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<http://solarscience.msfc.nasa.gov/feature3.shtml>

# 1 Oscillations and waves

## General

### Instability

A disturbance that is not stabilized by the resulting forces.

### Oscillations

Three types:

1. un-damped
2. damped
3. forced

### Waves

- Caused by any turbulence in a medium.
- Observable properties
  - Period
  - wavelength
  - amplitude
  - temporal/spatial signatures (shape of perturbation)
  - characteristic scenarios of evolution (e.g. damped?)

### Standing waves

Confined to a finite region of space → quantifies the frequency

- Fundamental (2 nodes, one on each end):

$$f_1 = \frac{1}{2L} \sqrt{\frac{F}{\mu}}$$

where  $L$  is the string length,  $F$  is the tension,  $\mu$  is the mass density

- 3 nodes:  $f_2 = 2f_1$
- 4 nodes:  $f_3 = 3f_1$

### Propagating waves

#### wave number

$k = 2\pi$  = number of waves in a unit distance.

### Phase velocity

$$v_p = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{c}{n}$$

where  $n$  = index of refraction ( $n = 1$  in vacuum).

$$k = \frac{2\pi}{\lambda}, \omega = \frac{2\pi}{T}$$

### Group velocity

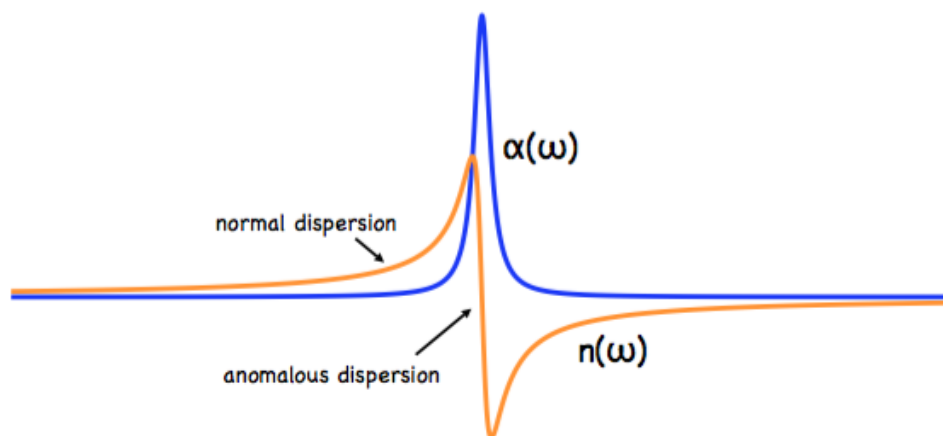
Direction of a wave's energy flow (Poynting vector for light).

$$v_g = \frac{d\omega}{dk}$$

### Dispersion

A mathematical term that applies to wave behavior; separation of a complex wave into its component parts according to a given characteristic (frequency, wavelength, etc.) "Dissipation" is a term that has to do with shock and heating, and can be applied to anything. Safer to use this one if you're not sure. **Dispersion** is when the distinct phase velocities of the components of the envelope cause the wave packet to "spread out" over time. The components of the wave packet (or envelope) move apart to the degree where they no longer combine to complete the envelope. Causes different components of the wave to have different phase velocities. Different kinds of dispersion:

- Normal dispersion: strictly increasing  $\text{Re } \epsilon(\omega)$  with increasing  $\omega$ .
- Anomalous dispersion: decreasing  $\text{Re } \epsilon(\omega)$  with increasing  $\omega$ .
- Resonant absorption: occurs in regions where  $\text{Im } \epsilon(\omega)$  is large.



No dispersion:  $v_{ph} = v_{gr}$ ; dispersion:  $v_{ph} \neq v_{gr}$ . A medium that is free from dispersion has index of refraction that is constant as a function of frequency, so all wavelengths are similarly affected.

Permittivity and permeability are functions of frequency. Non-dispersive waves have phase speed ( $v_{ph}$ , speed that wave actually travels through medium?) independent of wavenumber,  $k$ . All waves of *any*  $k$  propagate at the same speed. Uniform dissipation, resonant mode conversion, physical mechanisms vs. ? Dispersion *relation*:

- Relates the wavelength (or wavenumber) of a wave to its frequency.
- Describe the effect of dispersion in a medium on the properties of a wave traveling through that medium.

### waveguide

- Width  $\sim$  same order of magnitude as the wavelength of the guided wave.
- Pores extending up from the photosphere into the solar atmosphere can act as an MHD waveguide.
- Flux tubes are excellent waveguides

## Types of waves

### acoustic waves (aka sound waves)

- Longitudinal
- Isotropic
- Compressible (propagate by means of adiabatic compression and decompression)
- travel at sound speed (determined by the medium in which they were *created*, and maintain this speed even if they travel into medium with a different characteristic sound speed).
- Important quantities:
  - sound pressure
  - particle velocity
  - particle displacement
  - sound intensity
- exist because of a pressure restoring force: local compression (or rarefaction) sets up a *pressure gradient* in opposition to the motion
- carry energy away from source
- large enough amplitude  $\rightarrow$  shock wave, but usually small; ambient gas slightly disturbed
- exist in medium with low or non-existent magnetic field
- isotropic propagate equally in all directions. The phase and group velocities are both equal to the sound speed (hence, the part where acoustic waves are sound waves).

### longitudinal waves

waves in which the displacement of the *medium* is in the same direction as, or the opposite direction to, the direction of travel of the wave.

### transverse waves

(from wikipedia:) moving waves that *consist* of oscillations occurring perpendicular to the direction of energy transfer. If a *transverse* wave is moving in the positive x-direction, its *oscillations*

are in up and down directions that lie in the  $yz$  plane. Light is an example of a transverse wave. ( $\vec{B}$  and  $\vec{E}$  oscillate in directions perpendicular to the direction in which the light is actually traveling). With regard to transverse waves in matter, the *displacement of the medium* is perpendicular to the direction of propagation of the wave. Examples: A ripple in a pond and a wave on a string.

## Spherical harmonics

3 kinds of resonant modes of oscillation:

- p (pressure)
- g (gravity)
- f (?)

Numbers:

- $n$  - *order*; Number of nodes in radial direction
- $l$  - *harmonic degree*; number of node lines on surface  $\sim$  total number of planes slicing the sun.
- $m$  - *something*; number of surface nodal lines that cross the equator; phase  $-l \leq m \leq l$  (direction of waves is important); number of planes slicing longitudinally.

### pressure waves

P-modes are global acoustic oscillations. Longitudinal, fast, generated by turbulence near the photosphere, observed by measuring Doppler shift of absorption lines in the photosphere, spherical harmonics:

- $n$ : radial order, number of nodes in radial direction
- $l$ : angular degree, or harmonic degree, number of node lines on surface,  $\sim$  total number of planes slicing the sun
- $m$ : azimuthal number; number of surface nodal lines that cross the equator, number of planes slicing longitudinally.  $-l \leq m \leq +l$

### gravity waves

Generated in fluid medium or interface between two media when the force of gravity or buoyancy tries to restore equilibrium e.g. "wind waves" from between atmosphere and ocean.

## Magnetohydrodynamic (MHD) waves

- study of electrically conducting fluids (plasma)
- Theoretical foundation:
  - dispersion relation of MHD modes of a plasma cylinder
  - models: loops, prominence fibrils, plumes, various filaments
  - evolutionary equations
- Considerations of observed waves:
  - geometry: simple (slab or tube)? or more complex?
  - mode
    - \* longitudinal vs. transverse
    - \* compressible vs. incompressible

- \* oscillating vs. propagating
- \* fast vs. slow
- \* propagating vs. standing
- \* isotropic vs. anisotropic
- \* phase differences?
- Interpreting observations: distinguish between modes that are pressure driven (acoustic/slow magnetoacoustic) and magnetically driven (Alfvén)
- Relations between the (internal and external) characteristic speeds (Alfvén, sound, and tube speeds) *determine properties of MHD modes guided by the tube.*
- The behavior of linear perturbations of the form

$$\delta P_{tot}(r) \exp[i(k_z z + m\phi - \omega t)]$$

is governed by the following system of first order differential equations and algebraic equations:

$$D \frac{d}{dr}(r\xi_r) = (C_A^2 + C_s^2) \dots$$

- **Linearised equations**  
*Dissipation and Damping:*
- Dissipation of MHD waves is manifold:
  - Couple with each other
  - interact non-linearly
  - resonantly interact with the closed waveguide
  - develop non-linearly (e.g. solitons or shock waves can form)
- Inhomogeneous and magnetized plasma has two particular dissipation mechanisms of MHD waves:
  - Resonant absorption
  - Phase mixing

### Characteristic speeds of MHD

1. Sound speed:  $C_s = \sqrt{\frac{\gamma p_0}{\rho_0}} \approx 166 T_o^{1/2} \text{ m s}^{-1} = 200 \text{ km s}^{-1}$
2. Alfvén speed:  $C_A = \frac{B_0}{\sqrt{\mu_0 \rho_0}}$

$$V_A = 2.18 \times 10^{12} \frac{B_o}{\sqrt{n_o}} \text{ m s}^{-1} = 3000 \text{ km s}^{-1}$$

$$(B_o \sim 100 \text{ G}; n_o \sim 10^{16} \text{ m}^{-3})$$

3. Tube/Cusp speed:  $C_T = C_s C_A / (C_A^2 + C_s^2)^{1/2} = \text{combination of sound and Alfvén speeds}$

### magnetoacoustic waves

- A magnetosonic wave (also magnetoacoustic wave) is a longitudinal wave of ions (and electrons) in a magnetized plasma propagating perpendicular to the stationary magnetic field.
- compressible

- slow MHD wave; slow MA waves only have 1-3 oscillations before damping out, observed oscillations are manifestation of rapid damping due to radiative energy losses. Reduced  $\vec{B}$  regions = increased  $\rho \rightarrow$  rapid radiative losses. *Fast* MH waves radiate little because they're damped too slowly.
- collectively supported by the plasma environment, i.e., the wave mode acts across neighbouring magnetic field lines and across transverse plasma inhomogeneities.
- observed as disturbances of EUV (and possibly X-ray) emission

### kink modes

Fast kink waves:

- transverse (general property of fast waves)
- $v_{ph} = c_k = \sqrt{\frac{\rho_o V_{Ao}^2 + \rho_e V_{Ae}^2}{\rho_o + \rho_e}} \approx V_A \sqrt{\frac{2}{1 + \frac{\rho_e}{\rho_o}}}$  in the low- $\beta$  plasma.
- Period  $P = \frac{2\ell}{V_A} \sqrt{\frac{1 + \rho_e/\rho_o}{2}}$  where  $\lambda = 2\ell$  ( $\ell$  is the loop length). Typically,  $\ell \approx 60 - 600$  Mm in the corona.
- Period of global kink mode,  $P = \frac{2\ell}{c_K}$
- Important observation from which magnetic field strength can be derived.
- slab: phase speed =  $V_{Ae}$
- tube: kink speed:

$$c_K = \frac{B_i^2 + B_e^2}{\sqrt{\mu(\rho_i + \rho_e)}}$$

(mean Alfvénic speed). Slab (or tube) is moved laterally; little variation in cross-sec, density, or intensity.

Standing and propagating both rapidly damped. Much more observations of standing kink modes than propagating ('tadpole'-like structures moving downward; open magnetic structures).

Slow kink waves:

- longitudinal (velocity directed along magnetic field)
- compressible (variations in density and intensity)

Other (or simply unorganized):

- oblique (inclined with respect to the flow direction)
- *weakly compressible*, but could nevertheless be observed with imaging instruments as *periodic standing* or *propagating* displacements of coronal structures, e.g. coronal loops. The frequency of transverse or "kink" modes is given by the following expression:

$$w_K = \sqrt{\frac{2k_z B^2}{\mu(\rho_i + \rho_e)}}$$

In a cylindrical model of a loop, the parameter *azimuthal wave number*,  $m$ , is equal to 1 for kink modes.

- In the long wavelength limit, the phase speed of all but sausage fast modes tends to the so-called kink speed, which corresponds to the density weighed average Alfvén speed.
- Two instabilities of axisymmetric, current-carrying plasmas
  - sawtooth relaxations



- fishbone oscillations  
associated with instability of internal kink modes
- Both standing and propagating,  $T = \sim$  seconds-minutes.
- lowest spatial harmonics along field; global (fundamental) modes of coronal loops nodes of displacement at footpoints, maximum at apex.

### sausage modes

- $m = 0$
- The fast magnetoacoustic sausage mode is another type of localized, modified fast magnetoacoustic wave.
- Mainly transverse.
- Standing fast sausage modes have symmetric tube modes
- Has a long-wavelength cutoff (trapped sausage modes do not exist at longer wavelengths). Approaches a cut-off at the external Alfvén speed. (Under condition of *mode localization*). Effects of the sausage mode are easiest to observe in the radio regime (not the wave itself), where more of a “point” is observed, rather than extended...
- Main feature is the periodic fluctuation of the cross-subsectional area of the waveguide. This change is also associated with periodic fluctuations in density and temperature within the waveguide.
- Distinct sign of sausage oscillations is when periodic phenomena in cross-subsection and intensity are almost  $180^\circ$  out of phase  $\rightarrow$  strong signal. (Less distinct signal is when periodicities in pore size don't match any intensity variations).
- Produce perturbations in density and magnetic field strength, and the corresponding plasma motions cause pulsations in the tube cross-subsection.
- Associated with perturbations of the loop cross-subsection and plasma concentration.
- Perturbations of plasma in the radial direction are stronger than perturbations along the field.
- Mode conversion and absorption through Alfvén resonance cannot take place (slow resonance can still operate).
- Phase speed is in the range between the Alfvén speed inside and outside the loop.

### slow magnetoacoustic waves

- $C_{T_0} < C_{slow} < C_{s_0}$
- longitudinal
- essentially acoustic in a low- $\beta$  plasma and subject to reflection if the frequency is below the cutoff frequency.
- propagate slower than both  $V_A$  and  $C_s$  (but faster than the cusp speed).
- Sound waves are subject to nonlinear steepening and shocking as the density decreases with height through the chromosphere.

### standing acoustic oscillations

Characteristics:

- Pressure forces in opposition

- Period = 7–31 minutes (20 minutes from another source)
- Decay times = 5.7–36.8 minutes
- Peak velocity = 200 km/sec

Standing oscillations vs. propagating waves

- In loops, propagating waves damp before reaching opposite footpoint.
- Velocity and intensity are  $90^\circ$  out of phase for standing oscillations, and are in phase for propagating acoustic waves.
- Frequencies less than the cutoff are standing oscillations, waves with frequency greater than the cutoff propagate into the chromosphere.
- no loop shape change or displacement
- near footpoints.

#### propagating acoustic waves (slow)

- $v_{ph} < 150 \text{ km s}^{-1} \rightarrow$  slow
- longitudinal, compressive, anisotropic
- Parallel to  $\vec{B}$ , perturbation of  $\vec{B}$  is negligible.
- Generated impulsively at one end of a footpoint.
- Only penetrate  $\sim 10\%$  into loop before damped by thermal conduction
- weak dispersion in coronal conditions ( $V_A \gg c_s$ )
- 3 phases: periodic, QP, decay
- period = 3, 5, 10 minutes? Or 2–22 seconds? (see kink.1),
- velocity: 50–200  $\text{km s}^{-1}$
- $c_T = \sqrt{\frac{c_s^2 v_A^2}{c_s^2 + v_A^2}}$  propagate sub-sonically at  $c_T$ , which is less than  $c_s$
- “large” amplitude, max in top of chromosphere
- Observed using spectroscopy (intensity variations in EUV emission and Doppler shifts)

#### propagating acoustic waves (fast)

- $v_{ph} > 150 \text{ km s}^{-1} \rightarrow$  fast (or transverse standing waves).
- Quasi-isotropic
- Driven by magnetic forces + plasma pressure forces
- Compressive (magnetic sound wave)
- Speed:  $c_F = \sqrt{c_s^2 + v_A^2}$
- Moreton waves in the chromosphere
- Fast EUV waves in the corona

#### Alfvén waves

Alfvén waves are a type of MHD wave in which ions oscillate in response to a restoring force provided by an effective tension on the magnetic field lines. Low frequency (compared to the ion cyclotron frequency) traveling oscillation of the ion and the magnetic field. The ion mass density provides the inertia and the magnetic field line tension provides the restoring force.

They propagate in the direction of the magnetic field, although waves exist at *oblique* incidence

and smoothly change into the *magnetoacoustic* wave when the propagation is *perpendicular* to the magnetic field. The motion of the ions and the perturbation of the magnetic field are in the same direction and transverse to the direction of propagation. The wave is dispersionless.

Locally supported, phase and group velocities with magnitude equal to local Alfvén speed, directed along the magnetic field. Alfvén waves are easily excited by various dynamical perturbations of magnetic field lines, and are weakly dissipative (can propagate long distances, deposit energy and momentum far from source).

- $m=0$  (Axisymmetric, or azimuthally symmetric)
- transverse (shear) perturbations (perpendicular to  $\vec{F}_{res}$ ); plasma has characteristic elasticity.
- parallel to  $\vec{B}$  (the *group* velocity is strictly along  $\vec{B}$ , but the *phase* velocity doesn't have to be. The energy also propagates along the field lines, probably because the wave envelope is what carries all the information).
- (only) driving force: magnetic tension, the restoring force, which “snaps” the field back into a straight line, producing an Alfvén wave.
- Purely magnetic and incompressible in nature (in untwisted straight cylinder... twist may cause some compression).
- Displacement of plasma together with magnetic field frozen into it.
- velocity:  $v_A = \frac{B}{\sqrt{\mu_0 \rho}}$ ;  $\sim 1000 \text{ km s}^{-1}$  in the corona.
- Mode conversion: fast MHD to Alfvén.

How to observe:

- Only get Doppler shifts from *long*-period waves ( $>$  a few minutes).
- Measure additional (i.e. non-thermal) broadening of coronal emission lines; indirect way to observe short-period waves.
- Spatial variation in Doppler shift for long periods. Gyrosynchrotron emission in radio regime.
- $V_A$ : temporal resolution?

Effects of twisting (or *torsion*):

- Coupling of various MHD modes

(From *Priest*): Types of Alfvén waves:

- Shear or Torsional: No accompanying pressure or density changes (plasma).
- Compressional or Fast-Mode: Becomes a fast magnetoacoustic or fast-mode wave when pressure gradients are included (Note: these are often the preferred names, even when the pressure gradient is unimportant, so as to avoid confusion with shear Alfvén waves).

## EIT wave

## Moreton wave

Chromospheric signature of a large-scale coronal shock wave. Generated by flares,  $\sim$  fast-mode MHD waves.

## resonance

Periodic driving force frequency matches the wave frequency  $\rightarrow$  large amplitude.

## resonant absorption

- Mechanism of wave heating
- could damp kink mode oscillations.
- Loss of acoustic power in sunspots. Time scales:
  1. damping: collective mode  $\rightarrow$  local mode, *independent* of dissipation.
  2. dissipative damping of small scale perturbations of local mode.
- $\tau_1 \ll \tau_2$
- inherently non-linear

## torsional vibration

- angular vibration of an object  $\sim$  shaft along the axis of vibration

## Modes

A wave may be a superposition of lots of other waves. Each of those waves is a “mode” of the resultant wave (think of the foundation of Fourier Analysis: sums of sines and cosines). Modes with the lowest wave number are *global*, or *fundamental* modes.

- Different modes are driven by *different restoring forces*.

## Normal modes

Vibrational state of an oscillatory *system* where the frequency is the same for all elements. E.g. resonant frequencies: equally spaced multiples of the fundamental.

## trapped modes

For a specified geometry, uniqueness of the solution to a forcing problem at a particular frequency is equivalent to the non-existence of a trapped mode at that frequency. A trapped mode is a solution of the corresponding homogeneous problem and represents a free oscillation with finite energy of the fluid surrounding the fixed structure. For a given structure, trapped modes may exist only at discrete frequencies. Mathematically, a trapped mode corresponds to an eigenvalue embedded in the continuous spectrum of the relevant operator.

## flute modes

See *ballooning modes*

## ballooning modes

- $m > 1$
- Role not established yet

## leaky modes

- Waves are allowed to radiate into the external medium, i.e. the condition of mode localization is relaxed.
- complex eigenfrequencies eigenfrequency

- Bessel functions are replaced by Hankel functions in the dispersion relation. Can be the fundamental harmonic.
- Wavenumbers below cutoff value?
- Has electric field that decays monotonically for a finite distance in the transverse direction but becomes oscillatory everywhere beyond that finite distance.
- Mode “leaks” out of the waveguide as it travels down it, producing attenuation.
- Relative amplitude of oscillatory part (leakage rate) must be sufficiently small that the mode maintains its shape as it decays, in order to be called a mode at all.

### Global/Fundamental modes

- $y(x, t)$  governed by some PDE with no *explicit* time dependence.
- A global mode is a solution of the form  $y(x, t) = \hat{y}(x)e^{i\omega t}$
- PDE-dynamical system of infinitely many equations coupled together.

### Surface modes

evanescent behavior in both media (inside and outside cylinder). In the context of flux tubes and coronal seismology, surface waves are non-oscillatory inside the flux tube (body waves are oscillatory).

### Body modes

In the context of flux tubes and coronal seismology, body waves are oscillatory inside the tube (whereas surface waves are non-oscillatory inside the tube).

### mode conversion

### mode coupling

### phase mixing

- Cause decay of Alfvén waves, which suffer intense phase mixing
- Large gradients in Alfvén velocity.
- Part of resonant absorption, but is effective over entire range of frequencies, not just resonant ones; spread over volume rather than localized in narrow resonance layer.
- Occurs spatially (propagating wave) or in time (standing wave).
  - Space: produces effective wavenumber  $k_x^*$  which increases with  $z$ , so that the effective wavelength decreases. Eventually dissipation converts energy to heat.

$$k_x^* = \frac{dk_z}{dx} z$$

- Time: field lines become more and more out of phase.
- Not likely to operate in closed magnetic structures (e.g. coronal loops)

## 2 Observables

### polarization

- Doesn't apply to longitudinal waves, e.g. sound.
- Linear waves oscillate (transversely) in a single direction.

## Zeeman effect

## Hanle effect

- Reduction in polarization of light
- Used to indirectly measure  $\overline{B}$

## Emission measure

- $C_{A_0} < C_{fast} < C_{A_e}$
- highly dispersive.
- Kink modes
- Sausage modes
- propagate faster than both  $V_A$  and  $C_s$

## Differential Emission Measure (DEM)

$$EM = \int n_e^2 dV$$

$$DEM = \xi(T_e) = \int \frac{n^2(r)}{|\nabla T|} dS_T$$

or

$$DEM = \frac{d}{dT} \left[ \int n_e^2 dS \right]$$

in units of  $[\text{cm}^{-5} \text{ K}^{-1}]$ . DEM is the integral of electron density squared along an optically thin line of sight as a function of temperature. “Can predict the coronal bremsstrahlung flux component. . . bremsstrahlung correlates with EUV emission.”

## Fraunhofer lines

First observed in 1802 by William Hyde Wallaston, and carefully studied by Joseph von Fraunhofer (1787–1826), who catalogued the wavelengths in 1814. Ionized sodium: inert gas structure, no lines in visible regime. Dark absorption features in solar spectrum, in cooler layers of the atmosphere.

- H and K lines of ionized Calcium
- D line of neutral sodium (5890 Å, 5896 Å; resonance lines between  $n=1$  and  $n=2$ ).
- E (iron)
- C and F (hydrogen)

## 3 Analysis techniques

periodogram

wavelet analysis

WKB approximation

## 4 Solar features

## supergranules

Show up most clearly as a pattern of horizontal motions.

- Doppler measurements near limb
- local correlation tracking of granules near center

Above a supergranule cell

- 750 km active regions
- 1600 km quiet regions

Magnetic field spreads out to fill the chromosphere and form a horizontal canopy or partial canopy.

## Bright points

### active regions

Appear as bright *plages* (§??) of emission in the equatorial belt within  $\pm 30$  degrees of the equator and represent moderate concentrations of magnetic flux with mean field of 100 G or so.

## sunspots

Dark regions of intense magnetic field.

## spicules

Towers of gas observed in the chromosphere,  $\sim 500$  km above the photosphere. Last 5-10 minutes. Jets are about 500 km diameter and move upward from the photosphere at speeds of about 20 km/s. They are regions of intense energy, possibly caused by plasma being propelled by shock waves that originated in the interior. Acoustic waves from photosphere shock and heat plasma inside magnetic structures. Potential mass for solar wind and energy for corona.

Two types of spicules:

1. Type I spicules (dynamic fibers) are caused by shock waves. Fibrils driven by slow-mode magnetoacoustic shocks, sawtooth shape in chromospheric emission lines.
2. Type II spicules are caused by strong flows. Rapidly evolving.

## coronal holes

### coronal loops

- Main observational feature of the magnetic structure in the upper solar atmosphere.
- *Modelled* as flux tubes; probably consist of many flux tubes.

### coronal streamer

A wisp-like stream of particles traveling through the Sun's corona, visible in images taken with a coronagraph or during a total solar eclipse. Coronal streamers are thought to be associated with active regions and/or prominences and are most impressive near the maximum of the solar cycle. They are large scale magnetic structures of the equatorial solar corona.

## filament

- Viewed on disk ( $\equiv$  prominence against sky)

- Thin, cool, dark ribbons

Filament channel:

## flares

Rapid conversion of stored magnetic energy to particles acceleration and excess radiation in the corona.

## coronal mass ejections (CMEs)

Release of magnetic energy; reach Earth in a few days.

## flux tubes

- Formed deep in the convection zone.
- Rise by magnetic buoyancy in an  $\Omega$ -shaped loop.
- Magnetic field lines can be thought of as infinitely thin flux tubes.

## facula

Aka. "little torch"

- Appear in *photosphere*; same thing as plage (which appear in the chromosphere).
- Bright spots — reason why total brightness is higher at solar maximum.
- small scale bright points in the vicinity of sunspots; appear hours before the sunspots, but can remain for months after the sunspots are gone.
- visible only near limb.

## jets

Word for rapid burst of emission? Rapid upflows, plasma ejections... UV spectral emission requires high temperatures. For UV: flux from photospheric continuum is low. Chromosphere: temp is lower and background is higher (compared to ...?) so lines are in absorption. Low flux: radiative excitation doesn't occur. High temps allow for collisional excitation and emission upon returning to the ground state.  $\Delta v = \sqrt{v_{th}^2 + v_{Nth}^2}$ .

## solar cycle

Three stages:

1. Build-up of magnetic field
2. Turnover (maximum)
3. Reversal of poles

This takes a total of  $\sim 11$  years, followed by another 11 years of the same sequence, but with the poles reversed, for a total of 22 years.

## solar wind

Stream of energized, charged particles, primarily protons and electrons, flowing outward from the sun at  $v \leq 900 \text{ km s}^{-1}$  and  $T = 10^6 \text{ K}$ . Solar wind plasma originates in thin, intense flux tubes at granule and supergranule boundaries. Fast (steady) vs. slow (variable) wind.



## plage

- Appear in *chromosphere*; same thing as [faculae](#)
- Bright spots caused by light emitted by clouds of hydrogen or calcium (specifically H $\alpha$  and Ca H and K lines).

## pores

Small sunspots with an umbra, but no penumbra. Size on order of upper limit of magnetic bright points ( $\sim 1700$  km).

## prominence

- Viewed on the limb; same thing as filaments.
- May erupt sometime during its life and be associated with a CME

## plume

Apparently help to shield Earth from solar storms. They are long thin streamers that project outward from the Sun's north and south poles. We often find bright areas at the footpoints of these features that are associated with small magnetic regions on the solar surface. These structures are associated with the *open* magnetic field lines at the Sun's *poles*. The plumes are formed by the action of the solar wind in much the same way as the peaks on the helmet streamers.

# 5 Maths

## Bessel's equations

(From Boas p. 587): Bessel functions are damped sines and cosines. Solutions of differential equations can be represented by power series. Graphs, formulas; electricity, heat, hydrodynamics, elasticity, wave motion, quantum. . . , cylindrical symmetry. . .

Bessel's differential equation:

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - p^2) y = 0$$

where  $p$  is the *order* of the Bessel function  $y$  and is a constant, and  $y$  is the solution.  $a = \text{integer} \rightarrow \text{cylindrical}$ ,  $a = \text{half-integer} \rightarrow \text{spherical}$ .

$$x^2 y'' + xy' + (x^2 - p^2) y = 0$$

$$x(xy') + (x^2 - p^2)y' = 0$$

# 6 Other

## adiabatic index

The adiabatic index is the ratio of specific heats:

$$\gamma = \frac{c_P}{c_V}$$

## frozen-in flux

In a perfectly conducting material (i.e.  $\eta = 0$ ), Ohm's law goes from  $\vec{E} + \vec{v} \times \vec{B} = \vec{J}\eta$  to  $\vec{E} + \vec{v} \times \vec{B} = 0$ . Nothing can be perpendicular to the field lines ... See Alfvén's Theorem.

A property of a moving fluid which represents the potential for helical flow (i.e. flow which follows the pattern of a corkscrew) to evolve. Helicity is proportional to the strength of the flow, the amount of vertical wind shear, and the amount of turning in the flow (i.e. vorticity). Atmospheric helicity is computed from the vertical wind profile in the lower part of the atmosphere (usually from the surface up to 3 km), and is measured relative to storm motion. Higher values of helicity (generally, around 150 m<sup>2</sup>/s<sup>2</sup> or more) favor the development of mid-level rotation (i.e. mesocyclones). Extreme values can exceed 600 m<sup>2</sup>/s<sup>2</sup>.

## gyrosynchrotron radiation

Electromagnetic emission emitted by mildly relativistic electrons moving in a magnetic field (as opposed to synchrotron, with *ultrarelativistic* particles).

## polytropic index

# 7 Solar instruments

## SoHO

*The Solar and Heliospheric Observatory*

Instruments:

- *EUV Imaging Telescope* (EIT) discovered EUV waves in the corona.

## DKIST

**Visible Broadband Imager (VBI)** high spatial/temporal resolution of the atmosphere.

**Visible Spectro-Polarimeter (ViSP)** multi-line spectropolarimeter

**double Fabry-Pérot based Visible Tunable Filter (VTF)** high spatial resolution spectropolarimetry

**DL-NIRSP** fiber-fed diffraction-limited NIR spectropolarimeter

**Cryo-NIRSP** cryogenic NIR SP for coronal magnetic field measurements and on-disk observations of, e.g., the CO line at 4.7  $\mu$

## Hinode

### Interface Region Imaging Spectrograph (IRIS)

- Lockheed Martin Solar and Astrophysical Laboratory (LMSAL)
- Launched June 2013; sun-synchronous, low-Earth orbit.
- Goals: Understand how chromosphere is energized
- Revealed complexity, density/temperature contrasts in the interface region.
- 19 cm Cassegrain telescope
  - dual-range UV spectrograph (imaging) with 1 second cadence, 0.3" spatial resolution,  $< 1\text{\AA}$  spectral resolution.
  - slit-jaw imager (SJI) with four passbands:
    1. CII 1335 $\text{\AA}$  (transition region line)

2. SiIV 1403Å (transition region line)
  3. Mg II k 2796Å (chromospheric line)
  4. 2830Å (photospheric passband)
- “NASA Small Explorer developed and operated by LMSAL with mission operations executed at NASA Ames Research center and major contributions to downlink communications funded by ESA and the Norwegian Space Center.” (Bryans et al. 2016)

## RHESSI

### Skylab

Launched 1973

Instruments:

- Apollo Telescope Mount (ATM)

## Coronal Multichannel Polarimeter (CoMP)

## STEREO

### Solar Dynamics Observatory (SDO)

#### AIA

Wavelength (Å)	Ion(s)	Height	Temperature (K)
193	Fe	1 R <sub>☉</sub>	10 <sup>6</sup>

Table 1: Properties of AIA bassbands

#### EVE

#### HMI

## TRACE

The Solar Terrestrial Relations Observatory (STEREO) is a solar observation mission. Two nearly identical spacecraft were launched in 2006 into orbits around the Sun that cause them to respectively pull farther ahead of (STEREO A) and fall gradually behind (STEREO B) the Earth. This enables stereoscopic imaging of the Sun and solar phenomena, such as coronal mass ejections.

Instruments:

- Sun Earth Connection Coronal and Heliospheric Investigation (SECCHI) has five cameras: an extreme ultraviolet imager (EUVI), two white-light coronagraphs (COR1 and COR2), and two heliospheric imagers (called HI1 and HI2). The first three telescopes are collectively known as the Sun Centered Instrument Package (SCIP), and image the solar disk and the inner and outer corona. HI1 and HI2 image the space between Sun and Earth. The purpose of SECCHI is to study the 3-D evolution of Coronal Mass Ejections through their full journey from the Sun's surface through the corona and interplanetary medium to their impact at Earth.

- In-situ Measurements of Particles and CME Transients (IMPACT) will study energetic particles, the three-dimensional distribution of solar wind electrons and interplanetary magnetic field.
- PLASMA and SupraThermal Ion Composition (PLASTIC) will study the plasma characteristics of protons, alpha particles and heavy ions.
- STEREO/WAVES (SWAVES) is a radio burst tracker that will study radio disturbances traveling from the Sun to the orbit of Earth.