not twisted))
- type of MHD wave in which ions oscillate in response to a restoring force provided by an effective tension on the magnetic field lines.
- low frequency (compared to the ion cyclotron frequency) traveling oscillation of the ion and the magnetic field.
- propagates in the direction of the magnetic field, although waves exist at oblique incidence and smoothly change into the magnetosonic wave when the propagation is perpendicular to the magnetic field.
- possibly contribute to ion heating and acceleration in the solar wind via ion-cyclotron resonance with  $\nu > 10$  Hz, where  $R_m$  is very large; small scale structures down to resistive or kinetic scales.
- may be generated by tearing or interchange instabilities.
- locally supported, phase and group velocities with magnitude equal to local Alfvén speed, directed along the magnetic field.

- weakly dissipative (can propagate long distances, deposit energy and momentum far from source).
- easily excited by various dynamical perturbations of magnetic field lines.
- promising tool for heating and diagnostics of coronal magnetic structures.
- magnetic force *is* the restoring force. . . “snaps” back into a straight line, producing an Alfvén wave
- velocity is greater than the local sound speed in coronal conditions.
- questions/things to look up:
  - how often are they produced? how many?
  - present in magnetic structures? or everywhere?
  - three types (shear, torsional, fast magnetoacoustic)?

## B

### ballooning modes

- $m > 1$
- Role not established yet

### body modes

## C

### coronal loops

- Main observational feature of the magnetic structure in the upper solar atmosphere.

### cusplike speed

See *tube speed*

## D

### dispersion relations

- Relates the wavelength (or wavenumber) of a wave to its frequency.
- Describe the effect of dispersion in a medium on the properties of a wave traveling through that medium.

## E

## F

### fast waves

- $C_{A_0} < C_{fast} < C_{A_e}$
- highly dispersive.
- Kink modes
- Sausage modes
- propagate faster than both  $V_A$  and  $C_s$

### flute modes

See *ballooning modes*

### fundamental modes

See *global modes*

## G

### global modes

- $y(x, t)$  governed by some PDE with no *explicit* time dependence.
- A global mode is a solution of the form  $y(x, t) = \hat{y}(x)e^{i\omega t}$
- PDE-dynamical system of infinitely many equations coupled together.

### gravity waves

Generated in fluid medium or interface between two media when the force of gravity or buoyancy tries to restore equilibrium e.g. “wind waves” from between atmosphere and ocean. P-modes are global acoustic oscillations. (Note: *gravitational* waves are not the same thing; they have something to do with relativity).

**group velocity**

**gyrosynchrotron radiation**

Electromagnetic emission emitted by mildly relativistic electrons moving in a magnetic field (as opposed to synchrotron, with *ultrarelativistic* particles).

**H**

**I**

**instabilities**

A disturbance that is not stabilized by the resulting forces.

**J**

**K**

**kink modes/waves/oscillations**

(from wikipedia:) Kink (or transverse) modes, are oblique (inclined with respect to the flow direction) fast magnetoacoustic (also known as magnetosonic waves) guided by the plasma structure; the mode *causes the displacement of the axis of the plasma structure*. These modes are *weakly compressible*, but could nevertheless be observed with imaging instruments as *periodic standing* or *propagating* displacements of coronal structures, e.g. coronal loops. The frequency of transverse or “kink” modes is given by the following expression:

$$w_K = \sqrt{\frac{2k_z B^2}{\mu(\rho_i + \rho_e)}}$$

In a cylindrical model of a loop, the parameter *azimuthal wave number*,  $m$ , is equal to 1 for kink modes. This means that the cylinder is “swaying with fixed ends”...?

In the long wavelength limit, the phase speed of all but sausage fast modes tends to the so-called kink speed, which corresponds to the density weighed average Alfvén speed.

# L

## leaky modes

- Waves are allowed to radiate into the external medium, i.e. the condition of mode localization is relaxed.
- complex eigenfrequencies
- Bessel functions are replaced by Hankel functions in the dispersion relation. Can be the fundamental harmonic.
- Wavenumbers below cutoff value?
- Has electric field that decays monotonically for a finite distance in the transverse direction but becomes oscillatory everywhere beyond that finite distance.
- Mode “leaks” out of the waveguide as it travels down it, producing attenuation.
- Relative amplitude of oscillatory part (leakage rate) must be sufficiently small that the mode maintains its shape as it decays, in order to be called a mode at all.

## longitudinal waves

waves in which the displacement of the *medium* is in the same direction as, or the opposite direction to, the direction of travel of the wave.

# M

## magnetoacoustic waves

- A magnetosonic wave (also magnetoacoustic wave) is a longitudinal wave of ions (and electrons) in a magnetized plasma propagating perpendicular to the stationary magnetic field.
- compressible
- slow MHD wave
- collectively supported by the plasma environment, i.e., the wave mode acts across neighbouring magnetic field lines and across transverse plasma inhomogeneities.
- observed as disturbances of EUV (and possibly X-ray) emission
- both propagating and standing waves are observed

## magnetohydrodynamic (MHD) waves

- study of electrically conducting fluids (plasma)
- Dissipation of MHD waves is manifold:
  - Couple with each other

- interact non-linearly
- resonantly interact with the closed waveguide
- develop non-linearly (e.g. solitons or shock waves can form)
- Inhomogeneous and magnetized plasma has two particular *dissipation* mechanisms of MHD waves:
  - Resonant absorption
  - Phase mixing
- Theoretical modeling:
  - dispersion relations
  - evolutionary equations
- Theoretical foundation:
  - dispersion relations of MHD modes of a plasma cylinder
  - models: loops, prominence fibrils, plumes, various filaments
  - evolutionary equations
- Considerations of observed waves:
  - geometry: simple (slab or tube)? or more complex?
  - mode
    - \* longitudinal vs. transverse
    - \* compressible vs. incompressible
    - \* oscillating vs. propagating
    - \* fast vs. slow
    - \* propagating vs. standing
    - \* isotropic vs. anisotropic
    - \* phase differences?
  - damping: leakage by radiating modes? resonant abs? phase mixing?
  - corona: compare obs and theory to discover new things
- Interpreting observations: distinguish between modes that are pressure driven (acoustic/slow magnetoacoustic) and magnetically driven (Alfvén)
- Relations between the (internal and external) characteristic speeds (Alfvén, sound, and tube speeds) *determine properties of MHD modes guided by the tube.*
- The behavior of linear perturbations of the form

$$\delta P_{tot}(r) \exp [i(k_z z + m\phi - \omega t)]$$

is governed by the following system of first order differential equations and algebraic equations:

$$D \frac{d}{dr} (r \xi_r) = (C_A^2 + C_s^2) \dots$$

- Modes:
  - something

## modes

A wave may be a superposition of lots of other waves. Each of those waves is a “mode” of the resultant wave (think of the foundation of Fourier Analysis: sums of sines and cosines). Modes with the lowest wave number are *global*, or *fundamental* modes.

- Different modes are driven by *different restoring forces*.

## Moreton wave

Chromospheric signature of a large-scale coronal shock wave. Generated by flares,  $\sim$  fast-mode MHD waves.

## N

## Normal modes

Vibrational state of an oscillatory *system* where the frequency is the same for all elements. E.g. resonant frequencies: equally spaced multiples of the fundamental.

## O

## oscillations

Three types:

1. un-damped
2. damped
3. forced

## P

## phase mixing

- Large gradients in Alfvén velocity.
- Alfvén waves suffer intense phase mixing
- Cause decay of Alfvén waves.
- *Not* likely to operate in closed magnetic structures (e.g. coronal loops)

## Phase speed

$$v_p = \frac{\lambda}{T} = \frac{\omega}{k}$$
$$k = \frac{2\pi}{\lambda}, \omega = \frac{2\pi}{T}$$

## polarization

- Doesn't apply to longitudinal waves, e.g. sound.
- Linear waves oscillate (transversely) in a single direction.

## pressure waves

This is what pressure waves are ...

## R

### resonance

Periodic driving force frequency matches the wave frequency  $\rightarrow$  large amplitude.

### resonant absorption

- Mechanism of wave heating
- could damp kink mode oscillations.
- Loss of acoustic power in sunspots. Time scales:
  1. damping: collective mode  $\rightarrow$  local mode, *independent* of dissipation.
  2. dissipative damping of small scale perturbations of local mode.

$$\tau_1 \ll \tau_2$$

- inherently non-linear

## S

### sausage modes

- $m = 0$
- The fast magnetoacoustic sausage mode is another type of localized, modified fast magnetoacoustic wave.



- Mainly transverse.
- Standing fast sausage modes have symmetric tube modes
- Has a long-wavelength cutoff (trapped sausage modes do not exist at longer wavelengths). Approaches a cut-off at the external Alfvén speed. (Under condition of *mode localization*)
- Main feature is the periodic fluctuation of the cross-sectional area of the waveguide. This change is also associated with periodic fluctuations in density and temperature within the waveguide.
- Distinct sign of sausage oscillations is when periodic phenomena in cross-section and intensity are almost 180° out of phase → strong signal. (Less distinct signal is when periodicities in pore size don't match any intensity variations).
- Produce perturbations in density and magnetic field strength, and the corresponding plasma motions cause pulsations in the tube cross-section.
- Associated with perturbations of the loop cross-section and plasma concentration.
- Perturbations of plasma in the radial direction are stronger than perturbations along the field.
- Mode conversion and absorption through Alfvén resonance cannot take place (slow resonance can still operate).
- Phase speed is in the range between the Alfvén speed inside and outside the loop.

## slow waves

- $C_{T_0} < C_{slow} < C_{s_0}$
- longitudinal
- acoustic(?)
- propagate slower than both  $V_A$  and  $C_s$

## speeds

Characteristic speeds of MHD:

1. sound

$$C_s = \sqrt{\frac{\gamma p_0}{\rho_0}}$$

$$C_s \approx 166 T_o^{1/2} \text{ms}^{-1} = 200 \text{km s}^{-1}$$

2. Alfvén

$$C_A = \frac{B_0}{\sqrt{\mu_0 \rho_0}}$$

$$V_A = 2.18 \times 10^{12} \frac{B_o}{\sqrt{n_o}} \text{m s}^{-1} = 3000 \text{km s}^{-1}$$

$$(B_o \sim 100 \text{ G}; n_o \sim 10^{16} \text{ m}^{-3})$$

## spherical harmonics

3 kinds of resonant modes of oscillation:

- p (pressure)
- g (gravity)
- f (?)

Numbers:

- n - *order*; Number of nodes in radial direction
- l - *harmonic degree*; number of node lines on surface  $\sim$  total number of planes slicing the sun.
- m - *something*; number of surface nodal lines that cross the equator; phase  
 $-l \leq m \leq l$  (direction of waves is important); number of planes slicing longitudinally.

## standing waves

Confined to a finite region of space  $\rightarrow$  quantifies the frequency

- Fundamental (2 nodes, one on each end):

$$f_1 = \frac{1}{2L} \sqrt{\frac{F}{\mu}}$$

where L is the string length, F is the tension,  $\mu$  is the mass density

- 3 nodes:  $f_2 = 2f_1$
- 4 nodes:  $f_3 = 3f_1$

## surface modes

evanescent behavior in both media (inside and outside cylinder).

## T

## torsional vibration

- angular vibration of an object  $\sim$  shaft along the axis of vibration

## transverse waves

(from wikipedia:) moving waves that *consist* of oscillations occurring perpendicular to the direction of energy transfer. If a *transverse* wave is moving in the positive x-direction, its *oscillations* are in up and down directions that lie in the yz plane. Light is an example of a transverse wave. ( $\vec{B}$  and  $\vec{E}$  oscillate in directions perpendicular to the direction in which the light is actually traveling). With regard to transverse waves in matter, the *displacement of the medium* is perpendicular to the direction of propagation of the wave. Examples: A ripple in a pond and a wave on a string.

## trapped modes

For a specified geometry, uniqueness of the solution to a forcing problem at a particular frequency is equivalent to the non-existence of a trapped mode at that frequency. A trapped mode is a solution of the corresponding homogeneous problem and represents a free oscillation with finite energy of the fluid surrounding the fixed structure. For a given structure, trapped modes may exist only at discrete frequencies. Mathematically, a trapped mode corresponds to an eigenvalue embedded in the continuous spectrum of the relevant operator.

## tube speed

Also known as *cusp speed*, the tube speed is the combination of sound and Alfvén speeds.

$$C_T = C_s C_A / (C_A^2 + C_s^2)^{1/2}$$

## U

## W

## waves

- Caused by any turbulence in a medium.
- Observable properties
  - Period
  - wavelength
  - amplitude
  - temporal/spatial signatures (shape of perturbation)
  - characteristic scenarios of evolution (e.g. damped?)

## **waveguide**

- Width  $\sim$  same order of magnitude as the wavelength of the guided wave.
- Pores extending up from the photosphere into the solar atmosphere can act as an MHD waveguide.
- Flux tubes are excellent waveguides

## **wave number**

Number of waves in a unit distance.  $k = 2\pi$

**X**

**Y**

**Z**