Contents

1. Overview of ISM components and processes

-6- What can happen to an atom or molecule or dust grain sitting in the ISM?

- Can it absorb a photon? Energy levels $\leftrightarrow h\nu$ (Need cross-sections for dust grains)
- Can it collide with other particles? $\rightarrow$ collisional rates [s^{-1} cm^{-3}]
- Is there a magnetic field? Is the particle charged?
- Are there cosmic rays? These can penetrate dense gas.

Three possible sources of ionization (and excitation):

1. photons
2. collisions
3. cosmic rays

Typical collisional energies: kinetic energy. Translate $mv^2 \rightarrow kT \rightarrow h\nu \rightarrow eV$.

► 1.1 Neutral gas

-7- Overall, in the solar neighborhood, $n(HI) \simeq 1$ cm^{-3}. The *effective thickness* of the HI layer is defined as

$$\frac{N_{\perp}(HI)}{n(HI, z=0)} = \frac{\int_{-\infty}^{\infty} n(HI)\, \mathrm{d}z}{n(HI, z=0)} \sim 250 \text{ pc}$$

He will also be neutral in this medium, and makes up about 30% of the mass.

Spitzer discusses the existence of HI clouds with certain typicl properties; it is not clear how realistic the description is. For one, "spherical clouds" appear not to exist, but filamentary structures dominate. But his discussion does illustrate some of the basics of a clumpy medium embedded in a more diffuse substrate.

The typical temperature of HI gas is 50 - 150 K in "clouds", and up to $\sim$ 6000 K in warm neutral medium (WNM). WNM has low average density, in the range 0.05 - 0.2 cm^{-3}

► 1.2 Molecular gas

Molecular gas is typically much cooler ($T < 40$ K), as low as 10 K. Perhaps there is even a colder component, with $T \sim 3$ K, the microwave background temperature. This component would be very difficult to detect in emission (why?), but may have been seen in absorption (need reference). The molecular gas is warmer close to regions of star formation. Typical densities of the molecular gas are in the range $10^3 - 10^6$ cm^{-3}.

► 1.3 Dust

-8- Dust grains also occur in the neutral medium, and probably also in the (warm) ionized medium. Dust grains play an important role in various processes:

- extinction of starlight
- emission of absorbed energy in FIR
- formation of molecules often occurs on grain surfaces
- absorption of ionizing UV radiation and Lyα photons (reducing amount of ionizing radiation)
- heating of HI gas by *photoelectric* emission

The composition of dust grains is still a matter of debate, although certainly carbon and silicates (i.e. "sand grains".) Typical sizes are in the range 0.01 - 0.1 μm (How do we know? $\rightarrow$ shape of extinction curve). Grains as small as $\sim$ 60 atoms across discovered; evidence comes from emission lines in NIR and excess emission at 5 - 40 μm from dust in the ISM (excess over what is expected from the larger dust grains). While the larger dust grains have temperatures between 10 and 40 K, the small ones can be heated to higher temperatures due to the absorption of even a single UV photon. A promising candidate for small dust grains are so-called polycyclic aromatic hydrocarbons A good closely related analog is car soot!

It is not clear how well dust grains survive in hot environments: they may be destroyed by *sputtering*,
-9- where collisions of grains with other atoms, electrons, or molecules knock molecules off the grains. Since atoms and molecules stick to dust grains at low temperatures, grains can deplete the heavy elements of the ISM, producing apparently anomalously low abundances of these elements along certain lines of sight.

Dust contributes about 1% of the mass of the ISM in the solar neighborhood, mostly in the form of large grains.

► 1.4 Ionized gas

Three processes can ionize gas in the ISM:

a) Photoionization
b) Collisions with other particles
c) Cosmic rays

Photoionizatin is especially effective near hot stars, i.e. central stars in planetary nebulae, or white dwarfs in general, and massive OB stars in HII regions.

Shock ionization occurs due to expanding stellar winds, supernova explosions, and collisions among gas clouds.

Question: what collisionsal speeds are required to ionize hydrogen? What temperatures does this correspond to?

Planetary nebulae are similar to HII regions, except their central stars are typically hotter than those in
-10- HII regions, resulting in emission lines from more highly inized elements. The central stars, however, are
usually much *less luminous* than OB stars, producing much smaller, less lumnious nebulae.

Typical densities in HII regions are 10 - 10^4 cm^{-3}. In *compact* HII regions, densities can be even higher. Temperatures are in the range 5000 - 10,000 K, as we will see.

In addition to planetary nebulae and HII regions, we also recognize a much more diffuse ionized medium, which seems to fill a substantial portion of interstellar space. Most likely, this gas is also photo-ionized, although shocks are likely important in certain locations. Tracers/probes of this medium include pulsar dispersion measurements and Hα (and other Balmer emission lines).

Cosmic ray ionization can occur throughout most of the ISM, so cosmic rays can also produce a small amount of ionization in denser gas (recombination happens quickly, so not much of the gas is in ionized state at any given time).

-11- A typical example is the heating and resulting ionization produced by supernova explosions. The explosion creates a hot expanding bubble, initially $\sim 10^7$ K, which generally sweeps up a cooler shell of warm ($\sim 10^4$ K) ionized gas. The warm shell is largely ionized by shocks and shows a different optical spectrum than HII regions.

The hot gas shows up by:

- emission of free-free and line x-rays
- absorption lines of highly ionized species toward bright UV sources

Question: Kirchoff's laws tell us that absorption lines arise when we look through a cool gas toward a hotter source. The collisionally ionized gas showing up in OVI ($\lambda 1031.9 + \lambda 1037.6$ Å) is inferred to be several hundred thousand K, much hotter than the O star tward which these lines are seen in absorption. How can it be that we see this gas in absorption?

Typical densities of hot gas: 0.001 - 0.0001 cm^{-3}

The following table (page -12-) summarizes our knowledge of the local ISM, and is taken from Knapp (1989, Wyoming conference).

► 1.5 Magnetic fields and cosmic rays

-13- Typically, in the solar neighborhood, $\mathbf{B} = 2\text{–}5 \times 10^{-6}$ Gauss. This follows from measurements of *Faraday rotation*, giving $\langle n_e B_{\|} \rangle$ toward pulsars and radio sources. The random component of the B field is probably as large as the uniform component. In clouds, the B field can be much higher, $\sim 70\mu$G (from Zeeman effect splitting measurements).

The magnetic field is important for several reasons:

1. It aligns elongated grains, giving rise to polarization of starlight
2. It causes relativisitic electrons to emit synchrotron radiation, and almost certainly plays a role in accelerating electrons to relativistic velocities ("magnetic bottle", Fermi acceleration).
3. It provides pressure support against gravitational collapse of matter since it is frozen into the matter due to ionization heavy elements. It also seems to play an important role in solving the angular momentum problem in star formation.*

The cosmic rays that reach the Earth are mainly protons. At 1 GeV the number density ratio of protons to electrons is about 100. However, the electrons are more affected by energy loss mechanisms and it is not clear that this ratio holds in the ISM. The total energy density (or pressure) of cosmic rays in the solar neighborhood is $U_R \sim 1.3 \times 10^{-12}$ erg cm^{-3}.

-14- Why are cosmic rays important?

- They produce γ-rays through collisions with atoms and molecules. In fact, the observed γ-ray intensity from the ISM forms an excellent independent measure of the total amount of matter between stars. For example, γ-ray data have been used to calibrate the conversion factor of CO line intensity to H_2 mass. (*the X-factor?!?*)
- They also provide pressure against gravitational collapse

So we have five pressures that play an important role in supporting the ISM against gravitational collapse:

1. thermal: $P = nkT$
2. magnetic: $P = \dfrac{B^2}{8\pi}$
3. turbulent (bulk motion) ★
4. cosmic rays
5. radiation

★ There is a pressure associated with turbulent velocities in the ISM. The cloud to cloud velocity dispersion due to turbulence on various scales increases line widths [over?] thermal widths. From the table on page -12-, you will note rough thermal pressure euqilibrium between the components. This is not a coin-

* Shu et al.

cidence; in fact, some of this information was inferred by *assuming* pressure equilibrium. The argument is that if there were no equilibrium, the resulting perturbations would be wiped out on sound-crossing

-15- time scales, which are short compared to the time scales we wouuld consider the ISM to evolve over.

However, the actual evidence for equilibrium in the thermal pressure is scarse, and there are claims that it is not true in the very local ISM[*].

There seems to be a "cosmic conspiracy": the estimates for the thermal, magnetic, and cosmic ray pressure for the solar neighborhood give roughly equal numbers for all three. Thus it may be inappropriate to only consider the thermal pressure (the only one that can be measured with much certainty). Interestingly, the magnetic pressure number is also very similar to the energy density of the CMB. [†] [‡] Note that pressure and energy density have the same units if you convert them!

-16- Next section is sort of a shortened condensation of Draine's chapters 2 and 3. We may come back to specific topics discussed there in more detail.

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* See, e.g. Bowyer et al. *Nature* 1905

† Draine discusses possible reasons in section 1.3

‡ Supplemental info in Draine chapter 1.

2. Statistical physics in the ISM

► 2.1 The "big four" major laws of statistical physics

Maxwellian velocity distribution
Boltzmann distribution of energy levels in atoms and molecules
Saha equation for ionization equilibrium
Planck function for radiation

▷ 2.1.1 *Maxwell's velocity distribution*

Temperature is defined by the motion of the particles. Velocity is given by $\vec{\omega}$ (or $\vec{v}$ in Draine).

$f(\vec{\omega})\ \mathrm{d}\vec{\omega}$ = fractional number of particles whose velocity, $\vec{\omega}$, lies within the three-dimensional volume element $\mathrm{d}\vec{\omega} = \mathrm{d}\omega_x\ \mathrm{d}\omega_y\ \mathrm{d}\omega_z$, centered at $\vec{\omega}$.

In thermodynamic equilibrium, $f(\vec{\omega})$ is isotropic, so $\vec{\omega} \to \omega$.

$$f(\omega) = \frac{\ell^{3/2}}{\pi^{3/2}} e^{-\ell^2\omega^2}; \quad \ell^2 = \frac{m}{2kT} = \frac{3}{2\langle\omega^2\rangle}$$

$$f(\omega) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m\omega^2}{2kT}\right)$$

For two groups of particles with different masses, we replace ω by u, the relative velocity between the
-17- two groups, and m by the reduced mass $m_r = \dfrac{m_1 m_2}{m_1 + m_2}$. For H atoms colliding with particles of mass Am_H, Spitzer derives:

$$\langle u\rangle = \left[\frac{8kT}{\pi m_r}\right]^{1/2} = 1.46\times 10^4\sqrt{T}\left(1+\frac{1}{A}\right)^{1/2} \quad [\mathrm{cm\ s^{-1}}]$$

Exercise: Verify the above calculation, especially the numerical constant.

Note that there is a difference between the *speed* and *velocity* distribution. (Verify that $\langle\omega\rangle$ = 0.) But evidently, the mean *speed* is not 0. The speed distribution is given by:

$$f'(\omega')\,\mathrm{d}\omega' = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-m\omega'^2/2kT\right) \underbrace{4\pi\omega'^2\mathrm{d}\omega'}_{\text{volume in phase space}}$$

where $f'(\omega')\,d\omega'$ = fractional number of particles with speeds between ω' and $\omega' + d\omega'$ and $\omega' = |\omega|$.

The Maxwell velocity distribution is characterized by several speeds:

- Most probable speed: $\omega_o = \sqrt{\frac{2kT}{m}}$
- RMS speed: $\langle \omega^2 \rangle^{1/2} = \sqrt{\frac{3kT}{m}}$
- RMS velocity in one direction: $\langle \omega_x^2 \rangle^{1/2} = \sqrt{\frac{kT}{m}}$

▷ *2.1.2* *Boltzmann distribution*

-18- The *population of energy levels* in an atom or molecule is given by the Boltzmann distribution:

$$\frac{n_u}{n_l} = \frac{g_u}{g_l} e^{-(E_u - E_l)/kT} = \frac{g_u}{g_l} e^{-\Delta E/kT}$$

- $n_{u,l}$ = number densities
- $g_{u,l}$ = statistical weights
- $E_{u,l}$ = energies of the levels

▷ *2.1.3* *Saha equation*

The *ionization equilibrium* is described by the Saha equation:

$$\frac{n_{i+1}}{n_i} = \frac{g_{i+1} g_e}{g_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} \exp\left(-I/kt\right)$$

- I = ionization potential for an ion in the ground state and initial ionization state i (aka, the energy required to ionize from i to $i+1$)
- $g_e = 2$ (two spin conditions)

▷ 2.1.4 *Planck function*

The *radiation field* is specified by the Planck function: [*]

$$B(\nu) = \frac{2h\nu^3}{c^2}\frac{1}{\exp[h\nu/kT]-1}\begin{cases} \approx \dfrac{2\nu^2}{c^2}kT & \text{for } h\nu \ll kT \text{ (Rayleigh – Jeans)} \\ \approx \dfrac{2h\nu^3}{c^2}\exp[-h\nu/kT] & \text{for } h\nu \gg kT \text{(Wien)} \end{cases}$$

► 2.2 Validity of the four laws

-19- The four laws discussed above hold under thermodynamic equilibrium (TE). However, TE is not often the case for the ISM, for two reasons:

1. TE requires **detailed balancing**, i.e. each process is as likely to occur as its inverse. For example, consider the 3727 Å emission from O^+. This is a forbidden transition (actually a doublet). The excitation of the electron level occurs through collisions with electrons, in most conditions in the ISM. If detailed balancing were to hold, de-excitation should also occur by collisions. However, as we will see, under the low density conditions found in the ISM, collisions are rare, and de-excitation is more likely to proceed through emission of a photon, in spite of the fact that we are dealing with a forbidden transition. Thus [OII] emission can be quite strong, and by converting collisional (kinetic) energy into radiation, we actually have created a cooling mechanism for the gas.

2. The radition field is strongly diluted. A diluted radiation field is one in which the energy density does not match the color temperature. This concept is quite familiar; for example, the sun's photosphere is $\sim$ 6000 K, and at the surface the flux leaving the sun is approximately that of a blackbody of this temperature. However, the Earth is not 6000 K because by the time the radia-
-20- tion reaches us, it is diluted. For the solar neighborhood, the total energy density of the radiation field due to all stars in that volume is about 1 eV cm^{-3} (close to cosmic ray density, as mentioned before). When interpreted as an average temperature using the Stefan-Boltzmann law (energy density of a blackbody, u = aT^4), this energy density implies an equivalent temperature of $\sim$ 3 K. Yet the color temperature implied by the shape of the spectrum of this Interstellar Radiation Field (ISRF) is that of A and B stars (T $\sim 10^4$ K). So there is a **dilution factor** W given by:

$$W \approx \left(\frac{3}{10^4}\right)^4 \approx 0.25 \times 10^{-14}$$

[*] $e^x \approx 1 + x$ for $x \ll 1$
$e^{h\nu/kT} - 1 \approx 1$ for $h\nu \ll kT$
$e^{h\nu/kT} - 1 \approx e^{h\nu/kT}$ for $h\nu \gg kT$

We conclude that using the Planck law to describe intensities is not correct.

What about the other laws?

1. Maxwell velocity distribution

Good news! It is generally valid. Detailed balancing is possible for the elastic collisions that are generally occurring.* Because the Maxwellian distribution is a good description of the motions of the particles, we can define a *kinetic temperature* which describes the physical condition of the gas. Often, for a plasma, the kinetic temperature is equal to the electron temperature: $T_{ions} = T_e$. $T_{ions} \neq T_e$ may occur behind shocks.

2. Boltzmann distribution

-21- Rarely correct. If excitation and de-excitation occurs by photons, we may still not have a Boltzmann distribution because the photon distribution is not given by the Planck function. Often we do not even have detailed balancing. However, as we will see, sometimes the distribution of excited levels is not too different from Boltzmann distribution. This happens when collisions dominate excitation and de-excitation, while radiation is relatively unimportant.

To describe situations close to TE, Spitzer introduced the so-called b-factors (Draine calls them "departure coefficients").

$$b_j \equiv \frac{n_j(\text{true distribution}}{n_j(\text{LTE distribution})}$$

Example: in an HII region, the hightest excited levels of HI have $b_j \sim 1$. Some radiation does escape (producing radio recombination lines) but collisions dominate the level populations. Since motions of particles *are* described by a Maxwellian velocity distribution, whenever collisions dominate the level population they will closely follow a Boltzmann law.

-22- In general, a Maxwellian velocity distribution tends to set up a Boltzmann population for energy levels in the atoms/particles *if* transitions resulting from emission and absorption of photons are relatively unimportant, and collisional (de-)excitation is dominant. In the case of the highly excited levels in H mentioned before, collisions with electrons are dominant.

3. Saha equation

is generally not valid; there is no detailed balancing. Even though the ionization and recombination processes are each other's inverse
($h\nu + A \rightarrow A^+ + e^- \rightarrow h\nu + A$),
the ionization process is determined by the *photon* field in most cases, while the recombination process is determined by collisions between A^+ and e^-. The collision rate depends on $\{n_e, n_{A^+}\}$ and T_e (the electron temperature), but the ionization is dependent on $T_{radiation} (\neq T_e)$.

* See Draine section 2.3.3 for an analysis of "deflection times"; they tend to be short. (In dynamics, we speak of a "relaxation time".)

So in general, assume **statistical equilibrium**, where there is a balance between transitions one way and the other way, no matter what process caused each transition.

In level i, we have n_i atoms cm^{-3}, and R_{ij} is the rate coefficient (number of transitions from level i to level j for all possible processes) such that

$n_i R_{ij}$ [s^{-1}] = # transitions from level i to level j
$R_{ji} n_j$ [s^{-1}] = # transitions from level j to level i

$$\frac{\mathrm{d}n_i}{\mathrm{d}t} = \sum_j \left(-R_{ij} n_i + R_{ji} n_j\right); \quad i = 1, 2, \ldots$$

-23- In statistical equilibrium, $\mathrm{d}n_i/\mathrm{d}t = 0$. The rate factor R_{ij} includes *all* possible processes that would take the atom or molecule from level i to j. For example, if you consider excitation of an electron in an atom, R_{ij} could include absorption of photons, collisional excitation, etc.

In the worst case, you will have to include many processes to calculate the n_i values. This requires knowledge of a lot of physical input parameters, e.g. cross sections for particular processes, collisional rate coefficients *, etc.

In other cases, where only one or two processes matter, the situation can be very simple. We will encounter cases of each.

Before going more into Ch 2 and 3 in Draine, we will first discuss some basic radiative transfer.†

* Drane section 2.1
† RL Ch 1, Draine Ch 6, 7

3. Radiative Transfer

-24- (See Draine chapter 7)

Planck function (an intensity) $B_\nu(T) \equiv \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}\ \frac{dE}{dt d\nu d\Omega d\sigma}$

Photon occupation number $n_\gamma \equiv \frac{c^2}{2h\nu^3} I_\nu = \frac{1}{e^{h\nu/kt}-1}$ if $I_\nu = B_\nu$

Specific intensity $I_\nu \equiv \lim_{d\sigma, d\Omega, d\nu, dt \to 0}$

Flux at surface of a sphere $F_\nu = \pi B_\nu$ (for blackbody)

$F_\nu = \pi I_\nu$ (for isotropic emitting non-blackbody)

Flux at a distance r $F_\nu(r) = \pi I_\nu(\frac{R}{r})^2 = \frac{L_\nu}{4\pi r^2}$

where R = radius of body and

L_ν = luminosity of body [erg s^{-1} Hz^{-1}]

-25- The above are *monochromatic*: integration over frequency yields the total flux, etc.

$$I = \int_{r=0}^{\infty} I_\nu \mathrm{d}\nu; \quad F = \int_{\nu=0}^{\infty} F_\nu \mathrm{d}\nu; \text{ etc} \ldots$$

Energy density: $u_\nu = \frac{1}{c}\int I_\nu \mathrm{d}\Omega$ [erg cm^{-3} Hz^{-1}]

Radiation pressure: $P_\nu = \frac{1}{c}\int I_\nu \cos^2\theta \mathrm{d}\Omega$

► 3.1
Emission and absorption coefficients

▷ *3.1.1*
Spontaneous Emission

j_ν = volume emission coefficient [erg cm^{-3} sec^{-1} Ω^{-1} Hz^{-1}]

$j'_\nu = \dfrac{j_\nu}{\rho}$ = mass emission coefficient [erg g^{-1} sec^{-1} Ω^{-1} Hz^{-1}]

$j_\nu = \dfrac{\epsilon_\nu}{4\pi}$ for an isotropic emitter; ϵ_ν = emissivity [erg cm^{-3} s^{-1} Hz^{-1}]

▷ *3.1.2*
Absorption

Absorption is a bit more complicated than emission. It removes a fraction of the incoming radiation. Absorption also includes *stimulated* emission, aka "negative absorption". The entire process refers to the sum of "true absorption" + stimulated emission.

-26- κ_ν = volume absorption coefficient [cm^{-1}]

$\kappa'_\nu = \dfrac{\kappa_\nu}{\rho}$ = mass emission coefficient [g^{-1} cm^2]

Loss of intensity in a beam of light as it travels distance ds:

$$\mathrm{d}I_\nu = -\kappa_\nu I_\nu \mathrm{d}s$$

Microscopically:

$$\kappa_\nu = n\sigma_\nu$$

where n is the particle density and σ_ν is the cross section for the absorption process per particle.

optical depth: $\tau_\nu \equiv \int \kappa_\nu ds$

- $\tau \gtrsim 1$ optically *thick* emission
- $\tau < 1$ optically *thin* emission

mean free path: $\ell_\nu = \dfrac{\tau_\nu}{\kappa_\nu}$

so for condition $\tau = 1 \longrightarrow \ell_\nu = \dfrac{1}{\kappa_\nu} = \dfrac{1}{n\sigma_\nu}$

► 3.2 Radiative transfer equation

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu I_\nu + j_\nu$$

$$\mathrm{d}\tau_\nu = \kappa_\nu \mathrm{d}s$$

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = -I_\nu + \frac{j_\nu}{\kappa_\nu} = -I_\nu + S_\nu$$

where S_ν ≡ source function

Formal solution:

$$I_\nu(\tau_\nu) = I_\nu(o)\mathrm{e}^{-\tau_\nu} + \int_0^{\tau_\nu} \mathrm{e}^{-(\tau_\nu - \tau_\nu')} S(\tau_\nu')\mathrm{d}\tau_\nu'$$

-27- → attenuated incoming beam + contribution from gas itself.

Special cases:

1. Source function is constant throughout source:

$$I_\nu(\tau_\nu) = I_\nu(o)\mathrm{e}^{-\tau_\nu} + S_\nu(1 - \mathrm{e}^{-\tau_\nu})$$

   - optically thick emission: $I_\nu = S_\nu$
   - optically thin emission: $I_\nu = I_\nu(o)(1 - \tau_\nu) + \tau_\nu S_\nu$
2. Thermal radiation: $S_\nu = B_\nu(T)$ (The Planck function)
   - optically thick emission: $I_\nu = B_\nu$
   - optically thin emission: $I_\nu = \tau_\nu B_\nu$

Brightness temperature

For *radio* emission: I_ν is replaced by the *brightness temperature*, T_b, defined as:

$$I_{\nu,\mathrm{obs}} \equiv B_\nu(T_b)$$

In the Rayleigh-Jeans limit for B_ν we get:

$$I_{\nu,\mathrm{obs}} = \frac{2\nu^2 k T_b}{c^2} \quad \longrightarrow \quad I_b = \frac{I_\nu c^2}{2\nu^2 k}$$

Solution of the transfer equation in terms of T_b:

$$T_{b,\mathrm{obs}} = T_{b,o}\mathrm{e}^{-\tau_\nu} + T(1 - \mathrm{e}^{-\tau_\nu})$$

-28- T = thermal source temperature (*physical* temperature of the layer)
T_b = brightness temperature of the incident radiation

T_b is never greater than T!

► 3.3 Einstein coefficients

At the macroscopic level, Kirchoff's law gives $j_\nu = \kappa_\nu B_\nu(T)$. Einstein coefficients[*] describe reasons for this on a microscopic level. *They give the transition* probabilities *(per unit time)*.

Consider a system with two energy levels:

Three possible processes:

1. **Spontaneous emission**: Einstein A coefficient. A_{21} $\equiv$ transition probability for spontaneous emission per unit time per "system".

2. **Absorption**: requires photon of energy $\sim h\nu_0$ The mean intensity, J_ν is given by:

$$J_\nu \equiv \frac{1}{4\pi} \int I_\nu \, \mathrm{d}\Omega$$

-29- 3. **Stimulated emission:** $B_{21}\overline{J}$ = transition probability per unit time for stimulated emission. Process: incoming photon stimulates emission of another photon of exactly the same frequency and in the same direction (so the emitted photon is coherent with the photon that stimulated the emission).

   Note: if $\phi(\nu)$ is sufficiently sharp (approaches δ function), then $B_{21}J_{\nu_0}$ and $B_{12}J_{\nu_0}$ [etc.?] instead of $B_{21}J$ and $B_{12}J$

The Einstein coefficients are sometimes defined using energy density (u_ν) instead of intensity. This introduces factors $c/4\pi$ in the definition of B components.[†]

▷ *3.3.1 Relations between the Einstein coefficients*

The Einstein coefficients are derived from two-energy state system in thermodynamic equilibrium (TE), but prove to be valid under all conditions since they only refer to *atomic* properties.

In TE, the rate of transitions out of state 1 = rate of transitions into state 1 (both per unit volume). So if n_1, n_2 are number densities of atoms in levels 1 and 2, then:

$$n_1 B_{12}\overline{J} = n_2 A_{21} + n_2 B_{21}\overline{J}$$

$$\implies \overline{J} = \frac{A_{21}/B_{21}}{\left(\frac{n_1}{n_2}\right)\left(\frac{B_{12}}{B_{21}}\right) - 1}$$

-30- In TE, apply the Boltzmann law:

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[*] Draine, section 6.1

[†] Spitzer's notation

$$\frac{n_1}{n_2} = \frac{g_1}{g_2} e^{h\nu_0/kT}$$

where $h\nu_0 = \Delta E = E_2 - E_1$. Use this and the fact that $J_\nu = B_\nu(T)$ in TE, and that $\overline{J} = \int J_\nu \phi(\nu)\, \mathrm{d}\nu = B_\nu(T) \ldots$ true if $\phi(\nu)$ is narrow. This results in[*]

$$g_1 B_{12} = g_2 B_{21}; \quad A_{21} = \frac{2h\nu^3}{c^2} B_{21}$$

So knowledge of one of the Einstein coefficients is enough to know them all!

▷ *3.3.2*
Relations between Einstein coefficients and κ_ν & j_ν

Assumption: $\phi(\nu)$ is the same for emission and absorption.

$$j_\nu \ \mathrm{d}V \ \mathrm{d}\Omega \ \mathrm{d}\nu \ \mathrm{d}t = \frac{h\nu_{21}}{4\pi} n_2 A_{21} \phi(\nu) \ \mathrm{d}V \ \mathrm{d}\Omega \ \mathrm{d}\nu \ \mathrm{d}t$$

= amount of energy emitted in volume dV, solid angle dΩ, frequency range dν, and time dt. Each atom contributes energy $h\nu$; many atoms emit isotropically, so Hence, in general:

$$\boxed{j_\nu = \frac{h\nu_{kj}}{4\pi} \left(n_k \ A_{kj}\right) \phi(\nu)}$$

Similar reasoning leads to:

$$\boxed{\kappa_\nu = \frac{h\nu_{kj}}{4\pi} \left(n_j B_{jk} - n_k B_{kj}\right) \phi(\nu)}$$

-31- where $k > j$ for both. The second term here for κ_ν corresponds to *stimulated emission*.

► 3.4
Line profile function, $\phi(\nu)$

(See RL chapter 10.6 and Draine 6.4. In particluar, comments in Draine about the actual line width in some practical cases are useful!)

▷ *3.4.1*
Natural line width

Atom not moving, but still quantum effects. The natural line width is determined by lifetime of an excited state. It is described by a Lorentz profile, whose key feature is a narrow core and broad wings

[*] Note: See Draine equations 6.9 and 6.10; he defines B coefficients in terms of energy density, so differs by a factor of $4\pi/c$.

(not a Gaussian).

$$\phi(\nu) = \frac{4\gamma_{u\ell}}{16\pi^2(\nu - \nu_{u\ell})^2 + \gamma_{u\ell}^2}$$

- $\gamma_{u\ell}$ = damping constant = $1/\tau$
  (τ = lifetime of level)
- ν_0 = rest frequency

Example: if collisions are unimportant, and there is no stiumlated emission, then

$$\gamma_{u\ell} \equiv \gamma_{\ell u} = \sum_{E_j<E_u} A_{uj} + \sum_{E_j<E_\ell} A_{\ell j}$$

Key point: A small Einstein coefficient A results in a *narrow* line.

The natural line width of most transitions is quite small, and broadening due to other effects is more important.

▷ *3.4.2* *Doppler broadening*

- Thermal velocities
- Bulk motion (turbulence)

Thermal velocities:

$$\nu - \nu_0 = \frac{\nu_0}{c} v_z$$

Maxwell's distribution:

$$f(v)\,\mathrm{d}v_z \propto e^{-m_a v_z^2/2kT}\,\mathrm{d}v_z$$

$$v_z = c\left(\frac{\nu - \nu_0}{\nu_0}\right); \quad \mathrm{d}v_z = c\frac{\mathrm{d}\nu}{\nu_0}$$

$$\Rightarrow f(v)\,\mathrm{d}v_z \propto e^{-m_a c^2(\nu-\nu_0)^2/2\nu_0^2 kT}\,\mathrm{d}\nu$$

-32- and the profile function is

$$\phi(\nu) = \frac{1}{\Delta\nu_D\sqrt{\pi}} e^{-(\nu-\nu_0)^2/\Delta\nu_D^2}$$

where

$$\Delta\nu_D \equiv \frac{\nu_0}{c}\sqrt{\frac{2kT}{m_a}}$$

▷ *3.4.3*
Collisional broadening

~ Pressure broadening, which is not generally important in the ISM because the density is so low... mostly occurs in stellar atmospheres. This still produces a Lorenzian profile, but with:

$$\phi(\nu) = \frac{4\Gamma^2}{16\pi^2(\nu - \nu_o)^2 + \Gamma^2}$$

- $\Gamma = \gamma + \nu_{coll}/4\pi^2$
- ν_{coll} = frequency of collisions

Combined Doppler and Lorentz profile

$$\phi(\nu) = \frac{4\gamma_{u\ell}}{16\pi^2} \int_{-\infty}^{\infty} \frac{{\frac{m}{2\pi kT}}^{1/2} e^{-mv_t^2/2kT}}{\left(\nu - \nu_0 - \frac{\nu_0}{c} v_t\right)^2 + \gamma_{u\ell}^2} \, \mathrm{d}v_z$$

(includes everything). This can be written in terms of a *Voigt function*:

$$\phi(\nu) = \frac{1}{\Delta\nu_D} \frac{1}{\sqrt{\pi}} H(a, u)$$

$$H(a, u) \equiv \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2}}{a^2 + (u - y)^2} \quad \mathrm{d}y$$

$$a \equiv \frac{\gamma}{4\pi\Delta\nu_D} \qquad u = \frac{\nu - \nu_0}{\Delta\nu_D} \qquad \Delta\nu_D = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}$$

Note: in ISM, always $\Delta\nu_0 \gg \gamma$, so $a \ll 1$. In that case:

$$\begin{aligned} \phi(\nu) &\propto \left(e^{-u^2} + \frac{a}{\sqrt{\pi}u^2}\right) && \text{for } u \gg 1 (non - Gaussian wings) \\ \phi(\nu) &\propto e^{-u^2} && \text{for } u \ll 1 (Gaussian core) \end{aligned}$$

-33- So the core of the profile is Gaussian due to Doppler broadening, while the wings are much stronger than expected in a Gaussian profile, due to the intrinsic line width.

4. Neutral hydrogen (HI gas) in the ISM[*]

Wherever HI dominates the ISM, all atoms are found in the ground state (^{2}S)(n=1). The next excited level (^{2}P) is about 10 eV above the ground-level. This excitation is very rare and and quickly falls back to the ground level, so there is no significant population of this level. For example, consider potential excitation mechanism: collision of H-atom with cosmic ray particle (lots of energy, probability is once per 10^{17} seconds), ionizes the H-atom. Recombination results in some atoms winding up in ^{2}P state. But the Einstein A coefficient for spontaneous emission from $^2P \rightarrow {}^2S$ is 10^8 s^{-1}. Hence, this excitation process results in relative population of ^2P level of $10^{-8}/10^{17} = 10^{-25}$.

► 4.1 Possible tracers of HI gas

1. 21 cm HI transition (=hyperfine transition) in emission or absorption.
2. Lyman absorption lines against hot background stars.

Why only these two? Because only the ^{2}S level is populated. HI is hard to find in the ground state; fine structure → different angular momentum.[†]

▷ 4.1.1 *Excitation and radiative transport for the 21-cm line*

-34- Spin of proton and electron:

- Parallel (upper energy)
- Anti-parallel (lower energy)

(Spin is around particle's own axis, not to be confused with angular momentum). Motions specified by maxwellian velocity distribution, and collisions dominate the level populations (excite and de-excite).

Energy difference (very small):

$$h\nu = 9.4 \times 10^{-18}\ \text{erg}$$
$$\nu = 1420.4\ \text{MHz}$$
$$\lambda = 21.11\ \text{cm}$$

The probability for *spontaneous* emission to occur is *very* small:

$$A_{kj} = 2.86 \times 10^{-15}\ \text{sec}^{-1}$$

[*] (See Draine Ch 8, 29; Ch 17.1, 17.3.)

[†] See Draine Ch 4 (& 5) on notation of energy levels and atomic structure.

$$\rightarrow \text{lifetime} = 1.10 \times 10^7 \text{ years}$$

More frequently atoms will flip energy states by *collisions*.

$$\mathrm{H + H} \rightarrow \mathrm{H}_2^* \rightarrow \mathrm{H + H}$$

(H^* = excited H_2 molecule; not stable). Chance for collisional excitation per second: $\gamma n(H)$ [s^{-1}].

$\gamma[\times 10^{-11}\ \mathrm{cm^3 s^{-1}}]$	T [K]
0.23	10
3.0	30
9.5	100
16	300
25	1000

As long as $n(H^o) > 10^{-2}\ \mathrm{cm}^{-3}$, $\gamma n_H \gg A$, so collisions determine excitation *and* de-excitation $\longrightarrow$ implies detailed balancing, so Boltzmann distribution is valid for level populations.

$$\frac{n_1}{n_o} = \frac{g_1}{g_o}\exp(-h\nu/kT_k) \approx \frac{g_1}{g_o}\left(1 - \frac{h\nu}{kT_k}\right)$$

T_k = kinetic gas temperature.

-35- <u>Derivation of stats, coefficients, and radiative transfer</u>: Use "energy density" definition of Einstein B coefficients. *

Statistical equilibrium:

...

-36- ...

In general, in situations where stimulated emission and absorption can be neglected, we would obtain

$$\frac{b_1}{b_o} = \frac{1}{1 + \frac{A_{10}}{n_H \gamma_{10}}} \approx 1$$

<u>Note</u>: A potential other excitation process of upper level for HI is so-called *photon pumping*†: HI + Lyα photon $\rightarrow$ n=2 level (^2P). ^2P level might cascade down to upper ^2S hyperfine level. In principle, one

* Draine section 17.1

† D 17.3

might selectively populate the upper level this way. It turns out that, due to thermilization of the photons this anomalous population of the levels does *not* generally occur. (Lyα photons frequently scatter off A-atom again = LTE situation).*

-37- Since the b's are effectively 1, our main result is that

$$\frac{n_1}{n_0} = \frac{g_1}{g_0}\left(1 - \frac{h\nu}{kT_k}\right) \approx \frac{g_1}{g_0} = 3$$

$(h\nu \ll kT)$. So $n_0 = (1/4)n(H^o)$, $n_1 = (3/4)n(H^o)$

Are the HI levels always populated mainly by collisions? The following scenarios can cause this to not work as well:

HI gas is too warm The warm HI in our Galaxy has $n_H \sim 0.4$ cm $^{-3}$ and $T_k \approx 8000$ K. But T_s = spin temperature, of order $T_k/5$, if it weren't for excitation by Lyα photons. But it is true that the 3K background radiation field does *not* significantly disturb the equilibrium set up by collisions, as we discussed above.

n(H^0) drops too low If n_H drops to very low values, collisional excitation is ineffective. But even in that case, $T_s \approx T_k$ because of the above mentioned Lyα excitation.†

In any case, in general we now obtain for the emission coefficient:

$$j_\nu = \frac{3}{4}\, A\, \frac{h\nu}{4\pi}\, n(H^o)\, \phi(\nu)$$

which is *independent of temperature* in most circumstances. And for the absorption coefficient:

$$\kappa_\nu = \frac{h\nu}{c}\ [n_0 B_{01} - n_1 B_{10}]\ \phi(\nu)$$

-38- Using the relation between Einstein coefficients:

$$B_{10} = \frac{g_0}{g_1} B_{01}$$

$$n_1 = n_0 \frac{g_1}{g_0}\left(1 - \frac{h\nu}{kT_k}\right)$$

* This point is also discussed in Kushami & Heiles (in Gal. & Extragal. Radio astron., page 100)

† Kulkami & Heiles p. 100

now we *do* have to include this [1^e ?] order correction or $\kappa_\nu = 0$! So

$$\begin{aligned}
\kappa_\nu &= \frac{h\nu}{c}\left[n_0 B_{01} - n_0\left(1 - \frac{h\nu}{kT_k}\right)B_{01}\right] \\
&= \frac{h\nu}{c}\phi(\nu) n_0 B_{01}\frac{h\nu}{kT_k}B_{01} \\
&= \frac{(h\nu)^2}{c}\frac{n_0 B_{01}}{kT_k}\phi(\nu) \\
&\propto \frac{n(H^0)}{T_k}\phi(\nu) \\
&\propto \frac{1}{T_k}\phi(\nu)
\end{aligned}$$

Verify that $\dfrac{j_\nu}{\kappa_\nu} = B_\nu(T_k)$ as in thermal emission.

► 4.2 Simple case of a single layer of gas

Assume $\phi(\nu)$ = Gaussian profile

$$\begin{aligned}
I_\nu(A) &= \int_A^B j_\nu(x) e^{-\int_A^x \kappa_\nu(s)\,\mathrm{d}s}\,\mathrm{d}x \\
&= j_0 n(H^0)\phi(\nu)\int_A^B e^{-\tau_\nu(x)}\,\mathrm{d}x \\
&= \frac{j_0}{\kappa_0}T_k\left[1 - e^{-\tau_\nu(L)}\right]
\end{aligned}$$

$$j_0 \equiv \frac{3}{4}A\frac{h\nu}{4\pi} \qquad \kappa_0 \equiv \frac{(h\nu)^2}{c}\frac{B_{01}}{\kappa}\frac{1}{4} \qquad \tau_\nu(x) \equiv (x - x_A)\kappa_0 n(H^0)\frac{\phi(\nu)}{T_k}$$

τ_ν is max for $\nu = \nu_0$ because $\phi(\nu)$ is max there.

-39- Consider two cases:

(i) Optically thick: $\tau_{\nu_o}(L) \gg 1$

For the center of the line, we have:

$$I_\nu = \frac{j_0}{\kappa_0}T_k$$

The wings will be Gaussian, because τ_ν is still low there.

In general, the line profile of HI emission is entirely determined by the velocity of the atoms, so the assumption of a Gaussian profile is correct. In particular, Spitzer (page 43) presents the HI optical depth in practical terms: (yuck!)

$$\tau_\nu = \int \kappa_\nu \, \mathrm{d}s = 5.49 \times 10^{-14} \frac{H(HI)P(\omega)}{T_k}$$

$$\kappa_\nu \propto \frac{n(H_0)}{T_k}\phi(\nu)$$

$$\phi(\nu) \propto P(\omega)$$

$P(\omega)$ = Maxwellian velocity distribution

-40- We had before:

$$T_b = T_{b0}e^{-\tau_\nu} + T(1 - e^{-\tau_\nu}) = 0 + T(1 - e^{-\tau_\nu})$$

(no background source). Then

$$N(HI) = 1.823 \times 10^{13} \int T_b(\omega) \left[\frac{\tau_\nu}{1 - e^{-\tau_\nu}}\right] \, \mathrm{d}\omega$$

ω = velocity in cm s^{-1}.

(ii) Optically thin: $\tau_{\nu_o}(L) \ll 1$

In this case, we will always have

$$I_\nu = \frac{j_0}{\kappa_0} T_k \tau_\nu(L)$$

and since τ_ν has shape of $\phi(\nu)$, the line profile will look Gaussian.

$$N(HI) = 1.823 \times 10^{13} \int T_b(\omega) \, \mathrm{d}\omega$$

for uniform temperature T along the LOS! But generally true as long as total LOS is optically thin.

Aside: **How do we observe HI?**

1. Radio telescope
   - single dish
     - “single beam”: one spectrum per position 1×1 pixel!
     - Multiple beam system: get 7 = 30 beams (“ish”)
     - low angular resolution
     - sensitive to all the flux

   Make a map by many pointings. Point telescope, take spectra, repeat.

- interferometer One or $\sim$ few pointings per source. FOV is set by FWHM of beam of *one* telescope. Angular resolution set by the longest baseline (FOV still the same, regardless of D). In one pointing, can make a map of an area in the sky. $\theta = \dfrac{1.2\lambda}{D_{max}}$
  - High angular resolution
  - Missing baselines in array
    - ⋆ No zero-m spacing → no net flux
    - ⋆ Missing short spacings → negative bowl
    - ⋆ Missing gaps in baselines → grating rings

  Spectroscopic resolution is generally very good.

► 4.3 HI emission and absorption

-41- Distinguish two cases:

1. Absorption by an HI cloud of an extended background source, usually a continuum source (e.g. AGN)
2. HI self-absorption (enough of it at same velocity)

In the case of an extended background source, the radio continuum source is usually a compact, distant (extragalactic) radio source (but this does not have to be the case).

Reminder of radiative transfer: suppose that the HI layer has kinetic temperature T_s, uniform throughout. Then:

$$I_\nu = I_\nu(0)e^{-\tau_\nu} + B_\nu(T_s)\left[1 - e^{-\tau_\nu}\right]$$

Rayleigh-Jeans law:

$$B_\nu(T_s) = \frac{2\nu^2 k T_s}{c^2}, \quad h\nu \ll kT_s$$

-42- So in the direction toward the source ($T_{b,on}$) and immediately next to the source ($T_{b,off}$), we have

$$\begin{aligned} T_{b,on} &= T_{bo}e^{-\tau_\nu} + T_s\left(1 - e^{-\tau_\nu}\right) \\ T_{b,off} &= T_s\left(1 - e^{-\tau_\nu}\right) \end{aligned}$$

- T_b = observed brightness temperature (as function of ν)
- T_{bo} = brightness temperature of background source $\equiv T_c$ (continuum), which is essentially independent of frequency across narrow frequency range of 21 cm line.
- τ_ν = optical depth through HI layer, at frequency ν. $\tau_\nu \neq 0$ only right around the 21 cm line, where ν_0 = 1420.4 MHz.

(both for each individual frequency channel).

Some practical points:

1. Observe in a frequency *band* in a number of channels, together spanning a range in *velocity*.

$$\frac{\Delta v}{c} = \frac{\Delta \nu}{\nu}$$

   Total bandwidth is 100 - 1000 km s^{-1}, and the channel spacing, Δv is ~ 1 - 5 km s^{-1}.
2. The assumption here is that the "off source" position, the HI emission from the cloud is the same as in front of the continuum source (so the cloud is rather uniform). Evidently, this assumption works better if your radio telescope has high angular resolution $\longrightarrow$ need a large (single) dish.
3. A way to potentially avoid this problem is to have a background source that switches on and off (a pulsar). However, in practice, pulsars are generally too weak.
4. Do we really have only one HI cloud along the LOS, and is its temperature uniform? We really don't need only one cloud, provided the clouds have different *velocities*. However, the uniform T_s assumption is definitely a problem. (More about that later.) We can minimize the complexity in terms of many clouds along the LOS by observing clouds and background sources at high galactic latitudes.

-43- We observe at each frequency for which there is HI. Let the observed brightness temperature at source position after subtraction of continuum be given by

$$T_{b,on} - T_c \equiv T_{b,obs}$$

We get

$$T_{b,off} - T_{b,on} = -T_c e^{-\tau_\nu}$$

$$T_{b,off} - T_{b,obs} = T_c \left(1 - e^{-\tau_\nu}\right)$$

$$\left(1 - e^{-\tau_\nu}\right) = \frac{T_{b,off} - T_{b,obs}}{T_c}$$

Measure (observe) $T_{b,off}$, $T_{b,on}$, and T_c, then derive τ_ν. Once τ_ν is known, we can find T_s (the HI spin or kinetic temperature) because

$$T_s = \frac{T_{b,off}}{1 - e^{-\tau_\nu}}$$

Reminder of general point:

$$T_{b,off} = T_s \left(e^{-\tau_\nu}\right) = \begin{cases} \tau_\nu T_s & \text{for } \tau_\nu \ll 1 \\ T_s & \text{for } \tau_\nu \gg 1 \end{cases}$$

$T_{b,off} \leq T_s$ always!

There are many recent references that demonstrate the principles involved here; see, e.g. Dickey et al. ApJ 228, 465 (1979) and also Dickey and Lockman, Ann Rev of A & A 28 (1990).

-44- Plots

► 4.4 Conclusions

-45-

1. The absorption profile $(1 - e^{-\tau_\nu})$ is always sharper and simpler in structure than the emission line profile. Follows from Gauss analysis of profiles (so fit Gaussians).

$$T = \sum_{i=1}^{m} T_i \exp\left\{-\frac{1}{2}\left(\frac{v - v_i}{\sigma_i}\right)^2\right\}$$

   σ's are smaller for absorption profile. However, it is not clear as to what extent this is significant, except to note that absorption profiles are especially sensitive to cold cloud (cores) which presumably are narrower in velocity width. "You have to realize you are comparing apples & peas" *??* Given an HI distribution of column density $N(HI)$ = constant, say τT_s = 20 K (note that $\tau T_s \propto N(HI)$). Assume that T_s = 50, 100, 200 K. Then:

τT_s	T_s	τ	$1 - e^{-\tau}$	$T_B^\star$
20 K	50 K	.4	0.33	16.5
20 K	100 K	.2	0.18	18.0
20 K	200 K	.1	0.095	19.0

   ★ from $T_B = T_s(1 - e^{-\tau})$. So a variation of $\sim$ factor of 3 in absorption profile, $1 - e^{-\tau}$ corresponds to variation of only 16% in T_B. (Note: what would shape of $(1 - e^{-\tau})$ and T_B look like for single [layer cloud]?)

   Warm HI will only appear in emission, since the optical depth is too small to give significant absorption. Only recent deep observations begin to see absorption in warm HI.*

-46- 2. The variation in spin temperature, T_s, apparent in Fig 3-29, makes onewonder whether there is really one temperature in each cloud (there seems to be three in this profile). wonder whether there is really only one temperature in the cloud.

3. The range of T_s one finds from absorption line studies is 50 K $\leq T_s \leq$ 1000 K. However, the upper value is only a lower limit since we don't see absorption anymore for $T_s >$ 1000 K. So we have no handle on T_s above that.

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* Look for recent papers by Carl Heiles and by Jay Lockman.

4. One can compare T_{spin} with $T_{Doppler}$, the temperature derived from the velocity width of each line. It turns out that always $T_{spin} < T_{Doppler}$.

   $\longrightarrow$ Doppler motions that we see in a cloud generally do not correspond to thermal velocities; this is probably due to turbulence.

   Analysis of emission and absorption line profiles of HI across our Galaxy have led to a continuously evolving pictrue of the HI distribution that we have. The early desire to attribute all HI to "clouds" is understandable, but not always useful.[*]

► 4.5 Some relevant results

-47- 1. HI is *not* concentrated in a small number of giant clouds, as H_2 is; estimates of filling factor range from 20-90%?!

2. HI occurs roughly 50/50 in two important forms:
   - CNM = cold neutral medium $\sim$ 80 K
   - WNM = warm neutral medium $\lesssim$ 8000 K. About half of all HI gas is in the WNM phase. whose emission line profiles can be split into two components [†]:
     - narrow ($\sigma < 5$ km s^{-1})
     - wide ($5 < \sigma < 17$ km s^{-1})
       - ⋆ present in essentially all directions from solar neighborhood
       - ⋆ large vertical scale-geight, possibly up to 480 pc

3. Clouds are filaments and/or sheets, rather than spheres

4. At low galactic latitudes, it is not possible to distinguish between WNM and CNM because there is too much material along the line of sight.

5. **Problem with different temperatures along the line of sight!** Illustration: For one isothermal cloud:

$$T_s = \frac{T_B}{1 - e^{-\tau}}$$

   For two isothermal clouds, we have two spin temperatures: T_{s1} and T_{s2}.

-48- Let $T_n(v)$ be the (naively determined) spin temperature for n layers:

$$T_n(v) = \frac{T_{s1}(1 - e^{-\tau_1}) + T_{s2}(1 - e^{-\tau_2})e^{-\tau_1}}{(1 - e^{-\tau_1 - \tau_2})}$$

   We see that T_n depends on v and ranges from T_{s1} through T_{s2}, depending on τ_v. We would conclude that HI exists at temperatures intermediate to T_{s1} and T_{s2}!

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[*] Good summary of some of the main properties of MW HI distribution is given by Mulkami & Heiles in their 1988 review in Gal. & Extragal. radio astronomy

[†] Mebold, 1972

Special cases: Over velocity range where $N(HI)$ peaks:

1. $\tau_1 \gg 1, \tau_2 \ll 1 \longrightarrow T_n \approx T_{s1}$
2. $\tau_1 \ll 1, \tau_2 \gg 1 \longrightarrow T_n \approx T_{s1}\tau_1 + T_{s2}$. So we increased the cold temperature T_{s2} by a non-physical foreground temperature of warm gas. Happens a lot in practice!
3. $\tau_1 \ll 1, \tau_2 \ll 1 \longrightarrow T_n$ is a column density weighted *harmonic mean* temperature. Example: 80 K CNM + 8000 K WNM with equal column densities; then $T_n \approx 160$ K, so you *overestimate* T_{cool} and *greatly underestimate* T_{warm}.

► 4.6 Some additional points on HI[*]

Present research on Milky Way HI concentrates on:

1. Origin of high velocity clouds
2. Properties of smallest clouds/features
3. Search for HI clouds that may contain dark matter, but no stars

▷ *4.6.1 Determining distances*

-49- HI absorption measurements toward Galactic continuum sources provide (some) information on the distance to these sources, using the velocity of the HI absorption feature to derive a kinematical distance. The usual problem of distance ambiguity may still be a problem here as well, although the total HI column [density?] that is derived from the absorption measurements helps a little in distinguishing near and far distances.

▷ *4.6.2 Determining temperatures*

- WNM (two methods):
  1. HI absorption: complicated because τ is small, plus stray radiation. Also biases to low temperatures when cooler gas is along the LOS.
  2. UV absorption lines: local (< 200 pc) stars give $T \gtrsim 6000$ K for WNM

  So far, data are in agreement with 5000 K $\lesssim T \lesssim$ 8000 K, but more confirmation is needed.
- CNM: absorption spectra. We already pointed out correlation of T_{spin} with peaks in absorption profile (see fig. in handout, page 44). The interpretation is that HI clouds are *not* isothermal blobs. Usually we derive what we call T_{spin} of the cloud at the *center* of the absorption profile, so this is
-50- $T_{n,min}$. The absorption lines are *narrow*; for the velocity distribution/dispersion:

[*] Kulkami & Heiles

- ◦ Mode(σ_v) $\sim$ 0.75 km s^{-1}
- ◦ Mean(σ_v) $\sim$ 1.7 km s^{-1}

with a tail to higher velocities.
Cause of widths:

- ◦ Thermal motions: $\sigma_v(T) = \sqrt{T/212}$ km s^{-1}
- ◦ Turbulence

Emission lines typically show larger widths (not surprising, since absorption is most sensitive to the coldest cloud cores). Range in $T_{n,min}$: 20-250 K.

There seems to be a real distinction between the CNM and the WNM (although recent evidence may dispute this somewhat).

When does HI become opaque? Gaussian HI line profile, at center:

$$\tau = 5.2 \times 10^{-19} \frac{N_{HI}}{\Delta v T_s}$$

where $\Delta v = FWHM$ in km s^{-1}. Typically

- $T_s \gtrsim 50$ K, $\Delta v \sim 10$ km s^{-1}
- $\rightarrow N(HI) > 10^{21}$ cm^{-2} before $\tau > 1$

Present reasearch on MW HI concentrates on:

1. Origin of High Velocity Clouds
2. Properties of smalest clouds/features
3. Search for HI clouds that may contain DM but no stars

5. Atomic structure

-I 1- Purpose here is to give an overview of notation and the way atoms are built up, with structure of shells, transitions, etc.

► 5.1 Introduction

► 5.2 Hydrogen atom & hydrogen-like atoms (or ions)

-I 2- Example: energy of ground states:

- H = 13.6 eV
- He^{+} = 54.4 eV
- Li^{2+} = 122.5 eV

-I 3- We talked about the use of the reduced mass, μ, in the energy levels; this also produces a slight shift in the frquency of spectral lines of the same elements, but a different isotope. $\rightarrow$ *isotope effect* (ex: H, D, Tritium isotopes).

▷ 5.2.1 *Hydrogen spectrum*

Complications: Quantization of orbital angular momentum.

-I 4- Note: if central force field ≠ Coulomb force, then values of ℓ of each n are different, depending on the situation.

-I 5- For single electron atoms, these levels all have the same energy, but this is not exactly true because of relativistic corrections (which are very small).

Selection rules limit possible transitions. For electric dipole transitions: $\Delta\ell = \pm 1, \Delta m = 0, \pm 1$ has to be obeyed (follow from conservations of angular momentum).

► 5.3 Electron spin

-I 6- $\vec{J} = \vec{L} + \vec{S}$ = orbital angular momentum + electron spin angular momentum.

-I 7- But, since $\vec{J}_1$ and $\vec{J}_2$ can have different directions, there are different possible values of $\vec{J}$.

► 5.4
Spin orbit coupling

-I 8- Two possible orientations of electron spin with respect to orbital angular momentum lead to a doubling of energy levels of H-like atoms (except s-levels, where $\ell = 0$), also leading to doubling in the number of spectral lines.

Lines appear in pairs, close together, called *doublets*. Example: for Na D lines, λ = 5890Å, 5896Å. (Question: are atoms with filled shells + one electron also hydrogen-like? Doubling is due to spin-orbit coupling.)

Physically, the spin-orbit coupling produces an extra energy term for the electron, proportioned to $\vec{S} \cdot \vec{L}$.

We already discussed that $\vec{S}$ can have only two orientations with respect to $\vec{L}$, hence the doubling of the energy levels corresponding to $\vec{L}$ and $\vec{S}$ "parallel" (spin up) or "antiparallel" (spin down).

With spin-orbit coupling, we have quantum numbers ℓ, j, m, where m relates to J_z. Selection rules are: $\Delta\ell = \pm 1, \Delta j 0, \pm 1, \Delta m = 0, \pm 1$ ($\Delta j = 0$ are very weak since they require a reversal of spin to orbital angular momentum).

-I 9(a)- Question: what is the difference in energy levels "up" and "down"? (= W_{SL}).

Real life is yet more complicated; including a relativistic criteria, we would obtain:

$$W = W_n + \frac{|W_n| Z^2 \alpha^2}{n} \left(\frac{3}{4n} - \frac{1}{j + \frac{1}{2}} \right)$$

-I 9(b)- which implies that levels with equal n and j coincide: $n = 2$ $S_{\frac{1}{2}}$ and $P_{\frac{1}{2}}$ would coincide.

So the fine structure for H would look like:

► 5.5
Atoms with multiple electrons

-I 10- Since each electron influences the potential energy and "motions" of every other electron, we cannot talk about "the energy of this or that electron", but only about the energy of the entire atom.

▷ *5.5.1*
He-(like) atoms: two electrons

Already the Schrödinger equation is no longer exactly solvable.

-I 11- (Diagrams)

-I 12-

-I 13-

► 5.6
Electron configuration of atoms

-I 14-

► 5.7
Some more words about LS coupling

-I 15- (Table)

-I 16- Each term has a (slightly) different energy.

-I 17- Explanation for handouts:

-I 18- (printouts)

► 5.8
Transition rules

-I 19- Aka “selection rules”.

► 5.9
X-ray emission

-I 20- What if one were to excite an electron from the *closed* shell, instead of a valence electron? $\Rightarrow$ High energy transitions produce X-ray radiation. This does not typically occur in the ISM! Material would be ionized to high levels.

► 5.10
Zeeman effect

In a magnetic field, every spectral line of an atom with a single electron turns into a *triplet*, with three closely spaced components. For such atoms, the separation in frequency between the three lines is the same for all atoms and lines and is proportional to the magnetic field strength.

-I 21- This continues to be correct quantum-mechanically, implying also a quantization in the z-direction.

-I 22- Frequency difference…

-I 23- Parallel to the $\vec{B}$-field, only the two w_+ components are visible.…

-I 24- But, as we saw, as long as one only deals with spin, or orbital angular momentum of a single electron, it also applies.

-I 25- where $M_J = J_z$.

-I 26- A complete listing of such effects is given by…

-I 27- (Handout…)

-I 28- …

-I 29- …

6. HII regions

► 6.1 Introductory remarks

-51- Process: hot OB stars emit UV photons that can ionize the surrounding neutral H (and He) medium. In practice, this requires stars hotter than $\sim$ 30,000 K, aka. spectral type B0 or earlier. The physics for planetary nebulae (PN) is similar to that of HII regions, but central stars in PN may be much hotter and dimmer because there is "more stuff".

UV photons impart energy to gas by ionizing H and He. Excess kinetic energy of created free electrons is shared with ions and other electrons, *heating* the medium.

Relevant processes:

- photoionization
- electron-electron encounters
- electron-ion encounters
  - Excited ions
  - Recombination
  - Bremsstrahlung

H atom + photon $h\nu \longrightarrow p^+ + e^- + E_k$

$\Longrightarrow$ Since stars create a continuous stream of photons, a balance is reached between ionization and recombination.[*][†]

Field OB stars: were they born there or travel there somehow? Usually in groups (OB associations), dense clouds, etc.

Consequences: $\sim 10^4$ *K. Warm ionized plasma emitting various forms of radiation: recombination, free-free continuum, bree-bound continuum, 2-photon continuum, collisionally excited forbidden lines from "metals". If there is dust around, get MIR dust continuum. Runaway OB stars (like teenagers!)*

1 km ≡ 1 pc in 10^6 *years. (?)*

Q_o *[s^{-1}] = number of ionized photons emitted by OB star per second. Factor of 100 from O 9.5V to O 3V (luminosity classes).*

---

[*] Draine Ch 10, 13.1, 14, 15, 17, 18, 27, 28

[†] Osterbrock AGN^2

► 6.2
Strömgren Theory

-52- "It's only a model." Classical paper:Strömgren 1939 ApJ 89, 526

Basic result:

- Hot star in a uniform medium will ionize a spherical volume out to a certain radius, whose size is determined by:
  1. number of ionizing photons emitted by the star
  2. density of medium (determines recombination rate)
- There is a *sharp* boundary from the ionized to the surrounding neutral medium.

► 6.3
Simple derivation

Pure H nebula, uniform density. Hot star emits N_{Lyc} photons per second (Lyc = Lyman continuum), all at the *same* frequency, ν_o. The star will ionize the gas, but there is a balance:

$$\text{ionization} \longleftrightarrow \text{recombination}$$

Stationary ionization eqilibrium characterized by *degree of ionization*:

$$x(r) \equiv \frac{n_e(r)}{n_H}$$

where n_H = *total* hydrogen density (ions and neutral H atoms). $0 \leq x \leq 1$ ($x = 1 \rightarrow$ complete ionization; $x = 0 \rightarrow$ completely neutral). Problem: what is the shape of $x(r)$?

Important quantity: the Strömgren radius, R_{SO}, defined as

$$\frac{4}{3}\pi \alpha n_H^2 R_{SO}^3 \equiv N_{Lyc}$$

where α is the recombination coefficient (collisional process $\sim \rho^2 \rightarrow$ more encounters, see 6.3.2). The quantity N_{Lyc}[*] gives the total number of recombinations per unit time (all the magic is here!).[†]

or as I like to put it:

$$\frac{4}{3}\pi R_{SO}^3 = \frac{N_{Lyc}}{\alpha n_H^2}$$

The volume of the sphere is determined by the number of ionizing photons divided by the recombination coefficient.

---

[*] $N_{Lyc} = S_4$ in Spitzer's notation.
[†] S_4 formally should include contributions from the diffuse radation field.

▷ *6.3.1* *Description of photoionization equilibrium*

-53-

Overview:
Consider pure H nebula surrouding a single hot star. Ionization equilibrium:

$$n_{H^o} \int_{\nu_1}^{\infty} \frac{4\pi J_\nu}{h\nu} \sigma_\nu \mathrm{d}\nu = n_e n_p \alpha(H, T)$$

where

- n_{H^o} = neutral hydrogen density
- J_ν = mean intensity of radiation field
- $\sigma_\nu \approx 6 \times 10^{-18}$ cm^2 = ionization cross-section for H by photons with energy above threshold $h\nu_{01}$
- $\alpha(H, T) \approx 4 \times 10^{-13}$ cm^3 sec^{-1} = recombination coefficient
- n_e, n_p = electron and proton densities
- LHS: number of *ionizations* per second per cm^3
- RHS: number of *recombinations* per second per cm^3

To first order, not including radiative transfer effects:

$$4\pi J_\nu \;=\; \frac{R^2}{r^2} \pi F_\nu(0) \;=\; \frac{L_\nu}{4\pi r^2}$$

(where the far right term is the local radiation field). Order of magnitudes:

$$N_{Lyc} \;=\; \int_{\nu_1}^{\infty} \frac{L_\nu}{h\nu}\, \mathrm{d}\nu$$

- $N_{Lyc} \approx 5 \times 10^{48}$ sec^{-1} for O6 star

-54- Physical conditions: radiative decay from upper levels to $n = 1$ is quick $\longrightarrow$ nearly all neutral hydrogen will be in the ground level. Photoionization takes place from the ground level, and is balanced by recombination to excited levels, which then quickly de-excite by emission of photons.

The *photoionization cross-section* ($\sigma \propto \nu^{-3}$) is actually rather complicated to calcualte. Spitzer (who calls this erroneously the "absorption coefficient") gives:

$$\sigma_{f\nu} = \frac{7.9 \times 10^{-18}}{Z^2} \left(\frac{\nu_1}{\nu}\right)^3 g_1 f$$

where $g_1 f$ = Gaunt factor (from level 1 to free)*

Since $\sigma_\nu \propto \nu^{-3}$, higher energy photons will typically penetrate further into the nebula before being absorbed.

---

* see, e.g. Table 5.1 Spitzer (Draine Ch. 10); see also Fig 13.1 in Draine and Fig. 2.2 in handout (from Osterbrock).

▷ *6.3.2* *The recombination coefficient*

$\alpha_n(H,T)$ = recombination coefficient for *direct* recombination to level n (for hydrogen, H, at temperature T).

$$\alpha^{(n)} \equiv \sum_{m=n}^{\infty} \alpha_m$$

where m = other levels the electron cascades through to get to n.

Note: Ouside of level 1, higher energy not necessarily better for ionization. OIII is not necessarily from the same place as OII. Free electrons: there is a recombination coefficient for every level, depends on velocity of the electron.

-55- The *summed* recombination coefficient (α) is the relevant quantity because every recombination counts, regardless of the level n to which the electron recombines.

Larger velocities → smaller σ.

For a distribution $f(v)\mathrm{d}v$, $n_e f(v) v$ = number of electrons with velocity v passing through a unit area per second. Thus

$$\alpha_n = \int_o^\infty v\sigma_n(H,v) f(v)\mathrm{d}v$$

where $\sigma_n(H,v) \propto v^{-2}$ = recombination cross-section to the level n for electrons with velocity v.

$$f(v) = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 \mathrm{e}^{-mv^2/2kT}$$

(Maxwellian). Hence, $\alpha_n \propto \dfrac{1}{\sqrt{T}}$[*]

We are interested in the total recombination coefficient to all levels: However, for direct recombination to $n = 1$, a photon is generated which itself can ionize H again; so the recombination coefficient that we are really interested in is $\alpha^{(2)} \equiv \alpha_B$[†] *This refers to "case B": optically thick through Lyα emission. Almost always here! Most common situation. Case A: very low density gas.*

Neutral fraction *inside* HII region[‡]

-56- Consider pure H nebula, and "case B" (optically thick) recombination (see later).

- $h\nu$ = *Mean energy* of stellar ionization photons

---

[*] see e.g. Spitzer, Table 5.2

[†] see Draine section 14.2

[‡] see Draine section 15.3

- σ_{pi} = photoionization corss-section at freqeuncy ν
- $Q(r)$ = rate at which ionizing photons cross spherical surface
- Q_0 = rate at which ionizing photons are emitted by the star

In steady state:

$$Q(r) = Q_0 - \int_0^r n_H^2 \alpha_B x^2 4\pi (r')^2 \, \mathrm{d}r' = Q_0 \left[1 - 3 \int_0^{r/R_{SO}} x^2 y^2 \, \mathrm{d}y \right] \qquad (\bigstar)$$

- α_B = recombination rate for case B
- $x(r) \equiv \dfrac{n(H^+)}{n_H}$ = fractional ionization level = $\dfrac{n_e}{n_H}$ (x is a function of r).
- $y \equiv \dfrac{r}{R_{SO}}$ = radius in terms of the Strömgren radius.

At each point we need to balance recombinations per unit volume with rate of photoionizations per unit volume:

$$n_H^2 \alpha_B x^2 = \frac{Q(r)}{4\pi r^2} \, n_H \, (1-x) \, \sigma_{pi}$$

-57- Rewrite latter as:

$$\frac{x^2}{1-x^2} = \frac{Q(r)}{Q(0)} \frac{n_H \sigma_{pi} R_{SO}}{3y^2} = \frac{Q(r)}{Q(0)} \frac{\tau_{SO}}{3y^2}$$

where $\tau_{S0} = n_H \sigma_{pi} R_{SO}$ = 2880 $(Q_{0,49})^{1/3}$ $n_2^{1/3}$ $T_4^{0.28}$ for $\sigma_{pi} = 2.95 \times 10^{-18}$ cm^2. Since $\tau_{SO} \gg 1$, $1-x \ll 1$, so neutral fraction x is small and hence $x^2 \approx 1$.

$$1 - x \approx$$

For $x \approx 1$ in ($\bigstar$) on page -56- then:

and

At center of HII region:

At boundary of HII region:

HII regions have sharp boundaries (concept of Stromgren radius is useful!) Solving the equations iteratively actually shows that $Q(r)$ approaches 0 near $y \sim 1$ in exponential decay.* This is also solved by iteration.

* See page 59 for shape of $x(r)$ (fig 2.4 and 2.6).

-58- Comments:

1. Reality: include He, include diffuse radiation field generated from direct recombination to $n = 1$ (which can ionize an atom again!), realistic stellar energy spectrum below $\lambda = 912$Å; results are qualitatively the same.
2. Is assumption of a static ionization equilibrium realistic? No: HII region is overpressureized compared to the surrounding medium: T_e and n higher $\longrightarrow$ HII region expands*. Expansion of the ionization front happens rather slowly, $v \propto 1-2$ km s^{-1}, dynamical timescale. $\dfrac{R_{SO}}{v} \sim 10^7$ years, of order liftime of HII region. So ionization equilibrium does hold.
3. Recombination time scale = ionization time scale:

$$t_{ionization} \equiv \frac{\frac{4}{3}(R_{SO})^3 n_H}{Q_o} = \frac{1}{\alpha_B n_H} = \frac{1.22 \times 10^3}{n_2} \text{ years} = t_{recombination}$$

   ($n_2 = n_H$ in units of 100). The two are identical! So $t_{recombination} \propto \frac{1}{n_e}$ Key: for $n_H >$ 0.03 cm^{-3} $t_{recombination} <$ lifetime of massive stars ($\lesssim 5$ Myr).
4. As we will see later, each Lyc photon results in ~ 0.35 Hα photons begin produced in this ionization $\rightarrow$ recombination equilibrium.
5. Next: influence of dust on R_{SO}

-59- (plots)

-60- (printout)

-61- …

-62- …

► 6.4 The spectrum of an HII region

▷ 6.4.1 *Sources of continuum Radiation*

-63-
- Two-photon decay
- Free-free emission:
- Free-bound emission
- Dust particles

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* First discussed by Kahn (1954)

Two-photon decay:

Transition from $n = 2 \rightarrow n = 1$ ($\ell = 0 \rightarrow \ell = 0$) is strictly forbidden ($\Delta\ell \neq 0$). The actual process happens in two steps, with a virtual intermediate phase. Two photons are emitted whose joint energy adds up to the energy of a Lyα photon = 3/4 ionization energy of H. The two-photon continuum is symmetric about $\nu_{12}/2$ if expressed in photons per unit frequency instead.[*]

Free-free emission:

Thermal Bremsstrahlung[†]= continuous emission and absoprtion by thermal (Maxwellian velocity distribution) electrons due to encounters between electrons and positive ions.

Classical theory: electron emits a single narrow Em pulse in time, with no oscillation in E. $\rightarrow$ FT is broad, almost the FT of a δ function. $\rightarrow$ Emission coefficient j_ν is nearly independent of frequency, up to a *cutoff*
-64- frequency, which corresponds to the Maxwellian velocity distribution of electrons. So cutoff frequency is given roughly by $h\nu \sim kT_e$. (HII regions: $T_e \sim 10^4$ K, so $h\nu \approx 0.87$ eV and $\lambda \approx 1.4\mu$m). In practice, dust emission dominates at IR wavelengths, so it is unlikely to observe the free-free spectrum this far.

Emission coefficient:

$$\begin{aligned}
j_\nu &= \frac{8}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{Z_i^2 e^6}{m_e^{3/2} c^3 (kT)^{1/2}} g_{ff} n_e n_i e^{-h\nu/kT} \\
&= 5.44 \times 10^{-39} \frac{g_{ff} Z_i^2 n_e n_i}{\sqrt{T}} e^{-h\nu/kT} \quad [\text{erg cm}^{-3}\ \text{s}^{-1}\ \text{sr}^{-1}\ \text{Hz}^{-1}]
\end{aligned}$$

where $e^{-h\nu/kT}$ is the exponential cutoff due to kT_e and g_{ff} is the Gaunt factor for free-free transitions[‡].

Total amount of energy radiated in free-free transitions per cm^3 per second:

$$\begin{aligned}
\epsilon_{ff} &= 4\pi \int j_\nu \, d\nu \\
&= \left(\frac{2\pi kT}{3m_e}\right)^{1/2} \frac{2^5 \pi e^6}{3hm_e c^3} Z_i^2 \ n_e n_i \ \langle g_{ff} \rangle \\
&= 1.426 \times 10^{-27} \ Z_i^2 \ n_e n_i \ \sqrt{T} \ \langle g_{ff} \rangle \qquad \left[\text{erg cm}^{-3}\ \text{s}^{-1}\right]
\end{aligned}$$

Remember that the corresponding absorption coefficient κ_ν is related to j_ν by Kirchoff's law, since we

[*] See Osterbrock, pp 89-93 for more info.
[†] §3.5 Spitzer, Drane ch. 10
[‡] see Draine, figure 10.1

deal with *thermal* emission (*not* blackbody though!) $j_\nu = \kappa_\nu B_\nu(T)$ which leads to

$$\kappa_\nu = \frac{4}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{Z_i^2\, e^6\, n_e n_i\, g_{ff}}{c\, m_e^{3/2}\, (kT)^{3/2}\, \nu^2}$$
$$= 0.1731 \left[\left(1 + 0.130 \log \frac{T^{3/2}}{\nu}\right)\right] \frac{Z_i^2\, n_e n_i}{T^{3/2}\, \nu^2} \qquad \left[\mathrm{cm}^{-1}\right]$$

-65- Important in practical situations:

1. Free-free emission is generally observed in the *radio* regime for HII regions
2. For a source at constant T:

$$I_\nu = S_\nu \left(1 - e^{-\tau_\nu}\right)$$
$$S_\nu = \frac{j_\nu}{\kappa_\nu} = B_\nu(T_e)$$
$$\tau_\nu = \int \kappa_\nu \mathrm{d}s$$

   And if T_e is independent of path length, then

$$\tau_\nu \sim \int n_e n_i\, \mathrm{d}s \approx \int n_e^2\, \mathrm{d}s = EM \qquad \left[\mathrm{pc\ cm}^{-6}\right]$$

   where EM = emission measure. Also $\tau_\nu \propto \nu^{-2.1}$ (the 0.1 is due to the log term).

Optically thin emission

$$I_\nu = \tau_\nu S_\nu \propto \nu^{2.1} B_\nu(T_e)$$
$$B_\nu(T_e) = \frac{2\nu^2 kT}{c^2}$$
$$\boxed{I_\nu \propto \nu^{-0.1}}$$

Almost *flat* spectrum.

Optically thick emission

Since $\tau_\nu \propto \nu^{-2.1}$, the source will become optically thick at some low frequency:

$$I_\nu = S_\nu = B_\nu(T_e) \quad \text{for } \tau_\nu \to \infty$$

$$\boxed{I_\nu \propto \nu^{+2}}$$

See enclosed handout. The turnover frequency for HII regions is $\sim$ few GHz, depending on n_e and T_e.

-66- (Plots)

-67- We can derive the turnover frequency as follows: $\tau_\nu = \int \kappa_\nu \, \mathrm{d}s$. We had already derived an expression for

$$\kappa_\nu = 0.1731\{\ldots\}\frac{Z^2 n_e n_i}{T^{3/2}\nu^2} \quad [\mathrm{cm}^{-1}]$$

this can be rewritten as

$$\kappa_\nu \approx 0.21 a(\nu, T_e) T_e^{-1.35} \nu^{-2.1} n_e n(H^+ + H^+$$

for practical interesting cases in the ISM. This leads to

$$\tau\nu = 6.5 \times 10^{17} a(\nu, T_e) T_e^{-1.35} \nu^{-2.1} (EM)$$

- EM = emission measure = $\int n_e^2 \, \mathrm{d}s$
- $a(\nu, T_e)$ = slowly varying factor, $\leq$ 10% from unity

The turnover frequency is defined where

$$\tau_{\nu,to} \equiv 1$$

$$\rightarrow EM = 1.5 \times 10^{-18} \frac{1}{a} T_e^{1.35} \nu_{to}^{2.1}$$

So we can measure ν_{to} and EM, and then *derive* $\langle n_e \rangle$.

Also note that in case the emission is optically thick, the observed T_b is a direct measure of T_e! (See handout.)

At higher temperatures, Bremsstrahlung produces X-ray continuum emission.

Free-bound continuum radiation

-68- Produces source of (weak) continuum radiation in optical wavelength region (also in IR and radio, but free-free is much stronger there). Most important: free-bound emission from recombination of H and He^+.

HI free-bound continuum recombination of free electrons with velocity v to levels with $n \geq n_1$;

$$h\nu = \frac{1}{2} m v^2 + X_n; \qquad h\nu \geq X_{n_1} = \frac{h\nu_0}{n_1^2}$$

X_n = ionization potential of level n. The emission coefficient is given by

$$j_\nu = \frac{1}{4\pi} n_p n_e \sum_{n=n_1}^{\infty} \sum_{L=0}^{n-1} v \sigma_{nL}(H^0, v) f(v) h\nu \frac{\mathrm{d}v}{\mathrm{d}\nu}$$

σ_{nL} = recombination cross section.

This is fully discussed in Osterbrock, pages 86-88, who lumps free-free and free-bound optical continuum together in one function (which he confusingly also refers to as the emission coefficient), $\gamma_\nu(H^0, T)$ such that

$$j_\nu \equiv \frac{1}{4\pi} n_p n_e \gamma_\nu(H^0, T)$$

This optical continuum is quite weak, although the shape of γ_ν is interesting; look it up![*]

▷ *6.4.2* *Line Radiation*

-69- Two types:

1. Recombination lines, predominantly from H and He (which don't produce collisionally excited lines)
2. Collisionally excited lines in heavy elements. These are fine-structure lines, often forbidden, yet strong. (Recombination line spectra from heavy elements do exist, but are too weak)

Handout: spectra of M31, M33, M101 HII regions, with same telescope and instrument.

Recombination lines:

- emission coefficient: $j_\nu = \dfrac{A_{mn'} N_n h\nu_{nn'}}{4\pi} \phi(\nu)$
- absorption coefficient: $\kappa_\nu = \dfrac{h\nu_{nn'}}{c} \left[N_{n'} B_{n'n} - N_n B_{nn'} \right] \phi(\nu)$

<u>Question</u>: what are the level populations N_n and $N_{n'}$? Using the b-factors we can compare N_n with N_n^*, the level population in LTE, etc. We get:

$$b_n \equiv \frac{N_n}{N_n^*}$$

$$N_n^* = \left(\frac{h^2}{2\pi m k T_e} \right)^{3/2} n^2 \exp\left(\frac{\chi_n}{k T_e} \right) N_e N_p$$

for H (= Saha equation). χ_n = ionization potential for level n.

To find the values of N_n for all n we again use statistical equilibrium.

(I) Population that disappears from level n per second per cm^3:

[*] Figure 4.1 in Osterbrock (page 68-A in notes) and figure 10.2 in Draine

(II) Population that appears in level n per second per cm^3:

As you see, it can get quite complicated. By equating (I) and (II), we get an equation for each level n which is linear in the vector $\{N_n\}$. However, we need to solve for all levels at the same time, which leads to ∞ many equations.

-71- The calculation is still highly complicated. We can, however, derive some general properties of $\{N_n\}$

Radio recombination lines

Not very common, lines tend to be very faint.

-72- So:

Optical and IR recombination lines

For small n, $|E_{n+1} - E_n|$ is larger, so collisions become less important while the A coefficients get bigger.

Collisionally excited lines in heavy elements:

-79- Generally: electrons don't have enough energy to excite ions from the ground state to their first excited states. However, many ions have multiplets in their ground state, due to the coupling of the electrons individual (orbital and spin) angular momenta.

Many ions have incomplete "p-shells"; (there is not really a p-shell, but we talk about electrons which have $\ell = 1$, corresponding to p). Remember:

H	1s		
He	1s^2		
C	1s^2	2s^2	2p^2
O	1s^2	2s^2	2p^4

For $n = 2$, there is room for 6 p-electrons, since each $n\ell$ has a number of m_ℓ and m_s combinations:

$$\left.\begin{array}{l} m_\ell : 2\ell + 1 \\ m_s : \pm\frac{1}{2} \end{array}\right\} 2(2\ell + 1)$$

So as long as $\ell = 1$ is possible (meaning $n > 1$), we have 6 potential "p-shell" electrons, *always*.

The enclosed diagram shows the 3 most typical structures of energy levels for various elements in different ionization stages. For [OIII] we have, in particular: 2 electrons in "p-shell", $\ell_1 = \ell_2$, so $L = 0, 1, 2$

-81- One can set up 5 equations of statistical equilibrium for each of these levels. This shows that both the ^1S_o and the $^1D_2$ levels are populated essentially exclusively by collisions from ^3P level.

So, not surprisingly, the ratio

$$\frac{N(^1S_0)}{N(^1D_2)} \quad \propto \quad \exp\left(\frac{-\Delta E}{kT}\right)$$

where ΔE = energy difference between $^1S_0$ and $^1D_2$

Also, observed flux in λ4363 line is:

$$S(4363) \quad \propto \quad \frac{4}{3}\pi R^3 N('S_0) A_{4363} h\nu$$

and similarly for the 4959 and 5007 lines.

The net result, obtained by solving the equations of statistical equilibrium is:

$$\frac{S(4959) + S(5007)}{S(4363)} = 8.3 \exp\left(\frac{3.3 \times 10^4}{T_e}\right)$$

so this line ratio provides a direct measure of the electron temperature.

A similar relationship exists for [NII] (6548, 6583, and 5754) lines.

In general: Emission lines arising from ions, such as O^{++} and N^{+} that have upper energy levels that have considerable different energies are useful for estimating T_e. Vice versa, as we will see, ions with closely spaced upper enrgy levels provie little info on T_e, but can be used to derive n_e (examples: [SII], [OII] lines).

Notes about T_e determinations:

1. Look at
2. Collisional
3. $^1D_2 \rightarrow {}^3P_0$ transition

► 6.5 Types of HII Regions

-85- In general:

- HII regions are associated with molecular clouds and dark nebulae.
- Structure:
    - “Blister model” – cavity inside GMC
    - “Champagne model” – half cavity at edge of GMC

There are also *compact* HII regions, which are only visible at radio and FIR wavelengths. These are very young objects, often associated with H_2O and OH masers, infrared sources, molecular lines of complex molecules, etc.

The *range* in properties of HII regions is enormous*.

Modeling of HII region spectra: the probably best known code is CLOUDY, developed by G. Ferland. You can in put an ioinizing spectrum from a star or other, a gas distribution, metallicity, etc., and then calculate the detailed emission line spectrum for the region.

HII regions can be density-bounded or radiation-bounded. rad: run out of photons before running out of gas. density: run out of material before using up photons. No clear boundary.

* Kennicut, 1984

7. Interstellar absorption lines in stellar and quasar spectra

-89- Discovered in 1904, *before we were aware of dust, or that MW and M31 were separate galaxies.* Typical optical absorption lines discovered later include Ca^+, Na^o, K, Ti^+, and some molecular lines, e.g. CH^+, CH, CN. Lines from more abundant atoms and molecules (H, H_2, C, N, O, etc.) were only found after Copernicus was launched (early 70s) because $\lambda < 3000$Å for these lines, so they have to be observed from space.

Key: Since the absorption process is proportional to the incoming intensity, we can find very small column densities of ISM, provided the background source is strong enough.

Absorption of optical photons generally only occurs from the ground level of the atom, ion, or molecule, because only these levels are populated in most conditions (remember that kT = 0.86 eV for T = 10^4 K, or 8.6 eV for T = 10^5 K). This implies that in the formation of absorption lines, stimulated emission from the upper level does not play a role.

O^+ in HII region sits in ground state, can assume ions in ISM also stay in $n = 1$. Pure absorption foreground screen. $I = S(\lambda)(1 - e^{-\tau_\lambda})$, can leave the exponent out ($\rightarrow 0$).

This is not the case whenever excitation conditions are such that absorption lines do arise from excited atoms or molecular levels. Usually collisions would be responsibe for the excitation, so keep an eye on kT.

-90- Generally, interstellar absorption lines are *narrow* and *complex* in structure. So we require very high resolution to resolve them:
$\left(\frac{\lambda}{\Delta\lambda} > 3\times10^5\right) \Leftarrow$ ideal, but rarely achieved. *res: couple thousand*

> Remember that $\frac{\Delta\lambda}{\lambda} = \frac{\Delta v}{c}$ and for $\frac{\lambda}{\Delta\lambda} \geq 3\times10^5 \rightarrow v \leq 1$ km per second. Which implies $\Delta\lambda \sim 0.02$ Å at 6000 Å and $\Delta\lambda \sim 0.0033$ Å at 1200 Å

► 7.1 Theory of the formation of absorption lines in the ISM

Ideal background source: bright (especially UV), with a featureless continuum. $\Rightarrow$ Quasars and AGN or hot stars. Stars tend to be more complicated, though quasars sometimes have their own lines as well.

▷ *7.1.1*
Equivalent width

The high spectral resolution that is required to resolve a line is often unattainable. However, even if we cannot *resolve* the absorption line, we can still infer some important properties about the ISM. As long as the resolution is sufficient to separate absorption lines at different velocities, coming from different clouds along the line-of-sight, or if only one component is present, we can use the *area* of an absorption line, or the *equivalent width*, EW, the width of an absorption line which would absorb 100% everywhere, and which would have the same area as the hatched area. Note that EW is always defined relative to I_o; we *divide* by I_o.

-91- Note also that if we decrease the resolving power of the spectrograph, EW does not change.

Theoretical expression for EW:

$$\tau_\nu = \int_0^\infty \kappa_\nu \mathrm{d}r; \quad \kappa_\nu = h\nu_{u\ell}\frac{n_\ell B_{\ell u} - n_u B_{u\ell}}{c}\phi(\nu)$$

(using "energy density" definition of B coefficients).

- u = upper level
- ℓ = lower level.

If stimulated emission can be neglected, we have:

$$\kappa_\nu = \frac{h\nu_{u\ell}B_{\ell u}}{c}n_\ell\phi(\nu) \equiv \frac{\pi e^2}{m_e c} f_{\ell u} n_\ell \phi(\nu)$$

- $f_{\ell u}$ = *oscillator strength (historical terminology)*
- $\dfrac{\pi e^2}{m_e c}$ = "classical cross-section"* $\sim$ *Einstein coefficient.*

Use $|\mathrm{d}\nu| = \dfrac{c}{\lambda^2}|\mathrm{d}\lambda|$ and we obtain:

$$\begin{aligned} EW &= \int_o^\infty \left[1 - e^{-\tau_\nu}\right] \mathrm{d}\lambda \\ &= \frac{\lambda^2}{c}\int_o^\infty \left[1 - e^{-\tau_\nu}\right] \mathrm{d}\nu \\ &= \frac{\lambda^2}{c}\int_o^\infty \left[1 - \exp\left(-\int_0^\infty \left(\frac{\pi e^2}{mc}\right) f_{\ell u} n_\ell \phi(\nu) \mathrm{d}r\right)\right] \mathrm{d}\nu \end{aligned}$$

Now assume: $\phi(\nu)$ independent of r for the particular cloud that we are observing. Then:

$$EW = \frac{\lambda^2}{c}\int_0^\infty \left[1 - \exp\left(-\frac{\pi e^2}{mc} f_{\ell u}\phi(\nu) N_\ell\right)\right] \mathrm{d}\nu$$

with $N_\ell = \int n_\ell \,\mathrm{d}r$ = *column density* of atoms in level ℓ. The quantity $\phi(\nu)N_\ell$ is the key part of this equation.

-92- Note: This situation is very different from the discussion of HI 21-cm abs lines where the upper level *is* populated, and we had to include stimulated emission to first order.

We next assume a Gaussian velocity distribution (Maxwellian) to describe the distribution of radial velocities:

$$f(v_{rad}) = \frac{1}{\sqrt{\pi} b} \exp\left[-\left(\frac{v_{rad} - v_o}{b}\right)^2\right]$$

* see for example Rybicki and Lightman, section 3.6

where $b = \sqrt{\dfrac{2kT}{m}}$ (only valid for thermal motions) and $\phi(\nu)$ becomes:

$$\phi(\nu) = \frac{1}{\sqrt{\pi}} \frac{1}{\Delta\nu_b} H(a, u)$$

$$a \equiv \frac{\Gamma}{4\pi\Delta\nu_b}; \quad u = \frac{\nu - \nu*}{\Delta\nu_b}; \quad \Delta\nu_b \equiv b\frac{\nu_o}{c}; \quad \nu* = \nu_o(1 - \frac{\nu_o}{c})$$

ν^* is the central frequency of the line. $b^2 = 2\sigma^2$ where σ = velocity dispersion, and a is a measure of where the Gaussian profile levels off to the Lorentzian wings[*] *(transition of Gaussian line core to the damping wings)*.

► 7.2 The Curve of Growth

What is all this good for?

For a given spectral line, we can measure EW from observations. Free parameters: N_ℓ, b, and a. For a given cloud, b and a are specified, and we can plot EW as a function of N_ℓ. In practice, one plots $\log(W/\lambda)$ against $\log(N_\ell f_{\ell u}\lambda)$.[†] This is the *curve of growth*.

We can distinguish three regimes in a curve of growth.

-93- I. $EW \propto N_\ell$ (linear)
II. EW almost independent of N_ℓ
III. $EW \propto \sqrt{N_\ell}$

Turnover points between the regimes:

- Point A: location determined by N_ℓ/b
- Point B: location determined by value of a

▷ 7.2.1 *Regimes in the curve of growth*

I. Here $\tau_\nu \ll 1$ everywhere, which implies that all atoms see the same incoming continuum intensity. $1 - e^{-\tau_\nu} \approx \tau_\nu$ so

$$EW = \frac{\lambda^2}{c}\left(\frac{\pi e^2}{mc}\right) f_{\ell u} N_\ell$$

[*] see notes from before, p. 34

[†] example: figure 3.2 in Spitzer

and

$$\frac{EW}{\lambda} \propto N_\ell f_{\ell u} \lambda$$

II. Center of line becomes saturated, so absorption reaches 100%. Because the Voigt function has steep edges (not wings!) in the central part which is dominated by Doppler motions, EW increases only slowly with N_ℓ.

III. Now if we keep increasing N_ℓ, the optical depth in the wings starts playing a role. Here, $EW \propto \sqrt{N_\ell}$[*]

▷ 7.2.2
Turnover points in the curve of growth

-94- What determines the location of the turnover points?

- Point A (I $\rightarrow$ II): This is where the gas becomes optically thick, which is determined by column density N and the profile function $\phi(\nu)$, in particular for the line center $\phi(\nu*) \propto 1/b$, so $\tau \propto N_\ell/b$
  Example: What happens if b increases? τ becomes smaller because the velocity dispersion of the gas is bigger, so the line is not saturated as quickly. So the linear part of the curve of growth is larger, point A moves toward the upper right.
- Point B (II $\rightarrow$ III): This is mainly determined by the value of a, which specifies where the steep edges of the Lorentz profile go over into the extended wings. So essentially Γ, where $\Gamma \sim 1/\tau_0$ (τ_0 = lifetime of the level)

▷ 7.2.3
Growth curves in practice

What do observations tell us?

I. Suppose you only observe one interstellar line, but you managed to resolve it completely. Then you know $1 - e^{-\tau_\nu}$ and can find b and N_ℓ.

II. What if you can't resove the line and can only measure EW? EW is determined by N, b, a,
-95- and $f_{\ell u}$ (which is known from laboratory experiments). Without further knowledge, there is a lot of ambiguity since similar values of EW can be produced by different sets of N_ℓ, a, and b. However, if you know that your line is optically thin, a and b are not important, and your EW immediately gives N_ℓ, provided you know $f_{\ell u}$.

III. What if you are not sure that your line is optically thin? Generally a is very small, and the damping wings are weak; center of line profile is Gaussian, and point A is not influenced by the values of a, as we saw before.

[*] See, e.g. Mihalas, Stellar Atmospheres, section 10.3

Suppose now that we have two lines, originating from the same level E_j, with equivalent widths W_1 and W_2. N_ℓ and b would be similar for both. If we know both $f_{\ell u}$ values, we can derive N_ℓ and b.

Examples of such cases:

- Ca^+ H and K lines λ3934 and λ3968 from the same ground level $^2S_{1/2}$
- Na^o D lines near 5800Å and weak doublet near 3300Å

► 7.3 UV absorption lines from H and H_2

-96- H: $^2S_{1/2}$ ground level. Lyman α is the strongest line, 1216 Å. The center is "burned out" ($e^{-\tau_\nu} = 0$). Only very wide damping wings are visible, so we are in part III of the curve of growth.

Line profiles have been observed with rocket Hights, satellites* and on the ground in quasar absorption line systems that are redshifted into the visible and IR. We use the line shape, rather than the equivalent width, to derive N(HI) by fitting a curve.

$$\frac{I_\nu}{I_\nu(0)} = \exp\left[-\frac{e^2\lambda^4 f_{jk} N_\ell \Gamma_k}{m_e c^3 (\Delta\lambda)^2 4\pi}\right] \quad (\bigstar)$$

$N_\ell = N(^2S_{1/2}) = N(HI)$. This expression follows from

$$I_\nu = I_0 e^{-\tau_\nu}$$

$$\tau_\nu = \frac{\pi e^2}{mc} f N_\ell \phi(\nu)$$

$$\phi(\nu) = \frac{\frac{\Gamma}{4\pi^2}}{\left(\frac{\Gamma}{4\pi}\right)^2 + (\Delta\nu)^2}$$

(damping wings) and assuming that $\Delta\nu \gg \Gamma$ (which is the condition that $a \ll 1$, or Doppler width $\gg$ natural line width, which is the case in the ISM.)

-97- We obtain:

$$\phi(\nu) \simeq \frac{\Gamma}{4\pi^2} \text{ for } \Delta\nu \gg \Gamma$$

$$\Delta\nu = \nu_0 - \nu = \frac{c}{\lambda_0} - \frac{c}{\lambda} = c\left(\frac{\lambda - \lambda_0}{\lambda\lambda_0}\right) \approx c\left(\frac{\Delta\nu}{\lambda^2}\right)$$

so

$$\tau_\nu = \frac{\pi e^2}{mc} f N_\ell \frac{\Gamma}{4\pi^2}\frac{1}{c^2}\frac{\lambda^4}{(\Delta\lambda)^2} = \frac{e^2}{mc^3} f N_\ell \frac{\Gamma}{4\pi}\frac{\lambda^4}{(\Delta\lambda)^2}$$

---

* Copernicus, IUE, FUSE, and HST

which gives (★).

H_2 line from lowest level; ortho & para H_2, with rotational energy levels, with quantum number $J = 1$ (ortho) and $J = 2$ (para).[*] The actual transition is an electronic transition.

The molecule produces a number of lines shortward of Lyα; wavelengths:

- 1092.19 Å
- 1092.73 Å
- 1094.05 Å

"Lyman bands" (B $\longrightarrow$ X in figure).

8. Dust

-104- Size range: "standard large grains" follow exponential size distribution, extending from (theoretically) $a_{min} \approx 0.01\mu$m to $a_{max} \approx 0.25\mu$m (huge!)[†] $n(a) \propto a^{-3.5}$. A grain with size 0.1 μm = 1000 Å would contain $\sim 10^{10}$ atoms. Grains are really "solids"! *(behave like solids, even though tiny.)*

The size distribution was later extended downward to much smaller particles to account for the excess 12 μm and 25μm emission observed by IRAS; this excess indicated the presence of warm (several hundred K) dust, and it is not possible to heat large dust grains to such high temperatures (see below). It is also possible that at the smallest sizes, we have *complex molecules* rather than grains. A favorite candidate is "PAH", or polycyclic aromatic hydrocarbons, which resemble car soot. The small grains may be as small as 5 Å (0.0005 μm).

Grains can *scatter* or *absorb* incident UV and optical photons; at the FIR wavelengths at which they emit,
-105- the radiation is optically thin (so they do not *absorb* FIR photons effectively).

By absorbing a photon, a grain increases its energy and heats up from T_0 to T_1:

$$h\nu_{ph} = \frac{4}{3}\pi a^3 \int_{T_0}^{T_1} C_V(T)\mathrm{d}T$$

For small grains, the jump from T_0 to T_1 can be very large (several hundred K), depending on $h\nu_{ph}$, a, and C_V (the heat capacity per unit volume). These grains are said to be stochastically heated to high temperatures. As a result, the grains are found over a wide range in T.[‡] Standard large grains are not much affected by single photons. Instead, they reach a relatively constant equilibrium temperature when embedded in a specified radiation field.

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[*] J only for proton spin, see Draine Ch. 5

[†] Mathis, Rumple, and Nordsieck (1977)

[‡] handout, Fig. 2 from Draine (1990)

► 8.1 Absorption efficiency: the Q parameter

The absorption efficiency of grains is described by the *Q parameter*, defined as:

$$Q_\nu^{abs} = \frac{\sigma_\nu}{\pi a^2} \quad (= 1 \text{ for a blackbody})$$

σ_ν = effective absorption cross-section at frequency ν
πa^2 = geometrical area

Q = 1 for a blackbody. In practice, $\sigma_\nu^{abs} \sim \pi a^2$ at UV wavelengths, and $\sigma_\nu^{scat} \sim \sigma_\nu^{abs}$. For $\lambda \gg a$ (in IR), $\sigma_\nu^{scat} \to 0$.

-106- The optical depth becomes:

$$\tau_\nu = \int_R \alpha\nu \mathrm{d}r = \int_R n_d \pi a^2 Q_\nu \mathrm{d}r = \int_R \rho_d \kappa_\nu \mathrm{d}r$$

- κ_ν = mass absorption coefficient
- ρ_d = mass density of a grain

We have:

$$\kappa_\nu = \frac{n_d}{\rho_d}\pi a^2 Q_\nu$$

Typical numbers:

$\frac{M_d}{M_g} \sim 0.005 - 0.01$ (“dust-to-gas” ratio is $\sim$ 1%)

$\langle a \rangle \sim 0.1\mu\mathrm{m}$

$\rho_d \sim 3$ g cm^{-3}

$Q_{100\mu m} \sim \frac{1}{644}$ for particular model

We can also define the absorption of dust per H-atom:

$$\sigma_\nu^H \equiv \frac{\tau_\nu}{N_H} = \frac{1}{X(H)}\frac{M_d}{M_g}M_H\kappa_\nu \quad [\mathrm{cm}^2]$$

$M_d = n_d m_d$ = dust mass density

$M_g = \left(n_H + 2n_{H_2}\right)\frac{m_H}{X(H)}$ = gas density

X_H = fractional mass abundance of hydrogen (~ 0.7)

► 8.2
Calculating dust mass from FIR fluxes

The volume emission coefficient of dust grains is:

$$j_\nu = n_d \pi a^2 Q_\nu B_\nu(T_d) = \rho_d \kappa_\nu B_\nu(T_d) = \alpha_{nu} B_\nu(T_d)$$

(Kirchoff's law!)

-107- The emission is optically then, so

$$I_\nu = \int_R j_\nu \mathrm{d}r = \int \rho_d \kappa_\nu B_\nu(T_d) \mathrm{d}r$$

T_d is measured from $Q_\nu B_\nu(T_?)\ldots$?

What we measure is flux, $F_\nu = \int I_\nu \cos\theta \mathrm{d}\Omega \approx \int I_\nu \mathrm{d}\Omega$ for small θ.

For an optically thin, spherical cloud:

$$L_\nu = \int_{volume} \epsilon_\nu \mathrm{d}V; \quad \epsilon_\nu = \int j_\nu \mathrm{d}\Omega$$

$$F_\nu = \frac{1}{4\pi D^2} \int_\Omega \mathrm{d}\Omega \int_V \rho_d \kappa_\nu B_\nu(T_d) \mathrm{d}V$$

$$\boxed{F_\nu = \frac{M_d \kappa_\nu B_\nu(T_d)}{D^2}} \quad (\star)$$

So in practice:

- Measure F_ν at $\lambda = \lambda_1$
- Measure F_ν at $\lambda = \lambda_2$
- Derive T_d from fit to spectrum (see below)
- Derive M_d from $(\star)$

-108- ⇒ Temperature variations along the light of sight cause severe problems[*] But first some more comments on spectra emitted by dust grains. We saw

$$Q_\lambda = \frac{\sigma_\lambda}{\pi a^2} \equiv \frac{A_n}{\lambda^n}$$

- A_n = constant
- n = emissivity law exponent. In practice, n = 1 to 2.

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* See handout from Draine (1990).

Consequence: $\tau_\lambda \propto \lambda^{-n}$ (verify). And since emission is optically thin:

$$I_\lambda \propto \lambda^{-n} B_\lambda(T_d)$$

This is called a *modified blackbody spectrum*. *(Greenhouse effect!)*

In terms of frequency and luminosity:

$$I_\nu \propto \nu^{3+n} \frac{1}{e^{h\nu/kT} - 1} \propto F_\nu$$

$$L_{IR} \equiv 4\pi D^2 \int F_\nu \, \mathrm{d}\nu \propto \int_0^\infty \frac{\nu^{3+n}}{e^{h\nu/kT} - 1} \, \mathrm{d}\nu$$

$$x = \frac{h\nu}{kT} \rightarrow \mathrm{d}\nu = \frac{kT}{h} \mathrm{d}x$$

$$L_{IR} \propto \int_0^\infty T^{3+n} \frac{x^{3+n}}{e^x - 1} \, \mathrm{d}xT = T^{4+n} \int_0^\infty \frac{x^{3+n}}{e^x - 1} \, \mathrm{d}x$$

$$\boxed{L_{IR} \propto T^{4+n}}$$

-109- The luminosity rises steeply with dust temperature! How can we measure the value of n? At long wavelengths, $B_\nu(T_d) = \frac{2kT}{c^2}\nu^2$ (RJ tail) so $I_\nu \propto \nu^{2+n}$ or $I_\lambda \propto \lambda^{-(4+n)}$ Unfortunately, measuring the slope of this relation is difficult since we need measurements in the sub-mm (200 - 1000 μm), which are tough. Also, there is no guarantee that the same emissivity law holds over all wavelengths, since different grain sizes will have different temperatures, and emit at different wavelengths. But for the standard MRN grains it should work, if there are no problems of different temperatures along the line-of-sight.

► 8.3 Dust temperatures

If dust grains were bricks *(blackbodies)*, they would not be warm enough to emit in IRAS 60 and 100 μm bands: the energy density of the Interstellar Radiation field is comparable to that of the microwave background, so a brick would reach a temperature of 3-4 K in the ISM. Why are grains warmer? Because they are *not* blackbody emitters and absorbers, but emit a modified black body spectrum. One may
-110- think of it as a "greenhouse effect" operating on microscopic scales: dust is efficient at absorbing short-wavelength radiation, but inefficient at emitting in the FIR. Yet, given its temperature, it will emit in FIR. As a result, the grain is heated above the temperature of a brick before there is equilibrium between emission and absorption. In essence, the grain traps some heat.

Spitzer: energy gained by a grain heated by radiation:

$$G_r = c \int_0^\infty Q_a(\lambda) U_\lambda \, \mathrm{d}\lambda$$

- G_r = rate of energy gain per second per unit projected area of dust due to absorption of radiation
- c = speed of light
- $Q_a(\lambda)$ = absorption efficiency factor
- U_λ dλ = energy density of radiation field in the range dλ

The grain emits in the FIR according to a modified Planck law.

$$j_{\nu E} = n_d Q_a \sigma_d B_{nu}(T_s)$$

$$\sigma_d = geometrical cross - section$$

So Q_a is essentially a correction factor to the geometrical cross-section. In equilibrium: heating = cool-
-111- ing.

$$c \int_0^\infty Q_a(\lambda) U_\lambda \, \mathrm{d}\lambda = 4\pi \int_0^\infty Q_a(\lambda) B_\lambda(T_s) \, \mathrm{d}\lambda$$

If $Q_a(\lambda)$ is independent of λ, then T_s comes out to be $\sim$ 3.5 K ($T_s = [U/a]^{1/4}$) in the ISM. However, $Q_a(\lambda)$ does depend on λ, going somewhere as $1/\lambda$ or $1/\lambda^2$. This allows T_s to increase substantially, reaching 15 to 18 K.

► 8.4 Interstellar extinction

-116- The presence of dust was demonstrated by Trumpler, who detected the dimming and reddening of distant stars. *High-z proto-galaxies!*

Dust both scatters and absorbs light. The combined effect is **extinction**. For a point source (e.g. a star), the object is *dimmed* by extinction, since both scattered and absorbed light do not reach the observer.

For an extended source (e.g. a galaxy), some light may be scatter *into* the line of sight, so the extinction will generally be less than for point sources. On average, extinction is about 0.6 - 1 mag kpc^{-1} (reddening is about 0.3 mag kpc^{-1} in (B-V) in the plane of the Milky Way - see §8.5).

NGC 6240, a heavily obscured starburst nucleus, emits 10 times more power in the IR/FIR than in the optical, with peak near 60-100 μm. Large z: shifts into sub-mm. Led to detection of Ultra-Luminous IR Galaxies (ULIRGs), where "ultra-luminous" is $> 10^{12}\ L_\odot$. *More typical galaxies have* $L_{\text{FIR}} \sim L_{\text{optical}}$.

▷ *8.4.1* *The extinction law*

-117- Empirical result: reddening in magnitudes

$$r_\lambda = a + \frac{b}{\lambda}$$

where a and b are roughly constant. This is between two extreme cases:

1. grey extinction, r_λ independent of λ
2. Rayleigh scattering, $r_\lambda \propto \lambda^{-4}$

$\Rightarrow$ particle size is $\sim 10^{-5}$ cm = 0.1 μm *(= 100 nm)*.

$$\mathrm{d}I_\lambda = -I_\lambda n(x) \sigma_\lambda \mathrm{d}\lambda = -I \, \mathrm{d}\tau_\lambda$$

- $n(x)$ = number density of grains at position x
- σ_λ = extinction cross-section per dust grain at wavelength λ

The star is dimmed by

$$I_\lambda = I_\lambda^0 \mathrm{e}^{-\tau_\lambda}$$

where

$$\tau_\lambda = \sigma_\lambda \int_0^x n(x') \mathrm{d}x' = \sigma_\lambda N(x)$$

In magnitudes:

$$\begin{aligned} A_\lambda &\equiv -2.5 \log \left[\frac{I_\lambda}{I_\lambda^0} \right] \\ &= -2.5 \log \left[\mathrm{e}^{-\tau_\lambda} \right] \\ &= -2.5 \log \left[\mathrm{e}^{\ln(\mathrm{e}^{-\tau_\lambda})} \right] \\ &= -2.5 \ln(\mathrm{e}^{-\tau_\lambda}) \log [\mathrm{e}] \\ &= -2.5(-\tau_\lambda) \log [\mathrm{e}] \\ &= 1.086 \tau_\lambda \end{aligned}$$

Dependence on Galactic latitude:
Assume simplest model of uniform dust layer.

-118- In reality, an exponential z-dependence would be more realistic, while the dust is also clumped, just like the gaseous ISM.

► 8.5
Interstellar reddening

Reddening is caused by the wavelength dependence of extinction. We can define a **color excess**:

$$E(\lambda_1 - \lambda_2) \equiv \underbrace{(m_{\lambda 1} - m_{\lambda 2})}_{\text{observed color}} - \underbrace{(m_{\lambda 1,0} - m_{\lambda 2,0})}_{\text{intrinsic color}}$$

e.g.:

$$E(B-V) = (B-V) - (B-V)_0$$

<u>Question</u>: How is the reddening measured?

Reddening line is defined by the extinction law.

$$E(U-B) = \underbrace{0.72}_{\substack{\text{“standard}\\ \text{Galactic}\\ \text{reddening}\\ \text{law”}}} E(B-V) + \underbrace{0.05(B-V)}_{\text{minor correction}}$$

We can define *reddening free colors*, if the shape of the reddening law is known. For example, the following is reddening free:

$$Q \equiv (U-B) - \frac{E(U-B)}{E(B-V)}(B-V) = (U-B) - 0.72(B-V)$$

-119- In general:

$$m_i = m_i^0 + A(\lambda_i)$$
$$m_j = m_j^0 + A(\lambda_j)$$

$$C_{ij} \equiv m_i - m_j = C_{ij}^0 + E_{ij} = C_{ij}^0 + \left[A(\lambda_i) - A(\lambda_j)\right]$$

We define

$$R_V \equiv \frac{A_V}{E(B-V)} = \frac{\tau_V}{\tau_B - \tau_V}$$

For the standard Galactic reddening law, $R_V \approx 3.2 \pm 0.2$.

The interstellar extinction law is derived from comparing reddened and unreddened star's spectral energy distribution for stars of the same spectral type. There are some clever extrapolation tricks involved to get the zero point. The enclosed table[*] gives a good representation of the standard Galactic curve. Note that

$$\frac{A_\lambda}{E(B-V)} = \frac{E(\lambda - V)}{E(B-V)} + R_V$$

The broad, smooth curve is not the whole story. There are for example still several unidentified narrower absorption features which are called the diffuse interstellar band (DIBs). There are also many line features in the IR and the 2200Å bump (which is present in the table). Some IR features: 9.7 μm $\longrightarrow$ H_2O, NH_3?

-120- One can correlate the hydrogen column densities with dust extinctions (or optical depths) to find:

- N(HI) $\approx 4.8 \times 10^{21}$ cm^{-2} E(B-V)
- N(HI + H_2) = N(HI) + 2N(H_2) $\approx 5.8 \times 10^{21}$ cm^{-2} E(B-V)

The scatter in the second relation is considerably less. This relation, combined with grain theories[†] produce a dust-to-gas ratio of $\sim$ 1%.

Origin and composition of grains:[‡]

- oxygen-rich
- carbon-rich
- ice coatings?
  - smallest grains ($< 0.01\mu$m) may be too warm
  - larger grains, which produce UV and optical extinction, may have ice coatings

The grain formation and depletion of elements on grains can perhaps be understood as a condensation sequence (see earlier discussion on ISM absorption lines, & Draine figure 23.1).

[*] from Sauage & Mathis (1979), ARAA17, p 84
[†] e.g. Mathis, Rumple, & Nordsieck (1977) ApJ 217, 425 or Draine & Lee (1984) ApJ
[‡] See Draine chapter 23 for more details

9. Molecular hydrogen and CO

-124- Molecular gas is, with HI, the dominant phase of the ISM in galaxies, in its contribution to the total gas mass. The distribution of H_2, however, is very different from HI gas. The gas is much colder and denser, and as a result, fills only a small part of the volume of the Galactic disk. The gas is heavily concentrated to the midplane of the galaxy (scale height $\sim$ 60 pc) and is mostly found in Giant Molecular Clouds (GMCs), complexes of small clumps of molecular gas, gravitationally bound, with overall sizes $\sim$ 50 pc and masses of 10^4 - 10^6 $M_\odot$. All available evidence indicates that stars form from molecular gas. There is some diffuse molecular gas locally at higher latitude, so the scale height cited refers to the main GMC layer, not all molecular gas. *Forming molecules is not trivial; dust grains may act as a catalyst.*

Since H2 molecule is not easily observable (see later), we need a tracer. CO is the most abundant ($\sim$ one CO molecule for every 10^4 H2 molecules). In terms of observational techniques, the predominant tracer of molecular gas is the CO molecule, through rotational transitions in the mm wavelength range. *(2.6 mm, to be exact.)* This sets a very different set of observational challenges compared to the 21-cm line: smoother telescope surfaces, different receivers/noise demands, smaller primary beams, larger short spacings, problems in interferometer maps. *CO maps are harder to make than HI.*

► 9.1 Molecular gas and CO as a tracer

-128- H_2 molecule has no permanent dipole moment (because the center of mass and center of charge coincide). Electronic transitions in the UV (Lyman and Werner bands) have been detected in absorption, and rotational-vibrational transitions in IR (H_2 gas that is warm due to shocks) but these do not allow general maping of this component. $\Rightarrow$ Need a different tracer. CO has J=1-0 rotational transition at 2.6 mm (115.3 GHz) which is *easily excited by collisions with* H_2 ($h\nu/k$ = 5.5 K) even in cold gas. Critical density for significant excitation of the 2.6 mm line: $n_{H_2} \geq 3000$ cm^{-3} if there is a balance between CO-H_2 collisions and spontaneous decay. However, since lines are optically thick in reality, the critical density is reduced by $1/\tau$ (where τ is the optical depth)* The result is that $T_x = T_k$ for $n_{H_2} \geq 300$ cm^{-3} (the levels are "thermalized"). T_x is the excitation temperature and T_k is the kinetic temperature.

-129- Evidence that $J = 1 \rightarrow 0^{12}$CO is optically thick: ^{13}CO much less abundant, yet lines are of order 0.5 - 1.0 as strong as ^{12}CO.

Since the 2.6 mm lines is optically thick, $T_b = T_k$, provided appropriate corrections for cosmic background radiation, radio telescope beam inefficiencies, and beam dilution are made.

* See Draine 19.3.1, 19.3.2

► 9.2 CO luminosity

-130-

$$L_{CO} = 4\pi D^2 \int I \, \mathrm{d}\Omega$$

Cloud of surface area πR^2 occupies

$$\mathrm{d}\Omega = \frac{\pi R^2}{4\pi D^2}$$

so $L_{CO} \propto \pi R^2 T_{CO} \Delta v$ (assumes $T_{CO} \simeq T_A^* \ldots$). For gravitationally bound clouds:

$$\Delta v = \sqrt{\frac{G M_{H_2}}{R}}$$

$$\Longrightarrow \quad M_{H_2} = L_{CO} \sqrt{\frac{4}{3\pi G}} \frac{\sqrt{\rho}}{T_{CO}}$$

so

$$M_{H_2} \propto L_{CO} \frac{\sqrt{\rho}}{T_{CO}}$$

L_{CO} is measured (need to know distance). T_{CO} is not necessarily physical.[*]

Potential problems: How constant is the proportionality? For example, metallicity - abundance of H_2 over CO may differ. This is especially a problem for dwarf galaxies. Note: H_2 molecules mainly form on grains. Dissociation energy of H_2 and CO: 4.48 eV and 11.1 eV.

CO $\rightarrow$ H_2 conversion calibration methods:

- optical extinction for Galactic clouds $\leftrightarrow$ L_{CO} (A_V).
- γ-ray emission at "soft γ-ray energies" probes total gas column.
- The famous x-factor! Calibration of L_{CO} to M_{H_2}. Canonical value x (see below). Assuming gas-to-dust ratio of 1%.
- virial mass of molecular clouds

Results:

$$\frac{N(H_2)}{I_{CO}} = 2\text{–}3 \times 10^{20} \quad \mathrm{cm}^{-2}\,(\mathrm{km\ s}^{-1})^{-1}$$

-131- (plot)

-132-

[*] See Draine 19.6 and figure on reverse side of handwritten notes.

$$M(H_2) = 4 - 6 \times 10^6 L_{CO}(M_\odot)$$

In M82: ρ and T_{CO} scale such that even in that extreme starburst environment may be the same within a factor of 2. We may have two cloud populations:

$$\left.\begin{array}{ll} \text{warm, dense}: & T \sim 75K \\ & n_{H_2} \sim 5 \times 10^3 \text{ cm}^{-3} \\ & \\ \text{cool, diffuse}: & T \sim 10 - 20K \\ & n_{H_2} < 10^3 \text{ cm}^{-3} \end{array}\right\} \Delta\left(\frac{\sqrt{\rho}}{T}\right) \leq 2$$

This statement is still controversial (see paper by Maloney I later refer to), and more recent results.

Chapter 32 in Draine gives a good discussion of the properties of molecular clouds in the Milky Way. It also discusses the various calibration methods to onvert CO integrated intensity to molecular hydrogen column density. Read it!

-133- (plots)

-134-

-135- *Photocopy from book; good info about molecular lines and transitions here.*

-136-

-137-

-138-

-139-

-140-

10. Hot ionized gas

► 10.1 Collisional excitation and ionization

-142- We already talked about collisional excitation in:

HI the resulting level populations are given by Boltzman statistics, so we did not have to discuss the basics of the collision process

HII regions Introduced the rate coefficients, γ, which play a role in the statistical equilibrium equation.

Collisional excitation and ionization are also of crucial importance in hot gas ($T \sim 10^5 - 10^7$ K). As we saw, photoionization by OB stars produces HII regions with $T \sim 10^4$ K. The temperature of any ISM phase is determined by the heating and cooling rates, which we discuss later (see last section).

An O star cannot heat gas to much more than 10^4 K since most of the ionizing photons have energies that are only a few eV above the H ionization potential. More energetic photons ionize He, so again only a few eV is left (note that 1eV $\equiv 1.6 \times 10^{-12}$ erg $\equiv kT \rightarrow T \sim 16,000$ K). Electrons quickly excite forbidden lines which cool the ionized gas to lower temperatures. Similarly, to collisionally ionize H (13.6 eV) requires high temperatures (the tail of the Maxwellian velocity distribution matters). The main mechanism to create hotter gas is by shock ionization which occurs in supernova remnants, evidently the
-143- SN explosion, and any collision between gas clouds in which velocities in excess of 100 km/s are involved. (Shock models predict a temperature-shock velocity relationship of the form $T_s = 1.38 \times 10^5 v_s^2$ K for a fully ionized gas, where v_s = shock velocity in units of 100 km/s). There is not a lot (in terms of mass) of the hot gas around; it cools very efficiently by line radiation from collisionally excited lines, some recombination radiation, and by Bremsstrahlung (which would dominate at very high T…why?) if density is high. (What would typical wavelength be at which Bremsstrahlung radiation would be predominant? *x-ray*). The hot gas is sometimes referred to as "coronal" gas, in similarity to the sun's corona.

▷ *10.1.1 Calculation of collisional rate coefficients*

While we discussed the general effect of collisional excitation on HII region spectra, we have not in detail discussed how the collisional rate coefficients are calculated.[*] Evidently the principal factors to consider are:

1. Kinetic energy, or similarly (in collisions of entire clouds) the translation of velocity into a temperature regime (see expression for T_s above).

[*] Draine does this in chapter 2.

2. Probabilities for particular processes (ionization, excitation); the rate coefficients need to be determined from momentum physics and are then averaged over the relevant velocity distribution.*

Spitzer summarizes ionization by energetic particles as well. This includes thermal electrons and protons but also cosmic rays.

-144- At temepratures $T > 10^4$ K, ionization by thermal electrons (in the high speed tail of the Maxwell velocity distribution) play a role. Cosmic-ray protons at energies down to a few million eV can penetrate clouds and ionize H, as can X-ray photons with energies of 200 eV and above.

Once gas is shock heated, the (re-)ionization by thermal electrons is important and needs to be considered in the calculation of recombination time scales.

Spitzer considers the role of cosmic-ray particles in ionization of H in cool HI and H_2 clouds, and concludes that $\sim$ 1 to 4% ionization levels inside clouds may be possible.

The role of soft X-rays is likely not very important but is difficult to establish observationally due to the predominance of other ionization sources. It may play some role in inoizing warm HI (producing DIG) in galaxy halos and thick disks, but there also the extragalactic ionizing bckground plays a role.

Another possible source of ionization of DIG in halo are "transition surfaces" between one gas phase and another. We talk of "mixing layers" where shear (if there is a velocity difference) and /or hydro instabilities may mix cooler and hot gas.

Lastly, the bulk of the DIG is probably inoized by field OB stars, Lyman continuum photons "leaking" from density-bounded HII regions. Hot evolved stars (e.g. blue HB stars) may play a role in certain environments.

► 10.2 Properties of hot ionized gas and spectrum

-145- Not surprising, the key parameter is temperature. Just as in photoionization, each element is present up to its highest possible ionization stage,† given the temperature, i.e. kinetic energy, of the colliding electrons.

The presence of absense of a particular line of an ion, or a line ratio of two lines for different ions, then becomes an important diagnostic of electron temperature (e.g. table 5.6 in Spitzer).

Note: transitions of these highly ionized elements are (mostly) permitted lines in the UV.

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* Draine, 2.1

† As table 5.6 in Spitzer shows lower ionization stages are also still present in substantial fractional amounts at any given temeprature.

Note: the highly ionized species are important coolants, yet they have barely been observed in emission, since the medium is of low density, contributes little mass, and hence its total luminosity is small. The dominant evidence for the presence of this gas is from UV absorption line studies. This medium likely makes up most of the “missing baryons” in the Universe.

11. Heating and cooling

-151- Following Spitzer's general formalism[*]:

Γ = total kinetic energy *gained* [erg cm^{-3} s^{-1}]
Λ = total kinetic energy *lost* [erg cm^{-3} s^{-1}] (aka. the "cooling function")

For $\Lambda = \Gamma$ we reach equilibrium temperature, T_E. Since Λ and Γ may depend on T_E themselves, we can set up a time dependent differential equation (for a mono atomic gas):

$$\underbrace{n\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{3}{2}kT\right)}_{\text{rate of increase in thermal energy}} \;-\; \underbrace{kT\frac{\mathrm{d}n}{\mathrm{d}t}}_{\text{work done by gas (energy loss)}} \;=\; \underbrace{\sum_{\varrho,\eta}\left(\Gamma_{\varrho,\eta} - \Lambda_{\varrho,\eta}\right)}_{\text{Sum over interacting particles}} \;=\; \Gamma - \Lambda$$

(Note: in what follows, we generally ignore "work done by gas" component.)

For T$\lesssim 2 \times 10^4$ K, thermal conduction can be ignored. If we do have enrgy gain or loss by conduction (electrons), we add -$\vec{\nabla} \cdot \left(k\vec{\nabla}T\right)$ to the righthand side. *Conductivity is hard to figure out, but not important below 20,000 K.* ($\vec{B}$ fields prevent conductivity across field lines.)

$PV = nRT = NkT \rightarrow P\Delta V$ = work done by gas. $R = N_A k$, where N_A is Avagadro's number.

If a gas is not at its equilibrium temperature (T_E), we can define a "cooling time" t_T *(like a half-life, sorta)*:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{3}{2}kT\right) = \frac{3}{2}k\left(\frac{\mathrm{d}T}{\mathrm{d}t}\right) = \frac{3}{2}k\left(\frac{T - T_E}{t_T}\right)$$

where T_E = constant and T = variable. For T_E and t_T = constant, $T - T_E \propto \mathrm{e}^{-t/t_T}$ *(exponential decay*
-152- *for cooling). t obviously can't be negative $\rightarrow$ quasi-stable*. If $t_T < 0$, the situation is unstable and gas will cool or heat toward an entirely different T_E. This is relevant to the multi-phase ISM. All the physics is in Λ and Γ.

► 11.1 General sources

▷ *11.1.1 Primary heat source*

The primary heat source is (photo)ionization.

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[*] see also Draine chapters 27, 30, 34

E_2 = kinetic energy of ejected electron
E_1 = kinetic energy of recaptured electron

$\Delta E = |E_2 - E_1| \rightarrow$ *energy available. Ionization from the ground state (maximum energy) recombines with less energy.* Number of captures to level j of neutral atom:

$$n_e n_i \langle \omega \sigma_{cj} \rangle \quad [\mathrm{cm}^{-3}\ \mathrm{s}^{-1}]$$

where $\langle \omega \sigma_{cj} \rangle$ = recombination coefficient. The final net gain associated with electron-ion recombination:

$$\Gamma_{ei} = n_e n_i \sum_j \left(\underbrace{\langle \omega \sigma_{cj} \rangle \overline{E}_2}_{\text{ionization out of } j} - \underbrace{\langle \omega \sigma_{cj} E_1 \rangle}_{\text{recapture back to } j} \right)$$

$\langle \rangle$ = average over Maxwellian velocity distribution
$\overline{E}_2$ = average over all ionizing photon energies

As we have seen, all ionizations take place from the ground level $\rightarrow \overline{E}_2$ is independent of j; use recombination coefficient to all levels $\geq n$:

$$\alpha^{(n)} = \sum_n^{\infty} \alpha_m$$

and

$$\Gamma_{ei} = n_e n_i \left\{ \underbrace{\alpha \overline{E}_2}_{\equiv \alpha^{(1)}} - \underbrace{\frac{1}{2} m_e \sum \langle \omega^3 \sigma_{cj} \rangle}_{\text{kinetic energy}} \right\}$$

Main point:
-153- For *steady state* situation, where the number of ionizations is equal to the number of recombinations, Γ_{ei} is not dependent on ionization probability or radiation density

▷ *11.1.2* *Primary cooling source*

The primary cooling source is **inelastic collisions** (hence, excitation of energy levels). *Particles that undergo inelastic collisions will lose energy.*)

- $n_e n_i \gamma_{jk} [\mathrm{cm}^{-3}\ \mathrm{s}^{-1}]$ = number of excitations from level $j \rightarrow k$ for ions in ionization stage i and excitation state j.
- $E_{jk} = E_k - E_j$ = energy lost by colliding electrons *(in the form of emitted photons).*

Also: *de-exciting* collisions, which give energy gain to electrons *(give energy back to the nebula). Net cooling:*

$$\rightarrow \Lambda_{ei} = n_e \sum_{j<k} E_{jk} \left(\underbrace{n_{ij}\gamma_{jk}}_{\text{excitation}} - \underbrace{n_{ik}\gamma kj}_{\text{de-excitation}} \right)$$

γs are collisional rate coefficients. Assumption: all photons escape (not true for dense molecular clouds). Again, in most cases all ions are in the ground level, so we don't need to sum over j, and $n_{ij} = n_{i1} = n_i$ *(simpler)*

► 11.2
HII Regions

-154-

- Main heat sources: ionization of H and He
- Main cooling source: excitation of C, N, O, Ne
    - Heavy elements have low abundance. If they didn't, HII regions would cool to very low T_E!

HII Regions: Lyman photons didn't escape; they were all re-absorbed and turned into Balmer line. Photons absorbed by dust → cooling. Photons scattered → not *cooling. Keep in mind:* ***Do all photons make it out of the nebula?***

▷ 11.2.1
Heating of HII regions

$$\overline{E}_2 = \frac{\int_{\nu_1}^{\infty} h(\nu - \nu_1) s_\nu U_\nu \, \mathrm{d}\nu/\nu}{\int_{\nu_1}^{\infty} s_\nu U_\nu \, \mathrm{d}\nu/\nu}$$

- s_ν = cross section
- ν_1 = ionization limit for HI

Problem: *The radiation field energy density, U_ν* is determined by stellar radiation ($U_{s\nu}$), *ionizing photons from OB stars* and diffuse radiation ($U_{D\nu}$), *from direct recombination to n=1* and depends in turn on n_H, etc.

Spitzer discusses two simple cases:

1. Close to exciting star: $U_{s\nu}$ large, $U_{D\nu}$ negligible in comparison
2. Evaluate $\overline{E}_2$ for entire HII region.

Approximation: Use dilute blackbody of temperature T_c *(color temperature)* to describe the stellar radiation field at distances from the stars.

Define: $\psi = \dfrac{\overline{E}_2}{kT_c}$

- $\psi_0 = \psi(r \to 0)$ (close to star)
- $\langle\psi\rangle$ = average of star
- $\psi(r)$ = over entire HII region

-155-

$$\psi_0 : \quad U_\nu = \frac{4\pi B_\nu(T_c)}{c} = \frac{1}{c}\int I_\nu \, \mathrm{d}\Omega$$

Then it is possible to calculate table 6.1 (Spitzer), which lists values of ψ_0 (Draine, table 27.1) for various values of T_c.[*]

Result:					
	1.05	<	$\langle\psi\rangle$	<	1.65
	4000	<	T_c	<	64,000

We had that:

$$\Gamma_{ei} = n_e n_i \left\{ \alpha \overline{E}_2 - \frac{1}{2} m_e \sum \langle \omega^3 \sigma_{cj} \rangle \right\}$$

where second term = mean energy lost per recombining electron.

We have $\overline{E}_2$, now we need

$$\sum_{j=k}^{\infty} \langle \omega^3 \sigma_{cj} \rangle = \frac{2A_r}{\sqrt{\pi}} \left(\frac{2kT}{m_e} \right)^{3/2} \beta \chi_k(\beta)$$

- A_r = “recapture constant”[†]
- $\beta = \dfrac{h\nu_1}{kT}$

$\chi_k(\beta)$ are listed in table 6.2 in Spitzer. They are “energy gain functions” with values from $\sim$0.4 to 4.0.

▷ 11.2.2 Cooling of HII regions

Use directly the basic equation using

$$\frac{n_k}{n_j} = \frac{b_k}{b_j} \frac{g_k}{g_j} \exp\left(-\frac{h\nu_{ju}}{kT} \right)$$

[*] Spitzer also discusses how to calculate $\langle\psi\rangle$; too lengthy to do here.

[†] Spitzer, page 100

and

$$\frac{b_2}{b_1} = \frac{1}{1 + A_{21}/n_e\gamma_{21}}$$

(discussed for HI emission) for a two-state ion. 3 level ions are more complicated.

- $\frac{g_k}{g_j} \exp\left(-\frac{h\nu_{ju}}{kT}\right) \rightarrow$ Boltzmann equation
- $\dfrac{b_2}{b_1} \rightarrow$ "b correction", since we can't assume Boltzmann level populations

▷ *11.2.3* *Results*

-156- (See figures on handout, page -157-). Two groups of coolants:

1. meta-stable fine-structure levels in ground. Spectroscopic form of various ions.
    - $\mathrm{E}_{ex} < 0.1$ eV $\rightarrow$ IR radiation

   examples in Fig:

[OIII]	$^3P_0 -^3 P_1$	88.4 μm	**Weak T_E dependence**
	$^3P_0 -^3 P_2$	32.7 μm	

2. meta-stable other spectroscopic forms with excitation energies $\gtrsim$ 1 eV, giving rise to optical and UV lines. **Strong $\mathbf{T}_E$ dependence** of course! Act as thermostat $\rightarrow$ will keep $\mathrm{T}_e \sim 10,000$ K in HII regions.

<u>Note</u>: Figure is for (arbitrary) assumption that O, Ne, N are 80% singly and 20% double ionized. H is 0.1% neutral.

The *net effective heating rate*, G-L_R in the figure, is what we have been calculating (where G = photoionization and L_R = recombination emission for H and He). The intersection between heating and cooling gives T_E.

The optical depth τ_0 in the figures refers to *distance* from star. It is the optical depth at the ionization limit of HI, so it is proportional to N(H).

-158- Outer parts of nebula are hotter! This happens because the photoionization cross-section is proportional to ν^{-3}, so higher energy photons are absorbed *later*. T_E decreases at first, because $U_{s\nu}$ falls. Then, beyond $r = 0.6R_S$, T_E increases and is higher near $r = R_S$ than at $r = R_0$. Finally, it decreases again in the transition region where the ionized fraction drops to zero.

The second figure shows what happens if N_e is large enough that some excited levels of heavy ions are *collisionally* de-excited; cooling is less effective, so T_E increases.

ϵ_{ff}, the free-free loss rate, follows directly from (3-56) in Spitzer; it is not very important.

How fast does T change when $T \neq T_E$?
Close to T_E,

$$t_T \approx \frac{2\times 10^4}{n_p} \quad \text{[years]}$$

Compare to recombination time:*

$$t_r = \frac{1}{n_e \alpha} = \frac{1.54\times 10^3 \sqrt{T}}{Z^2 n_e \phi_2(\beta)} \quad \text{[years]}$$

(It would take 10^4 years for recombination to occur if the star in the HII region disappeared). So for $T \sim 10^4$ K, $t_R \gtrsim t_k \rightarrow$ cooling is faster than recombination. (Also, cooling time is much longer for $T \gg 10^4$ K, as we discussed before.)

-159- Consider the handout (cooling and heating in HII regions, taken from Osterbrock). What is missing in these diagrams *(if anything)*?

Cooling by free-bound and bound-bound HI and He? No; since each recombination is balanced by an ionization, these photons, even though they may well escape from the nebula, do not draw net heat from it. Rather, they draw heat from the star itself, not the nebula.†

Photon $h\nu$ is absorbed, ionizing atom, which creates electron with *(kinetic)* energy = $(1/2)mv^2 = h(\nu - \nu_0)$. Electron thermalizes its energy with ions and electrons and sets up temperature T_e. However, each electron recombines from energy $(1/2)mv'^2$, which produces photons with a total energy $h\nu' = (1/2)mv^{-2} - h\nu_0$

The *net* energy (or heat) gain from the photoionization is given by $(1/2)mv^2 - (1/2)mv'^2$ (notice that in general, $|v'| < |v|$). This is to be balanced against the *cooling*, which is predominantly due to emission from collisonally excited heavier elements in HII regions.

Pure H in the ISM: not many photons around; HII regions are fully ionized. Gas not generally in both phases, e.g. 90% neutral and 10% ionized.

► 11.3 HI gas

-160- (Draine, ch. 30) Not as simple as one might believe...

Simple considerations
HI is neutral, so there are few free electrons to share heat. Only elements with ionization potential (IP) < 13.6 eV will be ionized (and dust grains, but more on that later). However, there is still a dominant

* Spitzer 6-11

† Draine does not agree? He includes a recombination cooling rate in Fig. 27.2, 27.3; it is low. He keeps it in heating too though...so okay, fine. Whatever.

cooling line from CII (IP of C is 11.26 eV). So:

- $\frac{\Gamma_{ei}}{n_e}$ down by factor of $\sim$ 1/2000
- $\frac{\Lambda_{ei}}{n_e}$ similar below $T \sim 1000$ K

HI is naturally cool. However, we observe some very warm HI. General solution may well depend on T_e, n, chemical composition (i.e. depletion)...We will just list the general players.

▷ *11.3.1* *Cooling function*

Cooling function Λ:

- neutral atoms
- ions
- molecules

Excitation sources:

- electrons
- H atoms

-161- Some sources in particular:

1. CII and SiII excitation by collisions with H. Problem: *depletion*
2. Excitation of HI, [exp.?] $n = 2$ level in warm HI. Generally, $n = 2$ is not strongly populated, but if it does happen, it will cool:
   - Lyα photon $\longrightarrow$ dust $\longrightarrow$ IR
   - $\Lambda_{eH} = 7.3 \times 10^{-19} n_e n(HI) e^{-118,400/T}$ [erg cm^{-3} s^{-1}]
     ($T = h\nu/k$ = 118,400 for a 1216Å photon).

   Fate of Lyα photons: Scatter until absorbed by dust, can't do 2-photon emission, level 1 $\rightarrow$ 2p... or something.
3. H_2 molecules: Can be a gain or loss source
   - loss: excitation of rotational levels
   - gain: photon pumping of upper rotational levels, followed by collisional de-excitation. Also other molecular lines may cool (CO, CN, CH,...)
4. Collisions with dust grains (can both heat and cool): Spitzer's fig. 6-2 (page -172- in notes) shows cooling function, including HI and H^+ range. Generally, cooling time for HI gas:

   $$t_T \approx \frac{2.4 \times 10^5}{n_H} \quad \text{[years]}$$

   longer than for HII regions *(not many lines available)*.

▷ *11.3.2*
Heating function

The heating function for HI gas is very poorly known. Γ_{ei} from ionizing elements such as C is best known
-162- (i.e. photo-electric heating). However, using only CII heating would produce T_E = 16 K (Spitzer); clearly too cool. *Adding metals is not enough*

Other potential players

(a) Cosmic ray ionization of H, *no radiation, but this could do it too*.
(b) Formation of H_2 molecules on grains. This releases 4.48 eV. Goes into:
    - heating grain
    - overcome energy of adsorption to grain surface
    - excitation of new H_2 molecule
    - translational kinetic energy that H_2 molecule gets as it leaves the grain.
(c) Photoelectric emission from grains. Problem: what is efficiency, as a function of λ? Clearly, complicated problems; see Draine for a discussion.

How can HI become 6000K? Cooling is 10 times more efficient (from figure on page 172: $\frac{\Lambda}{n_H}$) than for cool HI. But, if n_H is lower, then:

- cosmic ray heating more efficient, but still problematic
- also grain - photoelectric heating

► 11.4
Few comments/additions from Spitzer

See figure 6.2, which sketches what happens at lower T (below 10^4K). Note that Spitzer talks about $\frac{\Lambda}{n_H^2}$

-166- Why are there two HI phases? (See typed notes here).

-167- This much about HI CNM and WNM phases. As mentioned, we can also consider the cold molecular medium as a further phase, “condensing” out of warmer HI CNM.

Finally, we have a “stable” hot phase, stable only when cooling time is sufficiently long that hot phase can exist for a long period (more on that later) or when it is continuously created and “destroyed” on a large scale (e.g. in a time averaged view of a galactic disk where SNe occur frequently enough to maintain a persistent hot phase, much like an HII region around OB stars). The “destruction” of the hot phase happens (only) through cooling of the gas. This cooling depends critically on T_e and n_e, as we will next explore.

► 11.5
Heating and cooling of hot gas

-168- Typed notes
. . .
-172-