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1. Introduction

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Discuss general viewpoint: What can happen to an atom or molecule or dust grain sitting in the ISM?

- Can it absorb a photon? Energy levels $\leftrightarrow h\nu$ (Need cross-sections for dust grains)
- Can it collide with other particles? \rightarrow collisional rates [$\text{s}^{-1} \text{ cm}^{-3}$]
- Is there a magnetic field? Is the particle charged?
- Are there cosmic rays? These can penetrate dense gas.

Three possible sources of ionization (and excitation):

1. photons
2. collisions
3. cosmic rays

Typical collisional energies: kinetic energy. Translate $mv^2 \rightarrow kT \rightarrow h\nu \rightarrow eV$.

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Dust grains also occur in the neutral medium, and probably also in the (warm) ionized medium. Dust grains play an important role in various processes:

- extinction of starlight
- emission of absorbed energy in FIR
- formation of molecules often occurs on grain surfaces
- absorption of ionizing UV radiation and Ly α photons (reducing amount of ionizing radiation)
- heating of HI gas by *photoelectric* emission

Composition: carbon and silicates. Typical sizes: 0.01 - 0.1 μm (How do we know? \rightarrow shape of extinction curve). Grains as small as ~ 60 atoms across discovered; evidence from emission lines in NIR and excess emission at 5 - 40 μm over what is expected from dust in the ISM. The larger dust grains have temperatures between 10 and 40 K, while the small ones can be heated to higher temps due to the absorption of even a single photon (smaller heat capacity, as volume $\propto r^3$). A promising candidate for small dust grains: polycyclic aromatic hydrocarbons (\sim car soot!)

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In hot environments, dust grains may be destroyed by *sputtering*, where collision of grains with other atoms, electrons, or molecules knock molecules off the grains. At low temperatures, molecules stick to dust grains, causing depletion of heavy elements along certain lines of sight (most dust in the plane). Dust contributes about 1% of the mass of the ISM in the solar neighborhood, mostly in the form of large grains.

Ionized gas

- a) Photoionization: especially effective near hot stars. Shock ionization

b) Cosmic rays: can occur throughout most of the ISM, so can also produce a small amount of ionization in denser gas (though recombination happens quickly, so not much of gas is in an ionized state at any given time).

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c) Collisions Hot ($\sim 10^7$ K) expanding bubble sweeps up shell of warm ($\sim 10^4$ K) ionized gas, which shows a different optical spectrum than HII regions. The hot gas shows up by:

- Free-free and line x-rays
- absorption lines of highly ionized species toward bright UV sources

Kirchoff's law: apparently can still see absorption lines when looking through a gas that is hotter than the source behind it!

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Magnetic fields and cosmic rays In the solar neighborhood, $\mathbf{B} \sim 2 - 5 \times 10^6$ Gauss. This follows from measurements of *Faraday rotation*, giving $\langle n_e B_{\parallel} \rangle$ toward pulsars and radio sources. The random component of the B field is probably as large as the uniform component. In clouds, the B field can be much higher, $\sim 70 \mu\text{G}$ (from Zeeman effect splitting measurements).

The magnetic field is important for several reasons:

1. It aligns elongated grains, giving rise to polarization of starlight
2. Causes relativistic electrons to emit synchrotron radiation, and most likely plays a role in accelerating electrons to relativistic velocities ("magnetic bottle", Fermi acceleration).
3. Provides pressure support against gravitational collapse of matter since it is frozen into the matter due to ionization heavy elements. It also seems to play an important role in solving the angular momentum problem in star formation.

Shu et al.

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Total energy density of cosmic rays in solar neighborhood: $U_R \sim 1.3 \times 10^{-12}$ erg cm^{-3} . Why are cosmic rays important?

- They produce γ -rays through collisions with atoms and molecules. The observed γ -ray intensity from the ISM forms an excellent independent measure of the total amount of matter between stars. For example, calibrating the conversion factor of CO line intensity to H_2 mass.
- Provide pressure against gravitational collapse

Five pressures that play an important role in supporting the ISM against gravitational collapse:

1. thermal $P = nkT$
2. magnetic $P = \frac{B^2}{8\pi}$
3. turbulent (bulk motion)*
4. cosmic ray
5. radiation

* The cloud to cloud velocity dispersion due to turbulence on various scales increases line widths [over?] thermal widths. From the table on page -12-, you will note rough thermal pressure equilibrium between the components. This is not a coincidence; in fact, some of this information was inferred by assuming pressure equilibrium. The argument is that if there were no equilibrium, the resulting perturbations would be wiped out on sound-crossing time scales, which are short compared to the time scales we would consider the ISM to evolve over.

However, the actual evidence for equilibrium in the thermal pressure is scarce, and there are claims that it is not true in the very local ISM*.

There seems to be a “cosmic conspiracy”: the estimates for the thermal, magnetic, and cosmic ray pressure for the solar neighborhood give roughly equal numbers for all three. Thus it may be inappropriate to only consider the thermal pressure (the only one that can be measured with much certainty). Interestingly, the magnetic pressure number is also very similar to the energy density of the CMB. ^{† ‡}

Next section is sort of a shortened condensation of Draine’s chapters 2 and 3. We may come back to specific topics discussed there in more detail.

2. Validity of the laws of statistical physics in ISM conditions

Four major laws of statistical physics:

1. **Maxwellian** velocity distribution
2. **Boltzmann distribution** of energy levels in atoms and molecules
3. **Saha equation** for ionization equilibrium
4. **Planck function** for radiation

Maxwellian T is defined by motion of particles. ($\vec{\omega}$ = velocity = \vec{v} in Draine). $f(\vec{\omega})d\vec{\omega}$ = fractional number of particles whose velocity lies within the three-dimensional volume element $d\vec{\omega} = d\omega_x d\omega_y d\omega_z$, centered at velocity $\vec{\omega}$.

In thermodynamic equilibrium, $f(\vec{\omega})$ is isotropic, so $\vec{\omega} \rightarrow \omega$.

$$f(\omega) = \frac{\ell^{3/2}}{\pi^{3/2}} \exp(-\ell^2 \omega^2); \quad \ell^2 = \frac{m}{2kT} = \frac{3}{2 < \omega^2 >}$$

$$f(\omega) = \left(\frac{m}{2\pi kT} \right)^{3/2} \exp \left(-\frac{m\omega^2}{2kT} \right)$$

*Bowyer et al. *Nature* 1905

[†]Draine discusses possible reasons in section 1.3

[‡]Supplemental info in Draine chapter 1.

-17- For two groups of particles with different masses, we replace ω by u , the relative velocity between the two groups, and m by the reduced mass $m_r = \frac{m_1 m_2}{m_1 + m_2}$. For H atoms colliding with particles of mass $A m_H$, Spitzer derives:

$$\langle u \rangle = \left[\frac{8kT}{\pi m_r} \right]^{1/2} = 1.46 \times 10^4 \sqrt{T} \left(1 + \frac{1}{A} \right)^{1/2} \text{ [cm s}^{-1}\text{]}$$

... Speed vs. velocity

The Maxwell velocity distribution is characterized by several speeds:

- Most probable speed: $\omega_o = \sqrt{\frac{2kT}{m}}$
- RMS speed: $\langle \omega^2 \rangle^{1/2} = \sqrt{\frac{3kT}{m}}$
- RMS velocity in one direction: $\langle \omega_x^2 \rangle^{1/2} = \sqrt{\frac{kT}{m}}$

-18- **Boltzmann distribution** gives the population of energy levels in an atom or molecule:

$$\frac{n_u}{n_l} = \frac{g_u}{g_l} \exp[-(E_u - E_l)/kT] = \frac{g_u}{g_l} \exp[-h\nu_o/kT]$$

where $n_{u,l}$ are the number densities, $g_{u,l}$ are the statistical weights, and $E_{u,l}$ are the energies of the levels. (Partition function: summing over all energy levels... chemical term).

Saha equation describes ionization equilibrium:

$$\frac{n_{i+1}}{n_i} = \frac{g_{i+1} g_e}{g_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} \exp(-I/kt)$$

where I is the ionization potential for an ion in the ground state and initial ionization state i (aka, the energy required to ionize from i to $i+1$). $g_e = 2$ (two spin conditions). Electrons affect whether and how easily atoms can be ionized.

Planck function specifies the radiation field: *

$$\begin{aligned}
 B(\nu) &= \frac{2h\nu^3}{c^2} \frac{1}{\exp[h\nu/kT] - 1} \\
 &= \sim \frac{2\nu^2}{c^2} kT \quad \text{for } h\nu \ll kT \text{ (Rayleigh - Jeans)} \\
 &= \sim \frac{2h\nu^3}{c^2} \exp[-h\nu/kT] \quad \text{for } h\nu \gg kT \text{ (Wien)}
 \end{aligned}$$

3. Statistical equilibrium

-19- **The four laws discussed above hold under thermodynamic equilibrium (TE).** However, this is not often the case for the ISM. Thermodynamic equilibrium requires **detailed balancing**, i.e. each process is as likely to occur as its inverse. Example: Consider the 3727 Å emission from O⁺. This is a forbidden transition (actually a doublet). The excitation of the electron level occurs through collisions with electrons, in most conditions in the ISM. If detailed balancing were to hold, de-excitation should also occur by collisions. However, as we will see, under the low density conditions found in the ISM, collisions are rare, and de-excitation is more likely to proceed through emission of a photon, in spite of the fact that we are dealing with a forbidden transition. Thus [OII] emission can be quite strong, and by converting collisional (kinetic) energy into radiation, we actually have created a cooling mechanism for the gas.

Another reason why TE does not hold is the strong **dilution** of the radiation field: the concept of a dilute radiation field is quite familiar. For example, the sun's photosphere is ~ 6000 K, and at the surface the flux leaving the sun is approximately that of a blackbody of this temperature. However, the Earth is not 6000 K because by the time the radiation reaches us, it is diluted. **A diluted radiation field is one in which the energy density does not match the color temperature.**

-20- For the solar neighborhood, the total energy density of the radiation field due to all stars in that volume is about 1 eV cm⁻³ (close to cosmic ray density, as mentioned before). When interpreted as an average temperature using the Stefan-Boltzmann law (energy density of a blackbody, $u = aT^4$), this energy density implies an equivalent temperature of ~ 3 K. Yet the color temperature implied by the shape of the spectrum of this Interstellar Radiation Field (ISRF) is that of A and B stars ($T \sim 10^4$ K). So there is a **dilution factor** W given by:

$$W \approx \left(\frac{3}{10^4}\right)^4 \approx 0.25 \times 10^{-14}$$

We conclude that using the Planck law to describe intensities is not correct.

* $e^x \approx 1 + x$ for $x \ll 1$
 $e^{h\nu/kT} - 1 \approx 1$ for $h\nu \ll kT$
 $e^{h\nu/kT} - 1 \approx e^{h\nu/kT}$ for $h\nu \gg kT$

What about the other laws?

1. Maxwell velocity distribution Good news! It is generally valid. Detailed balancing is possible for the elastic collisions that are generally occurring. Because the maxwellian distribution is a good description of the motions of the particles, we can define a kinetic temperature which describes the physical condition of the gas. Often, for a plasma, the kinetic temperature is equal to the electron temperature: $T_{ions} = T_e$. $T_{ions} \neq T_e$ may occur behind shocks.

-21- **2. Boltzmann distribution** is rarely correct. If excitation and de-excitation occurs by photons, we may still not have a Boltzmann distribution because the photon distribution is not given by the Planck function. Often we do not even have detailed balancing. However, as we will see, sometimes the distribution of excited levels is not too different from Boltzmann distribution. This happens when collisions dominate excitation and de-excitation, while radiation is relatively unimportant.

To describe situations close to TE, Spitzer introduced the so-called b-factors (Draine calls them “departure coefficients”).

$$b_j \equiv \frac{n_j(\text{true distribution})}{n_j(\text{LTE distribution})}$$

Example: in an HII region, the highest excited levels of HI have $b_j \sim 1$. Some radiation does escape (producing radio recombination lines) but collisions dominate the level populations. Since motions of particles *are* described by a Maxwellian velocity distribution, whenever collisions dominate the level population they will closely follow a Boltzmann law.

-22- In general, a Maxwellian velocity distribution tends to set up a Boltzmann population for energy levels in the atoms/particles *if* transitions resulting from emission and absorption of photons are relatively unimportant, and collisional (de-)excitation is dominant. In the case of the highly excited levels in H mentioned before, collisions with electrons are dominant.

3. Saha equation is generally not valid. There is no detailed balancing, and even though the ionization and recombination processes are each other’s inverse, the ionization process is determined by the photon field in most cases, while the recombination process is determined by collisions between A^+ and e^- . The collision rate depends on $\{n_e, n_{A^+}\}$ and T_e , but the ionization is dependent on $T_{radiation} (\neq T_e)$.

So in general, assume **statistical equilibrium**, where there is a balance between transitions one way and the other way, no matter what process caused each transition.

In level i , we have n_i atoms cm^{-3} , and R_{ij} is the rate coefficient such that

$$n_i R_{ij} [\text{s}^{-1}] = \# \text{ transitions from level } i \text{ to level } j$$

$R_{ji}n_j$ [s⁻¹] = # transitions from level j to level i

$$\frac{dn_i}{dt} = \sum_j (-R_{ij}n_i + R_{ji}n_j); \quad i = 1, 2, \dots$$

-23- In statistical equilibrium, $\frac{dn_i}{dt} = 0$. The rate factor R_{ij} includes *all* possible processes that would take the atom or molecule from level i to j . In the worst case, you would have to include many processes to calculate the n_i values. This requires knowledge of a lot of physical input parameters, e.g. cross sections for particular processes, collisional rate coefficients *, etc.

In other cases, where only one or two processes matter, the situation can be very simple. We will encounter cases of each.

Before going more into Ch 2 and 3 in Draine, we will first discuss some basic radiative transfer.[†]

4. Radiative Transfer

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Flux at surface of a sphere:

Flux at a distance r :

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$F_\nu = \pi B_\nu$ for blackbody

$F_\nu(r) = \pi I_\nu \left(\frac{R}{r}\right)^2 = \frac{L_\nu}{4\pi r^2}$

$F_\nu = \pi I_\nu$ for isotropic emitting non-blackbody

where R = radius of body and L_ν = luminosity of body [erg s⁻¹ Hz⁻¹]

4.1. Energy density of radiation

$$u_\nu = \frac{1}{c} \int I_\nu d\Omega \text{ [erg cm}^{-3} \text{ Hz}^{-1}\text{]}$$

4.2. Radiation pressure

$$P_\nu = \frac{1}{c} \int I_\nu \cos^2 \theta d\Omega$$

*Drane section 2.1

†RL Ch 1, Draine Ch 6, 7

4.3. Emission and absorption coefficients

4.3.1 Emission

- j_ν [erg cm⁻³ sec⁻¹ Ω⁻¹ Hz⁻¹] spontaneous emission coefficient
- $[j_\nu]$ = volume emission coefficient
- $j'_\nu = \frac{j_\nu}{\rho}$ [erg g⁻¹ sec⁻¹ Ω⁻¹ Hz⁻¹] = mass emission coefficient
- $j_\nu = \frac{\epsilon_\nu}{4\pi}$ (isotropic emitter); ϵ_ν = emissivity [erg cm⁻³ s⁻¹ Hz⁻¹]

4.3.2 Absorption

- κ_ν [cm⁻¹] volume absorption coefficient (includes *stimulated emission*, aka “negative absorption”).
- $\kappa'_\nu = \frac{\kappa_\nu}{\rho}$ [g⁻¹ cm²] = mass emission coefficient
- microscopically: $\kappa_\nu = n\sigma_\nu$

Loss of intensity in a beam of light as it travels distance ds :

$$dI_\nu = -\kappa_\nu I_\nu ds$$

The absorption process refers to the sum of “true absorption” + stimulated emission, (\sim net emission). It removes a *fraction* of the incoming radiation.

mean free path: ℓ_ν $\tau_\nu = \kappa_\nu \cdot \ell_\nu$, so for condition $\tau = 1$, $\ell_\nu = \frac{1}{\kappa_\nu} = \frac{1}{n\sigma_\nu}$

5. Radiative transfer equation

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu$$

Define optical depth (unitless):

$$\tau_\nu = \int \kappa_\nu ds$$

then

$$d\tau_\nu = \kappa_\nu ds$$

or

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{\kappa_\nu} \equiv S_\nu$$

where $S_\nu = \text{source function}$

- $\tau \gtrsim 1$ optically *thick* emission
- $\tau < 1$ optically *thin* emission

Formal solution:

$$I_\nu(\tau_\nu) = I_\nu(o)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S(\tau'_\nu) d\tau'_\nu$$

-27- \rightarrow attenuated incoming beam + contribution from gas itself.

Special cases

1. Source function is constant throughout source: $I_\nu(\tau_\nu) = I_\nu(o)e^{\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$
 - optically thick emission: $I_\nu = S_\nu$
 - optically thin emission: $I_\nu = I_\nu(o)(1 - \tau_\nu) + \tau_\nu S_\nu$
2. Thermal radiation: $S_\nu = B_\nu(T)$ (The Planck function)
 - optically thick emission: $I_\nu = B_\nu$
 - optically thin emission: $I_\nu = \tau_\nu B_\nu$

For *radio* emission: I_ν is replaced by the brightness temperature, T_b , defined as:

$$I_{\nu,\text{obs}} \equiv B_\nu(T_b)$$

In the Rayleigh-Jeans limit for B_ν we get:

$$I_{\nu,\text{obs}} = \frac{2\nu^2 k T_b}{c^2}$$

or

$$I_b = \frac{I_\nu c^2}{2\nu^2 k}$$

Solution of the transfer equation in terms of T_b :

$$T_{b,\text{obs}} = T_{b,o}e^{-\tau_\nu} + T(1 - e^{-\tau_\nu})$$

-28- where

- T is the thermal source temperature (*physical* temperature of the layer)
- T_b is the brightness temperature of the incident radiation

T_b is never greater than T !

5.1. Einstein coefficients

* *Transition probabilities per unit time.* Three possible processes:

1. Spontaneous emission: Einstein A coefficient
2. Absorption
3. Stimulated emission

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5.1.1 Relations between the Einstein coefficients

Valid under all conditions since they only refer to *atomic* principles (no collisions, just radiation). TE: rate of transitions out of state 1 = rate of transitions into state 1 (per unit volume).

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In TE, apply the Boltzmann law.

5.1.2 Relations between Einstein coefficients and κ_ν and j_ν

emission coefficient

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absorption coefficient The second term here for κ_ν corresponds to *stimulated emission*.

5.2. Line profile function, $\phi(\nu)$

†

5.2.1 Natural line width

Key point: A small Einstein coefficient A results in a *narrow* line.

The natural line width of most transitions is quite small, and broadening due to other effects is more important.

5.2.2 Doppler broadening

- Thermal velocities

*Draine, section 6.1

†See RL chapter 10.6 and Draine 6.4

- Bulk motion (turbulence)

-32- and the profile function is

5.2.3 Collisional broadening

~ Pressure broadening, which is not generally important in the ISM because the density is so low... mostly occurs in stellar atmospheres. This still produces a Lorentzian profile, but with:

$$\phi(\nu) = \frac{4\Gamma^2}{16\pi^2(\nu - \nu_o)^2 + \Gamma^2}$$

Can be written in terms of a *Voigt function*:

-33- So the core of the profile is Gaussian due to Doppler broadening, while the wings are much stronger than expected in a Gaussian profile, due to the intrinsic line width.

6. Atomic H in the ISM

* Wherever HI dominates the ISM, all atoms are found in the **ground state** (²S)(n=1). The next excited level (²P) is about 10 eV above the ground-level. This excitation is very rare and quickly falls back to the ground level, so there is no significant population of this level. For example, consider potential excitation mechanism: collision of H-atom with cosmic ray particle (lots of energy, probability is once per 10¹⁷ seconds), ionizes the H-atom. Recombination results in some atoms winding up in ²P state. But *A* coefficient for spontaneous emission from 2P → 2S is 10⁸ s⁻¹. Hence, this excitation process results in relative population of 2P level of 10⁻⁸/10¹⁷ = 10⁻²⁵!

ISM is too cool for collisions to happen often and cosmic rays are rare. Possible tracers of HI gas:

1. 21 cm HI transition (=hyperfine transition) in emission or absorption.
2. Lyman absorption lines against hot background stars.

Only the ²S level is populated. HI is hard to find in the ground state; fine structure → different angular momentum.

†

6.1. Excitation and radiative transport for the 21-cm line

Spin of proton and electron:

- Parallel (upper energy)
- Anti-parallel (lower energy)

(Spin is around particle's own axis, not to be confused with angular momentum). Motions specified by maxwellian velocity distribution, and collisions dominate the level populations (excite and de-excite).

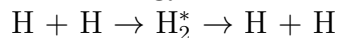
Energy difference (very small):

$$\begin{aligned} h\nu &= 9.4 \times 10^{-18} \text{ erg} \\ \nu &= 1420.4 \text{ MHz} \\ \lambda &= 21.11 \text{ cm} \end{aligned}$$

Spontaneous emission probability is *very* small:

$$\begin{aligned} A_{kj} &= 2.86 \times 10^{-15} \text{ sec}^{-1} \\ \rightarrow \text{lifetime} &= 1.10 \times 10^7 \text{ years} \end{aligned}$$

More frequently, atoms will flip energy states by *collisions*. These dominate transitions and cause energy levels to flip.



excited H_2 molecule (not stable). Chance for collisional excitation per second: $\gamma n(H)$ [s^{-1}].

$\gamma [\times 10^{-11} \text{ cm}^3 \text{ s}^{-1}]$	T [K]
0.23	10
3.0	30
9.5	100
16	300
25	1000

As long as $n(H^o) > 10^{-2} \text{ cm}^{-3}$, $\gamma n_H \gg A$, so collisions determine excitation and de-excitation. . . implies detailed balancing, so Boltzmann distribution is valid for level populations.

$$\frac{n_1}{n_o} = \frac{g_1}{g_o} \exp(-h\nu/kT_k) \approx \frac{g_1}{g_o} \left(1 - \frac{h\nu}{kT_k}\right) \approx \frac{g_1}{g_o}$$

where T_k = kinetic gas temperature.

Derivation:

Now assume that. . .

*Draine Ch 8, 29; Ch 17.1, 17.3

†See Draine Ch 4 (& 5) on notation of energy levels and atomic structure.

In general, in situations where stimulated emission and absorption can be neglected, we would obtain

$$\frac{b_1}{b_o} = \frac{1}{1 + \frac{A_{10}}{n_H \gamma_{10}}} \approx 1$$

Note: Potential other excitation process of upper level for HI: HI + Ly α photon \rightarrow n=2 level (2P). 2P level might cascade down to upper 2S hyperfine level.

In principle, one might selectively populate the upper level with this so-called photon pumping*. It turns out that, due to thermalization of the photons this anomalous population of the levels does not generally occur. (Ly α photons frequently scatter off A-atom again = LTE situation).

-37- Since the b's are effectively 1, our **main result** is that

Are the HI levels always populated mainly by collisions? No, if n(H^o) drops low enough, and if the HI is warm it doesn't work as well.

The warm HI in our Galaxy has $n_H \sim 0.4 \text{ cm}^{-3}$ and $T_k \approx 8000 \text{ K}$. But T_s = spin temperature, of order $T_k/5$, if it weren't for excitation by Ly α photons.

But it is true that the 3K background radiation field does *not* significantly disturb the equilibrium set up by collisions, as we discussed above.

If n_H drops to very low values, collisional excitation is ineffective. But even in that case, $T_s \approx T_k$ because of the above mentioned Ly α excitation.

In any case, in general we now obtain for the emission coefficient:

which is independent of T in most circumstances, and for the absorption coefficient:

-38- Using the relation between Einstein coefficients:

6.2. Consider simple case of a single layer of gas

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*D 17.3

Consider two cases

- (i) $\tau_{\nu_o}(L) \gg 1$ Optically thick
- (ii) $\tau_{\nu_o}(L) \ll 1$ Optically thin

In general, the line profile of HI emission is entirely determined by the velocity of the atoms, so the assumption of a Gaussian profile is correct.

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6.3. Observing brightness temperature

Aside: How do we observe HI?

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6.4. HI emission and absorption

Distinguish two cases:

1. Absorption by an HI cloud of an extended background source, usually a continuum source (e.g. AGN)
2. HI self-absorption (enough of it at same velocity)

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- $T_{b,\text{off}} = T_s(1 - e^{-\tau_\nu})$ in the direction immediately next to the source
- $T_{b,\text{on}} = T_{bo}e^{-\tau_\nu} + T_s(1 - e^{-\tau_\nu})$ directly toward the source

Some practical points

1. Frequency *band* in a number of channels, together spanning a range in velocity
2. The assumption here is that the
3. A way to
4. Do we really have only *one* HI cloud along the LOS, and is its temperature uniform?

-43- We observe at each frequency

$$T_{b,\text{off}} \leq T_s$$

-44- Discussion of some results Plots

-45- Conclusions:

1. The absorption profile $(1 - e^{-\tau_\nu})$ is always sharper and simpler in structure than the emission line profile.
2. The variation in spin temperature, T_s , makes one wonder whether there is really only one temperature in the cloud.
3. The range of T_s one finds from absorption line studies is $50 \text{ K} \leq T_s \leq 1000 \text{ K}$. However, the upper value is only a lower limit since we don't see absorption anymore for $T_s > 1000 \text{ K}$.
4. One can compare

Some relevant results:

1. HI is *not* concentrated in a small number of giant clouds, as H₂ is; estimates of filling factor range from 20-90%?!
2. HI occurs roughly 50/50 in two important forms:
 - CNM = cold neutral medium $\sim 80 \text{ K}$
 - WNM* = warm neutral medium $\lesssim 8000 \text{ K}$
3. Clouds are filaments and/or sheets, rather than spheres
4. At low galactic latitudes, it is not possible to distinguish between WNM and CNM because there is too much material along the line of sight.
5. Problem with different temperatures along the line of sight. Then:

Special cases:

1. $\tau_1 \gg 1$
2. $\tau_1 \ll 1$
3. $\tau_1 \ll 1, \tau_2 \ll 1$

Some additional points on HI[†]

- HI measurements toward Galactic continuum sources provide (some) information on the distance to these sources, using the velocity of the HI absorption feature to derive a kinematical distance. The usual problem of distance ambiguity may still be a problem here as well, although the total HI column [density?] that is derived from the absorption measurements helps a little in distinguishing near and far distances.
- **Temperature of WNM**... two methods:
 1. HI absorption
 2. UV absorption lines

So far: data in agreement with $5000 \text{ K} \lesssim T \lesssim 8000 \text{ K}$, but needs more confirmation.

- **Temperature of CNM**... from absorption spectra. The absorption lines are *narrow*.

Present research on Milky Way HI concentrates on:

1. Origin of high velocity clouds
2. Properties of smallest clouds/features

*Meibold, 1972

†Kulkarni & Heiles

3. Search for HI clouds that may contain dark matter, but no stars

-50a, Images and plots.
50b,
50c-
-I1-

7. Atomic structure

7.1. Introduction

7.2. Hydrogen atom & hydrogen-like atoms (or ions)

-I2- Example: energy of ground states:

- $H = 13.6 \text{ eV}$
- $He^+ = 54.4 \text{ eV}$
- $Li^{2+} = 122.5 \text{ eV}$

-I3-
-I4-
-I5-
-I6-

7.3. Electron spin

-I7- But, since \vec{J}_1 and \vec{J}_2 can have different directions, there are different possible values of \vec{J} .

-I8-

7.4. Spin orbit coupling

Two possible orientations of electron spin with respect to orbital angular momentum lead to a doubling of energy levels of H-like atoms (except s -levels, where $\ell = 0$).

Lines appear in pairs, close together, called doublets. Example: for Na D lines, $\lambda = 5890\text{\AA}$, 5896\AA . (Question: are atoms with filled shells + one electron also hydrogen-like? Doubling is due to spin-orbit coupling.)

Physically, the spin-orbit coupling produces an extra energy term for the electron, proportional to $\vec{S} \cdot \vec{L}$.

-I10-

7.5. Atoms with multiple electrons

7.6. Transition rules

7.7. X-ray emission

7.8. Zeeman effect

8. HII regions

8.1. Introductory remarks

Process: hot OB stars emit UV photons that can ionize the surrounding neutral H (and He) medium. In practice, this requires stars hotter than $\sim 30,000$ K, aka. spectral type B0 or earlier.

Physics for planetary nebulae is similar, but central stars may be much hotter (though they are also dimmer because there is “more stuff”).

UV photons impart energy to gas by ionizing H and He. Excess kinetic energy of created free electrons is shared with ions and other electrons, heating the medium. Relevant processes:

- photoionization
- electron-electron encounters
- electron-ion encounters
 - Excited ions
 - Recombination
 - Bremsstrahlung

H atom + photon $h\nu \longrightarrow p^+ + e^- + E_k$

Since stars create a continuous stream of photons, a balance is reached between ionization and recombination. * †

Field OB stars: were they born there or travel there somehow? Usually in groups (OB associations), dense clouds, etc.

Consequences: $\sim 10^4$ K. Warm ionized plasma emitting various forms of radiation: recombination, free-free continuum, free-bound continuum, 2-photon continuum, collisionally excited forbidden lines from “metals”. If there is dust around, get MIR dust continuum. Runaway OB stars (like teenagers!)

1 km \equiv 1 pc in 10^6 years. (?)

Q_o [s^{-1}] = number of ionized photons emitted by OB star per second. Factor of 100 from O 9.5V to O 3V (luminosity classes).

*Draine Ch 10, 13.1, 14, 15, 17, 18, 27, 28

†Osterbrock AGN²

8.2. Strömgren Theory

“It’s only a model.”

Classical paper*

Basic result:

- Hot star in a uniform medium will ionize a spherical volume out to a certain radius, whose size is determined by:
 1. Number of ionizing photons emitted by the star
 2. ρ of medium (determines recombination rate)
- There is a *sharp* boundary from the ionized to the surrounding neutral medium.

8.3. Simple derivation

Pure H nebula, uniform density. Hot star emits N_{Lyc} photons per second (Lyc = Lyman continuum), all at the *same* frequency, ν_o . The star will ionize the gas, but there is a balance:

ionization \longleftrightarrow recombination

Stationary ionization equilibrium characterized by *degree of ionization*:

$$x(r) \equiv \frac{n_e(r)}{n_H}$$

where n_H = *total* hydrogen density (neutral and ionized). $0 \leq x \leq 1$ ($x = 1 \rightarrow$ complete ionization; $x = 0 \rightarrow$ completely neutral). Problem: what is the shape of $x(r)$?

Important quantity: the Strömgren radius, R_{SO} , defined as

$$\frac{4}{3}\pi\alpha n_H^2 R_{SO}^3 \equiv N_{Lyc}$$

where α is the recombination coefficient (collisional process $\sim \rho^2 \rightarrow$ more encounters, see 8.3.2). The quantity N_{Lyc}^\dagger gives the total number of recombinations per unit time (all the magic is here!).[‡]

-53-

8.3.1 Description of photoionization equilibrium

Overview: Consider pure H nebula surrounding a single hot star. Ionization equilibrium:

$$n_{H^o} \int_{\nu_1}^{\infty} \frac{4\pi J_\nu}{h\nu} \sigma_\nu d\nu = n_e n_p \alpha(H, T)$$

*Strömgren 1939 ApJ 89, 526

[†] $N_{Lyc} = S_4$ in Spitzer’s notation.

[‡] S_4 formally should include contributions from the diffuse radiation field.

where

- n_{H^o} = neutral hydrogen density
- J_ν = mean intensity of radiation field
- σ_ν = ionization cross-section for H by photons with energy above threshold $h\nu_{01}$
- n_e, n_p = electron and proton densities
- $\alpha(H, T)$ = recombination coefficient
- LHS: number of *ionizations* per second per cm^3
- RHS: number of *recombinations* per second per cm^3

To first order, not including radiative transfer effects:

$$4\pi J_\nu = \frac{R^2}{r^2} \pi F_\nu(0) = \frac{L_\nu}{4\pi r^2}$$

(Local radiation field.) Order of magnitudes:

$$N_{Lyc} = \int_{\nu_1}^{\infty} \frac{L_\nu}{h\nu} d\nu$$

= number of Lyman continuum photons emitted per second.

- $N_{Lyc} \approx 5 \times 10^{48} \text{ sec}^{-1}$ for O6 star
- $\sigma_\nu \approx 6 \times 10^{-18} \text{ cm}^2$
- $\alpha(H, T) \approx 4 \times 10^{-13} \text{ cm}^3 \text{ sec}^{-1}$

-54- Physical conditions: radiative decay from upper levels to $n = 1$ is quick \rightarrow nearly all neutral hydrogen will be in the ground level. Photoionization takes place from the ground level, and is balanced by recombination to excited levels, which then quickly de-excite by emission of photons.

The *photoionization cross-section* ($\sigma \propto \nu^{-3}$) is actually rather complicated to calculate. Spitzer (who calls this erroneously the “absorption coefficient”) gives:

$$\sigma_{f\nu} = \frac{7.9 \times 10^{-18}}{Z^2} \left(\frac{\nu_1}{\nu} \right)^3 g_1 f$$

where $g_1 f$ = Gaunt factor (from level 1 to free)*

Since $\sigma_\nu \propto \nu^{-3}$, higher energy photons will typically penetrate further into the nebula before being absorbed.

8.3.2 the recombination coefficient

$\alpha_n(H, T)$ = recombination coefficient for *direct* recombination to level n .

$$\alpha^{(n)} \equiv \sum_{m=n}^{\infty} \alpha_m$$

*see, e.g. Table 5.1 Spitzer (Draine Ch. 10); see also Fig 13.1 in Draine and Fig. 2.2 in handout (from Osterbrock).

where m = all other levels stepped through to get to n .

Note: Outside of level 1, higher energy not necessarily better for ionization. OIII is not necessarily from the same place as OII. Free electrons: there is a recombination coefficient for every level, depends on velocity of the electron.

-55- The summed recombination coefficient (α) is the relevant quantity because recombination counts, irrespective of the level n to which it happens.

Larger velocities \rightarrow smaller σ .

For a distribution $f(v)dv$, $n_e f(v)v$ = number of electrons with velocity v passing through a unit area per second. Thus

$$\alpha_n = \int_0^\infty v \sigma_n(H, v) f(v) dv$$

where $\sigma_n(H, v) \propto v^{-2}$ = recombination cross-section to the level n for electrons with velocity v .

$$f(v) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT} \right)^{3/2} v^2 e^{-mv^2/2kT}$$

(Maxwellian). Hence, $\alpha \propto \frac{1}{\sqrt{T}}^*$

We are interested in the total recombination coefficient to all levels:

$$\alpha^{(n)} = \sum_{m=n}^{\infty} \alpha_m$$

$\alpha^{(1)}$: summed over all levels. However, for direct recombination to level 1, a photon is generated which itself can ionize H again; so the recombination coefficient that we are interested in is $\alpha^{(2)} \equiv \alpha_B^\dagger$ This refers to “case B”: optically thick through Ly α emission. Almost always here! Most common situation. Case A: very low density gas.

-56- Neutral fraction *inside* HII region ‡ : Consider pure H nebula, and case B recombination (see later). *Mean energy* of stellar ionization photons is $h\nu$.

-57- Rewrite latter as:

so neutral fraction x is small and hence $x^2 \approx 1$.

-58- Comments:

1. Reality: include He, include diffuse radiation field generated from direct recombination to $n = 1$ (which can ionize an atom again!), realistic stellar energy spectrum below $\lambda = 912\text{\AA}$; results are qualitatively the same.

*see e.g. Spitzer, Table 5.2

† see Draine section 14.2

‡ see Draine section 15.3

2. Is assumption of a static ionization equilibrium realistic? No: HII region is over-pressurized compared to the surrounding medium: T_e and n higher \rightarrow HII region expands*. Expansion of the ionization front happens rather slowly, $v \propto 1 - 2 \text{ km s}^{-1}$, dynamical timescale. $\frac{R_{SO}}{v} \sim 10^7$ years, of order lifetime of HII region. So ionization equilibrium does hold.
3. Recombination time scale = ionization time scale:

$$t_{ionization} \equiv \frac{\frac{4}{3}(R_{SO})^3 n_H}{Q_o} = \frac{1}{\alpha_B n_H} = \frac{1.22 \times 10^3}{n_2} \text{ years} = t_{recombination}$$

($n_2 = n_H$ in units of 100). The two are identical! So $t_{recombination} \propto \frac{1}{n_e}$ Key: for $n_H > 0.03 \text{ cm}^{-3}$ $t_{recombination} < \text{lifetime of massive stars} (\lesssim 5 \text{ Myr})$.

4. As we will see later, each Lyc photon results in ~ 0.35 H α photons begin produced in this ionization \rightarrow recombination equilibrium.
5. Next: influence of dust on R_{SO}

8.4. The spectrum of an HII region

-63-

8.4.1 Continuum Radiation

Sources:

- free-free[†] = thermal Bremsstrahlung
- free-bound[‡]
- two-photon decay[§]
- emission by dust particles

Two-photon decay: Transition from $n = 2 \rightarrow n = 1$ ($\ell = 0 \rightarrow \ell = 0$) is strictly forbidden ($\Delta\ell \neq 0$). The actual process happens in two steps, with a virtual intermediate phase. Two photons are emitted whose joint energy adds up to the energy of a Ly α photon = 3/4 ionization energy of H. The two-photon continuum is symmetric about $\nu_{12}/2$ if expressed in photons per unit frequency instead.

*First discussed by Kahn (1954)

[†]§3.5 Spitzer, Drane ch. 10

[‡]See previous footnote

[§]Osterbrock, pp 89-93

Free-free emission: Bremsstrahlung, continuous emission and absorption by *thermal* (Maxwellian velocity distribution) electrons due to *encounters* between electrons and positive ions. *Acceleration/deceleration* → *photons*.

Classical theory: electron emits a single narrow Em pulse in time, with no oscillation in E. → FT is broad, almost the FT of a δ function. → Emission coefficient j_ν is nearly independent of frequency, up to a *cutoff* frequency, which corresponds to the Maxwellian velocity distribution of electrons. So cutoff frequency is given roughly by $h\nu \sim kT_e$. (HII regions: $T_e \sim 10^4$ K, so $h\nu \approx 0.87$ eV and $\lambda \approx 1.4\mu\text{m}$).

In practice, dust emission dominates at IR wavelengths, so it is unlikely to observe the free-free spectrum this far.

Emission coefficient:

$$j_\nu = \frac{8}{3} \left(\frac{2\pi}{3} \right)^{1/2} \frac{Z_i^2 e^6}{m_e^{3/2} c^3 (kT)^{1/2}} g_{ff} n_e n_i e^{-h\nu/kT}$$

$$= 5.44 \times 10^{-39} \frac{g_{ff} Z_i^2 n_e n_i}{\sqrt{T}} e^{-h\nu/kT} \quad [\text{erg cm}^{-3} \text{ s}^{-1} \text{ sr}^{-1} \text{ Hz}^{-1}]$$

where $e^{-h\nu/kT}$ is the exponential cutoff due to kT_e and g_{ff} is the Gaunt factor for free-free transitions*.

Total amount of energy radiated in free-free transitions per cm^3 per second:

$$\epsilon_{ff} = 4\pi \int j_\nu d\nu$$

... (long-ass equation)

Remember that the corresponding absorption coefficient κ_ν is related to j_ν by Kirchoff's law, since we deal with *thermal* emission (*not* blackbody though!) $j_\nu = \kappa_\nu B_\nu(T)$ which leads to [another long-ass equation].

Free-bound emission

8.4.2 Line Radiation

Two kinds:

1. Recombination lines, predominantly from H and He
 - optical
 - radio
 - forbidden
2. Collisionally excited lines in heavy elements

*see Draine, figure 10.1

Recombination lines:

-79- **Collisionally excited lines in heavy elements:** Generally: electrons don't have enough energy to excite ions from the ground state to their first excited states. However, many ions have multiplets in their ground state, due to the coupling of the electrons individual (orbital and spin) angular momenta.

Many ions have incomplete "p-shells"; (there is not really a p-shell, but we talk about electrons which have $\ell = 1$, corresponding to p). Remember:

H 1s
 He 1s²
 C 1s² 2s² 2p² For $n = 2$, there is room for 6 p-electrons, since each $n\ell$ has a number
 O 1s² 2s² 2p⁴
 of m_ℓ and m_s combinations:

$$\left. \begin{array}{l} m_\ell : 2\ell + 1 \\ m_s : \pm \frac{1}{2} \end{array} \right\} 2(2\ell + 1)$$

So as long as $\ell = 1$ is possible (meaning $n > 1$), we have 6 potential "p-shell" electrons, *always*.

The enclosed diagram shows the 3 most typical structures of energy levels for various elements in different ionization stages. For [OIII] we have, in particular: 2 electrons in "p-shell", $\ell_1 = \ell_2$, so $L = 0, 1, 2$

-81- One can set up 5 equations of statistical equilibrium for each of these levels. This shows that both the ¹S₀ and the ¹D₂ levels are populated essentially exclusively by collisions from ³P level.

So, not surprisingly, the ratio

$$\frac{N(^1S_0)}{N(^1D_2)} \propto \exp\left(\frac{-\Delta E}{kT}\right)$$

where ΔE = energy difference between ¹S₀ and ¹D₂

Also, observed flux in $\lambda 4363$ line is:

$$S(4363) \propto \frac{4}{3}\pi R^3 N(^1S_0) A_{4363} h\nu$$

and similarly for the 4959 and 5007 lines.

The net result, obtained by solving the equations of statistical equilibrium is:

$$\frac{S(4959) + S(5007)}{S(4363)} = 8.3 \exp\left(\frac{3.3 \times 10^4}{T_e}\right)$$

so this line ratio provides a direct measure of the electron temperature.

A similar relationship exists for [NII] (6548, 6583, and 5754) lines.

In general: Emission lines arising from ions, such as O^{++} and N^+ that have upper energy levels that have considerable different energies are useful for estimating T_e . Vice versa, as we will see, ions with closely spaced upper energy levels provide little info on T_e , but can be used to derive n_e (examples: [SII], [OII] lines).

8.5. Types of HII Regions

-85- In general:

- HII regions are associated with molecular clouds and dark nebulae.
- Structure:
 - “Blister model” – cavity inside GMC
 - “Champagne model” – half cavity at edge of GMC

There are also *compact* HII regions, which are only visible at radio and FIR wavelengths. These are very young objects, often associated with H₂O and OH masers, infrared sources, molecular lines of complex molecules, etc.

The *range* in properties of HII regions is enormous*.

Modeling of HII region spectra: the probably best known code is CLOUDY, developed by G. Ferland. You can input an ionizing spectrum from a star or other, a gas distribution, metallicity, etc., and then calculate the detailed emission line spectrum for the region.

HII regions can be density-bounded or radiation-bounded. rad: run out of photons before running out of gas. density: run out of material before using up photons. No clear boundary.

*Kennicutt, 1984

9. Interstellar absorption lines in stellar and quasar spectra

-89- Discovered in 1904, before we were aware of dust, or that MW and M31 were separate galaxies. Typical optical absorption lines discovered later include Ca^+ , Na^o , K, Ti^+ , and some molecular lines, e.g. CH^+ , CH, CN. Lines from more abundant atoms and molecules (H, H_2 , C, N, O, etc.) were only found after Copernicus was launched (early 70s) because $\lambda < 3000\text{\AA}$ for these lines, so they have to be observed from space.

Key: Since the absorption process is $\propto I_{\text{incoming}}$, we can find very small column densities of ISM, provided the background source is strong enough.

General, important property: Absorption of optical photons generally only occurs from the ground level of the atom, ion, or molecule, because only these levels are populated in most conditions (remember that $kT = 0.86 \text{ eV}$ for $T = 10^4 \text{ K}$, or 8.6 eV for $T = 10^5 \text{ K}$). This implies that in the formation of absorption lines, stimulated emission from the upper level does not play a role.

O^+ in HII region sits in ground state, can assume ions in ISM also stay in $n = 1$. Pure absorption foreground screen. $I = S(\lambda) (1 - e^{-\tau_\lambda})$, can leave the exponent out ($\rightarrow 0$).

This is not the case whenever excitation conditions are such that absorption lines do arise from excited atoms or molecular levels. Usually collisions would be responsible for the excitation, so keep an eye on kT .

-90- Generally, interstellar absorption lines are *narrow* and *complex* in structure. So we require very high resolution to resolve them:

$$\left(\frac{\lambda}{\Delta\lambda} > 3 \times 10^5 \right) \Leftarrow \text{ideal, but rarely achieved. res: couple thousand}$$

Aside

Remember that $\frac{\Delta\lambda}{\lambda} = \frac{\Delta v}{c}$ and for $\frac{\lambda}{\Delta\lambda} \geq 3 \times 10^5 \rightarrow v \leq 1 \text{ km per second}$. Which implies $\Delta\lambda \sim 0.02 \text{ \AA}$ at 6000 \AA and $\Delta\lambda \sim 0.0033 \text{ \AA}$ at 1200 \AA

9.1. Theory of formation of (interstellar) absorption lines

Ideal background source: bright (especially UV), with a featureless continuum. \Rightarrow Quasars and AGN or hot stars (which tend to be more complicated, though quasars sometimes have their own lines).

The high spectral resolution that is required to resolve a line is often unattainable. However, even if we cannot *resolve* the absorption line, we can still infer some important properties

about the ISM. As long as the resolution is sufficient to separate absorption lines at different velocities, coming from different clouds along the line-of-sight, or if only one component is present, we can use the *area* of an absorption line, or the **equivalent width**, EW, the width of an absorption line which would absorb 100% everywhere, and which would have the same area as the hatched area. Note that EW is always defined relative to I_o ; we *divide* by I_o .

-91- Note also that if we decrease the resolving power of the spectrograph, EW does not change.

Theoretical expression for EW:

$$\tau_\nu = \int_0^\infty \kappa_\nu dr; \quad \kappa_\nu = h\nu_{ul} \frac{n_\ell B_{\ell u} - n_u B_{u\ell}}{c} \phi(\nu)$$

(using “energy density” definition of B coefficients). u = upper level and ℓ = lower level.

If stimulated emission can be neglected, we have:

$$\kappa_\nu = \frac{h\nu_{ul} B_{\ell u}}{c} n_\ell \phi(\nu) \equiv \frac{\pi e^2}{m_e c} f_{\ell u} n_\ell \phi(\nu)$$

where $f_{\ell u}$ = *oscillator strength* (historical terminology), and $\frac{\pi e^2}{m_e c}$ = “classical cross-section”^{*} \sim Einstein coefficient.

Use $|dv| = \frac{c}{\lambda^2} |d\lambda|$ and we obtain:

$$\begin{aligned} EW &= \int_o^\infty [1 - e^{-\tau_\nu}] d\lambda \\ &= \frac{\lambda^2}{c} \int_o^\infty [1 - e^{-\tau_\nu}] d\nu \\ &= \frac{\lambda^2}{c} \int_o^\infty d\nu \left[1 - \exp \left(- \int_0^\infty \left(\frac{\pi e^2}{mc} \right) f_{\ell u} n_\ell \phi(\nu) dr \right) \right] \end{aligned}$$

$\tau \sim n_e$

Now assume: $\phi(\nu)$ independent of r for the particular cloud that we are observing. Then:

$$EW = \frac{\lambda^2}{c} \int_0^\infty d\nu \left[1 - \exp \left(- \left(\frac{\pi e^2}{mc} \right) f_{\ell u} \phi(\nu) N_\ell \right) \right]$$

with $N_\ell = \int n_\ell d\nu$ = *column density* of atoms in level ℓ . The quantity $\phi(\nu) N_\ell$ is the key part of this equation.

-92- Note: This situation is very different from the discussion of HI 21-cm abs lines where the upper level *is* populated, and we had to include stimulated emission to first order.

We next assume a Gaussian velocity distribution (Maxwellian) to describe the distribution of radial velocities:

$$f(v_{rad}) = \frac{1}{\sqrt{\pi}b} \exp \left[- \left(\frac{v_{rad} - v_o}{b} \right)^2 \right]$$

^{*} see for example Rybicki and Lightman, section 3.6

where $b = \sqrt{\frac{2kT}{m}}$ (only valid for thermal motions) and $\phi(\nu)$ becomes:

$$\phi(\nu) = \frac{1}{\sqrt{\pi}} \frac{1}{\Delta\nu_b} H(a, u)$$

where $a \equiv \frac{\Gamma}{4\pi\Delta\nu_b}$; $u = \frac{\nu - \nu^*}{\Delta\nu_b}$; $\Delta\nu_b \equiv b\frac{\nu_o}{c}$; and $\nu^* = \nu_o(1 - \frac{v_o}{c})$, the central frequency of the line. $b^2 = 2\sigma^2$ where σ = velocity dispersion, and a is a measure of where the Gaussian profile levels off to the Lorentzian wings* (transition of Gaussian line core to the damping wings).

What is all this good for?

For a given spectral line, we can measure EW from observations. Free parameters: N_ℓ , b , and a . For a given cloud, b and a are specified, and we can plot EW as a function of N_ℓ .

\implies The Curve of Growth

In practice, one plots $\log\left(\frac{W}{\lambda}\right)$ against $\log(N_\ell f_{lu}\lambda)$.[†]

We can distinguish three regimes in a curve of growth.

- 93-
- I $EW \propto N_\ell$ (linear)
 - II EW almost independent of N_ℓ
 - III $EW \propto \sqrt{N_\ell}$

*see notes from before, p. 34

[†] example: figure 3.2 in Spitzer

10. Heating and cooling

-151- Following Spitzer's general formalism*:

Γ = total kinetic energy *gained* [$\text{erg cm}^{-3} \text{s}^{-1}$]
 Λ = total kinetic energy *lost* [$\text{erg cm}^{-3} \text{s}^{-1}$] (aka. the “cooling function”)

All the physics are contained in these two quantities; need to find out what they are.

For $\Lambda = \Gamma$ we reach equilibrium temperature, T_E . Since Λ and Γ may depend on T_E themselves, we can set up a time dependent differential equation:

$$\underbrace{n \frac{d}{dt} \left(\frac{3}{2} kT \right)}_{\text{rate of increase in thermal energy for a mono atomic gas}} - \underbrace{kT \frac{dn}{dt}}_{\text{work done by gas (energy loss)}} = \underbrace{\sum_{\varrho, \eta} (\Gamma_{\varrho, \eta} - \Lambda_{\varrho, \eta})}_{\text{Sum over interacting particles}} = \Gamma - \Lambda$$

(Note: in what follows we generally ignore “work done by gas” component.) Also ignored is thermal conduction, which is okay for $T \leq 2 \times 10^4$ K. If we do have energy gain or loss by conduction (electrons), we add $-\vec{\nabla} \cdot (k \vec{\nabla} T)$ to the righthand side. Conductivity is hard to figure out, but not important below 20,000 K. \vec{B} fields prevent conductivity across field lines.

$PV = nRT = NkT \rightarrow P\Delta V = \text{work done by gas}$. $R = N_A k$, where N_A is Avagadro's number.

If a gas is not at its equilibrium temperature (T_E), we can define a “cooling time” t_T (like a half-life, sorta):

$$\frac{d}{dt} \left(\frac{3}{2} kT \right) = \frac{3}{2} k \left(\frac{T - T_E}{t_T} \right)$$

where $T_E = \text{constant}$ and $T = \text{variable}$. For T_E and $t_T = \text{constant}$, $T - T_E \propto e^{-t/t_T}$ (exponential decay for cooling). t obviously can't be negative \rightarrow quasi-stable

-152- If $t_T < 0$, unstable situation and gas will cool or heat toward an entirely different equilibrium temperature. This is relevant to the multi-phase ISM. All the physics is in Λ and Γ .

General primary heat source: (photo)ionization -starlight.

E_2 = kinetic energy of ejected electron
 E_1 = kinetic energy of recaptured electron

$\Delta E = |E_2 - E_1| \rightarrow \text{energy available}$. Number of captures to level j of neutral atom [$\text{cm}^{-3} \text{s}^{-1}$]
 $= n_e n_i < \omega \sigma_{cj} >$ where $< \omega \sigma_{cj} > = \text{recombination coefficient}$. \rightarrow final net gain associated

* see also Draine chapters 27, 30, 34

with electron-ion recombination:

$$\Gamma_{ei} = n_e n_i \sum_j \left(\underbrace{\langle \omega \sigma_{cj} \rangle}_{\text{ionization out of } j} \bar{E}_2 - \underbrace{\langle \omega \sigma_{cj} E_1 \rangle}_{\text{recapture back to } j} \right)$$

$\langle \rangle$ = average over Maxwellian velocity distribution
 \bar{E}_2 = average over all ionizing photon energies

As we have seen, all ionizations take place from the ground level $\rightarrow \bar{E}_2$ is independent of j ;
 use recombination coefficient to all levels $\geq n$:

$$\alpha^{(n)} = \sum_n^\infty \alpha_m$$

and

$$\Gamma_{ei} = n_e n_i \left\{ \underbrace{\alpha \bar{E}_2}_{\equiv \alpha^{(1)}} - \underbrace{\frac{1}{2} m_e \sum \langle \omega^3 \sigma_{cj} \rangle}_{\text{kinetic energy}} \right\}$$

-153- Main point: Γ_{ei} not dependent on ionization probability, or on radiation density for steady state, where number of ionizations is equal to the number of recombinations.

Primary cooling source: inelastic collisions (hence excitation of energy levels).

- $n_e n_i \gamma_{jk}$ = number of excitations from level $j \rightarrow k$ [$\text{cm}^{-3} \text{ s}^{-1}$] of ions in ionization stage i and excitation state j .
- $E_{jk} = E_k - E_j$ = energy lost by colliding electrons (emit photons)

Also: de-exciting collisions, which give energy gain to electrons. Net cooling:

$$\rightarrow \Lambda_{ei} = n_e \sum_{j < k} E_{jk} \left(\underbrace{n_{ij} \gamma_{jk}}_{\text{excitation}} - \underbrace{n_{ik} \gamma_{kj}}_{\text{de-excitation}} \right)$$

γ s are collisional rate coefficients. De-excitation process gives energy back to the nebula. Assumption: all photons escape (not true for dense molecular clouds). Again, in most cases all ions are in the ground level, so we don't need to sum over j , and $n_{ij} = n_{i1} = n_i$ (simpler)

10.1. Heating and cooling of HI gas

-160-