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1. Introduction

2. Radiation from moving charges

2.1 Thomson scattering

Process: response of a free charge (electron) to an incident EM wave and subsequent scattering of light by the wave.

Assumption: if the charge oscillates at non-relativistic velocities, $v \ll c$, then the magnetic force is negligible ($\vec{F_L} = q(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$ and $|\vec{E}| = |\vec{B}|$ for EM wave).

Consider a linearly polarized incoming wave:

Does not appear to exist. . .

The incident flux of radiation is:

$$\langle S \rangle = \frac{c}{8\pi} E_0^2$$

Thomson cross section for scattering:

$$\sigma_{TH} = \frac{8\pi}{3} r_0^2$$

Note:

1. σ also follows from $P = \langle S \rangle \sigma$
2. σ is independent of frequency (any type: radio, optical, etc.); however, this is only true in the classical case, where $v \ll c$ (see later discussion of Compton scattering).
3. $\sigma \propto m^{-2}$, so for a proton, σ is about 10^6 times smaller than for an electron, hence in a plasma, electrons dominate.
4. Thomson scattered light introduces a degree of polarization, which will be a maximum at 90° to the incoming beam (if the incoming beam is

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*un*polarized). Degree of polarization:

$$\Pi = \frac{1 - \cos^2 \theta}{1 + \cos^2 \theta}$$

which is equal to 100% at 90°.

2.1.1   Optical depth and Thomson cross section

-55-   When can we still see an object centered on an ionized nebula, which scatters the radiation only through electron scattering?

2.1.2   Additional comments on Thomson scattering

-56-   Thomson scattering is a special case of the "collision" between a photon and an electron. It applies when the energy of the photon is small compared to the rest energy of the electron. When this is not the case, the situation is analyzed from the perspective of Compton scattering. A Compton scattering analysis also takes into effect quantum corrections, e.g. kinematic effects occur because the photon not only possesses energy but also momenutum.[*]

2.2   Eddington limit

-57-   Following R/L problem 1.4 parts a & c:

    a.) "Show that an optically thin cloud..."
    c.) "A minimum value for..."

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-59-   We essentially have...

Applications:

- There is a limit to how massive a star can be:

$$L_{Edd} \sim 1.25 \times 10^{38} \quad \left[\text{erg s}^{-1} \left(\frac{M}{M_\odot} \right) \right]$$

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[*]R/L chapter 7 discusses this in great detail.

If L for MS stars scales as $L \propto M^x$, this places a limit on the highest mass star we can have (~ 100 M$_\odot$).

- Accretion on black holes and accretion disks: note that L_{Edd} scales linearly with M, and is independent of r

2.3   Plasma effects

-60-   General issue: what happens to EM waves when they travel through a plasma?

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Application: **pulsar dispersion measurements**

A pulsar is a fast spinning neutron star emitting two beams of radiation (like a lighthouse). If there is an ionized plasma along the line of sight to the pulsar, radiation in the pulse of lower frequency will reach us later than high frequency radiation because it is slowed down more by the plasma.* (in a sense, it is scattered more, therefore travels a greater distance).

2.4   Faraday rotation

-69-   Basic idea: what happens to radiation (EM wave) travelling through a cold plasma if the plasma has a uniform magnetic field in it? In particular, what happens to a linearly polarized wave? As we will see, the direction of polarization changes!

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* see expression for v_{ph}

3.   Fourier transforms and convolutions

-F1-   References: Bracewell - *The FT and its applications*

3.1   Fourier transforms

Basic idea goes back to Fourier series, which approximate a general function as a sum of sines and cosines of different periods.

A simple example is a vibrating string, which can be represented as:

$$g(x) = a_0 + \sum_1^\infty [a_n \cos 2\pi n f x + b_n \sin 2\pi n f x]$$

which describes a periodic function $g(x)$ with period T and frequency f, where

$$a_0 = \frac{1}{T} \int_{-\frac{1}{2}T}^{+\frac{1}{2}T} g(x)\mathrm{d}x$$

$$a_n = \frac{2}{T} \int_{-\frac{1}{2}T}^{+\frac{1}{2}T} g(x) \cos 2\pi n f x \mathrm{d}x$$

etc. The general definition of a fourier transform (FT) is

$$F(s) = \int_{-\infty}^{+\infty} f(x)e^{-i2\pi x s}\mathrm{d}x$$

$$f(x) = \int_{-\infty}^{+\infty} F(s)e^{+i2\pi x s}\mathrm{d}s$$

The two transforms are not exactly equivalent because of the $\pm$ sign. If this were not present, we would not necessarily get back the original $f(x)$ in the second transform.[*]

Note here the link with the sine and cosine series, since

$$e^{i\theta} = \cos\theta + i\sin\theta$$

-F2-   Other conventions for where to put the 2π exist:

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[*]See Bracewell.

$$F(s) = \int_{-\infty}^{\infty} f(x)e^{-ixs}\mathrm{d}x \quad \longleftrightarrow \quad f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(s)e^{ixs}\mathrm{d}s$$

$$F(s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-ixs}\mathrm{d}x \quad \longleftrightarrow \quad f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)e^{ixs}\mathrm{d}s$$

If $f(x)$ and $F(s)$ are transform pair in our system, then $f(x)$, $F\left(\dfrac{s}{2\pi}\right)$ are

pair in second system and $f\left(\dfrac{x}{\sqrt{2\pi}}\right)$, $F\left(\dfrac{s}{\sqrt{2\pi}}\right)$ in third system.

3.1.1 Symmetry: oddness and evenness

-F3-
Symmetrical function: if $E(x) = E(-x)$, then $E(x)$ is symmetrical, or even. If $O(x) = -O(-x)$, then $O(x)$ is an odd function. Verify: sum of an even function and an odd function is typically neither even nor odd.

3.2 Complex conjugates

-F6-
The FT of. . .

3.3 Cosine and sine transforms

-F7-
The cosine and

3.4 Convolution

-F10-

3.5 Auto-correlation function

3.6 Impulse Symbol

-F15-
Another somewhat bizarre but useful function is the *impulse symbol*. By that

we mean an intense, unit-area pulse of length so short that it cannot be re-solved with a given piece of measuring equipment. Essentially, a pulse like this will show up only in integrals (hence its unit-area) and it is equivalent to a δ-function, which is the mathematical analog. δ-functions permeate (theoret-ical) physics: point masses, point charges, point sources, line sources, surface charges, etc. These things don't exist, but your measuring equipment doesn't know this, anything smaller than its resolution (in time, space, or frequency), acts as an impulse function, in a mathematical sense.

-F24- Properties related to the convolution theorem:

1. The area under a convolution is equal to the product of the areas under the factors:

$$\int_{-\infty}^{\infty} (f * g) \, \mathrm{d}x \quad = \quad \left[\int_{-\infty}^{\infty} f(x) \mathrm{d}x \right] \left[\int_{-\infty}^{\infty} g(x) \mathrm{d}x \right]$$

4. Signal processing and receivers: theory

5. Practical receiver systems

6.   Practical aspects of filled aperture antennas

7. Single dish observational methods

7.1 Atmosphere

7.2 Calibration procedures

7.3 Continuum observing strategies

7.4 Spectral line observations: additional requirements

8. Interferometers and aperture synthesis

9.   Radiation processes

9.1   Bremsstrahlung radiation

= free-free radiation[*]

Process: ionized plasma, encounters between electrons and ions causes deflec-
tions of electrons $\rightarrow$ electrons are accelerated and will radiate. Under most
conditions, velocity distributions of ions and electrons is *Maxwellian* - thermal
equilibrium (TE) distribution, so this is referred to as *thermal Bremsstrahlung*.
The word "Bremsstrahlung" means "braking radiation", which refers to the
slowing down (deceleration) of the electrons when approaching from a given
direction to the protons/ions. The approach will be classical, rather than
quantum mechanical; this is valid as long as $kT \sim mv_e^2 \gg h\nu$.

Note: collisions between the same particles produce *no net* dipole radiation
because dipole moment $= \sum e_i \vec{r}_i \propto \sum m_i \vec{r}_i$. The latter is the center of mass,
which is either at rest or in constant motion, but experiences no *net* acceler-
ation. In other words, each particle moves exactly opposite to its partner so
that their individual contributions to the time-varying electric fields at large
distances cancels[†].

9.1.1   Emission for single-speed electrons

Here we consider electron-ion collisions (or rather, *encounters*). In these
encounters, the electrons radiate primarily since $|\vec{a}| \propto \dfrac{1}{m}$ for equal charges.

Approximations:

1. ion mass $\gg$ electron mass $\rightarrow$ electron moves in *fixed* Coulomb field of
   ion.
2. electron speed is large; in a single encounter, the electron motion barely
   deviates from its original direction $\rightarrow$ small angle scattering regime (not

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[*]following: Rybicki & Lightman, chapter 5.
[†]Shu

essential approximation).

Take FT, note that[*]

$$FT(\ddot{\vec{d}}) = -\omega^2 \hat{\vec{d}}(\omega)$$
$$= -\frac{e}{2\pi} \int_{-\infty}^{\infty} \dot{\vec{v}} e^{i\omega t} \mathrm{d}t$$

Derive expressions for $\hat{\vec{d}}(\omega)$ in limits for large and small ω: collision time $\tau \equiv \dfrac{b}{v}$.

-147-   We had derived before the spectrum of dipole radiation:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega} = \frac{8\pi\omega^4}{3c^3} |\hat{d}(\omega)|^2$$

So now we find

No dependence on ω! (Impulse has flat spectrum). Clearly the task is to estimate $\Delta\vec{v}$. We use our approximation that the path of the electron is almost linear. $\Delta\vec{v}$ is mainly in $\perp$ direction.

(Use Coulomb force: $F = m_e \dfrac{\Delta v}{\Delta t} = \dfrac{Ze^2}{R^2}$.)

9.1.2   Determining the total spectrum

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We deal with the b_{min}, b_{max} issue, and the potential importance of quantum effects, by introducing the **Gaunt factor**, $g_{ff}(v, \omega)$, which includes these effects.

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[*]RL 3.3

9.1.3 Velocity distribution

9.2 Synchrotron radiation

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