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1. Introduction

1.1 Overview of ISM components and processes

-6- What can happen to an atom or molecule or dust grain sitting in the ISM?

- Can it absorb a photon? Energy levels $\leftrightarrow h\nu$ (Need cross-sections for dust grains)
- Can it collide with other particles? $\rightarrow$ collisional rates [s^{-1} cm^{-3}]
- Is there a magnetic field? Is the particle charged?
- Are there cosmic rays? These can penetrate dense gas.

Three possible sources of ionization (and excitation):

1. photons
2. collisions
3. cosmic rays

Typical collisional energies: kinetic energy. Translate $mv^2 \rightarrow kT \rightarrow h\nu \rightarrow eV$.

1.2 Dust grains

-8- Dust grains also occur in the neutral medium, and probably also in the (warm) ionized medium. Dust grains play an important role in various processes:

- extinction of starlight
- emission of absorbed energy in FIR
- formation of molecules often occurs on grain surfaces
- absorption of ionizing UV radiation and Lyα photons (reducing amount of ionizing radiation)
- heating of HI gas by *photoelectric* emission

Composition: carbon and silicates. Typical sizes: 0.01 - 0.1 μm (How do we know? $\rightarrow$ shape of extinction curve). Grains as small as $\sim$ 60 atoms across discovered; evidence from emission lines in NIR and excess emission at 5 - 40 μm over what is expected from dust in the ISM. The larger dust grains have temperatures between 10 and 40 K, while the small ones can be heated to higher temps due to the absorption of even a single photon (smaller heat capacity, as volume $\propto r^3$). A promising candidate for small dust grains: polycyclic aromatic hydrocarbons ($\sim$ car soot!)

In hot environments, dust grains may be destroyed by *sputtering*, where collisiosn of grains with other atoms, electrons, or molecules knock molecules
-9- off the grains. At low temperatures, molecules stick to dust grains, causing depletion of heavy elements along certain lines of sight (most dust in the plane). Dust contributes about 1% of the mass of the ISM in the solar neighborhood, mostly in the form of large grains.

1.3 Ionized gas

a) Photoionization: especially effective near hot stars. Shock ionization
b) Cosmic rays: can occur throughout most of the ISM, so can also produce a small amount of ionization in denser gas (though recombination happens quickly, so not much of gas is in an ionized state at any given time).
-11- c) Collisions: Hot ($\sim 10^7$ K) expanding bubble sweeps up shell of warm ($\sim 10^4$ K) ionized gas, which shows a different optical spectrum than HII regions. The hot gas shows up by:
   - Free-free and line x-rays
   - absorption lines of highly ionized species toward bright UV sources

   Kirchoff's law: apparently can still see absorption lines when looking through a gas that is hotter than the source behind it!

1.4 Magnetic fields and cosmic rays

-13- In the solar neighborhood, $\mathbf{B} \sim 2 - 5 \times 10^6$ Gauss. This follows from measurements of *Faraday rotation*, giving $< n_e B_{||} >$ toward pulsars and radio sources. The random component of the B field is probably as large as the uniform component. In clouds, the B field can be much higher, $\sim 70\mu$G (from Zeeman effect splitting measurements).

The magnetic field is important for several reasons:

1. It aligns elongated grains, giving rise to polarization of starlight
2. Causes relativisitic electrons to emit synchrotron radiation, and most likely plays a role in accelerating electrons to relativistic velocities

("magnetic bottle", Fermi acceleration).

3. Provides pressure support against gravitational collapse of matter since it is frozen into the matter due to ionization heavy elements. It also seems to play animportant role in solving the angular momentum problem in star formation.[*]

Total energy density of cosmic rays in solar neighborhood: $U_R \sim 1.3 \times 10^{-12}$
-14- erg cm^{-3}. Why are cosmic rays important?

- They produce γ-rays through collisions with atoms and molecules. The observed γ-ray intensity from the ISM forms an excellent independent measure of the total amount of matter between stars. For example, calibrating the conversion factor of CO line intensity to H_2 mass.
- Provide pressure against gravitational collapse

Five pressures that play an important role in supporting the ISM against gravitational collapse:

1. thermal $P = nkT$
2. magnetic $P = \dfrac{B^2}{8\pi}$
3. turbulent (bulk motion)*
4. cosmic ray
5. radiation

* The cloud to cloud velocity dispersion due to turbulence on various scales increases line widths [over?] thermal widths. From the table on page -12-, you will note rough thermal pressure euqilibrium between the components. This is not a coincidence; in fact, some of this information was inferred by assuming pressure equilibrium. The argument is that if there were no equilibrium, the resulting perturbations would be wiped out on sound-crossing
-15- time scales, which are short compared to the time scales we wouuld consider the ISM to evolve over.

However, the actual evidence for equilibrium in the thermal pressure is scarse, and there are claims that it is not true in the very local ISM[†].

There seems to be a "cosmic conspiracy": the estimates for the thermal, magnetic, and cosmic ray pressure for the solar neighborhood give roughly

---

[*] Shu et al.

[†] Bowyer et al. *Nature* 1905

equal numbers for all three. Thus it may be inappropriate to only consider the thermal pressure (the only one that can be measured with much certainty). Interestingly, the magnetic pressure number is also very similar to the energy density of the CMB. [*] [†]

-16- Next section is sort of a shortened condensation of Draine's chapters 2 and 3. We may come back to specific topics discussed there in more detail.

[*] Draine discusses possible reasons in section 1.3
[†] Supplemental info in Draine chapter 1.

2. Statistical physics in the ISM

2.1 The big four

The four major laws of statistical physics are:

1. **Maxwellian** velocity distribution
2. **Boltzmann distribution** of energy levels in atoms and molecules
3. **Saha equation** for ionization equilibrium
4. **Planck function** for radiation

These apply to stars, but not always to the ISM.

2.1.1 Maxwellian

T is defined by motion of particles. ($\vec{\omega}$ = velocity = $\vec{v}$ in Draine).

$f(\vec{\omega})\mathrm{d}\vec{\omega}$ = fractional number of particles whose velocity, $\vec{\omega}$, lies within the three-dimensional volume element $\mathrm{d}\vec{\omega} = \mathrm{d}\omega_x \mathrm{d}\omega_y \mathrm{d}\omega_z$, centered at $\vec{\omega}$.

In thermodynamic equilibrium, $f(\vec{\omega})$ is isotropic, so $\vec{\omega} \to \omega$.

$$f(\omega) = \frac{\ell^{3/2}}{\pi^{3/2}} e^{-\ell^2 \omega^2}; \quad \ell^2 = \frac{m}{2kT} = \frac{3}{2\langle \omega^2 \rangle}$$

$$f(\omega) = \left(\frac{m}{2\pi kT} \right)^{3/2} \exp\left(-\frac{m\omega^2}{2kT} \right)$$

For two groups of particles with different masses, we replace ω by u, the relative velocity between the two groups, and m by the reduced mass $m_r =$
-17- $\dfrac{m_1 m_2}{m_1 + m_2}$. For H atoms colliding with particles of mass Am_H, Spitzer derives:

$$\langle u \rangle = \left[\frac{8kT}{\pi m_r} \right]^{1/2} = 1.46 \times 10^4 \sqrt{T} \left(1 + \frac{1}{A} \right)^{1/2} \quad [\mathrm{cm\ s^{-1}}]$$

Exercise: Verify the above calculation, especially the numerical constant.

Note that there is a difference between the *speed* and *velocity* distribution. Verify that $\langle \omega \rangle = 0$. But evidently, the mean *speed* is not 0. The speed distribution is given by:

$$f'(\omega')\,\mathrm{d}\omega' = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-m\omega'^2/2kT\right) \quad \underbrace{4\pi\omega'^2\mathrm{d}\omega'}_{\text{volume in phase space}}$$

where $f'(\omega')\,\mathrm{d}\omega'$ = fractional number of particles with speeds between ω' and $\omega' + \mathrm{d}\omega'$ and $\omega' = |\omega|$.

The Maxwell velocity distribution is characterized by several speeds:

- Most probable speed: $\omega_o = \sqrt{\dfrac{2kT}{m}}$
- RMS speed: $< \omega^2 >^{1/2} = \sqrt{\dfrac{3kT}{m}}$
- RMS velocity in one direction: $< \omega_x^2 >^{1/2} = \sqrt{\dfrac{kT}{m}}$

2.1.2 Boltzmann distribution

-18- The Boltzmann distribution gives the population of energy levels in an atom or molecule:

$$\frac{n_u}{n_l} = \frac{g_u}{g_l} \exp\left[-\left(E_u - E_l\right)/kT\right] = \frac{g_u}{g_l} \exp\left[-h\nu_o/kT\right]$$

where

- $n_{u,l}$ are the number densities
- $g_{u,l}$ are the statistical weights
- $E_{u,l}$ are the energies of the levels

(Partition function: summing over all energy levels... chemical term).

2.1.3 Saha equation

The Saha equation describes ionization equilibrium:

$$\frac{n_{i+1}}{n_i} = \frac{g_{i+1}g_e}{g_i} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-I/kt\right)$$

where I is the ionization potential for an ion in the ground state and initial ionization state i (aka, the energy required to ionize from i to $i+1$). $g_e = 2$ (two spin conditions). *Electrons affect whether and how easily atoms can be ionized.*

2.1.4 Planck function

specifies the radiation field: *

$$\begin{aligned} B(\nu) = & \frac{2h\nu^3}{c^2}\frac{1}{\exp[h\nu/kT]-1} \\ & = \sim \frac{2\nu^2}{c^2}kT \quad \text{for } h\nu \ll kT \ (\text{Rayleigh} - \text{Jeans}) \\ & = \sim \frac{2h\nu^3}{c^2}\exp[-h\nu/kT] \quad \text{for } h\nu \gg kT(\text{Wien}) \end{aligned}$$

2.2 Validity of the four laws

-19- The four laws discussed above hold under thermodynamic equilibrium (TE). However, TE is not often the case for the ISM, for two reasons:

1. TE requires **detailed balancing**, i.e. each process is as likely to occur as its inverse. For example, consider the 3727 Å emission from O^+. This is a forbidden transition (actually a doublet). The excitation of the electron level occurs through collisions with electrons, in most conditions in the ISM. If detailed balancing were to hold, de-excitation should also occur by collisions. However, as we will see, under the low density conditions found in the ISM, collisions are rare, and de-excitation is more likely to proceed through emission of a photon, in spite of the fact that we are dealing with a forbidden transition. Thus [OII] emission can be quite strong, and by converting collisional (kinetic) energy into radiation, we actually have created a cooling mechanism for the gas.
2. The radition field is strongly diluted. A diluted radiation field is one in which the energy density does not match the color temperature. This

* $e^x \approx 1 + x$ for $x \ll 1$
$e^{h\nu/kT} - 1 \approx 1$ for $h\nu \ll kT$
$e^{h\nu/kT} - 1 \approx e^{h\nu/kT}$ for $h\nu \gg kT$

concept is quite familiar; for example, the sun's photosphere is ~ 6000 K, and at the surface the flux leaving the sun is approximately that of a blackbody of this temperature. However, the Earth is not 6000 K
-20- because by the time the radiation reaches us, it is diluted. For the solar neighborhood, the total energy density of the radiation field due to all stars in that volume is about 1 eV cm^{-3} (close to cosmic ray density, as mentioned before). When interpreted as an average temperature using the Stefan-Boltzmann law (energy density of a blackbody, u = aT^4), this energy density implies an equivalent temperature of ~ 3 K. Yet the color temperature implied by the shape of the spectrum of this Interstellar Radiation Field (ISRF) is that of A and B stars (T $\sim 10^4$ K). So there is a **dilution factor** W given by:

$$W \approx \left(\frac{3}{10^4}\right)^4 \approx 0.25 \times 10^{-14}$$

We conclude that using the Planck law to describe intensities is not correct.

What about the other laws?

1. Maxwell velocity distribution

Good news! It is generally valid. Detailed balancing is possible for the elastic collisions that are generally occurring.* Because the maxwellian distribution is a good description of the motions of the particles, we can define a *kinetic temperature* which describes the physical condition of the gas. Often, for a plasma, the kinetic temperature is equal to the electron temperature: $T_{ions} = T_e$. $T_{ions} \neq T_e$ may occur behind shocks.

2. Boltzmann distribution

-21- Rarely correct. If excitation and de-excitation occurs by photons, we may still not have a Boltzmann distribution because the photon distribution is not given by the Planck function. Often we do not even have detailed balancing. However, as we will see, sometimes the distribution of excited levels is not too different from Boltzmann distribution. This happens when collisions dominate excitation and de-excitation, while radiation is relatively unimportant.

* See Draine section 2.3.3 for an analysis of "deflection times"; they tend to be short. (In dynamics, we speak of a "relaxation time".)

To describe situations close to TE, Spitzer introduced the so-called b-factors (Draine calls them "departure coefficients").

$$b_j \equiv \frac{n_j(\text{true distribution}}{n_j(\text{LTE distribution})}$$

Example: in an HII region, the hightest excited levels of HI have $b_j \sim 1$. Some radiation does escape (producing radio recombination lines) but collisions dominate the level populations. Since motions of particles *are* described by a Maxwellian velocity distribution, whenever collisions dominate the level population they will closely follow a Boltzmann law.

-22- In general, a Maxwellian velocity distribution tends to set up a Boltzmann population for energy levels in the atoms/particles *if* transitions resulting from emission and absorption of photons are relatively unimportant, and collisional (de-)excitation is dominant. In the case of the highly excited levels in H mentioned before, collisions with electrons are dominant.

3. Saha equation
is generally not valid; there is no detailed balancing. Even though the ionization and recombination processes are each other's inverse
($h\nu + A \to A^+ + e^- \to h\nu + A$),
the ionization process is determined by the *photon* field in most cases, while the recombination process is determined by collisions between A^+ and e^-. The collision rate depends on $\{n_e, n_{A^+}\}$ and T_e (the electron temperature), but the ionization is dependent on $T_{radiation} (\neq T_e)$.

So in general, assume **statistical equilibrium**, where there is a balance between transitions one way and the other way, no matter what process caused each transition.

In level i, we have n_i atoms cm^{-3}, and R_{ij} is the rate coefficient (number of transitions from level i to level j for all possible processes) such that

$n_i R_{ij}$ [s^{-1}] = # transitions from level i to level j
$R_{ji} n_j$ [s^{-1}] = # transitions from level j to level i

$$\frac{\mathrm{d}n_i}{\mathrm{d}t} = \sum_j \left(-R_{ij}n_i + R_{ji}n_j\right) ; \quad i = 1, 2, \ldots$$

-23- In statistical equilibrium, $\frac{\mathrm{d}n_i}{\mathrm{d}t} = 0$. The rate factor R_{ij} includes *all* possible processes that would take the atom or molecule from level i to j. For example, if you consider excitation of an electron in an atom, R_{ij} could include absorption of photons, collisional excitation, etc.

In the worst case, you will have to include many processes to calculate the n_i values. This requires knowledge of a lot of physical input parameters, e.g. cross sections for particular processes, collisional rate coefficients *, etc.

In other cases, where only one or two processes matter, the situation can be very simple. We will encounter cases of each.

Before going more into Ch 2 and 3 in Draine, we will first discuss some basic radiative transfer.†

*Drane section 2.1
†RL Ch 1, Draine Ch 6, 7

3. Radiative Transfer

-24- (See Draine chapter 7)

Planck function (an intensity) $B_\nu(T) \equiv \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}\,\frac{dE}{dt d\nu d\Omega d\sigma}$

Photon occupation number $n_\gamma \equiv \frac{c^2}{2h\nu^3} I_\nu = \frac{1}{e^{h\nu/kt}-1}$ if $I_\nu = B_\nu$

Specific intensity $I_\nu \equiv \lim_{d\sigma, d\Omega, d\nu, dt \to 0}$

Flux at surface of a sphere $F_\nu = \pi B_\nu$ (for blackbody)
$F_\nu = \pi I_\nu$ (for isotropic emitting non-blackbody)

Flux at a distance r $F_\nu(r) = \pi I_\nu (\frac{R}{r})^2 = \frac{L_\nu}{4\pi r^2}$
where R = radius of body and
L_ν = luminosity of body [erg s^{-1} Hz^{-1}]

-25- The above are *monochromatic*: integration over frequency yields the total flux, etc.

$$I = \int_{r=0}^{\infty} I_\nu \mathrm{d}\nu \quad F = \int_{\nu=0}^{\infty} F_\nu \mathrm{d}\nu \text{ etc} \ldots$$

Energy density: $u_\nu = \frac{1}{c}\int I_\nu \mathrm{d}\Omega$ [erg cm^{-3} Hz^{-1}]

Radiation pressure: $P_\nu = \frac{1}{c}\int I_\nu \cos^2\theta \mathrm{d}\Omega$

3.1 Emission and absorption coefficients

3.1.1 *Spontaneous Emission*

j_ν = volume emission coefficient [erg cm^{-3} sec^{-1} Ω^{-1} Hz^{-1}]

$j'_\nu = \dfrac{j_\nu}{\rho}$ = mass emission coefficient [erg g^{-1} sec^{-1} Ω^{-1} Hz^{-1}]

$j_\nu = \dfrac{\epsilon_\nu}{4\pi}$ for an isotropic emitter; ϵ_ν = emissivity [erg cm^{-3} s^{-1} Hz^{-1}]

3.1.2 *Absorption*

Absorption is a bit more complicated than emission. It removes a fraction of the incoming radiation. Absorption also includes *stimulated* emission, aka "negative absorption". The entire process refers to the sum of "true absorption" + stimulated emission.

-26- κ_ν = volume absorption coefficient [cm^{-1}]

$\kappa'_\nu = \dfrac{\kappa_\nu}{\rho}$ = mass emission coefficient [g^{-1} cm^2]

Loss of intensity in a beam of light as it travels distance ds:

$$\mathrm{d}I_\nu = -\kappa_\nu I_\nu \mathrm{d}s$$

Microscopically:

$$\kappa_\nu = n\sigma_\nu$$

where n is the particle density and σ_ν is the cross section for the absorption process per particle.

optical depth: $\tau_\nu \equiv \int \kappa_\nu ds$

- $\tau \gtrsim 1$ optically *thick* emission
- $\tau < 1$ optically *thin* emission

mean free path: $\ell_\nu = \dfrac{\tau_\nu}{\kappa_\nu}$

so for condition $\tau = 1 \longrightarrow \ell_\nu = \dfrac{1}{\kappa_\nu} = \dfrac{1}{n\sigma_\nu}$

3.2 Radiative transfer equation

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu I_\nu + j_\nu$$
$$\mathrm{d}\tau_\nu = \kappa_\nu \mathrm{d}s$$
$$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = -I_\nu + \frac{j_\nu}{\kappa_\nu} = -I_\nu + S_\nu$$

where $S_\nu \equiv$ source function

Formal solution:

$$I_\nu(\tau_\nu) = I_\nu(o)\mathrm{e}^{-\tau_\nu} + \int_0^{\tau_\nu} \mathrm{e}^{-(\tau_\nu - \tau_\nu')} S(\tau_\nu')\mathrm{d}\tau_\nu'$$

-27- $\rightarrow$ attenuated incoming beam + contribution from gas itself.

Special cases:

1. Source function is constant throughout source:

$$I_\nu(\tau_\nu) = I_\nu(o)\mathrm{e}^{\tau_\nu} + S_\nu(1 - \mathrm{e}^{-\tau_\nu})$$

   - optically thick emission: $I_\nu = S_\nu$
   - optically thin emission: $I_\nu = I_\nu(o)(1 - \tau_\nu) + \tau_\nu S_\nu$

2. Thermal radiation: $S_\nu = B_\nu(T)$ (The Planck function)
   - optically thick emission: $I_\nu = B_\nu$
   - optically thin emission: $I_\nu = \tau_\nu B_\nu$

Brightness temperature

For *radio* emission: I_ν is replaced by the *brightness temperature*, T_b, defined as:

$$I_{\nu,\mathrm{obs}} \equiv B_\nu(T_b)$$

In the Rayleigh-Jeans limit for B_ν we get:

$$I_{\nu,\mathrm{obs}} = \frac{2\nu^2 k T_b}{c^2} \quad \longrightarrow \quad I_b = \frac{I_\nu c^2}{2\nu^2 k}$$

Solution of the transfer equation in terms of T_b:

$$T_{b,\mathrm{obs}} = T_{b,o}\mathrm{e}^{-\tau_\nu} + T(1 - \mathrm{e}^{-\tau_\nu})$$

-28- T = thermal source temperature (*physical* temperature of the layer)
T_b = brightness temperature of the incident radiation

T_b is never greater than T!

3.3 Einstein coefficients

At the macroscopic level, Kirchoff's law gives $j_\nu = \kappa_\nu B_\nu(T)$. Einstein coefficients[*] describe reasons for this on a microscopic level. They give the transition *probabilities* (per unit time). Three possible processes:

1. Spontaneous emission: Einstein A coefficient. $A_{21} \equiv$ transition probability for spontaneous emission per unit time per "system".
2. Absorption

-29- 3. Stimulated emission

3.3.1 *Relations between the Einstein coefficients*

Valid under all conditions since they only refer to *atomic* principles (no collisions, just radiation). TE: rate of transitions out of state 1 = rate of transitions into state 1 (per unit volume).

-30- In TE, apply the Boltzmann law.

3.3.2 *Relations between Einstein coefficients and κ_ν and j_ν*

emission coefficient
absorption coefficient
-31- The second term here for κ_ν corresponds to *stimulated emission*.

3.4 Line profile function, $\phi(\nu)$

See RL chapter 10.6 and Draine 6.4. In particluar, comments in Draine about the actual line width in some practical cases are useful!

---

[*]Draine, section 6.1

3.4.1 Natural line width

Atom not moving, but still quantum effects. Determined by lifetime of an excited state. Described by Lorentz profile, whose key feature is a narrow core and broad wings (not a Gaussian).

$$\phi(\nu) = \frac{4\gamma_{u\ell}}{16\pi^2(\nu - \nu_{u\ell})^2 + \gamma_{u\ell}^2}$$

Key point: A small Einstein coefficient A results in a *narrow* line.

The natural line width of most transitions is quite small, and broadening due to other effects is more important.

3.4.2 Doppler broadening

- Thermal velocities
- Bulk motion (turbulence)

-32- and the profile function is

3.4.3 Collisional broadening

$\sim$ Pressure broadening, which is not generally important in the ISM because the density is so low…mostly occurs in stellar atmospheres. This still produces a Lorenzian profile, but with:

$$\phi(\nu) = \frac{4\Gamma^2}{16\pi^2(v - v_o)^2 + \Gamma^2}$$

Can be written in terms of a *Voigt function*:

-33- So the core of the profile is Gaussian due to Doppler broadening, while the wings are much stronger than expected in a Gaussian profile, due to the intrinsic line width.

4. Neutral hydrogen (HI gas) in the ISM

[*] Wherever HI dominates the ISM, all atoms are found in the ground state (^{2}S)(n=1). The next excited level (^{2}P) is about 10 eV above the ground-level. This excitation is very rare and and quickly falls back to the ground level, so there is no significant population of this level. For example, consider potential excitation mechanism: collision of H-atom with cosmic ray particle (lots of energy, probability is once per 10^{17} seconds), ionizes the H-atom. Recombination results in some atoms winding up in ^{2}P state. But the Einstein A coefficient for spontaneous emission from $^2P \rightarrow {}^2S$ is 10^8 s^{-1}. Hence, this excitation process results in relative population of ^2P level of $10^{-8}/10^{17} = 10^{-25}$.

ISM is too cool for collisions to happen often and cosmic rays are rare.

4.1 Possible tracers of HI gas

1. 21 cm HI transition (=hyperfine transition) in emission or absorption.
2. Lyman absorption lines against hot background stars.

Why only these two? Because only the ^{2}S level is populated. HI is hard to find in the ground state; fine structure → different angular momentum.[†]

[*] Draine Ch 8, 29; Ch 17.1, 17.3

[†] See Draine Ch 4 (& 5) on notation of energy levels and atomic structure.

4.1.1 *Excitation and radiative transport for the 21-cm line*

-34- Spin of proton and electron:

- Parallel (upper energy)
- Anti-parallel (lower energy)

(Spin is around particle's own axis, not to be confused with angular momentum). Motions specified by maxwellian velocity distribution, and collisions dominate the level populations (excite and de-excite).

Energy difference (very small):

$$\begin{aligned} h\nu &= 9.4 \times 10^{-18}\,\mathrm{erg} \\ \nu &= 1420.4\,\mathrm{MHz} \\ \lambda &= 21.11\,\mathrm{cm} \end{aligned}$$

Spontaneous emission:
probability is *very* small:

$$A_{kj} = 2.86 \times 10^{-15}\,\mathrm{sec}^{-1}$$

$$\rightarrow \text{lifetime} = 1.10 \times 10^{7}\ \text{years}$$

Collisions:
are more likely to dominate transitions and cause the atoms to flip energy states.

$$\mathrm{H + H \rightarrow H_2^* \rightarrow H + H}$$

(excited H_2 molecule - not stable). Chance for collisional excitation per second: $\gamma n(H)$ [s^{-1}].

$\gamma[\times 10^{-11}\ \mathrm{cm^3 s^{-1}}]$	T [K]
0.23	10
3.0	30
9.5	100
16	300
25	1000

As long as $n(H^o) > 10^{-2}$ cm^{-3}, $\gamma n_H >> A$, so collisions determine excitation and de-excitation…implies detailed balancing, so Boltzmann distribution is valid for level populations.

$$\frac{n_1}{n_o} = \frac{g_1}{g_o}\exp(-h\nu/kT_k) \approx \frac{g_1}{g_o}(1 - \frac{h\nu}{kT_k}) \approx \frac{g_1}{g_o}$$

where T_k = kinetic gas temperature.

-35- Derivation:

-36- Now assume that…

In general, in situations where stimulated emission and absorption can be neglected, we would obtain

$$\frac{b_1}{b_o} = \frac{1}{1 + \frac{A_{10}}{n_H \gamma_{10}}} \approx 1$$

Note: Potential other excitation process of upper level for HI: HI + Lyα photon $\rightarrow$ n=2 level (^2P). ^2P level might cascade down to upper ^2S hyperfine level.

In principle, one might selectively populate the upper level with this so-called *photon pumping**. It turns out that, due to thermilization of the photons this anomalous population of the levels does *not* generally occur. (Lyα photons frequently scatter off A-atom again = LTE situation).†

-37- Since the b's are effectively 1, our main result is that

$$\frac{n_1}{n_0} = \frac{g_1}{g_0}\left(1 - \frac{h\nu}{kT_k}\right) \approx \frac{g_1}{g_0} = 3$$

(since $h\nu \ll kT$). So $n_0 = (1/4)n(H^o$, $n_1 = (3/4)n(H^o$

Are the HI levels always populated mainly by collisions? No, if $n(H^o)$ drops low enough, and if the HI is warm it doesn't work as well.

---

*D 17.3

† This point is also discussed in Kushami & Heiles (in Gal. & Extragal. Radio astron., page 100)

The warm HI in our Galaxy has $n_H \sim 0.4$ cm $^{-3}$ and $T_k \approx 8000$ K. But T_s = spin temperature, of order $T_k/5$, if it weren't for excitation by Lyα photons.

But it is true that the 3K background radiation field does *not* significantly disturb the equilibrium set up by collisions, as we discussed above.

If n_H drops to very low values, collisional excitation is ineffective. But even in that case, $T_s \approx T_k$ because of the above mentioned Lyα excitation.

In any case, in general we now obtain for the emission coefficient:

$$j_\nu = \frac{3}{4} A \frac{h\nu}{4\pi} n(H^o) \phi(\nu)$$

which is independent of temperature in most circumstances. *So we don't need temp to know how much HI there is!* And for the absorption coefficient:

$$\kappa_\nu = \frac{h\nu}{c} \left[n_0 B_{01} - n_1 B_{10} \right] \phi(\nu)$$

-38- Using the relation between Einstein coefficients:

Important result: high T_k (and T_s) and/or broad ϕ_ν both = less absorption. *Warm HI has less optical depth than cold HI for the same column density.*

4.2 Simple case of a single layer of gas

-39-

Consider two cases
(i) $\tau_{\nu_o}(L) >> 1$ Optically thick
(ii) $\tau_{\nu_o}(L) << 1$ Optically thin

In general, the line profile of HI emission is entirely determined by the velocity of the atoms, so the assumption of a Gaussian profile is correct.

-40-

4.3 Observing brightness temperature

Aside: **How do we observe HI?**

1. Radio telescope
   - single dish
     - "single beam": one spectrum per position 1×1 pixel!
     - Multiple beam system: get 7 = 30 beams ("ish")
     - low angular resolution
     - sensitive to all the flux

     Make a map by many pointings. Point telescope, take spectra, repeat.
   - interferometer One or $\sim$ few pointings per source. FOV is set by FWHM of beam of *one* telescope. Angular resolution set by the longest baseline (FOV still the same, regardless of D). In one pointing, can make a map of an area in the sky. $\theta = \dfrac{1.2\lambda}{D_{max}}$
     - High angular resolution
     - Missing baselines in array
       - No zero-m spacing $\rightarrow$ no net flux
       - Missing short spacings $\rightarrow$ negative bowl
       - Missing gaps in baselines $\rightarrow$ grating rings

     Spectroscopic resolution is generally very good.

4.4 HI emission and absorption

-41- Distinguish two cases:

1. Absorption by an HI cloud of an extended background source, usually a continuum source (e.g. AGN)
2. HI self-absorption (enough of it at same velocity)

Case 1:
Simple picture: the radio continuum source is usually a compact, distant (extragalactic) radio source (but this does not have to be the case).

Reminder of radiative transfer: suppose that the HI layer has kinetic temperature T_s, uniform throughout. Then:

$$I_\nu = I_\nu(0)e^{-\tau_\nu} + B_\nu(T_s)\left[1 - e^{-\tau_\nu}\right]$$

Rayleigh-Jeans law:

$$B_\nu(T_s) = \frac{2\nu^2 k T_s}{c^2}, \quad h\nu \ll kT_s$$

Toward the source, we then get:

$$T_b = T_{bo} e^{-\tau_\nu} + T_s \left(1 - e^{-\tau_\nu}\right)$$

where

- T_b = observed brightness temperature (as function of ν)
- T_{bo} = brightness temperature of background source $\equiv T_c$ (continuum), which is essentially independent of frequency across narrow frequency range of 21 cm line. *Constant, though technically still function of frequency.*
- τ_ν = optical depth ghrough HI layer, at frequency ν. Only non-zero right around the 21 cm line; ν_0 = 1420.4 MHz.

-42- In the direction immediately next to the source we have:

$$T_{b,off} = T_b e^{-\tau_\nu} + T_s \left(1 - e^{-\tau_\nu}\right)$$

The brightness temperature both on and off the source are for each individual frequency channel.

Some practical points

1. Observe in a *frequency band* in a number of channels, together spanning a range in velocity.
$$\frac{\Delta v}{c} = \frac{\Delta \nu}{\nu}$$
Total bandwidth is 100 - 1000 km s^{-1}, and the channel spacing, $\Delta\nu$ is $\sim$ 1 - 5 km s^{-1}.
2. The assumption here is that the "off source" position, the HI emission from the cloud is the same as in front of the continuum source (so the cloud is rather uniform). Evidently, this assumption works better if your radio telescope has high angular resolution. $\rightarrow$ need a large (single) dish.
3. A way to potentially avoid this problem is to have a background source that switches on and off (a pulsar). However, in practice, pulsars are generally too weak.

4. Do we really have only one HI cloud along the LOS, and is its temperature uniform? We really don't need only one cloud, provided the clouds have different *velocities*. However, the uniform T_s assumption is definitely a problem. (More about that later.) We can minimize the complexity in terms of many clouds along the LOS by observing clouds and background sources at high galactic latitudes.

-43- We observe at each frequency for which there is HI. Let the observed brightness temperature at source position after subtraction of continuum be given by

$$T_{b,on} - T_c \equiv T_{b,obs}$$

We get

$$\begin{aligned} T_{b,off} - T_{b,on} &= -T_c e^{-\tau_\nu} \\ T_{b,off} - T_{b,obs} &= T_c \left(1 - e^{-\tau_\nu}\right) \\ \left(1 - e^{-\tau_\nu}\right) &= \frac{T_{b,off} - T_{b,obs}}{T_c} \end{aligned}$$

Measure (observe) $T_{b,off}$, $T_{b,on}$, and T_c. Then derive τ_ν.

Once τ_ν is known, we can find T_s because

$$T_s = \frac{T_{b,off}}{1 - e^{-\tau_\nu}}$$

the HI spin or kinetic temperature.

<u>Reminder of general point</u>:

$$\begin{aligned} T_{b,off} &= T_s \left(e^{-\tau_\nu}\right) \\ &= \tau_\nu T_s \quad \text{for } \tau_\nu \ll 1 \\ &= T_s \quad \text{for } \tau_\nu \gg 1 \\ T_{b,off} &\leq T_s \end{aligned}$$

The last line is *always* the case!

Discussion of some results
Many recent references[*][†]

-44- Plots

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* Dickey et al. ApJ 228, 465 (1979)
† See also Dickey and Lockman, Ann Rev of A & A 28 (1990)

4.5 Conclusions

-45-

1. The absorption profile $(1 - e^{-\tau_\nu})$ is always sharper and simpler in structure than the emission line profile. Follows from Gauss analysis of profiles (so fit Gaussians).

$$T = \sum_{i=1}^{m} T_i \exp\left\{-\frac{1}{2}\left(\frac{v - v_i}{\sigma_i}\right)^2\right\}$$

σ's are smaller for absorption profile. However, it is not clear as to what extent this is significant, except to note that absorption profiles are especially sensitive to cold cloud (cores) which presumably are narrower in velocity width. "You have to realize you are comparing apples & peas" *??* Given an HI distribution of column density $N(HI)$ = constant, say τT_s = 20 K (note that $\tau T_s \propto N(HI)$). Assume that T_s = 50, 100, 200 K. Then:

τT_s	T_s	τ	$1 - e^{-\tau}$	T_B^*
20 K	50 K	.4	0.33	16.5
20 K	100 K	.2	0.18	18.0
20 K	200 K	.1	0.095	19.0

* from $T_B = T_s(1 - e^{-\tau})$. So a variation of $\sim$ factor of 3 in absorption profile, $1 - e^{-\tau}$ corresponds to variation of only 16% in T_B. (Note: what would shape of $(1 - e^{-\tau})$ and T_B look like for single [layer cloud]?) Warm HI will only appear in emission, since the optical depth is too small to give significant absorption. Only recent deep observations begin to see absorption in warm HI.*

-46- 2. The variation in spin temperature, T_s, apparent in Fig 3-29, makes onewonder whether there is really one temperature in each cloud (there seems to be three in this profile). wonder whether there is really only one temperature in the cloud.
3. The range of T_s one finds from absorption line studies is 50 K $\leq T_s \leq$ 1000 K. However, the upper value is only a lower limit since we don't see absorption anymore for $T_s >$ 1000 K. So we have no handle on T_s above that.

* Look for recent papers by Carl Heiles and by Jay Lockman.

4. One can compare T_{spin} with $T_{Doppler}$, the temperature derived from the velocity width of each line. It turns out that always $T_{spin} < T_{Doppler}$.
   $\rightarrow$ Doppler motions that we see in a cloud generally do not correspond to thermal velocities; this is probably due to turbulence.
   Analysis of emission and absorption line profiles of HI across our Galaxy have led to a continuously evolving pictrue of the HI distribution that we have. The early desire to attribute all HI to "clouds" is understandable, but not always useful.*

4.6 Some relevant results

-47-

1. HI is *not* concentrated in a small number of giant clouds, as H_2 is; estimates of filling factor range from 20-90%?!
2. HI occurs roughly 50/50 in two important forms:
   - CNM = cold neutral medium $\sim$ 80 K
   - WNM = warm neutral medium $\lesssim$ 8000 K
     About half of all HI gas, whose emission line profiles can be split into two components
     - narrow ($\sigma < 5$ km s^{-1})
     - wide ($5 < \sigma < 17$ km s^{-1})

     WNM = *wide* component
     - present in essentially all directions from solar neighborhood
     - large vertical scale-geight, possibly up to 480 pc
3. Clouds are filaments and/or sheets, rather than spheres
4. At low galactic latitudes, it is not possible to distinguish between WNM and CNM because there is too much material along the line of sight.
5. **Problem with different temperatures along the line of sight:** Illustration: consider just two isothermal clouds. For one cloud:

$$T_s = \frac{T_B}{1 - e^{-\tau}}$$

-48- For two clouds, we have two spin temperature: $T_{s1} and T_{s2}$. Then let $T_n(v)$ be the (naively determined) spin temperature:

$$T_n(v) = \frac{T_{s1}(1 - e^{-\tau_1}) + T_{s2}(1 - e^{-\tau_2})e^{-\tau_1}}{(1 - e^{-\tau_1 - \tau_2})}$$

* Good summary of some of the main properties of MW HI distribution is given by Mulkami & Heiles in their 1988 review in Gal. & Extragal. radio astronomy

We see that T_n depends on v and ranges from T_{s1} through T_{s2}, depending on τ_v. We would conclude that HI exists at temperatures intermediate to T_{s1} and T_{s2}.

Special cases: *compare τ_1 to τ_2 over velocity range where $N(HI)$ peaks*:

1. $\tau_1 \gg 1$, $\tau_2 \ll 1 \rightarrow T_n \approx T_{s1}$
2. $\tau_1 \ll 1$, $\tau_2 \gg 1 \rightarrow T_n \approx T_{s1}\tau_1 + T_{s2}$. So we increased the cold temperature T_{s2} by a non-physical foreground temperature of warm gas. Happens a lot in practice!
3. $\tau_1 \ll 1$, $\tau_2 \ll 1 \rightarrow T_n$ is a column density weighted harmonic mean temperature. Example: 80 K CNM + 8000 K WNM with equal column densities; then $T_n \approx 160$ K, so you *overestimate T_{cool}* and *greatly underestimate T_{warm}*.

4.7 Some additional points on HI

-49-

- HI absorption measurements toward Galactic continuum sources provide (some) information on the distance to these sources, using the velocity of the HI absorption feature to derive a kinematical distance. The usual problem of distance ambiguity may still be a problem here as well, although the total HI column [density?] that is derived from the absorption measurements helps a little in distinguishing near and far distances.
- **Temperature of WNM**... two methods:
  1. HI absorption: complicated because τ is small, plus stray radiation. Also biases to low temperatures when cooler gas is along the LOS.
  2. UV absorption lines: local (< 200 pc) stars give $T \gtrsim 6000$ K for WNM

  So far: data in agreement with 5000 K $\lesssim T \lesssim$ 8000 K, but needs more confirmation.
- **Temperature of CNM**... from absorption spectra. We already pointed out correlation of T_{spin} with peaks in absorption profile (see fig. in handout, page 44). Interpretation: HI clouds are not isothermal blobs. Usually we derive what we call T_{spin} of the cloud at the center of the
-50- absorption profile, so this is $T_{n,min}$ The absorption lines are *narrow*.

Present research on Milky Way HI concentrates on:

1. Origin of high velocity clouds
2. Properties of smallest clouds/features
3. Search for HI clouds that may contain dark matter, but no stars

-50a, Images and plots.
50b,
50c-
-I1-

5. Atomic structure

5.1 Introduction

5.2 Hydrogen atom & hydrogen-like atoms (or ions)

-I2- Example: energy of ground states:

- H = 13.6 eV
- He^{+} = 54.4 eV
- Li^{2+} = 122.5 eV

-I3-

-I4-

-I5-

-I6-

5.3 Electron spin

-I7- But, since $\vec{J}_1$ and $\vec{J}_2$ can have different directions, there are different possible values of $\vec{J}$.

-I8-

5.4 Spin orbit coupling

Two possible orientations of electron spin with respect to orbital angular momentum lead to a doubling of energy levels of H-like atoms (except s-levels, where $\ell = 0$).

Lines appear in pairs, close together, called doublets. Example: for Na D lines, λ = 5890Å, 5896Å. (Question: are atoms with filled shells + one electron also hydrogen-like? Doubling is due to spin-orbit coupling.)

Physically, the spin-orbit coupling produces an extra energy term for the electron, proportioned to $\vec{S} \cdot \vec{L}$.

-I10-

5.5 Atoms with multiple electrons

-I11-
-I12-
-I13-

5.6 Transition rules

-I14-
-I15-
-I16-
-I17-

5.7 *X*-ray emission

-I18-
-I19-

5.8 Zeeman effect

-I20-
-I21-

6. HII regions

6.1 Introductory remarks

Process: hot OB stars emit UV photons that can ionize the surrounding neutral H (and He) medium. In practice, this requires stars hotter than $\sim$ 30,000 K, aka. spectral type B0 or earlier. The physics for planetary nebulae (PN) is similar to that of HII regions, but central stars in PN may be much hotter and dimmer because there is "more stuff".

UV photons impart energy to gas by ionizing H and He. Excess kinetic energy of created free electrons is shared with ions and other electrons, *heating* the medium.

Relevant processes:

- photoionization
- electron-electron encounters
- electron-ion encounters
  - Excited ions
  - Recombination
  - Bremsstrahlung

H atom + photon $h\nu \longrightarrow$ p$^+$ + e$^-$ + E$_k$

$\Longrightarrow$ Since stars create a continuous stream of photons, a balance is reached between ionization and recombination. [*] [†]

Field OB stars: were they born there or travel there somehow? Usually in groups (OB associations), dense clouds, etc.

Consequences: $\sim 10^4$ *K. Warm ionized plasma emitting various forms of radiation: recombination, free-free continuum, bree-bound continuum, 2-photon continuum, collisionally excited forbidden lines from "metals". If there is dust around, get MIR dust continuum. Runaway OB stars (like teenagers!)*

1 km $\equiv$ *1 pc in* 10^6 *years. (?)*

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[*] Draine Ch 10, 13.1, 14, 15, 17, 18, 27, 28

[†] Osterbrock AGN^2

Q_o [s^{-1}] = number of ionized photons emitted by OB star per second. Factor of 100 from O 9.5V to O 3V (luminosity classes).

6.2 Strömgren Theory

-52- "It's only a model." Classical paper*

Basic result:

- Hot star in a uniform medium will ionize a spherical volume out to a certain radius, whose size is determined by:
  1. number of ionizing photons emitted by the star
  2. density of medium (determines recombination rate)
- There is a *sharp* boundary from the ionized to the surrounding neutral medium.

6.3 Simple derivation

Pure H nebula, uniform density. Hot star emits N_{Lyc} photons per second (*Lyc* = Lyman continuum), all at the *same* frequency, ν_o. The star will ionize the gas, but there is a balance:

$$\text{ionization} \longleftrightarrow \text{recombination}$$

Stationary ionization eqilibrium characterized by *degree of ionization*:

$$x(r) \equiv \frac{n_e(r)}{n_H}$$

where n_H = *total* hydrogen density (ions and neutral H atoms). $0 \leq x \leq 1$ ($x = 1 \rightarrow$ complete ionization; $x = 0 \rightarrow$ completely neutral). Problem: what is the shape of $x(r)$?

Important quantity: the Strömgren radius, R_{SO}, defined as

$$\frac{4}{3}\pi\alpha n_H^2 R_{SO}^3 \equiv N_{Lyc}$$

*Strömgren 1939 ApJ 89, 526

where α is the recombination coefficient (collisional process $\sim \rho^2 \rightarrow$ more encounters, see 6.3.2). The quantity N_{Lyc}[*] gives the total number of recombinations per unit time (all the magic is here!).[†]

or as I like to put it:

$$\frac{4}{3}\pi R_{SO}^3 = \frac{N_{Lyc}}{\alpha n_H^2}$$

The volume of the sphere is determined by the number of ionizing photons divided by the recombination coefficient.

6.3.1 Description of photoionization equilibrium

-53-

Overview:
Consider pure H nebula surrouding a single hot star. Ionization equilibrium:

$$n_{H^o} \int_{\nu_1}^{\infty} \frac{4\pi J_\nu}{h\nu} \sigma_\nu \mathrm{d}\nu = n_e n_p \alpha(H, T)$$

where

- n_{H^o} = neutral hydrogen density
- J_ν = mean intensity of radiation field
- $\sigma_\nu \approx 6 \times 10^{-18}$ cm^2 = ionization cross-section for H by photons with energy above threshold $h\nu_{01}$
- $\alpha(H,T) \approx 4 \times 10^{-13}$ cm^3 sec^{-1} = recombination coefficient
- n_e, n_p = electron and proton densities
- LHS: number of *ionizations* per second per cm^3
- RHS: number of *recombinations* per second per cm^3

To first order, not including radiative transfer effects:

$$4\pi J_\nu = \frac{R^2}{r^2}\pi F_\nu(0) = \frac{L_\nu}{4\pi r^2}$$

(where the far right term is the local radiation field). Order of magnitudes:

$$N_{Lyc} = \int_{\nu_1}^{\infty} \frac{L_\nu}{h\nu} \mathrm{d}\nu$$

[*] $N_{Lyc} = S_4$ in Spitzer's notation.
[†] S_4 formally should include contributions from the diffuse radation field.

- $N_{Lyc} \approx 5 \times 10^{48}$ sec^{-1} for O6 star

-54- Physical conditions: radiative decay from upper levels to $n = 1$ is quick $\rightarrow$ nearly all neutral hydrogen will be in the ground level. Photoionization takes place from the ground level, and is balanced by recombination to excited levels, which then quickly de-excite by emission of photons.

The *photoionization cross-section* ($\sigma \propto \nu^{-3}$) is actually rather complicated to calcualte. Spitzer (who calls this erroneously the "absorption coefficient") gives:

$$\sigma_{f\nu} = \frac{7.9 \times 10^{-18}}{Z^2} \left(\frac{\nu_1}{\nu}\right)^3 g_1 f$$

where $g_1 f$ = Gaunt factor (from level 1 to free)*

Since $\sigma_\nu \propto \nu^{-3}$, higher energy photons will typically penetrate further into the nebula before being absorbed.

6.3.2 The recombination coefficient

$\alpha_n(H, T)$ = recombination coefficient for *direct* recombination to level n.

$$\alpha^{(n)} \equiv \sum_{m=n}^{\infty} \alpha_m$$

where m = other levels the electron cascades through to get to n.

Note: Ouside of level 1, higher energy not necessarily better for ionization. OIII is not necessarily from the same place as OII. Free electrons: there is a recombination coefficient for every level, depends on velocity of the electron.

-55- The summed recombination coefficient (α) is the relevant quantity because every recombination counts, regardless of the level n to which the electron recombines.

Larger velocities $\rightarrow$ smaller σ.

For a distribution $f(v)\mathrm{d}v$, $n_e f(v) v$ = number of electrons with velocity v

*see, e.g. Table 5.1 Spitzer (Draine Ch. 10); see also Fig 13.1 in Draine and Fig. 2.2 in handout (from Osterbrock).

passing through a unit area per second. Thus

$$\alpha_n = \int_o^\infty v\sigma_n(H,v)f(v)\mathrm{d}v$$

where $\sigma_n(H,v) \propto v^{-2}$ = recombination cross-section to the level n for electrons with velocity v.

$$f(v) = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 \mathrm{e}^{-mv^2/2kT}$$

(Maxwellian). Hence, $\alpha_n \propto \frac{1}{\sqrt{T}}$*

We are interested in the total recombination coefficient to all levels: However, for direct recombination to $n = 1$, a photon is generated which itself can ionize H again; so the recombination coefficient that we are really interested in is $\alpha^{(2)} \equiv \alpha_B$† *This refers to "case B": optically thick through Lyα emission. Almost always here! Most common situation. Case A: very low density gas.*

-56- Neutral fraction *inside* HII region‡: Consider pure H nebula, and case B recombination (see later).

- $h\nu$ = *Mean energy* of stellar ionization photons
- σ_{pi} = photoionization corss-section at freqeuncy ν
- $Q(r)$ = rate at which ionizing photons cross spherical surface
- Q_0 = rate at which ionizing photons are emitted by the star

In steady state:

$$Q(r) = Q_0 - \int_0^r n_H^2 \alpha_B x^2 4\pi (r')^2 \mathrm{d}r' = Q_0 \left[1 - 3\int_0^{r/R_{SO}} x^2 y^2 \mathrm{d}y\right]$$

here:

...

-57- Rewrite latter as:

*see e.g. Spitzer, Table 5.2

†see Draine section 14.2

‡see Draine section 15.3

so neutral fraction x is small and hence $x^2 \approx 1$.

HII regions have sharp boundaries (concept of Stromgren radius is useful!) Solving the equations iteratively actually shows that $Q(r)$ approaches 0 near $y \sim 1$ in exponential decay.* This is also solved by iteration.

-58- Comments:

1. Reality: include He, include diffuse radiation field generated from direct recombination to $n = 1$ (which can ionize an atom again!), realistic stellar energy spectrum below $\lambda = 912$Å; results are qualitatively the same.
2. Is assumption of a static ionization equilibrium realistic? No: HII region is overpressureized compared to the surrounding medium: T_e and n higher $\longrightarrow$ HII region expands†. Expansion of the ionization front happens rather slowly, $v \propto 1 - 2$ km s^{-1}, dynamical timescale. $\frac{R_{SO}}{v} \sim 10^7$ years, of order liftime of HII region. So ionization equilibrium does hold.
3. Recombination time scale = ionization time scale:

$$t_{ionization} \equiv \frac{\frac{4}{3}(R_{SO})^3 n_H}{Q_o} = \frac{1}{\alpha_B n_H} = \frac{1.22 \times 10^3}{n_2} \text{ years} = t_{recombination}$$

   ($n_2 = n_H$ in units of 100). The two are identical! So $t_{recombination} \propto \frac{1}{n_e}$ Key: for $n_H > 0.03$ cm^{-3} $t_{recombination} <$ lifetime of massive stars ($\lesssim 5$ Myr).
4. As we will see later, each Lyc photon results in ~ 0.35 Hα photons begin produced in this ionization $\rightarrow$ recombination equilibrium.
5. Next: influence of dust on R_{SO}

6.4 The spectrum of an HII region

-63-

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* See page 59 for shape of $x(r)$ (fig 2.4 and 2.6).

†First discussed by Kahn (1954)

6.4.1 *Continuum Radiation*

Sources:

Two-photon decay:
Transition from $n = 2 \rightarrow n = 1$ ($\ell = 0 \rightarrow \ell = 0$) is strictly forbidden ($\Delta\ell \neq 0$). The actual process happens in two steps, with a virtual intermediate phase. Two photons are emitted whose joint energy adds up to the energy of a Lyα photon = 3/4 ionization energy of H. The two-photon continuum is symmetric about $\nu_{12}/2$ if expressed in photons per unit frequency instead.[*]

Free-free emission:
Thermal Bremsstrahlung[†]: continuous emission and absoprtion by thermal (Maxwellian velocity distribution) electrons due to encounters between electrons and positive ions.

Acceleration/deceleration $\longrightarrow$ photons.

Classical theory: electron emits a single narrow Em pulse in time, with no oscillation in E. $\rightarrow$ FT is broad, almost the FT of a δ function. $\rightarrow$ Emission coefficient j_ν is nearly independent of frequency, up to a *cutoff* frequency,
-64- which corresponds to the Maxwellian velocity distribution of electrons. So cutoff frequency is given roughly by $h\nu \sim kT_e$. (HII regions: $T_e \sim 10^4$ K, so $h\nu \approx 0.87$ eV and $\lambda \approx 1.4\mu$m).

Emission coefficient:

$$\begin{aligned} j_\nu &= \frac{8}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{Z_i^2 \mathrm{e}^6}{m_e^{3/2} c^3 (kT)^{1/2}} g_{ff} n_e n_i \mathrm{e}^{-h\nu/kT} \\ &= 5.44 \times 10^{-39} \frac{g_{ff} Z_i^2 n_e n_i}{\sqrt{T}} \mathrm{e}^{-h\nu/kT} \quad [\text{erg cm}^{-3}\ \text{s}^{-1}\ \text{sr}^{-1}\ \text{Hz}^{-1}] \end{aligned}$$

where $\mathrm{e}^{-h\nu/kT}$ is the exponential cutoff due to kT_e and g_{ff} is the Gaunt factor for free-free transitions[‡].

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[*]See Osterbrock, pp 89-93 for more info.

[†] §3.5 Spitzer, Drane ch. 10

[‡]see Draine, figure 10.1

Total amount of energy radiated in free-free transitions per cm^3 per second:

$$\epsilon_{ff} = 4\pi \int j_\nu \mathrm{d}\nu$$

...(long-ass equation)

Remember that the corresponding absorption coefficient κ_ν is related to j_ν by Kirchoff's law, since we deal with *thermal* emission (*not* blackbody though!) $j_\nu = \kappa_\nu B_\nu(T)$ which leads to [anther long-ass equation].

emission by dust particles
In practice, dust emission dominates at IR wavelengths, so it is unlikely to observe the free-free spectrum this far.

-65-

Important in practical situations

1. Free-free emission is generally observed in the *radio* regime for HII regions
2. For a source at constant T:

$$I_\nu = S_\nu \left(1 - e^{-\tau_\nu}\right)$$

$$S_\nu = \frac{j_\nu}{\kappa_\nu} = B_\nu(T_e)$$

$$\tau_\nu = \int \kappa_\nu \mathrm{d}s$$

   And if T_e is independent of path length, then

$$\tau_\nu \sim \int n_e n_i \mathrm{d}s \approx \int n_e^2 \mathrm{d}s = EM$$

   also $\tau_\nu \propto \nu^{-2.1}$ (the 0.1 is due to the log term).

Optically thin emission

Optically thick emission

-66- (Plots)

-67-

Free-bound emission

-68-

6.4.2 *Line Radiation*

-69- Two types:

1. Recombination lines, predominantly from H and He
   - optical
   - radio
   - forbidden
2. Collisionally excited lines in heavy elements; these are find-structure lines, often forbidden, yet strong. There are recombination lines from heavy elements as well, but they are too weak to observe.

Recombination lines:

- emission coefficient: $j_\nu = \dfrac{A_{mn'} N_n h\nu_{nn'}}{4\pi} \phi(\nu)$
- absorption coefficient: $\kappa_\nu = \ldots$

-71- The calculation is still highly complicated. We can, however, derive some general properties of $\{N_n\}$

Radio recombination lines
Not very common, lines tend to be very faint.

-72- So:

Optical and IR recombination lines
For small n, $|E_{n+1} - E_n|$ is larger, so collisions become less important while the A coefficients get bigger.

Collisionally excited lines in heavy elements:
-79- Generally: electrons don't have enough energy to excite ions from the ground state to their first excited states. However, many ions have multiplets in their ground state, due to the coupling of the electrons individual (orbital and spin) angular momenta.

Many ions have incomplete "p-shells"; (there is not really a p-shell, but we talk about electrons which have $\ell = 1$, corresponding to p). Remember:

H	1s		
He	$1s^2$		
C	$1s^2$	$2s^2$	$2p^2$
O	$1s^2$	$2s^2$	$2p^4$

For $n = 2$, there is room for 6 p-electrons, since each $n\ell$ has a number of m_ℓ and m_s combinations:

$$\left.\begin{array}{l} m_\ell : 2\ell + 1 \\ m_s : \pm\frac{1}{2} \end{array}\right\} 2(2\ell + 1)$$

So as long as $\ell = 1$ is possible (meaning $n > 1$), we have 6 potential "p-shell" electrons, *always*.

The enclosed diagram shows the 3 most typical structures of energy levels for various elements in different ionization stages. For [OIII] we have, in particular: 2 electrons in "p-shell", $\ell_1 = \ell_2$, so $L = 0, 1, 2$

-81- One can set up 5 equations of statistical equilibrium for each of these levels. This shows that both the ^1S_o and the $^1D_2$ levels are populated essentially exclusively by collisions from ^3P level.

So, not surprisingly, the ratio

$$\frac{N(^1S_0)}{N(^1D_2)} \quad \propto \quad \exp\left(\frac{-\Delta E}{kT}\right)$$

where ΔE = energy difference between $^1S_0$ and $^1D_2$

Also, observed flux in $\lambda 4363$ line is:

$$S(4363) \quad \propto \quad \frac{4}{3}\pi R^3 N('S_0) A_{4363} h\nu$$

and similarly for the 4959 and 5007 lines.

The net result, obtained by solving the equations of statistical equilibrium is:

$$\frac{S(4959) + S(5007)}{S(4363)} = 8.3 \exp\left(\frac{3.3 \times 10^4}{T_e}\right)$$

so this line ratio provides a direct measure of the electron temperature.

A similar relationship exists for [NII] (6548, 6583, and 5754) lines.

In general: Emission lines arising from ions, such as O^{++} and N^{+} that have upper energy levels that have considerable different energies are useful for estimating T_e. Vice versa, as we will see, ions with closely spaced upper enrgy levels provie little info on T_e, but can be used to derive n_e (examples: [SII], [OII] lines).

Notes about T_e determinations:

1. Look at
2. Collisional
3. ${}^1D_2 \rightarrow {}^3P_0$ transition

6.5 Types of HII Regions

-85- In general:

- HII regions are associated with molecular clouds and dark nebulae.
- Structure:
    - "Blister model" – cavity inside GMC
    - "Champagne model" – half cavity at edge of GMC

There are also *compact* HII regions, which are only visible at radio and FIR wavelengths. These are very young objects, often associated with H2O and OH masers, infrared sources, molecular lines of complex molecules, etc.

The *range* in properties of HII regions is enormous*.

Modeling of HII region spectra: the probably best known code is CLOUDY, developed by G. Ferland. You can in put an ioinizing spectrum from a star or other, a gas distribution, metallicity, etc., and then calculate the detailed emission line spectrum for the region.

HII regions can be density-bounded or radiation-bounded. rad: run out of photons before running out of gas. density: run out of material before using up photons. No clear boundary.

*Kennicut, 1984

7. Interstellar absorption lines in stellar and quasar spectra

-89- Discovered in 1904, *before we were aware of dust, or that MW and M31 were separate galaxies.* Typical optical absorption lines discovered later include Ca^+, Na^o, K, Ti^+, and some molecular lines, e.g. CH^+, CH, CN. Lines from more abundant atoms and molecules (H, H_2, C, N, O, etc.) were only found after Copernicus was launched (early 70s) because $\lambda < 3000$Å for these lines, so they have to be observed from space.

Key: Since the absorption process is proportional to the incoming intensity, we can find very small column densities of ISM, provided the background source is strong enough.

Absorption of optical photons generally only occurs from the ground level of the atom, ion, or molecule, because only these levels are populated in most conditions (remember that kT = 0.86 eV for T = 10^4 K, or 8.6 eV for T = 10^5 K). This implies that in the formation of absorption lines, stimulated emission from the upper level does not play a role.

O^+ in HII region sits in ground state, can assume ions in ISM also stay in $n = 1$. Pure absorption foreground screen. $I = S(\lambda)\,(1 - \mathrm{e}^{-\tau_\lambda})$, can leave the exponent out ($\rightarrow 0$).

This is not the case whenever excitation conditions are such that absorption lines do arise from excited atoms or molecular levels. Usually collisions would be responsibe for the excitation, so keep an eye on kT.

-90- Generally, interstellar absorption lines are *narrow* and *complex* in structure. So we require very high resolution to resolve them:
$\left(\dfrac{\lambda}{\Delta\lambda} > 3\times10^5\right) \Leftarrow$ ideal, but rarely achieved. *res: couple thousand*

Remember that $\dfrac{\Delta\lambda}{\lambda} = \dfrac{\Delta v}{c}$ and for $\dfrac{\lambda}{\Delta\lambda} \geq 3\times10^5 \rightarrow v \leq 1$ km per second.
Which implies $\Delta\lambda \sim 0.02$ Å at 6000 Å and $\Delta\lambda \sim 0.0033$ Å at 1200 Å

7.1 Theory of the formation of absorption lines in the ISM

Ideal background source: bright (especially UV), with a featureless continuum. ⇒ Quasars and AGN or hot stars. Stars tend to be more complicated, though quasars sometimes have their own lines as well.

7.1.1 *Equivalent width*

The high spectral resolution that is required to resolve a line is often unattainable. However, even if we cannot *resolve* the absorption line, we can still infer some important properties about the ISM. As long as the resolution is sufficient to separate absorption lines at different velocities, coming from different clouds along the line-of-sight, or if only one component is present, we can use the *area* of an absorption line, or the *equivalent width*, EW, the width of an absorption line which would absorb 100% everywhere, and which would have the same area as the hatched area. Note that EW is always defined relative to I_o; we *divide* by I_o.

-91- Note also that if we decrease the resolving power of the spectrograph, EW does not change.

Theoretical expression for EW:

$$\tau_\nu = \int_0^\infty \kappa_\nu \mathrm{d}r; \quad \kappa_\nu = h\nu_{u\ell} \frac{n_\ell B_{\ell u} - n_u B_{u\ell}}{c} \phi(\nu)$$

(using "energy density" definition of B coefficients).

- u = upper level
- ℓ = lower level.

If stimulated emission can be neglected, we have:

$$\kappa_\nu = \frac{h\nu_{u\ell} B_{\ell u}}{c} n_\ell \phi(\nu) \equiv \frac{\pi \mathrm{e}^2}{m_e c} f_{\ell u} n_\ell \phi(\nu)$$

- $f_{\ell u}$ = *oscillator strength (historical terminology)*
- $\frac{\pi \mathrm{e}^2}{m_e c}$ = "classical cross-section"* $\sim$ *Einstein coefficient.*

Use $|\mathrm{d}\nu| = \frac{c}{\lambda^2}|\mathrm{d}\lambda|$ and we obtain:

$$\begin{aligned} EW &= \int_o^\infty \left[1 - \mathrm{e}^{-\tau_\nu}\right] \mathrm{d}\lambda \\ &= \frac{\lambda^2}{c} \int_o^\infty \left[1 - \mathrm{e}^{-\tau_\nu}\right] \mathrm{d}\nu \\ &= \frac{\lambda^2}{c} \int_o^\infty \left[1 - \exp\left(-\int_0^\infty \left(\frac{\pi \mathrm{e}^2}{mc}\right) f_{\ell u} n_\ell \phi(\nu) \mathrm{d}r\right)\right] \mathrm{d}\nu \end{aligned}$$

Now assume: $\phi(\nu)$ independent of r for the particular cloud that we are observing. Then:

$$EW = \frac{\lambda^2}{c} \int_0^\infty \left[1 - \exp\left(-\frac{\pi \mathrm{e}^2}{mc} f_{\ell u} \phi(\nu) N_\ell\right)\right] \mathrm{d}\nu$$

with $N_\ell = \int n_\ell \, \mathrm{d}r$ = *column density* of atoms in level ℓ. The quantity $\phi(\nu) N_\ell$ is the key part of this equation.

-92- Note: This situation is very different from the discussion of HI 21-cm abs lines where the upper level *is* populated, and we had to include stimulated emission to first order.

* see for example Rybicki and Lightman, section 3.6

We next assume a Gaussian velocity distribution (Maxwellian) to describe the distribution of radial velocities:

$$f(v_{rad}) = \frac{1}{\sqrt{\pi}b} \exp\left[-\left(\frac{v_{rad} - v_o}{b}\right)^2\right]$$

where $b = \sqrt{\frac{2kT}{m}}$ (only valid for thermal motions) and $\phi(\nu)$ becomes:

$$\phi(\nu) = \frac{1}{\sqrt{\pi}} \frac{1}{\Delta v_b} H(a, u)$$

$$a \equiv \frac{\Gamma}{4\pi \Delta v_b}; \quad u = \frac{\nu - \nu*}{\Delta \nu_b}; \quad \Delta \nu_b \equiv b\frac{\nu_o}{c}; \quad \nu* = \nu_o(1 - \frac{v_o}{c})$$

ν^* is the central frequency of the line. $b^2 = 2\sigma^2$ where σ = velocity dispersion, and a is a measure of where the Gaussian profile levels off to the Lorentzian wings[*] *(transition of Gaussian line core to the damping wings)*.

7.2 The Curve of Growth

What is all this good for?

For a given spectral line, we can measure EW from observations. Free parameters: N_ℓ, b, and a. For a given cloud, b and a are specified, and we can plot EW as a function of N_ℓ. In practice, one plots $\log(W/\lambda)$ against $\log(N_\ell f_{\ell u} \lambda)$.[†] This is the *curve of growth*.

We can distinguish three regimes in a curve of growth.

-93- I. $EW \propto N_\ell$ (linear)
II. EW almost independent of N_ℓ
III. $EW \propto \sqrt{N_\ell}$

Turnover points between the regimes:

- Point A: location determined by N_ℓ / b
- Point B: location determined by value of a

---

[*] see notes from before, p. 34
[†] example: figure 3.2 in Spitzer

7.2.1 Regimes in the curve of growth

I. Here $\tau_\nu \ll 1$ everywhere, which implies that all atoms see the same incoming continuum intensity. $1 - e^{-\tau_\nu} \approx \tau_\nu$ so

$$EW = \frac{\lambda^2}{c} \left(\frac{\pi e^2}{mc} \right) f_{\ell u} N_\ell$$

and

$$\frac{EW}{\lambda} \propto N_\ell f_{\ell u} \lambda$$

II. Center of line becomes saturated, so absorption reaches 100%. Because the Voigt function has steep edges (not wings!) in the central part which is dominated by Doppler motions, EW increases only slowly with N_ℓ.

III. Now if we keep increasing N_ℓ, the optical depth in the wings starts playing a role. Here, $EW \propto \sqrt{N_\ell}$*

7.2.2 Turnover points in the curve of growth

-94- What determines the location of the turnover points?

- Point A (I $\rightarrow$ II): This is where the gas becomes optically thick, which is determined by column density N and the profile function $\phi(\nu)$, in particular for the line center $\phi(\nu*) \propto 1/b$, so $\tau \propto N_\ell/b$
  Example: What happens if b increases? τ becomes smaller because the velocity dispersion of the gas is bigger, so the line is not saturated as quickly. So the linear part of the curve of growth is larger, point A moves toward the upper right.
- Point B (II $\rightarrow$ III): This is mainly determined by the value of a, which specifies where the steep edges of the Lorentz profile go over into the extended wings. So essentially Γ, where $\Gamma \sim 1/\tau_0$ (τ_0 = lifetime of the level)

7.2.3 Growth curves in practice

What do observations tell us?

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* See, e.g. Mihalas, Stellar Atmospheres, section 10.3

I. Suppose you only observe one interstellar line, but you managed to resolve it completely. Then you know $1 - e^{-\tau_\nu}$ and can find b and N_ℓ.

II. What if you can't resove the line and can only measure EW? EW is determined by N, b, a, and $f_{\ell u}$ (which is known from laboratory ex-
-95- periments). Without further knowledge, there is a lot of ambiguity since similar values of EW can be produced by different sets of N_ℓ, a, and b. However, if you know that your line is optically thin, a and b are not important, and your EW immediately gives N_ℓ, provided you know $f_{\ell u}$.

III. What if you are not sure that your line is optically thin? Generally a is very small, and the damping wings are weak; center of line profile is Gaussian, and point A is not influenced by the values of a, as we saw before.

Suppose now that we have two lines, originating from the same level E_j, with equivalent widths W_1 and W_2. N_ℓ and b would be similar for both. If we know both $f_{\ell u}$ values, we can derive N_ℓ and b.

Examples of such cases:

- Ca^+ H and K lines $\lambda3934$ and $\lambda3968$ from the same ground level $^2S_{1/2}$
- Na^o D lines near 5800Å and weak doublet near 3300Å

7.3 UV absorption lines from H and H_2

-96-

8. Dust

-104- Size range: "standard large grains" follow exponential size distribution, extending from (theoretically) $a_{min} \approx 0.01\mu$m to $a_{max} \approx 0.25\mu$m (huge!)[*] $n(a) \propto a^{-3.5}$. A grain with size 0.1 μm = 1000 Å would contain $\sim 10^{10}$ atoms. Grains are really "solids"! *(behave like solids, even though tiny.)*

The size distribution was later extended downward to much smaller particles to account for the excess 12 μm and 25μm emission observed by IRAS; this excess indicated the presence of warm (several hundred K) dust, and it is not possible to heat large dust grains to such high temperatures (see below). It is also possible that at the smallest sizes, we have *complex molecules* rather than grains. A favorite candidate is "PAH", or polycyclic aromatic hydrocarbons, which resemble car soot. The small grains may be as small as 5 Å (0.0005 μm).

Grains can *scatter* or *absorb* incident UV and optical photons; at the FIR wavelengths at which they emit, the radiation is optically thin (so they do
-105- not *absorb* FIR photons effectively).

By absorbing a photon, a grain increases its energy and heats up from T_0 to T_1:

$$h\nu_{ph} = \frac{4}{3}\pi a^3 \int_{T_0}^{T_1} C_V(T)\mathrm{d}T$$

For small grains, the jump from T_0 to T_1 can be very large (several hundred K), depending on $h\nu_{ph}$, a, and C_V (the heat capacity per unit volume). These grains are said to be stochastically heated to high temperatures. As a result, the grains are found over a wide range in T.[†] Standard large grains are not much affected by single photons. Instead, they reach a relatively constant equilibrium temperature when embedded in a specified radiation field.

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[*]Mathis, Rumple, and Nordsieck (1977)

[†]handout, Fig. 2 from Draine (1990)

8.1 Absorption efficiency: the Q parameter

The absorption efficiency of grains is described by the *Q parameter*, defined as:

$$Q_\nu^{abs} = \frac{\sigma_\nu}{\pi a^2} \quad (= 1 \text{ for a blackbody})$$

σ_ν = effective absorption cross-section at frequency ν
πa^2 = geometrical area

$Q = 1$ *for a blackbody.* In practice, $\sigma_\nu^{abs} \sim \pi a^2$ at UV wavelengths, and $\sigma_\nu^{scat} \sim \sigma_\nu^{abs}$. For $\lambda \gg a$ (in IR), $\sigma_\nu^{scat} \to 0$.

-106- The optical depth becomes:

$$\tau_\nu = \int_R \alpha\nu \mathrm{d}r = \int_R n_d \pi a^2 Q_\nu \mathrm{d}r = \int_R \rho_d \kappa_\nu \mathrm{d}r$$

- κ_ν = mass absorption coefficient
- ρ_d = mass density of a grain

We have:

$$\kappa_\nu = \frac{n_d}{\rho_d} \pi a^2 Q_\nu$$

Typical numbers:

$\frac{M_d}{M_g} \sim 0.005 - 0.01$ ("dust-to-gas" ratio is $\sim 1\%$)

$\langle a \rangle \sim 0.1\mu$m

$\rho_d \sim 3$ g cm^{-3}

$Q_{100\mu m} \sim \frac{1}{644}$ for particular model

We can also define the absorption of dust per H-atom:

$$\sigma_\nu^H \equiv \frac{\tau_\nu}{N_H} = \frac{1}{X(H)} \frac{M_d}{M_g} M_H \kappa_\nu \quad [\mathrm{cm}^2]$$

$M_d = n_d m_d$ = dust mass density

$M_g = (n_H + 2n_{H_2}) \frac{m_H}{X(H)}$ = gas density

X_H = fractional mass abundance of hydrogen (~ 0.7)

8.2 Calculating dust mass from FIR fluxes

The volume emission coefficient of dust grains is:

$$j_\nu = n_d \pi a^2 Q_\nu B_\nu(T_d) = \rho_d \kappa_\nu B_\nu(T_d) = \alpha_{nu} B_\nu(T_d)$$

(Kirchoff's law!)

-107- The emission is optically then, so

$$I_\nu = \int_R j_\nu \mathrm{d}r = \int \rho_d \kappa_\nu B_\nu(T_d) \mathrm{d}r$$

T_d is measured from $Q_\nu B_\nu(T_?)\ldots$?

What we measure is flux, $F_\nu = \int I_\nu \cos\theta \mathrm{d}\Omega \approx \int I_\nu \mathrm{d}\Omega$ for small θ.

For an optically thin, spherical cloud:

$$L_\nu = \int_{volume} \epsilon_\nu \mathrm{d}V; \quad \epsilon_\nu = \int j_\nu \mathrm{d}\Omega$$

$$F_\nu = \frac{1}{4\pi D^2} \int_\Omega \mathrm{d}\Omega \int_V \rho_d \kappa_\nu B_\nu(T_d) \mathrm{d}V$$

$$\boxed{F_\nu = \frac{M_d \kappa_\nu B_\nu(T_d)}{D^2}} \quad (\star)$$

So in practice:

- Measure F_ν at $\lambda = \lambda_1$
- Measure F_ν at $\lambda = \lambda_2$
- Derive T_d from fit to spectrum (see below)
- Derive M_d from $(\star)$

$\Rightarrow$ Temperature variations along the light of sight cause severe problems*

-108- But first some more comments on spectra emitted by dust grains. We saw

$$Q_\lambda = \frac{\sigma_\lambda}{\pi a^2} \equiv \frac{A_n}{\lambda^n}$$

- A_n = constant

*See handout from Draine (1990).

- n = emissivity law exponent. In practice, n = 1 to 2.

Consequence: $\tau_\lambda \propto \lambda^{-n}$ (verify). And since emission is optically thin:

$$I_\lambda \propto \lambda^{-n} B_\lambda(T_d)$$

This is called a *modified blackbody spectrum*. *(Greenhouse effect!)*

In terms of frequency and luminosity:

$$I_\nu \propto \nu^{3+n} \frac{1}{e^{h\nu/kT} - 1} \propto F_\nu$$

$$L_{IR} \equiv 4\pi D^2 \int F_\nu \, \mathrm{d}\nu \propto \int_0^\infty \frac{\nu^{3+n}}{e^{h\nu/kT} - 1} \, \mathrm{d}\nu$$

$$x = \frac{h\nu}{kT} \rightarrow \mathrm{d}\nu = \frac{kT}{h} \mathrm{d}x$$

$$L_{IR} \propto \int_0^\infty T^{3+n} \frac{x^{3+n}}{e^x - 1} \, \mathrm{d}x T = T^{4+n} \int_0^\infty \frac{x^{3+n}}{e^x - 1} \, \mathrm{d}x$$

$$\boxed{L_{IR} \propto T^{4+n}}$$

-109- The luminosity rises steeply with dust temperature! How can we measure the value of n? At long wavelengths, $B_\nu(T_d) = \frac{2kT}{c^2}\nu^2$ (RJ tail) so $I_\nu \propto \nu^{2+n}$ or $I_\lambda \propto \lambda^{-(4+n)}$ Unfortunately, measuring the slope of this relation is difficult since we need measurements in the sub-mm (200 - 1000 μm), which are tough. Also, there is no guarantee that the same emissivity law holds over all wavelengths, since different grain sizes will have different temperatures, and emit at different wavelengths. But for the standard MRN grains it should work, if there are no problems of different temperatures along the line-of-sight.

8.3 Dust temperatures

If dust grains were bricks *(blackbodies)*, they would not be warm enough to emit in IRAS 60 and 100 μm bands: the energy density of the Interstellar Radiation field is comparable to that of the microwave background, so a brick would reach a temperature of 3-4 K in the ISM. Why are grains warmer? Because they are *not* blackbody emitters and absorbers, but emit a modified black body spectrum. One may think of it as a "greenhouse
-110- effect" operating on microscopic scales: dust is efficient at absorbing short-wavelength radiation, but inefficient at emitting in the FIR. Yet, given its temperature, it will emit in FIR. As a result, the grain is heated above the temperature of a brick before there is equilibrium between emission and absorption. In essence, the grain traps some heat.

Spitzer: energy gained by a grain heated by radiation:

$$G_r = c \int_0^\infty Q_a(\lambda) U_\lambda \, \mathrm{d}\lambda$$

- G_r = rate of energy gain per second per unit projected area of dust due to absorption of radiation
- c = speed of light
- $Q_a(\lambda)$ = absorption efficiency factor
- $U_\lambda \, \mathrm{d}\lambda$ = energy density of radiation field in the range $\mathrm{d}\lambda$

The grain emits in the FIR according to a modified Planck law.

$$j_{\nu E} = n_d Q_a \sigma_d B_{nu}(T_s)$$

$$\sigma_d = geometricalcross - section$$

So Q_a is essentially a correction factor to the geometrical cross-section. In
-111- equilibrium: heating = cooling.

$$c \int_0^\infty Q_a(\lambda) U_\lambda \, \mathrm{d}\lambda = 4\pi \int_0^\infty Q_a(\lambda) B_\lambda(T_s) \, \mathrm{d}\lambda$$

If $Q_a(\lambda)$ is independent of λ, then T_s comes out to be ~ 3.5 K ($T_s = [U/a]^{1/4}$) in the ISM. However, $Q_a(\lambda)$ does depend on λ, going somewhere as $1/\lambda$ or $1/\lambda^2$. This allows T_s to increase substantially, reaching 15 to 18 K.

8.4 Interstellar extinction

-116- The presence of dust was demonstrated by Trumpler, who detected the dimming and reddening of distant stars. *High-z proto-galaxies!*

Dust both scatters and absorbs light. The combined effect is **extinction**. For a point source (e.g. a star), the object is *dimmed* by extinction, since both scattered and absorbed light do not reach the observer. For an extended source (e.g. a galaxy), some light may be scatter *into* the line of sight, so the extinction will generally be less than for point sources. On average, extinction is about 0.6 - 1 mag kpc^{-1} (reddening is about 0.3 mag kpc^{-1} in (B-V) in the plane of the Milky Way - see §8.5).

NGC 6240, a heavily obscured starburst nucleus, emits 10 times more power in the IR/FIR than in the optical, with peak near 60-100 μm. Large z: shifts into sub-mm. Led to detection of Ultra-Luminous IR Galaxies (ULIRGs), where "ultra-luminous" is $> 10^{12}$ $L_\odot$. More typical galaxies have $L_{\rm FIR} \sim L_{\rm optical}$.

8.4.1 *The extinction law*

-117- Empirical result: reddening in magnitudes

$$r_\lambda = a + \frac{b}{\lambda}$$

where a and b are roughly constant. This is between two extreme cases:

1. grey extinction, r_λ independent of λ
2. Rayleigh scattering, $r_\lambda \propto \lambda^{-4}$

$\Rightarrow$ particle size is $\sim 10^{-5}$ cm = 0.1 μm *(= 100 nm)*.

$$\mathrm{d}I_\lambda = -I_\lambda n(x)\sigma_\lambda \mathrm{d}\lambda = -I\mathrm{d}\tau_\lambda$$

- $n(x)$ = number density of grains at position x
- σ_λ = extinction cross-section per dust grain at wavelength λ

The star is dimmed by

$$I_\lambda = I_\lambda^0 \mathrm{e}^{-\tau_\lambda}$$

where

$$\tau_\lambda = \sigma_\lambda \int_0^x n(x')\mathrm{d}x' = \sigma_\lambda N(x)$$

In magnitudes:

$$\begin{aligned}
A_\lambda &\equiv -2.5\log\left[\frac{I_\lambda}{I_\lambda^0}\right] \\
&= -2.5\log\left[\mathrm{e}^{-\tau_\lambda}\right] \\
&= -2.5\log\left[\mathrm{e}^{\ln(\mathrm{e}^{-\tau_\lambda})}\right] \\
&= -2.5\ln(\mathrm{e}^{-\tau_\lambda})\log[\mathrm{e}] \\
&= -2.5(-\tau_\lambda)\log[\mathrm{e}] \\
&= 1.086\tau_\lambda
\end{aligned}$$

Dependence on Galactic latitude:
Assume simplest model of uniform dust layer.

-118- In reality, an exponential z-dependence would be more realistic, while the dust is also clumped, just like the gaseous ISM.

8.5 Interstellar reddening

Reddening is caused by the wavelength dependence of extinction. We can define a **color excess**:

$$E(\lambda_1 - \lambda_2) \equiv \underbrace{(m_{\lambda 1} - m_{\lambda 2})}_{\text{observed color}} - \underbrace{(m_{\lambda 1,0} - m_{\lambda 2,0})}_{\text{intrinsic color}}$$

e.g.:

$$E(B-V) = (B-V) - (B-V)_0$$

<u>Question</u>: How is the reddening measured?

Reddening line is defined by the extinction law.

$$E(U-B) = \underbrace{0.72}_{\substack{\text{“standard}\\ \text{Galactic}\\ \text{reddening}\\ \text{law”}}} E(B-V) + \underbrace{0.05(B-V)}_{\text{minor correction}}$$

We can define *reddening free colors*, if the shape of the reddening law is known. For example, the following is reddening free:

$$Q \equiv (U-B) - \frac{E(U-B)}{E(B-V)}(B-V) = (U-B) - 0.72(B-V)$$

-119- In general:

$$m_i = m_i^0 + A(\lambda_i)$$
$$m_j = m_j^0 + A(\lambda_j)$$

$$C_{ij} \equiv m_i - m_j = C_{ij}^0 + E_{ij} = C_{ij}^0 + \left[A(\lambda_i) - A(\lambda_j)\right]$$

We define

$$R_V \equiv \frac{A_V}{E(B-V)} = \frac{\tau_V}{\tau_B - \tau_V}$$

For the standard Galactic reddening law, $R_V \approx 3.2 \pm 0.2$.

The interstellar extinction law is derived from comparing reddened and unreddened star's spectral energy distribution for stars of the same spectral type. There are some clever extrapolation tricks involved to get the zero point. The enclosed table* gives a good representation of the standard Galactic curve. Note that

$$\frac{A_\lambda}{E(B-V)} = \frac{E(\lambda - V)}{E(B-V)} + R_V$$

The broad, smooth curve is not the whole story. There are for example still several unidentified narrower absorption features which are called the diffuse interstellar band (DIBs). There are also many line features in the IR and the 2200Å bump (which is present in the table). Some IR features: 9.7 μm $\rightarrow$ H_2O, NH_3?

-120- One can correlate the hydrogen column densities with dust extinctions (or optical depths) to find:

- N(HI) $\approx 4.8 \times 10^{21}$ cm^{-2} E(B-V)
- N(HI + H_2) = N(HI) + 2N(H_2) $\approx 5.8 \times 10^{21}$ cm^{-2} E(B-V)

The scatter in the second relation is considerably less. This relation, combined with grain theories† produce a dust-to-gas ratio of $\sim$ 1%.

Origin and composition of grains:‡

- oxygen-rich
- carbon-rich
- ice coatings?
    - smallest grains ($< 0.01\mu$m) may be too warm
    - larger grains, which produce UV and optical extinction, may have ice coatings

The grain formation and depletion of elements on grains can perhaps be understood as a condensation sequence (see earlier discussion on ISM absorption lines, & Draine figure 23.1).

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* from Sauage & Mathis (1979), ARAA17, p 84

† e.g. Mathis, Rumple, & Nordsieck (1977) ApJ 217, 425 or Draine & Lee (1984) ApJ

‡ See Draine chapter 23 for more details

9. Molecular hydrogen and CO

-124- Molecular gas is, with HI, the dominant phase of the ISM in galaxies, in its contribution to the total gas mass. The distribution of H_2, however, is very different from HI gas. The gas is much colder and denser, and as a result, fills only a small part of the volume of the Galactic disk. The gas is heavily concentrated to the midplane of the galaxy (scale height $\sim$ 60 pc) and is mostly found in Giant Molecular Clouds (GMCs), complexes of small clumps of molecular gas, gravitationally bound, with overall sizes $\sim$ 50 pc and masses of 10^4 - 10^6 $M_\odot$. All available evidence indicates that stars form from molecular gas. There is some diffuse molecular gas locally at higher latitude, so the scale height cited refers to the main GMC layer, not all molecular gas. *Forming molecules is not trivial; dust grains may act as a catalyst.*

Since H2 molecule is not easily observable (see later), we need a tracer. CO is the most abundant ($\sim$ one CO molecule for every 10^4 H2 molecules). In terms of observational techniques, the predominant tracer of molecular gas is the CO molecule, through rotational transitions in the mm wavelength range. *(2.6 mm, to be exact.)* This sets a very different set of observational challenges compared to the 21-cm line: smoother telescope surfaces, different receivers/noise demands, smaller primary beams, larger short spacings, problems in interferometer maps. *CO maps are harder to make than HI.*

9.1 Molecular gas and CO as a tracer

-128- H_2 molecule has no permanent dipole moment (because the center of mass and center of charge coincide). Electronic transitions in the UV (Lyman and Werner bands) have been detected in absorption, and rotational-vibrational transitions in IR (H_2 gas that is warm due to shocks) but these do not allow general maping of this component. $\Rightarrow$ Need a different tracer. CO has J=1-0 rotational transition at 2.6 mm (115.3 GHz) which is *easily excited by collisions with* H_2 ($h\nu/k$ = 5.5 K) even in cold gas. Critical density for significant excitation of the 2.6 mm line: $n_{H_2} \geq 3000$ cm^{-3} if there is a balance between CO-H_2 collisions and spontaneous decay. However, since lines are optically thick in reality, the critical density is reduced by $1/\tau$ (where τ is the optical depth)* The result is that $T_x = T_k$ for $n_{H_2} \geq 300\ \mathrm{cm}^{-3}$ (the levels are "thermalized"). T_x is the excitation temperature and T_k is the kinetic temperature.

9.2 CO luminosity

-130-

$$L_{CO} = 4\pi D^2 \int I \mathrm{d}\Omega$$

Cloud of surface area πR^2 occupies

$$\mathrm{d}\Omega = \frac{\pi R^2}{4\pi D^2}$$

so $L_{CO} \propto \pi R^2 T_{CO} \Delta v$ (assumes $T_{CO} \simeq T_A^* \ldots$). For gravitationally bound clouds:

$$\Delta v = \sqrt{\frac{G M_{H_2}}{R}}$$

$$\implies \quad M_{H_2} = L_{CO} \sqrt{\frac{4}{3\pi G}} \frac{\sqrt{\rho}}{T_{CO}}$$

so

$$M_{H_2} \propto L_{CO} \frac{\sqrt{\rho}}{T_{CO}}$$

L_{CO} is measured (need to know distance). T_{CO} is not necessarily physical.†

Potential problems: How constant is the proportionality? For example, metallicity - abundance of H_2 over CO may differ. This is especially a problem for dwarf galaxies. Note: H_2 molecules mainly form on grains. Dissociation energy of H_2 and CO: 4.48 eV and 11.1 eV.

CO $\rightarrow$ H_2 conversion calibration methods:

- optical extinction for Galactic clouds $\leftrightarrow$ L_{CO} (A_V).
- γ-ray emission at "soft γ-ray energies" probes total gas column.
- The famous x-factor! Calibration of L_{CO} to M_{H_2}. Canonical value x (see below). Assuming gas-to-dust ratio of 1%.

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* See Draine 19.3.1, 19.3.2

† See Draine 19.6 and figure on reverse side of handwritten notes.

- virial mass of molecular clouds

Results:

$$\frac{N(H_2)}{I_{CO}} = 2{-}3 \times 10^{20} \quad \text{cm}^{-2}\,(\text{km s}^{-1})^{-1}$$

Good info about molecular lines and transitions on pages 135 - 141 (photocopy from book)

10. Hot ionized gas

10.1 Collisional excitation and ionization

-142- We already talked about collisional excitation in:

HI the resulting level populations are given by Boltzman statistics, so we did not have to discuss the basics of the collision process

HII regions Introduced the rate coefficients, γ, which play a role in the statistical equilibrium equation.

Collisional excitation and ionization are also of crucial importance in hot gas ($T \sim 10^5 - 10^7$ K). As we saw, photoionization by OB stars produces HII regions with $T \sim 10^4$ K. The temperature of any ISM phase is determined by the heating and cooling rates, which we discuss later (see last section).

An O star cannot heat gas to much more than 10^4 K since most of the ionizing photons have energies that are only a few eV above the H ionization potential. More energetic photons ionize He, so again only a few eV is left (note that $1\text{eV} \equiv 1.6 \times 10^{-12}$ erg $\equiv kT \rightarrow T \sim 16,000$ K). Electrons quickly excite forbidden lines which cool the ionized gas to lower temperatures. Similarly, to collisionally ionize H (13.6 eV) requires high temperatures (the tail of the Maxwellian velocity distribution matters). The main mechanism to create hotter gas is by shock ionization which occurs in supernova remnants, evidently the SN explosion, and any collision between gas clouds in
-143- which velocities in excess of 100 km/s are involved. (Shock models predict a temperature-shock velocity relationship of the form $T_s = 1.38 \times 10^5 v_s^2$ K for a fully ionized gas, where v_s = shock velocity in units of 100 km/s). There is not a lot (in terms of mass) of the hot gas around; it cools very efficiently by line radiation from collisionally excited lines, some recombination radiation, and by Bremsstrahlung (which would dominate at very high T…why?) if density is high. (What would typical wavelength be at which Bremsstrahlung radiation would be predominant? *x-ray*). The hot gas is sometimes referred to as "coronal" gas, in similarity to the sun's corona.

10.1.1 Calculation of collisional rate coefficients

While we discussed the general effect of collisional excitation on HII region spectra, we have not in detail discussed how the collisional rate coefficients are calculated.[*] Evidently the principal factors to consider are:

1. Kinetic energy, or similarly (in collisions of entire clouds) the translation of velocity into a temperature regime (see expression for T_s above).
2. Probabilities for particular processes (ionization, excitation); the rate coefficients need to be determined from momentum physics and are then averaged over the relevant velocity distribution.[†]

Spitzer summarizes ionization by energetic particles as well. This includes thermal electrons and protons but also cosmic rays.

10.2 Properties of hot ionized gas and spectrum

-145-

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[*]Draine does this in chapter 2.

[†]Draine, 2.1

11. Heating and cooling

-151- Following Spitzer's general formalism*:

Γ = total kinetic energy *gained* [erg cm^{-3} s^{-1}]
Λ = total kinetic energy *lost* [erg cm^{-3} s^{-1}] (aka. the "cooling function")

All the physics are contained in these two quantities; need to find out what they are.

For $\Lambda = \Gamma$ we reach equilibrium temperature, T_E. Since Λ and Γ may depend on T_E themselves, we can set up a time dependent differential equation:

$$\underbrace{n\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{3}{2}kT\right)}_{\text{rate of increase in thermal energy for a mono atomic gas}} - \underbrace{kT\frac{\mathrm{d}n}{\mathrm{d}t}}_{\text{work done by gas (energy loss)}} = \underbrace{\sum_{\varrho,\eta}\left(\Gamma_{\varrho,\eta} - \Lambda_{\varrho,\eta}\right)}_{\text{Sum over interacting particles}} = \Gamma - \Lambda$$

(Note: in what follows, we generally ignore "work done by gas" component.)

For T$\lesssim 2 \times 10^4$ K, thermal conduction can be ignored. If we do have enrgy gain or loss by conduction (electrons), we add -$\vec{\nabla} \cdot \left(k\vec{\nabla}T\right)$ to the righthand side. *Conductivity is hard to figure out, but not important below 20,000 K.* ($\vec{B}$ fields prevent conductivity across field lines.)

$PV = nRT = NkT \rightarrow P\Delta V =$ *work done by gas.* $R = N_A k$, *where* N_A *is Avagadro's number.*

If a gas is not at its equilibrium temperature (T$_E$), we can define a "cooling time" t_T *(like a half-life, sorta)*:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{3}{2}kT\right) = \frac{3}{2}k\left(\frac{\mathrm{d}T}{\mathrm{d}t}\right) = \frac{3}{2}k\left(\frac{T - T_E}{t_T}\right)$$

where T_E = constant and T = variable. For T_E and t_T = constant, $T - T_E \propto$ e$^{-t/t_T}$ *(exponential decay for cooling).* t *obviously can't be negative* $\rightarrow$ *quasi-stable*

* see also Draine chapters 27, 30, 34

-152- If $t_T < 0$, unstable situation and gas will cool or heat toward an entirely different equilibrium temperature. This is relevant to the multi-phase ISM. All the physics is in Λ and Γ.

11.1 General

11.1.1 Primary heat source

The primary heat source is (photo)ionization.

E_2 = kinetic energy of ejected electron
E_1 = kinetic energy of recaptured electron

$\Delta E = |E_2 - E_1| \rightarrow$ *energy available. Ionization from the ground state (maximum energy) recombines with less energy.* Number of captures to level j of neutral atom:

$$n_e n_i < \omega\sigma_{cj} > \quad [\mathrm{cm}^{-3}\ \mathrm{s}^{-1}]$$

where $< \omega\sigma_{cj} >$ = recombination coefficient. The final net gain associated with electron-ion recombination:

$$\Gamma_{ei} = n_e n_i \sum_j \left(\underbrace{< \omega\sigma_{cj} > \overline{E}_2}_{\text{ionization out of } j} - \underbrace{< \omega\sigma_{cj} E_1 >}_{\text{recapture back to } j} \right)$$

$<>$ = average over Maxwellian velocity distribution
$\overline{E}_2$ = average over all ionizing photon energies

As we have seen, all ionizations take place from the ground level $\rightarrow \overline{E}_2$ is independent of j; use recombination coefficient to all levels $\geq n$:

$$\alpha^{(n)} = \sum_n^\infty \alpha_m$$

and

$$\Gamma_{ei} = n_e n_i \left\{ \underbrace{\alpha \overline{E}_2}_{\equiv \alpha^{(1)}} - \underbrace{\frac{1}{2} m_e \sum < \omega^3 \sigma_{cj} >}_{\text{kinetic energy}} \right\}$$

Main point:

-153- Γ_{ei} is not dependent on ionization probability or radiation density for *steady state*, where the number of ionizations is equal to the number of recombinations.

11.1.2 Primary cooling source

The primary cooling source is **inelastic collisions**. (excitation of energy levels, *since particles that undergo inelastic collisions will lose energy*).

- $n_e n_i \gamma_{jk}$[cm^{-3} s^{-1}] = number of excitations from level $j \to k$ for ions in ionization stage i and excitation state j.
- $E_{jk} = E_k - E_j$ = energy lost by colliding electrons *(in the form of emitted photons)*.

This cooling is offset by *de-exciting* collisions, which give energy gain to electrons *(give energy back to the nebula). Net cooling:*

$$\to \Lambda_{ei} = n_e \sum_{j<k} E_{jk} \left(\underbrace{n_{ij}\gamma_{jk}}_{\text{excitation}} - \underbrace{n_{ik}\gamma kj}_{\text{de-excitation}} \right)$$

γs are collisional rate coefficients. Assumption: all photons escape (not true for dense molecular clouds). Again, in most cases all ions are in the ground level, so we don't need to sum over j, and $n_{ij} = n_{i1} = n_i$ *(simpler)*

11.2 HII Regions

-154-

- Main heat sources: ionization of H and He
- Main cooling source: excitation of C, N, O, Ne
    - Heavy elements have low abundance. If they didn't, HII regions would cool to very low T_E!

UV = radiation field energy density. Radiation is from OB stars (ionizing photons) and diffuse radiation in HII regions from direct recombination to n=1.

HII Regions: Lyman photons didn't escape; they were all re-absorbed and turned into Balmer line. Photons absorbed by dust $\to$ cooling. Photons scattered $\to$ not *cooling. Keep in mind:* **Do all photons make it out of the nebula?**

11.2.1 *Heating*

$$\overline{E}_2 = \frac{\int_{\nu_1}^{\infty} h(\nu - \nu_1) s_\nu U_\nu \mathrm{d}\nu / \nu}{\int_{\nu_1}^{\infty} s_\nu U_\nu \mathrm{d}\nu / \nu}$$

- s_ν = cross section
- ν_1 = ionization limit for HI

Problem: U_ν is determined by stellar radiation ($U_{s\nu}$) and diffuse radiation ($U_{D\nu}$), and depends in turn on n_H, etc.

Spitzer discusses two simple cases:

1. Close to exciting star: $U_{s\nu}$ large, $U_{D\nu}$ negligible in comparison
2. Evaluate $\overline{E}_2$ for entire HII region.

Approximation: Use dilute blackbody of temperature T_c (color temperature) to describe the stellar radiation field at distances from the stars.

Define: $\psi = \dfrac{\overline{E}_2}{kT_c}$

$\psi_0 = \psi(r \to 0)$, so close to star
$< \psi >$ = average of star
$\psi(r)$ = over entire HII region

-155-

$$\psi_0 : \quad U_\nu = \frac{4\pi B_\nu(T_c)}{c} = \frac{1}{c} \int I_\nu \mathrm{d}\Omega$$

Then it is possible to calculate table 6.1 (Spitzer), which lists values of ψ_0 (Draine, table 27.1) for various values of T_c. *

Result: $$\begin{array}{ccccc} 1.05 & < & < \psi > & < & 1.65 \\ 4000 & < & T_c & < & 64{,}000 \end{array}$$

We had that:

$$\Gamma_{ei} = n_e n_i \left\{ \alpha \overline{E}_2 - \frac{1}{2} m_e \sum < \omega^3 \sigma_{cj} > \right\}$$

where second term = mean energy lost per recombining electron.

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* Spitzer also discusses how to calculate $< \psi >$; too lengthy to do here.

We have $\overline{E}_2$, now we need

$$\sum_{j=k}^{\infty} < \omega^3 \sigma_{cj} > = \frac{2A_r}{\sqrt{\pi}} \left(\frac{2kT}{m_e} \right)^{3/2} \beta \chi_k(\beta)$$

A_r = “recapture constant”[*]
$\beta = \dfrac{h\nu_1}{kT}$

$\chi_k(\beta)$ are listed in table 6.2 in Spitzer. They are “energy gain functions” with values from $\sim$0.4 to 4.0.

11.2.2 Cooling

Use directly the basic equation using

$$\frac{n_k}{n_j} = \frac{b_k}{b_j} \frac{g_k}{g_j} \exp\left(-\frac{h\nu_{ju}}{kT} \right)$$

and

$$\frac{b_2}{b_1} = \frac{1}{1 + A_{21}/n_e \gamma_{21}}$$

(discussed for HI emission) for a two-state ion. 3 level ions are more complicated.

- $\frac{g_k}{g_j} \exp\left(-\frac{h\nu_{ju}}{kT} \right) \rightarrow$ Boltzmann equation
- $\dfrac{b_2}{b_1} \rightarrow$ “b correction”, since we can’t assume Boltzmann level populations

11.2.3 Results

-156- See figures on handout (page -157-).

Some main points:
two groups of coolants

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[*]Spitzer, page 100

1. meta-stable fine-structure levels in ground. Spectroscopic term of various ions.
   - $E_{ex} < 0.1$ eV $\rightarrow$ IR radiation

   examples in Fig:

$$\begin{array}{lll} [\text{OIII}] & {}^3P_0 - {}^3P_1 & 88.4\ \mu\text{m} \\ & {}^3P_0 - {}^3P_2 & 32.7\ \mu\text{m} \end{array}$$

   Weak T_E dependence
2. meta-stable other spectroscopic terms with excitation energies $\gtrsim 1$ eV, giving rise to optical and UV lines. Strong T_e dependence of course! Act as thermostat $\rightarrow$ will keep $T_e \sim 10,000$ K in HII regions.

Note:
Figure is for (arbitrary) assumption that O, Ne, N are 80% singly and 20% double ionized. H is 0.1% neutral.

Net effective heating rate
G-L_R in the fig. is what we have been calculating (where G is photoionization and L_R is recombination emission for H and He).

The intersection between heating and cooling gives T_E
Optical depth τ_0
in figures refers to *distance* from star. It is the optical depth at the ionization limit of HI, so it is proportional to N(H).

Outer parts of nebula are hotter!
-158- This happens because the photoionization cross-section is proportional to ν^{-3}, so higher energy photons are absorbed *later*. T_E decreases at first, because $U_{s\nu}$ falls. Then, beyond $r = 0.6R_S$, T_E increases and is higher near $r = R_0$. Finally, it decreases again in the transition region where the ionized fraction drops to zero.

The second figure shows what happens if Ne is large enough that some excited levels of heavy ions are collisionally de-excited; T_E increases, since cooling is less effective.

ϵ_{ff}, the free-free loss rate, follows directly from (3-56) in Spitzer; it is not very important.

How fast does T change when $T \neq T_E$?
Close to T_E,

$$t_T \approx \frac{2 \times 10^4}{n_p} \quad \text{[years]}$$

Compare to recombination time:*

$$t_r = \frac{1}{n_e \alpha} = \frac{1.54 \times 10^3 \sqrt{T}}{Z^2 n_e \phi_2(\beta)} \quad \text{[years]}$$

(It would take 10^4 years for recombination to occur if the star in the HII region disappeared). So for $T \sim 10^4$ K, $t_R \gtrsim t_k \rightarrow$ cooling is faster than recombination. (Also, cooling time is much larger for $T \gg 10^4$ K, as we discussed before.)

-159- Consider the handout (cooling and heating in HII regions, taken from Osterbrock). What is missing in these diagrams?

- Cooling by free-bound and bound-bound HI and He? No; since each recombination is balanced by an ionization, these photons, even though they may well escape from the nebula, do not draw net heat from it. Rather, they draw heat from the star itself, not the nebula.†
- photon $h\nu$ absorbed $\rightarrow$ ionizes atom $\rightarrow$ creates electron with *(kinetic)* energy $\frac{1}{2}mv^2 = h(\nu - \nu_0) \rightarrow$ electron thermalizes its energy with ions and electrons and sets up temperature T_e.
- However, each electron recombines from energy $\frac{1}{2}mv'^2$, which produces photons with a total energy $h\nu' = \frac{1}{2}mv^{-2} - h\nu_0$

The *net* energy (or heat) gain from the photoionization is given by $\frac{1}{2}mv^2 - \frac{1}{2}mv'^2$ (Notice that in general, $|v'| < |v|$). This is to be balanced against the *cooling*, which is predominantly due to emission from collisonally excited heavier elements (in HII regions).

*Spitzer 6-11

†Draine does not agree? He includes a recombination cooling rate in Fig. 27.2, 27.3; it is low. He keeps it in heating too though. . . so okay, fine. Whatever.

Pure H in the ISM: not many photons around; HII regions are fully ionized. Gas not generally in both phases, e.g. 90% neutral and 10% ionized.

11.3 HI gas

-160- (Draine, ch. 30)

Not as simple as one might believe. . .

Simple considerations
HI neutral → few free electrons to share heat. Only elements with ionization potential (IP) < 13.6 eV will be ionized (and dust grains, but more on that later). However, there is still a dominant cooling line from CII (IP of C is 11.26 eV). So:

- $\dfrac{\Gamma_{ei}}{n_e}$ down by factor of $\sim 1/2000$
- $\dfrac{\Lambda_{ei}}{n_e}$ similar below $T \sim 1000$ K

HI is naturally cool

Problem:
We observe some very warm HI. General solution may well depend on T_e, n, chemical composition (i.e. depletion). . .

We will just list the general players.

11.3.1 *Cooling function*

Cooling function Λ:
- neutral atoms
- ions
- molecules

Excitation sources:
- electrons
- H atoms

-161- Some sources in particular:

1. CII and SiII excitation by collisions with H. Problem: *depletion*
2. Excitation of HI, [exp.?] $n = 2$ level in warm HI. Generally, $n = 2$ is not strongly populated, but if it does happen, it will cool:

- Lyα photon $\rightarrow$ dust $\rightarrow$ IR
- $\Lambda_{eH} = 7.3 \times 10^{-19} n_e n(HI) \exp(-118,400/T)$ [erg cm^{-3} s^{-1}] ($T = h\nu/k$ = 118,400 for a 1216Å photon).

Fate of Lyα photons: Scatter until absorbed by dust, can't do 2-photon emission, level 1 $\rightarrow$ 2p. . . or something.

3. H_2 molecules: gain or loss source
   - loss: excitation of rotational levels
   - gain: photon pumping of upper rotational levels, followed by collisional de-excitation. Also other molecular lines may cool (CO, CN, CH,. . .)
4. Collisions with dust grains (can both heat and cool): Spitzer's fig. 6-2 (page -172- in notes) shows cooling function, including HI and H^+ range. Generally, cooling time:

$$t_T \approx \frac{2.4 \times 10^5}{n_H} \quad \text{[years]}$$

   longer than for HII regions *(not many lines available)*.

11.3.2 Heating function

Very poorly known. Γ_{ei} from ionizing elements such as C is best known (i.e. photo-electric heating).

-162- However, using only CII heating would produce T_E = 16 K (Spitzer); clearly too cool. *Adding metals is not enough*

Other potential players

(a) Cosmic ray ionization of H, *no radiation, but this could do it too*.
(b) Formation of H_2 molecules on grains. This releases 4.48 eV. Goes into:
   - heating grain
   - overcome energy of adsorption to grain surface
   - excitation of new H_2 molecule
   - translational kinetic energy that H_2 molecule gets as it leaves the grain.

(c) Photoelectric emission from grains. What is efficiency, as a function of λ? Clearly, complicated problems; see Draine for a discussion.

How can HI become 6000K? Cooling is 10 times more efficient (from figure on page 172: $\frac{\Lambda}{n_H}$) than for cool HI. But, if n_H is lower, then:

- cosmic ray heating more efficient, but still problematic
- also grain - photoelectric heating

11.4 Few comments/additions from Spitzer

See figure 6.2, which sketches what happens at lower T (below 10^4K). Note that Spitzer talks about $\frac{\Lambda}{n_H^2}$

-166- Why are there two HI phases?

-167- The "destruction" of the hot phase happens (only) through cooling of the gas. This cooling depends critically on T_e and n_e, as we will next explore.

-168- Heating and cooling of hot gas: