Contents

1. Introduction

-6- Discuss general viewpoint: What can happen to an atom or molecule or dust grain sitting in the ISM?

- Can it absorb a photon? Energy levels $\leftrightarrow h\nu$ (Need cross-sections for dust grains)
- Can it collide with other particles? $\rightarrow$ collisional rates [$\mathrm{s}^{-1}\ \mathrm{cm}^{-3}$]
- Is there a magnetic field? Is the particle charged?
- Are there cosmic rays? These can penetrate dense gas.

Three possible sources of ionization (and excitation):

1. photons
2. collisions
3. cosmic rays

-8- Typical collisional energies: kinetic energy. Translate $mv^2 \rightarrow kT \rightarrow h\nu \rightarrow eV$.

Dust grains also occur in the neutral medium, and probably also in the (warm) ionized medium. Dust grains play an important role in various processes:

- extinction of starlight
- emission of absorbed energy in FIR
- formation of molecules often occurs on grain surfaces
- absorption of ionizing UV radiation and Lyα photons (reducing amount of ionizing radiation)
- heating of HI gas by *photoelectric* emission

Composition: carbon and silicates. Typical sizes: 0.01 - 0.1 μm (How do we know? $\rightarrow$ shape of extinction curve). Grains as small as $\sim$ 60 atoms across discovered; evidence from emission lines in NIR and excess emission at 5 - 40 μm over what is expected from dust in the ISM. The larger dust grains have temperatures between 10 and 40 K, while the small ones can be heated to higher temps due to the absorption of even a single photon (smaller heat capacity, as volume $\propto r^3$). A promising candidate for small dust grains: polycyclic aromatic hydrocarbons ($\sim$ car soot!)

In hot environments, dust grains may be destroyed by *sputtering*, where collisiosn of grains with other atoms, electrons, or molecules knock molecules off
-9- the grains. At low temperatures, molecules stick to dust grains, causing de-

pletion of heavy elements along certain lines of sight (most dust in the plane). Dust contributes about 1% of the mass of the ISM in the solar neighborhood, mostly in the form of large grains.

Ionized gas

a) Photoionization: especially effective near hot stars. Shock ionization
b) Cosmic rays: can occur throughout most of the ISM, so can also produce a small amount of ionization in denser gas (though recombination happens quickly, so not much of gas is in an ionized state at any given time).

-11- c) Collisions Hot ($\sim 10^7$ K) expanding bubble sweeps up shell of warm ($\sim 10^4$ K) ionized gas, which shows a different optical spectrum than HII regions. The hot gas shows up by:

- Free-free and line x-rays
- absorption lines of highly ionized species toward bright UV sources

Kirchoff's law: apparently can still see absorption lines when looking
-13- through a gas that is hotter than the source behind it!

Magnetic fields and cosmic rays In the solar neighborhood, $\mathbf{B} \sim 2 - 5 \times 10^6$ Gauss. This follows from measurements of *Faraday rotation*, giving $< n_e B_{||} >$ toward pulsars and radio sources. The random component of the B field is probably as large as the uniform component. In clouds, the B field can be much higher, $\sim 70\mu$G (from Zeeman effect splitting measurements).

The magnetic field is important for several reasons:

1. It aligns elongated grains, giving rise to polarization of starlight
2. Causes relativisitic electrons to emit synchrotron radiation, and most likely plays a role in accelerating electrons to relativistic velocities ("magnetic bottle", Fermi acceleration).
3. Provides pressure support against gravitational collapse of matter since it is frozen into the matter due to ionization heavy elements. It also seems to play animportant role in solving the angular momentum problem in
Shu et al. star formation.

Total energy density of cosmic rays in solar neighborhood: $U_R \sim 1.3 \times 10^{-12}$
-14- erg cm^{-3}. Why are cosmic rays important?

- They produce γ-rays through collisions with atoms and molecules. The observed γ-ray intensity from the ISM forms an excellent independent measure of the total amount of matter between stars. For example, calibrating the conversion factor of CO line intensity to H_2 mass.

- Provide pressure against gravitational collapse

Five pressures that play an important role in supporting the ISM against gravitational collapse:

1. thermal $P = nkT$
2. magnetic $P = \dfrac{B^2}{8\pi}$
3. turbulent (bulk motion)*
4. cosmic ray
5. radiation

* The cloud to cloud velocity dispersion due to turbulence on various scales increases line widths [over?] thermal widths. From the table on page -12-, you will note rough thermal pressure euqilibrium between the components. This is not a coincidence; in fact, some of this information was inferred by assuming pressure equilibrium. The argument is that if there were no equilibrium, the resulting perturbations would be wiped out on sound-crossing time scales,
-15- which are short compared to the time scales we wouuld consider the ISM to evolve over.

However, the actual evidence for equilibrium in the thermal pressure is scarse, and there are claims that it is not true in the very local ISM[*].

There seems to be a "cosmic conspiracy": the estimates for the thermal, magnetic, and cosmic ray pressure for the solar neighborhood give roughly equal numbers for all three. Thus it may be inappropriate to only consider the thermal pressure (the only one that can be measured with much certainty). Interestingly, the magnetic pressure number is also very similar to the energy density of the CMB. [†] [‡]

-16- Next section is sort of a shortened condensation of Draine's chapters 2 and 3. We may come back to specific topics discussed there in more detail.

---

[*] Bowyer et al. *Nature* 1905

[†] Draine discusses possible reasons in section 1.3

[‡] Supplemental info in Draine chapter 1.

2. Validity of the laws of statistical physics in ISM conditions

Four major laws of statistical physics:

1. **Maxwellian** velocity distribution
2. **Boltzmann distribution** of energy levels in atoms and molecules
3. **Saha equation** for ionization equilibrium
4. **Planck function** for radiation

Maxwellian T is defined by motion of particles. ($\vec{\omega}$ = velocity = $\vec{v}$ in Draine). $f(\vec{\omega})\mathrm{d}\vec{\omega}$ = fractional number of particles whose velocity lies within the three-dimensional volume element $\mathrm{d}\vec{\omega} = \mathrm{d}\omega_x \mathrm{d}\omega_y \mathrm{d}\omega_z$, centered at velocity $\vec{\omega}$.

In thermodynamic equilibrium, $f(\vec{\omega})$ is isotropic, so $\vec{\omega} \to \omega$.

$$f(\omega) = \frac{\ell^{3/2}}{\pi^{3/2}} \exp(-\ell^2 \omega^2); \quad \ell^2 = \frac{m}{2kT} = \frac{3}{2 < \omega^2 >}$$

$$f(\omega) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m\omega^2}{2kT}\right)$$

For two groups of particles with different masses, we replace ω by u, the relative velocity between the two groups, and m by the reduced mass $m_r = \dfrac{m_1 m_2}{m_1 + m_2}$.

-17- For H atoms colliding with particles of mass Am_H, Spitzer derives:

$$< u >= \left[\frac{8kT}{\pi m_r}\right]^{1/2} = 1.46 \times 10^4 \sqrt{T} \left(1 + \frac{1}{A}\right)^{1/2} \quad [\mathrm{cm\ s^{-1}}]$$

…Speed vs. velocity

The Maxwell velocity distribution is characterized by several speeds:

- Most probable speed: $\omega_o = \sqrt{\dfrac{2kT}{m}}$
- RMS speed: $< \omega^2 >^{1/2} = \sqrt{\dfrac{3kT}{m}}$
- RMS velocity in one direction: $< \omega_x^2 >^{1/2} = \sqrt{\dfrac{kT}{m}}$

-18- **Boltzmann distribution** gives the population of energy levels in an atom or molecule:

$$\frac{n_u}{n_l} = \frac{g_u}{g_l} \exp\left[-\left(E_u - E_l\right)/kT\right] = \frac{g_u}{g_l} \exp\left[-h\nu_o/kT\right]$$

where $n_{u,l}$ are the number densities, $g_{u,l}$ are the statistical weights, and $E_{u,l}$ are the energies of the levels. (Partition function: summing over all energy levels. . . chemical term).

Saha equation describes ionization equilibrium:

$$\frac{n_{i+1}}{n_i} = \frac{g_{i+1}g_e}{g_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-I/kt\right)$$

where I is the ionization potential for an ion in the ground state and initial ionization state i (aka, the energy required to ionize from i to $i+1$). $g_e = 2$ (two spin conditions). Electrons affect whether and how easily atoms can be ionized.

Planck function specifies the radiation field: *

$$\begin{aligned} B(\nu) = &\ \frac{2h\nu^3}{c^2}\frac{1}{\exp[h\nu/kT] - 1} \\ &= \sim \frac{2\nu^2}{c^2}kT \quad \text{for } h\nu << kT \text{ (Rayleigh} - \text{Jeans)} \\ &= \sim \frac{2h\nu^3}{c^2}\exp[-h\nu/kT] \quad \text{for } h\nu >> kT \text{(Wien)} \end{aligned}$$

3. Statistical equilibrium

-19- The four laws discussed above hold under thermodynamic equilibrium (TE). However, this is not often the case for the ISM. Thermodynamic equilibrium requires **detailed balancing**, i.e. each process is as likely to occur as its inverse. Example: Consider the 3727 Å emission from O^+. This is a forbidden transition (actually a doublet). The excitation of the electron level occurs through collisions with electrons, in most conditions in the ISM. If detailed

* $e^x \approx 1 + x$ for $x << 1$
$e^{h\nu/kT} - 1 \approx 1$ for $h\nu << kT$
$e^{h\nu/kT} - 1 \approx e^{h\nu/kT}$ for $h\nu >> kT$

balancing were to hold, de-excitation should also occur by collisions. However, as we will see, under the low density conditions found in the ISM, collisions are rare, and de-excitation is more likely to proceed through emission of a photon, in spite of the fact that we are dealing with a forbidden transition. Thus [OII] emission can be quite strong, and by converting collisional (kinetic) energy into radiation, we actually have created a cooling mechanism for the gas.

Another reason why TE does not hold is the strong **dilution** of the radiation field: the concept of a dilute radiation field is quite familiar. For example, the sun's photosphere is ~ 6000 K, and at the surface the flux leaving the sun is approximately that of a blackbody of this temperature. However, the Earth is not 6000 K because by the time the radiation reaches us, it is diluted. A diluted radiation field is one in which the energy density does not match the color temperature.

-20- For the solar neighborhood, the total energy density of the radiation field due to all stars in that volume is about 1 eV cm^{-3} (close to cosmic ray density, as mentioned before). When interpreted as an average temperature using the Stefan-Boltzmann law (energy density of a blackbody, u = aT^4), this energy density implies an equivalent temperature of ~ 3 K. Yet the color temperature implied by the shape of the spectrum of this Interstellar Radiation Field (ISRF) is that of A and B stars (T $\sim 10^4$ K). So there is a **dilution factor** W given by:

$$W \approx (\frac{3}{10^4})^4 \approx 0.25 \times 10^{-14}$$

We conclude that using the Planck law to describe intensities is not correct.

What about the other laws?

1. Maxwell velocity distribution Good news! It is generally valid. Detailed balancing is possible for the elastic collisions that are generally occurring. Because the maxwellian distribution is a good description of the motions of the particles, we can define a kinetic temperature which describes the physical condition of the gas. Often, for a plasma, the kinetic temperature is equal to the electron temperature: $T_{ions} = T_e$. $T_{ions} \neq T_e$ may occur behind shocks.

-21- **2. Boltzmann distribution** is rarely correct. If excitation and de-excitation occurs by photons, we may still not have a Boltzmann distribution because the photon distribution is not given by the Planck function. Often we do not even have detailed balancing. However, as we will see, sometimes the distribution of excited levels is not too different from Boltzmann distribution. This happens when collisions dominate excitation and de-excitation, while radiation is relatively unimportant.

To describe situations close to TE, Spitzer introduced the so-called b-factors (Draine calls them "departure coefficients").

$$b_j \equiv \frac{n_j(\text{true distribution}}{n_j(\text{LTE distribution})}$$

Example: in an HII region, the hightest excited levels of HI have $b_j \sim 1$. Some radiation does escape (producing radio recombination lines) but collisions dominate the level populations. Since motions of particles *are* described by a Maxwellian velocity distribution, whenever collisions dominate the level population they will closely follow a Boltzmann law.

-22- In general, a Maxwellian velocity distribution tends to set up a Boltzmann population for energy levels in the atoms/particles *if* transitions resulting from emission and absorption of photons are relatively unimportant, and collisional (de-)excitation is dominant. In the case of the highly excited levels in H mentioned before, collisions with electrons are dominant.

3. Saha equation is generally not valid. There is no detailed balancing, and even though the ionization and recombination processes are each other's inverse, the ionization process is determined by the photon field in most cases, while the recombination process is determined by collisions between A^+ and e^-. The collision rate depends on $\{n_e, n_{A^+}\}$ and T_e, but the ionization is dependent on $T_{radiation}(\neq T_e)$.

So in general, assume **statistical equilibrium**, where there is a balance between transitions one way and the other way, no matter what process caused each transition.

In level i, we have n_i atoms cm^{-3}, and R_{ij} is the rate coefficient such that

$n_i R_{ij}$ [s^{-1}] = # transitions from level i to level j
$R_{ji} n_j$ [s^{-1}] = # transitions from level j to level i

$$\frac{\mathrm{d}n_i}{\mathrm{d}t} = \sum_j \left(-R_{ij} n_i + R_{ji} n_j\right); \quad i = 1, 2, \ldots$$

-23- In statistical equilibrium, $\frac{\mathrm{d}n_i}{\mathrm{d}t} = 0$. The rate factor R_{ij} includes *all* possible processes that would take the atom or molecule from level i to j. In the worst case, you would have to include many processes to calculate the n_i values. This requires knowledge of a lot of physical input parameters, e.g. cross sections for particular processes, collisional rate coefficients [*], etc.

In other cases, where only one or two processes matter, the situation can be very simple. We will encounter cases of each.

Before going more into Ch 2 and 3 in Draine, we will first discuss some basic radiative transfer.[†]

4. Radiative Transfer

-24-

Flux at surface of a sphere:

-25- $F_\nu = \pi B_\nu$ for blackbody

$F_\nu = \pi I_\nu$ for isotropic emitting non-blackbody

Flux at a distance r:

$F_\nu(r) = \pi I_\nu (\frac{R}{r})^2 = \frac{L_\nu}{4\pi r^2}$
where $R =$ radius of body and L_ν = luminosity of body [erg s^{-1} Hz^{-1}]

4.1 Energy density of radiation

$u_\nu = \frac{1}{c}\int I_\nu \mathrm{d}\Omega$ [erg cm^{-3} Hz^{-1}]

4.2 Radiation pressure

$P_\nu = \frac{1}{c}\int I_\nu \cos^2\theta \mathrm{d}\Omega$

[*] Drane section 2.1
[†] RL Ch 1, Draine Ch 6, 7

4.3 Emission and absorption coefficients

4.3.1 Emission

- j_ν [erg cm^{-3} sec^{-1} Ω^{-1} Hz^{-1}] spontaneous emission coefficient
- $[j_\nu]$ = volume emission coefficient
- $j'_\nu = \dfrac{j_\nu}{\rho}$ [erg g^{-1} sec^{-1} Ω^{-1} Hz^{-1}] = mass emission coefficient
- $j_\nu = \dfrac{\epsilon_\nu}{4\pi}$ (isotropic emitter); ϵ_ν = emissivity [erg cm^{-3} s^{-1} Hz^{-1}]

4.3.2 Absorption

-26-

- κ_ν [cm^{-1}] volume absorption coefficient (includes *stimulated emission*, aka "negative absorption").
- $\kappa'_\nu = \dfrac{\kappa_\nu}{\rho}$ [g^{-1} cm^{2}] = mass emission coefficient
- microscopically: $\kappa_\nu = n\sigma_\nu$

Loss of intensity in a beam of light as it travels distance ds:

$$dI_\nu = -\kappa_\nu I_\nu ds$$

The absorption process refers to the sum of "true absorption" + stimulated emission, ($\sim$ net emission). It removes a *fraction* of the incoming radiation.

mean free path: ℓ_ν $\quad$ $\tau_\nu = \kappa_\nu \cdot \ell_\nu$, so for condition $\tau = 1$, $\ell_\nu = \dfrac{1}{\kappa_\nu} = \dfrac{1}{n\sigma_\nu}$

5. Radiative transfer equation

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu$$

Define <u>optical depth</u> (unitless):

$$\tau_\nu = \int \kappa_\nu ds$$

then

$$d\tau_\nu = \kappa_\nu ds$$

or

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{\kappa_\nu} \equiv S_\nu$$

where S_ν = <u>source function</u>

- $\tau \gtrsim 1$ optically *thick* emission
- $\tau < 1$ optically *thin* emission

Formal solution:

$$I_\nu(\tau_\nu) = I_\nu(o)\mathrm{e}^{-\tau_\nu} + \int_0^{\tau_\nu} \mathrm{e}^{-(\tau_\nu - \tau'_\nu)} S(\tau'_\nu)\mathrm{d}\tau'_\nu$$

-27- $\rightarrow$attenuated incoming beam + contribution from gas itself.

Special cases

1. Source function is constant throughout source: $I_\nu(\tau_\nu) = I_\nu(o)\mathrm{e}^{\tau_\nu} + S_\nu(1 - \mathrm{e}^{-\tau_\nu})$
    - optically thick emission: $I_\nu = S_\nu$
    - optically thin emission: $I_\nu = I_\nu(o)(1 - \tau_\nu) + \tau_\nu S_\nu$
2. Thermal radiation: $S_\nu = B_\nu(T)$ (The Planck function)
    - optically thick emission: $I_\nu = B_\nu$
    - optically thin emission: $I_\nu = \tau_\nu B_\nu$

For *radio* emission: I_ν is replaced by the <u>brightness temperature</u>, T_b, defined as:

$$I_{\nu,\mathrm{obs}} \equiv B_\nu(T_b)$$

In the Rayleigh-Jeans limit for B_ν we get:

$$I_{\nu,\mathrm{obs}} = \frac{2\nu^2 k T_b}{c^2}$$

or

$$I_b = \frac{I_\nu c^2}{2\nu^2 k}$$

Solution of the transfer equation in terms of T_b:

$$T_{b,\mathrm{obs}} = T_{b,o}\mathrm{e}^{-\tau_\nu} + T(1 - \mathrm{e}^{-\tau_\nu})$$

-28- where

- T is the thermal source temperature (*physical* temperature of the layer)
- T_b is the brightness temperature of the incident radiation

T_b is never greater than T!

5.1 Einstein coefficients

[*] *Transition probabilities per unit time.* Three possible processes:

1. Spontaneous emission: Einstein A coefficient
2. Absorption

-29- 3. Stimulated emission

5.1.1 *Relations between the Einstein coefficients*

Valid under all conditions since they only refer to *atomic* principles (no collisions, just radiation). TE: rate of transitions out of state 1 = rate of transitions into state 1 (per unit volume).

-30- In TE, apply the Boltzmann law.

5.1.2 *Relations between Einstein coefficients and κ_ν and j_ν*

emission coefficient

-31- **absorption coefficient** The second term here for κ_ν corresponds to *stimulated emission.*

5.2 Line profile function, $\phi(\nu)$

[†]

---

[*]Draine, section 6.1

[†]See RL chapter 10.6 and Draine 6.4

5.2.1 Natural line width

Key point: A small Einstein coefficient A results in a *narrow* line.

The natural line width of most transitions is quite small, and broadening due to other effects is more important.

5.2.2 Doppler broadening

- Thermal velocities
- Bulk motion (turbulence)

-32- and the profile function is

5.2.3 Collisional broadening

$\sim$ Pressure broadening, which is not generally important in the ISM because the density is so low... mostly occurs in stellar atmospheres. This still produces a Lorenzian profile, but with:

$$\phi(\nu) = \frac{4\Gamma^2}{16\pi^2(v - v_o)^2 + \Gamma^2}$$

Can be written in terms of a *Voigt function*:

-33- So the core of the profile is Gaussian due to Doppler broadening, while the wings are much stronger than expected in a Gaussian profile, due to the intrinsic line width.

6. Atomic H in the ISM

[*] Wherever HI dominates the ISM, all atoms are found in the **ground state** (^{2}S)(n=1). The next excited level (^{2}P) is about 10 eV above the ground-level. This excitation is very rare and and quickly falls back to the ground level, so there is no significant population of this level. For example, consider potential

[*]Draine Ch 8, 29; Ch 17.1, 17.3

excitation mechanism: collision of H-atom with cosmic ray particle (lots of energy, probability is once per 10^{17} seconds), ionizes the H-atom. Recombination results in some atoms winding up in ^{2}P state. But A coefficient for spontaneous emission from 2P $\rightarrow$ 2S is 10^8 s^{-1}. Hence, this excitation process results in ralative population of 2P level of $10^{-8}/10^{17} = 10^{-25}$!

ISM is too cool for collisions to happen often and cosmic rays are rare. Possible tracers of HI gas:

1. 21 cm HI transition (=hyperfine transition) in emission or absorption.
2. Lyman absorption lines against hot background stars.

Only the ^{2}S level is populated. HI is hard to find in the ground state; fine structure $\rightarrow$ different angular momentum.

*

6.1 Excitation and radiative transport for the 21-cm line

-34-

Spin of proton and electron:

- Parallel (upper energy)
- Anti-parallel (lower energy)

(Spin is around particle's own axis, not to be confused with angular momentum). Motions specified by maxwellian velocity distribution, and collisions dominate the level populations (excite and de-excite).

Energy difference (very small):

$$h\nu = 9.4 \times 10^{-18}\,\text{erg}$$
$$\nu = 1420.4\,\text{MHz}$$
$$\lambda = 21.11\,\text{cm}$$

Spontaneous emission probability is *very* small:

$$A_{kj} = 2.86 \times 10^{-15}\,\text{sec}^{-1}$$

[*] See Draine Ch 4 (& 5) on notation of energy levels and atomic structure.

$$\rightarrow \text{lifetime} = 1.10 \times 10^7 \text{ years}$$

More frequently, atoms will flip energy states by *collisions*. These dominate transitions and cause energy levels to flip.
$H + H \rightarrow H_2^* \rightarrow H + H$
excited H_2 molecule (not stable). Chance for collisional excitation per second: $\gamma n(H)$ [s^{-1}].

$\gamma[\times 10^{-11}$ cm^{3}s^{-1}]	T [K]
0.23	10
3.0	30
9.5	100
16	300
25	1000

As long as $n(H^o) > 10^{-2}$ cm^{-3}, $\gamma n_H >> A$, so collisions determine excitation and de-excitation...implies detailed balancing, so Boltzmann distribution is valid for level populations.

$$\frac{n_1}{n_o} = \frac{g_1}{g_o} \exp(-h\nu/kT_k) \approx \frac{g_1}{g_o}(1 - \frac{h\nu}{kT_k}) \approx \frac{g_1}{g_o}$$

where T_k = kinetic gas temperature.

-35- Derivation:

-36- Now assume that...

In general, in situations where stimulated emission and absorption can be neglected, we would obtain

$$\frac{b_1}{b_o} = \frac{1}{1 + \frac{A_{10}}{n_H \gamma_{10}}} \approx 1$$

Note: Potential other excitation process of upper level for HI: HI + Lyα photon $\rightarrow$ n=2 level (2P). 2P level might cascade down to upper 2S hyperfine level.

In principle, one might selectively populate the upper level with this so-called photon pumping*. It turns out that, due to thermilization of the photons this anomalous population of the levels does not generally occur. (Lyα photons frequently scatter off A-atom again = LTE situation).

-37- Since the b's are effectively 1, our main result is that

Are the HI levels always populated mainly by collisions? No, if n(H^o) drops low enough, and if the HI is warm it doesn't work as well.

The warm HI in our Galaxy has $n_H \sim 0.4$ cm $^{-3}$ and $T_k \approx 8000$ K. But $T_s =$ spin temperature, of order $T_k/5$, if it weren't for excitation by Lyα photons.

But it is true that the 3K background radiation field does *not* significantly disturb the equilibrium set up by collisions, as we discussed above.

If n_H drops to very low values, collisional excitation is ineffective. But even in that case, $T_s \approx T_k$ because of the above mentioned Lyα excitation.

In any case, in general we now obtain for the emission coefficient:

which is independent of T in most circumstances, and for the absorption coefficient:

-38- Using the relation between Einstein coefficients:

6.2 Consider simple case of a single layer of gas

-39-

Consider two cases

(i) $\tau_{\nu_o}(L) >> 1$ Optically thick
(ii) $\tau_{\nu_o}(L) << 1$ Optically thin

In general, the line profile of HI emission is entirely determined by the velocity of the atoms, so the assumption of a Gaussian profile is correct.

-40-

*D 17.3

6.3 Observing brightness temperature

Aside: **How do we observe HI?**

-41-

6.4 HI emission and absorption

Distinguish two cases:

1. Absorption by an HI cloud of an extended background source, usually a continuum source (e.g. AGN)
2. HI self-absorption (enough of it at same velocity)

-42-

- $T_{b,\mathrm{off}} = T_s(1 - \mathrm{e}^{-\tau_\nu})$ in the direction immediately next to the source
- $T_{b,\mathrm{on}} = T_{bo}\mathrm{e}^{-\tau_\nu} + T_s(1 - \mathrm{e}^{-\tau_\nu})$ directly toward the source

Some practical points

1. Frequency *band* in a number of channels, together spanning a range in velocity
2. The assumption here is that the
3. A way to
4. Do we really have only *one* HI cloud along the LOS, and is its temperature uniform?

-43- We observe at each frequency

$T_{b,\mathrm{off}} \leq T_s$

-44- **Discussion of some results** Plots

-45- Conclusions:

1. The absorption profile $(1 - \mathrm{e}^{-\tau_\nu})$ is always sharper and simpler in structure than the emission line profile.
-46- 2. The variation in spin temperature, T_s, makes one wonder whether there is really only one temperature in the cloud.

3. The range of T_s one finds from absorption line studies is 50 K $\leq T_s \leq$ 1000 K. However, the upper value is only a lower limit since we don't see absorption anymore for $T_s > 1000$ K.
4. One can compare

-47- Some relevant results:

1. HI is *not* concentrated in a small number of giant clouds, as H_2 is; estimates of filling factor range from 20-90%?!
2. HI occurs roughly 50/50 in two important forms:
   - CNM = cold neutral medium $\sim$ 80 K
   - WNM[*] = warm neutral medium $\lesssim$ 8000 K
3. Clouds are filaments and/or sheets, rather than spheres
4. At low galactic latitudes, it is not possible to distinguish between WNM and CNM because there is too much material along the line of sight.

-48- 5. Problem with different temperatures along the line of sight. Then:

Special cases:

1. $\tau_1 >> 1$
2. $\tau_1 << 1$
3. $\tau_1 << 1, \tau_2 << 1$

-49- Some additional points on HI[†]

- HI measurements toward Galactic continuum sources provide (some) information on the distance to these sources, using the velocity of the HI absorption feature to derive a kinematical distance. The usual problem of distance ambiguity may still be a problem here as well, although the total HI column [density?] that is derived from the absorption measurements helps a little in distinguishing near and far distances.
- **Temperature of WNM**. . . two methods:
  1. HI absorption
  2. UV absorption lines

  So far: data in agreement with 5000 K $\lesssim T \lesssim$ 8000 K, but needs more confirmation.

-50-
- **Temperature of CNM**. . . from absorption spectra. The absorption lines are *narrow*.

---

[*] Mebold, 1972

[†] Kulkami & Heiles

Present research on Milky Way HI concentrates on:

1. Origin of high velocity clouds
2. Properties of smallest clouds/features
3. Search for HI clouds that may contain dark matter, but no stars

-50a, Images and plots.
50b,
50c-
-I1-

7. Atomic structure

7.1 Introduction

7.2 Hydrogen atom & hydrogen-like atoms (or ions)

-I2- Example: energy of ground states:

- H = 13.6 eV
- He^+ = 54.4 eV
- Li^{2+} = 122.5 eV

-I3-
-I4-
-I5-
-I6-

7.3 Electron spin

-I7- But, since $\vec{J}_1$ and $\vec{J}_2$ can have different directions, there are different possible values of $\vec{J}$.

-I8-

7.4 Spin orbit coupling

Two possible orientations of electron spin with respect to orbital angular momentum lead to a doubling of energy levels of H-like atoms (except s-levels, where $\ell = 0$).

Lines appear in pairs, close together, called doublets. Example: for Na D lines, $\lambda = 5890\text{Å}, 5896\text{Å}$. (Question: are atoms with filled shells + one electron also hydrogen-like? Doubling is due to spin-orbit coupling.)

Physically, the spin-orbit coupling produces an extra energy term for the electron, proportioned to $\vec{S} \cdot \vec{L}$.

-I10-

7.5 Atoms with multiple electrons

-I11-
-I12-
-I13-
-I14-

7.6 Transition rules

-I15-
-I16-
-I17-
-I18-
-I19-

7.7 X-ray emission

-I20-

7.8 Zeeman effect

-I21-

8. HII regions

8.1 Introductory remarks

Process: hot OB stars emit UV photons that can ionize the surrounding neutral H (and He) medium. In practice, this requires stars hotter than $\sim$ 30,000 K, aka. spectral type B0 or earlier.

Physics for planetary nebulae is similar, but central stars may be much hotter (though they are also dimmer because there is "more stuff").

UV photons impart energy to gas by ionizing H and He. Excess kinetic energy of created free electrons is shared with ions and other electrons, heating the medium. Relevant processes:

- photoionization
- electron-electron encounters
- electron-ion encounters
  - Excited ions
  - Recombination
  - Bremsstrahlung

H atom + photon $h\nu \longrightarrow$ p$^+$ + e$^-$ + E$_k$

Since stars create a continuous stream of photons, a balance is reached between ionization and recombination. * †

Field OB stars: were they born there or travel there somehow? Usually in groups (OB associations), dense clouds, etc.

Consequences: $\sim 10^4$ K. Warm ionized plasma emitting various forms of radiation: recombination, free-free continuum, bree-bound continuum, 2-photon continuum, collisionally excited forbidden lines from "metals". If there is dust around, get MIR dust continuum. Runaway OB stars (like teenagers!)

1 km $\equiv$ 1 pc in 10^6 years. (?)

---

*Draine Ch 10, 13.1, 14, 15, 17, 18, 27, 28

†Osterbrock AGN^2

Q_o [s^{-1}] = number of ionized photons emitted by OB star per second. Factor of 100 from O 9.5V to O 3V (luminosity classes).

-52-

8.2 Strömgren Theory

"It's only a model."

Classical paper*

Basic result:

- Hot star in a uniform medium will ionize a spherical volume out to a certain radius, whose size is determined by:
  1. Number of ionizing photons emitted by the star
  2. ρ of medium (determines recombination rate)
- There is a *sharp* boundary from the ionized to the surrounding neutral medium.

8.3 Simple derivation

Pure H nebula, uniform density. Hot star emits N_{Lyc} photons per second (*Lyc* = Lyman continuum), all at the *same* frequency, ν_o. The star will ionize the gas, but there is a balance:

$$\text{ionization} \longleftrightarrow \text{recombination}$$

Stationary ionization eqilibrium characterized by *degree of ionization*:

$$x(r) \equiv \frac{n_e(r)}{n_H}$$

where n_H = *total* hydrogen density (neutral and ionized). $0 \leq x \leq 1$ ($x = 1 \rightarrow$ complete ionization; $x = 0 \rightarrow$ completely neutral). Problem: what is the shape of $x(r)$?

Important quantity: the Strömgren radius, R_{SO}, defined as

$$\frac{4}{3}\pi\alpha n_H^2 R_{SO}^3 \equiv N_{Lyc}$$

*Strömgren 1939 ApJ 89, 526

where α is the recombination coefficient (collisional process $\sim \rho^2 \rightarrow$ more encounters, see 8.3.2). The quantity N_{Lyc}[*] gives the total number of recombinations per unit time (all the magic is here!).[†]

-53-

8.3.1 Description of photoionization equilibrium

Overview: Consider pure H nebula surrouding a single hot star. Ionization equilibrium:

$$n_{H^o} \int_{\nu_1}^{\infty} \frac{4\pi J_\nu}{h\nu} \sigma_\nu \mathrm{d}\nu = n_e n_p \alpha(H, T)$$

where

- n_{H^o} = neutral hydrogen density
- J_ν = mean intensity of radiation field
- σ_ν = ionization cross-section for H by photons with energy above threshold $h\nu_{01}$
- n_e, n_p = electron and proton densities
- $\alpha(H, T)$ = recombination coefficient
- LHS: number of *ionizations* per second per cm^3
- RHS: number of *recombinations* per second per cm^3

To first order, not including radiative transfer effects:

$$4\pi J_\nu = \frac{R^2}{r^2} \pi F_\nu(0) = \frac{L_\nu}{4\pi r^2}$$

(Local radiation field.) Order of magnitudes:

$$N_{Lyc} = \int_{\nu_1}^{\infty} \frac{L_\nu}{h\nu} \mathrm{d}\nu$$

= number of Lyman continuum photons emitted per second.

- $N_{Lyc} \approx 5 \times 10^{48}$ sec^{-1} for O6 star
- $\sigma_\nu \approx 6 \times 10^{-18}$ cm^2
- $\alpha(H, T) \approx 4 \times 10^{-13}$ cm^3 sec^{-1}

-54- Physical conditions: radiative decay from upper levels to $n = 1$ is quick $\rightarrow$

[*] $N_{Lyc} = S_4$ in Spitzer's notation.
[†] S_4 formally should include contributions from the diffuse radiation field.

nearly all neutral hydrogen will be in the ground level. Photoionization takes place from the ground level, and is balanced by recombination to excited levels, which then quickly de-excite by emission of photons.

The *photoionization cross-section* ($\sigma \propto \nu^{-3}$) is actually rather complicated to calcualte. Spitzer (who calls this erroneously the "absorption coefficient") gives:

$$\sigma_{f\nu} = \frac{7.9 \times 10^{-18}}{Z^2} \left(\frac{\nu_1}{\nu}\right)^3 g_1 f$$

where $g_1 f$ = Gaunt factor (from level 1 to free)*

Since $\sigma_\nu \propto \nu^{-3}$, higher energy photons will typically penetrate further into the nebula before being absorbed.

8.3.2 the recombination coefficient

$\alpha_n(H, T)$ = recombination coefficient for *direct* recombination to level n.

$$\alpha^{(n)} \equiv \sum_{m=n}^{\infty} \alpha_m$$

where m = all other levels stepped through to get to n.

Note: Ouside of level 1, higher energy not necessarily better for ionization. OIII is not necessarily from the same place as OII. Free electrons: there is a recombination coefficient for every level, depends on velocity of the electron.

-55- The summed recombination coefficient (α) is the relevant quantity because recombination counts, irrespective of the level n to which it happens.

Larger velocities $\rightarrow$ smaller σ.

For a distribution $f(v)\mathrm{d}v$, $n_e f(v) v$ = number of electrons with velocity v passing through a unit area per second. Thus

$$\alpha_n = \int_o^\infty v \sigma_n(H, v) f(v) \mathrm{d}v$$

*see, e.g. Table 5.1 Spitzer (Draine Ch. 10); see also Fig 13.1 in Draine and Fig. 2.2 in handout (from Osterbrock).

where $\sigma_n(H, v) \propto v^{-2}$ = recombination cross-section to the level n for electrons with velocity v.

$$f(v) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT}\right)^{3/2} v^2 \mathrm{e}^{-mv^2/2kT}$$

(Maxwellian). Hence, $\alpha \propto \frac{1}{\sqrt{T}}$*

We are interested in the total recombination coefficient to all levels:

$$\alpha^{(n)} = \sum_{m=n}^{\infty} \alpha_m$$

$\alpha^{(1)}$: summed over all levels. However, for direct recombination to level 1, a photon is generated which itself can ionize H again; so the recombination coefficient that we are interested in is $\alpha^{(2)} \equiv \alpha_B$† This refers to "case B": optically thick through Lyα emission. Almost always here! Most common situation. Case A: very low density gas.

-56- Neutral fraction *inside* HII region‡: Consider pure H nebula, and case B recombination (see later). *Mean energy* of stellar ionization photons is $h\nu$.

-57- Rewrite latter as:

so neutral fraction x is small and hence $x^2 \approx 1$.

-58- Comments:

1. Reality: include He, include diffuse radiation field generated from direct recombination to $n = 1$ (which can ionize an atom again!), realistic stellar energy spectrum below $\lambda = 912$Å; results are qualitatively the same.
2. Is assumption of a static ionization equilibrium realistic? No: HII region is overpressureized compared to the surrounding medium: T_e and n higher $\longrightarrow$ HII region expands§. Expansion of the ionization front happens rather slowly, $v \propto 1 - 2$ km s^{-1}, dynamical timescale. $\dfrac{R_{SO}}{v} \sim 10^7$ years, of order liftime of HII region. So ionization equilibrium does hold.

---

*see e.g. Spitzer, Table 5.2
†see Draine section 14.2
‡see Draine section 15.3
§First discussed by Kahn (1954)

3. Recombination time scale = ionization time scale:

$$t_{ionization} \equiv \frac{\frac{4}{3}(R_{SO})^3 n_H}{Q_o} = \frac{1}{\alpha_B n_H} = \frac{1.22 \times 10^3}{n_2} \text{ years} = t_{recombination}$$

($n_2 = n_H$ in units of 100). The two are identical! So $t_{recombination} \propto \frac{1}{n_e}$
Key: for $n_H > 0.03$ cm^{-3} $t_{recombination} <$ lifetime of massive stars ($\lesssim 5$ Myr).

4. As we will see later, each Lyc photon results in ~ 0.35 Hα photons begin produced in this ionization $\rightarrow$ recombination equilibrium.
5. Next: influence of dust on R_{SO}

8.4 The spectrum of an HII region

-63-

8.4.1 Continuum Radiation

Sources:

- free-free[*] = thermal Bremsstrahlung
- free-bound[†]
- two-photon decay[‡]
- emission by dust particles

Two-photon decay: Transition from $n = 2 \rightarrow n = 1$ ($\ell = 0 \rightarrow \ell = 0$) is strictly forbidden ($\Delta\ell \neq 0$). The actual process happens in two steps, with a virtual intermediate phase. Two photons are emitted whose joint energy adds up to the energy of a Lyα photon = 3/4 ionization energy of H. The two-photon continuum is symmetric about $\nu_{12}/2$ if expressed in photons per unit frequency instead.

Free-free emission: Bremsstrahlung, continuous emission and absoprtion by *thermal* (Maxwellian velocity distribution) electrons due to *encounters* between electrons and positive ions. Acceleration/deceleration $\longrightarrow$ photons.

---

[*] §3.5 Spitzer, Drane ch. 10
[†] See previous footnote
[‡] Osterbrock, pp 89-93

Classical theory: electron emits a single narrow Em pulse in time, with no oscillation in E. → FT is broad, almost the FT of a δ function. → Emission coefficient j_ν is nearly independent of frequency, up to a *cutoff* frequency,
-64- which corresponds to the Maxwellian velocity distribution of electrons. So cutoff frequency is given roughly by $h\nu \sim kT_e$. (HII regions: $T_e \sim 10^4$ K, so $h\nu \approx 0.87$ eV and $\lambda \approx 1.4\mu$m).

In practice, dust emission dominates at IR wavelengths, so it is unlikely to observe the free-free spectrum this far.

<u>Emission coefficient</u>:

$$\begin{aligned} j_\nu &= \frac{8}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{Z_i^2 \mathrm{e}^6}{m_e^{3/2} c^3 (kT)^{1/2}} g_{ff} n_e n_i \mathrm{e}^{-h\nu/kT} \\ &= 5.44 \times 10^{-39} \frac{g_{ff} Z_i^2 n_e n_i}{\sqrt{T}} \mathrm{e}^{-h\nu/kT} \quad [\text{erg cm}^{-3}\ \text{s}^{-1}\ \text{sr}^{-1}\ \text{Hz}^{-1}] \end{aligned}$$

where $\mathrm{e}^{-h\nu/kT}$ is the exponential cutoff due to kT_e and g_{ff} is the Gaunt factor for free-free transitions*.

Total amount of energy radiated in free-free transitions per cm^3 per second:

$$\epsilon_{ff} = 4\pi \int j_\nu \mathrm{d}\nu$$

. . . (long-ass equation)

Remember that the corresponding absorption coefficient κ_ν is related to j_ν by Kirchoff's law, since we deal with *thermal* emission (*not* blackbody though!) $j_\nu = \kappa_\nu B_\nu(T)$ which leads to [anther long-ass equation].

-68- **Free-bound emission**

8.4.2 Line Radiation

-69- Two kinds:

1. Recombination lines, predominantly from H and He
    - optical

*see Draine, figure 10.1

- radio
- forbidden

2. Collisionally excited liens in heavy elements

Recombination lines:

-79- **Collisionally excited lines in heavy elements:** Generally: electrons don't have enough energy to excite ions from the ground state to their first excited states. However, many ions have multiplets in their ground state, due to the coupling of the electrons individual (orbital and spin) angular momenta.

Many ions have incomplete "p-shells"; (there is not really a p-shell, but we talk about electrons which have $\ell = 1$, corresponding to p). Remember:

H	1s		
He	1s^2		
C	1s^2	2s^2	2p^2
O	1s^2	2s^2	2p^4

For $n = 2$, there is room for 6 p-electrons, since each $n\ell$ has a number of m_ℓ and m_s combinations:

$$\left.\begin{array}{l} m_\ell : 2\ell + 1 \\ m_s : \pm\frac{1}{2} \end{array}\right\} 2(2\ell + 1)$$

So as long as $\ell = 1$ is possible (meaning $n > 1$), we have 6 potential "p-shell" electrons, *always*.

The enclosed diagram shows the 3 most typical structures of energy levels for various elements in different ionization stages. For [OIII] we have, in particular: 2 electrons in "p-shell", $\ell_1 = \ell_2$, so $L = 0, 1, 2$

-81- One can set up 5 equations of statistical equilibrium for each of these levels. This shows that both the ${}^1S_o$ and the ${}^1D_2$ levels are populated essentially exclusively by collisions from ${}^3P$ level.

So, not surprisingly, the ratio

$$\frac{N({}^1S_0)}{N({}^1D_2)} \quad \propto \quad \exp\left(\frac{-\Delta E}{kT}\right)$$

where ΔE = energy difference between ${}^1S_0$ and ${}^1D_2$

Also, observed flux in $\lambda 4363$ line is:

$$S(4363) \quad \propto \quad \frac{4}{3}\pi R^3 N('S_0) A_{4363} h\nu$$

and similarly for the 4959 and 5007 lines.

The net result, obtained by solving the equations of statistical equilibrium is:

$$\frac{S(4959) + S(5007)}{S(4363)} = 8.3 \exp\left(\frac{3.3 \times 10^4}{T_e}\right)$$

so this line ratio provides a direct measure of the electron temperature.

A similar relationship exists for [NII] (6548, 6583, and 5754) lines.

In general: Emission lines arising from ions, such as O^{++} and N^{+} that have upper energy levels that have considerable different energies are useful for estimating T_e. Vice versa, as we will see, ions with closely spaced upper enrgy levels provie little info on T_e, but can be used to derive n_e (examples: [SII], [OII] lines).

8.5 Types of HII Regions

-85- In general:

- HII regions are associated with molecular clouds and dark nebulae.
- Structure:
  - "Blister model" – cavity inside GMC
  - "Champagne model" – half cavity at edge of GMC

There are also *compact* HII regions, which are only visible at radio and FIR wavelengths. These are very young objects, often associated with H2O and OH masers, infrared sources, molecular lines of complex molecules, etc.

The *range* in properties of HII regions is enormous*.

Modeling of HII region spectra: the probably best known code is CLOUDY, developed by G. Ferland. You can in put an ioinizing spectrum from a star or other, a gas distribution, metallicity, etc., and then calculate the detailed emission line spectrum for the region.

HII regions can be density-bounded or radiation-bounded. rad: run out of photons before running out of gas. density: run out of material before using up photons. No clear boundary.

*Kennicut, 1984

9. Interstellar absorption lines in stellar and quasar spectra

-89- Discovered in 1904, before we were aware of dust, or that MW and M31 were separate galaxies. Typical optical absorption lines discovered later include Ca^+, Na^o, K, Ti^+, and some molecular lines, e.g. CH^+, CH, CN. Lines from more abundant atoms and molecules (H, H_2, C, N, O, etc.) were only found after Copernicus was launched (early 70s) because $\lambda < 3000$Å for these lines, so they have to be observed from space.

Key: Since the absorption process is $\propto I_{incoming}$, we can find very small column densities of ISM, provided the background source is strong enough.

General, important property: Absorption of optical photons generally only occurs from the ground level of the atom, ion, or molecule, because only these levels are populated in most conditions (remember that kT = 0.86 eV for T = 10^4 K, or 8.6 eV for T = 10^5 K). This implies that in the formation of absorption lines, stimulated emission from the upper level does not play a role.

O^+ in HII region sits in ground state, can assume ions in ISM also stay in $n = 1$. Pure absorption foreground screen. $I = S(\lambda)\,(1 - e^{-\tau_\lambda})$, can leave the exponent out ($\rightarrow 0$).

This is not the case whenever excitation conditions are such that absorption lines do arise from excited atoms or molecular levels. Usually collisions would be responsibe for the excitation, so keep an eye on kT.

-90- Generally, interstellar absorption lines are *narrow* and *complex* in structure. So we require very high resolution to resolve them:

$\left(\dfrac{\lambda}{\Delta\lambda} > 3\times10^5\right) \Leftarrow$ ideal, but rarely achieved. res: couple thousand

Aside

Remember that $\dfrac{\Delta\lambda}{\lambda} = \dfrac{\Delta v}{c}$ and for $\dfrac{\lambda}{\Delta\lambda} \geq 3\times10^5 \rightarrow v \leq 1$ km per second.
Which implies $\Delta\lambda \sim 0.02$ Å at 6000 Å and $\Delta\lambda \sim 0.0033$ Å at 1200 Å

9.1 Theory of formation of (interstellar) absorption lines

Ideal background source: bright (especially UV), with a fetureless continuum. ⇒ Quasars and AGN or hot stars (which tend to be more complicated, though quasars sometimes have their own lines).

The high spectral resolution that is required to resolve a line is often unattainable. However, even if we cannot *resolve* the absorption line, we can still infer some important properties about the ISM. As long as the resolution is sufficient to separate absorption lines at different velocites, coming from different clouds along the line-of-sight, or if only one component is present, we can use the *area* of an absorption line, or the **equivalent width**, EW, the width of an absorption line which would absorb 100% everywhere, and which would have the same area as the hatched area. Note that EW is always defined relative to I_o; we *divide* by I_o.

-91- Note also that if we decrease the resolving power of the spectrograph, EW does not change.

Theoretical expression for EW:

$$\tau_\nu = \int_0^\infty \kappa_\nu \mathrm{d}r; \quad \kappa_\nu = h\nu_{u\ell}\frac{n_\ell B_{\ell u} - n_u B_{u\ell}}{c}\phi(\nu)$$

(using "energy density" definition of B coefficients). u = upper level and ℓ = lower level.

If stimulated emission can be neglected, we have:

$$\kappa_\nu = \frac{h\nu_{u\ell}B_{\ell u}}{c}n_\ell\phi(\nu) \equiv \frac{\pi \mathrm{e}^2}{m_e c}f_{\ell u}n_\ell\phi(\nu)$$

where $f_{\ell u}$ = *oscillator strength* (historical terminology), and $\frac{\pi \mathrm{e}^2}{m_e c}$ = "classical cross-section"* ∼ Einstein coefficient.

* see for example Rybicki and Lightman, section 3.6

Use $|\mathrm{d}v| = \frac{c}{\lambda^2}|\mathrm{d}\lambda|$ and we obtain:

$$\begin{aligned}
EW &= \int_o^\infty \left[1 - \mathrm{e}^{-\tau_\nu}\right] \mathrm{d}\lambda \\
&= \frac{\lambda^2}{c} \int_o^\infty \left[1 - \mathrm{e}^{-\tau_\nu}\right] \mathrm{d}\nu \\
&= \frac{\lambda^2}{c} \int_o^\infty \mathrm{d}\nu \left[1 - \exp\left(-\int_0^\infty \left(\frac{\pi \mathrm{e}^2}{mc}\right) f_{\ell u} n_\ell \phi(\nu) \mathrm{d}r\right)\right]
\end{aligned}$$

$\tau \sim n_e$

Now assume: $\phi(\nu)$ independent of r for the particular cloud that we are observing. Then:

$$EW = \frac{\lambda^2}{c} \int_0^\infty \mathrm{d}\nu \left[1 - \exp\left(-\left(\frac{\pi \mathrm{e}^2}{mc}\right) f_{\ell u} \phi(\nu) N_\ell\right)\right]$$

with $N_\ell = \int n_\ell \mathrm{d}\nu$ = *column density* of atoms in level ℓ. The quantity $\phi(\nu) N_\ell$ is the key part of this equation.

-92- Note: This situation is very different from the discussion of HI 21-cm abs lines where the upper level *is* populated, and we had to include stimulated emission to first order.

We next assume a Gaussian velocity distribution (Maxwellian) to describe the distribution of radial velocities:

$$f(v_{rad}) = \frac{1}{\sqrt{\pi} b} \exp\left[-\left(\frac{v_{rad} - v_o}{b}\right)^2\right]$$

where $b = \sqrt{\frac{2kT}{m}}$ (only valid for thermal motions) and $\phi(\nu)$ becomes:

$$\phi(\nu) = \frac{1}{\sqrt{\pi}} \frac{1}{\Delta v_b} H(a, u)$$

where $a \equiv \frac{\Gamma}{4\pi \Delta v_b}$; $u = \frac{\nu - \nu*}{\Delta \nu_b}$; $\Delta \nu_b \equiv b\frac{\nu_o}{c}$; and $\nu* = \nu_o(1 - \frac{v_o}{c})$, the central frequency of the line. $b^2 = 2\sigma^2$ where σ = velocity dispersion, and a is a measure of where the Gaussian profile levels off to the Lorentzian wings* (transition of Gaussian line core to the damping wings).

*see notes from before, p. 34

9.2 The Curve of Growth

What is all this good for?

For a given spectral line, we can measure EW from observations. Free parameters: N_ℓ, b, and a. For a given cloud, b and a are specified, and we can plot EW as a function of N_ℓ.

$$\Longrightarrow \text{The Curve of Growth}$$

In practice, one plots $\log\left(\frac{W}{\lambda}\right)$ against $\log\left(N_\ell f_{\ell u}\lambda\right)$.*

We can distinguish three regimes in a curve of growth.

-93- I $EW \propto N_\ell$ (linear)

II EW almost independent of N_ℓ

III $EW \propto \sqrt{N_\ell}$

-94-

9.2.1 Turnover points in the curve of growth

9.2.2 Growth curves in practice

9.3 UV absorption lines from H and H_2

-96-

---

* example: figure 3.2 in Spitzer

10. Dust

-104-

10.1 Far infrared emission from dust

10.1.1 General properties of dust

10.1.2 Absorption efficiency: the Q parameter

10.1.3 Calculating dust mass from FIR fluxes

10.1.4 Dust temperatures

10.2 Interstellar extinction

-116- The presence of dust was demonstrated by Trumpler, who detected the dimming and reddening of distant stars. High-z proto-galaxies!

Dust both scatters and absorbs light. The combined effect is **extinction**. For a point source (e.g. a star), the object is *dimmed* by extinction, since both scattered and absorbed light do not reach the observer. For an extended source (e.g. a galaxy), some light may be scatter *into* the line of sight, so the extinction will generally be less than for point sources. On average, extinction is about 0.6 - 1 mag kpc^{-1} (reddening is about 0.3 mag kpc^{-1} in (B-V) in the plane of the Milky Way - see §10.3).

NGC 6240, a heavily obscured starburst nucleus, emits 10 times more power in the IR/FIR than in the optical, with peak near 60-100 μm. Large z: shifts into sub-mm. Led to detection of Ultra-Luminous IR Galaxies (ULIRGs), where "ultra-luminous" is $> 10^{12}$ $L_\odot$. More typical galaxies have $L_{\rm FIR} \sim L_{\rm optical}$.

10.2.1 The extinction law

-117- Empirical result: reddening in magnitudes

$$r_\lambda = a + \frac{b}{\lambda}$$

where a and b are roughly constant. This is between two extreme cases:

1. grey extinction, r_λ independent of λ
2. Rayleigh scattering, $r_\lambda \propto \lambda^{-4}$

$\Rightarrow$ particle size is $\sim 10^{-5}$ cm $= 0.1$ μm (= 100 nm).

$$\mathrm{d}I_\lambda = -I_\lambda n(x)\sigma_\lambda \mathrm{d}\lambda = -I\mathrm{d}\tau_\lambda$$

- $n(x)$ = number density of grains at position x
- σ_λ = extinction cross-section per dust grain at wavelength λ

The star is dimmed by

$$I_\lambda = I_\lambda^0 \mathrm{e}^{-\tau_\lambda}$$

where

$$\tau_\lambda = \sigma_\lambda \int_0^x n(x')\mathrm{d}x' = \sigma_\lambda N(x)$$

In magnitudes:

$$\begin{aligned}
A_\lambda &\equiv -2.5\log\left[\frac{I_\lambda}{I_\lambda^0}\right] \\
&= -2.5\log\left[\mathrm{e}^{-\tau_\lambda}\right] \\
&= -2.5\log\left[\mathrm{e}^{\ln(\mathrm{e}^{-\tau_\lambda})}\right] \\
&= -2.5\ln(\mathrm{e}^{-\tau_\lambda})\log\left[\mathrm{e}\right] \\
&= -2.5(-\tau_\lambda)\log\left[\mathrm{e}\right] \\
&= 1.086\tau_\lambda
\end{aligned}$$

Dependence on Galactic latitude: Assume simplest model of uniform dust layer.

-118- In reality, an exponential z-depenence would be more realistic, while the dust is also clumped, just like the gaseous ISM.

10.3 Interstellar reddening

Reddening is caused by the wavelength dependence of extinction. We can define a **color excess**:

$$E(\lambda_1 - \lambda_2) \equiv \underbrace{(m_{\lambda 1} - m_{\lambda 2})}_{\text{observed color}} - \underbrace{(m_{\lambda 1,0} - m_{\lambda 2,0})}_{\text{intrinsic color}}$$

e.g.:

$$E(B-V) = (B-V) - (B-V)_0$$

<u>Question</u>: How is the reddening measured?

Reddening line is defined by the extinction law.

$$E(U-B) = \underbrace{0.72}_{\substack{\text{“standard} \\ \text{Galactic} \\ \text{reddening} \\ \text{law”}}} E(B-V) + \underbrace{0.05(B-V)}_{\text{minor correction}}$$

We can define *reddening free colors*, if the shape of the reddening law is known. For example, the following is reddening free:

$$Q \equiv (U-B) - \frac{E(U-B)}{E(B-V)}(B-V) = (U-B) - 0.72(B-V)$$

-119- In general:

$$m_i = m_i^0 + A(\lambda_i)$$
$$m_j = m_j^0 + A(\lambda_j)$$

11. Molecular hydrogen and CO

-124-

12. Heating and cooling

-151- Following Spitzer's general formalism*:

Γ = total kinetic energy *gained* [erg cm^{-3} s^{-1}]
Λ = total kinetic energy *lost* [erg cm^{-3} s^{-1}] (aka. the "cooling function")

All the physics are contained in these two quantities; need to find out what they are.

For $\Lambda = \Gamma$ we reach equilibrium temperature, T_E. Since Λ and Γ may depend on T_E themselves, we can set up a time dependent differential equation:

$$\underbrace{n\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{3}{2}kT\right)}_{\substack{\text{rate of increase in}\\ \text{thermal energy for}\\ \text{a mono atomic gas}}} - \underbrace{kT\frac{\mathrm{d}n}{\mathrm{d}t}}_{\substack{\text{work done by}\\ \text{gas (energy}\\ \text{loss)}}} = \underbrace{\sum_{\varrho,\eta}(\Gamma_{\varrho,\eta} - \Lambda_{\varrho,\eta})}_{\text{Sum over interacting particles}} = \Gamma - \Lambda$$

(Note: in what follows, we generally ignore "work done by gas" component.)

For T$\lesssim 2 \times 10^4$ K, thermal conduction can be ignored. If we do have enrgy gain or loss by conduction (electrons), we add -$\vec{\nabla} \cdot \left(k\vec{\nabla}T\right)$ to the righthand side. Conductivity is hard to figure out, but not important below 20,000 K. ($\vec{B}$ fields prevent conductivity across field lines.)

$PV = nRT = NkT \to P\Delta V$ = work done by gas. $R = N_A k$, where N_A is Avagadro's number.

If a gas is not at its equilibrium temperature (T$_E$), we can define a "cooling time" t_T (like a half-life, sorta):

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{3}{2}kT\right) = \frac{3}{2}k\left(\frac{\mathrm{d}T}{\mathrm{d}t}\right) = \frac{3}{2}k\left(\frac{T - T_E}{t_T}\right)$$

where T_E = constant and T = variable. For T_E and t_T = constant, $T - T_E \propto$ e$^{-t/t_T}$ (exponential decay for cooling). t obviously can't be negative $\to$ quasi-stable

-152- If $t_T < 0$, unstable situation and gas will cool or heat toward an entirely different equilibrium temperature. This is relevant to the multi-phase ISM. All the physics is in Λ and Γ.

---

* see also Draine chapters 27, 30, 34

12.1 General

12.1.1 *Primary heat source*

The primary heat source is (photo)ionization.

E_2 = kinetic energy of ejected electron
E_1 = kinetic energy of recaptured electron

$\Delta E = |E_2 - E_1| \rightarrow$ energy available. Ionization from the ground state (maximum energy) recombines with less energy. Number of captures to level j of neutral atom:

$$n_e n_i < \omega\sigma_{cj} > \quad [\mathrm{cm}^{-3}\ \mathrm{s}^{-1}]$$

where $< \omega\sigma_{cj} >$ = recombination coefficient. The final net gain associated with electron-ion recombination:

$$\Gamma_{ei} = n_e n_i \sum_j \left(\underbrace{< \omega\sigma_{cj} > \overline{E}_2}_{\text{ionization out of } j} \quad - \quad \underbrace{< \omega\sigma_{cj} E_1 >}_{\text{recapture back to } j} \right)$$

$<>$ = average over Maxwellian velocity distribution
$\overline{E}_2$ = average over all ionizing photon energies

As we have seen, all ionizations take place from the ground level $\rightarrow \overline{E}_2$ is independent of j; use recombination coefficient to all levels $\geq n$:

$$\alpha^{(n)} = \sum_n^{\infty} \alpha_m$$

and

$$\Gamma_{ei} = n_e n_i \left\{ \underbrace{\alpha \overline{E}_2}_{\equiv \alpha^{(1)}} \quad - \quad \underbrace{\frac{1}{2} m_e \sum < \omega^3 \sigma_{cj} >}_{\text{kinetic energy}} \right\}$$

-153- **Main point:** Γ_{ei} is not dependent on ionization probability or radiation density for *steady state*, where the number of ionizations is equal to the number of recombinations.

12.1.2 Primary cooling source

The primary cooling source is **inelastic collisions**. (excitation of energy levels, since particles that undergo inelastic collisions will lose energy).

- $n_e n_i \gamma_{jk}[\mathrm{cm}^{-3}\ \mathrm{s}^{-1}]$ = number of excitations from level $j \to k$ for ions in ionization stage i and excitation state j.
- $E_{jk} = E_k - E_j$ = energy lost by colliding electrons (in the form of emitted photons).

This cooling is offset by *de-exciting* collisions, which give energy gain to electrons (give energy back to the nebula). Net cooling:

$$\to \Lambda_{ei} = n_e \sum_{j<k} E_{jk} \left(\underbrace{n_{ij}\gamma_{jk}}_{\text{excitation}} - \underbrace{n_{ik}\gamma kj}_{\text{de-excitation}} \right)$$

γs are collisional rate coefficients. <u>Assumption</u>: all photons escape (not true for dense molecular clouds). Again, in most cases all ions are in the ground level, so we don't need to sum over j, and $n_{ij} = n_{i1} = n_i$ (simpler)

12.2 HII Regions

-154-

- Main heat sources: ionization of H and He
- Main cooling source: excitation of C, N, O, Ne
    - Heavy elements have low abundance. If they didn't, HII regions would cool to very low T_E!

UV = radiation field energy density. Radiation is from OB stars (ionizing photons) and diffuse radiation in HII regions from direct recombination to n=1.

HII Regions: Lyman photons didn't escape; they were all re-absorbed and turned into Balmer line. Photons absorbed by dust $\to$ cooling. Photons scattered $\to$ *not* cooling. Keep in mind: **Do all photons make it out of the nebula?**

12.2.1 Heating

$$\overline{E}_2 = \frac{\int_{\nu_1}^{\infty} h(\nu - \nu_1) s_\nu U_\nu \mathrm{d}\nu/\nu}{\int_{\nu_1}^{\infty} s_\nu U_\nu \mathrm{d}\nu/\nu}$$

- s_ν = cross section
- ν_1 = ionization limit for HI

Problem: U_ν is determined by stellar radiation ($U_{s\nu}$) and diffuse radiation ($U_{D\nu}$), and depends in turn on n_H, etc.

Spitzer discusses two simple cases:

1. Close to exciting star: $U_{s\nu}$ large, $U_{D\nu}$ negligible in comparison
2. Evaluate $\overline{E}_2$ for entire HII region.

Approximation: Use dilute blackbody of temperature T_c (color temperature) to describe the stellar radiation field at distances from the stars.

Define: $\psi = \dfrac{\overline{E}_2}{kT_c}$

$\psi_0 = \psi(r \to 0)$, so close to star
$< \psi >$ = average of star
$\psi(r)$ = over entire HII region

-155-

$$\psi_0 : \quad U_\nu = \frac{4\pi B_\nu(T_c)}{c} = \frac{1}{c}\int I_\nu \mathrm{d}\Omega$$

Then it is possible to calculate table 6.1 (Spitzer), which lists values of ψ_0 (Draine, table 27.1) for various values of T_c. *

$$\text{Result:} \quad \begin{array}{ccccc} 1.05 & < & <\psi> & < & 1.65 \\ 4000 & < & T_c & < & 64{,}000 \end{array}$$

We had that:

$$\Gamma_{ei} = n_e n_i \left\{ \alpha \overline{E}_2 - \frac{1}{2} m_e \sum < \omega^3 \sigma_{cj} > \right\}$$

where second term = mean energy lost per recombining electron.

---

* Spitzer also discusses how to calculate $< \psi >$; too lengthy to do here.

We have $\overline{E}_2$, now we need

$$\sum_{j=k}^{\infty} < \omega^3 \sigma_{cj} > = \frac{2A_r}{\sqrt{\pi}} \left(\frac{2kT}{m_e} \right)^{3/2} \beta \chi_k(\beta)$$

A_r = "recapture constant"[*]

$\beta = \dfrac{h\nu_1}{kT}$

$\chi_k(\beta)$ are listed in table 6.2 in Spitzer. They are "energy gain functions" with values from $\sim$0.4 to 4.0.

12.2.2 Cooling

Use directly the basic equation using

$$\frac{n_k}{n_j} = \frac{b_k}{b_j} \frac{g_k}{g_j} \exp\left(-\frac{h\nu_{ju}}{kT} \right)$$

and

$$\frac{b_2}{b_1} = \frac{1}{1 + A_{21}/n_e \gamma_{21}}$$

(discussed for HI emission) for a two-state ion. 3 level ions are more complicated.

- $\frac{g_k}{g_j} \exp\left(-\frac{h\nu_{ju}}{kT} \right) \rightarrow$ Boltzmann equation
- $\dfrac{b_2}{b_1} \rightarrow$ "b correction", since we can't assume Boltzmann level populations

12.2.3 Results

-156- See figures on handout (page -157-).

Some main points: two groups of coolants

1. meta-stable fine-structure levels in ground. Spectroscopic term of various ions.

[*]Spitzer, page 100

- $E_{ex} < 0.1$ eV $\rightarrow$ IR radiation

examples in Fig:

$$\begin{array}{lll} [\text{OIII}] & ^3P_0 -^3 P_1 & 88.4\ \mu\text{m} \\ & ^3P_0 -^3 P_2 & 32.7\ \mu\text{m} \end{array}$$

Weak T_E dependence

2. meta-stable other spectroscopic terms with excitation energies $\gtrsim 1$ eV, giving rise to optical and UV lines. Strong T_e dependence of course! Act as thermostat $\rightarrow$ will keep $T_e \sim 10,000$ K in HII regions.

Note: Figure is for (arbitrary) assumption that O, Ne, N are 80% singly and 20% double ionized. H is 0.1% neutral.

Net effective heating rate G-L_R in the fig. is what we have been calculating (where G is photoionization and L_R is recombination emission for H and He).

The intersection between heating and cooling gives T_E

Optical depth τ_0 in figures refers to *distance* from star. It is the optical depth at the ionization limit of HI, so it is proportional to N(H).

-158- **Outer parts of nebula are hotter!** This happens because the photoionization cross-section is proportional to ν^{-3}, so higher energy photons are absorbed *later*. T_E decreases at first, because $U_{s\nu}$ falls. Then, beyond $r = 0.6R_S$, T_E increases and is higher near $r = R_0$. Finally, it decreases again in the transition region where the ionized fraction drops to zero.

The second figure shows what happens if Ne is large enough that some excited levels of heavy ions are collisionally de-excited; T_E increases, since cooling is less effective.

ϵ_{ff}, the free-free loss rate, follows directly from (3-56) in Spitzer; it is not very important.

How fast does T change when $T \neq T_E$? Close to T_E,

$$t_T \approx \frac{2 \times 10^4}{n_p} \quad [\text{years}]$$

Compare to recombination time:*

$$t_r = \frac{1}{n_e \alpha} = \frac{1.54 \times 10^3 \sqrt{T}}{Z^2 n_e \phi_2(\beta)} \quad \text{[years]}$$

(It would take 10^4 years for recombination to occur if the star in the HII region disappeared). So for $T \sim 10^4$ K, $t_R \gtrsim t_k \rightarrow$ cooling is faster than recombination. (Also, cooling time is much larger for $T \gg 10^4$ K, as we discussed before.)

-159- Consider the handout (cooling and heating in HII regions, taken from Osterbrock). What is missing in these diagrams?

- Cooling by free-bound and bound-bound HI and He? No; since each recombination is balanced by an ionization, these photons, even though they may well escape from the nebula, do not draw net heat from it. Rather, they draw heat from the star itself, not the nebula.†
- photon $h\nu$ absorbed $\rightarrow$ ionizes atom $\rightarrow$ creates electron with (kinetic) energy $\frac{1}{2}mv^2 = h(\nu - \nu_0) \rightarrow$ electron thermalizes its energy with ions and electrons and sets up temperature T_e.
- However, each electron recombines from energy $\frac{1}{2}mv'^2$, which produces photons with a total energy $h\nu' = \frac{1}{2}mv^{-2} - h\nu_0$

The *net* energy (or heat) gain from the photoionization is given by $\frac{1}{2}mv^2 - \frac{1}{2}mv'^2$ (Notice that in general, $|v'| < |v|$). This is to be balanced against the *cooling*, which is predominantly due to emission from collisonally excited heavier elements (in HII regions).

Pure H in the ISM: not many photons around; HII regions are fully ionized. Gas not generally in both phases, e.g. 90% neutral and 10% ionized.

---

*Spitzer 6-11

†Draine does not agree? He includes a recombination cooling rate in Fig. 27.2, 27.3; it is low. He keeps it in heating too though... so okay, fine. Whatever.

12.3 HI gas

-160- (Draine, ch. 30)

Not as simple as one might believe...

Simple considerations HI neutral → few free electrons to share heat. Only elements with ionization potential (IP) < 13.6 eV will be ionized (and dust grains, but more on that later). However, there is still a dominant cooling line from CII (IP of C is 11.26 eV). So:

- $\frac{\Gamma_{ei}}{n_e}$ down by factor of $\sim 1/2000$
- $\frac{\Lambda_{ei}}{n_e}$ similar below $T \sim 1000$ K

HI is naturally cool

Problem: We observe some very warm HI. General solution may well depend on T_e, n, chemical composition (i.e. depletion)...

We will just list the general players.

12.3.1 Cooling function

Cooling function Λ:
- neutral atoms
- ions
- molecules

Excitation sources:
- electrons
- H atoms

-161- Some sources in particular:

1. CII and SiII excitation by collisions with H. Problem: *depletion*
2. Excitation of HI, [exp.?] $n = 2$ level in warm HI. Generally, $n = 2$ is not strongly populated, but if it does happen, it will cool:
   - Lyα photon → dust → IR
   - $\Lambda_{eH} = 7.3 \times 10^{-19} n_e n(HI) \exp(-118,400/T)$ [erg cm^{-3} s^{-1}] ($T = h\nu/k = 118{,}400$ for a 1216Å photon).

   Fate of Lyα photons: Scatter until absorbed by dust, can't do 2-photon emission, level 1 → 2p... or something.

3. H_2 molecules: gain or loss source
   - loss: excitation of rotational levels
   - gain: photon pumping of upper rotational levels, followed by collisional de-excitation. Also other molecular lines may cool (CO, CN, CH,. . .)
4. Collisions with dust grains (can both heat and cool): Spitzer's fig. 6-2 (page -172- in notes) shows cooling function, including HI and H^+ range. Generally, cooling time:

$$t_T \approx \frac{2.4 \times 10^5}{n_H} \quad \text{[years]}$$

longer than for HII regions (not many lines available).

12.3.2 Heating function

Very poorly known. Γ_{ei} from ionizing elements such as C is best known (i.e. photo-electric heating).

-162- However, using only CII heating would produce $T_E = 16$ K (Spitzer); clearly too cool. Adding metals is not enough

Other potential players

(a) Cosmic ray ionization of H, no radiation, but this could do it too.
(b) Formation of H_2 molecules on grains. This releases 4.48 eV. Goes into:
   - heating grain
   - overcome energy of adsorption to grain surface
   - excitation of new H_2 molecule
   - translational kinetic energy that H_2 molecule gets as it leaves the grain.
(c) Photoelectric emission from grains. What is efficiency, as a function of λ? Clearly, complicated problems; see Draine for a discussion.

How can HI become 6000K? Cooling is 10 times more efficient (from figure on page 172: $\frac{\Lambda}{n_H}$) than for cool HI. But, if n_H is lower, then:

- cosmic ray heating more efficient, but still problematic
- also grain - photoelectric heating

12.4 Few comments/additions from Spitzer

See figure 6.2, which sketches what happens at lower T (below 10^4K). Note that Spitzer talks about $\dfrac{\Lambda}{n_H^2}$

-166- Why are there two HI phases?

-167- The "destruction" of the hot phase happens (only) through cooling of the gas. This cooling depends critically on T_e and n_e, as we will next explore.

-168- Heating and cooling of hot gas: